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Preface

This volume, which constitutes the forty-fifth in the series of annual Transactions of the American Institute of Electrical Engineers, contains papers and discussions presented at three conventions and four regional meetings during the year 1926. The arrangement of the articles in the book is generally chronological and each paper or group of papers is followed directly by the discussion thereon. The contents are indexed both by title and author's name and are cross-indexed wherever possible. In addition to the index of authors, the names of all those taking part in the discussions are also included. The full text of each paper is given in the Transactions. All of the articles which are published here have also been printed in the Institute Journal, some having appeared there in abridged form, however. Reference to all papers printed in the Journal and not in the Transactions has been made in the Transactions Index, Journal publication date and page number being given in each case. It was necessary, as it has been for sometime past, to omit some of the large amount of material available for the Transactions; space does not permit the printing of many papers of more or less transitory interest. This book is more than two hundred pages longer than last year's volume, the total number of papers presented this year greatly exceeding the number delivered during 1925. The volume concludes with the annual report of the Board of Directors for the fiscal year ending April 30, 1926, together with the officers, committeemen, and Section and Branch officers for the corresponding year.

An Investigation of Transmission-System Power Limits

BY C. A. NICKLE[1] and F. L. LAWTON[1]
Associate, A. I. E. E. Associate, A. I. E. E.

***Synopsis.**—Results of theoretical analysis, verified by miniature-system tests of the power limits of transmission systems, are discussed, among the major conclusions being the following:*

The criterion for stability under all conditions is the steady-state power limit.

The charging kv-a. exercises marked detrimental effects on stability.

The characteristics of synchronous terminal apparatus are of great importance.

Improvements can be made by modifying present apparatus design.

Automatic voltage regulators, suitable exciters and fast relays are essential.

The mercury-arc rectifier, as an adjunct in excitation circuits, shows real advantages.

* * * * *

INTRODUCTION

THE subject of transmission-system power limits and related problems is one of utmost importance in present and future projects, as emphasized by recent papers.[2]

The present paper deals with the results of careful theoretical analysis and calculations, verified by extensive miniature-system tests, of the power limits of transmission systems. Theoretical studies have been checked by tests on a miniature system, equipped with synchronous apparatus, lines of variable constants, exciters, regulators, etc., and so chosen that 2300 volts and 180 kilowatts corresponded to 220,000 volts and 15,000 kw. on the actual system. The work covered was planned and directed by Messrs. R. E. Doherty and H. H. Dewey, and to a large extent outlined by them in a recent paper before the Institute.[3]

In addition to investigations relating to a basic knowledge of power limits of systems, attention has been focused on methods of improving their stability—in other words, increasing the power limits. In this connection, careful analysis has been made of special types of synchronous equipment, voltage-regulation schemes, induction generators, use of static condensers for the compensation of line inductive reactance, and many other devices. The work has been paralleled throughout by studies of actual projects.

In the computation of power limits, a thorough knowledge of the theory of synchronous machines is essential. It can be shown that, for all practical purposes, it is unimportant whether salient-pole or cylindrical-rotor theory be used in the calculation of all quantities but angular relationships. As steady-state power-limit studies made by the authors have not involved angular relationships, the use of either theory is justified. In transient problems, however, angular relationships are of fundamental importance and, hence, the appropriate theory must be used.

A large amount of work has also been done on the power-angle characteristics of synchronous machines. Papers dealing with these studies and synchronous-machine theory will, it is hoped, be presented before the Institute in the near future.

Among the important conclusions reached as a result of the investigations made by the authors and other interested engineers of the General Electric Company, may be mentioned:

1. Too great importance must not be attached to the transmission line as it is but one link in the circuit; the characteristics of terminal apparatus, such as synchronous machines, and methods of voltage regulation are of equal, if not greater, importance.

2. The only feature in which the stability of high-tension long-distance transmission and high-tension cable systems differs from that of any other type is in the limitation of the excitation of synchronous apparatus due to the charging kv-a. of the lines.

3. For slowly applied loads, automatic voltage regulators and suitable exciters permit stable operation up to the same limit which can be obtained with manual control.

4. The problem of obtaining greater power limits offers two avenues of approach:

   (*a*) Modification of the characteristics of existing synchronous apparatus and transmission lines.

   (*b*) The development of new methods of voltage regulation.

5. Extensive miniature-system tests indicated that no matter how fast load was applied, or in what manner, that under similar conditions of excitation or voltage, the power limit was always the same as the steady-state limit. In view of these results and experience, it appears that the fundamental criterion of power-system stability is the steady-state power limit.

6. Adequate speed of relaying is the vital factor in the maintenance of stability during system short circuits.

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[1] Both of General Electric Company, Schenectady, N. Y.

[2] Groups of papers presented at A. I. E. E. Convention at Philadelphia, February, 1924; New York, February, 1925; and Seattle, Sept., 1925.

[3] "*Fundamental Considerations of Power Limits of Transmissions Systems,*" R. E. Doherty and H. H. Dewey, JOUR. A. I. E. E., Vol. XLIV, October, 1925, p. 1045.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

7. Reliable methods of calculation of system power limits, for steady-state conditions, are available. For the more complicated networks, a-c. miniature systems give excellent results.

8. At present there are no satisfactory methods of computing system power limits, under transient conditions, for any but the simplest cases. However, it is possible to study completely all transients ensuing from disturbances which do not result in instability, by means of the electromechanical analyzer.

METHODS OF CALCULATION

It has been indicated that reliable methods of determining system steady-state power limits are now available. In general, it has been found that the method of calculation adopted by the authors gives results of good engineering accuracy, as shown by the computed and test values given in this paper. Furthermore, these results agree very well with those secured by independent investigators using other schemes of analysis.

A description of this method of calculation, and its application, is given in an Appendix, so comparatively little reference will be made to it in the body of the paper.

In developing methods of analysis, considerable attention has been devoted to a careful study of synchronous-machine theory. As previously noted, it is unimportant whether salient-pole or cylindrical-rotor theory is used in the calculation of all quantities other than angular relationships. Since power calculations made by the authors have been based on an excitation method, the use of either theory will be satisfactory. Inasmuch as in transient problems the angular relationships are of fundamental importance, it becomes necessary to use the appropriate theory.

Calculations of system steady-state power limits are comparatively easy for simple systems comprising, say, one generating station, a line or two, and one receiving station. Where several branches are involved, analysis becomes more difficult; partial solutions by graphical methods are combined to give a complete solution. When the system under consideration comprises many branches, computation by these graphical methods becomes practically impossible and resort must be had to analysis by miniature a-c. systems.[4] Steady-state stability problems can be solved with the aid of miniature a-c. systems of the same order of size as the d-c. short-circuit calculating table, which has been of such invaluable aid.

For the study of system transients arising from dropping a line, suddenly adding a load, etc., there is no practical scheme of analysis available for any but the simplest cases. The unwieldiness of the step-by-step method[5] increases very rapidly with the number of elements in the system, and it soon becomes impossible of application. However, by using the equivalent-circuit idea,[6] it is possible to study the electromechanical transients in any system, no matter how complicated, for loads below the power limit; that is, where the differential equations for the system are linear.

DESCRIPTION OF MINIATURE SYSTEM AND METHODS OF TESTING

When the investigation was commenced, it was realized that means should be available for verifying promising leads developed in analytical studies, and establishing the reliability of methods of calculation. Inasmuch as it was not feasible to use an actual system for this work, a miniature system of sufficient capacity to give reliable test information was set up in the factory. The equipment included several 225-kv-a. synchronous motors with direct-connected d-c. generators, units for setting up any type of line, exciters, standard vibrating-contact regulators, etc. By using the d-c. generator of a set as a motor supplied from the shop bus, and the corresponding unit of another set as a separately-excited generator supplying a water-rheostat load, it was possible to study systems comprising four synchronous units. The use of resistance loads prevented any possibility of hunting due to a feed back on the shop bus. For simulating an infinite generator, a 10,000-kv-a. alternator was available. Other equipment was added as required during the course of the investigation. Fig. 1 gives an idea of the test arrangement, the scale of which will be better realized when it is understood that the synchronous units were each rated at 225 kv-a., the lines being of sufficient range to give the desired constants and current-carrying capacity. Normal operating voltage was 2300 volts, but it was possible to operate at any voltage from about 1100 to 2900 volts. Sufficient metering and

FIG. 1—GENERAL VIEW MINATURE SYSTEM

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[4] "Artificial Representation of Power Systems," H. H. Spencer and H. L. Hazen, JOUR. A. I. E. E., Vol. XLIV, January 1925, p. 24.

[5] "Power System Transients," V. Bush and R. D. Booth, JOUR. A. I. E. E., Vol. XLIV, March 1925, p. 229.

[6] "Oscillographic Solution of Electromechanical Systems," by C. A. Nickle, JOUR. A. I. E. E., Vol. XLIV, December, 1925, p. 1277.

oscillographic equipment were provided for observations at all important points.

To correlate results obtained by miniature test with corresponding values for the actual system, it should be borne in mind that the miniature system was designed so that a voltage of 2300 corresponded to 220 kv. on an actual system; similarly, 180 kw. was equivalent to 150,000 kw.

Inasmuch as the method of analysis was based on the calculation of power-voltage or pull-out curves,[7] with excitation as parameter, a majority of the tests were run so as to obtain similar curves. In other words, for any given system, the machines were first synchronized and the excitations adjusted to the desired values. Holding both excitations and speed constant, the load was slowly increased by increasing the d-c. generator field excitation. At any desired value of load, observations were made of the meters indicating power, voltage and current in the a-c. circuit, and voltage and current in the load circuit. Behavior of the system at the point of breakdown, when the system load reached the power limit and the machines broke out of synchronism, was carefully noted. Where voltage regulators were used to maintain system voltage, the frequency was kept constant, and the load was slowly increased to the power limit.

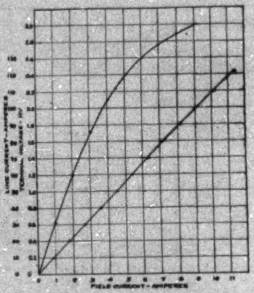

FIG. 2—OPEN AND SHORT-CIRCUIT CHARACTERISTIC CURVES OF 225-KV-A. SYNCHRONOUS MACHINES

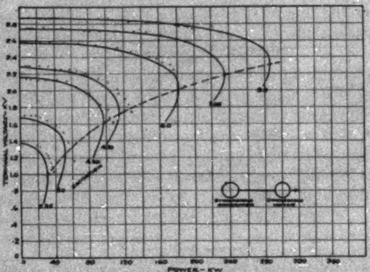

FIG. 3—CALCULATED CONSTANT-EXCITATION POWER VOLTAGE CURVES. EQUAL EXCITATION

. - - - Test points

Methods of carrying out the tests in which the load on some element in the system was suddenly changed are described in due course.

STEADY-STATE STUDIES

It is evident that any analysis of transmission-system power limits falls into two major divisions—one dealing with the limits obtaining for slowly applied loads, as in steady-state operation, and the other with the limits for transient-state, suddenly applied loads. Accordingly, the treatment of the subject in this paper follows more or less the above division.

Influence of Circuit Elements on Power Limits. The first system studied was a simple two-machine system consisting of two 225-kv-a. units connected directly together, electrically. The characteristic curves of these machines, which are identical in design with the other 225-kv-a. units, are given by Fig. 2.

Calculated power-voltage curves for this system, with excitation as parameter, are given by Figs. 3 and 4. Points determined by tests are also shown, giving an idea of the agreement between the computations and test results. The pull-out powers, or power limits, are given in Table I for equal excitations on both machines and in Table II for unequal excitations. The test values were obtained as outlined previously; that is,

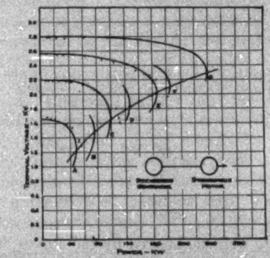

FIG. 4—CALCULATED CONSTANT-EXCITATION POWER-VOLTAGE CURVES. UNEQUAL EXCITATIONS

Curve	Generator Excitation	Motor Excitation
A	3.52 Amperes	2.35 Amperes
B	4.50 "	3.00 "
C	5.26 "	3.52 "
D	5.00 "	4.00 "
E	7.05 "	4.70 "
F	7.50 "	5.00 "
G	8.79 "	5.87 "

. - - - Test points

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7. Discussed later.

the machines were synchronized, the excitations adjusted to the desired values and maintained constant at those values, together with system frequency, as the load was slowly increased by increasing the d-c. generator excitation. The effect of excitation on the power limit is quite well emphasized by the curves; it will be discussed in detail later.

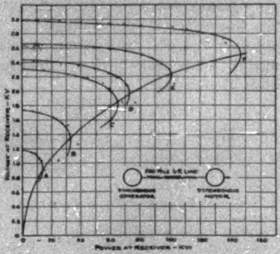

FIG. 5—CALCULATED CONSTANT-EXCITATION POWER-VOLTAGE CURVES FOR 500-MILE STRAIGHTAWAY SYSTEM

Curve	Initial Conditions at Receiver End	
A	1150 Volts	0 kw.
B	1725 "	0 "
C	2300 "	0 "
D	2300 "	50 "
E	2300 "	100 "
F	2880 "	100 "

The next step consisted in the introduction of lines of various constants, proportional to different lengths of actual 220-kv. transmission line. By computing the power-voltage curves for a system comprising an alternator, a line, and a synchronous motor, it was possible to determine the effect of the added impedance on the power limit. No effort was made to include the capacity susceptance of the actual line in the first analysis; this was done later. The constants of these lines, designated *L–R* lines, were as follows:

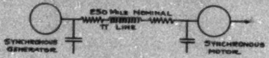

FIG. 6

125-mi. *L–R* line $z_L = 1.27 + j\ 9.25$ ohms
250-mi. *L–R* line $z_L = 2.54 + j\ 18.5$ ohms
500-mi. *L–R* line $z_L = 5.09 + j\ 37.0$ ohms

Calculated and test results for the case of "500-mile straightway transmission" are given by Fig. 5 and Table III.

Tests were made on the 500-mile system using standard regulators adjusted to hold various terminal voltages constant, with the results given by Table IV. These will be discussed under "Use of Voltage Regulators."

The 125-mi. *L–R* and 250-mi. *L–R* lines were substituted for the 500-mi. *L – R* line in the straightaway system. It was found that the power limit at normal excitations[8] was 85 kw. by calculation and 89 kw. by test, for the 250-mi. case, while with a 125-mi. *L–R* line, the corresponding values were 99 kw. (calculated) and 107 kw. (test). The normal regulator tests[9] for the various cases are summarized in Table V.

By replacing the alternator in the 250-mi. straightaway system by a 10,000 kv-a. alternator—practically an infinite bus—the normal-excitation power limit was increased to 125 kw.

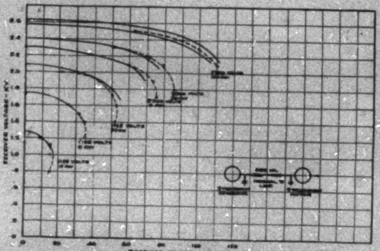

FIG. 7—CONSTANT-EXCITATION POWER-VOLTAGE CURVES FOR SYSTEM OF FIG. 6
——— Test
—·—·— Calculation

Investigation was also made of the effect of line capacity susceptance by studying the system shown in Fig. 6. The power-voltage curves calculated are given with test results in Fig. 7. A brief comparison of these values with those for the 250-mi. straightaway system with *L–R* line will show a decrease in the power limit from 85 to 76 kw., at the excitations giving normal voltage at no load, due entirely to the reduction of the machine excitations by the line charging current.

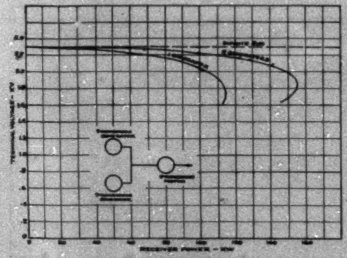

FIG. 8—CALCULATED CONSTANT-EXCITATION POWER-VOLTAGE CURVES
Normal excitation on all units
. . . . . Test points

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[8] At the excitations corresponding to 2300 volts at no load, *i. e.*, 4.7 amperes.

[9] Regulators set for 2300-volt operation.

A comprehensive idea of the relative effect of the various circuit elements on power limits can be gained by a study of Table V which summarizes representative results for the different systems discussed.

The values given in Table V are self-explanatory. They indicate, among other things:

1. The importance of voltage regulators if the maximum possible operating capacity is desired.
2. That transmission-line impedance plays a large part in reducing the maximum power limit but that the synchronous-apparatus characteristics are equally important.
3. That transmission-line capacity susceptance reduces the power limit considerably.

Sufficient has been said in the preceding discussion to indicate that a system with a number of machines operating in multiple should have a higher power limit than the simple straightaway transmission by virtue of the fact that the generating bus, or receiver bus, tends to become an infinite bus, a fact verified by investigation. Fig. 8 illustrates the effect of adding multiple generators while Table VI summarizes the power limits for a number of systems.

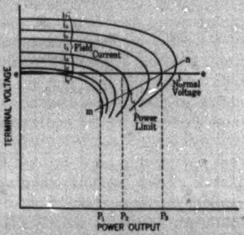

Fig. 9—Typical Constant-Excitation Power-Voltage Curves

Use of Voltage Regulators. A brief survey of the comparative power limits given in Table V will indicate the advantage of operating a system at normal terminal voltage rather than normal excitations. It is not, of course, intended to infer that systems are so operated but merely to establish bench-marks to indicate the advantage, with respect to power limit or stability, of constant-voltage operation. As noted, the ratio of the normal-regulator power limit to the normal-excitation power limit varies from 2.5 to about 1.6, depending on the relative impedance of the circuits.

In view of the importance of maintaining those excitations which correspond to normal terminal voltage at the load being carried, it seems well to discuss at this point the mechanism of the operation by which the regulator maintains constant terminal voltage. Referring to Fig. 9, a number of power-voltage curves, with excitation as parameters, are shown. Suppose the system to be operating with excitation (i_4) and load (p_1). An increase in load of about 20 per cent will cause pull-out at that power at which $\frac{dP}{dE} = 0$. To carry a load greater than that causing pull-out at excitation (i_4), it is necessary to increase the excitation to, say, (i_6). In this case pull-out will occur at a load (p_3). While it is possible to manually adjust the excitations for slowly increasing loads, this is not feasible where rapid load additions may occur. Resort must accordingly be made to some automatic device which will rapidly adjust the excitation to the value corresponding to the actual load and the desired operating voltage. Such a device is the vibrating-contact type voltage regulator. It will be interesting to examine its operation.

As well known, the voltage regulator is designed to maintain constant voltage at some point on a system—in this case, the supply and receiver busses. This is accomplished by increasing the excitation of the proper machine when the voltage at the regulated bus drops or vice versa. However, at any constant load, power factor and voltage, there is a definite excitation required—namely, that given by the intersection of the coordinates corresponding to power and voltage. In other words, the voltage regulator adjusts conditions by causing a transfer of operation from one constant-excitation (power-voltage) curve to another. It follows, then, that pull-out will take place at the intersection of the straight line representing the constant operating voltage with the locus of pull-out points, *i. e.*, power (p_3) and voltage (e), corresponding to excitation (i_6). This is the maximum power which can be delivered by the system at voltage (e). With imperfectly adjusted regulators, however, pull-out will take place at some power less than (p_3).

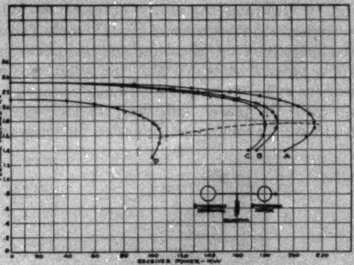

Fig. 10—Calculated Constant-Excitation Power-Voltage Curves for System of Fig. 11

Equal Excitations

Curve	Excitation	Reactor
A	7.6 Amperes	26.9 Ohms
B	6.8 "	36.8 "
C	6.6 "	39.2 "
D	4.7 "	106.5 "

. . . . Test points

It must be borne in mind that system operation with voltage regulators consists of nothing more than a succession of steady-states, each with some particular constant value of excitation. If, for any reason, the regulators are unable to adjust conditions to those corresponding to stable operation, at any load, pull-out will take place along the power-voltage curve corresponding to the particular excitations then obtaining.

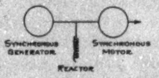

Fig. 11

The preceding discussion will emphasize the vital necessity for voltage regulators on systems which it is desired to operate at loads close to the maxium power at the system voltage. This point has been demonstrated by actual experience on systems.

Influence of Machine Nominal Voltages on System Stability. Analysis of the preceding investigations led to the conclusion that system stability could be improved by increasing the machine nominal voltages for any particular terminal voltage. Two simple methods of accomplishing this were available; namely, the use of static shunt reactors and synchronous reactors (condensers). Accordingly, the use of such devices was investigated and a number of tests made.

Fig. 10 illustrates the results obtained by utilizing shunt reactors in the system of Fig. 11, while Table VII is a comparison of the calculated and test-power limits. It will be noted that, with a 26.9-ohm reactor and equal excitations sufficient to give a terminal voltage of 2300 at no load, the power limit was 215 kw., compared with 115 kw. for similar excitations on the same system without the reactor. In other words, although the terminal voltage has not been raised, *the nominal voltage has been increased*, with a resultant increase in power limit of 87 per cent. If, however, the systems are compared on a basis of *equal* excitations, 4.7 amperes, calculation shows that a reactor of 26.9 ohms will reduce the power limit from 115 kw. to 75 kw.

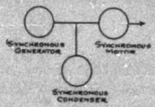

Fig. 12

The increase in power obtained by means of a shunt reactor is due to the greater excitation required to maintain a given terminal voltage for given power conditions. This increased excitation could be obtained more economically by lengthening the air-gap of the machine.

Similar results were obtained using an under-excited synchronous condenser in place of the shunt reactor, as in Fig. 12. The characteristics of this condenser were as shown by Fig. 13. In this case, the tests were made by adjusting the excitations of the generator and the motor to a desired value, and then changing the condenser excitation to give normal terminal voltage at no load. With constant field currents, the system was loaded until loss of synchronism occurred. Tabulated results are given in Table VIII. It will be noted that the power limit for generator and motor excitations of 6.8 amperes is practically identical with the corresponding case for a shunt reactor. The current taken by a synchronous condenser is the difference between the terminal voltage and the nominal voltage divided by the synchronous reactance. When the nominal voltage is zero, the current is just the terminal voltage divided by the synchronous reactance, and the current-voltage curve is identical with that of a static reactor. In the case cited, the condenser excitation was so low that the synchronous condenser was virtually a static reactor. The effect of increasing the condenser excitation, and decreasing the others, to maintain normal voltage (at no load), is well shown by the values in Table VIII. This point is of especial

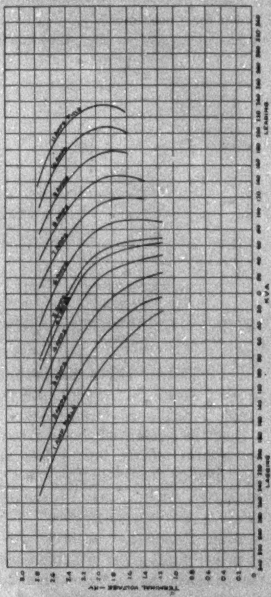

Fig. 13—Voltage-Kv-a. Characteristic Curves for Synchronous Condenser

interest because it illustrates the danger, to stable operation, of low synchronous-generator fields, a condition which may easily arise in actual practise.

Having shown the effect of reactors, either static or synchronous, in increasing machine nominal voltage, studies of systems with lines—Figs. 14 and 15—were undertaken. The static reactors considered were of two types:

(a) Unsaturated iron-core reactor
(b) Saturated iron-core reactor

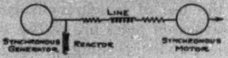

Fig. 14

In Fig. 16 characteristic curves for the actual reactors used are compared with a corresponding curve for the synchronous condenser.

The advantage of using a saturated iron-core reactor[10] lies in the fact that the rate of change of reactive kv-a. consumed with respect to terminal voltage is much greater than for the ordinary non-saturated reactor. In other words, at normal voltage, the kv-a. consumed by the two types of reactor is the same, but at a lower terminal voltage, the kv-a. taken by the saturated reactor is much less. Therefore, any disturbance in the system producing a voltage decrease will cause a reduction in the armature reaction of the synchronous equipment, due to the reactive kv-a. taken by the reactor, thus making an increase in machine excitation available for maintaining stability.

A somewhat similar feature, in connection with the design of synchronous equipment, is discussed later under the head of "Saturated Machines."

The actual tests were made by loading the motor to a desired point and then adjusting the machine excitations to give the required terminal voltage. With these excitations, and frequency, held constant, the system was loaded to the power limit. Computed and test results given in Table IX, when compared with the corresponding results for the simple systems without reactors, show the advantage of raising the machine nominal voltages.

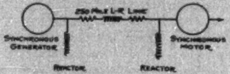

Fig. 15

With an appreciation of the effect of shunt reactors on system power limits, it is quite easy to see why the line capacity susceptance should reduce the power limit, as shown in a previous discussion. Consider the system of Fig. 6, which simulates an actual 250-mi. straightaway system in which the actual line is replaced by the "nominal π" line. The capacity susceptance is, so far as its effect on the machine nominal voltages is concerned, simply a *negative* shunt reactance of about 70 ohms, reducing the required excitation for a given terminal voltage.

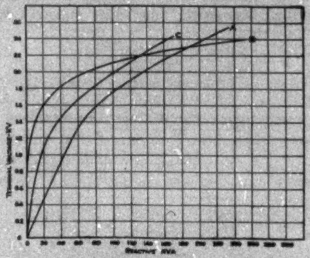

Fig. 16—Voltage-Reactive Kv-a. Characteristic Curves for Various Reactors

Curve *A*—184-kv-a. unsaturated reactor
" *B*—184 " saturated reactor
" *C*—225 " synchronous condenser with 1.0-ampere field current

In this connection, it is interesting to consider the effect of adding multiple lines between generating and receiving equipment of a given kv-a. rating. Due to the increase of charging kv-a. with numbers of lines, a point is soon reached where the decrease of machine excitations more than counterbalances the effective decrease in line reactance. When the number of parallel lines is increased beyond this point, a reduction in the power limit occurs. Fig. 17 illustrates this condition.

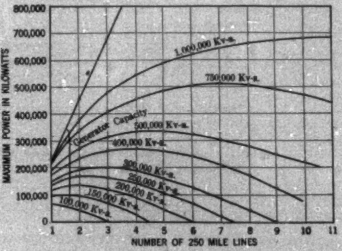

Fig. 17—Maximum Power Which can be Transmitted 250 Miles at 220,000 Volts, Shown as a Function of the Capacity of Synchronous Apparatus, and the Number of Transmission Circuits

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[10] "Theory of D-C. Excited Iron-Core Reactors and Regulators," by A. Boyajian, Trans. A. I. E. E., Vol. 43, 1924, p. 919.

"The Application of the Saturated-Core Reactor and Regulator," by D. K. Blake, Trans. A. I. E. E., Vol. 43, 1924, p. 937.

"Losses in Iron Under the Action of Superposed A-C. and D-C. Excitations," by O. C. Charlton and J. E. Jackson, Jour. A. I. E. E., Vol. 44, Nov. 1925, p. 1220.

It is evident that shunt reactors may be used to compensate for the effect of the leading kv-a. taken by the line capacity susceptance, and thus improve the power limit. Actual studies of reactors and synchronous condensers on large projects indicate that the reactors may be the more advantageous, on the score of economy. As previously pointed out, the same result can be gained by lengthening the air-gap of the synchronous apparatus.

"Saturated" Machines. It should be evident from the preceding discussion that *any measure by which the field current can be increased with the same terminal voltage will improve the stability.* Several methods of accomplishing this have been mentioned under the head of "Influence of Machine Nominal Voltages on System Stability." In addition to methods previously discussed, there is another—that is, increasing the degree of saturation of the poles.

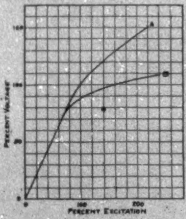

FIG. 18—OPEN-CIRCUIT CHARACTERISTIC CURVES
Curve *A*—Normal generator
Curve *B*—Saturated generator

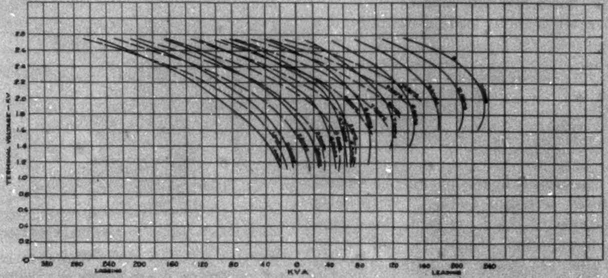

FIG. 19—VOLTAGE-KV-A. CHARACTERISTIC CURVES FOR SYNCHRONOUS CONDENSERS
——— "Normal" 225-kv-a. condenser ——— "Saturated" 225-kv-a. condenser

Consider the saturation curves of two synchronous machines, as shown in Fig. 18; one for a "normal" machine with operating range on the saturation curve somewhat below the knee, the other with normal voltage well above the knee. Furthermore, the machines will be of identical design except in so far as the cross-sectional area of the poles is concerned.

If the two machines be operated separately as generators with such field currents that normal terminal voltage occurs at no load, identical power increments on each will not cause the same lowering of the terminal voltage. In other words, *the saturated machine is stiffer.*

Such tests as were made indicate that the benefits obtained by the use of synchronous generators and motors with saturated poles are of the same order as could be obtained by lengthening the air-gap.

When synchronous condensers are used for voltage regulation, it is desirable that the rate of change of reactive kv-a. with terminal voltage be as large as possible. The ideal condenser would be one for which this rate of change is infinite—essentially an infinite bus. Fig. 19 shows how this rate of change can be increased by pole saturation.[11] Although this could be accomplished by lengthening the air-gap, nevertheless pole saturation is more effective.

Use of Induction Generators on Transmission Systems. Induction generators have been suggested, from time to time, for use in connection with long transmission lines but they have never come into practical use. During the course of the present investigation it was felt that there were certain merits in such use which should be carefully investigated. As known now, the power which can be transmitted over a line between two synchronous machines is limited by the phase relationships of the system. With the induction generator, however, the power delivered from the prime-mover shaft to the generator terminals depends upon the difference in speed of the prime mover and the terminal-voltage vector. With a prime mover and a rotor capable of transmitting any desired power from the prime-mover shaft to the stator circuit, the power limit will be dependent upon the angle between the rotor of the receiver-end motor and the air-gap flux of the generator. That is, the power limit of synchronous-to-synchronous transmission is a function of the angle between the machine rotors but with induction-synchronous transmission is dependent upon the angle between induction generator air-gap flux and synchronous motor rotor.

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[11] Discussion by Miss E. Clarke, TRANS. A. I. E. E., Vol. 43, 1924, p. 81.

Tests were made on the system of Fig. 20, in which the induction generator was rated at MT-6-200 h. p., 1200 rev. per. min., 2200 volts—a standard design of wound-rotor induction motor. Voltage regulators were provided for maintaining constant normal voltage at both ends. By running the synchronous machine at the supply end as an alternator carrying no load, it was possible to simulate condenser operation and yet maintain frequency. This was accomplished by loading the system, which threw load on the supply-end synchronous unit; then, by holding the frequency at its normal value and increasing the speed of the induction generator, it was caused to take load.

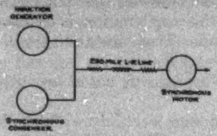

Fig. 20

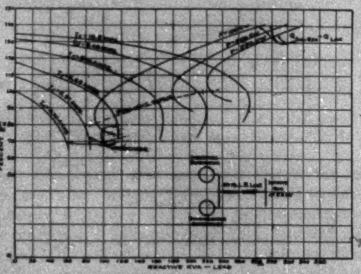

Fig. 21—Calculated Stability Curves

P = Power
Q = Quadrature kv-a.
I_f = Condenser field

The results were not striking because the power limit for the system of Fig. 20 was found to be 202 kw., whereas the limit for a 250-mi. *L–R* system with the same total kv-a. in *synchronous* apparatus at the supply end is 192 kw. These results were checked by calculations made for the actual test set-up and are summarized in Table X. In order to eliminate any possibility of error in method, calculations were also made utilizing the scheme employed by Messrs. Evans and Bergvall,[12] and others. In this case stability is determined by a comparison of available and required reactive kv-a. at the point under consideration. Terminal voltage enters as a variable, and power as a parameter. Curves calculated by this method are given by Fig. 21 and show a power limit of about 215 kw. at normal voltage, compared with 218 kw. by the method used by the authors.

Although economic studies may indicate a possible margin in favor of the induction generators, there is practically no advantage to be gained by their use, so far as stability is concerned.

"Loaded" Transmission Systems. In recent years much attention has been given to the possibility of increasing the stability of power systems by the use of synchronous condensers at intermediate points.[13] Investigations made by the authors indicate that greater gains have been ascribed to such use of synchronous condensers[14] than is actually the case.

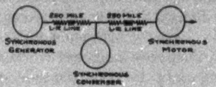

Fig. 22

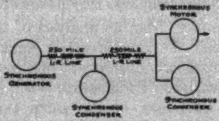

Fig. 23

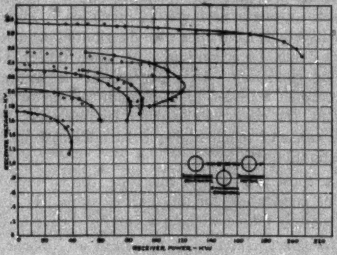

Fig. 24—Constant-Excitation Power-Voltage Curves for System of Fig. 22

—o— Calculated
+ Test points

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[12] "Experimental Analysis of Stability and Power Limitations," by R. D. Evans and R. C. Bergvall, Trans. A. I. E. E., Vol. 43, 1924, p. 39.

[13] "Voltage Regulation and Insulation for Large Power Long Distance Transmission Systems," by F. G. Baum. Trans. A. I. E. E., Vol. 40, 1921, p. 1017.

[14] "Some Theoretical Considerations of Power Transmission Systems," by C. L. Fortescue and C. F. Wagner, Trans. A. I. E. E., Vol. 43, 1924, p. 16.

"Power Limitations of Transmission Systems," by R. D. Evans and H. K. Sels, Trans. A. I. E. E., Vol. 43, 1924, p. 26.

The systems shown by Fig. 22 and Fig. 23 were used as a basis for computations, and verifying tests, of the effect of synchronous condensers used for "loading" a long time—that is, regulating for normal voltage at the "loading" point. Calculated and test power limits for the system of Fig. 22, with such constant excitations on the units as necessary to give equal line voltages simultaneously at any initial load, are given by Table XI, while the complete power-voltage curves are given in Fig. 24. With voltage regulators maintaining equal voltages at both ends of the line, as well as the midpoint, the power limits were those of Table XII. Corresponding data for the 500-mile system with a 225-kv-a. generator, a 225-kv-a. condenser at the midpoint and also one at the receiver end, together with a synchronous motor of a similar rating, are given by Tables XIII and XIV.

Data for several systems have been given in Table XV for similar conditions of operation, so that the effect of "loading" long lines may be appreciated.

It would appear, from an analysis of the values in Table XV, that the gain in power limit due to the use of synchronous condensers at intermediate points is quite appreciable. To illustrate —the addition of a synchronous condenser of the same rating as the terminal apparatus to the simple 500-mile L–R straightaway system increases the power limit as normal voltage from 110 kw. to 150 kw. or about 36 per cent. It must be remembered, however, that on actual systems, the synchronous condensers at loading points would ordinarily have a much smaller capacity than the terminal apparatus. Consequently, the percentage increase in power limit will be much less. Moreover, actual systems have lines with distributed capacitance, which tends to accomplish the same object as the loading condenser. In other words, to secure any appreciable increase in maximum power of actual systems, the condenser used for loading must have a kv-a. capacity much higher than indicated by early investigations.[14]

Like most problems of this type, the advantages of loading condensers, for any particular project, cannot be definitely stated without a careful economic study.

"Resistance" Loading of Transmission Systems. All the systems discussed so far have had synchronous-motor loads—in other words, typical shaft loads, which have constant kilowatt and variable reactive kilovolt-ampere consumption; that is, the power is independent of the voltage. No reference has been made to high power-factor admittance loads, such as lighting, industrial heating, etc. It will, therefore, be interesting to turn attention for a few moments to the question of such loads—"resistance" loads—for long transmission systems.

Consider two synchronous machines tied together through an impedance. It is well known that the limit of stability occurs at the point at which $\frac{dP}{dE} = 0$, a fact verified many times by test. The criterion for maximum power, $\frac{dP}{dE} = 0$, holds only where power is the independent variable, as in shaft loads. Suppose, however, that one of the synchronous machines in this combination is replaced by a dead impedance load. Then, instability will *not* occur at the point where $\frac{dP}{dE} = 0$, nor at any other point, since for a dead load there is nothing with which the generator can fall out of step. There will, however, be a *maximum power* point, for constant excitation on the generator, as illustrated by Fig. 25. If voltage is maintained at the load by a synchronous condenser, it can be shown that instability will occur where $\frac{dR}{dE} = 0$, instead of $\frac{dP}{dE} = 0$, where the resistance (R) of the dead load is the independent variable.

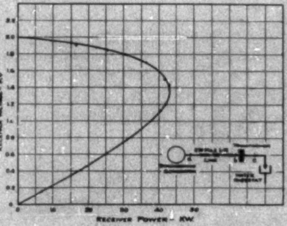

FIG. 25—CONSTANT-EXCITATION POWER-VOLTAGE CURVE. GENERATOR EXCITATION 3.8 AMPERES

The importance of choosing the proper criterion can be better appreciated if it is understood that a pure resistance load with a synchronous condenser for voltage regulation is equivalent to an infinite bus, as far as maximum power under steady-state conditions is concerned. This statement is true only when there is no other load on the system. To emphasize the importance of choosing the proper criterion, a system comprising both resistance and shaft loads may be cited.

In such a case, consider what happens when the shaft load is increased. Any increment of shaft load results in a decrease of terminal voltage and, hence, a decrease in power consumed by the resistance load. A set of typical power-voltage characteristics of a system with both shaft and resistance load is shown in Fig. 26. Curve (P) represents the variation of total power with voltage; (P_R), the variation of resistance power with voltage; and (P_s), the variation of shaft power with voltage. It is evident from the figure that the slope of the curve of total power becomes infinite—that is, $\frac{d\,P}{d\,E} = 0$—at that point where an increase of shaft power is exactly equal to the resultant decrease in resistance power. However, *the system is still stable,* inasmuch as the slope of the curve of shaft power has not yet become infinite, and (P_s) *is the independent variable.* Breakdown occurs where

$$\frac{d\,P_s}{d\,E} = 0,$$

and the total power is as shown by the ordinate ($a\,a'$). In other words, a system with such a composite load may be stable even though operation is on the under side of the total power-voltage curve.

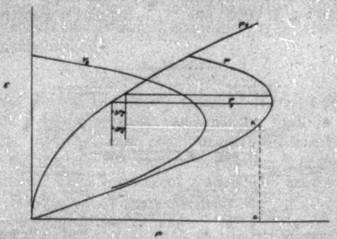

FIG. 26—TYPICAL POWER-VOLTAGE CHARACTERISTICS FOR SYSTEM WITH COMPOSITE SHAFT AND RESISTANCE LOAD

Compensation of Transmission-Line Inductive Reactance. Preceding discussion has shown the effect of the reactance of long transmission lines in limiting the amount of power which can be transmitted with stable operation. It is apparent, then, that any means by which this effect could be economically reduced would be advantageous. It is possible to reduce the line reactance by use of special line construction.[15] However, the decreased line reactance is gained at the expense of an increase in charging kv-a., which may partially offset or even more than offset the advantage gained.

However, by introducing series static condensers into the line,[16] it is possible to effectively neutralize or compensate for the effect of line inductive reactance without any increase in charging kv-a. There are certain difficulties attendent upon such use of static condensers, as protection from excess voltages during short circuit. Careful investigations show the feasibility of series static condensers for this service hinges almost entirely on over-all economy.

Influence of Methods of Voltage Regulation on Stability. Careful analysis of the effect of various circuit elements, and special types of apparatus, on transmission-system stability, as discussed elsewhere in this paper, will indicate that the real criterion of stability is the inherent slope of the power-voltage characteristic. In other words, instability occurs when the slope of the power-voltage curve is such that $\frac{d\,P}{d\,E} = 0$, for shaft loads. How can this occur in a power system the synchronous machines of which are provided with automatic voltage regulators?

As pointed out under "Use of Voltage Regulators," the regulator is merely a device for shifting the system from one constant-excitation condition to another, as dictated by the terminal voltage. Referring to Fig. 9, voltage is maintained by the regulator functioning in such a way that the system operates along the line (ee) by passing from one constant-excitation state to another. However, and this is the important point, the inherent stability of the system is not that indicated by the slope of the line (ee)—*i. e.,* $\frac{d\,E}{d\,P} = 0$—but by the slope of some constant-excitation curve. It may be any one of a number as shown, but it is always some one of the family of curves. It is in just this respect that regulated machines do not have the characteristic of an infinite bus with $\frac{d\,E}{d\,P} = 0$, the ideal type of voltage regulation.

In this connection, it will be of interest to examine the phenomena accompanying an increase of load on a piece of synchronous apparatus in a transmission system. Such an increase in load will result in a decrease in terminal voltage with a given excitation.

Now the voltage regulator is not responsive to the rate of change of voltage, but only to a value of voltage—the function itself; hence, it increases the excitation in the degree necessary to compensate for the increased armature reaction due to load only after the lapse of an appreciable time. That is, the field excitation is increased only after the voltage has dropped. Such investigations as made by the authors do not indicate that any appreciable gain in power limit can be secured by reducing the time constant of the excitation system below that of normal equipment. Compensation for increased load, and, hence, armature reaction, always lags behind the cause. This is important during transients and especially near the point of intersection of (ee) and (mn)—the point of pull-out. It is the reason why a system with regulated generating stations cannot be considered as having infinite supply busses and hence a power limit equal to that of the line itself.

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[15] "Output and Regulation of Long-Distance Lines," Percy Thomas, TRANS. A. I. E. E., Vol. 28, 1909, p. 615.

[16] Discussion by T. A. E. Belt on "Present State of Transmission and Distribution Developments," JOUR. A. I. E. E., Vol. 44, October, 1925, p. 1153.

Consider what happens when load is added to a d-c. generator provided with a compensating field winding. Simultaneously with the increase in armature reaction, a magnetomotive force is set up in opposition to it. Depending on the size of the compensating field, the effect of increased load may be entirely counterbalanced or compensated for—even over-compensated. A similar case is that of a circuit in which the inductive reactance is exactly balanced by an equal series capacitive reactance. What happens when the circuit load is increased? The voltage drops across both inductive and capacitive reactances increase simultaneously but being of opposite phase effectively neutralize each other. With respect to the terminals, there is no reactance drop.

Applying the principle enunciated above—i. e., the effective neutralization of the cause by making the effect take place simultaneously—to synchronous equipment, the great importance of it can be seen. The machines in effect would have infinite capacity, limited only by heating and mechanical considerations. To realize the significance of this, refer to Table V and related discussion, where it is pointed out that the power limit of a 250-mi. miniature *L-R* line by itself is 283 kw., but with normal machines, only 168 kw. That is, the synchronous reactance of the machines reduces the power limit about 41 per cent. In actual systems, the effect of the generating and receiving equipment might even increase this value to 50 per cent.

It is evident that if some device can be applied to synchronous apparatus which will prevent a reduction of the net ampere-turns tending to force flux through the magnetic circuit, such a device will effectively decrease the effect of armature reaction, which is so detrimental to the stability and power limits of long transmission systems. That is, as load builds up, the armature reaction builds up, but simultaneously and due to exactly the same causes, the field ampere-turns must build up with the right magnitude and space phase to compensate for the armature reaction. *The perfect simultaneity of cause and effect effectively neutralizes armature reaction.*

Of course, to obtain the ideal in voltage regulation, the inherent reactance of synchronous machines must also be neutralized—as by the use of series static condensers of suitable capacity—in the same way that the effect of line reactance may be overcome.

It is apparent that it is somewhat difficult to completely accomplish what has been outlined above—for the effective compensation of armature reaction necessitates the application of field ampere-turns in the proper space phase and varying in magnitude and time with the armature reaction. It was realized, however, that any device which would increase the excitation in exact proportion to the armature reaction—or line current—and in perfect simultaneity with it, should be of considerable advantage.

Accordingly, a number of devices and methods for accomplishing this object were carefully investigated. Finally, it was decided that ordinary mercury-arc rectifiers used as adjuncts in the otherwise normal excitation circuits of synchronous apparatus appeared to offer the greatest promise.

To determine the effectiveness of mercury-arc rectifiers for accomplishing the object in mind, comparative power-limit tests were made on a system comprising a 225-kv-a. synchronous generator, a 250-mi. *L-R* line and a 225-kv-a. synchronous motor. In the first case, the system was set up as in Fig. 27A, and equipped with standard automatic voltage regulators adjusted to maintain 2000 volts at the supply and receiver ends of the line. The power limit was determined in the usual way by slowly increasing the system load until pull-out occurred, at 120 kw. at the receiver end. Mercury-arc rectifiers were then added to the excitation systems as shown in Fig. 27B, and the power limit of the system, with 2000 volts maintained at both ends, was determined. The load was first brought to 120 kw. at 2000 volts. The regulators were then ineffective because the rectifiers were supplying sufficient excitation to maintain 2000 volts and the regulators had reduced the exciter terminal voltage to a negligible value. Consequently, the regulators were taken out of service although they could have been allowed to float on the system. The load was then slowly increased, voltage being held at 2000 volts by manually adjusting the rectifier shunts, to 154 kw. To repeat, with 2000-volt regulators, the system is stable up to 120 kw.; with mercury-arc rectifiers as adjuncts in the excitation circuits, the system is stable up to 154 kw., at 2000 volts.

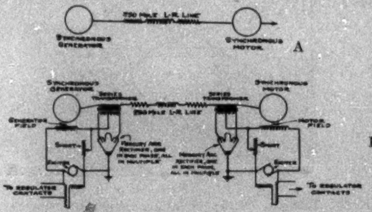

Fig. 27—Systems Used for Study of Effect of Mercury-Arc Rectifiers on Stability

Fig. 27 A—Simple system

Fig. 27 B—Simple system with addition of rectifiers as excitation-circuit adjuncts

Let the significance of this be realized. The power limit of a 250-mi. line of the miniature type used in these investigations, with infinite supply and receiver busses, is about 214 kw. at 2000 volts. The addition of two machines, a generator and a motor, reduces this to 120 kw. or by 43.8 per cent. By adding mercury-arc rectifiers to the excitation systems, the power limit is increased to 154 kw., or by 28 per cent. Furthermore, bear in mind that the rectifiers only supplied field excitation in rigid proportion to the line current in perfect simultaneity. The additional ampere-turns are not, however, supplied in the proper space phase.

It will be evident that it is difficult to calculate the performance of a system with rectifiers used in the manner described. This, of course, arises from the difficulty of knowing just how effectively the rectifier neutralizes the armature reaction. Assuming perfect compensation, the system reduces to one with a source of constant flux supplying power through the leakage reactance of the machines. Calculations have been made on this basis for a number of miniature systems actually tested.

The use of mercury-arc rectifiers has shown, in factory tests, that a very appreciable gain in stability and power limits can be secured. While practical application on power systems has not yet been made, it is confidently believed that, at least, a 50-per cent reduction in effective armature reaction can be secured. Moreover, a real scheme of voltage regulation, effective under steady and transient conditions, is for the first time made available.

TRANSIENT STUDIES

When the question of the stability of power-transmission systems recently came to the fore, it was thought that the worst condition of operation likely to be faced would be that obtaining during transient conditions. Such a transient state might arise from:

a. Sudden loss of a generating station

b. Sudden addition of load

c. Sudden loss of one or more multiple transmission lines

d. Major system short circuits

In fact, it was quite generally expected that the power limit for such transient conditions would be less than the steady-state pull-out power.

However, very complete analytical studies, verified by thorough tests on the miniature system used for the steady-state power-limit investigations previously discussed, have largely dispelled the doubts of successful operation of extensive power systems under heavy load transients. Briefly, the following important conclusions have been reached from the evidence furnished by analysis, miniature-system tests, and field experience.

1. The power limit of a system, when load is suddenly applied to some element in that system, is the same as the pull-out power for slowly applied loads, provided conditions of excitation and voltage are the same for the two cases. Certain transitory phenomena, discussed later, so affect system stability that *the fundamental criterion of stability appears to be the steady-state power limit.*

2. Power oscillations during a transient-state induced by the sudden application of load will not be excessive with units of comparable rating at the various points on the system. With an infinite generating unit, or an extremely large system interconnected with a relatively small one, the power "overshoot" may approach 100 per cent of the load increment.

3. Maintenance of stability during short circuits is mainly a matter of adequate relaying as duration of the short circuit is probably the most important single factor entering the problem.

During the existence of a transient state caused by a sudden change of load or a short circuit, a number of factors come into play which do not appear in steady-state operation. Among others, there are the kinetic energy of machines, the time element of electromagnetic circuits, as in excitation systems, and the load-time characteristic of prime-mover governors. For these reasons, the calculation of transient problems is much more involved than the computation of steady-state stability. However, rigorous mathematical methods and step-by-step analysis[5] can be applied to the simplest cases. The analysis of systems of any degree of complexity becomes almost hopelessly involved so that resort must be had to the equivalent-circuit idea[6], which is extremely useful for the study of transients involving loads below the steady-state power limit.

Effect of Suddenly Applied Loads. Inasmuch as the basic purpose of the transient studies undertaken by the authors and other interested engineers was the determination of the relation between steady-state and transient stability, and the factors affecting the latter, the miniature system previously described was utilized. It was possible to simulate the sudden loss of a generating station by opening the oil switch tying a generating unit to the system; the sudden addition of shaft load by closing a switch throwing a water-rheostat load on the d-c. generator coupled to a synchronous motor at the receiver end of a system, and the sudden addition of "resistance" load by switching a dead resistance load directly onto the system; the sudden loss of one or more multiple transmission lines by opening the oil switch in one of two 250-mi. *L–R* lines connecting a 225-kv-a. generator to a synchronous motor of identical rating.

The steady-state power limit of any particular system studied was first obtained for various constant-voltage or constant-excitation conditions. Then, by means of repeated trials, it was possible to determine how much load could be suddenly added to the system without causing loss of synchronism—instability. The results were tabulated for the various cases and, as mentioned, on analysis revealed the important fact that the power limit for transient state was essentially the same as that for steady state, under the same conditions of excitation or voltage.

Consider the curves shown by Fig. 28. These indicate the oscillations taking place in power and speed on the sudden addition of shaft load to the motor. They illustrate the relatively small overswing in power with units of comparable rating at either end of a transmission line.

When load is suddenly thrown on the motor in a system such as that in Fig. 28, a temporary drop in speed must occur before the motor can assume the new phase position corresponding to the load. Consequently, part of the increased demand is supplied by the change in kinetic energy of the motor. Similarly, the initial demand for increased power from the generator is also met by a decrease in its momentum. As a result, the units oscillate against each other until equilibrium between input and output is re-established. As the governor influences the mechanical input, it will be of interest to study Fig. 29, a set of typical steam-turbine governor time-load characteristic curves.[17]

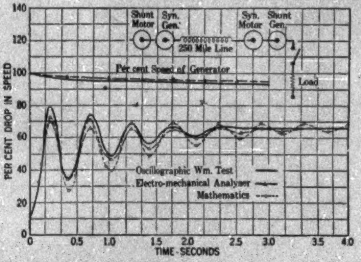

Fig. 28—Transient Conditions Following a Sudden Application of Shaft Load

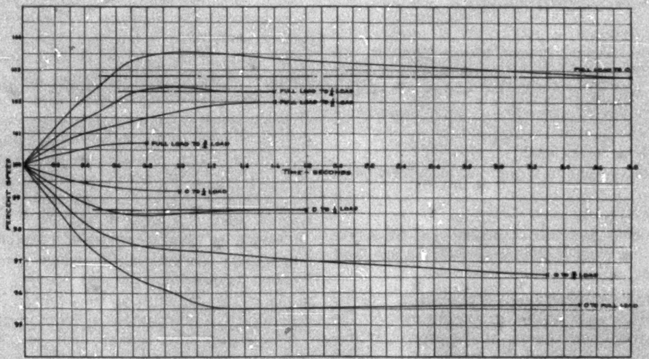

Fig. 29—Steam-Turbine Governor Time-Load Transient Characteristic Curves

The curves of Fig. 28 show close agreement for results obtained by actual test, mathematical computation, and by the electromechanical analyzer. It is particularly encouraging that the various methods employed should yield such a close comparison.

Fig. 30 illustrates what happens when a generating station is dropped from a small system tied to an extremely large one. The receiver-end units were each rated at 225 kv-a. and the "infinite generator" at 10,000 kv-a. The curve shows the power supplied to the system by the "infinite generator" before and during the transient. In this case, the power overswing was quite large. As previously noted, the limit of the power "overshoot" when the generating station has infinite capacity is 100 per cent of the load increment.

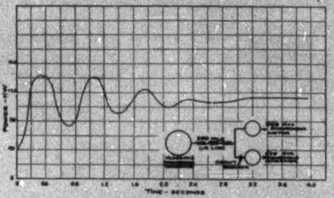

Fig. 30—Power Transient Following the Loss of a 225-Kv-a. Generator from a System Supplied by a 10,000-Kv-a. Generator

With units of comparable rating at either end of the system, as in Fig. 31, the transients are not of the same magnitude as with an infinite generator. In this case, generator No. 4 was dropped from the system, by opening its switch, with two complete lines in service and a load distribution as follows:

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[17] Attention is drawn to corresponding curves for waterwheel governors in an article entitled "Results of Seventy-Three Tests Made on Hydraulic-Turbine Governors," *Power*, Vol. 58, No. 15, Oct. 9, 1923, p. 560.

Generator No. 1	100,000 kw.
" " 2	280,000 kw.
" " 4	20,000 kw.
Synchronous Motor No. 3	400,000 kw.

The transients ensuing are shown by Fig. 32.

In a case such as this, in the first instant or two, the phase relationship of the machine rotors does not change, due to the inertia of the rotating masses. Consequently, only the electrical constants determine the power flow and electromagnetic torque of the various machines. As the transient dies away, however, the system assumes the state of equilibrium dictated by the governors of the prime movers.

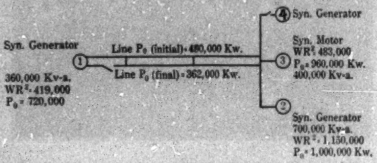

FIG. 31

Similar transients are caused by suddenly increasing the line reactance, as by opening a section of a line, or dropping one or more multiple lines. Fig. 33 shows the transients due to opening the line switch, in Fig. 31, under load. The load distribution was the same as before, with the exception of generator No. 4, which was disconnected from the system. The increase in line reactance causes a power oscillation during the time taken by the machine rotors in assuming their new relative positions.

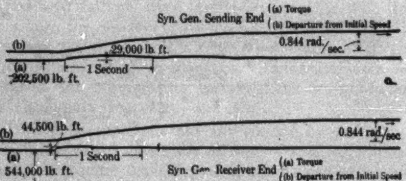

FIG. 32—TRANSIENTS DUE TO DROPPING GENERATOR NO. 4 FROM SYSTEM OF FIG. 31

It will be readily apparent from what has been said that any time delay in the various elements making up a system would play an important part in transient stability. There are two such elements:

1. The electromagnetic circuits
2. The prime-mover governors

As the first is quite fully discussed later, only the second will be considered here. Referring to Fig. 29, and remembering that the governors of hydraulic turbines are usually more sluggish than for steam units, it can be seen that it may be several seconds before the governors of a system assume their new positions. Consequently, in analysis of what happens during the first moment or two, constant flow (of water, or steam) is commonly assumed.

Short Circuits. Load transients apparently do not decrease the power limit of a system but the transient conditions obtaining during a major system short circuit radically affect stability. Not only do the inertias of machines and the time elements of governors and excitation systems come into play, but also the isolating effect of the short circuit. That is, no power flow can take place past the point of short circuit in the phases affected. For this reason, if the fault is not quickly cleared, a three-phase short circuit is likely to result in instability. Single-phase short circuits, whether line-to-line or line-to-neutral, are not so serious, inasmuch as partial flow of power is possible.

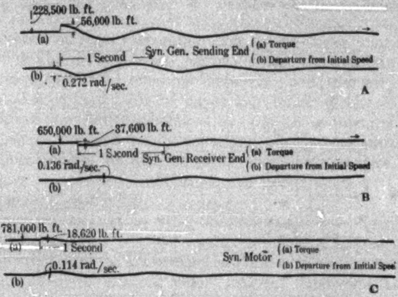

FIG. 33—TRANSIENTS FOLLOWING THE SWITCHING OUT OF A LINE SECTION, AS INDICATED IN FIG. 31

Brief consideration of what takes place in a system during a short-circuit disturbance may be illuminating. Assume a straightaway transmission with a single-phase line-to-line short circuit at the midpoint.

Just prior to the occurrence of the short circuit, the machine rotors will have a definite space-phase relationship fixed by the constants of the system, the load and the voltage. The instant the disturbance occurs the transfer of power between the machines changes, ceasing entirely in the faulty phase, and the rotors commence to assume a new relative position in space, depending on load and voltage conditions prior to the short circuit, the power flow, and the governor characteristics. Consider the generator. If the torque exerted by the system remains at the same value as previously, the net effect on the generator during the first instant will be zero. However, as the net flux is reduced and, hence, the voltage, this torque is likewise reduced and the machine accelerates, assuming no action on the part of the governor. Should the governor act, this acceleration will be reduced. The rate of change of net flux will be rapid in the first few instants, gradually decreasing with increasing time. This, of course, assumes no action on the part of the regulator tending to change the field ampere-turn loading. Should the torque imposed by short-circuit conditions be initially less than the load torque, the acceleration of the generator will, necessarily, be greater. It is evident, then, that the speed, the space-phase relationship of the generator relative to the initial position, and the magnitude of the net flux, hence, terminal voltage, are a function of the duration of the short circuit.

Inasmuch as similar conditions hold for the receiver-end synchronous apparatus, it is apparent that *the essential factor for maintenance of system stability during short-circuit disturbances is adequately fast relaying.*

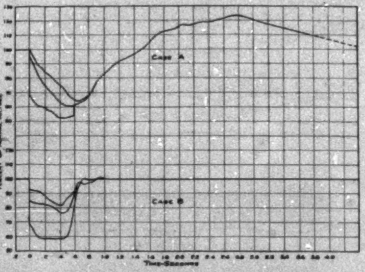

FIG. 34—VOLTAGE TRANSIENTS FOLLOWING A DEAD SINGLE-PHASE LINE-TO-LINE SHORT CIRCUIT AT MIDPOINT OF THE 250-MI. L-R LINE IN THE SYSTEMS OF FIG. 27A AND FIG. 27B. DURATION OF SHORT CIRCUIT ABOUT 0.5 SECOND.

Case A. System of Fig. 27A, with regulators. Load being carried at time of short circuit 37 kw.

Case B. System of Fig. 27B, with rectifiers as adjuncts in the excitation circuits. Load being carried at time of short circuit 35 per cent greater than in Case A.

Experience with actual systems has amply justified this statement.

Tests made by the authors on the miniature system with a 225-kv-a. generator supplying power to a synchronous motor of similar rating at the receiver-end of a 250-mi. *L–R* line have also justified the statement above. Careful tests have shown that this system can carry any load up to the steady-state power limit for a dead single-phase line-to-line fault at the midpoint which is practically instantaneously cleared. Increasing the duration to one-half second, the system could carry only 31 per cent of the steady-state limit, with automatic voltage regulators on both units. This limit was increased to 50 per cent, using the mercury-arc rectifier regulation scheme previously noted, and much less disturbance occurred, as illustrated by the comparative curves of Fig. 34. This, however, was entirely due to the fact that the armature reaction of the machines was effectively neutralized simultaneously with its occurrence.

The effect of a single-phase short circuit on power flow is shown by Fig. 35, for a fault at (*X*) on the system of Fig. 36. In this case, the short circuit was allowed to remain on the system. The similarity of these oscillations with those caused by a load transient is notable.

FIG. 35—POWER TRANSIENT FOLLOWING A SINGLE-PHASE SHORT CIRCUIT AT POINT X ON SYSTEM SHOWN IN FIG. 36

Experience on existing systems, tests and conservative calculations indicate that major system short circuits are the severest type of transient. Only by very fast relaying can stability be maintained for powers near the steady-state power limit.

Effect of the Excitation-System Time Element on Transient Stability. In the discussion of system stability under suddenly applied loads, it has been said that certain factors so strengthen or stiffen the system that it is inherently stable for any load up to the steady-state power limit. What are these factors?

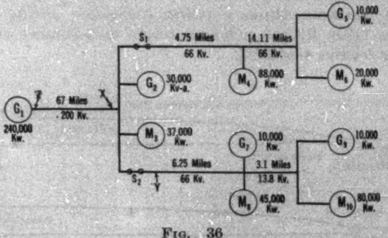

FIG. 36

Undoubtedly, the most important element in the *temporary stiffening* of a system during a disturbance is the *field transient.* Due to the inherent relationship between field and armature circuits, the magnetic flux linked with an alternator field cannot change in the first moment following the sudden addition of load.[18] The increased armature current induces a field m. m. f. tending to sustain a constant flux linkage. If there is no automatic device increasing the exciter voltage, the flux will gradually die down to a new steady-state condition. However, and this is the important point, *during the transient, the machine reactance is less than* *the synchronous reactance.* Initially, following the sudden application of load or short circuit, the transient reactance[19] is effective, and this gradually increases to the steady-state synchronous value. Furthermore, the inherent stiffening of the system enables the regulator to act, causing the exciter to build up. Briefly, it is a race between the rate at which the system assumes the new load and the rate at which the excitation can build up. Now, the period of oscillation of a system is of the order of one second, and, hence, the first power peak occurs in a half-second. The *field transient*, or the transient reactance, stiffens the system sufficiently to meet this condition. By the time the power peak occurs again, the excitation system has had considerable time in which to build up. In this respect, it will be interesting to note the relationship of the time constant $\left(\frac{L}{R}\right)$ to the volume of the exciter. Values for a large number of exciters, of different types and ratings, have been plotted in Fig. 37. In this case, the measure of volume used has been kilowatt per rev. per min., as this represents volume, for given current and magnetic densities. The actual exciters used in the experimental investigations are indicated by the points (ba), $(b'a')$, and $(b''a'')$.

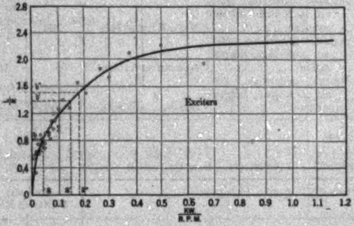

FIG. 37—CURVE INDICATING THE RELATION BETWEEN THE TIME CONSTANT AND THE SPEED AND RATING OF EXCITERS

It will be apparent that any scheme by which the resistance of the excitation circuit—that is, alternator field and exciter armature—can be reduced will tend to maintain constant flux linkages and so give the exciter a chance to build up. One method of accomplishing this would be the introduction of a *negative* resistance in the field circuit. A series exciter does function as a negative resistance. Its use in such a case has already been described.[8]

Neither the voltage regulator nor the series exciter, however, accomplishes the primary object of the ideal regulation scheme—*i. e.*, the effective neutralization of armature reaction at the time it occurs. As previously pointed out, the mercury-arc rectifier, as an adjunct in excitation systems, does accomplish, to a large degree, this purpose, and in so doing very materially stiffens any system so equipped, both in transient and steady states.

The inherent field transient so stiffens the average system that ordinary regulators and exciters are sufficiently responsive to maintain stability under heavy load transients. Mercury-arc rectifiers, or other devices accomplishing the same purpose, appear to be the most effective of all regulation schemes in reducing the effect of transients.

CONCLUSIONS

The preceding discussion has made it evident that the problem of stability is not confined to long-distance high-tension transmission lines, where economical considerations require operation close to the power limit. With the growth of the nation's power systems into widespread interconnected networks, and the consequent transmission of larger and larger blocks of power over short interstation tie lines, the problem of stability becomes vitally important, inasmuch as such short tie lines, on which depend reliability of service, may be operated too close to the power limit, with danger of instability.

In general, the investigations discussed have been most gratifying because they have indicated in what direction engineers interested in the design and operation of power systems should look for improvement. Furthermore, they have indicated that the real advances in obtaining additional stability lie in the application of certain comparatively cheap types of apparatus—namely, devices for voltage regulation which fulfill the conditions of the ideal voltage-regulation scheme—*i. e.*, $\frac{dE}{dP} = 0$. Improvement may be effected by changes in design of present synchronous apparatus—such as lengthening the air-gap or operating the pole structures at high magnetic densities.

To summarize, then, the following important conclusions have been reached from the studies undertaken by the authors:

1. The characteristics of synchronous terminal apparatus are of great importance in their effect on power-system stability. In fact, they may be of greater importance than the transmission-line characteristics.

2. Automatic voltage regulators and suitable exciters are essential to obtain, under all conditions, the same power limit as could be obtained with manual control and slowly applied loads.

3. The charging kv-a. of long transmission lines and extensive cable networks exercises more of a detrimental effect on stability than has been hitherto appreciated.

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[18] "A Simplified Method of Analyzing Short-Circuit Problems," by R. E. Doherty. TRANS. A. I. E. E., Vol. 42, 1923, p. 841.

[19] The transient reactance includes both armature and field leakage reactance.

4. Improvements in system stability can be readily made by modifications of present designs of apparatus. Investigations indicate that real advances in stability come from the adoption of methods of voltage regulation having inherent characteristics close to those of the ideal—*i. e.*, $\frac{d\,E}{d\,P} = 0$. One method studied—the mercury-arc rectifier as an adjunct in the excitation

TABLE I

POWER LIMITS FOR SYSTEM CONSISTING OF TWO 225-KV-A. MACHINES ON THE SAME BUS

EQUAL EXCITATIONS

Excitation in Amperes		Pull-out Power in Kw.	
Generator	Motor	Calculated	Test*
2.35	2.35	32	38
3.0	3.0	52	62
3.52	3.52	—	80
4.2	4.2	98	108
4.7	4.7	115	129
6.0	6.0	182	187
7.05	7.05	236	—
8.00	8.00	287	—

*Test Results are the average of several observations of the pull-out power.

TABLE II

POWER LIMITS FOR SYSTEM CONSISTING OF TWO 225-KV-A. MACHINES ON THE SAME BUS

UNEQUAL EXCITATIONS

Excitation in Amperes		Pull-out Power in Kw.	
Generator	Motor	Calculated	Test
3.52	2.35	48	59
5.26	3.52	98	112
7.05	4.7	167	180
8.79	5.87	239	—
2.35	3.52	48	54
3.52	5.26	98	111
4.7	7.05	167	—

TABLE III

POWER LIMITS FOR "500-MI. STRAIGHTAWAY TRANSMISSION."

Initial Conditions at Receiver End		Excitation on Each Unit	Pull-out Power in Kw.	
Volts	Kw.	— Amperes	Calculated	Test
1150	0	2.00	14	15
1725	0	3.15	33	38
2300	0	4.80	65	68
2300	50	5.20	73	77
2300	100	6.50	101	103
2880	100	9.40	147	137

TABLE IV

CONSTANT-VOLTAGE POWER LIMITS FOR 500-MI. SYSTEM.

	Pull-out Power in Kw.	
Terminal Voltage	Calculated	Test
1725	56	58
1900	68	67
2100	86	95
2300	110	104
2500	147	140

TABLE V

COMPARATIVE POWER LIMITS

		Pull-out Power—Kw.	
System	Excitation	Calculated	Test
1. Alternator and motor on same bus..........	Normal excitations	115	129
2. Alternator and motor on same bus..........	Normal regulators	280	—
3. Alternator, 125-mi. *L-R* line, motor..........	Normal excitations	99	107
4. Alternator, 125-mi. *L-R* line, motor..........	Normal regulators	—	260
5. Alternator, 250-mi. *L-R* line, motor..........	Normal excitations	85	89
6. Alternator, 250-mi. *L-R* line, motor..........	Normal regulators	168	155
7. Alternator, 500-mi. *L-R* line, motor..........	Normal excitations	65	68
8. Alternator, 500-mi. *L-R* line, motor..........	Normal regulators	110	104
9. Alternator, 250-mi. "nominal π" line, motor..........	Normal excitations	76	75
10. Alternator, 250-mi. "nominal π" line, motor..........	Normal regulators	144	134
11. Infinite generator, 250-mi. *L-R* line, infinite receiver..........	Normal regulators	283	—
12. Infinite generator, 250-mi. *L-R* line, normal motor..........	Normal excitations	—	125
13. Normal generator, 250-mi. *L-R* line, infinite receiver..........	Normal regulators	213	—

TABLE VI

EFFECT OF MULTIPLE UNITS

		Pull-out Power in Kw.	
System	Excitation	Calculated	Test
1. Alternator on same bus as motor..........	Normal excitations	115	129
2. Alternator and motor on same bus with addition of shunt synchronous condenser......	" "	140	145
3. Alternator supplying two motors on same bus*..........	" "	157†	145†
4. Two alternators‡ supplying one motor on same bus..........	" "	155†	145†
5. Alternator, 250-mi. *L-R* line, and motor......	Normal regulators	168	155
6. Two alternators‡ in multiple, 250-mi. *L-R* line, and motor......	" "	—	192

*Approximately equal division of load.

†Total load.

‡Equal division of load.

TABLE VII

POWER LIMITS FOR SYSTEM OF FIG. 11.

Excitation—Amperes		Reactor—	No-load	Pull-out Power—Kw.	
Generator	Motor	Ohms	Voltage	Calculated	Test
7.6	7.6	26.9	2300	215	195
6.8	6.8	36.8	2300	190	182
6.6	6.6	39.2	2300	181	171
4.7	4.7	106.5	2100	106	113
4.7	4.7	26.9	—	75	—

TABLE VIII

POWER LIMITS FOR SYSTEM OF FIG. 12.

Excitation—Amperes			Pull-out Power—Kw.	
Alternator	Motor	Condenser	Calculated	Test
6.8	6.8	1.0	181	—
6.1	6.1	2.0	164	174
5.5	5.5	3.0	149	162
5.0	5.0	4.0	142	155
4.0	4.0	6.0	122	134
3.2	3.2	8.0	110	120
2.3	2.3	10.0	86	103

TABLE XII

POWER LIMITS FOR 500-MI. SYSTEM WITH MIDPOINT SYNCHRONOUS CONDENSER.

CONSTANT VOLTAGES

	Power Limits—Kw.	
Voltage	Calculated	Test
1700	74	78
1900	95	90
2100	125	122
2300	150	136

TABLE IX

POWER LIMITS FOR SYSTEMS WITH SHUNT REACTORS

	Supply-End Reactor		Receiver-End Reactor		Excitation —Amperes		Initial Conditions		Power Limits—Kw.	
System	Type	Ohms	Type	Ohms	Generator	Motor	Volts	Kw. Motor Input	Calculated	Test
1. Alternator, 500-mi. *L-R* line, motor....................	Saturated	29.2*	—	—	9.9	4.8	2300	0	—	84
2. Alternator, 250-mi. *L-R* line, motor....................	Unsaturated	58.4	—	—	7.1	4.75	"	0	—	99
	"	"	—	—	7.15	5.2	"	50	—	111
	"	"	—	—	7.7	6.0	"	100	—	130
3. Alternator, 250-mi. *L-R* line, motor....................	"	"	Unsaturated	58.4	7.3	7.35	"	0	105	118
4. Alternator, 250-mi. *L-R* line, motor....................	"	29.2	—	—	4.7	4.7	—	0	—	62
	"	"	—	—	10.1	4.7	2300	0	108	108
	"	"	—	—	9.8	5.1	"	50	—	113
	"	"	—	—	10.3	5.9	"	100	—	132
5. Alternator, 250-mi. *L-R* line, motor....................	Saturated	29.2	—	—	2.15	2.	—	0	—	21
	"	"	—	—	9.55	4.8	2300	0	111	119
	"	"	—	—	10.2	5.1	"	50	—	135
	"	"	—	—	11.0	5.9	"	100	—	152
6. Alternator, 250-mi. *L-R* line, motor....................	"	29.2	Unsaturated	29.2	9.9	10.0	"	0	—	146
	Saturated	29.2	Unsaturated	29.2	9.95	10.3	2300	50	—	154
	"	"	"	"	10.8	10.8	"	100	—	165

* The saturated reactor has a reactance of 29.2 ohms at 2300 volts.

TABLE X

POWER LIMITS FOR 250-MI. SYSTEMS WITH INDUCTION AND SYNCHRONOUS GENERATORS AT 2300 VOLTS

	Power Limits—Kw.	
Equipment at Supply End	Calculated	Test
1. Single synchronous generator..........	168	155
2. Induction generator with synchronous condenser..........................	200	202
3. Two synchronous generators...........	—	192
4. Two synchronous generators*..........	238	—
5. Induction generator and synchronous condenser*..........................	218	—
6. Synchronous generator and synchronous condenser*..........................	217	—

* Infinite receiver bus.

TABLE XI

POWER LIMITS FOR 500-MI. SYSTEM WITH MIDPOINT SYNCHRONOUS CONDENSER.

CONSTANT EXCITATIONS

Excitation—Amperes			Initial Condition		Power Limits—Kw.	
Alternator	Motor	Condenser	Volts	Kw. Motor Input	Calculated	Test
3.1	3.1	3.15	1725	0	40	40
3.9	3.9	3.9	1725	60	61	59
4.8	4.95	4.8	2300	0	83	83
4.95	5.2	5.05	2300	50	91	87
5.8	6.0	6.0	2300	100	122	113
8.6	8.55	8.65	2880	0	208	—

TABLE XIII

POWER LIMITS FOR 500-MI. SYSTEM WITH SYNCHRONOUS CONDENSERS AT MIDPOINT AND AT RECEIVER END.

CONSTANT EXCITATIONS

Excitation—Amperes				Initial Condition		Power Limits—Kw.	
Alternator	M. P. Condenser	Motor	R-E. Condenser	Volts	Kw. Motor Input	Calculated	Test
3.1	3.2	4.7	1.5	1725	0	—	45
3.15	3.25	4.7	1.6	1725	15	—	56
4.75	4.9	4.7	4.8	2300	0	92	93
5.0	5.3	4.7	5.3	2300	50	—	101
6.1	6.4	4.7	6.3	2300	100	128	120

TABLE XIV

POWER LIMITS FOR 500-MI. SYSTEM WITH SYNCHRONOUS CONDENSER AT MIDPOINT AND AT RECEIVER END.

CONSTANT VOLTAGES.*

	Power Limits—Kw.	
Voltages	Calculated	Test
1900	92	103
2100	115	110
2300	145	143†

* Normal excitation on motor.

† System quite stable at 143 kw. but the power limit was not reached due to d-c. apparatus limitations.

TABLE XV
COMPARATIVE POWER LIMITS FOR STRAIGHTAWAY AND "LOADED" SYSTEMS

System	Condition of Operation	Power Limits—Kw.	
		Calculated	Test
1. 125-mi. straightaway....	normal regulators	—	260
2. 250-mi. straightaway....	" "	168	155
3. 250-mi. system with infinite receiver.........	" "	213	—
4. 250-mi. system with normal condenser at midpoint...............	" "	190	—
5. 250-mi. system with a "saturated" condenser at midpoint...........	" "	220	—
6. 500-mi. straightaway....	" "	110	104
7. 500-mi. system with a normal condenser at midpoint................	" "	150	136
8. 500-mi. system with one normal condenser at midpoint and one at the receiver end*.........	" "	—	143†

* Regulator on receiver-end synchronous condenser. Field of motor constant at 4.7 amperes.

† System stable at 143 kw. Power limit not reached due to d-c. apparatus limitations.

circuits of synchronous equipment in shop tests—showed real advantages.

5. The criterion of stability under all methods of load application appears to be the steady-state power limit.

6. To ensure stability during short-circuit disturbances, a well designed and adequately fast relaying system is essential.

7. Results of good engineering accuracy can be obtained in the computation of system steady-state power limits by the application of available methods of calculation, although in the more complicated cases resort must be had to solution by the a-c. miniature system.

Appendix I

DERIVATION OF CIRCLE DIAGRAMS FOR SYNCHRONOUS MACHINES

(As developed by Professor J. M. Bryant)

The characteristics of synchronous machines, when calculated by methods assuming constant synchronous reactance, are not sufficiently accurate when the machines possess an appreciable degree of saturation. In such cases the synchronous reactance may be considered as a variable and a function of the voltage but it is more convenient to apply a magnetomotive force method than to use the synchronous reactance method. If we let

P = per cent power (expressed as a fraction)
P_j = per cent reactive kv-a.
E_t = terminal voltage in per cent of normal
F_t = the per cent field excitation to give the voltage E on open circuit as given by the saturation curve
F_s = the per cent field excitation to give normal armature current on short circuit
F = the total field excitation
I = the per cent armature current

then

$$I = \frac{P}{E_t} + j\,\frac{P_j}{E_t} \qquad (1)$$

The field excitation to overcome the armature reaction and leakage reactance drop is:

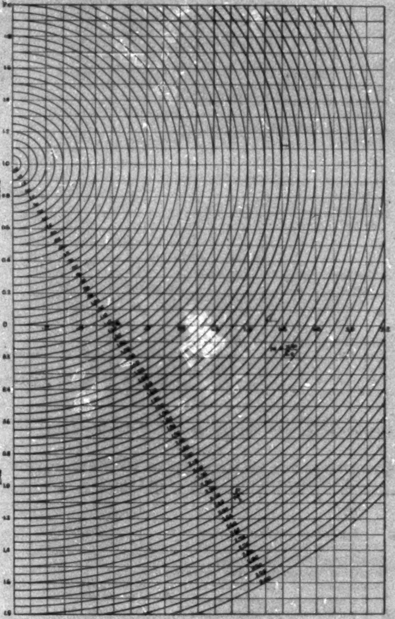

FIG. 38—CIRCLE DIAGRAM FOR SYNCHRONOUS APPARATUS

$$F_a = j\,F_s\left(\frac{P}{E_t} + j\,\frac{P_j}{E_t}\right) \qquad (2)$$

and the total field excitation is:

$$F = F_t + j\,F_s\left(\frac{P}{E_t} + j\,\frac{P_j}{E_t}\right) \qquad (3)$$

or

$$\frac{F}{F_t} = 1 - \frac{F_s\,P_j}{F_t\,E_t} + j\,\frac{F_s\,P}{F_t\,E_t} \qquad (4)$$

Reducing to absolute values

$$\left(1 - \frac{F_s P_j}{F_t E_t}\right)^2 + \left(\frac{F_s P}{F_t E_t}\right)^2 = \frac{F^2}{F_t^2} \qquad (5)$$

Letting $\frac{F_s}{F_t} = x$ (6)

$$\left(1 - \frac{x P_j}{E_t}\right)^2 + \left(\frac{x P}{E_t}\right)^2 = \frac{F^2}{F_t^2} \qquad (7)$$

This is the equation for a family of circles of radii $\frac{F}{F_t}$ having the center displaced upward unit distance on the $\frac{x P_j}{E_t}$ axis. These circles are given in Fig. 38.

This circle diagram in conjunction with the circle diagrams for the transmission lines may be used to obtain the power-voltage characteristics of the system.

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Discussion

For discussion of this paper see page 80.

Steady-State Stability in Transmission Systems
Calculation by Means of Equivalent Circuits or Circle Diagrams

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Synopsis.—*The maximum load on a proposed transmission system must be within the steady-state power limit of the system for stability of operation. Two methods of calculating steady-state stability are given in detail and illustrated by examples. (1) The given transmission system is replaced by a simple equivalent system; then the steady-state power limit of this equivalent system is determined graphically. (2) By means of a circle diagram the system is tested for stability with the maximum proposed load on the system.*

All formulas from published references necessary for the calculations are included and all calculations are given in full so that similar studies can readily be made by an engineer who has not previously made a study of the subject of stability.

THE object of this paper is to give two methods of determining the stability of operation of a proposed transmission system under steady-state conditions. The calculations will be given in detail so that the engineer who has not previously made a study of the subject will have no difficulty in applying the tests for stability to his system. The first method is by means of an equivalent circuit and the second by means of a circle diagram. Both methods are based on theorems which are exact, but in order to fit the transmission system to the theorems certain assumptions must be made. The results will therefore be approximate to the extent to which the assumptions approximate actual conditions.

Formerly when a transmission system was proposed, it was customary to make the line calculations for voltage regulation and losses for the maximum load conditions, and to select the generators, transformers and synchronous condensers to fit these conditions. There was nothing in such calculations to indicate that the system would be stable, but fortunately the length of line and maximum load have been such that cases of instability have been rare. At the present time when the tendency is for longer lines and greater loads, it is necessary to consider the question of stability both for steady-state and transient conditions. Steady-state stability only will be considered in this paper.

In steady-state stability the assumption is made that the load comes on in infinitesimal amounts so that the transient caused by one increment is over before the next increment is added. The criterion of steady-state stability is this: Assuming that the system is operating satisfactorily under the assumed load conditions, will it continue to operate satisfactorily if an increment of synchronous load is added and all excitations remain constant? When load is added there is an increase in current and a drop in voltage before there is any change in excitation. The voltage regulators then increase the excitations and normal voltage is obtained. If the load on the system is *just* the amount which can be transmitted at excitations which correspond to normal voltage, any increase in synchronous load will cause instability because the voltage must drop before the voltage regulators can increase the excitations and at the given excitations no more power can be transmitted. Therefore, when the voltage starts to drop it will continue to drop, for there is no voltage at which the load can be transmitted with those excitations.

When a generator and motor are on the same bus, at no-load, neglecting no-load losses, their induced voltages are in phase. Keeping the excitations on motor and generator constant as the motor is gradually loaded, the phase displacement between the excitation voltages of motor and generator increases with load until the machines fall out of step. The load at which the machines fall out of step is the maximum load and the angle is the maximum power angle. This angle and the power corresponding to it can be calculated. Power corresponding to an angle greater than the maximum power angle can also be calculated although it cannot be delivered.

In the simple transmission system consisting of a synchronous generator supplying power to a synchronous motor over a line having resistance and reactance, but no appreciable capacitance or leakance, the maximum power that can be transmitted over the system, and the angle between the generator and motor excitation voltages corresponding to maximum power, are not difficult to calculate. This simple system will be stable under a proposed load if the phase displacement between the synchronous generator and synchronous motor excitation voltages corresponding to the proposed load is less than the phase displacement which corresponds to maximum power on the shaft of the motor. When a transmission system consisting of generators, lines with distributed constants, and the usual station load can be replaced by the simple transmission system, the maximum power that can be transmitted over the system is easily calculated. In studying steady-state stability by means of equivalent circuits an attempt is made to replace the complicated transmission system by the equivalent simple system.

EQUIVALENT LINE

Dr. E. A. Kennelly[2] has shown that a line with distributed constants can be replaced by either an equivalent π or T line in which the constants are lumped. The π line consists of a line with a shunt at each end and

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[1] Control Station Engineering Dept., General Electric Co.

Presented at the Midwinter Convention of the A. I. E. E., New York, Feb. 8-11, 1926.

[2] "Application of Hyperbolic Functions to Electric Engineering Problems."

the *T* line of a line with a shunt at the center. The nominal π is formed by placing the total impedance of the actual line in the line or architrave of the π and one-half the admittance in each shunt or pillar. The nominal *T* is formed by placing one-half the total impedance of the actual line in each arm of the *T* and the total admittance in the shunt or staff. The nominal π and *T* lines are not exact equivalents of the actual line, but by applying connecting factors to them the equivalent π and *T* lines are obtained which are exact equivalents of the actual line.

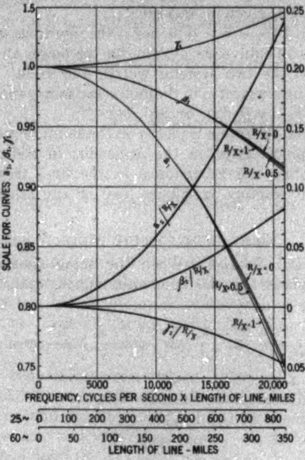

FIG. 1—COMPLEX HYPERBOLIC FUNCTIONS

$$\cosh \sqrt{Z Y} = 1 + \frac{Z Y}{2} + \ldots = a_1 + j\, a_2$$

$$\frac{\sinh \sqrt{Z Y}}{\sqrt{Z Y}} = 1 + \frac{Z Y}{2} + \ldots = \beta_1 + \beta_2$$

$$\frac{\tanh \sqrt{Z Y/2}}{\sqrt{Z Y/2}} = \gamma_1 + j\, \gamma_2$$

Correcting Factors for Converting the Nominal π or T Line Into the Equivalent π or T. The correcting factors which must be applied to convert the nominal π or *T* into the equivalent π or *T* are $\dfrac{\sinh \sqrt{Z Y}}{\sqrt{Z Y}}$ to be applied to the architrave of the π and the staff of the *T* and $\dfrac{\tanh (\sqrt{Z Y}/2)}{\sqrt{Z Y}/2}$ to be applied to the pillars of the π and the arms of the *T*.

Z is the total conductor impedance and *Y* is the total dielectric admittance. For short lines the nominal π or *T* line is the same as the equivalent π or *T*.

Fig. 1 gives the real and imaginary parts of these correcting factors. For convenience the real and imaginary parts of $\cosh \sqrt{Z Y}$ are also given in Fig. 1.

To Obtain the Correcting Factors from Fig. 1.

$$\text{Let } \cosh \sqrt{Z Y} = a_1 + j\, a_2$$

$$\frac{\sinh \sqrt{Z Y}}{\sqrt{Z Y}} = \beta_1 + j\, \beta_2$$

$$\frac{\tanh (\sqrt{Z Y}/2)}{\sqrt{Z Y}/2} = \gamma_1 + j\, \gamma_2$$

Calculate $f\,l$ and r/x, where f is frequency in cycles per second, l is length of line in miles and r and x are the resistance and reactance respectively.

Enter the chart using the product of frequency and length of line (or length of line at given frequency) as abscissa and read off the real parts a_1, β_1 and γ_1 of the three vectors corresponding to the calculated value of r/x, using the scale on the left. The imaginary parts a_2, β_2 and γ_2 have been divided by r/x. Read $a_2/(r/x)$, $\beta_2/(r/x)$ and $\gamma_2/(r/x)$, using the scale to the right. To obtain a_2, β_2 and γ_2 the values taken from the curves must be multiplied by r/x.

Fig. 1 was made by replacing the hyperbolic functions by their equivalent series, expressing *Z* and *Y* in terms of resistance, capacitance and inductance (leakance neglected), and giving the results in terms of r/x and $f\,l$, making use of the fact that the product of inductance and capacitance is practically constant for overhead lines. See Appendix B.

Power at a Point Within the Equivalent π. The π–line viewed from either end is an exact equivalent of the line with distributed constants but the current and voltage at any point in the architrave of the π do not correspond to current and voltage on the actual line. Let the π–line in Fig. 2 be the equivalent π of the line *A B* with distributed constants. Although the current at *d* is the same as the current at *B*, the current at *c* has no counterpart in the actual line. The power at *d* is the same as the power at *B*, but the power at *c* is the power at *d* plus the power lost in shunt Z_4. When the power is known at *b* and *c* it can be calculated at *a* and *d*.

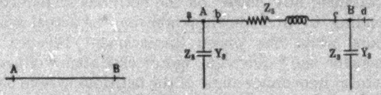

FIG. 2—A. LINE *A B* WITH DISTRIBUTED CONSTANTS
B. EQUIVALENT π = LINE

SYNCHRONOUS IMPEDANCE

It is understood that the synchronous impedance of a synchronous generator or motor is not constant. However, given the characteristics of the machine and the conditions of operation, it is possible to find an *equivalent* synchronous impedance which may be considered constant for the case under consideration. In the work which follows constant synchronous impedance refers to this equivalent synchronous impedance.

For the synchronous condenser it is necessary to find an equivalent excitation as well as an equivalent synchronous impedance. This is discussed at greater length in Appendix E.

Equivalent Generator

If a system consist of a synchronous generator and synchronous motor, both of constant synchronous impedance, connected by a line with a shunt at the generator end, this system can be replaced for all points beyond the shunt by a system unchanged beyond the shunt, but having a new generator whose impedance is the impedance of the generator and shunt in parallel and whose excitation voltage

$E_1' = E_1 \dfrac{Z_1 Z_s}{Z_1 + Z_s} / Z_1 = E_1 \dfrac{Z_s}{Z_1 + Z_s}$, where E_1 is the actual generator excitation voltage, Z_1 is the generator impedance and Z_s is the shunt impedance.

This will be an exact equivalent, for the power at breakdown on the two systems will be same. The proof is given in Appendix C (*b*).

Equivalent Motor

If a system consist of a synchronous generator and synchronous motor, both of constant synchronous impedance, connected by a line with a shunt at the motor end, this system can be replaced for all points in front of the shunt by a system unchanged up to the shunt, but having a new motor whose impedance is the impedance of the motor and shunt in parallel and whose excitation voltage $E_2' = E_2 \dfrac{Z_2 Z_s}{Z_2 + Z_s} / Z_2$

$= E_2 \dfrac{Z_s}{Z_2 + Z_s}$, where E_2 is the actual motor excitation voltage, Z_2 is the motor impedance and Z_s is the shunt impedance; but power on the fictitious system corresponding to breakdown power on the actual system occurs when the phase displacement between the generator and equivalent motor excitation voltages is the total impedance angle of the fictitious system plus a constant angle, $2(\theta_2 - \theta_2')$, where θ_2 is the impedance angle of the actual motor and θ_2' is the impedance angle of the equivalent motor. The proof is given in Appendix C (*b*).

Equivalent System and Voltage Regulators

If a system consists of a synchronous generator and synchronous motor, both of constant synchronous impedance, connected by a line with a shunt at each end, voltage being maintained at the ends of the line, this system can be replaced for all points of the line by a system consisting of the same line without shunts, a generator whose impedance is the impedance of the actual generator and shunt at the generator end in parallel and a motor whose impedance is the impedance of the actual motor and shunt at the motor end in parallel; but power on the fictitious system corresponding to breakdown power on the actual system occurs when the phase displacement between equivalent motor and generator excitation voltages is the total impedance angle of the fictitious system plus $2(\theta_2 - \theta_2')$ where θ_2 is the impedance angle of the actual motor and θ_2' the impedance angle of the equivalent motor. The proof is given in Appendix D.

When $2(\theta_2 - \theta_2') = 0$ deg., the fictitious system becomes an equivalent system, for the power at breakdown on the two systems will be the same. When resistance is neglected in the motor and motor end shunt $2(\theta_2 - \theta_2') = 0$ deg. or 360 deg.

Since no limitation is placed on the shunt impedances in the proofs given in the Appendix, in addition to representing the capacitance in the line, they may represent reactors, resistance load or any other dead load.

Equivalent Circuit Method

Graphical Solution. When the actual system has been replaced by the equivalent simple system, the maximum power which can be transmitted may be obtained graphically.

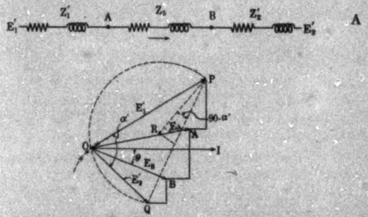

Fig. 3—A. Equivalent Transmission System
B. Graphical Determination of Maximum Power Over the Equivalent Transmission System

Let Fig. 3A represent the simple equivalent system, where

$Z_5 = r + jx =$ the impedance of the equivalent line.
$Z_1 = r_1 + jx_1 =$ the actual generator impedance.
$Z_1' = r_1' + jx_1 =$ the equivalent generator impedance formed by taking the shunts at the generator end of the line in parallel with the generator impedance.
$Z_2 = r_2 + jx_2 =$ the actual motor impedance.
$Z_2' = r_2' + jx_2' =$ the equivalent motor impedance formed by taking the shunts at the motor end of the line in parallel with the motor impedance.
$E_1 =$ excitation voltage of the actual generator
$E_2 =$ excitation voltage of the actual motor.
$E_1' =$ excitation voltage of the equivalent generator

$$= E_1 \frac{Z_1'}{Z_1}$$

E_2' = excitation voltage of the equivalent motor

$$= E_2 \frac{Z_2'}{Z_2}$$

E_A = terminal voltage at the generator end of the line.

E_B = terminal voltage at the receiver end of the line.

$Z_t = (r + jx + r_1' + jx_1' + r_2' + jx_2') = R_t + jX_t$
$= z_t e^{j\theta_t}$
= total impedance of the equivalent system.

$\theta_t = \tan^{-1} \dfrac{X_t}{R_t}$ = total impedance angle.

$\theta_2 = \tan^{-1} \dfrac{x_2}{r_2}$ = impedance angle of the actual motor.

$\theta_2' = \tan^{-1} \dfrac{x_2'}{r_2'}$ = impedance angle of the equivalent motor.

Calculate $\alpha' = \theta_t + 2\,(\theta_2 - \theta_2')$

Since there are no shunts in the equivalent or fictitious system the same current will flow in all parts of the circuit. Taking current as standard phase, lay off $Q\,B\,A\,P = I\;(r_2' + j\;x_2') + I\;(r + j\;x) + I\;(r_1' + j\;x_1')$ to any convenient scale, Fig. 3B. The value of this scale will be determined when the construction has been completed and the position of point O determined.

There are two conditions which determined the position of point O.

1. $\alpha' = \theta_t + 2\,(\theta_2 - \theta_2')$
2. The ratio between the magnitudes of E_A and E_B or between E_1' and E_2' is known. For a regulated line E_A and E_B are given, and for a line without voltage regulators E_1' and E_2' can be calculated from the known values E_1 and E_2.

Consider the case of the regulated line: To satisfy the first condition join P and Q and at P draw $P\,R$ making an angle $(90° - \alpha')$ with $P\,Q$. With R, the intersection of $P\,R$ with the perpendicular bisector of $P\,Q$, as a center and $R\,P$ as radius describe arc $P\,Q$. If point O lies on this arc, the first condition will be satisfied. To satisfy the second condition, find a series of points whose distances from A and B are in the ratio E_A/E_B and draw a curve through them. O will lie on the intersection of this curve with the arc $P\,Q$.

The scale of the vector diagram is determined, for $O\,B = E_B$. All voltage drops are now given in terms of E_B. E_B is known, therefore all voltage drops are known and the current can be calculated. The power factor angle, θ, can be measured.

Maximum Power at $B = E_B \,.\, I \,.\, \cos\theta$.

The impedances, voltages and currents may be expressed in ohms, volts and amperes respectively or in per cent, as is most convenient.

Algebraic Solution—Resistance Neglected. When resistance is neglected and equal voltages are maintained at the ends of the line a simple formula can be derived for the power delivered.

Let

X_1' = the equivalent generator reactance.
X_2' = the equivalent motor reactance.
X = the equivalent line reactance.
$V_A = V_B$ = magnitude of line terminal voltages, E_A and E_B.

Since resistance is neglected the total impedance angle is 90 deg. and this is the breakdown angle of the equivalent system. Laying off the voltage drops as described above, Fig. 4 is obtained. Since resistance is neglected, the power at all points of the circuit is the same. Since the line terminal voltages are equal, the power factor at S, the center of the line, is unity.

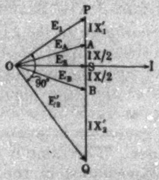

FIG. 4—GRAPHICAL DETERMINATION OF MAXIMUM POWER
Over the equivalent transmission system, when resistance is neglected

Maximum power = Power at $S = E_s \,.\, I \,.\, \cos\theta$ where E_s is the voltage at S, I = current and $\cos\theta = 1$.

$$E_s^2 = \overline{O\,S}^2 = \overline{Q\,S} \,.\, \overline{S\,P} = I^2 \left(X_1' + \frac{X}{2}\right)\left(X_2' + \frac{X}{2}\right)$$

$$E_s^2 = \overline{O\,S}^2 = \overline{O\,B}^2 - \overline{B\,S}^2 = V_B^2 - I^2 \left(\frac{X}{2}\right)^2$$

$$\therefore I^2 = \frac{V_B^2}{\left(\frac{X}{2}\right)^2 + \left(X_1' + \frac{X}{2}\right)\left(X_2' + \frac{X}{2}\right)}$$

and

$$\frac{E_s}{I} = \sqrt{\left(X_1' + \frac{X}{2}\right)\left(X_2' + \frac{X}{2}\right)}$$

$$P_{max} = E_s I = \frac{E_s}{I} I^2$$

$$= \frac{V_B^2 \sqrt{\left(X_1' + \frac{X}{2}\right)\left(X_2' + \frac{X}{2}\right)}}{\left(\frac{X}{2}\right)^2 + \left(X_1' + \frac{X}{2}\right)\left(X_2' + \frac{X}{2}\right)} \qquad (1)^*$$

When V_B = 100 per cent and the motor and generator have the same impedance ($X_1' = X_2'$), equation (1) becomes

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*This equation was first developed by Mr. C. A. Nickle.

$$P_{max} = \frac{X_1' + \dfrac{X}{2}}{\left(\dfrac{X}{2}\right)^2 + \left(X_1' + \dfrac{X}{2}\right)^2} \qquad (2)$$

When $X = O$, which is the case for a motor and generator on the same bus,

$$P_{max} = \frac{1}{X_1'} \qquad (3)$$

When $X_1' = X_2' = O$, which is the case for infinite busses at the ends of the line,

$$P_{max} = \frac{1}{X} \qquad (4)$$

APPLICATION OF THE FICTITIOUS OR EQUIVALENT SYSTEM TO STEADY-STATE STABILITY PROBLEMS

1. *Effect of Capacitance in the Line.* The effect of the capacitance in the line upon the stability of the system is apparent, for when the impedance of the shunt at the generator end of the equivalent π–line is taken in parallel with the generator impedance, the impedance of the equivalent generator is greater than the actual generator impedance. The same is true at the motor end. Therefore capacitance in the line, although it reduces the line impedance by the correcting factor β, has the effect of increasing the generator and motor impedances and thus may reduce the power that can be transmitted over the line.

Example: Take a 220-kv., 300-mile, 60-cycle line, connecting a synchronous generator supplying load to a synchronous motor. For simplicity neglect resistance in the line, generator and motor. The fictitious system then becomes the equivalent system.

Assume the following line constants per mile: $x = j\,0.813$ ohms per mile and $y = j\,5.22 \times 10^{-6}$mhos per mile.

The correcting factors, β and γ, from Fig. 2 corresponding to $f \times l = 60 \times 300 = 18{,}000$ and $r/x = 0$ are

$$\beta = 0.937 + j\,0 \text{ and } \gamma = 1.033 + j\,0$$

The equivalent π–line has the architrave impedance $0.937 \times j\,0.813 \times 300 = j\,229$ ohms and the pillar admittance $1.033 \times j\,5.22 \times 10^{-6} \times 300/2 = j\,0.00810$ mhos, or the pillar impedance $-\,j\,1233$ ohms.

Assume that voltage is held on the high side of the transformers at 220 kv. at each end, and express the above reactances as per cent on a 100,000-kv-a. base.

$X = 47.3$ per cent = line reactance

$X_3 = X_4 = -\,254$ per cent = shunt reactances

(a) Assume a 100,000-kv-a. generator and an equal motor, each of 100 per cent synchronous reactance.

Let X_1 = generator + transformer reactance = 112 per cent on 100,000-kv-a. base.

X_2 = motor + transformer impedance = 112 per cent on 100,000-kv-a. base.

$$X_1' = \text{equivalent generator reactance} = \frac{X_1\,X_3}{X_1 + X_3}$$

$$X_2' = \text{equivalent motor reactance} = \frac{X_2\,X_4}{X_2 + X_4}$$

Then

$$X_1' = X_2' = \frac{(112)\ (-\,254)}{(112 - 254)} = 200 \text{ per cent}$$

The maximum power by equation (2) is

$$P_{max} = \frac{X_1' + \dfrac{X}{2}}{\left(\dfrac{X}{2}\right)^2 + \left(X_1' + \dfrac{X}{2}\right)^2} = \frac{2.00 + 0.24}{0.055 + 5.00}$$

= 44.4 per cent on 100,000-kv-a. base = 44,000 kw.

b. Assume a 45,000-kv-a. generator and an equal motor, each of 100 per cent synchronous reactance on their own rating, and 45,000-kv-a. transformers of 14 per cent reactance. 114 per cent on 45,000 kv-a. = 254 per cent on 100,000 kv-a.

$$X_1' = X_2' = \frac{(254)\ (-\,254)}{(254 - 254)} = \infty, \text{ and } P = 0.$$

If there were no capacitance in this line X would be 50.4 per cent, and $P_{max} = \dfrac{2.54 + 0.252}{0.06 + 7.80} = \dfrac{2.79}{7.86} =$ 35.5 per cent on 100,000-kv-a. base = 35,500 kw.

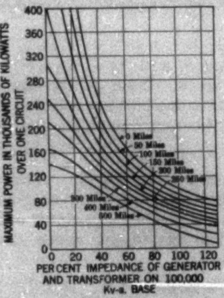

FIG. 5—MAXIMUM SYNCHRONOUS LOAD DELIVERED OVER 220-KV., 60-CYCLE, TRANSMISSION LINES OF VARIOUS LENGTHS

When resistance is neglected. Synchronous generators and motors assumed to have equal synchronous impedances. (Transformer impedance included in generator impedance)

CURVES FOR ESTIMATING MAXIMUM POWER

The curves in Fig. 5 were calculated for 60 cycles and various lengths of line, assuming reactance of 0.813 ohms per mile and capacity susceptance of 5.22×10^{-6} mhos per mile, 220 kv. was maintained at each end of the line, resistance was neglected in the line and in the generators and motors which were assumed of equal synchronous impedances.

These curves can be used as a first approximation. They give a rough idea of the power which can be transmitted. Resistance is neglected, so the values for that reason are optimistic, but the load is assumed all synchronous motor load, which is pessimistic. These values should be used for estimating purposes only, or to show qualitative effects on the system.

For example, given that the generator and motor impedance (plus transformers) are 112 per cent on 200,000 kv-a. or 56 per cent on 100,000 kv-a. from the curves in Fig. 5, 73,000 kw. can be transmitted over a 500-mile line and 118,000 kw. over a 250-mile line. These values indicate the order of magnitude of the maximum power which can be transmitted.

If two 500-mile lines, instead of one, connect the generator and motor in the above example, the generator plus transformer impedance per circuit is 112 per cent on 100,000-kv-a. base and only 18,000 kw. per circuit, or 36,000 kw. total, can be transmitted over the two lines as against 73,000 kw. over one line.

2. *Reactor Across the Generator Terminals.* If a reactor is placed across the generator terminals the effect is opposite to the effect produced by the capacity shunt. The reactor reduces the equivalent generator impedance so that more power can be transmitted over the system. It must be remembered that the excitation on the generator is increased by the use of a reactor, but when full field is not being used on the generator, a reactor increases the power that can be transmitted by the same amount that a generator of the same rating would do. Since reactors are cheaper than generators, a reactor of the size that would put full excitation on the generator can be used to advantage to increase the stability of the system.

3. *Power Limits of a Long Line.*

a. Limit of the line alone.

b. Limit of the line and transformers.

c. Limit of the line transformers and generator.

d. Limit of the system with various kinds of load.

1. Synchronous motors.
2. Lights and synchronous motors.
3. Lights, induction motors, synchronous motors and synchronous condensers.
4. Same as (3) but with a generator supplying local load.

A line with generator and transformers will be selected; then the maximum power will be obtained for the specified conditions.

Given:

A three-phase, 60-cycle, 250-mile line.

Line constants: r = 0.151 ohms per mile
x = 0.813 ohms per mile
y = 5.22 micro-mhos per mile
Leakance = O.

Step-up transformers: 270,000 kv-a. total
2 per cent resistance
12 per cent reactance
13,200—220,000 volts.

Step-down transformers: 240,000 kv-a. total,
2 per cent resistance
12 per cent reactance
210,000-13,200 volts

Generators: 270,000 kv-a. total
0.9 power factor, 13,200 volts
100 per cent synchronous impedance.

Voltage regulators will maintain 220 kv. and 200 kv. at the generator and receiver ends respectively on the low sides of the transformers (assuming a one-to-one ratio of transformation). The magnetizing current in the transformers will be neglected.

Conductor impedance $= Z = 250\ (0.151 + j\ 0.813) = (37.8 + j\ 203.2)$ ohms

Dielectric admittance $= Y = 250\ (O + j\ 5.22 \times 10^{-6}) = j\ 0.001305$ mhos

Step-up transformer impedance $= T_s = (3.59 + j\ 21.5)$ ohms, referred to high side

Step-down transformer impedance $= T_r = (3.67 + j\ 22.1)$ ohms referred to high side

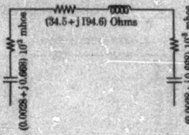
(34.5+j 194.6) Ohms
(0.0028+j 0.658) 10^{-3} mhos
(0.0028+j 0.658) 10^{-3} mhos

FIG. 6—EQUIVALENT π OF THE TRANSMISSION LINE ALONE

Generator impedance $= Z_1 = O + j\ 179.2$ ohms referred to high side.

From Fig. 1 corresponding to $f\,l = 15{,}000$ and $r/x = 0.186$

$$\cosh \sqrt{Z\,Y} = A^* = a_1 + j\,a_2 = 0.8700 + 0.0236$$

$$\frac{\text{Sinh} \sqrt{Z\,Y}}{\sqrt{Z\,Y}} = \beta_1 + j\,\beta_2 = 0.9563 + j\ 0.0080$$

$$\frac{\tanh \sqrt{Z\,Y}/2}{\sqrt{Z\,Y}/2} = \gamma_1 + j\,\gamma_2 = 1.0227 - j\ 0.0043$$

Since voltage will be held on the low sides of the transformers it will be necessary to express the line as an equivalent T–line, then adding in the transformers, obtain a T–line which represents line and transformers. This T–line can be replaced by its equivalent π–line.[3] A fair approximation can be obtained by adding the transformer impedances directly to the architrave of the equivalent π.

Figs. 6, 7, 8 and 9 give the equivalent π of the line alone, the equivalent T of the line alone, the equivalent T of the line and transformers, and the equivalent π

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*To be used for circle diagram following.

[3] "Application of Hyperbolic Functions," Appendix E.

of the line and transformers. Fig. 10 gives the equivalent π of the line alone with the transformer impedances added to the architrave impedance. The equivalent π of the line and transformers can also be obtained from the circuit constants representing the line and transformer used in constructing the circle diagram.

The limit of the line alone and of the line and transformers may be obtained by using equation (7) Appendix C (a).

$$P_{max} = \frac{V_1 V_2}{Z}\left(1 - \frac{V_2}{V_1}\cos\theta\right),$$ where V_1 and V_2 are terminal voltages at the generator and receiver ends respectively, Z is total impedance, and θ total impedance angle.

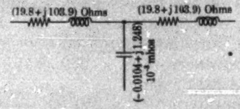
(19.8+j103.9) Ohms
(19.8+j103.9) Ohms
(−0.0104+j1.248) 10^{-3} mhos

FIG. 7—EQUIVALENT T OF THE TRANSMISSION LINE ALONE

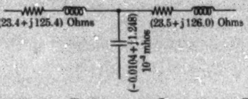
(23.4+j125.4) Ohms
(23.5+j126.0) Ohms
(−0.0104+j1.248) 10^{-3} mhos

FIG. 8—EQUIVALENT T OF THE LINE AND TRANSFORMER

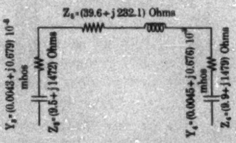
Z_5=(39.6+j232.1) Ohms
Y_3=(0.0043+j0.679) 10^{-3} mhos
Z_3=(9.5+j1472) Ohms
Y_4=(0.0045+j0.676) 10^{-3} mhos
Z_4=(9.9+j1479) Ohms

FIG. 9—EQUIVALENT π OF THE LINE AND TRANSFORMERS

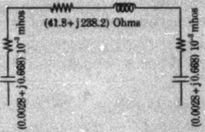
(41.8+j238.2) Ohms
(0.0028+j0.668) 10^{-3} mhos
(0.0028+j0.668) 10^{-3} mhos

FIG. 10—EQUIVALENT π OF THE LINE ALONE

With transformers impedances added to the architrave impedance (approximate)

a. Limit of the line alone with 220 kv. and 200 kv. maintained at the generator and receiver ends of the line respectively on the high sides of the transformers.

From the equivalent π of the line alone, Fig. 6.

$Z = 34.5 + j\,194.6$ ohms

$z = 198$ ohms, $\theta = 80$ deg.

$$P_{max} = \frac{3\,\dfrac{220}{\sqrt{3}} \times \dfrac{200}{\sqrt{3}}}{198}\left[1 - \frac{200}{220}(0.1736)\right]10^3$$

$= 187{,}000$ kw.

The power lost in the shunt must be subtracted to obtain the power delivered to the load.

$$\text{Power lost in shunt} = 3 \times \frac{200}{\sqrt{3}} \times \frac{200{,}000}{\sqrt{3}}(0.0000028)$$

$= 112$ kw.

Limit of the line alone = 187,000 kw.

b. Limit of the line and transformers with 220 kv. and 200 kv. maintained at the generator and receiver ends respectively on the low sides of the transformers (assuming a one-to-one ratio of transformation).

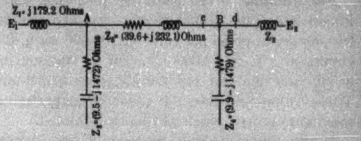
Z_1=j179.2 Ohms
E_1
A
Z_5=(39.6+j232.1) Ohms
c
B
d
E_2
Z_2
Z_3=(9.5−j1472) Ohms
Z_4=(9.9−j1479) Ohms

FIG. 11—EQUIVALENT π OF THE LINE AND TRANSFORMERS

With synchronous generator and motor. Impedances are in ohms referred to the high side.

From the equivalent π of the line and transformer, Fig. 9

$Z = 39.6 + j\,232.1$

$z = 235.6$ ohms, $\theta = 80.3$ deg.

$$P_{max} = \frac{220 \times 200}{235.6}\left[1 - \frac{200}{220}(0.1682)\right]10^3$$

$= 158{,}700$ kw.

Power lost in shunt $= 200 \times 200{,}000\ (0.0000045)$ $= 180$ kw.

Limit of the line and transformers = 158,500 kw.

c. Limit of the line, transformers and generator, with 220 kv. and 200 kv. maintained at the generators and receiver ends respectively on the low sides of the transformers (assuming a one-to-one ratio of transformation) is obtained by assuming a motor of zero impedance or an infinite bus at the receiver end.

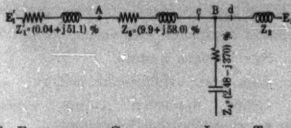
E_1'
A
Z_1'=(0.04+j51.1) %
Z_5=(9.9+j58.0) %
c
B
d
E_2
Z_2
Z_4=(2.48−j370) %

FIG. 12—EQUIVALENT CIRCUIT OF LINE, TRANSFORMERS, GENERATOR AND MOTOR

Impedances are in per cent on a 100,000-kv-a. base. 100 per cent voltage = 200 kv.

Fig. 11 represents the line with end shunts and a motor and generator each of constant synchronous impedance. The power at d, that is the power delivered to the load, will be the power at c minus the power lost in Z_4. Fig. 12 gives the equivalent circuit with the impedances in per cent on a 100,000-kv-a. base. 100 per cent voltage = 200 kv. The shunt Z_3 has been combined in parallel with the generator impedance. The shunt Z_4 has not been combined with the motor impedance.

The voltages and impedances in per cent from Fig. 12 are

E_A = 110 per cent, E_B = 100 per cent,

$Z_1' = (0.04 + j\,51.1)$ per cent, $Z_2' = O$

$Z_5 = (9.9 + j\,58.0)$ per cent,

$Z_5 + Z_1' + Z_2' = (9.9 + j\,109.1)$ per cent $= Z_t$

α' = maximum power angle $= \theta_t + 2\,(\theta_2 - \theta_2') = \theta_t$

$$= \tan^{-1}\frac{109.1}{9.94} = 84.7 \text{ deg.}$$

Making the graphical construction as described above, Fig. 13 is obtained. From Fig. 13

Since $O\,B$ = 100 per cent voltage, $I = \dfrac{\overline{B\,P}}{\overline{O\,B}}/z_t$

$$= \frac{1.845}{1.095} = 1.685 = 168.5 \text{ per cent current.}$$

Power factor at B = 0.84 lead.

Power at $c = 1 \times 1.685 \times 0.84 = 1.415$

= 141.5 per cent on 100,000-kv-a. base

= 141,500 kw.

Power lost in shunt Z_4 = 180 kw.

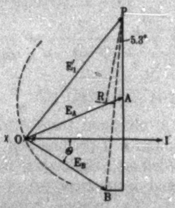

FIG. 13—GRAPHICAL DETERMINATION OF MAXIMUM POWER WITH AN INFINITE BUS AT THE RECEIVER END

Limit of the line, transformers and generators = 141,300 kw.

d. Limit of the system with various kinds of load.

1. Synchronous motor load of total capacity 170,000 kv-a., 85 per cent synchronous impedance.

Z_2 = motor impedance on 100,000-kv-a. base = 50 per cent

$$Z_2' = \frac{Z_2\,Z_4}{Z_2 + Z_4} = \text{impedance of equivalent motor}$$

$= (0.07 + j\,57.8)$ per cent

Fig. 14 gives the equivalent circuit.

Z_t = total impedance $= (10.0 + j\,166.9)$ per cent

θ_t = total impedance angle = 86.6 deg.

θ_2 = impedance angle of actual motor = 90 deg.

θ_2' = impedance angle of equivalent motor = 89.9 deg.

$2(\theta_2 - \theta_2')$ = 0.2 deg.

$\alpha' = \theta_t + 2\,(\theta_2 - \theta_2')$ = 86.7 deg.

Using the equivalent circuit given in Fig. 14 and making the graphical construction as described above, Fig. 15 is obtained.

Since $O\,B$ = 100 per cent voltage, I = 112.5 per cent current and

Power factor at B = 0.943 lead.

Power at $c = 1 \times 1.125 \times 0.943 = 1.06$

= 106,000 kw.

Power lost in shunt Z_4 = 180 kw.

Maximum power that can be delivered to the load = 106,000 kw.

2. Resistance load of 30,000 kw., synchronous motors of total capacity 170,000 kv-a. with 85 per cent synchronous impedance.

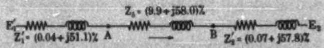

FIG. 14—EQUIVALENT CIRCUIT FOR SYNCHRONOUS MOTOR LOAD

R = resistance of the resistance load shunt

= 1333 ohms

= 333.3 per cent on 100,000 kv-a; base, 200 kv.

Z_4' = impedance of Z_4 and R in parallel $= (184 - j\,165)$ per cent

Z_2 = impedance of actual motor = 50 per cent

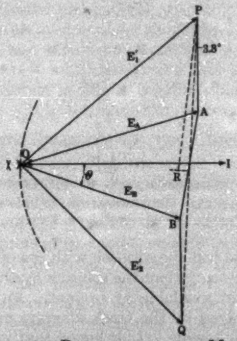

FIG. 15—GRAPHICAL DETERMINATION OF MAXIMUM POWER FOR A SYNCHRONOUS MOTOR LOAD

$$Z_2' = \frac{Z_2\,Z_4'}{Z_2 + Z_4'} = \text{impedance of equivalent motor}$$

$= (9.8 + j\,56.1)$ per cent

Z_t = total impedance of equivalent circuit $= (19.74 + j\,165.2)$ per cent

θ_t = 83.2 deg. = total impedance angle.

$\alpha' = \theta_t + 2\,(\theta_2 - \theta_2')$ = 83.2 deg. + 2 (90 − 80.1 deg.) = 103.0 deg.

The graphical construction is given in Fig. 16. Since $O\,B$ = 100 per cent

I = 138.5 per cent and power factor at B = 0.90 lead

Power at $c = 1 \times 1.385 \times 0.90 = 1.248$
$= 124{,}800$ kw.

Power lost in the line shunt $Z_4 = 180$ kw.

Maximum power that can be delivered to the load = 124,600 kw.

3. Lights, induction motors, synchronous motors and synchronous condensers.

4. Same as (3) but with generator supplying local load.

As voltage drops on an induction motor load, the power required by the load remains practically constant and the power factor becomes less lagging. A lightly loaded synchronous motor has the same characteristics. The induction motor load, therefore, may be replaced by an approximately equivalent synchronous motor.

A method of treating the synchronous condenser in the equivalent circuit is given in Appendix E.

Examples of type (3) can be solved by the equivalent circuit method but the solution with the circle diagram is more satisfactory than the solution by the equivalent circuit method available at present. An example of type (4) is solved by the circle diagram method. See example (3) following. It is hoped that systems more complicated than the one considered here as well as examples of types (3) and (4) will eventually be completely represented by equivalent circuits.

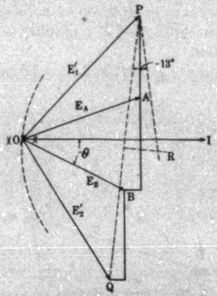

FIG. 16—GRAPHICAL DETERMINATION OF MAXIMUM POWER FOR A LOAD PARTLY RESISTANCE AND PARTLY SYNCHRONOUS

CIRCLE DIAGRAM METHOD

The test for stability by this method is to assume a slight increase in load and to determine if by a drop in receiver voltage this new load can be carried with the excitations corresponding to the original load.

When a system is operating at normal voltage and a load is added there is an increase in current at the receiver end and a drop in voltage. The power given to the additional load comes from the change in phase displacement between the sending and receiving end equipment. Due to the drop in voltage at the load, the original load does not require the same power it required at normal voltage if it is the average station load. The kw. and kv-a. taken by the original load changes with voltage. If voltage slightly less than normal is assumed at the receiver end and the kw. and kv-a. corresponding to the original load at this voltage plus a small load increment can be transmitted over the line with the given generator excitation, the system is stable, for the small increment is the contribution to the additional load made by the change in phase displacement of the system. For the limit of stability this increment approaches zero.

The method of obtaining the general circuit constants of a transmission system and the construction of the power circle diagram from these constants has been described by Mr. R. D. Evans and Mr. H. K. Sels in a paper[4] before the Institute.

GENERAL CIRCUIT CONSTANTS

The same line, transformers and generator used to illustrate the equivalent circuit method will be used for the circle diagram.

The constants of the line alone are:

$$A = D = \cosh \sqrt{Y\,Z} = a_1 + j\,a_2 = 0.8700 + j\,0.0236$$

$$B = Z\,\frac{\sinh \sqrt{Z\,Y}}{\sqrt{Z\,Y}} = Z\,(\beta_1 + j\,\beta_2) = 34.52 + j\,194.6$$

$$C = Y\,\frac{\sinh \sqrt{Z\,Y}}{\sqrt{Z\,Y}} = Y\,(\beta_1 + j\,\beta_2)$$

$$= (-\,0.0104 + j\,1.248)\,10^{-3}$$

(a_1, a_2, β_1 and β_2 obtained from Fig. 1.)

The circuit constants of the line and the step-up and step-down transformers, neglecting magnetizing current, are calculated from *Item* (*g*) of Messrs. Evans and Sels' paper[5]. To include magnetizing current *Item* (*j*) should be used instead of (*g*).

$A_0 = A + C\,Z_s = 0.8431 + j\,0.0279$
$B_0 = B + A\,(Z_s + Z_r) + C\,Z_s\,Z_r = 39.6 + j\,232.1$
$C_0 = C = (-\,0.0104 + j\,1.248)\,10^{-3}$
$D_0 = A + C\,Z_r = 0.8424 + j\,0.0280$

Z_s and Z_r are the transformer impedances at the sending and receiving ends respectively.

It is of interest to note that these constants can be obtained from the equivalent π including the line and transformer or the equivalent π can be obtained from these constants. Using the equations in Appendix F and referring to Fig. 9

$Z_5 = B_0 = 39.6 + j\,232.1$

$$Y_4 = \frac{D_0 - 1}{B_0} = \frac{(-\,0.1576 + j\,0.0280)}{39.6 + j\,232.1}$$

$$= (0.0043 + j\,0.679)\,10^{-3}$$

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[4] Power Limitation of Transmission Systems, TRANSACTIONS of the A. I. E. E. Vol. 43, 1924, page 33.

$$Y_4 = \frac{A_0 - 1}{B_0} = \frac{(-0.1569 + j\,0.0279)}{39.6 + j\,232.1}$$

$$= (0.0045 + j\,0.676)\,10^{-3}$$

The circuit constants including the generator as well as the line and transformers are

$$A_{00} = A_0 + Z_1\,C_0 = 0.6193 + j\,0.0260$$

$$B_{00} = Z_1\,D_0 + B_0 = 34.59 + j\,383.1$$

where A_0, B_0, C_0 and D_0 refer to the circuit constants of the line and transformers and Z_1 is the generator impedance.

CONSTRUCTION OF THE CIRCLE DIAGRAM

$$(P_R + l\,E_R^2)^2 + (Q_R + m\,E_R^2)^2 = n^2\,E_R^2\,E_s^2 *$$

is the equation of the receiver power circle diagram in volt-amperes. If the question is expressed in kilovolt-amperes and divided by E_R^4 it becomes

$$\left(\frac{P_R}{E_R^2} + l\,10^3\right)^2 + \left(\frac{Q_R}{E_r^2} + m\,10^3\right)^2 = \left(n\,10^3\,\frac{E_s}{E_r}\right)^2$$

This is a circle for receiver power in terms of the receiver voltage and the ratio between the sending and receiving voltages. The center of the circle is at the point $-\,l\,10^3$, $-\,m\,10^3$.

If

$$A_0 = a_1 + j\,a_2$$

and

$$B_0 = R_0 + j\,X_0$$

$$l\dagger = \frac{a_1\,R_0 + a_2\,X_0}{R_0^2 + X_0^2}$$

$$m = \frac{a_1\,X_0 - a_2\,R_0}{R_0^2 + X_0^2}$$

$$n = \frac{1}{\sqrt{R_0^2 + X_0^2}}$$

Two circle diagrams will be drawn; one for the line and transformers and the other for the line, transformers and generator. Since they are both for power at the receiver end in terms of receiver end voltage, they will be drawn on the same chart. The ratio of the voltage on the low side of the transformer at the generator end to the voltage on the low side at the receiver end for normal operation has been assumed 220/200 = 1.1. One circle with $E_s/E_r = 1.1$ will be drawn in the diagram for the line and transformers. A series of circles with the ratio of the excitation voltage of the generator to the receiver voltage having various values will be drawn for the line, transformers and generator.

For line and transformers:

$l \times 10^3 = 0.72$, $m \times 10^3 = 3.50$, $n \times 10^3 = 4.24$

The dotted circle in Fig. 17 has the center $-\,0.72$, $-\,3.50$ and the radius $1.1 \times 4.24 = 5.67$

For line, transformers and generator:

$l \times 10^3 = 0.21$, $m \times 10^3 = 1.60$, $n \times 10^3 = 2.60$.

The solid circles in Fig. 17 have their center at the point $-\,0.21$, $-\,1.60$. They are drawn for various ratios of the generator excitation voltage to the receiver voltage.

The ratio of the active and reactive power at the load in kilovolt-amperes to the square of the receiver voltage in kilovolts is given by the dotted circle at all loads for which the ratio of the sending to the receiver voltage in the circuit consisting of the line and transformers is 1.1. The ratio of the generator excitation voltage to the receiver voltage for these same loads may be read from the solid circles.

The problems already solved by means of the equivalent circuit will now be solved by the circle diagram.

b. The power limit of the line and transformers at the specified voltages is obtained from the dotted circle at the point where the tangent to the circle is vertical. At this point $\text{kw}_r/\text{kv}_r^2 = 3.95$, and $\text{kw}_r = 3.95 \times (200)^2 = 158{,}000$ kw.

c. The power limit of the generator, line and transformers is obtained from the dotted curve at the point where the tangent to the solid circle is vertical. At this point $\text{kw}_r/\text{kv}_r^2 = 3.53$, and $\text{kw}_r = 3.54 \times (200)^2 = 141{,}200$ kw.

The limit of the system with various kinds of load can not be determined directly by means of the circle diagram. A certain load must be assumed and then the system tested for stability with this load. If the system is stable, a larger load should be assumed, but if unstable, a test should be made with a smaller load. If this process is continued until a load is obtained for which the system is stable, but for which there is no margin, this will be the maximum power of the system.

TO TEST FOR STABILITY ON THE CIRCLE DIAGRAMS

Calculate $\text{kw}_r/\text{kv}_r^2$ for the given load at normal voltage and find the corresponding point on the dotted circle. The ratio of generator excitation voltage to receiver voltage is read from the solid circle cutting the dotted circle at this point, and the generator excitation voltage calculated. Assume a receiver voltage slightly less than normal and calculate the active and reactive power of the load corresponding to this voltage, assuming constant excitations. Divide the active and reactive power in kilovolt-amperes by the square of the assumed receiver voltage in kilovolts and locate the point on the diagram. Read E_1/E_r at this point and calculate E_1, the generator excitation voltage. If this value of E_1 is equal to or less than the excitation voltage calculated at normal voltage the system is stable. It is sometimes more satisfactory to select a receiver voltage slightly above normal as well as one below normal; then when the corresponding calculated generator excitation voltages are both higher than the excitation voltage corresponding to normal receiver voltage, the assumed load is the power limit.

1. Given: 170,000-kv-a. synchronous motor, 85 per cent synchronous impedance.

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*Equation (**28**), A. I. E. E. TRANSACTIONS, Vol. 43, page 36.

†Circle Diagrams for Transmission Systems, R. D. Evans and H. K. Sels, *Electric Journal*, December 1921.

It has been shown by the equivalent circuit method that 106,000 kw. is the maximum power that can be delivered to this motor. Testing by means of the circle diagram, Fig. 17, for a load of 106,000 kw., point *A*, located on the dotted circle for $kw_r/kv_r^2 = 2.65$, gives the generator excitation voltage from the solid circle passing through *A* as 242 kv. at normal receiver voltage. Point *A′* corresponds to 98 per cent receiver voltage and *A″* to 102 per cent receiver voltage. The corresponding generator excitation voltage in each case is just about 242 kv. which indicates that 106,000 kw. is very near the limit of stability. Fig. 18 was used to calculate the reactive kv-a. of the synchronous motor assuming constant excitation. This method of calculating will be given in greater detail under Example 3.

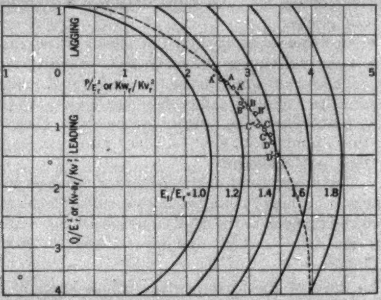

Fig. 17—Circle Diagram for the Transmission System

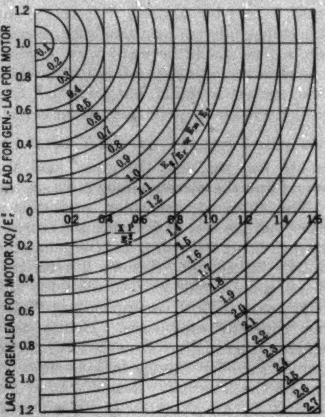

Fig. 18—Circle Diagram for Synchronous Motor or Generator of Constant Synchronous Impedance, 100,000-Kv-a. Base

2. Given: Resistance load of 30,000 kw., and the synchronous motor of example 1.

The power delivered to a resistance load varies as the square of the voltage. The power delivered to a shaft load is practically independent of voltage. Points *B*, *B′* and *B″* on the circle diagram give the generator excitation voltages corresponding to receiver voltages of 100 per cent, 98 per cent and 102 per cent respectively for a load of 120,000 kw., and points *C*, *C′* and *C″* the corresponding values for a load of 130,000 kw. The system is stable at 120,000 kw. but unstable at 130,000 kw.

3. Given: Total load of 180,000 kw. of which one-third is resistance load, one-third induction motor load and one-third synchronous motor load. The induction motors have an average power factor of 0.7 lag at normal voltage. The synchronous motors have 100 per cent synchronous impedance, are 75 per cent loaded and are operated at unity power factor. A 100,000-kv-a. synchronous condenser is placed at the load. The generators at the receiver end of the line, having total capacity of 100,000 kv-a. and synchronous impedance of 100 per cent, supply 45,000 kw. to the load and part of the reactive kv-a. needed for voltage regulation. Is the system stable?

100 Per Cent Receiver Voltage

E_r = 200 kv. = 100 per cent receiver voltage.

P_r = 135,000 kw. = total power over the line.

$\dfrac{P_r}{E_r^2} = \dfrac{kw_r}{kv_r^2} = 3.375$, determines location of point *D* on dotted circle, Fig. 17.

$\dfrac{Q_r}{E_r^2}$ = 1.28 read at point *D*, Fig. 17.

Q_r = 51,200 kv-a. = total reactive power over the line.

E_1/E_r = 1.385, obtained from solid circle passing through *D*.

E_1 = 277 kv. = excitation voltage of generator at sending end.

$P_m + Q_m = 60{,}000 + j\,O$ = synchronous motor load.

$R = 60{,}000 + j\,O$ = resistance load.

$P_1 - Q_1 = 60{,}000 - j\,61{,}000$ = induction motor load.

$Q_c = +j\,100{,}000$ = reactive kv-a. of condenser.

$-(P_g - j\,Q_g) = -(45{,}000 - j\,12{,}200)$ = load on generators at receiver end.

Sum $= 135{,}000 + j\,51{,}200$ = total load over line at 100 per cent voltage.

X_m = 125 per cent = per cent synchronous impedance of motors on 100,000-kv-a. base.

X_g = 100 per cent = per cent synchronous impedance of generators at receiver end on 100,000-kv-a. base.

$X_m \dfrac{(P_m + Q_m)}{E_r^2} = 0.75 + j\,O$ where P_m and Q_m are in per cent on 100,000-kv-a. base.

E_m/E_r = 1.25 read from Fig. 18.

E_m = 250 kv. = excitation voltage of synchronous motor.

$X_g \dfrac{(P_g + Q_g)}{E_r^2} = 0.45 - j0.122$ where P_g and Q_g are in per cent on 100,000-kv-a. base.

E_g/E_r = 1.21 read from Fig. 18.

E_g = 242 kv. = excitation voltage of generator at receiver end.

98 Per Cent Receiver Voltage

E_r = 196 kv. = 98 per cent.

R = 57,600 kw. = resistance load.

P_m = 60,000 kw. = watt component of synchronous motor load.

P_1 = 60,000 kw. = watt component of induction motor load.

P_g = 45,000 kw. = watt component of load on generator at receiver end.

P_r = 132,600 kw. = watt component of total load over the line at 98 per cent voltage.

$$\frac{P_r}{E_r^2} = 3.45$$

$$\frac{E_m}{E_r} = \frac{250}{196} = 1.275.$$

$$\frac{X_m\,P_m}{E_r^2} = \frac{1.25 \times .6}{(0.98)^2} = 0.78.$$

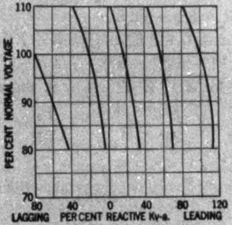

Fig. 19—Synchronous Condenser Characteristics

$\dfrac{X_m\,Q_m}{E_r^2} = 0.01$ read from Fig. 18 corresponding to E_m/E_r and $X_m\,P_m/E_r^2$

$$\frac{E_g}{E_r} = \frac{242}{196} = 1.235$$

$$\frac{X_g P_g}{E_r^2} = \frac{1.00 \times 0.45}{(0.98)^2} = 0.469$$

$\dfrac{X_g\,Q_g}{E_r^2} = 0.135$ read from Fig. 18 corresponding to E_g/E_r and $X_g\,P_g/E_r^2$

Q_c = $+\,j$ 103,000 from Fig. 19.

Q_m = $+\,j$ 770

Q_1 = $-\,j$ 59,500, from Fig. 20.

Q_g = $-\,(-\,j$ 13,000)

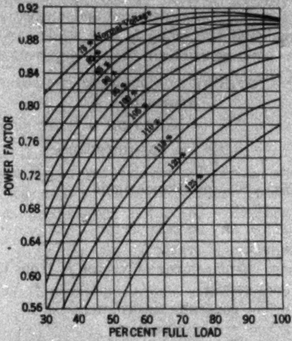

Fig. 20—Variation of Power Factor on an Average Induction Motor with Load and Voltage

Q_r = $+\,j$ 57,270 total reactive kv-a. over the line.

$$\frac{Q_r}{E_r^2} = +\,j\,1.49$$

$\dfrac{P_r + Q_r}{E_r^2} = 3.45 + j\,1.49$, determines location of point D'.

E_1/E_r = 1.41, obtained from solid circle passing through D'.

E_1 = 276 kv., excitation voltage of generator at the sending end.

Since E_1, the calculated generator excitation voltage, at 98 per cent receiver voltage is less than E_1 at 100 per cent receiver voltage for the same receiver load, the system is stable. These calculations do not indicate the load which can be added with stability maintained. They merely indicate that the system is stable under the assumed load conditions.

When variation of kw. and kv-a. at the load with receiver voltage is known, the calculations are greatly simplified. $(P_r + Q_r)/E_r^2$ is then calculated for various values of E_r and the corresponding values of E_1/E_r read from the circle diagram.

In the examples which have been considered a single generating station supplies power over one circuit to a single receiving station. In more complicated systems where it may be necessary to cut and try, the circle diagram can be used to advantage for the various parts of the system.

ACKNOWLEDGMENT

The idea of combining the shunts at the ends of the line with the synchronous apparatus is due to Mr. C. A. Nickle who proved that "neglecting resistance, a line with synchronous apparatus and reactance or capacitance shunts may be replaced by a line and equivalent synchronous apparatus with no shunts."

The writer wishes gratefully to acknowledge her indebtedness to Messrs. H. H. Dewey and R. E. Doherty for their encouragement and suggestions which have broadened the scope of this study, and to Mr. Nickle for his suggestions in the development of certain phases of the subject.

Appendix A

NOTATION

$Z \;= r + j\,x = z\,e^{j\theta}$ = any impedance

$\theta \;= \tan^{-1}\dfrac{x}{r}$ = impedance angle

$E \;= V\,e^{j\alpha}$ = any voltage vector

V = magnitude of vector E

α = phase displacement of vector E from the reference vector

$Z_1 = z_1\,e^{j\theta_1}$ = generator impedance

$Z_2 = z_2\,e^{j\theta_2}$ = motor impedance

$Z_3 = z_3\,e^{j\theta_3}$ = impedance of shunt at the generator end of the equivalent π–line

$Z_4 = z_4\,e^{j\theta_4}$ = impedance of shunt at the motor end of the equivalent π–line

$Z_5 = z_5\,e^{j\theta_5}$ = architrave impedance of the equivalent π–line

$Z_t = z_t\,e^{j\theta_t}$ = total impedance in the circuit

$Z_1{}' = z_1{}'\,e^{j\theta_1{}'} = \dfrac{Z_1\,Z_3}{Z_1 + Z_3}$

= equivalent generator impedance

$Z_2{}' = z_2{}'\,e^{j\theta_2{}'} = \dfrac{Z_2\,Z_4}{Z_2 + Z_4}$

= equivalent motor impedance

$Z_1 + Z_s = Z_{1s} = z_{1s}\,e^{j\theta_{1s}}$

$Z_2 + Z_s = Z_{2s} = z_{2s}\,e^{j\theta_{2s}}$

$Z_1\,Z_2 + Z_1\,Z_s + Z_2\,Z_s = Z_d = z_d\,e^{j\theta_d}$

$E_1 \;= V_1\,e^{j\alpha_1}$ = generator excitation voltage

$E_1{}' = V_1{}'\,e^{j\alpha_1{}'}$ = equivalent generator excitation voltage

$E_2 \;= V_2\,e^{j\alpha_2}$ = motor excitation voltage

$E_2{}' = V_2{}'\,e^{j\alpha_2{}'}$ = equivalent motor excitation voltage

E_A = terminal voltage at the generator end of the line

E_B = terminal voltage at receiver end of the line

E_s = sending voltage

E_r = receiver voltage

Y_3 = admittance of shunt at the generator of the equivalent π–line

Y_4 = admittance of the shunt at the motor end of the equivalent π–line

P = active power in kw. or per cent

Q = reactive power in kv-a. or per cent

P_{max} = maximum power

${}^{\alpha}B.D.$ = phase displacement between actual generator and motor excitation voltages at which the machines fall out of step.

${}^{\alpha'}B.D.$ = phase displacement between the excitation voltages of the equivalent system corresponding to maximum power on the actual system.

Appendix B

$$\cosh\sqrt{Z\,Y}$$

$$= \left(+\frac{Z\,Y}{2} + \frac{Z^2\,Y^2}{24} + \frac{Z^3\,Y^3}{720} + \frac{Z^4\,Y^4}{40320} + \;.\;.\;\right) = A$$

$$= a_1 + j\,a_2$$

$$\frac{\sinh\sqrt{Z\,Y}}{\sqrt{Z\,Y}} = \left(1 + \frac{Z\,Y}{6} + \frac{Z^2\,Y^2}{120} + \frac{Z^3\,Y^3}{5040}\;.\;.\;.\;\right) = \beta$$

$$= \beta_1 + j\,\beta_2$$

$$\frac{(\tanh\sqrt{Z\,Y})}{\sqrt{Z\,Y}} = \frac{(\sinh\sqrt{Z\,Y})/\sqrt{Z\,Y}}{\cosh\sqrt{Z\,Y}} = \frac{\beta_1 + j\,\beta_2}{a_1 + j\,a_2}$$

$Z \;= l\,(r + j\,x) = l\,x\left(\dfrac{r}{x} + j\right)$ = total conductor impedance

$Y \;= l\,(0 + j\,y) = l\;y\;(\;+ j)$ = total dielectric admittance

$$Z\,Y = l^2\,x\,y\left(-1 + \frac{j\,r}{x}\right)$$

Let $l^2\,x\,y = 4\,\pi^2 f^2 l^2\,L\,C = F$ and $\dfrac{r}{x} = K$

$Z\,Y = F\,(-1 + j\,K)$

$Z^2\,Y^2 = F^2\,(1 - K^2) - j\,2\,K$

$Z^3\,Y^3 = F^3\,\{\,(-1 + 3\,K^2) + j\,(3\,K - K^3)\,\}$

$Z^4\,Y^4 = F^4\,\{\,(1 - 6\,K^2 + K^4) - j\,(4\,K - 4\,K^3)\,\}$

Substituting for $Z\,Y$, $Z^2\,Y^2$ . . . in the above series

$$a_1 = 1 - \frac{F}{2} + \frac{F^2}{24}\,(1 - K^2) + \frac{F_3}{720}\,(-1 + 3\,K^2)$$

$$+ \frac{F_4}{40320}\,(1 - 6\,K^2 + K^4) + .$$

$$= \left(1 - \frac{F}{2} + \frac{F_2}{24} - \frac{F^3}{720} + \frac{F^4}{40320} - \;.\;.\;\right)$$

$$- K^2\left(\frac{F^2}{24} - \frac{F^3}{240} + \frac{F_4}{6720} - \;.\;.\;.\;\right)$$

$$+ K^4\left(\frac{F^4}{440320} + \;.\;.\;.\;\right) - \;.\;.$$

$$a_2 = K\left(\frac{F}{2} - \frac{F^2}{12} + \frac{F^3}{240} - \frac{F^4}{10080} + \;.\;.\;\right)$$

$$- K^3\left(\frac{F^3}{720} - \frac{F^4}{10080} + \dots\right) + \dots$$

$$\beta_1 = \left(1 - \frac{F}{6} + \frac{F^2}{120} - \frac{F^3}{5040} + \frac{F^4}{362880} + \dots\right)$$

$$- K^2\left(\frac{F^2}{120} - \frac{F^3}{1680} + \frac{F^4}{60480}\right)$$

$$- K^4\left(\frac{F^4}{362880} + \dots\right) - \dots$$

$$\beta_2 = K\left(\frac{F}{6} - \frac{F^2}{60} + \frac{F^3}{1680} - \frac{F^4}{90720} + \dots\right)$$

$$- K^3\left(\frac{F^3}{5040} - \frac{F^4}{90720} + .\right) + \dots$$

$$\frac{\tanh\sqrt{Z\,Y}/2}{\sqrt{Z\,Y}/2} = \gamma_1 + j\,\gamma_2$$

Tanh $\sqrt{Z\,Y}/2$ corresponding to a given value of $f\,l$ is the same as tanh $\sqrt{Z\,Y}$ corresponding to one-half of this value of $f\,l$. Tanh $\sqrt{Z\,Y}/2$ for a given value of $f\,l$ was obtained from the previously calculated values of $\frac{\sinh\sqrt{Z\,Y}}{\sqrt{Z\,Y}}$ and cosh $\sqrt{Z\,Y}$ corresponding to one-half of the given value of $f\,l$. For the range covered in calculating $\frac{\tanh\sqrt{Z\,Y}/2}{\sqrt{Z\,Y}/2}$ a_2, β_2 and γ_2 are so small in comparison with a_1, β_1 and γ_1 that the magnitude of these vectors is the same as the real parts and the angle in radians is equal to its tangent, *i. e.*, $\theta = \sin\theta = \tan\theta$, and $\cos\theta = 1$.

$$\frac{\tanh\sqrt{Z\,Y}/2}{\sqrt{Z\,Y}/2} = \gamma_1 + j\,\gamma_2 = \frac{\beta_1 + j\,\beta_2}{a_1 + j\,a_2}$$

$$= \frac{\beta_1}{a_1}\left\{1 - j\left(\frac{a_2}{a_1} - \frac{\beta_2}{\beta_1}\right)\right\}$$

The above equations give $a_1, a_2, \beta_1, \beta_2$ γ_1 and γ_2 in terms of $K = r/x$ and $F = f^2\,l^2\,(4\,\pi^2\,L\,C)$. Since $L\,C$ is practically constant for overhead lines, the values are known when $f\,l$ and r/x are given. In calculating these series $L\,C$ was taken as 0.029901×10^{-9} which corresponds to $S/D = 20$, where S is flat spacing in feet and D is diameter of the conductor in inches and $L = 0.0021421$ henrys and $C = 0.013960$ microfarads. The calculations for A and β were carried to five significant figures.

If the product $L\,C$ is different from this assumed value, the same percentage error will occur in F and approximately the same in the imaginary parts of all three factors. Consequently, correction for this error can be made. The errors in the real parts a_1 and β_1 are approximately $2\,\pi^2 f^2\,l^2\,(L\,C - 0.029901 \times 10^{-9})$ and $\frac{2}{3}\,\pi^2\,f^2\,l^2\,(L\,C - 0.29901 \times 10^{-9})$ respectively, but the percentage errors depend upon the size of F relative to unity.

Appendix C

Conjugate vectors will be used in the proofs which follow. A vector E, of magnitude V, making an angle α with the reference vector may be written $E = V(\cos\alpha + j\sin\alpha) = V\,e^{j\alpha} = a + j\,b$, its conjugate vector will be written $\bar{E} = V(\cos\alpha - j\sin\alpha) = V\,e^{-\alpha j} = a - j\,b$.

Represented by conjugate vectors it can be proved that Power $= \frac{1}{2}(\bar{E}\,I + E\,\bar{I})$

a. *Power Exchanged Between a Generator and Motor Connected by a Line Without Shunts.* Let Fig. 21 represent a synchronous motor and generator both of constant synchronous impedance connected by a line without shunts.

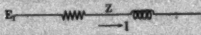

FIG. 21—SYNCHRONOUS GENERATOR AND MOTOR CONNECTED BY A LINE WITHOUT SHUNTS

$I = \frac{E_1 - E_2}{Z}$ where E_1 and E_2 are excitation voltages and Z is the total impedance between them.

$$2\,P = \bar{E}_2\,I + E_2\,\bar{I}$$

$$= \bar{E}_2\left(\frac{E_1 - E_2}{Z}\right) + E_2\left(\frac{\bar{E}_1 - \bar{E}_2}{\bar{Z}}\right)$$

If E_2 is the reference vector, then

$E_2 = \bar{E}_2 = V_2\,e^{jO} = V_2$

$E_1 = V_1\,e^{j\alpha}, \bar{E}_1 = V_1\,e^{-j\alpha}$

$Z = R + j\,X = z\,e^{j\theta}$

$$2\,P = \frac{V_1\,V_2}{z}\left(e^{j(\alpha-\theta)} + e^{-j(\alpha-\theta)}\right) - \frac{V_2^2}{z}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$P = \frac{V_1\,V_2}{z}\left\{\cos(\alpha - \theta) - \frac{V_2}{V_1}\cos\theta\right\} \quad (5)$$

With constant values of V_1 and V_2 maximum power occurs when $\cos(\alpha - \theta) = 1$

$$\therefore \alpha = \theta = \tan^{-1}\frac{X}{R} \text{ and} \quad (6)$$

$$P_{max} = \frac{V_1\,V_2}{z}\left(1 - \frac{V_2}{V_1}\cos\theta\right)^* \quad (7)$$

When V_1 and V_2 are in volts to neutral and z is in

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*"Calculation of Synchronizing Power," by E. S. Henningsen, *G. E. Review*, Nov. 1921.

ohms P will be in watts per phase, but if V_1, V_2 and z are in per cent P will also be in per cent.

Tie Line Between Two Stations. If V_1 and V_2 are the terminal voltages at the ends of a tie line, equation (7) gives the maximum power which can be transmitted across the tie.

With constant voltage, V_1, and constant phase displacement, α, the value of V_2 corresponding to maximum power obtained by differentiating equation (5) with respect to V_2 is

$$V_2 = \frac{V_1 \cos(\alpha - \theta)}{2 \cos \theta} \qquad (8)$$

if $\alpha = \theta$

$$V_2 = \frac{V_1}{2 \cos \theta} \qquad (9)$$

If V_1 has its maximum allowable value determined by the insulation voltage and V_2 can not be greater than V_1, equation (9) shows that for an impedance angle, θ, greater than 60 deg., maximum power is obtained when V_2 equals V_1, but for values of θ less than 60 deg. maximum power is obtained when V_2 is less than V_1.

b. *Equivalent Generator or Equivalent Motor.* Let Fig. 22 represent a synchronous motor and generator, both of constant synchronous impedance, connected by a line with a shunt.

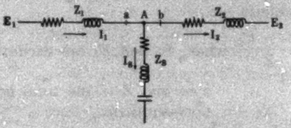

FIG. 22—SYNCHRONOUS GENERATOR AND MOTOR CONNECTED BY A LINE WITH A SHUNT

Z_1 = generator impedance or impedance from neutral of generator to point A

Z_2 = motor impedance or impedance from neutral of motor to point A

Z_s = shunt impedance

E_A = voltage at point A

To prove: (1) For all points beyond junction A, the actual system in Fig. 22 can be replaced by the equivalent system given in Fig. 23, where the shunt and generator have been combined in parallel into an equivalent generator of impedance $Z_1' = \dfrac{Z_1 Z_s}{Z_1 + Z_s}$ and excitation voltage $E_1' = E_1 \dfrac{Z_1'}{Z_1}$ and the power at breakdown on the actual system is the same as power at breakdown on the fictitious system; and (2) for all points in front of junction A, the actual system in Fig. 22 can be replaced by the equivalent system in Fig. 24, where the shunt and motor have been combined into an equivalent motor of impedance $Z_2' = \dfrac{Z_2 Z_s}{Z_2 + Z_s}$ and excitation voltage $E_2' = E_2 \dfrac{Z_2'}{Z_2}$ and power at breakdown on the actual system will be represented by the power on the fictitious system corresponding to a phase displacement between motor and generator excitation voltages equal to the total impedance angle of the fictitious system plus the constant angle $2(\theta_2 - \theta_2')$, where θ_2 is the actual motor impedance angle and θ_2' = the fictitious motor impedance angle.

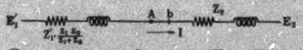

FIG. 23—GENERATOR AND SHUNT OF FIG. 22 REPLACED BY EQUIVALENT GENERATOR

Proof: *In the Actual System Represented by Fig. 22*

$$I_2 = \frac{E_A - E_2}{Z_2}$$

$$\text{Also } I_2 = I_1 - I_s = \frac{E_1 - E_A}{Z_1} - \frac{E_A}{Z_s}$$

$$E_A = \frac{E_1 Z_2 Z_s + E_2 Z_1 Z_s}{Z_1 Z_2 + Z_1 Z_s + Z_2 Z_s} \qquad (10)$$

$$I_2 = \frac{E_1 Z_s - E_2 (Z_1 + Z_s)}{Z_1 Z_2 + Z_1 Z_s + Z_2 Z_s} \qquad (11)$$

$$I_1 = \frac{E_1 (Z_2 + Z_s) - E_2 Z_s}{Z_1 Z_2 + Z_1 Z_s + Z_2 Z_s} \qquad (12)$$

Let P_2 = power on shaft of motor.

$$2 P_2 = \bar{E}_2 I_2 + E_2 \bar{I}_2$$

$$= \bar{E}_2 \frac{(E_1 Z_s - E_2 (Z_1 + Z_s)}{Z_1 Z_2 + Z_1 Z_s + Z_2 Z_s} + E_2 \frac{(\bar{E}_1 \bar{Z}_s - \bar{E}_2 (\bar{Z}_1 + \bar{Z}_s)}{\bar{Z}_1 \bar{Z}_2 + \bar{Z}_1 \bar{Z}_s + \bar{Z}_2 \bar{Z}_s}$$

Let $Z_1 Z_2 + Z_1 Z_s + Z_2 Z_s = Z_d = z_d e^{j\theta_d}$

$Z_1 + Z_s = Z_{1s} = z_{1s} e^{j\theta_{1s}}$

$Z_2 + Z_s = Z_{2s} = Z_{2s} e^{j\theta_{2s}}$

$$P_2 = V_1 V_2 \frac{z_s}{z_d} \cos(\alpha + \theta_s - \theta_d) - V_2^2 \frac{z_{1s}}{z_d} \cos(\theta_{1s} - \theta_d) \qquad (13)$$

For maximum power with constant excitations $\cos(\alpha + \theta_s - \theta_d) = 1$ and

$\alpha_{B.D.}$ = value of α at breakdown $= \theta_d - \theta_s$, and (14)

$$P_2 (\max) = V_1 V_2 \frac{z_s}{z_d} - V_2^2 \frac{z_{1s}}{z_d} \cos(\theta_d - \theta_{1s}) \qquad (15)$$

An inspection of equations (13) and (15) shows that increasing the excitation of the generator increases the power on the shaft of the motor, but increasing the motor excitation does not necessarily increase the power.

With constant generator excitation, E_1, and constant phase displacement α, the value of V_2 corresponding to maximum power obtained by differentiating equation (**13**) with respect to V_2 is

$$V_2 = \frac{V_1 z_s \cos(\alpha + \theta_s - \theta_d)}{2 z_{1s} \cos(\theta_d - \theta_{1s})} \qquad \textbf{(16)}$$

$$\text{if } \alpha = \theta_d - \theta_s$$

$$V_2 = \frac{V_1 z_s}{2 z_{1s} \cos(\theta_d - \theta_{1s})} \qquad \textbf{(17)}$$

Equation (**17**) gives the critical value of V_2 for a given value of V_1. If V_2 has this value, an increase in motor excitation will result in less power and the system will become unstable.

Consider the power at point a just in front of junction A.

P_a = power at point a, Fig. 22

$2 P_a = \bar{E}_A I_1 + E_A \bar{I}_1$

$$P_a = \frac{1}{z_d^2}[V_1^2 z_2 z_s z_{2s} \cos(\theta_2 + \theta_s - \theta_{2s}) - V_2^2 z_1 z_s^2 \cos\theta_1 + V_1 V_2 \{z_1 z_s z_{2s} \cos(\alpha + \theta_{2s} - \theta_1 - \theta_s) - z_s^2 z_2 \cos(\alpha + \theta_2)\}] \qquad \textbf{(18)}$$

The system falls out of step when $\alpha = {}^{\alpha}B.D. = \theta_d - \theta_s$; see equation (**14**). Substituting $\alpha = \theta_d - \theta_s$ in equation (**18**)

$$P_a(\max) = \frac{1}{z_d^2}[V_1^2 z_2 z_s z_{2s} \cos(\theta_2 + \theta_s - \theta_{2s}) - V_2^2 z_1 z_s^2 \cos\theta_1 + V_1 V_2 \{z_1 z_s z_{2s} \cos(\theta_d + \theta_{2s} - \theta_1 - 2\theta_s) - z_s^2 z_2 \cos(\theta_d - \theta_s + \theta_2)\}] \qquad \textbf{(19)}$$

1. *In the equivalent system represented by Fig. 23*

$$I = \frac{E_1' - E_A}{\dfrac{Z_1 Z_s}{Z_1 + Z_s}} = \frac{E_A - E_2}{Z_2} = \frac{E_1' - E_2}{Z_2 + \dfrac{Z_1 Z_s}{Z_1 + Z_s}}$$

$$\therefore E_A = \frac{E_1' Z_2 (Z_1 + Z_s) + E_2 Z_1 Z_s}{Z_1 Z_2 + Z_1 Z_s + Z_2 Z_s}$$

and

$$I = \frac{(Z_1 + Z_s), (E_1' - E_2)}{Z_1 Z_2 + Z_1 Z_s + Z_2 Z_s}$$

If E' is replaced by $E_1 \dfrac{Z_1 Z_s}{Z_1 + Z_s} / Z_1 = E_1 \dfrac{Z_s}{Z_1 + Z_s}$

$$E_A = \frac{E_1 Z_2 Z_s + E_2 Z_1 Z_s}{Z_1 Z_2 + Z_1 Z_s + Z_2 Z_s} \qquad \textbf{(20)}$$

and

$$I = \frac{E_1 Z_s - E_2 (Z_1 + Z_s)}{Z_1 Z_2 + Z_1 Z_s + Z_2 Z_s} \qquad \textbf{(21)}$$

Equations (**20**) and (**21**) are identical with (**10**) and (**11**); therefore the power-voltage curves at constant excitations, for points beyond junction A for the two systems will be identical.

From equation (**6**) breakdown on the fictitious system occurs when α', the angle between E_1' and E_2, is equal to the total impedance angle θ_t.

$$Z_t = Z_2 + \frac{Z_1 Z_s}{Z_1 + Z_s} = \frac{Z_1 Z_2 + Z_1 Z_s + Z_2 Z_s}{Z_1 + Z_s} = \frac{Z_d}{Z_{1s}}$$

$$= \frac{z_d}{z_{1s}} e^{j(\theta_d - \theta_{1s})} = z_t e^{j\theta_t}$$

$$\alpha' = \theta_t = \theta_d - \theta_{1s}$$

From equation (**7**) maximum power in the fictitious system on the shaft of the motor is

$$P_2(\max) = \frac{V_1' V_2}{z_t}\left(1 - \frac{V_2}{V_1'} \cos\theta_t\right)$$

If V_1' is replaced by $V_1 \dfrac{z_s}{z_{1s}}$, z_t by $\dfrac{z_d}{z_{1s}}$ and θ_t by $(\theta_d - \theta_{1s})$

$$P_2(\max) = V_1 V_2 \frac{z_s}{z_d} - V_2^2 \frac{z_{1s}}{z_d} \cos(\theta_d - \theta_{1s}) \qquad \textbf{(22)}$$

Equations (**15**) and (**22**) are identical; therefore the equivalent system of Fig. 23 can replace the actual system of Fig. 22 for points beyond junction A.

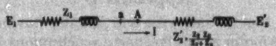

FIG. 24—MOTOR AND SHUNT OF FIG. 22 REPLACED BY EQUIVLENT MOTOR

2. *In the equivalent system represented by Fig. 34*

$$E_A = \frac{E_1 Z_2 Z_s + E_2' Z_1 (Z_2 + Z_s)}{Z_1 Z_2 + Z_1 Z_s + Z_2 Z_s}$$

$$I = \frac{(Z_2 + Z_s)(E_1 - E_2')}{Z_1 Z_2 + Z_1 Z_s + Z_2 Z_s}$$

if E_2' is replaced by $E_2 \dfrac{Z_2 Z_s}{Z_2 + Z_s} \Big/ Z_2 = E_2 \dfrac{Z_s}{Z_2 + Z_s}$

E_A and I are the same as E_A and I_1 of the actual line as given in equations (**10**) and (**12**); therefore the power-voltage curves, with constant excitations, for points in front of junction A for the two systems will be identical.

P_a = power at point a, Fig. 24.

$2 P_a = \bar{E}_A I + E_A \bar{I}$

$$P_a = \frac{1}{z_d^2}[V_1^2 z_2 z_s z_{2s} \cos(\theta_2 + \theta_s - \theta_{2s}) - V_2'^2 z_{2s}^2 z_1 \cos\theta_1 + V_1 V_2' \{z_1 z_{2s}^2 \cos(\alpha' - \theta_1) - z_2 z_s z_{2s} \cos(\alpha' + \theta_2 + \theta_s - \theta_{2s})\}] \qquad \textbf{(23)}$$

In equation (**23**) replace V_2' by $V_2 \dfrac{z_s}{z_{2s}}$ and α' by

$\theta_t + 2\,(\theta_2 - \theta_2')$

Since

$$Z_t = Z_1 + \frac{Z_2 Z_s}{Z_2 + Z_s} = \frac{Z_d}{Z_{2s}} = \frac{z_d}{z_{2s}}\, e^{j(\theta_d - \theta_{2s})} = z_t e^{j\theta_t} \text{ and}$$

$$Z_2' = \frac{Z_2 Z_s}{Z_2 + Z_s} = \frac{z_2 z_s}{z_{2s}}\, e^{j(\theta_2 + \theta_s - \theta_{2s})} = z_2'\, e^{j\theta_2'}$$

$$\alpha' = \theta_t + 2\,(\theta_2 - \theta_2') = \theta_d - \theta_{2s} + 2\,(\theta_{2s} - \theta_s) = \theta_d + \theta_{2s} - 2\,\theta_s$$

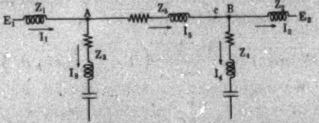

FIG. 25—LINE WITH END SHUNTS AND SYNCHRONOUS GENERATOR AND MOTOR

and equation (**23**) and equation (**19**) will be identical and the equivalent system of Fig. 24 can replace the actual system of Fig. 22 for points in front of junction A.

Appendix D

Proof that a line with end shunts, having a synchronous motor at one end and a synchronous generator at the other end, both of constant synchronous impedance, and voltage regulators at the ends of the line, can be replaced, for all points of the line, by a line without shunts and equivalent generator and motor. The impedance of the line will remain unchanged, the impedance of the equivalent generator will be the impedance of the given generator and the shunt at the generator end in parallel and the impedance of the equivalent motor will be the impedance of the motor and the shunt at the receiver end in parallel, and the power at breakdown on the actual system will be represented by the power on the fictitious system corresponding to a phase displacement between motor and generator excitation voltages equal to θ_t, the total impedance angle of the fictitious system plus the constant angle $2\,(\theta_2 - \theta_2')$, where θ_2 is the actual motor impedance angle and θ_2' the equivalent motor impedance angle.

Let Fig. 25 represent the actual system and Fig. 26 the equivalent system.

It must be shown that with voltages maintained at the ends of the line, the power at c' is the same as the power at c and the power at breakdown on the actual system is obtained on the fictitious system with an angle between equivalent generator and motor excitation voltages equal to θ_t, the total impedance angle $+ 2\,(\theta_2 - \theta_2')$.

Let α = the phase displacement between E_1 and E_2, the excitation voltages of the actual generator and motor in Fig. 25.

α' = the phase displacement between E_1' and E_2', the excitation voltages of the equivalent generator and motor in Fig. 26.

β = Phase displacement between E_A and E_B, the terminal voltages in Fig. 25.

β' = Phase displacement between E_A' and E_B', the terminal voltages in Fig. 26.

With voltages maintained at the ends of the line in both cases, the power at c as a function of β and the power at c' as a function of β' are given by equation (**5**). With $\beta = \beta'$ the power at c' is the same as the power at c, also the current in the line is the same in both systems. With β known, α is determined and with β' known, α' is determined. As power over the line increases, $\beta = \beta'$ increases, and α and α' increase. It will be shown that the difference between α and α' is constant for all loads, or $\alpha' = \alpha + \gamma$, where γ is a constant angle. When the actual system breaks down with an angle $^{\alpha}B\,.\,D\,.$ between motor and generator excitation voltages there is a certain definite power at c. For this same power at c' on the equivalent system $^{\alpha}B'\,.\,D\,.$ (value of α' corresponding to $^{\alpha}B\,.\,D\,.$) $= {}^{\alpha}B\,.\,D\,. + \gamma$

If $\alpha' = \alpha + \gamma$ for all loads

it will also be shown that $^{\alpha}B.\,D. + \gamma = \theta_t + 2\,(\theta_2 - \theta_2')$

To prove: (1) With maintained voltages E_A and E_B at the ends of the line and the same power at c and at c', $\alpha' = \alpha + \gamma$, where γ is a constant; and (2), $^{\alpha}B.\,D. + \gamma = \theta_t + 2\,(\theta_2 - \theta_2')$.

Proof: (1) Let I be the current in the line for both systems corresponding to any given load.

From Fig. 25

$$E_1 = E_A + Z_1\left(I + \frac{E_A}{Z_3}\right)$$

$$\therefore E_1 Z_3 = E_A\,(Z_1 + Z_3) + Z_1 Z_3 I \qquad (\mathbf{24})$$

From Fig. 26

$$E_1' = E_A + \frac{Z_1 Z_3}{Z_1 + Z_3}\, I$$

$$\therefore (Z_1 + Z_3)\, E_1' = E_A\,(Z_1 + Z_3) + Z_1 Z_3 I \qquad (\mathbf{25})$$

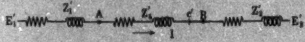

FIG. 26—EQUIVALENT SYSTEM FOR FIG. 25

From equations (**24**) and (**25**)

$$\frac{E_1'}{E_1} = \frac{Z_3}{Z_1 + Z_3} = \frac{Z_1 Z_3}{Z_1 + Z_3}\Big/ Z_1 = \frac{Z_1'}{Z_1} = \frac{z_1'}{z_1}\, e^{j(\theta_1' - \theta_1)} \qquad (\mathbf{26})$$

In like manner

$$\frac{E_2'}{E_2} = \frac{Z_4}{Z_2 + Z_4} = \frac{Z_2 Z_4}{Z_2 + Z_4}\Big/ Z_2 = \frac{Z_2'}{Z_2} = \frac{z_2'}{z_2}\, e^{j(\theta_2' - \theta_2)} \qquad (\mathbf{27})$$

From equation (**26**) E_1' leads E_1 by the angle $(\theta_1' - \theta_1)$ and from equation (**27**) E_2' leads E_2 by the angle $(\theta_2' - \theta_2)$

$\therefore \alpha' \quad = \alpha + (\theta_1' - \theta_1) - (\theta_2' - \theta_2) = \alpha + \gamma$

Where $\gamma = (\theta_1' - \theta_1) - (\theta_2' - \theta_2) =$ constant.

(2) From Fig. 25

$$I_2 = \frac{E_B - E_2}{Z_2} \tag{28}$$

$$I_5 = \frac{E_A - E_B}{Z_5} = \frac{E_1 - E_A}{Z_1} - \frac{E_A}{Z_3} = \frac{E_B - E_2}{Z_2} + \frac{E_B}{Z_4} \tag{29}$$

From (**29**) E_B can be expressed in terms of E_1 and E_2 and this value substituted in (**28**).

$$E_B = \frac{E_1 Z_2 Z_3 Z_4 Z_5 + E_2 Z_4 Z_5 (Z_1 Z_3 + Z_1 Z_5 + Z_3 Z_5)}{[Z_2 Z_4 + Z_5 (Z_2 + Z_4)]\,[Z_1 Z_3 + Z_5 (Z_1 + Z_3)] - Z_1 Z_2 Z_3 Z_4}$$

$$I_2 = \frac{E_1 Z_2 Z_3 Z_4 + E_2 Z_4 (Z_1 Z_3 + Z_1 Z_5 + Z_3 Z_5)}{Z_2 [Z_5 (Z_1 + Z_3)(Z_2 + Z_4) + Z_1 Z_3 (Z_2 + Z_4) + Z_2 Z_4 (Z_1 + Z_3)]} - \frac{E_2}{Z_2}$$

$$= \frac{E_1 Z_1 Z_2 Z_3 Z_4 + E_2 Z_1 Z_4 (Z_1 Z_3 + Z_1 Z_5 + Z_3 Z_5)}{Z_1 Z_2 (Z_1 + Z_3)(Z_2 + Z_4)\left(Z_5 + \dfrac{Z_1 Z_3}{Z_1 + Z_3} + \dfrac{Z_2 Z_4}{Z_2 + Z_4}\right)} - \frac{E_2}{Z_2}$$

$$\text{but } Z_5 + \frac{Z_1 Z_3}{Z_1 + Z_3} + \frac{Z_2 Z_4}{Z_2 + Z_4} = Z_t = z_t\, e^{j\theta t}$$

where $Z_t =$ the total impedance of the equivalent system, and $\theta_t =$ the total impedance angle, Fig. 26.

$$I_2 = \frac{E_1}{Z_1 Z_2 Z_t}\left(\frac{Z_1 Z_3}{Z_1 + Z_3}\right)\left(\frac{Z_2 Z_4}{Z_2 + Z_4}\right)$$

$$- \frac{E_2 (Z_1 + Z_3)(Z_4 + Z_5) + Z_1 Z_3}{Z_5 (Z_1 + Z_3)(Z_2 + Z_4) + Z_1 Z_3 (Z_2 + Z_4) + Z_2 Z_4 (Z_1 + Z_3)}$$

$2 P_2 = \bar{E}_2 I_2 + E_2 \bar{I}_2 = 2$ (power on shaft of motor)

$$P_2 \quad = V_1 V_2 \frac{z_1' z_2'}{z_1 z_2 z_t} \cos(\alpha + \theta_1' + \theta_2' - \theta_1 - \theta_2 - \theta_t) - V_2^2 \text{ (constant)}$$

For maximum power $\cos(\alpha + \theta_1' + \theta_2' - \theta_1 - \theta_2 - \theta_t) = 1$ and

$^{\alpha}B.\,D. = \theta_t - (\theta_1' + \theta_2' - \theta_1 - \theta_2)$

$\therefore\ ^{\alpha}B.\,D. + \gamma = \theta_t + 2(\theta_2 - \theta_2')$

and $^{\alpha'}B.\,D. = {}^{\alpha}B.\,D. + \gamma = \theta_t + 2(\theta_2 - \theta_2') \qquad$ (**30**)

Appendix E

Determination of the angular positions of the induced voltages of generator, synchronous motor and synchronous condenser when maximum power is delivered to the shaft of the motor and all excitations are held constant.

Fig. 27 represents a synchronous motor of impedance Z_2 and excitation voltage E_2 and a synchronous condenser of impedance Z_c and excitation voltage E_c having the same terminal voltage E_B. The total impedance from the neutral of the generator to this terminal is Z_1.

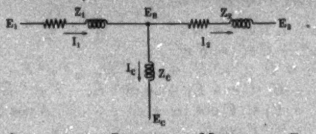

FIG. 27—SYNCHRONOUS GENERATOR, MOTOR AND CONDENSER

$$I_2 = \frac{E_B - E_2}{Z_2}$$

$$I_c = \frac{E_B - E_c}{Z_c}$$

$$I_1 = I_c + I_2 = \frac{E_1 - E_B}{Z_1} = \frac{E_B - E_2}{Z_2} + \frac{E_B - E_c}{Z_c}$$

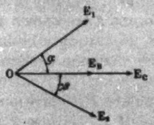

FIG. 28—PHASE RELATION OF EXCITATION VOLTAGES OF GENERATOR, MOTOR AND CONDENSER

$$\therefore E_B = \frac{E_1 Z_2 Z_c + E_2 Z_1 Z_c + E_c Z_1 Z_2}{Z_1 Z_2 + Z_1 Z_c + Z_2 Z_c}$$

$$I_2 = \frac{E_1 Z_c - E_2 (Z_1 + Z_c) + E_c Z_1}{Z_1 Z_2 + Z_1 Z_c + Z_2 Z_c}$$

$$I_c = \frac{E_1 Z_2 + E_2 Z_1 - E_c (Z_1 + Z_2)}{Z_1 Z_2 + Z_1 Z_c + Z_2 Z_c}$$

Let $P_c \quad$ = power on the shaft of the condenser.

$P_2 \quad$ = power on the shaft of the motor

$2 P_c = \bar{E}_c I_c + E_c \bar{I}_c = O$

$2 P_2 = \bar{E}_2 I_2 + E_2 \bar{I}_2$

Take E_c as reference vector, then

$E_c \quad = \bar{E}_c = V_c e^{jO} = V_c$

$E_1 \quad = V_1 e^{j\alpha}, \bar{E}_1 = V_1 e^{-j\alpha}$

$E_2 \quad = V_2 e^{-j\beta}, \bar{E}_2 = V_2 e^{j\beta} \qquad$ Fig. 28.

Let $$\frac{Z_1}{Z_1 Z_2 + Z_1 Z_c + Z_2 Z_c} = A\, e^{ja}$$

$$\frac{Z_c}{Z_1 Z_2 + Z_1 Z_c + Z_2 Z_c} = B\, e^{jb}$$

$$\frac{Z_2}{Z_1 Z_2 + Z_1 Z_c + Z_2 Z_c} = C\, e^{jc}$$

$$\begin{aligned}
&A\, e^{ja} + B\, e^{jb} = D\, e^{jd}\\
&A\, e^{ja} + C\, e^{jc} = F\, e^{jf}\\
&2\, P_c = \bar{E}_c\, (C\, e^{jc}\, E_1 + A\, e^{ja}\, E_2 - F\, e^{jf}\, E_c)\\
&\quad + E_c\, (C\, e^{-jc}\, \bar{E}_1 + A\, e^{-ja}\, \bar{E}_2 - F\, e^{-jf}\, \bar{E}_c)\\
&0 \;= V_1 V_c C \cos(\alpha + c) + V_2 V_c A \cos(\beta - a)\\
&\quad - V_c^2 F \cos f.
\end{aligned}$$

$$\therefore \cos(\alpha + c) = -\frac{V_2 A}{V_1 C} \cos(\beta - a) + \frac{V_c F}{V_1 C} \cos f \tag{31}$$

$$\begin{aligned}
2\, P_2 &= \bar{E}_2\, (B\, e^{jb}\, E_1 + A\, e^{ja}\, E_c - D\, e^{jd}\, E_2)\\
&\quad + E_2\, (B\, e^{-jb}\, \bar{E}_1 + A\, e^{-ja}\, \bar{E}_c - D\, e^{-jd}\, \bar{E}_2)\\
P_2 &= V_1 V_2 B \cos(\alpha + \beta + b) + V_2 C_c A \cos(\beta + a)\\
&\quad - D\, V_2^2 \cos d
\end{aligned} \tag{32}$$

Equation (**31**) gives α as an explicit function of β. If this value is substituted for α in equation (**32**) P_2 will be expressed in terms of β and known quantities. To find the value of β for which P_2 is a maximum it is necessary to differentiate this equation. This does not give a simple formula for β corresponding to maximum power, but by using equations (**31**) and (**32**) it is possible to determine whether a given system carrying a specified load is stable or unstable. For the specified load P_2, α and β are known. Since the excitations remain fixed, P_2 can only be increased by an increase in α and β. Assume a slightly larger value of $\alpha = \alpha'$. From equation (**31**) calculate β' and from equation (**32**) calculate P_2' corresponding to the new angles α' and β'. If P_2' is larger than P_2 then the system is stable under the load assumed. If P_2' is less than P_2 the system is unstable.

When the motor is a fictitious motor made by combining the impedances of the shunts at the receiver end in parallel with the impedance of the synchronous motor, it is necessary to determine whether the power given to the actual motor increases or decreases with an increase in α and β.

Neglecting resistance, equation (**31**) becomes

$$\cos(\alpha - 90\text{ deg.}) = -\frac{V_2 A}{V_1 C} \cos(\beta + 90\text{ deg.})$$

$$\sin \alpha = \frac{V_2 A}{V_1 C} \sin \beta$$

For a condenser at the center of a line connecting a generator and motor of equal impedances, $Z_1 = Z_2$, $A = C$ and $V_1 = V_2$.

$$\begin{aligned}
&\therefore \sin\alpha = \sin\beta \text{ and}\\
&\quad \alpha = \beta
\end{aligned}$$

$$P_2 = V_1^2 B \sin 2\beta + V_1 V_c A \sin \beta \tag{32}$$

$$\frac{d P_2}{d \beta} = 2\, V_1^2 B \cos 2\beta + V_1 V_c A \cos \beta.$$

for maximum power. $\dfrac{d P_2}{d \beta} = 0$ and

$$\cos\beta = -\frac{V_c A}{8\, V_1 B} \pm \sqrt{\left(\frac{V_c A}{8\, V_1 B}\right)^2 + \frac{1}{2}}$$

since $\dfrac{A}{B} = \dfrac{Z_1}{Z_c} = \dfrac{X_1}{X_c}$

$$\cos\beta = -\frac{V_c X_1}{8\, V_1 X_c} \pm \sqrt{\left(\frac{V_c X_1}{8\, V_1 X_c}\right)^2 + \frac{1}{2}} \tag{33}$$

Equations (**32**) and (**33**) may be used to obtain approximate values for the power transmitted over a line with a condenser at the center.

Z_c and E_c the synchronous impedance and excitation voltage of the condenser refer to the equivalent impedance and equivalent excitation voltage which can be obtained from the condenser characteristic at a given excitation.

Let E_c = actual excitation voltage
E_c' = equivalent excitation voltage
E_t = condenser terminal voltage
X_c = synchronous impedance at 100 per cent voltage and given excitation calculated from V curves.

$X_c' = \dfrac{d E_t}{d I_c}$ at 100 per cent voltage and given excitation.

When E_t is close to normal

$$\begin{aligned}
I_c &= \frac{E_c - 1}{X_c} + \frac{1 - E_t}{X_c'}\\
&= \left(\frac{E_c X_c' - X_c' + X_c}{X_c} - E_t\right) \frac{1}{X_c'}\\
&= \frac{E_c' - E_t}{X_c'}
\end{aligned}$$

$$\therefore E_c' = \frac{E_c X_c' - X_c' + X_c}{X_c} \text{ and } X_c' = \frac{d E_t}{d I_c}$$

Appendix F

The equivalent π may be obtained from the general circuit constants, or the general circuit constants may be obtained from the equivalent π.

Let A, B, C and D be the general circuit constants.

E_s = the sending-end voltage
E_r = the receiver-end voltage
Z_5 = the architrave impedance of the equivalent π
Y_3 = the admittance of the shunt at the generator end of the line.

Y_4 = the admittance of the shunt at the receiver end of the line.
I_3 = the current in the generator end shunt
I_4 = the current in the receiver end shunt
I_s = current at the sending end
I_r = current at the receiver end

Using equivalent circuit constants,

$$E_s = A\,E_r + B\,I_r \tag{34}$$

$$I_s = C\,E_r + D\,I_r \tag{35}$$

From the equivalent π,

$$\begin{aligned} E_s &= E_r + (I_r + I_4)\,Z_5 \\ &= E_r + (I_r + E_r\,Y_4)\,Z_5 \\ &= E_r\,(1 + Y_4\,Z_5) + Z_5\,I_r \qquad (36) \end{aligned}$$

$$\begin{aligned} I_s &= I_r + I_3 + I_4 \\ &= I_r + E_r\,Y_4 + E_s\,Y_3 \\ &= I_r + E_r\,Y_4 + Y_3\,E_r\,(1 + Y_4\,Z_5) + Y_3\,Z_5\,I_r \\ &= I_r\,(1 + Y_3\,Z_5) + E_r\,(Y_3 + Y_4 + Y_3\,Y_4\,Z_5) \qquad (37) \end{aligned}$$

Equating the coefficients in equations (34) and (36) and in equations (35) and (37),

$$\begin{aligned} A &= 1 + Y_4\,Z_5 \\ B &= Z_5 \\ C &= Y_3 + Y_4 + Y_3\,Y_4\,Z_5 \\ D &= 1 + Y_3\,Z_5 \\ Z_5 &= B \end{aligned}$$

$$Y_3 = \frac{D - 1}{B}$$

$$Y_4 = \frac{A - 1}{B}$$

Discussion

For discussion of this paper see page 80.

Practical Aspects of System Stability

BY ROY WILKINS[1]

Associate, A. I. E. E.

***Synopsis**—During the past few years there has been much discussion regarding the behavior of long transmission lines under transient conditions, such as flashovers, short circuits, arcs and grounds which would tend to make them unstable, but unfortunately this discussion has been largely theoretical due to the absence of any actual operating data upon which to base assumptions. It has only been recently that an opportunity has been afforded to make field tests on one of the two existing 220-kv. systems and the results of such a series of tests made on the system of the Pacific Gas and Electric Company are presented in the paper.*

This is the first instance where tests of this nature have been attempted and the lack of proper testing equipment proved a serious handicap. It was necessary to develop a special high-speed oscillograph wattmeter, a high-speed oscillograph filmholder and a pilot generator. Moreover the technique of testing was developed so that it was possible to secure oscillographic records 200 mi. apart by telephone signal.

The tests established the following important facts:

1. System stability as a problem is inextricably entangled with operating economics, and cannot be handled solely as a problem in design, except for very simple cases. 2. For any adequate conclusions to be reached much more fundamental data are necessary. 3. Requisite equipment for obtaining such data is not now available. 4. Studies of models and artificial transmission lines are not adequate because too little is known about the relative importance of the several factors to allow intelligent duplication. 5. Proper relay equipment and action is vital. 6. Oil-switch operation is an important factor. 7. Only a certain part of the stored energy of a system is available in any given case of trouble. 8. Operating distribution of excitation current is one of the major problems.

As transmission networks have grown in economic importance and kilowatt capacity, the problem of stable operation has assumed an increasingly greater importance, culminating in 1922-23 in studies of the proposed long distance transmission of large blocks of power to important market centers in the Atlantic States. In the consideration of these proposals a detailed study of system stability was considered essential. Preliminary theoretical analysis of the several problems encountered was presented at the Midwinter Convention of the Institute in 1924, and showed a decided lack of agreement on the methods of attack and the results obtained.

Opportunity was afforded to carry out tests on one of the two existing transmission systems having long 220-kv. transmission lines feeding a large load network; consequently a series of tests were carried out early in 1925 on the transmission system of the Pacific Gas and Electric Company to secure definite operating data on several of the points in question. The data accumulated on the test were an attempt to determine what was required on a network in order to predict its characteristics with respect to stability. It cannot be too strongly emphasized that data secured are for a specific condition on a given system and that such lack of agreement as evidenced in the calculations made in the past is due not so much to differences in mathematical treatment as to differences in assumptions made in regard to the relative importance of the several factors.

In the tests these factors are naturally included in their proper proportion and place, and any calculation

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made for the same conditions, to be at all adequate, must presuppose a thorough knowledge of all of the factors as well as of their relative importance. Artificial lines and miniature equipment cannot be substituted without such knowledge, for at best the results are only a reflection of the assumptions made. The difference between values in tests on an actual system and calculated or miniature equipment tests represents the numerical value of errors due to such assumptions.

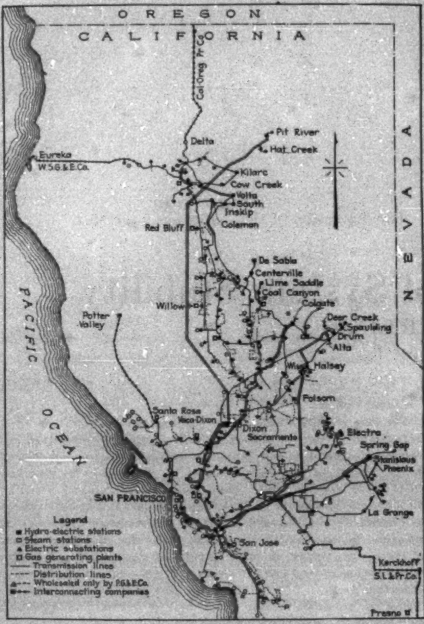
OREGON
CALIFORNIA
NEVADA
PACIFIC OCEAN
Cal.Oreg. Pr. Co.
Delta
Pit River
Hat Creek
Eureka
W.S.G.&E.Co.
Kilarc
Cow Creek
Volta
South
Inskip
Coleman
Red Bluff
De Sabla
Centerville
Lime Saddle
Coal Canyon
Colgate
Willow
Potter Valley
Deer Creek
Spaulding
Drum
Alta
Halsey
Folsom
Santa Rosa
Vaca-Dixon
Dixon
Sacramento
Electra
Spring Gap
Stanislaus
Phoenix
SAN FRANCISCO
La Grange
San Jose
Kerckhoff
S.L.&Pr.Co.
Fresno
Legend
Hydro-electric stations
Steam stations
Electric substations
Gas generating plants
Transmission lines
Distribution lines
Wholesaled only by P.G.&E.Co.
Interconnecting companies

Fig. 1—System Diagram, Pacific Gas and Electric Company

Given enough fundamental data on the network during trouble, the effect of such trouble may, with much patience and persistence, be calculated for very simple cases, but at least for the present the actual quantitative values for this fundamental data must be measured to insure any degree of accuracy in the final result. The tests as carried out are an attempt to evaluate at least a part of the unknown quantities, in order that the necessary assumptions for calculation may be at least reasonably correct.

Outline of Tests

The points on the system of the Pacific Gas and Electric Company selected for the tests were Pit No. 1 Power-House, the generator end of the two 202-mi., 220-kv. Pit transmission lines, and Vaca-Dixon Substation, the receiver end of these lines (see Fig. 1). Physical data on the system network at the time of the test are given in Table I.

TABLE I

Network Data at Time of Test

202 mi. of two-circuit 220-kv. line connected to three power-houses and one substation.
607 mi. of 110-kv. line connected to six power-houses and 10 substations.
2039 mi. of 60-kv. line, connected to 19 power-houses and 162 substations.
8434 mi. below 60-kv. line connected to 100 substations.
160 mi. of underground.
27—Hydro plants having 69 units and 317,975-kv-a. capacity.
4—Steam plants having 12 units and 142,000-kv-a. capacity.
2—20,000-kv-a. synchronous condensers.
2—12,500-kv-a. synchronous condensers.
1—7500-kv-a. synchronous condensers.

There were also, connected and operating in parallel, the Great Western Power Company, having

196 miles of 165-kv. line.
150 mi. of two circuit; 30 mi. of one circuit of 100-kv. line
520 mi.—below 60 and above 20-kv. line, with;
2—Hydro plants having nine units, 131,000 kv-a.
4—Steam plants having nine units 30,800 kv-a.
3—Synchronous Condensers having three units, 60,000 kv-a.

The California Oregon Power Company having 200 mi. of 66 kv.

513 mi. of 60 and 38 kv. with
10—Hydro plants having 16 units, 56,160 kv-a.

The Truckee River Power Company with five hydro plants and 8650 kv-a.

The Snow Mountain Power Company with

107 mi. of line below 60 kv. and above 30 kv.
1 Hydroelectric plant having 1 unit and 6400 kv-a. together with several smaller distributing networks to which the Pacific Gas and Electric Company wholesales power.

There was on the Pacific Gas and Electric Company system approximately 1,500,000 h. p. connected load, of which the average yearly distribution is:

35.0 per cent—Commercial and domestic lighting and heating.
13.5 per cent—Agricultural power (mostly centrifugal pumps).
3.0 per cent—Mining-power induction motors.
21.0 per cent—Manufacturing power.
8.0 per cent—Railway power.
19.5 per cent—Miscellaneous power.

The interconnected companies have roughly the same connected load per kv-a. generating capacity and the same general distribution of load.

For the Pacific Gas and Electric Company the monthly load factor in December is about 61 per cent, for August about 73 per cent, and the yearly load factor 62.5. The daily load factor is 65-79.

Pit No. 1 Power-House has installed two 35,000-kv-a., 3-phase, 60-cycle, 11,000-volt generators driven by 40,000-h. p. Francis turbines, each having a $W R^2$ of 7,365,000 lb-ft. On the same 11,000-volt bus is also connected the two Hat Creek plants, each having one 12,500-kv-a. turbine-driven generator, with 2,800,000 $W R^2$, 6600-volt, 3-phase, 60-cycle connected through 6600- to 60,000-volt transformer banks and approximately five mi. of double-circuit tower lines to a step-down bank 60,000/1100 at Pit No. 1.

Connecting Pit No. 1 with Vaca Substation is a 202-

mi. double-circuit 220-kv. transmission line,[2] through two 11,000- to 220,000-volt delta Y-connected 50,000-kv-a. transformer banks at Pit No. 1 and through two 200,000/110,000/10,460-volt 50,000-kv-a. banks at Vaca Substation. The Vaca transformer banks are Y-connected auto-transformers with a delta-connected tertiary from each of which is operated a 20,000-kv-a., 11,000-volt synchronous condenser. At Vaca the power enters the 110-kv. network as shown in Fig. 1. At the time of test, one line was operating at 220 kv. and one at 125 kv., one bank at Pit No. 1 being temporarily reconnected and the line going to the 110-kv. terminal of the bank at Vaca instead of the 220-kv.; the 110-kv. is designated as No. 1 and the 220-kv. as No. 2.

The following is an outline of the schedule of tests.

I. *a.* Start condenser at Vaca.
  b. Trip one condenser at Vaca.
II. Run both generators at Pit, one leading and one lagging.
  a. Trip one.
III. Running both generators at Pit.
  Trip one at ¼ load.
  “ “ “ ½ “
  “ “ “ ¾ “
IV. Line switching at Pit.
  a. Switch out No. 2 line at Pit at 220 kv.
  b. “ “ “ “ “ “ Vaca at 220 kv.
  Vary and repeat.
V. Ground No. 2 line through fuse first tower out from Vaca bus.
  Repeat.

Test Equipment

A description of the equipment used in making the tests is given in Table II. The six-element oscillograph used at Pit No. 1 was a Westinghouse portable permanent magnet type equipped with a special film holder using 6½-in. films 24 ft. long and running up to speeds of about 1 in. per cycle. The 3-element was also a Westinghouse portable permanent magnet type with a special holder, handling films 3¼ in. by 15 ft. at speeds

TABLE II

Test Equipment

Pit River No. 1 Power-House:
One 6-Element Oscillograph
One 3-Element Oscillograph
One 3-Element, 3-phase, oscillographic type wattmeter
One Motor-Generator Set for measuring absolute change in phase position
One Generator for showing rotor phase angle position
One Mechanical recording device to show governor action
Vaca-Dixon Substation:
One 3-Element Oscillograph
Two Recording Wattmeters
One Motor-Generator same as that used at Pit No. 1
Claremont Substation:
One Esterline Wattmeter

Measuring the Following Quantities

Line Switching—Angular Relations

Pit No. 1 Power-House:
Six-Element Oscillograph
One—Pilot Governor
Two—Motor Generator Set Voltage
Three—Generator Internal Voltage
Four—Vaca Voltage
Same zero line
Five—Generator Current
Six—No. 1 Line Current
Same zero line
Three-Element Oscillograph
One—Hat Creek Current
Two—Generator Terminal Voltage
Three—No. 2 Line Current

High Speed Wattmeter

1—Generator Power
2—Hat Creek Power
3—Line No. 1 Power

Esterline Wattmeter

1—Line No. 2 Power
Vaca Substation:
Three—Element Oscillograph
One—100-kv. Bus Voltage
Two—Condenser Current on Line No. 1
Three—Single-Phase Wattmeter-No. 1 Condenser Power
Four—Differential 100-kv. Bus—Motor Generator Voltage

Esterline Wattmeter

One—Power Condenser on Line No. 2
Two—Drum line
Claremont Substation:
Esterline Wattmeter

Line Switching-Flux Relations

Pit No. 1 Power-House:
Six—Element Oscillograph
One—Pilot Generator
Two—Generator Terminal Voltage
Three—Generator Internal Voltage
Four—Generator Current
Same zero line
Five—Field Current
Six—Field Voltage
Same zero line
Three—Element Oscillograph
One—Motor Generator Voltage
Two—Vaca Voltage
Three—Generator Internal Voltage

High Speed Wattmeter

One—Generator Power
Two—Hat Creek Power
Three—Line No. 1 Power

Esterline Wattmeter

One—Line No. 2 Power
Three—Element Oscillograph

One—100-kv. Bus Voltage
Two—Condenser Current on Line No. 1
Three—Single-Phase Wattmeter—No. 1 Condenser Power
Four—Differential 100-kv. Bus—M. G. Set

Esterline Wattmeters

One—Power Condenser on Line No. 2
Two—Drum Line
Claremont Substation:

Esterline Wattmeter

Single Phase—Short Circuit

Pit No. 1 Power-House:
Six-Element Oscillograph

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2. See Transactions A. I. E. E., 1924. page 1148, Corona Loss Test.

One—Pilot Generator Voltage Two—Motor Generator Voltage Three—Generator Internal Voltage	Same zero line
Four—Generator Current Five—No. 1 Line Current Six—Hat Creek Current	Same zero line

Three-Element Oscillograph

One—Residual Current
Two—Generator Terminal Voltage
Three—No. 2 Line Current

High Speed Wattmeter

One—Generator Power
Two—Hat Creek Power
Three—No. 1 Line Power

Esterline Wattmeter

One—No. 2 Line Power

Vacu Substation:

Three-Element Oscillograph

One—Residual Current—3—220-kv. Bushing Transformer
Two—Positive sequence voltage
Three—Single-phase wattmeter—el No. 1 and el No. 4
Four—Residual Voltage

Esterline Wattmeters

One—Power of Condenser on Line No. 2
Two—Power on Drum Line

Claremont Substation:

Esterline Wattmeter

up to 1 in. per cycle. The oscillograph wattmeter was a moving coil instrument having three single-phase elements operating one mirror with straight line characteristics and a natural period of about $^1/_{20}$ second, developed by the Westinghouse Company for the test, see Fig. 2, calibration Fig. 3.

There were three of these 3-phase meters in one case, all working on the same film. Timing was accomplished by interrupting the beam of light from one meter for a short time every $^1/_{10}$ second by means of a small synchronous motor, giving indications shown as small gaps in the record of one meter on the film. The films were 3¼ in. by 55 in. and had a speed up to 15 in. per second. For measuring the absolute change in phase position a small motor-generator set comprised of a d-c. motor and an alternator with a fly-wheel was run from the station storage battery. There was

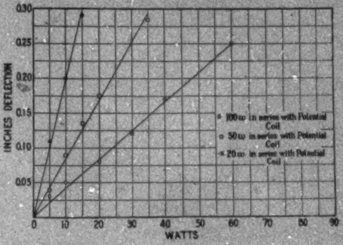

FIG. 3—CALIBRATION OF SAME (SINGLE-PHASE)

FIG. 2—POLYPHASE ELEMENT OF OSCILLOGRAPHIC WATTMETER

FIG. 4—PILOT GOVERNOR

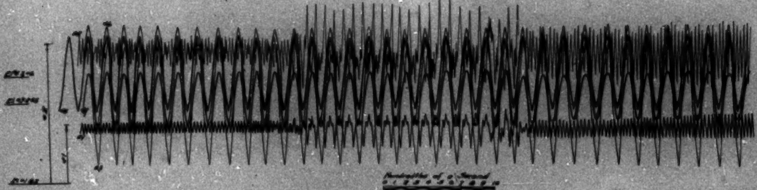

FIG. 5—SECTION OF 24-FT. FILM OF A 6-ELEMENT OSCILLOGRAPH

Stability tests at Pit River Power-House No. 1, June 18, 1925, 11:40 p.m.

El. No. 1—Field Voltage	El. No. 4—Rotor Voltage
No. 2—Field Current	No. 5—Gen. Terminal Voltage
No. 3—Generator Current	No. 6—Gen. Internal Voltage

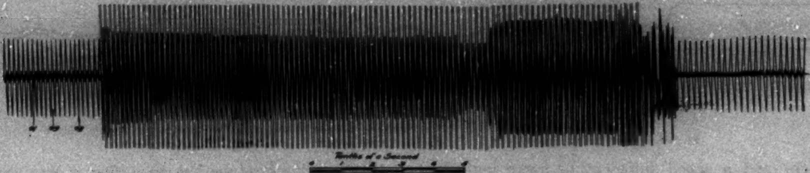

FIG. 6—SECTION OF 15-FT. FILM OF A 3-ELEMENT OSCILLOGRAPH AT PIT NO. 1 DURING FLASHOVER
Stability Tests at Pit River Power-House No. 1, June 18, 1925
El. No. 1—Residual Current No. 2—Gen. Voltage No. 3, No. 2 Line Current, $B\phi$

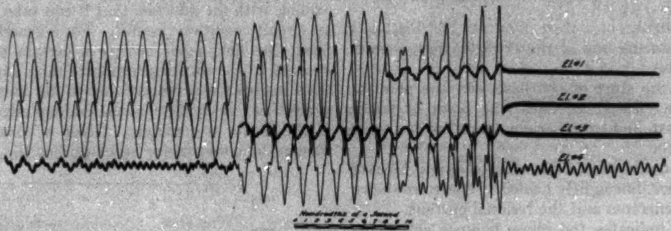

FIG. 7—FILM OF OIL-CIRCUIT BREAKER ACTION
Stability Tests at Pit River Power-House No. 1, June 18, 1925, 10:35 p. m.
El. No. 1—No. 2 Line Current $C\phi$ El. No. 3, No. 2 Line Current, $A\phi$
No. 2—No. 2 Line Current $B\phi$ No. 4 Residual Current

mounted on the generator, with its rotor mounted on the end of the generator shaft, a remodeled "Ford" magneto having the same number of poles as the main generator, (see Fig. 4). This magneto, having no load, gave a true record of the field or rotor position at all times and its wave is referred to as the pilot generator voltage. There was also installed a mechanical recording device on the turbine governor giving governor travel.

The 3-element oscillograph used was a Westinghouse electromagnet portable oscillograph, having in addition to the three elements, a single-phase wattmeter using the fourth prism and the same film. The film was 3¼ in. by 55 in. and was run at speeds up to 15 in. per second. There were also two special polyphase graphic wattmeters using high speed charts driven by synchronous motors. All of the data at Vaca was taken by telephone signal from Pit No. 1 as was also the switching. This was satisfactory enough that the two sets of equipment 200 mi. apart were able to catch each test on the standard film within one-quarter of its travel.

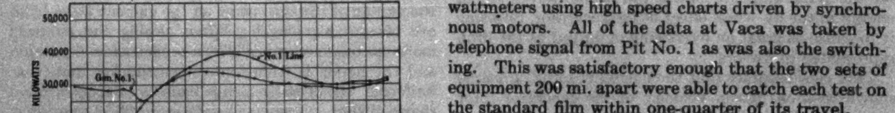

FIG. 8—OSCILLOGRAPHIC WATTMETER RECORD OF OPENING ONE LINE
Stability Test at Pit River Power-House No. 1, June 17, 1925, 11:40 p. m.
El. No. 1—No. Gen. Power No. 2—Hat Creek Power No. 3—No. 1 Line Power

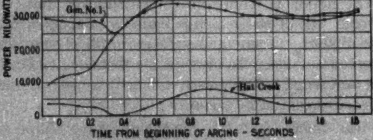

FIG. 9—FIG. 8 PLOTTED TO A COMMON SCALE FOR ALL ELEMENTS

CHARACTER OF DATA

It was necessary that the preliminary data be taken comparatively cheaply and without undue disturbance to normal operation, that the oscillographic records have a length in time sufficient to cover all of the

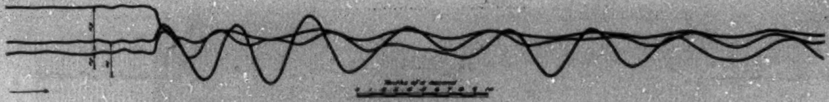

FIG. 10—OSCILLOGRAPHIC WATTMETER RECORD OF CLOSING ONE LINE
Stability Tests at Pit River Power-House No. 1, June 18, 1925, 12:12 a. m.
El. No. 1—No. 1 Gen. Power No. 2—Hat Creek Power No. 3—No. 1 Line Power

transient and a speed sufficient to make individual cycles available for angular measurement.

Fig. 5 represents that portion of a 24-ft. film during which switching took place, for opening a 220-kv. line at Pit No. 1.

Fig. 6 is a portion of a 15-ft. film taken at Pit No. 1 Power-House during one of the artificial flashovers at Vaca Substation.

This shows the start of the flashover, the point at which the line cleared at Vaca, the point at which Pit No. 1 cleared, and on the original film 7 or 8 seconds record thereafter.

Fig. 7 shows a portion of a film of an oil circuit breaker opening a 220-kv. line lightly loaded. There are shown the three line currents and the residual current.

This record indicates the phase balance action between phases when opening a polyphase circuit; the current ruptured by the last circuit breaker shows nearly four times the original phase current.

Fig. 8 is the record by the oscillographic wattmeter of switching out the 220-kv. line at Pit No. 1 carrying 24,000 kw., thereby increasing the load on the 110-kv. line from 12,000 to 35,000 kw. The generator instead of dropping from 32,000 kw. to 12,000 kw. drops only about 3000 kw., and that only after $^3/_{10}$ second.

Fig. 9 shows these wattmeter records plotted on a common scale.

Fig. 10 shows the reverse process, *i. e.*, closing the 220-kv. line at Pit No. 1 and picking up load from the 110-kv. line. This action is much more severe, takes place faster and causes more outside disturbance than the opening of a line.

Fig. 11 shows this plotted to a common scale.

Previous articles[3] on this subject have considered that when one of two parallel lines was opened, the output of the station feeding them dropped instantly to the load on the remaining line while the prime mover stored energy in the fly-wheel effect of the generator, releasing it to cause as great an overswing of output as the drop represented in kw-hrs.

This view assumes that the energy absorbed must be shown in a change in velocity of the generator rotor caused mechanically by the prime mover, and that there is no damping action. The curves demonstrate that such is not the case and also that closing a line more nearly approximates it than opening one.

In the writer's opinion the same effect is obtained by changing the power-factor of the generator load because this gives the same shift of the generator rotor and rotating armature field as a change in rotor velocity would, with the addition that it can take place as fast as circuit conditions permit. For a generator carrying load, the field poles occupy a certain position with respect to the rotating armature flux, any increase in load causes the armature flux to lag behind the field poles, *i. e.*, to come from the trailing edge of the pole if the power is constant. Given a constant load, a change in power-factor accomplishes the same result.

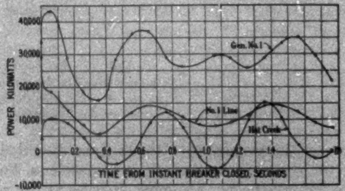

FIG. 11—FIG. 10 PLOTTED TO A COMMON SCALE FOR ALL ELEMENTS

With such a pilot generator as used at Pit No. 1, these changes may be observed on an oscillograph. Any such shift in armature flux position absorbs from or delivers energy to the field structure rotating as a fly-wheel and it must take place before there is any tendency to change the speed of the prime movers.

The rate at which energy can be absorbed depends on the electrical characteristics of the circuit involving a great number of variables among which is oil-circuit breaker action. No circuit breaker at present in use interrupts a circuit instantly, neither do they interrupt all three phases simultaneously because they interrupt the circuit at the zero point on the current wave of an arc of variable length. When closing the contacts make contact mechanically at a predetermined position and the action is, therefore, much quicker and more uniform. All of these things cause a change in wave shape in both current and voltage in all of the interconnected circuits. In most of the tests described, the speed and length of film were sufficient to permit angular measurement to be taken.

It was found that the change in wave shape was sufficient to materially affect the results.

On one of the tests, shown by Fig. 12, the several

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[3] See J. P. Jollyman, JOURN. A. I. E. E., Sept., 1925, p. 950, Fig. 4. C. L. Fortescue, JOUR. A. I. E. E., Sept., 1925, p. 955, Fig. 4.

TABLE III

HARMONICS PRESENT DURING ARTIFICIAL FLASHOVER AT VACA

(See Figs. 12 to 17)

Wave	Per Cent Harmonics	Angle from Actual Wave
Terminal voltage before transient	99.8 1st	−1.2
	3.5 3rd	12.0
	5.1 5th	7.9
Terminal voltage 0.3 sec. after transient	99.4 1st	1.0
	7.9 3rd	18.6
	6.9 5th	−3.8
Terminal voltage 0.95 sec. after transient	99.2 1st	−1.5
	10.4 3rd	20.9
	7.4 5th	−7.1
Terminal voltage 1.2 sec. after transient	99.3 1st	0
	9.4 3rd	16.6
	7.6 5th	−4.7
Generator current before transient	100 1st	+2.8
	.5 3rd	−12.5
	.4 5th	38.5
Hat Creek current before transient	99.6 1st	−2.8
	3.8 3rd	35.9
	7.9 5th	4.0
Hat Creek current 0.40 sec. after transient	98.1 1st	−11.9
	4.8 3rd	27.4
	18.8 5th	18.9
Hat Creek current 0.7 sec. after transient	99.6 1st	7.1
	5.3 3rd	−27.6
	7.1 5th	−3.0
Hat Creek current 1.3 sec. atter transient	99.6 1st	−3.2
	1.7 3rd	22.8
	8.2 5th	27.2

waves have been analyzed for fundamental, third and fifth harmonics and the angular position of the fundamental with respect to the actual wave determined.

Fig. 13 gives a polar diagram of the generator terminal voltage before the transient; Fig. 14, the same wave $^{7}/_{10}$ second after the transient.

Fig. 15, Hat Creek current before the transient. Fig. 16, $^{7}/_{11}$ second after and Fig. 17 1 $^{3}/_{10}$ seconds after the transient.

Table III, by S. B. Griscom of Westinghouse Electric and Manufacturing Company, gives the numerical values of these harmonics and angles in this particular test.

Fig. 18 shows an oscillographic wattmeter record of an artificial flashover on the 220-kv. line under normal operating conditions caused by closing an air switch; Fig. 19, on a string of insulators over which a 10-ampere fuse had been placed.

This gave an arc from one phase to ground over an insulator string cleared by relays in the normal manner under actual operating conditions.

Oscillograms of current, voltage, etc., were taken both at Pit No. 1 Power-House and Vaca during such a flashover. Fig. 20 and Fig. 21 are illustrations of such an arc.

At Vaca, in addition to the oscillographic record of current and voltage, there was a single-phase wattmeter record of the residual power, *i. e.*, voltage across the corner of an open delta and the common return of the three current transformers connected in Y.

Fig. 22 shows the record for two such flashovers.

Too strong emphasis cannot be placed on the state-

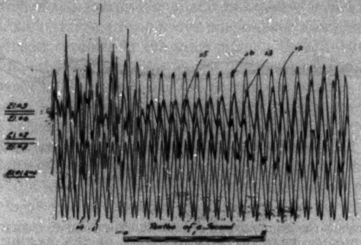


FIG. 12—SECTION OF 24-FT. FILM OF A 6-ELEMENT OSCILLOGRAPH AT PIT NO. 1 FOR ARTIFICIAL FLASHOVER AT VACA

Stability Tests at Pit River House No. 1, June 19, 1925, 1:20 a. m.

El. No. 1—M. G. Set Voltage
No. 2—No. 1 Line Current
No. 3—Generator Current
El. No. 4—Rotor Voltage
No. 5—Vaca Voltage
No. 6—Gen. Int. Voltage

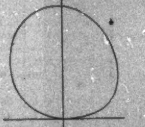


FIG. 13—POLAR DIAGRAM OF GENERATOR TERMINAL VOLTAGE BEFORE TRANSIENT

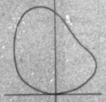


FIG. 14—THE SAME AS FIG. 13 BUT 7/10 SECOND AFTER THE TRANSIENT

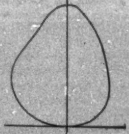


FIG. 15—POLAR DIAGRAM OF HAT CREEK CURRENT BEFORE TRANSIENT

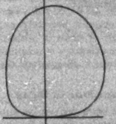


FIG. 16—SAME AS FIG 15 BUT 7/10 SECOND AFTER THE TRANSIENT

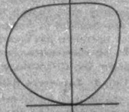


FIG. 17—SAME AS FIG. 15 BUT 1 3/10 SECONDS AFTER THE TRANSIENT

ment that any such data are for a network in operating condition, and that there are a great number of unknown variables involved, the determination of any one of which is a problem of major importance, worthy of a complete discussion in itself.

The stability problem is inextricably enmeshed with system operating economies and in the greater number of cases the limitations are as much operating difficulties as engineering design. It is not possible to economically build and operate a complete superpower network for ideal conditions at the present time. Such networks grow naturally from the demand for more and better power service, and are made up of interconnected existing networks too valuable to junk outright. They must be adapted to operate together.

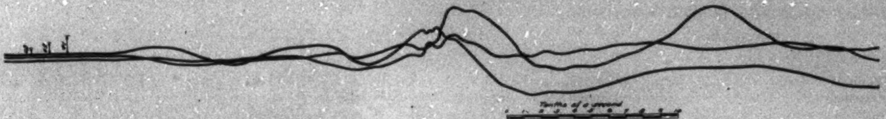


FIG. 18—AN OSCILLOGRAPHIC WATTMETER RECORD OF AN ARTIFICIAL FLASHOVER ON THE 220-KV. LINE
Stability Tests at Pit River Power-House No. 1, June 19, 1925, 2:15 a. m
El. No. 1—No. 1 Gen. Power No. 2—Hat Creek Power No. 3—No. 1 Line Power

TABLE IV
ANALYSIS OF RELAY OPERATION ON THE 220-KV., 110-KV., AND 60-KV. TRANSMISSION NETWORK OF THE PACIFIC GAS AND ELECTRIC COMPANY FOR 1923 AND 1924

1		2		3		4		5		6	
Total operations involving relays		Correct relay operations		Relay failures		Incorrect relay operations				Questionable relay operations	
						Due to relay		Due to external causes			
Year	No.	No.	Per Cent	No.	Per Cent	No.	Per Cent	No.	Per Cent	No.	Per Cent
1923	1,593	1,490	93.6	24	1.5	11	.7	40	2.5	28	1.7
1924	1,884	1,699	90.2	31	1.6	3	.2	108	5.7	43	2.3

Col. 1—Relay Failures (No Operation of Switch)

Under this heading are listed all cases where subsequent investigation has proved that conditions at the point in question were such as to permit correct relay operation but the relays failed to function. No case is listed as a relay failure if investigation proved the trouble to be due to a break, ground or other fault in the direct-current tripping circuit, or to mechanical defect in the switch or auxiliary apparatus external to the relay itself.

Col. 2—Total Relay Operations

This tabulation includes all cases of relay operation causing the opening of oil switches and is a summation of Cols. 3, 4 and 5. (Note—Cases of relay operation due to accidental shorting or closing of contacts are not included in this report).

Col. 3—Correct Relay Operations

Under this heading are listed all relay operations which have been proved to be correct under the conditions existing at the time. This tabulation includes many cases which were not entirely satisfactory from an operating standpoint, but where, owing to load, voltage and power-factor conditions, it was necessary to give the relays credit for correct operation, even though the results were not precisely as desired. There are also included in this table a number of cases in which unsatisfactory relay operation has occurred due to inadequate equipment. (See Col. 4.) In all these cases, however, the relays themselves functioned correctly under the circumstances.

Col. 4—Incorrect Relay Operations Due to Relay

This tabulation includes all cases that have been proved by subsequent investigation and test to be due to faults, either mechanical or electrical, in the relays themselves. (Note—This classification does not include relay operations which were apparently faulty because of inadequate equipment or improper connections.)

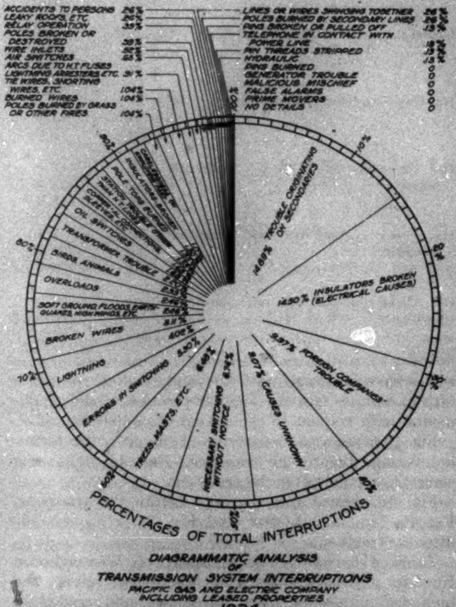
Col. 4—Incorrect Relay Operations Due to External Causes

Under this classification are listed all cases of relay operation which investigation has shown to be the result of faults or improper connections in auxiliary apparatus external to the relays themselves.

Col. 3—Questionable Relay Operations

This table covers relay operations which it has been found impossible to classify under Col. 3 or 4. In most cases the operation has been unsatisfactory, but owing to the lack of evidence it was considered unfair to classify it as incorrect, particularly as a test made after the trouble showed the relays to be operating correctly so far as could be determined.

TABLE V
ANALYSIS OF SYSTEM TROUBLES, PACIFIC GAS AND ELECTRIC COMPANY



NECESSARY CONSIDERATIONS

Trouble, Relays and Switching. On any major line in a network similar to the one on which these tests were made, it is *absolutely essential* to clear arcs quickly; so quickly in fact that neither governors nor automatic voltage regulators of the present type can act sufficiently to make any material change before the line is cleared. With proper relay action a line can be cleared of an arc over an insulator string so promptly that it has been almost impossible to locate the point of trouble. If an arc is left unchecked for anything like the time required for the network to reach a stable condition, the wire in trouble is melted or sufficiently annealed to cause it to part mechanically. Arcs on high tension lines generally strip the top and bottom units of an insulator string first and then cause a burn sufficient to drop a conductor. On any line of 220 kv. the conductor size and stringing tension make it necessary to have heavy equipment, usually gasoline-driven trucks, to handle any work done on the conductors. For this reason it is necessary to clear positively and quickly any trouble. Failure to do this means not a momentary interruption but hours, or more probably days, until the necessary heavy equipment can be assembled at the point of trouble for line repairs.

Experience has shown that the only practical procedure is to clear the smallest practical section of the network around the trouble in the shortest possible time.

In Table IV is given the high-tension relay operations for 1923 and 1924, and in Chart V is given the segregation of all high-tension troubles for the year 1924.

Of manual switching for 182,500 switching operations on the Pacific Gas and Electric Company system in a year, 23 caused trouble.

The greater portion of the high voltage troubles are phase-to-ground arcs caused by conditions fairly well known and to a considerable extent avoidable.

Insulation in any reasonable amount is not a positive insurance against direct strokes of lightning, severe storms or certain birds and no amount is proof against occasional man-made failures.

STORED ENERGY AND GOVERNING

The stored energy or fly-wheel effect of the rotating equipment on the network varies from 2350 ft-lb. per kv-a. in the larger hydroelectric units to 1195 ft-lb. per kv-a. in some of the smaller high speed units and typical steam turbo alternators. Typical steam turbo alternators have about 5500 ft-lb. per kv-a.

FIG. 19—AIR-SWITCH ARRANGEMENT FOR CLOSING A GROUND ON THE 220-KV. LINE

The effect of the rotating equipment connection as load is very difficult to determine but it was in the order of 20 per cent of the effect of the generating equipment on the network.

At any given point on the network only a certain portion of this energy is available in a case of switching or trouble, varying for 3-phase, single-phase and phase-to-ground troubles in the same manner that the short circuit kv-a. varies as far as the distribution over the network is concerned, but the energy delivered is dependent on the rate of change of speed and also on the line and equipment characteristics.

Experience has demonstrated that for a load of 300,000 kw., a drop from 60 cycles to 59 cycles will drop the load about 3.5 per cent, while an increase of from 60 to 61 cycles increases the load about 4 per cent. This is due to the character of the load, some of which varies with the cube of the speed and some of which has no variation with speed. The actual percentages vary

FIG. 20—PHOTOGRAPH OF A 220-KV. ARC DURING THE ARTIFICIAL FLASHOVER

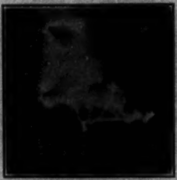


FIG. 21—SAME AS FIG. 20 FROM A DIFFERENT DIRECTION

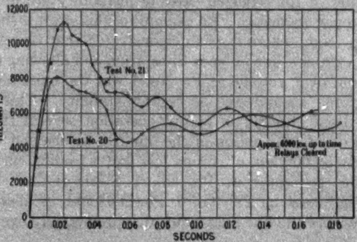


FIG. 22—RESIDUAL POWER IN ARTIFICIAL FLASHOVER

slightly therefrom with the season. The network is somewhat self-governing and the actual governing is done on one unit in a designated plant with all the remaining plants carrying block loads.

There are two methods of accomplishing this result:

1. Setting all governors on a flat speed characteristic to reject load at approximately ½ cycle above normal speed with a load limit set for a block load on all but the one governor used for regulation on turbines or a given nozzle opening on impulse wheels. Then as the speed varies too near the limit of the governing unit add to or reduce the block load to keep the governing unit in action.

2. Set all governors with a drooping speed characteristic and change the governor setting as load increases or decreases in the manner of the synchronizing motor on a steam turbine.

The Pacific Gas and Electric Company network uses the first on hydro units and the second on steam, although various combinations have been used in the past.

On the network described above with the present system of relaying,[4] the speed does not drop enough during switching or trouble for the governors to act until after the switching is over. This is true for several reasons:

First, it is not possible to deliver sufficient kv-a. at any given point in the network to drop the speed fast enough to get governor action during the time it takes to clear trouble.

Second, governors, particularly on hydro units, have an operating time determined by the equipment controlled, *i. e.*, turbines, relief valves and penstocks, and this is of necessity a matter of several seconds.

All governors in general use today are used for holding speed and nothing else, and are, therefore, of necessity actuated only by speed. Particularly on hydroelectric units not having a means of by-passing the unit, the governors are definitely limited in action by penstock pressure rise, and are, therefore, a compromise between the cost of generators to stand overspeed and overvoltage and penstocks to stand over-pressure.

CHARGING CURRENT

Charging current on high-voltage lines plays an important part in system stability, and a considerable portion of the reported cases of instability are in fact improper operation. An example of this effect is shown in the tests where it was demonstrated that switching out a 220-kv. line at Vaca Substation and allowing it to trip on over voltage at Pit No. 1 was the most severe condition encountered as regards stability. This has also been checked in actual operation several times. It is due to the action of the line free at one end with nearly 30,000 kw. of corona loss and a leading current in the generator at Pit No. 1 Power-House. The voltage rises at Pit No. 1 Power-House in about 5 cycles and the line relays out in approximately over ¾ second under such conditions. In this connection must be considered the excitation characteristics of the generators and the manner and direction from which the charging current is supplied, *i. e.*, either from the system network and receiving-end condensers or from the generators themselves.

a. Corona. Corona load on long 220-kv. lines under certain switching conditions may be several thousand kilowatts and must be considered in stability problems.

b. Phase Balancing. On all 220-kv. switches so far installed and operated the difference in opening time of the three phases varies from 5 to 20 cycles and allows an interchange of power between phases as well as a damping action in the arc.

c. Impedance. Intimately connected with all network disturbances is the system impedance to the point in trouble. This, to be accurate, must include all the connected equipment with its proper characteristics, in most cases transient characteristics, because present-day operating practise developed by trial has demonstrated that the relay equipment must be able to separate equipment and lines in trouble in a time measured in seconds or parts of a second, or the damage done will permanently disable such line or equipment.

Very little is known about transient impedance values, particularly single-phase or phase-to-ground impedances. For most networks such data are not available nor has any practical method been developed for obtaining it.

The oscillographic wattmeter used on the flashover tests in this connection is believed to be an innovation in this respect.

All changes of load caused either by switching or grounds and short circuits are taken up in the network by a transient condition causing a change in speed followed by a readjusted steady state either at a new speed or at the same speed and new loads on one or more units.

The amount of speed change is dependent on the rate at which the stored energy can be absorbed at the point of trouble, and this in turn is dependent on the impedance which is varied by all of those things that affect the short-circuit problem, together with the character of the trouble, the excitation on the rotating equipment, charging current, corona and those things which go to make up the speed load characteristics of the network.

CONCLUSION

These tests represent the initial attempt to secure data of this character on a large network in actual operation. The manner of using such data and the results obtained therefrom are given in the companion paper by Evans and Wagner who were instrumental in furnishing both personnel and equipment for the tests thus far made.

Discussion

For discussion of this paper see p. 80.

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[4] See *Electrical World*, November 22, 1924.

Studies of Transmission Stability

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Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis:—*Stability may be defined as the capacity of a power system to remain in equilibrium under steady load conditions, and its ability to regain a state of equilibrium after a disturbance has taken place. The lack of stability first manifested itself in the cases of overloaded machines and high impedance tie lines. The transmission of large blocks of power over long distances has presented the problem in a new form. Attention was directed to this problem in a group of papers before the Institute at the Midwinter Convention of 1924.*

These papers gave a general discussion of the stability problem and pointed out the necessity of considering the limitations imposed not only by the line alone but by the transformers, rotating machines and load. Extensive and pertinent discussions followed which emphasized the importance of the limitations imposed on power transmission by stability conditions.

The papers and discussions at the 1924 Midwinter Convention established a method for the determination of power limits under steady load conditions assuming fixed excitation. The limit so determined is due to the inherent characteristics of machines and does not take into account the possibility of changes in excitation due to the action of voltage regulators. The possibility of exceeding the "inherent stability limits" by the operation of the voltage regulators and exciters was pointed out. This condition of "artificial stability" was not at that time believed to be attainable. It was recognized that under the actual operating conditions on a transmission system instability would occur because of short circuits or other disturbances at a point considerably below the maximum static limit.

Subsequently extensive studies of stability conditions were made to determine the feasibility and economics of a number of large transmission projects. These studies emphasized the necessity of determining the maximum permissible load under the most severe operating conditions which obviously arise at the time of system disturbances, such as switching operations or flashovers with the attendant switching.

Transmission stability has been the subject of a number of articles in the technical press and of papers before the Institute, the principal ones of which are listed in the bibliography. C. L. Fortescue's paper† before the Seattle Convention in September 1925 serves as an introduction to the present paper, presenting in a qualitative manner results of recent investigations, whereas this paper presents methods for the quantitative determination of system oscillations.

During the early part of 1925 extensive stability tests including switching operations and single-phase faults to ground were conducted on the system of the Pacific Gas and Electric Company. These tests will be described in a companion paper by Roy Wilkins.‡

The present paper first deals with the principal elements entering into the stability problem, such as the action of generators and exciters during disturbances, effect of dissymmetry produced by single-phase short circuits, simplification of the load end network and methods for combining these various factors in the determination of the electromechanical oscillations of the system following major disturbances. Results of calculations by these methods are compared with the results of tests on the system of the Pacific Gas and Electric Company. The paper concludes with a discussion of various methods of improving stability.

INTRODUCTION

FUTURE power developments in this country and in Canada will involve the transmission of larger blocks of power over greater distances than hitherto have been realized in actual practise. Attention is being directed to the possibilities of hydroelectric developments located remote from load centers. In order to compete, on an economical basis, with high efficiency steam plants located near load centers, it is necessary to transmit large amounts of power per circuit. However, electrical considerations show that the power limits per line closely approach the limits determined by economical considerations. For these reasons the stability of transmission systems is particularly important.

The present investigation of the stability problem has for its object the determination of power limits for a transmission system under the various conditions that arise in actual operation. The results which may be expected of such an investigation include (1) the determination of the proper basis of design of machines and control apparatus for future power developments involving long transmission lines, (2) the development of methods of analysis and testing equipment to determine the performance of existing systems, (3) the improvement of operating methods with a view of reducing the effects of disturbances. This should follow naturally as a result of the better understanding of what takes place during and following a disturbance.

Stability may be defined as the capacity of a power system to remain in equilibrium under steady load conditions and its ability to regain a state of equilibrium after a disturbance has taken place. The first part of this definition is referred to as "static stability" and the second part as "transient stability." It should be noted that after a disturbance, the system will not necessarily seek the original state of equilibrium.

The maintenance of stability on a transmission system is obviously of the utmost importance since a line deficient in stability is inoperative. Normal loads on a system must, of course, lie well below the static limit. It is recognized that stability cannot reasonably be expected under all abnormal operating conditions. However, provision should be made for the more frequently occurring conditions including normal switching operations, reasonable increase of load and single-phase line to ground faults.

Reference should be made to the paper by C. L. Fortescue,* presented at the Seattle Convention in

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* Both of the Westinghouse Electric & Mfg. Company.

† Bibliography, Item 1.

‡ Bibliography, Item 2.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

*Bibliography, Item 2.

1925; this gives a general discussion of static stability and presents a picture of the phenomena taking place during transients. Mr. Fortescue's paper serves as an introduction to the present paper, as it presents in a qualitative manner results of recent investigations whereas this paper presents methods for the quantitative determination of system oscillations.

Static and Transient Stability. In previous papers on transmission stability the subject of "static" stability has received most consideration. Approximate methods for the determination of the static limit have been developed on the assumption of constant field current in synchronous machines.† These methods do not take into account the increase in the limit which may arise from the action of regulators, exciters, inertia effects, governors, etc. The actual limit obtained with these factors taken into account will be in excess of that shown by the above mentioned approximate methods. It was recognized that the permissible load on a power system should be appreciably below the actual static limit in order to prevent loss of synchronism at times of system disturbances.

There has been some discussion as to the relative importance of the transient and static stability limits of transmission systems and on this account it seems desirable to review briefly their essential points. It is, of course, impossible to operate a system with a load in excess of the static limit. In investigating a disturbance on a power system at least two circuit conditions must be considered; the original condition before the disturbance and the condition during the disturbance; and in addition a third circuit condition, in case the disturbance is cleared. The static limit corresponding to each circuit condition would perhaps not have been exceeded had the changes taken place at a sufficiently slow rate to permit the system to adjust itself. During the disturbance the mechanical inertia comes into play causing the system to overshoot the new point of equilibrium to such an extent as to produce instability. It follows, therefore, that the transient limit which takes into account these mechanical oscillations has a lower value than the static limit and is therefore of greater practical importance to the operating engineer. The static limit merely gives a theoretical maximum value of power which can be transmitted under ideal conditions and therefore is chiefly of academic interest.

The importance of transient limits depends largely upon the importance attached to the interruption in power supply over the transmission line. If the switching out of a section of line for inspection or repair at times of heavy load would always lead to loss of synchronism, the operating conditions would be intolerable. If the increases in load which normally take place on a system would result in loss of synchronism, the operating conditions would also be intolerable. If any kind of a fault on a transmission system would always lead to a loss of synchronism, the operating conditions would be unsatisfactory. It is recognized that stability cannot be maintained under all abnormal conditions, but the layout should be such as to prevent loss of stability under the more frequently occurring conditions, such as the addition of a reasonable block of load, normal switching operations, and the majority of single-phase line to ground faults. The decision as to the standard of service is important and, in our opinion, nothing less than that suggested above will be tolerable for the superpower transmission systems which have been contemplated.

While theoretical considerations have indicated that some transmission systems were being operated close to their static limits it was not until quite recently that an actual instance of the static limit being exceeded has been made public. H. A. Barre* described a case in which the transmission line of the Southern California Edison system between Big Creek and Laguna Bell became unstable. The load at Big Creek built up slowly to about 183,000 kw. and at that point the system lost synchronism.

It is known that in several cases the transient limit of transmission systems has been reached. At times of faults, loss of synchronism has occurred between generating stations connected by tie lines and between generating systems and load centers connected by transmission lines. An interesting case occurred recently on a system connected by a double-circuit, high-voltage transmission line between the generating stations and distribution network in which a short circuit in the network caused the system to drop load and this occurred for a load below the static limit. Such an experience justifies the stand taken that transient stability limits are of greater practical importance than static limits.

Another factor which tends to give more importance to static limits than they deserve is the fact that published methods for investigating static limits have invariably assumed that regulators and exciters did not have time to become effective before the limit is reached. This assumption is not justified as will be shown in the following section.

Static and Artificial Stability. The published methods for determining the static stability limits have employed the assumption of constant field current in synchronous machines. It is to be noted that the limits so calculated are dependent only upon inherent characteristics of the machines and the other parts of the system. E. B. Shand† pointed out the theoretical possibility of maintaining a condition of "artificial" stability in which the inherent static limits could be exceeded by operation of regulators and exciters. It was then generally believed and not at that time contradicted that the speed of commercial regulators and exciters was insufficient to permit the attainment of artificial stability on actual power systems. Adequate methods of transient analysis were not then available, and only by means of transient analysis is it possible to prove that artificial stability may be obtained on commercial systems. Artificial stability may be defined as the condition of stable operation of power systems which is attainable only through the operation of automatic voltage regulators and exciters. The time available for the functioning of this apparatus is dependent upon the rate of change of rotor position and the time constants of machines. The natural period of mechanical oscillation of a power system under light load conditions is of the order of a second or less, but in the vicinity of the static limit the natural period becomes longer. At the static limit, the time constants of machines become the controlling factors in determining rotor movements. On this account sufficient time may be available for important changes in excitation as a result of the operation of voltage regulators. Assume the case of a transmission system operating at its inherent static limit and that a slight increase in the angular displacement is made; instability should occur on the assumption of constant field current. However, as the system tends to drift apart, the terminal voltage drops and the regulators in seeking to maintain terminal voltage increase the excitation tending to restore the system to the original point of equilibrium. In this connection it may be pointed out that regulators automatically increase the internal voltage with increasing load to maintain a constant voltage at machine terminals or on the high voltage lines. This fact tends to increase the possibility of securing a condition of artificial stability.

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†Group of papers and discussions at the 1924 Convention, Bibliography, Items 3 to 9 inclusive.

*Bibliography, Item 13.

†Bibliography, Item 8.

The matter of artificial stability will be discussed in a more quantitative manner and illustrations will be given after the method of calculating transient stability has been developed.

METHOD OF CALCULATION OF SYSTEM DISTURBANCES

System disturbances set up oscillations which are essentially the same phenomena regardless of the method of initiation. Consequently, the same general methods of calculation may be applied to all of them. The performance of a system during and following a disturbance will involve the following problems: generators and exciter characteristics, simplification of the load-end network, inertia effect of rotating machines, governor characteristics, dissymmetry due to single-phase short circuits, and methods of combination of electrical and mechanical oscillations into one final result.

Generator Characteristics. Generator characteristics also affect the stability of a system. For steady-state conditions with fixed excitation these characteristics can be represented by plotting the power output against reactive kv-a. for different values of terminal voltage. The resultant curves resemble distorted receiving end circle diagrams. However, when the excitation is permitted to vary by means of a voltage regulator, this method is no longer applicable. During transients the internal reactions within the generator can best be analyzed by the Blondel* two-reaction method in which the flux is resolved into two components, a direct component in phase with the field poles and linking the field windings and a cross component in quadrature with the field poles. Referring to Fig. 1, I is the armature current, E_t terminal voltage, and E_a internal voltage. This latter voltage is produced by rotation of the flux ϕ_a. The vector $O\,A$ is in phase with the axis of the field poles and represents the m. m. f. produced by the field current and $A\,B$ represents the m. m. f. produced by armature current. The total flux ϕ_a may be resolved into the direct component ϕ_d in phase with the axis of the field poles and the cross component ϕ_c in quadrature with the axis of the field poles. The flux ϕ_d is produced by the m. m. f. equal to $O\,A$ minus $A\,B \sin \phi$ and ϕ_c by $A\,B \cos \phi$. In calculations, however, it has been found more convenient to resolve the internal voltage E_a into two components and consider E_d as produced by ϕ_d and E_c by ϕ_c. Due to the very slight change in rotor speed during most transients it is sufficiently accurate to consider E_d and E_c proportional and in quadrature to ϕ_d and ϕ_c respectively. If certain special cases arise involving greater speed changes, this error can be corrected. E_d can be read directly from the no-load saturation curve as the voltage produced by i minus $K_1\,I \sin \phi$ where i is the field current, and K_1 is the field current equivalent of unit armature current. E_c is equal to $K_2\,I \cos \phi$. In a turbo generator the air gap is uniform and the reluctance to the cross flux is equal to the reluctance to the direct flux. Hence the value of the constant K_2 can be obtained by reading from the no-load saturation curve the voltage produced by the field current equivalent to unit armature current. In salient pole machines greater reluctance is offered to the cross flux than to the direct flux, the ratio† being a function of the ratio of

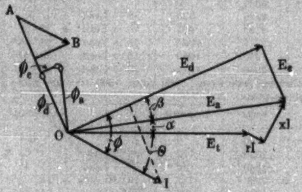

FIG. 1—GENERATOR VECTOR DIAGRAM

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*Bibliography, Item 17.

† In Lawrence's notation this ratio is equal to $\frac{K_D}{K_C}$.

See Bibliography, Item 17.

pole pitch to pole arc. In normal machines this ratio is about 1.75. Since practically all of the m. m. f. is required in the air gap the straight portion of the saturation curve can be used to determine the voltage produced by unit armature current. The constant K_2 for salient pole machines is then equal to $\frac{1}{1.75}$ times the terminal volts produced per armature ampere at zero power factor leading.

As a preliminary to the use of this method for calculation of transient conditions it was deemed advisable to check the angular relation of the terminal voltage and rotor position for steady-state conditions. Tests were made on a turbo generator and on a salient pole machine and this angle observed. The close agreement between observed and calculated results for different loads is indicated in Table I. The check for the salient pole machine is the more remarkable in that the terminal voltage contained harmonics, necessitating an analysis for the phase position of the fundamental.

TABLE I

COMPARISON OF TEST AND CALCULATED VALUES OF ROTOR PHASE SHIFT

Kw.	Volts	Amperes	θ	Observed	Calculated
0	2350	0		0	0
339	2320	325	88°.5 Lead	0	0
2350	2283	657	25°.3 Lead	15° 20′	15° 41′
3980	2378	1005	15°.8 Lead	24° 15′	23° 4′
4040	2280	1080	18°.9 Lead	26° 45′	26° 13′
4130	2346	1133	26°.5 Lead	28° 55′	26° 47′

Steam turbine: 6000-kv-a.; 2200 volts, 1500 rev. per min., two-pole, 25 cycles, three-phase.

Kw.	Volts	Amperes	θ	Observed	Calculated
0	2200	0		0	0
34	2020	112	84°.9 Lag	1° 33′	1° 32′
34	2040	120	85°.4 Lead	11° 42′	7° 42′
196	2230	52	19°.4 Lead	13° 7′	12° 1′
295	2240	88	29°.5 Lead	21° 32′	21° 43′
297	2130	84	16°.3 Lead	22° 54′	21° 44′
331	2040	105	26°.7 Lag	20° 57′	19° 35′
368	2230	112	31°.8 Lead	28° 28′	29° 20′
370	2180	117	32°.9 Lead	28° 47′	30° 10′

Salient pole machine: 750-kv-a.; 2200 volts, 300 rev. per min., 24-pole, 60 cycles, three-phase.

A sudden change in armature current will cause E_d and E_c to change, but with different decrements. In both salient pole and turbo machines ϕ_d has associated with it heavy field windings. Any change in $K_1 I \sin \phi$ induces a corresponding change in the currents flowing in the field windings, armortissuers, and pole faces which tend to prevent a change in flux. The magnitude of these currents is such as to make the change in flux very slow. Fig. 2 shows the change in terminal voltage of a 6000-kv-a. turbo generator as a 4850-kv-a. lagging load is thrown off. The terminal voltage which, after switching, is a measure of internal flux increases very slowly. Curve *a*, Fig. 3 is a similar curve obtained on a 750-kv-a. salient pole machine in which 389 kv-a. at approximately zero power factor lagging is thrown off. This is a high impedance machine used for testing purposes but the field structure is sufficiently representative to illustrate the point. In a salient pole machine, the rate at which E_d varies can be calculated sufficiently accurately by considering only the current induced in the field windings and neglecting the field leakage, eddy currents and inductance of the exciter; *i. e.*, by assuming perfect coupling between the armature and field and considering armature leakage reactance as external. If $I \sin \phi$ is increased suddenly, the induced current in the field windings can be calculated from the equivalents of field and armature currents. The voltage to circulate the additional field current is furnished by collapse of the generator field, the rate being of just sufficient magnitude to supply the additional resistance drop. In most cases it is sufficiently accurate to assume constancy of direct component of voltage but certain cases arise for which it is desirable to know the manner in which E_d varies.

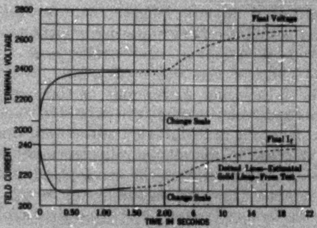

FIG. 2—VARIATION IN FIELD FOR TURBO GENERATOR

4859 kv-a. 0 per cent p. f. lagging dropped from 6000 kv-a., 2200-volt, 1575-ampere, two-pole, 25-cycle machine

Let

i = field current at any instant.
r = sum of the field and the external resistances.
ϕ_d = flux per pole.
N = number of poles.
n = turns per pole of field winding.
C_1 = flux per pole per generator terminal volt.
e_x = exciter voltage at any instant.

The instantaneous value of field current i is equal to $K_1 I \sin \phi$ plus field current required to produce E_d. The voltage supplied by the collapse of the field is then equal to

$$e_x - i\,r = n\,N \frac{d\,\phi_d}{d\,t} \tag{1}$$

$$= n\,N\,C_1\,10^{-8} \frac{d\,E_d}{d\,t}$$

$$\frac{d\,E_d}{d\,t} = \frac{10^8}{n\,N\,C_1}\,(e_x - i\,r) \tag{2}$$

This expression enables one to calculate the rate of change of E_d from the values of the exciter voltage and field current at that particular instant. In this way, the variation of E_d can be taken into consideration in the step-by-step method employed by the authors in calculating transients. The variation of terminal voltage indicated by Curve *b* in Fig. 3 was calculated in this manner and indicates that the assumptions employed for the salient pole machines are sufficiently justified.

While it is true that some of these assumptions are rather bold, it must be remembered that in many cases it is sufficiently accurate to assume E_d constant, the exciter voltage building up sufficiently rapidly to warrant this assumption. The calculation of $\frac{d E_d}{d t}$ is more on the order of a correction and need not be very accurate.

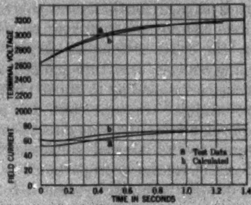

FIG. 3—VARIATION IN FIELD FOR SALIENT POLE MACHINE

389 kv-a., 8 per cent p. f. lagging dropped from 750-kv-a., 2200-volt, 197-ampere, 24-pole, 60-cycle machine

The calculation of the rate of change of ϕ_c and E_c is quite different. The path for induced currents tending to prevent the change in flux is not so well defined and the analysis becomes too involved for calculation. Furthermore, the phenomena are different in a turbo generator from those in a salient pole machine. In the former, ϕ_c is restrained from changing by the presence of heavy dampers and solid masses while in the latter this effect is very small. Tests were made to determine the rate of change of these quantities. A contactor mounted on the shaft of the turbo generator was used to measure the phase angle between the terminal voltage and rotor as 4040 kw. at 94.8 per cent power factor leading was thrown off a 6000-kv-a. machine. The results of these tests are shown in Fig. 4 and while the curve seems to be very irregular, the tendency is very distinct and indicates that the cross component changes quite slowly and is of the same order as that of the variation of ϕ_d. For salient pole machines the change is much more rapid. To determine this rate of change a testing machine of high reactance was chosen because the field structure was representative and because of the presence of two duplicate generators in addition to requisite driving motors on the same shaft. A load was placed on one machine of such power factor that the current was substantially in phase with E_d. The other machine was used as a reference vector to indicate the phase position of the rotor. By measuring the difference between the terminal voltage and the reference voltage the phase displacement due to loading was obtained very accurately. An oscillogram was taken as 368 kw. was thrown off, the phase angle being measured in the manner indicated. The results of this test are shown in Fig. 5. The oscillations superposed on the curve are due to mechanical oscillations of the shaft and rotor. The angles indicated also subtend the leakage reactance drop which in this case was about 14 deg. for the initial steady-state condition. These curves indicate that the change in cross flux is quite rapid.

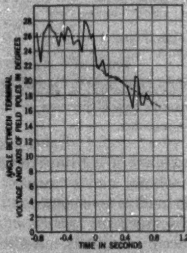

FIG. 4—VARIATION OF CROSS FLUX IN STEAM TURBO GENERATOR

Dropping 4040-kw., 94.8 per cent p. f. leading from 6000 kv-a., 2200-volt, two-pole, 25-cycle machine

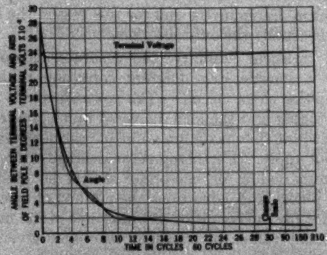

FIG. 5—VARIATION IN CROSS FLUX IN SALIENT POLE MACHINE

Dropping 360-kw., 85 per cent leading from 750 kv-a., 2200-volt, 197-ampere, 24-pole, 60-cycle machine

Taking into consideration the fact that in practise the change in $I \sin \phi$ will be more gradual than the condition imposed by test and that the angle between the internal voltage and the terminal voltage will be a small part of the total angle, the assumption for a salient pole machine that E_c changes in conformity with $I \sin\phi$ without appreciable time lag in a manner similar to reactance drop is well justified.

In the foregoing treatment of the salient pole machine the field leakage was neglected and except for the flux involved in the impedance known as "armature leakage reactance" perfect coupling between the armature and field windings was assumed. In effect this reduces the problem to one of considering a perfectly coupled machine with an external series reactance. The value of "armature leakage reactance" can be obtained by assuming constant permeability and determining the linkages associated with the flux which, with unit armature current and zero current in the rotating field winding, does not cut any of the field windings.

For transient stability problems it might be expected that "tranisent" reactance rather than "armature leakage" reactance should be employed. However, transient reactance is not a constant quantity and varies with the power factor and with the rotor position which effects the coupling with the different phases. Transient reactance has been used principally for the determination of short-circuit currents and the values have been computed for the condition which gives the highest value of short-circuit current in one phase for a three-phase short circuit. The maximum value of transient reactance occurs for the same condition which produces maximum dissymmetry and for this reason the highest value is usually the only one given.*

For transient stability, the generator is operating under a considerably different condition than for a three-phase short circuit. The power factor is usually much higher and the load delivered much greater. On this account the maximum value of transient reactance does not apply. Furthermore, an average rather than a maximum value for one-phase should be used. For single-phase fault conditions the method of calculation provides for the condition of unbalance in the machine. The usual values of transient reactance given range from 25 to 50 per cent greater than the conventional armature leakage reactance but the value applicable to the transient stability calculations should be about 10 to 15 per cent greater than the armature leakage reactance. The calculations made on the salient pole machine employed a single value of leakage reactance for the determination of both steady-state and transient conditions. The close check obtained indicates that there is very little difference in the reactance to be used for steady-state conditions and the value of reactance which should be used for transient conditions. The simplest way to include the effect of machine reactance is to consider a machine with perfect coupling between armature and field with external series reactance equal to about 110 per cent of armature leakage reactance. While this method is not rigorous, it is both convenient and sufficiently accurate. With the above assumptions an analytical expression can be obtained representing the conditions within a salient pole machine. The problem is peculiar in that the cross component of voltage while being of the nature of an impedance drop cannot strictly be treated as such. A derivation of this expression is contained in Appendix 1 in which the components of current are defined in terms of the constants of the machine, the direct component of voltage and the terminal voltage.

Summarizing, the method of calculating machine characteristics is as follows: For turbo generators, to assume the internal voltage fixed relative to the rotor, perfect coupling between armature and field, variation in cross component the same as for the direct component of flux, and, in addition, a series reactance corresponding to armature leakage; for the salient pole machine, to consider a direct component of voltage fixed with respect to the rotor, the cross component varying instantly, perfect coupling between armature and field and with a series reactance corresponding to about 110 per cent of the armature leakage. In some cases it is permissible to use the approximation that the direct component of voltage is equal to the internal voltage and that the cross component of voltage is proportional to the power delivered. For large receiving systems it is frequently permissible to assume a constant internal voltage in synchronous machines and to neglect the effects of demagnetization and cross-magnetization.

TABLE II

NEGATIVE SEQUENCE IMPEDANCE OF SALIENT POLE ALTERNATOR

	In ohms per phase	Imped. angle in deg.	In ohms per phase
1. Application of negative sequence voltage.			
(a) *With residual excitation only			
Star value of applied voltage = 228	2.16	81.0	$0.338 + j\,2.13$
= 466	2.17	81.5	$0.321 + j\,2.14$
= 465	2.16	81.0	$0.338 + j\,2.08$
(b) With equal positive and negative voltages at 237 per phase	2.14	77.0	$0.482 + j\,2.08$
2. Sustained short circuit across two phases	2.14	78.5	$0.426 + j\,2.09$
3. Loading with one phase open			
(a) Kilowatt load	2.15	77.0	$0.484 + j\,2.10$
	2.07	76.5	$0.482 + j\,2.01$
(b) Reactive load	2.07	81.0	$0.324 + j\,2.045$

750-kv-a., 2200-volt, 197-amperes, 24-pole, 60-cycle machine.
Positive sequence armature leakage reactance per phase = 3.17 ohms.

*Indicates most accurate method.

The above discussion of generator impedances referred to balanced conditions. Additional constants are required to determine the performance under the conditions of single-phase short circuits. The constants required by the method described subsequently are the "negative phase sequence" and the "zero phase sequence" impedances. Negative sequence impedance is the ratio of the voltage to the current obtained when balanced voltages of the rotation opposite to that of the rotor are applied to the terminals of the machine. Tests were made by this method and by loading on a single phase. The data were analyzed for negative sequence impedance and the results are indicated in Table II. It will be noted that substantially the same values were obtained by the different methods.

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*Bibliography, Items 23 and 24.

As pointed out in Appendix II, the zero sequence impedance is usually unimportant in transmission stability problems. Additional information relative to both the negative and the zero sequence impedances will be found in the appendix.

Excitation Systems. Excitation systems serve to supply the m. m. f. required in synchronous machines to produce the direct component of voltage of such value as to maintain any desired terminal voltage. As the load is changed, the field current must be varied to bring the system voltage to normal. This is accomplished by changing the voltage applied to the field terminals of the machine either by hand or automatically. With hand-operated systems, the excitation remains constant during the transient unless changed by the operator, but with automatic regulators, changes in excitation normally take place. In a later part of the paper it will be shown how for steady loads, these regulators make possible a condition of "artificial" stability. Automatic voltage regulators are of two principal types, the vibrating or Tirrill type, and the rheostatic type.

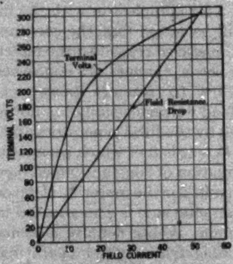

FIG. 6—EXCITER CHARACTERISTIC FOR CONSTANT RESISTANCE LOAD

The vibrating regulator functions by cutting a block of resistance "in" or "out" of the exciter field circuit at such a rate as to maintain the proper mean value of exciter voltage to supply the required field current for the main machine. The standard exciters adjusted so as to vary between 120 and 300 (or 60 and 150) volts as limits, but never reaching these values unless the contacts remain either open or closed. While the exciter voltage varies within large limits, the field current of the main generator is restrained from following because of the inductance of the field winding. For the case of single-phase, line-to-ground faults, it is important to know the rate of rise in exciter voltage, as a knowledge of this variation is required for the calculation of the change in E_d. Fig. 6 represents the voltage-field current characteristics of a normal exciter with a constant resistance load taking into consideration armature reaction and armature resistance drop. If the field resistance of the exciter is adjusted to produce 300 volts with contacts closed, the straight line through the curve at 300 volts represents the exciter field resistance drop. If the exciter voltage had been (e) volts before closing the contacts, it must, due to the inductance of its field, start from that value in ascending. A terminal voltage of (e) volts requires (i) amperes exciter field current. This produces a drop of ($i\,r$) volts in the field winding, but since the exciter field is connected directly across its terminals, the difference in voltage ($e - i\,r$) represents the inductive drop due to change in flux through the exciter field winding.

$$e - i\,r = n\,N\,\frac{d\,\phi}{d\,t}\,10^{-8} \qquad (3)$$

where

N = Number of poles

n = Number of turns per pole

ϕ = Flux per pole

but

$e = k\,\phi$

where

k = Voltage produced by rotation of unit flux

therefore

$$\frac{d\,e}{d\,t} = \frac{k}{N\,n}\,(e - i\,r) \qquad (4)$$

This equation expresses the relation between the rate of change in terminal voltage of the exciter with time as a function of e and i, and enables one, by a step-by-step method, to determine the exciter voltage as a function of time. It will be noted that as the voltage increases from 120 volts the rate of change increases for a while and then decreases, becoming zero at the intersection of the two curves at 300 volts. A similar curve can be obtained for the descending voltage condition by using the exciter voltage curve resulting from the higher resistance load and the resistance drop line corresponding thereto. Within the operating range, the exciting voltage fluctuates between the two limits as shown by the exciter voltage to the left of zero time in Fig. 18, ascending and descending on the curves described above. In analyzing disturbances it is necessary to know the lower limit. This limit is a function of the period of vibration of the regulator, being low for long periods and nearer the average value for short periods. The rate of vibration of the regulator varies from about ten vibrations per second near the middle of the operating range to zero at 120 and at 300 volts for 250 volt exciters. For definite rates of vibration and definite exciter characteristics, the lower limit of exciter voltage can be determined.

With the rheostatic type of regulator the exciter voltage remains constant but the resistance in the main field of the alternator is automatically varied by means of a motor-operated, face-plate rheostat, controlled by the regulator. It may be noted that with this type of regulator the operation occurs in the field circuit of the main machine, whereas with the vibrating type the operation occurs in the field circuit of the exciter. With the vibrating regulator, there is a time lag in the building up of the voltage across the machine terminals due to the time constants of the exciter field, whereas with the rheostatic regulator a certain amount of time is required for the movement of the motor-operated rheostat.

The Load-End Network. A transmission line will usually deliver power to a load-end system of considerable extent and having other sources of power. Frequently this "load-end network" will have connected to it much greater capacity than the output of the transmission line. The load-end network constitutes a problem in stability studies as rigid analytical methods are frequently impractical on account of the complexity of the network and it is difficult to find a network that is equivalent to the actual network and at the same time sufficiently simple to be handled analytically.

The load-end network includes synchronous generators and condensers in addition to the load whose principal components are synchronous motor, synchronous converter, induction motor and lighting load. E. J. Amberg has given the following segregation of load on a particular system, which segregation appears to be quite typical:

TABLE III

TYPICAL SEGREGATION OF PEAK LOAD

Type	Percentage
Induction Motor	60
Synchronous Motor	10
Synchronous Converter	10
Lighting	20

The various types of load have different characteristics: With synchronous and induction motors the true power demand may be assumed to remain constant with variation in voltage, whereas the lighting and synchronous converter load will vary as the square of the voltage. The reactive power varies greatly with the type of load and in the curves of Fig. 7 are shown the variations of reactive power with variation in terminal voltage.

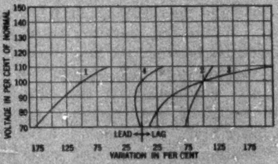

FIG. 7—VARIATION OF REACTIVE KV-A. WITH VOLTAGE

1. Synchronous motor—75 per cent load, 85 per cent p. f. lead (at normal voltage)
2. Induction motor, 75 per cent load, 79 per cent p. f. lag (average at normal voltage)
3. Transformer exciting kv-a.
4. Synchronous converter (reactive kv-a. in per cent of machine rating)

It is impractical to consider a multitude of individual loads, and it therefore becomes necessary to use some such composite load characteristic as shown in Fig. 8. This load characteristic must take into account the effect of the local supply lines, transformer impedances and admittances. Such a composite curve is difficult to handle analytically except by step-by-step methods.

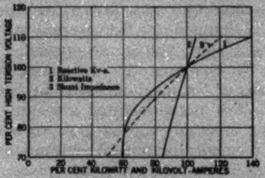

FIG. 8—TYPICAL LOAD CHARACTERISTIC CURVES

An approximation of the load characteristics that is very convenient is the assumption of a constant impedance shunt load. The variations in real and reactive power using this assumption are plotted on the curves of Fig. 8 for comparison with the more accurate values. It will be noted that the constant impedance method is optimistic in regard to the effect on stability so far as true power changes are concerned, and pessimistic in regard to reactive power changes. The variations in load voltage usually are not great because it is maintained by the local generators and synchronous condensers. It is this fact which permits the relatively crude approximation of constant impedance load to give satisfactory results for the majority of cases.

The load characteristic is also affected by changes in system frequency. Quite definite information on this point is available from actual operating experience. J. P. Jollyman* has stated that a reduction in frequency from 60 to 59 cycles will reduce the real power demand by 3.5 per cent.

The various types of load-end networks and the corresponding equivalent networks will now be discussed. Probably the best classification of the load-end networks is based on the number of separate sources of e. m. fs. that must be considered and the discussion is arranged under four principal headings as follows:

1. Load-end network with a single voltage source.
2. Load-end network with two voltage sources with fixed angular displacement.
3. Load-end network with two or more voltage sources with variable angular displacement.
4. Load-end network of very large capacity.

Load-End Network with a Single Voltage Source. If the receiver end system merely involved a single synchronous motor there would be no problem of how to handle the load-end network, as it would be possible to apply rigid methods of analysis. In general, it would be necessary to take into account the finite inertia of the synchronous motor and also the variations in direct and cross component of flux.

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*Bibliography, Item 15.

If the load consisted of a synchronous motor and a shunt impedance, the load-end problem would be handled in a similar manner. The shunt impedance load at the receiver would be combined to form a part of a single equivalent network between supply and receiver ends as indicated in Figs. 9A and 9B.

In case of a radial type receiving system as indicated in Fig. 10, it would be permissible to use the same method provided the voltages of the various machines have the same phase relation at all times. This would require that the regulation of the various branches, and also the product of the effective impedance and the change in power per electrical degree of rotor movement, be the same in order to have the same natural period of mechanical oscillation in the different branches.

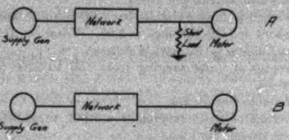

Fig. 9

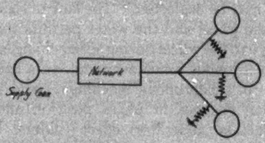

Fig. 10

It should perhaps be pointed out that all the methods described in this section have an accurate analytical basis and permit of taking into account the finite inertia and the changes in direct and cross components of synchronous machines. It may be noted that the shunt impedance when used to represent non-synchronous load is, however, an approximation.

Load-End Network with Two-Voltage Sources at Fixed Angular Displacement. To employ the assumption that the voltages of all load and synchronous machines are in phase is clearly a crude approximation in many cases, as for example, in the case of a synchronous motor and a synchronous condenser, or of a load-end generator and the motor of a large frequency changer set. The problem of analyzing the operation of three synchronous machines, one at the supply end and two at the load end, taking into account the inertia effects and variation in direct and cross component of voltages in each machine, is quite complicated. This is due in part to the fact that the two machines at the load end have, in general, different natural periods and the resulting oscillation may be considered as a principal oscillation upon which is superposed a minor oscillation of higher frequency. The minor oscillation is of relatively little importance in the stability problem and this suggests the desirability of making a simplifying assumption which would eliminate the minor oscillation. Fortunately, this assumption is simple to apply. It requires merely that the two machine voltages maintain a constant angular relation between each other. In Appendix II is derived a method for replacing two networks connected in parallel, each with a source of voltage by a single equivalent network with a single equivalent supply voltage. The assumption of constant angle between machines is, in reality, quite accurate; for the initial condition there is, of course, no error as the angle is obtained from this condition; for the steady-state conditions subsequent to the disturbance, the phase angle is approximately the same because the power distribution is about the same as initially. During the transient conditions the approximation is relatively good but is in error because the oscillation represents that of the two machines operating as a unit, instead of permitting them to move with respect to each other. Of course, the method gives the correct initial accelerations and decelerations of the various machines and is best suited for systems comprising turbo generators in which the internal voltage may be considered constant. The method may be applied to the case of a receiver system having a shunt impedance load, it being necessary only to include the shunt impedance as part of the equivalent network.

The case of a load-end network with three or more voltage sources having fixed angular relationships may be solved in a similar manner by the use of the single equivalent network.

Load-End Network with Several Voltages having Variable Angular Relations. In case of a load-end network with two or more voltage sources with variable angular relations, the calculations become quite involved. This method may be handled by the use of the single equivalent network in a step-by-step solution. For each step it is necessary to obtain the voltage power and angle relations for the equivalent network and then the corresponding quantities for the individual machines. The rate of change in voltage and angular position for the individual machines may then be estimated. For the next point, at the end of the arbitrary time interval, voltage and power relation can be determined by the use of a single equivalent network with a new equivalent supply voltage. The estimated rate of change in rotor position and voltage used throughout the interval may now be checked with the values at the beginning and end of the interval.

Load-End Network for Very Large Systems. On large receiving systems the variations in power demand and in demagnetizing and cross-magnetizing actions on a particular machine due to a disturbance on the transmission system are likely to be relatively small. Consequently, the approximation of constant internal voltage is quite accurate and the variation in phase angle of the load-end network becomes small in comparison with the total variation in phase angle. Hence it is permissible to represent an extensive transmission network with many sources of voltage by a single equivalent network with a single machine and a single shunt impedance representing non-synchronous load. In large systems the total inertia effect becomes very large with respect to the inertia of the generator supplying the transmission line and for this condition the inertia of the equivalent machine for the load-end network may be assumed, without serious error, as being infinite. If desired, a correction for the effect on the natural period of the mechanical oscillation may be made by increasing the inertia of the generators supplying the transmission line as indicated below:

$$\frac{1}{M_0} = \frac{1}{M_s} + \frac{1}{M_L} \qquad (51)$$

where M_s and M_L represent the actual inertia of the machines at supply and receiver ends respectively, and where M_0 is the equivalent inertia at the supply end with the receiver inertia considered infinite.

It is to be emphasized that high accuracy in the representation of the load-end network is usually not required because the effect on the stability of the transmission system is relatively small. In general, where the load-end network is simple, it is necessary to employ more accurate methods than where the system is large, which relation is fortunate from the standpoint of calculation.

Mechanical System Data. The stored mechanical energy in the synchronous rotating machinery is important in determining the period of system oscillations and the rate of change of angular positions of rotors, and hence internal voltages in response to changes in input or output. Because of their inertia the rotors cannot change phase position immediately. During the first instant of a disturbance the redistribution of power can be calculated with the internal voltages in the same position as at the instant immediately preceding a disturbance. Before the beginning of the transient the input and output are equal,—the rotor being in equilibrium,—but with the redistribution of power, the equilibrium is disturbed and the rotors accelerate or decelerate, the rate being determined by the following equation:

$$a = \frac{180\,f}{E}\,\Delta P \qquad (6)$$

where

a = Acceleration in electrical degrees per sec. per sec.

f = System frequency.

E = Stored mechanical energy at synchronous speed in kw. sec.

ΔP = difference between input and output in kw.

By assuming sufficiently small time intervals and assuming the acceleration constant during the interval, or by assuming larger intervals and correcting for average acceleration, the rotor movements can be traced by a step-by-step method of calculation throughout the period of disturbance.

The stored energy is dependent upon the total mass, the distribution of this mass, and speed. The following table indicates the stored energy in kw-sec. per kw. output for a large number of different units of each type of machine.

TABLE IV

STORED ENERGY OF MACHINES

1. *Water-Wheel Generators.*
   Average of units ranging from 1500 to 35,000 kw. = 2.40 kw-sec. From an examination of a small amount of data the fly-wheel effects of the water-wheels appear to be in the neighborhood of 10 to 20 per cent of the generator fly-wheel effects.
2. *Steam Turbine-Generator Units.*
   Average of units ranging from 1500 to 35,000 kw. Generators = 5.32, turbines = 5.65. Total = 10.97 kw-sec.
3. *Rotary Converters (Railway & Edison).*
   Average of units ranging from 750 to 3250 kw. = 2.10 kw-sec.
4. *Synchronous Condensers.*
   Average of units ranging from 1000 to 40,000 kv-a. = 1.48 kw-sec. per kv-a.

In consideration of water-wheels, the inertia effect of the water may be neglected. The lack of rigid coupling between the wheel and the water permits the latter to slip relative to the wheel during speed changes.

The synchronous machinery and, to a lesser extent, the eddies in synchronous machinery tend to cushion any sudden changes in phase position and voltage. This induction generator effect differs from the synchronous effect in that it is responsive to speed changes rather than angular space changes. However, the effect is small and may be neglected.

Governor Characteristics. The time constants of governors are also important in their effect upon system stability during certain types of disturbances. The acceleration or deceleration of the rotor determining the phase position of the internal voltage depends upon the difference between generator input and output, the input being regulated by the governor. In present day practise, the governor is a device which functions on speed. This point is important as large angular displacements might take place before the governing device begins to function.

Normally, steam turbine governors require approximately two to three seconds traversing time for full stroke with 100 per cent change in load, but more recent developments have reduced this time to one second. In addition to the traversing time, time is required for the speed to change and to start corrective action. This time will be greater for small changes than for large changes.

Time constants of water-wheel governors are also of this same order of magnitude. Tests* made on units of the Pennsylvania Water & Power Company indicate that the "dead time" between load change and first movement of the gates is about 0.3 seconds for all load changes. Maximum speed of gate movement occurs when full load is thrown off. If the rate of change at the beginning of the motion were continued, full stroke would be covered in 2.7 seconds. For smaller load changes, this time is much larger, rising to ten seconds for a 15 per cent change in load. These data are based on the performance of relatively low head units. For higher head plants, rapid changes in the velocity of the water may cause excessively high pressures which must be avoided. This is accomplished by the installation of surge tanks and by by-passing the flow through relief valves and subsequently closing the valves slowly. For Pelton wheels, another method is also available, namely, the use of stream flow deflectors which merely interpose a barrier between the nozzle and the buckets, and deflect a portion of the stream.

The method of system operation also influences the manner in which the governors affect stability. In the governing system of the Pacific Gas & Electric Company which they have found satisfactory for their type of system, governors are set with a flat characteristic to govern at one-half cycle above normal frequency, the load dispatcher directing one particular plant to act in a governing capacity. The governors, when operating "against a block," merely act as overspeed devices.

Before complete analysis of stability conditions on new projects are made, proper cognizance must be taken of the method of operation and adequate information relative to the time constants of the particular governors must be obtained.

In general, since governors react only after the speed change and since these changes are extremely small during switching operations, the variations in gate opening can be neglected for these conditions. The action of the governor during single-phase short circuits will depend upon the change in output and the resultant speed change. For relatively small changes of the order of 25 per cent of rated load, consideration of the 0.3 seconds "dead time" and the relatively slow traversing rate of about six seconds would justify neglecting any change in gate position due to governor action. In general, one must look into the individual case and determine whether or not the effect of governors can be neglected. If their effect must be taken into account, it will be found convenient in the analysis to plot the variation in speed of machines and in gate opening as functions of time. These curves enable one to determine the interaction between rotor velocity and gate opening throughout the transient condition.

Single-Phase Short Circuits. The calculation of single-phase, short-circuit currents on a transmission network is complicated because the currents and voltages in the different phases are unsymmetrical. Each phase is inductively coupled with the other phases in transmission lines, transformers and rotating machines. In addition, rotating machines, including synchronous generators, motors and condensers, and also induction motors, provide a distinct phase-balancing action, tending to restore symmetry in voltage and current. Because of the fact that no suitable method had previously been published, the authors found it necessary to develop a method for the solution of the single-phase, short-circuit problem. This method is essentially a combination of two well-known methods of network solutions; namely, phase sequence components, and general circuit constants. In the phase sequence method developed by C. L. Fortescue,† the voltages and currents of a three-phase grounded system are resolved into three components, namely, the positive sequence, the negative sequence, and the zero sequence. This results in an important simplification for the case we are considering of balanced polyphase systems, as the different sequences do not react one upon another. For normal balanced loads, only positive sequence voltages and currents are present. In case of a line-to-line fault, only the positive and negative sequence components are present; while in the case of a fault to ground, all three components are present. Synchronous machines generate only positive sequence e. m. fs., and the negative and zero sequence voltages appearing at machine terminals are due to the voltage drops produced by negative and zero sequence currents, respectively. Since polyphase synchronous machines generate only positive sequence voltages, it follows that machine decrements involve only positive sequence voltages and currents, and the constants of the machines and the network to which they are connected. In other words, it is possible to compute the decrements for single-phase short circuit by the methods applicable to three-phase short-circuits if the positive sequence voltages and currents of the various machines are considered.

The most convenient methods for handling transmission networks employ the general circuit constants developed by Evans and Sels.* In this connection, it may be pointed out that the recent revision of the book "Electrical Characteristics of Transmission Lines," by William Nesbit† gives a very complete series of tables of the general circuit constants for transmission lines covering the commercial ranges of conductors and spacings for lines from 50 to 300 miles in length. The method of general circuit constants was originally developed for the solution of balanced three-phase systems where only positive sequence components of currents and voltages are present. The method, however, can readily be extended for negative and zero sequence components.

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*Bibliography, Item 18.

†Bibliography, Item 19.

*Bibliography, Item 6.

†Bibliography, Item 20.

In Appendix II is given in detail the theory of the method for the solution of the single-phase short circuit. The method presented is one of determining an equivalent network with equivalent voltages and general circuit constants for positive, negative and zero phase sequences to represent the complicated network under consideration. The steps in the solution include the replacing of two network branches operated in parallel, each with a source of voltage, or two networks in series by a single network with equivalent supply voltage and equivalent general circuit constants. This method may readily be extended to cover almost any type of network. The final form of the solution is that of a single equivalent network having an equivalent supply voltage and equivalent general circuit constants for the three-phase sequence components. Formulas are then presented for the calculation of the short-circuit currents for both line-to-line and line-to-neutral faults. It should perhaps be pointed out that the method may be applied to calculating boards when the assumptions usually made in short-circuit calculations are employed. Reference may be made to an article‡ by one of the authors describing the method in a simplified form, considering only the reactance of the various circuits.

The application of the phase sequence method permits the determination of the voltages and currents in the different phases at all points in the transmission system for the case of a fault from line to line, or from line to ground. However, the authors have found it both permissible and advantageous to replace the short circuit from line conductor to ground by a symmetrical shunt impedance between the line conductors at the point of short circuit, the magnitude of this impedance being so chosen as to produce throughout the entire transmission network the identical values of positive sequence voltages and currents that occur in the actual network. The value of the equivalent shunt impedance is equal to the sum of the negative and zero sequence impedances of the transmission network, as calculated at the point of short circuit. A derivation of this method is given in Part III of the Appendix. By the use of this equivalent network, the line-to-ground short circuit problem is reduced to a symmetrical three-phase problem for which the more familiar methods of solution may readily be employed. Also, it provides the solution in the form most convenient for analyzing stability; that is, the positive sequence solution. A similar equivalent network may be developed for the case of a line-to-line short circuit.

The single-phase, line-to-ground fault causes current to flow through ground connections, both at the fault and the transformer neutrals. The resistance of this path is speculative, as it depends upon local conditions as to the location of fault, method of grounding neutral, character of soil, etc. The resistance from transformer neutral to the faulty conductor may vary from substantially zero in the case of the transformer bushing failure to a very high value in case of a ground on certain rocky soils. The magnitude of this resistance is an important factor in the stability problem because upon it depends the tendency to accelerate or decelerate the rotors of generators or other machines at the time of a fault to ground. The effect on generator speed may be enormous as may be realized from the fact that a fault current of the order of 4000 amperes, which may be expected on some of the projects under consideration, would cause an energy absorption in ground connections of 16,000 kw. per ohm. Only a limited amount of information is available at the present time in regard to the resistance of earth connections when carrying heavy currents. The data obtained in a couple of cases where tests have been made on large power systems indicate that the value of this resistance is likely to be sufficiently high to cause the generators to slow down on the occurence of a fault and then overswing when the faulty section of line is cleared.

Combination of Factors. The various individual factors entering into the problem of stability have been discussed and the matter of their combination for the calculation of system oscillations will now be considered.

Single Generator and Motor with Reactive Tie Lines. In order to make the essential steps in the determination of system oscillations stand out more clearly, a number of simplifying assumptions will be made. Consider the case of two identical salient pole machines—one a generator and the other a motor—tied together by two tie lines as indicated in Fig. 11. Let it further be assumed that the resistance in the circuit and the capacity of the tie-line is negligible, the load a shaft-brake load having essentially constant power characteristics, the governor setting constant during the transient, and the initial field currents on the two machines equal. The system will be symmetrical about the midpoint of the line with current and voltage in phase at this point, and, as the generator accelerates or decelerates, the motor will decelerate or accelerate an equal amount. This simplifies the problem to the consideration of the action of only one unit.

The initial conditions of operation with both tie lines in service can be determined by the usual methods of calculation including phase angle between rotor positions, direct component of machine voltages and excitation.

Switching out a section of line, as would occur, for example, by the opening of breaker "a" shown in Fig. 11, is the equivalent of a sudden increase in the reactance of the tie lines. The opening of a breaker introduces certain unsymmetrical electrical transients of very short duration, which may be neglected. The increase in the reactance of the tie line requires the system to find a new position of equilibrium with a greater phase angle between rotors. The system cannot instantly assume the new position of equilibrium at normal voltage because of the inertia of the rotating masses and the inductance of machine fields. At the first instant after breaker opening, the generator output is less than its input and the motor input is less than its output and consequently the generator initially tends to accelerate and the motor to decelerate by equal amounts as determined by the method discussed previously. At the same time, a change occurs in the demagnetizing effect of armature current from which, by methods previously described, the rate of change of the direct component of voltage can be calculated. Choosing an interval of time and assuming the rate of change constant during this interval, the final values of E_d and θ at the end of the interval can be calculated. Continuing this method by the step-by-step process in which the acceleration and $\dfrac{d\,E_d}{d\,t}$ are assumed constant during the intervals, it is possible to plot the variation in rotor angle and other quantities throughout the system oscillations. By employing sufficiently short time intervals, any desired degree of accuracy may be obtained. The step-by-step method may be carried out more accurately by estimating the average rate of change of the various quantities and using it throughout the arbitrary time interval. This average rate of change can be approximated by extending the curve through the succeeding interval.

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‡Bibliography, Item 21.

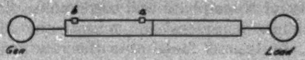

FIG. 11

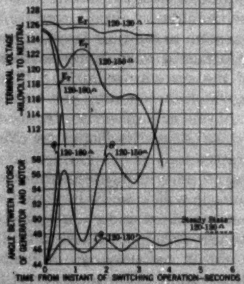

FIG. 12—SWITCHING TRANSIENTS CAUSED BY CHANGING REACTANCE OF TRANSMISSION LINE

Connecting alternator and similar motor from 120 ohms to values indicated on curves 220-kv. 250,000 kv-a. system

Fig. 12 indicates the results of calculations of this nature in which the reactance of the tie line was increased from 120 to 130, 140 and 160 ohms. It will be noted that the system remained stable for the change from 120 to 130 ohms only, the other cases resulting in loss of synchronism as indicated by the continuous increase in angle and the drop in terminal voltage.

For the case of a synchronous generator and motor connected by a general network, the method of calculating system oscillations is very similar to that described above. The initial conditions are first determined by the usual method. After the disturbance takes place, the initial changes in acceleration and in the direct component of machine voltages may be computed from the power, voltage and angle relations given in Appendix I. The same step-by-step solution is then employed but with the difference that, on account of the losses in a general network, the motor input is less than the generator output, which makes it necessary to compute separately for each machine the change in acceleration and in direct component of voltage and exciter voltage.

The calculation of system oscillations for other types of networks is essentially the same as that described above, the difference arising only in the method required to give the power, angle and machine loads. In the case of two or more machines at the receiving end of the system the same procedure may be followed if the receiver end of the system is handled in the manner described in the section on *Load End Network*. In the case of two or more machines at the supply end, a similar method may be followed.

APPLICATION OF METHODS TO VARIOUS TYPES OF SYSTEM DISTURBANCES

The methods of calculating system oscillations described in the preceding section will now be discussed with respect to their application to the various types of system disturbances. The principal conditions which may give rise to important disturbances on a power system are as follows:

1. Sudden increases in load.
2. Switching operations.
3. Faults.

For a general discussion of the phenomena accompanying these various types of disturbances, reference should be made to the paper by Mr. Fortescue.*

Sudden Increases in Load. The supply of power from long-distance transmission lines is usually supplemented by local steam or hydroelectric power plants. In ordinary operation, sudden increases in load do not occur except in the case of the loss of a load-end generator as the result of a breaker operation. For this condition, however, the load is taken up largely by the deceleration of the parallel units which initially tend to contribute power in inverse proportion to their connecting impedances. The increased demand for power will require a new position of equilibrium and before reaching it, the system, on account of mechanical inertia, will tend to overshoot and go out of step if the transient limit is too low. If the system withstands the first overswing, the two ends of the system will usually stay in step and slow down together. The reduction in system frequency will lead to a simultaneous reduction in load which, on a typical system as pointed out by J. P. Jollyman, is of the order of 3½ per cent for a reduction of frequency of from 60 to 59 cycles. The increased load will cause a reduction in system frequency and in machine voltages which will bring the governors and voltage regulators into action, tending to restore the system to normal.

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*Bibliography, Item 2.

For the investigation of the stability condition of the system for sudden increases of load, the calculations are carried out for the condition corresponding to the loss of the largest generating unit at the receiving end as the result of a switching operation. In general, however, the limiting condition from the standpoint of stability will not arise from sudden increases in load, though, of course, consideration should be given to this problem in any specific project.

Switching Operations. The switching operation most likely to be important from the standpoint of stability of a transmission system is the opening of a section of line under load. The switching out of a section of line will cause the system to seek a new position of equilibrium, and the system, in moving to this new point, will overswing to such an extent as to produce instability if the transient limits are too low. If the system does not pull out of step on the first overswing it is unlikely to do so later because sufficient time will usually be available for regulators to increase the excitation of the machines.

In making stability studies, the most severe switching condition is the one which arises from the opening of the longest section of line or the section most remote from sources of maintained voltage. In such studies, it is usually permissible to assume constant internal voltages or constant direct components of voltages. In connection with such studies, the effect of the charging current of the section of line disconnected at one end should be taken into account.

Under certain conditions it is possible for switching operations to lead to very severe system disturbances. The conditions favorable to producing severe disturbances include very long sections of line, operation very close to the corona limits, and small generating capacity as would occur in initial developments. In the case of the opening of a breaker at the remote end of the section from the generating station, as breaker *a* in Fig. 11, the corona loss on the disconnected section may rise to a relatively high value. In case this corona loss would be greater than the output of the generating station, the power would be supplied from the load-end network. If the breaker *b* in Fig. 11 were opened, the system must swing forward through a large angle to reach a new point of equilibrium, and the resulting system oscillation would be of very considerable magnitude.

Faults. From the standpoint of maintaining stability, faults produce most severe conditions occurring in ordinary operating experience.

Three different types of faults may occur; namely, three-phase, single-phase, line-to-line, and the single-phase, line-to-ground. High-voltage transmission systems are normally designed with neutral grounded and with relatively large clearances between conductors. There is a distinct tendency to employ single circuit tower lines and horizontal spacing of conductors on account of ice conditions. Greater attention is being given to substation layout in order to minimize the possibility of three-phase and single-phase, line-to-line faults. These considerations indicate that the occurrence of any type of fault other than the single-phase line-to-ground is relatively remote. Data on some of the high-voltage transmission circuits indicate that over 90 per cent of the faults that have occurred were from line to ground. In view of these facts, it appears that the layout should be such as to provide a reasonable margin of stability in the case of single-phase, line-to-ground faults, but that it is not essential in the case of three-phase and single-phase, line-to-line faults.

A fault on a transmission line will normally give rise to three circuit conditions; namely, the original condition before the fault, the condition during the fault, and the condition after the fault is cleared; and in addition, another intermediate step if more than one breaker is required to clear the fault. The time interval between these changes in circuit conditions is insufficient for the system to readjust itself, consequently the changes may occur at such times as to augment the magnitude of the system oscillation. The description of the way in which the opening of a breaker to clear a line to neutral fault may give rise to very large system oscillation, is given in the paper by Mr. Fortescue previously referred to.

In analyzing stability conditions from the standpoint of short circuit, it will usually be found necessary to investigate faults located at two or more points on the system and to use different values of fault resistance, in order to obtain an adequate conception of the operation of the system. The most severe fault is likely to occur on the high-voltage line close to a bus or sectionalizing point. The value of the resistance at the fault and in the neutral connections should be estimated as closely as possible, taking into account all the available information in regard to the particular project as to location, character of soil and type of construction.

In determining the time for the opening of the circuit breaker, the operating range should be estimated, and the time within this interval which would produce the most severe disturbance should be selected. Similar assumptions should be made for the opening of the second breaker.

Studies of faults should not be limited to the high-voltage lines alone, but should include the distribution system and also the low-voltage bus at generating stations or at substations.

ARTIFICIAL STABILITY

Artificial stability can best be considered as a series of periodic transients and for this reason will be investigated by the methods of transient analysis. The theoretical possibility of attaining a condition of artificial stability has been considered for some time but no calculations have hitherto been presented in substantiation.

The case chosen for investigation consisted of a synchronous generator and a similar synchronous motor connected by a reactive tie line. To simplify calculations, the effect of resistance and line capacity were neglected. The governor setting at the generator end and the shaft load at the receiving end were assumed constant. The inherent static limit with the excitation adjusted to produce normal terminal voltage was calculated to be 250,000 kw. This value is the absolute limit for constant excitation and to prove the possibility of artificial stability it is only necessary to show that the system can deliver power at normal terminal voltage in excess of 250,000 kw.

An arbitrary load of 285,000 kw. was assumed. The excitation system was so chosen as to operate at a mean value of 250 volts in order to maintain normal voltage on the terminals of the main machine at 285,000 kw. The system is, of course, in equilibrium at this point but the slightest unbalance would lead to a loss of synchronism with fixed excitation. In order to show that artificial stability can be maintained it would be necessary only to displace the system an arbitrary amount and follow the system oscillations and show that the condition does not become unstable. The rotors were displaced a total of two degrees, other conditions such as governor setting at the generating end and the shaft load at the receiving end remaining unchanged. The variations in the direct and cross component of internal voltage and the variation in exciter voltage and currents and the acclerations of the rotors were calculated as explained in the previous sections. An arbitrary assumption was necessary to fix the time at which the regulator contacts opened and closed. The contacts of the regulator were opened and closed as terminal voltage of the main machine passed through normal.

The results of the calculations are indicated in the several curves of Fig. 13. The significant factor to note is the degree to which the terminal voltage under the influence of the excitation system adhered to the normal value of 127 kv. The mechanical oscillation was so slow that the voltage regulator had time to open and close several times during an oscillation. As the actual static limit is approached the natural period increases, permitting still greater opportunity for the excitation system to function. No consideration was paid to the damping factor so that the actual result should indicate smaller amplitudes for each successive oscillation of the rotor.

For operation under heavy load conditions, the adjustment or lack of adjustment of the voltage regulators may become the controlling factor from the standpoint of stability. Special attention must be paid to voltage regulators when a system is operating in the range of artificial stability.

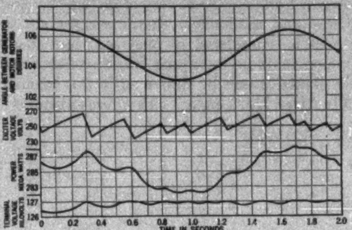

FIG. 13—THE EFFECT OF THE EXCITATION SYSTEM IN MAINTAINING ARTIFICIAL STABILITY

ANALYSIS OF STABILITY TESTS ON THE SYSTEM OF THE PACIFIC GAS AND ELECTRIC COMPANY

Cooperative stability tests were made during the early part of 1925 on the transmission system of the Pacific Gas & Electric Company. Such tests furnish accurate information as to the operation of a system at times of disturbances and are valuable for planning future expansion. In addition, these tests, which were the first of their kind, afford an opportunity to check methods of calculations and to analyze test results on an actual transmission system.

These tests are described in detail in the companion paper by Mr. Wilkins. The present discussion will be restricted to the comparison of test results and calculations. Two representative tests, one a switching operation and the other a single-phase fault, were selected for analysis.

Switching Operation. The layout of the part of the Pacific Gas & Electric Co. system involved in these tests is indicated in Fig. 14. One transmission line was operated at 110 kv. and the other at 220 kv. In test No. 12—one selected for analysis—a switching operation was performed by opening the high-voltage breaker in the 220-kv. line at the Pit River No. 1 power house, transferring all of the load to the 110-kv. line. A relatively severe switching disturbance is produced in this manner as the load on the 110-kv. line is increased to over 300 per cent of the initial value.

The application of the methods of calculation previously described to the particular problem will now be discussed. The load-end network consists of the 110-kv. and lower voltage distribution system, connected loads and local generators and the synchronous condensers at the Vaca-Dixon substation. This type of network may be considered as involving two sources of e. m. f. having a fixed angular relation and a shunt impedance load and may be analyzed by the use of one of the equivalent load-end networks. The supply-end network involves three power plants in parallel, Pit River No. 1, Hat Creek No. 1 and No. 2. Since Hat Creek No. 1 and No. 2 are very close together and their electrical and mechanical characteristics are similar, they can be considered as one unit, but due to the difference in impedance and inertia effects the Hat Creek plants cannot be combined in this manner with the Pit River No. 1 plant. This necessitates a set-up similar to that shown in Appendix I, Fig. 20, consisting of two salient pole machines at the supply end and a single equivalent machine at the load end with a shunt impedance branch representing non-synchronous load. The solution is then obtained by the step-by-step method, assuming constant gate opening for the supply end, constant direct component of voltage at the supply end generator, and constant internal voltage at the load-end machines.

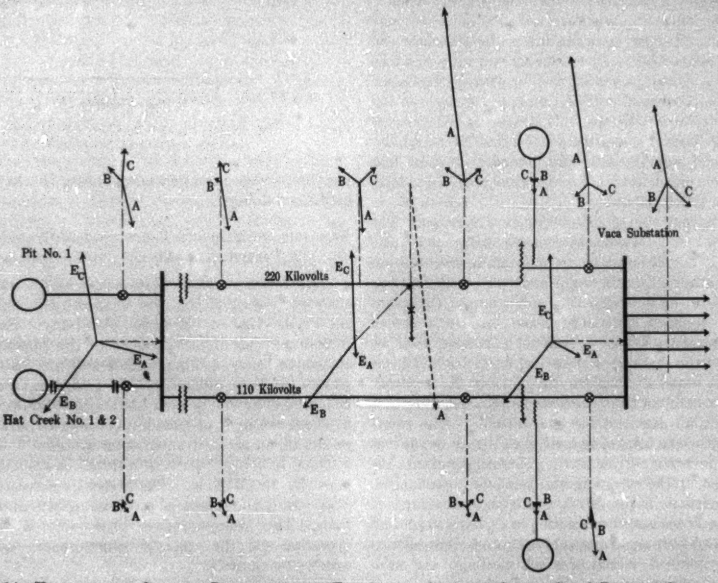

FIG. 14—VOLTAGE AND CURRENT DISTRIBUTION ON TRANSMISSION SYSTEM OF PACIFIC GAS & ELECTRIC COMPANY

With a single-phase fault to ground on 220-kv. line near Vaca substation—calculated. Voltage 30,000 volts per in.; current, 500 amperes per in., 110-kv. base

Results of the calculations of the switching operation are shown by a number of curves of power, voltage and angle plotted as functions of time from the beginning of the disturbance. In the curves of Fig. 15 are shown the calculated and observed values of power measured at Pit River No. 1. Similar curves for the generator voltage at Pit River No. 1 and the 110-kv. bus voltage at Vaca are given in Fig. 16. Fig. 17 shows the instantaneous observed and calculated angles between the rotor of the Pit generator and the voltage of the 110-kv. bus at Vaca.

It will be noted that the curves show good agreement between calculated and observed values as to general magnitude and trend. It should be explained at this time that the breaker did not open the circuit completely at the beginning of the switching disturbance. This accounts for the time displacement of about 0.2 seconds in the observed results and also for the fact that the observed oscillation is of somewhat smaller magnitude.

Tests were also made and the transients recorded when the same breaker was closed. The results of tests and calculations of power variations for this condition are shown in Fig. 18. In this case the effect of the non-simultaneous action of the different poles does not enter and a more satisfactory check is obtained.

Single-Phase Fault to Ground. The system layout for the single-phase fault to ground was the same as for the switching test previously described. The fault was applied to the 220-kv. line two towers from the Vaca-Dixon substation.

An analysis was made to determine the initial distribution of currents and voltages over the entire transmission system by the methods outlined in Appendix II. The simplified network identical to that used for the calculations of switching operation was used with the "equivalent symmetrical network" connected at the point of short circuit. The conditions of tests were unusual in that the closely coupled transmission lines were operating at different voltages.

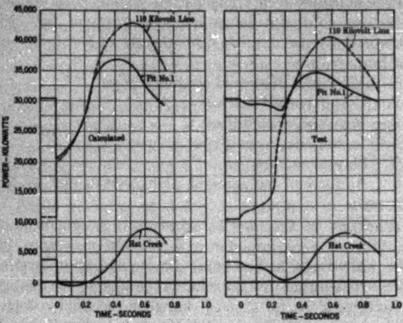

FIG. 15—COMPARISON OF TESTS AND CALCULATIONS OF POWER—OPENING ONE LINE

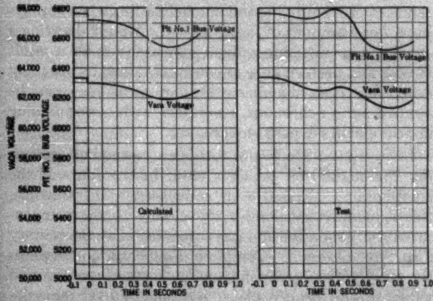

FIG. 16—COMPARISON OF TESTS AND CALCULATIONS FOR VARIATIONS IN VOLTAGE OF PIT NO. 1 BUS AND VACA 110-KV. BUS—OPENING ONE LINE

The results of calculations are shown in Fig. 14 which gives the vector distribution of the currents and voltages at different points in the system. In this figure all the vectors are given on a common voltage basis of 110 kv. and to obtain the actual values of current and voltages in any particular part of the circuit, it is necessary to take into consideration the transformer ratios and phase shifts due to the star-delta transformations. The diagram assumes the normal direction of power flow from the Pit end toward the load. The fault actually occurred on phase *B* but for convenience in calculation it was assumed to occur on phase *A*. It will be noted that at the time of fault the negative phase sequence current supplied from the condensers at Vaca are greater than the positive sequence components. The pronounced distortion of the current vectors is due principally to the relatively low value of the transmitted load currents at the time of the tests.

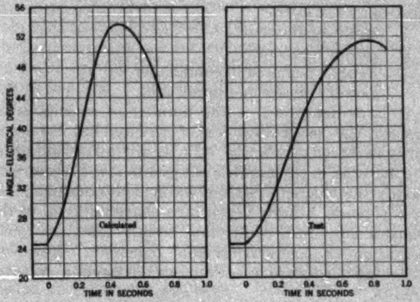

FIG. 17—COMPARISON OF TESTS AND CALCULATIONS OF ANGLE BETWEEN VACA BUS VOLTAGE AND ROTOR OF PIT NO. 1 GENERATOR—OPENING ONE LINE

In Table V is given the comparison of calculated voltages and currents with the corresponding quantities observed during the tests. The calculated values are based on the instantaneous symmetrical value of short circuit current and the test results are based on the values during the fourth cycle after the application of the fault. The close agreement, considering the factors involved, serves as a sufficient check upon the general method.

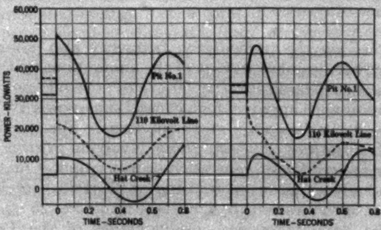

FIG. 18—COMPARISON OF TESTS AND CALCULATIONS OF POWER—CLOSING ONE LINE

METHODS OF INCREASING STABILITY

Discussion of the methods of determining stability under various operating conditions leads directly to the consideration of methods of increasing stability.

The most evident method of increasing the stability limit of a transmission system for a given voltage is to decrease the series impedance. This may be accomplished by the use of:

1. Additional circuits.
2. Lower frequency.
3. Percy Thomas split conductor.
4. Lower transformer impedance.

It is evident that if the number of circuits is increased, the power limit is also increased but not necessarily in direct proportion. To secure full advantage of the increase in the number of circuits it is necessary to modify the characteristics of the synchronous machines at both ends, otherwise the high charging kv-a. of the transmission lines will not permit adequate field excitation of the synchronous machines. However, switching surges are less severe with a larger number of circuits.

TABLE V

COMPARISON OF MEASURED AND CALCULATED INSTANTANEOUS SYMMETRICAL VOLTAGES AND CURRENTS FOR FAULT TO GROUND AT VACA-DIXON SUBSTATION

	Measured	Calculated
1. Residual current in circuit breaker at Vaca	1020	1130
2. Voltage, "A" phase at Vaca	60,000	59,300
3. Residual voltage, 110-kv. bus at Vaca	14,200	13,300
4. Residual current in 220-kv. line at Pit No. 1	* 140	138
5. Current, "B" phase, 220-kv. line at Pit No. 1	235	280
6. Current "A" phase, Pit No. 1 generator*	2780	4150
7. Voltage, "A" phase, Pit No. 1 generator	4840	4996
8. Current, "A" phase, 110-kv. line low side at Pit No. 1	1070	1030

*Discrepancy unexplained.

Lowering the transmission frequency also increases the power limit but in view of the unquestionable trend toward standardization at 60 cycles[*] the scheme does not meet with approval for a general superpower system with its attendant interconnections. For peculiar local situations, such as those involving existing 25-cycle apparatus for generation or utilization, it may be found advantageous to transmit at 25 cycles especially if part of the transmitted load is to be used at 25 cycles.

The advantages and disadvantages of the Percy Thomas split conductor scheme[†] have been discussed at various times.

The possibilities of reducing the impedance of transformers are limited, due to the small reduction obtainable at reasonable increase in cost.

A scheme closely related to those mentioned above is one providing additional sectionalizing stations in order to reduce the increase in impedance arising from the disconnection of a section of line. Such a scheme involves the ordinary bussing arrangements and the necessary automatic circuit breakers and relays. This scheme is most useful in increasing stability for switching operations.

Another important general method for increasing the power stability of a transmission system is to employ measures to maintain or increase terminal voltages. This may be accomplished by the use of the following methods:

1. Machines of special characteristics.
2. Compensated machines.
3. High speed excitation.
4. Intermediate condenser stations.
5. Shunt reactors.
6. Additional tie lines at the receiver end of the system.

Machines of Special Characteristics. A synchronous machine of zero impedance inherently maintains its terminal voltage at the value determined by its excitation. The larger this impedance the smaller the change in reactive kv-a. for a given change in terminal voltage, or the greater the change in voltage of the line to which the machine is connected for a given change in load or in the network impedance. Thus it can be seen that a machine's ability to supply lagging kv-a. with drop in voltage is a measure of its desirability from the standpoint of maintaining stability. To supply this large amount of lagging kv-a. with drop in voltage, a synchronous machine should have low leakage reactance, as this is initially the limiting factor and in addition should have a "strong field and weak armature" which tends to prevent reduction in machine flux when supplying a large amount of lagging kv-a. Merely derating a synchronous machine will not give as favorable a result in cost and performance as can be obtained by specially designing the machine. Similar measures can be applied to synchronous generators with corresponding improvements in performance.

Compensated Machines. Another method of improving the regulation presents itself in the form of the compensated alternator or condenser.[*] In principle, this scheme corresponds very closely to the compensated d-c. generator and was suggested in an elementary form by Latour.[†] Complete compensation requires the supplying of compensating currents of the required value in the proper phase position to overcome the effects of armature reaction, both demagnetizing and cross-magnetizing. The design of the compensating winding may be proportioned to compensate for armature reaction or to include the effect of leakage reactance or even to over-compensate to any desired degree. The machine may be protected from short circuits by designing the series transformers so as to saturate without seriously affecting the compensation for ordinary changes in load conditions. In the event of a short circuit between the alternator or condenser and the series transformers, the machine will act like an ordinary high-impedance unit.

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[*] Bibliography, Item 22.

[†] Bibliography, Item 25.

[*] Bibliography, Item 2.

[†] U. S. Patent, No. 869185.

The compensated synchronous condenser is a particularly attractive possibility. Since the condenser is required to supply only reactive kv-a., it is not necessary to provide compensation except for demagnetization. Thus the compensated condenser merely requires an additional field winding whose d-c. exciting current varies in direct proportion to armature current of the condenser. This may be accomplished by means of a suitable series transformer and rectifier arrangement.

Recent investigations of the stability problem appear to make desirable a reconsideration of compensated machines, the development of which was checked by the introduction of the Tirrill regulator.

While no inherent defect is apparent in such machines, a certain amount of time will be required to develop their possibilities.

High Speed Excitation. Instead of increasing excitation by inherent action as in the compensated machine mentioned above, it is possible to do this by automatic devices. The ordinary automatic voltage regulator does it to some extent, but there is an important time lag involved in the response of the regulator relays and in the building up of the exciter voltage. However, it is possible to accelerate this corrective action so as to actually increase the total machine flux. The time required for the regulator and excitation system to function, therefore, becomes very important. Because of the high time constant of large a-c. machine fields it is a relatively difficult matter to increase the main machine flux. Consequently the principal object of a high-speed excitation system is to maintain the main machine flux during any disturbance tending to pull the system out of step.

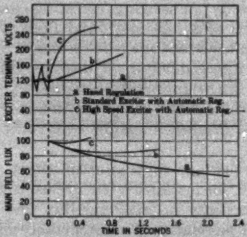

FIG. 19—EFFECT OF HIGH SPEED EXCITER IN MAINTAINING FLUX IN ALTERNATOR WHEN ARMATURE CURRENT IS SUDDENLY INCREASED FROM ZERO TO 100 PER CENT LAGGING 90 DEG.

In order to reduce the time of response of the regulator and exciter in case of a disturbance, there has been developed a high speed system of excitation involving special relays and exciters of low time constants. Tests were made on a 10,625-kv-a. 100-rev. per min. machine equipped with standard and special exciters. A test was made by throwing off leading kv-a. load, so as to simulate the condition of a sudden demand for supplying lagging kv-a. such as would occur in the case of a short circuit. In Fig. 19 are shown certain curves prepared from the test data obtained, which indicate the change in exciter terminal voltage and in the flux of the main alternator field, when the armature current is increased from zero to rated value at zero per cent power factor. Curves marked *a* are for constant excitation; curves marked *b* are for the standard excitation system; and curves marked *c* are for the high speed excitation system.

As has been pointed out previously, actual static limits depend upon the rate of change of voltage with respect to time, and the critical point depends upon angular movements which are slow for relatively high loads. It has been shown in connection with "artificial" stability that for steady loads standard exciters are sufficiently fast. For greater demand in reactive kv-a. such as occasioned by system disturbances faster excitation systems are required. High-speed excitation should be capable of preventing any important reduction in machine flux during transient conditions, and test results have indicated that it is permissible with this type of excitation system to assume that the direct component of machine flux remains substantially constant during the disturbance, that is, the only drop within a machine during a disturbance is that due to leakage reactance.

Intermediate Condenser Stations. The scheme of placing synchronous condenser stations at intermediate points between the supply and receiver ends of a transmission system has been discussed before and it constitutes a vital element in the Baum system of power transmission. With a condenser station capable of maintaining voltage at the point of connection to the transmission line, the static limit of the system approaches the limits of the individual sections. With commercial apparatus, the limit is determined by the degree to which the excitation system is capable of maintaining this voltage in spite of the reactive drop in transformers and condensers. Because of the factors which tend to bring out a condition of artificial stability previously discussed, the static limits of systems provided with intermediate condenser stations are relatively high. From the standpoint of transient conditions such as switching operation and short circuit, there is initially a marked tendency to maintain machine flux which is supplemented by the effect of the excitation system.

These natural tendencies of the intermediate condenser station may be improved by the use of condensers of special inherent characteristics, particularly with low leakage reactance by transformers of low reactance, and by high-speed excitation systems. The compensated condenser seems to offer good promise by taking from the intermediate synchronous condenser station its chief limitation, that due to the inherent regulation of the condenser and transformer.

Shunt Reactors. The use of shunt reactors* has been proposed as a means for increasing the stability of transmission systems. Such reactors are operated in conjunction with over-excited synchronous generators or condensers. In case of a drop in terminal voltage, the lagging kv-a. drawn by the reactor decreases rapidly, thus producing in effect a large increase in leading kv-a. with drop in terminal voltage. In connection with this scheme, it is to be pointed out that under normal operating conditions the shunt reactor draws a large amount of reactive kv-a. which must be supplied in additional condenser capacity or increased capacity of generators.

Additional Tie Lines. The interconnection of local power networks at the receiving end or at other points of a transmission system, tends to increase the stability because the local power network tends to prevent a drop in the transmission voltage by supplying an increased amount of leading kv-a. The effect may be increased by "stiffening" the tie lines in the local power network, and for the interconnection. This method is, in effect, one of improving the load characteristics of the transmission system.

Governor Control. Instead of employing various devices to increase power limits either continuously or for a short time following a disturbance, there is a possibility of decreasing the power input for a short time following a disturbance. The reduction in power input to the transmission system would tend to reduce the system frequency, which would cause the governors on the prime movers at the receiving end of the system to operate in an effort to maintain the frequency. Thus the power transmitted over the transmission lines would be reduced, but the two ends of the system would stay in step and slow down together. After the disturbance, the power input at the supply end would be increased automatically, thus returning the system to its normal operating condition.

The reduction in input would be accomplished by the use of special relays which would act upon the governor and start corrective action in advance of the change in speed normally used to actuate the governor mechanism. The relay could, for example, be actuated from ground current and would tend to reduce the power input from the beginning of the fault condition, whereas an ordinary governor would initially tend to increase the input. Another scheme would be to employ a relay in which a torque proportional to gate opening is balanced against the torque of a wattmeter element measuring generator output.

It is believed that the double transients which occur at times of faults to ground are of sufficient duration to give the special governor control time to make an important reduction in gate opening and in power input.

In a transmission system in which the governor is operated "against a block," the special governing relay would take control of gate opening in a manner similar to that now in use for preventing excessive overspeed.

General. In order to increase the stability of a transmission system in the event of a single-phase short circuit, a number of rather special measures have been suggested and a discussion of them is useful in explaining the action taking place. Some of the suggestions are given below:

1. Layouts arranged to limit short-circuit current
2. Neutral impedances
3. High-speed circuit breakers
4. Artificial control of generator speed
5. Single-pole circuit breakers

The layout of a transmission system may be modified so as to limit the magnitude of short-circuit currents. This can be accomplished by the use of reactors or transformers, and the avoidance of the bussing arrangements ordinarily employed. This method is open to the objection that it is not sufficiently flexible to meet the requirements of changing operating conditions.

In the event of a line-to-ground fault the reduction of short-circuit current may be obtained by the use of neutral impedance devices. However, this method is not viewed with approval, because experience has shown that the solidly grounded neutral system provides the best insurance against unfavorable operating experiences.

If the short circuit were interrupted in a very small period of time, for instance a few cycles, it would follow that the transients in machine flux and mechanical inertia would be relatively unimportant, and the short-circuit condition would reduce to that of a switching operation. At the present time, there are no high-voltage breakers capable of operating within any such period of time.

As has been brought out in the discussion of the single-phase faults, the action may be to accelerate or decelerate the generator rotors by a very large amount, and the mechanical transients thus produced may become controlling factors from the standpoint of stability. To avoid this condition, it has been proposed to place automatic control on the load delivered by the generator or on the governor supplying power to the generator unit, so as to prevent large variations in the generator output.

The use of single-pole breakers, with suitable relay systems for disconnecting the faulty conductor instead of the faulty section of line, has been proposed. Subsequently, the remaining conductors of the faulty section would be disconnected automatically. By this means, the stability of the system would be increased temporarily until the effects of the mechanical transient have been spent, and the fields of the synchronous machines could be increased.

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*Bibliography, Item 16.

The authors wish to take this opportunity to express their appreciation of the cooperation and assistance of the Engineers of the Pacific Gas & Electric Company, especially Messrs. J. P. Jollyman and Roy Wilkins, in affording opportunity of investigating stability conditions on a large high-voltage power system. In addition, they also wish to acknowledge the assistance of their associates, particularly Mr. S. B. Griscom, who has made important contributions to the methods presented.

CONCLUSIONS

Stability is an important limitation in the transmission of large amounts of power per circuit over long distances. Provision for securing stability should be made for both steady state and the more frequently occuring abnormal conditions. The limiting condition, as indicated by these studies, is the single-phase fault-to-ground, followed by the disconnection of the faulty section of line. Normal switching operations, except in a few special cases, will not give rise to severe system disturbances. Under emergency conditions when a large proportion of the lines are out of service, the system may approach the limit of static stability.

The static limit of systems operating without voltage regulators is determined by the inherent characteristics of the machines and lines. With voltage regulators, a condition of artificial stability may be obtained on commercial systems and the machines tend to operate on a constant voltage rather than a constant excitation characteristic.

Methods have been presented for the analysis of stability for the various conditions that arise in system operation. The principal new features are the treatment of machine characteristics, the simplification of load-end networks, and the handling of single-phase faults.

Increase in the power or stability limits may be obtained by the use of a number of methods discussed in the paper. Of particular importance are high-speed excitation and machines of special design capable of delivering a relatively large increase in lagging kv-a. with drop in voltage. Mention should also be made of special governor control.

The underlying requirement for securing stability of a system delivering a large amount of power is one of obtaining good regulation of line and terminal equipment. Improved stability conditions may be obtained by methods providing good regulation, either inherently as by compensated machines, or automatically by high-speed excitation systems.

The use of intermediate condenser stations permits the transmission of the largest amount of power per circuit. The effectiveness of this scheme may be considerably increased by the use of specially designed condensers or compensated condensers, or by high-speed excitation.

Stability is important not only for high-voltage long-distance transmission but also for low-voltages and interconnections where relatively large amounts of power are handled per circuit.

The method of operating a power system, including setting of relays and circuit breakers, sequence of switching operations, governor adjustments, etc., has an important bearing on stability. An analysis of these factors as they affect stability should lead to the selection of the operating method which would minimize system disturbances.

Appendix I

INCLUSION OF CROSS FLUX IN TRANSMISSION CALCULATIONS

It has been shown that in salient pole, synchronous machines without dampers or with high resistance dampers, it is sufficiently accurate to assume that the cross component of flux changes instantly. With this assumption and a knowledge of the phase position and magnitude of E_d in all the machines and the constants of the network, it is possible to calculate the current and power distribution.

In the vector notation used in this section of the Appendix, the ordinary vector will be designated by an inverted caret, as $\check{E}$, and its conjugate by the ordinary caret, as $\hat{E}$.

As before stated, the scalar value of E_c is equal to $K_2\, I \cos \phi$, which in vector notation is equal to

$$\frac{K_2}{E_d}\left(\frac{\check{E}_d\,\hat{I} + \hat{E}_d\,\check{I}}{2}\right)$$

Since $\check{E}_c$ lags $\check{E}_d$ by 90 deg., its direction is determined by $-j\,\dfrac{\check{E}_d}{E_d}$. $\check{E}_c$ is then completely determined by the following expression:

$$\check{E}_c = -j\,\frac{\check{E}_d}{E_d}\,\frac{K_2}{E_d}\left(\frac{\check{E}_d\,\hat{I} + \hat{E}_d\,\check{I}}{2}\right) \qquad (1)$$

The total internal voltage, $\check{E}_a$, is equal to the sum of $\check{E}_d$ and $\check{E}_c$.

$$\check{E}_a = \check{E}_d - j\,\frac{K_2}{2}\,\frac{\check{E}_d}{E_d^2}\,(\check{E}_d\,\hat{I} + \hat{E}_d\,\check{I}) \qquad (2)$$

But $\check{E}_a$ is also equal to $\check{E}_t + \check{Z}\,\check{I}$
where
$\check{E}_t$ is the terminal voltage, and
$\check{Z}$ the impedance of the machine.
Equating these values of $\check{E}_a$

$$\check{E}_t + \check{Z}\,\check{I} = \check{E}_d - j\,\frac{K_2}{2}\,\frac{\check{E}_d}{E_d^2}\,(\check{E}_d\,\hat{I} + \hat{E}_d\,\check{I}) \qquad (3)$$

Now let $\check{I} = a + j\,b$ $\hat{I} = a - j\,b$
Substituting and collecting terms:

$$a\left\{\check{Z} + j\,\frac{K_2}{2}\,\frac{\check{E}_d}{E_d^2}\,(\check{E}_d + \hat{E}_d)\right\}$$

$$+ j\, b \left\{ \check{Z} + j \frac{K_2}{2} \frac{\check{E}_d}{E_d^2} (\hat{E}_d - \check{E}_d) \right\} = \check{E}_d - \check{E}_t$$

or

$$\check{M}\, a + j\, \check{N}\, b = \check{E}_d - \check{E}_t \qquad (4)$$

where

$$\check{M} = \check{Z} + j \frac{K_2}{2} \frac{\check{E}_d}{E_d^2} (\check{E}_d + \hat{E}_d) \qquad (5)$$

$$\check{N} = \check{Z} + j \frac{K_2}{2} \frac{\check{E}_d}{E_d^2} (\hat{E}_d - \check{E}_d) \qquad (6)$$

Let $\check{E}_d = E_d\, \epsilon^{j\delta}$ (7)

Then $\hat{E}_d = E_d\, \epsilon^{-j\delta}$ (8)

and $\check{M}$ and $\check{N}$ reduce to the following expressions:

$$\check{M} = \check{Z} + j\, K_2 \cos \delta\, (\cos \delta + j \sin \delta) \qquad (9)$$

$$\check{N} = \check{Z} + K_2 \sin \delta\, (\cos \delta + j \sin \delta) \qquad (10)$$

The above expression defines the components of current in terms of the constants of the machine, the direct component of voltage and the terminal voltage.

The application of this formula will be illustrated in two of the more frequently occurring cases. Consider first the system shown in Fig. 20 in which the generating station, consisting of salient pole machines, is connected to the load through a transmission line and transformers, and the load-end network is replaced by a shunt load and a synchronous turbo generator with damper windings and internal voltage equal to E_{a2}. Regardless of the complexity of the network between the load-end generator and the supply generator, provided only that the impedances are constant, as shown in Appendix II, the following relation exists:

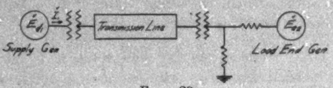

FIG. 20

$$\check{E}_{t1} = \frac{1}{\check{D}} \check{E}_{a2} + \frac{\check{B}}{\check{D}} \check{I}_1 \qquad (11)$$

where

$\check{E}_{t1}$ = Terminal voltage at supply generator

$\check{I}_1$ = Current in supply generator

and $\check{B}$ and $\check{D}$ are general circuit constants of the network between $\check{E}_{a2}$ and $\check{E}_{t1}$.

The generator conditions furnish the following relation:

$$\check{M}_1\, a_1 + j\, \check{N}_1\, b_1 = \check{E}_{d1} - \check{E}_{t1} \qquad (12)$$

Eliminating $\check{E}_{t1}$ and combining terms, the following equation results:

$$\left(\check{M}_1 + \frac{\check{B}}{\check{D}}\right) a_1 + j \left(\check{N}_1 + \frac{\check{B}}{D}\right) b_1 = \check{E}_{d1} - \frac{1}{\check{D}} \check{E}_{a2} \qquad (13)$$

Since both $\check{E}_{d1}$ and $\check{E}_{a2}$ will be known in phase position and magnitude at any instant, the equation can be expanded, real and imaginary components equated, respectively, and solutions for a_1 and b_1 obtained.

The next illustration consists of two hydraulic power houses supplying power to a transmission line, the tie between the plants being sufficiently short to render their charging currents negligible. For example, suppose plant No. 1 in Fig. 21 tied directly to the high-tension line through transformers and plant No. 2 located, say, thirty miles away and connected to the same line. As in the previous case, the transmission line feeds into a network which can be replaced by an equivalent shunt load and an equivalent generator having an internal voltage $\check{E}_3$.

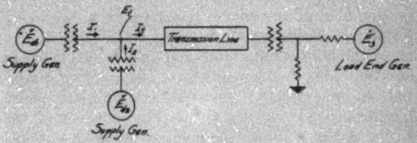

FIG. 21

Let

$\check{E}_t$ = Voltage at point of tie.

$a_1 + j\, b_1$ = Current in generator No. 1.

$a_2 + j\, b_2$ = Current in generator No. 2.

$\check{I}_t$ = Total current in line at supply end.

$\check{E}_{d1}$ = Direct component of voltage in generator No. 1.

$\check{E}_{d2}$ = Direct component of voltage in generator No. 2.

$\check{A}, \check{B}, \check{C}$, and $\check{D}$ = General circuit constants, for network between $\check{E}_t$ and $\check{E}_3$.

From generator conditions:

$$\check{M}_1\, a_1 + j\, N_1\, b_1 = \check{E}_{d1} - \check{E}_t \qquad (14)$$

$$\check{M}_2\, a_2 + j\, \check{N}_2\, b_2 = \check{E}_{d2} - \check{E}_t \qquad (15)$$

and from line conditions:

$$\check{I}_t = \frac{\check{D}_3}{\check{B}_3} \check{E}_t - \frac{1}{\check{B}_3} \check{E}_3 \qquad (16)$$

$$\check{I}_t = (a_1 + j\, b_1) + (a_2 + j\, b_2) \qquad (17)$$

Eliminating $\check{I}_t$ and $\check{E}_t$, the following expressions are obtained:

$$\left(\frac{\check{D}_3}{\check{B}_3} \check{M}_1 + 1\right) a_1 + a_2 + j \left(\frac{D_3}{\check{B}_3} \check{N}_1 + 1\right) b_1 + j\, b_2$$

$$= \frac{\check{D}_3}{\check{B}_3} \check{E}_{d1} - \frac{\check{E}_3}{\check{B}_3}$$

$$a_1 + \left(\frac{\check{D}_3}{\check{B}_3} \check{M}_2 + 1\right) a_2 + j\, b_1 + j \left(\frac{\check{D}_3}{\check{B}_3} N_2 + 1\right) b_2$$

$$= \frac{\check{D}_3}{\check{B}_3} \check{E}_{d2} - \frac{\check{E}_3}{\check{B}_3}$$

Let

$$\frac{\check{D}_3}{\check{B}_3}\check{B}_1+1=p_1+j\,q_1;K_1\frac{\check{D}_3}{\check{B}_3}=r_1+j\,s_1;K_2\frac{\check{D}_3}{\check{B}_3}=r_2+j\,s_2 \tag{18}$$

$$\frac{\check{D}_3}{\check{B}_3}\check{M}_1+1=p_1+j\,q_1+j(r_1+j\,s_1)\,(\cos\delta_1+j\sin\delta_1)\cos\delta_1$$

$$=A_1+j\,A_2 \tag{19}$$

$$\frac{\check{D}_3}{\check{B}_3}N_1+1=p_1+j\,q_1+(r_1+j\,s_1)\,(\cos\delta_1+j\sin\delta_1)\,\sin\delta_1$$

$$=B_1+j\,B_2 \tag{20}$$

$$\frac{\check{D}_3}{\check{B}_3}\check{M}_2+1=p_2+j\,q_2+j(r_2+j\,s_2)\,(\cos\delta_2+j\sin\delta_2)\cos\delta_2$$

$$=C_1+j\,C_2 \tag{21}$$

$$\frac{\check{D}_3}{\check{B}_1}\check{N}_2+1=p_2+j\,q_2+(r_2+j\,s_2)\,(\cos\delta_2+j\sin\delta_2)\,\sin\delta_2$$

$$=D_1+j\,D_2 \tag{22}$$

Further, let

$$\frac{\check{D}_3}{\check{B}_3}\,\check{E}_{d1}-\frac{\check{E}_3}{\check{B}_3}=Y_1+j\,Y_2 \tag{23}$$

$$\frac{\check{D}_3}{\check{B}_3}\,\check{E}_{d2}-\frac{\check{E}_3}{\check{B}_3}=Y_3+j\,Y_4 \tag{24}$$

Substituting, the following equations are obtained:

$$(A_1+j\,A_2)\,a_1+a_2+j(B_1+j\,B_2)\,b_1+j\,b_2=Y_1+j\,Y_2 \tag{25}$$

$$a_1+(C_1+j\,C_2)\,a_2+j\,b_1+j(D_1+j\,D_2)\,b_2=Y_3+j\,Y_4 \tag{26}$$

Equating real and imaginary components:

$$A_1\,a_1+a_2-B_2\,b_1+O=Y_1 \tag{27}$$

$$A_2\,a_1+O+B_1\,b_1+b_2=Y_2 \tag{28}$$

$$a_1+C_1\,a_2+O-D_2\,b_2=Y_3 \tag{29}$$

$$O+C_2\,a_2+b_1+D_1\,b_2=Y_4 \tag{30}$$

These equations can be expanded directly by means of determinants or the a_1 terms can be eliminated and the remainder expanded.

While this method might appear extremely laborious, examination of the expressions defining the constants A_1, A_2, B_1, etc., shows that they consist of a constant term plus a variable that depends upon the angle only. With but little labor, curves can be plotted from which the values of the constants can be taken. The solution of the four simultaneous equations is also simpler than would at first appear because of the fact that several of the coefficients are equal to zero or unity. After a_1, a_2, b_1 and b_2 have been obtained, the voltage and power distribution over the system is readily calculated.

Appendix II

CALCULATION OF SINGLE-PHASE, SHORT-CIRCUIT CURRENTS ON TRANSMISSION NETWORKS

The determination of single-phase, short-circuit currents on extensive transmission networks is different from the customary methods of short-circuit calculation in two principal ways, because of (1) the dissymmetry of the circuits, and (2) because the effects of charging currents and load currents cannot well be ignored on extensive transmission systems possessing high reactance. The method here presented is the combination of two well-known methods of network solution, namely phase sequence components and general circuit constants.

On three-phase, four-wire or grounded neutral systems, the voltages (or currents) of the line conductors may be represented by three vector expressions. It has been shown by C. L. Fortescue* that these voltages can be replaced by three symmetrical components of voltage designated "positive sequence," "negative sequence" and "zero sequence." The positive sequence system of voltages consists of three voltages of equal magnitude displaced 120 deg. in phase position, as indicated in Fig. 22A. The negative sequence system of voltages consists of three voltages also equal in magnitude and displaced 120 deg. in phase position as indicated in Fig. 22B. It will be noted that the sequence of voltage maxima of the negative sequence system is opposite to that of the positive sequence system. This must not be confused with the direction of rotation of the vectors which is counter-clockwise for all vectors. The zero sequence system of voltages consists of three voltages of equal magnitude, and displaced zero degrees from each other, as indicated in Fig. 22C. It is to be understood that the different sequence systems of voltages are, in general, unrestricted as to magnitude or phase relation with respect to each other. By a suitable combination of these three systems of voltages, any combination of voltages on the three-phase system can be obtained; or conversely, any combination of three-phase voltages can be resolved into these three-phase sequence components. The above discussion applies to currents as well as to voltages.

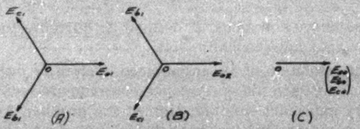

FIG. 22—SEQUENCE VOLTAGE DIAGRAM

A. Positive sequence voltage. B. Negative sequence voltage. C. Zero sequence voltages. Voltages from line to neutral in phases *a*, *b*, and *c*.

The relationship between the phase-to-neutral voltages and the phase sequence components may be expressed as follows:†

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*Bibliography, Item 19.

†In this section of the Appendix, voltages, currents and general circuit constants will be considered as complex quantities without special designation; and the numerical subscripts will indicate the different phase sequences; 1—positive sequence; 2—negative sequence, and 0—zero sequence.

$$E_a = E_{r0} + E_{r1} + E_{r2} \qquad (1)$$

$$E_b = E_{r0} + \alpha^2 E_{r1} + \alpha E_{r2} \qquad (2)$$

$$E_c = E_{r0} + \alpha E_{r1} + \alpha^2 E_{r2} \qquad (3)$$

In the above equations, the phase-to-neutral voltages are given by E_a, E_b and E_c; positive, negative and zero sequence components of voltage by E_{r1}, E_{r2}, and E_{r0} respectively;

$$\alpha = -0.5 + j\frac{\sqrt{3}}{2}$$

and

$$\alpha^2 = -0.5 - j\frac{\sqrt{3}}{2}$$

The above equations may be solved for the phase sequence components with the following results:

$$E_{r1} = \frac{E_a + \alpha E_b + \alpha^2 E_c}{3} \qquad (4)$$

$$E_{r2} = \frac{E_a + \alpha^2 E_b + \alpha E_c}{3} \qquad (5)$$

$$E_{r0} = \frac{E_a + E_b + E_c}{3} \qquad (6)$$

The above equations, if defined as currents instead of voltages, apply equally well.

The resolution of an unbalanced system of voltages or currents into phase sequence components results in a very important simplification of the problem for the important case we are considering, in which the impedances and admittances are symmetrical in each phase. This simplification resides in the fact that each phase-sequence component may be analyzed independently and the solutions combined. This arises from the fact that one sequence component of voltage at one point is proportional to the same component of voltage at another point added to a voltage drop due to that component of current flowing from the second point to the first. In other words, for symmetrical impedances, the voltage drops produced by currents of one sequence are not affected by the other sequences. These relations may be more accurately and concisely stated by the following equations:

$$E_{x1} = A_{x1} E_{r1} + B_{x1} I_{r1} \qquad (7)$$

$$I_{x1} = C_{x1} E_{r1} + D_{x1} I_{r1} \qquad (8)$$

$$E_{r1} = + D_{x1} E_{x1} - B_{x1} I_{x1} \qquad (9)$$

$$I_{r1} = - C_{x1} E_{x1} + A_{x1} I_{x1} \qquad (10)$$

where E_{x1}, E_{r1}, I_{x1} and I_{r1} are respectively the positive sequence, supply and receiver voltages and currents for the two terminals of any network, and where A_{x1}, B_{x1}, C_{x1} and D_{x1} are the constants of the network for positive sequence. The use of four constants in this manner to define network conditions was developed by Evans and Sels‡ and the constants are termed general circuit constants to indicate that they apply to any type of network having constant impedance and constant admittance branches. Similar expressions may be written for the other phase sequences; in fact, it is necessary only to change the numerical subscripts. The general circuit constants have, in general, different numerical values for the different phase sequences.

The particular way in which the method has been worked out is one of determining an equivalent network with an equivalent voltage and general circuit constants to represent the complicated network under consideration. As steps in this method, formulas are developed for combining in parallel two network branches, each with a source of voltage, and also for combining two networks in series. By a suitable combination, complicated networks of many branches may be reduced to a single network having a single equivalent supply voltage and a single set of general circuit constants for the positive, negative and zero sequences. Formulas will then be derived for the determination of the short-circuit currents on the equivalent network.

Simple General Network. A simple general network will now be considered. As will be shown later, the equivalent network will be identical in form, and the notation that is employed will be the same as that to be used with the equivalent network. Reference may be made to Fig. 22 for the diagram of connections and the notation employed. The fault is assumed to be at the receiver end of the network, but the notation is of general application if the subscript r indicates the point on the three-phase system at which the fault occurs. The general equations for a single network branch with a single source of voltage such as a generator may be written as follows:

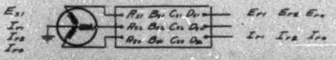
E_{x1}
I_{x1}
I_{x2}
I_{x0}
A_{x1} B_{x1} C_{x1} D_{x1}
A_{x2} B_{x2} C_{x2} D_{x2}
A_{x0} B_{x0} C_{x0} D_{x0}
E_{r1} E_{r2} E_{r0}
I_{r1} I_{r2} I_{r0}

FIG. 23—EQUIVALENT GENERAL NETWORK

General network with single source of voltage with

E_{x1} = Equivalent supply voltage, positive sequence

I_{x1}, I_{x2}, I_{x0} = Currents at supply end, positive sequence, negative sequence and zero sequence respectively

E_{r1}, E_{r2}, E_{r0} = Voltages at receiver end, positive sequence, negative sequence and zero sequence respectively

I_{r1}, I_{r2}, I_{r0} = Currents at receiver end, positive sequence, negative sequence and zero sequence respectively

A_{x1}, B_{x1}, C_{x1}, D_{x1} = Positive sequence general circuit constants

A_{x2}, B_{x2}, C_{x2}, D_{x2} = Negative sequence general circuit constants

A_{x0}, B_{x0}, C_{x0}, D_{x0} = Zero sequence general circuit constants

Machine impedances are included as part of network $_x$, and the supply voltage considered is the internal voltage of the synchronous machine supplying the network.

$$E_{x1} = A_{x1} E_{r1} + B_{x1} I_{r1} \qquad (11)$$

$$O = A_{x2} E_{r2} + B_{x2} I_{r2} \qquad (12)$$

$$O = A_{x0} E_{r0} + B_{x0} I_{r0} \qquad (13)$$

Since symmetrical synchronous machines generate only positive sequence e. m. fs., the supply voltage for this network has no negative or zero sequence components. On this account, it is convenient to consider the point of short circuit as the source of negative and zero sequence voltages which circulate currents of negative and zero sequence respectively. The equations (**11**) to (**13**) may be written in the form shown below:

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‡Bibliography, Item 6.

$$I_{r1} = \frac{E_{x1}}{B_{x1}} - \frac{A_{x1}}{B_{x1}} E_{r1} \tag{14}$$

$$I_{r2} = - \frac{A_{x2}}{B_{x2}} E_{r2} \tag{15}$$

$$I_{r0} = - \frac{A_{x0}}{B_{x0}} E_{r0} \tag{16}$$

It will be noted that the solution given above is expressed in terms of receiver voltages and currents, and equivalent supply voltage and general circuit constants. Also it will be noted that the receiver currents are expressed in terms of receiver voltages and four ratios which will be computed for actual networks as shown subsequently.

Two Networks in Parallel. The case of two networks in parallel, each having a source of voltage, will now be considered. Reference may be made to Fig. 24 for the diagram of connections and the notation employed. The notation is made more definite by the equations employing the general circuit constants as given below:

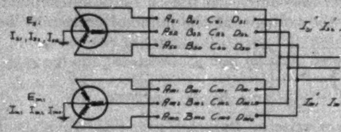

FIG. 24—TWO GENERAL NETWORKS IN PARALLEL. (REFER TO FIG. 22 FOR EXPLANATION OF NOTATION)

$$E_{s1} = A_{s1} E_{r1} + B_{s1} I_{s1} \tag{17}$$

$$O = A_{s2} E_{r2} + B_{s2} I_{s2} \tag{18}$$

$$O = A_{s0} E_{r0} + B_{s0} I_{s0} \tag{19}$$

$$E_{m1} = A_{m1} E_{r1} + B_{m1} I_{m1} \tag{20}$$

$$O = A_{m2} E_{r2} + B_{m2} I_{m2} \tag{21}$$

$$O = A_{m0} E_{r0} + B_{m0} I_{m0} \tag{22}$$

$$I_{r1} = I_{s1} + I_{m1} \tag{23}$$

$$I_{r2} = I_{s2} + I_{m2} \tag{24}$$

$$I_{r0} = I_{s0} + I_{m0} \tag{25}$$

The above equations may be solved for receiver currents as indicated below:

$$I_{r1} = \left(\frac{E_{s1}}{B_{s1}} + \frac{E_{m1}}{B_{m1}} \right) - \left(\frac{A_{s1}}{B_{s1}} + \frac{A_{m1}}{B_{m1}} \right) E_{r1} \tag{26}$$

$$I_{r2} = - \left(\frac{A_{s2}}{B_{s2}} + \frac{A_{m2}}{B_{m2}} \right) E_{r2} \tag{27}$$

$$I_{r0} = - \left(\frac{A_{s0}}{B_{s0}} + \frac{A_{m0}}{B_{m0}} \right) E_{r0} \tag{28}$$

It will be noted that the above equations are similar in form to those given by equations (**14**) to (**16**) and reduce to the identical form when the four ratios are computed as indicated below:

$$\frac{E_{x1}}{B_{x1}} = \frac{E_{s1}}{B_{s1}} + \frac{E_{m1}}{B_{m1}} \tag{29}$$

$$\frac{A_{x1}}{B_{x1}} = \frac{A_{s1}}{B_{s1}} + \frac{A_{m1}}{B_{m1}} \tag{30}$$

$$\frac{A_{x2}}{B_{x2}} = \frac{A_{s2}}{B_{s2}} + \frac{A_{m2}}{B_{m2}} \tag{31}$$

$$\frac{A_{x0}}{B_{x0}} = \frac{A_{s0}}{B_{s0}} + \frac{A_{m0}}{B_{m0}} \tag{32}$$

This process may be extended so as to apply to three or more networks in parallel, and the forms of the equations are symmetrical with those indicated above.

Two Networks in Series. Reference should be made to Fig. 25 for diagram of connections and notation employed. The supply-end network should be considered as an equivalent network with an equivalent supply voltage and network constants given in the form of equations (**14**) to (**16**). The general circuit equations for the first network may be written as follows:

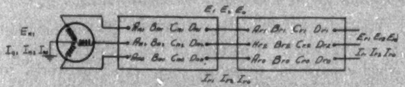

FIG. 25—TWO GENERAL NETWORKS IN SERIES. (REFER TO FIG. 22 FOR EXPLANATION OF NOTATION)

$$I_1 = \frac{E_{n1}}{B_{n1}} - \frac{A_{n1}}{B_{n1}} E_1 \tag{33}$$

$$I_2 = - \frac{A_{n2}}{B_{n2}} E_2 \tag{34}$$

$$I_0 = - \frac{A_{n0}}{B_{n0}} E_0 \tag{35}$$

The expressions for voltages and currents at the intermediate point in the two networks may be written in the form of equations (**7**) to (**10**) as given below:

$$I_1 = D_{r1} I_{r1} + C_{r1} E_{r1} \tag{36}$$

$$I_2 = D_{r2} I_{r2} + C_{r2} E_{r2} \tag{37}$$

$$I_0 = D_{r0} I_{r0} + C_{r0} E_{r0} \tag{38}$$

$$E_1 = A_{r1} E_{r1} + B_{r1} I_{r1} \tag{39}$$

$$E_2 = A_{r2} E_{r2} + B_{r2} I_{r2} \tag{40}$$

$$E_0 = A_{r0} E_{r0} + B_{r0} I_{r0} \tag{41}$$

The currents and voltages at the intermediate point may be eliminated from the equations (**33**) to (**41**) with the following results:

$$I_{r1} = \frac{E_{n1}}{A_{n1} B_{r1} + D_{r1} B_{n1}} - \frac{A_{n1} A_{r1} + B_{n1} C_{r1}}{A_{n1} B_{r1} + D_{r1} B_{n1}} E_{r1} \tag{42}$$

$$I_{r2} = -\frac{A_{n2}\,A_{r2} + B_{n2}\,C_{r2}}{A_{n2}\,B_{r2} + D_{r2}\,B_{n2}}\,E_{r2} \qquad (43)$$

$$I_{r0} = -\frac{A_{n0}\,A_{r0} + B_{n0}\,C_{r0}}{A_{n0}\,B_{r0} + D_{r0}\,B_{n0}}\,E_{r0} \qquad (44)$$

In order to combine two equivalent networks in series the following form will be found convenient.

$$I_{r1} = \frac{\dfrac{E_{n1}}{B_{n1}}}{\dfrac{A_{n1}}{B_{n1}}\,B_{r1} + D_{r1}} - \frac{\dfrac{A_{n1}}{B_{n1}}\,A_{r1} + C_{r1}}{\dfrac{A_{n1}}{B_{n1}}\,B_{r1} + D_{r1}}\,E_{r1} \qquad (45)$$

$$I_{r2} = -\frac{\dfrac{A_{n2}}{B_{n2}}\,A_{r2} + C_{r2}}{\dfrac{A_{n2}}{B_{n2}}\,B_{r2} + D_{r2}}\,E_{r2} \qquad (46)$$

$$I_{r0} = -\frac{\dfrac{A_{n0}}{B_{n0}}\,A_{r0} + C_{r0}}{\dfrac{A_{n0}}{B_{n0}}\,B_{r0} + D_{r0}}\,E_{r0} \qquad (47)$$

Formulas have been developed for the combination of two networks in parallel or two networks in series which enable one to reduce complicated networks to a single equivalent network.

Fault to Ground. Having shown that a complicated transmission network may be simplified to the form of a single branch having an equivalent supply voltage, and equivalent general circuit constants of the three-phase sequences, formulas will be developed for the determination of the currents that flow in case of a fault to ground. Reference should be made to Fig. 26 for the diagram of connections and the notation.

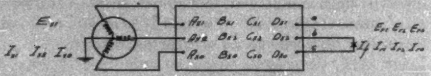
E_{s1}
I_{s1} I_{s2} I_{s0}
A_{s1} B_{s1} C_{s1} D_{s1}
A_{s2} B_{s2} C_{s2} D_{s2}
A_{s0} B_{s0} C_{s0} D_{s0}
a
b
c
E_{r1} E_{r2} E_{r0}
I_f I_{r1} I_{r2} I_{r0}

FIG. 26—FAULT TO GROUND ON PHASE *a* AT POINT ON EQUIVALENT NETWORK

The general equations for this network are given by equations (**14**) to (**16**). The equations arising from the condition of a fault to ground are as follows:

$$E_a = E_{r1} + E_{r2} + E_{r0} = 0 \qquad (48)$$

$$I_b = \alpha^2 I_{r1} + \alpha\, I_{r2} + I_{r0} = 0 \qquad (49)$$

$$I_c = \alpha\, I_{r1} + \alpha^2 I_{r2} + I_{r0} = 0 \qquad (50)$$

Equation (**48**) states that the voltage on phase *a* from line to neutral is zero. Equations (**49**) and (**50**) state that no current is flowing to the fault in phases *b* and *c*. It should be noted that these equations do not require that no current flows in phases *b* and *c* in other parts of the network in the case of two actual networks in parallel. By subtracting equation (**49**) from equation (**50**) we obtain $I_{r1} = I_{r2}$ and by substitution in equation (**49**) we have further that

$$I_{r1} = I_{r2} = I_{r0}.$$

The fault current I_f is equal to $I_{r1} + I_{r2} + I_{r0}$.

Equations (**14**) to (**16**) may be solved for E_{r1}, E_{r2} and E_{r0} which give:

$$E_{r1} = \frac{E_{s1}}{B_{s1}} \cdot \frac{B_{s1}}{A_{s1}} - \frac{B_{s1}}{A_{s1}}\,I_{r1} \qquad (51)$$

$$E_{r2} = -\frac{B_{s2}}{A_{s2}}\,I_{r2} \qquad (52)$$

$$E_{r0} = -\frac{B_{s0}}{A_{s0}}\,I_{r0} \qquad (53)$$

The value of the currents may now be computed which give:

$$I_{r1} = I_{r2} = I_{r0} = \frac{I_f}{3} = \frac{\dfrac{E_{s1}\,B_{s1}}{B_{s1}\,A_{s1}}}{\dfrac{B_{s1}}{A_{s1}} + \dfrac{B_{s2}}{A_{s2}} + \dfrac{B_{s0}}{A_{s0}}} \qquad (54)$$

The currents at the point of fault can be determined from equation (**54**) and with the currents determined, the value of the voltages at the point of fault may be calculated. With the voltages and currents at the point of fault determined, the value of voltages and currents for all points in the network may then be computed.

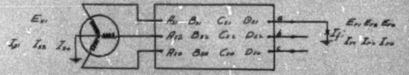
E_{s1}
I_{s1} I_{s2} I_{s0}
A_{s1} B_{s1} C_{s1} D_{s1}
A_{s2} B_{s2} C_{s2} D_{s2}
A_{s0} B_{s0} C_{s0} D_{s0}
a
b
c
E_{r1} E_{r2} E_{r0}
I_f I_{r1} I_{r2} I_{r0}

FIG. 27—FAULT BETWEEN PHASES *b* AND *c* ON EQUIVALENT NETWORK

Line-to-Line Short Circuit. For the case of a line-to-line fault, reference may be made to Fig. 27 for the diagram of connections and notation. The general equations for the network are given by the equations (**14**) to (**16**). For the case of a fault between phases *b* and *c*, the equations representing this condition are given below:

$E_b = E_c$ or

$$\alpha^2 E_{r1} + \alpha\, E_{r2} + E_{r0} = \alpha\, E_{r1} + \alpha^2 E_{r2} + E_{r0} \qquad (55)$$

$$I_a = I_{r1} + I_{r2} + I_{r0} = 0. \qquad (56)$$

$I_b = -I_c$ or

$$\alpha^2 I_{r1} + \alpha\, I_{r2} + I_{r0} = -(\alpha\, I_{r1} + \alpha^2 I_{r2} + I_{r0}) \qquad (57)$$

Equation (**55**) states that the voltages of phases *b* and *c* are equal: that is, that there is no difference in voltage between them. Equation (**56**) states that no current flows directly to the fault in phase *a*, and equation (**57**) states that the fault current flowing in phase *b* returns in phase *c*. From equation (**55**)

$$E_{r1} = E_{r2}$$

Also from equations (**56**) and (**57**) we have that

$$I_{r2} = -I_{r1},$$

and from equation (**56**)

$$I_{r0} = 0.$$

Substitution of the above relations in equations (**14**) and (**16**) gives

$$\frac{E_{x1}}{B_{x1}} \cdot \frac{B_{x1}}{A_{x1}} - I_{r1}\frac{B_{x1}}{A_{x1}} + I_{r1}\frac{B_{x2}}{A_{x2}} = 0. \qquad (\mathbf{58})$$

Hence,

$$I_{r1} = -I_{r2} = \frac{\dfrac{E_{x1}}{B_{x1}} \cdot \dfrac{B_{x1}}{A_{x1}}}{\dfrac{B_{x1}}{A_{x1}} + \dfrac{B_{x2}}{A_{x2}}} \qquad (\mathbf{59})$$

Remembering that $I_{r0} = O$ for this case, we have the expression for the line-to-line fault current as follows:

$$I_f = (\alpha^2 - \alpha)\frac{\dfrac{E_{x1}}{B_{x1}} \cdot \dfrac{B_{x1}}{A_{x1}}}{\dfrac{B_{x1}}{A_{x1}} + \dfrac{B_{x2}}{A_{x2}}} \qquad (\mathbf{60})$$

The value of the positive and negative sequence voltages at the point of fault can be computed since the currents at this point are defined by equation (**60**). With the voltages and currents of both phase sequences known at the point of fault, the currents and voltages throughout the network can then be calculated.

Calculation of General Circuit Constants

The method of calculating the value of general circuit constants for positive, negative and zero sequence will now be considered.

Positive Sequence General Circuit Constants. The positive sequence general circuit constants are, of course, identical with those constants normally used to determine the performance of transmission systems. Convenient formulas for deriving these constants for the cases of various networks are given in the paper by Evans and Sels* previously referred to. Analytical methods of network solution will be found of considerable assistance in the determination of these constants.† A very complete set of tables of the general circuit constants for transmission lines of all standard sizes of conductors for a considerable number of spacings and for lengths of circuit from 50 to 300 miles, and five- to ten-mile intervals, has recently been published.‡

Negative Sequence General Circuit Constants. The negative sequence general circuit constants are derived in the same manner as the positive sequence, and differ from them only because of the fact that machine impedances have different values for positive and negative sequence.

The negative sequence reactance of the synchronous machine may usually be assumed as about seven-tenths the conventional value of armature leakage reactance. C. M. Laffoon in a recent paper§ has shown that synchronous machines will initially develop about equal short-circuit currents for a three-phase short circuit, and for a single-phase short circuit from line-to-line terminals. The expressions for the value of short-circuit current for three-phase short circuit, and single-phase line-to-line, as derived in the previous section of the paper, are given below, in which the notation is self-explanatory.

$$I_3\varphi = \frac{E}{X_1} \qquad (\mathbf{61})$$

$$I_1\varphi_{L-L} = \frac{\sqrt{3}\,E}{X_1 + X_2} \qquad (\mathbf{62})$$

In case the short-circuit current for the two conditions are equal, then

$$X_2 = (\sqrt{3} - 1)\,X_1 = 73\%\,X_1 \qquad (\mathbf{63})$$

It may be pointed out that the negative sequence resistance is higher than the positive sequence because of the loss produced by the double-frequency current in the rotor.

The negative sequence impedance of an induction motor may be measured by driving the rotor at synchronous speed in the opposite direction to that which the rotor would normally revolve, and circulating current through the primary windings. The negative sequence impedance is approximately the same as the standstill impedance.

Zero Sequence General Circuit Constants. The general circuit constants for zero sequence are, in general, radically different from the constants for positive or negative sequence. It is convenient to set up a new network for zero sequence which is usually much simpler than the network for positive or negative sequence, because the absence of a neutral wire or ground connection will prevent the flow of zero sequence current in a number of branches. The point of fault should be considered as the source of zero sequence voltage, and the general circuit constants derived on this basis.

The method of determining the general circuit constants for zero sequence is identical to that used for positive or negative sequence. The only difference arises from the fact that the impedances and admittances of the various component branches are usually different for zero sequence than for the other sequences. For zero sequence, the three-line conductors should be considered as connected in parallel with the neutral wire or earth as the return circuit. If a ground wire is employed connected in parallel with the earth, its effect should also be included as part of the return circuit.

In solving zero sequence problems, it is possible to carry through the calculations considering either the total zero sequence current and the impedances and admittances of the three phases in parallel, or considering the current in the individual phases and the equivalent impedances and admittances of the different phases from line to line. The latter method is preferable, and it is necessary to multiply the combined impedances by three, and to divide the total admittances by three in order to combine them with the other impedances and admittances. The method for determining the impedances and admittances of various component branches of a transmission network will now be considered.

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*Bibliography, Item 6.

†Bibliography, Item 26.

‡Bibliography, Item 20.

§Bibliography, Item 23.

Series Impedances. Series impedances in the three phases have the same value of impedance to positive, negative and zero sequence, provided there is a neutral wire or ground connection through which zero sequence current can flow. If there is no neutral connection, the impedance to zero sequence is infinite.

Series impedance in a neutral wire or ground connection is, of course, wholly zero sequence. The value of this impedance should be divided by three to obtain the equivalent value for phase to neutral.

Shunt Admittances. Shunt admittances have the same value for positive, negative and zero sequence if there is a neutral connection. If there is no neutral connection, the admittance is open-circuited to zero sequence.

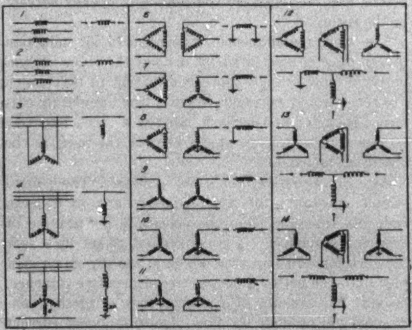

Fig. 28—Method of Setting up Zero Sequence Network

Transformers. The zero sequence impedance of the transformer is either the same as the positive or negative sequence impedance per phase, or the value is infinite, due to the lack of neutral connection.

The setting up of a zero sequence network may be facilitated by the use of the conventions illustrated in Fig. 28. These conventions as to connections may be stated as follows: A neutral ground or neutral wire should be considered as a closed circuit, and lack of them as an open circuit; star connections to the three-phase lines as closed circuits, delta-connections as open circuits; in a transformer a delta winding is a closed path with respect to the transformer and an open circuit with respect to the side to which the delta winding is connected.

For the three-winding transformer, the equivalent circuit* should be employed, and the conventions as to connections for zero sequence are illustrated in Fig. 28.

Machine Impedances. The zero sequence impedance of a synchronous machine depends largely upon winding arrangement and chording factors, and its magnitude may be very low. Of course, the zero sequence impedance of the machine is infinite unless the neutral point is grounded or connected to a neutral wire through which zero sequence current can flow. In the paper by C. M. Laffoon† previously referred to, it is shown that, for commercial machines, the initial value of line-to-neutral short circuit current will normally vary from 1.5 to 1.73 times the three-phase short circuit current. The value of the three-phase and single-phase line-to-neutral short-circuit currents as derived in the previous part of this Appendix may be stated as follows:

$$I_{3\varphi} = \frac{E}{X_1} \qquad (64)$$

$$I_{1\varphi L-N} = \frac{3\,E}{X_1 + X_2 + X_0} \qquad (65)$$

in which the notation is self-explanatory. The value of the zero sequence reactances on this basis is from zero to about 25 per cent of the positive sequence reactance if the negative sequence reactance is taken as about seven-tenths of the positive sequence reactance.

The zero sequence resistance is higher than the positive sequence resistance because of the loss produced by the double frequency current in the rotor.

It should perhaps be pointed out that the zero sequence impedance of a synchronous generator usually does not enter into the problem of transmission line faults to ground because there is no path for the flow of zero sequence current through the generator due either to the lack of a generator neutral ground or the interposition of a delta winding of the transformer between the generator and the transmission line.

Transmission Line. The zero sequence general circuit constants of a transmission line are computed in a manner similar to that used for these constants for positive or negative sequence. The constants are calculated from the following equations:

$$A = \cosh \sqrt{Z\,Y} = \left(1 + \frac{Z\,Y}{2} + \frac{Z^2\,Y^2}{24} \ldots\right) \qquad (66)$$

$$B = \sqrt{\frac{Z}{Y}}\,\sinh \sqrt{Z\,Y} = Z\left(1 + \frac{Z\,Y}{6} + \frac{Z^2\,Y^2}{120} \ldots\right) \qquad (67)$$

$$C = \sqrt{\frac{Y}{Z}}\,\sinh \sqrt{Z\,Y} = Y\left(1 + \frac{Z\,Y}{6} + \frac{Z^2\,Y^2}{120} \ldots\right) \qquad (68)$$

In the above equations, *Z* and *Y* are the series impedances and shunt admittances per mile multiplied by the number of miles.

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*Bibliography, Items 27 and 26.

†Bibliography, Item 23.

The path for zero sequence current is through all the transmission conductors in parallel as one side of an imaginary circuit with earth as the return. The formula for the zero sequence reactance of a transmission line is as follows:

$$X_0 = 0.278 \log_{10} \frac{1000 \times 12}{r_0} \text{ ohms per mile at 60 cycles} \qquad \textbf{(69)}$$

The above expression assumes that the return current flowing in the earth produces the effect of being concentrated at a point 1000 ft. below the transmission conductors. This assumption is derived from the results of tests made for this purpose. In the above equation, r_0 is the equivalent geometric mean radius of the conductors expressed in inches, which, for a single round conductor, is 77.9 per cent of the physical radius. The equivalent radius for n conductors is equal to the n^2 root of the product of n^2 terms composed of the equivalent geometric mean radius of each of the n conductors and the distances of each conductor to every other conductor. The equivalent radius for three similar conductors with triangular spacing reduces to the following formula:

$$\sqrt[3]{0.779\, r\, d^2}$$

where d is the spacing between conductors and r is the physical radius, the dimensions being expressed in inches. The value of reactance calculated by the formulas given above must be multiplied by three to secure the line-to-neutral values per phase.

The corresponding value of the zero sequence resistance is equal to the resistance of the conductors of one phase.

The zero sequence admittance of the transmission line is computed in a similar manner. All line conductors are considered as one side of the circuit with ground as the other side. The capacity to ground of a system of conductors may be computed accurately by the method of capacity coefficients. The following approximate formula may be used.

$$b_0 = \frac{14.63 \times 10^{-6}}{\log_{10} \dfrac{h}{r_{eq}}} \text{ mhos per mile at 60 cycles} \qquad \textbf{(70)}$$

In the above expression, h is the geometric mean distance of the line conductors to their images, considering the surface of the earth as the ground plane. The term r_{eq} is the equivalent radius of all of the line conductors, the value of which is computed in a manner similar to that given previously for r_0 except that the geometric radius of a single conductor is equal to the physical radius so far as capacity effects are concerned. The total susceptance obtained in the manner outlined above should be divided by three to reduce the value to the line-to-neutral quantity per phase.

The equivalent conductance for zero sequence is equal to the actual conductance from one phase wire to ground.

Appendix III

EQUIVALENT SYMMETRICAL NETWORK FOR SINGLE-PHASE SHORT CIRCUIT

The method of calculating short-circuit currents on transmission networks by the use of phase sequence components has been described in the previous section of the appendix. An extension of this method is, however, pertinent to the discussion of stability and leads to a considerable simplification from the analytical point of view as well as a reduction in the numerical work including, incidentally, calculations for step-by-step solutions.

This extension in method permits the replacement of the actual network having a short circuit on one phase by the original network without the short circuit, but with a symmetrical shunt impedance connected across the three phases at the point of short circuit. The value of this impedance is chosen so as to produce a positive sequence voltage across it and a positive sequence current through it identical with the corresponding quantities produced by the short circuit on the actual network. The possibility of making such a substitution arises from the fact that the different phase sequence components, while bearing certain relations to each other, depending upon the nature of the short circuit, do not react upon each other at other points in a symmetrical network, assuming that no machines generate negative or zero sequence voltage. On account of this fact, the effects will be the same whether the different sequence components of current flow in the same conductors or whether they flow in separate impedances. This relation may be understood more readily by a study of the equations which will be given for the determination of the value of the equivalent shunt impedance.

In case of a single-phase, line-to-neutral short circuit on the phase A at the point r, the solution may be expressed in terms of the notation used in the previous section of the appendix as follows:

$$I_{r1} = I_{r2} = I_{r0} = \frac{\dfrac{E_{x1}}{B_{x1}} \cdot \dfrac{B_{x1}}{A_{x1}}}{\dfrac{B_{x1}}{A_{x1}} + \dfrac{B_{x2}}{A_{x2}} + \dfrac{B_{x0}}{A_{x0}}} \qquad \textbf{(1)}$$

$$E_{r1} = \frac{B_{x1}}{A_{x1}} \left[\frac{E_{x1}}{B_{x1}} - \frac{\dfrac{E_{x1}}{B_{x1}} \cdot \dfrac{B_{x1}}{A_{x1}}}{\dfrac{B_{x1}}{A_{x1}} + \dfrac{B_{x2}}{A_{x2}} + \dfrac{B_{x0}}{A_{x0}}} \right] \qquad \textbf{(2)}$$

The value of the equivalent symmetrical shunt impedance to be connected across the network at the point r is determined as follows:

Equivalent impedance (in ohms per phase) is equal to

$$\frac{E_{r1}}{I_{r1}} = \frac{B_{z2}}{A_{z2}} + \frac{B_{z0}}{A_{z0}} \qquad (3)$$

which is obtained by dividing equation (**2**) by equation (**1**).

Therefore, the value of the equivalent shunt impedance for a line to neutral short circuit is equal to the sum of the negative and zero sequence impedances as measured from the point of short circuit. Since the equivalent shunt impedance produces the same value of positive sequence voltage and current at the point of short circuit as occurs in the actual network, and since negative and zero sequence current do not react on the positive sequence, it follows that the magnitude of the positive sequence current in the remainder of the network is the same as that in the actual network. Hence the equivalent network may be used instead of the actual network with the short circuit on one phase for determining the positive sequence solution.

It may be pointed out that a similar equivalent symmetrical shunt impedance may be employed to represent a single-phase, line-to-line short circuit. For this case, the value of the equivalent impedance per phase is equal to $\frac{B_{z2}}{A_{z2}}$, the effective value of the negative sequence impedance as determined from the point of short circuit.

The use of the equivalent symmetrical network permits a great simplification in the analysis of stability under single-phase short-circuit conditions as it reduces the problem to that of an equivalent symmetrical system with an additional shunt connected branch.

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Discussion

PAPERS ON TRANSMISSION STABILITY

(NICKLE AND LAWTON, CLARKE, WILKINS, EVANS AND WAGNER)

NEW YORK, N. Y., FEBRUARY 8, 1926

R. D. Evans: Miss Clarke's paper dealing with the use of equivalent circuits for analyzing static stability is of very considerable interest. Probably the most important contribution in the paper is the method of using an equivalent circuit to obtain the angle between generator and motor for maximum power. I believe this general method will find extensive use.

In carrying out the calculations, Miss Clarke replaces a complicated network with shunt admittances, by an equivalent network of a single series impedance, but with equivalent supply and receiver voltages. Sometime ago we had occasion to solve a transmission network in which many of the branches involved a long transmission line, and this led to a somewhat similar method of calculation. It is not convenient to go into the mathematical analysis at this time, but in the written discussion we will submit an alternative proof of the method of using equivalent networks and equivalent voltages, which method is based on the use of general circuit constants and is of very simple form. I might say that some of the formulas used by Miss Clarke have been independently derived by us and that, for example, the transformation of the general network to the equivalent pi and vice versa will appear in the revision of the "Electrical Characteristics of Transmission Circuits" by William Nesbit, which is now in press.

On the middle of the first page, Miss Clarke states that she takes the characteristics of synchronous machines into account by assuming a definite value of synchronous impedance and a definite value of excitation voltage. These assumptions are used to determine the static stability limit. These assumptions apply when the excitation is fixed but do not apply when the excitation varies under the control of automatic voltage regulator.

The determination of the proper method of representing synchronous machines for the calculation of static limits is beyond the scope of Miss Clarke's paper, but since it affects the static limit, it seems pertinent to comment upon this phase of the problem. The methods described by Miss Clarke are not applicable to the determination of the effects of voltage regulators since they do not permit the analysis of the time variation in excitation and of rotor movement. In the paper by Evans and Wagner, it is pointed out that the use of automatic regulators will actually increase the static limit over the value obtainable under hand control. These considerations do not destroy the usefulness of Miss Clarke's methods which apply for hand regulated system. The methods may also be of value for systems with automatic voltage regulator in case an assumption can be made as to a definite value of machine impedance and a definite value of machine voltage. For example, if the regulator were fast enough to maintain terminal voltage, then machine impedance could be neglected, or if the regulator were merely fast enough to maintain constant flux then leakage reactance should be used. The true static limit would be determined, however, by a transient analysis in the manner indicated in the Evans and Wagner paper.

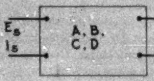

FIG. 1

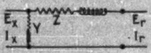

FIG. 2

FIG. 1—GENERAL NETWORK
FIG. 2—SIMPLIFIED NETWORK

(By letter): In the verbal discussion, it was not practical to go into a mathematical derivation of an alternative proof for the method of equivalent networks given in Miss Clarke's paper. This derivation follows:

Referring to Fig. 1, the general equations for this network maybe written as follows:

$$E_s = A\,E_r + B\,I_r \qquad (1)$$

$$I_s = C\,E_r + D\,I_r \qquad (2)$$

where the notation for voltages and currents are indicated in Fig. 1 and A, B, C, and D are the general circuit constants. Following the notation used in the paper, complex quantities will be understood without any designating mark but a conjugate will be indicated by a bar over the symbol.

Let us divide equation (1) and multiply Equation (2) by the circuit constant A which gives:

$$\frac{E_s}{A} = E_r + \frac{B}{A}\,I_r \qquad (3)$$

$$A\,I_s = A\,C\,E_r + A\,D\,I_r \qquad (4)$$

Consideration of the above equations will show that they correspond to the network shown in Fig. 2 below. The equations for the network of Fig. 2 are as follows:

$$E_x = E_r + Z\,I_r \qquad (5)$$

$$I_x = Y\,E_r + (1 + Y\,Z)\,I_r \qquad (6)$$

By comparing equations (5) and (6) with equations (3) and (4), it will be seen that:

$$E_x = \frac{E_s}{A} \text{ and } I_x = A\,I_s$$

Also

$$Z = \frac{B}{A} \text{ and } Y = A\,C$$

These considerations show that any network with constant impedance branches may be replaced by the network of the form shown in Fig. 2, using an equivalent supply voltage. It will be clear that the conditions at the receiver end for the equivalent network are identical with the conditions at the receiver end actual network. At the supply end, however, the voltage should be divided by A and the current multiplied by A. This simplification is obtained by the use of a transformation constant A, which constant is a complex number and necessitates a phase shift correction for power quantities as indicated below.

$$P_x + j\,Q_x = E_x\,\overline{I}_x = \frac{E_s}{A}\,\delta\,\overline{A}\;\overline{I}_s = \frac{\overline{A}}{A}\,E_s\,\overline{I}_s$$

By consideration of the simplified network of Fig. 2, it is possible to obtain the receiver power and the corresponding angle between supply and receiver voltages by consideration of only the series impedance Z and the phase shift in the equivalent supply voltage from the equivalent network.

Of course, the actual network might be divided in such a way as to permit the use of equivalent voltages at either end of the actual network.

We wish to emphasize the simple manner here employed to find an equivalent network which consists merely in rewriting the equations of the general network in the desired simplified form, and finding the equivalent network that corresponds thereto. The above method has been employed on a few occasions, but the phase shift introduced by the use of the equivalent voltage has been found to be somewhat confusing. The actual limiting angles between supply and receiver voltages may be computed directly from the general circuit constants for the over-all network, and the power corresponding to the limiting angle may be obtained graphically as is brought out in the accompanying discussion by Mr. Wagner.

H. W. Smith: A cursory review of Miss Clarke's and Mr. Nickle's papers indicates that methods are now available for the calculation of the steady-state power limit. These methods substantially agree for all practical purposes. Mr. Nickel's paper conveys the impression that the criterion of stability is the steady-state power limit although on the sixteenth page of his paper, tests are mentioned in which a dead single-phase, line-to-line short circuit, continued for half a second, reduces the power limit to 31 per cent of the steady-state power limit.

I think that all operating men will agree that if this is true, over-emphasis has been placed on the steady-state power limit. In any actual operating system, even though the best known relaying system is utilized, faults will occur, and on high-voltage systems they cannot be cleared in less than one-half second. It thus appears that the study of stability under fault conditions is extremely important.

Mr. Evans' and Mr. Wagner's paper is an extremely valuable contribution. They have developed a method of analysis of stability applied to an extensive transmission system, and this has been checked against actual tests which have been described in Mr. Wilkins' paper. The close agreement between the calculated results and test results seems to indicate that the method of analysis and assumptions made are correct enough for all practical cases. This paper indicates that for fault conditions, the transient stability is less than the steady-state limit. It would be very convenient if we could express this in terms of per cent of the steady-state power limit. This, however, can not be done, and for each system it will be necessary to determine the transient stability by a method of analysis indicated in Mr. Evans' paper.

In deciding on the limit to which systems can be worked the service standards must be considered. It may be permissible to have a system pull out of step under a fault condition that may occur at rare intervals, but if operation must be satisfactory for all fault conditions, the power limit must be considerably less than the steady-state limit.

H. H. Dewey: The general problem of power limits of transmission systems or stability has been before us only for the last year or two actively, and the papers that have been presented so far have left the subject in a somewhat confused state.

The problem as a whole is one that is assuming greater importance, as Mr. Wilkins points out, due to the interconnection of our power systems and the fact that we are endeavoring to make more use of our transmission lines. When we actively started the study of what limitations were important and what to do about it, we naturally started in a number of different directions and there was no very concerted effort made to direct our investigations along the lines that would produce immediate results.

The investigations that were made by the manufacturing companies with their artificial lines brought about certain basic results that were of immense importance. We found out things that we could have easily calculated if we had taken time enough and had got the idea but in our artificial transmission systems these suggestions came up as we noted the results of our tests.

When we had the first papers on this subject at Philadelphia two years ago, practically all of the authors, and I believe most of the men who discussed the papers, were willing to concede that stability depended very greatly on the excitation of our synchronous machines. They were also willing to concede that there was not very much hope of doing anything with our regulators or with artificial changes that could be made in the excitation during a transient. Starting out with that as a basis, we reached limitations of transmission that were surprisingly low. We found that it was difficult to get full rated output out of a generator when its excitation was limited by that necessary to produce normal terminal voltage under the conditions with which we started. That is one point that we come to a conclusion on on our artificial transmission tests that regulators were of extreme importance, and that due to certain facts such as the time element required to put on additional load or the rate at which transients come on, we could make use of artificial regulators and could thus increase the stability of a generator or a generator and its transmission line including transformers and all of the circuits in between.

So that, we feel, is an important point that we have fairly well settled on and I believe that the investigators fairly well agree that artificial regulation is possible and important.

There are a number of things that come before us in the study, some of which make us revamp some of our ideas on transmission. One point that I have voiced a few times and have been questioned on is the effect of charging current, the effect of capacity in our transmission line. It is in effect an absolute detriment. We have always in the past been in the habit of thinking that a long line introduced extra reactance and it was a bad thing to introduce such reactance and cause a voltage drop. We have felt it was compensated for by charging current of the line which had a tendency to hold up the voltage. That is true so far as regulation under specific conditions is concerned but when we are trying to get the last drop out of the transmission system from the generator to the motor, if there is any charging current in between, that current reduces the excitation required on the generator and the excitation required for the synchronous load and it is an absolute detriment, therefore, because it reduces excitation and decreases the maximum power that can be transmitted. That is a point we dislike to give up but it is there just the same.

I was very much interested in Mr. Wilkins' paper on the test on a practical transmission system and fully agree with him in his statement that artificial transmission systems are of some use in studying the problem but the proof of the pudding is in the eating and we must apply our studies to a practical system and the points that have to be taken into consideration are so many and varied that we cannot reproduce them on an artificial system. We can get the basic principles, however, and those basic principles have been fairly well settled on by our studies on artificial lines and we hope there will be many cases of analyses made such as that on the Pacific Gas and Electric Company's system. The more complicated the system, the more difficulty we are going to encounter in making such a study because there are so many side issues coming in,—points Mr. Wilkins brought out, such as the speed of the relays, the action of the governors, the characteristics of the particular machines you have and the complicated number of machines of varied characteristics.

There are certain things that we know will increase stability. We know that a generator having low reactance helps to increase stability. The characteristics of transformers and characteristics of transmission lines themselves must be taken into consideration. The problem is one we are just starting on and one that is worthy of the combined efforts of all of our best people who have the opportunity to study these problems.

In general, we have not felt that the stability problem is one that was bothering us to any great extent, but usually when we have trouble there are so many things happening that we do know about that we concentrate on the analysis and cure of those particular troubles, and if we knock the system out of step we think it is another happening and one we have let go. Since we have started to study this problem, we have found case after case of pure power limitations that a few years ago we should have passed over as being an incident rather than the real cause of our difficulty.

The main problem before us is the question of the effect of short circuits. It has been pointed out by the authors of the papers that steady-state stability is something that is fairly definite, something we can increase to some extent by careful design of our system and regulating equipment and so forth, but the problem of short circuits is one that we know very little about. We know that we ride through certain types of short circuits. We know that other types break up our system. The analysis of the difference between these varied kinds of short circuits is one that requires more data before we can come to definite conclusions.

On high-voltage lines, we are much more likely to have grounds than we are to have line-to-line short circuits and I believe that has been the history of the 220,000-volt systems in California and the study made of their short circuits last fall brought me to the conclusion that with reasonably fast relaying, we can ride through a ground. It is a pretty difficult thing, however, to run through a phase-to-phase short circuit.

Mr. Wilkins pointed out some of their experiences and on the Southern California System I found they have had many more grounds on the high-tension line than Mr. Wilkins has had due to the different conditions, but their record of relaying and holding in step has been excellent during those times. But with short circuits on the 66,000-volt system which is veritable network, the short circuits are of the order of 1,000,000 kv-a. when they are phase-to-phase. They have had great difficulty in keeping the system in step. They have dropped out of step fifteen times from the first of January to the first of October. That is an extremely serious condition when the drop is a matter of 300,000-kw. loads during those times.

That, you probably will say, is poor operating but operation hasn't much to do with it; it is the limitations we are getting into in the big system where we get short circuits right in the center of the system.

H. H. Spencer: I think the electrical engineering profession should be most grateful to Miss Clarke for having reduced so complicated a problem to a slide rule and arithmetical basis from the more complicated methods which we have been assumed to use during the past two or three years that the problem has been confronting us. Perhaps the most obvious difference between Miss Clarke's method of calculation and some of the other methods which have been presented, is her choice of a constant generator reactance. Other investigations have presented methods of analysis which take into account the variation in synchronous reactance in accordance with the Blondel or similar diagram. I think it would be most interesting if Miss Clarke would point out what method is used in the selection of this constant synchronous reactance.

Another point which is perhaps not entirely obvious in the equivalent circuit method of calculation is that it is possible for the receiver network to which a transmission line is tied, to fall out of step *per se*. That is to say, that although a transmission line feeding a network may of itself be stable, the network which is fed may go into instability by virtue of its connection with the transmission line. Just how that works out can be seen by looking at Miss Clarke's example No. 3 on the 11th page of her paper.

Suppose the receiver system had consisted of 100,000 kv-a. of synchronous generators at the receiver end and these generators had been loaded not to 45,000-kw. but to a load approximating their rating, say, 80,000-kw., allowing 20,000 kw. for spare capacity. Under such a set-up, it is quite possible that line would be able to transmit the calculated load of 132,600 kw., but that a reduction in the voltage at the receiver end while not producing instability between the receiver load and the sending end generators, would produce instability between the synchronous load and the receiver end synchronous generators.

In regard to the difference between steady-state and transient power limits, methods of analysis are certainly developed at the present date to the point where given the same assumptions, two engineers will come out with the same answer. The difference between the answers which are obtained from transient stability analysis lies, I believe, in the difference in the fundamental assumptions in regard to the flux relationships in the synchronous apparatus at the two ends of the line.

Now the problem of flux relationships in synchronous apparatus is not one of novelty, but on the contrary is one which has been dealt with for a long time and with consistent success by designing engineers. It is necessary for the engineer who would calculate stability, or power limits, simply to compute the flux relationships which exist under certain known conditions by means of perfectly definite methods which are in common use. Having disposed of the problem of flux relationship, the difficulty of the stability problem very largely disappears and the power limits are readily calculable.

If we concede that during a particular transient disturbance the flux in a generator or a synchronous motor remains essentially unchanged, then I think any engineer would have a tendency to feel that the power of limitations imposed during operation under the conditions which led to the development of the initial flux would obtain throughout the disturbance. Thus switching operations such as the dropping of a generator or one of two parallel transmission circuits will introduce no power limitations below those imposed by steady-state operation since disturbances of this sort have no tendency to reduce materially the flux in the synchronous generators. Short circuits, on the other hand, which do reduce the flux in the synchronous apparatus may very possibly impose power limits appreciably below these encountered in steady-state operations.

Several single-phase, short-circuit analyses which have been studied during the past few months have shown that in a system of ordinary design, the steady-state stability limit is very markedly reduced by single-phase short circuits to ground. However, by means of changing the circuit set-ups, introducing a zero sequence reactance in the transformer windings, or using neutral reactors, the steady-state stability limit of the transmission system can be very closely approached.

H. K. Sels: In order that I may present clearly and concisely a number of points in some chronological order, I am taking the liberty of reading a prepared discussion. I do not wish to deal in personalities but to present such criticisms as I have from the purely abstract viewpoint of an operating engineer for a large and extensive system—particularly since the papers have not been in my hands long enough to fully acquaint myself with their respective contents. After three years of theoretical and experimental investigations I am more interested in the questions,—what has been accomplished and whither are we bound? No single organization seems to be responsible for the headway that is being made and I offer that as a hint to some of our several transmission committees.

The question of stability is as old as the alternating current art itself. We have always been confronted with the problems of synchronizing power between large generating stations, "hunting" of synchronous motors and the stability of rotary converters. A large number of papers have been presented before the Institute relating to these problems. The impression one gets from the papers which have been presented on the subject of stability is that we have discovered something radically new, some vital thing in which our power systems are unduly weak. Such is not the case and our methods of electric distribution are no weaker than they have been, by virtue of this discovery. However, what we do have that is new is a broader vision of the problem and the relation of its elements, and new methods of attack, and we hope, after all the facts have been analyzed, that we will obtain a solution which will extend the power limits of transmission to the ultimate.

Fundamentally, what we are really interested in is the power limits of a transmission system. A large number of factors determine the power limits of a system. It may be the current carrying capacity of the wire; it may be the economical amount of power that can be transmitted over a given line before it pays to build additional lines and maintain the same continuity of service, or it may be the question of stability of operation, which are the subjects of today's papers.

Psychologically I think it would be better for us to speak more in terms of the power limit, remembering that stability is only one of the important factors involved.

I believe everyone is over-emphasizing the relation of stability to high-voltage transmission. Of course it is true that stability is more likely to fall within the economic power limits of transmission at the higher voltages, but this does not reduce the importance of stability at all voltages and on all systems. Every operator is familiar with the losing of load or the dropping out of synchronous apparatus during system disturbances and fundamentally this is nothing more than the effect of instability. Therefore any improvements for stability apply equally well throughout the system.

Just to refresh my memory on the general trend of opinion on the various factors entering into the stability question, I reviewed recently all the papers and discussions that have been presented before the Institute. From an operating viewpoint I was very much impressed with the technicalities and, to me, the impractical viewpoints which were expressed. I believe that the average engineer would be hopelessly lost in attempting to follow through calculations on anything more than a very simple transmission system rather than a large interconnected system where the attempt is made to keep short circuits as low as possible by using synchronizing buses with reactors between individual generators and other complications used in the layout of the modern system for dispatching load; in fact it is very much like the mathematical conundrum of trying to solve a problem having more unknown variables than equations.

In addition to the factors outlined by the various authors, let me indicate for a moment some of the multitudinous conditions which the operating engineer must consider in making any calculations on stability. First, today we have a certain distribution of power stations serving a given load in a given territory connected up with a certain arrangement of transmission. Tomorrow we have a more efficient generating station coming into operation, a new arrangement of transmission to new load centers, which entirely alters the fabrication of the system. In actual operation the load conditions obtained hour by hour, day by day and year by year, change on account of the relative efficiency of the generating units on the system, the variation in load and different set-ups of lines in and out of service due either to failure or ordinary maintenance. All of these features only serve to make an intricate problem more involved. In attempting to determine a method of calculation which can be used with some degree of accuracy, I believe we should use those assumptions which we find agree the closest to actual operating tests.

Today's papers indicate more than ever that the methods of calculation used by the different engineers do not differ greatly in theory, but there does seem to be some difference of opinion as yet on the assumptions to be made and whether the steady state or the transient state is more important. These differences apparently account for the divergence in the results of calculations by different engineers. As I have already brought out, the problem is so intricate that at best the solution is a cut and try process in which your methods of calculation and assumptions must conform with the results obtained by experience. For this reason I believe that for the present at least we shall consider all calculations only qualitatively and not accept calculations by separate engineers as comparative unless made on the basis of the same assumptions. For example, I might agree with the general shape and relationship of the curves in Fig. 17 in the paper by Messrs. Nickle and Lawton, but not agree with the maximum power scale due to some difference in my calculations.

With regard to the relative importance of steady-state and transient conditions, I believe it is evident that in cases dealing with high voltage, long distance transmission where the stability limit falls within the economic power limit of the line the steady state performance of the line will be of primary importance, because with such a large amount of capital tied up in the transmission system every means possible will be used to make failures on the transmission practically unknown. While the transient state does not enter so much into economics of transmission, it should be expected that continuity of service to local loads as well as the extreme importance of keeping a large high voltage system operating continuously as a whole will make this a factor not to be overlooked. In the case of lower voltage and shorter distance transmission, we can point to case after case of both major and minor importance in which some disintegration of the system takes place during a system short circuit. In actual operating experience I have yet to hear of the case where a system designed to operate in parallel has fallen apart due to steady-state conditions except where the static limit was reached in the Southern California Edison system within what we might say is the economic emergency capacity of the line. However, I should like to ask what experience shows here that transient stability is not an important factor. Before too much discussion is entered into on steady-state and transient stability, it would be apropos to have the terms defined, and we may find more of us in agreement than is apparent at first. Personally I classify the steady-state stability limit as that load which breaks down a system when slowly applied and the transient stability limit as that load which breaks down a system when suddenly applied or during switching operations including short circuits. It is conceivable that under certain conditions of small load increments or minor switching operations, the two limits will be the same when so defined. Summing up the relative importance of steady-state and transient stability, here again I believe we must resort to actual operating experience to tell us where our weaknesses lie. It is commended that the various investigators review the interruption records of some of the operating companies.

If we must rely so much upon actual operating experience, you may then ask, "What are we gaining by all our calculations?" As an operating engineer, I turn to the manufacturers of electrical equipment for the answer to this question. There is relatively little that can be done in changing the characteristics of our transmission systems so as to get more power per circuit over them, but there is apparently much that can be done in the development of terminal equipment. As already indicated, calculations are of more use as a qualitative measure than as a quantitative measure, particularly in reducing the annual bickering as to the relative merits of designing equipment.

There is a point to which I have been leading in this connection and that is as to whether the design engineers of terminal equipment are making its characteristics to suit special cases or whether the improvements are to take the form of a general design which will be suitable to meet the ever-changing conditions in a developing system. For example, I believe these improvements in design should be of a general nature because if we picture the developments of a twenty- to thirty-year period in a large interconnected system, the various load centers in the network will pass through a number of stages in which machine characteristics of only a general nature will apply. Of course for periods of a longer time, we can, by the usual methods of accounting, charge off old equipment and buy new of suitable character. If we take a broad vision of the problem at this time, however, we have an opportunity for obtaining economies which otherwise might be lost.

I should like to call your attention to the related problem of short circuits. Broadly speaking, a short circuit on a system is merely a certain type of load, of which the characteristics are rather elusive, but we can assume conditions covering a wide range in the hope of representing the actual conditions and apply the same methods of calculation carried out in studying stability. Here, then, we have a new method of investigating short circuits which takes into account the various phase relations of the prime movers, giving us a better picture of the duty required of our oil circuit breakers. From one standpoint we may be pessimistic in making calculations on the assumption that the sources feeding the short circuits are all in phase, but on the other hand I believe we have been optimistic in the way our circuit breakers have been performing in interrupting large short circuits, the value of which has been obtained from short-circuit calculation on the d-c. tables. It may seem somewhat unfair to add to the criticisms of our circuit breaker performance but it is a question which is closely allied to the stability problem. The whole thing is like burning the candle at both ends. On one side, we are trying to make our systems more stable which, in a sense, means greater concentrations of power and greater short circuits, and on the other we are trying to keep our concentrations of power to a minimum and thereby relieve our short circuit requirements. How can we have our cake and eat it? The only possibility in view along this line is to lay out small individual systems interconnecting them for diversity and reserve purposes only, but this step is in direct opposition to present tendencies in operating economies.

It is easy to sit here, and I may as well add, in our offices, luxuriously basking in the sunshine of a successful convention or conference, and dispose of this and that theoretical possibility with some oratorical gesture, but we must remember that experience often reverses the best of our intentions. I might illustrate this by an actual happening. The continuity of service on a given substation bus having two independent power station sources and two independent feeders from each of these was questioned and the statement was made that a shutdown was well nigh impossible. Approximately six hours later the impossible happened and the substation had an interruption measured in hours and the trouble was not a bus fault. We must remember that there are a number of links in the chain between our design tables and the distribution of power to the ultimate consumer. We are dependent upon Tommy Riley, Buck Jones, Ted Flinn and the rest of the boys on the firing line, who, in weather like this, keep maintenance on switching to the point where we may expect fast relaying. The more we can incorporate improvements in the inherent characteristics of the apparatus itself the more we will remove the personal element and for that reason we should proceed slowly in getting greater stability with additional equipment that may not function at the critical moment. It is easy to set up operating conditions but often quite another matter to obtain them. Therefore, I believe it is more practical and economical to first improve the characteristics of equipment now in general use and then turn to the possibility of adding other auxiliary equipment.

C. A. Powel: There are two points mentioned by Mr. Wilkins, and also by Messrs. Evans and Wagner, upon which I wish to comment. The first is the character of system oscillations at times of a fault to ground. Ordinarily, one thinks of a short circuit as reducing the energy output of a generator. However, in the case of a fault to ground on a large transmission system, the effect may be to actually increase the generator power output. This is due to the resistance in the ground connections from the faulty conductor to the transformer neutral.

Three records have come to my notice, and in each of these cases the generator output was actually increased, and in two of them the prime mover input was also increased by the opening of the gates. While this evidence should not be construed as meaning that all faults to ground will increase the power output of generators, it certainly does substantiate the position which has been taken that the resistance of faults to ground must be carefully considered.

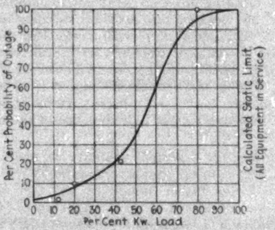

Fig. 3—Probability of Outage of Transmission System as a Function of Load in Case of Fault

The second point is the fact brought out that there is no particular value for the transient limit of a transmission system. Loss of synchronism at times of fault is dependent not only upon the system layout, but also on the load being carried and on certain speculative elements as to location and character of the fault. The point seems to me important, because it involves the whole question of rating of transmission systems. If the operation is such that it is permissible to pull apart every time a fault occurs, then obviously it will be possible to carry a load not far from the static limit. If, however, the transmission system is expected to ride through the majority of troubles, the power transmitted must be kept considerably lower.

In view of these considerations, it is desirable to obtain data as to outages and to plot the results in the form of a probable outage curve as a function of the load. Such a curve is shown in Fig. 3. All faults on the transmission line, and in addition, faults on secondary systems which produce outages, should be considered. It will be noted that the curve shows a definite but small probability of outage when the system is carrying zero load; such outage, for example, might be due to an operating error. The curve also approaches 100 per cent outage as an asymptote at the static limit. It becomes very steep in the middle portion.

The smooth curve, which has been drawn is based on the limited amount of data which has been obtained for a particular system. While more complete data will probably change the exact points through which the curve would pass, it will not change its general shape.

The shape of the load curve on the transmission system must also affect its rating. If the power transmitted varies so that its peak value is only of short duration, the maximum rating can be kept higher with the same risk of outage, because the probability of faults is independent of the load.

Stability and not the losses in the lines determines the rating of a transmission system. Increasing the size of conductors will not appreciably increase the stability, and consequently the question of rating is of prime importance when deciding on the size of conductors to be used in a transmission system. The expenditure for extra heavy conductors may not always be justified.

R. E. Doherty: I think that the Institute is to be congratulated on having the papers which have been presented at this session, particularly the one of great importance, Mr. Wilkin's paper, which, for the first time, gives some actual operating data regarding these problems.

I should like to say a few words with respect to methods of calculations which have been referred to in various papers. A new subject, or a new phase of an old subject, came up recently, (within the last few years), and independent investigators attacked that problem. New nomenclature appeared, and each man has his own notion about those matters. Until they are standardized, we always shall have with us the well grounded objection of the next-to-the-last speaker, that he doesn't know what every one is talking about, and I gathered that he doubted whether the authors did. I have sympathy with his bewilderment.

Each one of us who studies this problem must make some simplifying assumptions. I think that there is one point on which we are all agreed; that a complicated power system network is not amenable to rigorous treatment by mathematical methods as we know them now, and I doubt that it ever will be. It is therefore necessary to make certain simplifying assumptions. Mr. A may make one assumption, Mr. B another assumption, and so on; but before he acts on it, you can rest assured that he is going to have some data or actual experience, some basis for his conclusion before he takes any step toward actually applying it. And if their conclusions on which action may be based are in substantial agreement, it seems of little avail, and certainly misleading, to publicly emphasize trifling disagreement of experts on certain details. While I may appear to be letting the cat out of the bag, so far as this apparent disagreement is concerned, I wish to state that when it comes down to brass tacks on a given proposition, to a study of some proposed system, the agreement among engineers seems to be pretty complete on what can be done and what cannot be done—which indicates to me that the more important factors are generally understood and agreed upon. I am sure all of you who are not daily living with this problem would rather have those who are discuss and settle these disputed details over a table rather than here on the floor of the convention. I think one of the first things that could be settled in that way is nomenclature, and I propose that it should be done.

Just one word about this discussion of static stability and transient stability. I don't like to talk about stability. I share the view of a previous speaker, that "power limit" is a perfectly definite thing. It happens that the maximum power in some cases is also the limit of stability, but it is not in all cases. You can have a maximum power which is not the limit of stability, but I think all engineers are agreed on this: That there is a definite maximum power which can be transmitted over a given system under steady state at normal voltage. It is a perfectly definite thing, and everybody will agree on the value of that limit. And if so, let's talk about that limit.

We have found on investigation that if one throws on any load up to that limit, and the system is controlled by an ordinary vibrating commercial type regulator, the system will carry it. It will not lose synchronism. There may be such cases where synchronism would be lost, but we haven't found them. So long as the power to be carried is not greater than the static limit under the new condition—for instance, when a line section has been switched out—you can get away with it. In connection with this, I agree with the conditions mentioned at the top of page 2, second column, in the Evans and Wagner paper. The supposition is that if you have adequate relaying system and can promptly clear a line-to-ground fault, it is possible to carry through it.

In connection with this, Mr. Nickle states in his paper that the static limit appears to be a criterion for the study or determination of power systems, and I submit that this is a reasonable position to take. It does not say that there are no other criteria, and that if it is satisfied, the system can not fall out of synchronism. It does say, however, that if it so happens, during a transition from condition *A* to condition *B*, the system is so regulated that it could carry condition *B* under steady state, then it can, by virtue of that regulation and the inherent electrical and mechanical characteristics of the synchronous machines, carry through the transition and remain in synchronism. Now if this were true in all cases, the steady-state limit for the worst condition *B* would be a comprehensive criterion. Our conclusion is that with proper regulation and, excluding short circuits, it would be generally true. It is probably true in the usual line-to-ground short circuit. However, whether or not we speak of the steady-state limit as a criterion, I earnestly submit that it is a definite figure which is characteristic of any given system, and is a quantity which any informed engineer with the same data will arrive at. It therefore constitutes a sort of a bench mark for reference.

I wish to add a word about high-speed excitation. Two elements are essential in an excitation system in order to increase the maximum power beyond the steady-state limit to which I have just referred. One is an exciter of sufficiently high magnetic speed. The other is an automatic regulator which would properly control the exciter. The high-speed exciter is easily obtained. However, no commercial regulator within our knowledge at the present time has the required characteristics to thus utilize such an exciter. The mercury-arc rectifier scheme[1] which was discussed at the Seattle Convention has the necessary inherent qualities. With proper regulator control there should be no discontinuity between the steady-state power limit obtained with the standard exciters as ordinarily regulated, and the limit obtained by the extremely high-speed excitation of the rectifier, so long as the increased exciter speed is properly applied. And while I reiterate that inherent characteristics of present day commercial vibrating regulators prevent their accomplishing this, it is nevertheless hoped that a regulator of proper characteristics may presently be available.

To sum up my discussion, it is certainly gratifying that all engineers who are studying these matters seem to be coming to the same general conception of this whole problem. There are certain trifling matters concerning which they are not yet in complete agreement because they have started out from slightly different bases, some thinking one assumption is more important than another. But these are not of general concern. The whole problem, I say, is sifting down to a narrower range in which we can all view it from the same angle and agree, provided we are careful about defining the terms we use.

C. L. Fortescue: I want to express my appreciation of the ingenious method Miss Clarke has developed to determine the static stability limit. I also wish to say, however, that the methods that have been used before are in reality no more complicated than Miss Clarke's method but the apparent simplicity of Miss Clarke's method is due largely to the able way in which it has been presented.

Regarding the paper by Messrs. Nickle and Lawton, I want to congratulate them upon this paper. It is an extremely readable paper. I want to emphasize in discussing this paper not the apparent differences in opinion between our group of papers and this one, but more the points on which we have come to an agreement. There is a tendency in discussions to pick out the points of disagreement and discuss them only. As a result of that, those who read the discussions say, as I heard Mr. Roper remark, "Well, we can't make head nor tail of this matter. Some say this is black and others say it is white." The static stability limit is a very definite limit. One might say it is an invariant of a given system. It is quite a definite value and can be determined without difficulty. I shall go Mr. Doherty one better and say the static stability limit as defined by constant excitation is one of the criteria of the stability of a system—not the criterion, but one of the criteria.

In connection with this static stability limit, in considering ways and means to raise this limit, we found with high speed automatic regulators we were able to get quite an appreciably larger amount of power over the circuit.

Now I am bringing out this point not to emphasize the point of difference, but because this artificial stability has an element of hope in it. If you can do it with a regulator today, you may be able to get a regulator that will do it still better tomorrow. You will surely be able to get more power over a given line tomorrow. Maybe the automatic regulator of special design will be the answer; maybe an inherent regulator scheme will be the answer. We don't know now, but one of these days we will know.

Now regarding the transient stability as one of the criteria of operation, I want to say that after reading over the conclusions in the paper I was rather struck by the fact that transient stability wasn't considered of great importance, but on reading over the paper itself, I found data presented there that led me to say that transient stability was just as much of a criterion as the static stability.

One statement in the paper, page 16, calls attention to single-phase faults and points out if the fault is removed instantaneously, the load can be carried right up to the stability limit. Now if you remove a fault instantaneously, you are doing exactly the same thing as if you cut out a section of line on a loaded transmission line instantaneously. And we know if you cut out sections of loaded lines, the effect on the load limit is very small, so I think that is the answer to that.

As a remedy, mention is made of quick operating relays; that is part of the remedy but we must go a step or two further than that. We must have quickly operating circuit breakers and we must have adequate voltage regulating systems, either high speed or inherent, and we may possibly have to help out with the governing system and also consider the effect of resistance of the fault. I wish to point this out because I myself reached somewhat different conclusions from those of the authors from the data they present in their paper. We can't say that one limit is more important than another limit. We have several limits. We have the static stability limit, and if you raise that, you also raise the transient stability limit as a general rule, so that we have several loopholes to follow in order to solve this problem.

As to methods of calculating transient stability, I may say that we have found it practical to calculate even relatively complicated systems by the point-by-point method. It is of course a long drawn out tedious method. There is no doubt that a technique will be developed in the course of time to enable engineers to calculate power transients in more complicated types of systems. In other words, the present methods are cumbersome and long drawn out but they will be improved and perfected in the course of time.

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[1] *Fundamental Considerations of Power Limits of Transmission Systems*, by Doherty and Dewey, A. I. E. E. JOURNAL, October, 1925, p. 1045.

We do not dissent from the opinion that a dynamic model may be evolved, but we insist that such a model must take into account the essential facts, the most outstanding of which is that the differential equation of the motion is not linear. The condition with which we are concerned in transient stability is that unless we get instantaneous removal of the fault, it does not permit the use of approximately linear form in analysis. In fact, the problem is similar to that of a pendulum and the difference between the two problems is the difference between the cycloidal pendulum and the simple pendulum, between the harmonic motion and the elliptical.

I bring this point up because I think Mr. Nickle in the course of time will undoubtedly get the right kind of dynamic model to find the answer to the problem in the proper terms.

As to the effect of machine characteristics, the importance of machine characteristics has been emphasized by the group of engineers to which I belong. Machines designed according to present practise must incorporate the necessary stiffness, to use the authors' term. This may be done by designing machines with lower synchronous impedance, that is, with long air-gaps or saturated poles. This applies to generators and synchronous condensers. In addition to this, we must have voltage regulators quickly responsive to changes in voltage.

An alternative method which may develop is the compensated machine which may be a generator or a condenser. Partial compensation in the case of a generator may be considered of advantage. Investigations on compensated machines are being carried out now and data are being obtained which will be available in due course.

In regard to the intermediate condenser, the authors have indicated a substantial gain with sufficient condenser capacity. While the capacity requirements as represented by them are large, it need not worry us as there is no doubt that in due time the right kind of machine which will give the desired results with smaller capacity will be developed.

W. P. Dobson: Mr. Wilkins evidently relied upon telephone signals to obtain simultaneous oscillograph records at widely separated stations. In 1912 and 1913 the writer, during an investigation of transients on 60- and 110-kv. systems, made use of an automatic attachment to an oscillograph to accomplish this result. The closing of a switch on the oscillograph table operated the high-tension breaker, the film-motor and shutter in proper sequence to obtain the desired record on the film. Circuit breakers in distant stations were connected to the oscillograph control by means of the system telephone circuits. It was possible to obtain records of transients of short duration on a standard 12-in. film and satisfactory records of transients of over one second's duration were obtained with a 42-in. film. The apparatus was designed by Professor H. W. Price of the University of Toronto and was used on the lines of the Toronto Power Co. and the Hydro-Electric Power Commission of Ontario over a period of 18 months. Several hundred records were taken with a negligible number of failures. It is believed that this was the first instance of the application of automatic control of an oscillograph for this purpose.

H. B. Dwight: On the 11th page of Miss Clarke's paper, there is a diagram in which leading reactive kv-a. are plotted downward. This is typical of the papers by several authors. There is also a group of writers who draw this type of diagram with leading reactive kv-a. plotted upward, the same way that leading reactive current would be plotted. It is an inconvenience to have these two opposite methods in use, and as diagrams of kw. and reactive kv-a. are of somewhat frequent occurrence for various purposes, it seems that the time has come when it is appropriate for the Institute to give a decision as to whether leading reactive kv-a. should be considered positive or negative, and should be plotted upward or downward in diagrams.

J. W. Legg: Mr. Wilkins deserves much credit for recognizing the value of reliable high-speed records, and for insisting on having apparatus capable of giving reliable graphs at these high speeds. The Esterline graphic instruments used in these tests were too sluggish of movement to warrant a high-speed chart until they were reconstructed with greatly increased restoring torque and more than 50 times normal power input. This necessitated a battery of special transformers and special switching schemes to throw the graphic instruments out of the circuit after operating less than half a minute, to prevent excessive heating.

In 1914, while still a student, the writer conceived a truly portable oscillograph to operate with a low-voltage incandescent lamp, and to have wattmeter elements, and other effective-value elements, as well as the standard elements for instantaneous values. The ease of obtaining truly high-speed graphs, freely crossing one another, appealed to the writer as being worth the inconvenience of photographic development. A daylight-loading film holder was conceived then but not developed and perfected until 1921. In spite of this improvement in film-holders, all effort was expended to develop high-speed graphic instruments requiring no photographic film. Progress has been made in this line, but the demand for higher and higher speed records (with correspondingly higher-speed movements) has surpassed all improvements in standard graphic instruments. This is shown very clearly in Mr. Wilkins' paper. The standard high-speed (so-called) graphic instruments were of practically no value for these important tests until remodelled and forced at fifty times normal input. C. F. Wagner saw the limitations of these instruments, after all the improvements he had made, and made up several oscillographic wattmeters to be placed inside our standard, portable oscillographs, and in separate cabinets using standard oscillograph lamps, lenses, film holders, etc. These confirmed the writer's predictions and proved very successful in these tests.

Several years ago the writer figured on an instantaneous, polyphase, reflecting, wattmeter-element, with a natural period of approximately 2000 cycles per second. A single-phase wattmeter element, with a natural period of 5000 cycles per second, would be very simple, but not particularly valuable on account of the high (double) frequency oscillation of the power wave on a-c. lines. The quick-acting (but not instantaneous) elements, pushed through for these tests, proved so successful that they will be commercialized, in an improved form, very soon.

A nine-element portable oscillograph, considerably smaller than its three-element predecessor described in the JOURNAL of February 1923, has been designed to give nine simultaneous records on one 7-in. width of film. Three of these oscillograph elements may be replaced by high-speed wattmeter elements, or by other effective-value elements when they are developed. Such an instrument will be ideal for stability tests and for operation on chance disturbances. A daylight-loading long-film holder has been developed to take films 6½ in. wide and either 3, 6, 8, 16, or 24 ft. long. Twenty-four-foot films were used for the first time in these stability tests, passing through in less than six seconds. For lower speeds the film-holder can be made to operate quite reliably for daylight-loading and daylight-unloading.

During the discussion of the writer's paper, in 1923, on "Expansion of Oscillography," J. R. Craighead seemed to doubt if the oscillograph could be made to function, automatically, quickly enough to show what happened before the oil circuit-breakers functioned. All that the writer claimed then, and more too, has been done since then. Our portable oscillographs have been set up in substations and have operated, repeatedly, on chance disturbances, and have given valuable information as to the disturbance and breaker operation. Furthermore the new single-element oscillograph (the OSISO) can be set up to record chance disturbances within less than one-thousandth of a second after the closing of a quick-acting relay. The 4-volt lamp is initially excited by a condenser charged to 110 volts, d-c. The lighting of the lamp is quick enough to obtain a perfect oscillogram of the chance rupture of the arc in a quick-acting circuit breaker on d-c. lines. With the proper quick-acting relay, operating on the steepness-of-wave-front principle, the oscillograph will function perfectly within two-thousandths of a second after the very start of a short circuit, before the quick-acting breaker can reduce the short-circuit current. The photographic film must be kept rotating all the time, for such extremely quick operation.

The advantages of such apparatus are being better and more universally recognized, and oscillography will continue to expand and be a greater and greater help to others besides the electrical manufacturer.

Svend Barfoed: In designing transmission systems to safely carry a specified load, I have in the past used methods that are very direct. They involve the use of diagrams in which the various factors affecting the power limit of a transmission system appear together and may thus be studied more easily. As yet I have not found any necessity for transmutation of the diagram and method into others which, as far as can be seen, are the exact equivalent. The accompanying diagrams illustrate such a study.

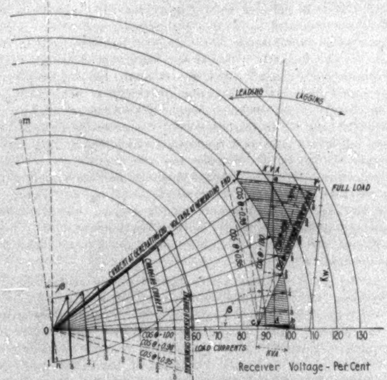

FIG. 4—TYPICAL REGULATION DIAGRAM FOR HIGH-CAPACITY TRANSMISSION LINE

Fig. 4 has been described several times. The load lines are shown advancing by eights up to full load; to scale they are measured in position in kw. The magnetizing power is measured parallel to the load lines to the same scale in kv-a., being for example *c F* at unity power factor of load at full load or *F′ F* at a power factor of 0.95. The magnetizing effect of the charging current is given by *a c*. Thus all factors affecting the transmission limit of power of the line and transformers are had at a glance. It is at once apparent that all factors are very definite and do not admit of discussions of a qualitative nature. For any given relation the line and the transformers have characteristics which are fixed and quantitatively known. It can easily be seen that the smaller the resistance compared to a given reactance the higher the power limit, and highest for zero resistance. In other words, the more nearly vertical line *c e* is, the higher the power limit. On the other hand, with nearly zero reactance the power limit would be very low indeed. The criterion is the tangency of a load line to any one of the circles shown on which it may be desired to operate. Tangency represents the static load transmission limit. Swinging the unity power-factor line *c e* around *c* as center to the right, representing a designed decrease in reactance for the same resistance, makes the maximum load lines become tangent to the circles at smaller and smaller loads and when finally the load lines become nearly vertical the ability of the system to transmit power almost vanishes. It is thus seen that in order to transmit power with alternating current the proper relation between resistance and reactance must prevail and it is evident that low resistance lines are highly desirable since the reactance of the line can be changed but little due to clearance limitations. Similar reasoning holds for the transformers, but with them it is also desirable that the reactances are no greater than required for the strength of the mechanical structure during short circuit. At previous meetings when power transmission limits have been discussed, the characteristics of the synchronous machinery have been included, but in no very definite manner. It is with respect to the synchronous machines that it is ventured to hoist a warning signal. It is seldom that a large power plant contains but one machine. If there are several it is imperative that they have such characteristics that they will operate properly in parallel under all conditions of load and can readily be synchronized. There must be no hunting between them. If for the sake of increased power transmission limit the synchronous reactance is very much reduced, hunting will surely occur between generators running in parallel in the same station, and stability, possibly, may only be secured again by adding reactors external to the machines.

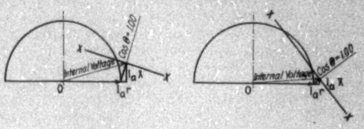

FIG 5 FIG.. 6

FIGS. 5-6—GENERATOR DIAGRAMS FOR DIFFERENT RATIOS $\dfrac{I\,r}{I\,X}$

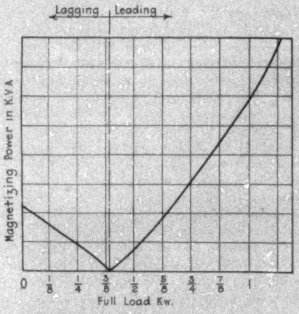

FIG. 7—MAGNETIZING POWER REQUIRED FROM SYNCHRONOUS MACHINES ON SYSTEM (FROM DIAGRAM SIMILAR TO FIG. 4)

This is an old story and is visually shown in Figs. 5 and 6. In Fig. 7 the ratio $\frac{I_a r}{I_a x}$ permits the proper synchronizing force to develop between machines due to a large amount of power possible of transmission between them. In Fig. 6 the ratio $\frac{I_a r}{I_a x}$ is such that the limit to the synchronizing force is very quickly reached with the machines liable to fall out of step.

The effect of the load power factor is given at once by these diagrams, and there is nothing vague about the amount of magnetizing power required to compensate for power factor, either for voltage regulation or for a given power transmission limit. The synchronous condenser is a very admirable machine to give or consume magnetizing power in the required amount. It will perform exactly as desired if the V curve of zero power load coincides with the magnetizing power curve of Fig. 7.

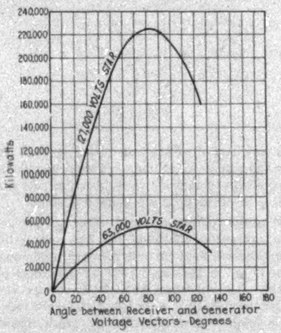

Fig. 8—Static Stability Curves—200 Miles of Line (Plotted from Diagrams Similar to Fig. 4)

From the above it is seen that the line and transformer characteristics and the charging current do not lend themselves to a discussion of power transmission limits. They are to be considered known and fixed within narrow limits. As long as we transmit power with alternating current, the charging current cannot be removed and to say that it is detrimental to a higher power limit is like saying that the current flowing in a coil of wire is undesirable because it heats the coil.

The power limit of a given transmission system is therefore influenced chiefly by the characteristics of synchronous apparatus. I accept the statement that the machine designers know how to design machines for maximum transmission of power. That being so, the transmission line designer would like to know in what manner this is done. He would like to know how the synchronous reactance is apportioned between self induction and armature reaction, how compensated for, whether means to affect a quick response to excitation power is counteracted by means to affect damping of power swings, and whether a machine designed for maximum power limit is a suitable machine for installation in a plant where there are several which must safely operate in parallel on the same bus bars.

It has been said that power factor has much to do with the power transmission limit. To be sure it has, but in a perfectly definite manner. Power factor lagging means that magnetizing power must be furnished by synchronous machines either at one end of the line or both; power factor leading means that magnetizing power must be absorbed by synchronous machines or by some other variable tractor. If the rate of change in magnetizing power is such that the excitation power of the field structures cannot follow at the same rate, oscillations will be started. Tests on actual systems cannot reveal much in this connection since the magnetic structures cannot be changed. Such tests belong on the test floors of the manufacturer. With machines of conventional design it appears that transmitting power at 220 kv. up to 250 mi. is safe when the lines are loaded to one ampere per 2200 cir. mils and condensers located only at the receiving end. This can be improved by suitable engineering of the line itself. A discussion of this would, however, lead too far at this time. Only this,—there must not be added further apparatus in power houses, substations or elsewhere until circuit breakers for 220-kv. operation have been improved to the point where they no longer constitute the present limit to transmission of power. The circuit breaker must perform its function without destroying itself too soon and it must open the circuit at such a rate that a power arc on the line will not have time to seriously injure the conductor. The circuit breaker must be relied upon to transfer without distress the energy from a line section in trouble to a parallel line. The normal energy to be transferred is of the order of 130,000 kw. It is comparatively easy to compute power swings and to provide a sufficient margin of safety. It is far more difficult to design economically a system where disturbances causing power oscillations will inherently be reduced to a minimum. The first problems to solve are 220-kv. circuit breakers and line insulation.

F. L. Lawton: It is certainly in order to compliment Mr. Wilkins and his associates on the courage and initiative displayed by them in attempting investigations of system stability on a power system of the magnitude described. Further, the addition of field tests of this type to the theoretical studies undertaken in the past few years affords an opportunity of passing judgment on the assumptions made in those studies.

It is indeed encouraging to note the real agreement between the conclusions reached by Mr. Wilkins and those presented by Mr. Nickle and myself, today, and also those in a recent paper[4].

The writer disagrees with Mr. Wilkins' observations anent the place filled by the use of artificial lines and miniature equipment. It is, of course, true that test data, whether obtained on an actual network or on miniature systems as used by Mr. Nickle and myself, apply only to specific conditions on given systems. However, and this is important, *it is virtually impossible to assign the right values to the factors affecting the stability of a real network*, of anything like the complexity of the Pacific Gas and Electric Co.'s transmission and distribution system. With the miniature system, methods of analysis can be studied, actual cases[5] checked, and *the methods of analysis may then be applied with real confidence in the soundness thereof to the solution of stability problems.* In this respect, the miniature systems with synchronous apparatus up to a few hundred kv-a. capacity have been of great assistance but it is not suggested that a 225-kv-a. system be used for the solution of stability—or rather, power limit—problems. For these there are available:

(a) The miniature system of Spencer and Hazen[6].
(b) The equivalent-circuit method of Nickle[7].
(c) Proved methods of computation.
(d) Experience.

When anyone conversant with the difficulties involved in analyzing system power limits by theoretical methods visualizes the physical data applying to the system in Fig.1, he will have a better appreciation of what a miniature-system method of stability analysis means.

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[4] *Fundamental Considerations of Power Limits.* R. E. Doherty and H. H. Dewey. A. I. E. E. Journal, October, 1925, p. 1045.

[5] Such as the case described by H. A. Barre in the *Electric Journal* for June, 1925, and discussed by R. D. Evans, A. I. E. E. Journal, January, 1926, p. 70-71 and by R. E. Doherty, p. 75-76.

[6] *The Artificial Representation of Power Systems.* H. H. Spencer and H. L. Hazen, A. I. E. E. Journal, Jan., 1925, p. 24.

[7] *Oscillograph Solution of Electro-Mechanical Systems.* C. A. Nickle, A. I. E. E. Journal, Dec., 1925, p. 1277.

When results of good engineering accuracy, directly applicable to proposed or existing power networks, can be obtained relatively easily and expeditiously by use of a miniature-system method and proved principles, it is presumed that they will be used and "actual quantitative values for this fundamental data" will *not* need to be "measured." Such miniature systems and principles, it would appear, are already available.

Regarding the contention that system stability, as a problem, is inextricably entangled with operating economics, this is largely true but nevertheless good progress can be made in determining the power limits of transmission systems under the majority of circumstances likely to arise.

The results obtained by Mr. Wilkins when the 220-kv. line at Pit No. 1 was switched out under a load of 24,000 kw. agree quite closely with those obtained by Mr. Nickle and the writer. I think that a little closer consideration will convince Mr. Wilkins that the cause of the greater disturbance on closing the line was entirely due to the effect of the line charging current in changing the phase and magnitude of the "open line" voltage relative to that on the generator side of the closing circuit breaker.

In connection with the above, it might be added that an oscillographic wattmeter with characteristics similar to that described was used throughout the major part of the miniature-system tests previously referred to.

To conclude, it appears that the first extensive investigation of power limits on an actual system of a fairly high degree of complexity leads to conclusions in substantial agreement with those presented by Mr. Nickle and myself, as well as others, entirely from the results of miniature-system studies, coupled with field experience and office analysis.

A. P. Mackerras: A pleasing feature of this paper is the use of graphical methods. Graphical solutions are usually sufficiently accurate for practical purposes; and, in addition to their rapidity, they give a better physical conception of the problem than algebraic formulas, and indicate more readily the effect of changes in the quantities involved.

In the construction of Fig. 3B, it has been pointed out that the locus of points at which $P\ Q$ subtends a constant angle is a circle with centre R. But it may also be of interest to note that the locus of points the distances of which from A and B are in a fixed ratio is also a circle. If the line $A\ B$ is divided internally at N and externally at N' in the given ratio E_A / E_B, this circle has $N\ N'$ for its diameter.

In Appendix C the equation

$$\text{Power} = \frac{1}{2}\,(E\,I + E\,\bar{I})$$

is given. The truth of this equation may readily be seen from the accompanying diagram. Let $O\ A$ be standard phase, and let E and I be the voltage and current vectors the magnitudes of which are e and i. Then the conjugate vectors E and $\bar{I}$ will also have magnitudes of e and i, and their angles with $O\ A$ will be equal and opposite to those of E and I. To multiply the vector E by the vector I, we multiply their magnitudes and add (with due regard to sign) the angles they make with $O\ A$. The vector $E\,\bar{I}$ will therefore have a magnitude of $e\,i$ and will make an angle ϕ above $O\ A$, where ϕ is the angle between E and I. Similarly the vector $\bar{E}\,I$ will have a magnitude of $e\,i$ and make an angle ϕ below $O\ A$. Then clearly, the vector sum $\bar{E}\,I + E\,\bar{I}$ will be equal to $O\ B$, or $2\,e\,i\cos\phi$, which is twice the power in the circuit.

Sidney Withington (communicated after adjournment): Since the limit of power transmission is a function of the system frequency, it is entirely conceivable that in the transmission of large amounts of power over long distances it may be found desirable to adopt relatively lower frequencies than the standard 60 cycles; for instance 25 cycles or even lower. In this case there would be large frequency-changing substations at the centers of distribution. The frequency-changer sets might perform also the function of synchronous condensers, and the increment of cost would be relatively small compared to the total transmission line cost.

If this occurs and a low frequency is adopted for long distance transmission, it is entirely within the range of possibility that it would be economical for the electrified railroads of the country to obtain their power direct from these major transmission lines without the medium of rotary apparatus, perhaps balancing the load among the phases in sections. This is significant, considering the possibilities of railroad electrification, and may be a controlling factor in developing a standard system.

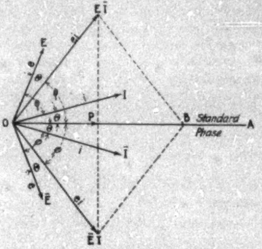

FIG. 9

C. F. Wagner (communicated after adjournment): Miss Clarke has presented methods for the calculation of static stability for constant field current in a clear and concise manner. It should be noted in passing, however, that these limits do not represent the ultimate when automatic voltage regulators are used to maintain voltage. Miss Clarke uses the logical device of representing loads where possible by static networks. This results in considerable simplification.

Using the various devices pointed out by Miss Clarke, the most general case in which the problem can still be reduced to a single synchronous machine at each end is that shown in Fig. 9, herewith, in which the three rectangles represent general networks, E_a and E_d the internal voltages and b and c the points at which the voltages are known.

Messrs. Evans and Sels in the July 1921 number of the *Electric Journal* have indicated how three general networks in series can be combined into one equivalent, so that the following expression is obtained.

$$\breve{e}_a = \breve{A}_0\,\breve{e}_d + \breve{B}_0\,\breve{I}_d \tag{1}$$

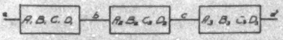

FIG. 10

where $\breve{A}_0$ and $\breve{B}_0$ are the general circuit constants for the three combined networks, e_a and $\breve{e}_d$ the star voltages at a and d respectively and $\breve{I}_d$ the current at d.

Transposing equation (1)

$$\breve{I}_d = \frac{1}{\breve{B}_0}\,\breve{e}_a - \frac{\breve{A}_0}{\breve{B}_0}\,\breve{e}_d \tag{2}$$

The power at d

$$P_d + j\,Q_d = 3\,\breve{e}_d\,\hat{I}_d = \frac{1}{\hat{B}_0}\,\hat{E}_a\,\breve{E}_d - \frac{\hat{A}_0}{\hat{B}_0}\,E_d^2 \tag{3}$$

when E_a and E_d are phase volts at a and d respectively. Now let

$$\check{E}_d = E_d \, \epsilon^{j0} \text{ and } \check{E}_a = E_a \, \epsilon^{j\phi} \tag{4}$$

$$P_d + j\, Q_d = \frac{1}{\hat{B}_0} E_d \, E_a \, \epsilon^{-j\phi} - \frac{\hat{A}_0}{\hat{B}_0} E_d^2 \tag{5}$$

The real component of the above expression P_d becomes a maximum when

$$\phi = \tan^{-1} \frac{b_0}{a_0}$$

where a_0 and b_0 are the components of $\check{B}_0 = a_0 + j\, b_0$. This determines the angle ϕ between a and d for which maximum power is delivered at d. It is necessary to calculate only the $\check{B}_0$ constant for the combined network.

$$\check{B}_0 = \check{A}_1 (\check{B}_3 \check{A}_2 + \check{D}_3 \check{B}_2) + \check{B}_1 (\check{B}_3 \check{C}_2 + \check{D}_3 \check{D}_2)$$

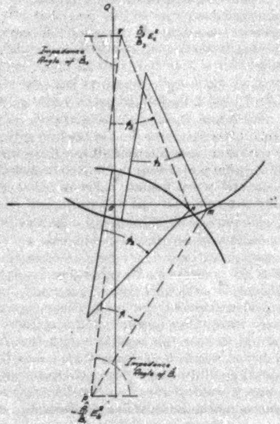
Q
r
$\frac{\hat{D}_3}{\hat{B}_3} E_c^2$
Impedance Angle of $\check{B}_3$
ϕ_3
ϕ_2
o
n
m
ϕ_1
ϕ
Impedance Angle of $\check{B}_1$
p
$-\frac{\hat{A}_1}{\hat{B}_1} E_b^2$

FIG. 11

Having obtained the maximum over-all angle corresponding to maximum power delivered, the actual value of power delivered can be determined by the following graphical means. From a knowledge of A_2, B_2, C_2, D_2 and the voltages at b and c, the sending and receiving-end power circle diagrams for the middle section can be drawn as shown by the heavy lines in Fig. 10. For any angle ϕ_2 between the voltages at b and c the power flow at these points is indicated by the points m and n. Every point such as m on the supply circle must also represent a point on a receiving circle for the first section. This latter circle is represented by the expression

$$P_b + j\, Q_b = \frac{1}{\hat{B}_1} E_a \, E_b \, \epsilon^{-j\phi_1} - \frac{\hat{A}_1}{\hat{B}_1} E_b^2$$

The center of this circle is located at $-\frac{\hat{A}_1}{\hat{B}_1} E_b^2$. It will be noted that all of these quantities are known and can be plotted at the point p. Now while the value of the voltage E_a is unknown (and incidentally will not be necessary to determine) the reference vector from which the angle ϕ_1 is measured can be drawn making an angle with the horizontal equal to that of $\frac{1}{\hat{B}_1}$ which angle is the same as the angle of $\check{B}_1$, *i. e.*, $\tan^{-1} \frac{b_1}{a_1}$ where $\check{B}_1 = a_1 + j\, b_1$. As stated previously, the point m must lie on the circle, therefore, and angle ϕ_1 between

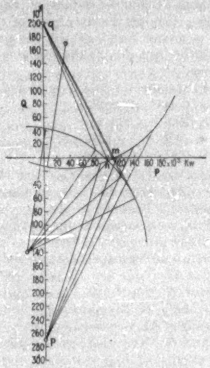

FIG. 12

$p\, m$ and the reference line indicates the angle between the voltages at a and b for the power transmitted at c corresponding to point n.

A similar construction applies to the receiving-end network. The power at c into this network is expressed

$$P_c + j\, Q_c = \frac{\hat{D}_3}{\hat{B}_3} E_c^2 - \frac{1}{\hat{B}_3} E_c \, E_d \, \epsilon^{+j\phi_3}$$

The center of the circle q is determined by $\frac{\hat{D}_3}{\check{B}_3} E_c^2$ and the reference line makes an angle with the horizontal equal to that of $-\frac{1}{\hat{B}_3}$, *i. e.*, $\tan^{-1} \frac{b_3}{a_3}$ where $\check{B}_3 = a_3 + j\, b_3$.

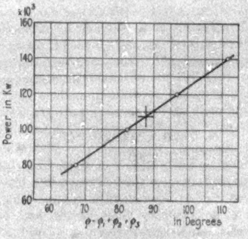

FIG. 13

The angle ϕ_3 between $n\, q$ and the reference line indicates the angle between the voltages at c and d. Therefore $\phi_1 + \phi_2 + \phi_3$ gives the total angle between the voltages at a and d for the power conditions at c corresponding to the point n. After a few trials the point *n* can be determined for which $\phi_1 + \phi_2 + \phi_3 = \phi$. This value of power indicates the maximum that can be transmitted at *c*.

The general method can best be illustrated by means of a solution of a particular case. The example chosen is that indicated on the eighth page of Miss Clarke's paper which involves the determination of the steady-state limit for a 270,000-kv-a. generator, transformers, a 250-mi. line and a 170,000- kv-a. synchronous motor. Complete details of line and machine characteristics are given on the sixth page of the paper. The voltage at the generator terminals will be maintained at an equivalent of 220 kv. and that at the receiving end at 200 kv. by hand regulation.

The combined constants for transformers and line are:

$$\begin{aligned} A_2 &= 0.8431 + j\,0.0279 \\ B_2 &= 39.6 + j\,232.1 \\ C_2 &= (0.0104 + j\,1.248)\,10^{-3} \\ D_2 &= 0.8428 + j\,0.0280 \end{aligned}$$

These constants enable one to construct the sending and receiving power circle diagrams in the ordinary way. These are indicated by the full lines in Fig. 11.

The center of the receiving circles for network 1 is located at $-\frac{\hat{A}_1}{\hat{B}_1} E_b^2$. Since this network contains only the generator reactance

$$\begin{aligned} \check{A}_1 &= 1.0 \\ \check{B}_1 &= 0 + j\,X_1 \end{aligned}$$

and

$$\frac{\hat{A}_1}{\hat{B}_1} = \frac{1}{-j\,X_1}$$

$$-\frac{\hat{A}_1}{\hat{B}_1} E_b^2 = -j\,\frac{E_b^2}{X_1}$$

$$= -j\,\frac{220 \times 220{,}000}{179.2} = -j\,270{,}000 \text{ kv-a.}$$

Incidentally, this is equal to the sustained short circuit kv-a. at 220 kv. This point is indicated by the letter *p* on Fig. 3. The reference line coincides with the axis of reactive power.

The center of sending circles for network 3 is obtained in a similar manner.

$$\begin{aligned} \check{D}_3 &= 1.0 \\ \check{B}_3 &= 0 + j\,X_3 \end{aligned}$$

$$\frac{\check{D}_3}{\check{B}_3} = \frac{1}{j\,X_3}$$

The center is located at

$$\frac{\check{D}_3}{\check{B}_3} E_c^2 = \frac{E_c^2}{-j\,X_3}$$

$$= j\,\frac{200 \times 200{,}000}{200} = j\,200{,}000$$

This point is plotted at *q*, the reference line being the axis of reactive power.

Now give ϕ_2 an arbitrary value, say 34 deg., and determine *m* and *n* for this value. Draw *m p* and *n q* and measure ϕ_1 and ϕ_3.

$$\begin{aligned} \phi_1 &= 22.3 \text{ deg.} \\ \phi_3 &= 26.0 \text{ deg.} \\ \phi_1 + \phi_2 + \phi_3 &= 82.3 \text{ deg.} \end{aligned}$$

The total angle between internal voltages is then equal to 82.3 deg. for 100,000 kw. transmitted at receiving end. Choose a different value of ϕ_2 and repeat. By this means the curve shown in Fig. 4 can be obtained.

Substituting in the equation for B_0, this constant is found equal to

$$B_0 = 28.6 + j\,506.7$$

$$\phi = \tan^{-1} \frac{506.7}{28.6}$$

$$= 518.7$$

$$= 86.7$$

From Fig. 4 the maximum power, which occurs for $\phi = 86.7$ deg., is equal to 106,500 kw. which value checks the result obtained by Miss Clarke.

C. A. Nickle: There seems to be a misunderstanding of what the authors intended to convey by their use of the terms "steady-state power limit" and "transient-load power limit." To make this matter clearer, let us consider three cases of load change which may be classified as transient changes.

First, let us consider the change involved in throwing on a load. Before the load is thrown on, the line will have a certain voltage and the synchronous machines will have their magnetic circuits excited to give this voltage. Under this adjustment, the line will carry a definite maximum load of the type to be thrown on. What the authors intended to convey was that it makes very little difference in the ability of the system to carry the load whether this load is thrown on gradually or rapidly or even instantaneously.

Second, consider dropping a section of the line. We do not intend to say that the steady-state power limit to be used is that of the line before the section is dropped. The steady-state power limit to be used is the limit of the line with the section out and our tests again indicate that we can drop a section of a line which is carrying power so as to leave the remaining section with a load equal to its steady-state value and that this may be done gradually or quickly.

Third, consider the case of short circuits. In the case of three-phase short circuits, of course, no power can be transmitted beyond *the short circuit; that is, the final steady-state power has been reduced to zero*, and the load at that point will be dropped unless the relays relieve the condition very quickly. In the case of single-phase short circuits, the line has been weakened in its ability to carry steady-state power but the amount will be a definite value. If we take this same line with the short circuit permanently on and calculate its steady-state power limit,—that is, for loads slowly applied,—this will give the same value as when the short circuit is suddenly applied, the magnetic adjustment of the synchronous machines remaining unchanged.

It is very important that we choose the proper steady-state power limit. The reason single-phase short circuits are severe is because they reduce the value of the steady-state power which the system can carry. When we were able to carry 31 per cent of the steady-state power of the line before short circuit it meant that the steady-state value to be used was the value with short circuit on the line. If we increase the steady-state power limit by means of special excitation systems, such as mercury arc and other schemes, we also increase the transient power limit to the same value. In every case where we have tried this on the miniature system, we have been able to create any condition suddenly that we could produce slowly and still maintain synchronous operation.

Edith Clarke: Mr. Evans and Mr. Wagner have stated in their discussions that the methods used in my paper to obtain the maximum power which can be transmitted over a given transmission system under steady state are applicable for constant field currents and when the voltage is regulated by hand, but do not apply when there is an automatic voltage regulator on the system. Mr. Spencer has asked how the constant generator reactance, which is to be used in the calculations for maximum power, is selected.

In order to answer satisfactorily, it will be necessary to define constant field or fixed excitation and hand regulation and to explain what is meant by *equivalent* synchronous impedance.

For simplicity, assume a synchronous generator and motor on the same bus with constant and equal fields on both machines; then increase the load until the machines fall out of step, plotting power delivered *vs.* terminal voltage. If this is done for a number of different field currents, it will be noted that there is one curve in which maximum or breakdown power occurs at normal terminal voltage. On the curves corresponding to lower field currents, maximum power occurs at voltages below normal, while on curves corresponding to higher field currents, maximum power occurs at voltages above normal. With given fixed excitations maximum power will not in general occur at normal voltage. When hand regulation is used and load is slowly added so that normal voltage is maintained, the maximum power limit is usually understood to mean the power corresponding to that fixed excitation for which breakdown power occurs at normal terminal voltage.

It has been found by tests that with regulators such as are used commercially today, it is not possible to transmit more than the maximum power limit as defined above by hand regulation. I agree with Mr. Evans that if regulators were fast enough to maintain terminal voltage, the machine impedance could be neglected; or if fast enough to maintain constant flux, then leakage reactance could be used; but until something is done to increase their speed of operation, the maximum power limit with voltage regulators will be the limit obtained by hand regulation.

Equivalent synchronous impedance is a fictitious value used for convenience to obtain maximum power. The field current corresponding to it is a fictitious current and does not represent actual field current. The justification for using *equivalent* synchronous impedance is that by its use the calculations have been greatly simplified and the calculated maximum power has been found to give a satisfactory check upon test values.

For a given machine, the sustained impedance which replaces leakage reactance and armature reaction is not constant, except for a machine of zero saturation, but varies with terminal voltage, power factor and load. The *equivalent* synchronous impedance likewise is not constant for a given machine but depends upon terminal voltage, power factor and load. In steady-state stability problems a voltage regulator is assumed; therefore maximum power is obtained at normal terminal voltage. The power factor on the generator for long lines heavily loaded is not far from unity. The load on the generator may be estimated after one approximation. Therefore, terminal voltage is known and power factor and load can be estimated.

When the leakage reactance, the armature reaction, and the saturation curve of a generator are given, the following *equivalent* synchronous impedance has been found to give a good approximation for maximum power.

$$\text{Equivalent synchronous impedance} = (\text{leakage reactance}) \text{ plus } \left(\text{armature reaction} \times \frac{\text{slope of saturation curve}}{\text{slope of air gap line}}\right).$$

The leakage reactance will be given in per cent values. The armature reaction should be expressed in per cent of the air-gap ampere-turns corresponding to no-load normal voltage. The slope of the saturation curve should be obtained at a point corresponding to virtual voltage. Virtual voltage depends upon terminal voltage, power factor and load, and may be estimated, but in general the slope of the saturation curve at normal terminal voltage may be used.

With voltage regulators or with hand regulation, the maximum or breakdown power will occur at normal terminal voltage so that the same value of *equivalent* synchronous impedance will be used and the same value for maximum power will be obtained in either case. With fixed excitations, however, the conditions are different since the voltage at which maximum power occurs is not given. The procedure is not so direct as where normal voltage is maintained but *equivalent* synchronous impedance may be estimated and maximum power obtained after one or two approximations.

R. D. Evans and C. F. Wagner: Mr. Sels makes a plea for a definition of stability limits, so that we may all know what the other fellow is talking about. We might refer to the definition as given in our paper in which we say that the stability may be defined as the capacity of a power system to remain in equilibrium under steady-load conditions and its ability to regain a state of equilibrium after a disturbance has taken place. The first part of the definition is referred to as static stability and the second part as transient stability.

Mr. Sels believes that the static limit is of the utmost importance. He believes that in the future it will be possible so to improve insulation that insulator failures will be very rare. We hope that such will be the case, but we are living in the present, not the future. For that reason, we believe that transient stability is of more importance at the present time than static stability.

Mr. Sels also stated that we have the stability problem present in all metropolitan systems. In general, however, the stability calculations for metropolitan systems are much more difficult and complicated than for long-distance transmission systems. In the latter the line constitutes the greater part of the impedance between the two points considered, the impedance of the terminal equipment acting more in the nature of a correction factor. In addition, generators at each end of the transmission line can be grouped together and considered as units. In metropolitan districts the connecting impedances are much more complicated and in most cases it is not permissible to simplify the problem by grouping the generators into two groups. The effect of faults and changes in voltages is more problematic.

The curve presented by Mr. Powel showing the relation between transmitted load and the probability of outage is both interesting and important. It would seem desirable to obtain similar data on the operation of other existing systems. Such data might be analyzed in a number of ways, one of which would be the plotting of the number of outages for different loads. Such a curve would take into account the load cycle under which the system is operated, and also the variation in the probability of failure during different parts of the day. Ater a considerable amount of data were available, it would be possible to plot curves for each of the various types of disturbances, such as low-tension faults, three-phase faults, and flashovers to ground; and from a study of these curves to determine the amount of consideration which should be given to each type of disturbance in making additions to existing systems, or in laying out new projects. In order to compare data on different systems, it would be desirable to plot the outage curve in terms of the angle between supply and receiver ends. This function would form the basis of comparison of operation of different systems. The collection of such system-operating data can best be obtained by the use of such recording instruments as described in the paper by Mr. Wilkins and in the discussion by Mr. Legg.

In connection with Mr. Doherty's discussion, we agree with him that the static limit is a more definite quantity than the transient limit. The point which we wish to emphasize is that system stability under disturbances is the more important problem and that the transient stability limit is the more important limit. We agree that with fixed excitation in machines, the maximum limit is quite a definite quantity. However, this limit is not the same as the limit of a system operating with voltage regulators for the case of slowly increasing loads. Under this condition of operation, the true static limit will be in excess of the limit determined by fixed excitation. This arises because of the phenomenon which, in our paper, is termed "artificial stability." It is stated in the paper by Nickle and Lawton, and is endorsed by Mr. Doherty, that the only function of the regulator is to increase automatically the excitation as the load is applied, the actual limit being the same as that for hand control. Our paper presents results of analytical work on this problem which indicate that a condition of artificial stability in which the system is held together merely by the action of regulators, is possible. Since this paper was written, these results have been verified by experiments. The set-up consisted of a d-c. motor, a-c. generator, artificial line, an a-c. synchronous motor, and a d-c. generator loaded on resistors. With hand regulation, increasing the excitation to maintain constant terminal voltage as the load was slowly applied produced a limit of 65.1 kw., and with voltage regulators 79.6 kw., an increase of some 20 per cent. Mr. Doherty states that tests which he has carried out have not shown the existence of stable operation for loads beyond the static limit determined by hand control. He does not deny the possibility of such operation, but merely states that he has not found such a condition. It is possible that this disagreement may be ascribed to differences in the mechanical inertia and in the time constants of machines, particularly of exciters.

While we point out that this additional limit is available, we do not wish to imply that we advise working to this limit in ordinary operation. It should be recognized that this limit is available, and that advantage could be taken of it for emergency operation. This matter of artificial stability, however, is largely one of academic interest. We do not wish to emphasize it too strongly. The real limit of practical importance which we have pointed out, and have tried to emphasize, is the transient limit. It is gratifying to find that the tests of Messrs. Nickle and Lawton on miniature systems bear out this contention.

Prof. Dwight has called attention to the difference in the method of plotting reactive kv-a. in power-circle diagrams. In the absence of an A. I. E. E. standard governing this practise, the various investigators have used the convention that seemed to them most logical and convenient. We heartily endorse Prof. Dwight's plea for A. I. E. E. standardization ruling on this point.

Mr. Barfoed brings up the matter of line insulation and circuit-breaker performance for 220-kv. systems, as affecting the general stability problem. In addition, Mr. Wilkins has called attention to the fact that 220-kv. circuit breakers may have a small time lag between the opening of the three poles. This difference in time in the test described by Mr. Wilkins was due to the failure to secure simultaneous mechanical opening of the three poles of the circuit-breaker. This is evidenced by the fact that the time required for opening the breaker was closely the same and independent of the load carried in the various tests. The oscillograph records also showed the time of arcing was approximately the same on the three poles, and could not be responsible for the period of about 0.2 seconds from the beginning of arcing on the first pole to the final interruption of the circuit. The 220-kv. circuit breakers can be adjusted to open the contacts on the several poles in about a cycle, as measured by the cycle counter. Furthermore, it may be pointed out that in the new breakers for 220-kv. service, further improvements have been made which secure more positive action, and facilitate adjustment in the field. Tests of a type similar to that described in the paper have been made subsequently, and in these it was found that the difference in time of operation at the three poles was less than two cycles.

Mr. Withington has called attention to the fact that the power limits at 25 cycles are considerably higher than at 60 cycles. This is a pertinent point in connection with the selection of the frequency to be used for railway electrification and general power purposes.

Mr. Roy Wilkins: It might be worthwhile to give the actual operating experience to reassure those who contemplate building such lines that they can be operated. One line has been in operation for twenty-two months and the other about nine months. There have been twenty-six total cases of trouble, of which two were mechanical, four were caused by birds (which was definitely known since the birds were killed), two by lightning, and five were located but the cause not known. The remainder have never been located. Four of these interruptions, of which two were mechanical and two caused by lightning, caused interruptions of one line of longer than three minutes. The balance were momentary line interruptions and the line went back without any change whatever. These lines are normally charged from the generating plant at reduced frequency, usually thirty-five cycles, in which they build up with the field circuit open, to the normal voltage and little above normal current. The condenser is then put on at the receiving end and they are brought up to full frequency and paralleled at the receiving end. Thereafter the load is pulled up and they are paralleled at the sending end. Customarily, this is carried out without any dispatching whatever, and the whole procedure has been carried out in approximately one minute. In times of great stress, for instance, if one line goes out over the peak when the load is high, we charge the full 202 miles back to the system and parallel at the power house. This gives a voltage rise at the substation end of 10 per cent and is not customarily carried out.

The two lines, by the ordinary method of analysis, will carry about 198,000 kw. each. We plan never to run over 120,000 kw. on them. At such time that the load increases above this point, segregating points will be added in the line. When development requires, there will be four complete lines.

The question has been brought up about the oscillographic wattmeter and meters. This oscillographic wattmeter is more or less of a novelty. It is felt that it is reasonably accurate. It requires so little power that the ordinary bushing-type transformer will supply it without serious error in ratio and it will record disturbances lasting as short a time as about $^1/_{20}$ sec.

Transmission Systems with Over-Compounded Voltages

BY H. B. DWIGHT[1]

Member, A. I. E. E.

Synopsis.—A usual method of calculating a transmission line with transformers, in which the voltage is held constant at both ends by synchronous condensers, is to use the circle diagram method. It is often advisable to use "over-compounded" voltage instead of constant voltage; that is, to increase the voltage as the load increases. Methods of calculation are given in this paper for two cases, first with over-compounded generator voltage and constant receiver voltage, in which case the diagram is a circle, and second, with over-compounded voltage at both generator and receiver, in which case the diagram is an ellipse. Examples are given and the diagrams are shown in Fig. 2.

A short discussion is given of the advantages and limitations of using over-compounded voltage.

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It is a very usual practise to control the station voltages of transmission systems by automatic voltage regulators, which adjust the field currents of generators and synchronous condensers so that the desired voltages are obtained. In such cases, very great advantages are obtained in many stations by maintaining a higher voltage at a time of heavy load than at a time of light load. This may be called "over-compounded voltage," and can be easily accomplished by means of a simple arrangement with an automatic voltage regulator, which is often called line drop compensation. One of the most common methods of producing line-drop compensation is to put a few turns of winding, carrying current from series transformers, on the magnet of the relay which controls the voltage. These series ampere-turns usually amount to not more than about 25 per cent of the ampere-turns of the shunt winding, which is the main winding. Therefore, the result is that, at no load, the main winding operates the regulator so as to produce a certain voltage. With load conditions and the series winding opposing the shunt, a higher voltage is necessary before the regulator will operate; therefore, a higher voltage than at no load is maintained.

A somewhat similar arrangement to give over-compounded voltage is frequently provided with induction voltage regulators.

ADVANTAGES OF OVER-COMPOUNDED HIGH-TENSION VOLTAGE

The statement is sometimes made that the voltage supplied to a certain customer or city by a transmission system should have not more than a specified amount of variation, say five per cent. Where such a statement is made, it practically always refers to "under-compounded" voltage; that is, the voltage variation is caused by the load and the voltage is low at times of heavy load. It is evident that, if a voltage of 12,000 volts or higher is under-compounded, then circuits of 2300 or 115 volts will be still more under-compounded due to the drop in the intervening transformers and lines at full load. If the higher voltage is over-compounded, then the drop in transformers and lines tends to decrease the variation in the distribution voltage instead of increase it.

It can be stated that, in general, twice as much over-compounding as under-compounding of a circuit of 12,000 volts or higher can be allowed for the same amount of inconvenience and for the same necessity of using feeder voltage regulators. In reality, the circuits which feel the disadvantage of voltage variation most are 115-volt circuits, where changes in voltage are noticeable at once in the changed brightness of lamps. Voltage variation in 2300-volt circuits is often inconvenient mainly because it causes variation in the connected 115-volt circuits. Slightly over-compounded voltage on a motor circuit is not very disadvantageous, for low voltage is most liable to occur out of working hours, when there is light load on the power system. Such an arrangement is the least troublesome, and further, is economical, for the core loss of transformers and motors is reduced during the night when they are giving what is often practically only stand-by service. Good speed and good starting torque and pull-out torque in induction motors are especially desirable when loads are heavy, and these are all helped by over-compounded voltage.

It may be said that over-compounded voltage on circuits operating at greater than 115 volts is more desirable than constant voltage or under-compounded voltage, in almost every case. This is true for practically any customer, city, or place where there are generators, synchronous condensers or induction regulators to give control of the voltage; but it is particularly true of the generating end of long transmission lines. Where there is an appreciable amount of charging current, at no load the equivalent voltage at the generator terminals must be lower than at the load end with no synchronous condensers connected, due to the reactance of the transformers and of the line. However, it would not be economical to operate the generators always at this low voltage, since this would involve greater resistance loss and a larger rating of synchronous condensers if such are used for voltage regulation. The generators at full

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[1] Mass. Inst. of Technology.

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load should be operated at full rated voltage. It is also inadvisable, in many cases, to operate the generators at constant full rated voltage, for this produces too high a voltage at the distant end at no load, necessitating delay when starting to put the synchronous condensers on the line for even a small amount of load, or for paralleling with other lines. Unduly high voltage increases strain on the insulation and produces trouble in the generating station in handling the charging current. Accordingly, if automatic voltage regulators are used, as is generally the case with long transmission lines, they should, if possible, be adjusted for a moderate amount of over-compounding of the voltage rather than for constant voltage. It is fortunate that the over-compounding is practically proportionate to the true kilowatt load, and is not appreciably affected by the quadrature charging current.

Where there are several generators in a station often only one or two are used to carry the load in the middle of the night. At such time each generator should have less amount of compounding than it would have when the entire station is operating. However, since the over-compounding or line-drop compensation feature can be reduced or cut out by merely turning a dial on the voltage regulator, it is not difficult to do this at times when the number of generators operating in the station is changed. It is also possible to take the current for the series coils of the regulators from a current transformer carrying the total load of the entire station. This will give the same over-compounding for the station regardless of the number of generators in operation.

MOST ADVISABLE AMOUNT OF OVER-COMPOUNDING

The question as to how much over-compounding of the voltage should be used in a transmission system is not so much a question of design as of operation, for the over-compounding will be limited in most cases to a comparatively small percentage by operating considerations.

Transmission lines over 100 miles long cost considerably more than the synchronous condensers used to control their voltage. Consequently, it pays to install synchronous condensers sufficient to maintain the line at nearly rated voltage in all parts when it is fully loaded. If the line current at the load end has a very low leading power factor, its resistance loss and, what is more important, its stability will usually be improved by putting part of the synchronous condensers in an intermediate substation.

The voltages to be maintained with long lines at light load and no load will depend mainly on the method considered desirable for starting up the transmission system; whether the generators are to be thrown on the line alone or on sections of it; whether the synchronous condensers are to be started and adjusted for lagging power factor before connecting on any load; or whether the generators are to be brought up from zero speed, together with the synchronous condensers.

With lines shorter than 100 miles, the cost of synchronous condensers becomes of greater relative importance, because the cost of the lines is proportionately less. The cost of condensers is usually compared with the cost of more or heavier transmission circuits.

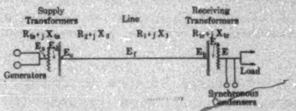

FIG. 1—SCHEME OF CONNECTIONS OF TRANSMISSION SYSTEM

The question of the amount of voltage variation to be allowed on low voltage circuits depends on the standard set in different localities for different classes of load. This standard is maintained in practical operation by installing more circuits, transformers and induction feeder regulators in the lower voltage parts of the system, as well as by improving conditions in the higher voltage parts.

A certain amount of over-compounding on the higher voltage circuits will generally help to maintain the above standard. How far this can be carried is a matter on which only a small amount of data has been collected or published. An investigation on actual systems as to how much over-compounding can be used under different conditions before real practical disadvantages are encountered would be valuable.

Where there is no load to be supplied close to the generators, over-compounding up to about 25 per cent at the generators can be used. If loads are supplied directly from the generator bus-bars, a smaller amount of over-compounding must be used, the amount depending on whether the power passes once or twice through transformers before it is used by motors or lamps, and on whether the load curve of the local load throughout the day corresponds closely with the load curve of the generator load.

In conclusion, it is generally advisable to use as much over-compounding at any point as experience shows can be used without interfering with the accepted standards of good maintenance of voltage in the different low-voltage circuits of the system. A small percentage of over-compounding can be used on high-tension circuits at load points and a larger percentage, amounting in some cases to as much as 25 per cent, can be used at generating stations.

CALCULATION OF LINES HAVING OVER-COMPOUNDED VOLTAGE

A very convenient method of calculating the electrical behavior of a transmission line or system provided with synchronous condensers is to draw the circle diagram showing the reactive kv-a. required from the synchronous condensers at various loads, the step-up and step-down transformers being taken into account[2]. In the present paper, the method of calculation is extended to cover the case of over-compounded generator voltage and constant receiver voltage, for which case the diagram is a circle. An extension of the calculation is also given to cover the case of over-compounded voltage at both generator and receiver and in this case, the diagram is not circular but elliptical. Examples are given and the resulting diagrams are shown in Fig. 2.

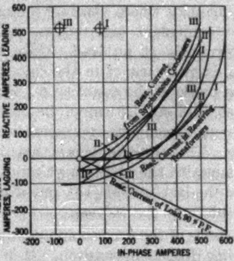

FIG. 2— I. OVER-COMPOUNDING AT GENERATOR END
II. OVER-COMPOUNDING AT BOTH ENDS
III. CONSTANT VOLTAGE AT BOTH ENDS

A solution for the case of over-compounded generator voltage and constant receiver voltage has been published using the method of the equivalent π line[3].

CALCULATION FOR LINE AND TRANSFORMERS, WITH OVER-COMPOUNDED GENERATOR VOLTAGE AND CONSTANT RECEIVER VOLTAGE

Numerical values, except for $(P + j\,Q)$ which always appears, are to be found for the following quantities:

Current in secondary of receiving transformers:

$$I_a = P + j\,Q + P_c \text{ amperes per conductor} \qquad (1)$$

Voltage induced in receiving transformers:

$$E_a = E + \frac{1}{2} I_a\,(R_{tr} + j\,X_{tr}) \text{ volts to neutral} \qquad (2)$$

Current in primary of receiving transformers:

$$I_b = I_a + E_a\,(G_{tr} + j\,B_{tr}) \text{ amperes per conductor} \qquad (3)$$

Voltage at receiving end of transmission line:

$$E_b = E_a + \frac{1}{2} I_b\,(R_{tr} + j\,X_{tr}) \text{ volts to neutral} \qquad (4)$$

Voltage at a point f where the conductor or spacing is changed but not where the voltage is controlled:

$$E_f = E_b\left(1 + \frac{Y_1 Z_1}{2} + \frac{Y_1^2 Z_1^2}{2 \times 3 \times 4} + \ldots\right) + I_b Z_1\left(1 + \frac{Y_1 Z_1}{2 \times 3} + \frac{Y_1^2 Z_1^2}{2 \times 3 \times 4 \times 5} + \ldots\right) \text{ volts to neutral} \qquad (5)$$

Current at point f:

$$I_f = I_b\left(1 + \frac{Y_1 Z_1}{2} + \frac{Y_1^2 Z_1^2}{2 \times 3 \times 4} + \ldots\right) + E_b Y_1\left(1 + \frac{Y_1 Z_1}{2 \times 3} + \frac{Y_1^2 Z_1^2}{2 \times 3 \times 4 \times 5} + \ldots\right) \text{ amperes per conductor} \qquad (6)$$

Y_1 and Z_1 are the admittance and the impedance of the line from b to f.

Voltage at supply end of transmission line:

$$E_c = E_f\left(1 + \frac{Y_2 Z_2}{2} + \frac{Y_2^2 Z_2^2}{2 \times 3 \times 4} + \ldots\right) + I_f Z_2\left(1 + \frac{Y_2 Z_2}{2 \times 3} + \frac{Y_2^2 Z_2^2}{2 \times 3 \times 4 \times 5} + \ldots\right) \text{ volts to neutral} \qquad (7)$$

Current at supply end of transmission line:

$$I_c = I_f\left(1 + \frac{Y_2 Z_2}{2} + \frac{Y_2^2 Z_2^2}{2 \times 3 \times 4} + \ldots\right) + E_f Y_2\left(1 + \frac{Y_2 Z_2}{2 \times 3} + \frac{Y_2^2 Z_2^2}{2 \times 3 \times 4 \times 5} + \ldots\right) \text{ amperes per conductor} \qquad (8)$$

Y_2 and Z_2 are the admittance and the impedance of the line from f to c.

Voltage induced in supply transformers:

$$E_d = E_c + \frac{1}{2} I_c\,(R_{ts} + j\,X_{ts}) \text{ volts to neutral} \qquad (9)$$

Current in primary of supply transformers:

$$\begin{aligned} I_d &= I_c + E_d\,(G_{ts} + j\,B_{ts}), \text{ amperes per conductor.} \\ &= C + j\,D = \text{current at generator terminals.} \end{aligned} \qquad (10)$$

Voltage at generator terminals:

$$\begin{aligned} E_s &= E_d + \frac{1}{2} I_d\,(R_{ts} + j\,X_{ts}) \text{ volts to neutral} \\ &= A + j\,B \\ &= E_{ss} + L_s\,I_d \end{aligned} \qquad (11)$$

Equation for circle diagram:

$$\begin{aligned} E_{ss} &= E_s - L_s\,I_d \text{ volts to neutral} \\ &= E' + j\,E'' + (P + j\,Q)\,(R' + j\,X') \end{aligned} \qquad (12)$$

Quantities required for above equations (referring to a three-phase system):

$P + j\,Q$ = Load current + reactive current from synchronous condensers. This is always

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2. "Electrical Characteristics of Transmission Systems," by H. B. Dwight, TRANS. A. I. E. E., Vol. 41, 1922, page 781.

3. "Regulator Settings for Long Lines," by L. F. Woodruff, *Electrical World*, August 30 and September 6, 1924.

expressed by letters in the above calculation.

P_c = Current for average loss in synchronous condensers.

E = Equivalent high-tension voltage at low-tension side of receiving transformers, in volts to neutral.

R_{tr} = Equivalent high-tension resistance from line to neutral of receiver transformers.

If the resistance is given in per cent,

$$R_{tr} = \frac{\text{Per cent resistance} \times E^2}{100{,}000 \times \text{kv-a. per phase}} \qquad (13)$$

X_{tr} = Equivalent high-tension reactance from line to neutral of receiver transformers. If the reactance is given in per cent,

$$X_{tr} = \frac{\text{Per cent reactance} \times E^2}{100{,}000 \times \text{kv-a. per phase}} \qquad (14)$$

It is assumed in equations (**2**) and (**4**) that, so far as the magnetizing current is concerned, the transformer impedance is equally divided between the primary and secondary. If the division is more accurately known, the actual values of primary and secondary impedance can be used.

$G_{tr} + j\,B_{tr}$ = Admittance (equivalent high-tension) for core loss and magnetizing current of receiver transformers at average voltage. If these characteristics are given in per cent,

$$G_{tr} = \frac{\text{Per cent core loss}}{100} \times \frac{1000 \times \text{kv-a. per phase}}{E^2} \qquad (15)$$

and

$$B_{tr} = -\frac{\text{Per cent magnetizing current}}{100} \times \frac{1000 \times \text{kv-a. per phase}}{E^2} \qquad (16)$$

B_{tr} is a negative quantity.

The characteristics of the supply transformers are denoted by the letters $t\,s$ as in R_{ts}, etc.

The point f denotes a place in the transmission line where the line characteristics change due to different conductor or spacing. A constant load may be assumed for this point without changing the form of the calculation. However, it is not to be assumed that the voltage is controlled at this point, for if it were, a circle diagram would have to be calculated as far as f, and another diagram for the remainder of the transmission system. Other points g, h, etc., similar to f, can readily be included in the calculation.

$Y_1 = G_1 + j\,B_1$, the admittance of the line from b to f.

$Z_1 = R_1 + j\,X_1$, the impedance of the line from b to f.

The subscript (**2**) denotes the characteristics of the line from f to c, as in Y_2, etc.

The series in $Y\,Z$ are very convergent at commercial frequencies and can be quickly calculated. It may be noted that

$$1 + \frac{Y_1 Z_1}{2} + \frac{Y_1^2 Z_1^2}{2 \times 3 \times 4} + \ldots = \cosh \sqrt{Y_1 Z_1}$$

and

$$1 + \frac{Y_1 Z_1}{2 \times 3} + \frac{Y_1^2 Z_1^2}{2 \times 3 \times 4 \times 5} + \ldots = \frac{\sinh \sqrt{Y_1 Z_1}}{\sqrt{Y_1 Z_1}}$$

E_{gn} is a quantity whose absolute value is equal to that of the equivalent high-tension voltage at the generator terminals at no load. Only the absolute value of E_{gn} is made use of in the calculation in this paper. The phase of the quantity E_{gn} changes as the load changes, and so E_{gn} cannot be considered as a constant nor as a complete representation of the no-load voltage. The reason for this is that the automatic regulator maintains the absolute value of the voltage according to the last part of (**11**), but it has no power to alter the phase of the voltage.

L_s is a measure of the over-compounding of the voltage, or the line drop compensation. It is equal to the rise in equivalent high-tension voltage produced by one ampere of current (equivalent high tension) in phase with E_s.

Circle Diagram. Since equation (**12**) is of the same form as the equation for a constant-voltage transmission line, a circle diagram may be drawn for the transmission system with over-compounded voltage at the generator end. Since E, the load voltage, is constant, the values of current may be multiplied by

$$\frac{3\,E}{1000}$$

to give kw. and reactive kv-a. as in Reference 1. However, where the voltage is varying, it is desirable to keep account of the current and voltage rather than the kv-a., and so the circle diagram equations are given here in terms of amperes.

The center of the circle is the point (a', b') where

$$a' = -\frac{E'\,R' + E''\,X'}{R'^2 + X'^2} \text{ amperes} \qquad (17)$$

$$b' = +\frac{E'\,X' - E''\,R'}{R'^2 + X'^2} \text{ amperes} \qquad (18)$$

The radius is

$$c' = +\frac{E_{gn}}{\sqrt{R'^2 + X'^2}} \text{ amperes} \qquad (19)$$

The circle shows the reactive current at the receiver end of the line plotted on values of in-phase current at the receiver end.

In order to plot the reactive current required from the synchronous condensers, first draw a straight line at angle θ below the base line, where $\cos\theta$ is the power factor, lagging, of the load. If the power factor is not the same at all loads, the line will not be straight, but will be a curve showing the reactive current of the load from no load to full load. By means of a pair of dividers add the reactive current of the load to the corresponding ordinate of the circle, thus plotting the curve of current required from the synchronous condensers. This curve is an ellipse when $\cos\theta$ is constant. (See Fig. 2.)

Circle Diagram Limit of Load.

$$\text{Maximum Load} = c' + a' \text{ in-phase amperes} \qquad (20)$$

This is numerically less than c' when a' is a negative quantity, and greater than c' when a' is positive. It may be read from the circle diagram, as it is the farthest distance to the right reached by the circle. It is a useful point to locate on the diagram before drawing the circle, as is also the bottom point of the circle, $(a', b' - c')$. It assists in drawing the circle to have these two points through which the circle is to pass.

The "stability limit" of load is usually somewhat less than the circle diagram limit, and of course, in the operation of a transmission line, neither limit must be reached.

Calculated Value of Reactive Current.

A direct calculation of the reactive current is more precise than a reading from the circle diagram, and is less work than a trial and error method. The value of the reactive current in the circuit, Q, for a given in-phase current P, may be found from the following equation:

$$(b' - Q)^2 = c'^2 - (P - a')^2 \qquad (21)$$

First, find the value of the right hand side of the equation. Then take the square root and subtract b'. The reactive current required from the synchronous condensers is equal to

$$Q + P\frac{\sin\theta}{\cos\theta} \text{ amperes} \qquad (22)$$

where the in-phase current is P amperes and the lagging power factor is $\cos\theta$. It should be remembered that b' is a positive quantity and a' may be positive or negative. It is worth while checking the results of equations (21) and (22) by drawing the circle diagram and obtaining the same results graphically.

Concentric Circles. Since a' and b' which give the center, are independent of E_s and since the radius c' is directly proportional to E_s, a number of circles corresponding to different values of E_s may be drawn about the same center.

Total Losses. The losses in the transmission system for a given power load are

$$\frac{3}{1000}(A\,C + B\,D - E\,P) \text{ kw.} \qquad (23)$$

P is determined by the power load. Then Q must be found from the circle diagram or equation (21). Then A, B, C and D can be found from equations (10) and (11), for which numerical coefficients have already been obtained.

Efficiency of the transmission system

$$\text{Efficiency} = \frac{100\,E\,P}{A\,C + B\,D} \text{ per cent} \qquad (24)$$

Kw. at supply end of system

$$\frac{3}{1000}(A\,C + B\,D) \text{ kw.} \qquad (25)$$

Kv-a. at supply end

$$\frac{3\,E_s\sqrt{C^2 + D^2}}{1000} \text{ kv-a.} \qquad (26)$$

Power factor at supply end

$$\frac{100\,(A\,C + B\,D)}{E_s\sqrt{C^2 + D^2}} \text{ per cent} \qquad (27)$$

Reactive kv-a. at supply end

$$\frac{3}{1000}(A\,D - B\,C) \text{ kv-a.} \qquad (28)$$

When this quantity is positive the reactive kv-a. and the power factor are leading, and when it is negative they are lagging.

OVER-COMPOUNDED VOLTAGE AT BOTH SUPPLY AND RECEIVER ENDS OF SYSTEM

The receiver voltage can be over-compounded when there are synchronous condensers, by using exactly the same kind of apparatus as that used to provide over-compounded voltage at the supply end. The diagram in this case is not a circle, but an ellipse, and so values of Q for different values of P are found by calculation.

If L_r is the rise in equivalent high-tension voltage at the receiver end produced by one ampere of current (equivalent high-tension) in phase with E, then

$$E_s = E - L_r\,P - j\,L_r\,Q$$

as in eq. (12). The phase of E_s in the above expression changes as the load changes. Its absolute value is equal to that of the equivalent high-tension voltage at the receiver end at no load. P is in phase with E.

The absolute value of E_s is equal to

$$E - L_r\,P + \frac{(L_r\,Q)^2}{2\,(E - L_r\,P)} \qquad (29)$$

$L_r\,P$ is usually less than 25 per cent of E for any load that the line is to carry. If $L_r\,Q$ is 20 per cent of rated voltage, the third term of (29) is less than three per cent of E. It is therefore seen that the quadrature current Q has very little effect on the voltage, and the over-compounding of the voltage or the line-drop compensation is practically proportional to the in-phase current or the true kilowatt load.

In the calculations in this paper the third term of (**29**) is neglected, and we write

$$E = E_s + L_r P \qquad (\mathbf{30})$$

Now evaluate expressions (**1**) to (**12**). The quantity $(P + j\,Q)$ will not be a factor as it was in the previous case, but P and Q will now have different numerical coefficients. The final expression as given by (**12**) is

$$E_{os} = E_s - L_s\, I_d$$

If a numerical value is assigned to P, this becomes an equation in complex quantities involving Q. $E_{os}{}^2$ is equal to the square of the real part of the right hand side plus the square of the imaginary part. This is equivalent to multiplying each side of the equation by its conjugate, that is, by a complex quantity equal to itself except that the sign of the imaginary part is changed. The result is an ordinary quadratic equation in Q, of the form

$$a\,Q^2 + b\,Q + c = 0.$$

Q is given by

$$Q = \frac{-b - \sqrt{b^2 - 4\,a\,c}}{2\,a}$$

The minus sign of the radical is to be used so as to give the normal working part of the curve, corresponding to the lower part of the circle diagram for constant receiver voltage.

EXAMPLE I

Draw the circle diagram for the following line and transformers, with constant receiver equivalent high-tension voltage 200,000 volts between conductors, no-load equivalent high-tension voltage 183,000 volts between conductors at generators, and 220,000 volts at generators with in-phase current of 355 amperes.

Length of line = 200 miles.
Frequency = 60 cycles.
$R + j\,X = 23.2 + j\,160$ ohms.
$Y = +j\,0.00106$ mho

$$1 + \frac{YZ}{2^s} + \ldots = 0.91637 + j\,0.01195$$

$$1 + \frac{YZ}{2.3} + \ldots = 0.97197 + j\,0.00403$$

$P_e = 8.66$ amperes
$R_{tr} + j\,X_{tr} = 1.33 + j\,24.0$ ohms.
$G_{tr} + j\,B_{tr} = 0.000{,}022{,}5 - j\,0.000{,}187{,}5$ mho
$R_{ts} + j\,X_{ts} = 1.61 + j\,29.0$ ohms.
$G_{ts} + j\,B_{ts} = 0.000{,}018{,}6 - j\,0.000{,}155{,}0$ mho.

The calculation is the same as for Example I, Reference 2, as far as the equation,

$$E_s = 107{,}050 + j\,3340 + (P + j\,Q)\ (23.9 + j\,204.4)$$

Then the absolute value of $E_{os} = 105{,}850$ volts to neutral.

$$L_s = \frac{127020 - 105850}{355} = 59.8$$

$$E_{os} = 106{,}310 - j\,1610 + (P + j\,Q)\ (-31.3 + j\,203.6)$$
$$= E' + j\,E'' + (P + j\,Q)\ (R' + j\,X')$$

The center of the circle is the point (a', b') where

$$a' = +86.2 \text{ amperes}$$

and

$$b' = +509.2 \text{ amperes}$$

The radius of the circle is

$$c' = +513.8 \text{ amperes.}$$

See Curve I, Figure 2.

EXAMPLE II

Draw the diagram for the same line and transformers as in Example I, with the same over-compounding of the voltage at the generator end, and with over-compounding at the load end from 182,000 volts at no load to 200,000 volts at 346 in-phase amperes.

$$L_r = \frac{115{,}470 - 105{,}000}{346} = 30.2$$

$E = 105{,}000 + 30.2\,P$
$I_a = P + j\,Q + 8.7$
$E_a = 105{,}000 + j\,100 + P\,(30.9 + j\,12.0) + j\,Q\,(0.674 + j\,12.0)$
$I_b = P\,(1.003 - j\,0.006) + j\,Q\,(1.002 + j\,0.0002) + 11.1 - j\,19.7$
$E_b = 105{,}240 + j\,220 + P\,(31.6 + j\,24.0) + j\,Q\,(1.3 + j\,24.0)$
$E_c = 99{,}740 + j\,2760 + P\,(51.5 + j\,178.4) + j\,Q\,(22.8 + j\,178.0)$
$I_c = P\,(0.894 + j\,0.039) + j\,Q\,(0.894 + j\,0.013) + 9.7 + j\,90.5$
$E_d = 98440 + j\,2970 + P\,(51.6 + j\,191.4) + j\,Q\,(23.3 + j\,191.0)$
$I_d = P\,(0.925 + j\,0.035) + j\,Q\,(0.924 + j\,0.013) + 12.04 + j\,75.2$
$E_s = 97360 + j\,3200 + P\,(51.8 + j\,204.8) + j\,Q\,(23.8 + j\,204.4)$
$E_{os} = 96640 - 3.4\,P - 203.6\,Q - j(1300 - 202.7\,P + 31.4\,Q)$

In calculating the point on the curve for $P = 300$,

$$E_{os} = 95620 - 203.60\,Q + j\,(59510 - 31.4\,Q)$$

Absolute value of $E_{os} = 105{,}850$.

Dividing by 100 and then squaring the real part and the imaginary part,

$$4.244\,Q^2 - 4267\,Q + 148{,}000 = 0$$

$$Q = \frac{4267 - 3961}{8.488}$$

$$= +36.0 \text{ amperes.}$$

This is plotted on Curve II in Fig. 2.

EXAMPLE III

Draw the circle diagram for the same line and transformers as in Example I, except that there is to be constant voltage at both ends. This is the same as Example I, Reference 2, except that the results are expressed in amperes. The result is Curve III, Figure 2 of this paper.

Discussion

L. F. Woodruff: One of the chief problems encountered in the calculating of the most advantageous amount and phase of compounding for the generator-end voltage of a transmission line is the determination of the settings required to locate the receiver power circle center at a predetermined point. This problem is more involved than that of finding the center of the circle for given regulator settings, and while it is possible of explicit solution, such a solution is so cumbersome and involved that it is practically useless. Yet the logical approach to the problem is to try different circles of load power until the one is found which affords most economical operation, considering line and transformer losses, synchronous reactor losses and costs, voltage rise on the line, and stability. Then, having located the circle from economic considerations, the problem becomes that of finding the regulator settings required to make this the operating circle of the system.

Mapping of Power Coordinate Plane to Show Effect of Various Degrees of Compounding. I have found that the best method of solving this last problem is to draw on the load-power coordinate scale curves which map out the region in which the center is to be located in terms of parameters which indicate the amount of compounding. Certain features of these curves render their calculation and construction very much simpler than is at first apparent.

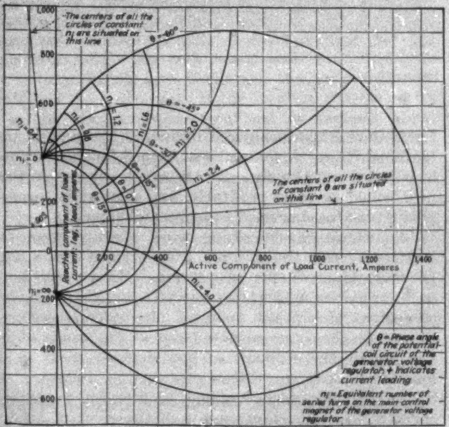

Fig. 1—Chart Showing Amount and Phase of Compounding at Generator End Required to Locate the Center of the Load Current or Power Circle at Any Desired Point. Load Voltage is Constant

Fig. 1 herewith shows the amount of compounding required to locate the center of the load-power circle at any desired point, for a transmission circuit consisting of a 250-mi. line with terminal transformers. This figure has already been published[1], and the details of line, transformer and regulator constants may be found in the original references. In connection with it now I wish to point out these facts:

1. The center of the load-power circle, with constant load voltage and with generator voltage compounded, may be located anywhere on the chart.

2. For a given amount of compounding, the location of the center depends largely on the phase of the compounding.

3. The curves of constant amounts of compounding, but various phase angles, comprise a family of circles whose centers, for a constant load voltage, all lie on a straight line passing through the center of the circle diagram for ordinary constant-voltage operation, and another point in the third quadrant corresponding to theoretically infinite compounding. The curves of constant phase angle, but varying amounts of compounding, comprise another family of circles passing through the two points just mentioned. The phase of the compounding can be adjusted by putting a reactance and resistance in series with the potential coil of a voltage regulator, instead of only a resistance, and while this has not been done in practise, it offers opportunity of improvements in line control.

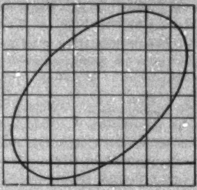

Fig. 2—Original Ellipse Diagram of Power at Load End of Line with Voltage Componded at Both Ends

Voltages at Both Ends Compounded—Load Power Ellipse Transformed to Circle. Since graphical methods are so advantageous in shortening or eliminating calculation, and since the main advantage of circle diagrams over others which could present the same information lies in their easy preparation, a method of transforming the "ellipse diagram" which Professor Dwight derived, into a circle diagram, should be of some interest and value.

Fig. 2 represents an ellipse diagram plotted on rectangular axes. To transform this to a circle diagram, let one of the scales be such as to make the major axis of the ellipse lie along the 45-deg. line, unless it already conforms to this condition. If the original angle of this axis with the horizontal is θ degrees, the change to 45 deg. may be accomplished by using a vertical scale equal to the horizontal scale multiplied by $\tan \theta$. Now suppose the major and minor axes of the new ellipse to be of lengths A and B respectively. By skewing the axes as shown in Fig. 3, so that the angle between them in the first quadrant is $2 \tan^{-1}\left(\dfrac{A}{B}\right)$, the ellipse is transformed into a perfect circle, and so may be drawn at once with a compass.

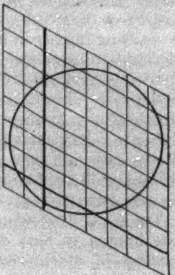

Fig. 3—Ellipse Diagram of Load Power Transformed to a Circle Diagram by Skewing the Axes

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[1] L. F. Woodruff, Regulator Settings for Long Lines, *Elec. Wld.* 1924. Also, Principles of Electric Power Transmission (Wiley), 1925.

Dielectric Absorption and Theories of Dielectric Behavior

BY J. B. WHITEHEAD[1]
Fellow, A. I. E. E.

Synopsis.—*The first report of the Committee on Electrical Insulation of the Division of Engineering and Industrial Research of the National Research Council laid a plan for a review of existing data and information and its coordination, looking to a comprehensive plan of experimental attack. The whole field was divided into a number of divisions which have since been assigned to several groups of investigators. The present paper constitutes the second report of the Committee on Electrical Insulation, and in accordance with the plan described presents the results of an extensive review of the literature on dielectric absorption and theories of dielectric behavior.*

The paper presents briefly the fundamental classical theory of dielectrics as based on the work of Cavendish, Faraday and Maxwell, and devotes its principal attention to the anomalies or departures from that theory. The principal experimental facts of anomalous behavior under continuous potential are reviewed and classified, together with the various theories proposed to account for them. These theories are divided into four classes: (1) those in which the fundamental magnetic equations are retained and the anomalies attributed to anomalies of the structure of the dielectric, (2) those in which the anomalies are attributed to special laws of dielectric displacement, without explanation of underlying mechanism, (3) those in which explanation of displacement and its anomalies is traced to the motion of electrons within the atom, and (4) those invoking anomalies of conductivity in explanation of dielectric behavior.

The behavior of dielectrics under alternating electric force is treated as a separate division of the report. The history of the observation and study of these losses is reviewed and the most reliable experimental indications for the variation of loss with voltage, frequency, and temperature are collected and reviewed. The review of the work of those who have attempted to find an explanation of dielectric loss indicates that from the time of Hopkinson it has been suspected that dielectric absorption would account for the major portion of dielectric loss. The most recent work has confirmed this idea. The paper closes with a brief summary and review presenting conclusions of the study.

I. INTRODUCTION

Of the various component parts entering into electrical systems of all characters, the insulation is the least susceptible to exact computation and design. In few if any cases of even simple and pure materials are the dielectric properties, resistivity, dielectric strength, specific inductive capacity, either constant or uniform; and in the cases of composite and fabricated insulations of manufacture the variations are extremely wide. As results, in all cases liberal factors of safety to cover the worst probable conditions must be allowed, resulting further in increased size and cost, and in undesirable magnitudes of other properties, such as dielectric loss and phase difference, volume and surface conductivity, circuit capacity and conductance, etc. Little if any attempt has been made to control the inherent characteristics of simple dielectrics, or to study their influence in combinations. Physicists appear to have all but forgotten the unsolved problems of dielectric behavior, or perhaps to have given them up. The control of manufactured insulation appears to be limited to heat treatment, principally for the purpose of elimination of moisture—a sufficiently important object—and to the obtaining of pure raw materials. Studies of the properties of these materials in their bearing on those of the composite final form of the insulation have not appeared in any quantity.

The Committee on Electrical Insulation of the Division of Engineering and Industrial Research of the National Research Council was formed for the purpose of pointing out the directions in which research in this field would be most profitable, and if possible to propose a plan for concerted experimental attack.

The literature describing work already done on dielectrics, from the points of view of both theory and practise, is enormous in volume. Many important data are hidden in this mass, as well as much careful and competent theoretical analysis. In order, therefore, to have as clearly as possible before it the results of all this work, the Committee has deemed it its first important duty to review the entire literature of dielectrics and insulation and to attempt to gather together the important results of the work of others into a series of reviews; and further, to draw from these reviews conclusions as to the most promising directions for future work.

It will be recalled that the Committee's first report (JOURNAL A. I. E. E., June 1923) divided the entire field to be studied into seven divisions based generally on the more common properties of insulating materials. Two of these divisions, both entrusted to the present writer, were "Dielectric Absorption" and "Theories of Dielectric Behavior." Of a number of important attempts to give theoretical explanations of the properties of dielectrics, nearly all concern themselves primarily with the anomalous property of dielectric absorption. Therefore, the two topics, "Theories" and "Dielectric Absorption," separated in the first report, will be considered concurrently in the following review.

II. HISTORICAL OUTLINE

Dielectric theory may be said to have had its origin in the work of Cavendish* (1773). Although looking for different values of induced charges amongst various substances, and not apparently noting the distinction between conductors and dielectrics, he nevertheless showed clearly the influence of the specific inductive capacity of the latter, and his measurements give a scale of relative values.

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[1] Professor of Electrical Engineering, Johns Hopkins University, Baltimore, Md.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

Faraday's elaborate studies on the exact equality between inducing and induced charges on concentric conductors, with and without dielectrics and conductors between them, led to his famous conception of a point-to-point transference through the medium of the influence of an electric charge, at a point, or on a conductor, at a distance. Following immediately therefrom are his well-known ideas of lines of force, tubes of induction, and dielectric polarization. Maxwell's matchless genius appreciated the importance of Faraday's work and theory, and saw the applicability of the theory of potential, as already developed by Poisson for the magnetic field. Maxwell introduced the conceptions of volume distribution of charge—the polarized molecule—as the active part of the medium demanded by Faraday, electric displacement and its magnetic effect, and built up the theory of dielectric behavior which has ever since constituted, without rival, the foundation of all subsequent physical research and of all electrical engineering application.

The outstanding features of the theory thus briefly outlined, are the idea of dielectric polarization as developed by Faraday, and Maxwell's generalized analysis of the dielectric field. The treatment built up assumes constant values for dielectric susceptibility and specific inductive capacity, and it is interesting to note that these assumptions are common today in elementary text-books, and even in more advanced treatises, in spite of our long knowledge of the limitations pertaining to such a view. The explanation is apparently found in the facts that a few substances show little or no dielectric absorption, and thus constant values of susceptibility, and that with care in preparation, the error arising from dielectric absorption may be reduced for many substances to small value. In other words, it seems to be generally assumed that the phenomenon of dielectric absorption, which is not accounted for by fundamental theory, is an anomalous property of certain dielectrics, which would probably disappear if sufficient care were taken in the elimination of impurities. As a matter of fact, however, there are few substances which show no absorption, and many show it in conspicuous degree. Absorption, therefore, should rather be considered a normal property of all dielectrics, although not yet rigidly susceptible to expression in terms of fixed constants or definite functions.

Dielectric absorption made its appearance in the very earliest days of electrostatic experiment in the residual charge of the Leyden jar (1746). It is described at some length in the experiments of Benjamin Franklin (1748). It was thus well recognized by Faraday and Maxwell. In fact, as well known, Maxwell gives us the first attempt at a theoretical explanation, in terms of the normal constants of specific resistance and specific inductive capacity, as described in further detail below. Since Maxwell's time there have been many studies of the property of absorption, mostly by physicists, and for a long period it was apparently considered an interesting anomalous property, of interest chiefly in its bearing on theories of the nature of dielectric behavior. Its practical importance as probably the most important factor determining dielectric loss in an alternating field is just beginning to be realized. The use of static condensers in alternating circuits quickly developed (Siemens, 1864) the substantial magnitude of the energy losses in the dielectrics of condensers. For some time it was generally considered that these losses were of the nature of a hysteresis similar to that in magnetic materials, but in recent years with further investigation, the difficulties in the way of this explanation have led to its abandonment. In its place, we have substantial proof that the phenomenon of absorption is sufficient to account for the principal facts in the matter of dielectric loss. In fact, it is a principal conclusion of this report that since in many cases the failure and life of insulation is certainly due to heat arising in internal losses, and since these losses are probably directly related to the magnitude of the dielectric absorption, and since there are numerous methods open to us for the study of absorption, the most promising field for experimental study looking to improvement in all forms of composite insulation is that of dielectric absorption and its control.

III. DIELECTRIC ABSORPTION AND RELATED PHENOMENA

We may conveniently describe the phenomenon of dielectric absorption and other allied properties of dielectrics in terms of the behavior of a simple parallel plate condenser. In accordance with fundamental theory, in which a dielectric is considered as subject to polarization but free from conductivity, if a constant potential difference E_0 is applied at time $t = O$ to the terminals of the condenser, there flows in the connections a "normal charging" current $i_1(t)$ which varies with the time. The time variation of this normal charging current may be determined experimentally or may be computed from the well-known differential equation:

$$L \frac{d^2 i_1}{d t^2} + R \frac{d i_1}{d t} + \frac{i_1}{C} = O \qquad (1)$$

L and R being respectively the self-induction and resistance of the connections, and C the capacity of the condenser as computed from its dimensions and the specific inductive capacity of the dielectric. The

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1. A bibliography of all work referred to, and classified by subject, will be found at the end of the paper.

solution of the equation shows that $i_1(t)$ is either a periodic function with decreasing amplitude, or decreases to zero with aperiodic amplitude, according to well-known relations between L, R, and C. With the usual small values of L and R it always results that the normal charging process is practically ended after a very small fraction of a second.

Since it is probable that no dielectric or insulator exists which is entirely free from conductivity, it has been customary, since Maxwell's classical analysis, to consider that in all dielectrics the polarization and conduction currents are superposed on each other. We may, therefore, extend our definition of a "normal" dielectric to include also the property of conductivity. Therefore, in the above case we must also recognize the presence of a conduction current, i_e, added to the charging current $i_1(t)$. The value of i_e depends on the instantaneous value of the potential difference on the plates of the condenser, and on the dimensions and specific conductivity of the dielectric. On discharge of the condenser by short circuit, since $E_0 = O$, there is no conduction current in the dielectric, and the normal discharge or displacement current i_2 follows the same law as i_1 but in opposite direction, *i. e.*, $i_2(t) = - i_1(t)$.

The theoretical variation of the charge and discharge currents, described above, is observed, however, in only very few actual cases, as for example if the dielectric is either a vacuum or a gas. In the cases of fluid and solid dielectrics usually the charge and discharge currents show not only the initial sharp decrease, ending within a very short interval, but the currents persist as so-called "residual" currents, in some cases for days, or even for months. In these cases the stationary condition as regards the *potential* of the plates is reached following the same formula as in the earlier case. That is, the charge and discharge currents due to the polarization or displacement in the dielectric, follow in a practically indefinitely short time either as highly damped oscillations or usually as periodically damped currents, these currents then merging into the relatively much smaller slowly diminishing residual currents referred to above. Hopkinson has shown that for a certain flint-glass condenser 97 per cent of its total charge was discharged in 0.000059 sec., that this amount was not changed up to 0.0125 sec., and that the specific inductive capacity of the glass was constant up to 5000 volts per mm. Hopkinson states that Sir William Thomson found a similar result. Thus, as based on the initial transient charge or discharge, the condenser has a constant value of capacity and so the dielectric has a definite value of specific inductive capacity. This capacity is generally referred to as the "geometric" capacity of the condenser. Thus, even in dielectrics showing the residual charge and discharge currents mentioned, the specific inductive capacity corresponding to a rapid charge has a definite value, although the residual currents may show wide variations among several samples of the same material. We shall see later in the discussion of the influence of the residual currents on the *apparent* capacity under alternating voltage, that with increasing frequency this capacity approaches the "geometric capacity" as limiting value.

These residual or "anomalous" currents occur following both charge and discharge, dying away more or less slowly in each case. We will designate the "anomalous" currents on charge and discharge respectively by $i_1^1(t)$ and $i_2^1(t)$. The total currents on charge and discharge are thus given by:

$$I_1(t) = i_1(t) + i_e + i_1^1(t)$$
$$I_2(t) = i_2(t) + i_2^1(t) \qquad (2)$$

According to the type of relation between the "anomalous" charging current $i_1^1(t)$ and the "anomalous" discharge current $i_2^1(t)$, von Schweidler distinguishes two types of anomalous currents:

(*a*) the "irreversible anomalous current" in which case $i_2^1(t)$ vanishes, and (*b*) the "reversible anomalous current," for which we have the relation $i_2^1(t) = - i_1^1(t)$, as for the normal displacement current.

However, it often happens that both these types of anomalous current are present in the same material. The various dielectrics show the several types of normal and anomalous current in different amounts. Granier has made their behavior as regards these currents the basis of another classification of dielectrics as follows:

(*a*) These discharge instantaneously the total quantity they have taken up on charge. Examples: air, sulphur.

(*b*) On discharge these give up their total charge, but slowly. Examples: certain samples of mica and ebonite.

(*c*) These give up only a portion of the quantity of electricity taken up in charging.

This classification does not emphasize the fact that the conductivity in dielectrics is of two types, one, best observed in liquids, but also in some solids, not obeying Ohm's law; and the other, found in solids only, following Ohm's law. With this addition the classification is in accord with that of von Schweidler. Apparently there are few if any dielectrics having conductivity and no residual effects.

Dielectric Absorption as Observed in Experiment. Referring to our elementary condenser, the application of continuous voltage is followed by the instantaneous charging of the geometric capacity, current $i_1(t)$, a conduction current, i_e, which may or may not obey Ohm's law, and a slowly decaying "anomalous," or "absorption," or "residual" current, $i_1^1(t)$. If this current, $i_1^1(t)$, is all of "reversible" type as is usually the case in solid dielectrics, it builds up the stored or residual charge, $Q_1 = \int_0^t i_1^1(t) \, . \, dt$, which after the initial discharge of the geometric capacity, appears as the building up of a difference of potential between the plates if the condenser stands open, or as a slowly decaying discharge current giving a charge, $Q_2 = \int_0^t i_2(t)\,dt$, if the condenser is short-circuited. In such cases (*i. e.*, if $Q_2 = Q_1$), $i_1^1(t)$ will be called the "reversible absorption current." In liquid dielectrics there is also a steadily decreasing residual current $i_1^1(t)$, but this does not in general obey the same laws as in the case of solids, and in nearly all cases there is no corresponding discharge current $i_2^1(t)$. In such cases $i_1^1(t)$ will be called the "irreversible" absorption current.

The Reversible Absorption Current. This is the absorption current commonly observed in good solid insulators, free from moisture, and there is good experimental evidence that the charge and discharge currents have the same form, *i. e.*, in equation (**2**) $i_1^1(t) = i_2^1(t)$. Systematic and reliable studies of the mode of the variation of the absorption currents of charge and discharge, may be said to have begun with Kohlrausch (1854). They received a great stimulus in the experiments of Sir John Hopkinson who made extensive studies in the period 1876 to 1897, and in which he appears to have had the benefits of Maxwell's advice (Paper No. 19). Maxwell was deeply interested in the phenomena in dielectrics and treats them at great length.

Kohlrausch, Hopkinson, Giese, J. Curie, Schweidler, Shuddemagen, Jordan, Tank and others have found that the time variation of the reversible absorption current may be expressed with close accuracy by the empirical formula:

$$i_1^1(t) = a\,t^{-m} \qquad (3)$$

Others have found that the following formulas express their observations more closely:

By J. Curie, Malcles, Wagner, Steinmetz:

$$i_1^1(t) = a\,e^{-bt^n} \qquad (4)$$

By J. Curie, von Schweidler, H. A. Wilson:

$$i_1^1(t) = \frac{a}{1 + b\,t} \qquad (5)$$

By Trouton and Russ:

$$i_1^1(t) = \frac{a}{b + t} + c \qquad (6)$$

By Fleming and Dyke, Thornton:

$$i_1^1(t) = a\,e^{-bt} + c \qquad (7)$$

Among these various formulas, (**3**) appears to express the results of experiment in a single term better than any of the others. On the other hand, it has often been noted that by using the sum of several terms of the form of equation (**4**), all with $n = 1$ and with different values of the other constants, an expression may be written for any set of observations (Hopkinson, Schweidler, Wagner, Steinmetz). Probably a similar result could be found using formula (**3**). When a single expression is needed in discussions of theory both (**3**) and (**4**) are used—(Tank, Lahousse), the latter generally with two or three terms but rarely more. Maxwell's theory of dielectric absorption explains the exponential form of (**4**), and so the latter is generally used by supporters of Maxwell's theory. Several observers (Fleming and Dyke, Thornton, Schreiber) have noted that the charging current curve has a region of linear change with the time, gradually changing into an asympotic decrease.

General Formula. With change in the value of the applied voltage, or of the thickness of the dielectric, the time rate of variation of the current is unchanged, but the values are increased in proportion to the voltage gradient (Curie, Wilson). The stationary final value of the charging current also obeys Ohm's law in most cases, and so may be considered as a normal conduction current. In a few cases, this conduction current is either absent or negligibly small, although the absorption may be quite large; *e. g.*, ebonite (Gaugain, Malclés). Generally, therefore, we may write:

$$i_1^1(t) = B \, . \, C\,E_0 = \varphi(t) \qquad (8)$$

in which B is a constant and $\varphi(t)$ a definite function for the material of the dielectric, and C the geometric capacity proportional to the area and inversely as the thickness.

Temperature has a marked influence on the absorption current. Hopkinson was one of the earliest investigators, and he showed clearly that the absorption in glass increases with the temperature. It has been generally supposed from the work of von Schweidler, Wagner, Zeleny and others that in formula (**8**) B increases with the temperature but that the form of the function $\varphi(t)$ suffers little or no change. Lately Wagner (Die Isolierstoffe, Schering), proceeding from Maxwell's theory of a layer dielectric (see below) and observations on a paper condenser, concludes that the effect of temperature is to increase the rate of decay of $i_1^1(t)$, at the same time increasing its initial values so that the amount of absorbed charge is unaffected. Granier notes the same effect but further evidence is needed to support the generality of this conclusion. There is universal agreement, however, that temperature increases both the currents $i_1^1(t)$ and $i_2^1(t)$, as well as the final conduction current.

Other Facts. Other facts noted in connection with $i_1^1(t)$, some of which need further study, are the following: Some substances (*e. g.*, ebonite—Gaugain, Malclés) show notable residual or absorption currents, and no final conduction current. If the electrodes are separated from the dielectric the final charge on the electrodes is less than if an equal thickness of metal is substituted for the dielectric, indicating that absorption is not due to conductivity alone. In this case the absorbed charge is not removed by exposure to radioactive substances. A dielectric withdrawn from its electrodes, affects an electroscope, but shows zero total charge in a Faraday cylinder, thus indicating a polarized condition.

The Principle of Superposition. One of the earliest features noticed in the phenomenon of residual charge was the occasional reversal of its sign as related to the foregoing charge. Hopkinson (Papers No. 18 and 19) studied this property in glass at great length. Applying a succession of charges and discharges at irregular intervals and polarities, he showed that the discharge curve had widely different forms, and frequent reversals of polarity as related to the type and succession of the foregoing charging periods. He quotes Sir Wm. Thomson (Lord Kelvin) as saying: "The charges come out of the glass in the inverse order in which they go in," and developes an equation for residual potential analogous to the equation for the angular twist of a torsion thread under a succession of couples of various magnitudes and direction. Each couple, applied for some time, is supposed to cause a slow time yield, *i. e.*, increase of angular displacement, and the decay of these is reflected in subsequent twisting couples. He states that this view was suggested to him by Maxwell; it is essentially the same as used by Boltzmann for the "afterworking" of any mechanical strain. From many experiments he concludes that his equations are only roughly approximated, but that "it seems safe to infer that the effects on a dielectric of past and present electric forces are superposable." The principle thus proposed by Hopkinson was confirmed beyond question by J. Curie in a careful and accurate study of the specific inductive capacities and the conductivities of quartz, a number of other crystals, and of mica, sulphur, gypsum, ebonite, porcelain, etc. Working with a piezoelectric cell as a balancing source of charge, he developed a zero method permitting the measurement of the absorption current from 0.1 sec. after the application of the charging potential. Three brief laws announced by Curie are worth repeating:

1. The ordinates of the curve of charging current as function of the time are rigorously proportional to the electromotive force.

2. For the same e. m. f. the ordinates of the curve of charging current are inversely proportional to the thickness of the dielectric.

3. Law of Superposition. Each change in the value of the e. m. f. applied to the two faces of a dielectric layer, acts as if it were the only one, adding to any preceding curve its own curve, *i. e.*, the curve it would produce if it acted alone.

The following more exact statement of the principle of superposition is due to von Schweidler. After any change, ΔE, in the voltage, the actual observed current takes on an additional term, and is made up of a term representing the undisturbed variation of the original current, and a superposed current which so varies as though a voltage of the absolute value of the change in voltage, ΔE, had been applied to the uncharged condenser. If, therefore, at time $t = 0$, the voltage E_0 is applied, and then at times t_1, t_2, t_3, etc., any positive or negative changes of voltages $\Delta_1 E$, $\Delta_2 E$, $\Delta_3 E$ are applied then we have as the value of i_1^1 at any instant t.

$$i_1^1(t) = B\,C\,[E_0\,\varphi(t) + \Delta_1 E\,.\,\varphi(t - t_1) + \Delta_2 E\,.\,\varphi(t - t_2) + \,.\,.\,] \qquad (\mathbf{9})$$

Obviously the conduction current is not included in the above expression. Thus, if the final conduction current is subtracted from the charging current, it is seen that the equality of the reversible charge and discharge currents ($i_1^1 = -\, i_2^1$) is merely a special case of the principle of superposition.

Still more generally it follows, if the principle of superposition be true, that for any continuous variation of the voltage as expressed by the function $E(t)$, the reversible absorption current is given by:

$$i_1^1(t) = B\,C\,.\int_0^t \frac{d}{d\,u}(E(u))\,.\,\varphi(t - u)\,d\,u \qquad (\mathbf{10})$$

in which t is the instant at which $i_1^1(t)$ is measured and u is the elapsed time controlling the variation of E. We shall see that formula (**10**) is of great importance as a means for computing the value of dielectric loss due to alternating electric stress. It also permits analysis of the rate of loss of charge of an open condenser, after brief charge, or after brief short circuit following a long charge, as investigated experimentally by Kohlrausch, and analytically in another manner by Maxwell. The bibliography contains references to the important work of many others, but in studying the phenomenon of absorption one can not afford to omit any part of the beautifully simple papers of J. Curie.

The Irreversible Absorption Current. In equation (**2**) the component i_0 of the charging current is pure conduction current and, therefore, is not present when the condenser is discharged by short circuit. It is, therefore, irreversible, and flows only on charging. The irreversible absorption current also flows only on charging, but differs from i_0 in that it varies with the time, decreasing usually very slowly (often expressed $i_1^1(t) = b\,.\,t^{-m}$). Its final constant value is reached earlier and earlier as the voltage is higher and higher. This final constant value may not be considered as a normal conduction current since it does not obey Ohm's Law. With different values of E the final value of the current is not proportional to E but generally increases more slowly than the voltage. In some cases an apparent saturation current is observed, and on the other hand in a very few instances the final current appears to increase more rapidly than in proportion to the voltage. This type of irreversible absorption current is observed particularly in liquid dielectrics. Sometimes, however, it is noted in composite solids, as for example built-up laminated insulation, impregnated paper, etc., particularly in the undried state. Experimental investigations of this current in liquids are numerous, but very few in the case of solids, beyond the simple observation of a difference between the values and shapes of the currents of charge and discharge. A notable exception is the work of Evershed on the influence of moisture on the conductivity of paper.

His results show several of the properties of the irreversible absorption current as mentioned above, and are reviewed in greater detail in another paragraph. The general trend of the conclusions from the studies on liquids is that this current may be due in part to dissociated impurities (sometimes eliminated permanently by prolonged flow of current), but more generally to ions more or less free to move, and which obey the laws of gaseous conduction. A more extended description of the phenomena in liquids appears to lie outside our present purpose of discussing the phenomena in the commoner forms of electrical insulation. We leave the subject therefore with a list of references in the bibliography, and in pointing out that so far as the irreversible absorption current in solids is concerned, there is good evidence that it is associated with the presence of moisture, or of impurities giving it an electrolytic character, and that with the progressive elimination of both, this type of current is reduced to very small proportions if not completely eliminated. Some further discussion of this type of conduction will be found in the paragraph dealing with anomalous conductivity as a theory in explanation of absorption.

Phenomena in Rotating Electric Fields. If a cylindrical block of dielectric be suspended with its axis normal to a rotating electric field, it will turn about its axis. If it be set in oscillation about its axis its rate of damping will be increased if it is subjected to an electric field normal to the axis. These facts have been attributed to dielectric absorption; expressions for the torque acting on the block have been derived, and a number of investigators have used the method for studying the phenomenon of absorption itself. However, it may be shown that a normal dielectric, *i. e.*, one possessing conductivity and dielectric susceptibility, will also develop similar turning moments under similar conditions. As it is not easy, in such a method, to separate the two influences, many of the experimental results are open to question. The bibliography contains a number of references to work in this field.

IV. THEORIES OF DIELECTRIC PHENOMENA

Even the simplest natural phenomena leave us groping when we seek their ultimate understanding as a sequence of cause and effect. Such questions soon lead us into the realm of metaphysical speculation. For the most part, therefore, in the field of physical phenomena, we are usually content if we can express them in terms of the fundamental mechanical relations of force, mass, motion, and energy. And in the special fields of electricity and magnetism it is only by means of their manifestations through the laws of mechanics, that we are at all able to bring within the science and art of engineering application, these most mysterious of all natural phenomena—save only that of life itself. Explanations and theories are thus relative terms only, and we are usually satisfied to consider that we have a theory of any new natural phenomenon if we can explain it in terms of more fundamental phenomena whose laws are well recognized, even though they may not themselves be completely understood. Thus, since the earliest discoveries of the properties of dielectrics many explanations of their origin and nature have been suggested. Few if any of these suggestions, however, have risen to the dignity of a theory substantiated by quantitative experimental test.

Specific inductive capacity is one of the simplest of electrical phenomena. The phenomenon of induced charge as affected by the medium, was noted by several early investigators, chief among them being Cavendish, but it was not until Faraday's time that the complete influence of the medium was recognized, and the apparent action at a distance explained by a point-to-point transference through the ultimate particles of the medium. Thus there is introduced the idea of a separation of the charges on the ultimate particles of an insulating medium, by the inductive action of the electrodes. This is known as "polarization" and the internal separated charges contribute to the total force. These ideas are sufficient to explain all normal dielectric properties, and are the basis on the classical development by Maxwell with the aid of Poisson's theory of potential (1823). Of earlier suggestions of the nature of the structure of the medium we need mention only one or two. Mossotti and Clausius assumed that a dielectric contains a certain number of spherical conducting particles, which leads to a value of K depending on the volume density of the particles. Helmholz proposed that the molecules contain charged particles which are drawn out of their natural positions by the external field—thus forecasting (1870) the electron theory. Another suggestion (Debye and Schrodinger) was that the molecules are already electrically polarized, and that they are oriented in an external field.

Faraday's idea was completely developed by Maxwell whose theory forms the basis of all subsequent electrical theory and speculation. The development and consequences of Maxwell's theory are so well known as to need no comment here. It may be mentioned, however, that so far as the nature of specific inductive capacity is concerned Maxwell pictured an electrically elastic ether pervading all space and matter. The electric field displaces the medium, the displacement is opposed by the elastic forces, and these latter are different in the presence of the molecules of matter, thus explaining the varying dielectric properties of the latter.

The fact that many dielectrics have a measurable conductivity was also recognized early, and Maxwell assumes that dielectric displacement and conductivity are contemporaneous phenomena. The conductivity is treated as though it were similar to that in conductors; little if any suggestion appears in the fundamental theory as to its origin or the laws of its variation. We now know, however, that so far as a definite specific conductivity, similar to that in metallic conductors, is concerned, it is probable that no dielectric possesses it, for the laws governing variations in the two cases are quite different, as appears in other sections of this paper.

We thus see that even in the matters of the so-called normal properties of dielectrics we have very little certain fundamental theory upon which to proceed. Modern physical theory has added little or nothing in these particulars to the ideas of Maxwell's time; for while it has greatly extended our knowledge of the structure of the atom, and its relation to the great phenomenon of the electromagnetic radiation arising in the motion of the electrons, it has not yet uncovered knowledge warranting even an approximate explanation of such familiar fundamental phenomena as electric conduction and induction. If, therefore, these older fundamental properties of a dielectric have so far withheld the secrets of their origin, it would appear well nigh hopeless to attempt a theoretical explanation of the anomalous properties described above. The case is not quite so bad as this, however, if we recall that an explanation in terms of simpler and more familiar phenomena is often to be accepted as a theory.

Theories of Dielectric Absorption. Ever since the phenomenon of residual charge was recognized there have appeared suggestions and hypotheses as to the general nature of the underlying processes. Beginning with mere suggestions in the early days, as for example, the slow penetration or "soaking in" of the charge, (readily shown to be untenable) they have increased in elaboration and complexity as further knowledge of the phenomenon has been gained. In reviewing these theories today we are presented with such pictures as a viscous yielding of the dielectric, frictional motion of molecules and electrons, infinitesimal conducting particles embedded in insulating sheaths, the free motion of electrolytic ions, dielectric hysteresis, the capillary motion of water, etc. It is impossible within a limited space to give a complete view of all these theories. It appears best, therefore, to attempt their classification into a few groups and to give one or two conspicuous examples in each group, with some attempt to outline the reasoning and evidence in support.

Most physicists offering explanations of absorption have apparently considered that the observed phenomena in anomalous dielectrics are not consistent with the fundamental equations of the electromagnetic field, and have built other equations based on new special properties of the dielectric, not embraced in the older theory. There is, however, one conspicuous instance, that of Maxwell, in which the fundamental equations are taken as a basis. We may, therefore, select two of our groups in accordance with these two views, and add a third to include those looking to present theories of the internal structure of the atom as bases for the explanation of dielectric behavior. There is also a possible fourth group as proposed by von Schweidler, in which anomalies of conductivity are invoked, but the evidence in support is not so strong as in the other groups.

Our classification of theories of dielectric absorption is then as follows:

1. Those in which the fundamental magnetic equations are retained, and the anomalies of dielectric behavior are attributed to anomalies of the structure of the dielectric medium.
2. Those in which the departures from the fundamental laws are attributed to anomalies of dielectric displacement without reference to underlying mechanism. Dielectric displacement is not proportional to field strength, but dependent on the preceding state of the dielectric.
3. Explanation of displacement and its anomalies is traced to the motion of electrons within the atom.
4. Explanations based on anomalies of conductivity, such as the free motion of ions, electrolytic dissociation, water in bulk or in capillary filaments.

GROUP I

Maxwell. The great outstanding exponent of this group is J. Clark Maxwell. Others have followed him, but always as supporters and with adherence to the simple principles laid down by him. His theory is clearly sufficient to account for the ordinary phenomena of absorption and residual charge. While it does not account for all the observed anomalies, it nevertheless has far more adherents and much firmer support than any other single theory, and always constitutes the starting point of modern dielectric discussion.

All students of dielectric theory are familiar with Maxwell's treatment of absorption. He starts with the assumption that all dielectrics have both specific inductive capacity and conductivity as we know them in normal dielectrics, and that under electric force they function simultaneously and independently of each other. The assumption is justified by the experimental facts that conductivity may be observed in even the best insulators, and that poor insulators with very high conductivity also manifest specific inductive capacity. No further assumptions, as for example as to the origin of these properties, are necessary in Maxwell's development. He then shows by means of Poisson's equation $\Delta^2 V + 4 \pi \rho = 0$, and the equation of continuity of current flow, that in a *homogeneous* dielectric—*i. e.*, one in which r, the specific resistance, and K, the specific inductive capacity, are uniform and constant throughout—there can appear no residual charge. After charging, standing open, and then on short circuit, such a condenser discharges in accordance with the normal circuit equation, and no residual charge can appear.

For simplicity he then assumes a dielectric as built up of a number of plain strata of different materials of thickness a_1, a_2, etc., stating that a medium formed of a conglomeration of small pieces of different materials would behave in the same way, although the case is not susceptible of exact analysis. The obvious assumption then is that every dielectric which shows absorption consists of a mixture of two or more different materials, even though under our closest examination it may appear to be homogeneous. Considering unit cross section, let X_1, X_2, etc., be the electric intensities in the several strata, f_1, f_2, etc., the displacements, k_1, k_2, etc., the reciprocals of the specific inductive capacities, r_1, r_2, etc., the specific resistances, p_1, p_2, etc., the conduction currents, and we have at any instant:

$$p_1 = \frac{X_1}{r_1} \tag{11}$$

$$f_1 = \frac{X_1}{r \pi k_1} \tag{12}$$

and

$$u = \frac{X_1}{r_1} + \frac{1}{4 \pi k_1} \frac{d X_1}{d t} \tag{13}$$

in which u is the current in the outer circuit and so in each layer. Similar equations hold for the other layers. The total e. m. f. on the condenser is the sum of X_1, X_2, X_3, etc. From equations (**13**) the X's may be evaluated as similar functions of u, in terms of the different values of r and k, and so an expression may be had for u in terms of E, *i. e.*, the charging current of the condenser as function of E, the constants of the material and the time t. Maxwell does not derive this expression, but states that if there are n layers of material having *different values of the ratio* $r : k$, the combined general equations (**13**) will form a linear differential equation of the nth order with respect to E, and the $(n - 1)$th order with respect to u, t being the independent variable. He also shows that if the ratio $r : k$ is constant for all layers the case reduces to that of a homogeneous dielectric.

Maxwell thus does not derive the equation of the reversible absorption current $i_1{}^1$ of equation (**2**). For the case of two layers only, however, it may readily be shown (Wagner), following Maxwell's method, that

$$i_1{}^1 = A\, e^{-\frac{t}{T}} \tag{14}$$

in which A is its initial value and T a function of the constants of the material. Thus, according to Maxwell the absorption current follows the negative exponential relation to the time.

The treatment is more complete for the building up of a residual voltage. Suppose that E_0 is suddenly applied, then

$$Q = \int u\, d t = \frac{X_1}{r_1} d t + \frac{X_1}{4 \pi K_1} \tag{15}$$

and in the first instant since X_1 is finite the first term on the right is negligible, and:

$$E_0 = 4 \pi Q \left(k_1 a_1 + k_2 a_2 + \quad . \quad . \quad . \quad .\right) \tag{16}$$

that, is the instantaneous charge is that corresponding to the geometric capacity, and the initial division of the total e. m. f. over all the layers is that corresponding to the series connection of the geometric capacities. If now E_0 is applied for a long time, the unequal conduction currents in the several layers, due to initial e. m. fs. determined by the geometric capacities, cause charges to accumulate on the surface of separation until finally an equilibrium condition is reached in which, by formula (**13**), the division of the total e. m. f. over the several layers is determined only by their conductivities. The absorption currents of charge and discharge are thus seen to be due to the change from a voltage division due to the different specific inductive capacities, to one due to the different values of resistance or vice versa. The treatment shows by similar methods that after charge the instantaneous discharge is that due to the geometric capacity, and that if then opened the condenser will build up an e. m. f.

$$E = E_0 \left\{ \left(\frac{a_1 r_1}{R} - 4 \pi a_1 k_1 C \right) e^{-\frac{4\pi k_1}{r_1}} + \left(\frac{a_2 r_2}{R} - 4 \pi a_2 k_2 C \right) e^{-\frac{4\pi k_2}{r_2}} + \quad . \quad . \quad . \quad . \quad . \right\} \tag{17}$$

in which R is the total resistance and C over all geometric capacity. The exponential form of the variation of the component terms is obvious. The instantaneous discharge at any instant t will be $E\ C$. If the ratio $r : k$ is the same for all strata, the exponents are all equal, and the total of the other terms is zero (in two series) and thus $E = 0$, *i. e.*, there is no residual e. m. f. or charge.

Experimental evidence in favor of Maxwell's theory is very meager, and chiefly limited to indirect and broadly qualitative confirmatory observations. Cohn and Arons tested the assumption that polarization and conductivity occur independently by means of parallel condensers of different dielectrics and found good agreement. Mixtures of xylol and anilin showed a 10,000-fold variation of resistance with only a ⅓ variation of dielectric constant. Rowland and Nichols showed that in perfect samples of calcite and possibly quartz, probably the most nearly homogeneous substances available, there is no absorption. Hertz showed that benzine, a homogeneous fluid, when impure shows absorption, which disappears on purification. Arons claims that carefully purified paraffin shows no absorption; this is disputed however by Dessau and others. Wagner finds extremely low values of power factor at 5000 cycles for ceresin and paraffin, but that for a 50 per cent-50 per cent mixture the power factor was increased several times. Muraoka by careful purification found no absorption in parraffin oil, petroleum, resin oil, turpentine and xylol. For layers of air and paraffin absorption appears. Many observers have found that the observed curves of charge and discharge currents, while not obeying the exponential law with a single term, can nevertheless be represented readily by several such terms, as called for in Maxwell's most general case. The principle of superposition as observed by Hopkinson and Curie has been shown by von Schweidler to be a necessary consequence of Maxwell's theory. However, its firmest basis is found in the fact that it introduces no new phenomena nor assumptions, but relies only on properties of matter already well known, and on fundamental electrodynamic equations. This, more than any experimental confirmation, accounts for the firm hold that this theory has on the mind of the physicist of today.

The chief disadvantage that the theory suffers is that it not only has had no quantitative nor exact experimental confirmation, but many experiments appear to offer actual contradiction. Many observers have found other expressions than the exponential for the absorption current, of whom particular mention may be made of Curie, Fleming and Dyke, and Trouton and Russ. The phenomenon of conduction in a dielectric may be based on ionic conductivity (Bialobjeske), and so follow a law different from that of metallic conduction. Anderson and Keane have shown that a movement of free electrons in the medium, accumulating at the positive electrode, and returning to their equilibrium distribution on discharge, will account for absorption and residual charge. Further, it has been shown that the conductivity of dielectrics may be increased by exposing them to the influence of ionizing radiation. Muraoka found that solutions and certain liquid mixtures show no absorption. Particularly in the behavior of dielectrics under alternating stress, discussed in later paragraphs, noticeable discrepancies have been found by Curtis and others. Comparatively few serious attempts have been made to check Maxwell's theory quantitatively. This is largely due to the very general, if not sketchy, form in which it is developed by its author. In particular, attempts to extend his equations to the alternating case encounter great mathematical difficulties.

It should, however, be possible to extend the few efforts which, in the light of Maxwell's analysis, have been made to study the case of two simple substances, carefully controlled as to purity, and carefully examined as to electrical properties separately and as a combination. Such studies are particularly desirable in view of the occurrence of pronounced absorption in many substances which can have in them only relatively small amounts of impurities. While the further removal of these often lowers the absorption, there is some doubt that it ever completely disappears and it appears remarkable that a combination of such extremely disproportionate amounts of two substances should still function in accordance with the theory. Further, if these small amounts so function should not greater amounts function in increasing degree? If the amounts of impurities sufficient to account for absorption are too small to control or even observe, then there is little hope that the theory will ever be rigidly confirmed.

Wagner. Maxwell's development assumes successive layers of different dielectrics, each having different values of r and K. Many layers must be assumed to account for the results of experiment, and this leads to mathematical difficulties. The charging curve for two substances obeys the negative exponential relation to the time as already noted. Using this relation, K. W. Wagner examines the curves taken on various substances, and states that only a very few exponential terms are necessary in any case to account for the curves of experiment, and moreover that the time constants of these terms group themselves more or less closely about a principal value T_0. He is therefore led to attempt a picture of a model dielectric which provides the variation in conductivity and in specific inductive capacity called for by Maxwell, but which avoids the simultaneous occurrence of each in a single dielectric substance. He proposes a pure dielectric medium without conductivity, but with conducting spheres imbedded in it, small compared with their distances apart. He investigates this model mathematically at great length; the different time constants of the several terms in the absorption curve are provided by different sets of conducting spheres, mixed with the surrounding dielectric in different proportions. He arrives at equations for the electric displacement, residual charge, time constant, etc., which agree with those of Maxwell's theory in their general form but which involve fewer terms. He also gives an expression for the grouping of these terms involving only two constants of the dielectric medium. He considers the behavior of the model under change of temperature and states that it conforms to the results of observation. Wagner states that such a model has been built by Schering and Schmidt, but does not report their results. The development is highly speculative and pictures a highly specialized interrelation of dielectric and imbedded conductor. It seems doubtful whether it could ever be subjected to satisfactory experimental confirmation, and its chief value seems to lie in its offering of a mental picture of a type of structure accounting for Maxwell's assumptions, while retaining the idea of a homogeneous dielectric free from conductivity.

GROUP 2. THEORIES BASED ON ANOMALOUS DISPLACEMENT

As the phenomena of anomalous charge and discharge were known long before the discovery of the electron and ionic conductivity, it is natural that early theories as to their causes should have taken the form of analogies with other elastic and viscous phenomena. Thus, one of the earliest, that of Hopkinson, adopted the relations found in the elastic residual properties of ordinary materials under mechanical distortion. Electric displacement is assumed analogous to mechanical displacement. But electric displacement in the fundamental theory is proportional to the electric intensity, and not so in anomalous dielectrics, and so practically all the theories in this class assume a more or less complicated relation between the field E and the displacement D, and of such a kind that D is determined not alone by the instantaneous value of E, but also by the foregoing condition of the dielectric. Such relations exist between deformation and force in elastic media, and between induction and field strength in magnetic media.

Hopkinson developed the Boltzmann formula for elastic residual phenomena into the following general form:

$$D(t) = \frac{K}{4\pi} E(t) + \int_0^\infty E(t-u) \, . \, \varphi(u) \, . \, du \qquad (18)$$

that is, the actual displacement $D(t)$, at any instant (t) is determined first by the instantaneous value of the field $E(t)$, but also by the residual effects of all foregoing values of E, considered as a function of an elapsed time u. The expression, therefore, has the important property of showing the variation of the displacement when the electric field strength has varied in the past in accordance with any given function of the time $E(t)$. Every such residual effect of foregoing values of E decreases with the elapsed time u in accordance with a definite function $\varphi(u)$ and the single residual effects are superimposed on each other. Thus, the principle of superposition, observed experimentally, is a fundamental assumption of Hopkinson's theory. He makes no special assumption as to the particular form of the continually decreasing function $\varphi(t)$. From his results he concludes that two exponential terms are not sufficient, but that possibly a greater number of such terms might be sufficient to express them.

Houllevigue has extended Hopkinson's theory so as to place the apparently viscous behavior of the dielectric in the relation between the molecule and the ether. He assumes the displacement in an imperfect dielectric to consist of two parts—(a) an instantaneous displacement of the ether until the elastic forces between the ether and the molecules are in equilibrium, and (b) an additional gradual yielding of the molecules under this tension, so that there is added to (a) the displacement of the mean position of the molecules. The first part (a) is the normal displacement of a perfect dielectric as called for by Maxwell's theory. The second part involves a frictional reaction dependent on the velocity with which the displacement occurs. This latter leads to the heating of the dielectric under alternating force, and Houllevigue developed his equations for both the continuous and the alternating cases. Grover examines this theory in the light of his own observations on condensers. He finds that for increasing frequency the theory calls for a minimum value of the apparent capacity, smaller than the geometric capacity, increasing thereafter. As this is contrary to the results of his observations and those of others the theory appears to be untenable.

Pellat also considers the displacement as divided into two parts. The first is that of the fundamental theory:

$$D^1(t) = \frac{K}{4\pi} E(t) \qquad (19)$$

the second part $D''(t)$ is assumed to obey the equation

$$\frac{d\,D''(t)}{dt} = \alpha\,[D''(\infty) - D''(t)]$$

$$= \alpha \left[\eta \frac{K}{4\pi} E(t) - D''(t) \right] \qquad (20)$$

that is, that D'' tends to a final value $D(\infty)$, proportional to E, and the rate of change of D'' is always proportional to its difference from the final value. Pellat calls D^1 the "fictitious" and D'' the "true" polarization; Schweidler, who has developed this theory further, prefers the terms "normal" and "viscous" displacement.

Thus for constant E_0 and $t > 0$ we have:

$$D''(t) = \eta \frac{\overline{K} E_0}{4\pi} (1 - e^{-\alpha t}) \qquad (21)$$

$$i_1{}^1(t) = \frac{d\,D''(t)}{dt} = \alpha \eta \frac{K}{4\pi} E_0 e^{-\alpha t} \qquad (22)$$

and the function showing the time variation of the reversible anomalous current takes the simple negative exponential form.

By integration of the above equation, we have for any type of variation of the electric force $E(t)$, remembering the principle of superposition

$$D''(t) = e^{-\alpha t} \int_\infty^t a \eta \frac{K}{4\pi} E(u) e^{-\alpha u} \, . \, du \qquad (23)$$

$$= \alpha \eta \frac{K}{4\pi} \int_0^\infty E(t - \omega) e^{-\alpha\omega} \, . \, d\omega$$

and so:

$$D(t) = \frac{K}{4\pi} E(t) + \alpha \eta \frac{K}{4\pi} \int_0^\infty E(t-\omega) e^{-\alpha\omega} \, . \, d\omega \qquad (24)$$

Thus, the variation of the displacement satisfies the principle of superposition, and the theory of Pellat is seen to be a special case of that of Hopkinson, in which $\varphi(\omega)$ is proportional to $e^{-\alpha\omega}$.

The simplicity of Pellat's assumption as to the variation of the displacement, and the close approximation to observations which results in the form of the anomalous charging current, make a strong appeal, in spite of the absence of all suggestion of underlying explanation. In order to supply this deficiency, von Schweidler has extended Pellat's proposal in considerable elaboration, with the aim first to bring it more nearly into accord with observation, and second to present a picture of underlying mechanism.

von Schweidler. It has already been noted that many observations show a departure of the anomalous charge and discharge currents from the simple exponential relation with the time. von Schweidler, therefore, modifies the Pellat theory through the assumption that the anomalous displacement D'' consists of a number of terms, each one of which satisfies the Pellat differential equation, but for each of which η and α have different values. This leads to expressions for the displacement and current which contain sums of terms. The dielectric is characterized by the normal constants K and r, and in addition by a so-called "distribution function" defining the mode of occurrence of the constants of η and α. The analysis, therefore, has the character of a mathematical fiction and is somewhat analogous to the expression of a periodic function by means of a Fourier series. In order to give the theory a physical interpretation he proposes that, in addition to high frequency oscillations of molecular charges involved in the phenomena of dispersion and absorption of electromagnetic waves, there are also present some molecules the oscillation of whose charges is aperiodically damped, and which even in relatively slowly changing fields return to their equilibrium positions with a noticeable time lag; moreover that there are as many different kinds of molecules as are required by the "distribution function." In order to simplify the determination of the distribution function he proposes the method used by Wiechert in a theory of elastic "after working," and which was utilized by Wagner in his extension of Maxwell's theory already described. By this expedient the various sets of constants group themselves about a single predominating set, and the number of sets and their proximity to the central set is indicated by a single constant of the material. von Schweidler states that in general only three sets of constants, *i. e.*, three terms of the Pellat type, are sufficient for a calculation of all measured values both continuous and alternating.

As regards experimental confirmation Grover, with a-c. studies of paper condensers concluded that of several theories examined, the Pellat theory as modified by von Schweidler was the only one that could be made to give quantitative results in agreement with the observations. The quantities studied were changes in capacity and phase difference, with frequency and temperature. It is to be noted, however, that Grover did not examine Maxwell's theory as extended by Wagner, which appeared later, and which involves the same type of variation of the anomalous charging current, and an entirely analogous method of assuming a number of terms and of studying their grouping. It appears certain that an equally good agreement would have been obtained from Wagner's equations. In fact, it is safe to say the same of any theory providing for the medium a sufficient number of terms, all obeying a continually decreasing function $\varphi(t)$ of relatively simple form, but with different values of the constant terms. It appears not improbable that Wagner, not caring for von Schweidler's idea of slow period molecular oscillations, set out to picture a structure of an anomalous dielectric which would involve only the fundamental properties of specific inductive capacity and conductivity, thus adhering to Maxwell's ideas.

Dielectric Hysteresis. The term *hysteresis* has apparently never been clearly defined. If it be assumed to apply generally to any phenomenon in which the value of the effect of a force depends not only on the value of the force but on the preceding history of the system, then there are certain grounds for using the term to describe anomalous dielectric behavior. For this property is possessed by magnetic, dielectric, and elastic media. The term as applying to dielectrics was apparently first used by Steinmetz shortly after his famous work on magnetic hysteresis. It has been so generally used ever since as to make it evident that there is wide feeling that the two phenomena are similar in underlying character. The grounds for this are probably (*a*) the close parallel between the fundamental theories of magnetic and pure dielectric phenomena μ and k); (*b*) the cyclic electrification of anomalous dielectrics often showing the relation between E, the electric force, and Q, the total charge, as a closed curve; and (*c*) the dielectric loss per cycle being proportional to E^n, n in certain cases having been reported as low as 1.6.

But if the word is understood to describe the complete phenomenon as it occurs in magnetic material, then it should not be used as applying to dielectrics, for the behavior of the latter differs in important particulars from that of the former. The work of Hess, Beaulard, Germanischskaja and others has shown the following essential differences in dielectrics as compared with magnetic materials.

(*a*) There is no saturation region, D being proportional to E, up to breakdown (*b*) The stationary *final* value of the displacement is always proportional to the field strength, for both ascending and descending values, although a time element is involved. (*c*) The dielectric loss per cycle varies widely with the frequency, while the magnetic loss does not (except in the extreme upper range). (*d*) There is no dielectric *permanent* residual displacement nor coercive force. (*e*) The anomalous charging current may not be explained on the basis of hysteresis. Hess discusses the difference between hysteresis and viscosity and considers that the phenomena in dielectrics have the properties of the latter rather than of the former. In hysteresis there are two values of B for every value of H, dependent only on the direction from which H is approached and on H_{max}. They are independent of the rapidity of the cycle of H within a wide range. In a viscous medium, however, there can be any number of B's for the same H, dependent on the rapidity of the cycle of H, but for very slow cycles there is only one value of B for each value of H. Thus hysteresis is clearly evident under slow cycles in iron, but not in dielectrics.

GROUP 3. THEORIES BASED ON THE STRUCTURE OF THE ATOM

The idea that electric displacement is due to the movement of charges within the molecule or atom is an old one dating back to Helmholz (1870), and has frequently been invoked in a general way to explain anomalous displacement. Thus, as already noted, Anderson and Keane have proposed the drift of electrons to the electrodes as the explanation of anomalous charge and discharge currents; and von Schweidler used the idea of the damped motion of charges within the molecules; the notion of dielectric hysteresis involves the motion of charges within the molecule. These, however, are little more than suggestions, as compared with the single outstanding theory of this class, viz., that of Décombe.

Décombe, in seeking an explanation of the heating of condensers, notes the remarkable and very general fact that most thermodynamic modifications are inseparable from noticeable electric phenomena. Thus, mechanical deformations are always accompanied by both heat and electrification (tribo-, and piezo-electric effects); capillary deformations, shock, cleavage, etc., all result in both heat and electric manifestations. Similar effects are noticeable in chemical relations, as in the thermal variations of crystals, the thermo-electric cell, the Thomson and Peltier effects, etc. He thus concludes that dielectric absorption and losses are also to be explained in terms of motions or deformations of electrons within the atom, and he bases an interesting and extended development of this theory on the general electron theory of Lorenz:

Separating the displacement of the ether from that of the material, as in Maxwell's theory:

$$Q = \frac{E}{4\pi} + P \qquad (25)$$

Q being the total charge, and P the "polarization." P, he assumes to be due to electron displacement and it therefore obeys Lorenz's equation:

$$E = a\frac{d^2 P}{d t^2} + b P + c\frac{d P}{d t} \qquad (26)$$

E being the electric intensity and a, b, and c constants of the reactionary forces of acceleration, elasticity, and friction respectively. He assumes further that for frequencies less than those of light the first term on the right is negligible, and so

$$E = b P + c\frac{d P}{d t} \qquad (27)$$

He shows that **(27)** obeys the principle of superposition, and also that it is satisfied by a value of alternating polarization computed from oscillograms taken by Hochstadter on high tension cables. He shows further that Maxwell's equations as extended by Hess, when applied to the alternating case, reduce to the same form as his own, and so by inverse reasoning, account is made for residual charge. Moreover, he shows that Pellat's arbitrary expression for the variation of the displacement follows immediately from his own equations. He shows also that under alternating e. m. f. the equations lead to a loss proportional to the square of the polarization current $\left(\frac{d P}{d t}\right)$, and a loss per period independent of the frequency, as shown experimentally by Steinmetz, Hochstadter and others. The general conclusion is that all residual effects are due to the term $c\,\frac{d P}{d t}$, representing a frictional force (electric) within the atom, *i. e.*, to a viscous property of the atom.

The theory is very striking in its agreements with the results of observation and in its simplicity. It is strengthened by the firm basis now attained by the electron theory, and its explanation of many phenomena in the upper ranges of frequency. It leaves something to be desired, however, in its explanation of residual charge, and in the extent of the experimental confirmation, notably in the matter of loss. Moreover, it is to be noted that theories in this and in the foregoing class do not account for the pronounced effect, on both dielectric absorption and loss, of very small quantities of moisture and impurities.

GROUP 4. THEORIES BASED ON ANOMALOUS CONDUCTIVITY

There are no theories in this class sufficiently developed to merit the name. Yet there are so many significant facts in the matter of dielectric conductivity, and they have been discussed so widely in their bearing on other anomalous properties, that any discussion of possible underlying causes of absorption must give them a conspicuous place. It has often been suggested that dielectric absorption is in fact due to a varying conductivity giving rise to the motion of movable charges of "ions" within the dielectric. Experimental investigations of this idea as applied to solids are very difficult, but the irreversible anomalous current of a poorly conducting liquid offers very much wider opportunity. It will be recalled that this current does not obey Ohm's law but the apparent conductivity falls off with increasing voltage, just as in the case of an ionized gas. This has led to the suggestion that the conductivity of dielectrics is of the same nature as that in gases. Qualitatively this gives a satisfactory agreement with the results observed in liquids. With the continued application of electric force the ions are swept to the electrodes and the current falls off; with greater e. m. f. ions are drained off more and more rapidly as compared with their rate of formation, and so the final conductivity falls off. The theory applies especially well to liquids having very low conductivity, in which an increased conductivity is induced by radio-active influence. In this case the liquid shows the characteristic behavior of ionized gases, *e. g.*, saturation current, etc., and methods have been worked out for the determination of the important quantities, *e. g.*, mobilities of positive and negative ions, coefficients of recombination, etc. The case is not so clear for the "inherent conductivity" of liquids, as this shows marked departures from the laws of gases. These departures are at times found to be due to an influence of the electrodes, to dissolved or free impurities, but even with these removed it has been found necessary in attempts to bring these liquids into line, to assume different types of ions, having different mobilities. An excellent review of the work in this field is found in the article by von Schweidler in Graetz; "Handbuch der Elek. u. Mag.," 1; 157, 1918.

Efforts have been made also to extend the same explanation (*i. e.*, ionic conduction) to the anomalous charge and discharge currents of solids, but with little success. von Schweidler has maintained that in an ionized medium the reversible anomalous current can not arise; also that the principle of superposition cannot be explained in this way, and offers as proof the absence of residual charge in liquid dielectrics. On the other hand, Anderson and Keane have shown that the drifting of free electrons to the positive electrode will result in a space variation of charge sufficient to account for residual charge in accordance with Maxwell's theory, and have checked their conclusions with observations on sulphur. The behavior of glass is especially significant in this connection. The final conductivity of glass has undoubtedly an electrolytic character. Sodium is deposited out of glass on the electrodes, with resulting decrease of conductivity, and the latter may be maintained by providing an anode of sodium amalgam. Lithium may also be used, but apparently no other metals of the chemical group. Moreover the electrolytic action is in accordance with Faraday's laws. It must be remembered, however, that glass is not a homogeneous substance and in fact is generally considered to be a solid solution, and so of highly special character. On the other hand similar behavior has been observed in quartz, certainly not a solution, but in which small quantities of sodium and lithium are usually present. Mercury, quartz, and sodium form an electrolytic cell giving about 0.5 volts. H. H. Poole has shown the relation $x = e^{a+bX}$ for the conductivity of glass, a being the thickness and X the field intensity; *i. e.*, the conductivity increases with the field strength. Gunther-Schulze sees in this an evidence of ionic conductivity and ionization by collision as in gases. Thus, while we may not be sure that glass presents a behavior typical of all anomalous dielectrics, it unquestionably is significant as showing the possibility of ionic conductivity in solids.

Perhaps one of the most significant facts under this heading is the close relationship which exists between the final conductivity and the anomalous charging current curve, as affected by electromotive force and temperature. In all dielectrics well freed of moisture, these two properties appear to vary absolutely in unison. The final conductivity is obtained by subtracting from the charging current curve corresponding ordinates of the discharge curve; the result is generally a horizontal line representing the constant conductivity of the material. Under variation of e. m. f. proportionate changes in both absorption and conductivity result. Under temperature variation the final conductivity approximately obeys the relation, $\lambda_\theta = \lambda_0 e^{a\theta}$, with corresponding change in the ordinates of the absorption curve. In these cases the evidence is very direct that absorption and conductivity are definitely related properties.

The Influence of Moisture. Many porous and fibrous dielectrics absorb water from the surrounding atmosphere. Their conductivities are markedly influenced thereby, and in many cases their absorptive properties as well. Once taken up, and doubtless due to its retention in capillary spaces, the moisture leaves the insulation but slowly and only under elevation of temperature. Hence fibrous insulation exposed to the air and operating at low temperatures is apt to deteriorate steadily. Curtis has shown that between 90 per cent and 25 per cent saturation of the air, and standing at the latter 40 days, the resistances of bakelite, fiber, marble, and celluloid all increase from 5 to 300 times according to the material, and a steady condition not yet reached. Shellac was an exception showing no change of resistance though taking up considerable water.

Cable paper absorbs up to 10 per cent or even 20 per cent by weight of moisture, rapidly at first and then more slowly. Under continuous voltage when comparatively dry (2 per cent to 3 per cent moisture) it shows a typical absorption curve in accordance with formula (**3**), $i_1^1(t) = a\,t^{-m}$. With increasing moisture the ordinates increase, and the curve flattens out, becoming horizontal (except for a short initial descending portion) at about 7 per cent moisture. Above this the curve of charging current with time rises gradually (Lübben). These increases are all of the nature of conduction current, for the discharge current does not take on corresponding increases, and follows equation (**3**) in all cases. Thus, the difference between charge and discharge currents, which usually measures the insulation resistance, increases continually with time after the application of voltage (Wagner). This indicates that the continued application of voltage causes a decrease in resistance. Moreover, the final steady value of resistance is found to depend on the voltage; so that we have the following approximate relations for the resistance r, the conductance g, and the conduction current i, in a fibrous dielectric containing moisture:

$$r = \frac{A}{\sqrt{E}}; \quad g = g_0 E^m; \quad i = i_0 t^p \qquad \textbf{(28)}$$

E being the voltage and A, g_0, m, i_0, p constants. Thus equation (2) becomes

$$I_1(t) = i_1(t) + i_g t^p + i_a t^{-m} \qquad (29)$$

The second term is the conduction current due to the moisture, often negligible for low moisture content. The third term is the absorption current of the dry material, and this is completely obscured at high moisture content. The discharge current does not contain the second term.

An ingenious explanation of many of the above relations has been offered by Evershed who supposes that the water is in part contained in the capillary tubes of the fibers of the material. The water is separated by air bubbles, but the walls of the tube surrounding the bubbles are wet with a thin film of water. These films constitute the principle resistance of the complete water path. Under the electric field, water is forced from the drops into the films making their walls thicker and so increasing their conductivity. In the thin state of the films they are very sensitive to slight water addition, but less so as the walls get thicker, corresponding to the observations of experiment. Evershed constructed a model containing a large number of glass capillary tubes, and studied its behavior under the microscope. He found that it gave the typical resistance, voltage, time relations found for fibrous insulation, and that the water films behaved as already stated.

D. DuBois has suggested a somewhat different mechanism for the behavior of water in dielectrics. He assumes that the moisture in dielectrics exists in the form of small globules or drops, and that under the electric field these globules stretch out into filaments supposedly in openings or fissures in the dielectric. Sometimes these filaments stretch the entire distance between electrodes, and sometimes they are of limited length. Since they are conducting they carry current, consume energy, and in the slow process of stretching cause a slow variation in the distribution of electric flux. Proceeding on this picture he shows that they should account for the anomalous charging current, residual charge, dielectric loss as function of the voltage and temperature, and several other phenomena as observed in the experiment. The agreements here have been largely qualitative only, and the interesting suggestions would seem to require further experimental confirmation.

J. Curie showed the important part water may play in materials containing no fibers but which are porous. By maintaining porcelain at different degrees of moisture he reproduced different types of charging current curve, among them typical absorption curves as observed for dry substances. Moreover, this moist porcelain polarized up to several hundred volts, gave typical discharge curves and in fact obeyed the principle of superposition. Curie suggests in explanation the linking up of a number of internal individual electrolytic cells due to water and local conducting impurities.

V. DIELECTRIC BEHAVIOR UNDER ALTERNATING ELECTRIC FORCE

General. A condenser containing a perfect insulator as dielectric, if subjected to alternating e. m. f., would consume no power, its current leading the applied e. m. f. by exactly one-quarter of a period. The only condensers approaching these conditions are those with gases as dielectrics. All solid dielectrics have some conductivity, and so even if free from absorption, they consume some energy. A condenser containing such a dielectric should show a value of capacity independent of the frequency and a power factor (or phase difference) varying inversely proportionally to the frequency. Such dielectrics, however, are unknown, and condensers made with the best insulating materials, most carefully prepared, all show power factors much higher than that corresponding to their conductivities, and capacities much higher than their geometric values. Moreover, the power factors do not vary inversely as the frequency, but very much less rapidly (sometimes even increasing) and the capacity decreases slowly with the frequency.

The alternating losses in condensers were first noted by Siemens (1864), and since then they have been studied by many observers. In fact the problems of the laws governing these losses and their explanations have probably received greater attention than any others in the field of dielectric behavior. The volume of the literature is enormous, embracing the attacks of physicists working with pure materials, and research engineers extending the ranges of voltage and frequency in the effort to improve the qualities of commercial insulation.

For a long time and even up to the present, many have assumed that the cause of these losses is to be found in some special and unknown property of dielectrics, usually called dielectric hysteresis and supposedly arising in some undiscovered molecular phenomenon, similar to that in magnetic materials. It is remarkable that this illusion should have maintained so persistent a hold, for it is easy to see that the phenomenon of absorption causes a lag of charge behind e. m. f. and so is sufficient to account for the energy component of current.

Theories Based on Absorption. *Beaulard* in 1894 and *Hess* in 1895 gave convincing arguments against the idea of hysteresis, and *Hopkinson* in 1897 made alternating measurements, attempting to link up absorption with the values of capacity and loss currents. Ever since then careful students of dielectric behavior have sought to explain the losses in dielectrics in terms of the phenomenon of absorption as observed with continuous potentials. Naturally this has led to points of view and analyses differing in accordance with the several theories of the nature of the phenomenon of absorption.

Rowland extended Maxwell's theory of absorption to the case of a sinusoidal e. m. f. and derived expressions showing variations with the frequency, of both the capacity and the phase difference. He developed a sensitive electrodynamometer, many special types of bridge connection, and with his co-workers made numerous studies of dielectric loss, failing however to find close agreement with the Maxwell theory. The measurements of Curtis in 1910 also failed to agree with this theory.

von Schweidler. Grover in 1911 made a similar effort not only with the Maxwell-Rowland expressions, but also by extending the theories of Houllevigue, Pellat, von Schweidler, and Hopkinson, to the alternating case, and checking them with measurements on a number of condensers for frequencies up to 1000 cycles, and temperatures between 10 deg. and 35 deg. He found that von Schweidler's extension of Pellat's theory was the only one which could be made to give quantitative results in agreement with the observations. It will be remembered that this theory assumes the presence of aperiodically damped molecules in the dielectric, and that in order to conform to experimental observations of absorption, it is necessary to assume special laws of distribution for several types of such molecules. Grover finds that three types of distribution, *i. e.*, three sets of constants in the formulas, are sufficient to account for his observations.

It is to be noted that Pellat's theory results in the same type of function for the variation of the anomalous charging current with the time as called for by Maxwell. The equations of the latter are not easy to handle, however, for the case of several different substances or impurities in the dielectric, and von Schweidler's idea of the oscillation of molecules was apparently adopted largely because of the possibility it offers for mathematical manipulation. Thus, von Schweidler's picture of a number of distributions of aperiodically damped molecules may be described as a mathematical fiction used for the solution of Maxwell's equations for a dielectric containing an equal number of different substances.

Wagner. Much the same may be said of the analysis of Wagner, who has always adhered to Maxwell's view that differences of resistivity and specific inductive capacity are sufficient to account for absorption, but who, as already shown, finds it necessary to picture a series of special types of conducting particles imbedded in the dielectric in order to account for the results of observation on anomalous charging current.

Wagner has also extended his picture of the Maxwell dielectric to the alternating case in a series of papers dating from 1913, and of which the latest and perhaps the clearest is the opening chapter in Schering's Die Isolierstoffe der Elektrotechnik (Julius Springer, 1924). He assumes the simplest possible case of a two-layer Maxwell dielectric, and further that the conduction and displacement currents flow simultaneously, that the total currents are equal in each dielectric, and that the alternating electromotive force on the condenser is the vector sum of the e. m. f's. on the two layers. This leads at once to a complex expression for the impedance of the condenser, from which the following values of the variable dielectric constant (and so the capacity) and the variable angle of phase difference appear at once:

$$K\,\omega = K\left[1 + \frac{k}{1 + \omega^2 T^2}\right] \qquad (\mathbf{30})$$

$$\tan \delta = k\,\frac{\omega T}{1 + K + \omega^2 T^2} \qquad (\mathbf{31})$$

in which K is the specific inductive capacity corresponding to the geometric capacity, T the time constant in the exponential expression of the anomalous charging current, k a simple function of the electric constants of the two materials, and $\omega = 2\,\pi\,f$.

These two equations are in some degree in qualitative accord with the results of experiment. The former (**30**) indicates a capacity starting at a finite value and decreasing with the frequency to the geometric value at infinite frequency. This behavior is universally found in experiment. The latter equation (**31**) indicates a phase difference starting at zero, at 0 frequency, and with increasing frequency passing through a maximum and then steadily decreasing toward zero at infinite frequency. This general behavior has also been frequently observed. At times the maximum value of tan δ may not appear, the frequency at which it occurs being determined by the constant k, and often lying outside the available range. On the other hand many other observations do not follow the comparatively simple relations indicated by (**30**) and (**31**). In order to account for these variations Wagner introduces the idea, already described above, of a homogeneous dielectric with small conducting particles imbedded in it. In an extended mathematical discussion he arrives at the conclusion that such a combination behaves as does the Maxwell model with certain modifications of the constants. New expressions are derived for $K\,\omega$ and tan δ, in general similar to (**30**) and (**31**), but also involving the relative quantity of the imbedded conducting material, *i. e.*, impurity. Several such impurities may be present. Each will contribute its own effect, and their total may show curves differing markedly from those of (**30**) and (**31**). In order, therefore, to bring theory and observation into line it is only necessary to assume a sufficient number of impurities present, and Wagner has been able to do this in several apparently complex cases, claiming that generally only a few sets of constants (*i. e.*, different impurities) will be necessary. Wagner's chief contribution is his extended discussion of the number of terms necessary to account for the results of observation. Much the same types of discussion are possible with von Schweidler's picture, and also in adhering to Maxwell's original equations. In fact Wagner's equations in several cases are readily transformed into those derived much earlier by Rowland, Curtis, and Grover.

Décombe has extended his electron theory of the anomalous behavior of dielectrics, already described in connection with absorption, to the explanation of alternating losses. Noting that the element of absorbed energy in a condenser is $E\,d\,q$, substituting for q and E their values in terms of the polarization p, he finds the energy loss per cycle

$$W = \int_0^T C\left(\frac{d\,P}{d\,t}\right)^2 d\,t \qquad \textbf{(32)}$$

that is, proportional to the square of the polarization current and, therefore, to the square of the e. m. f. Noting that in conformity with the results of Hochstadter the polarization must lag behind E by a phase angle θ, and substituting its value in the above integral, he reaches the conclusions that the loss per period is proportional to the phase difference θ and independent of the frequency. The last conclusion, as to the influence of frequency, although in agreement with early observations of Steinmetz and Hochstadter is not confirmed by more recent work, which indicates that the loss per period decreases with increasing frequency.

F. Tank has studied the Pellat-Schweidler theory for the alternating case, computing the loss as due to absorption, measuring the latter and also measuring the loss, and comparing with the computed value. The method used for computation is typical in the main of the methods used by several others (Wagner, Lahousse) and is briefly as follows: The alternating current due to an e. m. f., $E = E_0 \sin \omega\,t$, flowing in a condenser circuit may be expressed

$$i = (t) = a \sin \omega\,t + b \cos \omega\,t; \qquad \textbf{(33)}$$

further by the principle of superposition, see formula (**10**), the anomalous charging current is:

$$i\,(t) = \beta\,C \int_{-\infty}^{t} \left(\frac{d\,E\,(u)}{d\,u}\right) \varphi\,(t-u)\,d\,u \qquad \textbf{(34)}$$

t being the instant at which the current is considered, and u the elapsed time since the application of E. Equating the two values of $i\,(t)$, the coefficients a and b may be obtained thus showing the influence of absorption on both the capacity and the apparent conductance of the condenser. This operation depends on the possibility of evaluating the integral in equation (**34**). Tank examines the von Schweidler-Wagner forms of the function $\varphi\,(t)$ which, it will be recalled, embrace a number of sets of constants, but concludes that in his work it is sufficiently represented by $\varphi\,(t) = \beta\,t^{-n}$, as often indicated (Hopkinson, Curie, et al). Of special interest is his investigation of the accuracy of this assumption for very small values of t, *i. e.*, comparable to the duration of an alternating cycle. By a pendulum method he measured residual voltage for time intervals between 0.32×10^{-3} and 0.22 secs., and found that in three out of six condensers studied the current closely follows the relation above, values of n being 0.73 – 0.82, 0.73 – 0.78, 0.62 – 0.68; agreement in the other cases was not good.

Adopting this simple form of $\varphi\,(t)$, the integral is readily evaluated and the loss component of the current computed. Tank found a remarkable agreement between the measured and computed losses and currents, the per cent differences for the former in the six cases being 3.8, 4.1, – 0.4, 1.6, 13.8 and 23 per cent. He concludes from his work that the alternating losses in solid dielectrics are almost entirely accounted for by absorption, resistance loss being less than 1 per cent of the total, and no evidence of other losses. Owing to the simple form assumed and measured for the function $\varphi\,(t)$, this work may hardly be considered a confirmation of von Schweidler's theory as claimed by the author. However, it is apparently the only effort so far made for a direct check between measured loss and loss computed from measured absorption.

Lahousse adopts a slightly different method of approach for computing the loss from the absorption. If E is the electric intensity, I the polarization, and k the dielectric susceptibility ($K = 1 + 4\,\pi\,k$),

$$I = k\,E\,(t) + \int_{-\infty}^{t} k\left(\frac{d}{d\,u}\,E\,(u)\right) \varphi\,(t-u)\,d\,u \qquad \textbf{(35)}$$

For a closed cycle, the loss per cycle, w, is:

$$= \int E\,d\,I = -\int I\,d\,E \qquad \textbf{(36)}$$

Substituting $E = E_0 \sin \omega\,t$, and introducing two new constants a and β whose values are determined by the integral in (**35**);

$$I = K\,E_0\,[\,\sin \omega\,t + a \sin\,(\omega\,t - \beta\,)\,] \qquad \textbf{(37)}$$

This is the equation of an ellipse between I and E_0, a result independent of the form of $\varphi\,(t)$. Such ellipses have been shown experimentally by Granier. (See below.)

Further, from (**36**) and (**37**)

$$w = \pi\,k\,a\,E_0^2 \sin \beta \qquad \textbf{(38)}$$

or the loss is proportional to the square of the applied e. m. f. as often observed. Both the foregoing relations are independent of the form of $\varphi\,(t)$, which is contained in β. If $\varphi\,(t) = m\,e^{-nt}$ (Maxwell, Curie, et al), and if n is small as usually observed, the loss per cycle becomes

$$W = \frac{2\,k\,\omega\,m\,E_0^2}{\omega^2 + n^2} = \frac{2\,k\,m\,E_0^2}{\omega} \qquad \textbf{(39)}$$

and the total loss

$$W = \frac{k\,m\,E_0^2}{\pi} \qquad \textbf{(40)}$$

that is, the loss per cycle is inversely proportional to the frequency, and the total loss independent of the frequency. As indicated below, these relations have some apparent support, but the weight of the evidence of experiment shows a loss increasing with the frequency.

Lahousse has also extended the Pellat theory of anomalous displacement to the alternating case and shows that it leads to the same values of W and w as given above, for moderate values of the electric intensity, but that for high values the loss would increase a little less than as the square of the voltage. And he shows further that expressions for the loss given Hess, Pellat, and Beaulard are substantially of the same form as his own.

J. Granier has investigated in very elegant manner the losses due to absorption alone for alternating frequencies between 0.3 and 150 cycles. He interrupts the steady alternating excitation at different points on the cycle, and by a careful zero method obtains the total residual charge. This permits the plotting of the absorption-voltage relation for a complete cycle at each frequency. Plotted in rectangular coordinates the curves are found to be almost perfect ellipses, as called for by theoretical analysis (c. f. Lahousse above). The areas of the ellipses, representing the loss per cycle, continually decrease with increasing frequency, also in general accord with theory. The ellipses become quite flat within the range mentioned, and the study of the influence of frequency is continued up to 1500 cycles, using bridge methods. Within the entire range the total loss increases, but less than in proportion to the frequency; the relation found is very consistent over the entire range and for several condensers studied is

$$W = m f^n \qquad (41)$$

W being the loss per second, f the frequency and m and n constants. The following values of n were found: paraffined paper, 0.4; wax paper, 0.5; glass, 0.78; gutta-percha, 1 (in this case the phase difference being very small even at low frequencies). The results also show both the capacity and phase difference decreasing steadily over the whole range of frequency in each case.

VI. COMPARISON OF ALTERNATING THEORY AND EXPERIMENT

The above review indicates that almost without exception, students of the theory of alternating dielectric losses agree in attributing these losses to the phenomenon of dielectric absorption. Moreover, although there are various theories as to the ultimate nature of absorption, they all conform sufficiently to the results of experiment as to make it difficult to make choice among them. But it is really unnecessary to do so, if we are only concerned with an explanation of the origin of alternating dielectric losses. For the results of the analyses of all the authors cited above agree quite closely as to the general laws governing loss, phase difference, and capacity as influenced by voltage, frequency and temperature, although they start in one or two cases from quite different premises. Thus, Wagner starts from Maxwell's theory which says that absorption is purely a matter of conductivity and specific inductive capacity. This leads, however, to the exponential form of the anomalous charging current, and as this form also follows from Pellat's theory, the alternating analyses of Wagner and Lahousse are in close agreement. Tank's analysis is also based on Pellat's theory, although he assumes a different type of variation of the anomalous charging current ($i = a\,t^{-n}$). Nevertheless, he arrives at expressions coinciding quite well with the results of observation. Granier's work is based on the same assumptions as that of Lahousse, adds experimental confirmation, and proves beyond question that alternating dielectric losses in solids arise almost entirely in dielectric absorption.

So far as agreement between theory and the results of experiment goes, it will have been gathered that such agreement is approximate only, and is to be found only in the mode of variation of the several dielectric properties with the variation of external electric and other conditions. Absorption itself cannot be controlled or predicted, and so it is not to be expected that definite expressions and constants may be laid down for the behavior under alternating electric intensity. With this limitation in mind the following agreements between theory and experiment appear to place beyond question the seat of the principal part of the dielectric loss in solid dielectrics in the fundamental phenomenon of dielectric absorption.

Loss-Voltage. There is almost universal theoretical agreement that the rate of loss varies as the square of the electric intensity. Many observers have studied this relation (see bibliography), and the extreme range of the exponent of the electric force appears to be from 1.3 to 2.7. However, the great mass of the evidence centers about the value 2. It is difficult to account for the values lying below 2, but those lying above permit quite probable explanation. As already noted most dielectrics contain some moisture and probably possess some electrolytic conductivity. The latter decreases with increasing voltage and the former causes a conductivity increasing with the voltage, and each would cause a corresponding negative or positive increment to a normal absorption loss varying as E^2. The total loss is extremely sensitive to temperature change, and as the loss increases the temperature, this doubtless accounts for many of the higher values of the exponent. In built-up insulation internal ionization also adds an additional loss, and it has been shown that the prevention of this loss reduces the exponent from higher values toward the value 2. One striking exception should be noted in the work of M. A. Frigon on impregnated paper, which shows the exponent decreasing from 2.7 at 15 deg. to 1.9 at 118 deg. cent.; this work should be confirmed, as there is no obvious explanation in the light of the above theories.

Loss-Frequency. In the matter of frequency the several theoretical developments are not in entire accord. Wagner shows the loss proportional to both frequency and phase difference and as the latter first increases and then decreases with the frequency, a uniform variation in the loss is not always to be expected. Lahousse deduces a loss per cycle inversely proportional to the frequency and thus a loss per second independent of the frequency, for all but very low values. The experiments of Granier show a loss per cycle decreasing with the frequency up to 150 cycles, and a loss per second increasing only as the 0.5th and 0.6th power of the frequency, thus seeming to lend some force to Lahousse's conclusion. M. A. Frigon reports the losses in impregnated paper increasing nearly in proportion to the frequency between 20 and 60 cycles. H. J. MacLeod working on such good insulators as glass, pyrex, paraffin, cresin, mica, and with unusually careful conditions finds the losses varying as the 0.85th to 0.9th power of the frequency. Many other observers have reported losses increasing with the frequency but usually in less than direct proportion. Addenbroke reports three types of variation, (*a*) direct proportion, to be found only in the best insulators such as crown glass, guttapercha; (*b*) loss independent of frequency, for insulators containing much moisture; and (*c*) a relation lying between these two, to be found in materials having low insulation resistance, *e. g.*, celluloid.

On the other hand, Wagner has examined the variation of the power factor with frequency over a wide range, and finds that the maximum value of the phase difference may occur for different substances in the range of 4 to 1000 cycles per second, and higher. This means that within the commercial range and far beyond, the phase difference may either increase or decrease with the frequency. This is probably the chief cause of the general confusion to be found in attempting to coordinate the results of different observers, and the reason why no simple empirical law has appeared expressing the influence of frequency on dielectric loss. Wagner has studied the combined influence of temperature and frequency on the phase difference, and finds that temperature variation provides a very convenient means for bringing out the comparatively slow frequency change. Using this method he has reported some remarkable experimental checks with the theoretical expressions already given. It will be seen, therefore, that a general simple relation between loss and frequency is not to be expected, and observations at different frequencies can only be interpreted through knowledge of other properties such as phase difference, temperature and moisture content.

Loss-Temperature. The influence of temperature on dielectric loss is very great, the loss rapidly increasing with the temperature. But here, too, it does not appear possible to expect a definite law owing to the indirect influence of temperature on other properties. For solids in which the temperature has no marked effect on the structure, *e. g.*, glass, the losses are apparently proportional to some power (from 3rd to 4th) of the absolute temperature, and leakage is still negligible. Substances about to melt but still pasty show large increase in loss due to absorption but not that due to leakage. Passing into the liquid state the losses due to absorption are transformed into those due to conductance. These facts are very significant in their suggestion that the influence of temperature is chiefly that of increasing the mobility of electric charges or ions in the dielectrics; first in limited paths, and afterwards in the liquid state in paths quite unrestricted. This view is still further borne out by the behavior of impregnated paper, which does not pass over into the conducting state when the compound is liquified, the paper acting as a barrier to the passage of the ions.

The only considerable attempt to account for the influence of temperature from a theoretical standpoint is that of Wagner. He carries it back to its effect on the time constant of the material, *i. e.*, the factor multiplying *t*, the time, in the exponential variation of the anomalous charging current. This enables him to bring it through into the alternating case, in its effect on phase difference, and so on the loss. He reports some remarkable general agreements with the results of observation, and his work stands out as a striking support, in the matter of temperature, of Maxwell's theory.

VII. SUMMARY AND CONCLUSIONS

1. Dielectric absorption is a conspicuous but obscure and little understood phenomenon. Its general character is well known as shown by the decay, with time, of the charging current, residual charge, etc. However, exact and definite forms of even the empirical laws are still lacking.

2. Only solids show the complete absorption phenomena of charge and discharge. Liquids often show an apparent absorption in charging but no residual phenomena. Nearly all solid dielectrics show some absorption. In some substances in a very pure state, *e. g.*, sulphur, quartz, paraffin, it is very small, if not negligible, in amount.

3. Large changes in the absorption in solids may be caused by extremely small changes in composition. Impurities and moisture in very small amounts may cause large changes in absorption.

4. The charging absorption current merges into a final steady conduction current. Both are strongly increased by increase in temperature, the absorption finally disappearing or changing into conduction.

5. The alternating losses in solid dielectrics are due almost entirely to absorption. This is shown by theoretical analysis, and confirmed by experiment. The losses due to conductivity are usually very small compared with those due to absorption, and there is no evidence of losses of other types. There is nothing to indicate a hysteresis loss of the character pertaining to magnetic materials.

6. Theories of the ultimate nature of the phenomenon of absorption are: (*a*) That it arises in the mixture of two or more dielectrics, and depends only on the known quantities, conductivity and specific inductive capacity. This is the theory of Maxwell. (*b*) That it is due to anomalous relation between electric displacement and electric force, the seat of which is within the molecule or atom. Pellat, (c) That it may be

explained by Lorentz's theory of electron motion within the structure of the atom. Décombe. (*d*) That it is due to water in capillaries or interstices in the body of the dielectric.

7. The most satisfactory theory is that of Maxwell though it is far from firmly established. The evidence is mostly indirect and it still needs quantitative proof. Probably its firmest support is found in the extension to the alternating case in which it explains a number of experimental observations. However, the theories of class (*b*) are susceptible to much the same extension.

8. From the standpoints of both theory and practise there is great need and fine opportunity for further study of the phenomenon of absorption. Thoroughgoing and careful efforts to test Maxwell's theory have been remarkably few in number. It should not be difficult to plan a comprehensive series of experiments for that purpose. Much the same may be said as to the question of the presence of electrolytic conduction in dielectrics, and its importance as a factor in explaining absorption. Moisture is difficult to eliminate completely from many dielectrics. It influences profoundly the permanent conductivity, and probably the absorption. Whether or not it is a definite factor in all absorption is a question needing, and apparently susceptible of, experimental solution.

On the practical side, speaking generally, absorption is a conduction phenomenon, and therefore in all cases of varying applied electric force, occasions a current component in phase with that force. This means an energy loss in the dielectric. The two extremes of disadvantage of the presence of this loss are the danger it occasions to the life of high voltage insulation, and the attenuation and damping it causes in the high frequency-impulses and signals of wire and cable communication of telegraphy and telephony. In both cases the situation is aggravated by the fact that composite insulation is used in nearly all such cases, and such insulation always possesses high absorption. It is remarkable that so little attention has been given the property of dielectric absorption, in the efforts which have been made to improve the qualities of commercial insulation. Studies in this direction are certain to produce new knowledge leading to improvements and economies. One difficulty and so an immediate problem is the development of a convenient method for measuring absorption. And beyond this the mere suggestion of such questions as the absorption of the various component materials entering into composite insulation, as dependent on their methods of preparation, and particularly the value of the absorption of the final product, as dependent on the values for the constituent parts, indicate without further example the wide field of study apparently still untouched in our efforts to control the most vulnerable and troublesome of all the materials used by the electrical engineer.

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Discussion

For discussion of this paper see p. 134.

Theory of Absorption in Solid Dielectrics

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***Synopsis.**—Most solid dielectrics are imperfect in the sense that when a constant d-c. voltage is suddenly applied, a displacement of electricity first takes place almost instantly to a certain value, and then continues to increase asymptotically towards an ultimate magnitude. Accordingly, an initial electric charge and a greater final charge may be distinguished, with the corresponding values of initial and final permittivities. The purpose of the present investigation is to establish certain general properties of the function which expresses the increase in the initial electric displacement with the time. The initial and the final leakage conductivities of the material are also taken into consideration.*

An assumption is made that the law of relaxation of electric displacement in the individual particles of the dielectric is a simple exponential function of time, but that the exponent varies from particle to particle. In some "non-viscous" particles the final displacement takes place instantly, in some others it occurs infinitely slowly, while for a great majority of the particles the relaxation proceeds at various finite rates. A "distribution function" for the numbers of different particles is introduced and the general conditions which this function must satisfy are established.

The results are then applied to a particular form of distribution function which has a large enough number of parameters for representing experimental data on a given dielectric with sufficient accuracy. Integrations are carried out for the cases of direct and sinusoidal applied voltages. With direct voltage, expressions are deduced for the permittivity and conductivity as functions of time. Since experimental curves of these two quantities can be obtained, a comparison with the theoretical formulas will permit a determination of the numerical values of the parameters in the assumed distribution function. With alternating voltages, the apparent permittivity and the apparent conductivity are expressed as functions of the frequency. Since experimental values of these quantities may be computed from measurements of capacitance and dielectric loss, another possibility is thus afforded for checking an assumed distribution function.

The ultimate aim of the theory (as now developed and as may be improved in the future) is to make it possible to correlate and to mutually check a vast amount of experimental data on absorption and on dielectric loss, and to enable one to predict these quantities within the range of voltages and frequencies for which no test figures are available.

* * * * *

WHEN a constant continuous voltage is suddenly applied to an imperfect solid dielectric, a phenomenon, shown in Fig. 1, takes place. Let the piece of dielectric be in the form of a slab of thickness a and let the applied voltage be E. Then the *voltage gradient* in the material is

$$G = E/a \tag{1}$$

By assumption, this gradient remains constant, as shown by a horizontal line in Fig. 1. Let the cross-section of the slab in the plane normal to the lines of force be A, and let the total quantity of electricity displaced through the slab be Q. Then the density of displacement, or the *dielectric flux density*, is

$$D = Q/A \tag{2}$$

Experiment shows that D does not reach its final value D_f at once, but continues to increase from an initial value $D_0 = O\,a$ to $D_f = O\,b$ over an appreciable interval of time. Accordingly, a charging current is produced,

$$i = d\,Q/d\,t \tag{3}$$

which continues to flow into the dielectric. Since D refers to unit cross-section of the slab, we shall also introduce the *current density*

$$u = d\,D/d\,t \tag{4}$$

instead of the total current i. At $t = o$, the density u is equal to zero; then it rapidly rises to a value $O\,c$ determined by Eq. (4), and begins to decrease. We are not interested in the part $O\,d$ of the curve, since it is determined by the inductance of the circuit, and we shall assume that point d is reached practically at $t = o$. If the applied voltage is of very high frequency instead of being continuous, the inductance of the circuit, including the magnetic field in the dielectric itself, may become a factor of appreciable magnitude. If the conductivity of the dielectric is zero, the current finally becomes equal to zero. Otherwise, the current density asymptotically reaches a steady small value, $u_f = h\,f$.

The above described phenomenon is known as *dielectric absorption*, and is present to some degree in most solid dielectrics, especially in those whose molecular structure is non-homogeneous or complicated. Absorption affects both the charge and the discharge of a condenser, causes the true capacitance to be a somewhat indefinite quantity and, with alternating voltages, causes dielectric loss.

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A study of absorption is of interest not only from the point of view of the nature of dielectric phenomena, but also in application to charge and discharge of condensers and to dielectric loss.

In view of our meager knowledge of the atomic structure and electronic forces in solids, the time for a rational theory of dielectric absorption has not arrived. It is therefore necessary to establish a semi-empirical theory, with functions chosen by trial to represent the general character of the curves shown in Fig. 1 and with constants determined by tests. Nevertheless, such a theory is useful in permitting us to interpret some observed phenomena and to predict numerical values not actually measured. The theory given below is a further extension of that given by K. W. Wagner.[2] A somewhat different approach to the problem, based on the idea of "stratified" dielectrics, was given by Maxwell[3] and recently further developed by Grünewald.[4]

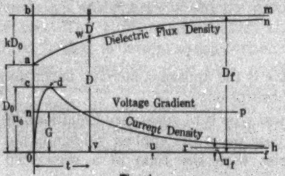

FIG. 1—VARIATIONS IN THE DIELECTRIC FLUX AND CURRENT DENSITY WITH THE TIME

When a constant d-c. voltage is applied suddenly

The character of the curves shown in Fig. 1 suggests the presence of three kinds of paths or "fibers" between the electrodes (Fig. 2); namely, (a) portions of a perfect dielectric, in which an electric displacement faithfully and instantly follows the applied voltage; (b) portions of an imperfect dielectric in which the final orientation of molecular charges requires some time and is opposed by some kind of viscous reactions; and (c) conducting paths. Of course, all these portions are of irregular shapes and intermingled in series and in parallel, the three distinct fibers being shown merely for the sake of convenience. Such a dielectric would behave as indicated in Fig. 1.

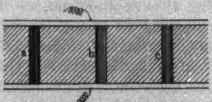

FIG. 2—THREE ASSUMED KINDS OF FIBERS OR PARTICLES IN AN IMPERFECT DIELECTRIC

Viz., a. perfectly elastic, b. viscously elastic, c. conducting

A mechanical analog to the behavior of an imperfect dielectric is shown in Fig. 3. $A\,B$ is a stationary plate and $C\,D$ is an identical plate connected to it by two springs, S_1 and S_2. A vessel V, filled with viscous liquid, is attached to $A\,B$. A plate q, supported by the spring C, is immersed in the liquid. Two weights, G_1 and G_2, rest on supports h_1 and h_2, and are attached to $C\,D$ by strings over the stationary pulleys P_1 and P_2. The lower ends of the springs S_1 and S_2 are attached to the stems K_1 and K_2 fitted into the board $A\,B$ with considerable friction.

Assume the apparatus to be initially in equilibrium with no tension in the springs and with the weights resting on their supports. Now let h_1 and h_2 be suddenly withdrawn, causing the weights to pull $C\,D$ upward and to stretch the springs S_1, S_2 and C. Consider the stems K_1 and K_2 to be so tight as not to move, and assume the motion of q to be negligible at first, due to the viscosity of the liquid. The tension is then initially taken up by the three springs. As the pull continues, the plate q moves upward, relieving the tension of C so that ultimately the whole tension is taken up by S_1 and S_2 only. This subsequent increase in the tension and displacement of the springs S_1 and S_2 illustrates the effect of absorption. Let now the supports K_1 and K_2 be slightly yielding. Then the motion of $C\,D$ will never stop, thus giving a picture of a conduction current.

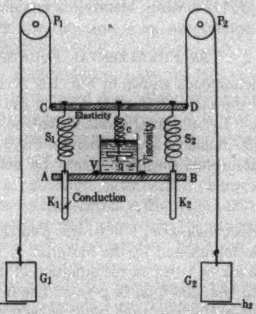

FIG. 3—A MECHANICAL ANALOG OF AN IMPERFECT SOLID DIELECTRIC

If the stress be suddenly changed before the final stage has been reached, the additional stress put on C depends upon the position of the plate q. Thus, the amount of absorption depends upon the previous history of the sample.

If the weights be suddenly removed (corresponding to a short-circuit), there will be an instant position of equilibrium at which the tension of the springs S_1 and S_2 will balance the compression of the spring C. Then, gradually, the bar $C\,D$ will go down to a final position of equilibrium. If, however, the short circuit be removed before the final position has been reached, and the bar $C\,D$ locked (to correspond to an open circuit) a readjustment of stresses will take place, with a residual charge. Thus, later, upon releasing $C\,D$, another movement will take place without an application of the weights.

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[2] *Annalen der Physik*, 1913, Vol. 40, p. 817. Wagner's work is based on earlier investigations by Pellat, von Schweidler, etc., references to whose writings will be found in his article.

[3] Electricity and Magnetism, Vol. 1, p. 452.

[4] Archiv für Elektrotechnik, 1923, Vol. 12, p. 96.

The influence of temperature can be taken into account by changing the viscosity of the liquid in V; for instance, by heating it.

Of course, this analog cannot be followed in detail, but it may facilitate an understanding of the general nature of phenomena taking place in an imperfect dielectric. The writer has also designed a kinematic linkage with a spring and a dashpot, to illustrate the ideas discussed in the above mentioned article by Grünewald. See also Maxwell's model, *ibid.*, p. 462.

2. THE FUNDAMENTAL EQUATIONS

The phenomenon shown in Fig. 1 is primarily determined by the ratio of D_0 to D_f and by the shape of the curve $a\,n$. Let the final displacement be $1 + k$ times the original displacement. Then, in Fig. 1, $a\,b = k\,D_0$, and the *absorption coefficient* k may be defined by the equation

$$k = (D_f - D_0)/D_0 \tag{5}$$

For a perfect dielectric, $k = 0$; the greater k, the more absorption manifested by the material.

Assume $a\,b = 1$ and let the equation of the curve $a\,n$, referred to the axis $b\,m$, be $\phi\,(t)$. The function $\phi\,(t)$ may be called the *relaxation function,* as it characterizes the rate at which the absorbed charge is being released with the time. Let κ_0 be the *initial permittivity* of the material; that is, the permittivity that would be obtained from a ballistic test at the first instant of voltage application. In other words, let

$$D_0 = \kappa_0\,G \tag{6}$$

and

$$D_f = (1 + k)\,\kappa_0\,G \tag{7}$$

At any instant, t, the *deficiency* $D' = w\,s$ between the final displacement D_f and the actual displacement D is

$$D' = k\,D_0\,\phi\,(t) = k\,\kappa_0\,G\,\phi\,(t) \tag{8}$$

Hence

$$D = D_f - D' = \kappa_0\,G\,[(1 + k) - k\,\phi\,(t)] \tag{9}$$

According to Eq. (4),

$$u = -\,\kappa_0\,\kappa\,G\,d\,\phi\,(t)/d\,t \tag{10}$$

In these equations, the function $\phi\,(t)$ is as yet arbitrary, except that it must correspond to a curve the general shape of which is that of the curve $a\,n$, and must satisfy the terminal conditions

$$\phi\,(o) = 1 \text{ and } \phi\,(\infty) = 0 \tag{11}$$

The factor k may be expressed through the *final permittivity* of the material, κ_f, as follows: After the sample has been subjected to a constant d-c. voltage for a long time, we have, by analogy with Equation (6):

$$D_f = \kappa_f\,G \tag{12}$$

so that

$$(D_f - D_0)/D_0 = (\kappa_f - \kappa_0)/\kappa_0 \tag{13}$$

Comparing this expression with equation (5), it will be seen that

$$k = (\kappa_f - \kappa_0)/\kappa_0 \tag{14}$$

and

$$\kappa_0\,k = \Delta\,\kappa = \kappa_f - \kappa_0 \tag{14a}$$

where $\Delta\,\kappa$ is the apparent increase in the permittivity, due to absorption.

The next step is to generalize Equations (9) and (10) for the case when the applied voltage is not constant but varies with the time in a prescribed manner. Let us begin with the simplest case when a constant stress G_1 (Fig. 4) is applied from $t = 0$ to $t = \tau$, and is then increased by an amount G_2. The new stress, $G_1 + G_2$, may be assumed to last indefinitely. The flux density curve, $a\,w$, is similar to that in Fig. 1 as long as G_1 is acting alone. At the instant of application of G_2 the flux density is suddenly increased by an amount $w\,w'$ and then continues to increase asymptotically towards a new limit $b'\,m'$.

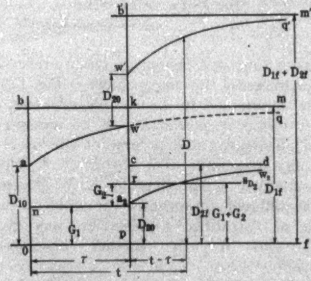

FIG. 4—VARIATIONS IN THE DIELECTRIC FLUX DENSITY

When a constant d-c. voltage is applied in two steps

It is impossible to deduce, theoretically, the relationship between the displacement $p\,w$, existing at the instant τ and the resultant displacement $p\,w'$ an instant later, due to a suddenly increased voltage. We shall therefore make the simplest assumption; namely, that the added voltage G_2 produces its own displacement curve, just as if G_1 did not exist, and that the displacements due to G_1 and G_2 are simply added together. According to this principle of superposition (due to Hopkinson), the curve $a\,w$ continues in the range $w\,q$ as if G_2 did not exist, while the stress G_2 causes a new curve, $a_2\,w_2$, the initial ordinate of which is $D_{20} = \kappa_0\,G_2$, and the final value is $D_{2f} = (1 + k)\,\kappa_0\,G_2$; Equations (**6**) and (**7**). The ordinates of the curve $a_2\,w_2$ added to those of $w\,q$ will give the curve of resultant displacement, $w'\,q'$. This is a mathematical hypothesis, justified only by the possibility of deducing on this basis a set of formulas which can be made to fit observed curves to a sufficient degree of accuracy. From a physical point of view, we know that the a fibers (Fig. 2) satisfy the law of superposition because, in a perfect condenser, the charge is strictly proportional to the voltage. The c fibers also satisfy the law of superposition in that a current is proportional to the applied voltage. Therefore, it is only necessary to grant that the viscous or b fibers also obey this law. In a viscous fluid, at low velocities, the velocity is proportional to the applied force, so that the superposition hypothesis seems reasonable for these fibers as well.

Thus, for any instant $t > \tau$, we may write

$$D = \kappa_0\,(1 + k)\,(G_1 + G_2) - \kappa_0\,k\,[G_1\,\phi\,(t) + G_2\,\phi\,(t - \tau)] \qquad \textbf{(15)}$$

Let now the total stress G_t, which exists at the instant t, be gradually built up in small increments $\Delta\,G_\tau$, each of which is applied at a different instant τ. Generalizing Equation (**15**), we get

$$D = \kappa_0\,(1 + k)\,G_t - \Delta\,\kappa\,\Sigma\,\Delta\,G_\tau\,\phi\,(t - \tau) \qquad \textbf{(16)}$$

where

$$\Sigma\,\Delta\,G_\tau = G_t \qquad \textbf{(17)}$$

and $\Delta\,\kappa$ is used in place of $\kappa_0\,k$, according to equation (**14a**).

In the limit, let the applied voltage vary continuously; then, Equation (**16**) becomes

$$D = \kappa_0\,(1 + k)\,G_t - \Delta\,\kappa \int_{t_0}^{t} \left(\frac{d\,G_\tau}{d\,\tau}\right) \phi\,(t - \tau)\,d\,\tau \qquad \textbf{(18)}$$

where t_0 is the instant at which the voltage is first applied to the sample. In changing from Equation (**16**) to Equation (**18**), $\Delta\,G_\tau$ is divided and multiplied by $\Delta\,\tau$, and, in the limit, Δ is replaced by d.

In Equation (**18**), ϕ is an assumed function, while G_τ is the impressed stress. It is more convenient to have a derivative of ϕ than of G_τ. Integrating in parts, we get

$$\int_{t_0}^{t} \left(\frac{d\,G_\tau}{d\,\tau}\right) \phi\,(t - \tau)\,d\,\tau = G_\tau\,\phi\,(t - \tau)\Big|_{t_0}^{t} - \int_{t_0}^{t} G_\tau\,d_\tau\,\phi\,(t - \tau) \qquad \textbf{(19)}$$

But

$$G_\tau\,\phi\,(t - \tau)\Big|_{t_0}^{t} = G_t\,\phi\,(o) - G_{t0}\,\phi\,(t - t_0) \qquad \textbf{(20)}$$

According to Equation (**11**), $\phi\,(0) = 1$; moreover, $G_t = 0$, since, by assumption, t_0 is the instant at which the voltage begins to be applied. Hence, the value of the right-hand side of Equation (**20**) is simply G_t. Using this value in Equation (**19**) and substituting the value of the integral in Equation (**18**), gives

$$D = \kappa_0\,G_t + \Delta\,\kappa \int_{t_0}^{t} G_\tau\,d_\tau\,\phi\,(t - \tau) \qquad \textbf{(21)}$$

Using this expression for D in Equation (**4**), we get the following expression for the current density:

$$u = \kappa_0\left(\frac{d\,G_t}{d\,t}\right) + \Delta\,\kappa \int_{t_0}^{t} G_\tau\left[\frac{d^2\,\phi\,(t - \tau)}{d\,t\,d\,\tau}\right] d\,\tau + \Delta\,\kappa\,G_t\left[\frac{d\,\phi\,(t - \tau)}{d\,\tau}\right]_{\tau = t} \qquad \textbf{(22)}$$

When taking a total derivative of the definite integral in Equation (**21**), with respect to t, two terms are obtained; viz., one assuming t to be a parameter under the sign of integral, and the other assuming t to be a variable upper limit. Equations (**21**) and (**22**) are the fundamental equations in the theory under consideration. In so far as no particular form has been ascribed to the function ϕ, these equations are perfectly general. The next step is to specify the function ϕ in accordance with the observed behavior of solid dielectrics.

3. An Exponential Law of Relaxation

In Fig. 1, the simplest assumption which can be made regarding the shape of the curve, $a\,n$, is that the "deficiency" $w\,s$ is an exponential function of time; that is

$$\phi\,(t) = \epsilon^{-\alpha t} \qquad \textbf{(23)}$$

where α is a *relaxation factor* which characterizes the sample under consideration. Equation (**23**) proposed by Pellat[5], satisfactorily represents the general shape of the curve $a\,n$ and also satisfies Equation (**11**). On the other hand, substances which show considerable absorption are usually too complicated to be characterized by a single parameter α. Therefore, von Schweidler proposed a generalization of the foregoing formula to

$$\phi\,(t) = (\Sigma\,N\,\epsilon^{-\alpha t})/\Sigma\,N \qquad \textbf{(24)}$$

Here the material is supposed to be heterogeneous and to consist of N_1 particles of constant α_1, N_2 particles of constant α_2, etc.

Wagner has further extended this idea and proposed for the relaxation function an expression of the form

$$\phi\,(t) = \int_{0}^{\infty} F\,(\alpha)\,\epsilon^{-\alpha t}\,d\,\alpha \qquad \textbf{(25)}$$

the integration being with respect to α, and not t. His idea was that a piece of heterogeneous dielectric may have some particles with a relaxation constant $\alpha = 0$ (perfect dielectric, no relaxation) and others with $\alpha = \infty$ (conducting particles, instant relaxation). For most particles, α lies between zero and infinity, and the number of particles which have a definite value of α is a function of α. In other words, for a fairly uniform material, α ranges within rather narrow limits for a great majority of particles, and there are comparatively few particles with very large or small values of α, due to admixtures and impurities. A specific form of $F\,(\alpha)$

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[5] **For literature references see the above mentioned article by Wagner.**

will be considered in the next section. We shall first deduce some general expressions which hold true for any function of α. The function $F(\alpha)$ may be called the distribution function, because it shows the relative number of particles having different values of α.

From the physical significance of $F(\alpha)$, we judge that this function remains finite between $\alpha = 0$ and $\alpha = \infty$. Therefore the second Equation (**11**) is satisfied. In order that the first Equation (**11**) be satisfied, the function $F(\alpha)$ must satisfy the condition

$$\int_0^\infty F(\alpha)\, d\alpha = 1 \qquad \textbf{(26)}$$

We shall now apply expression (**25**) to equations (**21**) and (**22**) and deduce some special cases.

General Case of Variable Voltage.

Equation (**25**) gives

$$\phi(t-\tau) = \int_0^\infty F(\alpha)\, \epsilon^{-\alpha(t-\tau)}\, d\alpha \qquad \textbf{(27)}$$

so that

$$d_\tau\, \phi(t-\tau) = \int_0^\infty \alpha F(\alpha)\, \epsilon^{-\alpha(t-\tau)}\, d\alpha\, d\tau \qquad \textbf{(28)}$$

and

$$[d^2 \phi(t-\tau)/d t\, d\tau]\, d\tau = -\int_0^\infty \alpha^2 F(\alpha) \epsilon^{-\alpha(t-\tau)}\, d\alpha\, d\tau \qquad \textbf{(29)}$$

Substituting expression (**28**) in Equation (**21**), gives

$$D = \kappa_0 G_t + \Delta\kappa \int_{t_0}^{t} G_\tau\, d\tau \int_0^\infty \alpha F(\alpha)\, \epsilon^{-\alpha(t-\tau)}\, d\alpha \qquad \textbf{(30)}$$

The integration with respect to time is independent of that with respect to α. Changing, therefore, the order of integration, the foregoing equation becomes

$$D = \kappa_0 G_t + \Delta\kappa \int_0^\infty \alpha F(\alpha)\, \epsilon^{-\alpha t} \left[\int_{t_0}^{t} G_\tau\, \epsilon^{\alpha\tau}\, d\tau \right] d\alpha \qquad \textbf{(31)}$$

Substituting expressions (**28**) and (**29**) in Equation (**22**), gives

$$u = \kappa_0 \frac{d G_t}{d t} - \Delta\kappa \int_{t_0}^{t} \int_0^\infty G_\tau\, \alpha^2 F(\alpha)\, \epsilon^{-\alpha(t-\tau)}\, d\alpha\, d\tau$$

$$+ \Delta\kappa\, G_t \int_0^\infty \alpha F(\alpha)\, d\alpha \qquad \textbf{(32)}$$

Equation (**32**) may also be deduced directly from Equation (**31**) by differentiation; see Equation (**4**).

Continuous Constant Voltage. In this case, $G_t = G_\tau = G$; assume also $t_0 = 0$. We then have for the second integral in Equation (**31**)

$$\int_{t_0}^{t} G_\tau\, \epsilon^{\alpha\tau}\, d\tau = G \int_0^t \epsilon^{\alpha\tau}\, d\tau = \left(\frac{G}{\alpha}\right)(\epsilon^{\alpha t} - 1) \qquad \textbf{(33)}$$

Therefore, the first integral in Equation (**31**) becomes

$$G \int_0^\infty F(\alpha)\, \epsilon^{-\alpha t}\, (\epsilon^{\alpha t} - 1)\, d\alpha$$

$$= G \int_0^\infty F(\alpha)\, d\alpha - G \int_0^\infty F(\alpha)\, \epsilon^{-\alpha t}\, d\alpha \qquad \textbf{(34)}$$

According to Equation (**26**), the value of the first integral on the right hand side is 1. Substituting from Equation (**34**) in Equation (**31**) and dividing by G, gives

$$\kappa_t = \frac{D}{G} = (\kappa_0 + \Delta\kappa) - \Delta\kappa \int_0^\infty F(\alpha)\, \epsilon^{-\alpha t}\, d\alpha \qquad \textbf{(35)}$$

Here κ_t may be considered to be the *apparent permittivity* of the sample at the instant t. When $t = 0$, $\kappa_t = \kappa_0$; when $t = \infty$, $\kappa_t = \kappa_0 + \Delta\kappa = \kappa_f$.

The current density u may be obtained directly by differentiating the right-hand side of Equation (**35**) with respect to t, and multiplying the result by G. We then get

$$u = \Delta\kappa\, G \int_0^\infty \alpha F(\alpha)\, \epsilon^{-\alpha t}\, d\alpha \qquad \textbf{(36)}$$

Let γ be the final conductivity of the sample, that is, the conductivity of the fibers c in Fig. 2. Then the *apparent conductivity* at the instant t is

$$\gamma_t = \frac{u}{G} + \gamma_f \qquad \textbf{(37)}$$

The first term on the right-hand side corresponds to the displacement current and the second to the real conduction current. Substituting in Equation (**37**) the value of u from Equation (**36**), gives

$$\gamma_t = \Delta\kappa \int_0^\infty \alpha F(\alpha)\, \epsilon^{-\alpha t}\, d\alpha + \gamma_f \qquad \textbf{(38)}$$

The initial conductivity, at $t = 0$, is

$$\gamma_0 = \Delta\kappa \int_0^\infty \alpha F(\alpha)\, d\alpha + \gamma_f \qquad \textbf{(39)}$$

Thus the function $F(\alpha)$ must satisfy the condition

$$\int_0^\infty \alpha F(\alpha)\, d\alpha = \frac{\gamma_0 - \gamma_f}{\kappa_f - \kappa_0} \qquad \textbf{(40)}$$

where the four quantities on the right-hand side characterize the material.

Alternating Voltage. For a sinusoidal applied voltage we can generally put

$$G_\tau = G_m\, \epsilon^{j\omega\tau} \qquad \textbf{(41)}$$

More particularly, at the instant t,

$$G_t = G_m\, \epsilon^{j\omega t} \qquad \textbf{(41a)}$$

where G_m is the amplitude of the stress and ω its frequency. It is convenient to put $t_0 = -\infty$ because then the transient term disappears. The second integral in Equation (**31**) becomes

$$G_m \int_{-\infty}^{t} \epsilon^{(j\omega+\alpha)\tau}\, d\tau = \frac{G_m\, \epsilon^{(j\omega+\alpha)t}}{(j\omega + \alpha)} \qquad \textbf{(42)}$$

so that, using in Equation (**31**) the value of G_t from Equation (**41a**), we get

$$D = G_m\, \epsilon^{j\omega t} \left[\kappa_0 + \Delta\kappa \int_0^\infty \frac{\alpha F(\alpha)}{(j\omega + \alpha)}\, d\alpha \right] \qquad \textbf{(43)}$$

Multiplying the numerator and the denominator of this equation by $(\alpha - j\omega)$, and separating the real from the imaginary part, we get

$$D = G_m\, \epsilon^{j\omega t} \left[\kappa_0 + \Delta\kappa \int_0^\infty \frac{\alpha^2 F(\alpha)\, d\alpha}{(\alpha^2 + \omega^2)} \right.$$

$$- j\,\omega\,\Delta\,\kappa \int_0^\infty \frac{\alpha\,F(\alpha)\,d\,\alpha}{(\alpha^2+\omega^2)} \Bigg] \tag{44}$$

The sample under consideration may be thought of as a combination of an ideal capacitance, C, in parallel with a conductance, g. The latter is *not* the conductance due to leakage, but the apparent conductance caused by absorption. For such a combination, the current

$$I = E\,\epsilon^{j\omega t}\,(j\,\omega\,C + g) \tag{45}$$

Consequently, the charge varies according to the law

$$Q = E\,\epsilon^{j\omega t}\left(C - \frac{j\,g}{\omega}\right) \tag{46}$$

because a derivative of Equation (**46**) with respect to t gives Equation (**45**). Equation (**44**) corresponds to Equation (**46**) for unit volume of the dielectric. Therefore, comparing the two equations term by term, we find that

$$\kappa_\omega = \kappa_0 + \Delta\,\kappa \int_0^\infty \frac{\alpha^2\,F(\alpha)\,d\,\alpha}{(\alpha^2+\omega^2)} \tag{47}$$

$$\gamma_\omega = \Delta\,\kappa\,\omega^2 \int_0^\infty \frac{\alpha\,F(\alpha)\,d\,\alpha}{(\alpha^2+\omega^2)} + \gamma_f \tag{48}$$

In the latter expression, the term γ_f is added to account for the actual leakage conductivity (fibers c in Fig. 2). Thus, with alternating voltages, a dielectric with absorption shows a permittivity and a conductivity both of which are functions of the frequency. Equations (**47**) and (**48**) show that the relationship between κ_ω, γ_ω, and ω is quite involved. Nevertheless, it is this relationship that should permit to compare the theory with observed results.

Differentiating Equation (**44**), or directly on the basis of Equation (**45**), we may write for the current density u:

$$u = G_m\,\epsilon^{j\omega t}\,(j\,\omega\,\kappa_\omega + \gamma_\omega) \tag{49}$$

where κ_ω and γ_ω are determined by Equations (**47**) and (**48**).

In Equation (**47**), κ_0, by definition, is the initial or instant permittivity; therefore, it is the permittivity which the dielectric should show at an infinite frequency. In fact, by putting $\omega = \infty$, Equation (**47**) gives

$$\kappa_\infty = \kappa_0 \tag{50}$$

At zero frequency, that is, when $\omega = 0$, Equation (**47**) gives

$$\kappa_{\omega=0} = \kappa_1 + \Delta\,\kappa = \kappa_f \tag{51}$$

which also agrees with the assumptions made above.

The variable term in Equation (**48**) becomes equal to zero both at $\omega = 0$ and at $\omega = \infty$. This agrees with the physical conception of this term, as giving the apparent time effect of absorption. The power loss per unit volume is $G_{eff}^2\,\gamma_\omega$. Subtracting from this the true conduction loss $G_{eff}^2\,\gamma_f$, gives the dielectric loss per unit volume at frequency ω:

$$P_\omega = G_{eff}^2\,(\gamma_\omega - \gamma_f) \tag{52}$$

Thus, the observed power loss corrected for true conduction and plotted against ω as abscissas gives some information in regard to the shape of the function $F(\alpha)$.

For the circuit represented by Equation (**45**), we have

$$\tan\psi = \frac{g}{\omega\,C} \tag{53}$$

where the *loss* or *imperfection angle* ψ is complementary to the phase angle between the voltage and the current. Knowing ψ, the value of sin ψ can be found, and this value is the power factor of the condenser. Applying Equation (**53**) to our case, we have

$$\tan\psi_\omega = \gamma_\omega/(\omega\,\kappa_\omega) \tag{54}$$

where κ_ω and γ_ω are determined by Equations (**47**) and (**48**). Knowing ψ_ω, we find,

$$\text{Power factor} = \text{Sin}\,\psi_\omega \tag{55}$$

Dielectric loss increases considerably with temperature[6]. In the present investigation, the temperature of the sample is assumed to be the same throughout its thickness and, in checking experimental data with theoretical formulas, it is of importance to see that this condition has been at least approximately fulfilled during the tests.

Transient Voltages. The theory developed above for an established direct voltage may be extended to include a short d-c. impulse, or a combination of such impulses, by using the principle of superposition. For example, if a voltage E is applied at the instant t_1 and removed at the instant t_2, the conditions are the same as if the voltage E were continued from $t = t_1$ to $t = \infty$ and a voltage $-E$ applied from $t = t_2$ to $t = \infty$. It is also possible that the Heavisidian operational calculus may be applied to the solution of this problem.

If the applied transient voltage is of the nature of a decremental sine wave, Equation (**41**) may be used for G_τ, in which ω is a complex quantity. The lower limit of τ is no longer $-\infty$ and Equation (**42**) has to be integrated anew. Of course, Equations (**31**) and (**32**) can be used directly for any transient voltage provided that G_τ is given as a function of τ.

4. A DISTRIBUTION FUNCTION $F(\alpha)$ AND THE CONDITIONS WHICH ITS PARAMETERS MUST SATISFY

A reasonable shape of the function $F(\alpha)$ for a fairly homogeneous dielectric is shown in Fig. 5. A large number of particles have values of α within the range near the maximum of the curve. There are fewer and fewer particles as one chooses values of α more remote from this range. The curve passes through the origin and may or may not be tangent to the axis of abscissas there; it should approach the axis of abscissas asymptotically at infinity. Probably several analytical functions would give a curve of this shape, and the selection of a particular function, in so far as there are no more valid theoretical grounds, is mainly determined by the following considerations:

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[6] See, for example, L. Dreyfus, Mathematische Theorien für den Durchschlag fester Isoliermaterialien; *Schweiz. Elektrot. Verein, Bulletin*, 1924, Vol. 15, p. 321.

a. General simplicity.

b. Ease in the integration of various expressions such as Equations (**38**), (**47**), etc.

c. A sufficient number of parameters to fit the observed behavior of a given dielectric.

As a fundamental empirical expression, we shall take

$$F(\alpha) = H\,\alpha^{m-1}\,\epsilon^{-n\alpha} \qquad \textbf{(56)}$$

where H, m, and n, are constants independent of α. These constants are different for different materials and, for a given material, are functions of temperature.

The physical dimension of $\phi(t)$ is a numeric; see its definition, Equation (**8**). According to Equation (**52**), the physical dimension of α is (time)$^{-1}$, because the product $\alpha\,t$ must be a numeric. Consequently, the physical dimension of $F(\alpha)$ must be "time," because the product $F(\alpha)\,d\,\alpha$ must be a numeric. In Equation (**56**), $n\,\alpha$ must be a numeric, hence the dimension of n is "time." The exponent m is a numeric, and the dimension of H is (time)^m. It will be seen later that H is eliminated from the final computations.

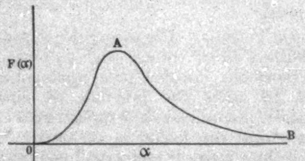

FIG. 5—THE GENERAL CHARACTER OF THE DISTRIBUTION FUNCTION $F(\alpha)$

Taking a derivative of $F(\alpha)$ with respect to α, we get

$$d\,F(\alpha)/d\,\alpha = H\,\alpha^{m-2}\,\epsilon^{-n\alpha}\,(m-1-n\,\alpha) \qquad \textbf{(57)}$$

With the aid of Equations (**56**) and (**57**), the limiting values of m and n may be determined as follows:

(a) As α approaches zero, $\epsilon^{-n\alpha}$ approaches unity for any finite value of n. Hence, α^{m-1} should approach zero in order that the curve pass through the origin. This gives the first limiting condition, namely, $m > 1$. But for values of m less than 2, Equation (**57**) gives infinite values of the slope, showing that the curve is tangent to the axis of ordinates. This does not agree with the assumed character of the phenomenon. At $m = 2$, the value of the right-hand side of Equation (**57**) is H.

This may be accepted (at least provisionally) as a possible value, so that

$$m \geqq 2 \qquad \textbf{(58)}$$

(b) At infinity, the term α^{m-1} tends to an infinitely great value, while $\epsilon^{-n\alpha}$ tends to zero, provided that n is positive. Moreover, since an exponential expression ϵ^{α} tends to infinity at a higher rate than any positive power of α, the ratio $\alpha^{m-1}/\epsilon^{n\alpha}$ approaches zero when $\alpha = \infty$.

Equation (**57**) gives a zero slope at $\alpha = \infty$. Thus, the second limiting condition is

$$n > 0 \qquad \textbf{(59)}$$

(c) The last factor in Expression (**57**) becomes zero when

$$\alpha = (m-1)/n \qquad \textbf{(60)}$$

At this value of α, the function $F(\alpha)$ reaches its maximum. Equation (**60**) may have some bearing upon the selection of values of m and n, or upon the interpretation of experimentally obtained values.

(d) Substituting expression (**56**) in Equation (**26**) and integrating, we get[7]

$$H \int_0^\infty \alpha^{m-1}\,\epsilon^{-n\alpha}\,d\,\alpha = H\,\Gamma(m)/n^m = 1 \qquad \textbf{(61)}$$

from which

$$H\,\Gamma(m) = n^m \qquad \textbf{(62)}$$

where Γ stands for the so-called Gamma Function[8].

Thus, the quantities H, m, n, are not entirely independent of one another, but must satisfy equation (**62**).

(e) Substituting expression (**56**) in Equation (**40**) and integrating gives

$$\begin{aligned} H \int_0^\infty \alpha^m\,\epsilon^{-n\alpha}\,d\,\alpha &= H\,\Gamma(m+1)/n^{m+1} \\ &= (\gamma_0 - \gamma_f)/(\kappa_f - \kappa_0) = \Delta\,\gamma/\Delta\,\kappa \end{aligned} \qquad \textbf{(63)}$$

from which

$$\begin{aligned} H\,\Gamma(m+1) &= n^{m+1}\;(\gamma_0 - \gamma_f)/(\kappa_f - \kappa_0) \\ &= n^{m+1}\,\Delta\,\gamma/\Delta\,\kappa \end{aligned} \qquad \textbf{(64)}$$

This condition imposes another limitation upon the choice of the quantities H, m, and n. There is an additional connection between Equations (**62**) and (**64**) because

$$\Gamma(m+1) = m\,\Gamma(m)\;\text{[9]} \qquad \textbf{(65)}$$

Dividing Equation(**64**) by Equation (**62**), and using expression (**65**), we get

$$m/n = (\gamma_0 - \gamma_f)/(\kappa_f - \kappa_0) = \Delta\,\gamma/\Delta\,\kappa \qquad \textbf{(66)}$$

The function (**56**) is different from that chosen by Wagner. He used the so-called probability function for the distribution of values of relaxation time[10]. There is no valid theoretical reason for selecting this function in preference to any other, even for a chemically simple and strictly homogeneous material. With actual substances, of complicated molecular structure and with impurities, such a function is totally inadequate, since it has not a sufficient number of parameters to fit experimental curves. Therefore, Wagner

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[7] B. O. Peirce, Short Table of Integrals, formula 493.

[8] For properties and tables of numerical values of Gamma Function see an advanced treatise on the Integral Calculus, for example B. Williamson's.

[9] Equation (65) may be extended as follows:

$$\begin{aligned} \Gamma(m+1) &= m\,(m-1)\,\Gamma(m-1) \\ &= m\,(m-1)\,(m-2)\;\;\Gamma(m-2) = \text{etc.} \end{aligned} \qquad \textbf{(65a)}$$

[10] Loc. cit. p. 831.

had to form a function consisting of a sum of a "finite number" of probability functions of different parameters, each giving a different most probable value of the relaxation time. However, if one has to resort to such a summation, then any other function of reasonable shape, such as Equation (**56**), will do just as well.

The probability function leads to quite complicated integrations, and Equation (**56**) offers considerable advantages in this respect, although two of the integrals also lead to infinite series when m is not an integer. In any event, it seems advisable to carry the theory through on the basis of Equation (**56**), for such practical applications and interpretations of tests as it may lead to.

5. FOUR INTEGRALS CONTAINING $F(\alpha)$

In order to apply Equation (**56**) to the general theory given in Section 3 above, it is necessary to evaluate the following four integrals:

$$A = \int_0^\infty F(\alpha)\, \epsilon^{-\alpha t}\, d\,\alpha \qquad \textbf{(67)}$$

$$B = \int_0^\infty \alpha\, F(\alpha)\, \epsilon^{-\alpha t}\, d\,\alpha \qquad \textbf{(68)}$$

$$V = \int_0^\infty \alpha\, F(\alpha)\, d\alpha/(\alpha^2 + \omega^2) \qquad \textbf{(69)}$$

$$W = \int_0^\infty \alpha^2 F(\alpha)\, d\alpha/(\alpha^2 + \omega^2) \qquad \textbf{(70)}$$

These integrals enter in the following equations: (**35**), (**36**), (**38**), (**44**), (**47**), and (**48**); indirectly, they enter in some other expressions derived from these. Substituting the value of $F(\alpha)$ from Equation (**56**) in Equation (**67**) and using formula 493 in Peirce's Table, we get:

$$A/H = \int_0^\infty \alpha^{m-1}\, \epsilon^{-\alpha(n+t)}\, d\,\alpha = \Gamma\,(m)/(n+t)^m \qquad \textbf{(71)}$$

Similarly,

$$B/H = \Gamma\,(m+1)/(n+t)^{m+1} \qquad \textbf{(72)}$$

The V integral becomes

$$V/H = \int_0^\infty \alpha^m\, \epsilon^{-\alpha n}\, d\,\alpha/(\alpha^2 + \omega^2) \qquad \textbf{(73)}$$

Put

$$m = p + q \qquad \textbf{(74)}$$

where p is the nearest smaller *even* integer and q is a regular or irregular positive fraction smaller than 2. For example, if $m = 5.72$, then $p = 4$ and $q = 1.72$. By long division, we obtain:

$$\alpha^m/(\alpha^2+\omega^2) = \alpha^{m-2} - \omega^2\,\alpha^{m-4} + \omega^4\,\alpha^{m-6} - \text{etc.}$$
$$\pm\, \omega^{p-2}\,\alpha^q \mp \omega^p\,\alpha^q\,/(\alpha^2+\omega^2) \qquad \textbf{(75)}$$

Therefore, Equation (**73**) becomes

$$V/H = \Gamma\,(m-1)/n^{m-1} - \omega^2\,\Gamma\,(m-3)/n^{m-3}$$
$$+\,\omega^4\,\Gamma\,(m-5)/n^{m-5} - \text{etc.} \pm \omega^{p-2}\,\Gamma\,(q+1)/n^{q+1}$$
$$\mp\, \omega^p\, R\,(q, n, \omega) \qquad \textbf{(76)}$$

where the function

$$R\,(q, n, \omega) = \int_0^\infty \alpha^q\, \epsilon^{-n\alpha}\, d\,\alpha/(\alpha^2 + \omega^2) \qquad \textbf{(77)}$$

Factoring out the first term on the right-hand side of Equation (**76**) and keeping in mind the general relationship (**65a**), we get

$$V/H = [\Gamma\,(m-1)/n^{m-1}]\,[1 - \theta^2/\{(m-1)\,(m-2)\}$$
$$+\, \theta^4/\{(m-1)\,(m-2)\,(m-3)\,(m-4)\} - \text{etc.}$$
$$\pm\, \theta^{p-2}/\{(m-1)\,.\;.\;.\;.\;(q+2)\}] \mp \omega^p\, R\,(q, n, \omega) \qquad \textbf{(78)}$$

where, for the sake of abbreviation, the product $\omega\, n$ is denoted by θ, so that

$$\theta = \omega\, n \qquad \textbf{(79)}$$

By analogy, from Equations (**70**) and (**79**),

$$W/H = [\Gamma\,(m)/n^m]\,[1 - \theta^2/\{m\,(m-1)\}$$
$$+\,\theta^4\,/\{m\,(m-1)\,(m-2)\,(m-3)\} - \text{etc.}$$
$$\pm\, \theta^{p^1-2}/\{m\,.\;.\;.\;.\;(q^1+2)\}] \mp \omega^{p^1}\, R\,(q^1, n, \omega) \qquad \textbf{(80)}$$

where $R\,(q^1, n, \omega)$ is again expressed by Equation (**77**), but for another value of q. In Equation (**80**) we put

$$m + 1 = p^1 + q^1 \qquad \textbf{(81)}$$

where p^1 is the nearest smaller *even* integer to $m + 1$; q^1 is the remainder. Let for example $m = 8.2$. Then

$p = 8 \quad q = 0.2$
$p^1 = 8 \quad q^1 = 1.2$

On the other hand, let $m = 9.2$. Then

$p = 8 \quad q = 1.2$
$p^1 = 10 \quad q^1 = 0.2$

Thus, p^1 is either equal to p or is greater than p by 2. The remainder, q^1, may be either greater or smaller than q by 1.

For a complete evaluation of the functions V and W, it is necessary to derive an expression for the function $R\,(q\;n, \omega)$ given by the integral in Equation (**77**). A solution in finite form can be obtained only when m is an integer, and this case will be considered separately.

Case I, m is an integer. When m is an integer, q can have only one of the two values, 0 or 1. Therefore, for m *even*

$$R_e\,(q, n, \omega) = \int_0^\infty \epsilon^{-n\alpha}\, d\,\alpha/(\alpha^2 + \omega^2) \qquad \textbf{(82)}$$

and for m *odd*

$$R_0\,(q, n, \omega) = \int_0^\infty \alpha\, \epsilon^{-n\alpha}\, d\,\alpha/(\alpha^2 + \omega^2) \qquad \textbf{(83)}$$

Integrated, this gives[11]

$$R_e\,(q, n, \omega) = \frac{1}{\omega}\,[\sin\theta\, C\,i\,\theta$$
$$+ \cos\theta\,(0.5\,\pi - S\,i\,\theta)] \qquad \textbf{(84)}$$

$$R_0\,(q, n, \omega) = -\cos\theta\, C\,i\,\theta + \sin\theta\,(0.5\,\pi - S\,i\,\theta) \qquad \textbf{(85)}$$

where $\theta = n\,\omega$ according to Equation (**79**), and the symbols $S\,i$ and $C\,i$ stand for the functions known as the *integral sine* and *integral cosine* respectively. In numerical applications, $R_e\,(q, n, \omega)$ and $R_0\,(q, n, \omega)$ may be computed in one of the following three ways:

(a) By using Glaisher's tables of $S\,i\,\theta$ and $C\,i\,\theta$[12]

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[11] Bierens de Haan, *Noubelles Tables d' Intégrales Définies*, Leyden, 1867, p. 133, formulas (**7**) and (**8**).

[12] Royal Society of London, *Philosoph. Trans.* 1870, Vol. 160, p. 367.

(b) By using Steinmetz' tables[13] for col θ and sil θ, where

$$\text{col}\,\theta = 0.5\,\pi - S\,i\,\theta \tag{86}$$

$$\text{sil}\,\theta = -\,C\,i\,\theta \tag{87}$$

Equations (**84**) and (**85**) then become

$$R_e\,(q, n, \omega) = \frac{1}{\omega}\,(-\sin\theta\,\text{sil}\,\theta + \cos\theta\,\text{col}\,\theta) \tag{88}$$

$$R_0\,(q, n, \omega) = \cos\theta\,\text{sil}\,\theta + \sin\theta\,\text{col}\,\theta \tag{89}$$

(c) If the foregoing expressions for $R_e\,(q, n, \omega)$ and $R_0\,(q, n, \omega)$ are to be used to any considerable extent, it may save time to prepare tables or curves of $\omega\,R_e\,(q, n, \omega)$ and $R_0\,(q, n, \omega)$, within the desired range of values of $\theta = n\,\omega$.

Case II, m is not an integer. Equation (**77**) can be integrated only in the form of an infinite series. The details of transformations are as follows: Let x be an auxiliary variable; then we may write the following identity:

$$\int_0^\infty \epsilon^{-\alpha x}\cos\omega\,x\,d\,x = \alpha/(\alpha^2 + \omega^2) \tag{90}$$

This expression may be derived from Equation (**420**) in Peirce's tables by putting $n = 1$ and substituting the limits. Using Equation (**90**) in Equation (**77**), we get

$$R\,(q, n, \omega) = \int_0^\infty \alpha^{q-1}\,\epsilon^{-n\alpha}\,[\int_0^\infty \epsilon^{-\alpha x}\cos\omega\,x\,d\,x]\,d\,\alpha \tag{91}$$

Since the integrations with respect to α and with respect to x are independent of each other, their order may be changed and Equation (**91**) rewritten in the form

$$R\,(q, n, \omega) = \int_0^\infty \cos\omega\,x\,[\int_0^\infty \alpha^{q-1}\,\epsilon^{-(n+x)\alpha}\,d\,\alpha]\,d\,x \tag{92}$$

But

$$\int_0^\infty \alpha^{q-1}\,\epsilon^{-(n+x)\alpha}\,d\,\alpha = \Gamma\,(q)/(n + x)^q \tag{93}$$

See, for example, Peirce, formula (**493**). Therefore, Equation (**93**) becomes

$$R\,(q, n, \omega)/\Gamma\,(q) = \int_0^\infty \cos\omega\,x\,d\,x/(n + x)^q \tag{94}$$

In order to expand the right-hand side of this expression, we write the identity

$$\begin{aligned}\int \cos\omega\,x\,(n + x)^{-q}\,d\,x &= \omega^{-1}\sin\omega\,x\,(n + x)^{-q} \\ &- \omega^{-2}\,q\cos\omega\,x\,(n + x)^{-(q+1)} \\ &- \omega^{-2}\,q\,(q + 1)\int \cos\omega\,x\,(n + x)^{-(q+2)}\,d\,x\end{aligned} \tag{95}$$

The right-hand side of this identity has been derived from the left-hand side by integrating the latter in parts and again integrating the result in parts. However, the truth of Equation (**95**) can be readily demonstrated by a direct differentiation. Substituting the limits 0 and ∞, this expression becomes:

$$\begin{aligned}\int_0^\infty \cos\omega\,x\,d\,x/(n + x)^q &= \omega^{-2}\,q\,n^{-(q+1)} \\ &- \omega^{-2}\,q\,(q + 1)\int_0^\infty \cos\omega\,x\,d\,x/(n + x)^{(q+2)}\end{aligned} \tag{96}$$

This means that the integral in Equation (**94**) which contains $(n + x)$ to the power of q may be expressed through a similar integral in which $(q + 2)$ enters in place of q. This process can be continued indefinitely, so that Equation (**96**) is the basis for expanding the right-hand side of Equation (**94**) into an infinite series. Introducing $\theta = n\,\omega$, according to Equation (**79**), we get

$$\begin{aligned}R\,(q, n, \omega) = \Gamma\,(q)\,.\,[q/(\theta^2\,n^{q-1})\,]\,[1 - (q + 1) \\ (q + 2)/\theta^2 + (q + 1)\,(q + 2)\,(q + 3)\,(q + 4)/\theta^4 - \text{etc.}]\end{aligned} \tag{97}$$

This formula is useful only if the series happens to be fairly rapidly converging for the particular desired values of q and θ.

6. Applications of the Monomial Function $F\,(\alpha)$

The function $F\,(\alpha)$, defined by Equation (**56**), makes it possible to express by formulas the apparent permittivity κ_t of an imperfect dielectric, Equation (**35**), and its apparent conductivity, γ_t, Equation (**38**), as functions of time. Should such theoretical expressions, with properly chosen constants, agree with experimental curves for the same material, the next step is to substitute the function $F\,(\alpha)$, with the same constants, in Equations (**47**) and (**48**). Should the observed curves of variation of capacitance and dielectric loss agree at least approximately with the values derived from these formulas, then the monomial form (**56**) may be said to be adequate for that particular material, and may be used to predict its behavior in a range beyond experimental data. However, should there be a contradiction between the computed and measured data with alternating voltages, the simple monomial term and the three parameters which it contains are insufficient to account for the properties of that particular material (because of impurities or of complicated molecular structure). The next step is to use a binomial empirical formula of the form

$$F\,(\alpha) = H_1\,\alpha^{m_1-1}\,\epsilon^{-n_1\alpha} + H_2\,\alpha^{m_2-1}\,\epsilon^{-n_2\alpha} \tag{98}$$

which is an extension of Equation (**56**). This formula contains six parameters, and it would seem that it could be made to fit observed curves of widely differing shapes, unless the accepted theory is fundamentally wrong.

We shall first consider the possibilities of the simple monomial expression (**56**), on the assumption that there are materials for which it is adequate. Substituting Equation (**56**) in Equation (**35**), and using the result (**71**), we get:

$$\kappa_t = (\kappa_0 + \Delta\,\kappa) - H\,\Delta\,\kappa\,\Gamma\,(m)/(n + t)^m \tag{99}$$

or, with the aid of Equation (**62**),

$$\kappa_t = \kappa_f - \Delta\,\kappa\,[n/(n + t)]^m \tag{100}$$

This expression may also be rewritten in the form

$$[n/(n + t)]^m = (\kappa_f - \kappa_t)/\Delta\,\kappa \tag{101}$$

Let experimental data be available from which values of $(\kappa_f - \kappa_t)/\Delta\,\kappa$ may be computed for various values of

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[13] C. P. Steinmetz, *Transient Electric Phenomena*, Third Edition, Appendix.

t. Then values of m and n may be found by trials which satisfy Equation (**101**) to a fair degree of accuracy. These values of m and n must also satisfy Equation (**66**). The simplest method may be to choose for m an integer, such as $m = 2, 3, 4$, etc., to find the corresponding values of n from Equation (**66**), and then try these values in succession in Equation (**101**)[14]. Even if none of the integers fit the curve, the trials will show between which two integers the best value of m lies.

In a similar manner, from Equations (**38**), (**62**), (**64**), and (**72**) we derive

$$\gamma_t = \gamma_f + \Delta \gamma \, [n/(n + t)]^{m+1} \qquad \textbf{(102)}$$

The numerical values of m and n, which have been chosen to satisfy Equations (**66**) and (**101**), should also satisfy Equation (**102**). In fact, the values should be chosen to satisfy as well as possible all the three equations. Should this be impossible, this would be an indication that the simple form of function $F(\alpha)$ is inadequate, and a function of the form given by Equation (**98**) should be tried.

Assuming that values of m and n have been found to satisfy the d-c. curves of conductivity and permittivity, we substitute Equation (**56**) in Equation (**47**), using Equation (**80**). This will give, after simplification

$$\begin{aligned}\kappa_\omega = \kappa_f - \Delta \kappa \, [\theta^2/m\,(m-1)]\,[1 - \theta^2/(m-2)\,(m-3) \\ + \text{etc.} \mp \theta^{p^1-4}/(m-2)\,(m-3) \;.\;.\;.\; (q^1+2)] \\ \mp \Delta \kappa \, \theta^{p^1} n^{q^1-1} R\,(q^1)/\Gamma\,(m) \qquad \textbf{(103)}\end{aligned}$$

Similarly, Equation (**48**) becomes

$$\begin{aligned}\gamma_\omega = \gamma_f + \omega^2 \Delta \kappa \, [\theta^2/n\,(m-1)]\,[1 - \theta^2/(m-1)\,(m-2) \\ + \theta^4/(m-1)\,(m-2)\,(m-3)\,(m-4) - \text{etc.} \pm \\ \theta^{p-2}/(m-1) \;.\;.\;.\; (q+2)] \mp \Delta \kappa \, \theta^p \omega^2 n^q R\,(q)/\Gamma\,(m) \\ \textbf{(104)}\end{aligned}$$

For values of R see Equations (**84**), (**85**), (**88**), (**89**), and (**97**).

In Equations (**103**) and (**104**) the sign $\mp$ should be carefully distinguished from $\pm$. Thus, in Equation (**103**), if the term θ^{p^1-4} comes out with the minus sign, the term θ^{p^1} should also have a minus sign, and vice versa. On the other hand, in Equation (**104**) the terms θ^{p-2} and θ^p should have opposite signs.

From Equations (**103**) and (**104**) the values of κ_ω and γ_ω may be computed for the frequencies for which experimental data on permittivity and dielectric loss are available. This permits to determine if the simple formula (**56**) applies to the a-c. dielectric phenomena in the material in question.

The experimental data are supposed to consist of the charging current and dielectric loss at various frequencies. The capacitance of the condenser is then determined from the formula

$$I = \omega\, C\, E \qquad \textbf{(105)}$$

and knowing C and the dimensions of the sample, κ_ω may be computed. Knowing the dielectric loss per unit volume and the stress to which the sample has been subjected, the experimental value of γ_ω may be computed from Equation (**52**), knowing the constant final conductivity γ_f with d-c. voltage.

The author wishes to acknowledge gratefully the assistance rendered by Mr. J. J. Kehoe, Jr., of the General Engineering Laboratory, General Electric Company, who read the manuscript and carefully checked the formulas.

LIST OF SYMBOLS

A	cross-section of a slab of dielectric, perpendicular to the lines of force
a	thickness of a slab of dielectric
$A, B,$	values of the integrals defined by Equations (**67**) and (**68**)
C	ideal capacitance
$C\,i$	integral cosine
col	defined by Equation (**86**)
D	dielectric flux density, or charge per unit area
D'	deficiency in the density of displacement
E	applied voltage
e	a subscript meaning "even"
$F(\alpha)$	distribution function defined by Equation (**25**)
f	a subscript meaning "final" or at the time $t = \infty$
G	voltage gradient or stress
g	apparent conductance due to absorption
H	a parameter defined by Equation (**56**)
I, i	current
j	$= \sqrt{-1}$
k	absorption coefficient
m	a subscript meaning "maximum value" or amplitude
m	an exponent in Equation (**56**)
N	number of particles
n	an exponent in Equation (**56**)
o	a subscript meaning "initial," or at the instant $t = t_0$; in application to R means "odd"
P	dielectric loss per unit volume
p, q	quantities defined by Equation (**74**)
p^1, q^1	quantities defined by Equation (**81**)
Q	dielectric flux or electric charge
R	an integral defined by Equation (**77**)
$S\,i$	integral sine
sil	defined by Equation (**87**)
t	time
u	current density
V, W	values of the integrals defined by Equations (**69**) and (**70**)
x	an auxiliary variable which first enters in Equation (**90**)
α	relaxation factor defined by Equation (**23**)
Γ	gamma function
γ	conductivity of the material

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[14] The problem of fitting the coefficients of an equation to agree with given experimental data is a large subject by itself, and cannot be treated in this investigation. See, for example, Engineering Mathematics, Chapter VI, by C. P. Steinmetz, and special works on the method of the least squares and on fitting of empirical curves.

Δ	a finite increment
θ	an auxiliary quantity defined by Equation (**79**)
κ	permittivity or dielectric constant
τ	instant of application of a new stress
$\phi(t)$	relaxation function of time
ψ	loss angle or imperfection angle, defined by Equation (**53**)
ω	electric angular velocity, that is, 2π times the frequency.

Discussion

DIELECTRIC ABSORPTION AND THEORIES OF DIELECTRIC BEHAVIOR

(WHITEHEAD)

AND

THEORY OF ABSORPTION IN SOLID DIELECTRICS

(KARAPETOFF)

NEW YORK, N. Y., FEBRUARY 8, 1926

A. E. Kennelly: In Faraday's time, the properties of substances under magnetic stress or magnetization, and the properties of dielectric materials under dielectric stress or electric induction, were studied and were regarded as of great interest by him and by his followers.

It seems remarkable that, using the relatively feeble magnetic stresses and magnetic fluxes of ordinary dynamoelectric machinery according to Ewing's theory, it is not necessary to go deeper perhaps than the molecules of the iron or steel in order to account for the main features of magnetic induction. No doubt when we come to consider much more powerful magnetic stresses such as in the Zeeman effect it may be necessary to go deeper than merely into the molecule, or into groups of molecules. On the other hand, however, in order to account for the behavior of dielectric material under electric stresses, it seems that even with the relatively feeble stresses of electric signalling, a consideration of groups of molecules or even of molecules themselves is inadequate, and it seems necessary to descend to the subatomic state to explain the phenomena.

Strange to say, in the early history of electrical engineering, going back to the time when the only electrical engineers were the telegraph engineers, the study of dielectric absorption was found necessary for practical purposes, in order to specify the insulation resistance of a given length of any kind of solid insulated conductor. It was necessary as our text-books show, as far back as the 1850's, to take some account of dielectric absorption, and it was known that the apparent insulation resistance at the beginning of the first quarter-minute was different from what would be obtained at the first half-minute or at the first minute. It became necessary, I think, in the 1860's, to define the measured insulation as that obtainable at a certain definite interval of time, let us say one minute or three minutes after the application of the continuous dielectric stress.

W. A. Del Mar: I have run into the same difficulty that Professor Whitehead mentioned; namely, that the physicists who are working on the theory of electrons and the ultimate nature of matter have not reached a stage of progress in their work where they can help us very materially in our theories of absorption in electric strength, and I don't believe that very great progress will be made until the physicists who are dealing in those ultimate matters make some further steps forward.

A few years ago I would have thought that the experimental physicists and those who are dealing with such matters as electric strength and absorption were the ones who would have to take the principal steps in order to reach a rational theory of the behavior of dielectric, but I have now come to the view that the pure scientists are the ones upon whom we are waiting.

We must be careful, in studying such a subject as dielectric loss and power factor, not to put a word in place of a phenomenon and take refuge behind the word. There is a little danger that we may do that with the word "absorption". It is taken to cover a multitude of sins, and when one reads through Professor Whitehead's admirable summary, one has to stop occasionally and think: What is the meaning of this "absorption?" Is it not just a word which is being used in a general way to cover a great variety of phenomena?

One might perhaps get the impression, by reading Professor Whitehead's summary, that there are several theories which might be chosen from, to explain the phenomenon, whereas more likely all of these theories will be required in some measure to explain it in full.

R. E. Marbury: Dr. Whitehead has given us a very complete review and valuable bibliography of what has been done on dielectric absorption and related phenomena.

Our experience has indicated that Maxwell's theory can explain dielectric absorption and a-c. losses of the low frequency type. Maxwell's theory assumes that the s. i. c. and resistivity of the dielectric remain fixed. This is generally the case in solids, but with semi-liquid dielectrics, such as oil-impregnated paper, there is evidence of changes in either s. i. c., resistivity, or both, with voltage, or temperature. This may be illustrated by plotting the residual voltage under a fixed set of conditions, against various applied voltages. If the law of superposition holds and if the resistivity and s. i. c. remained fixed throughout, a straight line relation would be found. This is not the case, as we find that the residual does not increase proportional to voltage. In some specimens the applied voltage may be doubled with little effect on the residual. The conditions may be changed so as to modify this relation. For example, if the duration of charge is made very small, such as 0.0015 sec., the resulting curve may be a straight line and appear to follow the law of superposition. This, among other things, leads us to believe that this departure from the law of superposition is caused by a movement of moisture, and when the duration of charge is made too quick for the moisture, the dielectric acts like a solid, as is to be expected.

A dielectric which shows this non-proportional residual possesses other interesting characteristics. For example, the losses will decrease with increased voltage. Such a dielectric will show a marked decrease in losses with time after the application of voltage. The latter effect may be seen by measuring the losses at five-sec. intervals, starting five sec. after the voltage is applied.

It is now possible to predict quite definitely certain characteristics of a condenser from absorption tests of the proper type. These characteristics are:

1. Whether the losses will decrease or remain constant with voltage.
2. How the capacity will vary with frequency.
3. Completeness of drying process.
4. Magnitude of loss variation with temperature.
5. Whether the condenser will give good performance on alternating current.
6. Whether the condenser has characteristics desirable for d-c. operation.

We can predict less accurately the actual losses, and can calculate a curve which will have a shape closely following the real curve which will be found if losses are plotted against voltage.

It appears that a complete interpretation of absorption curves will make possible a closer control of dielectric characteristics than has heretofore been possible.

R. W. Atkinson: I wish to remark that great increase in our knowledge of dielectric phenomena is certain to be made in the next few years on account of the interest and attention given these problems by the leading physicists in the faculties of the colleges and universities of this country. The present and other recent contributions are an important part of the recent advance, especially as leading toward a clearer understanding of dielectric phenomena.

When Mr. Fisher, for our company, first began measuring dielectric losses, before I was associated with him and also during the early part of my work, we found it useful to measure both quantities, dielectric absorption and dielectric loss, and a very close relation between them was observed, and for a long time both measurements were faithfully made on all materials.

Later, the large increase in the number of dielectric loss measurements necessitated that attention should be concentrated on the loss end of the problem. But I believe it is important to have further investigation of the absorption side, and have no doubt it will lead to valuable knowledge of the subject.

I believe that the greater part of the losses in paper cable dielectrics at high temperature can be quite largely accounted for by specific numerical application of Maxwell's method.

E. S. Lee: If you will look into the bibliography which Dr. Whitehead has prepared, you will find some three hundred and fifty items, and yet we have the work of Tank—one man—given to us as "apparently the only effort so far made for a direct check between measured loss and loss computed from measured absorption."

I think that is a condition about which we ought to think and I think that it is largely because of that condition that Professor Karapetoff worked out a formula which we could use in this regard. In other words, if, for any insulation, we shall determine the curve of current against time and determine it well,—that is, allow the insulation to retain its initial condition between each measurement and obtain the curve with which we are familiar and then obtain the constants *M* and *N*, which you will find in Professor Karapetoff's paper, from that curve,—then the dielectric power loss of the insulation may be calculated from the equations given. This value may then be compared with measured values, and we shall be able to determine whether or not the equations which Professor Karapetoff has derived are applicable to the work which we have at hand.

We have tried to do this, somewhat, and have found, thus far, that out of 13 articles in the literature, only one had data which we could substitute in these equations.

So I make a plea to those of us who are interested enough to obtain data, that we shall obtain it in such a manner that we may make proper substitutions into the formula which Professor Karapetoff has derived in order to determine whether or not it applies. When you consider all of the work that Dr. Whitehead and his Committee have done, and when you find the meager amount of data that are available for substitution directly into such formulas as we have, it seems as though we had overlooked something in connection with the work, which should be rectified in the future.

D. W. Roper: Dr. Kennelly, I believe, stated that our knowledge of this subject was far from complete.

And Mr. Lee apparently has somewhat the same idea, because, after a careful study of the paper, he picks out one sentence which sums up the knowledge that he was able to use on the subject.

Now, if there is all that difficulty in getting some real knowledge about the very simplest forms of insulation, how much more difficult and how much more remote is the chance of our getting some fundamental knowledge regarding such a very complex insulation as impregnated paper, such as is used in our lead-covered cables. Dr. Whitehead touches upon that point a little in his final paragraph.

I have been endeavoring, with a very great effort, to find some method of measuring the quality of the insulation without destroying the insulation, and apparently there is no such method. We don't know enough about insulation to measure the quality, without destroying it, and so what we do in practise is to measure the quality of some samples of insulation and then include in our cable or our machinery some insulation which is similar to the samples we have tested; and we hope that the insulation which we use will be just as good as the samples. Until we can get more fundamental knowledge of the properties of the insulation, we shall not be able to replace that hope with knowledge.

J. Slepian: Equation No. (10) in Dr. Whitehead's paper and No. (21) in Professor Karapetoff's, written with a little different notation

$$D = K_0 E + \int_0^t E(u)\, \phi(t-u)\, d u \qquad (1)$$

is a very interesting one and gives a complete account of the electrical behavior of a dielectric in those cases in which it applies. Professor Karapetoff has called the function ϕ a relaxation function. Another viewpoint is to call the function a memory function, as this function indicates how much the dielectric remembers of the voltage which has been impressed upon it in the past. As will be seen in Fig. 1, if $E(u)$ represents the voltage gradient which has been applied to the dielectric from the time $u = 0$, then at the time $u = t$ the displacement of the dielectric according to the above equation depends not only upon the value of E at the time $u = t$ but on all the preceding values. However, the preceding values are not taken with equal weight but are multiplied by a factor which diminishes as the time recedes further into the past. The function $\phi(t-u)$ represents this weighing factor. In other words, it shows how the dielectric has a diminishing recollection of past voltages.

The Equation (1) follows directly from the principle of superposition, as both Professor Karapetoff and Professor Whitehead bring out. It is also true that solutions of linear differential equations with constant coefficients can always be expressed in the form of equation (1). The simplest memory function is probably that which corresponds to a first order linear differential equation and that is a simple exponential. The memory functions of higher order differential equations are made up of sums of exponential functions. If a number of electrical systems, each satisfying a first order linear differential equation and therefore having a simple exponential memory function, are coupled together in any way, the resulting system is characterized by a linear differential equation of high order, with a more complicated memory function. Likewise a given system with a complex memory function is equivalent in its action to a large number of simple systems coupled together.

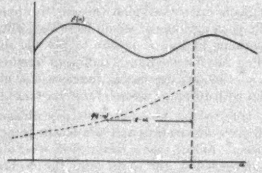

FIG. 1

The resolution of the complicated system into simple systems is quite indeterminate and can be effected in a variety of ways. Referring specifically to the problem of the dielectric, it has been attempted to resolve the dielectric with complex memory function into an aggregate of dielectrics with simple memory functions, but such resolutions are indeterminate and usually of little value unless properties other than checking ordinary electrical measurements are considered. Maxwell, Wagner, and others suppose the dielectric made up of an assemblage of materials, each having a simple dielectric constant and ohmic resistivity. Karapetoff makes the dielectric consist of an aggregate of small pieces of dielectric, each piece having a simple memory function, without explaining how it comes to have that property. Von Schweidler assumes various kinds of molecules each with its own memory function combining to give a more complex memory function to the dielectric.

All of these theories lead to the same result,—namely, the Equation (1)—and say no more than that the principle of superposition applies to the dielectric. Maxwell & Wagner's theories will have value only in those cases in which portions of the dielectric can actually be found which have simple dielectric constant and ohmic conductivity. Karapetoff's theory likewise will have value only if he can actually determine by means independent of the ordinary electrical measurements, the existence of portions of dielectric having simple memory functions. Von Schweidler too must show independent evidence of the existence of his various kinds of molecules. All these theories are sufficient to account for the principle of superposition. None of them is necessary. It seems to be a choice of being satisfied with the idea of the homogeneous substance having complex dielectric properties or a complex assemblage of materials having simple dielectric properties. Unless, however, there is physical evidence of some kind for the existence of the complex assemblage of simple substances, I do not believe that anything is gained over the assumption of the homogeneous substance with complex characteristics. As far as agreeing with electrical measurements, all these theories are on a par, provided they accurately represent the memory function of the dielectric.

That these theories without other physical evidence do not add to our knowledge of the dielectric seems to be intimated by Professor Whitehead when on page 11 he says of Von Schweidler's theory, "The analysis has the character of a mathematical fiction," and again further along on the same page, in speaking of the check which Grover made upon the Pellat theory, "It appears certain that an equally good agreement would have been obtained from Wagner's equations; in fact it is safe to say the same of any theory providing for the medium a sufficient number of terms, all obeying a continually decreasing function $\phi(t)$ of relatively simple form but with different values of the constant terms."

While it is true that a heterogeneous assemblage of simple dielectrics will show absorption and satisfy the principal superposition, it does not follow that a material showing absorption is necessarily heterogeneous, and to my mind the physical evidence on that point is far from conclusive. Very pure substances show little absorption, but, at the same time, they usually show a tremendously high resistance. When two pure substances are mixed they frequently show absorption, but such mixtures are hardly to be described as heterogeneous and the absorption which arises frequently is not consistent with the numerical values of the dielectric constant and resistivity of the pure materials themselves. For example, a mixture of pure water and pure sulphuric acid shows a great conductivity which cannot be accounted for on the theory that it is a heterogeneous assemblage of the pure substances, each of which has a tremendously high resistance.

How universal is the principle of superposition for dielectrics? The existence of the irreversible current which Dr. Whitehead points out shows at once that it does not always apply. Last year Mr. Marbury, in his paper on oil condensers, showed phenomena which contradicted the principle of superposition. The departure from the law of superposition may take place in various ways. First the memory function may change with time of application of voltage as, for example, when water in the pores of a material is displaced gradually by the electric field. The memory function may change with amplitude of voltage as it undoubtedly does near the dielectric breakdown point of the material, or finally the properties of the material may not be characterizable by a memory or relaxation function at all.

The paragraphs of Professor Whitehead on electric hysteresis, I think, are exceedingly good and to the point. The phenomena of dielectric absorption and magnetic hysteresis are too unlike in character to be confused by a similar name. In the language of my preceding discussion, whereas a dielectric has memory of past electric fields which diminishes with lapse of time, a piece of iron has an unchanging memory of the fields which have acted upon it. You may put a piece of iron through magnetic cycles today and tomorrow the iron will be ready to go on from where it left off, no effect of the magnetic fields of the previous day being lost.

The work of Debye and Schrodinger, I believe, deserves more than the casual mention which Professor Whitehead gives. Their polarized molecules have an actual physical reality, since, in their work on dielectric constants of gases, they are able to explain the dielectric properties by assuming that all the molecules of the gas are polarized with moments of the order of magnitude consistent with other phenomena and are able to predict correctly the effect of temperature by calculations from the kinetic theory of gases; thus Debye and Schrodinger give a complete theory of the dielectric properties of an un-ionized gas, the only case in which such complete theory has been given.

I wish to take issue with Professor Whitehead with respect to what he calls the fundamental equations of electromagnetism. In his classification on page 7 he implies that some of the theories given involve deviations from the fundamental laws. I believe the common usage is to regard the following equations as given by Maxwell fundamental:

$$\text{Curl } E = -\frac{d^B}{d\,t} \tag{1}$$

$$\text{Curl } H = 4\,\pi\frac{d\,D}{d\,t} + 4\;C\,\pi \tag{2}$$

$$\text{Div. } B = O \tag{3}$$

$$\text{Div. } D = -\,4\,\pi\,\zeta \tag{4}$$

These equations correspond respectively, the first to Faraday's law of induction, the second to the manner in which conduction and displacement currents give rise to magnetic fields. The third equation states the fact that free magnetic charge does not exist, and the fourth, that lines of electric flux terminate in electric charge. These four equations are not sufficient to determine the quantities E, H, B and D, but further relations must be found connecting B and H and D and E, depending upon the properties of the material considered. In the simplest theories, it is customary to take $B = \omega\,H$ and $D = \epsilon\,E$. These last equations are the ones which are not adopted by some of the theories which Professor Whitehead mentions. However, they are not fundamental in electromagnetism; they are only a very special case in electromagnetism.

C. A. Adams: As I was originally responsible for the organization of the Insulation Committee, of which Dr. Whitehead's excellent paper is a one-man report, I am naturally interested in its progress.

But the attack which I had in mind was a very much more vigorous one that is now being conducted largely by voluntary services.

A more thorough understanding of dielectric phenomena in solid and semi-solid dielectrics constitutes the most important problem in the whole field of electrical engineering at this time. But unfortunately it is a terrifically complicated and difficult problem from the standpoint of modern theory. It has been almost completely ignored or shirked by our American physicists, and the very suggestive and interesting first steps taken by two or three foreign physicists are buried in the proceedings of the more highly scientific organizations.

This most important problem is not going to be solved by engineers. We need a comprehensive fundamental research involving the cooperation of the best physicists and chemists that the world affords, and backed financially by the great corporations who either manufacture or use insulating material.

The cost to any individual corporation would be small, if they all took part, particularly if the cooperation were extended to other countries, as it should be in a problem of such fundamental scientific and commercial importance.

Much research work in this field is now being conducted in this country, but mostly by engineers, who, owing to lack of thorough knowledge of modern theory, are only skimming the surface. Moreover, there is no thorough cooperation, and much duplication of effort.

If the A. I. E. E. wishes to stand for cooperation and progress, here is certainly a supreme opportunity.

S. L. Gokhale: I have nothing to contribute directly to the topic under discussion; my purpose is merely to supplement the very suggestive remarks of Dr. Kennelly by an interesting experiment of ours in the General Engineering Laboratory.

Dr. Kennelly has suggested that the dielectric phenomena are perhaps sub-atomic in their nature; he also referred to the magnetic phenomena as being analogous to the dielectric phenomena, so that any insight into the one phenomenon might help to give us a better understanding of the other. The experiment I am going to describe refers to magnetics, and I am describing it in the hope that it may prove of use to those who are trying to study corresponding phenomena in dielectrics.

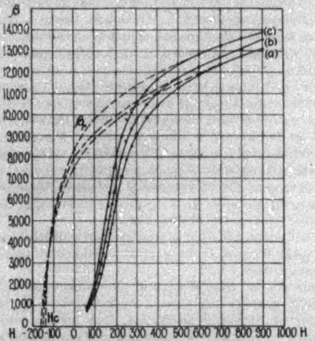

FIG. 2—COBALT MAGNET STEEL (SAMPLE NO. 16-1) CHANGE OF MAGNETIC CHARACTERISTICS APPARENTLY DUE TO STRONG MAGNETIZING FORCE

The sample under test was a toroid ring sample of cobalt magnet steel, wound for a magnetizing force of about $H = 1000$ g. The resulting curves are shown in Table I. The curve (a) is the result of the first test; the point for $H = 900$ gave at first $\beta = 13{,}000$, but the reading could not be repeated on second and subsequent tests, and on going back it was found that other points could not be duplicated also. The whole test was therefore repeated, and gave the curve (b). At $H = 900$, the curve developed the same anomaly as before. The whole test was therefore repeated once more, which gave the curve (c). A fourth test gave a further shift of about 50 gausses with a corresponding increase of deflection of half a millimeter. The sample seems to have reached a steady state now. The hysteresis curve (dotted) is conjectural, but the cardinal points βr and H_c are observation points. It seems that during the test the sample was softening, probably under the influence of the strong magnetizing force. It was probably not a heat effect, as the precaution against heating had been tried out in some previous tests, and was found to be quite satisfactory; no thermic measurements were made in this particular case.

In trying to interpret the phenomenon, my first guess was that there was some change in the structure of the molecular

COBALT MAGNET STEEL
Ring-Form Sample No. P-16-1

Test (a)

H	β	βr	$H c$
60	750		
80	1050		
100	1500		
120	2080		
130	2450		
140	2850		
150	3300		
160	3800		
170	4400		
180	4850		
190	5350		
200	5950		
225	7100		
250	7900		
275	8520		
300	9100		
350	9880		
400	10450		
500	11250	7250	
600	11850	7350	
700	12400	7400	162
800	12800	7500	162
900	13100	7500	(Reading not duplicated on repeating)
1000		No Test	

Test (b)

60	800		
80	1200		
100	1750		
120	2500		
130	2960		
140	3480		
150	4020		
160	4600		
170	5250		
180	5800		
190	6350		
200	6950		
225	8020		
250	8780		
275	9350		
300	9780		
350	10480		
400	11000		
500	11750	7750	
600	12250	7750	
700	12700	8000	149
800	13100	8000	149
900	13550	8150	(Reading changed after first reading)
1000	No test		

Test (c)

60	920		
80	1400		
100	2100		
120	3050		
130	3700		
140	4350		
150	5050		
160	5700		
170	6350		
180	6950		
190	7500		
200	8000		
225	8950		
250	9750		
275	10100		
300	10550		
350	11150		
400	11650		
500	12400	8240	140.0
600	12850	8350	141.0
700	13250	8350	141.4
800	13600	8350	141.8
900	13900	8350	142.0
1000	14100	8350	142.0

systems which are supposed to be oriented during magnetization. This explanation implies a modification of Weber's theory, which refers to orientation of the molecules. On trying to discuss the question with my colleagues in the Research Laboratory, I was informed that in view of the recent researches about the structure of the crystals, as revealed by the X-Ray spectroscope, there is no intermediate step between an atom of iron and a complete crystal. The phenomenon of magnetism should therefore be regarded as an atomic or sub-atomic phenomenon, which leaves no room for Weber's theory of molecular orientation, and even less room for my theory of molecule-system orientation.

I do not claim to have understood the phenomenon; I am not even convinced that it is sub-atomic. I am mentioning it here in the hope that it might help in some way towards a better understanding of the process of magnetic polarization in the first place, and of dielectric polarization by analogy.

J. B. Whitehead: All writers on the theory of dielectric absorption have discussed the form of the curves of current of residual charge and discharge. A number of these are set forth in my present paper. All such workers have recognized that a single term of any of the forms mentioned will rarely represent the experimental curve. Hopkinson found for his samples that two terms were not enough, and Maxwell gave general expressions for n terms as required in a mixture of $n + 1$ materials. A number of observers have found that for some simple substances one term is closely sufficient.

Professor Karapetoff's paper is based on the assumption that a great many, perhaps an infinite number, of such terms are necessary. The Germans, Wagner and Von Schweidler, have also made this assumption and have attempted analytical developments which should in effect permit us to assign definite constants to particular materials which would fit the relative importance of this large number of terms and so define the behavior of the material. Professor Karapetoff has gone somewhat further, first in selecting a function fixing the distribution of the series of terms which is somewhat easier to handle, second in allowing us to follow more intimately his ingenious mathematical manipulation, and particularly in evaluating the changes which may be looked for under alternating voltage, in the values of conductivity and permittivity as affected by his "generalized absorption."

But while we may admire the persistence and skill manifested in these developments, I question seriously whether they constitute the necessary path by which we will ultimately reach a position in which we can control and predict the performance of dielectrics for the following reasons:

It has never been shown that $\phi\,(t)$ is uniform or the same for any given material. Why discuss $F\,(\alpha)$ if it may have widely different forms for the same material prepared in different places? The absorption of any material may be continually reduced by greater and greater care in purification. Hopkinson showed that while two terms were not sufficient to account for his curves, not many more would be required. Steinmetz showed that even in so highly absorbent a dielectric as cable insulation, three terms were sufficient to very closely account for the observed curve. There are many instances where one term appears sufficient. F. Tank has studied $\phi\,(t)$ for very small values of t, has found it of simple form, *i. e.*, one term, and that the use of a single term in developing the expression for loss under alternating voltage gives values in accord with experiment.

Professor Karapetoff states that the time for a rational theory of dielectric absorption has not arrived. One must ascribe this statement to momentary forgetfulness, induced by deep interest in the theory of Pellat, and his elaborate development thereof. I am sure that he will agree with me that Maxwell's theory of dielectric absorption, relying as it does only on fundamental electromagnetic theory, and in no wise invoking the structure of the atom, is in every respect a rational theory. Maxwell, as everyone knows, explains dielectric absorption entirely in terms of the specific conductivities and specific inductive capacities of different substances when mixed together. It accounts qualitatively at least for most of the important phenomena to be observed in the dielectrics of practise, and does this quite as satisfactorily as any other theory. If it is not subject to exact experimental corroboration, it has certainly never been shown that this is not due to the impossibility of obtaining strictly pure and simple materials or mixtures. Maxwell himself recognized the probability of this difficulty and extended his expressions so as to embrace any number of different materials. Followers of his theory have always assumed that the several terms often needed to account for the shape of the curves of charge and discharge, are due to the presence of other substances or impurities throughout the mass of the principal dielectric. Wagner is a follower of Maxwell, and in his development of the distribution function he has attempted to picture the type and method of mixture of the conducting impurities in the mass of the dielectric. If this view is correct, and there seems to be no satisfactory contradiction of it, it would appear that the evaluation of distribution functions such as assumed by Von Schweidler and Wagner, and now by Professor Karapetoff, resolves itself into an effort to define the particular ways in which impurities may occur in a fundamental material. Obviously there is no reason to suppose that on this basis the distribution function can ever be the same in any two cases, even for the same fundamental material.

These questions have long excited the discussion of those interested in the theory of dielectric theory and behavior. The following are a few experimental problems the solution of which would go far toward solution of some of the open questions:

To fix the definite curves of particular substances, and the accuracy with which they can be reproduced.

To determine the importance of their departures from simple curves.

To study the control of these curves.

To study their behavior in combination in different substances.

To study the relation between absorption and loss for some of the simpler cases.

In my opinion careful work in these directions will be found to greatly simplify the present confusion as to the nature and behavior of dielectric absorption, and further, will result in enormous simplification of the problem of engineering design and insulating mediums.

H. L. Curtis (by letter): Dr. Whitehead has reviewed a large amount of literature and has made an excellent summary of our present knowledge of the anomalous behavior of dielectrics. It is a fair and impartial discussion of all the theories that have been proposed to explain this most baffling phenomenon. It should serve as a starting point in the extension of our knowledge of dielectrics.

I agree with the author that, from a theoretical standpoint, the most satisfactory theory of dielectric absorption is that of Maxwell's stratified dielectric. However, I do not agree that there is any chance that it can explain all the observed facts. One illustration will suffice. The application to alternating current phenomena of Maxwell's theory or any of the modifications proposed by Rowland, Wagner, and others, invariably leads to an equation for the phase difference which is equivalent to equation (**31**). For high frequencies, this equation takes the approximate form

$$\tan \delta = \frac{k}{\omega\,T}$$

so that for such frequencies the phase difference is inversely proportional to the frequency. Now there are abundant data to show that this is not the case in many dielectrics. Hence in such dielectrics it appears impossible to explain the phenomena on the basis of this theory.

However, there are other dielectrics where the observed phenomena are largely explainable by Maxwell's theory. Moreover, from the fundamental assumptions of this theory it must play some part, however small, in most absorption phenomena, since homogeneous dielectrics are extremely rare. The conclusion must therefore be drawn that the final explanation of dielectric absorption will include at least two theories of which Maxwell's stratified dielectric will be one.

Professor Karapetoff has given a very clear derivation of the integral equation (Eq. No. **21**) which results when the principle of superposition is applied to an absorbing dielectric in an alternating electric field. Wagner's relaxation function (Eq. **25**) is then inserted in this integral equation and the solution carried to a point where both the change of capacitance with frequency and the power factor are expressed in terms of integrals which involve both the relaxation function and the frequency. An empirical expression (Eq. **56**) for the relaxation function is then assumed and the integrals evaluated. The results (Eqs. **103** and **104**) are given in terms of ascending powers of the frequency. There is, however, no discussion of the convergence of these series. It is evident that, for high frequencies, they either diverge or become impracticable as methods of representing the function.

The value of the paper would have been greatly increased had it contained some numerical examples. At best the results as given are applicable only to low frequencies. Whether this includes frequencies that are of any practical or theoretical value can not be judged from the equations as given. Until the paper is completed by the application of the equations to typical sets of data, one should withhold judgment as to the value of the results obtained.

J. Katzman: Professor Whitehead's paper reviewing and bringing together the many theories of dielectric absorption and indicating the behavior of dielectrics under various conditions as found by former experimenters is certainly of inestimable value to all those engaged in this work. However, the divergence of views, theories, and results obtained makes it impossible, as yet, to predict effects from given causes. Qualitatively predictions can more readily be made, provided the knowledge of accumulated facts is had. Effect of temperature may be prophesied if it is known and remembered that according to Hopkinson and others, absorption current increases with temperature, and that according to Wagner, rate of absorption and decay is increased with temperature and the total absorbed charge remains unaltered. From Lahousse's equations it may be surmised that a loss in a dielectric will vary as the square of the voltage. Similarly, effects of frequency on loss per cycle, on loss per minute, and on capacity, and the effect of voltage on absorbed charge, etc., may be foretold if results of former experiments are remembered, or resorted to.

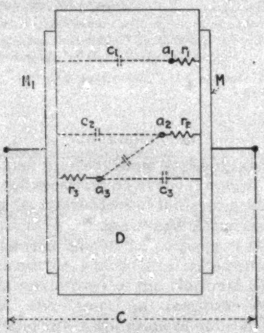

Fig. 3

To be able to say, even approximately, what may be expected when a dielectric is made to undergo a change, is often of great value, and this may be deduced from a consideration of the properties of conductors and specific inductive capacity only. Assume the dielectric to have conducting particles embedded in it and the dielectric itself to have conductivity. Fig. 3 is a diagram of a condenser having a dielectric *D*, the metallic plates M_1, M_2 on the opposite faces of the dielectric, and some of the conducting particles a_1, a_2, a_3; r_1, r_2, r_3 are the resistance of the dielectric between respective plates and particles as shown. Each of the particles will form a sub-condenser with one of the plates or with each other as indicated. By forming a mental picture of such a condenser, all of the facts mentioned before concerning dielectrics can be reasoned out. Thus when the applied e. m. f. is altered, the charging current of each of these sub-condensers is proportionally altered and therefore $I_3^2\, r_1$, $I_1^2\, r_1$, etc., varies as the square of the voltage. The summation of these losses being assumed to produce the absorption loss, it is seen that absorption loss will vary as the square of the applied e. m. f.

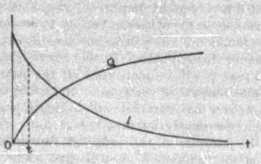

Fig. 4—Curves of Variation of Charge and Charging Current with Time

With but very few exceptions when temperature is increased the resistivity of solid insulators reduces.[1] Hence resistances r_1, r_2, etc. reduce. The effect is therefore to reduce the time constants $c_1\, r_1$, $c_2\, r_2$, etc. of the sub-condensers, thus increasing the rate of absorption and decay, but does not alter the total absorbed charge, in accordance with Wagner's conclusions. The absorption current having increased, $I_1^2\, r_1$, etc. increases even though r_1 does decrease. The result to be expected then is an increase in power loss due to absorption.

When the frequency is increased the time available for charging the sub-condenser is decreased, and as can be seen from Fig. 4, if this time is t_1, the condensers will be only partially charged, with the result that absorption loss per cycle is decreased since the loss per cycle is $\int_0^{t_1} I^2\, r\, d\, t$. From the shape of curve *I*, it is obvious that the greatest loss occurs only at the beginning and therefore the reduction in loss is less in proportion as the frequency is increased. It follows then that the loss per second is increased by an increase of frequency. It is readily conceivable that the sub-condensers increase the capacity *C* by increasing the ratio Q/E, where *Q* is the total charge and *E* the applied e. m. f. At very high frequencies, however, the charge on the sub-condensers becomes practically zero, as can be seen from the curve Fig. 2, and hence the capacity of the condenser becomes equal to the geometric capacity. In other words the capacity of the condenser approaches the geometric capacity as the frequency is increased.

By similar reasoning, and these involving only elementary principles, other predictions may be deduced.

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[1] Dietric in Phys. Zs. 11, p. 187, 1910 shows that $R_t = R_0\, \epsilon^{\frac{-qt}{273(273+t)}}$ *q* being a constant depending on the material and *t* the temperature.

A further development of this idea is under preparation.

E. R. Le Ghait: Professor Karapetoff's mathematical analysis will certainly be of great interest to all those occupied with the behavior of dielectrics. He does not make any assumption on the physical nature of the phenomena. The work we have done with fibrous insulating materials has led us to the opinion that Maxwell's ideas are, in that case, quite capable of explaining the observed phenomena, provided however, one takes into consideration the complicated structure of the dielectric. We believe that the absorption as well as the dielectric losses at commercial frequencies are due for the greater part to the presence of very small quantities of moisture imprisoned in the capillary tubes of the fibers. If we then consider the dielectric as composed of a very great number of very thin paths crossing from one electrode to the other, and apply Professor Karapetoff's analysis, his function $F(\alpha)$ will be the law of distribution of the moisture among those paths. If the curve $F(\alpha)$ has a very pronounced peak it will indicate that all the paths have pretty much the same amount of moisture. If on the contrary $F(\alpha)$ is a comparatively flat curve it will indicate that some paths contain much more moisture than others.

If then the moisture is the cause of the dielectric loss the knowledge of the function $F(\alpha)$ will give us an indication of whether the loss is uniformly distributed all over the dielectric or whether it is for the greater part concentrated in some of the paths, these paths constituting possibly weak spots on account of the deterioration of the insulation due to an exaggerated loss in a given point.

Dr. Whitehead, in his comments of Maxwell's theory, calls attention to the fact that pronounced absorption is observed in substances that can have in them only small amounts of impurities. This is probably due to the peculiar shape of the lines of force in a dielectric containing materials of different specific inductive capacities.

Our own experience agrees with Dr. Whitehead's opinion that dielectric losses are closely related to absorption. In fact, in the case of unimpregnated paper, starting from residual voltage curves obtained with a dielectric previously left charged at 100 volts d-c. for a certain time and then left short-circuited for a very short time, we are able to compute 60-cycle losses with less than five per cent error.

In the case of oil-impregnated paper, greater difficulties are encountered, as in that case the law of superposition does not hold true (this being probably due to movements in the oil under the influence of the electric field, of moisture and other components of high specific inductive capacity). In correlation with this, the power factor at a given frequency varies widely with the value of the applied voltage. It may, however, prove to be possible in the future to make a low voltage d-c. absorption test enabling one to predict with sufficient accuracy the a-c. losses at a voltage many times higher, and perhaps even to predict the curve of variation of power factor with applied voltage.

Ionization Studies in Paper-Insulated Cables—I.

BY C. L. DAWES[1]
Member, A. I. E. E.
and
P. L. HOOVER[2]
Associate, A. I. E. E.

Synopsis.—*A research investigation of the ionization phenomena which occur in paper-insulated, high-voltage power cables is being made at the Harvard Engineering School under the auspices of the Impregnated Paper-Insulated Cable Research Committee. The paper presents some of the preliminary results which have been obtained, certain tentative conclusions that are suggested by the data and a description of the method developed for making the measurements. It is a preliminary report and in subsequent papers it is intended to record the progress of the investigation as it proceeds.*

The paper consists essentially of four parts as follows:

(a) A number of curves of power, power factor and capacitance taken on samples of cable at two frequencies and over a wide range of temperature. These curves show well-known characteristics but in order to exaggerate these effects, cable models were made up to simulate the general conditions in a cable—one model consisting of glass and air only and another of glass, air and paper. Tests of these cable models gave very interesting results, particularly with reference to power factor. They also show rather clearly the baffling action of paper and the effects on power, power factor and capacitance when this baffling action is eliminated. In practically all the curves, the power and power factor begin to increase rapidly at a lower voltage gradient than the capacitance.

(b) Discussion of the results obtained and certain tentative conclusions which may be drawn from the results so far obtained and reported herein.

1. Ionization in the dielectric of an air condenser increases its capacitance slightly at first and then rapidly as the electron is separated from the atom. In our measurements we accordingly found that the power and power factor increase rapidly at a lower voltage gradient than the capitance.

2. Ionization which occurs within air spaces in a dielectric may be called "restricted ionization" in that the current is limited because of the remainder of the dielectric in series.

3. With "restricted ionization," the voltage across each air space reaches a constant finite value with indefinite increase in the over-all voltage. The resistance of the ionized space, therefore, must be inversely as the current.

4. Consequently when this condition is reached, the ionization loss may be proportional to the charging current.

5. The increase of power factor with increase in voltage gradient with subsequent decrease of power factor is due to the fact that the capacitance of the solid dielectric, which is substantially constant, is in series with ionized air spaces whose resistances are inversely as the current. A simple mathematical analysis of this type of electric circuit shows that power-factor curves should have the form obtained in the measurements. Hence power factor alone is not a criterion of the degree of the completeness of saturation by compound.

6. Ionization by its bombarding action may destroy the baffling action of paper.

7. Ionization may produce potential gradients tangential to the surface of the layers of paper which, in conjunction with the bombarding action, may be the cause of the so-called "tree designs."

(c) Discussion of methods for measurements of this sort and a description of a new type of bridge which was devised for the measurement of dielectric losses at these extremely low power factors with the necessary high degree of accuracy. A large air condenser used as a standard and a vibration galvanometer of unusually high sensitivity and wide range of tuning were designed for use as a detector. The bridge is quickly and accurately balanced by varying a resistance and a mutual inductance. The angle of defect of the standard air condenser, although extremely small, is nevertheless very important in measurements of this character. This angle was measured by a substitution method and corrections made accordingly.

(d) Four appendices discussing respectively (1) method of measuring defect angle of the standard air condenser, (2) mathematical analysis of a condenser of composite dielectric consisting of air and solid homogeneous material, (3) measurement of the high-voltage ratio, and (4) measurements of the capacitance and other electrical constants of an ionized air space in series with a solid dielectric.

INTRODUCTION

THIS paper presents some of the results of research work being carried on at the Harvard Engineering School on the subject of Ionization Phenomena in the Insulation of High-Voltage Paper Cables. The work is conducted under the auspices of the Cable Research Committee which is a subcommittee of the appropriate committees of the National Electric Light Association, the American Institute of Electrical Engineers, and the Association of Edison Illuminating Companies. The National Electric Light Association provides the Research Fellow for the work. The work is under the immediate supervision of a Cable Research Committee made up of members of the Faculty of the Harvard Engineering School, and one of the authors is Chairman of this Committee.

A summary of the more important results, which have been obtained up to the present time, is presented together with suggested explanations based on the modern theory of ionization. However, an important object in presenting these results at this time is to invite discussion, from which it is hoped to obtain ideas which will aid in the further prosecution of the investigation.

PROCEDURE

The fact that ionization or break down of occluded air and other gaseous films exists in laminated insulation, such as impregnated paper as used in cables, has been known for some time. However, opinions and theories differ as to the exact nature of this ionization and its effects on the insulation of the cable as a whole. A fundamental object of our investigation is to determine, so far as possible, the exact nature of these ionization phenomena and to suggest in explanation a rational theory which can be substantiated by experimental data.

Our first procedure, naturally, was to obtain ex-

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perimental data on samples of commercial cables under various conditions of temperature, of frequency and of voltge gradient. From the analysis of these data it was hoped that some of the laws of the ionization taking place within the cables could be determined, and a theory suggested to account for them.

In order to study such laws, it is necessary to make measurements with a degree of precision much higher than is required for commercial testing. To obtain high precision in measuring such losses is not easy, especially at the present time when cables are designed particularly to have low dielectric loss and low power factor. For example, most of the cables which we have tested have power factors of one-half of one per cent and even less, at voltages below the ionization voltage and at ordinary room temperature. In addition, the fact that our measurements needed to be made at voltages as high as 70 kv., and over a wide range of frequencies, made the development of the method of measurement still more difficult. Accordingly, before beginning the work, it was necessary to devise a satisfactory method of making the measurements. The authors investigated the various methods which have been described up to the present time and tried many of them. However, they were all found to be unsatisfactory because of their lack of precision or the difficulty in manipulation. After considerable experimenting, during which two or three promising methods were discarded, the one shown in the diagram of Fig. 1 was found to be the most satisfactory as regards accuracy, simplicity and ease of manipulation.

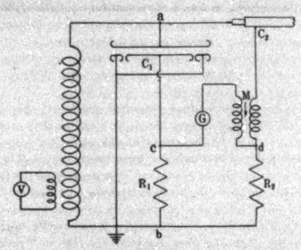

Fig. 1

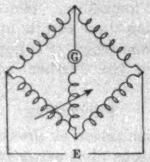

Fig. 2

Although the authors were not aware of it at the time it was designed and developed, this method happens to be a modified Heaviside[3] bridge, in which there is mutual inductance between one arm of the bridge and the galvanometer circuit, Fig. 2. A simplified diagram of the bridge is given in Fig. 3. Referring to Fig. 1, the cable and a resistance R_2 of the order of 50 to 100 ohms for a 10-ft. length of cable, form one side of the bridge, while a standard air condenser C_1 and a resistance R_1 of the order of 1000 ohms form the other side. The primary of a mutual inductance M is inserted in series with the cable. The galvanometer G is connected in series with the secondary of the mutual inductance between points c d of the bridge. The voltage drops across R_1 and R_2 are small, being of the order of one volt, so that no difficulty is experienced from the effects of the capacitance to ground of the detector. A simplified diagram of this bridge is given in Fig. 3. The cable is represented by the capacitance C_2 and its equivalent series resistance r_2. The standard air condenser is represented by the capacitance C_1, and its equivalent series resistance is shown as r_1. The resemblance of this bridge to the simple Heaviside bridge, shown in Fig. 2, may be seen by comparing the two diagrams. In both, the balance is obtained by the adjustment of the impedance in one of the bridge arms and a mutual inductance between one bridge arm and the galvanometer circuit. The vector diagram for the low-voltage portion of the bridge in Figs. 1 and 3 is shown in Fig. 4.

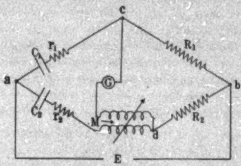

Fig. 3

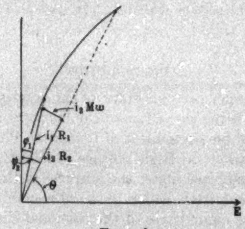

Fig. 4

The self-inductance L of the primary of the mutual inductance M is approximately 10 millihenrys, so that its impedance is small compared with that of the cable.

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[3] Heaviside: On the Use of the Bridge as an Induction Balance. *The Electrician*, Vol. 16, p. 489.

Hence in developing the equations for this bridge, the impedance of the primary of the mutual inductance M is neglected. In the equations for the bridge, let

$$\eta_1 = (R_1 + r_1)\, C_1\, \omega = \operatorname{Tan} \psi_1$$
$$\eta_2 = (R_2 + r_2)\, C_2\, \omega = \operatorname{Tan} \psi_2 \quad \text{(See Fig.4.)}$$

A simple calculation gives for the conditions of balance:

$$C_1 R_1 = C_2 R_2 \frac{(\eta_1{}^2 + 1)}{(\eta_1 \eta_2 + 1)} \tag{1}$$

and

$$\frac{M \omega}{R_2} = \frac{\eta_2 - \eta_1}{\eta_1 \eta_2 + 1} \tag{2}$$

Since η_1 and η_2 are small compared to unity, the second order terms may be neglected and:

$$C_1 R_1 = C_2 R_2 \tag{3}$$

and

$$\operatorname{Tan} \psi_2 - \operatorname{Tan} \psi_1 = \frac{M \omega}{R_2}$$

or

$$\operatorname{Tan} \psi_2 = \frac{M \omega}{R_2} + \operatorname{Tan} \psi_1 \tag{4}$$

If the condenser C_1 is a perfect air condenser, $\operatorname{Tan} \psi_1 = 0$, and ψ_2 is the angle of phase defect of the cable. If ψ_1 is not zero, the correction is simple. For example, we found that with our air condenser, $\operatorname{Tan} \psi_1 = 0.00240$ and $\psi_1 = 8.3'$. The correction, therefore, was merely a matter of adding $\operatorname{Tan} \psi_1$ to $\frac{M \omega}{R_2}$ in order to find the total angle of phase defect of the cable, as shown in Fig. 4.

Careful check measurements showed that our air condenser, although carefully insulated and guarded, did not have a phase angle of 90 deg. When it is realized that an angle of phase-defect of only 10 minutes in the air condenser will introduce an error of 60 per cent in the power and power factor when the power factor is 0.5 per cent, the importance of determining the angle of the air condenser and of correcting for it becomes apparent. After considerable difficulty, the authors were able to obtain this angle at the lower voltages by a substitution method which is described in Appendix I. It was not possible for the writers to determine the angle at the higher voltages as they did not have a high voltage air condenser whose loss would remain constant when its capacitance was varied. The angle of phase-defect measured at the lower voltages was assumed to remain unchanged at the higher voltages. Although this is probably not strictly true, the fact that the power factor increases rapidly with increase of voltage reduces the percentage of error.

The bridge was balanced by varying R_1 and M, R_2 remaining constant. If the air condenser be assumed perfect

$$\operatorname{Tan} \psi_2 = \frac{M \omega}{R_2} = \sin \psi_2 \text{ (Fig. 4).}$$

The power factor of the cable = cosine θ = $\sin \psi_2$ which is sensibly equal to $\tan \psi_2 = \frac{M \omega}{R_2}$, since ψ_2 is a small angle. Hence, for approximate work with the higher power factors, the mutual inductance may be calibrated directly in power factor for any given frequency. This is a distinct advantage both in research work and in routine testing.

Considerable difficulty was experienced in obtaining a suitable detector. The detector must not only be sensitive but must have moderately high impedance in order to work properly with the bridge. The various standard types of vibration galvanometer were tried, but none was found satisfactory. Therefore, it became necessary to design and construct one. The moving iron-vane type, having an electromagnet for excitation, was found to be the most satisfactory. This type of galvanometer has two distinct advantages. The detecting coil may be wound for very high impedance, which is not possible for the moving-coil galvanometer. The tuning may be easily accomplished by controlling with a rheostat the current in the field coils of the electromagnet. If the bridge is out of balance, the deflections of the galvanometer may be kept on the scale by slightly detuning the galvanometer. This galvanometer is far more sensitive than any which we were able to purchase. The control is very flexible and accurate. Its natural frequency of vibration is readily adjusted from 20 cycles to 300 cycles per second and throughout an even greater range, if desired.

With low voltages, or with small samples, the detector itself would not give the desired sensitivity. We, therefore, designed and constructed a three-stage resistance-coupled vacuum tube amplifier, which increases the sensitivity by something like 1000 times.[4]

With the foregoing apparatus it is possible to balance the bridge to measure phase angles to 0.001 per cent and detect even smaller changes. In view of the fact that the dielectric properties of the insulation which were measured are not constant, the foregoing precision is much greater than is necessary.

The voltage was measured by means of a voltmeter coil in the high-voltage winding of the 100-kv., 15-kv-a., 60-cycle transformer which supplied the power. The multiplying factor of the voltmeter coil was determined by a null method described in Appendix III.

The standard condenser consisted of two polished copper plates with rounded edges and corners. The low-voltage plate is guarded as is indicated in Fig. 1. The area of the working plate is 4 ft. × 5 ft. (1.219 m. × 1.525 m.). The distance between plates can be adjusted to suit the voltage, hence constant sensitivity is obtained.

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[4] See TRANS. A. I. E. E., 1925, p. 122, Discussion by C. L. Dawes.

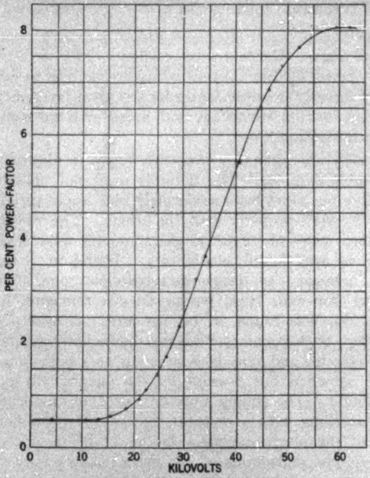

Fig. 5—5,000,000-Cir. Mils, Single Conductor Cable

20/32 inch insulation. 1/8-inch lead
20 deg. cent. 60 cycles per second

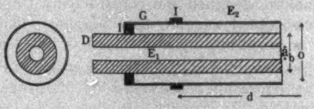

Fig. 6

E_1	Mercury electrode
E_2	Brass tube
I	Bakelite rings
G	Guard Rings
D	Glass tube
a	3.1 mm.
b	9.3 mm.
c	14.3 mm.
d	31.5 cm.

The capacitance C_1 of the standard condenser was measured with a precision capacitance bridge. Knowing this capacitance and the resistance R_1, the capacitance C_2 of the cable is determined. It is then a simple matter to calculate the power, since the power

$$P = E^2 C_2 \omega \sin \psi_2$$

where ψ_2 is the angle of phase-defect of the cable. If ψ_2 is small, the power

$$P = \frac{E^2 C_2 M \omega^2}{R_2}$$

If an ammeter be connected in series with the cable, then knowing the voltage and power factor, the power may be readily calculated.

A 15-kv-a., 220-volt, 60-cycle alternator known to have a reasonably good sine wave[5] was used in all these measurements. Its voltage was controlled with a motor-driven field rheostat.

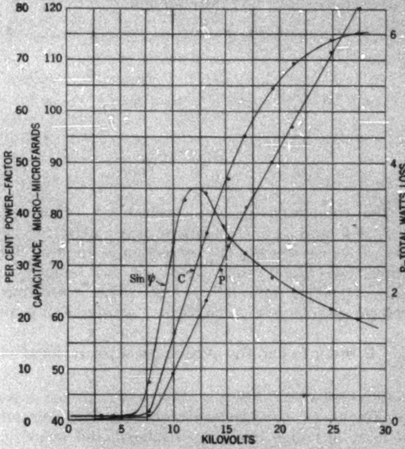

Fig. 7—Cable Model. Glass-Air Dielectric

20 deg. cent. 60 cycles per second

Measurements. Measurements were made at first on single-conductor cables having walls of insulation of approximately 20/32 in. thickness (16 mm.). Such cables would ordinarily be rated at 33 kv. Fig. 5 shows a typical power-factor curve obtained with a 500,000-cir. mils. cable having this thickness of wall. We noticed that at 50 kv. the rate of increase of the power factor became less. This has been observed by others.[6] At 60 kv. it apparently reached a maximum and then decreased slightly. This decrease of power factor in high-voltage cables after a maximum is reached has been known for some time among engineers.[7] So far as we know, however, no satisfactory explanation has been given for this character of power-factor curve. In fact, this decrease in power factor appeared

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5. For an oscillogram of this voltage wave, see "Artificial Transmission Lines," Kennelly & Lieberknecht, Trans. A. I. E. E., Vol. XXXI, 1912, p. 1138.

6. C. F. Proos. "Some Considerations on the Dielectric Losses in High-tension Cables." Report to Ass. of Managers of Electrical Works in the Netherlands, 1921. (Proos plotted P/E^2, or equivalent conductance, against voltage which gives the same character of curves as power factor against voltage, provided the capacitance of the cable does not change very much.)

7. D. W. Roper. Trans. A. I. E. E., 1925, p. 116. Curve *E*, Figure 4. Also *Electric World*, Feb. 21, 1925, p. 399.

to be contrary to the well-known explanations heretofore given to account for the increasing power factor with increasing potential gradient. That is, the ionization commences in the air spaces adjacent to the conductors and occurs progressively in the air spaces from the center outwards as the potential is raised. In view of these uncertainties, it appeared to us that further study of this phenomenon might give some additional information as to the nature and effects of ionization in cables.

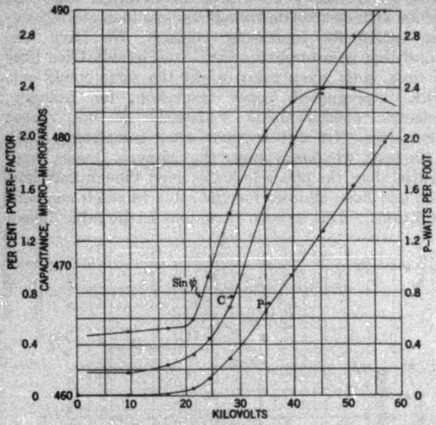

Fig. 8—Impregnated Paper Cable. No. 00 Single Conductor

9/32-inch insulation. 20.4 deg. cent. 60 cycles per second

Schrader[8] also found with small air spaces in flat dielectrics that the power factor increased to a maximum and then decreased to a low value with increase of voltage.

For purposes of study, we made a small cable model, Fig. 6, in which the ionization effects occurring in actual cables would be exaggereated. A glass tube having an inside diameter of 3.1 mm. and an outside diameter of 9.3 mm. was filled with mercury, the mercury forming the center electrode. A brass tube having an inside diameter of 14.3 mm. was placed concentric with this glass tube and insulated from it. Suitable guard rings were used, as shown in Fig. 6. The length of the brass tube between guard rings was 31.5 cm. Fig. 7 shows the curves of power factor, capacitance and watts loss obtained with this cable model at room temperature and at a frequency of 60 cycles per second. It will be observed that the power-factor curve begins to increase rapidly at 6.5 kv. and the other two curves at 8 kv. Also the power factor rises rapidly to a maximum of 45 per cent and then actually does begin to decrease, as we had anticipated. After passing its maximum, the rate of decrease at first is high, but becomes less with increase of voltage until at 27.5 kv. the power factor is only 20 per cent. The capacitance is constant below the ionization voltage. It increases rapidly just beyond the ionization voltage. At first the increase is linear, but at higher voltages the increase is less rapid, the curve drooping slightly. This is the character of curves which we expected the cables would have.

In order that we might obtain sufficiently high voltage gradients in cables to produce similar effects, it was found desirable to use smaller cable. This enabled us to use lower maximum voltages, thus making it easier to insulate the cable ends, and also reducing the probability of errors due to inevitable small corona formation in our air condenser. Accordingly, a 00 A. w. g. (diameter = 0.418 in. = 10.6 mm.) single-conductor cable with a 9/32 in. (7.1 mm.) wall of insulation was used. In accordance with ordinary American practise, this cable would probably operate at 12 kv. The cable was guarded at its ends and end bells filled with impregnating compound were fastened to the ends to prevent the compound from oozing out and air entering with change in temperature. The net length of cable tested was 7½ ft. (2.3 m.). Curves taken for this cable under various conditions are shown in Figs. 8 to 16, inclusive.

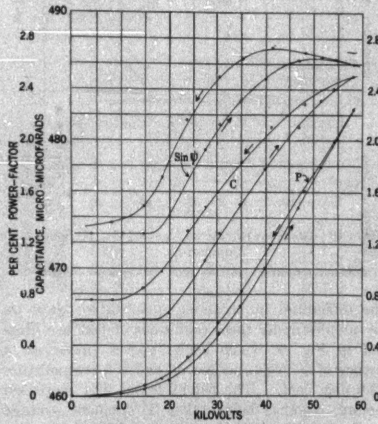

Fig. 9—Impregnated Paper Cable. No. 00 Single Conductor

9/32-inch insulation. 53.8 deg. cent. 60 cycles per second

In Fig. 8 curves of power factor, capacitance and watts per foot are given for a frequency of 60 cycles per second and at a temperature of 20.4 deg. cent.

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[8] Schrader. "Corona in Air Spaces in a Dielectric." Trans., A. I. E. E., Vol. 41, 1922, p. 583.

as shown. At each reading sufficient time was allowed for the reading to become constant. It will be noted that the power factor after reaching a maximum value of 2.4 per cent actually does begin to decrease, as had been anticipated. The capacitance begins to increase at 10 kv., corresponding to an apparent potential gradient of 22 kv. per cm. at the conductor surface. The rate of increase of capacitance is a maximum at 30 kv., at which potential the capacitance curve has a point of inflection. Beyond this potential the rate of increase diminishes.

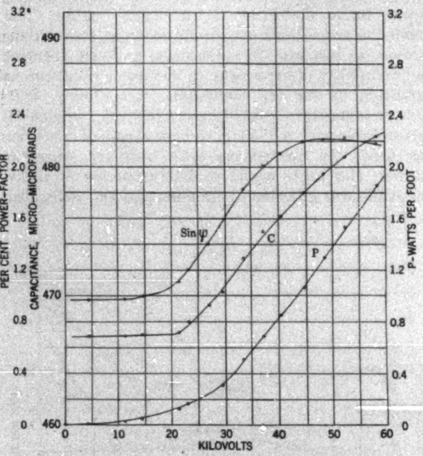

FIG. 10—IMPREGNATED PAPER CABLE. No. 00 SINGLE CONDUCTOR

9/32-inch insulation. 45.9 deg. cent. 60 cycles per second

The difficulties in obtaining quantitative data on cable samples under these conditions of test are illustrated in Fig. 9, which shows curves taken at a frequency of 60 cycles per second and at a temperature of 53.8 deg. cent. The potential was first increased as before, readings being taken at frequent voltage intervals, and only after conditions had apparently become constant. When 58 kv. was reached the potential was decreased and readings taken at intervals to zero voltage. These readings, however, were taken immediately following each adjustment of voltage, a pause of only two or three minutes being necessary for taking each reading. Hence the dielectric was not given time to attain constant condition. It will be noted that the values of power factor, watts per foot and capacitance are all considerably greater for decreasing voltage than for increasing voltage. It should also be noted that with increasing voltage the watt curve and the power-factor curve begin to increase at a considerably lower voltage than the capacitance curve. Moreover, with decreasing voltage the curves reach constant values at lower values of voltage than they do with increasing voltage. Before taking each reading with decreasing voltage, if sufficient time for conditions to become constant be allowed, the values obtained are in most cases only slightly greater than the corresponding results with increasing voltage. In some instances, however, it has been found even when steady conditions are reached that the values with decreasing voltage are less than corresponding values for increasing voltage. We have also found that measurements vary considerably from time to time, even when taken under the same conditions. With impregnated paper insulation, for example, dielectric measurements frequently do not repeat themselves. On one occasion after completing a 60-cycle test, we found that the 25-cycle losses were greater than the 60-cycle losses even though sufficient time had been allowed for the cable to reach constant conditions. Due to such variations in the cable dielectric, great care must be taken if quantitative data are to be of value.

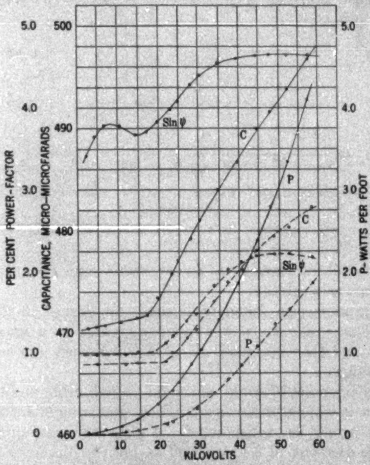

FIG. 11 —IMPREGNATED PAPER CABLE. No. 00 SINGLE CONDUCTOR

9/32-inch insulation. 60 cycles per second. 88 deg. cent. (Dotted curves are Fig. 10)

In the opinion of the authors, one reason why with decreasing voltage the values of power factor, watts loss and capacitance are greater than with increasing voltage is that a large number of free ions still persist in the air spaces of the dielectric. The effect of dielectric absorption may also be present. Hence, the ionization effects will be greater for any given potential gradient than is the case with increasing voltage.

Other contributing causes of the higher values with decreasing voltage are the slightly higher internal temperature and the fact, which is well known, that a dielectric which has been over-stressed shows lower insulation resistance and greater dielectric loss than it does before being overstressed.

Fig. 10 gives curves taken at a frequency of 60 cycles per second and at a temperature of 45.9 deg. cent. At higher temperature, the ionization effects begin at lower potential as would be expected on the basis of present-day ionization theories. In Fig. 10 also both the power and the power factor curves commence to increase rapidly at a lower potential gradient than the capacitance curve. We find this effect to be true in every case except that of Fig. 8, in which the capacitance curve commences to increase rapidly at a lower potential gradient than either the power or the power factor. Fig. 8 is the only exception to this effect. Our records show that the data from which Fig. 8 was plotted were taken shortly after another test had been completed. Probably in this instance sufficient time was not allowed for the cable to reach its stable condition.

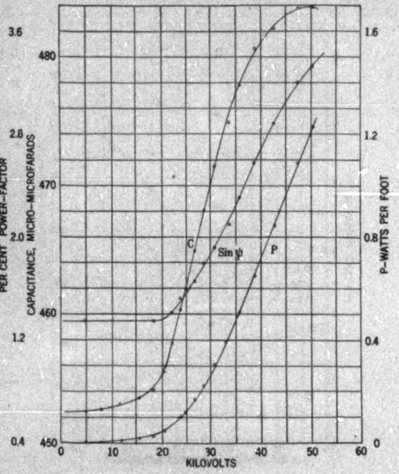

Fig. 12—Impregnated Paper Cable. No. 00 Single Conductor

9/32-inch insulation. 32 cycles per second. 20 deg. cent.

In Fig. 11 the solid lines are the curves of power factor and capacitance taken at 60 cycles but at a temperature of 88 deg. cent. It will be noted that the power factor increases rapidly when the potential is increased from 2 to 8.5 kv., at which point it reaches a first maximum. It then decreases, reaching a minimum at 15 kv. and again increases in the ordinary manner. This peculiar effect cannot be due to any errors in measurement, since we checked this portion of the curve carefully. Moreover, this effect occurs only at high temperatures. It appears again in Fig. 13 taken at a temperature of 98.5 deg. cent. and at a frequency of 32 cycles per second. The authors are not prepared at this time to offer a definite explanation of this sudden variation in the power factor. They feel, however, that it must be due to physical changes in the dielectric, either in the paper or in the impregnating compound, or possibly in both. Since we did not have any sample of the materials used in the manufacture of this cable, we were unable to make further analysis, but we intend to study this effect at a later date.

It will be observed at the higher temperatures that the range of maximum power factor is greater than at lower temperatures; also the power factor does not begin to decrease at as low a voltage as it does at the lower temperatures, and further the decrease in power factor is not so marked.

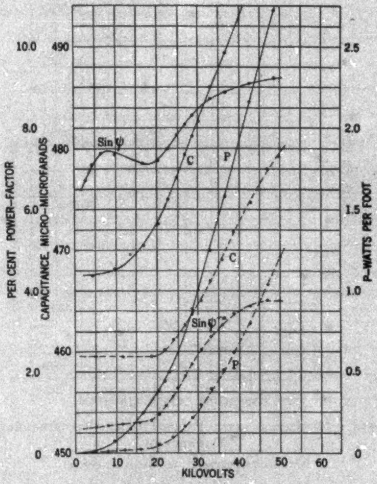

Fig. 13—Impregnated Paper Cable. No. 00 Single Conductor

9/32-inch insulation. 32 cycles per second. 98.5 deg. cent. (Dotted curves are Fig. 12)

Fig. 12 gives the three typical curves taken at a frequency of 32 cycles per second and a temperature of 20 deg. cent. It will be observed that the watts per foot are less for any given potential, but that the power factor and capacitance are greater than those obtained at a frequency of 60 cycles per second and approximately the same temperature (Fig. 8).

In Fig. 13 the solid lines are curves taken at a frequency of 32 cycles per second and a temperature of 98.5 deg. cent. It will be observed that this power-factor curve has the same peculiar characteristic which appears in the 60-cycle curve, Fig. 11. For purposes of comparison, the curves of Fig. 12 are shown dotted.

Fig. 14 shows a set of curves in which the watts per foot are plotted as a function of temperature, each curve being taken at a constant voltage. The frequency throughout is 60 cycles per second.

Fig. 15 shows curves of power factor as a function of temperature, taken at various voltages. We feel that the reason the 60-kv. curve crosses the 50-kv. curve at 40 deg. cent. is the fact that the dielectric in the cable was beginning to weaken under the repeated high gradients at which we were operating. Fig. 16 gives similar curves for capacitance. These curves show consistent change in capacitance in the neighborhood of 45 deg. cent. at the lower voltages. Clark and Shanklin[9] obtained similar curves for the relative permittivity of cables. We feel that this is a property of the dielectric but cannot give at this time a conclusive explanation.

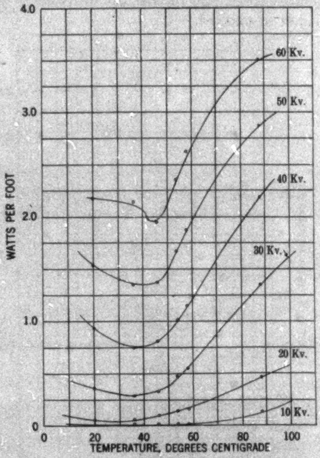

Fig. 14—Impregnated Paper Cable. No. 00 Single Conductor

9/32-inch insulation. 60 cycles per second

The curves, Figs. 14, 15, and 16, are similar to curves which have been shown in the past, in which the power factor and watts per foot reach minimum values and then begin to increase with further decrease in temperature. Owing to the fact that samples of the compound which was used for impregnating this cable were not available, the authors are not prepared to state definitely at this time the reason why the cables reach a minimum loss and power factor per foot at some temperature in the neighborhood of 40 deg. to 43 deg. cent. However, it has occurred to us that this minimum might possibly be due to a change in the physical structure of the impregnating compound at these temperatures. Hence, a cooling curve was taken for a sample of paper-cable impregnating compound which we happened to have in our laboratory. It was found that if its temperature be plotted as a function of time, a change in the slope of the temperature curve occurs at 40 deg. cent. The authors conclude that a physical change, involving a change in energy relations in the molecule, probably occurs at this temperature. This effect might well account for points of minimum power factor which have so frequently been observed by others and for which thus far no explanation has been given. The authors intend to study this effect further at a later date.

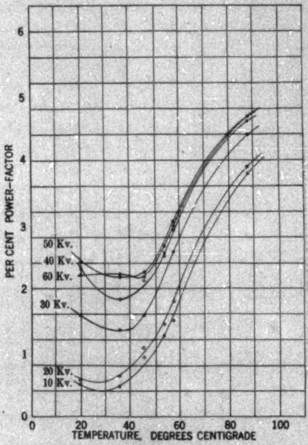

Fig. 15—Impregnated Paper Cable. No. 00 Single Conductor

9/32-inch insulation. 60 cycles per second

In a measure at least, the foregoing results are explainable on the basis of ionization. However, we feel that this explanation needs to be supplemented by further analysis of the effects of ionization.

Analysis of Ionization Phenomena

Since the curves taken with our glass-tube cable model (Fig. 7) have all the dielectric characteristics of the actual cable, it seemed as if further data taken with such models might be of considerable assistance in the analysis of cable phenomena, particularly in view of the fact that the composite dielectric of glass and air is much simpler than that of paper, a compound of low viscosity, and air. Accordingly, another model similar to that of Fig. 6 was made and curves taken, Fig. 17. The glass of this model was slightly different from that of the first model, the curves for which are shown in Fig. 7. Aside from slight quantitative differences, these curves are similar to those of Fig. 7.

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[9] Clark and Shanklin, "Insulation Characteristics of High-Voltage Cables," Trans. A. I. E. E., Vol. XXXVI, 1917, p. 447 (Figure 10).

In attempting to analyze our own results, as well as those of Schrader, we found it necessary to know the effect of corona formation on the voltage gradient in air spaces and on the capacitance of air spaces. With the formation of corona about cylindrical conductors, it has frequently been assumed that corona increases the effective diameter of the electrode.[10] This may be true, but certainly this assumption must be qualified to the extent that this corona formation is not equivalent to increasing the diameter of the electrode in the sense that the voltage across the corona formation is negligible. Peek,[10] later in his book, recognizes this fact for on page 85 he states: "Hence, corona seems to be either in *effect* a 'series resistance,' or it grades or distributes the flux density."

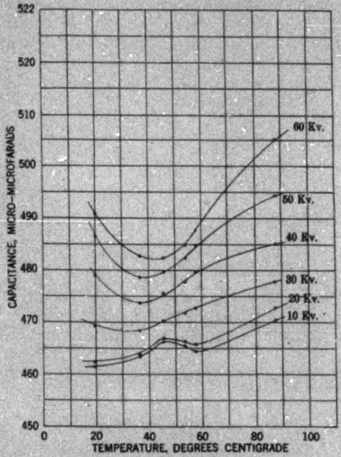

FIG. 16—IMPREGNATED PAPER CABLE. NO. 00 SINGLE CONDUCTOR

9/32-inch insulation. 60 cycles per second

If, for example, in a composite dielectric of solid material and air, the voltage across the air spaces were neglected after corona is formed, the capacitance would increase abruptly at the critical gradient and immediately attain a nearly constant value. With our glass cable models, the capacitance should behave in almost this same manner when the gradient is sufficient to cause corona throughout the air space between the glass tube and brass cylinder. The outer and the inner radii of the air space (Fig. 6) are in the ratio of 1.54 to 1, or very much less than the critical ratio of 2.718 (Napierian base ϵ). Therefore, our glass tubes, so far as this effect is concerned, are equivalent to flat air spaces and the capacitance should increase suddenly and reach almost immediately a constant value. A study of Figs. 7 and 17 shows that this is not true.

In order to obtain additional information on the effect of corona on the voltage gradient in air spaces and on the change of capacitance, the variation of power factor and capacitance with voltage was determined for a concentric-cylinder condenser with air only as a dielectric. The outer cylinder was provided with guard rings and the working section between guard rings was 55 in. (140 cm.) long. The center electrode was a smooth No. 14-A. w. g. wire having a diameter of 64 mils (0.1635 cm.) and the inner diameter of the outer cylinder was 7.34 cm. This gives 45 to 1 as the ratio of outer to inner diameter, which is so high as to give ample opportunity for corona to form before sparkover occurs. The curves taken for this condenser are shown in Fig. 18.

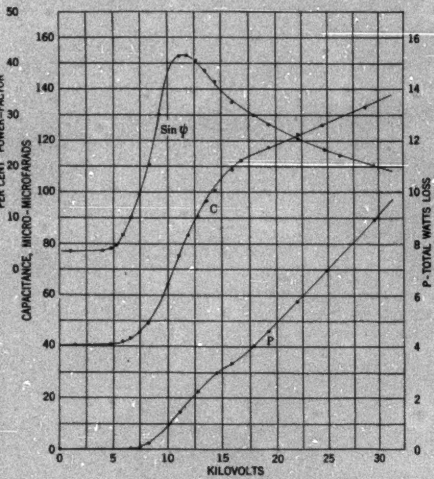

FIG. 17—CABLE MODEL. GLASS-AIR DIELECTRIC

20 deg. cent. 60 cycles per second

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[10] F. W. Peek, Jr., "Dielectric Phenomena in High-Voltage Engineering," p. 27.

M. F. Gardner, "Corona Investigations on an Artificial Line," JOUR. A. I. E. E., Aug. 1925, p. 813.

J. J. Ryan and H. H. Henline, "The Hysteresis Character of Corona Formation," JOUR. A. I. E. E., Sept. 1924, p. 826. (The statement, however, is omitted in the TRANSACTIONS.)

The capacitance is given by

$$C = \frac{0.555\ L}{\log_e \frac{R}{r}}\ \mu\,\mu\,f.$$

where R is the inner radius of the outer cylinder, r the radius of the center conductor and L the length in cm.

If it be assumed that corona increases the effective radius of the inner electrode to a value r_1, at which radius the gradient again becomes equal to the breakdown gradient of the air, the ratio $\frac{R}{r_1}$ cannot exceed 2.718.

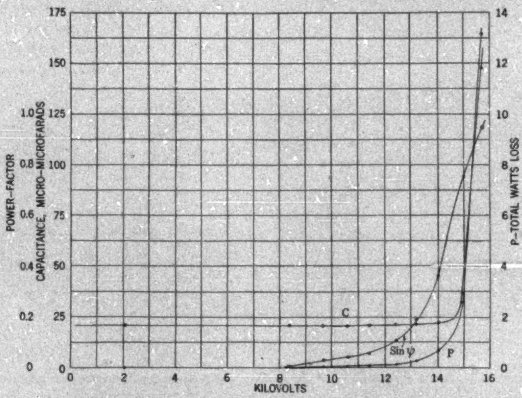

Fig. 18—Concentric Cylinders. Air Dielectric

4 = 7.34 cm. r = 0.1625 cm. Length = 140 cm.

Were the effect of corona to increase the effective diameter of the inner electrode, the capacitance of the foregoing condenser cannot exceed

$$C = 0.555 \times 140 = 77.7\ \mu\,\mu\,f.$$

A study of Fig. 18 shows that a capacitance of more than twice this value actually was obtained before sparkover occurred at 15.9 kv. On the basis of the assumption that the corona increases the effective diameter of the center electrode, this would give a diameter of 4.37 cm. at sparkover, or an apparent ratio of outer to inner radius (R/r_1) of 1.68. The increase of capacitance, therefore, would indicate either that the gradient through corona formation is not small, but is comparatively large, or that ions in the air space increase the capacitance of the cylinder by a large amount. In the opinion of the author, these two effects occur simultaneously.

It is well known that ionization increases the capacitance of a gaseous medium.[11] In the initial stages of corona this increase is probably due to the doublet action within the atom, and in later stages the space charge effect increases the capacitance.

Faraday, in attempting to explain the greater dielectric constant of solid material, was the first to assume that the molecules of the solid became polarized, due to induction. This shortens the tubes of force and hence increases the capacitance. Faraday also showed that these polarized molecules (or doublets, as they are now called) may be compared with small insulated globular conductors. Mossotti put this theory on a mathematical basis. Jeans replaced the idea of conducting spheres with clustered electrons, and in this manner uses Mossotti's hypothesis to reconcile dielectric phenomena with modern views as to the structure of matter. Essentially, Jean's assumption is to consider the surface of the molecule as an equipotential surface.

In the first stages of ionization the electron probably moves only from an inner to an outer orbit, thus absorbing energy. In so doing, it increases the doublet action of the molecule and thus tends to increase capacitance slightly. When the gradient has become sufficient to cause the electron to leave the molecule, free charges are separated to a finite distance. The presence in the gaseous dielectric of ions and electrons separated by finite distances will increase the capacitance since it will cause a shortening of some of the lines of force. Hence, when the voltage gradient becomes sufficient to cause the electrons to leave the atoms and to produce further ionization by collision, a rapid increase of capacitance should result.

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[11] See J. J. Jeans, "Electricity and Magnetism," Molecular Action in a Dielectric, p. 126.

Harris J. Ryan. (Discussion) Trans. A. I. E. E., Vol. XXXII, 1913, p. 1827.

Our results appear to bear out this theory. In practically every curve, Figs. 7 to 13, inclusive, and in Figs. 17, 18, and 23, the power and the power factor begin to increase before the capacitance increases appreciably. That is, at first the electron, in moving from an inner to an outer orbit, absorbs energy and increases the capacitance slightly. It gives out this energy as heat and visible corona, in falling back to its initial orbit. This would account for an increase of power with but a small increase in capacitance. When, however, the gradient reaches a value sufficient to cause electrons to leave the atoms, the capacitance increases rapidly, due to the space charge effect.

So far as is known, no attempt has been made to determine the law connecting change in capacitance and ionization. Accordingly, some preliminary investigations of this phenomenon have been made by the authors and some of the results are given in Appendix IV. These results show that with a flat air space 2 mm. thick the capacitance of the air space when ionized reaches a value which is 28 times the capacitance for the non-ionized condition.

In addition to the increase in capacitance in the air films, when ionization takes place, the relation of voltage gradient to current also changes. To show in more detail the effects of this relationship, a reproduction of some curves given on page 267 of Townsend's "Electricity in Gases," is shown in Fig. 19. These curves show the current-voltage relation of air with parallel plate electrodes. The curves are taken for spacings of 2, 1, and 0.5 cm. The pressure in each case was 1.1 mm. of mercury and the initial ionization was produced by X-rays. These curves represent some of the first data taken in experimental work on ionization phenomena and are now considered quite elementary. Nevertheless, they are important in that they appear to offer explanations of some of the ionization phenomena which we have observed.

At low voltages, a current which, although very small, is finite flows through the air space. This current is due to the motion of the free ions and is practically independent of the voltage. In air, under ordinary conditions, this so-called "saturation current" is very small, and is practically negligible.

With increasing voltage the velocity of the moving ions increase and ionization by collision begins. As a result, the current increases rapidly, but there is no discontinuity in the curve. The curves have sharp bends, however, and after a certain voltage gradient is reached, the current density increases so rapidly with slight increases in voltage that we usually regard these curves as discontinuous and consider that the air has a definite breakdown gradient. It is important to remember that although the curves are not discontinuous with voltage gradients beyond the bend, the current density increases enormously with further slight increase in gradient. In the limit the current density becomes so great that sparkover or arc discharge results and the curve then takes a negative slope. With air films in solid dielectrics, the current is restricted by the solid dielectric in series with the air film, and the discharge does not go beyond the glow or corona formation, unless rupture of the entire dielectric occurs.[12]

From the curves, Fig. 19, it is apparent that for each spacing of electrodes the voltage gradient *increases* with the ionization current density until a limiting voltage gradient, which cannot be exceeded, is reached. With the 2-cm. spacing, Fig. 19, this gradient is approximately 180 volts per cm. and with the 1-cm. spacing it is approximately 300 volts per cm. With air, under ordinary conditions of temperature and pressure, this gradient is 30 kv. per cm. (maximum), except with very thin films.[13] With any current whose magnitude is consistent with values of currents existing in dielectrics, such as are given in Fig. 19, the voltage gradient in the air is never zero. On the contrary, the gradient increases with increasing current density, approaching a definite maximum gradient for that particular air film. These effects have been verified by the tests made on ionized air, the results of which are given in Appendix IV.

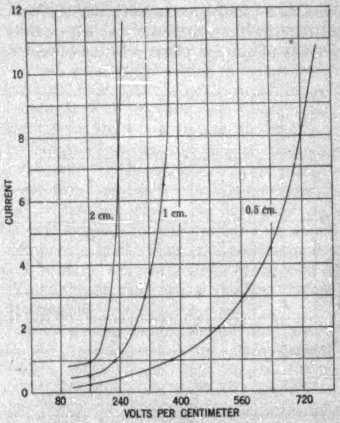

FIG. 19—CURRENT-VOLTAGE CURVES FOR AIR. PARALLEL ELECTRODES

1.1 mm. Hg. (Data from Townsend "Electricity in Gasses," page 267.)

Returning now to the cable models, let e_1 be the potential across the air dielectric, and e_2 the potential across the glass dielectric. Until the ionization voltage is reached, the potentials e_1 and e_2 will be inversely proportional to the normal capacitances c_1 and c_2 of the air and glass. Beyond this voltage, however, the potential e_1 cannot increase appreciably but must remain substantially constant. Hence, any further increase in the total voltage must be taken by the capacitance c_2. Curves illustrating this effect are given in Fig. 20, in which the voltages e_1 and e_2 are plotted as functions of the total voltage E. Each of the voltages e_1 and e_2 increases proportionately to E until 7.5 kv. is reached, beyond which point e_1 remains constant. For voltages greater than this critical voltage, any increase in E produces in equal increase in e_2. This assumes that e_1 and e_2 are substantially in phase. This is not strictly true, particularly at large values of E, as e_1 tends to come in phase with the current because of ionization losses in the air space. This would only change the slope of e_2 without changing its general character.

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[12] A. L. Atherton, "1922 Developments in Autovalve Lightning Arresters," TRANS. A. I. E. E., Vol. XLII, (1923) p. 179.

[13] F. Dubsky, "The Dielectric Strength of Air Films Entrapped in Solid Insulation," TRANS. A. I. E. E., Vol. XXXVIII, 1919, p. 537.

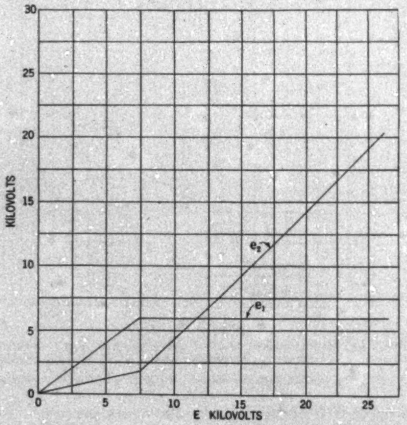

FIG. 20—VARIATION OF e_1 WITH TOTAL VOLTAGE, E

In Fig. 29, Appendix IV, are shown similar curves which were obtained experimentally for a combination of glass in series with a flat air space. These curves are of the same character as those of Fig. 20.

When the air spaces are ionized, their capacitance decreases, and due to energy loss, the current is probably almost in phase with the voltage across the air space, or in phase with e_1, Fig. 20. Hence, the power lost in the air space should be proportional to the current. We find this to be nearly true with the power curves in both Figs. 7 and 17. If correction be made for the increase of loss in the glass which is included in these power curves, the agreement is still closer. In order to avoid confusion, we have not plotted the current in these figures. It is easily determined, however, from the capacitance curves and total voltage E, since $I = E\,C\,\omega$.

We feel the foregoing relationship to be important, since it suggests a method by which the actual ionization loss in a gaseous dielectric may be separated from the loss in solid or non-ionized dielectrics. It also goes to substantiate the fact of the voltage gradient across ionized air spaces remaining substantially constant after corona forms.

It seems reasonable, therefore, to conclude that corona in gaseous films produces a definite limiting gradient in the film which depends on the nature of the gas, the pressure, the temperature and the geometry of the film. The current density in such an ionized gas does not depend on the voltage across the film, but rather on the impressed voltage and the remainder of the dielectric circuit. That is, the solid or non-ionized dielectric will limit the current density and it cannot increase indefinitely, as is indicated by the curves of Fig. 19, until rupture of the entire dielectric occurs.

This gives a condition which may be considered as *restricted ionization*.

Decreasing Power Factor. Schrader,[5] in explaining the decreasing power factor with increasing voltage, which he obtained, states that it "indicates that saturation of ionization is approached." Our ordinary understanding of the statement "saturation of ionization" is that practically all the molecules have become ionized and the resulting ions and electrons are acting as carriers of electricity across the gaseous film. Calculation shows that this condition cannot exist in such air films. For example, if it be assumed that there is but one electron per molecule carrying the ionization current, the current density through the ionized gas would be several amperes per sq. cm. Obviously, no such current density is possible in air films within dielectrics.

Proos[6] plotted conductance against kilovolts and obtained curves very similar to power-factor curves, but consisting of three straight lines. He explains the greater slope which occurs just above the ionization voltage, and the lesser slope which occurs at a higher voltage, as follows:

> "When the ionization voltage is exceeded, the air directly around the conductor is ionized first because here the intensity of the electric field is the highest. As the voltage is increased still further the air farther away from the conductor is also ionized and when we reach the second step in the voltage curve nearly all the air in the cable seems to be ionized. The remaining particles of the air will be ionized gradually as a higher voltage is applied. It is very probable that the last part of the voltage curve will be horizontal again when all the air in the cable has been ionized or when the remaining non-ionized air particles are so small that their

> ionization cannot affect the results in a measurable way." (Translated.)

If every molecule of air in the cable were ionized, the current density would be greater than could possibly exist in insulation. Proos possibly meant to say "when the corona *is formed* in every air space" rather than "when all the air in the cable has been ionized." Moreover, in our curves the maximum values of power factor occur at a potential gradient of approximately 110 kv. (r. m. s.) per inch or practically twice that of the breakdown of air under ordinary conditions—53 kv. (r. m. s.) per inch. Hence, we do not feel that this explanation is an adequate one, as will be shown later.

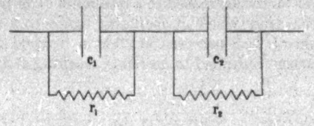

Fig. 21

Clark and Shanklin[9] plotted "specific resistivity (effective)" or P/E^2, or the equivalent shunt resistance of the cable, against kilovolts. (This is the reciprocal of the quantity which Proos plotted.) They explain the lesser rate of decrease of their curves as follows:

> "Gradually a point of saturation is reached and the energy consumed by ionization approaches more nearly a true ohmic loss."

Shanklin and Matson[14] explain similar characteristics as follows:

> "Complete saturation is not yet reached, but the curve is flattening out, showing that the majority of the gas spaces have been ionized."

Although these statements are in a sense correct, we do not feel that they are adequate explanations of the phenomena. Even though ionization is a contributing cause, the character of these curves is not determined solely by the degree of ionization, but rather by the changes, due to ionization, in the constants of the series condensers consisting of the solid dielectric and the air films.

The equivalent circuit of the solid dielectric and air space is shown in Fig. 21, in which c_1 is the capacitance of the air space and r_1 is its equivalent shunted resistance; c_2 is the capacitance of the solid dielectric and r_2 is its equivalent shunted resistance. It can be shown that the sine of the angle of phase-defect of the combination is

$$\sin\psi = \frac{e_2 \sin\psi_2 + e_1 \sqrt{1 - \dfrac{(e_1 c_1)^2}{(e_2 c_2)^2}}}{E} \qquad (5)$$

where e_1 and e_2 are the voltages across c_1 and c_2, E is the total voltage and ψ_2 is the angle of phase-defect of the solid dielectric. (See Appendix II.)

Below the ionization voltage, $e_1 c_1 = e_2 c_2$ since e_1 and e_2 are substantially in phase with each other, hence the radical is equal to zero and the power factor equals

$$\sin\psi = \frac{e_2 \sin\psi_2}{E}.$$

Above the ionization voltage, e_1 and c_2 remain nearly constant. c_1 and e_2 both increase very rapidly (Figs. 20, 29 and 30). From our experimental data with both the cable models and with flat air spaces (Appendix VI) we know that immediately above the ionization voltage e_2 increases more rapidly than c_1. Hence, the expression under the radical sign increases rapidly, which causes a rapid increase in sine ψ or in the power factor of the combination. Also, when ionization commences, the left-hand term becomes small in comparison with the right-hand term. At higher values of the voltage E, c_1 commences to increase very rapidly also (see Fig. 30), and the rate of increase of the right-hand expression becomes less than the rate of increase in E. As a result, with further increase in E, sin ψ and hence the power factor of the combination commences to decrease. Therefore, above the ionization voltage, sin ψ at first increases rapidly, reaches a maximum and then decreases. In the limit e_2 is nearly equal to E, and $\sin\psi = \sin\psi_2$, the power factor of the solid dielectric. The ordinary cable would puncture before this limit was reached.

We realize that in a cable the air films are not all ionized simultaneously, but the films nearest the conductor probably ionize first, due to the higher gradient there, and a greater and greater number further from the conductor become ionized as the voltage is increased. Hence, the power factor is determined by the summation of the effects of the separate films. This results in power-factor curves which rise less rapidly and which have more gradual changes in slope, as is shown by comparing the curves taken with actual cables with those of model cables.

The results of Proos may be analyzed in a similar manner. For example,

$$\frac{P}{E^2} = C\,\omega\,\frac{e_2 \sin\psi_2 + e_1 \sqrt{1 - \dfrac{(e_1 c_1)^2}{(e_2 c_2)^2}}}{E} \qquad (6)$$

which is similar to (**5**) multiplied by C. Proos did not carry his gradients to sufficiently high values to obtain decreasing values for (**6**). The curves of Clark and Shanklin and of Matson and Shanklin are merely the reciprocal of (**6**), and are subject to the same analysis. Their gradients were not carried to sufficiently high values to obtain increasing values for the reciprocal function of (**6**).

Determining as closely as possible the constants of our glass cable models, we have calculated the power factor for the model whose curves are shown in Fig. 7, using the foregoing equation. We assumed that the ionization voltage began at a definite value of 7.5 kv. In Fig. 22 the experimental curve and the calculated curve are plotted. In our opinion, the agreement between the curves is unusually good, particularly for dielectric phenomena of this character. These curves show that the foregoing theories are correct not only in a qualitative sense, but substantially so in a quantitative sense as well. They also show that the decreasing power factor is not caused by such phenomena as saturation of ionization but is merely the natural result of a combination of series capacitances, of this character. We conclude from this that when ionization exists in cables the power factor alone cannot be used to determine the degree of ionization in any one cable or to compare the ionization characteristics of different cables.

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[14] Shanklin and Matson, "Ionization of Occluded Gases in High-Tension Insulation," Trans. A. I. E. E., Vol. XXXVIII, 1919, p. 489—See page 494.

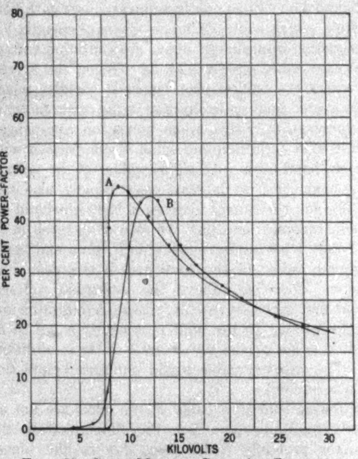

FIG. 22—CABLE MODEL. GLASS-AIR DIELECTRIC

A. Calculated
B. Experimental

It is apparent that it is possible to calculate with a fair degree of precision the effects which may be expected in solid and gaseous dielectrics, provided we know the constants of the solid medium and the constants of the gaseous films.

It has been recognized for some time past that, although the paper itself has lower dielectric strength than the impregnating compound, the dielectric strength of the cable is increased by using paper in combination with the compound. This is probably due to a considerable extent to the fact that the paper acts as a barrier,[15] which prevents or restricts the motion of the ions through the insulation. We felt it worth while to study these effects with our cable models.

The glass tube of the model whose characteristics are shown in Fig. 17 was wrapt tightly with dry manila paper, such as is used for insulating cables. The brass tube was then slid over the paper. Thus we produced a cable model having a composite insulation of glass, air and paper.

Fig. 23 shows the curves of power factor, watts loss and capacitance plotted with kilovolts which we obtained with this model cable. The ionization point started at a lower voltage than with air and glass alone as the dielectric. This would be expected. The power factor increased to a first maximum of 24 per cent at 7 kv. When the voltage was increased above 7 kv. the power factor decreased to a minimum of 18 per cent at 16 kv., after which it increased again to a second maximum of 27.5 per cent at 22 kv. Beyond this the power factor continued to decrease, reaching at 32.5 kv. nearly a constant value. The capacitance curve increases steadily with increased kilovolts, but shows changes in slopes corresponding to the maxima and minima of the power-factor curve. The watts loss curve begins to increase with the first increase of power factor and then increases suddenly when the power-factor curve begins to increase for the second time at 16 kv. For comparison, the curves of Fig. 23 and of Fig. 17 are plotted together in Fig. 24.

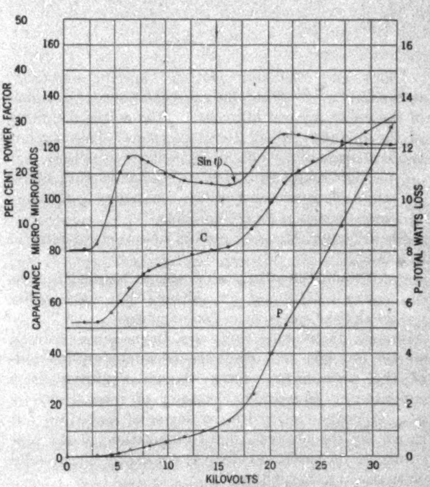

FIG. 23—CABLE MODEL. GLASS-AIR-PAPER DIELECTRIC

20 deg. cent. 60 cycles per second

Fig. 23 indicates that this composite cable has a second ionization voltage. The paper was removed and examined, and found to be pierced throughout by numerous fine pinholes. This effect could not be found unless the voltage were carried up beyond the second ionization voltage. The explanation of the second ionization voltage is now obvious.

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[15] William A. Del Mar, "Electric Cables," p. 84.

As the impressed voltage is increased, the voltage across the air spaces reaches its maximum value and does not increase further. Hence the gradient in the paper and the gradient in the glass increase more rapidly with further increase in impressed voltage. When the gradient in the paper becomes sufficiently high, the ions striking its surface are drawn through causing perforations. A number of ions concentrating at one point causes a pinhole. The restricted ionization current density prevents dynamic rupture. Because of the perforations, the dielectric and the barrier actions of the paper are in part destroyed. Any increase in impressed voltage now occurs almost entirely across the glass tube and the power factor begins to increase and then decrease again in the manner which was discussed in connection with Figs. 7 and 17.

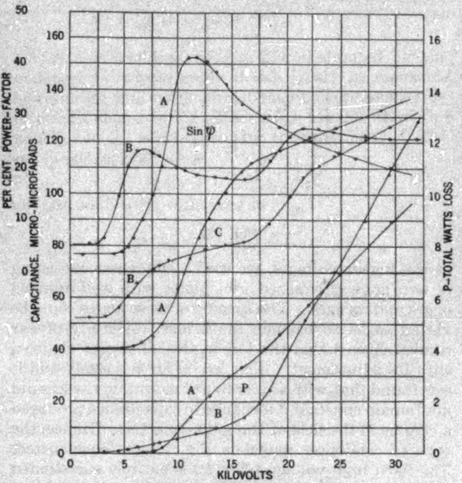


Fig. 24—Cable Models

20 deg. cent. 60 cycles per second
A. Glass-air dielectric
B. Glass-air-paper dielectric

This suggests that one of the requisite qualities of paper which is to be used for cable insulation may be its barrier action under conditions similar to the foregoing.

If the voltage is carried sufficiently high, in these paper-glass cable models, a temperature is obtained which causes a slight carbonizing of the paper to a brown color. In addition, with a slightly lesser voltage than this, it is possible to produce charred spots at numerous places over the surface of the paper.

The authors have not as yet made a study of this barrier action with paper tape impregnated with compound, which naturally approximates more nearly working conditions.

The foregoing experiments and the resulting analyses suggest hypotheses which may explain the origin of "tree designs" and the incipient causes of breakdown. Fig. 25 shows a magnified and perhaps exaggerated section of paper insulation in which two spaces, one on each side of a paper tape, happen to occur directly opposite each other, due to the tapes not touching. Owing to the fact that the tapes do not quite touch and the compound does not quite fill all the voids so formed, gas films exist in spaces $Q'\,Q$ and $Q'\,Q''$. At low voltages, below the ionization voltage, the potential gradient across the air films and the corresponding layer of paper will be equal in most cases. Hence no potential difference will exist between points P and Q, P' and Q' P'' and Q''. Above the ionization voltage the gradient through the paper increases. The gradient through the air-spaces cannot increase beyond a limiting value as already demonstrated. This causes a potential difference to exist between points P and Q, P' and Q', P'' and Q''. Therefore, there will be voltage gradients tangential to the surfaces of the paper which may cause creepage or tangential currents which carbonize the surfaces and ultimately result in "tree designs."

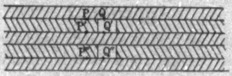


Fig. 25

This creepage may be followed by discharges over the edges of the tape, and these discharges are likely to be more or less oscillating in character and of high frequency. These discharges cause further local heating and further destruction of insulation. The barrier action of the paper at Q' may also be destroyed, as with the authors' paper-glass models, causing increase in the watts loss (Fig. 23), and ultimately producing hot spots. Hence, these effects are all cumulative, and eventually rupture will occur at this point.

Conclusions

1. When corona occurs in air spaces, the increase in energy loss begins at a lower voltage gradient than the increase in capacitance.

2. With ionization in a gas the voltage gradient must always be greater than zero.

3. The gradient in an ionized gaseous dielectric is finite but cannot exceed a definite maximum value. This maximum value is the breakdown gradient for each particular condition.

4. The ionization current is limited or restricted by the solid dielectric.

5. When the voltage gradient in cables has reached

a value sufficiently high to ionize all the gas films in the dielectric, the ionization loss becomes proportional to the cable charging current.

6. The power factor is a direct function of the relative capacitance of the solid dielectric and air spaces in series. Hence, power factor of itself does not show the degree of ionization which exists in a cable.

7. The power factor due to a combined solid and ionized gaseous dielectric can be calculated if the constants of the two dielectrics are known.

8. Restricted ionization effect has been called "the barrier action of paper" in high-voltage cables and a method is suggested whereby it can be measured and the relative quality of cable papers be determined.

9. "Tree designs" are probably due to tangential gradients set up in the cable as a direct result of ionization.

10. Ionization causes progressive deterioration due to the high-frequency oscillations accompanying these internal discharges. The increased local temperatures or hot spots, due to ionization, tend to exaggerate these effects.

The authors are indebted to the members of the Electrical Engineering staff of the Harvard Engineering School for their cooperation in this research and particularly to Professor H. E. Clifford for his suggestions in the preparation of the paper.

Appendix I

In practically all bridge methods used for measuring dielectric loss, the air condenser is assumed to have zero loss or its angle of phase defect is zero. The angle of phase defect of such air condensers is not zero and at very low power factors this assumption may cause large percentage errors. In order to eliminate, so far as possible, errors due to the phase defect of our air condenser, we measured its angle up to 10 kv. by means of a substitution method. This method is not new, but has been used frequently for low-voltage condensers. In its application to high voltages, however, difficulties appear which do not exist with low voltages.

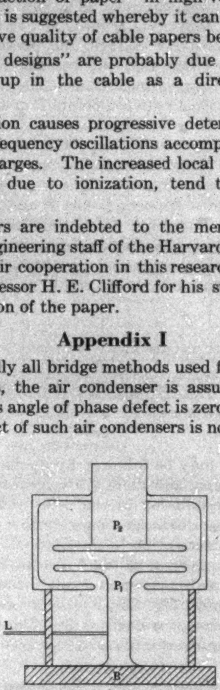
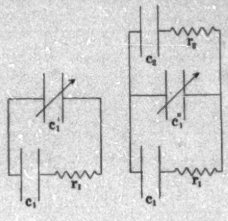


FIG. 26

B	Insulating base and support
L	High voltage lead
P_1	High voltage electrode
P_2	Low voltage electrode mounted on large, accurately cut screw

This substitution method is applicable to any type of capacitance bridge. It is necessary to have one air condenser other than the condensers which are used in the bridge. This condenser must be variable and its loss must remain constant at different settings. In attempting to measure the phase angle of our condensers at the higher voltages, we used a condenser consisting of two large galvanized iron plates with well-rounded edges and corners. The spacing of these plates could be varied so as to balance the bridge. A study of our results showed that the loss in this condenser changed with its adjustment. This led to further study and it was found that with any ordinary mounting the loss did not remain constant if a change in capacitance produced a change in the field of the solid dielectric. Hence, the type of condenser shown in Fig. 26 was constructed. The fixed high-voltage plate P_1 is entirely surrounded by the low-voltage plate P_2. It is supported by a brass pedestal P_1 extending outside the condenser which in turn is mounted on a base of bakelite B. The capacitance of this condenser is varied by moving the low-voltage plate P_2 by means of a threaded support. It is seen that a movement of P_2 will produce no change in the dielectric field in the insulating base B. Because of the expense, we have not as yet constructed such a condenser for voltages exceeding 10 kv. The method of measurement is first to balance the bridge with this variable condenser as one arm. Calculate the difference ψ' in the angles of phase-defect between this variable condenser and the condenser which must form one arm of the bridge. The variable condenser is then shunted by the test condenser and the bridge is re-balanced by decreasing the capacitance of the variable condenser so that the total capacitance of the bridge does not change, making any other adjustments that are necessary. Again calculate the difference ψ'' in the angles of phase-defect between the combination (variable and fixed condenser) and the condenser which must form one arm of the bridge.



(a) (b)

FIG. 27

To derive the equation, let the equivalent circuit of the variable condenser be represented by the circuit, Fig. 27A, consisting of a fixed capacitance C_1 in series with a resistance r_1 shunted by a perfect condenser C_1'. The equivalent capacitance of this combination, neglecting terms of higher order, is

$$C' = C_1 + C_1'$$

The equivalent series resistance

$$R' = \frac{r_1 C_1^2}{(C')^2}$$

$$\text{Tan } \psi' = R' C' \omega = \frac{r_1 C_1^2 \omega}{(C')} \qquad (1)$$

When this combination (Fig. 27A) is shunted by the condenser C_2 whose angle it is desired to measure, the resulting equivalent circuit may be represented by the circuit shown in Fig. 27B. Now assume that the capacitance C_1' is decreased by the amount C_2. The equivalent capacitance is

$$C'' = C_1 + C_1'' + C_2 = C'$$

and the equivalent resistance is

$$R'' = \frac{r_1 C_1^2 + r_2 C_2^2}{(C')^2}.$$

Therefore

$$\text{Tan } \psi'' = R'' C'' \omega = \frac{(r_1 C_1^2 + r_2 C_2^2)\ \omega}{(C')} \qquad (2)$$

Subtracting (1) from (2)

$$\text{Tan } \psi'' - \text{Tan } \psi' = \frac{r_2 C_2^2 \omega}{C'}$$

or

$$r_2 C_2 \omega = \frac{C'}{C_2} (\text{Tan } \psi'' - \text{Tan } \psi') = \text{Tan } \psi \qquad (3)$$

Equation (3) gives the product $r_2 C_2 \omega$, of the condenser whose angle of phase defect it is desired to measure, in terms of a ratio of two capacitances multiplied by the difference of the tangents of the two angles ψ'' amd ψ'.

Appendix II

Consider a condenser of composite dielectric, a part of which is air and the remainder some solid homogeneous dielectric. This may be considered as two condensers, c_1 and c_2 in series. Let c_1 be the capacitance of the air portion and c_2 the capacitance of the solid dielectric part. Fig. 21 shows the electrical equivalent of the combination. Assume that (1) Ionization begins at a definite voltage; (2) c_1 has zero loss up to the ionization voltage; (3) the angle of phase defect of c_2 is independent of voltage; (4) above the ionization voltage e_1 is constant. The total voltage across the combination

$$E = e_1 + e_2 \qquad (4)$$

Assume that E is given. Up to the ionization point e_1 and e_2 can be calculated in the ordinary manner. As soon as the ionization voltage is reached, e_1 becomes constant and e_2 is then found from equation (4). (See Fig. 20.) With parallel plate electrodes, the ionization voltage will be the breakdown voltage for that particular thickness of air.

The power loss in condenser c_2 is

$$P_2 = e_2 I \sin \psi_2$$

where I is the charging current through the combination, or

$$P_2 = e_2^2 c_2 \omega \sin \psi_2 \qquad (5)$$

Below the ionization voltage, the power factor of the combination c_1 and c_2 is

$$\text{Sin } \psi = \frac{P_2}{E I}$$

or

$$\text{Sin } \psi = \frac{e_2}{E} \sin \psi_2 \qquad (6)$$

However, above the ionization voltage, the power loss in c_1 is

$$P_1 = e_1 I \sin \psi_1 \qquad (7)$$

but

$$I = e_2 c_2 \omega \qquad (8)$$

and

$$\text{Sin } \psi_1 = \frac{i_r}{I} = \frac{I^2 - i_c^2}{I}$$

Where i_r is the energy current and i_c the quadrature current through c_1.

$$\therefore \sin \psi_1 = \sqrt{1 - \frac{(e_1 c_1)^2}{(e_2 c_2)^2}} \qquad (9)$$

Substituting (9) and (8) in (7)

$$P_1 = e_1 e_2 c_2 \omega \sqrt{1 - \frac{(e_1 c_1)^2}{(e_2 c_2)^2}} \qquad (10)$$

The total power is equal to the sum of (10) and (5) or

$$P = P_1 + P_2$$

$$= e_2^2 c_2 \omega \sin \psi_2 + e_1 e_2 c_2 \omega \sqrt{1 - \frac{(e_1 c_1)^2}{(e_2 c_2)^2}}$$

The power factor of the combination is

$$\text{Sin } \psi = \frac{P}{E I}$$

or

$$\text{Sin } \psi = \frac{e_2 \sin \psi_2 + e_1 \sqrt{1 - \frac{(e_1 c_1)^2}{(e_2 c_2)^2}}}{E} \qquad (11)$$

Equation (11) gives the value of the power factor of this combination of capacitances. Before ionization occurs, the right-hand term equals zero, and the equation still gives the circuit power factor. Correction should be made for the phase difference between e_1 and e_2, but for a first approximation this is not necessary.

The use of these formulas to check experimental data has been shown in Fig. 22.

Appendix III

The voltage was measured by means of a voltmeter coil in the high-voltage winding of the transformer. The ratio of the high-voltage winding and the voltmeter coil was determined by the bridge method of Fig. 28. A bridge of this type has been described by Atkinson.[16]

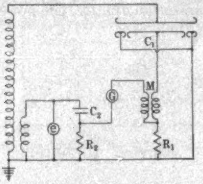


Fig. 28

However, the bridge (Fig. 28) was in use in our work before the publication of Atkinson's paper.

The equations of balance are:

$$\frac{E}{e}\,\frac{C_1}{C_2} = \frac{R_2}{R_1} \qquad (\mathbf{12})$$

$$\frac{M\,\omega}{R_1} = \text{Tan}\,\psi \qquad (\mathbf{13})$$

where ψ is the difference of the phase angles of the currents in the *two halves* of the bridge.

In order to calculate the ratio $\frac{E}{e}$, equation (**12**) is sufficient when both capacitances and resistances are known. In our work C_1 was a large air condenser, having approximately 50 $\mu\,\mu\,f.$ capacitance, and C_2 was a 0.05 $\mu\,f.$ secondary standard, mica condenser. The resistances R_1 and R_2 were about 100 ohms each.

Equation (**13**), nevertheless, gives a method of calculating the phase difference between the voltage of the high voltage side of the transformer and that of the voltmeter coil, if the losses in C_1 and C_2 are known.

Appendix IV

A knowledge of the electrical properties of ionized air is very important since these properties determine in such a large measure the behavior of dielectrics containing occluded gas. With flat air-spaces it is practically impossible to determine these properties without having a solid dielectric in series, since, when the critical voltage gradient is reached, the current increases very rapidly with but a very slight increase in voltage gradient (Fig. 19) and complete rupture of the air space immediately follows. However, by having a solid dielectric in series, the current in the air space may be limited to any desired value and the properties of the air space studied under those conditions. The discharge in the air space then takes the form of true corona or a glow discharge.

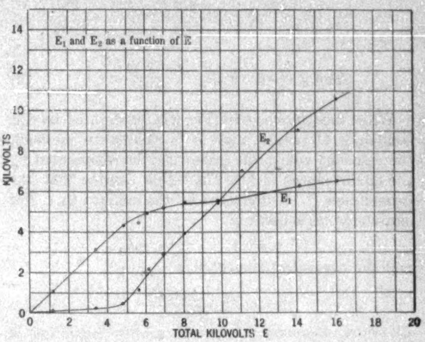


Fig. 29

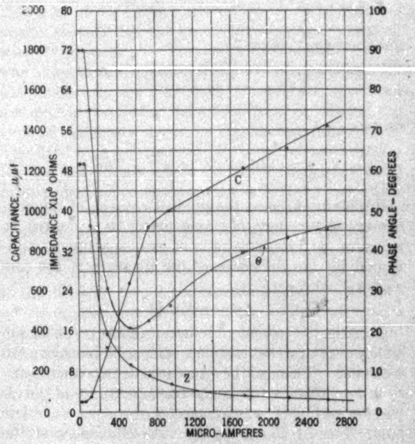


Fig. 30—Dielectric Properties of Air. Parallel Plate Electrodes

t = 0.2 cm. Area = 117 sq. cm.
20 deg. cent. 60 cycles per second

In order to determine quantitatively some of these characteristics, measurements were made on a flat air space, 2 mm. thick, in series with a flat glass plate, 2 mm. thick. The glass plate was first silvered on both sides. The active electrodes were circles having diameters of 12.2 cm. which gives an area of 117 sq. cm.

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[16] "A High-Tension Bridge for Measurement of Dielectric Loss in Cables." *The Electrical Journal*, Vol. 22, Feb. 1925, p. 57.

One circle was surrounded by an annular guard ring having a radial thickness of 1.4 cm. The dielectric properties of the glass were then determined. The silver was then removed from one side and a brass plate electrode spaced 2 mm. from the non-silvered side was used in determining the dielectric properties of the combination of air and glass.

The constants of the combination of glass and air were then measured. Knowing the constants of the glass alone, the constants of the air alone could be calculated by the equations

$$I = \frac{E}{Z} = \frac{E_1}{Z_1} = \frac{E_2}{Z_2} \qquad (20)$$

$$Z = Z_1 + Z_2 \qquad (21)$$

and

$$E = E_1 + E_2 \qquad (22)$$

where I is the current, E and Z the voltage and impedance of the combination, E_1 and Z_1 the voltage and impedance of the air, and E_2 and Z_2 the voltage and impedance of the glass.

These quantities are all vectors and the operations must be made vectorially.

Fig. 29 and 30 give the results of the measurements on the air. The current voltage curve of Fig. 29 is the same character as the curves of Fig. 20 and shows that above the ionization voltage, the voltage across the ionized gaseous film remains practically constant. The solid dielectric then takes most of the voltage stress.

Fig. 30 shows the increase of the capacitance, and the change in the impedance and the phase angle of the ionized gaseous film as functions of the current. Below the ionization voltage the capacitance of the air space is 50 $\mu\,\mu f$. Above the ionization voltage it increases rapidly to 900 $\mu\,\mu f$. at 700 micro-amperes. It then increases more slowly but uniformly until at 2600 micro-amperes its value is 1430 $\mu\,\mu f$. This curve then consists essentially of three straight lines. At 2600 micro-amperes the capacitance of the air space is *28 times* its non-ionized value. Hence the dielectric constant of the air under these conditions is 28.

The curve of the power-factor angle θ has a peculiar shape. After the ionization voltage is reached the angle decreases rapidly from 90 deg. to 20 deg. and then commences to increase more slowly, the rate of increase decreasing with increase of current.

After the ionization is reached the impedance drops very rapidly to approximately one-fifteenth its initial value and then decreases very slowly with further increase of current.

Discussion

A. E. Kennelly: Turning to the paper by Dawes and Hoover, it seems to me that the great force of that contribution lies in the fact that a bridge has been developed which employs a vibration galvanometer, with all the delicacy and precision that the vibration galvanometer affords, so that instead of having to employ a relatively great length of cable dielectric for test, it is sufficient to have only, say, a couple of meters, provided that the particular couple of meters shall represent fairly well the whole length of a cable under consideration.

E. W. Davis: One of the most important problems which confronts the manufacturer of high-voltage cables is that of dielectric loss, its fundamental cause and its relation to the other necessary properties of a well designed cable.

The paper by Messrs. Dawes & Hoover adds one more to the numerous theories concerning the nature of dielectric loss and power factor. Unfortunately many of the papers published to date, while excellent mathematical treatises on the subject, are not founded on any scientific research work or based on data of many tests. In fact, outside of a very few isolated experiments, many of the theories cannot be checked at all.

In a recent paper presented before the British Institution of Electrical Engineers, "Dielectric Problems of High-Voltage Cable", by Percy Dunsheath, an attempt is made to connect dielectric loss with dielectric absorption, dielectric hysterisis, and dielectric conduction. Much data and many curves are given to substantiate the theory and there is included a rather unique discussion of the relation between a-c. and d-c. losses.

The paper by Dunsheath and the paper by Dawes & Hoover offer two widely divergent theories for dielectric loss, and yet both of them contain features that may be readily checked.

Fig. 1 herewith shows a few typical power factor voltage curves that we have obtained with various impregnating compounds and at various temperatures. These tests were made in a special oil testing condenser and with a high-voltage, 60-cycle bridge. The curves are quite similar to those shown in the Dawes-Hoover paper, one in particular showing the peculiar knee found at high temperatures by these investigators.

Fig. 2 shows two curves calculated by means of the formula (**17**) of the Dawes & Hoover paper. By slightly varying the assumptions as to location and dimensions of the air films, both types of curves are obtained. One interpretation of such tests would indicate that compound ordinarily contains more or less air distributed through it and exhibits the same phenomena of restricted ionization as laminated insulation even though the barrier effect of the paper is not present. Another phenomenon exhibited by cable insulation, but not discussed by Dawes & Hoover, is the *V* curve of power factor and voltage. Numerous samples tested show a decrease of power factor with increasing voltage and then as higher stresses are reached, a rapid increase.

In built-up insulation, there are invariably minute air films. The loss through these films depends upon the conduction through and conduction around the film along the paper. The conduction loss at low stresses would follow Ohm's law and at high voltages would depend upon the degree of ionic saturation or ionization. If we consider a very low-pressure air film between two paper tapes, the full ionic saturation would be reached at a very low stress and, therefore, the leakage current would not increase directly with the voltage. The dielectric loss, therefore, would not increase directly with the voltage and this would mean a decrease of power factor with increasing stress. This phenomenon holds until a point is reached where ionization of the higher-pressure air in the insulation is complete. If the ionization took place at the same voltage for a large majority of the air films, we would expect a sharp increase of power factor. It is possible that the ionization point for various films may differ so much that a gradual rather than a sharp increase of power factor occurs. This effect is of course governed by the relative amount of air to insulation, the average pressure of these air films and the leakage characteristic of the paper insulation. We should not expect to find this phenomenon in a cable the paper insulation of which had quite high leakage characteristics.

Tests on Impregnated-Paper Sheets. Power-factor tests on sheets of vacuum-dried and impregnated paper were made with a high-voltage bridge in order to check the previous conclusions. It was thought that tests on single sheets of paper in very close contact with the electrodes would show the characteristics of the paper insulation itself. Then, using more than one sheet, tests could be made with air films between.

The curves in Fig. 3 show the results of these tests.

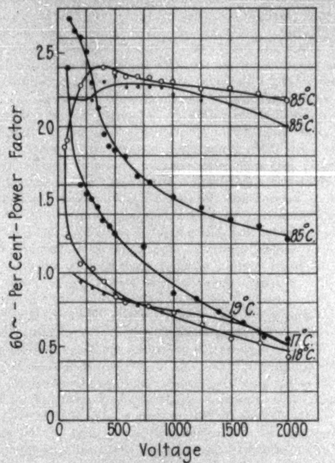

Fig. 1—Power Factor Versus Voltage Various Impregnating Compounds

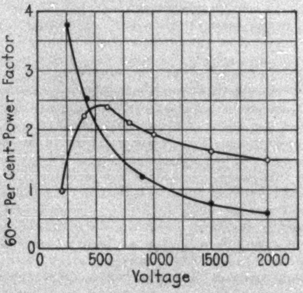

Fig. 2—Calculated Power Factor-Voltage Curves Impregnating Compounds

In no instance was a *V* curve obtained with a single sheet of paper. Two and three sheets of paper, when placed loosely together and tested, gave decided *V* curves of both the gradual and the sharp type. With three and four sheets of paper, sufficient stress could not be obtained with the voltage available to reach the critical point in the power-factor curve but the start of the *V* curve was obtained in each instance. The application of pressure to the upper electrode when testing multiple sheets changes the character of the curves very much. In Fig. 3, when two sheets are tested under pressure, the *V* curve is partially eliminated and the voltage at which ionization takes place is increased. With three sheets under pressure, the *V* curve is eliminated completely. In the case of four and five sheets, sufficient pressure was not applied to remove the *V* curve completely but the dotted curves shown seem to indicate the tendency for this to take place.

W. A. Del Mar: The paper by Dawes and Hoover contains a wealth of experimental material which requires considerable detailed study, and it is difficult to comment upon the results without knowing more about the materials worked upon. For instance, cables impregnated with different types of compounds would probably show different characteristic curves. I think that the value of the paper would be increased very much if information were given regarding the type of compound used in the cables for which curves are given—for instance, whether the compounds are solid or liquid at the temperatures at which the experiments were made, and so on.

I notice that most of the experiments, if not all of them, were made with single-conductor cables. Some rather interesting work which Mr. Hanson has been doing and also some which has been done in England and recorded in the current Journal of the Institution of Electrical Engineers, England, indicates that in three-conductor cables, a large proportion of the loss which occurs at the higher voltages occurs in the fillers, and if the fillers are omitted or replaced by some material which has not the same characteristics as the ordinary paper crepe, those losses can practically be eliminated and the ionization curve,—that is to say, the voltage-power factor curve,—becomes virtually a horizontal line between stresses of 20 volts and 100 volts per mil, which are about the limits in Fig. 5 of the paper.

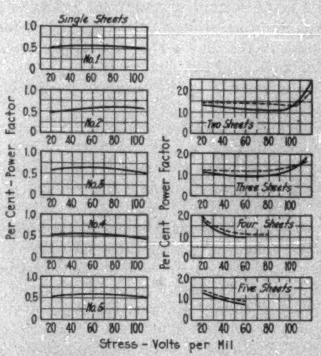

Fig. 3—Sixty-Cycle Power Factor Versus Voltage Impregnated-Paper Sheets. Dotted Curves are Results of Tests with Pressure on Upper Electrode

It would therefore seem that the work which the authors have done needs to be somewhat amplified by an analysis of just the parts of the cable in which the principal losses occur, at the different voltages. It is possible that by such an analysis, steps could be taken to reduce these losses at the higher voltages.

Anyone who has done any laboratory work of this kind will appreciate the immense amount of painstaking, careful work that Prof. Dawes and Mr. Hoover have done in this paper. Nevertheless, I am not entirely convinced that their curves represent insulation characteristics to the exclusion of test-apparatus idiosyncrasies. When, however, you look at the curves and compute the number of experimental points they have made, the work appears to be really prodigious, and even if some corrections should have to be made, the authors are to be congratulated upon their results.

R. W. Atkinson: I have been interested in bridge measurements ever since I became connected with the cable industry, and last year published in the *Electric Journal* a description of the method we are now using and gave a brief description of it in a discussion before the Institute. The bridge that has been shown tonight, and the one I described are about as different in general construction and circuit as two instruments both of the bridge type could well be and yet serve the same purpose. Yet I am interested to note that they both have certain fundamental characteristics and attain final results equivalent in many respects.

The sensitivity of the two types is nearly the same, so far as I can judge from the figures given in the paper, and the voltage across the measuring parts of the bridges is comparable. These are fundamental parts of the whole problem of precision measurements of dielectric losses. As Prof. Dawes has pointed out, it is very desirable indeed to have the voltage drop across the resistance in series with the measured condenser very low so that no difficulty will be experienced from the effects of the capacitance to ground of the detector and certain other parts of the circuit. His instrument and mine are closely alike in this respect and have a very much lower drop than in the case of any other dielectric-loss-measuring device of comparable sensitivity that has been described.

Some fundamental differences between the two bridges are these:—My bridge uses an a-c. galvanometer as a detector; Prof. Dawes uses a vibration galvanometer with vacuum-tube amplifiers to get the same result.

The vibration galvanometer requires accurate frequency which is not always convenient where a commercial circuit supplies the power. The a-c. galvanometer requires an auxiliary source of current to supply the field at the same frequency as that supplying the measured condenser.

An advantage of our bridge over most other devices for measuring dielectric loss is that it is so arranged that when balance is obtained, the power factor may be read directly from the dials.

There is one difference between the two instruments, which is a very important one and is in favor of the a-c. galvanometer type. In using the vibration galvanometer, it is necessary to secure a very accurate capacity balance before the power-factor balance can be obtained. With the a-c. galvanometer type the bridge is first made sensitive almost entirely to capacity and then made sensitive almost entirely to the power factor, by changing the relation of the field so that the capacity balance does not have to be made nearly so accurately as it would otherwise have to be. This is very significant because it permits measurements to be made much more rapidly than with the other type.

E. S. Lee: With reference to the Dawes-Hoover paper, I think there are several things of great interest to those who have to work with cables and make measurements thereon. In the first place, we find that we have one more method added to the already long list and large number of different means by which we measure dielectric power loss. This is a bridge method. As Mr. Atkinson has said, about a year ago he described his bridge, and just a year ago from this same platform we were told of work which had been done with the Schering bridge, which work has been carried on for this past year with great success.

Prof. Dawes says: "On one occasion after completing a 60-cycle test, we found that the 25-cycle losses were greater than the 60-cycle losses even though sufficient time had been allowed for the cable to reach constant conditions." Such conditions are not new to those who are making these measurements continually. Also, if you will observe the curves of his Figs. 11, 13, and 16 you will observe "humps" which might be looked upon as indication of error. But Prof. Dawes has stated that they have checked those portions and they find those humps should be there in the curves. These also are found by other observers.

Thus, we are glad to find that as new investigators take up this work, their results are in agreement with what has been found in factory testing and experimentation.

I want to call attention to one point, which may not be necessary, particularly to those who are familiar with the work, but I think it is quite necessary, particularly for those who are not so familiar with the work. Some reference was made to the fact that these are characteristic curves, and I grant you that they are, except I would call to your attention that the largest portion of these curves is outside of the operating range; that is, the curves in Figs. 8 to 16 are for a cable which we would say would be normally rated at 12 kv. If you will look along the abscissas or on the curves for 12 kv., you will notice that within that value the dielectric power loss is quite small; the power factor is very low, and the change in these values is small. It is well beyond this voltage that the changes occur. In the present-day cable, the same conditions exist. We have to go beyond the operating values, considerably, in order to get these changes.

There is one other thing to which I would like to call attention; when I first looked at these curves, I thought the change in capacitance was enormous, but if you will note the scales, you will find it is not. It is only about 5 per cent. The change in the power factor is of the order of 300 to 400 per cent, the change in dielectric power loss is of the order of about 3000 per cent. So one has to observe these scales carefully in order to obtain correct conceptions. The explanation offered by the authors for the shape of these curves is good and of value, though if tests are made on cables with different treating compounds and with liquid fillers, it will be found that these same curves are not characteristic for those cables but that another set of characteristic curves will be obtained. Their shape likewise has to be explained.

I believe we are largely forgetting what power factor is. We talk about it quite a bit. I believe I am right when I say that it is the ratio between the dielectric power loss and the product of the voltage and the current. In other words, the variations in these three quantities govern the variation of the power factor; therefore, a discussion of the variation of power factor must include the variation of these factors. It is really the dielectric power loss that we are interested in. If we talk in terms of power factor, then our results become more directly comparable without regard to volume of the dielectric, but I think if we remember that the loss varies quite nearly as the square of the voltage, that the capacitance does not change greatly, hence current is proportional to voltage, we can work out the ratio of these factors and see just about how the power factor will vary. If air ionization is present, however, conditions are quite different from those when no ionization exists, particularly since the dielectric power loss varies at a faster rate than the square of the voltage.

I was interested in the agreement between calculated results and measured results, and the statement also that if we knew the constants, we could very readily calculate. I think that is true but the constants are often difficult to determine.

W. B. Kouwenhoven: The authors state that the air condenser that they employed in their bridge had a phase angle. In measurements that I have made with the Schering bridge I have never been able to detect any phase angle in the air condenser used. Slight errors, however, may creep in due to insufficient shielding of the leads and apparatus, and these will cause discrepancies which are usually assigned to a phase angle in the air condenser. Proper shielding will often eliminate these errors.

In some work that I have been doing with Dr. Whitehead we have found it necessary to shield the air condenser and its leads as shown in Fig. 4 herewith, which shows the air condenser side of a high-voltage bridge. The pipe shield on the lead to the low-voltage plate of the air condenser must be maintained at the same voltage as the lead, otherwise charging currents between this lead and its shield will introduce errors in the results. Any change that is made in the bridge resistance in series with the condenser lead must be accompanied by a proportional change in the resistance in series with the shield, otherwise an error will be produced in the phase angle as determined in the bridge. The shielding of the specimens should be carried out in a similar manner.

In measuring losses at high voltages with the Schering bridge or with the quadrant electrometer at Johns Hopkins University we have followed the practise of determining the accuracy of the apparatus. In making this check we first measure the loss in a specimen and then insert a known non-inductive resistance in series with the specimen and measure the loss again. We determine the current in the specimen circuit and calculate the loss in the added resistance. If the apparatus is functioning correctly the loss in the added resistance should equal the difference between the first and second loss measurements.

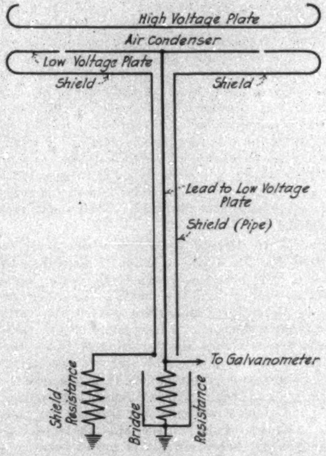

Fig. 4

W. F. Davidson: I desire to call attention to certain limitations of the method used by Dawes and Hoover and, indeed, by most other investigators. We are dealing here with a composite insulation which is stressed to such values that ionization reaches considerable proportions. As a consequence, such "constants" as capacity and resistance cease to be true constants and not only vary with the effective value of the impressed voltage but even vary cyclically. Whitehead, Peek, and others have called attention to the distorted current wave of corona, which is merely another way of stating that the capacity and resistance are not constant.

Under these conditions the observed data, and with them the computed data, will depend upon the method of measurement—a wattmeter method will give one result and bridge methods other results.

To illustrate this, assume that the impressed voltage is not distorted in any way and that we have a reference "no-loss" circuit as well as the circuit containing the test specimen. The voltage and the two currents may be written in the form:

$e = E\,[A_1 \sin \omega t + B_1 \cos \omega t]$

$i_1 = I_1\,[a_1 \sin \omega t + b_1 \cos \omega t]$

$i_2 = I_2\,[\alpha_2 \sin \omega t + \beta_2 \cos \omega t + \ldots \alpha_n \sin n \omega t + \beta_n \cos n \omega t]$

and

$A_1^2 + B_1^2 = 1, \quad a_1^2 + b_1^2 = 1, \quad \alpha_2^2 + \beta_2^2 + \ldots \alpha_n^2 + \beta_n^2 = 1$

In the test circuit the true power will be

$$W = \frac{E\,I_2}{2}\,(A_1\,\alpha_2 + B_1\,\beta_2)$$

since the over-tones in the current wave have no corresponding element in the voltage wave and hence do not contribute toward the average power.

If the measurements be made with a wattmeter (dynamometer or electrostatic) with an ammeter and voltmeter, the observed and computed results will be:

Potential difference $= E/\sqrt{2}$ r. m. s. volts

Current $= I_2/\sqrt{2}$ r. m. s. amperes

Power $= \dfrac{E\,I_2}{2}\,(A_1\,\alpha_2 + B_1\,\beta_2)$ watts

Power factor $= A_1\,\alpha_2 + B_1\,\beta_2$

Resistance $= \dfrac{E}{I_2}\,(A_1\,\alpha_2 + B_1\,\beta_2)$

Capacity $= \dfrac{I_2}{E\,\omega\,\sqrt{1 - (A_1\,\alpha_2 + B_1\,\beta_2)^2}}$

If the measurements be made with a bridge using a tuned vibration galvanometer or a dynamometer with fields excited at the fundamental frequency, the results would be as follows if the measured voltage and standard capacity are used:

Potential difference $= E/\sqrt{2}$ r. m. s. volts

Current $= \dfrac{I_2}{\sqrt{2}}\,\sqrt{\alpha_2^2 + \beta_2^2}$

Power $= \dfrac{E\,I_2}{2}\,(A_1\,\alpha_2 + B_1\,\beta_2)$

Power factor $= \dfrac{A_1\,\alpha_2 + B_1\,\beta_2}{\sqrt{\alpha_2^2 + \beta_2^2}}$

Resistance $= \dfrac{E}{I_2}\,\dfrac{A_1\,\alpha_2 + B_1\,\beta_2}{\sqrt{\alpha_2^2 + \beta_2^2}}$

Capacity $= \dfrac{I_2\,(\alpha_2^2 + \beta_2^2)}{E\,\omega\,.\,\sqrt{(\alpha_2^2 + \beta_2^2) - (A_1\,\alpha_2 + B_1\,\beta_2)^2}}$

These results depend on the fact that the bridge will only balance the fundamental component of i_2 against the reference circuit. In a bridge measurement, securing balance with a hot-wire or other untuned detector, it will be impossible to secure an exact balance and the observed values will differ in many points from those above.

In closing, I should like to suggest that we have reached a point in our researches where it seems essential that we consider instantaneous value $\dfrac{\partial E}{\partial I}$ and not average values. Fortunately the means for doing this are already at our disposal in the cathode-ray oscillograph. The Journal of the Institution of Electrical Engineers for November, 1925, contains three excellent papers dealing with this powerful tool of the modern investigator into dielectric phenomena.

G. B. Shanklin: Several years ago I made a special study of the subject of "ionization in cables," and as a co-author submitted some of the findings in Institute papers. Our work dealt largely with "cause and effect" from a practical standpoint. No quantitative nor qualitative analysis was attempted. The work, in conjunction with that of contemporary workers, resulted in recognition of the cause and importance of ionization in cables and established methods of eliminating or reducing it.

Recent advancement in the voltage rating of cables is based on the principles demonstrated by this early work.

A more expert and systematic study by qualified physicists and research engineers familiar with the electron theory has long been needed. This excellent paper by Messrs. Dawes and Hoover represents, I hope, a beginning which will result in eventually establishing the theory of ionization in cables on a firm quantitative basis.

Their present results are not only a distinct advancement but are in good agreement with our previous work, as far as it went. By using a length of ordinary impregnated paper cable with a special loose sheath, we obtained curves very similar to those in Fig. 7 of the paper. The curves were not, as theirs are, subject to accurate analysis, for ionization of the large air space directly under the loose sheath was obscured by simultaneous ionization through the thickness of paper insulation. The use of a model cable sample enabled them to obtain much more accurate data.

Change or shifting of ionization curves, as shown in Fig. 9, is, as they state, a function of time. I agree that residual or free ions, after the voltage is reduced, are the main contributing cause of this, but do not agree that absorption or higher internal temperature plays an important part. Instead, there appears to be a factor discovered by J. J. Thomson and called "ionization pressure," which is important. When the gas spaces are first ionized, this "ionization pressure" pushes the softened compound back, resulting either temporarily or permanently in enlarged gas spaces. A particular cable might operate indefinitely at normal voltage without ionization, but if a momentary over-voltage, such as produced by a switching surge, is applied, ionization might persist for some time after the voltage drops back to normal. This is, in effect, exactly the same characteristic shown in Fig. 9.

The irregular shape of the first section of the high temperature-power factor curves, as shown by Figs. 11 and 13, is, I have believed for some time, due to the liquid nature of the compound. Some form of ionic conduction through this liquid is apparently responsible, for these peculiar shapes are obtained only with liquid filling compounds. I have seen the same type of curves obtained at room temperature when a thin oil is used for impregnation.

On the fourteenth page the authors make the statement: "It has been recognized for some time past that, although the paper itself has lower dielectric strength than the impregnating compound, the dielectric strength of the cable is increased by using paper in combination with compound." This statement is misleading. They evidently mean that cellulose fiber in paper form and in combination with air has less strength than when in combination with compound. This is evident since compound has greater strength than air. Actually, cellulose in pure form is an excellent insulator, and this is why paper, acting as a barrier, improves the dielectric strength of the compound.

Their theory of the formation of tree designs seems very plausible. Certainly, it must be due to tangential stresses along and between the surfaces of the paper wrappings. The high-frequency nature of this discharge is also well established. It might be pointed out that impregnated paper insulation has something like one-tenth the dielectric strength longitudinally between surfaces of wrappings, that it has radially or perpendicular to these surfaces. The absence of fiber barriers in this direction accounts for this.

It is sincerely hoped that the authors will continue their valuable work to a final conclusion.

J. B. Whitehead: We have had some difficulties which Professor Dawes doesn't mention, and which I hope he hasn't had. There are also some that he has had which we have not. Dr. Kouwenhoven has called attention to one of our greatest difficulties, that is, the screening of the test electrode and maintaining the guard electrode at exactly the same potential. On our Schering bridge, we found we could get power-factor measurements in the same range that Professor Dawes is working, that would differ by 50 to 100 per cent simply by a variation in the resistance in the tail of the guard circuit. I presume he has solved that difficulty, or perhaps his bridge is not so sensitive to it as our own.

As to the phase difference of his air condenser, I am astonished at the value that he finds. We have found nothing to indicate such a difference in our condenser. We have split it into two halves, put one-half in each side of the bridge, and have added small resistances in the tail circuit of the side we are measuring, thus checking by successive increments of power factor, to see whether there is any initial value of phase difference. We have not detected any, and I hope that Professor Dawes will tell us what he considers to be the source or cause of the phase difference he finds. In this connection I have the following references to phase difference in air condensers:

Giebe and Zickner, in the Archive fur Elektrotechnik, find that in standard air condensers of the Reichsanstalt no phase difference can be detected by methods measuring the same to an accuracy of from one to two seconds of arc. These are small air condensers, and I think the maximum voltage upon them was about 300 volts, and the measurements were directed only at the possibility of a phase difference due to the normal ionization of the air. This is the only source that I can think of for a phase difference, unless one has actual point discharges between the plates of the condenser, in which case you could get ionization losses of any description. I assume that Professor Dawes' condenser was free from error of this type.

E. Möller also finds that radium rays and copious brush discharge in the neighborhood of an air condenser have no effect on its phase difference, but he is working at frequencies up to 180,000 and above, so perhaps his figures cannot be taken.

C. H. Willis who is working in my laboratory has computed the value of the phase difference due to the normal ionization in the air. This was for a condenser of our dimensions, in which the space separation up to about 35,000 or 40,000 volts is two inches. The ratio of charging current to conduction current is based on the normal velocity of ions in the air, and the potential gradient and the frequency, and so on, a very simple calculation. The value of the phase difference in this case works out to be 4×10^{-6}. This is far below the figures which Messrs. Dawes and Hoover have given and indicates a negligible phase difference.

I do not feel that it is necessary to invoke the structure of the atom in explanation of the behavior of this type of insulation. I believe that the change in the power factor, as it rises beyond the ionization point, can be accounted for by the saturated conditions in these air layers, if it be remembered that this ionization is extinguished at every half wave, as the voltage goes down below the critical value. We have every reason to suppose that at such frequencies as we are using here, there is complete extinction every half wave. As a consequence there arises the question of the recombination of the ions, and I think it is clearly evident from the very interesting and beautiful experiments on the artificial cable with an air layer, that there is a normal saturated condition due to a limited air area, beyond which you do not get any further ionization and therefore no further loss, as you go on up in voltage.

Another thing that we have to note here is that there are two possible sources of loss in ionization: one due to the actual process of ionization itself, the actual collision or the knocking apart of component parts, and then on top of that such conductivity as exists due to the passage of the ions across the air space. I think it is very easy to picture a saturated condition and if we have that, it is very easy to see that as the voltage curve goes up, the power factor may reach a maximum.

I also was particularly interested in the difference between the ascending and descending curves, and I would like to ask whether these curves repeat themselves accurately. Nothing has been said here about the effect of the presence of ionization on the cable structure itself. It is a most powerful destructive agent. If you have any layer in which there is active ionization going on, I wouldn't say that it would remain in the same condition for more than a few seconds. I think it probable that much of the trouble in cable insulation is due to the fact that it is rapidly destroyed as soon as we let ionization start. I venture the prediction that in a few years' time we are going to see some curves of this kind which will be flat, and when you do that, and not until you do, will you have cable that will be satisfactory. When ionization goes on, the cable is going to pieces, and it seems to me that it is only a question of thorough and complete impregnation. If we can get that, I don't see how we can fail to straighten these curves out and get rid of the principal cause of this trouble.

C. A. Adams: Referring to the destructive effect of internal corona, I have photomicrographs which show the fusing by corona of mica laminae in the insulation of a 25,000-kv-a. turbo alternator. There was in this case conclusive evidence of what might be called a "corona blast," namely the reaction of the corona currents on the magnetic field due to the load currents in the armature conductors, the corona current being perpendicular to the magnetic field.

C. L. Dawes: Mr. Del Mar requested some more data on the cable. These are in the hands of the Committee. We have the cable numbered and I believe we know the manufacturer, but we we do not know the compound. He states that he hopes we determine in which part of the cable the loss occurs. At the present time we are analyzing the cable layer by layer, finite thicknesses from the center out, those thicknesses being so small that they are practically surfaces. Our one source of error is that we must assume that the air spaces are equally distributed through the insulation. If that is not true, our work obviously will be in error.

He states that we have done a lot of work and determined so many experimental points. I merely wish to add to that and state that these are just a few representative data. There wouldn't be room in an ordinary journal to include all the curves which we could put in.

We are very glad Mr. Lee pointed out the fact that we were carrying the operating range of the cable above the ordinary operating voltage. It is necessary to do this, however, in order to study the phenomena. We are studying ionization itself, and we are not attempting to obtain the ordinary commercial curves which have been published or which are readily obtainable in many manufacturers' laboratories.

Mr. Davidson brought out the fact that the current is distorted by the variable factors in the circuit. We have had that in mind for some time, and Mr. Hoover and I have discussed the possibility of obtaining oscillograms of the charging current through the cable. That is one of the details which we have set aside until a more opportune time comes to make an investigation in that particular direction. I will state, however, that in some of our early experiments the man who was doing the work at that time told me that he had obtained a power factor of something like 0.1 per cent. I questioned it very much, and on further investigation I found that he was using as a source the commercial lighting circuit, and he had not become familiar with the galvanometer, and inadvertently he tuned in for the third harmonic; in other words, he was balancing up the third harmonic and in his calculations he obtained a power factor which was one-third of what he should have obtained.

P. L. Hoover: I will reply first to Mr. Atkinson's statement that in balancing a bridge "it is necessary to secure a very accurate capacity balance before the power-factor balance can be obtained." It can be shown mathematically that this effect is inherent in the bridge and does not depend on the type of measuring instrument used. Furthermore, with the Wien and Schering bridges it can also be shown that the power factor, as determined by a minimum deflection of the detecting instrument, may be in error by 50 or 100 per cent if the capacity balance is out by about one per cent. This effect does not enter in the bridge used for this investigation. With this new bridge an accurate capacity balance can not be obtained if the power-factor balance is out very far. The two balances must be obtained simultaneously.

Furthermore, an amplifier is not necessary with this bridge although it helps considerably at the lower voltages if the sample is small. At higher voltages, above 25 kv., the amplifier is not used.

The question raised by Professors Kouwenhoven and Whitehead concerning the grounding of the condenser shield through a resistance is interesting in that it points out another advantage of this new bridge over the Wien and Schering bridges. Since, with the latter bridges, it is necessary to make the capacity balance to extraordinary precision in order to get a fair precision in the power-factor balance, it is essential to maintain the guard ring or shield at exactly the same potential as the low-voltage electrode of the condenser. Otherwise the effective capacity of the condenser will change due to the change in the potential distribution within the condenser. This effect is largely eliminated in this new bridge since the power-factor balance does not depend upon such a critical balance of the capacitances.

It should also be noted that the resistance in the shield circuit is not a constant, neither is it equal to the resistance in the other arms of the bridge. If C_1 and C_2 represent the capacitances of the low-voltage electrode and the shield respectively, and R_1 and R_2 the resistances in series with these capacitances, then, for the shield to be at the same potential as the low-voltage electrode the shield resistance must be $R_1 = R_2 (C_1 \; C_2)$. Consequently the resistance in the shield must be changed every time the bridge is changed. With the Wien and Schering bridges this effect is very important and accounts for Prof. Whitehead's observation that a small change in the shield resistance may change the power factor by 50 to 100 per cent.

Check measurements made since the writing of this paper have shown that the angle of our air condenser was due to insufficient shielding of the lead from the low-voltage electrode of our air condenser as Prof. Kouwenhoven has pointed out. In our particular case there was magnetic coupling as well as electrostatic. Shielding corrected the difficulty and our air condenser now has no measurable loss.

Operating Performance of a Petersen Earth Coil-II

BY J. M. OLIVER[1] AND W. W. EBERHARDT[1]
Associate, A. I. E. E. Associate, A. I. E. E.

***Synopsis.**—At the A. I. E. E. Spring Convention, Pittsburg, April 24-26, 1923, a paper was presented by the authors on the operation of a Petersen earth coil installed by the Alabama Power Company on its Lock 12-Vida 44,000-volt system. That paper reported an excellent performance record for the coil, although several unaccounted-for bus insulator flashovers occurred at Lock 12, indicating that further investigations on the action of the coil were necessary.*

This paper presents eleven months' additional operating experience with the Earth Coil, which shows that over-voltage disturbances can be eliminated by doing all line switching operations with the Petersen Coil out of service, that is, with the system neutral solidly grounded. It is believed that the same results can also be obtained by under-tuning the coil.

The Lock 12 Coil is no longer in service, as the system has outgrown the need of such equipment; additional power sources have been added to the system and reliable service can be maintained without the use of flashover suppressing devices.

It appears that the application of Petersen Earth Coils is limited to comparatively low voltage lines (66,000 volts and less) of moderate length with a single source of power supply. In such an application, the use of a Petersen Coil between system neutral and ground has the advantage over grounded neutral operation by reducing line outages due to flashovers and over isolated neutral operation by eliminating arcing grounds.

* * * * *

THE purpose of this paper is to present eleven months' additional operating experience with the Alabama Power Company's Petersen earth coil installed on its Lock 12-Vida 44,000-volt system, shown in Fig. 1. A detailed description of this installation is given in the writers' earlier paper[2] presented at the A. I. E. E. Spring Convention, Pittsburgh, April 24-26, 1923. A complete operating log of the Lock 12 coil for the period extending from October 12, 1921 to March 1, 1923 was also given in that paper, which showed an excellent performance for the coil, the number and duration of interruptions due to lightning flashovers having been reduced by 83.5 and 94 per cent respectively, by its use. There were, however, several unaccounted-for bus insulator flashovers at Lock 12, which indicated that further investigations on the action of the coil were necessary.

The bus insulator flashovers in question occurred when the line switch at Lock 12 opened automatically or was opened by hand while the Petersen coil was in service. It was the consensus of opinion of those who discussed the Pittsburgh paper that the high-voltage flashovers were caused by the non-simultaneous opening of the three poles of the line breaker. This is a very plausible explanation of the trouble because at that time the coil was overtuned—that is, it had excess reactance—and the disconnection of one line conductor ahead of the other two would change the line capacitance in the right direction to produce a condition of resonance in the series circuit, in which case there would be nothing to limit the rise of voltage except the system losses. Assuming that this is the correct explanation for the high-voltage disturbances, there are two remedies which can be applied; namely (a) to undertune the coil, and (b) to electrically interlock the control circuit of the grounding switch with the overload protective relay scheme on the line switch so that the grounding switch is always automatically closed before the line switch opens to clear phase-to-phase short circuits.

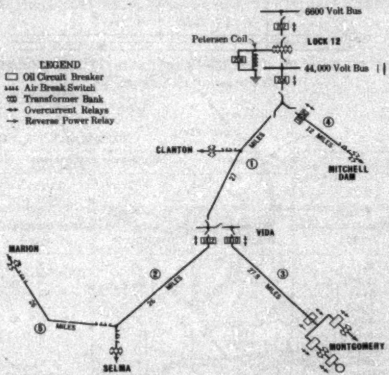

FIG. 1—LOCK 12—VIDA 44,000-VOLT SYSTEM

Of these two methods, (a), if successful, would prove the more advantageous since no change in the normal control connections is required. Method (b) has the disadvantage of introducing complications in the control scheme and also slows up the relay operations on phase-to-phase short circuits, since additional timing must be allowed for the closing of the ground switch. Method (b), however, had already been applied and was in operation when the theory of undertuning was advanced so it was decided to continue operation with the interlocking feature between the ground and line

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[1] Both of the Alabama Power Co., Birmingham, Ala.

[2] A. I. E. E. TRANSACTIONS, 1923, page 435-45.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

switches during the 1923 lightning season, and then perhaps operate the coil under-tuned during the 1924 season. Unfortunately, the coil had to be taken out of service before the 1924 lightning season for reasons stated in the latter portion of this paper, and the under-tuning principle of operation thus far has not been given a trial.

OPERATION WITH INTERLOCK BETWEEN LINE AND GROUND SWITCHES

Fig. 2 is a connection diagram of the complete control equipment on the Lock 12 Petersen coil, the interlocking of the grounding switch control circuit with the protective relay scheme on the line switch being accomplished by the use of the two type *H G* auxiliary relays shown in the lower left hand corner of the diagram. Operators were also given instructions to always close the neutral grounding switch; that is, to take the Petersen coil out of service before doing any line switching. The Lock 12 coil operated under this new operating procedure from February 24, 1923 to January 26, 1924, during which time no further high-voltage disturbances were experienced, indicating that a satisfactory solution to the problem had been found.

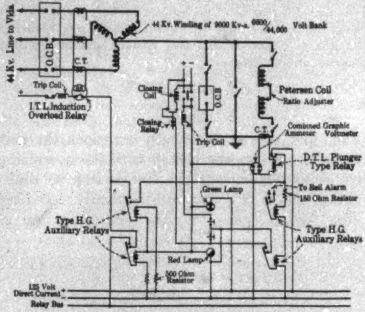
44 Kv. Line to Vida
O.C.B.
C.T.
44 Kv. Winding of 9000 Kv-a. 6600/44,000 Volt Bank
Trip Coil
I.T.L. Induction Overload Relay
Closing Coil
Closing Relay
O.C.B.
Trip Coil
Petersen Coil
Ratio Adjuster
C.T.
Combined Graphic Ammeter Voltmeter
D.T.L. Plunger Type Relay
Green Lamp
To Bell Alarm
150 Ohm Resistor
Type H.G. Auxiliary Relays
Type H.G. Auxiliary Relays
Red Lamp
500 Ohm Resistor
125 Volt Direct Current
Relay Bus

FIG. 2—LOCK 12—PETERSEN COIL INSTALLATION—SCHEMATIC DIAGRAM OF CONNECTIONS

A complete log of all the operations was taken during the period mentioned and summary of these operations is given in Table I which shows the following results:

Flashovers. There was a total of 109 insulator flashover indications recorded on the combined graphic ammeter and voltmeter, 94 of which, or 86 per cent, did not result in line interruptions and are considered perfect operations of the Petersen coil.

On the remaining fifteen flashover indications the line switch opened causing an interruption to service. In each case, however, the line was promptly put back into service, O. K., indicating that the trouble was not a solid ground, but rather a flashover which the Petersen coil failed to clear. These operations are therefore considered faulty.

Solid Grounds and Phase-to-Phase Short Circuits. There was a total of five solid grounds and 21 phase-to-phase short circuits, due to line and transformer troubles. These cases of trouble were outside of the operating sphere of the Petersen coil, but in each case the line overload relays functioned properly and promptly to clear the disturbance.

Interruptions to Lock 12-Vida Line Due to all Causes. Table II is a classification of service interruptions to the Lock 12-Vida line from March 1, 1923 to January 26, 1924, exclusive of prearranged interruptions and those due to trouble on other parts of the Alabama Power Company's system. This tabulation shows that there was a total of 40 interruptions to service with a total outage of 221 minutes for this eleven months' period, which is a far better service record than was obtained for any previous period of similar duration.

TABLE I

SUMMARY OF PETERSEN COIL OPERATIONS.[1]

Month	Flashover Indications				Transformer Trouble		Line Trouble	
1923	Under 6 Amperes	Over 6 Amperes	Faulty Operation of Coil Line Switch Opened	Switch 208 Closed but no Interruption	Phase-to-ground	Phase-to Phase	Phase-to-Ground	Phase-to-Phase
March......	—	2	1	–	1	–	–	–
April......	—	2	2	—	–	1	–	2
May.......	4	5	3	6	–	–	1	5
June.......	—	4	—	—	–	1	–	3
July.......	—	4	—	—	–	–	2	3
August.....	15	22	7	—	–	–	–	5
Sept........	2	9	1	4	–	–	–	1
Oct........	—	4	—	4	–	–	–	–
Nov........	—	—	1	1	–	–	–	–
Dec........	2	2	—	—	–	–	–	–
1924								
Jan.	—	1	—	1	–	–	1	–
Total	23	55	15	16	1	2	4	19
			Total number of operations..........135					

1. Copies of the complete log from which this Summary was compiled may be obtained from the authors on request.

This service record is especially remarkable for a long radial feeder when it is taken into consideration that 144 minutes of the total time out were due to a single case of trouble when a tree fell into the line.

TABLE II

INTERRUPTIONS ON LOCK 12-VIDA 44-KV. SYSTEM MARCH 1, 1923 TO JAN. 26, 1924.

(Exclusive of Interruptions due to Trouble on Other Parts of Alabama Power Cmpany's System.)

	Lightning		Trees Falling Into Line		Insulator on Lightning Arrester		Transformer		Unknown		Total	
Month	No. of Cases	Time Out Min.	No. of Cases	Time Out Min.	No. of Cases	Time Out Min.	No. of Cases	Time Out Min.	No. of Cases	Time Out Min.	No. of Cases	Time Out Min.
1923												
March	1	2	–	–	–	–	1	2	–	–	2	4
April	3	5	1	6	–	–	1	2	–	–	5	13
May	1	2	–	–	1	2	–	–	7	11	9	15
June	2	4	1	2	–	–	1	2	–	–	4	8
July	2	3	1	2	–	–	–	–	2	3	5	8
Aug.	11	23	–	–	–	–	–	–	1	2	12	25
Sept.	2	2	–	–	–	–	–	–	–	–	2	2
Oct.	–	–	–	–	–	–	–	–	–	–	–	–
Nov.	–	–	–	–	–	–	–	–	1	2	1	2
Dec.	–	–	–	–	–	–	–	–	–	–	–	–
1924												
Jan.	–	–	1	144	–	–	–	–	–	–	1	144
Total	22	41	4	154	1	2	3	6	11	18	41	221

Since the function of the Petersen coil is to reduce the interruptions due to lightning, a comparison of service interruptions to the Lock 12-Vida line chargeable to lightning for the years 1922, 1923 and 1924 proves very interesting as the following tabulation will show:

Year	Method of Operation	Number of Lightning Storms Over Line	Interruptions due to Lightning	
			Number	Total Time Out
1922	System neutral solidly grounded	82	43	230 minutes
1923	Neutral grounded through Petersen Coil	97	7	14 minutes
1924	Ditto	80	22	41 minutes

This tabulation shows that although approximately the same number of lightning storms have occurred in the vicinity of Lock 12 each year, the interruptions due to lightning were greatly reduced while the Petersen coil was in service, proving its effectiveness in suppressing flashovers.

LIMITATIONS IN USE OF THE LOCK 12 COIL

The Lock 12 coil was taken out of service and the system neutral solidly grounded January 26, 1924, because

(a) At that time the line mileage of the Lock 12-Vida system was practically doubled by the addition of new lines, making the total line charging current greater than the Petersen coil was designed for and to take care of which would have required new taps on the coil, and

(b) A generating station was added at Upper Tallassee which was connected to the system through a delta-star transformer bank with a solidly grounded neutral on the 44,000-volt winding.

Of course, a second Petersen coil could have been introduced between the Upper Tallassee neutral and ground since on a Petersen coil system a reactor must be installed at each grounding point; but with two sources of power supply to Lock 12-Vida system, it was felt that reliable service could be maintained at Montgomery without the use of Petersen coils and their expense therefore was not justified. It was also foreseen that the Lock 12-Vida system would be further extended and linked with the Alabama Power Company's main 44,000-volt network, which is solidly grounded at Warrior, Bessemer, Leeds, Jackson Shoals and Anniston substations. This interconnection was completed in the

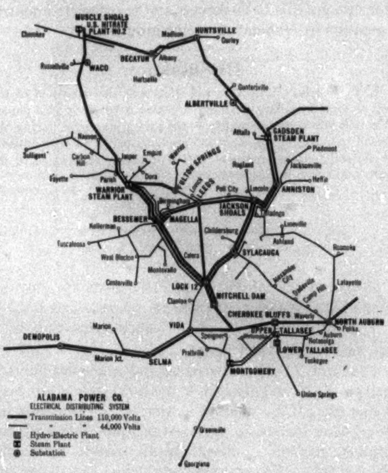
MUSCLE SHOALS U.S. NITRATE PLANT NO.2
Cherokee
Madison
HUNTSVILLE
Gurley
DECATUR
Albany
Hartselle
Russellville
WACO
Guntersville
ALBERTVILLE
Attalla
GADSDEN STEAM PLANT
Piedmont
Jacksonville
Heflin
ANNISTON
Nauvoo
Sulligent
Carbon Hill
Jasper
Empire
Fayette
Parrish
Dora
FULTON SPRINGS
LEEDS
Pell City
Ragland
Lincoln
WARRIOR STEAM PLANT
Birmingham
JACKSON SHOALS
Talladega
BESSEMER
MAGELLA
Kellerman
Tuscaloosa
West Blocton
Montevallo
Centerville
Childersburg
Lineville
Ashland
SYLACAUGA
Roanoke
Calera
LOCK 12
Alexander City
Dadeville
Camp Hill
Lafayette
MITCHELL DAM
Clanton
CHEROKEE BLUFFS
Waverly
NORTH AUBURN
Opelika
VIDA
Marion
UPPER TALLASEE
Auburn
Notasulga
Prattville
LOWER TALLASEE
Tuskegee
DEMOPOLIS
Marion Jct.
SELMA
MONTGOMERY
Union Springs
Greenville
Georgiana
ALABAMA POWER CO.
ELECTRICAL DISTRIBUTING SYSTEM
Transmission Lines 110,000 Volts
" " 44,000 Volts
Hydro-Electric Plant
Steam Plant
Substation

FIG. 3

latter part of 1924 and the 44,000-volt system network now exists, as shown in Fig. 3.

CONCLUSIONS

The Alabama Power Company's experience with the Petersen earth coil indicates that it will greatly reduce the number of interruptions due to insulator flashovers during lightning storms.

All high-voltage disturbances which were experienced when the coil was first placed in service have been entirely eliminated by making provisions to do all line switching, both hand and automatic, with the coil out of service—that is, with the system neutral solidly grounded.

The application of Petersen coils appears to be limited to comparatively low-voltage lines (of 66,000 volts and less) of moderate length, with a single source of power supply. On an interconnected network, such as the Alabama Power Company now operates, several sources of supply are maintained to all principal load centers and good service can be carried on without the use of flashover suppressing devices. In other words the expense and complications of a Petersen coil installation are justified only on radial feeder systems where it is desired to improve service to an important load center which is connected to the power source by only a single line. The Petersen coil in such an application has the advantage over grounded neutral operation by reducing line outages due to flashovers and over isolated neutral operation by eliminating arcing grounds.

Discussion

L. P. Ferris: I wish to ask what was the condition of the coil as regards tuning during the period covered by this paper? Was it the same or different from the tuning employed during the period covered by the authors' previous paper on this subject? If the coil remained in the condition of tuning mentioned in the previous paper, overtuned by some 23 per cent., perhaps that will explain to some extent why the coil failed to prevent the tripping of the line switch in some of those fifteen cases listed in Table I. Under this detuned condition, the coil does not limit the fault current to as low a value as it would if it were tuned more exactly. With 23 per cent overtuning (*i. e.*, with 1207 ohms reactance), the measurements reported in Mr. Lewis' paper of 1923 indicate a fault current of about 4.8 amperes, whereas the same paper gives measured currents of approximately 2.4 amperes for the coil setting nearest resonance (982 ohms). It is conceivable that, under some fault conditions, the arc might be maintained if the current were not reduced below the first figure, but would be extinguished if the current were limited to the smaller value corresponding to exact tuning. Perhaps Messrs. Oliver and Eberhardt could say how large a factor this detuning might be in accounting for some of these fifteen cases? As brought out in the discussion of the earlier papers, it would seem desirable if the coil were operated slightly detuned, that it be operated somewhat undertuned rather than overtuned, although the direction in which it deviates from the exact tuning would not seem to affect its ability to quench an arc.

W. W. Lewis: In this country the trend of operation away from the isolated neutral and toward the grounded neutral has limited the field of the Petersen coil. Nevertheless, the report of Messrs. Oliver and Eberhardt is valuable in clearly pointing out what can be done with this device under suitable circuit conditions.

In Europe, the tendency has been to design the reactor with an iron core to operate normally off resonance, but to be pulled into resonance by saturation when one conductor becomes grounded. As an alternative, the reactance is designed to be outside a certain band on each side of the resonance condition at all times. Both of these designs are intended to prevent certain overvoltage troubles anticipated when the three poles of an oil circuit breaker do not operate simultaneously. The modified devices have found much favor and have been very successful.

W. W. Eberhardt: I shall answer Mr. Ferris' question by stating that the coil, during the period reported in this paper, was overtuned approximately 23 per cent. Towards the end of the operating period, the coil was slightly undertuned but that was after the end of the lightning season and no operating experience was obtained with the undertuned setting.

Theory of the Autovalve Arrester

BY JOSEPH SLEPIAN[1]
Member, A. I. E. E.

***Synopsis.**—The advantage of valve type arresters for high-voltage, power-system protection is briefly discussed. The theory of the autovalve arrester is given.*

I. Induction and Regulation of Surges

In the last few years, considerable light has been shed on the manner of induction of high voltage on power systems by lightning,[2] and some estimates of the magnitudes involved have been made. Charges on clouds produce electrostatic fields extending down to the ground, which induce charges on power lines. The vertical gradient at the earth's surface due to these fields has been estimated[3] to be of the order of 100 kv. per ft. When the inducing charges on the clouds disappear suddenly by a lightning discharge, the induced charges on the power lines produce voltages to ground equal to the height of the lines multiplied by the inducing gradient, and, therefore, of the order of hundreds of kilovolts. The induced charges may be a few miles in extent.[4]

The power line becomes then a source of voltage so high as to be dangerous to connected machines, and the very important question arises when considering the possibility of relief by lightning arresters as to what is the regulation of this source of voltage. It is now well recognized[5] that voltage due to a free charge of this type on a power line regulates like a generator having an internal resistance of a few hundred ohms. Hence the voltage can be quickly materially reduced if, and only if, sufficient current is drawn from the line. An arrester will be effective, if, and only if, it draws nearly two amperes per kilovolt of induced surge[6]. An arrester then must be able to discharge hundreds of amperes with only a moderate rise of voltage.

II. Energetics of Surge Dissipation

The large current which must pass through an arrester if it is to be really effective introduces great difficulties in the design of arresters of the arc-resistance type for higher voltage circuits. This is not due to the energy of the surge itself, which is only moderate in amount because of its short duration, but to the energy supplied by the normal working voltage, which lasts throughout the whole arcing period, and may be from one-half cycle to several seconds. For example, a surge 1.86 mi. long will discharge for only 1/100,000 sec. If an arrester connected to a line, the normal voltage of which to ground is 10,000 volts, discharges 900 amperes from this surge and reduces the voltage thereby to 30,000 volts, the energy involved will be only 30,000 by 900 by 1/100,000, or 280 watt-seconds. On the other hand, if the normal line voltage discharges 300 amperes for one half-cycle, or 0.0083 second, the energy will be 10,000 by 300 by 0.0083 or 24,900 watt-seconds, or more than 90 times the energy due to the surge alone.

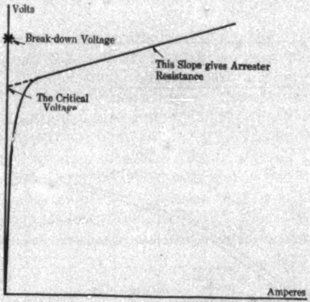

Fig. 1—The Valve Characteristic

Valve type arresters, being built up of elements having the characteristic shown in Fig. 1, are not subject to the disadvantage of disposing of this large draft of energy from the normal voltage. With the passing of the surge, and the restoring of normal voltage, the discharge ceases. It is, therefore, entirely practical to construct valve type arresters with adequate discharge capacity for even the highest voltages.

Some illuminating calculations made for an arrester set at the center of a freed charge as shown in Fig. 2 are given in the curves of Fig. 3. The initial surge voltage is taken as 250 kv. The normal voltage to

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[1] Research Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

[2] A. L. Atherton, Trans. A. I. E. E., Vol. XLII, p. 179 (1923); E. E. F. Creighton, Trans. A. I. E. E., Vol. XLI, p. 52 (1922); D. W. Roper, Trans. A. I. E. E. Vol XXXIX, p. 1895 (1920); C. P. Steinmetz, Trans. A. I. E. E., Vol. XXXIX, p. 1941 (1920).

[3] F. W. Peek, Trans. A. I. E. E., Vol. XLIII, (1924); H. Nörinder, *Electrical World*, Feb. 2, 1924; E. E. F. Creighton, Trans. A. I. E. E., Vol. XLIII, (1924).

[4] H. Nörinder, loc. cit.; F. W. Peek, loc. cit.

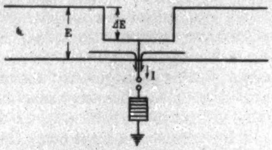
[5] H. Rudenberg, "Elektrische Schaltvorgange," Berlin 1924, p. 330.

[6] E. E. F. Creighton, Trans. A. I. E. E., Vol. XLII, p. 179 (1923).

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

ground is taken to be 8660 volts, and the line surge impedance, 400 ohms. The heavy line curves show the reduction in voltage for varying resistance in an arrester of the arc type and one of the valve type with critical voltage, 10,000 volts. Evidently suitable protection is not obtained until the arrester resistance is less than 50 ohms.

The light lines show the energy dissipated in the arrester. In order to get the curve for the valve type well into the picture, it is necessary to consider a surge 400 mi. long; the more reasonable surge length, 4 mi., would be barely visible on the scale chosen.



ΔE = 200 x I

Voltage Reduction = 200 x Arrester Current

FIG. 2—BASIS OF CALCULATION OF FIG. 3

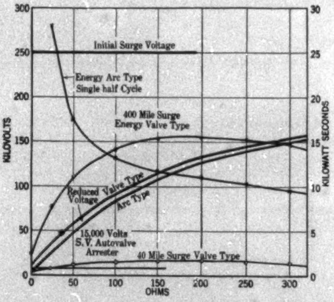


FIG. 3—ENERGETICS OF SURGE DISSIPATION

The energy relations illustrated in Fig. 3 have proved determining factors. The only practical arresters with adequate discharge capacity for high-voltage lines are the valve type arresters.

III. MODERN VALVE TYPES

The so-called valve characteristic, shown in Fig. 1, has been frequently discussed, and the terminology indicated in the figure is generally used. Conducting systems having this characteristic are numerous in nature. However, in most of these systems the critical voltage is too low (as in contacts or electrolytic polarization cells) or the current which may be carried is too small, (as in thermionic or low-pressure gas tubes), to be useful for lightning arresters for power lines. So far the only systems which have been found to possess the valve characteristic to a necessary degree have been certain films which are made conducting by application of sufficient voltage, but which are subject to constant repair action, requiring the continued application of the high voltage for the maintenance of the conductivity and in which the original resistivity is restored when the voltage is reduced. Three practically used arresters have been developed utilizing films of this type.

The Electrolytic Arrester. The film in this arrester consists of a layer of gas-laden aluminum oxide, which forms on an aluminum anode in a suitable electrolyte. Application of a few hundred volts breaks this film down, but the flow of current brings about a repairing electrolytic action.

The Oxide Film Arrester. In this arrester, the film is initially a layer of varnish, which, in use, is gradually replaced by litharge, PbO. The repairing action lies in the thermal effect of the current upon lead peroxide, which reduces it to litharge at the points of breakdown of the film.

The Autovalve Arrester. Here the film is a thin layer of air next to a cold cathode which is the seat of the cathode drop in a glow discharge. With the application of sufficient voltage, this air film becomes highly ionized, but the discharge of these ions into the electrodes and recombination quickly restore the normal resistivity when the voltage is reduced.

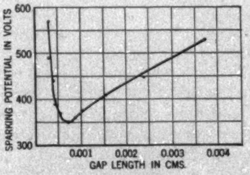


FIG. 4—SPARKING POTENTIAL FOR SHORT AIR-GAPS

IV. THE BREAKDOWN OF AIR BETWEEN PARALLEL PLANE ELECTRODES

Since the active element in the autovalve arrester is air, any explanation of the arrester's design and functioning must include a discussion of the properties of air with respect to electrical breakdown and resulting conductivity. Fig. 4 shows the relation between breakdown voltage and distance between parallel plane electrodes. A striking feature of this curve is the minimum at electrode separation of 0.001 cm., so that shorter separation than this requires increased voltage for breakdown. This remarkable fact is readily explained by the current theory of ionization by collision[7].

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[7] J. J. Thomson, "Conduction of Electricity in Gases," p. 381. J. S. Townsend, "Electricity in Gases," Chap. VIII and IX.

The existence of a minimum breakdown potential of about 350 volts for short gaps may seem contradictory to experience. For example, a widely used type of telephone protector consisting of two small carbon blocks separated by 0.002 in. will usually break down at 200 volts, or sometimes even less. In this case, however, the breakdown is due to the lining up of carbon dust particles in the intense electric field so that a conducting bridge which starts an arc is formed. If precautions are taken to prevent contacts taking place in this or any other manner, the existence of a minimum breakdown voltage may be shown experimentally.

In the autovalve arrester, as will be explained later, it is necessary to use gaps between resistance material electrodes with a breakdown voltage of little more than 350 volts. The curve of Fig. 4 indicates that an electrode separation of 0.0003 in. is necessary for this. At first sight, so minute an electrode separation would appear impracticable in a commercial arrester.

This difficulty was overcome in the early experimental autovalve arresters by merely placing the resistance electrode disks in contact. Due to the resistivity of the electrode material, the gap would not be short-circuited at the contacts. At the same time, in the neighborhood of each contact, there would be points at which the electrode separation was 0.0003 in., and there, breakdown at 350 volts would occur.

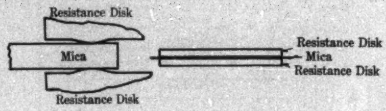

Fig. 5—The Autovalve Gap

A better solution now used was found later and consists in using a mica spacer, 0.003 to 0.005 in. thick, placed directly between the disks. Fig. 5 shows the standard autovalve element (right) and section at the mica spacer, highly magnified (left). The mica, having a dielectric constant of six to seven, distorts the electrostatic field, very much as if it were conducting. Thus there is a concentration of electrostatic stress at the corners of the mica. Due to inherent variations in the nature of the surface of the resistance disk and its contact with the mica spacer, the total voltage applied is not expended symmetrically between the two disks and the edges of the mica, but at some points nearby, all the voltage appears between one resistance disk and the adjacent mica edge, and at other points between the other resistance disk and adjacent edge. Hence, when a little more than 350 volts is applied, these highly stressed points break down and precipitate the discharge of the whole gap. Numerous tests have shown that in commercial autovalve arresters, the breakdown of the column of disks is less than 400 volts per gap.

This expedient of using the electrostatic influence of the mica spacer to precipitate the discharge at low voltage is not practically useful if metal electrodes are used. The discharge must start at the mica, and if it is permitted to concentrate at its point of origin, as with metal electrodes, the mica is quickly destroyed. In the autovalve arrester relatively high resistivity electrodes are used, which limit the intensity of the discharge next to the mica. Hence, thousands of discharges may be sent through it with no deterioration of the mica.

V. Electric Discharges in Air

Arc Discharge. The breakdown of a gap is due to the ionization produced by the high electrostatic gradient. If the gap is to remain conducting for the duration of a discharge, this ionization must somehow be maintained. If the cathode remains cold the ionization is effected by the field becoming so distorted that with carbon electrodes approximately 350 volts are impressed across a layer of air 0.001 cm. thick next to the cathode. The discharge then takes the form of a glow. If, however, the cathode becomes sufficiently hot for thermionic emission, much less voltage need be expended at the cathode and the discharge takes the form of an arc.

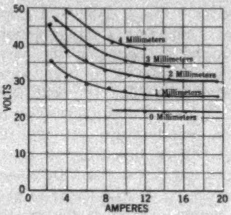

Fig. 6—Arc Characteristics for Copper

In the arc discharge the voltage expended at the cathode must be sufficiently great to maintain the cathode at a temperature sufficiently high for thermionic emission, and also must be sufficiently great, and concentrated on a sufficiently small space next to the cathode, so that the electrons liberated thermionically will ionize by collision. Twenty volts is sufficient for both these purposes for most electrode materials.

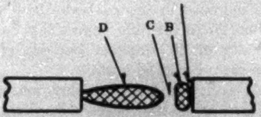

Fig. 7—The Glow Discharge (magnified)

In addition to the voltage expended at the cathode some must be expended in carrying current in the remainder of the discharge. This additional voltage is found to vary inversely as the current strength, so that characteristics such as shown in Fig. 6 are obtained.

It is now clear why an arc discharge is not suitable for giving the valve characteristic to a gap. Aside from the melting or burning of the cathode due to its necessary high temperature the voltage of the arc discharge is too low, (about 20 volts for short gaps) in comparison with the lowest reliable gap breakdown of about 350 volts.

Glow Discharge. When the cathode is too cold for thermionic emission the glow form of discharge takes place in which the ionization is primarily produced by ionization through collision due to high gradient in a film of air about 0.001 cm. thick next to the cathode. The voltage expended in this film is about 350 volts for carbon electrodes. Fig. 7 shows on a magnified scale the appearance of such a glow between copper electrodes. This was obtained by reducing the current to a few milliamperes by means of a high series resistance. When the current is so small, the heating of the cathode is insufficient for an arc, and so the glow discharge may be maintained indefinitely. *A*, the cathode dark space, is the 0.0003-in. film which is kept broken down by the high gradient due to the 300 volts (cathode drop for copper) across it; *B*, the cathode glow is a highly ionized blue region, about 0.005 in. thick. *C*, the Faraday dark space, is also highly conducting, and is about 0.010 in. thick. *D*, the pink positive column, extends to the anode.

So long as the cathode is not completely covered by the discharge, the cathode drop and the cathode current density are approximately independent of current, the cathode glow simply increasing or decreasing in area as the current is varied. The cathode drop and current density do vary with the nature of the cathode; for carbon, the cathode drop and current density are respectively about 350 volts, and 10 amperes per cm^2.

When the cathode is completely covered by glow, further increase of current must of course increase the current density. An increase of current density causes a moderate increase in the cathode drop. Extrapolation of a theoretical formula shows that an increase in current density of 25 amperes per sq. cm. increases the cathode drop by 37½ volts.

The conductivity of the blue glow and the Faraday dark space is so great that little voltage is consumed in these parts. In the pink column, however, the resistivity is greater, and for small currents, the gradient in it may amount to over 5000 volts per cm. However, this gradient decreases as the current increases.

Volt-ampere characteristics of a glow between copper electrodes are shown in Fig. 8. The longer gap lengths show a falling characteristic due to the properties of the pink positive column. For the 0.1-mm. gap, the positive column is completely eliminated and so a flat characteristic is obtained.

Transition from Glow to Arc Following Sparkover. At the moment a spark-gap is broken down by high voltage, the electrodes and, in particular, the cathode are, of course, cold. It follows, then, that the discharge must begin as a glow. After a short but finite time, the energy input at the cathode, due to the glow, heats some spot to such a degree that thermionic emission begins there. The cathode drop then falls to about 20 volts and the current concentrates at the point to sufficient degree to maintain the point hot. For a definite time, then, immediately following a sparkover, the discharge is in the form of a glow.

Heating of Autovalve Disk. It would seem, then, that if the time taken between the sparkover of a gap and the transition from the resulting glow into an arc is long compared to the duration of a surge, it should be very easy to make a lightning arrester which would only discharge in glow form. It is easy to get an estimate of the time involved. The formula given on page 98 of "Mathematical Theory of Heat Conduction," Ingersoll and Zobel, Ginn & Co., may be readily transformed into

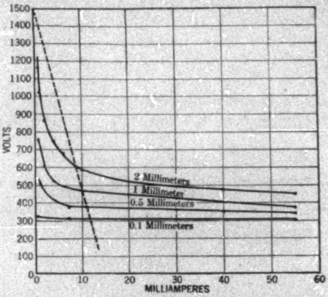

FIG. 8—GLOW CHARACTERISTICS FOR COPPER

$$T = \frac{W\sqrt{t}}{4.18\sqrt{\pi k c \delta}}$$

where *k*, *c* and δ are heat conductivity, heat capacity and density, respectively, of the material making up an infinite solid; *W* is the power input per unit area on an infinite plane surface in that solid, and *T* is temperature rise at that surface. Units are deg. cent., small calories, grams, centimeters and watts.

For the resistance material electrodes used in the autovalve arrester, $c = 0.185$ cal. per gr., $k = 0.016$ cal. per cm^2. per deg. cent. per cm^3. $\delta = 2.0$ gr. per cm^3. For a glow discharge at normal current density we have approximately 10 amperes per cm^2. at 350 volts, giving $W = 3500$. Using these numerical values, we get the curves shown in Fig. 9. Remembering that each one-thousandth of a second corresponds to 186 miles of surge, it is evident that the heating of the electrode surface is so slow that in any surge of practical length the temperature rise will be only a few degrees; hence the discharge will still be in the glow form when the surge has ended, and if the normal line voltage per disk is less than glow voltage, the discharge will stop when the surge voltage disappears.

Current Concentration at Inhomogeneities. So far, it would appear from the formula for the surface temperature rise under a glow that the most desirable materials for electrodes would be those having the highest thermal conductivity and capacity. Metals, then, would seem to be particularly suitable, and one would expect to find them superior to the resistance material used in autovalve arresters. However, when put to test contrary results are obtained. With metal electrodes, a heavy discharge only ten microseconds long will usually end as an arc, whereas, with autovalve arrester disks, it will still be a glow for a discharge 100 times as long. Some as yet unconsidered factor is playing a part here. This factor is the great current concentration in the glow which takes place at any points of the metal surface which happen to have a lower cathode drop than the rest of the metal surface.

It has been mentioned before that the cathode drop in a glow discharge is a function of the material making up the cathode, being 350 volts for carbon, 300 volts for copper, 250 volts for iron, etc. However, like other properties of surfaces, these cathode drops are not absolute constants of the material, but vary somewhat depending upon the state of the surface. Films of absorbed gas or moisture change the cathode drop by a small amount. Dust particles or adhering impurities of any kind may lower the cathode drop enormously. The alkali metals and their oxides are particularly effective in this respect, and may lower the cathode drop to less than 150 volts.

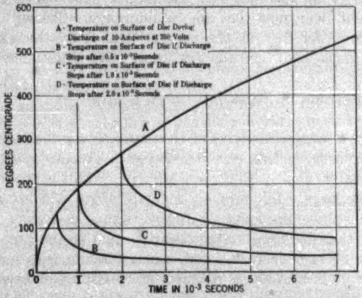

FIG. 9—TEMPERATURE RISE OF RESISTANCE MATERIAL CATHODE

Now imagine on a cathode carrying a glow discharge a point at which the cathode drop is a few volts less than that of the rest of the surface. It is evident that instead of a uniform current distribution over the cathode surface at the moderate density of ten amperes per cm²., there will be concentration of current at the point of low cathode drop. This point will heat up very much faster than is indicated by Fig. 9, and the transition from glow to arc will take place in a much shorter time.

Another effect which is even more important for very short gaps in hastening the transition from glow to arc, is the lining up of minute conducting dust particles, under the intense electrostatic field. In the cathode dark space, the electric gradient is of the order of 350,000 volts per cm. The mechanical force on conducting particles in such fields is relatively enormous, and there is a tendency for these particles to form into chains almost instantly. In very short gaps, five mils or less, these chains bridge the electrodes and start arcs by the current concentration in them with resulting rise in temperature.

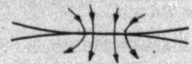

FIG. 10—CURRENT FLOW AT A CONTACT

Temperature of a Contact. The effects of surface inhomogeneities and conducting bridges in causing premature heating of some cathode surface point and striking of an arc is combated in the autovalve arrester by giving the electrode material sufficient resistivity. We may say that if at any point less voltage is consumed in the discharge or gap space than at other points, then this difference of voltage will be expended on the resistance of the path offered to current immediately behind this point in the electrode material. Conditions are then very similar to those which take place at a point of contact between electrodes as illustrated diagrammatically in Fig. 10, the arrows indicating the lines of flow of current. It is not difficult to calculate the approximate temperature rise at such a contact, if the contact area is assumed circular. It is

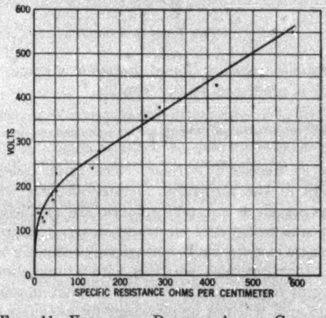

FIG. 11—VOLTAGE TO DRAW ON ARC BY CONTACT

$$T = \frac{E^2}{33\,k\,\rho}$$

where E is the voltage on the contact and

k and ρ are the thermal conductivity and electrical resistivity, respectively, of the electrode material. Because of the very small thermal capacity of the contact, this temperature rise is almost instantaneous. If copper electrodes were used with $k = 1.0$ and $\zeta = 10^{-6}$, a surface inhomogeneity or conducting bridge which would throw 10 volts onto the electrode material at a point would give a temperature rise of

$$T = \frac{100}{33 \times 1 \times 10^{-6}} = 3 \times 10^{6} \text{ deg. cent.,}$$

so that an arc would form instantly. Autovalve electrodes on the other hand with $k = 0.016$ and $\rho = 20$ give

$$T = \frac{100}{33 \times 0.016 \times 20} = 9.4 \text{ deg. cent.,}$$

or a practically negligible increase in the tendency to strike an arc.

Experiments on Arc Drawing at a Contact. The considerations given above show that the voltage necessary to start an arc by contact, increases with the resistivity of the electrode material. The results of experiments confirming this are shown in Fig. 11. The parabolic relation found between voltage and resistivity is predicted by the formula,

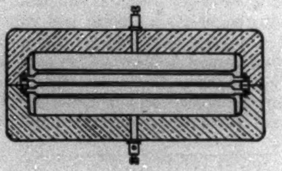

FIG. 12—THE THOMAS ARRESTER

$$T = \frac{E^2}{33\, k\, \rho}$$

Design of Autovalve Arrester. The three essential parameters in the autovalve arrester element which determine its electrical performance are gap length, electrode resistivity, and electrode area. A desirable characteristic is a discharge voltage only little less than the breakdown voltage, and this with a minimum of resistivity in the electrode material so that impractically large area will not be necessary for adequate discharge capacity.

From Fig. 8 it would appear that if the current density is sufficiently limited, a discharge voltage nearly equal to the breakdown voltage may be obtained. Thus the valve characteristic may be obtained for any gap length if only electrodes of sufficiently high resistivity are used. It is very interesting that this principle was understood and described seventeen years ago by P. H. Thomas in his U. S. Patent No. 882,218. Fig. 12 shows his proposed arrester. Quoting from his patent, "It is an essential characteristic of my invention that there shall be an opportunity for a large number of independent static discharges between the discharge plates and that each discharge path shall have such resistance that the dynamo current cannot follow the static discharge."

However, it is only by using very short gaps with a few hundred volts breakdown, that resistivity sufficiently low for a practical arrester may be used. To see this, consider again the curves of Fig. 8 for glow discharge between copper electrodes. Suppose that by using mica spacers or otherwise, a 2-mm. gap could be broken down by 1500 volts. Then the current and voltage of the discharge would be determined by the intersection of the 2-mm. glow curve, and the straight line drawn from 1500 volts on the voltage axis, with a slope equal to the resistance of the circuit. The line shown in the figure corresponds to a resistance of 102,500 ohms, and with this resistance, the discharge voltage is only 600 volts. If the discharge voltage is to be raised up to 20 per cent less than 1500 volts, that is 1200 volts, the line must be swung around until it corresponds to a resistance of 440,000 ohms. At its point of intersection with the 2-mm. glow curve the current will now be one milliampere. Taking 10 amperes per cm². as the current density in the glow this gives 0.001 cm². as the area. The resistance in an electrode up to such an area is given by $\frac{0.261}{a}\rho$ where ρ is the resistivity of the material, a is the radius of the area, in this case 0.0056 cm. Thus, $\rho \times 46.6 = 440{,}000$ or $\rho = 9400$ ohms per cm³. This is prohibitively large. If, however, the gap length and breakdown voltage are lessened, the resistance necessary to keep the discharge voltage nearly equal to the breakdown voltage decreases, and when 0.1-mm. gap with 350-volt breakdown is reached, no resistance is necessary for keeping up the discharge voltage.

For a practical arrester, then, it is necessary to use a gap so short that its glow volt ampere curve is substantially flat. This will occur if the glow has no positive column and from the dimensions given in connection with Fig. 18, this means a gap length not over 0.015 inch. In the commercial autovalve arrester, gap length of not over 0.005 inch is used to ensure breakdown at little more than 350 volts.

With gap lengths less than 0.015 inch the resistivity of the electrode material must be made only high enough to take care of surface inhomogeneities and the partial contacts due to bridging particles. These are not calculable, and the permissible low limit resistivity must be determined by test. The low limit for resistivity in commercial autovalve arresters at present is 20 ohms per cm³., giving a total resistance of only a fraction of an ohm per disk.

The gap length and resistivity being thus given, the area of the electrodes will determine the discharge rate of the arrester. In the *S V* type of autovalve arrester the area has been chosen to give a discharge rate equal to that of the electrolytic arrester.

Discussion

K. B. McEachron: The comparison made by Dr. Slepian between the energy to be handled by the valve-type arrester and the arc-resistance type for equal protection is important. With the arc type, if the flow of system energy following the impulse is not to be excessive, it is necessary to employ considerable series resistance with the result that the arrester's ability to discharge the impulse is seriously impaired. To prevent system current from following the discharge of an impulse through an arrester, it is necessary that the voltage across the arrester, as the impulse ceases, be equal to or greater than the system crest voltage. The modern valve-type arresters are designed to operate on this principle.

In his discussion of the glow discharge Dr. Slepian states that with an electrode spacing of slightly less than 0.001 cm. a voltage of 350 volts is required to maintain the glow. The theory given states that the voltage required to start the glow is less than 400 volts and the current density in the glow is about 10 amperes per sq. cm. until the disk area is covered, after which the gap voltage increases with increasing current becoming 387½ volts with a current density of 25 amperes per sq. cm. Since the effective area inside the mica spacer is of the order of 15 sq. cm. the voltage drop across the air-gap based on Dr. Slepian's figures should be constant at 350 volts with a current of 150 amperes or less increasing to 387½ volts with a current of 375 amperes.

A series of tests covering a period of two years made on these disks have led the writer to suspect that the voltage across the air gap during a discharge was not maintained at approximately 350 volts during a discharge.

It was found, for instance, that with 280 volts (crest) applied that current would follow an impulse so timed as to strike near the crest of the 60-cycle wave. This seemed to indicate a voltage much less than 350 volts following the impulse since 280 volts was sufficient to cause current flow. With steep-wave-front impulses, applied voltages as high as 1200 volts were indicated across a single gap which showed that the breakdown might be more than three times the glow voltage.

More recently we have been able, using the cathode-ray oscillograph, to determine accurately the volt-ampere characteristic of disks with spacers even when the discharge lasted but a few millionths of a second. With this oscillograph photographs have been taken showing wave fronts of less than a microsecond duration with oscillations whose frequencies were of the order of 100,000,000 cycles showing so plainly that their wave shape could be determined. With this instrument, three volt-ampere curves were taken on a stack of sixteen disks with fifteen mica spacers, each different volt-ampere curve having a different crest value of current. The disks were then coated with copper on each side and stacked up without the spacers and a volt-ampere curve taken from which the voltage drop in the disks was determined for any particular value of current. By this means it was found that the average resistance of a single disk was 3 ohms at 10 amperes and 2 ohms with 150 amperes flowing. With 450 amperes the average disk resistance is 1.1 ohms.

The volt-ampere curves of the air gaps for the three different currents were obtained by subtracting the disk volt-ampere curves from the disk-plus-spacer volt-ampere curves. Using Dr. Slepian's statement that with current densities less than 10 amperes per sq. cm. that the area of discharge is proportional to current, it is clear that the disk voltage is constant for currents less than 150 amperes, and is numerically equal to the voltage when 150 amperes are flowing through the disk. Using this method of calculation of disk resistance the curves shown in Fig. 1 herewith were obtained. The theoretical glow voltage according to Dr. Slepian is also given. It is to be noticed that a voltage of from 600 to 700 volts was required to start current flow through the gap with the particular wave front used. As the current increases to about five amperes the voltage rises to 710, 810, and 830 respectively for the three current waves. With both of the higher current curves the voltage tends to remain constant at about 800, decreasing, however, for currents in excess of about 100 amperes. When the current begins to decrease the voltage falls off until at low current values the voltage is less than 100 volts which seems to indicate the formation of an arc. It must be remembered that the shape of these curves is dependent on the amount of resistance material involved at any particular current and if the author's theory of the spreading of the discharge is incorrect then the values of resistance which have been used are not correct. The volt-ampere curve obtained across the stack of disks is a true characteristic, and the volt-ampere relations in the gap will be those given if the proper correction has been made for the resistance of the disks.

If the discharge had the characteristics attributed to it by Dr. Slepian, the voltage across the gap would be represented by 350 volts plus the additional 37½ volts as the current increased to 375 amperes. With decreasing current the relation between the voltage and the current should still be the same so that the volt-ampere curve will be represented by a single line instead of the loop which is actually found.

With all three volt-ampere curves shown, the applied voltage rose to the gap breakdown voltage in a time close to one microsecond, while the respective currents rose to their crest values in 20, 35, and 48 microseconds. From these results it would seem that the time-current relations are important in the gap discharge for with the same current different voltages were found on each of the three different volt-ampere curves.

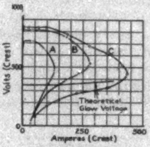

Fig. 1

The effect of rate of voltage application on the breakdown of the gap was not discussed by Dr. Slepian but, as with any phenomena involving ionization, time is required for the gap to reach breakdown conditions. The results of tests on sixteen disks with mica spacers are as follows:

Time-Voltage Relations of Autovalve Gap

Time to Rise to Breakdown	Gap Voltage (Crest)
1.5 microseconds	665 volts
2.5 "	530 "
30 "	390 "
60 cycles	400 " variable

These results show that in discussing the characteristics of the discharge between two electrodes as in the autovalve gap, the effect of time both before the discharge begins and after it has started cannot be neglected.

The test results given in this discussion show that discharges having quite different volt-ampere characteristics from those given by Dr. Slepian are actually to be found in the air gap between autovalve disks. These results are useful not only in connection with the theory of the glow discharge but also are of importance when applied to the actual arrester performance in service. So far as I know these results are the first to be actually obtained under impulse conditions where the time, voltage, and current relations are fully determined.

On the sixth page Dr. Slepian gives a formula for the determination of the temperature rise at the surface of a disk when a certain amount of energy is liberated at the disk surface. I believe that several factors have been left out of this formula.

J. Slepian: The paper which I have presented was written over a year ago, and in that time there have been some important additions to our knowledge of the theory of the autovalve arrester which I would like to mention briefly before taking up Mr. McEachron's welcome account of his researches.

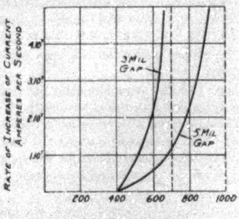

FIG. 2

The autovalve arrester in its functioning depends on the well established fact that a heavy current discharge between electrodes of resistivity of the order of ten ohms per cu. cm. remains in a high-voltage form for a relatively long time, whereas such a discharge between metal electrodes changes into a low-voltage form within less than a microsecond after its initiation. These two forms of discharge have long been known as glow and arc, respectively, and their properties, at least in the steady state, are fairly well known. As stated in the paper, it is generally believed that the arc discharge takes place when the cathode is hot enough for thermionic emission. Then a difficulty arises because it appears that the density of energy input into a metal cathode surface from a glow is not great enough to heat a spot on the electrode to a high temperature in a time so short as a microsecond. It seems necessary to assume almost immediate convergence of current into some spot before that spot has become hot. The resistivity of the autovalve electrodes opposes this concentrating tendency.

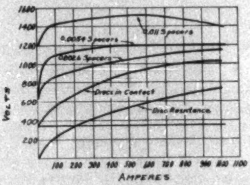

FIG. 3

Until recently, it was assumed that this concentration of current took place at inhomogeneities of the metal surface. Now, however, a new theory has been developed which seems to fit the facts quantitatively, and eliminates the need for inhomogeneities to start the current concentration. This theory is that in the heavy-current glow discharge at atmospheric pressure, the air immediately adjacent to the cathode becomes heated almost immediately to a temperature high enough for thermal ionization, and with metal electrodes this thermal ionization causes the glow to be unstable and to converge into an arc. Further particulars may be found in a paper which appeared in the *Journal* of the Franklin Institute for January, 1926.

Another advance which has been made is in our understanding of the factors which determine the rate of spreading of the discharge in the autovalve gap from its point of initial breakdown next to the mica spacer. We have known, from our tests on the protective valve of the arrester, that this spreading is very rapid, but now we have information as to the influence of electrode spacing, and also know how this factor influences the volt-ampere characteristic for impulse voltages.

The points of lowest breakdown in the autovalve gap are next to the mica spacer, the breakdown voltage there being usually less than 400 volts. If the mica spacer were not there, the breakdown voltage would be much higher, depending on the width of the gap, and for a 5-mil gap would be about 1000 volts. Now it is clear, that with such a 5-mil autovalve gap, if 1000 volts are applied, the discharge does not need to start at the mica, but the gap will break down at all points together. For such a voltage then, the discharge may be said to spread instantly over the electrode surfaces. For a lower voltage the rate of spreading while still very fast will no longer be instantaneous. Since the rate of spreading of the discharge determines the rate at which the discharge current builds up in the arrester, we may expect a curve such as in Fig. 2 herewith, with an asymptote at 1000 volts for the 5-mil gap. For a 3-mil gap the asymptote will be at about 700 volts.

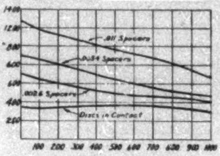

FIG. 4

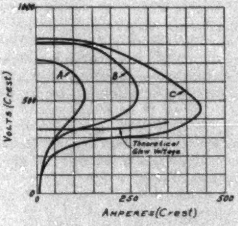

FIG. 5

When an impulse current is sent through an autovalve gap, the total voltage while the current is increasing is determined by the rate at which the current is increasing. If the current is increasing slowly, the voltage will be about 400, but if the current builds up instantly the total voltage may be 1000 for the 5-mil gap and 700 for the 3-mil gap. The total voltage here includes not only the discharge voltage but also the resistance drop through the disks.

These ideas are largely the result of a study by G. F. Harrington of the impulse volt-ampere characteristics of arresters with different sizes of spacers. These were taken by determining the maximum volts and maximum amperes in a discharge by means of sphere gaps. Fig. 3 shows results obtained. It is evident that with the disks in contact, the voltage obtained corresponds to the steady-state glow, but that when the disks are separated, there is an additional rise in voltage which is necessary to cause the discharge to spread sufficiently rapidly over the disk faces.

The lowest curve in Fig. 3 is the impulse volt-ampere curve of the disk resistance, obtained by copper-plating both faces of each disk and stacking them in series. On subtracting this curve from the others in Fig. 3 we get the curves of Fig. 4. Disks in contact give a curve which lies very close to the 350-volt line, but with increasing thickness of spacer, the voltage on impulsive discharge increases rapidly.

Coming now to the results which Mr. McEachron has obtained with the cathode-ray oscillograph and which are shown in his Fig. 1, we see that as regards the magnitudes of the voltage for increasing current, there is general agreement with the results obtained by Harrington and shown in Figs. 2 and 3. It would seem from these curves that Mr. McEachron used spacers at least 0.005 in. thick, which is the upper limit for spacer thickness tolerated in the commercial autovalve arrester. With thinner spacers even better performance would be obtained. However, I believe that because of the incorrect value which he has taken for the ohmic drop through the disks, the lower parts of Mr. McEachron's curves are in error.

First of all, the current density in the discharge is only approximately 10 amperes per sq. cm., and may actually be somewhat larger. It is probably also not an absolute constant, and may vary with slight changes in disk composition. But taking this value, although the whole disk area is not uniformly covered for current less than 150 amperes, when the current decreases to less than this value, the discharge breaks up into a large number of very small spots which are distributed over the whole face of the disk. With this distribution of discharge the ohmic resistance is that of the whole disk and not that of only part of the disk as Mr. McEachron figured. Therefore I have redrawn Mr. McEachron's curves with the correct ohmic drop in Fig. 5.

These curves show three more or less distinct portions—first there is a higher voltage part where the current is spreading over the disk face. Then comes a drop to approximately 350 volts, where the curve tends to become horizontal. The slight departure from 350 volts is readily accounted for due to uncertainty in the disk resistance, since the current distribution in plated disks is different from that with discharge, and also uncertainty in the proper value to take for the current density. Lastly there is a low-voltage part during the last 50 amperes of the discharge which is undoubtedly due to vague contacts and lining up of particles in the gap by the intense field.

These vague contacts which follow an intense discharge explain Mr. McEachron's finding a current flow at 280 volts. By raising the resistivity of the disks this leakage current may be reduced to as small a value as we please. In the commercial autovalve arrester this leakage is usually not detectable on an ordinary oscillograph.

Mr. McEachron's experiments on the impulse breakdown of autovalve gaps are very interesting. They show that the voltage rises to less than double for very fast discharges. When the voltage reaches the value corresponding to the gap length without distortion due to the spacer, we should expect speeds equal to that of a sphere gap

Current Limiting Reactors
with Fire-Proof Insulation on the Conductor

BY F. H. KIERSTEAD[1]
Associate, A. I. E. E.

Synopsis.—*In a previous paper, tests were described which proved conclusively that if conducting material were lodged between the turns of a reactor having bare conductor, the reactor would flashover at the instant a failure occurred on the circuit in which the reactor was placed. In this paper, tests are described which were made during the development of a proper insulation for the conductor of reactors.*

Short-Circuit Tests. *Reactors tested consisted of one reactor with enameled conductor and two reactors with asbestos insulated conductor; one having a thin covering of asbestos; the other a thicker covering.*

The reactor with enameled conductor flashed over during the first short-circuit test. That with a thin covering of asbestos stood one short circuit and arced over in the second short circuit. The reactor with the thick covering stood many short circuit tests without any failure or sign of distress.

These tests established the fact that thin insulation on the conductor will not prevent such failures, even though it has sufficient dielectric strength to withstand the voltages placed across it for the reason that the magnetic force exerted on iron and steel objects will cause them to break through thin insulation. On the other hand, thick insulation will adequately protect the reactor from failure due to foreign substances.

Thermal Tests. *Thermal tests on the asbestos insulation established the following facts:*

First: That this insulation does not smoke excessively at temperatures below 350 deg. cent.

Second: That it does not burn even at temperatures of melting copper.

Third: That its insulation and mechanical strength are not appreciably affected when heated rapidly as high as 350 deg. cent.

Thermal Capacity. *The thermal capacity of the conductor is affected by the insulation, as follows:*

First: Under the effects of extremely high short circuit currents for a very brief interval, the thermal capacity is not affected by the asbestos.

Second: With a moderate short-circuit current for a longer length of time, the thermal capacity of the insulated conductor is increased due to the storage of heat in the insulation.

Third: During normal operation, at rated current, the temperature rise of conductor is increased due to the drop in temperature in the insulation.

Costs. *The asbestos insulation increases the cost of the reactor directly by the addition of the cost of the insulation itself and indirectly by making it necessary occasionally to increase the size of the conductor. However, this increase in cost is not a large percentage of the total cost of the reactor.*

* * * * *

In a previous paper entitled "The Design, Installation and Operation of Current Limiting Reactors" presented by Kierstead and Stephens at the Annual Convention of the A. I. E. E., July 1924, short-circuit tests upon reactors were described. These tests proved conclusively that if conducting material were lodged between the turns of a reactor having bare conductor, the reactor would flashover at the instant a failure occurred on the circuit in which it was placed. In other words, if a piece of metal, such as a nut, bolt, washer, or screw drops or is magnetically drawn into a reactor and becomes lodged between two of its turns, there may be no indication of its presence in the reactor during the normal operation of the circuit but at the instant of a fault on this circuit, the voltage between these turns jumps to many times its previous value, incipient arcs shoot out at the points where the metal bridges between turns and this is followed instantly by a complete flashover of the reactor. Since that paper was presented, an investigation has been carried out to determine a suitable insulation for the conductor of reactors to prevent such foreign conducting material causing reactors to flashover. The purpose of this paper is to describe the tests which were made to determine the kind and thickness of insulation to be used.

The primary requisites of such insulation are as follows:

First: It must have sufficient dielectric and mechanical strength to prevent short circuits between turns by foreign conducting materials.

Second: It must conform to the well established practise of using only fire-proof materials in the construction of current-limiting reactors.

The first part of this paper is devoted to a description of the short-circuit tests made upon reactors to determine the insulation to be used, while the second part describes thermal tests made to determine the fire-proof characteristics of the insulation.

SHORT-CIRCUIT TESTS

Enamel and asbestos were chosen as the most suitable insulations because of their fire-proof characteristics. One reactor was built with enameled conductor and two reactors with asbestos-insulated conductors; one having a thin covering of asbestos, the other a thicker covering. The reason for making tests on reactors with these different insulations was to determine the thickness of insulation necessary.

The short-circuit tests made upon the reactors consisted of bringing up the terminal voltage on a 26,700-kv-a., 25-cycle generator to 13,200 volts, and then

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Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

short-circuiting two of its phases through the reactor under test. All three reactors were the "cast-in concrete" type rated 60 cycles, 68 kv-a., 229 volts, 300 amperes, and were designed for use in introducing 3 per cent reactive drop in a 13,200-volt circuit. A diagram of the connections used in the tests is shown in Fig. 1. The tests made upon the different reactors are described under the following headings: (A) Reactor with Enameled Conductor, (B) Reactor with a Thin Covering of Asbestos Insulation on the Conductor, (C) Reactor with a Thick Covering of Asbestos Insulation on the Conductor.

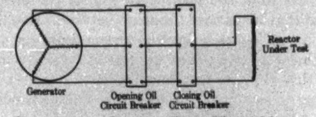
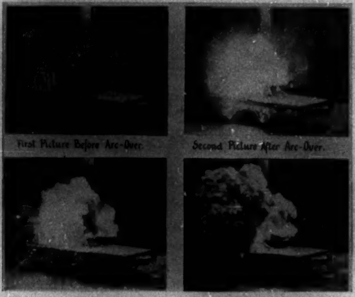


FIG. 1—DIAGRAM OF CIRCUIT USED WHEN MAKING SHORT-CIRCUIT TESTS

A. Reactor with Enameled Conductor. The conductor of the reactor on which these tests were made was enameled and the purpose of the tests was to determine whether the enamel would afford sufficient insulation to prevent a flashover if conducting material bridged between the turns.

The test was made by placing a steel nut between two adjacent turns of the reactor midway between the top and the bottom and then short-circuiting the generator through the reactor. The reactor as it appeared immediately before the test is shown in the upper left hand corner of Fig. 2 which is an enlargement of a portion of the motion pictures taken during these tests. The nut is painted white and is clearly visible in the photograph.



First Picture Before Arc-Over Second Picture After Arc-Over

Third Picture After Arc-Over Sixth Picture After Arc-Over

FIG. 2—REPRODUCTION FROM MOTION-PICTURE FILM TAKEN DURING SHORT-CIRCUIT TESTS ON REACTOR WITH ENAMELED CONDUCTOR

Arcing due to steel nut. *Enamel did not afford* sufficient insulation.

Although the reactor had previously been tested several times under short circuit (without giving any indication of distress), after the nut was placed in it, it flashed over during the first half-cycle of the first short-circuit test. This test proved that enamel is not sufficient insulation to prevent a flashover, due to foreign conducting material.

The nut was thrown violently from the reactor to a distance of about 30 ft. where it struck a board and dented it to a depth of ⅛ in. In short-circuiting a portion of the reactor winding, the nut carried a current in opposition to the main current in the reactor and the magnetic force between these opposing currents was probably the propelling force which expelled the nut. In this respect, foreign conducting material partakes of lifelike characteristics in that it may cause a lot of damage and then clear out and leave no clue as to the cause of the damage.

B. Reactor with a Thin Covering of Asbestos on the Conductor. The reactor on which the tests under this heading were made had a thin wall of asbestos covering on its conductor. It received the same tests as those applied to the preceding reactor, except that in this case a nail was tied to the conductor so as to span between the outside turns of the upper two layers. The reactor with the nail attached is shown in the upper left hand picture of Fig. 3 which is a reproduction of a portion of the motion pictures taken during this test. This reactor stood the first short-circuit test without showing any visible signs of distress but flashed over during the first half-cycle of the second test. The other pictures shown in Fig. 3 immediately followed the instant of flashover.

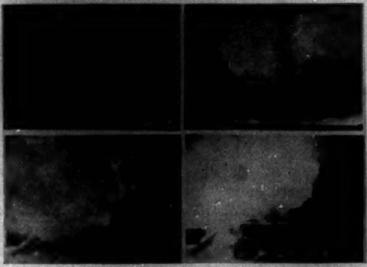


FIG. 3—REPRODUCTION FROM MOTION-PICTURE FILM TAKEN DURING SHORT-CIRCUIT TESTS ON REACTOR WITH CONDUCTOR INSULATED WITH THIN ASBESTOS

Arcing due to nail. Asbestos *was not thick enough* to afford sufficient insulation.

These tests showed that while the thin asbestos covering afforded more protection than the enamel (since it was able to pass through one short-circuit test without failure) still it was not thick enough to adequately protect the reactor from foreign conducting material.

C. Reactor with a Thick Covering of Asbestos Insula-

tion on Its Conductor. The reactor tested next had thicker asbestos insulation on its conductor than that of the preceding reactor. Many short-circuit tests similar to those previously described were made on this reactor with steel bolts and nuts variously located in an endeavor to make the tests as severe as would be encountered in actual service. In this reactor, however, the insulation on the conductor was sufficient to afford the conductor adequate protection from the foreign conducting material to which it was subjected. Some of the reproductions of the motion pictures taken during these tests are of interest, for the reason that having been taken at the rate of 125 pictures per second they show the movement of loose steel objects around and in the reactor. Fig. 4 shows four pictures taken when a loose bolt was placed on the winding midway between the top and the bottom. The pictures are consecutive in the order in which they are numbered. The first picture shows the bolt resting on the winding. The next picture taken 1/125 of a second later, shows one end of the bolt raised up by magnetic force so that it is striking against one of the turns above while the other end is resting on one of the turns below. The pictures also show nails which had been strewn on the floor near the reactor being lifted by the magnetic field. Fig. 5 shows a similar group of pictures except that the bolt is first resting on top of the winding but is later pulled down so as to span between the top and next adjacent layers. The picture again shows nails being lifted from the floor. In Fig. 6, the bolt was placed so that it spanned the space between the top and next adjacent layers before the test and did not move during the test. The chief interest in these latter pictures is that they show nails being lifted from the floor and being drawn into the windings with considerable force.

FIG. 4—REPRODUCTION FROM MOTION-PICTURE FILM OF SHORT-CIRCUIT TESTS ON REACTOR WITH CONDUCTOR INSULATED WITH THICK ASBESTOS INSULATION. SHOWING BOLT IN WINDING BEING RAISED AND NAILS BEING DRAWN TOWARD REACTOR BY MAGNETIC FIELD. ARROWS INDICATE NAILS

FIG. 5—SIMILAR TO FIG. 4 BUT SHOWS BOLT BEING DRAWN BY MAGNETIC FIELD FROM TOP LAYER TO A POSITION BRIDGING BETWEEN THE TWO TOP LAYERS. ARROWS INDICATE NAILS BEING DRAWN TOWARD REACTOR

The reproductions of the motion pictures show that the magnetic field of a reactor exerts a considerable force on magnetic substances. Therefore, the insulation on the conductor not only must be strong enough to withstand the electric stress which may be placed upon it, but must also be strong enough mechanically to withstand, without injury, the cutting or piercing action of iron and steel objects when drawn against it by the magnetic force. It was this latter action that was the primary cause of the failures of the first two reactors.

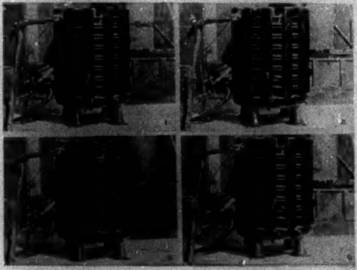


FIG. 6—SIMILAR TO FIG. 5 BUT SHOWS NAILS BEING DRAWN FROM FLOOR INTO REACTOR BY MAGNETIC FIELD. ARROWS INDICATE NAILS

The thicker insulation on the last reactor tested had sufficient resilience to resist the blows delivered to it by the steel objects without being cut and in addition, being thicker, could be indented to a greater depth without being pierced through to the conductor. Therefore, it was able to withstand the many short-circuit tests it was subjected to, without failure.

Summarizing, the foregoing tests established the following facts:

First: That a reactor with bare conductor will flashover during a short circuit if conducting material is lodged in its winding.

Second: That thin insulation on the conductor will not prevent such failures, even though it has sufficient dielectric strength to withstand the voltages placed across it, for the reason that the magnetic force exerted on iron and steel objects will cause them to break through thin insulation.

THERMAL TESTS

Having established the thickness of asbestos insulation required to prevent electrical failures in reactors, it was next required to determine if this insulation was fire-proof.

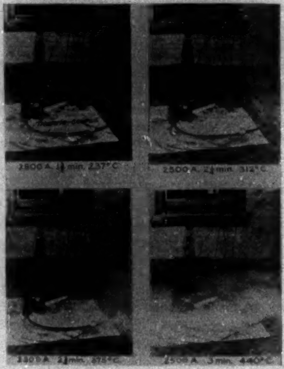


FIG. 7—SUCCESSIVE PICTURES TAKEN DURING HIGH TEMPERATURE TEST ON ASBESTOS INSULATED CABLE

Tests to determine whether the insulation would burn were made on 5-ft. lengths of 250,000-cir. mil. cable insulated with asbestos insulation. The equipment used in making these tests is shown in Fig. 7 and is comprised of a transformer for use in obtaining high current, a current transformer for measuring the current, and a thermocouple placed in the center of the cable. The tests consisted of holding 2500 amperes in the conductor until it melted. The amount of smoke emitted from the cable during these tests is more clearly shown by Fig. 7 than can be described. The figure shows that smoke just began to make its appearance at 312 deg. cent. while it became most dense at 440 deg. cent. As the temperature increased above this figure, the denseness of the smoke gradually decreased and had discontinued before the melting point was reached. At no time did the insulation show any tendency to burn. After the cable had been raised to the melting point of copper, the binding materials which held the asbestos fibers together had been destroyed and as a result the mechanical strength of the asbestos was practically zero.

The next tests were made to determine to what temperatures the cable could be raised without damaging the asbestos insulation.

It was felt that the insulation should be cabable of resisting the effects of having the conductor rapidly heated up to a temperature of 350 deg. cent. without injury to its mechanical or electrical strength. Therefore, a test was made by holding 2500 amperes in the cable until 350 deg. cent. was reached. This required about 2½ minutes. The insulation showed no indication that its mechanical strength had been injured by this test and the puncture tests showed that its insulation strength had not deteriorated.

Summarizing, the foregoing high temperature tests have established the following facts:

First: That this asbestos insulation does not smoke excessively at temperature below 350 deg. cent.

Second: That it does not burn even at temperatures of melting copper.

Third: That its insulation and mechanical strength is not appreciably affected when heated rapidly to temperatures as high as 350 deg. cent.

In actual service with the reactor carrying full short-circuit current, there will be no tendency for the reactor to smoke or the insulation to be injured for the reason that the current will be interrupted by the circuit breaker long before the temperature of the conductor has risen to 350 deg. cent.

The question quite naturally arises as to what effect the asbestos insulation will have on the thermal capacity of the conductor. Of course, if a short circuit is maintained for such a brief period of time that practically all the heat is stored in the copper then the insulated conductor will rise to practically the same temperature as the bare conductor. On the other hand, if the short circuit continues until an appreciable part of the heat generated is dissipated, there will be a period during which the flow of heat from the insulated conductor into its insulation will be more rapid than the flow of heat from the bare conductor into the air. During this period, the insulated conductor will be cooler than the bare conductor.

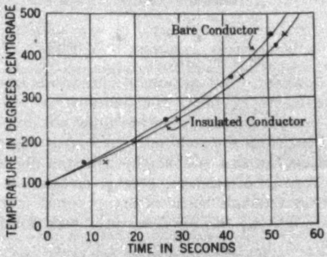


FIG. 8—COMPARATIVE RATE OF HEATING OF BARE AND INSULATED CONDUCTOR FOR SHORT PERIODS

The accuracy of the above statement was checked by comparative tests on bare and insulated 250,000-cir. mil. cable. The tests consisted in holding 4000 amperes in each cable and measuring the time required for the conductors to rise to a given temperature. The temperature of the conductor was measured by thermocouples soldered to the conductor. In Fig. 8, the results of these tests are plotted. It wil be noted that in all the tests the bare conductor was the hottest.

During the normal operation of the reactor when it is carrying rated current continuously, all the heat generated is of course, dissipated and naturally the insulated conductor becomes the hottest. It has been found, however, that although asbestos is one of the best insulators of heat, the asbestos covering used on the conductors for reactors does not impede the flow of heat from the conductor to a great degree. The explanation for this is that the asbestos fibers are so densely formed around the conductor that the rate of conduction through them is many times more rapid than through the forms of asbestos used for heat insulation.

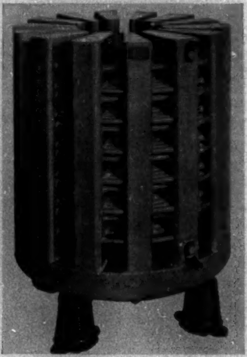


Fig. 9—"Cast-In Concrete" Reactor with Conductor Insulated with Asbestos Covering

As has been shown in the preceding pages, the use of thick asbestos insulation removes the danger of flashover that a reactor without such insulation is subject to and yet does not affect its fire-proof qualities or the simplicity of its construction. Fig. 9 shows a reactor with asbestos insulation on its conductor. It will be noted that it is of the same general construction that has typified the "cast-in concrete" type of reactors for the past ten years. The asbestos fibers are closely and firmly woven on the conductor. They are treated with a compound which makes them very strong and able to resist cutting or tearing very tenaciously.

Insulating the conductors of reactors with asbestos increases their cost directly by the addition of the cost of the insulation itself and indirectly by making it necessary occasionally to increase the size of the conductor to compensate for the reduction in the rate of heat dissipation caused by the insulation. However, the cost of the insulation is not a large percentage of the total cost of the reactor and the additional cost of a larger conductor is greatly offset by the reduced operating charges resulting from the reduction in losses occasioned by use of a larger conductor. Furthermore, as stated above, the asbestos covering has not been found to impede the flow of heat from the conductor to a great degree. These cost increases, therefore, are slight and will make no material difference in the present economies of a system which includes reactors.

In conclusion, attention is again called to the fact that the tests described in this paper have proved that any conductor insulation which will afford adequate protection to a reactor from foreign conducting material not only must have sufficient insulation strength to stand the voltage stresses placed upon it, but also must have sufficient mechanical strength to withstand without injury the cutting and piercing action of iron or steel objects that may be drawn against it by the magnetic field of the reactor. Furthermore, the tests have shown that the asbestos insulation which has been developed for reactors affords protection from foreign conducting materials without sacrificing the fire-proof qualities which modern central station practise demands.

The author wishes to acknowledge the valuable assistance which Mr. L. P. Burgess rendered when the tests herein described were made.

Discussion

V. M. Montsinger: The general tendency today is to make all electrical apparatus as fool-proof as possible. This is especially true of apparatus applied to protect other apparatus. In other words, when the safety of other apparatus worth many thousands of dollars is dependent upon the proper functioning of a reactor it becomes doubly important to make the reactor as nearly perfect as possible. Mr. Kierstead's paper shows a marked advance in the art of making a dependable current-limiting reactor.

I well remember some of the early designs a few years ago, which were built by spacing bare copper cables by means of wooden supports notched to hold the conductors in place. Obviously a reactor having its turns supported by a combustible material, like wood, would not stand a very high temperature on short circuit, certainly not over 150 to 175 deg. cent. It was soon realized that a reactor of this design would not meet the severe conditions demanded of it. When the cast-in concrete type was developed and perfected it was thought that the problem of building a perfect reactor was solved.

But it was not long before someone left a bolt or a nut near a reactor and when a short circuit came on, the loose metal was drawn into the reactor by the magnetic forces set up, and the reactor failed to perform its function. A new problem had appeared which must be solved. This problem was not an easy one to solve because it was a difficult matter to obtain an insulation with sufficient toughness to resist the cutting or bruising action of a bolt hurled against it and at the same time to withstand a temperature of 300 to 400 deg. cent. without becoming charred. Some of the early forms of asbestos insulation were not satisfactory because they contained an appreciable amount of cotton, which soon burned out and weakened the fabric. Mr. Kierstead's efforts have succeeded in obtaining practically a 100 per cent asbestos insulation which is entirely satisfactory and we now have once again a reactor which appears to be practically perfect.

It is to be hoped that operating engineers who are dependent upon these reactors to protect large and expensive apparatus will appreciate Mr. Kierstead's efforts in producing a product so nearly proof against failures.

S. I. Oesterreicher: While I am in hearty agreement with many of Mr. Kierstead's statements regarding the insulation of current-limiting reactor conductors, still I regret very much not being able to give him full credit for his interesting conclusions.

As far as covering of reactor conductors is concerned, originality belongs to Mr. Philip Torchio and Mr. H. R. Woodrow. As far back as 1912, these two gentlemen designed and put in service current-limiting reactors in which the conductors were insulated partly with asbestos and partly with a rubberized tape.

Due to the increased generator rating and voltage of the circuit on which some of these first reactors were installed, they became obsolete and we had occasion to dismantle and inspect them.

On some of them the insulation after about 12 years of service was not impaired in the least and would have been good for many more years of service.

There is no question in my mind about the necessity of covering the reactor conductor with an insulation if reliability is sought. I believe that under normal operating conditions, if foreign materials are dropped or drawn into a bare conductor of a reactor, the effect will be far more serious than Mr. Kierstead states. If reactor turns or layers are short-circuited within the reactor, they act as secondaries in a transformer and the currents in the two circuits are inversely in proportion to the number of turns in these circuits. Therefore, the heating due to the increased currents in the short-circuited sections will make itself manifest in a very short time and greatly to the detriment of the reactor.

Mr. Kierstead attaches great importance to the absolute fire-proof quality of the insulation upon the reactor conductor. He gives data of tests carried up to 350 deg. cent. without any distress of the asbestos insulation.

However, the limitation of an arrangement as Mr. Kierstead describes it is not the absolute fire-proof construction but the fact that the temperature coefficients of expansion between copper and concrete are so greatly different and the temperature stresses caused by the copper in the concrete so out of proportion that cracking of the concrete starts long before any shortcomings of the insulation becomes dangerous.

We have in operation throughout the country over 2500 reactors with all kinds of fibrous insulations and do not seem to have trouble on account of the kind of insulation over the conductor.

I do agree with the statement that the insulation should be mechanically strong to be able to withstand injury. From time to time, we have received complaints from one or another of our clients that suddenly a reactor started to rattle. Investigation proved that in practically all cases the trouble was due to foreign bodies becoming lodged in the reactors.

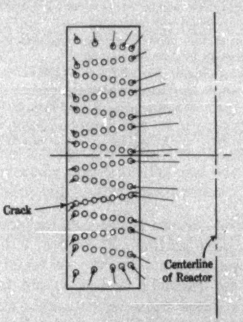
Thus it would seem that as long as the reactor conductor insulation has ample dielectric and mechanical strength it is only a convenience of design or cost as to what kind should be used.

A. E. Kennelly: In connection with Mr. Kierstead's paper, it may be worth while to point out what is certainly known although not perhaps very generally known, that in a uniform magnetic field, parallel lines of magnetic force so to speak, a small sphere of iron or steel is not subjected to any mechanical pull. That is contrary to popular belief on that subject. If a small sphere of steel is supported in a uniform magnetic field, no matter how strong the field, it will pull one way as much as the other. But if the magnetic field, instead of being uniform, is divergent in one direction, or convergent in the other, then the ball of iron or steel inserted in the field will be pulled in the direction of convergence and the pull is proportional to the flux density and to convergence. Mathematically it may be expressed as proportional to $B \cdot \frac{dB}{dc}$

W. W. Lewis: Reactors as well as other apparatus have their share of failures; some of these in the past have been due to mechanical weakness, others have been attributed to high voltage and others to so-called foreign metallic material.

The theory of foreign material has been amply substantiated in practise. In one installation 9 reactor failures occurred. Inspection after the first and last of these disclosed nails lying across some of the turns. In two other cases the reactors had been previously damaged. In the five other cases no definite cause was found for the failure. After the first failure, however, a quantity of bolts, nuts, washers and nails was cleaned out of the reactors. It may well be that some of these articles were missed in the inspection as indicated by the nail found after the last failure.

It seems obvious that live conductors so closely spaced as in reactors, are a menace to operation if left uninsulated and Mr. Kierstead is to be commended for removing this source of trouble.



SECTION THROUGH A REACTOR, SHOWING HOW THE MAGNETIC FORCE EXERTS A COMPRESSIVE STRESS UPON THE CONCRETE

F. H. Kierstead: Mr. Oesterreicher is correct in his statement that when the conductor is raised to 350 deg. cent. the concrete may crack, but his inference that failure of the reactor follows this cracking is not correct. The magnetic field of the reactor exerts a force on the winding that is transmitted as a compressive force on the concrete which forces the cracked surface closer together as is indicated in the illustration shown herewith. Because of this fact the reactor continues to perform its function just as well after the concrete has cracked as before. The principle involved here is very much the same as that in a reenforced concrete building. The concrete of a building cracks due to expansion and contraction but it causes no concern because its compressive strength is not impaired by the cracks while the steel reenforcing resists the tension forces. The two cases are similar except that in the reactor the forces are all compressive and thus the reenforcing steel is not required.

These statements have been verified by twelve years of experience with this type of reactor in service.

Temperature Rise and Losses in Solid Structural Steel

Exposed to the Magnetic Fields from A-C. Conductors

BY O. R. SCHURIG* and H. P. KUEHNI*
Member, A. I. E. E. Non-member

Synopsis.—*Experimental investigations were made for the determination of the temperature rise and the losses occurring in structural steel members exposed to the fields from a-c. conductors. The work originated from the demand among the designing engineers of an electrical manufacturing company for practical data which would enable them to estimate, at least roughly, the temperature rise and the heat losses in the more common cases of structural steel members passing near the conductors. In the paper presented here are given the results of the investigations in question. Examples are given illustrating the method of procedure in estimating temperature rise and losses in structural steel. The method of calculation applied is, in most cases, empirical. The major factors affecting temperature rise and losses are discussed on the basis of the test data. An understanding of these factors will often permit a designer to avoid iron heating without sacrifice of space or of economy. Thus a good part of the data presented apply to cases for which the temperature rise of the steel would ordinarily be considered not seriously objectionable.*

Iron members passing between going and return conductors in adjacent-phase layouts may reach very high temperatures; on the other hand, iron members passing at right angles to, but outside of, a group of going and return conductors will usually heat less than an iron member crossing a single conductor, when the current and minimum spacings are the same.

The temperature rise of structural iron beams in large rectangular magnetic frames surrounding a straight conductor (as in isolated-phase layouts) and having sides larger than, say, 5 ft. may be calculated separately for each individual beam disregarding the other three sides of the frame, even if the conductor passes near the corner at a point as close as 8 in. from the nearest part of each of the two adjacent sides. Small closed structural steel frames or ducts having sides of the order of two ft. or less, when surrounding a conductor must however be expected to heat considerably more than a single straight iron member passing at right angles to a conductor, for equal spacings and currents.

A long and narrow steel member, such as an I-beam or a channel, will ordinarily heat much less when running parallel to a conductor than when crossing it at right angles, for the same spacings and currents. This is because in the parallel arrangement the iron portion of the flux path around the conductor is usually a much smaller part of the complete flux path than in the case of a steel member crossing at right angles to the conductor. It will, therefore, often be advantageous, from the standpoint of iron heating, to lay out stations with conductors parallel rather than at right angles to the iron members.

For various sizes of straight long solid structural iron members crossing at right angles to a conductor and having a thickness not less than 1/8 in., when placed with respect to the conductor at any one center spacing within the range from, say, 6 to 16 in., the temperature rise was practically independent of the size, or cross-sectional area or shape of the iron member. In the same tests the losses of the various solid structural iron members not thinner than 1/8 in. were proportional to the perimeter of the iron cross section regardless of shape or volume. The iron members in question ranged in size from a 1 1/4-in. standard pipe to a 12-in. by 10-in. I-beam.

The results presented were found to be in good agreement with other investigators' theoretical results relating to skin effect and losses in solid iron.

* * * * *

INTRODUCTION

IN electric stations heavy a-c. conductors are often run in close proximity to steel structural members.

It is well known that the steel members when exposed to a-c. magnetic fields will tend to heat on account of induced eddy currents and hysteresis losses. While a local temperature rise of a few degrees here and there in the steel will generally not be seriously objectionable, the iron losses must not reach an amount sufficient to raise materially the ambient temperature in the vicinity of the conductors themselves or in the vicinity of other electrical apparatus. Moreover undesirable expansion of the building steel and cracking of the concrete may sometimes result from excessive heating of the steel. It is even possible that, unless consideration is given to the iron losses, they may seriously add to the station losses.

While all of the above matters have been known, at least qualitatively, there has been but a scant amount of data available for designers to estimate the temperature rise and the losses in question.

OBJECT AND SCOPE OF PAPER

Since there has been an increasing demand for data on heating and losses in solid structural steel, especially in view of the more frequent use of the isolated-phase system of station layout, with its more wide-spread stray magnetic fields, an experimental investigation was made by an electrical manufacturing company to obtain, for their designers and engineers, practical data on the temperature and the losses in a number of the more common cases of structural steel members passing in the vicinity of heavy-current a-c. conductors. Actual measurements of temperature rise and of losses on full-sized steel members were made in the laboratory. Naturally only a limited variety of structural arrangements could be represented. Nor was it possible to set up a general or fundamental formula, for practical use, for the calculation of the temperature rise of solid iron members. Hence the application of the results obtained is primarily limited to the range of conditions covered by the tests. Nevertheless, it has

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*General Engineering Laboratory, General Electric Co.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

been possible, by suitable choice of test conditions, to obtain a considerable amount of useful data, which have been effectively applied for a year or more, in practical cases of station steel design and of conductor layout. The principal results obtained are summarized in the following paper. The major factors affecting temperature rise in structural steel are reviewed and discussed from a practical, rather than theoretical, standpoint. An understanding of the factors affecting heating will often permit a designer to avoid iron heating without sacrifice of space or of economy. Thus a good part of the data presented apply to cases for which the temperature rise of the steel would ordinarily be considered safe and not seriously objectionable. No recommendations are made, however, concerning the safe permissible temperature rise of the steel members or the maximum allowable losses.

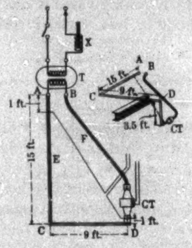

FIG. 1—CIRCUIT ARRANGEMENT FOR HEATING AND LOSS TESTS OF SOLID STRUCTURAL IRON MEMBERS

T Transformer 550/27.5 volts, 150 kv-a.

X adjustable air and iron core reactor

C. T. Current transformer. 6000 amperes, ratio 1200:1

Circuit *A C D* to current transformer consisting of four 4 in. by ¼ in. copper bars in multiple; circuit from *C T* to *B* consisting of three 2,000,000-cir. mil cables in multiple.

The magnetic field intensity at points below the center of *C-D* and up to 16 inches from the center of *C D* was approximately the same as that for a long straight conductor running along the line of *C D* with return conductors remote. The busbar currents had a wave shape approaching closely a sine wave.

Results are given of measurements of temperature rise and of losses in structural steel *I*-beams, channels, pipes, plates, ducts and of temperature rise in rods, sheets and meshes when the iron members were exposed to the fields from 60-cycle, 40-cycle or 25-cycle conductors carrying currents as high as 5800 amperes. Special tests on riveted and bolted steel frames placed within the field from a high-current circuit simulating isolated-phase construction were made. Experimental data were obtained to serve as a basis for designing copper sleeves to minimize heating and losses in structural iron members crossing at right angles to conductors.

It was, however, not attempted, in this work, to establish fundamental methods of calculating losses and temperature rise for the most frequent case of a straight magnetic member crossing at right angles to a straight conductor, because of the considerable variation of the m. m. f. along the iron. For this case a detailed analysis of the magnetic, electrical and thermal phenomena occurring within the steel and in the space nearby, when exposed to a-c. fields, would be extremely complex, since a thoroughgoing calculating procedure of this kind would call for consideration of not only the varying m. m. f. along the iron, but also of the varying heat distribution along the iron. The effect of these factors would have to be studied in addition to such factors as the varying permeability both at different surface flux densities and at different depths; the varying degree of flux and current penetration in thick and in thin iron members; the heat dissipation characteristics of the iron, including the effects of surrounding space; the electrical resistivity; the frequency; the hysteresis losses at both low and high densities. It is therefore highly questionable whether a rigorous analysis of the problem of solid structural steel heating would lead to practical results in the common case of non-uniform m. m. f. along the iron member. Hence in the investigation reported here, the experimental method was preferred to the theoretical analysis because it was felt that the ex-

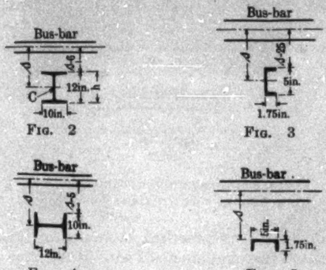

FIG. 2 FIG. 3

FIG. 4 FIG. 5

FIGS. 2-5—TEST POSITIONS OF IRON MEMBERS (*I*-BEAM AND CHANNEL BEAM, SAMPLES NO. 1 AND 3 RESPECTIVELY IN TABLES I AND II)

With respect to conductor crossing at right angles to the iron

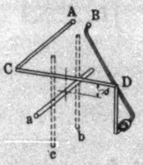

FIG. 6—1¼-IN. STANDARD SWITCHBOARD IRON PIPES (ITEM NO. 7 IN TABLES I AND II)

Crossing at right angles to conductor *C-D* of Fig. 1

perimental procedure would give sufficiently close results for practical use with a minimum of expense and time.

On the other hand, in cases where a uniform distribution of m. m. f. along the iron path exists or is closely approached, as, for instance, in the case of a solid iron circular ring surrounding a straight conductor, or even in the case of a uniform square magnetic frame with a straight conductor passing approximately through the center of its plane, the results of theoretical analyses made by previous investigators* were found to be applicable to practical estimates of losses and temperature rise, and are dealt with in the paper.

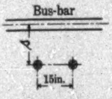

Fig. 7—1¼-in. Standard Switchboard Iron Pipes Crossing at Right Angles to a Straight Long Conductor

Classification of Cases Investigated

The cases of iron heating investigated were of three kinds:

1. Straight magnetic members running parallel to or crossing at right angles to the conductor (or conductors). The flux produced by the conductor current passes partially through the air and partially through the iron (see Fig. 1). This case is commonly met, since heavy-current conductors very frequently run parallel with or at right angles to steel floor beams, columns, reinforcing rods, pipe supports of switchboards, etc. When such iron members pass at right angles to a conductor, higher iron temperatures will generally be obtained than in the case when the iron members run parallel to the conductor.

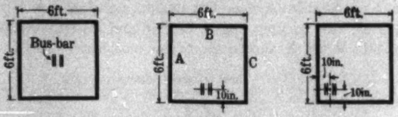

Fig. 8 Fig. 9 Fig. 10

Figs. 8, 9, 10—Test Arrangements of 6-ft. by 6-ft. Closed Channel Iron Frame (Item No. 11 in Table I)

With respect to a straight conductor passing through the frame at right angles to its plane

2. Magnetic members forming a closed frame or loop ("magnetic loop") around the conductor, so that there is a closed magnetic path in a plane at right angles to and surrounding the conductor, as in Figs. 8, 9, 10. This case occurs, for instance, in isolated-phase layouts when large closed structural steel frames (see Fig. 19) are used in the station buildings, but is generally of more serious importance in respect to heating *when the frame is small*, having sides of the order of, say, 1.5 to 2 ft., or less as, for example, pipe frames in switchboard construction (see Fig. 13).

3. Closed loop of structural steel in a plane parallel to the conductor so that the steel frame acts as a short-circuited transformer secondary ("electric loop") carrying an induced circulating current due to the flux set up by the conductor and passing through the loop. (See Fig. 12).

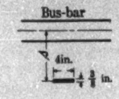

Fig. 11—4-in. by $^3/_8$-in. Steel Plate (Item No. 4 in Tables I and II)

Crossing at right angles to a straight long conductor

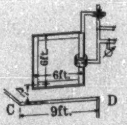

Fig. 12—Test Arrangement of "Electric Loop" Consisting of a 6-ft. by 6-ft. Channel Iron Frame, (Items No. 11 and 12 in Table I)

Conductor C D being in the plane of the frame

Among the cases not dealt with here are those of steel plates or sheets through which a conductor is passed at right angles, and of steel sheets located in a plane which passes through the conductor.

Method of Test

The iron structural members to be tested—in most cases full-sized pieces—were placed in a variety of positions with respect to heavy-current test circuits as indicated in Figs. 1-21. In a good many of the tests, isolated-phase construction was simulated by having the return conductors relatively remote from the samples under test, so that the simple case of one infinitely long, straight conductor, with the return conductor infinitely remote, was closely approached. In other tests the resultant field, at the iron member under test, due to all conductors in the circuit had to be con-

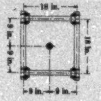

Fig. 13—Closed 18-in. by 18-in. Pipe Frame (Item No. 15 in Table I)

Surrounding a straight 2,000,000-cir. mil cable passing through the center of the frame sidered. Moreover, in some circuit arrangements the adjacent-phase construction was deliberately imitated. The effects of conductor bends and loops on nearby iron members was also investigated. All tests were made at an ambient temperature of from 20 to 25 deg. cent.

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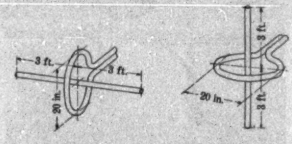
*See Bibl. 1, 2, 3, 5, 12.

In Tables I and II are listed the magnetic members tested, their dimensions and electrical as well as magnetic characteristics.



A. Pipe horizontal B. Pipe Vertical

FIG. 14—1¼-IN. STANDARD SWITCHBOARD IRON PIPE (ITEM NO. 7 IN TABLES I AND II)

Passing at right angles through the center of a circular loop formed by a 2,000,000-cir. mil cable

The temperature measurements were made with mercury thermometers attached with putty to the iron samples under test, or by thermocouples in the case of some of the smaller samples such as reinforcing rods. In most cases the sustained final temperatures, or temperature rise, of the steel for constant loads were obtained.

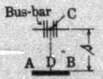


FIG. 15—LONG IRON PLATE OF WIDTH *A-B* RUNNING PARALLEL TO A LONG STRAIGHT CONDUCTOR

The currents in the test circuit were of substantially sine-wave shape at frequencies of either 60, 40 or 25 cycles per sec. In the different tests both multiple rectangular bar conductors and single or multiple insulated copper cables were used to carry the currents supplied to the test circuit through stepdown, high-current transformers. Current measurements were made on portable indicating ammeters with the aid of high-range current transformers. The magnitudes of current were as high as 5800 r. m. s. amperes.

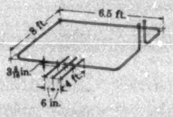


FIG. 16—CIRCUIT ARRANGEMENT FOR HEATING TESTS ON REINFORCING RODS (ITEMS NO. 8, 16, 17 IN TABLES I AND II)

The conductor circuit consisting of one 2,000,000-cir. mil cable

In a number of cases loss measurements were made. The method was to obtain the total input into the busbar system and to subtract therefrom the copper losses. A sensitive wattmeter of the reflecting dynamometer

FIG. 17—SHEET IRON DUCT 18 IN. BY 18 IN. OF 1/8-IN. SHEET STEEL (ITEM NO. 14 IN TABLE I)

Enclosing a 3000-ampere busbar

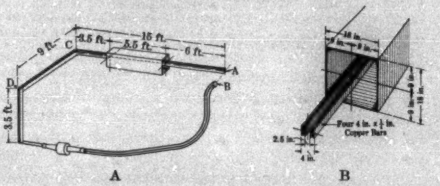


FIG. 18—CIRCUIT ARRANGEMENT AND DIMENSIONS IN HEATING AND LOSS TEST OF SQUARE SHEET IRON DUCT (ITEM NO. 14 IN TABLE I)

Enclosing a 3000-ampere conductor

TABLE I
DIMENSIONS AND DESCRIPTION OF MAGNETIC MEMBERS TESTED

Item No.	Name	Dimensions and description	Length ft.	Cross-sectional circumference in.	Cross-sectional area sq. in.
1	Girder *I*-beam	10-in. flange, 12-in. web. Thickness of flange 1 in. maximum. Thickness web 3/8 in.	12	64	18.75
2	Girder beam flange	10-in. flange, 1 in. thick	12	21	7.5
3	5-in. channel iron	5-in. web, 3/16 in. thick. 1¾ in. flanges. 7/16 in. thick	6	16.5	1.88
4	Blue annealed machinery steel plate	⅜ in. by 4 in.	8	8¾	1.5
5	Blue annealed sheet steel	⅛ in. thick 18 in. wide	5½	36	2.25
6	1/16-in. blue annealed sheet iron	1/16 in. thick 18 in. wide	6	36	1.125
7	Standard switchboard 1¼-in. iron pipe, painted black	1¼ in. inside diam. 1 5/8 in. outside diam.	6	5.2	0.85
8	Round reinforcing rods	½ in. diameter	8	1.57	0.196

TABLE I—Continued

Item No.	Name	Dimensions and Description
9	Expanded metal mesh	6 ft. sq., 3/32 in. thick, 1 in. rhombic mesh holes
10	Galvanized fine iron wire mesh	6 ft. sq., 1/32 in. thick wire. Hexagonal mesh, openings of ½ in. sides.
11	Square frame of channel iron	Consisting of four 5 in. channel irons 6 ft. long of 1¾ in. flanges and 5 in. web (same as No. 3 above). From one to four ½ in. bolts in each joint. Joints not painted.
12	Square frame of channel iron	The same as Item 11, but four ½ in. rivets in each joint (driven hot). Joints were painted with red lead before assembly.
13	Rectangular iron pipe frame	6 ft. 11 in. by 5 ft. 2 in., consisting of 1¼ in. black painted iron pipes joined by standard pipe clamps for right-angle pipe cross-overs.
14	Horizontal square sheet iron duct, blue annealed sheet iron	Consisting of 4 sheets of 1/8 in. thick iron of 5 ft. 6 in. length by 18 in. wide mounted on 1½ in. angle irons in the corners and fastened with 3/16 in. iron screws. (See Fig. 18).
15	Square frame of iron pipes	Consisting of four 1¼ in. standard switchboard iron pipes each 18 in. long, joined by standard threaded pipe clamps. (See Fig. 13)
16	Square reinforcing rods	½ in. by ½ in. square reinforcing rods with 1/32 in. high ridges every 5/8 in.
17	Round reinforcing rods	½ in. round reinforcing rods with 1/32 in. high ridges every 1 inch and with 2 longitudinal ribs 1/8 in. high on either side.
18	Cylindrical closed iron ring	Consisting of a 4 in. by ¼ in. machinery steel bar of 62.5 in. length bent into a circular ring of 20 inches diameter, ends welded together.

TABLE II
MAGNETIC AND ELECTRIC PROPERTIES OF STRUCTURAL STEEL SAMPLES

Samples were taken from Item No.*	Test Sample No.	Specification of test sample	Maximum permeability	Flux density at maximum permeability	Permeability at B = 10,000 gausses	Flux density at magnetizing force H = 25 gilbert per cm.	H at B = 15000 gausses	Resistivity
				gausses		gausses	gilberts per cm.	microhm -cm.
1	1	Cut off from 12-in. by 10-in. girder beam flange, at right angles to rolling direction	1700	6600	1450	14400	35	15.9
4	2	Cut off from 4-in. by 3/8-in. blue annealed steel plate	1500	6200	1300	14200	38	13.2
5	3	1/8-in. thick blue annealed sheet iron	1300	7000	1170	14300	33	14.3
6	4	1/16-in. thick blue annealed sheet iron	4265	6350	4000	16000	10	24.9
7	5	Standard switchboard 1¼-in. iron pipe	1260	6300	1000	13800	44	13.8
8	6	½-in. round reinforcing rod samples	1210	7800	1150	15200	28	15.4
16	7	½-in. by ½-in. square reinforcing rod sample	450	7000	400	10200	74	25.3
18	8	Cut off from ¼-in. machinery steel plate parallel to rolling direction	1800	6500	1500	14900	26	13.4

*These item numbers refer to the items listed in Table I.

type was used whose current coil was connected to the line-current transformer and whose potential circuit was connected to two points of the busbar system embracing all the flux which produces losses. The copper losses were measured separately in the absence of the iron members.

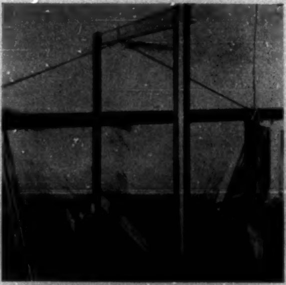


FIG. 19—CHANNEL-IRON FRAME 6 FT. BY 6 FT. (ITEM NO. 11 IN TABLE I)

Surrounding a 3000-ampere conductor

The results obtained by this method are subject to error, the iron losses being the difference of two large numbers. Another error might come in due to changes in the current distribution in the busbars under the influence of the adjacent iron. Hence the copper loss measured together with the iron losses probably differed somewhat from that measured in the absence of the iron. The latter error, however, was minor, because the spacing between busbar and iron was greater than the width of the copper bars, and because any change in the relative impedances of the individual conductor laminations on account of the iron would be largely local. The measurement of a known loss (of the order of magnitude of the iron losses) produced by a current induced in a short-circuited coil by the flux from the busbar circuit AB (Fig. 1) showed an over-all accuracy of loss measurements of better than 10 per cent.

Short-circuited copper windings of one or more turns were placed around the iron member in several cases so as to embrace the flux passing through the iron member and thus to reduce the iron loss. Measurements of the circulating current in the short-circuit windings were made for the purpose of obtaining a relation between iron heating (and losses) and the ampere turns in the short-circuited copper. A convenient method for measuring the circulating currents in copper bands placed around the iron is afforded by the use of Rogowski's* m. m. f. meter. The device—used for measuring the total m. m. f. in a closed magnetic circuit, *i. e.*, line integral of $H\,d\,l$ along a closed path—consists of a helical (single-layer) coil of uniformly spaced turns whose axis forms a closed loop. The voltage induced in the winding is a measure of the total m. m. f. around the path formed by the axis of the coil. If, for instance, the coil is placed such that its axis forms a closed, say circular, path linking an a-c. conductor, the voltage induced in the coil is a measure of the current flowing in the conductor. One of the coils used had an axial length of 36 in., 1600 turns of No. 24 B & S double-cotton-covered wire were wound around a 36-in. length of rubber gas tubing, of 0.5-in. outside diameter, in one layer having approximately 40 turns per inch axial length. The voltage induced in the coil was measured by a sensitive reflecting d-c. galvanometer with the aid of a 25-milliampere thermocouple. Calibrations were made with the Rogowski coil placed around a conductor carrying adjustably variable known currents.

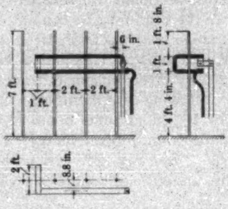


FIG. 20—CONDUCTOR AND IRON PIPE LAYOUT FOR IRON HEATING TESTS WITH ADJACENT-PHASE CIRCUIT

RESULTS OF TESTS

I. Straight Magnetic Members

In the case of straight magnetic members passing *at right angles* to conductors higher maximum temperatures at the iron may generally be expected than in the case of the iron running parallel to the conductor. Hence the case of straight iron members passing at right angles to conductors will be treated in detail.

TEMPERATURE RISE OF STRAIGHT MAGNETIC MEMBERS CROSSING AT RIGHT ANGLES TO CONDUCTORS

The effect, on temperature rise, of each of the following variables will be summarized below: Conductor current, spacing between conductor and iron member, frequency, shape, thickness and mass of iron, and fields from more than one conductor. Unless otherwise stated final values of maximum temperature rise of the iron are given for constant currents in the conductors. In the first paragraph below temperature rise data for a standard 1¼-in. switchboard iron pipe will be given to show of what order the temperatures are in a typical case.

TEMPERATURE RISE IN TYPICAL CASES

The temperature rise of a standard 1¼-in. bare switchboard iron pipe, 1.66-in. outside diameter and 6 ft. long, placed horizontally at 4-in. center spacing from and at right angles to a long straight 1500-ampere 60-cycle conductor (return conductor remote) had a temperature rise of 21 deg. cent. measured by thermometer at the hot spot, *i.e.*, at the point nearest to the conductor. A similar bare pipe (at A in Fig. 21) placed vertically half way between the going and return conductors of a long loop, 8 in. wide, so that the pipe was at 4-in. center spacing from either conductor, had a hot-spot temperature rise of 82 deg. cent. by test when 2000 amperes at 60 cycles per sec. were flowing through the conductor loop of Fig. 21.

EFFECT OF CURRENT ON TEMPERATURE RISE

The final temperature rise at the surface of the steel member at the point nearest to the conductor was found to vary as the 1.7th power (average value) of the conductor current. The value given is an average exponent derived from test results obtained with the structural arrangements of Figs. 1, 16, 20 and 21, covering a current range from 1000 to 5800 amperes, center spacings between conductor and iron member from 2.5 to 16 in., and values of iron temperature rise from 10 to 175 deg. cent. Values of the exponent obtained from the various tests ranged from as high as 2 (for low values of maximum flux density and temperature rise) to 1.4 (for the highest values of temperature rise tested). See also below under discussion of "Effects of Fields from Several Conductors." This exponent cannot, of course, be expected to be strictly a constant, since the permeability of the steel is not constant and since the hot-spot temperature is materially affected

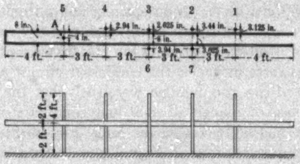


FIG. 21—CONDUCTOR AND IRON PIPE LAYOUT IN IRON HEATING TESTS

With pipes passing outside of conductor group as well as between going and return conductors in adjacent-phase circuit by the distribution, along the steel, of the heat generated and by the heat dissipation characteristics of the iron and surrounding space. Typical curves of hot-spot temperature rise vs. current are shown in Fig. 23.

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*Bibliography No. 10.

TEMPERATURE RISE AT DIFFERENT SPACINGS FOR STRAIGHT IRON MEMBER CROSSING AT RIGHT ANGLES TO A SINGLE STRAIGHT CONDUCTOR.

The effect on temperature rise of spacing alone, in the absence of material effects of return conductors, is indicated in Fig. 22, for the case of a long, straight 3000-ampere, 60-cycle conductor crossing at right angles to a long, straight, horizontal, bare, solid, structural iron member. (It will be shown later that the shape and size of the iron member, within certain limits, do not materially affect the hot-spot temperature rise for a given center spacing and current). All spacings are from center of conductor to the center of the iron member. The data for Fig. 22 were obtained (1) from tests in which the conductor circuit and the position of the iron member were such that the fundamental case of a single straight long conductor crossing at right angles to a straight long iron member was closely approached, as in Figs. 1 and 16, and (2) from tests in which the temperature rise at the iron hot spot was materially affected by the return conductors but in which the effect of these conductors could be allowed for in accordance with the results given below under the heading "Effect of Fields from Several Conductors," items *a* and *b*.

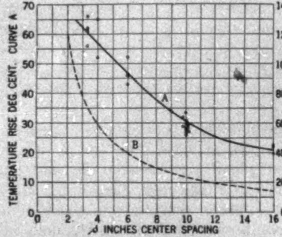

FIG. 22—TEMPERATURE RISE VS. SPACING (CURVE A)

For long straight bare structural iron members crossing at right angles to a straight 3000-ampere, 60-cycle conductor, return conductors remote.

Including data obtained from tests of the following steel members: items No. 1, 2, 3, 4, 5, 7, 8, 16, 17

Curve A: Final maximum temperature rise at iron hot spot for a constant conductor current

Curve B: Air field intensity vs. spacing for a single 3000-ampere straight long conductor

Center spacing measured from center of conductor to center of iron member

According to Fig. 22 it is seen that increasing the spacing from 6 to 10 in. reduced the maximum temperature rise of the iron 35 per cent, a further increase of spacing to a total of 16 in. gave a temperature rise about half that for the 6-in. spacing.

EFFECT OF FREQUENCY ON TEMPERATURE RISE

Tests for temperature rise at 25, 40 and 60 cycles per sec. gave values of temperature rise roughly in the ratio of 1, 1.3, and 1.6, currents and center spacings being the same. The tests were made on 1¼-in. standard switchboard bare iron pipes (item No. 7 in Tables 1 and 2) in the circuit of Fig. 14, and on bare ½-in. round and ½-in. square iron reinforcing rods (items No. 8, 16, 17 in Tables I and II) in the circuit of Fig. 16.

EFFECTS OF SIZE, SHAPE, AND THICKNESS ON TEMPERATURE RISE

Various kinds of straight iron members crossing at right angles to a conductor, and not less than, say, ⅛ in. thick and of cross-sectional dimensions (in a direction perpendicular to the conductor as dimension *h* in Fig. 2) up to 12 in. at *center spacings* from 10 in. to 16 in. from the conductor, had substantially equal values of temperature rise at any given busbar current, as indicated by tests of the following iron members: 12-in. by 10-in. I-beam, 5-in. by 1¾-in. channel beam, 4-in. by ⅜-in. by 8-ft. steel plate, 10-in. *I*-beam flange,* 1¼-in. switchboard steel pipe. See Fig. 24. In other words, the temperature rise under these conditions was *independent of the size, or cross-sectional area or shape for a given center spacing between conductor and iron*, regardless of the distance from the conductor to the nearest part of the iron. Variations of temperature rise within this rule were found to be not higher than 30 per cent and usually much less.

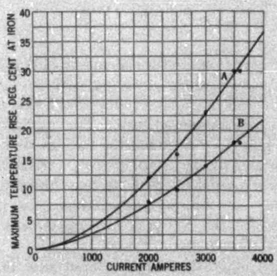

FIG. 23—RELATION BETWEEN MAXIMUM TEMPERATURE RISE OF SOLID STRUCTURAL IRON MEMBERS AND CONDUCTOR CURRENT

Bare, vertical, 1¼ in standard switchboard iron pipes in circuit of Fig. 21.

Curve A for pipe No. 3 at center spacing $s = 2.63$ in. from nearest conductor

Curve B for pipe No. 6 at center spacing $s = 3.9$ in. from nearest conductor

For Curve A the temperature rise is proportional to the 1.65 power of current

For Curve B the temperature rise is proportional to the 1.55 power of the current.

Frequency is 60 cycles per sec.

The temperature rise of straight, compact† iron members crossing at right angles to a conductor may therefore be estimated from the current magnitude and the *center* spacing between conductor and iron member, regardless of its size, cross-sectional area or shape, provided the iron thickness is not less than, say, $\frac{1}{8}$ in. and the ratio R representing the fraction

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*Cut from the 12 in. I-beam already mentioned.

†Meaning members in which the metal is continuous such as plates, I-beams, channels, pipes, sheets; but not including members in which the metal is scatttered or not continuous such as meshes and screens.

$$\frac{\text{center spacing between conductor and iron member}}{\text{cross-sectional dimension of the iron in a direction perpendicular to the conductor}}$$

is larger than 0.8 (minimum ratio R for the above I-beam is $\frac{10}{12} = 0.83$). On the other hand, when the ratio R is 0.8 or less (as may occur for instance in the case of large beams) the maximum temperature rise should be estimated by considering the nearest part of the iron member as a plate (or sheet as the case may be) and reducing the temperature rise of this equivalent plate by, say, 30 per cent, the 30 per cent reduction being a rough allowance for the heat carried off by the iron adjoining the plate.

If the iron members have a thickness of the order of, or less than, two or three times the equivalent depth of flux penetration‡ (calculated for relatively thick members,) or say less than $\frac{1}{8}$ in., it will have a smaller temperature rise than thicker iron members because a smaller volume of iron per unit of surface will participate in the production of loss. This fact is indicated by a 21-deg. cent. temperature rise of a $^1/_{16}$-in. horizontal black iron sheet 18 in. wide and 5 ft. long against 28 deg. cent. of a $\frac{1}{8}$-in. black iron sheet, 18 inches wide and 5 ft. 6 in. long, both at the same center spacing of 10 in. from the busbar and for 3000 amperes, 40-cycle busbar current.

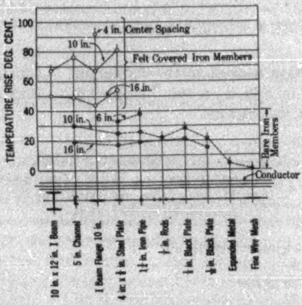

FIG. 24—COMPARISON OF VALUES OF MAXIMUM TEMPERATURE RISE

For different iron members crossing at right angles to conductor.
Circuit layout of Fig. 1, iron members horizontal
Constant busbar current of 3000-amperes at 40 cycles per sec.
Figures on plot indicate center spacing between conductor and iron members

A 5-ft. by 6-ft. expanded metal mesh of 1-in. rhombic openings and $^3/_{32}$-in. square mesh ribs exposed to the field of a 3000-ampere, 40-cycle busbar current and at 10 in. from the center of the busbar had a maximum temperature rise of only 5 deg. cent.

A 6-ft. by 6-ft. fine galvanized wire mesh of $^1/_{32}$-in. wire diameter (chicken mesh) under the same test conditions as above, had a negligible temperature rise.

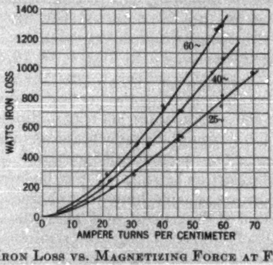

FIG. 25—IRON LOSS VS. MAGNETIZING FORCE AT FREQUENCIES OF 25, 40, AND 60 CYCLE PER SEC.

For solid iron ring 20 in. diameter, of 4 in. by ¼ in. steel (item No. 18 in Tables I and II)
M. m. f. applied by means of a uniformly distributed winding of 453 turns of No. 10 B & S stranded double-cotton covered copper wire. Sine-wave voltages were applied to the terminals of the magnetizing winding
For all three frequencies the iron loss is proportional to the 1.6 power of the m. m. f.

It is seen that the temperature rise of the meshes tested (cross-sectional dimensions of mesh wire less than $\frac{1}{8}$-in.) was not over 25 per cent of the temperature rise for the compact iron members tested.§ Meshes composed of members of more than $\frac{1}{8}$-in. cross-section may be expected to have a temperature rise greater than that of the meshes tested, but not more than that of a compact iron member, under otherwise similar conditions. Iron meshes made from material having cross-sectional dimensions within the range from ⅛ in. to ½ in. were not tested.

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‡Bibliography 2. Also equation (5) below.

§The reduced temperature rise observed in the meshes is affected by the following factors:

The most important one is probably the good ventilation of the mesh, the air being absolutely free to circulate around the mesh ribs.

Moreover, if two sides of the mesh openings are parallel to the flux, the other two sides (at, say 90 deg. to the flux) will carry heat away from the loss-producing ribs and serve to increase the radiating surface, thus further reducing the average temperature rise.

In the case where none of the mesh ribs is parallel to the magnetizing force, the flux and loss per unit metal surface of the mesh ribs will also be less than that of an unbroken iron sheet.

A further reduction of the heating of iron meshes will occur when the mesh ribs have a thickness of less than, say, $\frac{1}{8}$ in.

There is, of course, a factor tending to increase the heating of a mesh, namely the I^2R losses due to circulating currents caused by the flux passing through the mesh openings. The heating due to these circulating currents will generally occur at points away from the hot spot nearest to the conductor. These additional I^2R losses prevent somewhat the heat flow from the hot spot and therefore will tend to increase the maximum temperature rise. However, judging from the test data, it appears that this last effect is largely outweighed by the cooling effects stated above.

EFFECT OF FIELDS FROM SEVERAL CONDUCTORS ON TEMPERATURE RISE OF IRON

In a great many practical cases, there is more than one conductor contributing to the field at the hot spot of the iron. Often the return conductors are nearby, or the conductor in question has bends, as in the case of current-transformer or circuit-breaker connections, so that there are several conductors, perhaps carrying different currents, to be taken into account. The general case of a multiplicity of conductors affecting the iron heating has not been solved nor experimentally investigated. However, data were obtained for the circuit arrangements shown in Figs. 1, 14, 16, 20, and 21.

At once the question arises as to what the relation is between the temperature rise and the field intensity at the iron, say at the section of the hot spot. Since only part of the flux path around a conductor crossing a magnetic member is shunted (magnetically speaking) by the iron, and since the portion so shunted may be large or small depending on the spacing between iron and conductor, the calculation of the true m. m. f. along the iron would be quite difficult, even if only one straight, long conductor were involved. However, in most ordinary cases, the "air field intensity" at any point along the axis of the iron may be determined. It gives a simple measure of the resultant field intensity due to several conductors. By "air field intensity" is meant the calculated resultant intensity, due to all conductors, in air assuming no magnetic bodies present, the intensity for the present purpose being calculated along the line of the longitudinal center axis of the iron member at the section through the hot spot, *i. e.*, usually at the point where the axis is at the shortest distance from the nearest conductor. The air field intensity was calculated for all test data taken with the circuit arrangements of Figs. 1, 14, 16, 20, 21 and the following results were obtained.

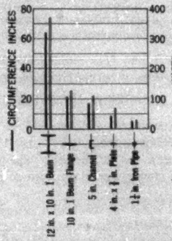

FIG. 26—COMPARISON OF TOTAL IRON LOSS WITH CROSS-SECTIONAL PERIMETER OF VARIOUS IRON MEMBERS CROSSING AT RIGHT ANGLES TO CONDUCTOR

Test circuit of Fig. 1. Each iron member was tested separately in the position of the I-beam shown.

Center spacing between iron member and conductor 10 in.

Test current 3000 amperes at 40 cycles per sec.

a. At any given center spacing from iron member to nearest conductor, the hot-spot temperature rise varied as the 1.4 to the 2.0 power (see Table III) of the air field intensity calculated for the center point of the cross-section nearest to the conductor, as for instance, point C in Fig. 2—a relation similar to that already given for the current.

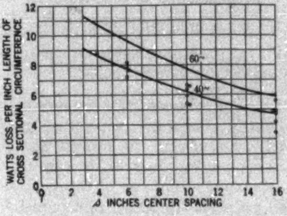

FIG. 27—IRON LOSS PER INCH LENGTH OF IRON CROSS-SECTIONAL PERIMETER VS. SPACING

Test circuit of Fig. 1, closely approaching, as far as iron heating and losses are concerned, the arrangement of a single, long, straight conductor crossing at right angles to a straight long iron member. Busbar current 3000 amperes.

Based on tests of the following structural steel members: items No. 1, 2, 3, 4, 7.

Center spacing *s* measured from center of conductor to center of iron member.

TABLE III

VALUES OF EXPONENT n IN EQUATION (2) FOR CALCULATION OF TEMPERATURE RISE OF SOLID STRUCTURAL IRON MEMBERS CROSSING AT RIGHT ANGLES TO CONDUCTORS

H r. m. s. gilberts per cm. air field intensity.	n
20 or less	2.0
50	1.7
100	1.5
150	1.4

b. An approximate relation useful in a variety of cases for estimating the temperature rise of iron members exposed to fields from more than one conductor was found to be the following: at any given center spacing between an iron member and the conductor contributing the major component of the resultant field (usually this is the conductor passing nearest to the iron) the maximum hot-spot temperature rise, for any particular value of air field intensity (due to all conductors), at the center of the iron section through the hot spot, was independent of the arrangement of conductors—within certain limits. A material departure from this rule was found, as would be expected, for those cases in which the different arrangements of conductors involved considerable changes in the distribution of the m. m. f. along the iron member (*i. e.*, on either side of the hot spot).

The application of rule *b* over a wide variety of structural arrangements is favored, in cases where substantial values of temperature rise occur, by the fact that for high flux densities (above say 8000 gausses for structural steel), the permeability decreases with increasing m. m. f. Thus rule, *b*, applied quite well within experimental error to all cases in which the principal components of the resultant field at the iron member were additive (*i. e.*, of the same sign) as in the case of iron sample *A* in Fig. 21 or in the case of the pipe in Fig. 14.

c. However, for iron members outside the loop of Fig. 21, such as pipes No. 3 and 6, and for the iron samples in Fig. 20, the experimental values of temperature rise were from 20 to 30 per cent less than those estimated from rulings *a* and *b* above, the reduction allowing for the fact that in the cases in question the m. m. f. and hence the iron losses fall off much more rapidly along the iron (*i. e.*, on one or on both sides of the hot spot) than the m. m. f. and the losses in a steel member crossing at right angles to *one* straight conductor only, center spacings being equal.

The application of these relations to practical cases of estimating temperature rise is illustrated in a few examples below and is shown to give results closely approximating the observed values of temperature rise.

IRON MEMBERS RUNNING PARALLEL TO CONDUCTOR

A long and narrow steel member, such as an I-beam, a channel, or a pipe, will ordinarily heat much less when running parallel to the conductor than when crossing it at right angles, for the same spacings and currents as already mentioned. This is because the iron portion of the flux path around the conductor is a much smaller part of the complete flux path than in the case of a steel member crossing at right angles to the conductor. The following test results may be used for rough estimates of temperature rise of iron members running parallel to a conductor: When the ratio of width to spacing, *i. e.*, of the width *A B* (in Fig. 15) of the iron member (measured in a direction perpendicular to the plane through the conductor and the longitudinal axis of the iron member) to the spacing *C D* (in Fig. 15) was from 0.5 to 1.0, the temperature rise was only from 20 to 30 per cent as much as that obtained when the same member was crossing at right angles to the conductor at the same spacing. When the ratio of width to spacing, that is, the ratio $\frac{AB}{CD}$ was from 2 to 3, the temperature rise was from 50 to 75 per cent as much as that obtained when the same iron member was crossing at right angles to the conductor.

OTHER FACTORS

Both the *permeability of the iron and its resistivity* affect its temperature rise when exposed to a-c. fields. Special tests to determine the separate effects of these variables were not made, *i. e.*, samples of chiefly one grade of steel were investigated. According to Table II, the permeability and resistivity of the structural steel samples tested did not cover a sufficiently wide range to indicate the effects of these variables on temperature rise. (For the *I*-beam, channel beam, switchboard pipe, round reinforcing rods and steel plates tested the resistivity ranged from 13.2 to 15.9 microhm-cm., and the maximum permeability from 1210 to 1800 at densities of from 6200 to 7800 gausses). In so far as structural steel members are concerned (having values of resistivity and permeability within the range of those given) approximate estimates of temperature rise may be made on the basis of the test data given here (in accordance with the procedure outlined in the examples below), without further consideration of permeability and resistivity.

However, when permeability or resistivity, or both, are considerably different from the values obtained here, proper allowance must be made. Previous investigators* have dealt both theoretically and experimentally with the effects of permeability and resistivity on flux penetration and losses in solid iron. The effect of these factors on losses may be roughly predicted, for a wide variety of cases, from Rosenberg's approximate formula for eddy-current losses in solid iron, discussed in Appendix A.

The effects, on temperature rise, of embedding the iron member (as, for instance, in the case of reinforced concrete construction) was not investigated except in one test with concrete-embedded steel rods. A slab of concrete was made, 48 in. long, 18 in. wide and 5 in. thick, containing three ½-in. reinforcing rods 6 in. apart and running lengthwise half way between the top and bottom of the slab. With a single straight conductor carrying 1500 amperes at 60 cycles and crossing the rods at 3 $^5/_{16}$-in. spacing from the center plane of the rods they reached only half as large a temperature rise as similarly placed rods (cut from the same pieces as those embedded) in open air, (see Fig. 16). At the time of the test the concrete was two weeks old, having been kept at room temperature in the laboratory during this period. The average values of observed temperature for the three rods were 12 deg. cent. at the embedded rods, and 24 deg. cent. at the bare rods. However, it may be expected that embedded iron members having a cross-sectional circumference of an order of magnitude approaching more nearly (or exceeding) that of the concrete, will reach materially higher temperatures than corresponding members, in open air, similarly placed with respect to the conductors. In order to indicate the effect of a very considerable amount of heat insulation a few measurements of temperature rise of felt-covered samples were made; a 5-in. by 1¾-in. channel beam, a 4-in.

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*See Bibliography 1, 2, 3, 5.

by ⅜-in. steel plate (the respective samples being listed in tables I and II as items No. 3 and 4) when closely wrapped with one layer of ½-in. felt, had values of hot-spot temperature rise about three times as large as those obtained with the samples bare.

Losses in Straight Iron Members Crossing at Right Angles to Conductors

A few loss measurements were made to indicate their general order of magnitude in a few cases, and their relation to the current magnitude, frequency, to the cross-sectional dimensions of the solid iron members, and to the spacing between iron and conductor. A complete investigation of losses to cover the wide field of the many customary structural layouts was not made. However, the few data taken will be presented here though their application is restricted, because they should be helpful in some of the simpler cases (of straight structural steel members crossing at right angles to a straight conductor). It is hoped that this work will stimulate further and more thorough investigations. For additional data on flux penetration and losses in solid iron members, the reader is referred to the valuable work done† by Rayleigh, Rosenberg, Kennelly, Steinmetz, Lang and others. The Rosenberg formula for eddy-current losses in solid iron members is discussed in Appendix A.

Effect of Frequency on Losses

The relation between frequency and total loss in iron was investigated in the case of a closed iron ring 4 in. wide, ¼ in. thick and 20 in. average diameter. The results of the loss measurements, at 25, 40 and 60 cycles per sec., given in Fig. 25 show that the loss varied roughly as the square root of the frequency, as indicated in Table IV.

TABLE IV

Average ratio of loss at 60 cycles per sec. to loss at 40 cycles per sec. by test	1.24
$\sqrt{\frac{60}{40}}$	1.22
Average ratio of loss at 60 cycles per sec. to loss at 25 cycles per sec. by test	1.6
$\sqrt{\frac{60}{25}}$	1.55

While it is true that the loss distribution along an iron member crossing at right angles to a conductor is not uniform, falling off rapidly on either side of the "hot spot," it is believed that the frequency-loss relations given in Table IV for a ring uniformly magnetized along its circumference, apply fairly closely to the case of solid iron members crossing at right angles to a conductor, because the ratio of losses at different frequencies did not vary beyond the magnitude of experimental errors for the full range of m. m. fs. covered by the data of Fig. 25.

Effect of Current on Losses

Loss measurements at currents ranging from 1500 to 3000 amperes were made for a 10-in. by 12-in. steel I-beam, a 5-in. by 1¾-in. channel, a 4-in. by ⅜-in. steel plate, and a 1¼-in. standard switchboard iron pipe, when passing at right angles to conductor *C D* in Fig. 1. For example, a straight 10-in. by 12-in. I-beam crossing at 10-in. center spacing from a straight, 3000-ampere conductor (return conductors remote) was found to have a loss of the order of 370 watts at 40 cycles per sec., or 450 watts at 60 cycles per sec. The losses for this I-beam varied as the 1.8 to 1.9 power of the current, at constant spacing from the conductor. For other samples tested, this exponent was as low as 1.6.

Effects, on Losses, of Cross-Sectional Area Circumference and Shape of Solid Iron Members

For iron members crossing at right angles to a conductor and not less than ⅛ inch thick and of cross-sectional dimensions (in a direction perpendicular to the conductor such as dimension *h* in Fig. 2) up to 12 in. at any given center spacing within the range from 6 to 16 in., the iron losses at any given current were actually found by test (see Figs. 26 and 27) to vary only with the circumference of the iron section and not with its area for the following iron members tested: 12-in. by 10-in. I-beam, 5-in. channel iron, 1¼-in. iron pipe, 4-in. by ⅜-in. steel plate, 10-in. I-beam flange.* Moreover, at any given spacing and conductor current, the loss for these iron members when crossing at right angles to a conductor was approximately proportional to the circumference of the iron cross-section, as indicated in Figs. 26 and 27. Deviations from this rule were less than 30 per cent (based on the average of the measured values) for the iron members tested. This result is in good agreement with the behavior of solid iron plates in respect to flux penetration and eddy-current loss, as worked out by previous investigators.†

Effect of Spacing

The relation between loss and spacing for straight iron members crossing at right angles to a straight conductor is indicated in Fig. 27. The curves show the average total measured loss per inch length of cross-sectional perimeter for center spacings from 4 in. to 16 in. from a 3000-ampere conductor at 40 cycles and at 60 cycles. It is seen that increasing the spacing from 6 in. to 10 in. reduced the loss roughly 20 per cent, and a further increase of spacing to 16 in. gave a loss about 40 per cent less than that at 6 in.

By means of this curve (Fig. 27) and the relation between current and losses to the effect that the loss varies as the 1.6 to 1.9 power of the current, it is possible to estimate the loss in a solid iron member crossing at right angles to a conductor and not less than ⅛ in. thick when the iron cross-sectional dimensions are within the range covered by the samples tested. The largest sample tested was a 10-in. by 12-in. I-beam having a 64-in. cross-sectional perimeter. Other samples tested had perimeters as low as 5.2 in.

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†See Bibliography.

*Cut from the 12-in. I-beam already mentioned.

†Bibliography 2, 5, 7.

II. Closed Iron Loops Around Conductor

TEMPERATURE RISE AND LOSSES

To simulate *large frames of structural steel*, the circuit arrangements of Figs. 8, 9 and 10, with a 6-ft. by 6-ft. square frame of four 5-in. by 1¾-in. structural channel irons riveted or bolted together at the corners, were tested at busbar currents ranging from 1500 to 3000 amperes (see also the illustration, Fig. 19.) For none of the three cases of Figs. 8, 9, and 10 did any part of the frame reach materially higher temperatures than the corresponding single straight iron member at the same separation from the conductor. Even when the conductor was near one of the corners of the frame and was at 10 inches center spacing from each of the two channel beams forming the rectangular corner, meaning 5½-in. clearance between the edge of the busbar and the nearest part of the channel, the temperature rise at the steel was not measurably higher (for values of 30 deg. cent. rise) than that observed for a *single straight* channel beam crossing at the same spacing from the conductor. In other words, the temperature rise of iron beams in *large* rectangular magnetic frames surrounding a straight conductor and having sides larger than, say, 5 ft. may be calculated separately for each individual beam disregarding the other three sides of the frame, even if the conductor passes near the corner at a point as close as 8 in. from the nearest part of each of the two adjacent sides.

The losses in the entire closed 5-inch channel-iron loop 6 ft. by 6 ft. square surrounding the busbar (see Figs. 8, 9 and 10) were roughly from 60 per cent to 300 per cent more than the losses of a single straight 5-in. channel iron crossing the busbar, at the same center spacing as the nearest channel of the frame. These relations are, of course, approximate since in the test circuit the flux at the various parts of the frame and the losses were somewhat affected by the return conductors. The 60 per cent excess loss was measured in the case where the busbar was passing through the loop at a point 10 in. from the center of a channel iron side (Fig. 9), 100 per cent excess loss occurred when the busbar was passing through the loop at a point near one corner and 10 in. (center spacing) from each of the two adjacent sides of the frame (Fig. 10). The 300 per cent excess loss applied, of course, when the conductor passed through the middle of the frame (Fig. 8).

Small Frames. Both calculations and experimental observations indicate that small, closed uniform magnetic frames having sides of the order of, say, 2 ft. or less when surrounding a conductor must be expected to heat considerably more than a single straight iron member passing at right angles to a conductor, for equal spacings. Thus a small closed square frame having 18-in. sides of standard 1¼-in. iron pipe (item No. 15, Table I), when tested with a 1500-ampere, 60-cycle conductor passing at right angles through its center as shown in Fig. 13, had a maximum temperature rise 50 per cent in excess of that for a single straight 1¼-in. pipe, 6 ft. long placed at 9-in. center spacing from the same conductor.

Calculations. As already indicated, the losses and temperature rise in a uniform, square magnetic frame may be determined (with certain approximations and restrictions) with the aid of Rosenberg's formula* for eddy-current loss when a straight conductor passes approximately through the center of the frame. Several examples are given below to illustrate the procedure in the calculations in question and to show that good agreement between experimental and calculated results was obtained.

DUCTS OF IRON ENCLOSING A STRAIGHT CONDUCTOR

In order to demonstrate experimentally that relatively high values of temperature rise and of loss may be obtained for sheet iron ducts completely enclosing an a-c. conductor, the 18-in. by 18-in. square iron duct of Figs. 17 and 18 was tested and gave a maximum temperature rise of 78 deg. cent. at the iron for a 3000-ampere, 40-cycle† busbar current, and an iron loss of 600 watts per foot of duct length.

Replacing the two horizontal iron sheets of the duct by non-magnetic sheets, but retaining the two iron side sheets, reduced the maximum temperature rise of the iron to 20 per cent; the total iron loss was reduced to about 10 per cent.

III. Closed Iron Frames Parallel to Conductor

Tests of two kinds of "electric" loops of structural iron were made (by an electric loop is meant a loop of structural iron which is linked by flux from an a-c. conductor; a typical electric loop is a structural-iron rectangle with a straight conductor running near the rectangle and in the plane of the rectangle):

a. A square frame with 6-ft. sides of 5-in. channel iron both with bolted and with riveted joints. (see Fig. 12).

b. A similar square frame of 1.25-in. standard switchboard iron pipes.

Each loop (of bare iron) was tested vertically with a 3000-ampere, 40-cycle bus running parallel to and spaced as near as 10 in. (center spacing) from one of the iron sides of the frame, the bus being outside the frame but in the same plane with it.

In the case of the channel-iron frame (item 12 in Table I) the iron-loop current was as high as 80 amperes, with 3000 amperes in the bus, but gave no noticeable temperature rise with tightly bolted or riveted joints not even when one bolt or one rivet per joint was used. A loosened one-bolt joint (½-in. bolt without a nut), however, rose to a temperature 10 deg. cent. above the ambient temperature.

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*Discussed in Appendix A.

†60-cycle values of temperature rise and losses approximately 20-30 per cent higher.

The 1.25-in. iron pipe frame, 6 ft. 11 in. by 5 ft. 2 in. (item No. 13 in Table I) assembled with the usual clamps for right-angle cross-overs gave *zero* current in the frame, the paint on the pipes and fittings keeping the joints open-circuited. The open-circuit voltage induced in the loop was 0.5 volt at 3000 amperes 40-cycle bus current and for a 10-in. center spacing from bus to the nearer parallel 6-ft. 11-in. side of the loop. When the joints of the pipe frame at all four corners were short-circuited by ¼-in. stranded copper bonds, 70 amperes flowed in the pipes, without causing an appreciable temperature rise. It should be pointed out here that other tests have shown that it is quite possible for pipe joints made of standard pipes and fittings to conduct current at low voltages without the use of electrical bonds.

While the observed currents flowing in the pipe frame (with bonded joints) as well as in the channel frame, under these test conditions, were small (less than 100 amperes) considerably larger currents of, say, several hundred amperes may flow in heavy iron frames of isolated-phase stations when the iron loops are large (say 15 or 20 ft. on a side) and especially when a heavy-current bus follows two sides of the iron frame,‡ or when the flux linking the loop is materially increased by the effects of other conductors. However, on account of the large surface over which the losses due to circulating currents in closed frames of large structural beams are distributed, their temperature rise will commonly be of minor consequence.

COPPER SHORT-CIRCUIT RINGS

To Reduce Heating in Magnetic Members Crossing at Right Angles to Conductor. Copper loops around iron members crossing at right angles to heavy-current conductors may be used effectively to reduce the losses and temperature rise of the iron members; but when designed with too small a current-carrying capacity the copper loops may themselves seriously overheat.

In two typical cases, one with an iron girder beam 10 in. by 12 in., another with a 1¼-in. iron pipe, a copper sleeve or short-circuit winding having an ampere-turn capacity of 30 per cent of the busbar current reduced the iron losses and temperature rise to less than 10 per cent of their original values. In both cases the center spacing between the busbar and the iron member (crossing at right angles to the busbar) was of the order of from 6 to 10 inches, the busbar current ranging from 1500 to 3000 amperes. Under conditions similar to those in question, a smaller current carrying capacity for the copper short-circuit ring is not recommended, on account of increased heating in the copper ring itself. When the structural-iron magnetic circuit is bent, say at right angles around a conductor, each side of the bend will, of course, have to be protected separately by a short-circuit winding having an ampere-turn rating of the order of 25 per cent of the busbar current, (combined ampere-turn ratings of the two windings being 50 per cent of the busbar current).

The short-circuit winding may consist of one or more turns of heavy copper or may be in the form of several separate short-circuited loops; in any case the design is to be based on ampere turns of the short-circuit winding.

The best position for the short-circuit winding is the point on the iron member nearest to the conductor.

It was shown that the use of a particular iron mesh of "expanded metal"—having a wire section of $^3/_{32}$ in. by $^3/_{32}$ in. and 1-in. rhombic mesh openings—the mesh completely surrounding a 10-in. by 12-in. girder beam crossing at right angles to a 3000-ampere conductor, caused a 30 per cent reduction of the girder beam temperature rise. The temperature rise of the mesh was, however, more than half of the (reduced) girder beam temperature rise. The saving in total losses due to the mesh was not measured, and cannot be expected to be as high as 30 per cent since there is a material loss in the mesh itself. It appears, therefore, that an iron mesh surrounding a structural steel member and of the type commonly used to hold the concrete together cannot be relied on as a means for effectively reducing the total iron losses when heavy-current conductors are fairly close to the iron member.

EXAMPLES

In examples 1, 2 and 3 the temperature rise in bare solid structural iron members crossing at right angles to one or more conductors is estimated from the empirical data presented in the paper.

Examples 4 and 5 illustrate the method of estimating losses from the empirical loss data for cases of straight structural iron members crossing at right angles to a conductor.

The last three examples (No. 6, 7 and 8) are intended to illustrate how the losses and temperature rise in *closed uniform magnetic frames* of solid steel may be approximately calculated from the dimensions of the structure, and the electrical and magnetic properties of the iron frame, when a conductor passes through the center of the frame (at right angles to its plane) and when the thickness of the iron is not less than ⅛ in.

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‡The circulating currents may be estimated for the more common types of frames if the magnitudes and relative phases of the currents in the nearby conductors, the dimensions of the frame, and its position with respect to the conductors, are known. The calculation in question involves determining the mutual inductance between the frame and the conductors, the self inductance and the skin-effect resistance of the frame. For skin-effect resistance calculations the reader is referred to Bibliography 2. From the loss, determined by the circulating current and the effective frame resistance, the order of magnitude of the temperature may be obtained from loss vs. temperature rise relations such as those given in Bibliography 12.

Example 1. Estimate the temperature rise of a horizontal, bare, 1¼-in. switchboard iron pipe, having the characteristics of Sample No. 7 in Tables I and II, when placed at a center spacing $s = 4$ in. from a long straight 1500-ampere, 60-cycle conductor crossing at right angles to the pipe.

The procedure is to obtain from the curve in Fig. 22 the reference temperature rise $T\,R_r$ for 4-in. center spacing for the 60-cycle reference current of 3000 amperes. This temperature rise is 57 deg. cent. from Fig. 22. The reference air field intensity is $H_r = 59$ gilberts per cm. The temperature rise for 1500 amperes may now be calculated from the relation:

$$\frac{T\,R}{T\,R_r} = \left(\frac{I}{I_r}\right)^n \qquad (1)$$

where n is 1.7 in accordance with Table III. Thus

$$T\,R = 57\left(\frac{1500}{3000}\right)^{1.7} = 57 \times 0.31 = 18 \text{ deg. cent.}$$

and the desired temperature rise of the pipe in question is 18 deg. cent. The temperature rise by test was 21 deg. cent., indicating good agreement with the estimated value.

NOTE: The calculations for this example could have been made according to the procedure of example No. 2, using equation (**2**) in place of equation (**1**). When only one straight long conductor is involved, the air field intensities need not be calculated.

Example 2. Estimate the temperature rise of a vertical bare 1¼-in. switchboard iron pipe having the characteristics of sample No. 7, in Tables I and II, when placed in position A in the circuit of Fig. 21, with 3600 amperes at 60 cycle per sec. flowing in the loop.

The procedure is to obtain

$T\,R_r = 57$ deg. cent. from Fig. 22 and from the same

$H_r = 59$ gilberts per cm.

Since there are two conductors (rather than a single one) contributing to the m. m. f. along the pipe, the temperature rise has to be calculated by the formula* involving air field intensities.

$$\frac{T\,R}{T\,R_r} = \left(\frac{H}{H_r}\right)^n \qquad (2)$$

From the dimensions of the conductor the resultant air field intensity H at A (at the center of the pipe between the busbars) is 144 gilberts per cm., for 3600 amperes. This high value of H calls for an exponent $n = 1.45$ in accordance with Table III. Therefore

$$T\,R = 57\left(\frac{144}{59}\right)^{1.45} = 207 \text{ deg. cent.}$$

is the desired temperature rise. The value obtained by test was 177 deg. cent. or 15 per cent lower than that calculated.

Example 3. Estimate the hot-spot temperature rise of a square, vertical, bare channel-iron frame, having 6-ft. sides consisting of 5-in. by 1¾-in. bare channels, item No. 11 in Tables I and II, when a 3000-ampere, 60-cycle conductor passes at right angles through the plane of the frame at a point 10 in. above the center of the lower horizontal channel (Fig. 9). The conductor circuit is that of Fig. 1, the frame being placed so that its lower horizontal side occupies the position of the I-beam shown, and that the channel section through the hot-spot is located as shown in Fig. 3 with respect to the conductor, s being 10 inches.

In estimating the hot-spot temperature rise it will be remembered that in large frames of this kind the maximum temperature is not affected by the presence of the three channel iron sides $A\ B\ C$; in other words the hot-spot temperature rise may be estimated by following the procedure given for single-straight iron members in examples 1 and 2.

Accordingly the first step is to obtain the reference temperature rise $T\,R_r$ from Fig. 22 for a straight horizontal iron member crossing at 10-in. center spacing from a long, straight 3000-ampere, 60-cycle conductor, as in examples 1 and 2. Thus

$T\,R_r = 31$ deg. cent.

and

$H_r = 23.6$ gilberts per cm.

Calculating H, the air field intensity for $s = 10$ in. at the center of the channel section through the hot spot, the value

$H = 24.8$ gilberts per cm.

is obtained for a 3000-ampere conductor current for the circuit of Fig. 1. Then the hot-spot temperature rise in the channel is found from (**2**)

$$\frac{T\,R}{T\,R_r} = \left(\frac{H}{H_r}\right)^n$$

where n in this case is 1.85, interpolated from Table III. Substituting the values

$$T\,R = 31\left(\frac{24.8}{23.6}\right)^{1.85} = 31 \times 1.095$$

Hence

$$T\,R = 34 \text{ deg. cent.}$$

i. e., the desired final hot-spot temperature rise for a constant 3000-ampere current is estimated to be 34 deg. cent.

The nearest test condition to compare with this calculated result is a 40-cycle, 3000-ampere test with the frame of Fig. 9, and the circuit of Fig. 1, which test gave a 40-cycle hot-spot temperature rise of 29.5 deg. cent. by thermometer. Estimating from the calculated 60-cycle temperature rise of 34 deg. cent., the corresponding 40-cycle temperature rise by the relation:

$$\frac{T\,R \text{ at 40 cycles}}{T\,R \text{ at 60 cycles}} = \frac{1.3}{1.6}$$

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*Based on relations (*a*) and (*b*) established above under the heading, "Effect of Fields from Several Conductors on Temperature Rise of Iron."

a 40-cycle value of $\frac{34}{1.23} = 28$ deg. cent. is calculated, which is in good agreement with the test value of 29.5 deg. cent.

Example 4. Estimate the loss in a 4-in. by ⅜-in. steel plate (item No. 4 in Tables I and II) when crossing at right angles to conductor $C\,D$ in Fig. 1, in the position occupied by the I-beam shown. The data are as follows:

$s = 6$ in.
$I = 3000$ amperes
$f = 60$ cycles per sec.
$P = 8.75$ in.

From Fig. 27, at $s = 6$ in.,

$L_0 = 9.6$ watts per inch length of perimeter

Then

$$L = P\,L_0 \tag{3}$$

or

$L = 8.75 \times 9.6 = 84$ watts.

Thus the loss in the plate is 84 watts. The loss measured at 40 cycles per sec. was 68 watts, which is in good agreement with the estimated 40-cycle value of

$$\frac{84}{1.25} = 67 \text{ watts.}$$

Example 5. Estimate the loss in a structural 12-in. by 10-in. I-beam (item 1 in Tables I and II) placed as shown in Fig. 1, when

$s = 16$ in.
$I = 2600$ amperes
$f = 60$ cycles per sec.
$P = 64$ in.

From Fig. 27, at $s = 16$ in.,

$L_0 = 5.9$ watts per inch length of perimeter for a 3000-ampere reference current. Then the total loss at $I = 2600$ amperes is estimated from

$$L = P\,L_0 \left(\frac{I}{I_r} \right)^n \tag{4}$$

where $n = 1.75$, an average value. Hence

$$L = 64 \times 5.9 \times \left(\frac{2600}{3000} \right)^{1.75} = 295 \text{ watts}$$

Thus the total loss in the I-beam is estimated at 295 watts. The loss measured at 40 cycles per sec. with 2600 amperes was 230 watts, which is in good agreement with the estimated 40-cycle value of $\frac{295}{1.25} = 235$ watts.

Example 6. Estimate the loss per unit length and the temperature rise of a ⅛-in. sheet iron duct (item No. 14 in Tables I and II) as shown in Figs. 17 and 18. The duct is square having sides of 18 in., a 3000-ampere 60-cycle conductor passing through the center of the duct.

The circumference of the duct is 72 in. or 183 cm.

The eddy-current losses will be calculated by Rosenberg's formula* (Appendix A) on the assumption that the square duct is equivalent to a circular cylinder of 183 cm. circumference, or 29.1 cm. radius. The eddy-current loss per sq. cm. surface L_e on each side of the sheet is, then, by equation (**6**)

$L_e = 2 \times 10^{-4} \sqrt{\rho\, f\, N^3\, B}$ watts per sq. cm.

where

$\rho = 14.4 \times 10^{-6}$ ohm-cm.
$f = 60$ cycles per sec.
$N = 16.4$ ampere-turns per cm.
$B = 14700$ gausses, maximum

Then

$$L_e = 2 \times 10^{-4} \sqrt{14.4 \times 60 \times 16.4^3 \times 14700 \times 10^{-6}}$$
$$= 0.0475 \text{ watts per sq. cm. surface.}$$

Since the total loss-producing surface per foot length of duct is 11,100 sq. cm. the estimated eddy-current loss per foot of duct is

$$11{,}100 \times 0.0475 = 530 \text{ watts per ft. of duct.}$$

Hysteresis losses may for a first approximation be estimated from empirical hysteresis-loss curves (see Bibliography No. 12, p. 971) for which the hysteresis-loss coefficient η is taken as 0.004 and the volume of iron exposed to the loss is based on the equivalent depth of flux penetration†

$$d = \sqrt{\frac{10^9\,\rho}{0.4\,\pi\,\mu\,\omega}} \text{ cm.} \tag{5}$$

For the sheets in question the measured permeability μ at the maximum flux density $B = 14700$ gausses was $\mu = 500$. Thus the penetration depth d is 0.078 cm. at 60 cycles per sec. giving a volume of

$$\frac{2 \times 0.078 \times 12 \times 72}{2.54} = 53 \text{ cu. in. per ft. of duct}$$

exposed to hysteresis losses. At 14,700 gausses the hysteresis loss in watts per cu. in. per cycles per sec. is 0.031 from the curve for $\eta = 0.004$ on p. 971 of Bibliography 12. Thus the total 60-cycle hysteresis loss by this rough method is

$L_H = 0.031 \times 53 \times 60 = 98$ watts per ft. length of duct.

Hence eddy-current and hysteresis losses combined are: 530 + 98 or, say, 630 watts per ft. length of duct at 60∼.

A 40-cycle loss measurement gave a value of 600 watts per ft. of duct under the conditions of this problem. The above calculating procedure applied to the 40-cycle case gives eddy-current and hysteresis losses per ft. of 435 and 80 watts respectively or a total of 515 watts per ft. or about 15 per cent less than the measured value.

The temperature rise due to the iron loss of 515 watts per ft. of duct may be estimated from temperature rise vs. loss per unit area curves‡. The radiating surface per foot of duct length (outside surface only) is 864 sq. in. Hence the calculated 40-cycle loss per sq. in. surface is

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*For magnetic sheets thinner than ⅛ in., the Rosenberg formula is ordinarily not applicable.

†Bibliography No. 2 and 3.

‡Bibliography 12, p. 757.

$$\frac{515}{864} = 0.6 \text{ watt per sq. in.}$$

For this unit loss, the heating curves* indicate a temperature rise of 50-80 deg. cent. The corresponding measured value was 78 deg. cent. including the effect of the heating due to the enclosed busbar copper loss. The corresponding 60 ~ temperature rise, estimated from the calculated loss of 630 watts per ft., is approximately 20 per cent higher than the 40 ~ temperature rise, or of the order of 60-95 deg. cent.

Example 7. Estimate the losses and temperature rise in the 18-in. by 18-in. bare iron pipe frame of standard 1¼-in. switchboard iron pipe, illustrated in Fig. 13 when a straight 1500-ampere, 60-cycle conductor passes at right angles centrally through it.

The square 18-in. by 18-in. frame will be assumed to be equivalent to a circular ring of pipe bent into a ring of 72 in. (183 cm.) circumference. Then for 1500 amperes, H along the center axis of the pipe is

$H = 10.3$ gilberts per cm.

and

$N = 8.2$ r. m. s. ampere-turns per cm.

From magnetic tests, the corresponding maximum cyclic flux density is

$B = 11900$ gausses.

Also

$\rho = 13.9 \times 10^{-6}$ ohm-cm.

$f = 60$ cycles per sec.

Then the estimated eddy-current loss by Rosenberg's formula (**6**)† is

$$L_e = 2 \times 10^{-4} \sqrt{13.9 \times 10^{-6} \times 60 \times 8.2^3} \times 11900$$
$$= 0.015 \text{ watts per sq. cm. surface}$$

giving a total eddy-current loss of 36 watts for the pipe frame.

To estimate the hysteresis loss, as in the preceding examples, the equivalent depth of flux penetration d is found from (**5**) which gives, for $\mu = 820$,

$$d = \sqrt{\frac{10^8 \times 13.9 \times 10^{-6}}{0.4\,\pi \times 820 \times 2\,\pi \times 60}} = 0.061 \text{ cm.}$$

Hence the total equivalent volume exposed to hysteresis loss is 8.9 cu. in., and the total hysteresis loss from the curve for $\eta = 0.004$ is (see curve p. 971, Bibliography 12):

$$L_H = 0.022 \times 8.9 \times 60 = 12 \text{ watts}$$

Thus the entire iron loss in the pipe loop is $36 + 12 = 48$ watts, giving a loss per square inch surface of $\frac{48}{380}$ $= 0.13$ watt per sq. in. The corresponding temperature rise for the pipe frame is, in accordance with Bibliography 12, p. 757, of the order of 12–20 deg. cent. The average measured temperature rise of the pipe frame under the conditions of this problem was 15 deg. cent.

Example 8. Estimate the loss in a solid iron ring 20 in. average diameter, ¼ in. thick and 4 in. wide (item No. 18 in Tables I and II) when a straight 6000-ampere conductor passes through the center of the ring.

As in the preceding two examples, the eddy-current loss will be computed by equation (**6**) and the hysteresis loss by the method of Bibliography 12, p. 970-971, with the aid of equation (**5**) for the depth of flux penetration.

For the ring, whose perimeter is 62.8 in. (159 cm.), the m. m. f. along its surface, due to the 6000-ampere conductor, is

$H = 47.2$ r. m. s. gilberts per cm.

and

$N = 37.6$ r.m. s. ampere-turns per cm.

From magnetic tests, the corresponding maximum cyclic flux density, (assuming sine-wave shape), is

$B = 16800$ gausses

at a permeability

$\mu = 250$

Also

$\rho = 13.4 \times 10^{-6}$ ohm-cm.

$f = 60$ cycles per sec.

Then, the eddy-current loss, by equation (**6**) is

$$L_e = 2 \times 10^{-4} \sqrt{13.6 \times 10^{-6} \times 60 \times 37.6^3 \times 16800}$$
$$= 0.170 \text{ watts per sq. cm.}$$

giving a total eddy-current loss of 580 watts.

For the hysteresis-loss calculation, the equivalent depth of flux penetration by equation (**5**), at maximum surface flux density, is

$$d = \sqrt{\frac{10^8 \times 13.4 \times 10^{-6}}{0.8\,\pi^2 \times 250 \times 60}} = 0.11 \text{ cm.}$$

giving an equivalent flux-carrying volume of 22 cu. in. (360 cu. cm.) or about one-third of the entire volume of iron of ¼ in. thickness. Accordingly the total hysteresis loss, computed by Bibliography 12, p. 971, with the aid of the curve for $\eta = 0.004$, is

$$L_H = 0.038 \times 22 \times 60 = 50 \text{ watts}$$

The combined calculated eddy-current and hysteresis losses are $580 + 50 = 630$ watts. The measured losses were 635 watts (from Fig. 25), the magnetizing force of 37.6 ampere turns per cm. being applied in the test by a coil uniformly wound around the iron ring, and directly connected to a sine-wave voltage.

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†Both the Rosenberg eddy-current loss formula as well as the flux penetration formula (5) are based on the assumption of thick wide magnetic sheets. Cylindrical solid flux-carrying members obey somewhat different laws (see Bibliography 11.) However, for a rough approximation, the formulas based on the theory of skin effect for wide plates are preferable because of their simplicity and because (as Kennelly has shown in Bibliography No. 2, p. 159) the simple flat-plate theory was not seriously in error, as far as flux and current penetration are concerned, for a magnetic tube of 50 per cent smaller diameter than that in the problem of example 7.

SYMBOLS

B	gausses, maximum cyclic flux density at surface of solid iron.
d	cm. equivalent depth of flux penetration in solid iron.
η	hysteresis loss coefficient for estimating hysteresis losses with the aid of the loss curves given in Bibliography No. 12, p. 971.
f	frequency, cycles per sec.
H	r. m. s. gilberts per cm. in air, resultant air field intensity, due to all conductors, in the absence of iron members, calculated at the point occupied by the center of the iron cross-section through the hot spot and in the direction of the longitudinal axis of the iron.
H_r	r. m. s. gilberts per cm. reference air field intensity at spacing s inches from a long straight 3000-ampere conductor.
I	r. m. s. amperes, conductor current for which iron heating or losses are desired.
I_r	r. m. s. amperes, reference current of 3000 amperes flowing in a long straight conductor.
L	watts total loss in solid iron member.
L_0	watts loss per inch length of perimeter P, for reference current I_r.
L_e	watts eddy-current loss per sq. cm. surface by Rosenberg formula.
L_H	watts hysteresis loss.
N	r. m. s. ampere-turns per cm., the m. m. f. exerted along the surface of a solid iron member.
μ	magnetic permeability.
n	exponent of current or of air field intensity in temperature-rise calculations by equations (**1**) or (**2**).
ω	$= 2 \pi f$.
P	inches perimeter of cross-section of solid iron member.
ρ	ohm-cm. resistivity of solid iron.
s	inches, center spacing between iron member and nearby conductor.
$T R$	deg. cent. hot-spot temperature rise at solid iron for current I and spacing s.
$T R_r$	deg. cent. reference temperature rise at spacing s for current I_r.

SUMMARY

1. Results are given of measurements of temperature rise and of losses in structural steel I-beams, channels, pipes, plates, ducts and of temperature rise in rods, sheets and meshes when the iron members were exposed to the fields from 60-cycle, 40-cycle or 25-cycle conductors carrying currents as high as 5800 amperes. Experimental data were obtained to serve as a basis for designing copper sleeves to minimize heating and losses in structural-iron members crossing at right angles to conductors. Special tests were made on riveted and bolted steel frames placed within the field from a high-current circuit simulating isolated-phase construction.

2. A long straight bare structural steel 5-in. by 1¾-in. channel beam crossing at right angles to a 3000-ampere, 60-cycle conductor (return conductors remote) has a final hot-spot temperature rise of the order of 30 deg. cent. and a loss of 130 watts when placed at 10 in. center spacing from the conductor. The hot spot is at the point nearest to the conductor.

3. The final hot-spot temperature rise at the surface of structural steel members crossing at right angles to a straight conductor varied as the 1.7 power (average value) of the current, for conductor currents up to 5800 amperes and for center spacings ranging from 2.5 to 16 inches between conductor and iron member.

4. Various sizes of straight long iron members crossing at right angles to a conductor and having a thickness not less than, say, ⅛ inch when placed with respect to the conductor at any one center spacing (from center of conductor to center of iron cross-section) within the range from, say, 6 to 16 in., had substantially equal values of temperature rise for any given busbar current, as indicated by tests on structural-iron members of the following cross-sectional dimensions: 12-in. by 10-in. I-beam, 5-in. by 1¾-in. channel beam, 4-in. by ⅜-in. steel plate, switchboard iron pipe of 1¼ in. nominal inside diameter, and ⅛-in. by 18-in. steel sheet. In other words, the temperature rise under these conditions was independent of the size, or cross-sectional area or shape of the iron member for a given center spacing between conductor and iron and for any given conductor current. Variations of temperature rise, within this rule, were found to be not higher than 30 per cent and usually much less.

5. The values of temperature rise of structural iron members at the frequencies of 25, 40 and 60 cycles per sec. were roughly in the ratio of 1:1.3:1.6; *i. e.*, the 60-cycle temperature rise was 60 per cent higher than that at 25 cycles per sec.

6. Increasing the center spacing between a straight conductor and a straight structural steel member crossing at right angles to the conductor, from 6 to 10 inches reduced the maximum temperature rise of the iron 35 per cent; a further increase of center spacing to 16 inches gave a temperature rise about half of that for the 6-in. spacing.

7. Empirical data and a simple procedure are given in the paper for estimating the temperature rise of straight steel members exposed to the magnetic fields from more than one conductor. Iron members passing *between* going and return conductors in adjacent phase layouts may reach very high temperatures. Iron members passing at right angles to, but outside of, a group of going and return conductors will usually heat less than an iron member crossing a single conductor, when the currents and minimum spacings are the same.

8. A long and narrow steel member, such as an I-beam or a channel, will ordinarily heat much less (from

30-80 per cent less in the cases tested) when running parallel to a conductor than when crossing it at right angles, for the same spacings and currents. This is because in the parallel arrangement the iron portion of the flux path around the conductor is usually a much smaller part of the complete flux path than in the case of a steel member crossing at right angles to the conductor. It will, therefore, often be advantageous, from the standpoint of iron heating, to lay out stations with conductors parallel rather than at right angles to the iron members.

9. The loss in a straight structural-steel 12-in. by 10-in. *I*-beam of 64-inch cross-sectional perimeter when crossing at right angles to a straight 3000-ampere 60-cycle conductor at 15 inches, center spacing from the conductor is of the order of 400 watts.

For straight solid structural-iron members of a wide variety of sizes and shapes, but not thinner than 1/8 in., when crossing at right angles to a straight conductor at any given center spacing from 6 to 16 inches, the measured iron losses at any given current were approximately proportional to the perimeter of the iron cross-section.

10. Test data indicate that the temperature rise of structural-iron beams in *large* rectangular magnetic frames surrounding a straight conductor (as in isolated-phase layouts) and having sides larger than, say, 5 ft. may be calculated separately for each individual beam disregarding the other three sides of the frame, even if the conductor passes near the corner at a point as close as 8 in. from the nearest part of each of the two adjacent sides.

11. Small closed structural steel frames having sides of the order of 2 ft. or less, when surrounding a conductor must, however, be expected to heat considerably more than a single straight iron member passing at right angles to a conductor, for equal spacings and currents.

12. The losses and temperature rise in uniform square magnetic frames may be determined, (with certain approximations and restrictions) with the aid of Rosenberg's formula (given in Appendix A) for eddy-current losses in solid iron when a straight conductor passes approximately through the center of the frame. However, in cases when the m. m. f. along the solid iron member is far from uniform (as in the most frequent cases, for instance when a long straight iron member crosses at right angles to a conductor) the *empirical, rather than analytical, method* of determining losses and temperature rise is used.

13. The maximum temperature of an 18-in. by 18-in. square steel duct of 1/8-in. annealed sheets completely enclosing a 3000-ampere, 60-cycle conductor is of the order of 100 deg. cent. in a room of 20 deg. cent. ambient temperature. The iron loss per ft. of duct is more than 0.5 kw.

14. The circulating current in a 6-ft. by 6-ft. channel iron frame was of the order of 100 amperes when a straight 3000-ampere conductor was run parallel to one side of the frame and at 10 inches center spacing from the nearest channel iron, the conductor being outside the frame but in the same plane with it. The temperature rise of the iron was negligible. While the observed current in the 6-ft. by 6-ft. frame was small, considerably larger currents of, say, several hundred amperes may flow in heavy structural-iron frames of isolated-phase stations when the iron loops are large (say 15 or 20 ft. on a side) and especially when a heavy-current bus follows two sides of the frame or when the flux linking the iron loop is materially increased by the effects of other conductors.

15. Short-circuited copper loops or bands around iron members crossing at right angles to heavy-current conductors may be effectively used to reduce the losses and temperature rise of the iron; but when designed with too small a current-carrying capacity, the copper loops themselves may seriously overheat. In two typical cases a copper sleeve, or short-circuit winding, having an ampere-turn capacity of 30 per cent of the busbar current reduced the iron losses and temperature rise to less than 10 per cent of their original values.

Acknowledgment. The authors are indebted to Mr. D. Basch for the use of iron-heating data obtained under his direction in a number of the earlier tests.

Appendix A

ROSENBERG FORMULA FOR EDDY-CURRENT LOSS IN SOLID IRON

An approximate practical formula for estimating the eddy-current loss per unit area of surface for solid iron members has been derived by Rosenberg (Bibliography 5), on the assumptions that (1) the thickness of the iron is materially more than twice the equivalent depth of a-c. flux penetration, (2) current and flux penetration follow the theory developed for skin effect in wide plane plates,* (3) the flux is of sine-wave shape, (4) the maximum surface flux density is fairly high, say above the "knee" of the saturation curve (*i. e.*, well above the density corresponding to maximum permeability). The Rosenberg formula for the loss per sq. cm. of surface of solid iron is:

$$L = 2 \times 10^{-4} \sqrt{\rho f N^3 B} \text{ watts per sq. cm. surface} \qquad \textbf{(6)}$$

where ρ is the ohm-cm. resistivity of iron; f the frequency in cycles per sec.; N the m. m. f., r. m. s. ampere-turns per cm., exerted along the surface of the iron; and B the maximum flux density at the surface in gausses. The results obtained by this formula have been found by Rosenberg and others to be fairly closely equal to the total loss including hysteresis loss, for high surface flux densities in a number of cases. (See Bibliography 2, 4, 5, 6, 13.)

The Rosenberg formula is a valuable aid in the calculation of losses and heating of structural iron members in those cases where the m. m. f. along the iron member is known. However, in a great many cases of structural iron members exposed to a-c. magnetic fields (as for instance in the common case of a straight structural-iron member crossing at right angles to a conductor) the application of equation (**6**) is greatly complicated by the fact that the m. m. f. varies along the iron and is rather difficult to determine.

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Discussion

H. C. Forbes: Engineers who have been concerned with the layout of isolated-phase generating stations will be especially interested in the paper of Mr. Schurig and Mr. Kuehni. In general, this type of station construction is likely to lead to a condition such that heavy currents will be flowing through a busbar which may be in fairly close proximity to the steel work of the station while the return circuit for this current is comparatively remote.

Some tests were made by The New York Edison Company to determine how serious the heating might be under these conditions and to ascertain what measures might be taken to reduce it. The results of these tests are too voluminous to be presented in their entirety, but it may be of interest to discuss some of them and to note the agreement with those given in the paper.

The structure tested was a rectangular steel loop about 6 in. by 10 in. Tests were made with the current-carrying conductor passing through the closed loop and then were repeated with one side of the loop removed and finally with the conductor passing at right angles to a single member of the loop.

It was found that with the conductor approximately 30 ft. from the steel work, the loss in the case of the single member was about 60 per cent of the loss under similar conditions with the loop completely closed. If the steel loop were larger, this difference would undoubtedly become less pronounced and for the construction used in most generating stations, the losses and resultant heating would be largely independent of whether or not the steel work formed a closed loop around the conductor. The temperature rise in the members would depend chiefly upon the proximity of the conductor, and the value of the current flowing.

Most of our tests were made at 60 cycles, but a few comparative tests were made at both 25 and 60 cycles. These gave a ratio of temperature rise of about 1.7 which I would consider a very good check on the value of 1.6 as given in the paper, particularly in view of the limited number of our own tests covering this point.

It was realized that the losses might be reduced by putting short-circuited copper turns around the steel members and tests showed that such a loop, when placed around the steel member at the point nearest the conductor, reduced the losses by 40 per cent in the case of a single steel member at right angles to the conductor and 60 per cent in the case of the closed steel frame. The use of more short-circuited loops than one at the point nearest to the conductor gave but little further reduction in the losses.

It appears from the tests which have been made that some caution should be observed in station design to avoid heating in the steel work and the paper which has been presented will serve as a very valuable guide. However, if care is taken to place conductors which are to carry heavy currents, say from 2500 to 4000 amperes at a distance of 2.5 ft. or more from the steel work, the heating will not ordinarily be serious and it is doubtful if the use of short-circuited copper loops would be justified.

A. E. Kennelly: We know that such physical conditions as described in this paper are very complicated and that it is very difficult to find a simple formula to cover a great range of such multitudinous conditions, but the practical way in which the results are tabulated and indicated seems of great value.

O. R. Schurig: I agree with Mr. Forbes that for spacings as large as 2.5 ft. between conductor and iron members crossing the conductor the temperature rise in the iron may be expected to be only a few degrees for currents of the order of 3000 or 4000 amperes even if the return conductors are quite remote as in isolated-phase construction. That follows from the curves shown in the paper and there is good agreement on this point as well as on the others brought out by Mr. Forbes. Also let me say that it would help a great deal if data obtained by the operating companies could be published in the discussion of this paper or in separate papers. Doubtless a good deal of the work we did could have been saved if the data and experience of other investigators had been available. We certainly hope such information will be published.

We agree with Professor Kennelly that the calculations from the dimensions and physical constants are very complicated and probably not practical at the present stage. However, efforts would doubtless be productive of good results in the course of time. This problem may be a good one for some of the college research laboratories.

Carrying Capacity of 60-Cycle Busses for Heavy Currents

BY TITUS G. LE CLAIR[1]

Associate, A. I. E. E.

***Synopsis.**—Up to the present time it has seldom been necessary to design busses for carrying capacity above 2000 or 3000 amperes. Within the last few years we have passed this mark, and we shall soon be required to design busses for very much larger capacities.*

For simple geometrical designs there are formulas from which we may calculate the capacities of large busses. These simple designs cannot be easily mounted, and for this reason we must resort to styles which are easier to construct. These types cannot be calculated readily by the mathematics available to the ordinary engineer. This paper is presented with the idea of giving a ready reference for determining bus capacities without involved calculations.

Curves are given showing the carrying capacity of a few types which are proposed as standards, and, in addition, a few curves compiled from tests showing the distribution of current in busses to show the necessity of this type of design. By a little careful study of these curves, the average designer may quickly choose the type of bus which will best meet his requirements for carrying capacity and allowable space. All busses are designed on the basis of 30-deg. cent. temperature rise, and their ratings may be proportionately increased if the conditions warrant a 40-deg. temperature rise.

As the usefulness of electric power becomes more and more widespread and its uses more diversified, large blocks of power are frequently required in a small space. This is especially true in factories where there may be a great many machines on a small floor area with individual motor-drive, or for electric furnace work. In consequence, we find it necessary to supply large blocks of power at low voltage, with correspondingly heavy currents.

As the transmission system grows in size the energy of short circuit on the high-tension system is so great that the cost of protective apparatus, as well as the expense of insulating for high voltage, requires that all power be supplied from large, well protected transformer banks in fire-proof vaults. This means that very heavy currents must be brought out from the transformer bank to the distribution switchboard or to the furnace through a single low-tension bus. Due to this rapid development it has become necessary within the last few years to design busses far beyond the old limits of 2000- or 3000-ampere capacity, with the time not far in the future when we shall need to carry 10,000 amperes or more on a single low-tension bus.

THEORY

In any d-c. circuit, be the conductor solid, laminated, or stranded, the current divides in all parts in proportion to the resistance, which means that with a conductor of homogeneous material, the current is practically the same in all parts. The same condition does not hold true, however, for alternating current. In addition to the resistance drop, an alternating current introduces an alternating flux surrounding any element of the conductor. This alternating flux generates a voltage which tends to oppose the flow of current in the conductor element. When we consider a large conductor, it is obvious that the lines of force caused by an element in the outer part surround the entire conductor, but, for a central element, some of the flux does not cut the outer element. The result is that the effective impedance of an element in the central part of the copper is higher than in the outer edge, thereby forcing most of the current to the outer surface, producing the so-called skin effect. In very large conductors this can be carried so far as to have the current in the center of the bus very nearly in the opposite direction to the current in the outer part of the bus, as well as being smaller in magnitude.

When the phases are placed close together there is, in addition to the skin effect, a voltage induced by the flux from an opposite phase which is not uniform over the entire conductor and forces current toward the near side. This is called the proximity effect. Both the skin effect[2] and proximity effect[3] can be calculated for cylindrical or tubular conductors from formulas developed by H. B. Dwight.

Unfortunately, it does not often pay, due to the difficulty of mounting and of making connections, as a practical problem to use a circular conductor for very large bus work. Former practise has been to build the bus of laminated copper bars for the required capacity. For a bus of this shape, it is impracticable, if not utterly impossible, to calculate the distribution of current in order to obtain the losses and temperature rise on alternating-current circuits. To further complicate the problem nearly all high capacity busses are three-phase and not single-phase, which makes it more difficult.

TESTS

Due to the demand for large increases in bus capacity and the impossibility of making calculations, we have just completed a series of tests to determine, if possible, an efficient and practicable type of bus construction for very high currents.

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[1] Of the Commonwealth Edison Co., Chicago.

Presented at the Midwinter Convention of the A. I. E. E., to be held in New York, Feb. 8-11, 1926.

[2] *Skin Effect in Tubular and Flat Conductors*, H. B. Dwight, A. I. E. E. Vol. XXXVII, p. 1379.

[3] *Skin Effect and Proximity Effect in Tubular Conductors*, H. B. Dwight, JOURNAL A. I. E. E., Vol. XLI, p. 189.

In order to make these tests applicable to heavy currents and three-phase circuits, a three-phase bus 20 ft. long was set up and connected in the circuit between the transformer and three of the rings of a 3900-kw., 230-volt rotary converter. (See Fig. 1.) This converter could be operated in parallel with another machine on the d-c. side and permitted a ready control of the current up to quite high values. Practically all tests were run with constant current to determine the ultimate temperature rise of the various types of bus construction.

Fig. 1—Test Bus

Consisting of four 8-in. by ¼-in. copper bars per phase, arranged in rectangular form. Phases on corners of an equilateral triangle, 17-in. sides.

Temperature measurements were made by means of thermocouples connected to the centers of the various bars in this 20-ft. section. A number of checks were made during the progress of the test with temperature measurements at points other than the center of the bus to be sure that no contact resistance or other variables were affecting the results. Impedance voltage drop was measured by means of straight leads perpendicular to the bus, carried far enough away to be out of the influence of the magnetic circuit.

In addition to these measurements, the current in various parts of individual conductors was also determined by a method similar to that described by C. F. Wagner.[4] The leads described running parallel to the bars were No. 24 enameled wire. In order to be positive that there was no space between the wire and the bus surface and no sag, this wire was cemented to the surface of the copper with asbestos cement. To eliminate end effects, only the central 15 ft. of bus were used. In the lower right-hand corner of Fig. 1 may be seen a four-inch bar with five leads cemented on.

We have found a number of things which enter into the construction of a bus that are not constant for all circuits. For example, the current distribution will be very different on single-phase from that on three-phase circuits. Also, there is an extremely great difference between isolated-phase and group-phase busses due to proximity effect.

The majority of engineers are not particularly interested in the exact distribution of current in various parts of the bus. The primary question which enters the designer's mind is rather how much copper he must use and how he may ~~best~~ use it. A concrete example of the distribution of current will, however, clarify ideas of the results of skin effect and proximity effect and help a great deal in deciding for a particular case what form should be used. In Fig. 2 is shown a three-phase bus, each phase consisting of four 8-in. by ¼-in. bars per phase with phases set in an equilateral triangle on ten-inch centers. It will be noted that the current in *A* phase bus is practically all in the very bottom edge of the bars, and the usefulness of the upper half of the bar is more in the nature of a radiator than a conductor. For this

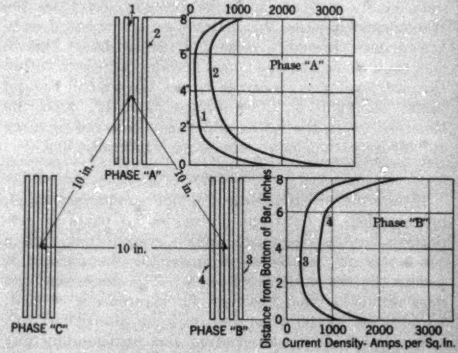

Fig. 2—Current Distribution in a Three-Phase Bus

Consisting of four 8-in. by ¼-in. copper bars per phase, at 4000 amperes per phase.

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[4] "Current Distribution in Multi-Conductor, Single-Phase Busses," C. F. Wagner, *Electrical World*, March 18, 1922.

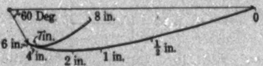

Fig. 3—Polar Diagram of Current Distribution in Outer Surface of 8-In. by ¼-In. Bar

Figures refer to distances from bottom edge of bar The surface considered is the same as Curve 2 in Fig. 2.

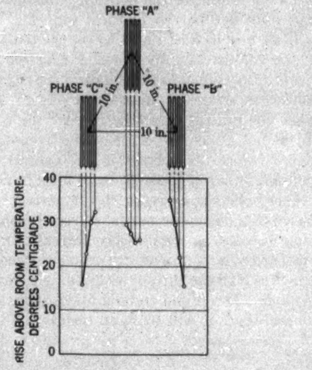

Fig. 4—Temperature Rise on a Three-Phase Bus

Consisting of four 8-in. by ¼-in. bars per phase under a continuous load of 4000 amperes per phase. Bus mounted in still air and not enclosed.

reason a bus of similar construction, except made of four-inch bars in the same layout, that is, four bars wide and two bars high, would not be nearly so effective because the upper set of bars would not carry much current, and in addition would have poor heat connection to the lower bars. Hence, they would not serve as good radiators.

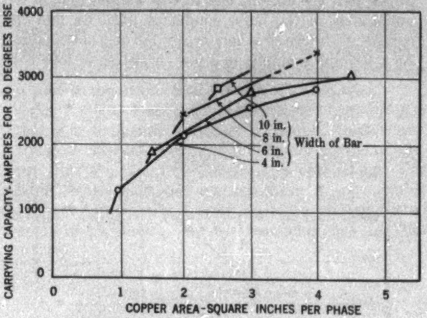

Fig. 5—Electric Power Club Standards

Ratings of laminated busses on grouped phases up to 3000 amperes at 60 cycles. Centers on a straight line with 8 in. between phases; ¼-in. spacing between laminations.

It may also be interesting to note in Fig. 3 the relation of the phase angle of current in various parts of the bar.

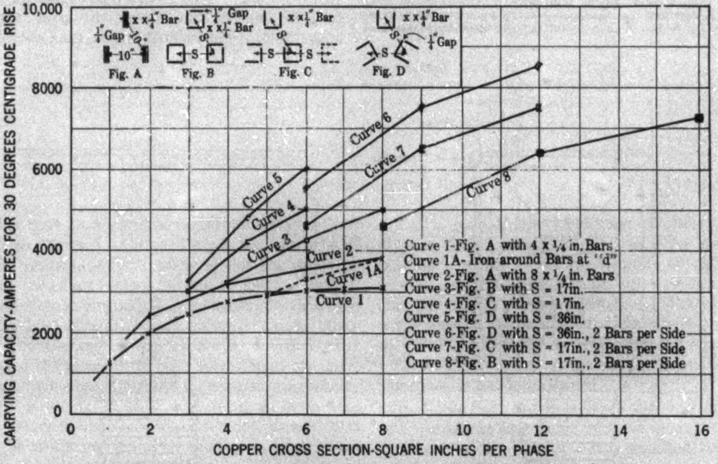

Fig. 6—Ratings of Busses on Grouped Phases Above 3000 Amperes at 60 Cycles

All bars are ¼ in. thick by 4 in., 6 in. or 8 in. wide. For instance, the center point on Curve 6 represents six bars of 6-in. by ¼-in. copper and the upper point on the same curve means six bars of 8-in. by ¼-in. copper.

It will be seen that the current at the minimum point lags 60 deg. behind that in the lower edge and that the point of minimum current is 6 in. instead of 4 in. from the bottom of the bar. As we go nearer to the center of the bus, the current lags further behind that in the outer edge of the outer conductor until we find the current at the center is very nearly 180 deg. out of phase. As a striking example, in a bus consisting of four bars laid on the sides of a rectangle, the addition of a fifth bar in the center actually increased the losses and temperature rise for the same current. In Mr. Wagner's article,[4] for single-phase busses he draws the curve from the outer edge to the center of the bar, and assumes that the same condition holds from the center to the opposite edge of a bar. This is perfectly true in some cases but not at all true in others, and the electrical center, or the point of minimum current in a bar, may not be the physical center.

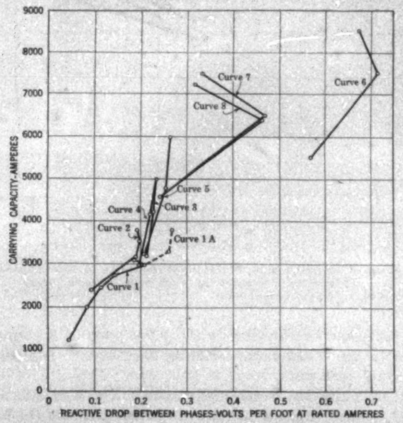

FIG. 7—REACTIVE VOLTAGE DROP ON THREE-PHASE BUSSES

Curve numbers and points refer to corresponding curve numbers and points for busses shown in Fig. 6. With phases arranged with centers on the same straight line, the average drop will be about 25 per cent higher and the drop in the two outer phases will be higher than that in the center phase.

If we now look at Fig. 4 to get the temperature rises of the various bars and consider that the temperature rise is proportional to the square of the current, we get a somewhat erroneous impression due to the fact that while the central bars may be carrying some current, this is not a measure of their effectiveness, because as stated before the current in the central bar may be sufficiently out of phase to be of little or no value.

For moderate currents the important item in the design of a bus is the matter of ventilation. When we come to consider very heavy currents, this matter of ventilation is of minor importance, since ventilation is useless if all the current is carried in a small portion of the bus. The prime consideration, then, is to put the copper where it will be most useful.

BUS CAPACITIES

There are very few data available at this time on the carrying capacity of busses for heavy current. For example, some operating companies have for many years used the standard of 1000 amperes per square inch of copper section. This gives ample copper for busses up to 2000 amperes, but beyond this point the rule no longer holds. For isolated phases, the General Electric

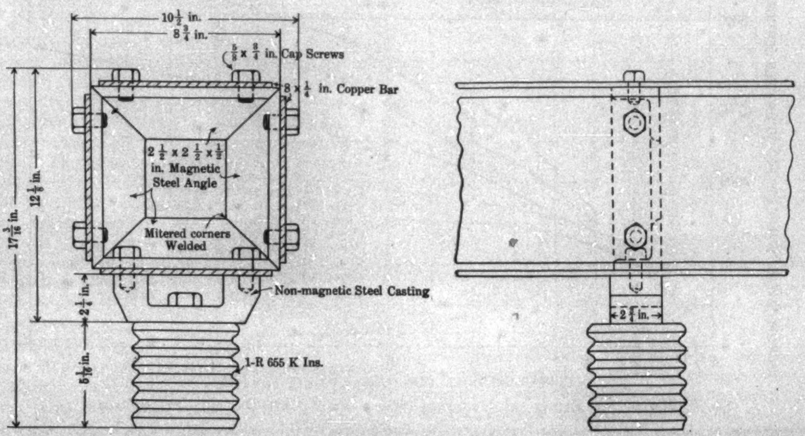

FIG. 8—FORM OF BUS SUPPORT FOR RECTANGULAR BUSSES REQUIRING DRILLING OF COPPER ON JOB. NO CLAMPS REQUIRED

Company has worked out a system of busses[5] which is not at all difficult to mount and is very satisfactory for currents up to 7400 amperes at 60 cycles. Due to the proximity effect as shown in the test data, it is obvious that this type of bus construction would be of little value for group phases on very close centers since the current would all be thrown into one corner of the bus and would cause it to run very hot. The Electric Power Club has given as a standard Fig. 5, which is for group phases up to 3000 amperes. Beyond this point we have compiled from our test data the curves in Fig. 6 for special types of bus construction. The curves do not, of course, give all the data required for any particular installation, but they give the points necessary for determining what should be used.

In Fig. 6, Curve 7 gives the rating of C on 17-in. centers. If the phases are separated further, the ratings of these busses will be raised somewhat but not so high as Curve 6. Curve 6 gives the rating of D on 36-in. centers, and the rating of this bus would be lower on the closer centers but not so low as Curve 7. In Curve 8 the rating may be very materially increased by one or both of two methods, either by a wider opening of the corners of the rectangle, or by further separating the phases. If both these things are done, we may obtain a bus of perhaps higher carrying capacity than that given in Curve 6. When the space limitations or the allowable reactive drop does not demand that phases be on the corners of an equilateral triangle, the dotted arrangement shown in C is practically equivalent in carrying capacity to that of D.

Up to about 3000 amperes the reactance of the bus is not of very great importance, but beyond this point and especially for low voltages it becomes quite a serious item. For instance, on a bus carrying 6000 amperes at 230 volts and only 30 ft. long, it may be readily seen from Fig. 7 that the reactive drop may be from 3 per cent to 8 per cent of line voltage. This, in addition to the reactance of feeders, may cause a very poor voltage regulation at low power factors unless care is used in the selection of the bus.

Another important point in the matter of reactance is that it is frequently necessary to parallel transformer banks of different sizes or with different lengths of copper from the banks to the point of paralleling. For large transformers the bus reactance is considerable compared to the transformer reactance and up to the point of paralleling, the bus reactance may be considered a part of the transformer reactance so far as division of load is concerned. This effect is especially noticeable where transformer banks of different capacities are paralleled, because the percentage reactance per foot is very much higher for large than for small busses. Sometimes it even becomes necessary to install a reactance in series with the smaller transformer bank to prevent overheating of one bank when the other is not fully loaded.

In Curve 1-A, Fig. 6, is shown the carrying capacity of a laminated bus, balanced with magnetic steel, as proposed by Mr. Wagner. It is true that the addition of this magnetic steel does increase the carrying capacity. However, the amount of iron to be added must be determined by the cut-and-try method for any in-

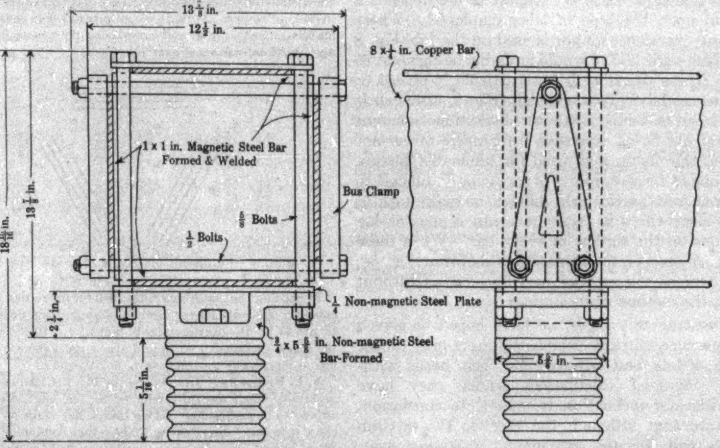

FIG. 9—FORM OF BUS SUPPORT FOR RECTANGULAR BUSSES REQUIRING NO JOB DRILLING OF COPPER

stallation and the cost of adding this balancing becomes prohibitive, especially when the class of labor usually employed knows nothing of what is to be done.

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[5] *General Electric Bulletin No. 87000-D*, Sept. 1924, p. 26.

The principal objection to the use of a rectangular form of bus is the same as one of the objections to the tubular form of bus; that is, the difficulty of mounting. However, a little careful consideration will show that the rectangular bus is not particularly difficult to mount since flat clamps may be used which are very similar to those used for an ordinary laminated bus.

The manner of making taps is little more difficult than with the ordinary laminated bus because all the copper surfaces are flat. Especially is this true if the corners of the rectangle are left open, making room to handle bolts inside. The necessary bracing between phases may be attached to the clamps in practically the same way as is done with laminated busses. We have added two preliminary sketches of supports, (Figs. 8 and 9). The type which is cheapest to use depends a good deal upon the type of labor employed. Where skilled and experienced labor is used on the job, Fig. 8 requires less material and would probably be cheapest to install. Where the labor is not particularly skilled or fast, or where labor costs are high, Fig. 9, although it requires more material, takes very little time to mount and would not be as expensive per ampere of current carried as the clamp now used for laminated busses. Neither one of these supports is particularly difficult to handle, nor is it particularly difficult to make taps to this bus since there is room for strap copper or lug connections on the surface of every bar. When these supports are used on the form of bus shown in Fig. 6C, the bar nearest the support may be omitted without changing the method of mounting.

In presenting this paper we have hoped to give a ready reference through which engineers may choose the type of bus best adapted for their needs without any involved calculations which they have neither time nor inclination to make. In conclusion, let us state that although the Electric Power Club Standards call for the maximum of 30-deg. cent. temperature rise and the busses given are designed on this basis, nevertheless, a number of years' experience has shown that a 40-deg. temperature rise gives no trouble due to oxidation when reasonably good connections are made. Especially is this true when the ambient temperature is nearer to 25 deg. than to 40 deg. cent.

Discussion

H. B. Dwight: Usually it is found undesirable to place copper straps across lines of magnetic flux, but they should be placed parallel to the magnetic field as much as possible. Accordingly, where the phases are separated, I should suggest placing the straps parallel to the sides of an equilateral triangle, as in Fig. 1 of this discussion. If arrangement has not been tried out, it should be tested in comparison with the arrangements shown in Fig. 6 of the paper.

It is not always necessary to separate widely the conductors of different voltages. If the voltage is not over 250 volts, parallel straps connected + − + − etc., or A, B, C, A, B, C, etc., give extremely good results. In this way, currents up to 50,000 amperes at about 200 volts have been carried without trouble from skin effect, proximity effect, or reactance drop, as is well known in heavy electric-furnace work.

C. F. Wagner: The problem of current capacity of busses, by its very nature, is largely empirical. Mr. LeClair has contributed valuable experimental data to the general fund of knowledge on the subject.

I wish to question the statement made by Mr. LeClair on the fourth page of his paper which reads as follows: "In Mr. Wagner's article for single-phase busses he draws the curve from the outer edge to the center of the bar and assumes that the same condition holds from the center to the opposite edge of a bar. This is perfectly true in some cases but not at all true in others; and the electrical center, or the point of minimum current, in a bar may not be the physical center." While this statement is perfectly true, Mr. LeClair draws the inference that I would probably have made the same assumption, and, of course, been wrong, had I had a different set-up. The fact is that my assumption was correct for the particular case chosen, and for other cases in which the distribution was unsymmetrical I determined the distribution in the entire bar. Perhaps I am unjustified in interpreting the statement in this light and considering the matter merely one of an unfortunate choice of expression.

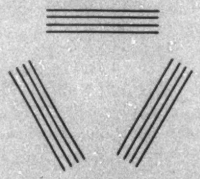

Fig. 1—Proposed Arrangement of Bus Bars

Regarding the use of magnetic steel for increasing the carrying capacity, the cut-and-try method of application does not appear to me to be an insurmountable difficulty. In connection with the magnetic balancers I should also suggest the use of radiating fins to carry away the iron loss in the balancers.

A. E. Kennelly: In the paper by Mr. LeClair, skin effect and proximity effect are fully referred to, but edge effect does not appear to be mentioned. If we take a flat strip of copper and allow it to carry direct current (the return conductor remote), we know, except for temperature variations, that the current density will be the same in all parts of the cross-section. However, when the strip carries an alternatng current, there may be negligible skin effect because the strip is thin. But the current density will not be uniform; it will tend to be much greater at the edges. That effect can be eliminated completely by bending up the strip edges so as to form a tube. The current density then becomes uniform everywhere, and the edge effect disappears.

That is a well known phenomenon and what we have in Mr. LeClair's paper is a mixture of edge effect and proximity effect. For example in the case of Fig. 2, you see that there is edge effect, but there are also proximity effects at the edges which are near to the other conductors. It is exaggerated, so to speak, by the vicinity of the neighboring conductors. But if the bus bars were removed or separated by a considerable distance, there would still be edge effect and the linear resistance of that conductor would be greater than that which would be obtained even though the skin effect were extremely small.

When the conductor is bent round, we tend to eradicate the edge effect, but we do not get rid of the proximity effect, and Curve 3, which represents Figure B, I think has an advantage in carrying capacity over Curves 1 and 2, corresponding to constructions where the edge effect is more pronounced. As Mr. LeClair has pointed out so well, it is very difficult to make measurements upon the resistance of bus bars owing to their size and cross-section. The linear resistance of the bars microhms per meter is so small that it is very often necessary, as he says, to infer the resistance from the temperature observations because where the point of resistance goes up including skin or edge effect, the temperature also will go up. On the other hand, we have to remember that all these effects of extra current density depend upon the resistivity and where the temperature goes up, the resistivity goes up too and modifies the effect. We thus get only an approximate measurement. The changes of temperatures involved will alter the distribution of current density.

S. W. Mauger: This subject began to interest the writer many years ago when rather exhaustive tests were made to determine a practicable and efficient method of carrying heavy currents. As Mr. LeClair states, the old standard of 1000 amperes per sq. in. and the practise of simply adding another parallel bar of copper to obtain increased capacity had become inadequate, especially for 60-cycle current. After trying many schemes, the writer suggested the plan now standard with the General Electric Company and referred to by Mr. LeClair in foot-note No. 5. One of the features of this scheme is the increased ventilation obtained and another is the so-called "box" arrangement which takes care of the skin effect. The first feature was very useful in carrying heavy d-c. currents and it was found that two sets of 4-in.-wide bars in vertical relation with 2-in. vertical space between them would carry at least as much current as one set of 10-in. bars, thereby saving 20 or more per cent of copper.

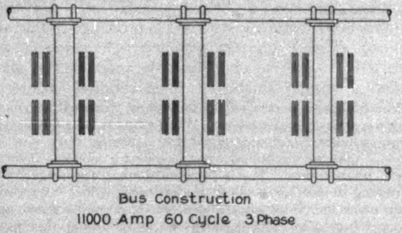

FIG. 2—BUS CONSTRUCTION, 10,000-AMPERE, 60-CYCLE, 3-PHASE

It may be inferred from Mr. LeClair's statement in the first paragraph on the fifth page of his paper that the General Electric Company scheme is suitable only for isolated phases and for currents not exceeding 7400 amperes at 60 cycles. Such an inference would be incorrect as the scheme allows the phases to be on close centers and by using wider bars with slightly more space between bars for increased ventilation, it has been possible to carry much greater currents than 7400 amperes, although this is as high as the published table goes. When there are long runs of bars for heavy a-c. current, transposition can be resorted to which overcomes any difficulty resulting from unequal distribution.

It would seem that with Mr. LeClair's proposed arrangement there would be difficulty in taking off connections from the bus bars except those of small capacity.

Mr. LeClair mentions the matter of permissible temperatures of bus bars. The rule of 30-deg. cent. rise is, of course, based on an ambient temperature of 40 deg. cent. to give a maximum total of 70 deg. cent, the total being what we must have in mind. Naturally, if the ambient is lower, the rise may be higher, but for a standard rule, it is not safe to consider varying ambient temperatures. We must also remember that switching devices are designed on the basis of the A. I. E. E. rule of 30-deg. cent. rise and bars connected to these devices must not have a higher temperature. Oxidation is much more rapid above 70 deg. cent. than below it, and it does not seem wise to consider an ambient of less than 40 deg. cent.

E. G. Bern (communicated after adjournment): In Mr. LeClair's contribution is a statement from which may be inferred that 7400 amperes is the feasible limit for a 60-cycle, three-phase bus with double-tier vertical laminations.

When dealing with currents of such magnitudes, the question of supporting the bus structure and its connections to withstand abnormal magnetic stresses usually demands a more liberal spacing of the bus phases. This is, of course, usually limited by available space, and by the permissible reactance, as mentioned by Mr. LeClair. By properly proportioning the design, it has been found entirely practicable to use this construction for very much higher capacities without neglecting any of the above factors, and at the same time to make connections in a convenient manner. Fig. 2 herewith shows this construction successfully applied to a bus of 11,000-ampere capacity, which, however, should not be considered as the practical limit. Under some conditions a transposition scheme of equalizing the load in the different sections of the bus has worked out to good advantage without undue complications.

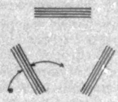

FIG. 3

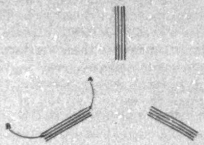

FIG. 4

T. G. LeClair: At first thought it would seem undesirable to put copper straps across the lines of magnetic flux, and it would appear advisable to set up the three-phase bus with the bars arranged as shown in Fig. 3 herewith. However, after a careful examination of test results, we come to the conclusion that, with this arrangement, the ratio of currents at points A and B is practically the same as the ratio of currents at points A and B in Fig. 4 herewith. The particular case on which we have the most information from our test results is a bus consisting of four bars of 8-in. by ¼-in. copper, in arrangements which approximate Figs. 1 and 2. In this case, it appears that the current at point A is nearly three times the current at point B for either arrangement. This means that with the arrangements shown in Fig. 3 most of the current will be carried by the bar facing the center of the triangle and the other bars will carry very little. In Fig. 4, the outer edge of each of the bars will be carrying much less current than the inner edge, but they will reduce the temperature because they act in the nature of radiators, which is not the case with the unused copper of Fig. 3.

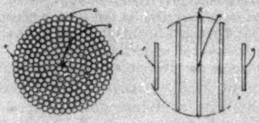

FIG. 5

I do not wish to convey the impression that Mr. Wagner was making a mistaken assumption in his previous article. Some warning is, however, necessary because the majority of busses will be set up on an arrangement wherein the distribution of current will be unsymmetrical.

On the matter of balancing currents by means of magnetic balancers around the bars, there are two different points of view to take. One is that of the manufacturer of switchboards who has factory conditions to deal with and close contact with the men doing the work. In this case magnetic balancers may possibly be used to advantage. Out in the field we have different conditions, and usually the class of men doing the work is one unfamiliar with what is required. The contact between the engineer and the construction men is not so close, and the expense of the cut-and-try balancing is altogether out of reason. Some other method of balancing is much less expensive and more satisfactory for all cases.

Dr. Kennelly has brought up an important point in the use of the terms "edge-effect" and "skin-effect." The difference in the significance of these terms should be made more clear. Fig. 5 herewith will probably explain my reasons for using the term skin-effect rather than the term edge-effect. This figure represents a single-phase circuit in which one of the busses is made up of laminated copper bars and the other of copper strands in the form of the cable. In the left-hand bus we all agree that due to skin-effect the current in the outer surface at C is much greater than the current at the center, D. Due to proximity effect the current at the point E is greater than the current at the point F, and by following the same reasoning in the right-hand bus, which, for the sake of clarity, I have made circular, we should still say that skin-effect is the cause of a greater current density at G than at H, and that proximity effect is the cause of the greater current density in bar J than in bar K. Now, if we arbitrarily take out the center bar of this bus and consider it separately we say that it is no longer skin-effect but edge-effect which causes the greater density at the point G than at point H. As I see it, edge-effect is a special application of the word skin-effect. Skin-effect should include any uneven distribution of the current in a bus or cable due to the flux created by the current in this bus, while proximity effect should include any uneven distribution caused by flux in an adjacent conductor. Regardless of the shape of the conductor, whether it be round, flat, or of any other shape, the uneven distribution due to the current in itself then causes skin-effect, and changing the shape changes the amount of this effect but does not change it from one type of effect to another. I do not wish to state that this is the best nomenclature to use for all cases, but I think it is time that we settle upon one definition by which we could all understand each other.

We did not make any test on an arrangement of bars exactly like that described by Mr. Mauger and by Mr. Bern, but it appears that they would have some trouble due to proximity effect unless the busses were spaced a reasonable distance apart. Of course, so long as we can increase the spacing between phases and between individual bars of the phases, we may increase the carrying capacity indefinitely. When we come to the greater current, however, the weights of copper to be used become of considerable importance, and it becomes advisable to study carefully which arrangement will give the best copper economy. Perhaps it will pay to use a slightly more expensive support in order to use less copper. The type of construction illustrated in Mr. Bern's discussion is very interesting, but unfortunately no dimensions are shown with which to make comparisons of the copper economy to be obtained with this or another arrangement.

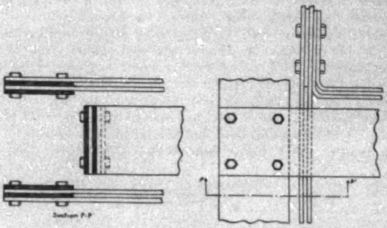

Fig. 6

Mr. Mauger's points on the matter of permissible temperature rise are very well chosen. When a bus is to be connected to some switching device designed for a 30-deg. cent. rise, it is obvious that the bus copper must operate at the same, if not at a lower, temperature. It is impossible to make contacts for use in switching devices as good as a stationary bolted contact, and they must be protected. There are, however, a great many places where a bus is not connected to switching devices, for instance, between a transformer and rotary converter, and in such cases experience has shown that considerably higher temperatures may be maintained without any trouble from oxidation.

Fig. 6 herewith shows a sample top on a bus for high capacity. Tops in other directions can be made with equal ease.

Supervisory Systems for Electric Power Apparatus

BY CHESTER LICHTENBERG[1]

Member, A. I. E. E.

Synopsis.—*A general survey and description of the various types of supervisory systems for control and indication of remotely located electrical apparatus is given in the paper, the author first of all comparing the better known remote-control system with the supervisory system in general practise today. Description is given of the selector, distributor, audible, code-visual, synchronous-relay-visual and the carrier-current systems, the principles and features of each being discussed. The subject of telemetering is also included, together with an expression of the author's ideas of future possibilities for each of the systems above enumerated.*

INTRODUCTION

SUPERVISORY systems for the control and indication of remotely located electric power apparatus can best be introduced by comparing them with the better known and more widely applied remote control systems in general application today.

The usual remote control scheme employs at least one continuous individual metallic connection between each device to be controlled and the controlling switches or keys. Supervisory systems use no individual and one, two, three or more common metallic connections between the devices to be controlled and the controlling switches or keys for as many as fifty or sixty or even more devices. Remote control systems ordinarily use control currents of the order of magnitude of one to ten amperes. Supervisory systems usually use currents of the order of magnitude of three to ten milliamperes. Remote control systems require a definite and usually very short time interval to elapse between the closing of the control switch or key and the closing of the contacts at the remote point. Supervisory systems require an appreciable and variable time to elapse between the operation of the control switch or key and the closing of the contacts at the remote point. Remote control schemes are usually designed into the electrical operating sequence of the power apparatus. Supervisory systems are invariably superimposed upon the usual sequence. Briefly, supervisory systems provide improved means for the supervision of electric power transmission and distribution.

HISTORY

Supervisory systems were developed to meet the requirements of the railway and central station industries as these expanded. The grouping of generating stations under the direction of a centrally located load dispatcher made it desirable that the dispatcher have prompt and accurate information concerning the electric power system. The widespread application of automatic switching to railway substations made it important that the power director have immediate and correct information in regard to his substations at all times.

One of the first supervisory systems was installed between the Sherman Creek Station of the United Electric Light & Power Co. at 201st Street and Broadway, New York City, and the load dispatcher's office of the New York Edison Co. at 38th Street and First Avenue, New York City, in 1915. It used a form of step-by-step selector relay developed about that time for automatic telephone application. It gave indication at the dispatcher's office of the open and closed position of about one hundred oil circuit breakers using a total of six wires between the generating station and the load dispatcher's office.

The next step was made when the Receiver of the Des Moines City Railway expressed a desire for some means of isolating his automatic railway substations in case of an emergency such as a fire or an accident. This demand was met by the development of the selector supervisory system and followed promptly by the development of the distributor supervisory system. Next in order came the code visual, the audible, the synchronous relay and lastly the carrier current supervisory system. Beside these, developed and standardized by the manufacturing companies in this country, there has been a number of systems developed by the employees of many corporations not only in the railway field but also in the central station and industrial fields. These, however, have usually been limited to a definite field of application and have not been extended to become a commercial article of trade.

SELECTOR

The selector system was the first one developed for commercial application which is still on the market. It uses the essential elements of the telephone train dispatching call system modified for the control and indication of remote electric power apparatus. An installation made in 1920 is still in active service.

A key, a selector, some indicating lamps, and three line wires form the essential elements. The key when turned and then released makes and breaks an electric circuit through two of the wires to which all of the selectors in the outlying stations are connected in multiple. The key in operating sends out a predetermined number of electric impulses spaced in a definite time sequence thus forming a code. The selectors are provided with contacts mounted so as to make an electric circuit when a ratcheted wheel moves a given number of steps. The code sent out by the key starts all of the selectors simultaneously.

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[1] Engineering Dept., General Electric Co., Schenectady, N. Y.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

Only that selector, however, which is arranged to respond to the code sent out by the key closes its contacts. The other selectors drop out successively as the code being sent fails to give their combination. Thus, there is usually one hand-operated key at the controlling station for each selector contact at the outlying station. For the return indication there is provided at the outlying station a motor operated key for each group of four or less pairs of functions at that station to be indicated. Upon the operation of any supervised device in an outlying station, auxiliary contacts upon it start the motor key. This then sends back to the controlling station a code similar to that sent out from the controlling station to perform an operation. The selector located at the controlling station which is set to close its contacts upon the receipt of the code operates indicating lamps to show the position of the device supervised.

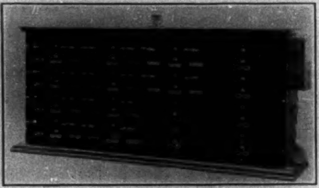

Fig. 1—Lamp and Key Case for Dispatcher's Office of Selector Supervisory System

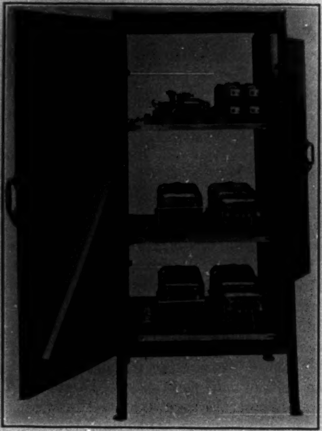

Fig. 2—Sending Apparatus in Cabinet of Selector Supervisory System

All of the outlying stations of a selector supervisory system are connected to one controlling station by three line wires. At each outlying station, the equipment is multipled to two of the wires as they pass by while the third wire loops into each station. The two wires as before mentioned are used for sending out the code for causing an operation to be performed and for transmitting back the code which results in giving an indication by lamps of the position of the equipment supervised. The third wire is used to lock out all the stations connected to the system excepting the one being controlled or the one sending in a code for causing an indication to be shown.

An arbitrary maximum of fifteen outlying stations may be connected by the usual selector systems to the controlling station. Each outlying station may have a maximum of eight devices to be supervised if there are as many as fifteen stations, or if there are fewer, each outlying station may have a maximum of twelve or fifteen. The maximum number of devices, however, which can be most economically supervised by a single selector system depends largely on circumstances but under present conditions should never exceed fifty or sixty.

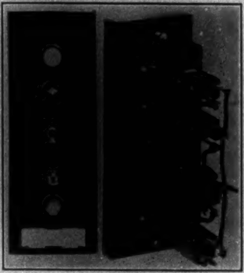

Fig. 3—Lamp and Key Unit for Distributor Supervisory System

The selector system requires about nine seconds to send out each code and complete an operation. Therefore, if ten outlying devices were to be operated it would take about one and two-thirds minutes to send out the code and operate these devices.

Distributor

The distributor system was the next type of supervisory system to be developed. It is an adaptation of the automatic printing telegraph in commercial use today on practically all the principal telegraph circuits. An installation of this type of supervisory system made in 1921 is still in active service.

A key, a distributor, a polarized relay, some indicating lamps, and four line wires form the elements. The key is usually a two-position one and is the equivalent of a double-throw switch. When turned to either position it makes a circuit between a common wire and either the positive or negative side of a source of power and a segment on the distributor. The distributor consists essentially of three sets of coaxial segments insulated from each other. Each segment of one set is connected individually to its control key. Each segment of the second set is connected to the coil of a two-position polarized relay which is associated with the appropriate indicating lamps. A set of segments midway between these two is used for synchronizing the distributor in the outlying station with the distributor in the dispatcher's office, synchronism being checked each five segments or ten times per revolution.

Fig. 4—Panel Containing Lamp and Key Units

For the control and indication of fifty circuit breaker equivalents in distributor supervisory system

The distributors are each provided with three sets of brushes which rotate over the coaxial segments and make connections successively between each individual segment and a continuous segment in each set thus permitting a momentary electric circuit to be made through the distributor. The brush arms of each distributor are driven through a reduction gearing by a direct-current motor from a trickle-charged storage battery and are provided with a centrifugal governor for maintaining constant speed within quite narrow limits. The brush arm has a speed of about twelve rev. per. min.

At the controlling station are located the control key, indicating lamps, one distributor for each group of fifty or less devices to be supervised, and one relay for each device to be supervised, as well as suitable battery or other power equipment. At the outlying station are located a similar distributor for each group of fifty or less devices to be supervised in that station, one relay for each device to be supervised and the necessary battery or other power equipment.

The system operates continuously. This means that the positions of all devices which are supervised are automatically checked twelve times per minute or once each five seconds. Should any device change its position the change is indicated at the controlling station within five seconds of the time the operation has occurred. If several devices or the entire group connected to the system change position simultaneously, the lamps at the controlling station will give correct indication of the new position within five seconds of the time the change has occurred. Besides, the workability of the system is indicated continuously in much the same manner as the pulse indicates the general health of a human being.

Fig. 5—Relay and Distributor Cabinet

For control and indication of fifty circuit breaker equivalents in the distributor supervisory system

To open or close an oil circuit breaker or perform any similar function it is merely necessary to turn first one key and then a master key. This changes the polarity of a segment on the distributor at the controlling station and within five seconds momentarily energizes a corresponding segment on the distributor at the outlying station. This causes the position of the polarized relay connected to that segment at the outlying station to change in accordance with the impulse sent out from the controlling station and the operation desired is performed. Immediately the operation is completed, auxiliary switches on the device supervised change the polarity on an associated segment on the distributor at the outlying station. This, within a maximum time of five seconds, changes the polarity of a corresponding segment on the distributor at the controlling station which in turn changes the position of associated polarized relays located there. The relays then change the lamp combinations to indicate the new positions of the devices supervised at the outlying station.

AUDIBLE

The audible system is an adaptation of the automatic telephone being applied in many large and small telephone systems today. Its essential elements are a dial, a telephone line, telephone lamps, selectors and a receiver or loud speaker. Operation is secured by dialing as in calling a number by an automatic telephone. The first dialing selects the station to be supervised, it being possible to connect as many as six to a single controlling station by a single pair of telephone lines. The stations not selected are locked out by the operation of suitable devices in their equipment.

FIG. 6—DISPATCHER'S SENDING STATION FOR WESTINGHOUSE AUDIBLE SUPERVISORY SYSTEM

FIG. 7—DISPATCHER'S SENDING CABINET FOR G. E. AUDIBLE SYSTEM

The station selected sets up a series of impulses indicated at the controlling station by a series of tones in code through a receiver or loud speaker located at that point.

The next dialing causes the desired operation to occur in the outlying station by setting up a suitable path in a relay combination finally closing the contacts of an operating relay. When the operation directed has been performed, auxiliary contacts on the device actuate other relays which in turn set up impulses indicated at the control station by a series of tones in code as before.

CODE VISUAL

The code visual system is similar in principle to the selector system but differs quite markedly in its detailed design. It consists essentially of a key, with its associated lamps, groups of relays at the controlling station and similar groups at the outlying station. It uses two common wires between the control station and all of the outlying stations and in addition one individual wire from the controlling station to each outlying station. Thus, if there were three outlying stations, there would be five wires starting from the controlling station and running to the first outlying station, four running to the second outlying station and three to the third outlying station.

FIG. 8—LAMP AND KEY UNIT
For code visual supervisory system

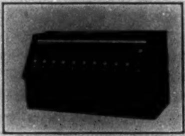

FIG. 9—DISPATCHER'S LAMP AND KEY CABINET
For code visual supervisory system

An operation is performed by moving a three-position key to either of the two extreme positions. This completes an electric circuit which causes groups of relays at the control station to send out a code of successive impulses. These impulses select suitably coded groups of relays at the outlying station. If the code is not received correctly at the outlying station, the controlling station then again starts the code out and continues to send it at intervals until correct code is received at the outlying station. If the code, however, is repeated correctly at the outlying station, then an impulse is sent out from the controlling station which causes the operation desired to be completed.

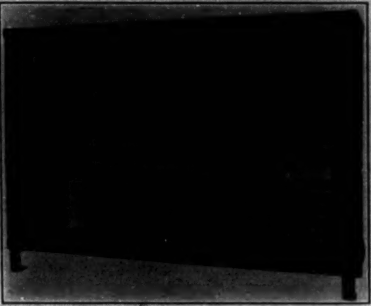

Fig. 10—Relay Equipment in Dispatcher's Office
For code visual supervisory system

When the operation of a supervised device has occurred an auxiliary switch connected to it causes a code to start from the outlying station. This is formed and transmitted to the control station in the same fashion as is the operating sequence from the controlling station. When the correct code is checked the lamp signals at the controlling station are changed to indicate the new position of the supervised device.

This system operates devices successively and it requires about nine seconds per device to cause a series of operations. If, therefore, ten devices were to be operated it would take about ninety seconds to operate the devices.

Synchronous Relay Visual

The synchronous relay visual supervisory system is an all relay system which uses the principle of step-by-step synchronous selection. It consists essentially of a two-position key with its associated lamps, a start key and relays in the controlling station, four wires between the controlling station and the outlying station, and relays in the outlying station.

When an operation is to be performed, the two-position key is turned from the position it occupies to the other position and the start key is pushed. This immediately causes a simultaneous operation of relays in the control station and the outlying station until the point is reached in the sequence in the outlying station corresponding to the position of the key in the controlling station. Operating current then passes through the signaling circuit from the control station to the outlying station and causes the desired operation to be performed. When the device supervised has had its position changed, auxiliary contacts on it transmit a return signal in a similar fashion to the controlling station causing the lamps located there to change in accordance with the changed position of the apparatus in the outlying station.

Two of the four wires between the control station and the outlying station are used for keeping the signaling relays at the two stations in step as they are moved from point to point in a definite sequence. The other two wires are switched by the operation of these relays from one device to another as occasion demands to cause operation or indication of that device.

Carrier Current

Carrier current has been developed for use with the selector and code visual systems. Both systems use codes of impulses for causing an operation to be performed and for causing an indication of that operation to be registered. The carrier current equipment consists of the necessary tubes, reactors, condensers and resistors for generating high frequency at the controlling and outlying stations.

Fig. 11—Cabinet Containing Lamp and Key Units and Ammeter
For the control and indication of thirty circuit breaker-equivalents by the synchronous relay supervisory system

In general, the code is transformed from pulsating alternate polarity impulses to high frequency impulses and is transmitted over one line and ground or a pair of lines which may at the same time be used for communication or power transmission purposes. The carrier current equipment is connected to the transmission circuit by a condenser or other suitable coupling.

Carrier current for use with supervisory systems takes the place of the special line wires required where direct-current impulses are employed. It permits the use of practically the same equipment, so far as the controlling station and outlying stations are concerned, as is used for direct current. The only difference is that where direct current is employed, special line wires are required, while where carrier current is employed, any existing wires can usually be used.

WIRELESS

Wireless has been considered for use with supervisory systems but to date no commercial development or application is known by the writer. The art seems not yet to have developed sufficiently.

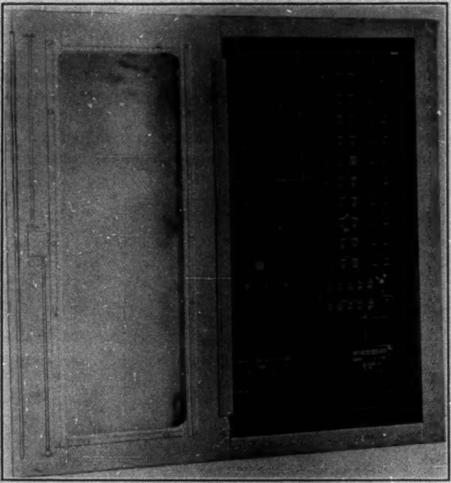

FIG. 12—RELAY CABINET FOR DISPATCHER'S OFFICE, SYNCHRONOUS RELAY SUPERVISORY SYSTEM

CONTROL STATION

The control station equipments provided for the various types of supervisory systems which have been described vary one from another.

The selector system uses two keys, one to perform a closing or similar operation, the other to perform an opening or opposite operation. The key is quite similar to that used in the call stations of the Western Union Telegraph Company. The key is turned but nothing happens in the external circuit until it is released. The turning of it winds up a helical spring. As soon as the key is released the helical spring rotates it in the reverse direction under the control of a fly ball governor in order to obtain practically constant speed. In rotating back to its original position it opens and closes contacts in a definite sequence. If these contacts are suitably connected in an electric circuit they thereupon form a code of successive impulses.

As before stated, two keys are required for each pair of functions. Associated with these two keys are two telephone lamps, capped with suitable lenses. These indicate the open or closed position of the device supervised. In place of indicating the position of a device if it is desired to indicate the position of water level or the position of a gate then the lamps only are used and as many of them are employed as positions to be indicated are desired.

The distributor system uses a combination of three telephone lamps and one two-position key to obtain control and indication. The two-position key performs the same connections as a single-pole double-throw switch having no mid position. Associated with it are three telephone lamps. One is capped with a red lens, one with a green lens and one with a white lens. The red and green capped lamps indicate the closed and open position of the device supervised or any other indication usually desired. The white capped lamp is lit only during the time that a function has been called for and is put out immediately after that function has been completed and a return indication showing the completion of this function recorded at the controlling station. It is also lit should a device supervised at the outlying station automatically change its position, in which case it remains lit until the key in the controlling station is turned to acknowledge the operation at the outlying station. For each group of fifty two-position keys with their associated lamps there is one master key. The master key is turned after the two-position keys have been placed in the positions corresponding to the one which it is desired that the outlying supervised devices take.

The audible supervisory system employs a dial similar to that used for automatic telephones. In one commercial scheme, this dial is identical with that used for automatic telephones; in another commercial scheme this dial is similar but has had its speed altered to make it correspond to that of the selectors used. No lamps are used with the audible system, the operator depending upon hearing certain code tones in a receiver or loud speaker for indications.

The code visual supervisory system uses a three-position key associated with three telephone lamps capped by red, green and white lenses in much the same fashion as does the two-position key with its associated lamps in the distributor system.

The synchronous visual supervisory system employs a dispatcher's equipment consisting of a two-position key, a locking-type push button and three telephone lamps capped by red, green and white lenses. The two-position key is to fix the operation while the push button is to stop the synchronous relay control equipment at any point.

Various methods have been employed for mounting the controlling lamps and keys. They may be placed on vertical panel boards, set into desk tops or even placed in a system diagram. The mounting best suited for any particular application depends to a large measure upon the individual requirements.

APPLICATION

The application of supervisory systems to any existing or new electric power transmission or distribution systems requires careful study in order that a maximum benefit may be derived. It is necessary to know intimately the method of operating the network of lines, feeders, and machines as well as the electric power traffic requirements. For congested districts where an emergency may at times require the rapid re-establishment of service, the speed of operation of a supervisory system is quite essential. On interurban railway projects, however, where automatic substations are located at intervals along a narrow strip of territory and where there are only three or four devices or functions to be supervised in each station, speed is not as important as line maintenance so another type of supervisory system is usually chosen. In some cases, the revenue derived from a station warrants only the most inexpensive supervisory system. Hence, here would be a place for the so-called audible type where the operator checks manually and at intervals determines by sound the condition of the equipment in the outlying station. Again, on a high-tension transmission system where distances often exceed one hundred miles the carrier current supervisory system is the best choice.

In applying supervisory systems it must be clearly borne in mind at all times that they are quite different in their performance as contrasted with the usual types of remote control systems. Each has its limitations. Remote control systems are in general limited to distances of several miles while the supervisory systems may be used for distances up to several hundred miles. Supervisory systems require an appreciable time interval averaging about five seconds for the completion of an operation through their medium while remote control systems operate practically instantaneously. So in applying supervisory systems, care must be exercised to take into account all of the electric power system characteristics as well as the limitations of the supervisory system.

INSTALLATION

The correct installation of a supervisory system is quite as important as its selection if the best results are to be obtained. Supervisory systems use currents of telephonic magnitude while remote control systems use currents of power magnitude. Hence, insulation and current leakage require very careful attention for the successful installation of a supervisory system, telephone practise being the standard toward which the installation of these systems must tend. The steam railroads have recognized these requirements when installing the selector type telephone train dispatching call system. More and more of them are going to the higher grade lead-covered paper-insulation telephone cable for their train dispatching circuits. The same cable and the same type of insulation are generally recommended as best practice for supervisory system installations.

CHECKING

Supervisory systems from a checking standpoint may be classified into two groups. One group requires checking at intervals; the other group is automatically checked at intervals.

All systems excepting the distributor system require manual checking at intervals. The distributor system has a distinct advantage in that it automatically checks the position of all of the devices connected to it at frequent intervals besides indicating its readiness to serve continuously.

MAINTENANCE

The maintenance of a supervisory system depends upon its initial design and installation. All supervisory systems may for this comparison be divided into two classes. One class uses telephone relays and auxiliaries throughout. The other class uses telegraph relays and similar devices throughout.

The telephone relays and devices have been designed with the fundamental idea that someone would be present not only at the subscriber's station for operating the equipment but also at the central office for supervising and constantly inspecting the equipment. This results in a tacit assumption by the designers that if anything should go wrong either the subscriber or the wire chief or operator would detect the fault and correct it, no harm being done excepting possibly an interruption of service from one subscriber's station. Such relays particularly require inspection at least daily in order that the best results may be obtained with them.

The systems using telegraph devices may be typified by the selector system. Here the devices are large and substantial and so designed as not to require attention for years at a time. To be sure, there is quite a difference in the cost of the individual devices since the telegraph devices are made in very much less quantity than are the telephone devices. Contrasted, however, one with the other is the fact that the telephone type of devices require more maintenance and adjustment than do the telegraph devices.

PROTECTION

Supervisory systems operating over special wires require protection particularly where these wires are subjected to lightning, inductive and similar high voltage disturbances. In general, isolating transformers or drainage coils cannot be used because these transformers would interrupt the flow of signals and the drainage coils would drain off the signal current. So far as is known, the best type of protection is for the conductors to be placed in cables. For this purpose paper-insulated lead-covered cable having an insulation which will withstand 1000 volts between conductors and 3500 volts between conductors and lead sheath is usually recommended.

The question of protection is an active one at the present time and many of the operating companies as well as the manufacturers would be interested to hear what other operators are doing to maintain their supervisory systems with a maximum of protection and a minimum of interruption.

TELEMETERING

A discussion of supervisory systems would not be complete without brief mention being made of telemetering. Telemetering furnishes the concluding link in the development of the art of supervising remotely located power apparatus.

Telemeters have been developed for transmitting the readings of ammeters, voltmeters, watthour meters, wattmeters and power factor meters over telephone lines the same distance as supervisory systems can be operated. They use practically the same type of relays at the originating and receiving ends of the telephone lines and in certain designs it has been found possible to combine the telemetering and supervisory functions on the same wires. In one particular installation recently made, two incoming lines, two synchronous converters and nine feeders were controlled and indicated and the readings of two watthour meters transmitted back to the controlling station, all over two pairs of telephone lines between the controlling station and the outlying station. This is not a limiting number of things which could be done over these four wires but represents only about one-third of the functions and readings which could have been obtained if desired.

FUTURE

Supervisory and telemetering systems are just about being recognized by the engineering public. They are relatively new although a number of installations are now in operation. Nevertheless, it is believed that only an initial advantage has been taken of them and that they are not yet as thoroughly a part of electric power transmission and distribution engineering as are transformers, turbines, and the more commonly accepted pieces of electrical apparatus.

It is my firm belief that supervisory and telemetering systems in combination with automatic stations represent as distinct an advance in the electric art as has the development of the automobile in the transportation problem. It is also believed that a very great change will be effected in electrical engineering practise during the next ten years as a result of the economic application of supervisory and telemetering systems.

Discussion

C. E. Stewart: I should like, briefly, to call attention to one or two things of interest. In the first place, supervisory systems have passed the experimental stage. We have had actual service out of the systems so that we know that they are successful in the field for which they are intended. By the end of this year, systems put out by the General Electric Company alone will have supervision over some 2500 oil circuit breakers or their equivalent. We shall have in service a distributor type whereby a dispatcher at a central point will have supervision over breakers located in twenty-two out-lying stations.

Another very interesting application of supervisory systems will be made on an electrified railroad division sixty miles long between Chicago and South Bend, whereby a load dispatcher in Chicago will control eight automatic stations by means of a carrier selector system operating over one wire and ground-rail return. He will be able to start and stop any station, open and close the feeders going out from the station, and also control incoming lines in some cases.

R. J. Wensley: Mr. Lichtenberg says that telephone relays are not suited for supervisory control service. With this statement I do not agree. I would like to call attention to several installations in the New York area which depend entirely on telephone relays for reliability in operation. The equipments in actual operation seem to be the best refutation of Mr. Lichtenberg's condemnation of telephone relays.

The Long Island Railroad has one of the simple audible types in service controlling a remote 600-volt switch house. It is controlled by a dial and a howler gives an audible answer. It works over two wires and is the least expensive type of control.

This same company has a more elaborate equipment of the code-visual type in service at Amityville controlling one 2000-kw. converter in 600-volt railway service.

The New York Edison Company has a remote selective metering equipment in service at their Forty-First Street substation which transmits readings of volts and amperes over standard telephone cable.

The New York and Queens Electric Light and Power Company has a very comprehensive set of relay-visual equipment in the Woodhaven substation. With it they can control oil breakers, adjust induction regulators, bring in spare regulator and transformer when needed, and make necessary simultaneous readings of a-c. volts and amperes. These functions are all accomplished over telephone cable with relatively few wires.

The most extensive equipment in this area is that on Staten Island. The equipment is used jointly by the Staten Island Rapid Transit Company and the Staten Island Edison Company. It is of the all-relay visual type. Five railway substations and one power-distribution substation are controlled from one dispatching point. Two separate-control desks use the same relay equipment to handle the business of the two operating companies.

H. C. Don Carlos (communicated after adjournment): In the operation of two generating stations under supervisory control, the Hydro-Electric Power Commission has encountered, among others, some very interesting protective problems. Although the final step in remedying certain troubles has not yet been taken, the following comments on them may be of interest to the profession.

The stations in question are near Campbellford, Ontario, on the Trent river, a navigable stream on which they form three successive developments with navigable reaches intervening. Ranney Falls or No. 10, the upstream station, is the point of control for Nos. 9 and 8, the total distance between plants 10 and 8 being three and a half miles. In No. 9 there are three vertical generators, each of 1400-kv-a. capacity, and in No. 8, three vertical generators of 2000-kv-a. capacity each. In common with a number of other stations, they supply power to a 44,000-volt, 60-cycle network, complicated by a large number of loops.

Since the navigation regulations limit the permissible variation of the level of the small reaches between the plants to about two inches, there is practically no storage available, and consequently very exact information is required at the control point on reach levels, plant output, gate openings (in tenths) of all turbines, as well as switch positions and various other control data. For each of the stations, something over fifty supervisory operations are performed in addition to the transmission of three graphic records. A twenty-pair standard, lead-covered telephone cable, located on a separate pole line, 25 ft. from a 44,000-volt power line, connects No. 10 to No. 9, and a ten-pair cable continues on to No. 8.

The No. 8 generating station went into operation under manual control, using automatic equipment, September 11, 1924, and under remote supervisory control, January 13, 1925. A great deal of trouble was at first experienced, which required patient study and careful analysis to eliminate. With few exceptions the equipment itself is now performing very well and gives promise of ultimate success if the disturbing effects of power short circuits and lightning trouble can be overcome.

Soon after the supervisory cable went into operation, trouble began to develop in it, affecting one or more conductors each time, and occasionally actual burn-outs occurred. The more serious faults were traced down and eliminated, but after a time all spare conductors had been used, and finally we were left without sufficient clear wires to operate.

Induced disturbances due to the proximity of the 44,000-volt power line were at first suspected as the cause of our trouble, but theoretical considerations showed that since the sheath was grounded at all three stations, inductive effects could not be responsible. The use of insulating joints in the cable sheath, splitting it into numerous sections grounded at each end, was one of the proposals most frequently advanced by those who were not thoroughly familiar with the problem.

For various reasons, the sheath must be grounded to the apparatus ground of all three stations. The main trouble is caused by differences in potential between the so-called grounds (*i. e.*, the prevailing potential of the ground leads and steel work within the building) of any two of the three stations due to our inability to obtain a sufficiently high-capacity and low-resistance "ground."

This potential difference is uniformly distributed over the cable sheath (which carried a portion of the fault current) between the two stations. The conductors, however, are normally at uniform potential from end to end, and their insulation is quite unable to withstand the stresses due to the potential drop along the sheath, which may be upwards of 8000 volts.

This surprising figure has a simple explanation. The ground resistance at each of the three stations is in the order of two ohms each, and the cable-sheath resistance is nearly 10 ohms. The possible fault current is in the order of 3000 amperes. Duly considering the return distribution of fault current, calculations and tests indicate a probable maximum potential difference of 8000 volts.

It should be remarked that the frequency of our troubles is attributed to the use at No. 10 of a common ground for the power neutral and the station apparatus (the neutrals at Nos. 9 and 8 are not grounded), so that a 44,000-volt fault anywhere on the system causes a difference in potential between the station ground at No. 10 and those at Nos. 9 and 8. This, of course, occurs despite the fact that the power neutral is grounded at two other locations. To remedy this condition, a separate ground for the power neutral has recently been installed at a considerable distance from the station ground, as a result of which the station ground, in large measure, is freed from variations in potential due to fault currents in the neutral. When again in operation, the supervisory cable will thus be undisturbed by faults other than those at one of the three stations in question.

Among the numerous remedies considered may be mentioned:

a. The adoption of a-c. supervisory operation with transformers at each end of the cable completely isolating the conductors from the station ground, or from any apparatus connected thereto.

b. The use of a stabilizing ground conductor of sufficient admittance to limit the possible voltage difference between the three station grounds to an amount which would not prove injurious to the cable.

c. The use of transformers, the primary of which would be connected between the cable sheath and the station ground, and the secondary of which would consist of all the other conductors of the cable wound in close coupling and of such ratio (approximately 1:1) as to cause the conductors to follow approximately the same potential gradient as the sheath.

In addition to the foregoing, the idea of installing a highly insulated cable has been advanced, but this would not help matters since the supervisory equipment, which, in itself, will not withstand high potentials, would be still subjected to the severe stresses and cannot, according to the manufacturer, be protected against trouble of the nature described without blowing fuses which render it inoperative until an attendant makes replacements.

Of the three schemes mentioned, (a), which was developed independently by the Commission and the manufacturer, appears to have merit, but it is expensive and involves the use of additional equipment in the automatic train, and the consequent additional care and maintenance and risk of failure is objectionable. Plan (b) is also expensive, and lacking in some of the merits of (a), although free from its shortcomings. On the whole, (c), proposed by R. W. Osborne, telephone engineer of the Commission, seems to offer the greatest return for the investment, and the greatest promise of satisfactory ultimate operation. Preliminary tests of its behavior under transient disturbances are now pending.

It is interesting to note the manufacturer's contention that conditions exist in this installation which have not been encountered before, yet we are all agreed that the trouble is due to differences in potential between the ground conductors of any two of the stations. So far as I am aware, our ground resistances are not high for isolated plants and are not capable of any considerable improvement consistent with a reasonable outlay which can be relied upon the year round. In fact, the expensive proposition of ground stabilization by means of an interconnecting cable (scheme b) would be cheaper, and, if feasible, far more positive than an attempt to overcome the trouble by further improving the station grounds.

The question of how usual or unusual the conditions are can be decided by any interested engineer who has a general knowledge of the dependable minimum resistances of ground connections, and if he has any particular case in mind and knows the ohmic resistance of the grounds and the maximum fault current, he can calculate the probable resulting potentials in a supervisory installation from causes such as I have described.

In his remarks under the heading "Protection," Mr. Lichtenberg has shown a very open mind and has stated his interest in the experience of operating companies. Some of his views seem to be at variance with the conclusions drawn in the foregoing, and in support of these conclusions I can only add that practical operating experience as well as theory seem to be in agreement—a circumstance which always carries weight.

F. R. McBerty (communicated after adjournment): Mr. Lichtenberg's contribution on the subject of the maintenance of supervisory systems composed of telephone relays (which is in substance a repetition of his remarks before the meeting reported in the JOURNAL of August, 1925, page 879) demands some qualification in the records of the Institute. Mr. Lichtenberg's experience with such relays must have been both limited and unfortunate. Having participated in the design of the telephone relays most extensively used throughout the world and being now the manufacturer of an "all-relay" supervisory system for power equipments, I can perhaps offer definite and accurate information on these matters.

Mr. Lichtenberg is in error in his statement as to the fundamental ideas and assumptions present in the minds of the designers of telephone relays. The designers as a matter of fact did not work under the idea that the relays would be or could be inspected frequently or that some person would be present at one end or the other of a line equipped with them to note their failures and ensure their operation. On the contrary they had in mind that such relays would be used in great numbers, would be operated by persons intent on telephonic communication, would be operated in constantly shifting combinations which would make discovery of individual failure difficult, and would largely determine the reliability of telephone service. The designing and making of relays which should be both sufficiently cheap in cost and sufficiently reliable in service was so well done that there are now scores of millions of them in service throughout the world, of which a large part have been in operation for a quarter century performing thousands of millions of operations per day with a percentage of failure so slight that of all common causes of defect in the operation of telephone systems, the failures of telephone relays form a negligible factor.

Among manual-exchange systems composed of such relays there is no daily inspection, and in most cases not even an annual nor a decennial inspection or test; in machine-switching systems the faults due to relays are a trifling factor in comparison with the faults caused by step-by-step or power-driven moving switch mechanism.

The supervisory system illustrated in Fig. 12 of Mr. Lichtenberg's paper is composed of telephone relays of a type used for performing selection in the automanual city telephone exchange systems and in automatic or dial-controlled private exchanges. As an example, a single unit has performed on test more than fifty million circuit closings without failure or significant change in contact resistance. There are large city exchanges in operation in which the initial selections for all originating telephone calls are made by such relays comprising a total of about sixty thousand relays. some of which have been in continual operation for fifteen years, which are never inspected or tested and which collectively show failures or defects in operation numbering no more than two hundred cases per year in the performance of about five hundred million selective operations. In a particular large exchange the ratio of failures to selections is about one in five million. These relays have never been given routine tests and have had only occasional ones and in some cases, no inspection since installation.

Reference was made by Mr. Lichtenberg, in his paper dealing with the same subject matter reported in the JOURNAL of last August, to the wide range of temperature under which supervisory systems must operate. Relay selection gear fulfills these requirements far better than any other form of selecting mechanism in existence. As an example, there is an unattended remote-controlled all-relay telephone exchange in Ohio, which gives continual local and long-distance service to about 140 subscribers, situated at a distance of sixteen miles from a city and operated over a pair of control wires from the city, housed in a small building without any means of controlling temperature or humidity. The relay selection equipment has developed only one fault in fourteen months, has had only casual inspection without any adjustment or repair whatever in seven months, and has been without even a visit from an inspector for ten weeks during the winter. During this time there have been no cases of interruption of service or of failures in selection traceable to the relay switching equipment.

The conspicuous merits of the relay selection systems composed of telephone relays, as compared with step-by-step, ratchet-driven and with rotary power-driven selective gear, are as follows:

The moving parts of the relays are pivoted armatures and long flexing flat springs only, with small and fixed ranges of movement with ample factor of safety in power working against relatively unchanging frictional resistance. The contacts themselves, when multiplied, are for all practical purposes free from failure. When properly constructed the relays never afterward require adjustment or cleaning. The complications of the selective gear lie in the circuits which, being in the form of soldered and cabled wire net works, are unchanging and permanent.

Such relays could readily be made much heavier and even more ornamental, but so far as I am aware there are no reasons other than personal taste to warrant the added expense. After all, the function of a relay is to move a bit of bronze or platinum against another bit, and telephone relays do that very well.

On the contrary, step-by-step or constantly moving selective mechanism is in all cases dependent upon the operation of relays, subject to whatever failures may characterize relays, with the additional handicap of being itself composed of numerous small moving parts—ratchets, pawls, retractile springs and brushes—among which accurate, minute, mechanical adjustments must be closely maintained under driving forces varying widely and against frictional resistance and inertia also varying widely. Its complications lie largely in nice relations of moving parts and forces. In order to be operative such mechanism universally requires rather closely regulated voltage, must be adjusted, lubricated, and kept free from dust and oxidized metal—conditions which change with use, atmospheric conditions and time.

Chester Lichtenberg: Supervisory systems, as mentioned in the paper, are new. Only a few of them have been in service sufficiently long to obtain experience with them. The discussion presented by Mr. Don Carlos is, therefore, exceedingly valuable since it presents in most thorough fashion an operating problem together with three suggested solutions of it.

The discussions of Messrs. Stewart, Wensley and McBerty present data which supplement that given in the paper and express opinions relative to operation. These opinions are indeed very valuable, since designers of supervisory systems will no doubt examine and take from them such suggestions as will prove of material benefit to the art.

A discussion of supervisory systems cannot be concluded without again making clear the fact that these systems are quite different from remote-control systems and, therefore, their application must be carefully made if they are to be successful.

A Contribution to Research on the Experimental Determination of the Losses in Alternators

BY EDOUARD ROTH[1]
Associate, A. I. E. E.

Synopsis.—*In this paper the author deals with one of the questions with which the International Electrotechnical Commission is at present occupied, namely, the determination by simple tests of the actual losses in alternators. The author first reviews the existing rules and then states the variation of the losses in alternators as a function of the load and as a function of the power factor. His paper contains records of experimental researches which he has made and which serve on the one hand as a basis for discussion of the theoretical considerations which he develops and on the other hand to compare the actual losses with those obtained by the methods given in the rules at present in force. The author does not present definite conclusions since, in his opinion, these investigations ought to form the basis for a program of future researches and ought to serve as the basis for discussions in the Rating Committee of the International Electrotechnical Commission.*

* * * * *

INTRODUCTION

THE separate loss method is still often considered as sufficient for the experimental determination of the efficiency of electric machines. In this method a no-load test at the normal voltage gives the mechanical losses, friction and windage losses, as well as the core or magnetic losses. The electrical losses due to the current in the armature and the excitation circuit are calculated on the basis of the resistances, when hot, of the various circuits and the currents in these circuits. This method gives, of course, good results in the case of small machines and it is not desired to supersede it, but in the case of large machines, and especially in large alternators, it gives absolutely incorrect results. In fact, this method does not take into account either the actual electrical losses, or the fact that the iron losses correspond to an internal voltage higher than that at the terminals, and it totally excludes the phenomena by which the no-load conditions differ from the conditions under load. In short, it neglects the stray load-losses.

Therefore, the modern tendency is to investigate methods by which, with simple tests, the actual losses in the machines may be determined with great accuracy when carrying full load.

The American Institute of Electrical Engineers was the first of the electrical associations in any of the various countries to establish elaborate rules on this subject.

In what follows the excitation loss will not be taken into consideration since this can be determined with sufficient accuracy by simple tests. In fact, rules established by these associations give methods which make it possible to calculate the exciting current on load from the no-load and the short-circuit characteristics. Besides, it is always possible to get the exact value of the excitation loss by a test on load. It is much more difficult to determine the magnetic loss and particularly the electric loss in the armature.

With regard to core loss, the Standards of the A. I. E. E. (1925 revision) read as follows:

"7-468 Core losses.—Drive the machine from an independent motor, the output of which shall be suitably determined. The brushes shall be in contact, and the machine shall be excited, so as to produce at the terminals a voltage corresponding to the calculated internal voltage for the load under consideration. The difference between the output obtained by this test and that obtained by test under paragraph 7-467 shall be taken as the core loss.

The internal voltage of synchronous machines shall be determined by correcting the terminal voltage for the resistance drop only."

To determine the losses on load, these Standards propose to augment the core losses by the stray load-losses. In regard to the stray load-losses they read as follows:

"7-470 Stray Load Losses.—These include iron losses, and eddy-current losses in the copper, due to fluxes varying with load and also to saturation.

Stray load-losses shall be determined by operating the machine on short-circuit and at rated-load current. This, after deducting the windage and friction and $I^2 R$ loss, gives the stray load-loss for polyphase generators and motors."

This method of obtaining the stray load-losses from the short-circuit tests has likewise been adopted in France. However, the following method may also be used alternatively:

> "With the rotor removed and the stator winding carrying normal current at rated frequency, the power input is measured.
>
> This value, after deduction of the $I^2 R$ losses, represents the supplementary losses."

Several comments will be made on these rules; first, it is easy to see that the geometrical addition of the ohmic drop to the normal voltage has no influence whatever on the value of the iron loss to be introduced in the calculation of the efficiency. However, this rule shows that a certain correction should be made to the normal voltage to account for the actual core loss on

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[1] **Engineer-in-Chief, Société Alsacienne de Constructions, Mécaniques, Belfort, France.**

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

load. One of the main objects of this study is to determine the precise amount of this correction.

Then, with regard to losses other than the core loss, that shall be accounted for on load, two questions should be answered; first, do the two tests give the same result and second, in what proportion is this result to the actual losses?

It is well known that in turbo alternators or in general, in high armature reaction alternators, the *short-circuit losses* measured are very much too high; the tests reported hereafter were made to determine whether or not this is the case with salient pole alternators of small size and small armature reaction. It is essential to know the value of the armature reaction in determining the accuracy of the short-circuit test as a method of measuring the stray load-loss. Several authors[2] have already given reasons for this experimental feature in the case of turbo alternators, and it is needless to go into them here. However, these reasons cannot be applied to salient pole alternators; but contrary to what is generally admitted, it seems that the iron loss on short circuit cannot be neglected on account of the well-known shape of the field curve. See Fig. 1 as an example of such curves.

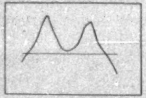

FIG. 1—FIELD CURVE ON SHORT-CIRCUIT

On the contrary, the results of the test with *rotor removed* seem to be nearer actual conditions, even in the turbo alternators. One objection to this test is that it is carried out with a very low power factor which implies a lack of accuracy in the readings of the wattmeters, especially if current transformers are used.

Reviewing the situation, it may be stated that the core loss determined according to the A. I. E. E. Standards is certainly too small, while the electrical losses resulting from the short-circuit test are often too large. However, it may be possible, as already expressed at the Hague, that the sum of these two indications is sufficiently close to the actual value of the losses on load. In the event that this could be proved for a sufficient number of very different machines, the prescribed rules of the Standards would hold good.

We trust that the tests reported hereafter will shed some light on these various questions. These tests have been carried out at the works of the Société Alsacienne de Constructions Mécaniques, at Belfort. These measurements are very delicate, and in order that definite conclusions be reached other manufacturers should make similar tests. We consider the present study more as a basis for discussion and as a program for further investigations.

Definitions. As in the plain separate loss test the losses will still be segregated into mechanical, magnetic and electric. No special comment is necessary with regard to the first; these include the sum P_0 of the friction losses in the bearings, the friction losses of the brushes, etc., as well as the windage and the ventilation losses. By *magnetic* losses is meant those due to flux. These are developed principally in the active iron; however, they can develop elsewhere. They include, for example, the losses at the surface of the pole pieces, those in the fastening bolts and in the end-bells or end-plates, where they are generated by the leakage flux of the inductor. They vary with the load and are in consequence a function of both the current I and the voltage U. By *electric* losses are meant the losses proportional to the square of the current. They appear not only in the copper of the armature but in every part of the machine where currents are induced by the main current, such as end-bells, end-plates, dampers, etc.

Neglecting, as stated above, the excitation loss in the machine, the total losses, P, can be represented by the following equation:

$$P = P_0 + f(U, I) + a\,I^2 \qquad (1)$$

The above definitions do not mention the stray load losses as these are included either in the magnetic or electric losses.

Electric losses. The coefficient a in the expression $a\,I^2$ is equal to the resistance r of the winding, multiplied by a factor k greater than unity.

$$a = k\,r$$

It should be noted here that the electric losses are not necessarily proportional to the square of the current. When the solid parts subjected to the induced currents become saturated by flux, it may happen that some of these terms vary according to another law. But, as stated above, we will neglect such variation and assume the law of the square.

In what follows we will assume that for a given temperature of the machine and consequently for a well determined value, r, of the resistance, the value of k is independent of the value of the current and of its phase angle.

Variation of the magnetic loss with the load. Curve I of Fig. 2 represents the no-load losses of an alternator running as a synchronous motor at no-load, with minimum current. This curve we shall call the "no-load loss characteristic." It is desired to find on this curve the value of the virtual internal voltage corresponding to the magnetic loss on load, for a given current and a given phase angle, which voltage we will call the "magnetic loss voltage."

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[2] S. F. Barclay, "The Mechanical Design and Specification of the Turbo Alternator Rotor." *Jour.* I. E. E., 1919, p. 483.

E. Roth, "Les pertes supplémentairesdans les machines électriques." *Bulletin de la Société Francaise des Electriciens*, 1923, p. 497.

E. Roth, "Les alternateurs de 40,000-kw. construits par la Société Alsacienne de Constructions Mécaniques pour la Centrale de Gennevilliers de l'Uniond 'Electricité." *Revue Générale de l'Electricité*, 24 Février 1922, XIII, p. 311.

Let $O\,A$ (Fig. 3) represent the normal voltage U_0 at the terminals, or else the flux in the machines; add geometrically to $O\,A$ the ohmic drop $A\,B = k\,r\,I$, also the inductive drop $D\,B = x\,I$, due to the leakage flux, to obtain the total electromotive force $O\,D$ (or $O\,D'$ in the case of a leading angle of phase when running as a generator).

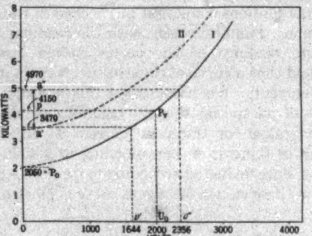

FIG. 2—CHARACTERISTICS OF CORE LOSSES

It would be very simple, as it has been proposed, to adopt for the value of the magnetic loss voltage that corresponding to the vector $O\,D$, as the inductive drop $B\,D$ due to leakage can be measured with sufficient accuracy without difficulty.[3]

This method has already been proposed at the Hague and it may be possible that the results of the studies now in progress will lead to its insertion in the rules.

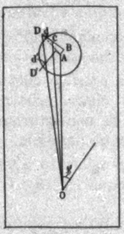

FIG. 3

However, there are several reasons why the voltage $O\,D$ does not represent the magnetic loss voltage.

First, the inductive drop $B\,D$ due to the leakage is composed of two parts: the one $B\,C$ corresponding to the leakage in the end turns of the coils, the other $C\,D$, due to the leakage in the slots. But the flux in the core is only $O\,C$, so that taking the voltage $O\,D$ as the magnetic loss voltage will, therefore, give too high values.

Further, the leakage flux in the slots tends to saturate the teeth which separate the slots belonging to two different phases when the alternator runs on a capacitive load, and tends to weaken the flux in the teeth when it runs on an inductive load, which increases the losses in the first case and reduces them in the second.[4]

Obviously, it may be inferred that it would not be correct to add vectorially the total inductive drop $B\,D$ to obtain the magnetic loss voltage; it seems that a virtual inductive drop $x_p\,I = D\,d$ should be added, (less than $B\,D$), to obtain this voltage. Running with a lagging phase angle, the losses would be smaller than those corresponding to the inductive drop $O\,D$; while with a leading angle of phase they would be higher since $O\,d'$ is greater, in that case, than $O\,D'$.

But a very important experimental fact invalidates these conclusions in great proportion. It is known that the field curve along the surface of the poles of salient-pole alternators varies considerably with the phase angle of the current.

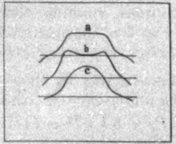

FIG. 4—FIELD CURVES OF ALTERNATOR
a. No load b. Lagging c. Leading

This curve inflects with lagging armature current (field over-excited) and swells, becoming nearly a sine wave with leading current (field under-excited).

Fig. 4 is an example of such curves. Fig. 4A represents a field curve at no-load and Figs. 4B and 4C correspond to lagging and leading phase angles respectively. But the curves of the type of Fig. 4B give higher losses than those of the shape of Fig. 4C; this is proved theoretically and confirmed by tests. Thus the points d and d' draw nearer again to D and D^1. See Fig. 3. It is proposed to determine precisely, by suitable tests, whether, and by how much, the virtual inductive drop $B\,d = x_p\,I$, which we call the "inductive drop of loss," differs from the inductive drop of leakage $B\,D$.

Owing to the secondary phenomena we have just mentioned there is nothing to indicate that the reactance of loss, x_p, is independent of the phase angle for a given current. However, in the following developments we will assume that the reactance x_p is constant. On this assumption, the current being constant, when the phase angle varies, the point d describes a circle around A

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[3] E. Roth, "Critique de quelques méthodes de mesure de la resistance due aux fuites de l'induit des machines électriques rotatives a courants alternatifs." *Revue Générale de l'Electricité* du 7 Juin 1925, XVII, p. 217.

[4] E. Roth, "Les pertes supplementaires dans les machines electriques," *Bulletin de la Société Francaise des Electriciens*, 1925, p. 495.

as a center, with $A\,d$ as a radius. This assumption as well as that of the constancy of the factor k shall be justified by tests.

Determination of the constants k and x_p from measurements of the losses, the machine being run as a synchronous motor at no-load, with variable current:[5] It is possible to run an alternator approximating normal conditions at no load as a synchronous motor, either over-excited or under-excited at nearly zero power factor, the current being the same in each case. The power input in these two tests represents the losses, and will not only give the reactance of loss x_p and consequently the magnetic loss, but also the electric losses and, therefore, the factor k.

The conclusions that can be drawn from these tests amply justify the assumptions, for these are the limiting values of the secondary phenomena and particularly of the variation of the shape of the field curve.

In fact, for the over-excited conditions, the total losses P'' may be expressed for a three-phase alternator by

$$P'' = P_0 + 3\,k\,r\,I^2 + f\,(U'') \tag{2}$$

r being the resistance per phase. U'' is the loss voltage corresponding to these running conditions (Fig. 2) and as shown in Fig. 5 is approximately:

$$U'' = O\,N'' = U_0 + x_p\,I\,\sqrt{3} \tag{3}$$

x_p being the reactance of loss per phase.

Similarly, in the under-excited conditions:

$$P' = P_0 + 3\,k\,r\,I^2 + f\,(U') \tag{4}$$

where

$$U' = O\,N' = U_0 - x_p\,I\,\sqrt{3} \tag{3 a}$$

Subtracting (**4**) from (**2**)

$$\begin{aligned} p &= P'' - P' = f\,(U'') - f\,(U') \\ &= a'' - a' \text{ taken on Fig. 2.} \end{aligned} \tag{5}$$

The question is then to obtain by successive approximations, which are easily made, two values of U' and U'', symmetrical with regard to U_0, and such that the difference of the losses measured on the loss characteristic (Fig. 2) is equal to p.

Then:

$$x_p = \frac{U'' - U_0}{I\,\sqrt{3}} = \frac{U_0 - U'}{I\,\sqrt{3}} \tag{6}$$

Substituting this value of x_p in equation (**3**) gives U'', and this, when substituted inequation (**2**), determines $K\,r$. Now, since r is known, K is obtained.

Knowing the values of k and x_p, the loss at any load may be calculated. The magnetic loss voltage $O\,M$ for any running condition may be determined from the diagram of Fig. 5; and knowing this, the magnetic losses may be obtained from the loss curve, to which the $k\,r\,I^2$ losses are to be added and also the mechanical losses P_0.

It is to be noted that this method, as in the case with the rotor removed, involves the disadvantage of being carried out at a low power factor. However, with certain precautions, this method can be carried out successfully, as the following tests indicate. If wattmeters gave correct readings at low power factor, this method could be extended to the calculation of the losses in any running condition, provided, of course, that the above assumptions be justified by tests.

Study of the curve of the losses as a function of current when running as a synchronous motor at no-load. The above assumptions could first be checked in the following manner: Simultaneously with the readings for the V curve, readings of the corresponding losses are taken, and then a curve of these losses is drawn in terms of the current. We have called this a comet curve because of its special shape. Readings of the losses are then taken for the same current I_1 on the two branches of the curve, corresponding to which, values of x_p and k are deducted according to the above method and from these values the comet curve is plotted. The coincidence of the calculated curve and that obtained from test will justify the assumptions.

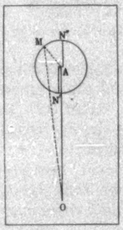

FIG. 5

This calculation is possible only when the equation of the no-load loss characteristic is known. It is particularly simple in non-saturated machines where this curve is a quadratic parabola.

In this case it is possible to avoid the cut-and-try method by solving equation (**5**); $f\,(U)$ is then of the form, $\dfrac{U^2}{\rho}$. The analytical solution is given by the equation:

$$p = \frac{U''^2}{\rho} - \frac{U'^2}{\rho} \tag{7}$$

since:

$$(U''^2 - U'^2) = (U'' - U')\,(U'' + U')$$

$$U' + U'' = 2\,U_0 \tag{8a}$$

$$U'' - U' = 2\,x_p\,I_1\,\sqrt{3} \tag{8b}$$

Thence:

$$p = \frac{(U'' + U')\,(U'' - U')}{\rho} = \frac{4\,U_0\,x_p\,I_1\,\sqrt{3}}{\rho}$$

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[5] In other words, readings of the power input are taken when the alternator is run as a synchronous motor for the V-curve test.

which gives:

$$x_p = \frac{\mathrm{p}\,\rho}{4\,U_0\,I_1\,\sqrt{3}} \tag{9}$$

ρ can then be determined by setting up the following expression for the losses corresponding to the voltage U_0—see Fig. 2.

$$\frac{U_0^2}{\rho} = P_v - P_0 \text{ and } \rho = \frac{U_0^2}{P_v - P_0}$$

k is then deducted and thus it is possible to calculate the electric losses and to plot the comet curve for the various values of the current.

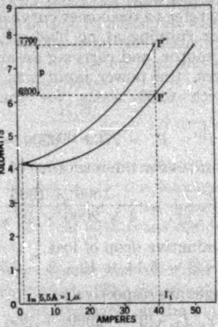

Fig. 6—Comet Curve

Were it possible to assume the no-load loss characteristic to be a parabola of the second order, the above mentioned method could be employed to give a first approximation of the value of x_p, later to be revised by a cut-and-try process.

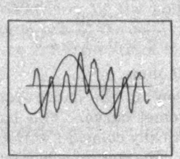

Fig. 7

Formulas (8a) and (8b), and the conclusions drawn therefrom, have been based on the assumption of zero power factor. This condition is fulfilled even for small values of current, and it is only for currents near the minimum of the V curve that the method indicated fails.

If it is assumed that the no-load loss curve is a quadratic parabola, it is possible to calculate the comet curve for the exact value of the angle of phase; however, to give this study a practical instead of a theoretical aspect, we will not present this development here. The curve of Fig. 6 has thus been calculated, and for the current I, the corresponding losses are P'' and P'.

Even when variable power factor is taken into account near minimum current, the points of the comet curve do not correspond with those of the curve obtained from the tests. It is known that the wave form of minimum current of a synchronous motor is very distorted (Fig. 7). The minimum current read will be higher than that corresponding to the voltage and the power. This explains why the test results give the dotted part of the comet curve shown in Fig. 6.

The comet curve has a peculiar characteristic which may appear paradoxical at first. This feature is of no practical interest. However, it may confirm our hypothesis. It will be noticed on Fig. 6 that the minimum loss does not correspond to the minimum current. The losses decrease in a certain part of the curve when the current increases. This minimum loss occurs when the conditions are such that the increase of the electrical losses $k\,r\,I^2$ is less important than the corresponding decrease of the magnetic loss. Hence, this minimum occurs for a certain ratio between x_p and $k\,r$.

The existence of this minimum is easily explained analytically in the simple case of the quadratic parabola. Since this minimum can only occur in under-excited conditions, formula (4) reads as follows:

$$P' = P_0 + k\,r\,I^2 + \frac{(U_0 - x_p\,I\,\sqrt{3})^2}{\rho}$$

which gives:

$$p' = \left(P_0 + \frac{U_0^2}{\rho}\right) - \frac{2\,x_p\,U_0\,\sqrt{3}}{\rho}\,I + 3\left(\frac{x_p^2}{\rho} + k\,r\right) I^2$$

as the equation of the lower branch of the comet curve, on the assumption of zero power factor, which as stated, is permissible, even for small values of current.

This parabola gives a minimum corresponding to a current:

$$I_u = \frac{U_0}{\sqrt{3}}\,\frac{x_p}{(x_p^2 + k\,r\,\rho)} \tag{12}$$

and a power:

$$P_u = \left(P_0 + \frac{U_0^2}{\rho}\right) - \left\{\frac{U_0^2}{\rho}\right\}\left\{\frac{x_p}{x_p^2 + k\,r\,\rho}\right\} \tag{13}$$

$$P_u = \left(P_0 + \frac{U_0^2}{\rho}\right) - \frac{x_p}{\rho}\,U_0\,I_u\,\sqrt{3}$$

which, remembering that the term $P_0 + \dfrac{U_0^2}{\rho}$ represents the no-load loss P_v at the pressure U_0, therefore, can be written

$$P_u = P_v - \frac{x_p}{\rho}\,U_0\,I_u\,\sqrt{3} \tag{13a}$$

This minimum power has no physical existence unless the value of the minimum current I_m taken by the motor be less than the current I_u corresponding to the minimum of the losses

$$I_m < I_u \qquad (14)$$

It can be shown that this corresponds to the condition

$$\frac{P_v}{P_v - P_0}\, k\, r < x_p \qquad (15)$$

or

$$\frac{\text{Total no-load losses}}{\text{No-load core losses}}\, k\, r < x_p$$

Experimental researches. A series of tests has been carried out on a 320-kv-a., 3000-volt, 61.6-ampere, 50-cycle, 26-pole synchronous motor. The results of these tests have been translated into the curves of Figs. 1, 2, 4, 6 and 8. The comet curve of Fig. 6 and the field curve of Fig. 4 have not been obtained at the normal voltage of 3000 but at 2000 volts, as the no-load loss characteristic at the latter voltage very nearly follows the law of the square, and also because the differences in the field curves are more pronounced when the alternator is not saturated.

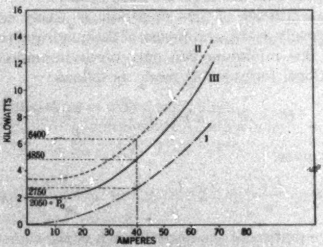

FIG. 8—CHARACTERISTICS OF ELECTRICAL LOSSES

I—Losses, measured, rotor removed

II—Short circuit losses

As already explained, this characteristic has been obtained from the value of the power input with the machine running as a synchronous motor, at no-load and at the minimum current. The motor became unstable below 620 volts. Therefore, the dotted part of the curve had to be assumed in order to determine the mechanical losses; $P_0 = 2050$ watts. The total losses P_v on no-load at $U_0 = 2000$ volts are 4150 watts, which gives as the iron loss $4150 - 2050 = 2100$ watts.

ρ will be obtained from

$$2100 = \frac{2000^2}{\rho}$$

thence, $\rho = 1905$ ohms.

The iron losses at 2356 volts and 1644 volts are:

$4970 - 2050 = 2920$ watts and

$3470 - 2050 = 1420$ watts, respectively.

In order to determine whether the law of the square is satisfied, at 2356 volts we find

$$\frac{2356^2}{1905} = 2905 \text{ watts against } 2920 \text{ watts;}$$

and at 1644 volts

$$\frac{1644^2}{1905} = 1418 \text{ watts against } 1420 \text{ watts}$$

These results are practically identical. At 3000 volts, on the contrary, the curve considerably deviates from the quadratic parabola.

Let us now refer to the comet curve of Fig. 6. With the alternator running at no load as an over-excited synchronous motor, and carrying an armature current of 40 amperes, the power input was 7700 watts as compared with 6200 watts for the under-excited conditions.

Then $p = P'' - P' = 7700 - 6200 = 1500$ watts

Formula 9 gives for the reactance of loss,

$$x_p = \frac{p\,\rho}{4\,U_0\,I_1\sqrt{3}} = \frac{1500 \times 1905}{4 \times 2000 \times 40\sqrt{3}} = 5.14 \text{ ohms}$$

and for the inductive drop of loss,

$$x_p\,I_1\sqrt{3} = 5.14 \times 40\sqrt{3} = 356 \text{ volts}$$

The loss voltages are respectively:

$$U'' = U_0 + x_p\,I_1\sqrt{3} = 2000 + 356 = 2356 \text{ volts}$$

$$U' = U_0 - x_p\,I_1\sqrt{3} = 2000 - 356 = 1644 \text{ volts}$$

Equation (**2**),

$P'' = P_0 + f(U'') + 3\,k\,r\,I^2$ becomes

$$7700 = 4970 + 3\,k\,r\,I^2$$

whence $3\,k\,r\,I^2 = 2730$ watts.

The resistance r per phase is

$$r = 0.52 \text{ ohms}$$

which gives

$$k = \frac{2730}{3 \times 0.52 \times 40^2} = 1.094$$

or $k\,r = 0.569$ ohms.

Thus, we know all the data required for the calculation of the comet curve. This has been traced on Fig. 6 and practically coincides with the curve obtained from the tests. The coincidence is particularly marked for the minimum P_u of the losses. The existence of this minimum is evidenced by the verification of the following inequality:

$$\frac{\text{Total no-load losses}}{\text{No-load iron losses}}\, k\, r < x_p$$

which gives in our case;

$$\frac{4150}{2100} \times 0.569 < 5.14$$

$$1.13 < 5.14$$

The corresponding current I_u for minimum input is, according to (**12**),

$$I_u = \frac{U_0}{\sqrt{3}} \times \frac{x_p}{x_p^2 + k\,r\,\rho} = \frac{2000}{\sqrt{3}} \times \frac{5.14}{5.14^2 + 0.569 \times 1905}$$

I_u = 5.35 amperes.

The minimum power P_u is, according to (**13a**),

$$P_u = P_v - \frac{x_p}{\rho} U_0 I \sqrt{3}$$

$$P_u = 4150 - \frac{5.14}{1905} 2000 \times 5.35 \sqrt{3}$$

$$P_u = 4100 \text{ watts}$$

this being 50 watts less than the total no-load losses.

The minimum current I_m measured was 2.1 amperes. This high value results from the distortion of the current wave (Fig. 7). If the current wave were a sine, the actual current should be:

$$\frac{4150}{2000\sqrt{3}} = 1.2 \text{ amperes}$$

The problem is now to determine, first, the ratio of the reactance of loss x_p to the leakage reactance x and then the ratio of the actual losses to the losses given, either by the test with rotor removed or by the short-circuit test.

The ratio of reactances was obtained by a method proposed by the writer[6]. The rotor being removed, a rectangular coil was arranged on the stator core, two sides being parallel with the axis of the machines and the two other sides running along the ends of the core. This coil embraced one pole pitch and consisted of one or more wires.

The stator winding was then supplied with a current I_s of frequency f and readings were taken of the impressed voltage U_s, and also of the voltage u_s across the ends of the rectangular coil.

Let v be the number of turns of the coil, r_s and r_v respectively the resistances of the coil and the voltmeter across the terminals of the coil. The flux across the cylindrical space inside the stator is:

$$\Phi_s = \frac{u_s \times 10^8}{4.44\, f\, v} \left(1 + \frac{r_s}{r_v}\right) \text{Maxwell's} \qquad (\mathbf{16})$$

corresponding to a pressure at the terminals of a three-phase machine of

$$U_s = 2.13 \sqrt{3}\, f\, N\, \Phi_s\, 10^{-8} \text{ volts} \qquad (\mathbf{17})$$

N being the number of conductors in series per phase.

The reactance x due to the leakage of the stator is then:

$$x = \frac{U_s - U_s}{I_s \sqrt{3}} \qquad (\mathbf{18})$$

For a current I_s = 67 amperes, the following voltages have been obtained,

$$U_s = 861 \text{ volts}, \quad u_s = 0.441 \text{ volts}$$

at f = 50 cycles per second.

The coil was made up of but one wire making $v = 1$; r_s and r_v were

$$r_v = 4.7 \text{ ohms}, \; r_s = 0.17 \text{ ohms}$$

corresponding to

$$\frac{r_s}{r_v} = 0.036$$

Hence,

$$\Phi_s = \frac{0.441 \times 1.036 \times 10^8}{4.44 \times 50} = 0.207 \times 10^6$$

and for N = 728 conductors per phase,

$$U_s = 2.13 \sqrt{3} \times 50 \times 728 \times 0.207 \times 10^{-2} = 278 \text{ volts}$$

Therefore,

$$x = \frac{861 - 278}{67\sqrt{3}} = 5.02 \text{ ohms}$$

which corresponds nearly to the value of 5.14 ohms found for x_p.

In this case it is permissible to take as the value of the inductive drop of leakage of the stator that of the inductive drop of loss.

While the above test was made at a variable current, readings of the stator losses were taken, (Curve *I*, Fig. 8) giving, for 40 amperes, a loss of 2750 watts, which is practically equal to the above loss,

$$3\,k\,r\,I^2 = 2730 \text{ watts}$$

The losses on short circuit were measured as follows: The machine was driven by a direct-current motor the power input of which was measured with the alternator running on short circuit. Curve *II*, Fig. 8, gives the input to this motor in terms of the current in the alternator.

For the calibration of the motor the alternator was run on no-load, at a variable voltage, the input to the d-c. motor also being measured. Curve *II*, Fig. 2, has thus been obtained. As Curve *I* on the same figure represents the actual power input to the machine under test, it is possible to obtain the exact losses of the direct-current motor. These are measured by the difference of the ordinates of the two curves.

It is thus possible to determine the curve of the short-circuit losses of the machine as shown by Curve *III*, Fig. 8. For a current of 40 amperes, the total losses on short-circuit are 4850 watts, of which P_0 = 2050 watts, should be subtracted, leaving

$$4850 - 2050 = 2800 \text{ watts}$$

This does not differ greatly from the 2730 watts found above for $3\,k\,r\,I^2$.

Therefore, according to the prescribed rules, the no-load loss P_v, at the normal pressure U_0, should be added to the losses on short-circuit. This gives a total value of

$$4150 + 2800 = 6950 \text{ watts}$$

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[6] **Edouard Roth**, *Revue Générale de l'Electricité*, 7 February, XVII, p. 217.

as compared with the values 6200 and 6700 watts obtained when the machine is run at the same current as a synchronous motor, either over- or under-excited.

It should be noted that the method of the comet curve, the method with rotor removed and the short-circuit test give the same result for the electric losses. Further, we have seen that the inductive drop of leakage represents the voltage drop of loss.

NOTE: The tests which have been carried out are relatively simple, as they only involve running the machine at no load. The conditions under which they were made do not enable us to determine the losses that may exist on load at power factors other than zero. Presently another series of tests will be carried out that will enable us to determine the value of these losses for full load and to compare them with those found by the separate loss method.

As already mentioned in this article, it is desirable that other manufacturers make similar tests before definite conclusions are established.

Discussion

W. F. Dawson: In an article published in 1920[1] I called attention to the fact that in many cases load losses indicated during short-circuit test were greatly reduced when the machine was run at full voltage and rated power factor.

I have since run many other tests and while I am not prepared to lay down any definite rule, I am prepared to suggest that the present A. I. E. E. rule (paragraph 470, No. 7) which charges all of the load losses measured during short circuit against the losses of the machine, is probably unfair. Since the publication of my article in 1920, improved methods have been developed. At that time I intimated that electric heaters delivering a known quantity of energy into the discharge duct could be employed as a calibrating device. I had tried them in a rather amateurish way. Since then we have perfected our electrical air heaters and we have also perfected resistance thermometers for measuring the inlet air, the outlet air, and the temperature of the air after it has passed the electric heater. It is now a comparatively simple matter to measure the actual losses of a machine having an enclosed ventilating system, such as turbine alternators, under any condition of load.

We have satisfied ourselves that the question of convection, at least on high-speed machinery, introduces no appreciable error, even though we have not been able to establish an exact emissivity constant. This may possibly vary from 0.0125 watts per sq. in. per deg. cent. rise of the frame surface to 0.007 watts per sq. in. per deg. cent. rise; but when it is realized that even with the higher figure the convection loss amounts to less than two per cent of the total loss on an alternator as small as 3750 kv-a. and the total loss in turn is only 4.2 per cent of the rating (on 3000 kw.), the total loss due to convection will be only about 0.08 per cent of the rating.

It has been my pleasure, during the last year, to build and test a turbine alternator rated three-phase, 480 volts, 7518 amperes, 0.8 power factor. Check tests at full load, 0.8 power factor, were made on the customer's premises a few weeks ago. The factory test consisted of a zero-field (no excitation), open-circuit heat run at normal voltage and a short-circuit heat run at normal current, the calorific method being used to estimate the losses. The aggregate of these losses, assuming the same losses under operating condition as during short-circuit test, was 279 kw. On the customer's premises a check was made under contract conditions, except that the phases were slightly unbalanced; the average armature current was slightly over the rating and the armature voltage slightly less. The normal rating of 5000 kw. was maintained accurately, the field current held constant and the power factor, within one per cent of the 0.8 guaranteed.

What was the result? The generator was equipped with an air cooler. We installed flow meters to measure the quantity of cooling water, thermometer wells, and calibrated mercury thermometers reading to within 0.1 deg. fahr. We also installed numerous thermometers for measuring the temperature of the (inlet and outlet) air. Assuming that the factory measurement of quantity of ventilating air was still correct value, total losses by temperature gain of the ventilating air were 266 kw., while for the temperature gain of the cooling water, the value indicated was 270 kw., a check within 1.5 per cent. An average between the 270 kw. and the 266 kw. (between the air and the water method) gave 268 kw., which was 11 kw. less than the aggregate shown by the factory measurement.

B. L. Barns: What I have to offer has reference to the general subject of the determination of losses in alternators.

We have built in Canada some very large vertical shaft alternators which are installed in the Queenston Power House of the Hydro-Electric Power Commission. In making the acceptance tests it was not possible to determine the load losses by any of the methods described in the A. I. E. E. rules, and for the efficiency results an assumption was made regarding the value of these losses. Naturally in view of the size of these generators we were curious to know just what was the true value of these losses. In reviewing the work that had been done it was noticed that in 1913 Mr. H. M. Hobart had suggested what he called the calorimeter method. Later, in 1920, Mr. Dawson described the method which he, himself, has mentioned. Mr. Dawson's method involved heating the air that passes through the machine and under the conditions existing at the Queenston Power House it was quite difficult to take care of this feature. Furthermore it was very difficult to obtain an accurate measurement of the air passing through the generator.

Mr. Hobart's suggestion and Mr. Dawson's experiments offered a clue as to a method that might be used. An accurate determination had been made of the windage and friction and the open-circuit core loss on which we had consistent checks. It was reasoned that since the value of the core loss was known, it would be possible to determine by test the temperature rise of the air passing through the generator due to the core loss; then, having determined this constant, the losses under any other condition of load could be determined by measuring the temperature rise of the air. Thus, if it were found that the temperature rise of the air at full load was twice or three times the rise due to core loss alone it would be reasonable to conclude that the total loss was two or three times the core loss. Accordingly, a series of tests of this nature has been carried out on two of the generators. The tests consisted of measuring the temperature rise of the air under four conditions of load; windage and friction, open-circuit core loss, short-circuit core loss, and full rated load. It was necessary to separate the windage and friction from the open-circuit losses because the losses in the thrust and guide bearings were not taken up by the cooling air but were carried away by the cooling water and circulating oil. The difference in the temperature rise of the cooling air under conditions of open-circuit, normal-voltage core loss and windage and friction represents, therefore, the temperature rise for a known loss which is the core loss and the field copper loss. These results permit the establishment of a constant representing the kilowatts loss per degree rise of the cooling air. Having obtained the value of this constant, other runs at full load or on short circuit may be made and the temperature rise of the air measured and the value of the total losses determined.

This method of test of course involves the measurement by suitable test of the windage and friction and open-circuit core loss so that a known loss may be used for determining the temperature-rise constant. These losses may be determined by running the alternator as a synchronous motor and measuring the input, or by a retardation test. Tests by these two methods were made on one of the Queenston generators and remarkably consistent results were obtained. Our first trial of this method was made in June 1924, but, due to some unforeseen conditions, we were not well satisfied with the results and they were later repeated with quite satisfactory results which were described by the speaker in a paper which was read before the Toronto Section of the A. I. E. E., in February 1925. This method of test is briefly described by Mr. Roth in his paper[2] on the 60,000-kv-a. turbo alternators for Genevilliers.

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[1] *G. E. Review*, Feb. 1920, Page 161.

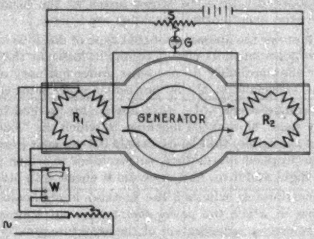
This method of test has certain limitations and obviously it cannot be used on all types of machines. No doubt it could be used with success in the case of most turbo alternators and enclosed alternators, synchronous motors, and condensers. The generator setting at the Queenston Power House is particularly well adapted to this method of test in that it is enclosed in a chamber having walls of hollow tile so that the radiation and convection are reduced to so small an amount that the error would be of a very small order. (A description of these generators and their setting is published in volume XLI, pp. 472-499, of the A. I. E. E. TRANSACTIONS). In carrying out a test of this kind it is essential that the true mean temperature of the air entering and leaving the machine should be obtained and several methods of doing this have been proposed. We have used a large number of thermometers with 0.1-deg. graduations together with an ordinary copper wire resistance grid, and the average readings of the thermometers have checked closely with the grid. I wish to call attention to one necessary precaution in making this test; each run should be continued long enough to obtain a constant temperature of the coils and iron of the machine and at the same time the temperature of the cooling air should be constant because a rising or falling temperature of the cooling air will introduce a considerable error even though a constant temperature rise of the air passing through the machine may be observed for considerable time. These precautions apply to the methods described by Mr. Dawson as well as to the method which I have described.

P. A. Borden: In most of the methods which have been used for the determination of losses by measurement of temperature rise in the cooling air, it would appear that there exists the necessity of either determining the quantity of air supplied to the machine, or of maintaining that quantity at a very steady rate of flow. In the testing of large units, exact measurement of the air flow is usually very difficult, while the maintenance of a uniform rate is seldom practicable.

While the following method is not offered as a panacea for all the troubles attending such a measurement, it would seem that the fact of its not being inherently dependent in its results upon volume or rate of air flow or upon temperature of the air supply would tend to remove many of the objections to the thermal system of loss determination.

Basically, the method, like a number of others, consists of introducing into the inlet and outlet air passages resistance grids having the same temperature coefficient, connected as two arms of a Wheatstone bridge. The bridge is first balanced in what we choose to call the "zero" position. After a steady condition of losses is established, the unbalance of the bridge due to the temperature gradient in the air column is corrected; but the restoring of the balance is not accomplished by manipulation of the circuit of the bridge, but by "artificially" raising the temperature of the cooler grid to meet that of the other. Measurement is then made of the power necessary to be introduced into the cool arm of the bridge to equalize the balance; and this value, expressed in watts, should bear a constant proportionality to the heat energy communicated to the air by the losses in the machine.

The elementary parts of the system are shown in the accompanying sketch, where R_1 and R_2 represent the resistance grids in the air inlet and outlet, respectively. These grids, as units, form two arms of a bridge circuit and are balanced on the galvanometer G by means of the slidewire S. The grid in the incoming air passage is itself made up in the form of a balanced Wheatstone bridge, so that the auxiliary heating current may be passed though it without disturbing the balance of the main bridge circuit. This current is derived from an auxiliary source (preferably alternating), and supplied to equipotential points of the grid R. It will be seen that with this arrangement neither the voltage drop due to the auxiliary current nor the resistance of the auxiliary circuit has any direct effect upon the balance of the main bridge. The other grid, while not necessarily a duplicate, is preferably of a similar construction and of approximately equal resistance. Upon the establishment of a temperature difference between the two grids, with a consequent unbalancing of the bridge, current from the auxiliary source is fed into R_1 and adjusted until the balance is restored. A reading on the wattmeter W of the amount of power thus supplied to the grid then gives a measure of the heat dissipated by the machine under test and communicated to the outgoing air. Calibration of the set-up is accomplished by making one or more determinations with a known value of losses dissipated by the machine, thus establishing the relationship between the measured value of the auxiliary watts and the quantity under investigation.



BALANCE METHOD FOR DETERMINING LOSSES IN TOTALLY ENCLOSED MACHINES

The controlling factor in the acceptability of this method of test would appear to lie in the constancy of ratio between the values of the auxiliary watts and the watts dissipated as losses in the machine; and there has not been as yet (Feb. 1926) an opportunity to study the soundness of the system on an actual power house installation[3]. Laboratory tests have been independently made, however, by the writer and by Mr. Baker of the Ontario Power Company, using set-ups of dissimilar design and following different courses of procedure, but both with a view to establishing the constancy of the ratio under varying conditions, principally of air flow and watts loss.

While at the present time a compilation of the results of these tests is rather premature, it has been possible to arrive at conclusions which would appear to justify further investigation under actual service conditions; and it will be of great interest to learn of others following out a similar application of the principle described.

W. J. Foster: It is interesting to note that the author points out what many of us have been conscious of all along; that our method of determining the total losses of machines is approximate.

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[2] *Three-phase, 66,000-Kv-a., Turbo generators for Gennevilliers,* by E. Roth, A. I. E. E. JOURNAL, September 1925, page 927.

[3] Since presentation of the above discussion this method has been tried out in a large hydroelectric station with very promising results.

I should like to revert to the first attempt made to estimate load losses and speak of the two men whom I think had more to do with the exact formula than any others. I refer to Mr. Lamme and Dr. Steinmetz. They both regarded the determination of losses under short circuit as giving excessive values for load losses due to the very abnormal conditions the worst possible distortion of the flux existing. Consequently, they made an estimate of one-third of the short-circuit load losses as the load losses.

It soon became evident to all designers that that was not sufficient. On the occasion referred to by two or three of the speakers, when so many papers were prepared, determinations had been made by input and output method on a number of machines, and it was found that on the machines selected, the addition of the entire short-circuit losses to the other segregated losses gave efficiencies that agreed very closely with the over-all efficiency. That is how it happened to be agreed upon.

Mr. Roth speaks of the method, I think, as applying to turbo generators. We are all aware that there is a difference in the type of generator. Turbo generators have almost invariably distributed field windings; that is, the field windings are in several slots, with practically uniform air-gap, and with the end windings secured by binding bands usually containing magnetic material. These bands are at almost the same radius, standing out in the line of the air-gap. Thus the determination of short-circuit losses undoubtedly involves larger losses at the heads of the rotor than exist under normal operating conditions.

In like manner the operation of that type of machine which has only a few poles on short-circuit results in losses at the heads of the stator that probably do not exist under ordinary operating conditions, especially at the 80 per cent power factor condition.

C. J. Fechheimer: Mr. Roth applies his method to a comparatively small, slow-speed machine, in which the "load losses" are almost negligible; that is, $k\,r$ is only 9.4 per cent greater than r. Has he applied his method to machines in which the load losses are high, and if so, how close did it check with other tests?

It is questionable whether the method is applicable to the general case in which the power factor does not approach zero. Usually the cross-magnetization of the armature greatly distorts the flux, with the result that the magnetic inductions in the teeth, and to some extent in the core, are far from uniform. The iron losses may be considerably augmented thereby. It is believed that this statement is especially applicable to the machines described in a recent paper by Mr. Roth[3]. For those machines the armature ampere-turns are higher than are needed to excite the machine for no-load normal voltage.

Mr. Roth assumes the equation of the iron-loss curve to be a quadratic parabola (Fig. 2 in the paper). Inasmuch as when saturation is approached, that loss increases faster than the square of the voltage, so that a simple exponential form of equation can no longer be applied, he uses his method at 2000 volts instead of the normal 3000, in the machine he tested. How does Mr. Roth propose to extend his method so as to make it applicable at the normal voltage of the machine? If he assumes that the value of k is the same at normal voltage as at the reduced voltage, and if he allows for the incremental iron losses due to saturation by extending the tests as plotted in Fig. 2 to normal voltage plus the internal impedance drop added vectorially, it is questionable whether the method is correct. A large number of tests would have to be made to confirm the accuracy of the method.

In 1920 a method of separating the iron loss from the mechanical losses was suggested.[4] That did not require the assumption that the iron loss is proportional to the square of the voltage. The method has frequently been used at the Westinghouse Company, and Mr. Roth may find it useful. The machine is run idle as a synchronous motor with minimum excitation, just as as Mr. Roth does for Fig. 2. The calculations are, however, different.

A considerable part of the discussion has centered upon the thermal method of measuring losses. Many of us have attempted to measure losses that way, but it is rather difficult to obtain accurate results. The check that should be used is to run the machine without mechanically-connected load or driving machine, and measure the input with calibrated wattmeters. Then, if one is assured that the testing equipment is accurate, the load conditions may be fixed. It is expected that the results of tests along that line will be reported in an early paper by one of the Westinghouse engineers.

E. H. Freiburghouse: Mr. Roth points out, just as others have done, that there is considerable difficulty involved in measuring power input to a synchronous motor running light at low-power factor. It is more difficult to measure the power input to a stator of an alternator with the rotor removed. As stated by Mr. Roth, the input to the alternator running as a synchronous motor can be measured quite accurately by determining the rate of flow and temperature rise of the air which cools it. In other words, it is quite possible to measure the entire losses of the alternator at low power factor. The method which he proposes, however, for determining the constants k and p depends upon the assumption that the electric losses of the alternator as a synchronous motor are the same when it is over-excited as when under-excited. I do not believe this assumption is justified since it is well known that the stray losses in the end structure of the stator are very much less when it is over-excited than when it is under-excited. These losses as defined by Mr. Roth are electrical, since they are known to be generated by the induction of the stator winding.

This is the principal criticism which I have to make of the method which he has proposed. It is rather ingenious and obtains results which seem to agree quite closely with the tests which he made upon the three different alternators. He found that in all cases the losses as determined by his method checked within 4.3 per cent with the losses as determined by the Rules of the A. I. E. E. and the French Rule.

Mr. Fechheimer called attention to the function of the voltage which appears in the equation on the sixth page. Of course that has been assumed as a square function; that is, as the square of the voltage. He assumed that and used that part of the no-load loss curve in which the relationship of loss to voltage was a square curve.

Mr. Fechheimer also called attention to the distorted form of Fig. 7 in relation to the loss. That is an insignificant influence because it really is the condition which the author states exists at the minimum input current, which is, of course, an extremely small current as compared with the rated current of that generator.

There is one other point; that is with reference to the cross magnetization. I think he takes account of that in the terminal voltage as a function of power factor. That is, one can determine the voltage which causes the loss by combining the fluxes for terminal voltage and that of the reactance voltage.

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[4] *"A Method for Separating No-Load Losses in Electrical Machinery,"* by Carl J. Fechheimer. A. I. E. E. Transactions, 1920, p. 291.

No-Load Copper Eddy-Current Losses

BY THOMAS SPOONER[1]

Member, A. I. E. E.

***Synopsis.**—One of the factors which is sometimes responsible for very considerable losses in rotating machines is that of eddy-current losses in the copper conductors resulting from slot leakage flux produced by the main flux. These losses occur at no-load as well as under load and should not be confused with the copper losses resulting from the slot leakage flux produced by the load current. These no-load copper losses occur in salient pole machines, both d-c. and a-c., and in induction motors.*

This paper is an attempt to remove the calculation of no-load copper eddy-losses from the field of empiricism where it has previously been and to place it on a firm theoretical foundation. Test results are presented which show that the theoretical formulas which have been developed are correct.

Some of the consequences of this analysis are rather unexpected where the frequencies are sufficiently high to produce large skin effect. For instance, laminating the copper may produce increased or decreased losses, depending upon the conditions.

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INTRODUCTION

THE design of electrical apparatus is becoming so standardized, and the efficiencies so high, that further improvements in performance will probably come largely through attention to details which previously have been considered of somewhat minor importance. This paper considers one of these details which has, perhaps, not received as much attention as its importance under certain conditions would warrant.

When a copper conductor in the slot of a rotating machine carries alternating current there are produced slot leakage fluxes which may induce sufficiently large eddy currents in the conductor copper to result in very appreciably increased losses. This type of loss has been quite adequately dealt with by Field, Gilman, and others. The type of copper eddy-current loss with which this paper deals has a different origin and is the result of slot leakage fluxes caused by the main flux. These losses are often nearly as large at no-load as at full load, and are of importance in connection with both salient pole machines and induction motors.

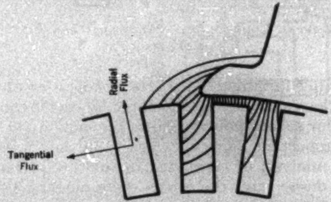

FIG. 1—SLOT LEAKAGE FOR SALIENT POLE MACHINE

CAUSE AND NATURE OF LOSSES

Fig. 1 illustrates the nature of the slot-leakage flux for a salient pole machine. As a slot comes out from under the pole tip there is a rapid decrease in the radial component of slot-leakage flux and a rapid generation and then dying out of the tangential component of slot-leakage flux as shown by Fig. 2, which is from test data obtained on a small d-c. railway motor with unsymmetrical pole tips. It will be seen that the radial leakage flux consists chiefly of the fundamental, while the tangential flux has a larger third harmonic than it has fundamental. Some of the other harmonics for the tangential flux are also quite large. These results are typical.

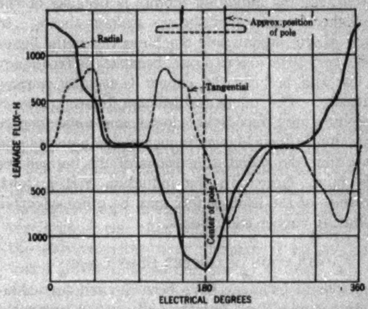

FIG. 2—SLOT LEAKAGE FLUXES FOR D-C. RAILWAY MOTOR

Fig. 3 shows the nature of the slot-leakage fluxes for an induction motor. It will be seen that there are leakage fluxes in the rotor slots having both radial and tangential components. The radial components pulsate in amplitude but do not change sign while under a given pole. The tangential components reverse in sign twice during the interval required for a rotor tooth to

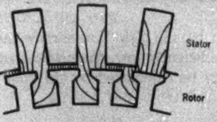

FIG. 3—SLOT LEAKAGE FOR INDUCTION MOTOR

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[1] Research Engineer, Westinghouse Electric & Mfg. Co.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

pass over a distance equal to one stator slot pitch. The slot-leakage fluxes, both tangential and radial, therefore, have a frequency equal to the number of stator teeth per pair of poles, multiplied by the rotor frequency, the latter frequency being in general nearly equal to the applied fundamental frequency. These leakage fluxes are small in magnitude but high in frequency and are often of importance. In the ordinary construction, consisting of nearly closed rotor slots, the pulsations of leakage flux in the stator slots are usually negligible.

These slot-leakage fluxes are appreciable only when the teeth begin to saturate. In salient pole machines, tooth inductions of the order of 120 kilolines per sq. in. are necessary before noticeable eddy losses in the copper conductors begin to be produced. For induction motors the tooth inductions must reach about 70 kilolines per sq. in. before copper eddy losses of noticeable magnitude appear. The reason for the lower inductions for the induction motors is because of the much higher frequency of the slot-leakage fluxes. In the case of the salient pole machines the radial fluxes produce very little loss as a rule, because the frequency is so low and because the copper is usually narrow tangentially. The tangential leakage fluxes, however, may produce very considerable losses, and consequently heating, due to their higher-frequency components and the fact that the radial dimensions of the copper are usually fairly large. In the induction motor, both components of the leakage flux may be quite effective in producing copper eddy losses.

CALCULATION OF COPPER EDDY LOSSES

A number of formulas have been devised for calculating this type of loss, but they all contain empirical constants and all neglect certain essential factors. For instance, one of Mr. Lamme's formulas,[2] which was designed for use with large salient pole machines and was very satisfactory for the particular conditions, gave calculated losses which were from 6 to 10 times the true values when applied to a certain small railway motor. The formulas which will be proposed in this paper are based on fundamental considerations only and were devised without reference to any test data.

If the copper bars in a slot are narrow radially or the frequency is sufficiently low, the no-load eddy losses in the copper may be calculated by the following simple formula:

$$W_e = \frac{\pi^2}{6} \times \frac{W^2}{\rho} f^2 H^2 10^{-7} \qquad (1)$$

(See list of symbols below.)

If, however, the bars are wide (this refers to radial width when considering tangential slot leakage flux) or the frequency high, the above formula will no longer apply, due to the fact that the eddy currents in the copper produce a damping effect on the flux, thus making the losses less than would otherwise occur. This is the well-known skin effect. Under these conditions the flux in the region of the center of the bars is damped out. (See Fig. 4.) The eddy currents are greater in magnitude as the bottom and top of the bars are approached and there are no appreciable currents in the center; that is, if the center portions of the copper were removed there would be no change in the eddy losses.

Some time ago we developed the following formula for calculating eddy losses under the conditions of appreciable skin effect:

$$W_e = \frac{\pi^2 f^2 H^2}{m^3 \rho W 10^7} \frac{\sinh m W - \sin m W}{\cosh m W + \cos m W} \qquad (2)$$

The method of developing this formula was similar to that used by Latour for his formulas dealing with losses in laminated steel at radio frequencies.[3] Dr. J. Slepian assisted in the development of this formula. For no skin effect the results are identical with those obtained by the simple formula (**1**). In the use of either formula, the eddy losses have to be calculated for each frequency; namely, for salient pole machines the magnitude of the various important harmonics has to be determined by test and analysis or estimated in some way from the machine design and the losses corresponding to each harmonic calculated as if the others did not exist. The sum is the total eddy loss. In the case of the induction motor, it is generally satisfactory to consider only the pulsating frequency corresponding to the number of stator teeth, without considering harmonics.

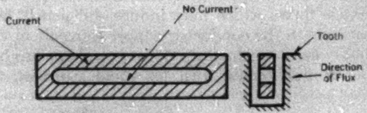

FIG. 4—COPPER EDDY-CURRENT LOSSES, ILLUSTRATING SKIN EFFECT

When it was attempted to apply formula (**2**) to actual machines it was found that in many cases the experimental and calculated results did not correspond as closely as was expected. In order to determine whether the discrepancies were due to inaccuracies in the formula or to the neglect of certain factors having to do with the conditions in the actual machines, the fundamental tests which are about to be described were made.

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[2] Iron Losses in Direct Current Machines. Electrical Engineering Papers by B. G. Lamme, p. 499.

[3] Notes on Losses in Sheet Steel at Radio Frequencies; Marius Latour. Inst. of Radio Engineers *Journal*, February 1919, p. 61.

TEST METHODS AND APPARATUS

The method of test was suggested by a similar method devised by Churcher.[4] As shown by Fig. 5, a laminated yoke was constructed having pole pieces which were adjustable to the right and left in order that the air-gap could be varied. This air-gap was about 6 inches by 2 inches and its width between pole pieces could be adjusted from 7/8 inch to 0. The yoke was provided with windings as shown. High-frequency current up to 1000 cycles was obtained from a generator through an ammeter *A*. In general, the samples consisted of two copper bars as indicated. Two small holes were drilled near the center of one of the bars about 1/8 inch apart and thermocouple wires of copper and advance soldered into the holes. The bars were separated from each other by a small distance and the couple wires were insulated from the bars except at the junction. The wires were brought out from between the bars at one end, thus keeping a considerable length adjacent to the copper in order to prevent loss of heat at the junction. An exploring coil wound on a rectangular bakelite form was provided for determining the flux density in the air-gap. The coil was approximately 6 inches by 1/2 inch and had 21 turns.

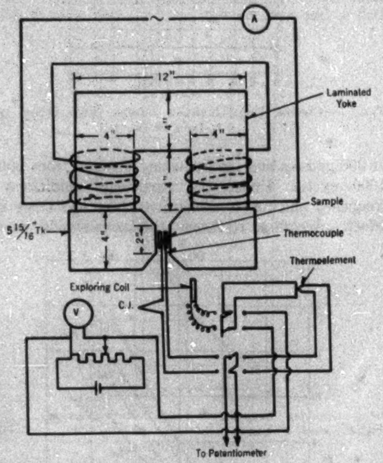

FIG. 5—COPPER EDDY-CURRENT LOSSES—DIAGRAM OF CONNECTIONS

The method of test was as follows: The air-gap was adjusted to the desired width and alternating current of the desired frequency applied to the field winding. Assuming a sine wave of flux in the air-gap, the voltage which would be induced in the exploring coil for the desired induction was calculated. The voltmeter *V* was then adjusted to this value, the current element of a sensitive vacuum thermocouple was connected to the terminals of the voltmeter by a suitable manipulation of the switches and the terminals of the thermo-element connected to a Wolff potentiometer. The e. m. f. of the thermo-element was then read. Next the exploring coil was placed in the air-gap, the switches manipulated, and the magnetizing current adjusted by control of the high-frequency generator field until the thermo-element produced the same reading on the potentiometer as was obtained from the d-c. voltage. This gave the required induction in the air-gap. The thermocouple attached to the sample was then connected to the potentiometer. The cold junction of this couple was placed in a thermos bottle which was at any convenient temperature, preferably near room temperature. A copper sample which should be at or below the temperature in the air-gap was then placed in the gap, the high-frequency current switched on, and the ammeter adjusted to its original value without the sample. The potentiometer was set to some convenient reading corresponding to a value slightly above room temperature. Due to the eddy losses in the sample the temperature immediately began to rise. When this temperature corresponded to the setting on the potentiometer, the potentiometer galvanometer went through *O* and a stop-watch was started. The potentiometer was then set to a higher reading. When the galvanometer went through *O* again the watch was stopped and another started by means of a simple arrangement of levers. This was repeated five or six times in order to obtain sufficient data for plotting a curve between millivolts and time interval. By extrapolating back to the millivolts corresponding to the temperature at the air-gap (with corrections for cold junction temperature) the rate of temperature rise in seconds (ΔT_i) for a given change in millivolts could be obtained. Knowing the millivolts per degree for the thermocouple at this temperature, the rate of temperature rise per degree could be calculated.

The eddy losses were calculated as follows, assuming a temperature of 25 deg. cent.,

$$W = Q \frac{d\,t\,e}{d\,t} \text{ watts}$$

$$Q = 4.186 \times .0917 \times G$$

where

Q is the thermal capacity of the conductor in watt-sec. per deg. cent.,

G is the weight of conductor in grams

$\frac{d\,t\,e}{d\,t}$ is the rate of temperature rise, namely, the deg. cent. rise per sec. on the assumption that there is no heat lost or gained from the outside. Since the rate was determined corresponding to the temperature of the surroundings, there was no correction for dissipated heat; 4.186 is the thermal capacity of water in joules per gram calorie; 0.0917 is the specific heat of copper with reference to water at 25 deg. cent.

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[4] The Measurement of Temperature in a Rotating Armature by Means of Thermocouples, B. G. Churcher. *Journal of Sci. Inst. V. I.*, July, 1924, p. 310.

TEST RESULTS

In order to keep the rate of temperature rise down to a reasonable value, very low air-gap inductions were used; namely, of the order of a few hundred gausses. Under these conditions there was no appreciable heating of the yoke material and the reluctance of the iron path was practically negligible. Therefore, for a given air-gap the induction was practically constant for a definite exciting current for a wide range of frequencies as shown by test results not given here.

If the sample had appreciable skin effect, the counter magnetomotive force of the eddy currents in the copper had the effect of increasing the reluctance of the air-gap. Therefore, when the copper was introduced into the gap the exciting current increased. It was found that if the exploring coil was placed in the air-gap just above the copper, the magnetizing current had to be brought back to its original value in order to bring the air flux in the gap above the copper back to its original value. Provided the current was so adjusted, it was immaterial whether the m. m. f. was measured with or without the sample in position.

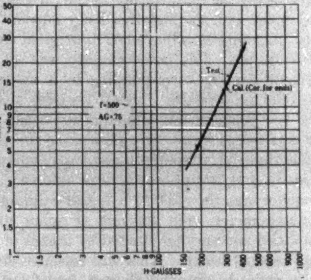


Fig. 6—Copper Eddy-Current Losses, Two Bars ¾-in. x ¼-in. x 10-in.

In order to obtain data for the purpose of determining the validity of our method of calculating eddy losses, a number of copper samples were tested under various conditions. Fig. 6 shows the results of the first set of tests. Two bars ¼ by ¾ by 10 inches long were placed in the air-gap (0.75 inches) with the shortest dimension parallel to the flux. Losses were determined at 500 cycles for three values of m. m. f., namely, 190, 291 and 395 gausses, and the results plotted on double log paper. The m. m. f. and flux density, namely, gilberts per centimeter and gausses are used interchangeably since in the c. g. s. system they are identical. For comparison, values as calculated by formula 2 are shown. The test and calculated values check very well with respect to magnitude and slope. As indicated by theory, the losses increased as H^2.

The next set of tests are shown by Fig. 7. The results are for the same set of bars but in this case H was held at 300 gausses and the frequency changed from 200 to 1000 cycles. Under the particular conditions of frequency and copper dimensions there was large skin effect. According to theory, the losses should, in this case, increase as the square root of the frequency. It will be seen that within the experimental errors this was the case.

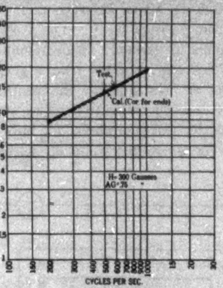


Fig. 7—Copper Eddy-Current Losses, Two Bars ¾-In. x ¼-In. x 10-In.

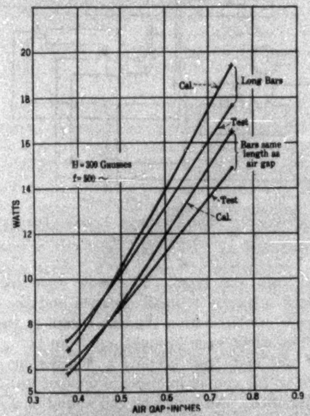


Fig. 8—Copper Eddy-Current Losses, Two Bars 1/8-In. x 1.0-In. x 10-In., Two Bars 1/8-In. x 1.0-In. x 5 15/16-In.

It will be noted that in the case of Figs. 6 and 7 the copper bars were considerably longer than the length of the air-gap. This is, of course, the condition in rotating machines. In several cases tests were made with long bars and then the bars were cut off to the length of the air-gap and again tested in order to determine the amount of the end effect. It was found in all cases tried that if the effective length of the bar for the long bars was taken as equal to the air-gap length plus a length equal to the vertical dimension of each bar (see Fig. 5) the end effects would be very closely compensated for. This explains the notation in Figs. 6 and 7 ("corrected for ends").

Fig. 8 shows the relation between air-gap and loss for an H of 300 gausses and a frequency of 500 cycles. In this case there were two bars, ⅛ by 1 by 10 inches. The results are also given for the same bars cut to the length of the air-gap. The test results follow the theoretical values within reasonable limits, the test values being a little less for the large air-gaps and a little greater for the small air-gaps. Obviously for solid copper in the gap (100 per cent space factor) the gap would be 0.25 inch.

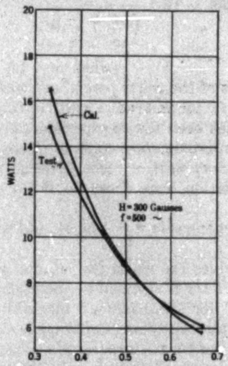


FIG. 9—COPPER EDDY-CURRENT LOSSES, TWO BARS 1/8-IN. x 1/10-IN. x 5 15/16-IN.

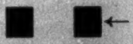
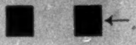
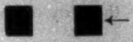
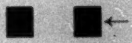
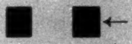
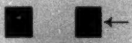
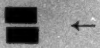
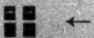
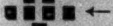
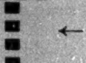
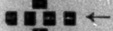
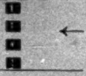
Fig. 9 shows the same data for the short bars only plotted against t/σ (ratio of copper thickness to gap width or the space factor of the copper).

Table I gives some further comparisons between test and calculated results and includes the data of the previously mentioned curves. In most cases the average test values have been converted to a standard m. m. f. of $H = 300$ according to the square law, although the tests were made at other m. m. fs. The dimensions in the table for the copper are approximate values. The actual values were used for the calculations.

DISCUSSION OF RESULTS

Except in two or three cases the test and calculated results are as close as could be reasenably expected. When the space factor is low and the bars are wide, there is a tendency for the calculated results to be high. In the case of the four bars side by side with the larger cross sectional dimension vertical there is a tendency for the calculated values to be low. We may say that for all practical purposes and under the conditions of these experiments the calculated values are sufficiently reliable. All ranges have been covered from the simple case of no skin effect (formula 1) to the case of large skin effect. Moreover, multiple bars have been used and the results check practically as well as for a single pair of bars.

TABLE I

H gausses	f	$A\ G$	Watts Test	Watts Cal.		Remarks
2 Bars ¼ in. by ¾ in. by 10 in. (Approximate Dimensions).						Flux
395	500	0.75	23.8	23.0		
291	500	0.75	12.9	12.5		
190	500	0.75	5.31	5.35		
300	200	0.75	8.5	8.4		
300	500	0.75	13.7	13.3		
300	1000	0.75	18.3	18.8		
2 Bars ¼ in. by ¾ in. by 5-15/16 in.						
300	500	0.75	11.9			
2 Bars 1/8 in. by 1 in. by 10 in.						
					t/σ	
300	500	0.75	17.6	19.4	0.333	
300		0.50	10.33	10.6	0.497	
300		.375	07.33	6.88	0.665	
2 Bars 1/8 in. by 1 in. by 5-15/16 in.						
300	500	0.73	14.82	16.5	0.333	
300		0.50	8.77	9.00	0.497	
300		0.375	6.09	5.85	0.665	
2 Bars 0.299 in. by 0.484 in. by 9.281 in.						
300	500	0.75	10.95	11.75		
300	200	0.75	7.35	7.45		
300	200	.75	4.54	5.10		
4 Bars 1/8 in. by ½ in. by 8 in.						
300	500	0.75	30.1	36.4		
300	500	0.50	20.1	19.8		
300	500	0.75	14.9	12.8		
300	500	0.75	5.26	5.11*		
300	1000	0.75	20.2	18.1		
300	1000	0.75	20.5	19.3		

*5.25 by formula (1)

Unfortunately, in the case of rotating machines, the conditions are not as simple as in the ideal case. In the present tests the effective width of air-gap is the actual width of gap. In the case of rotating machines the corresponding actual width of slot is not the effective width which has to be used in the formula. By referring to Fig. 8 or 9, for instance, it will be seen that the larger the width of the gap the greater the copper loss for a given air-leakage flux, (assuming the conductors have large skin effect). For no skin effect the losses are independent of the gap width. Now in the case of rotating machine teeth the teeth themselves are saturated to some extent, at least, or there would be no appreciable slot leakage fluxes. This means that their permeability is low and that an appreciable m. m. f. is required to force the flux through them. Again the applied m. m. f. which causes the leakage flux to pass across the slot may partly, at least, traverse a second slot and the machine air-gap. This means that due to the higher actual m. m. f. applied over that necessary to cause flux to cross the particular slot in question, the eddy currents in the copper would not be as effective in damping out the flux as would otherwise be the case, and therefore the losses are higher. This means that the value of σ in formula (**2**) should be increased by some more or less uncertain amount.

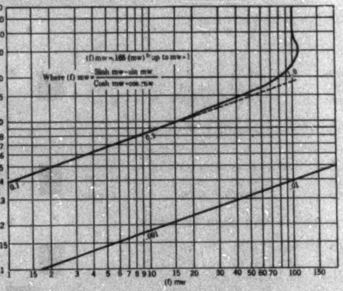


FIG. 10—COPPER EDDY-CURRENT LOSSES, SHOWING HYPERBOLIC FUNCTION IN EXPONENTIAL FORM

In the majority of machines which we have tested, σ should be multiplied by 1.5 on the average, but in some cases it has to be much higher than this and in a few cases lower. Before it would be possible to accurately predict the necessary corrections for all types of machines, much more experimental data must be obtained.

CHARACTERISTICS AND CALCULATION OF EDDY LOSSES

An analysis of the test results and of formulas (**1**) and (**2**) reveal some interesting relations. If the function

$$\frac{\sinh m\,W - \sin m\,W}{\cosh m\,W + \cos m\,W}$$

is plotted against $m\,W$ on double log paper, there is obtained the relation shown by Fig. 10. It will be seen that for any value of $m\,W$ up to 1, the function increases approximately as the cube of $m\,W$. Above 2 the function is nearly equal to 1. The reason that the function reduces to 1 is that for large values the sinh and cosh terms are practically equal and increase very rapidly in magnitude, thus making the sine and cosine values negligible. When this function reduces to 1, the losses are obtained from the first part of formula (**2**), namely

$$W_e = \frac{\pi^2 f^2 H^2}{m^3 \rho\, W\, 10^7} \qquad (\mathbf{3})$$

In order to further simplify this expression, assume that $\rho = 1.8$ by 10^3 (1.8 microhm centimeters corresponds to 31 deg. cent).

$$W_e = 1.688 \times 10^{-7} \frac{f^{1/2} H^2}{\left(\frac{t}{\sigma}\right)^{\frac{3}{2}} W} \qquad (\mathbf{4})$$

(when $\rho = 1.8$ by 10^3)

The variations of the watts per cu. cm. with frequency, air-gap, space factor and radial width of copper are now evident. It has been shown experimentally that these relations are approximately correct. They apply only when the product $m\,W$ exceeds 2 (large skin effect). Using this relation, care must be taken to note the causes of the changes in $\frac{t}{\rho}$ and W and the fact that this formula gives the watts per cu. cm. and not the total watts. If t/σ is changed by increasing the width of gap (or slot for the machine), but with the copper the same, the total eddy losses will vary as $(t/\sigma)^{-3/2}$ for a given H as shown by the formula or as $\sigma^{3/2}$ (slot width). Now if t/σ is altered by changing the total thickness of copper the weight is also changed and therefore the total loss varies at $(t/\sigma)^{-1/2}$, namely, for a given slot the total loss varies inversely as the square root of the tangential copper thickness per slot.

Again suppose the width of the copper (W) is changed by increasing the width of the individual bars; then, since an increase in width means an increased total volume of copper, the losses are independent of the copper width W (for large skin effect). This is obvious from the fact that there are no eddy currents near the middle of the bars (see Fig. 4). If, however, W is changed by laminating but keeping the total weight of copper the same, the total losses are, as shown by the formula, inversely proportional to W; but as the laminating is continued the skin effect is reduced and soon the losses begin to decrease as the square of W or according to formula (**1**). Fig. 11 illustrates such a condition. It will be seen that for the particular conditions the losses increase rapidly with increasing lamination and then decrease. These are calculated values but test results would show the same thing. The main point is that laminating may increase or diminish copper eddy losses depending upon the conditions.

Now in using this formula, m should first be calculated as shown above, namely $m = 2\,\pi \sqrt{\frac{f\,t}{\sigma\,\rho}}$ or

$m = 0.148 \sqrt{f \times \frac{t}{\sigma}}$ for $\rho = 1.8 \times 10^3$. Then if

the product $m\,W$ is less than I, formula (**1**) or some modification may be used, or if desired, formula (**2**) may be used. If $m\,W$ is over 2, or for accurate work over 4, formulas (**3**) or (**4**) may be used. For intermediate values use formula (**2**) or use formulas (**3**) or (**4**) and multiply by $(f)\ m\,W$ as obtained from Fig. 10. Assuming

$\rho = 1.8 \times 10^3$ formula (**1**) becomes

$$W_e = 0.915 \times 10^{-10} f^2\,W^2\,H^2 \qquad \text{(when } \rho = 1.8 \times 10^3\text{)} \qquad (5)$$

and may be used for approximate results up to values of $m\,W$ of 1.5.

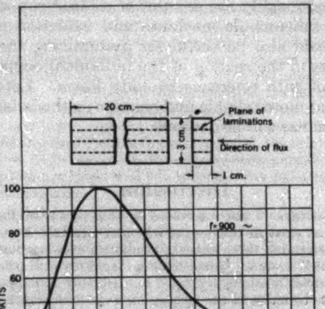


FIG. 11—COPPER EDDY-CURRENT LOSSES—VARIATION OF LOSS DUE TO LAMINATING

APPLICATION OF FORMULAS

As pointed out above, formula (**2**) or its modifications will, in general, give results which are of the right order of magnitude if σ the slot width is increased by 50 per cent before using in the formula. This method of calculation may be used for salient pole machines. In this case the tangential slot-leakage curves must be analyzed into their harmonics and the copper losses calculated for each harmonic. For low frequencies, formula (**1**) will usually apply, but for the higher harmonics some form of formula (**2**) must be used. In the case of wound-rotor induction motors the no-load copper eddy losses may be calculated by assuming a single frequency corresponding to the tooth-pulsation frequency. The radial as well as the tangential slot-leakage fluxes may be of importance in this case.

As a first approximation if test data are not available the radial slot-leakage flux may be assumed to be equal to the m. m. f. necessary to magnetize the teeth adjacent to the slot and the tangential slot-leakage flux to the difference in m. m. f. between the two adjacent teeth. Of course, tooth taper introduces complications but this can be readily taken care of approximately. As pointed out, the chief difficulty is in determining the total effective m. m. f. where skin effect is large. In calculating the total loss the volume of copper is that in the slot plus an amount corresponding to an added length of copper equal to W.

SUMMARY OF FORMULAS

For $m\,W$ less than 1 (approximately correct to 1.5),

$$W_e = \frac{\pi^2}{6} \times \frac{W^2}{\rho} f^2\,H^2\,10^{-7} \qquad (1)$$

$$W_e = 0.915 \times 10^{-10} f^2\,W^2\,H^2 \qquad \text{(when } \rho = 1.8 \times 10^3\text{)} \qquad (5)$$

For $m\,W$ between 1 and 4,

$$W_e = \frac{\pi^2 f^2 H^2}{m^3\,\rho\,W\,10^7}\ \frac{\sinh m\,W - \sin m\,W}{\cosh m\,W + \cos m\,W} \qquad (2)$$

For $m\,W$ greater than 4 (approximately correct for $m\,W$ above 2),

$$W_e = \frac{\pi^2 f^2 H^2}{m^3\,\rho\,W\,10^7} \qquad (3)$$

$$W_e = 1.688 \times 10^{-7} \frac{f^{1/2}\,H^2}{\left(\frac{t}{\sigma}\right)^{\frac{3}{2}} W} \qquad \text{(when } \rho = 1.8 \times 10^3\text{)} \qquad (4)$$

The above formulas, when ρ is included in the constant, are based on copper at 31 deg. cent. For copper at 65 deg. cent. ρ should be taken as 2.0×10^3 instead of 1.8×10^3. For copper or aluminum alloys, use the formulas with ρ.

For cool machines on no-load test, formulas (**4**) or (**5**), in general, will be satisfactory. For hot machines they may be modified, as indicated above but this refinement will usually be found unnecessary due to other much larger uncertainties. Formula (**4**) is to be used when $m\,W$, is greater than 2 and formula (**5**) when $m\,W$ is less than 1.5. For intermediate values of $m\,W$, corrections according to Fig. 10 may be applied or Fig. 10 may serve merely to indicate the possible errors which will result by neglecting this correction.

The above assumes that the effective slot-leakage flux is the tangential component. For the radial component, copper dimensions at right angles must be used.

SYMBOLS

$$m = 2\pi\sqrt{\frac{f\,t}{\rho}}$$

$$= 0.148\,f \times \frac{t}{\sigma} \quad (\text{when } \rho = 1.8 \times 10^3)$$

W_e = eddy loss in watts per cu. cm. of bar material.
f = cycles per second.
t = thickness of copper bar (tangential) multiplied by the number of bars side by side in a slot.
σ = slot width
ρ = resistivity of bar material in abohms (equals microhm − cm. by 10^3).
W = radial width of individual bars in cm.
H = slot leakage m. m. f. in gilberts per cm. or slot leakage flux in gausses (no bars).

CONCLUSIONS

The above-described experiments indicate that the formulas for no-load copper eddy-current losses due to the slot-leakage fluxes are correct to within 10 or 15 per cent for all of the conditions tried which cover a wide range. In the case of rotating machines, however, we have the saturation of the teeth, other gap reluctances and other factors which make it very difficult sometimes to estimate accurately the total m. m. f. applied to the slot-leakage magnetic circuit. In general, fairly reliable corrections may be made by adding 50 per cent to the slot width. When skin effect is small no appreciable errors will result since the loss is independent of the gap reluctance, assuming a given leakage flux. This analysis gives the following interesting relations.

For small skin effect, $m\,W$ less than 1 (low frequency or narrow bars), the eddy losses for a unit volume of copper vary as $f^2\,W^2$. For large skin effect, $m\,W$ greater than 2 (high frequency or wide bars), the eddy losses for a unit volume of copper vary as $f^{1/2}$, $(t/\sigma)^{-3/2}$, W^{-1} and H^2. For variations in total copper eddy current losses consideration must be given, as previously mentioned, to the way in which t/σ and W are varied.

In the case of salient pole machines the wave shape of the slot leakage fluxes must be analyzed or the components estimated and the losses calculated for each component frequency. In the case of induction motors the tooth-pulsation frequency is to be used.

Even though these formulas often can not be used directly without considerable labor, they should serve a very useful purpose in indicating quickly the result of changing the laminating of the copper and changing the slots. The interesting fact has been pointed out that in some cases increased laminating of the copper may increase the losses, while in others it decreases them.

SUMMARY

Check tests are presented, showing that our theoretical formulas for calculating no-load copper eddy-current losses are reliable provided all of the conditions are known. The actual application to rotating machines may yield erroneous results due to difficulties in estimating the total reluctance of the flux-leakage paths. These formulas in their present form should be useful for calculating roughly the amount of no-load copper eddy losses in salient pole machines and induction motors. They should also be useful for determining the effect of changes in the section of the individual copper bars in the slot with reference to eddy losses. Later it is hoped that more reliable and simpler methods for using these formulas will be devised.

Discussion

W. J. Foster: I am interested in this paper on the eddy-currents that exist in the copper on open circuit. A very easy way of determining these losses is to make a core-loss test before the machine is wound. Sometimes it interferes with production if a commercial machine is used. Occasionally it happens that a test has to be made at the factory on a machine which is to be shipped that involves knocking it down. In such a case, the core loss can be run off with the machine completely assembled, then after the winding is removed from the armature, it can be repeated.

I notice the author shows reasons why eddy losses exist in the slot, the cross fluxes changing as the rotor advances, etc. There are also conditions at the head of the armature due to the fringing of the flux that are responsible for some losses.

I remember in the case of a few machines many years ago learning something about how to sectionalize conductors by making use of different windings in the same machine, and finding quite a difference in the open-circuit core loss. Possibly others have had similar experiences.

In the matter of determining the losses, as Mr. Dawson and Mr. Barns have described, I believe their method to be about the only feasible method for certain large enclosed machines. However, we must look out for a number of things;—such as the specific gravity of the air. There is the matter of radiation. If it is a very high-speed machine, occupying small space, probably we can neglect entirely the heat that passes off by radiation, but in the case of slower-speed machines that occupy large space and have very large surfaces, some allowance should be made.

P. L. Alger (communicated after adjournment): It has been my opinion for some time that a large part of the observed core loss in a squirrel-cage induction motor is due to circulating currents in the squirrel-cage bars caused by the pulsations of flux in the rotor teeth. These pulsations are due to the variations of reluctance caused by the stator teeth, and are especially large when open slots are used in the stator. Also, these pulsations in the rotor teeth are very large when the ratio of rotor to stator teeth much exceeds unity, they are negligible when the ratio of slots is nearly equal to one, and they are fairly large when the ratio is considerably less than unity, but greater than one-half.

As I understand it, Mr. Spooner now suggests that the loss due to these pulsations of flux in the rotor teeth is not so much due to the circulating currents induced in the winding as to the eddy-current losses produced by the flux forced across the slots as a result of saturation of the teeth. Of course, the flux across the rotor slots is the same when a given number of ampere-turns are present, whether due to current in the rotor bars, or due to saturation in the rotor teeth, other things being equal. Thus, whether the pulsation of reluctance, due to the stator teeth, is counterbalanced by eddy currents and saturation ampere-turns produced in the teeth, as is the case with an open-circuited rotor winding, or is counterbalanced by the ampere-turns of a current induced in the squirrel-cage bars, the same eddy-current loss in the copper should result. But, in the former case, the ampere-turns in the rotor teeth are produced as a result of flux pulsations in the teeth, and these pulsations must be quite large to make many ampere-turns due to saturation. And, in the other case, the ampere-turns in the squirrel-cage winding can be produced by only a small change in flux, corresponding to the voltage required to supply the impedance drop of the winding. Therefore, in the former case, the flux pulsations in the rotor are large, the ampere-turn pulsations in the rotor teeth are not large, and the reflected pulsations of flux in the stator teeth are small; while, in the latter case, the flux pulsations in the rotor teeth are small, the ampere-turn pulsations in the rotor winding are large, and the reflected flux pulsations in the stator teeth are large.

From all this, in the case of a squirrel-cage motor with straight slots and considerably more squirrel-cage bars than stator slots, there can be a very small loss of the type described by Mr. Spooner, as the saturation ampere-turns in the rotor teeth are nearly constant, except for a slip-frequency variation. Therefore, in such cases, one ought to be able to calculate the tooth-frequency core loss as the sum of the pole-face loss, the copper loss in the rotor bars due to the induced currents based on the high-frequency bar resistance, and the flux pulsation loss produced in the stator as a result of the induced rotor currents. Hence, I came to the conclusion that Mr. Spooner's theory does not account for the losses in the ordinary squirrel-cage induction motor with straight slots, and, if he obtains numerical agreement between his calculations and test results, this agreement must be due in some measure to chance.

The foregoing remarks are not intended as a criticism of the paper, but as giving a line of reasoning parallel to Mr. Spooner's which should bring from him some interesting additional ideas. As far as slip-ring induction motors and direct-current machines are concerned, I think Mr. Spooner's theory should give a sound basis for core-loss calculations. It is interesting in this connection to note that, on the basis of Mr. Spooner's ideas, by using a high-grade steel in a d-c. machine, a greater core loss may be obtained than with a low-grade steel, as the extra saturation ampere-turns may cause more losses in the winding than are due to the saving in iron loss itself. On this basis, by subdividing the winding to avoid eddy currents, it should be possible to use a better grade of steel, with the result that a double gain in losses can be secured.

T. Spooner: Referring to Mr. Foster's remarks, we have tested a number of experimental and commercial machines, both d-c. and a-c., according to the method suggested by him, namely, with the windings in position in the slots and with the windings removed. The differences in losses caused by the eddy currents in the copper began to be appreciable at about 70 kilolines per sq. in. mean-tooth induction for induction motors and at about 120 kilolines per sq. in. for salient-pole machines. Often the test losses check the calculated losses quite closely, but sometimes there are rather wide differences. For small-diameter machines where there is considerable tooth taper, it is sometimes difficult to estimate accurately the slot leakage fluxes. This is perhaps one of the chief causes of the observed discrepancies.

We have made no investigation of losses caused by leakage fluxes in the neighborhood of the end windings.

Because of the following considerations, I am unable to agree with Mr. Alger's line of reasoning in connection with the eddy-current losses in the rotor bars of squirrel-cage induction motors. In other words, I am still convinced that there exist eddy-current losses in the squirrel-cage bars, after connecting the end rings, which are equal to or greater than those which exist with the bars open-circuited.

Suppose we consider two adjacent teeth, No. 1 tooth having a smaller air-gap reluctance than No. 2 tooth at a given instant. Now, with the bars open-circuited, flux will flow across the slot through the bar from tooth No. 2 to tooth No. 1, giving radial and tangential components of leakage flux. If now the bars are short-circuited, sufficient current will flow in them to bring the mean flux in tooth No. 2 nearly up to that in No. 1. Since the fluxes in the two teeth are practically equal but the air-gap for one of these is less than for the other, the m. m. f. acting on tooth No. 2 must be greater than that acting on No. 1. In other words, the current in the bars will decrease the m. m. f. acting on tooth No. 1 and increase the m. m. f. acting on tooth No. 2. The tangential slot-leakage flux should therefore be increased toward the top of the slot and decreased toward the bottom, the total remaining about constant. At the same time the radial slot leakage flux will be decreased on the side toward tooth No. 1 and increased toward tooth No. 2.

High-frequency circulating currents in squirrel-cage windings, assuming an effective resistance as calculated by Field's formula, do not, in the cases which I have observed, account for the increased losses due to the presence of the copper bars unless the pulsation losses in the stator teeth due to the high-frequency circulating currents in the rotor windings are much greater than those which existed in the rotor teeth before closing the squirrel-cage windings. This seems improbable since the pulsation losses with closed squirrel-cage windings show no tendency toward decreasing as the induction increases due to tooth saturation, as would be the case if they were due to high-frequency iron losses in the stator teeth.

Mr. Alger's point concerning the danger of using high-silicon steel for the cores of machines operating at high-tooth inductions is well taken. Since at high inductions the permeability of 4 per cent silicon steel may be from ½ to $^1/_3$ of that of low-silicon steel, the slot-leakage fluxes will be greatly increased for the former. Since the eddy-current losses increase as the square of the slot-leakage fluxes, the substitution of high- for low-silicon steel may increase very considerably the copper eddy losses. The remedy is, of course, a greater subdivision of the copper, as suggested by Mr. Alger.

Mechanical Force Between Electric Circuits

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Associate, A. I. E. E.
and
R. H. PARK
Non-member

Synopsis.*—A general equation is developed for the mechanical force exerted by a system of n electric circuits, upon any part of that system. The electric circuits are assumed to contain resistances, and the reluctance of the several magnetic circuits is assumed to be a function of both the currents and the relative positions of the circuits. The equation is therefore applicable to circuits involving saturated iron.*

Special cases of a single circuit and of n circuits are treated, in which a method of graphical solution is given. Also an approximation is made, which results in fairly simple analytical expressions for the force in the case of circuits involving saturated iron.

The results are preliminary to an investigation of the forces existing in synchronous machines under the condition of short circuit, which subject will be treated in a forthcoming paper.

I—Discussion of Problem and Method of Solution

Although methods have been available for calculating the force of an electromagnet, or between simple circuits of constant inductance, such as reactance coils containing no iron, nevertheless general methods have not been developed, to the authors' knowledge, for calculating the force exerted between dissimilar circuits which contain iron and which therefore have magnetic reluctances that may vary both with the magnetic density and the relative position of the circuits. It is the purpose of the present paper to offer methods for these cases. The work presented here, as well as that in a forthcoming paper in the near future, is largely the result of an investigation made by one of the authors in 1922 on the mechanical forces in synchronous machines under short circuit conditions.

HISTORICAL

The basic contributions regarding the mechanical force exerted by electromagnetic circuits were, of course, made by Ampere[2], Kelvin and Maxwell[3]; and the greater part of the subsequent development of the subject has been the application of their fundamental equations to particular cases.

Maxwell's equation for the "mechanical action between two circuits" is, in different notation,

$$f_x = \frac{1}{2} I_1^2 \frac{d L_1}{d x} + I_1 I_2 \frac{d M}{d x} + \frac{1}{2} I_2^2 \frac{d L_2}{d x} \qquad \textbf{(56)}$$

where I_1 and I_2, L_1 and L_2 are the respective currents and inductance coefficients of the two circuits; M, the mutual inductance; f_x, the component of force in the direction of x, "one of the geometrical variables on which the form and relative position depend."

Quoting Maxwell, "If the motion of the system corresponding to the variation of x is such that each circuit moves as a rigid body, L_1 and L_2 will be independent of x and the equation will be reduced to the form,"

$$f_x = I_1 I_2 \frac{d M}{d x} \qquad \textbf{(57)}$$

Although it is clear from the above statement and his original assumptions that the electrical circuits are not considered to be in proximity to iron or other magnetic material, nevertheless the equation holds, as will appear later, for circuits which do enclose, or are near, iron provided there is no saturation.

If there is a single circuit only, such as an electromagnet, all terms of (1) drop out except the first, thus reducing to the well-known expression,

$$f_x = \frac{1}{2} I^2 \frac{d L}{d x} \qquad \textbf{(58)}$$

Thus for any circuit or pair of circuits in which the inductances are functions of shape or position only, Maxwell's equation applies.

Now there is another classical method of analysis according to which the force is obtained by integrating over the circuit member in question, the force on each current element, due to the magnetic density in which the element exists. In some cases this is more convenient to apply than the foregoing equations. It depends, of course, upon the nature of the problem. An excellent example of the application of the latter method is Dwight's[4] work regarding the electromagnetic force on switches.

Other authors, as listed in the bibliography, have also treated special cases, and in one or two of these, where saturation is considered, an additional step of fundamental nature has been taken. In 1911, Steinmetz[5] treated the special cases of electromagnets and transformers in accordance with existing theory, but in addition, considered the case of a single-circuit electromagnet containing iron and involving saturation. While his results were not carried further than the detailed expression of the *energy* relations which obtain during a change in position[6], this nevertheless constituted the initial step toward the derivation of a general equation for *force* of a single circuit.

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[1] Consulting Engineer, General Electric Co., Schenectady, N. Y.

[2] Clerk Maxwell, Electricity and Magnetism, Vol. II, Arts. 502 to 528.

[3] Clerk Maxwell, Electricity and Magnetism, Vol. II, Art. 583.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

[4] H. B. Dwight, "Magnetic Force on Disconnecting Switches," TRANS. A. I. E. E., 1290, Vol. 39, Part II, pp. 1337-1355.

[5] Mechanical Forces in Magnetic Fields, TRANS. A. I. E. E., 1911, Vol. 30, pp. 357-413.

[6] This case is reviewed by Rudolph Richter in his *Elektrische Maschinen,* in which the equations are interpreted graphically.

SCOPE OF PAPER

Thus, the development to the present appears to include the solution of the cases of one and two circuits involving inductances which are not functions of the current, and of the case of a single circuit involving saturation, to the extent of the mathematical expressions for the various terms in the energy equation. It is the purpose here to give the general equation for a system of n circuits which may contain iron, either saturated or not, but assumed to have no hysteresis, and thus having inductances which are functions of both position and current; and in a subsequent paper in the near future, to apply the results to the case of mechanical forces in synchronous machines under short-circuit condition.

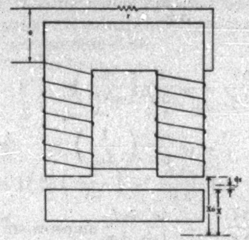

FIG. 1

Two cases will be taken up in order; single circuit, and n circuits.

SINGLE CIRCUIT

The magnet in Fig. 1 is assumed to be excited to a point a on the saturation curve Fig. 2.

Let

ω = magnetic linkages $\times 10^{-8} = L\,i$

Ω = value of ω at which the force is to be determined; viz., $\Omega = L\,I$

L = inductance in henrys, defined as *linkages-per-ampere* $\times 10^{-8}$, and is a function of the current.

i = current in amperes.

I = value of i at which force is to be determined.

f_x = mechanical force exerted *on* the armature *by* the magnetic field, in the direction of x.

x = distance in cm. from arbitrary reference.

Although the electromagnet in Fig. 1 is a special case involving, as a result of symmetry, only one space coordinate, the general case is fully treated in Appendix A, equations (**5**) and (**6**).

Applying the principle of *virtual displacement* to the system in Fig. 1, let there be a displacement $d\,x$ of the armature, thus allowing the force f_x to do work. Then, referring to Fig. 2, the curve $o\,a$ corresponds to position x; curve $o\,b$, to position $x + d\,x$:

(a) Energy increment from electrical source during the position change $d\,x$ (I and Ω being variable during the change) $= I\,d\,\Omega$ = area $a\,b\,c\,d$, where $d\,\Omega$ is the increment due to the change $d\,x$.

(b) Total energy storage at position x and current $I = \int_0^{\Omega} i\,d\,\omega$ = area $o\,a\,d$ where $d\,\omega$ is the increment in ω, due to increment in current at constant x.

By conservation of energy, the increment in energy received from the electrical source during the change $d\,x$ must equal the sum of the mechanical work plus increment in stored magnetic energy. Thus,

$$I\,d\,\Omega = f_x\,d\,x + d\int_0^{\Omega} i\,d\,\omega$$

from which, by equations in Appendix A,

$$f_x\,d\,x = -\int_0^{\Omega} \frac{\partial i}{\partial x}\,d\,x\,d\,\omega \qquad \text{joules} \qquad (\mathbf{59})$$

In Fig. 2,

$$\frac{\partial i}{\partial x}\,d\,x = h$$

Thus,

(c) Mechanical work $= f_x\,d\,x = o\,a\,b'$

In equation (**59**), $d\,x$ is a constant in the integration. Hence

$$f_x = -\int_0^{\Omega} \frac{\partial i}{\partial x}\,d\,\omega \qquad \text{joules/cm.} \qquad (\mathbf{60})$$

which is the same as equation (**5**), Appendix A.

Or,

$$f_x = -\,22.5\int_0^{\Omega} \frac{\partial i}{\partial x}\,d\,\omega \qquad \text{lb.} \qquad (\mathbf{60a})$$

This is the *general equation* for force by a single circuit in which the inductance is a function of both position and magnetic flux, and is therefore applicable to electromagnets involving saturation.

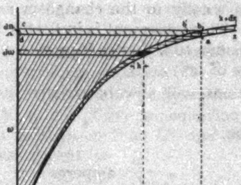

FIG. 2

Equation (**60**) is applied as follows:

Let,

$$i = f\,(\omega, x)$$

With ω as parameter, plot

$$i = F(x)$$

as in Fig. 3.[7] From this, plot

$$\frac{\partial i}{\partial x} = \varphi(\omega)$$

corresponding to X, as in Fig. 4. Then by equation (**60**), the force corresponding to the position X, current I, and linkages Ω, is the shaded area in Fig. 4.

Special Case. An interesting and important simplification is made possible by an assumption which will apply approximately to many actual cases. In Fig. 3, if no saturation exists, and neglecting the magnetic reluctance of the iron,

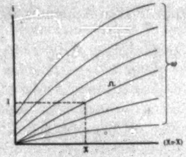

FIG. 3

$$i = 0 \text{ at } x = x_0$$

that is, when the air-gap $(x_0 - x) = 0$, since obviously no current would be required for zero reluctance. However, at values of ω which involve saturation, i has a definite value when the air-gap $(x_0 - x) = 0$, namely, the value required to sustain the magnetic flux in the saturated iron. But if, as in Fig. 3, the change in x is made at constant flux linkages, that is at approximately constant magnetic flux, the magnetic reluctance of the iron path should remain unchanged. Under this assumption *there is no change in that component of current which sustains the flux in the iron.* Thus the change in current due to such a change in x would be due wholly to the change in reluctance of the air path. This increment is indicated by h in Fig. 2, and corresponds to a change from x to $x + d\,x$, at a constant value of ω.[8]

These relations will now be written into equation (**60**).

By definition,

$$i = \frac{\omega}{L} \text{ amperes} \qquad (\mathbf{61})$$

and

$$L = \frac{N^2}{\mathcal{R}} \text{ henrys} \qquad (\mathbf{62})$$

where

N = number of series turns.

$$\mathcal{R} = \mathcal{R}_0 + \mathcal{R}_i \text{ turns}^2/\text{henry} \qquad (\mathbf{63})$$

$\mathcal{R}$ = magnetic reluctance coefficient, which is made up of $\mathcal{R}_0$, applying to the air portion of the magnetic circuit; $\mathcal{R}_i$, to the iron portion.

Substituting (**62**) and (**63**) in (**61**) and differentiating partially with respect to x, taking ω constant,

$$\frac{\partial i}{\partial x} = \frac{\omega}{N^2}\frac{\partial}{\partial x}(\mathcal{R}_0 + \mathcal{R}_i)$$

But under the assumption,

$$\frac{\partial \mathcal{R}_i}{\partial x} = 0$$

Hence,

$$\frac{\partial i}{\partial x} = \omega \frac{\partial}{\partial x}\left(\frac{\mathcal{R}_0}{N^2}\right)$$

$$= \omega \frac{\partial}{\partial x}\left(\frac{1}{L_0}\right)$$

Thus,

$$\frac{\partial i}{\partial x} = -\frac{\omega}{L_0^2}\frac{\partial L_0}{\partial x} \text{ amperes/cm} \qquad (\mathbf{64})$$

Substituting (**64**) in (**60**),

$$f_x = \int_0^\Omega \frac{1}{L_0^2}\frac{\partial L_0}{\partial x}\,\omega\,d\,\omega$$

Making the assumption that in this integration

$$L_0 \text{ and } \frac{\partial L_0}{\partial x}$$

are not functions of ω, they may then be placed outside of the integral sign[9]. The integral then becomes

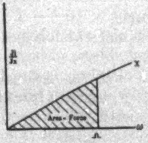

FIG. 4

$$f = \frac{1}{2}\frac{\Omega^2}{L_0^2}\frac{\partial L_0}{\partial x} \text{ joules/cm} \qquad (\mathbf{65})$$

Thus, for those cases where the above assumptions approximately hold, the force is expressed in the following known and convenient terms: the actual magnetic linkages corresponding to the existing current I and position X; the constant (with respect to ω) inductance L_0 corresponding to the straight part of the (ω, i) curve, Fig. 1; and the rate at which that inductance changes with respect to x.

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[7] It is more convenient in the particular case of Fig. 1 to plot i as $F(x_0 - x)$ instead of $F(x)$.

[8] In calculating force, it makes no difference in the value whether it is assumed that the change $d\,x$ is made at constant linkages, as above, or at constant current. Referring to Fig. 2, the difference in mechanical work is represented by the second order area $a\,b\,b'$, which vanishes in the limit.

[9] Thus, equation (**65**) holds only in so far as R_0 and the ratio $\frac{d\,\mathcal{R}_0}{d\,x}$ are independent of ω.

Equation (**65**) can be written in the following form:

$$f = \frac{1}{2} I_f^2 \frac{\partial L_0}{\partial x} \text{ joules/cm.} \qquad \textbf{(66)}$$

where I_f is the fictitious current,

$$I_f = \frac{\Omega}{L_0} = I \frac{L}{L_0} \text{ amperes} \qquad \textbf{(67)}$$

thus smaller than the actual current I by the ratio of the inductance at the actual current, to the inductance corresponding to no saturation.

Or in still another form,

$$f = \frac{1}{2} \left(\frac{L}{L_0} \right)^2 I^2 \frac{\partial L_0}{\partial x} \text{ joules/cm.} \qquad \textbf{(68)}$$

Or, in pounds

$$f = 11.25 \left(\frac{L}{L_0} \right)^2 I^2 \frac{\partial L_0}{\partial x} \text{ lb.} \qquad \textbf{(66a)}$$

Thus if,

$$L = L_0$$

that is, with no saturation,

$$f = \frac{1}{2} I^2 \frac{\partial L_0}{\partial x} \qquad \textbf{(69)}$$

This is the well-known expression for a single circuit with no saturation. Comparison of (**68**) and (**69**) shows that saturation decreases the force, at the same current I, by the ratio

$$\left(\frac{L}{L_0} \right)^2$$

It is interesting to note that under the assumptions contained in equation (**65**), equation (**60**) also reduces to the familiar form

$$f_x = \frac{\beta^2 A}{8\pi} 10^{-7} \qquad \text{joules/cm.}$$

when applied to the special case of a magnet with relatively small uniform air-gap.

Let the total magnetomotive force for the magnetic circuit corresponding to the linkages ω be

$$N i = F(\omega) + \frac{\omega (x_0 - x) 10^8}{0.4 \pi N A}$$

where A is the cross-section in sq. cm., β is the magnetic density in lines per sq. cm. of the air path, and $F(\omega)$ represents the saturation curve of the iron alone. Thus, differentiating partially with respect to x,

$$\frac{\partial i}{\partial x} = - \frac{\omega 10^8}{0.4 \pi N^2 A}$$

Substituting this in (**60**) and integrating,

$$f_x = \frac{10^8}{0.8 \pi A} \left(\frac{\Omega}{N} \right)^2 = \frac{\beta^2 A}{8\pi} 10^{-7} \text{ joules/cm.}$$

n CIRCUITS

General Case. From Appendix B, the general equation for f_x for the case of n circuits is given in the following two forms:

$$f_x = - \sum_1^n \int_0^{\Omega_u} \frac{\partial i_u}{\partial x} d\,\omega_u \qquad \text{joules/cm.} \qquad \textbf{(9)}$$

and

$$f_x = \sum_1^n \int_0^{I_u} \frac{\partial \omega_u}{\partial x} d\,i_u \qquad \text{joules/cm.} \qquad \textbf{(11)}$$

The application of (**9**) may be illustrated as follows:

As discussed in Appendix B, any functional relation between the currents or between the linkages may be assumed during the integration indicated, provided that relation brings all currents to their given values, which are the limits of the integrals. Thus, assume the currents are brought up, proportional to each other, the proportionality constants being the ratios of the final values I_1, I_2, I_3, etc. Then, for any value of i_u, all other currents and values of ω, at a given value of x, are fixed. Thus, for each circuit, following the same procedure as indicated for the case of a single circuit, equation (**60**), Figs. 3 and 4,

$$i_u = f_u(\omega_u, x)$$

with ω_u as parameter, plot

$$i_u = F_u(x)$$

for each circuit. From these, plot the corresponding curves,

$$\frac{\partial i_u}{\partial x} = \phi_u(\omega)$$

corresponding to the particular value of x. Then the force is the sum of the areas corresponding to that in Fig. 4.

A similar procedure may be followed in the application of the other form, equation (**11**).

Special Case: Inductance Independent of Current. From Appendix C, one of the two equations derived for f_x, under the assumption that L's and M's are independent of the current, is

$$f_x = \sum_1^n {}_u \sum_1^n {}_v \frac{I_u I_v}{2} \frac{d M_{uv}}{d x} \text{ joules/cm.} \qquad \textbf{(21)}$$

where M_{uv} = mutual inductance in henrys of circuit v upon circuit u.[10]

Thus for $n = 3$, for illustration,

$$f_x = \frac{1}{2} I_1^2 \frac{d L_1}{d x} + \frac{1}{2} I_2^2 \frac{d L_2}{d x} + \frac{1}{2} I_3^2 \frac{d L_3}{d x}$$

---

[10] When $u = v$, self induction is indicated. Thus $M_{11} = L_1$, $M_{22} = L_2$, etc.

$$+ I_1 I_2 \frac{d M_{12}}{d x} + I_1 I_3 \frac{d M_{13}}{d x} + I_2 I_3 \frac{d M_{23}}{d x}$$

$$\text{joules/cm.} \qquad (70)$$

For two circuits only, equation (**70**) reduces to Maxwell's equation, as given in (**56**).

The other equation for f_x for this case is

$$f_x = - \sum_{1}^{n}{}_{u} \sum_{1}^{n}{}_{v} \frac{\Omega_u \Omega_v}{2} \frac{d K_{uv}}{d x} \text{ joules/cm.} \qquad (26)$$

where K_{uv} is a "coefficient of magnetization," discussed in Appendix C, and defined in general by

$$I_u = \sum_{1}^{n}{}_{v} K_{uv} \Omega_v \text{ amperes} \qquad (24)$$

Approximation. Making the same simplifying assumptions as contained in equations (**64**) and (**65**), discussed under "SINGLE CIRCUIT," saturation may be approximately taken into account in the equations.

As explained in Appendix *D*, the above mentioned assumptions reduce equation (**9**) to the form

$$f_x = - \sum_{1}^{n}{}_{u} \sum_{1}^{n}{}_{v} \frac{\Omega_u \Omega_v}{2} \frac{d K_{uv}}{d x} \qquad (32)$$

where Ω_u and Ω_v are the linkages of circuits u and v corresponding to the actual currents I_1, I_2, I_3, etc., under the existing condition of partial saturation of the iron paths of the magnetic circuit; and where K_{uv} is the coefficient of magnetization, discussed in Appendix C, applying to the *non-saturated* condition. In other words, equation (**32**) is of exactly the same form—indeed, the same equation—as (**26**), which applies to the *non-saturated* case. Thus, under the assumptions, different expressions for the force are encountered for the saturated and non-saturated conditions, only when the force is expressed in terms of *currents*; but the same equation is obtained for either case when it is expressed in terms of linkages.

In Appendix E, expressions have been worked out for K_{uv} for 1, 2, 3, and 4 circuits in terms of the self and mutual inductances without saturation, namely L_u and M_{uv}.

To illustrate the application of (**32**), take the case of a single circuit. Here,

$$K_{uv} = K_{11} = \frac{1}{L_0}$$

Then

$$f_x = \frac{\Omega_1^2}{2} \left(\frac{d}{d x} \frac{1}{L_0} \right)$$

or

$$f_x = \frac{1}{2} \frac{\Omega_1^2}{L_o} \frac{d L_o}{d x}$$

It will be noted that this is the same as equation (**65**). Thus, under the assumption, if the values of $\Omega_1, \Omega_2, \Omega_3$, etc., are known in any system involving saturated iron, the force is approximately determined by (**32**), using K_{uv} as given in Appendix E.

It is possible to express the force, in the case of n circuits, and under the above assumptions, as follows: Equations (**21**) and (**26**) express exactly the same result for the non-saturated case. But equation (**32**), which is the same as equation (**26**), gives the force for the saturated case. Therefore, equation (**21**) *will apply also to the saturated case, under the assumptions, provided such fictitious current values are used as will cause the actual existing values of* $\Omega_1, \Omega_2, \Omega_3$, etc.

Thus, knowing the values of Ω_1, Ω_2, etc., n simultaneous equations may be written for the n values of current, in terms of L's and M's as follows:

$$\left.\begin{aligned} \Omega_1 &= I_1 L_1 + M_{12} I_2 + M_{13} I_3 + \ldots \\ \Omega_2 &= I_2 L_2 + M_{21} I + M_{23} I_3 + \ldots \\ \Omega_2 &= I_3 L_3 + M_{31} I_1 + M_{32} I_2 + \ldots \end{aligned}\right\} \frac{\text{Linkages}}{10^8} \qquad (71)$$

The force is then obtained by substituting in equation (**21**) the fictitious values of current I_1, I_2, I_3, etc., as found by solving equation (**71**). The solution for these currents may be readily obtained by substituting in equation (**24**) the known values of linkages Ω_v and the values of K_{uv} as determined by the equations given in Appendix E.

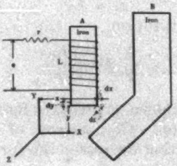

FIG. 5

The authors gratefully acknowledge the valuable suggestions and encouragement of V. Karapetoff, and the assistance of J. M. Bryant, D. S. Snell and R. M. Ryan.

Appendix A

GENERAL EQUATIONS FOR A SINGLE CIRCUIT

Let Fig. 5 represent any electromagnetic circuit with negligible electrostatic capacity, but containing a resistance r, and having a magnetic reluctance which is a function both of the form and position of the circuit, and also of the current i. That is, the electrical circuit may be interlinked with, or near to, saturated iron. It is required to obtain a general expression for the mechanical force exerted *by* the circuit *on* some part or element of the circuit system, say A, Fig. 5. The total force f is equal to the vector sum of the three rectangular components f_x, f_y, and f_z.

Applying the principle of *virtual displacement*, let one of the components, say f_x, do work by allowing a displacement $d\,x$ of the element A in the time $d\,t$.

Then, by conservation of energy, the increment of energy, during the time $d\,t$, from the electrical supply must equal the sum of the energies expended in heat, in mechanical work and in increasing the magnetic energy storage. Letting I be the value of current i, and Ω, the value of magnetic linkages ω under which the change[11] $d\,x$ is made, the energy equation is

$$p\,d\,t = I^2\,r\,d\,t + f_x\,d\,x + d\,E \qquad (1)$$

where

p = power in watts
I = particular value of current i in amperes
f_x = x – component of force, joules/cm.
E = magnetic energy storage, joules
t = time in seconds
r = resistance in ohms.

But at the time t,

$$p = e\,I$$

and

$$e = I\,r + \frac{d\,\Omega}{d\,t}$$

Substituting these relations in (1) the $I^2\,r$ terms drop out, leaving

$$I\,d\,\Omega = f_x\,d\,x + d\,E \qquad \text{joules} \qquad (2)$$

Now the energy storage at any current I and magnetic linkage Ω is

$$E = \int_0^\Omega i\,d\,\omega \qquad \text{joules} \qquad (3)$$

Substituting this in (2), and differentiating,[12]

$${}_x = I\,\frac{d\,\Omega}{d\,x} - \frac{d}{d\,x}\int_0^\Omega i\,d\,\omega, \text{ joules/cm.} \qquad (4)$$

$$= I\,\frac{d\,\Omega}{d\,x} - \left[\int_0^\Omega \frac{\partial\,i}{\partial\,x}\,d\,\omega + I\,\frac{d\,\Omega}{d\,x}\right]$$

Thus,

$$f_x = -\int_0^\Omega \frac{\partial\,i}{\partial\,x}\,d\,\omega$$

Analogously,

$$f_y = -\int_0^\Omega \frac{\partial\,i}{\partial\,y}\,d\,\omega \qquad \text{joules/cm.} \qquad (5)$$

and

$$f_z = -\int_0^\Omega \frac{\partial\,i}{\partial\,z}\,d\,\omega$$

Equation (4) may be integrated by parts, as follows, thus obtaining a different form for the force.

The integral is,

$$\int_0^\Omega i\,d\,\omega = I\,\Omega - \int_0^I \omega\,d\,i$$

and

$$\frac{d}{d\,x}\int_0^\Omega i\,d\,\omega = I\,\frac{d\,\Omega}{d\,x} + \Omega\,\frac{d\,I}{d\,x} - \int_0^I \frac{\partial\,\omega}{\partial\,x}\,d\,i - \Omega\,\frac{d\,I}{d\,x}$$

Substituting this in (4)

$$f_x = \int_0^I \frac{\partial\,\omega}{\partial\,x}\,d\,i$$

Likewise,

$$f_y = \int_0^I \frac{\partial\,\omega}{\partial\,y}\,d\,i \qquad \text{joules/cm.} \qquad (6)$$

and

$$f_z = \int_0^I \frac{\partial\,\omega}{\partial\,z}\,d\,i$$

Appendix B

GENERAL EQUATIONS FOR n CIRCUITS

Let Fig. 6 represent any system of n electromagnetic circuits, each comprising a supply voltage e, a constant resistance r, and a magnetic reluctance R, assumed to be variable with respect to both position (x, y, z) and magnetic linkages ω. Electrostatic capacity is neglected. Let the subscript u indicate any particular circuit, and let capital letters indicate particular values.

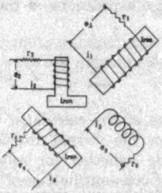

FIG. 6

With the currents

$$i_1 = I_1 \qquad i_2 = I_2 \qquad i_3 = I_3, \text{ etc.}$$

and the magnetic linkages

$$\omega_1 = \Omega_1 \qquad \omega_2 = \Omega_2 \qquad \omega_3 = \Omega_3, \text{ etc.}$$

it is required to determine the force which the system of circuits, as a whole, exerts *on* the circuit u, or on any particular group of the circuits.

The total force f exerted on the circuit, or group of circuits, is equal to the vector sum of the three rectangular components f_x, f_y, f_z. To determine any component, say f_x, let there be a displacement $d\,x$ from the position (x, y, z), in the time $d\,t$, of that circuit, or group of circuits, whose force is required.

Then by conservation of energy,

---

[11] Both I and Ω may be functions of x.

[12] Whittaker and Watson, Modern Analysis, Chapter IV, Art. 4.2

A	$=$	B	$+$	C	$+$	D (7)
sum of energy increments from all electrical sources e_1, e_2, e_3, etc.		mechanical work done *by* the system on the circuit, or group of circuits, displaced.		sum of increments in magnetic energy storage		sum of energy loss in $i^2 r$

Thus,

$$(A) = \sum_1^n p_u \, d\, t$$

but

$$p_u = e_u I_u$$

and

$$e_u = r_u I_u + \frac{d\, \Omega_u}{d\, t}$$

Substituting these relations in the first term,

$$(A) = \sum_1^n I_u^2 r_u \, d\, t + \sum_1^n I_u \, d\, \Omega_u$$

$$(B) = f_x \, d\, x$$

$$(C) = \sum_1^n d\, E_u = \sum_1^n d \int_0^{\Omega_u} i\, d\, \omega$$

$$(D) = \sum_1^n I_u^2 r_u \, d\, t$$

Substituting these expressions in the energy equation (**7**), the force is

$$f_x = \sum_1^n I_u \frac{d\, \Omega_u}{d\, x} - \sum_1^n \frac{d}{d\, x} \int_0^{\Omega_u} i_u \, d\, \omega_u \text{ joules/cm.} \quad (\mathbf{8})$$

Now the value of the calculated force is the same regardless of how the virtual displacement $d\, x$ is assumed to be made—whether, for instance, "at constant currents," or "at constant linkages." However, equation (**8**) will be applied to each assumption, since being in different forms, one result may be more convenient in application than the other, depending upon the problem.

At Constant Linkages. If the change $d\, x$ is made at constant linkages, the first summation on the right hand side of equation (**8**) drops out. Moreover, since, under this assumption, the limits in the integrals of the remaining term are not functions of x, one may differentiate partially under the integral sign. Thus, (**8**) becomes

$$f_x = - \sum_1^n \int_0^{\Omega_u} \frac{\partial\, i_u}{\partial\, x} d\, \omega_u \text{ joules/cm.} \quad (\mathbf{9})$$

At Constant Current. For this case it is convenient to put (**8**) in a different form. The second term on the right hand side may be integrated by parts. Thus,

$$\sum_1^n \frac{d}{d\, x} \int_0^{\Omega_u} i_u \, d\, \omega_u = \sum_1^n \frac{d}{d\, x} \left[I_u \, \Omega_u - \int_0^{I_u} \omega_u \, d\, i_u \right]$$

Substituting this in (**8**),

$$f_x = \sum_1^n I_u \frac{d\, \Omega_u}{d\, x} - \sum_1^n \frac{d}{d\, x} \left[I_u \, \Omega_u - \int_0^{I_u} \omega_u \, d\, i_u \right] \text{ joules/cm.} \quad (\mathbf{10})$$

At constant current[13] with respect to x, this becomes

$$f_x = \sum_1^n \int_0^{I_u} \frac{\partial\, \omega_u}{\partial\, x} d\, i_u \text{ joules/cm.} \quad (\mathbf{11})$$

General Equation. Thus the general equation for mechanical force is given, for convenience, in two forms:

$$f_x = - \sum_1^n \int_0^{\Omega_u} \frac{\partial\, i_u}{\partial\, x} d\, \omega_u \text{ joules/cm.} \quad (\mathbf{9})$$

and

$$f_x = \sum_1^n \int_0^{I_u} \frac{\partial\, \omega_u}{\partial\, x} d\, i_u \text{ joules/cm.} \quad (\mathbf{11})$$

The other components f_y and f_z are, analogously, of the same form and may be written by merely substituting y or z for x in the above equations.

In order to apply, or to graphically interpret, equations (**9**) and (**11**), it is necessary to specify the functional relation between i_1, i_2, i_3, etc., or between $\omega_1, \omega_2, \omega_3$, etc., during the integration. Considering the physical conditions, it is evident that, neglecting hysteresis, the space distribution of magnetic density, magnetic intensity and current throughout the system is the same regardless of the particular manner in which the currents are brought to their final values I_1, I_2, I_3, etc., at which the linkages are $\Omega_1, \Omega_2, \Omega_3$, etc. Thus the energy distribution and total energy are also the same regardless of the history of the currents, so long as the final values are the same. It is therefore permissible to assume any convenient functional relation between the currents, or between the magnetic linkages, which will finally bring these quantities to the specified final values.

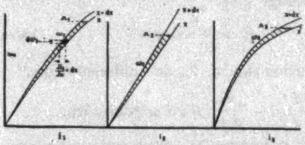

FIG. 7

Thus to interpret equation (**9**), let $\omega_1, \omega_2, \omega_3$, etc., bear the same relation to each other during the integration as exists between the final values $\Omega_1, \Omega_2, \Omega_3$, etc. This means, of course, that there must also be corresponding definite, though not proportional, values of i_1, i_2, i_3, etc. Hence, for any value of i_1 there must be definite values of all other currents and linkages. One may therefore plot two curves, as in Fig. 2, for each circuit corresponding respectively to the positions x and $x + d\, x$. Thus, in Fig. 7,

---

[13] In connection with the present study, this form has been obtained also by V. Karapetoff, using Calculus of Variations.

$$\text{Position } x \quad \begin{cases} \omega_1 = a\,(i_1) \\ \omega_2 = b\,(i_2) \\ \omega_3 = c\,(i_3) \text{ etc.} \end{cases}$$

$$\text{Position } x + d\,x \quad \begin{cases} \omega_1 = A\,(i_1) \\ \omega_2 = B\,(i_2) \\ \omega_3 = C\,(i_3) \end{cases}$$

It is interesting to note that at any particular value of ω, say ω_{10}, the following relation exists:

$$\omega_{10} = a\,(i_{10}) = A\left[\,i_{10} + \left(\frac{\partial\, i_1}{\partial\, x}\right)_{\omega} d\,x\right]$$

It will be noted that, multiplying both sides of (**9**) by $d\,x$, the mechanical work done by the force component f_x is represented by the sum of the shaded areas Fig. 7. Thus

$$f_x\, d\,x = \sum_1^n \int_0^{\Omega_u} \left(\frac{\partial\, i_u}{\partial\, x}\, d\,x\right)_{\omega} d\,\omega_u \text{ joules}$$

Appendix C

THE FORCE f_x EXERTED BY A SYSTEM OF n CIRCUITS WHICH ARE NOT SUBJECT TO SATURATION

The energy storage in a system of n circuits may be evaluated by integrating the electrical energy input to the system, exclusive of ohmic losses, during the establishment of the final currents.

That is,

$$E = \sum_1^n{}_u \int_0^T i_u \frac{d\,\omega_u}{d\,t}\, d\,t$$

$$= \sum_1^n{}_v \int_0^{\Omega^u} i_u\, d\,\omega_u \qquad \text{joules} \qquad (\mathbf{21})$$

provided that the integration is effected with every part of the system at rest.

The stored magnetic energy in a medium not subject to hysteresis depends only on the flux density at each point and therefore on the instantaneous values of the exciting currents, and is independent of the manner in which these values were obtained.

We may therefore assume for convenience in calculation that all the currents $i_1, i_2 \ldots i_n$ increase uniformly and in constant proportion from zero to the values $I_1, I_2, \ldots I_n$ respectively. Then at any instant the current in circuit u may be expressed in terms of the current in circuit 1 by the relation

$$i_u = \frac{I_u}{I_1}\, i_1 \text{ amperes} \qquad (\mathbf{13})$$

If there is no saturation the flux, linkages ω_u in any circuit u are a linear function of the exciting currents. It follows that ω_u will increase in the same proportion as the currents. Thus

$$\omega_u = \frac{\Omega_u}{I_1}\, i_1 \frac{\text{linkages}}{10^8} \qquad (\mathbf{14})$$

Then the energy is given as

$$E = \sum_1^n{}_u \frac{I_u\, \Omega_u}{I_1^2} \int_0^{I_1} i_1\, d\, i_1$$

$$= \sum_1^n{}_u \frac{I_u\, \Omega_u}{2} \text{ joules} \qquad (\mathbf{15})$$

That is, the magnetic energy storage in a system of n circuits not subject to saturation is given as one-half the sum of the product of current and linkages in each circuit.

The x-component of force on any part of the system is given as

$$f_x = \sum_1^n{}_u I_u \frac{d\,\Omega_u}{d\,x} - \frac{d\,E}{d\,x}$$

$$= \sum_1^n{}_u \left[I_u \frac{d\,\Omega_u}{d\,x} - \frac{1}{2} \frac{d}{d\,x}\left(I_u\, \Omega_u\right)\right]$$

$$= \sum_1^n{}_u \left[\frac{I_u}{2} \frac{d\,\Omega_u}{d\,x} - \frac{\Omega_u}{2} \frac{d\,I_u}{d\,x} \right]$$

$$\text{joules/cm.} \qquad (\mathbf{16})$$

The linkages Ω_u in any circuit u may be expressed in the condensed notation

$$\Omega_u = \sum_1^n{}_v M_{uv}\, I_v \qquad (\mathbf{17})$$

wherein M_{uv} is the mutual inductance between circuits u and v, and M_{uu} is the self inductance of circuit u.

The force is then

$$f_x = \sum_1^n{}_u \frac{I_u}{2} \frac{d}{d\,x} \sum_1^n{}_v M_{uv}\, I_v$$

$$- \sum_1^n{}_u \sum_1^n{}_v M_{uv} \frac{I_v}{2} \frac{d\;I_u}{d\,x}$$

or

$$f_x = \sum_1^n{}_u \sum_1^n{}_v \frac{I_u\, I_v}{2} \frac{d\,M_{uv}}{d\,x}$$

$$+ \sum_1^n{}_u \sum_1^n{}_v \frac{I_u\, M_{uv}}{2} \frac{d\,I_v}{d\,x}$$

$$- \sum_1^n{}_u \sum_1^n{}_v \frac{I_v\, M_{uv}}{2} \frac{d\,I_u}{d\,x} \text{ joules/cm.} \qquad (\mathbf{18})$$

It may be readily shown on the basis of the conservation of energy that

$$M_{uv} = M_{vu} \qquad (\mathbf{19})$$

Then, since it is legitimate to vary the order of summation, the last term of (**18**) may be written,

$$\sum_1^n{}_u \sum_1^n{}_v \frac{I_v\, M_{uv}}{2} \frac{d\,I_u}{d\,x} = \sum_1^n{}_v \sum_1^n{}_u \frac{I_v\, M_{vu}}{2} \frac{d\,I_u}{d\,x} \qquad (\mathbf{20})$$

This form is seen to be identical with the second term in (**18**). These two terms therefore cancel and the final result becomes

$$f_x = \sum_1^n{}_u \sum_1^n{}_v \frac{I_u I_v}{2} \frac{d M_{uv}}{d x} \text{ joules/cm.} \qquad (\mathbf{21})$$

An alternative expression for the force f_x in a system not subject to saturation may be obtained as follows:

Without saturation the linkages in the n circuits are related to the corresponding currents by a system of linear equations.

$$\begin{aligned} \Omega_1 &= \sum_1^n{}_v M_{1v} I_v \\ \Omega_2 &= \sum_1^n{}_v M_{2v} I_v \frac{\text{linkages}}{10^8} \qquad (\mathbf{22}) \\ \Omega_n &= \sum_1^n{}_u M_{nv} I_v \end{aligned}$$

The solution of these equations then gives rise to a corresponding system of linear equations.

$$\left.\begin{aligned} I_1 &= \sum_1^n{}_v K_{1v} \Omega_v \\ I_2 &= \sum_1^n{}_v K_{2v} \Omega_v \\ &\cdots\cdots\cdots \\ I_n &= \sum_1^n{}_v K_{nv} \Omega_v \end{aligned}\right\} \text{ amperes} \qquad (\mathbf{23})$$

That is, the current in any circuit u is related to the linkages in the whole number of circuits by a relation of the form

$$I_u = \sum_1^n{}_v K_{uv} \Omega_v \qquad \text{amperes} \qquad (\mathbf{24})$$

where K_{uv} is a quantity which is independent of the various values of flux linkages, but is a function of the relative position in space of the separate circuits, and which is to be determined either by the solution of equation (**22**), by calculation, or experiment. In this way K_{uv} is a coefficient exactly analogous to M_{uv}, the mutual inductance of the two circuits u and v. Completing the analogy, the coefficient K_{uv} may be termed the "coefficient of magnetization" of circuits u and v.

The force f_x on any part of the system is given by equation (**16**) as

$$f_x = \sum_1^n{}_u \left[\frac{I_u}{2} \frac{d\, \Omega_u}{d x} - \frac{\Omega_u}{2} \frac{d\, I_u}{d x} \right] \qquad (\mathbf{16})$$

Introducing equation (**24**), this becomes

$$f_x = \sum_1^n{}_u \sum_1^n{}_v \frac{K_{uv}\, \Omega_v}{2} \frac{d\, \Omega_u}{d x}$$

$$- \sum_1^n{}_u \sum_1^n{}_v \frac{\Omega_u}{2} \frac{d}{d x} \Big(K_{uv}\, \Omega_v \Big) \qquad (\mathbf{25})$$

which may be reduced by the same process as in (**18**), (**19**), (**20**) to the result

$$f_x = - \sum_1^n{}_u \sum_1^n{}_v \frac{\Omega_u\, \Omega_v}{2} \frac{d\, K_{uv}}{d x} \text{ joules/cm.} \qquad (\mathbf{26})$$

Appendix D

An Approximate Solution for the Force on any Part of a System of n Circuits Subject to Saturation

The x-component of force is

$$f_x = - \sum_1^n{}_u \int_0^{\Omega_u} \frac{\partial\, i_u}{\partial\, x}\, d\, \omega_u \qquad (\mathbf{9})$$

During the integration, let the linkages increase uniformly and proportional to each other throughout the system. That is,

$$\omega_u = \frac{\Omega_u}{\Omega_1}\, \omega_1 \frac{\text{linkages}}{10^8} \qquad (\mathbf{27})$$

Then equation (**9**) may be written

$$f_x = - \sum_1^n{}_u \frac{\Omega_u}{\Omega_1} \int_0^{\Omega_1} \frac{\partial\, i_u}{\partial\, x}\, d\, \omega_1 \qquad (\mathbf{28})$$

The total excitation i_u in any circuit u may be broken up into two parts: i_{ua}, which accounts for the magnetic potential drop in the air, and i_{ui}, which accounts for the drop in the iron.

Then suppose, first, that the distribution of lines of force in the air does not change as the amount of flux through each circuit is increased, that is, the intensities of the field in air at various points are fixed in relative amount. Then the intensity of the field at every point will be proportional to the value of the flux linkages in any circuit, for example, the linkages ω_1 in circuit 1. Then in any circuit, that part of the excitation i_{ua} which is required on account of the magnetic potential drop in the air will be always directly proportional to ω_1; and the factor of proportionality is obviously the same as that which applies at low densities, when saturation is absent.

Thus, $i_{ua} = \Sigma\, K_{uv}\, \omega_v$

$$= \left[\sum_1^n{}_v K_{uv} \frac{\Omega_v}{\Omega_1} \right] \omega_1 \text{ amperes} \qquad (\mathbf{29})$$

Suppose, secondly, that in the circuit u the part of the excitation i_{ui} consumed in iron depends only on the linkages and is independent of the position of the circuit.

Then,

$$\frac{\partial\, i_{ui}}{\partial\, x} = 0 \qquad (\mathbf{30})$$

and we have

$$\frac{\partial\, i_u}{\partial\, x} = \frac{\partial}{\partial\, x} \Big[i_{ua} + i_{ui} \Big] = \frac{\partial\, i_{ua}}{\partial\, x}$$

$$= \sum_1^n{}_v \frac{\Omega_v}{\Omega_1} \frac{d\, K_{uv}}{d\, x}\, \omega_1 \text{ amperes/cm.} \qquad (\mathbf{31})$$

Substituting in (**9**),

$$f_x = -\sum_1^n {}_u \sum_1^n {}_v \frac{\Omega_u\,\Omega_v}{\Omega_1^2}\,\frac{d\,K_{uv}}{d\,x}\int_0^{\Omega_1}\omega_1\,d\,u$$

$$= -\sum_1^n {}_u \sum_1^n {}_v \frac{\Omega_u\,\Omega_v}{2}\,\frac{d\,K_{uv}}{d\,x}\text{ joules/cm.} \qquad \textbf{(32)}$$

subject to the assumptions indicated.

It is not the intention here to give special applications of these equations. The proper use of such approximations always requires a knowledge of circuit conditions of the particular case, in order to determine whether the assumptions are justified. In other words, the assumptions and corresponding equations are given, and their proper application must, of course, be left to judgment in particular cases.

Appendix E

The coefficients of magnetization are evaluated for 1, 2, 3 and 4 circuits.

According to equation (**24**)

$$I_u = \sum_1^n {}_v\, K_{uv}\,\Omega_v$$

That is, K_{uv} is the coefficient of Ω_v in the solution for I_u. With I_u expressed in determinant form, K_{uv} is the first minor of the term Ω_v.

a. *One circuit*

$$L_0\,I = \Omega$$

$$I = \frac{1}{L_0}\,\Omega$$

$$K_{11} = K = \frac{1}{L_0}\;\frac{\text{amperes}}{\text{linkages}} \times 10^8 \qquad \textbf{(33)}$$

b. *Two circuits*

$$L_1\,I_1 + M\,I_2 = \Omega_1$$
$$M\,I_1 + L_2\,I_2 = \Omega_2$$

$$I_1 = \frac{\begin{vmatrix} \Omega_1 & M \\ \Omega_2 & L_2 \end{vmatrix}}{D}$$

$$I_2 = \frac{\begin{vmatrix} L_1 & \Omega_1 \\ M & \Omega_2 \end{vmatrix}}{D}$$

$$D = \begin{vmatrix} L_1 & M \\ M & L_2 \end{vmatrix}$$

$$K_{11} = \frac{L_2}{D} \qquad \textbf{(35)}$$

$$K_{12} = K_{21} = -\frac{M}{D} \qquad \textbf{(36)}$$

$$K_{22} = \frac{L_1}{D} \qquad \textbf{(37)}$$

c. *Three circuits*

$$L_1\;I_1 + M_{12}\,I_2 + M_{13}\,I_3 = \Omega_1$$
$$M_{12}\,I_1 + L_2\;I_2 + M_{23}\,I_2 = \Omega_2$$
$$M_{13}\,I_1 + M_{23}\,I_2 + L_3\;I_3 = \Omega_3$$

Solving these equations in determinant form, the denominator is

$$D = \begin{vmatrix} L_1 & M_{12} & M_{13} \\ M_{12} & L_2 & M_{23} \\ M_{13} & M_{23} & L_3 \end{vmatrix} \qquad \textbf{(38)}$$

and

$$K_{11} = \frac{\begin{vmatrix} L_2 & M_{23} \\ M_{23} & L_3 \end{vmatrix}}{D} \qquad \textbf{(39)}$$

$$K_{12} = K_{21} = -\frac{\begin{vmatrix} M_{12} & M_{13} \\ M_{23} & L_3 \end{vmatrix}}{D} \qquad \textbf{(40)}$$

$$K_{13} = K_{31} = \begin{vmatrix} M_{12} & M_{13} \\ L_2 & M_{23} \end{vmatrix} \qquad \textbf{(41)}$$

$$K_{22} = \frac{\begin{vmatrix} L_3 & M_{13} \\ M_{13} & L_1 \end{vmatrix}}{D} \qquad \textbf{(42)}$$

$$K_{23} = K_{32} = -\frac{\begin{vmatrix} L_1 & M_{13} \\ M_{12} & M_{23} \end{vmatrix}}{D} \qquad \textbf{(43)}$$

$$K_{33} = \frac{\begin{vmatrix} L_1 & M_{12} \\ M_{12} & L_2 \end{vmatrix}}{D} \qquad \textbf{(44)}$$

d. *Four Circuits*

$$L_1\,I_1 + M_{12}\,I_2 + M_{13}\,I_3 + M_{14}\,I_4 = \Omega_1$$
$$M_{12}\,I_1 + L_2\,I_2 + M_{23}\,I_3 + M_{24}\,I_4 = \Omega_2$$
$$M_{13}\,I_1 + M_{23}\,I_2 + L_3\,I_3 + M_{34}\,I_4 = \Omega_3$$
$$M_{14}\,I_1 + M_{24}\,I_2 + M_{34}\,I_3 + L_4\,I_4 = \Omega_4$$

Thus

$$D = \begin{vmatrix} L_1 & M_{12} & M_{13} & M_{14} \\ M_{12} & L_2 & M_{23} & M_{24} \\ M_{13} & M_{23} & L_3 & M_{34} \\ M_{14} & M_{24} & M_{34} & L_4 \end{vmatrix} \qquad \textbf{(45)}$$

and

$$K_{11} = +\frac{\begin{vmatrix} L_2 & M_{23} & M_{24} \\ M_{23} & L_3 & M_{34} \\ M_{24} & M_{34} & L_4 \end{vmatrix}}{D} \qquad \textbf{(46)}$$

$$K_2 = K_{21} = -\frac{\begin{vmatrix} M_{23} & L_3 & M_{34} \\ M_{24} & M_{34} & L_4 \\ M_{12} & M_{13} & M_{14} \end{vmatrix}}{D} \qquad \textbf{(47)}$$

$$K_{13} = K_{31} = +\frac{\begin{vmatrix} M_{24} & M_{34} & L_4 \\ M_{12} & M_{13} & M_{14} \\ L_2 & M_{23} & M_{24} \end{vmatrix}}{D} \qquad \textbf{(48)}$$

$$K_{14} = K_{41} = -\frac{\begin{vmatrix} M_{12} & M_{13} & M_{14} \\ L_2 & M_{23} & M_{24} \\ M_{23} & L_3 & M_{30} \end{vmatrix}}{D} \tag{49}$$

$$K_{22} = +\frac{\begin{vmatrix} L_3 & M_{34} & M_{13} \\ M_{34} & L_4 & M_{14} \\ M_{13} & M_{14} & L_1 \end{vmatrix}}{D} \tag{50}$$

$$K_{23} = K_{32} = -\frac{\begin{vmatrix} M_{34} & L_4 & M_{14} \\ M_{13} & M_{14} & L_1 \\ M_{23} & M_{24} & M_{12} \end{vmatrix}}{D} \tag{51}$$

$$K_{24} = K_{42} = +\frac{\begin{vmatrix} M_{13} & M_{14} & L_1 \\ M_{23} & M_{24} & M_{12} \\ L_3 & M_{34} & M_{13} \end{vmatrix}}{D} \tag{52}$$

$$K_{33} = +\frac{\begin{vmatrix} L_4 & M_{14} & M_{24} \\ M_{14} & L_1 & M_{12} \\ M_{24} & M_{12} & L_2 \end{vmatrix}}{D} \tag{53}$$

$$K_{34} = K_{43} = -\frac{\begin{vmatrix} M_{14} & L_1 & M_{12} \\ M_{24} & M_{12} & L_2 \\ M_{34} & M_{13} & M_{23} \end{vmatrix}}{D} \tag{54}$$

$$K_{44} = +\frac{\begin{vmatrix} L_1 & M_{12} & M_{13} \\ M_{12} & L_2 & M_{23} \\ M_{13} & M_{23} & L_3 \end{vmatrix}}{D} \tag{55}$$

LIST OF SYMBOLS

x, y, z = coordinates in cm. of the position of any part of the circuit.

f_x = the mechanical force in the direction x exerted by the magnetic field on that part of the circuit under consideration. Units, joules/cm.

i_u = the general symbol for current in amperes in circuit u.

I_u = the particular current in circuit u at which the force is measured.

ω_u = the general symbol for linkages in circuit u. Units, magnetic linkages $\times 10^{-8}$.

Ω_u = the particular value in circuit u, of linkages ω_u at which the force is measured.

N = the number of effective series turns linking the magnetic flux of the circuit.

E_u = magnetic energy storage in joules in circuit u.

p_u = power input to circuit u in watts.

t = time in seconds.

r_u = resistance of circuit u in ohms.

L_u = M_{uu} = the self inductance in henrys of circuit u without saturation.

M_{uv} = the mutual inductance in henrys of circuits u and v without saturation.

K_{uv} = the "coefficient of magnetization" of circuits u and v, without saturation. See Appendix E.

L = the self-inductance in henrys of a single circuit with saturation.

L_o = L_1 = the self-inductance in henrys of a single circuit without saturation.

$\mathcal{R}$ = reluctance coefficient for the entire magnetic circuit. Units, turns/henrys.

$\mathcal{R}_o$ = reluctance coefficient for the air path of the magnetic circuit.

$\mathcal{R}_i$ = reluctance coefficient for the iron path of the magnetic circuit.

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Is opposed to the "action-at-a-distance" method and develops a method called the energy method.

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Discussion

J. Slepian: The relation which is perhaps most frequently used by electrical engineers for calculating mechanical forces between circuits is that due to Maxwell, which states that, in any displacement in which currents are kept constant, the mechanical work received during the displacement is equal to the increase in magnetic energy of the system. This relation is readily derived from the principle of conservation of energy by taking into account the electrical input during the displacement. As Professor Karapetoff puts it, there is a "fifty-fifty" rule here; half the electrical input goes to increasing the magnetic energy and half is given up to mechanical work.

However, this fifty-fifty rule does not apply when there is iron in the neighborhood, subject to saturation, and Messrs. Doherty and Park do well to bring out this fact strongly. In my own experience I have seen several cases where this rule led to completely erroneous results when applied to practical machines. In one instance, a complete change in sign of the force was involved, a repulsion was predicted, whereas actually an attraction was found. The magnetic circuit shown in Doherty and Park's Fig. 1 is one for which application of the fifty-fifty rule would give the wrong sign if the iron is saturated. With constant current, motion of the armature upward *decreases* the magnetic energy, and yet the armature is attracted and not repelled.

The great contribution of Doherty and Park in this paper lies in showing that application of the principle of conservation of energy in another way leads to a relation which is universally applicable, saturation or no saturation. Namely, if the mechanical displacement is effected, not with constant currents but with constant flux linkages, then the mechanical work received is equal to the *decrease* in magnetic energy. The authors further add to the value of this result by showing how the calculation on this basis may be carried out, when the curves connecting flux linkages with currents for various positions of the moving parts are known.

I have a suggestion to make as to the formulas (*9*) and (*11*) which the authors give, and the corresponding expression for magnetic energy. While it is principally a matter of notation, I think it is important because it makes such a difference in the clarity of ideas. For simplicity, considering only two circuits, the authors would write the magnetic energy,

$$E = \int_0^{\Omega_1} i_1 \, d\,\omega_1 + \int_0^{\Omega_1} i_2 \, d\,\omega_2$$

To bring out more definitely that i_1 and i_2 are functions of both variables, w_1 and w_2 I shall write this as,

$$E = \int_0^{\Omega_1} i_1 \, (\omega_1 \, \omega_2) \, d\,\omega_1 + \int_0^{\Omega_2} i_2 \, (\omega_1 \, \omega_2) \, d\,\omega_2$$

Now ω_2 is not a constant in the first integral, but may be varied as ω_1 varies, and ω_1 is not a constant in the second integral. However, whatever relation is adopted between ω_2 and ω_1 in the first integral, the same relation must be used in the second if the correct value for magnetic energy is to be obtained.

Now integrals of this kind are of frequent occurrence in mathematical physics, and they are usually denoted by line integrals written thus:

$$E = \int_{\substack{\omega_1 = 0 \\ \omega_2 = 0}}^{\substack{\omega_1 = \Omega_1 \\ \omega_2 = \Omega_2}} i_1 \, d\,\omega_1 + i_2 \, d\,\omega_2$$

This single integration is to be carried out over a curve in the $\omega_1 \, \omega_2$ plane, joining the point $\begin{pmatrix} \omega_1 = 0 \\ \omega_2 = 0 \end{pmatrix}$ to point $\begin{pmatrix} \omega_1 = \Omega_1 \\ \omega_2 = \Omega_2 \end{pmatrix}$.

In general, the value of such a line integral depends upon the path chosen in going from the initial point to the final point, but in this case since the magnetic energy is the same, however the state $\begin{pmatrix} \omega_1 = \Omega_1 \\ \omega_2 = \Omega_2 \end{pmatrix}$ is reached, the integral is independent of the path. Mathematicians have shown that in this case it follows that

$$\frac{\partial\, i_1}{\partial\, \omega_2} = \frac{\partial\, i_2}{\partial\, \omega_1}.$$

This may be looked upon as a generalization for the case of saturation, of the well-known fact for the linear case, that the mutual inductance of circuit No. 1 upon circuit No. 2 is equal to the mutual inductance of circuit No. 2 upon circuit No. 1.

On the fourth page first column of their paper, the authors give a number of formulas applying to their Fig. 1, and state that the quantity L_0, which appears there, is the inductance corresponding to no saturation. However, in their derivation of these formulas, if I understand their derivation correctly, L_0 is taken as the inductance corresponding to the air gap alone. That is, L_0 is the inductance corresponding to infinite permeability, and not to no saturation. There thus seems to be an omission in the authors' proof, which might be rectified as follows:

Let

L_0 = inductance corresponding to air-gap alone: *i. e.* inductance corresponding to infinite permeability.

Let

L_0 = inductance corresponding to no saturation,

Let

L_1 = inductance corresponding to iron alone with no saturation, *i. e.*, inductance with zero air-gap and no saturation.

Then,

$$\frac{1}{L_0'} = \frac{1}{L_0} + \frac{1}{L_1}$$

Hence, since L_1 is constant,

$$\frac{\partial}{\partial\, x}\left(\frac{1}{L_0'}\right) = \frac{\partial}{\partial\, x}\left(\frac{1}{L_0}\right)$$

Hence, since L_0 appears in the authors' formulas only in the factor

$$\frac{\partial}{\partial\, x}\left(\frac{1}{L_0}\right)$$

therefore L_0' may replace L_0 throughout.

R. H. Park: In order to test the ease of applicability of the formulas developed and the agreement of calculated and observed results, a calculation and test was made in a particular case.

The circuit employed (see Fig. 1 herewith) consisted of two magnetic yokes separated by a brass strip and excited by direct current in two coils connected in series.

In order to determine the attractive force of the yokes, a weight was attached to the lower yoke and the exciting current was reduced until the yokes separated.

In order to calculate the force, the flux linkages ω were measured with a ballistic galvanometer for different air-gap and excitation. This data are shown in Fig. 2.

From the saturation curves, the curves of $\dfrac{\partial\, i}{\partial\, x}$ at constant ω were computed (see Fig. 3) as outlined in the paper. The forces were then calculated from the areas under these curves.

The agreement of observed and calculated results is shown in Fig. 4. The smooth lines in the figure represent the calculated, and the small circles, the observed results.

The difference between observed and calculated results is about 4 or 5 per cent, the observed results being, in general, less than the calculated. The tendency for the observed results to be low may be accounted for by unavoidable disagreement in the positions of the center of moment of the load and of the attractive force resulting in less effective magnetic pull, or by the effect of vibration.

From a theoretical point of view, we know that until the saturation of the core is sufficient to change appreciably the distribution of the field in air, the curves of $\dfrac{\partial\, i}{\partial\, x}$ should be linear.

It may be observed that the experimental results agree with this requirement, the curves of $\dfrac{\partial\, i}{\partial\, x}$ being linear except at very high saturation.

This fact indicates that the approximate expression for force [equation (**65**) of the paper] holds good in this case until extremely high values of saturation are encountered.

Thus, the experimental results obtained agree with those theoretically deduced in the paper.

R. E. Doherty: The authors wish to thank Dr. Slepian for his suggestion regarding notation. It is undoubtedly a somewhat clearer representation of quantities than that used by them.

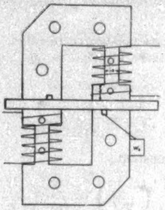

FIG. 1

With respect to his other suggestion, however, the authors have not made the omission in logic which he alleges.

They have treated two different *special cases* where saturation is taken into account, and in the results of both cases, the term L_0 appears. The symbol L_0 is defined in the list of notations as "the self-inductance in henrys of a single circuit without saturation," and it therefore, in general, includes the effect of the reluctance of the iron as well as of the air. Dr. Slepian's point is that the effect of the reluctance in the iron, when saturation is absent, is not taken into account.

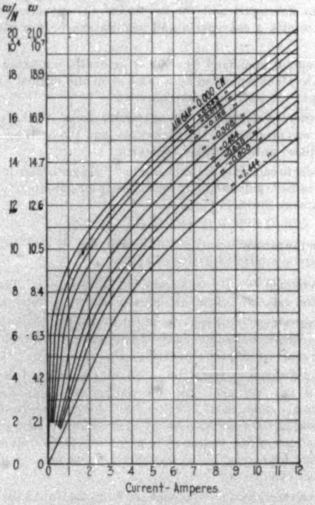

FIG. 2

The two cases treated are

a. "Special Case," in connection with Fig. 3 of the paper. Here it is specifically stated that, so far *as* L_0 is concerned, the magnetic reluctance of the iron is neglected. Therefore L_0 in this special case in the inductance corresponding to zero reluctance in the iron. Hence Dr. Slepian's discussion, referring to the equations based on this assumption, is not pertinent.

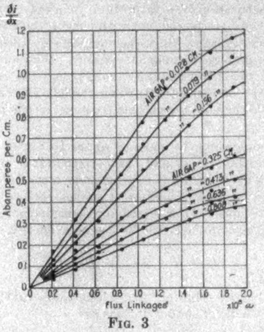

FIG. 3

b. Another case, Appendix D, in which no qualifying assumption is made with respect to L_0 except those given in the definition, quoted above, from the list of symbols. And, therefore, if it is desired to consider the case which Dr. Slepian has in mind, he should use the equations for this case given in Appendix D.

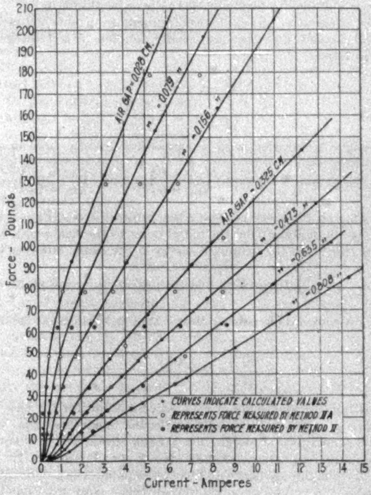

FIG. 4

Ventilation of Turbo Alternators—Concluding Study

Multiple Path Radial Systems

BY C. J. FECHHEIMER[1] and G. W. PENNEY
Fellow, A. I. E. E. Associate, A. I. E. E.

***Synopsis.**—In 1924 two papers were presented before the A. I. E. E. on Turbo Alternator Ventilation. In one of the papers tests on two models for two methods of ventilation were described, and data from the tests were given. The other paper contained a mathematical treatment for one system of ventilation, which was based upon the data obtained from the tests. It was recognized that the tests were not sufficiently accurate to evaluate the loss coefficients, nor was it possible to obtain data on the distribution of volumes for the intake vents. For that system, (see Fig. 1), it was found that the influence of rotation upon total volumes and their distribution could be neglected; consequently the investigation could be continued on stationary models. Those tests, the methods of determining the losses and the equations derived therefrom are given in this paper.*

Since the effect of rotation could be neglected, the test could be reduced to a model which represented only one axial row of vent ducts. On this model the stator vent ducts were imitated by square brass tubes with a plaster of paris restriction cast in one end to imitate the vent duct restriction so that the pressure drop obtained for either direction of flow was approximately the same as in a stator vent. A long steel channel or duct represented the section of the air-gap. Some of these brass tubes, representing intake vents, lead from a large sheet metal box to the gap channel. Other tubes, representing discharge vents, lead from the gap channel to the atmosphere. The gap channels could be interchanged readily to represent various sizes of air-gaps. Any desired number of intake and discharge tubes could be used so as to represent any desired layout of the machine.

The volumes in individual intake tubes were measured by reading on a manometer the drop in pressure from the intake chamber to a point a little way down the tube. For the discharge tubes small impact tubes were employed. Each tube was carefully calibrated before making any other tests; a thermal volume meter was used for calibrating an orifice, and the orifice was used for the tube calibrations. Various difficulties arose, and the means of overcoming them are given.

Tests were made with a group of intake vents, and the losses were separated into: (1) those in the tubes; (2) those accompanying a right angle turn; (3) those due to a stream of one velocity impinging upon a stream of another velocity; (4) those due to sudden increase in velocity; (5) those due to surface frictions. Similar tests on the discharge side showed that the losses there could be separated into: (1) those in the tubes; (2) those accompanying a right angle turn; (3) those due to sudden decrease in velocity; (4) those due to surface friction.

The losses were put, for the most part, into comparatively simple expressions. They were then combined in order to obtain final solutions. There was no difficulty on the intake side in obtaining a differential equation which admitted ready integration. On the discharge side, however, it was necessary to use approximations. With the aid of the final equations the total volumes and their distribution can be calculated for a given pressure drop. One difficulty is that the equations include trigonometric and hyperbolic functions of quantities involving the distance between the point where the gap velocity is maximum to the points where the gap velocity is zero. The latter point, called the "balance point", is not known, and a simultaneous solution of the transcendental equations needed for its determination is impossible. Suggestions for a direct simple approximate solution are given, which may be followed by trial and error methods. In most of the applications, only one or two trials were required.

The equations were checked for accuracy by comparison with tests made on the tubes, on the turbo model of 1922 and 1923, and with those on an actual machine. The agreement in total volumes was very close, and is considered to be quite good for distribution also.

* * * * *

INTRODUCTION AND PURPOSE

At the 1924 Midwinter Convention of the American Institute of Electrical Engineers, two companion papers were presented on the general subject of Ventilation of Turbo Alternators.[2] One dealt with a description of two models on which the experiments were made, the methods of conducting tests, together with representative data and the important conclusions. The other paper (by Bratt) was mathematical, the equations being derived for an ideal system, in which all losses in the air-gap were neglected; he increased the losses empirically on the discharge side, so that the measured and calculated total pressure drops agreed. Bratt's equations applied to the system of ventilation in which the air flows axially in the air-gap, shown schematically in Fig. 1. That system was the one adopted by the Westinghouse Company, as tests on the turbo models indicated its superiority over the other system in which the air flows circumferentially in the air-gap.

The experimental data obtained from the large wooden models were too inaccurate to enable an evaluation to be made of the losses of head at the turns and other parts of the air circuit so as to incorporate them in the equations. Furthermore, no method was then or is now available for measuring the volumes in the individual intake vent ducts. It was recognized then that it was an erroneous assumption to consider that all of the additional losses were on the discharge side, as some were on the intake side as well. In order to enable the designer to calculate the volumes of air and their distribution, further experiments were made as described in this paper.

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[1] Research Engineer, Westinghouse Electric & Mfg. Co.

[2] "An Experimental Study of Ventilation of Turbo Alternators" by C. J. Fechheimer, and "Multiple-Radial System of Cooling Large Turbo Generators" by Donald Bratt, TRANS. A. I. E. E., 1924, pp. 476 and 467.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

Perhaps the most important conclusion reached as a result of the earlier experiments was that with a ventilation system as shown in Fig. 1, the influence of rotation upon the total volume of air is nearly negligible. Thus, the total volumes with the rotor vents closed and with pressures of the order of five inches of water were reduced about 4 per cent by changing the rotor speed from 0 to 3600 revolutions per minute (surface velocity of 24,600 feet per minute). With the rotor vents open, and with higher pressures, there was even a smaller reduction in volume; consequently the change between standstill and normal speed could be ignored in machines as are ordinarily built, with pressures of 10 inches of water or more.

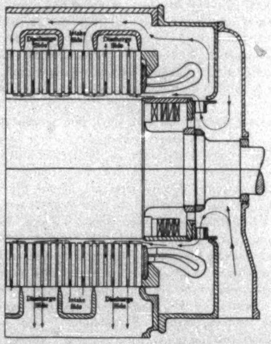

Fig. 1—Scheme of Ventilation of Turbo Alternator

A second important conclusion was that on the discharge side the axial distribution of volumes in the vent ducts was made somewhat more uniform by the influence of rotation. The influence is less the larger the air gap, and for the air gaps usually employed, the effect of rotation upon axial distribution is quite small. (See Figs. 34 and 35 on pp. 495 and 496 of the 1924 A. I. E. E. Transactions.) On the intake side, undoubtedly there is likewise a modification in axial volume distribution due to rotation, but now from theoretical considerations there are probably greater differences between maximum and minimum vent duct velocities than at standstill.[3] But the change is doubtless quite small with the usual size of air-gap. In the large machines, the coils are sunk, so that there is an axial passage above them in which the flow is not disturbed appreciably by rotation. (This may be seen for the outlet vents from a comparison of Figs. 33 and 34 in the 1924 paper.) Also, the tests on the turbo model were made at the reduced total end-bell pressure of about 3.3 inches of water. In the large modern machines, the end-bell pressure is considerably higher, 10 inches and more. Then all the velocities in the vents and gap are greater, and the influence of rotation upon vent duct volume distribution becomes still less. Certainly, whatever the influence of rotation is, it is sufficiently small to justify the assumption that it may be neglected.

This meant that the study could be continued on stationary models. They could be of such reduced size that the tests could be made in a laboratory instead of in a factory, and thus not conflict with production. The conditions that obtain in a machine with axial flow in the gap are the same for all positions circumferentially, so that a study of flow with one set of vents would be sufficient. The construction of the laboratory apparatus and the manner of conducting the experiments have been fully justified by the close agreement between the wooden turbo model and machine data, and the laboratory data.

Apparatus

The vent ducts were imitated by square brass tubes, two of which are shown in Fig. 2. From a knowledge of the pressure drops in vent ducts obtained from earlier laboratory tests on small ducts, it was possible to determine how to shape the sections within the square tubes by calculation, so that the pressure drop in the individual tube would approximate the drop in a vent duct for either direction of flow. A special mandrel was made, and plaster of paris was cast about this mandrel on the inside of the tubes.[4] It was found that because of lack of uniformity at the ends of the plaster of paris, there were many irregularities in the test results. Consequently, small brass insets were made as shown in Fig. 2, and the plaster of paris behind them was formed accurately to dimensions.

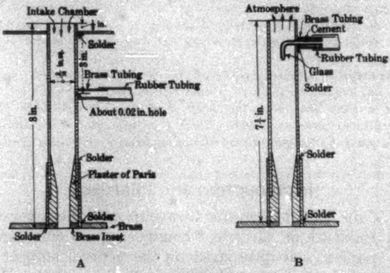

Fig. 2—Construction of Square Tubes used in the Experiments

(a) Tube representing intake vent duct
(b) Tube representing discharge vent duct

Fig. 2A shows one of the intake tubes and Fig. 2B one of the outlet tubes. The air enters the intake tubes from a relatively large chamber and there is a loss of head at, and just beyond, the entrance of air to the tube of approximately ½ of a velocity head ($\frac{1}{2}V^2/2g$). In addition, there is the velocity head itself ($V^2/2g$) so that the total difference in pressure from the large intake chamber to a point a little distance down the tube is approximately $1.5\,V^2/2g$. Hence, if a manometer is connected between the large intake chamber and a point down the tube the readings should be of such order that they could be observed with considerable accuracy. That was the principle used for measuring the volumes in the intake tubes, the construction being indicated in Fig. 2 A. A sample tube was tried out before all the tubes were made and the method was found to be satisfactory.

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[3] This is discussed more fully in the Appendix.

[4] Metals of various kinds had been tried, but had been abandoned.

On the outlet side, the air discharges from the individual tube directly into the atmosphere. Neglecting what small friction drop there is in the tube, the static pressure a short distance from the end is nearly the same as that of the surrounding atmosphere. Consequently, the method used for measuring volumes on the intake side could not be applied on the discharge side. The construction that was adopted is shown in Fig. 2B. Inside the large square brass tube was a small impact tube which was connected by the usual rubber tubing to a manometer, the other side of the manometer being open to the atmosphere. In the construction which was first used steel impact tubes were employed, but these rusted, broke, and differed so much from one another that they were all removed, and replaced by small glass tubes. Those were bent 90 degrees, and the impact ends were ground to fine edges. Holes were drilled through the square brass tubes large enough to permit of the insertion of the glass tubes. Brass tubes of 0.25-inch outside diameter were soldered to the square tubes and the space between the glass and the round brass was filled with a cement which hardened and supported the glass. Then the glass was cut off flush with the end of the round brass tube.

In using the intake tubes, the manometer was connected by means of rubber tubing between the round brass tube soldered to the square tube and the relatively large intake chamber in which latter the velocities were so low that the influence of velocity head could be neglected. On the discharge side, the manometer was connected to the impact tube. Each tube was carefully calibrated before any other tests were made. The calibration differed slightly from one another, due to small errors in construction, which necessitated referring to the calibration data when the final data were used.

All tubes were soldered into brass plates about 0.25 inch thick, and were assembled in several combinations. There were two groups of eight tubes each for the intake side. For the discharge side there were two groups of four tubes each, two of six tubes each, and one group of eight tubes, making a total of 28 discharge tubes. Channels of various sections to imitate different air gaps were made to bolt to the brass plates, rubber gaskets being used to eliminate leakage. Groups of tubes could be joined together at their ends, for which suitable flanges and bolts were provided. The channels were made of fairly heavy steel, accurately machined, as sheet metal which was first tried, distorted and introduced uncertainties.

There are three set-ups, two of which are shown clearly in the illustrations, Figs. 3 and 6. In Fig. 3, which shows the first set-up, intake conditions only are imitated. In Fig. 4 it is shown schematically. The second set-up imitated the discharge only, and it is shown schematically in Fig. 5. In Fig. 6, for the third set-up, the ventilating system of one-half of a generator, with one intake chamber at either side of the center line, is simulated. In the first set-up, the data taken enabled the loss coefficients on the intake side to be determined. From the data taken with the second set-up, the discharge loss coefficients were

FIG. 3—FIRST SET-UP, IMITATING THE INTAKE SIDE OF A TURBO ALTERNATOR

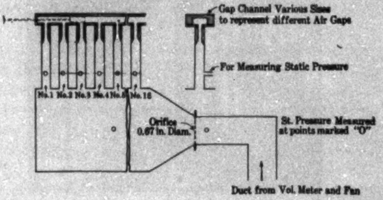
Gap Channel Various Sizes to represent different Air Gaps
For Measuring Static Pressure
No.1
No.2
No.3
No.4
No.5
No.16
Orifice 0.67 in. Diam.
St. Pressure Measured at points marked "O"
Duct from Vol. Meter and Fan

FIG. 4—SCHEME OF FIRST SET-UP, IMITATING THE INTAKE SIDE OF A TURBO ALTERNATOR

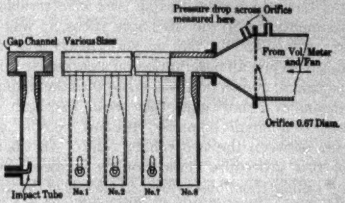

FIG. 5—SCHEME OF SECOND SET-UP, IMITATING THE DISCHARGE SIDE OF A TURBO ALTERNATOR

calculated. The third set-up combined the first two, and in addition, air was admitted directly to the air gap. All 44 tubes were then used, which corresponds for the most part to more than would be employed in the equivalent part of a well-designed turbo-alternator ventilating system. In Fig. 6, the tubes at the left simulate vent ducts adjacent to the center of a machine. Those at the right imitate vents adjacent to the end bell. The intake vents are between these two sets of discharge vents.

In any set-up, the number of tubes in any part could be altered at will, thus imitating generators of different lengths. To this end, special square rubber stoppers were made to close individual discharge tubes. Several may be seen in Fig. 6. On the intake side it was found more convenient to remove the particular section of the gap channel, and close individual tubes with wax.

In order to connect the manometer with any of the intake tubes, a brass pipe with closed ends was equipped with a large number of cocks, and rubber tubing joined a particular cock with its corresponding intake tube (Figs. 3 and 6). The brass pipe and connections were tested for leaks before starting, and at intervals during experiments. For the discharge tubes, it was found more convenient to put on short lengths of rubber tubing. When an observation on a particular tube was to be taken, connection was made with the manometer by means of a short glass tube which had been drawn down to a small diameter, thereby completing the circuit between the two rubber tubes. On the intake side, this procedure could not be followed, because while taking those observations, one side of the manometer was kept connected to the intake chamber, and had the other side of the manometer been opened to the atmosphere, the liquid would have been blown out of the manometer.

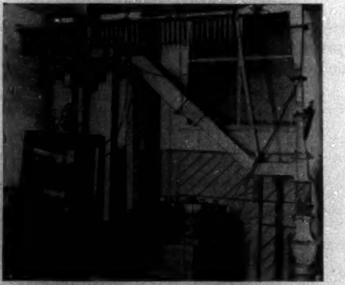
FIG. 6—THIRD SET-UP, IMITATING THE VENTILATING SYSTEM OF HALF OF A TURBO ALTERNATOR

Two small thermal volume meters were made, the principle of which was similar to the well-known one of Prof. Carl Thomas. They differed in that instead of resistance coils groups of thermocouples in series were used to measure the temperature rise. One meter was 1.5 inches square, and the other 3.5 inches square, these being inside dimensions. The small volume meter appears in Figs. 3 and 6, and the larger one in Fig. 6, and they may be readily distinguished by the fact that wires are connected to them. The small volume meter was used to calibrate the tubes and was also used in all the set-ups. In calibrating the tubes, it was found simpler to use an orifice which in turn had been calibrated by means of the small volume meter. In the first set-up, the orifice was in a thin plate and was placed between the flanges of the diverging duct leading to the intake chamber and the connecting square sectioned pipe (see Figs. 3 and 4). The velocities in the sheet metal ducts were very low, so that readings of pressure were accurate. The calibration of the tubes is further dealt with under "Difficulties in Measurement."

With the small volume meter, volumes from about 15 to 50 cubic feet per minute could be measured. The larger meter could be used up to about 200 cu. ft. per minute. To determine the accuracy of these volume meters, which were smaller than any that had previously been made, they were connected in series with a volume meter that had been in use for some time, and in which the discharged air was forced to pass around parts of the outside to avoid the escapement of heat. In such small volume meters the heat stored in the walls is sufficient to necessitate the delay of several minutes before taking an observation. Other possible sources of error will be described in an early paper. One of them has been known to be poor distribution of volumes[5]. That was the reason why the duct just below the small volume meter in Fig. 3, and the larger meter in Fig. 6, has curved sides.

The blower does not appear in the photographs, but it is just below the fairly large duct near the extreme right. It was capable of delivering up to about 200 cu. ft. per min. at 10 inches of water. Its outlet was joined to the other ducts by means of soft rubber sheet, thereby eliminating vibrations from the blower that would otherwise have been transmitted to the rest of the apparatus. Cheese cloth and parts of an air filter were introduced just above the blower outlet to cleanse the air.

DIFFICULTIES IN MEASUREMENT

1. Calibration of Tubes. It was found from the check on the thermal volume meters that they were liable to be in error for small volumes, and further that the time required for the steady state to be reached for small volumes was too long to make its use for calibrating individual tubes practical. The causes of error will be gone into in another paper. The error was negligible for volumes above 15 cu. ft. per minute, but the calibrations were to be for volumes of about 5 to 10 cu. ft. per minute.

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[5] For derivation of equation of error, due to poor distribution, see Appendix of paper by C. J. Fechheimer, A. S. M. E. *Transactions*, 1924, "Centrifugal Fans for Electrical Machinery."

Four tubes were connected in parallel and then the volume through the volume meter was from 20 to 40 cu. ft. per minute, which could be read quite accurately. Then the orifice and the channel imitating the air gap path were removed, because the former caused too large pressure drop, and the presence of the latter would have introduced inequalities in distribution other than imposed by unequal resistances to flow. When the orifice was in place, the air flowed through single tubes. The volume for the individual tube was taken to be proportional to the square root of the pressure drop across the orifice, and the sum of the volumes for four tubes was that given by the small volume meter. By proportioning the total volume (given by the volume meter) according to the square roots of the pressure drop across the orifice, the volume in the individual tube was determined with accuracy. It was, of course, necessary to maintain the same pressure in the intake chamber in a series of such tests. As a final check other combinations of tubes were tried, and the results were found to agree with those calculated from the first calibrations.

For the discharge tubes, it was necessary to keep the gap channel in place, resulting in unequal division between tubes. The orifice had previously been calibrated with the intake tubes. However, it was believed that the drops across the orifice for those conditions with a diverging channel beyond the orifice, probably would not be quite the same as when a converging channel was used. (See Figs. 4 and 5.) Considering the high velocities through the orifice and the very low velocities before and after, the first orifice calibrations were probably not very different from those for the second set-up. They could be used to determine the approximate flow when single discharge tubes were open. Then, using those approximate calibrations with several discharge tubes open, and with the orifice removed, the volume meter was read. Its reading checked within about one per cent of the sum of the volumes from the approximate individual tube calibrations. The final calibrations of the discharge tubes were obtained by correcting the approximate calibrations by the amount thus found. The orifice calibration with the converging duct, as used for the discharge tubes, was also modified so that those data could be used in other tests.

Due to slight irregularities in the areas at the smallest sections, the maximum difference in volumes in individual tubes for a given pressure was found to be about 4 per cent. This applies to intake as well as to discharge tubes.

It was at first thought that the impact tube method of measuring volumes, as used for the discharge tubes, would be more reliable than the entrance pressure drop method used for the intake tubes, since impact measurements are usually more dependable than are those for static pressure. It was found, however, that the reverse was the case in these experiments. Thus, when taking observations of discharge volume distribution with several tubes open, some of the points did not lie on a smooth curve, which was not the case for the intake tubes. The discrepancy was of the order of 3 or 4 per cent. Four small impact tubes were supported in position just beyond one of the square discharge tubes from which the glass impact tube had been removed. The ratio of readings taken with them did not remain fixed, showing that the character of flow changed, and the other impact tubes undoubtedly were similarly affected. Apparently, the highly turbulent state in the gap channel, and immediately after entrance to the square tube was not entirely eliminated before the impact tube was reached. It is well known that with some turbulent states, the flow may change and be periodic. These tests also proved that the character of flow in the tube is affected by the magnitude of the gap velocity. This was indicated by the fact that the volume in a duct depended somewhat upon the gap velocity, as well as upon the square root of the static pressure difference. Subsequent reference to this will be made. For the intake tubes the velocity in the large chamber was negligible, so that any flow there did not influence the drop to the point where the readings were taken.

2. *Pressures in the Air Gap.* In order to determine where the losses of head occurred, it was necessary to know the static pressure in the air-gap as well as the average velocity there. Anyone who has attempted to measure static pressures in a high-velocity, or turbulent, stream will appreciate how difficult it is. As originally planned, the plunger in the gap channel, shown dotted in Fig. 4, was to be ahead of those square tubes that were closed by some other means, such as by rubber stoppers. The handle of the plunger was a round brass tube in which a small hole was drilled near the plunger head. There were small clearances around the plunger head, permitting air to pass back and enter the hole, so that if the tube handle were connected to a manometer its indication would be the pressure in the gap channel where the hole was drilled. It was believed that the reading thus obtained would be the static pressure immediately beyond the other end of the plunger. Thus, in Fig. 4, if the square tubes 1, 2 and 3 were closed, the velocity immediately in front of the plunger head would be zero, so that the static there could be read with accuracy. Subsequent tests indicated that the method was accurate. The objection to the plunger method was that static pressures in the gap could not be read, except in regions where there was no flow, and it was important to obtain readings in other parts where the velocities were of considerable magnitude.

The tests made by the "toy balloon" method, shown in Fig. 37 and described on page 496 in the 1924 A. I. E. E. TRANSACTIONS, indicated that a reasonable accurate measure of the gap static pressure could be obtained by closing a vent and then observing the pressure in the vent some distance from the gap. That method was tried in these tests also. To determine the air gap pressure at a given tube, the two adjacent tubes were closed at their far ends. The manometer readings were then the static pressures in the gap. The average of these readings for the two adjacent tubes was taken to be the desired static pressure. That this method was satisfactory was proved by subsequent tests. It was used chiefly for the discharge tubes, but it was found to check on the intake side also.

3. Elimination of Leakage. Experimental studies with air are not accurate unless leakage is eliminated. In these investigations, after all parts were fastened with rubber gaskets at all joints, the blower and duct joined thereto were disconnected from the rest of the apparatus, and a plate with suitable gasket was bolted over the entrance to the volume meter. With a manometer connected at a point in the system, the pressure was increased by blowing in at another point, after which the latter was closed. If the system were leakless, the manometer reading would not change. Much time was spent until this condition was attained, and a preparation, consisting largely of bees' wax, was used to seal up the joints. In Figs. 3 and 6, some of the flange joints appear to be very thick; this was because melted wax was poured on them to seal them effectively A suggestion is offered to others contemplating similar work that the flange material be heavy, around 0.125 in., which prevents springing out of shape between bolts, as was the case with the sheet material used in these tests.

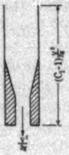
DETERMINATION OF THE LOSSES

A. On the Intake Side

The air streams from the intake vents discharge into a common channel (the air-gap), turn a right angle, flow axially through the air-gap, and then enter those parts of the air-gap that are in the discharge zone. In the first set-up, they were discharged into the atmosphere. It seemed evident that the losses of head (representing energy loss) could be divided into four groups: (1) Loss within the radial vent duct; (2) loss accompanying the turning of a right angle; (3) loss due to sudden increase in velocity when an incoming stream impinged upon a stream normal to it; and (4) loss due to surface friction. There must be, of course, an additional allowance for the velocity head, ($V^2/2g$) as the stream leaves the intake and enters the discharge zone in the air gap (or discharges into the atmosphere), in order to check the total pressure. These losses will now be considered in the same order:

1. Loss Within the Radial Vent. This was determined at the time the tests on the turbo-model were made, in 1922 and 1923, by means of a full-size model imitating a single vent duct, the pressure drop for various velocities being measured. As the air entered the model from a relatively large reservoir and discharged into another, the velocities in them were very low; consequently, the pressure drops could be measured with accuracy. As given at that time, the coefficient for the radial vent was called $C\,i$, as defined by the equation: $\frac{p}{\gamma} = C\,i\,\frac{V^2}{2\,g}$ (p = pressure drop, γ = density, V = velocity at the minimum section, and g = acceleration of gravity). For the tubes, the value of $C\,i$ was again measured by removing the gap channel, and allowing the intake tubes to discharge into the atmosphere. The value of $C\,i$ thus found agreed very well with the previous tests. As the velocity head at discharge from the intake vent is $V^2/2\,g$, the loss within the vent is evidently

$$(C\,i - 1)\,\frac{V^2}{2\,g}. \qquad (1)$$



2. Loss Accompanying the Turning of a Right Angle. It was found that this loss could be broken into two parts: (a) that accompanying the turning of a right angle, the velocity before and after turning being the same; and (b) that accompanying the shock when a stream of one velocity impinges upon a stream of a different velocity.

a. The loss corresponding to the turning of the corner without change of velocity is called $K_L\,\frac{V^2}{2\,g}$, where V is the velocity at the smallest section of the radial duct. Referring to Fig. 4, consider that the tests are to be made on tube or duct No. 16, and that enough ducts preceding No. 16 feed into the gap channel to secure substantially the same velocity there after passing that duct as obtains at the minimum section of No. 16. The gap velocity was controlled by the plunger, shown dotted in Fig. 3. Then, if $\frac{p}{\gamma}$ is the head in the intake chamber, and as stated previously, $(C\,i - 1)\,\frac{V^2}{2\,g}$ is the loss in the radial duct or tube, by Bernouilli's theorem:

$$\frac{p}{\gamma} = (C\,i - 1)\,\frac{V^2}{2\,g} + K_L\,\frac{V^2}{2\,g} + \frac{V^2}{2\,g} \quad \ldots\ldots \quad (2)$$

The last term corresponds to the velocity head as the air issues from the gap channel. As all quantities are known or measured except K_L, its value can be readily determined. It was found that K_L depended upon the size or width of the air gap channel, being greater the smaller the channel. This was as one might expect, as the wider channel permits larger radii for the fluid to turn.

b. The usual case is when the duct and gap velocities are not equal. Then there is the additional loss due to the high velocity stream impinging upon the stream in the gap. The magnitude of this loss was determined by taking observations with the last tube open, and the gap velocity was controlled by varying the number of tubes feeding into the gap. This was done readily by altering the position of the plunger (See Fig. 4). The intake pressure was controlled by the speed of the blower. That simplified tests, and made results more certain, because the static pressure just beyond the last tube was substantially equal to that of the atmosphere. The loss in the tube, given by $C\,i\,\frac{V^2}{2\,g}$, and the loss for turning a right angle having been previously determined, the additional loss sought was readily obtained. In all of the tests, the velocity V, at the minimum section of the square tube, was never less than the velocity in the gap, v, just beyond the last tube. That corresponds to conditions in the actual generator in which the length of an intake belt is so chosen with regard to the areas that those velocity relations obtain. The loss due to the impinging stream was found to be:

$$\frac{V^2 - v^2}{2\,g} \qquad (3)$$

This expression is only approximately true. In some cases a little velocity head may be converted into static. However, the loss thus given plus that given by $K_L\,\frac{V^2}{2\,g}$ accounts for the total in turning the right angle almost within the degree of accuracy of measurement.

If the expression for these losses is given as

$$K_L\,\frac{V^2}{2\,g} + \frac{V^2 - v^2}{2\,g},$$

it is found that for the particular case when $v = 0$, or approaches zero (for the first vent feeding into the gap) the loss is $(K_L + 1)\,\frac{V^2}{2\,g}$ which is more than a velocity head. This is not correct. For the determination of K_L, V and v were made equal, so that v may be substituted for V. The loss is then:

$$K_L\,\frac{v^2}{2\,g} + \frac{V^2 - v^2}{2\,g} \qquad (4)$$

Then if $v = 0$, the loss is $\frac{V^2}{2\,g}$ which is substantially correct. Returning to equation (2) for evaluating K_L, the expression becomes:

$$\frac{p}{\gamma} = (Ci - 1)\,\frac{V^2}{2\,g} + K_L\,\frac{v^2}{2\,g} + \frac{v^2}{2\,g} \qquad (5)$$

Using the set-up shown schematically in Fig. 4, and remembering that the volumes, and therefore the velocities, were determined with the use of the tube calibrations previously obtained, the total calculated pressure drops were compared with many conditions of set-ups and the agreements were very close. The expression used is:

$$\frac{p}{\gamma} = (C\,i - 1)\,\frac{V^2}{2\,g} + K_L\,\frac{v^2}{2\,g} + \frac{V^2 - v^2}{2\,g} + \frac{v^2}{2\,g} \qquad (6)$$

3. *Loss Due to Sudden Increase in Gap Velocity.* As a stream from a vent duct impinges upon the stream in the gap, the velocity of the gap stream increases suddenly due to the increase in volume. Just as for the loss due to the high-velocity stream impinging upon the stream of lower velocity, the expression was the difference of the squares of the velocities, so for sudden increases in velocities the form should be similar. Having made that assumption, its validity was checked by tests. The form of the equation was then:

Loss due to sudden increase in gap vel.

$$= K\,\frac{v_2^2 - v_1^2}{2\,g} \qquad (7)$$

Here v_2 and v_1 are respectively the gap velocities after and before the sudden increase, and K is a coefficient to be determined experimentally.

The following was the method used for evaluating K. The plunger was removed and many ducts fed into the gap (see Fig. 4). Then the velocity in the duct (No. 1 in Fig. 4) was very low, and the drop from the intake chamber to the gap at duct No. 1 was small. The gap static pressure at duct No. 1 was nearly equal to the intake chamber pressure. Although a correction was made for the drops in that tube, the static pressure in the gap at No. 1 was known quite accurately. If equation (7) holds, and K is the same for all points in the gap, the loss of head for the full length of the gap is $K\,\frac{v_0^2 - v_1^2}{2\,g}$. (Here v_0 is the final discharge gap velocity, at atmospheric pressure, and v_1 is the gap velocity at duct No. 1.) This expression must be equal to the total pressure drop, the final atmospheric pressure being zero. Thus K could be evaluated. This takes account not only of the loss due to sudden increase in velocity, but of the friction drop in the gap as well.

4. *Loss Due to Surface Friction.* The friction losses are small, and have been included among other losses. In the vent ducts (tubes) they are included in the coefficient Ci. In the gap they are taken as part of the coefficient K as explained in the preceding paragraph.

Combination of the Equations on the Intake Side

Call P the pressure in the intake chamber, and p_0 the pressure in the gap at its discharge points; (this was atmospheric pressure in the tests described above, or taken as zero for reference). By Bernouilli's equation,

$$\frac{P}{\gamma} = \frac{p_0}{\gamma} + \frac{v_0^2}{2g} + \text{sum of losses from discharge}$$

point back to the intake chamber.

Or

$$\frac{P}{\gamma} = \frac{p_0}{\gamma} + \frac{v_0^2}{2g} + (C\,i - 1)\ \frac{V^2}{2g} + \frac{V^2 - v^2}{2g} + K_L\,\frac{v^2}{2g} + K\,\frac{v_0^2 - v^2}{2g} \qquad (8)$$

To test the accuracy of equation (**8**), (Ci, K_L, and K having been previously determined), tests were made by measuring the volume of air passing through each vent duct and the total calculated pressure drop was checked for 63 combinations, involving various numbers of vent ducts and sizes of gaps. In every test there were as many parallel paths for the air as there were vent ducts. In Table I are given the final results and per cent differences between calculated and test pressures for one test. Pressures are in inches of water. The agreement between tests and calculations is undoubtedly close.

TABLE I

I Vent Tube Number	II Vel. V in Vent.	III Total Calc. Press.	IV Total Meas. Press.	V Per Cent Difference
3	9480	8.48	8.18	+3.7
5	7280	8.25	8.18	+0.9
6	6070	8.29	8.18	+1.4
7	5090	8.33	8.18	+1.8
8	4100	8.09	8.18	−1.1
9	3400	8.11	8.18	−0.9
10	2650	8.12	8.18	−0.7
11	2300	8.17	8.18	−0.1
12	2140	8.20	8.18	+0.2
13	1860	8.18	8.18	0.0
14	1620	8.17	8.18	−0.1
15	1480	8.15	8.18	−0.4
16	1420	8.15	8.18	−0.4

DETERMINATION OF THE LOSSES

B. *On the Discharge Side*

On the discharge side, the air streams flow from the air gap through parallel vent ducts out to a common discharge chamber. In these tests they discharged into the atmosphere. The losses of head were divided into four groups: (1) loss within the radial vent ducts; (2) loss accompanying the turning of a right angle; (3) loss due to sudden decrease in velocity arising when some of the fluid discharged through a vent duct; (4) loss due to surface friction. For the particular set-up there was also a small loss in the converging duct leading to the gap channel, but this loss does not appear in the machine, in the parts that are fed from the intakes. There is an entrance loss whose magnitude depends upon the entrance conditions in machines at the end-bell ends. Approximate data obtained in the turbo-model tests in 1923 permit evaluation of the latter loss with sufficient accuracy.

1 and 2. Loss Within the Radial Vent, and Loss Accompanying the Turning of a Right Angle. As for the intake vents, the losses in the radial vents were determined at the time the tests on the turbo-model were made in 1922 and 1923. They were made with the same apparatus but the direction of flow was reversed. The co-efficient was called C_d, as defined by the equation $\frac{p}{\gamma} = C_d\,\frac{V^2}{2g}$. The meanings of the symbols are the same as before. It was recognized that the conditions in the machine, or in this model, might be different from what they were in the small set-up of 1923, because the entrance conditions were different. The loss accompanying the turning of a right angle could not readily be separated from the entrance conditions, and there was little to be gained, if that could have been done without great difficulty. Consequently those two losses were combined, and the new values of the coefficient C_d take account of the turning of a right angle, of the entrance loss to the vent duct, and the loss in the vent itself.

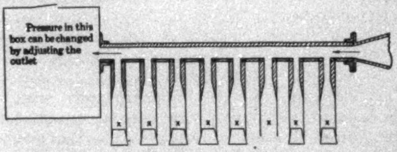


FIG. 7—SCHEMATIC DIAGRAM, SHOWING THE METHOD IN WHICH TESTS WERE MADE FOR EVALUATING LOSS COEFFICIENTS ON THE DISCHARGE SIDE

One vent duct was left open. All other vents were closed by rubber stoppers. The pressure in the gap can be changed by adjusting the outlet from the box at the end of the gap so that any combination of velocity and pressure can be obtained. Impact tubes located at points marked "x" give vent duct velocity head if the vent duct is open, or static pressure in the gap if the vent duct is closed by a rubber stopper.

To determine the value of C_d and the gap loss, a special set-up, shown schematically in Fig. 7, was devised. Only one vent duct (tube) was open, and most of the air passed through the air gap into a relatively large box with an adjustable outlet. By adjusting the size of this outlet, any desired combination of gap velocity and static pressure could be secured. The pressure in the gap was determined by the method previously described: the pressures in the adjacent ducts were measured, and the mean taken as the gap pressure at the vent duct. With only one vent duct open, the losses of head from that vent to the box were the velocity head at discharge from the gap channel and the friction drop therein, both of which could be calculated with reasonable accuracy. The two methods of determining the static pressure in the gap checked quite well.

In Fig. 8 will be found a plot for one test. The air flowed in the gap channel from right to left, and No. 3

tube was the only one open. The points indicated as ⊙ were the measured static pressures as read in the closed vent ducts. The points indicated as + were calculated by figuring the friction drop back from the box with adjustable outlet in which the measured static pressure was 2.4 inches of water. The point indicated as ⊗ is the average static at the two extremes of vent No.3. The value of C_d is then given by

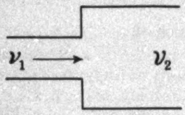
$$C_d = \frac{p}{\gamma} \bigg/ \frac{V^2}{2g} \qquad (9)$$

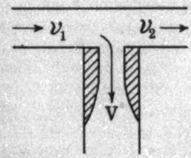
The value of C_d is not constant but is dependent upon the gap velocity.

3. Loss Due to Sudden Decrease in Velocity. It was shown largely by theory and checked by experiment that when there is a sudden enlargement of cross-section, the loss of head is:

$$\frac{(v_1 - v_2)^2}{2g} {}^* \qquad (10)$$

v_1 and v_2 are the velocities before and after the enlargement, as given by the volume divided by the cross-sectional area. It may, without much difficulty, be shown that the decrease in velocity in the gap due to the abstraction of the air through the vent duct is comparable with the case of sudden enlargement, and probably should be given by the same equation. Equation (**10**) was found to apply with considerable accuracy, except for the case when $v_2 = o$. Even then the loss considered as part of the total was not great. To check equation (**10**) tests were made in a similar manner to those for the individual tubes, and the gap static pressure on either side of the particular tube was measured. The latter method was better than the other because near the end of gap channel the velocities were low and the static pressures could be measured with accuracy.





4. Loss Due to Surface Friction. From the curve in Fig. 8 it will be seen that the friction loss is by no means negligible. It is given by the well-known formula to be found in any standard work on hydraulics:

$$\text{Friction drop} = f\,\frac{L\,R}{A}\,\frac{v^2}{2g} \qquad (11)$$

wherein.

f = coefficient of friction
L = length
R = perimeter of duct
A = area of cross-section of duct
v = velocity
g = acceleration of gravity

If v is feet per minute, and the density is taken as 0.074 lb. per cu. ft., the friction drop in inches of water is

$$f\,\frac{L\,R}{A}\left(\frac{v}{4030}\right)^2$$

f = about 0.0054.

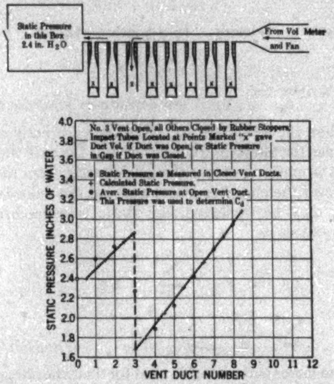


Fig. 8—Example of Plotted Data Showing Method of Determining Static Pressure for Evaluating $C\,d$

Combination of the Equation on the Discharge Side. Considering the case where the air enters the discharge part of the air-gap from the intake side (that is, not the case where the air enters the air-gap directly from the end bell; see Fig. 1), then the static pressure in the gap at the division point between intake and discharge is p_0. The velocity in the gap at that point is v_0. Following the path of a stream of air from that point to some other point in the gap, the equation, by Bernouilli's theorem, is:

$$\frac{p_0}{\gamma} + \frac{v_0^2}{2g} = \frac{p}{\gamma} + \frac{v^2}{2g} + \sum \text{losses from the}$$

division point to the point in the gap considered.

$$\frac{p_0}{\gamma} + \frac{v_0^2}{2g} = \frac{v^2}{2g} + C\,d\,\frac{V^2}{2g} + \sum \frac{(v_1 - v_2)^2}{2g} + \sum f\,\frac{L\,R}{A}\,\frac{v^2}{2g} \qquad (12)$$

---

*See Gibson: "Hydraulics and Its Applications." Second Edition. pp. 82 and 83. The equation is quite different from what one might at first glance expect:

$$\frac{v_1^2 - v_2^2}{2g}$$

Here $\sum \frac{(v_1 - v_2)^2}{2\,g}$ means the summation of all losses due to sudden decrease in velocity from the division line up to the particular vent duct considered; and $\sum f \frac{L\,R}{A} \frac{v^2}{2\,g}$ is the summation of friction drops between the same points, account being taken of the changes in velocity. In equation (**12**), $C_d \frac{V^2}{2\,g}$ has been written for $\frac{p}{\gamma}$.

When the air enters the gap channel directly, as was the case for the second set-up, or as is the case when the air in a machine enters the gap directly from the end-bell, and the chamber is so large that the velocity head there may be neglected, the static pressure being P, the equation is

$$\frac{P}{\gamma} = \frac{v^2}{2\,g} + \frac{p}{\gamma} + \sum \frac{(v_1 - v_2)^2}{2\,g} + \sum f \frac{L\,R}{A} \frac{v^2}{2\,g} \qquad (13)$$

This latter equation was used for final checking of the discharge loss coefficients. In Table II the results of one test are summarized.

TABLE II

I Vent Tube Number	II Meas. Stat. Press.	III Avg. Meas.* Stat. Press.	IV Avg. Calc. Stat. Press.	V Per Cent Difference
8	7.03			
7		7.32	7.15	2.3
6	7.62			
5		7.84	7.73	1.4
4	8.06			
3		8.20	8.10	1.2
2	8.32			
1			8.25	

*Average of measured pressures on each side of open duct.

Derivation of Equations

In order to obtain final expressions which will be useful to the designer, the general method used by Bratt in his 1924 A. I. E. E. paper was followed. The assumption is made that there are an infinite number of vent ducts each of infinitesimal width, instead of a finite number, each of finite width. This assumption is comparable with the one usually made in polyphase alternating current machinery: there is a sine wave distribution of magnetomotive forces and fluxes.

INTAKE SIDE

Let S be the vent duct area at the minimum section per unit length axially. Then for a short length $d\,x$ the vent area $= S\,d\,x$ (Fig. 9). If the vent duct velocity is V at the position x, the volume of air passing through the vent area is $V\,S\,d\,x$. This volume augments the previous volume in the air-gap, causing an increment in air-gap velocity $d\,v$. If a is the cross sectional area of the air-gap, the incremental volume is $a\,d\,v$. Actually on the intake side the volume increases as x decreases, (see Fig. 9). Consequently the equation of continuity may be written as:

$$S\,V\,d\,x = -\,a\,d\,v.$$

or

$$V = -\,\frac{a}{S}\,\frac{d\,v}{d\,x} \qquad (14)$$

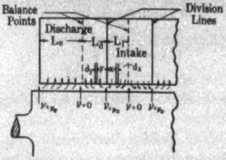


FIG. 9—SCHEMATIC DIAGRAM OF THE VENTILATING SYSTEM IN A TURBO ALTERNATOR, SHOWING MEANINGS OF SYMBOLS USED

Combining terms from equation (**8**),

$$\frac{P}{\gamma} = \frac{p_0}{\gamma} + C_i \frac{V^2}{2\,g} + (1 + K) \frac{v_0^2}{2\,g} - (1 + K - K_L) \frac{v^2}{2\,g} \qquad (15)$$

Differentiating with respect to x, and simplifying,

$$C_i\,V\,\frac{d\,V}{d\,x} - (1 + K - K_L)\,v\,\frac{d\,v}{d\,x} = 0$$

From equation (**14**),

$$\frac{d\,V}{d\,x} = -\,\frac{a}{S}\,\frac{d^2 v}{d\,x^2}$$

which substituted in the above equation gives:

$$C_i \left(\frac{a}{S} \right)^2 \frac{d\,v}{d\,x} \cdot \frac{d^2\,v}{d\,x^2} - (1 + K - K_L)\,v\,\frac{d\,v}{d\,x} = 0$$

or,

$$\frac{d^2\,v}{d\,x^2} - \alpha^2\,v = 0 \qquad (16)$$

where

$$\alpha = \frac{S}{a} \sqrt{\frac{1 + K - K_L}{C_i}} \qquad (17)$$

A solution is:

$$v = A \cosh \alpha\,x + B \sinh \alpha\,x \qquad (18)$$

where A and B are arbitrary constants.

The terminal conditions are that $v = 0$ for $x = L_i$, and $v = v_0$ for $x = 0$. Then,

$$v = v_0 \frac{\sinh \alpha\,(L_i - x)}{\sinh \alpha\,L_i} \qquad (19)$$

Substituting in (**14**), and differentiating,

$$V = v_0\,\alpha\,\frac{a}{S}\,\frac{\cosh\alpha\,(L_i - x)}{\sinh\alpha\,L_i} \qquad (\mathbf{20})$$

The total pressure is obtained by substituting the values of v and V in equation (**15**). The value thus found holds for all values of x, and may be taken at either of the boundaries. There is little difference, as regards simplification, which boundary is used. Taking $x = L_i$ the result is:

$$\frac{P}{\gamma} = \frac{p_0}{\gamma} + \frac{v_0^2}{2\,g}\left[\frac{1 + K - K_L}{\sinh^2\,\alpha\,L_i} + (1 + K)\right] \qquad (\mathbf{21})$$

Discharge Side

Referring to Fig. 9, if, as before, V is the vent duct velocity at position y, the volume of air passing a vent duct of width $S\,d\,y$ is $V\,S\,d\,y$. This volume decreases the previous volume in the air-gap by $a\,d\,v$. As the air-gap volume decreases with increasing y, this quantity is negative. So $V\,S\,d\,y = -\,a\,d\,v$. Or,

$$V = -\,\frac{a}{s}\,\frac{d\,v}{d\,y} \qquad (\mathbf{22})$$

The equation with losses (number **13**) can, with the use of (**22**), be reduced to the form of a differential equation. Thus, by considering the losses over a short length $d\,y$, the summation signs can be replaced by integrals. Then the whole reduces to a differential equation by taking the derivative. The form of that equation was rather complicated, and although a solution was obtained, it was so involved as to be impracticable. It was decided, therefore, to make approximations which would alter the form of equation (**13**), and still be close enough for general application. The assumption was made that there were no gap losses, that C_d was constant and that its value corresponded to the highest gap velocity. This was the highest value for C_d, so that compensated for having ignored the gap losses. For $y = 0$, the assumptions are the same as for the actual conditions, since the value of C_d used corresponds to that point, and the gap losses there are zero. At points where the gap velocity is lower, the assumed value of C_d is too high, but since then the gap losses are neglected, the static pressure, as calculated by Bernouilli's equation, is high. As the vent duct velocity is dependent upon the quotient of the gap pressure and C_d, and as both are high, the error is quite small.

With these approximations, Bernouilli's equation for the gap channel on the discharge side may be written as:

$$\frac{p}{\gamma} + \frac{v^2}{2\,g} = \text{total head} = \text{a constant.}$$

Here p is the pressure and v the velocity at any point in the air-gap.

Making use of equation (**9**)

$$C_d\,\frac{V^2}{2\,g} + \frac{v^2}{2\,g} = \text{constant} \qquad (\mathbf{23})$$

Substituting for V from (**22**) and differentiating,

$$\frac{d^2\,v}{d\,y^2} + \frac{1}{C_d}\left(\frac{S}{a}\right)^2 v = 0 \qquad (\mathbf{24})$$

A solution will be recognized as:

$$v = A\,\sin\beta\,y + B\,\cos\beta\,y \qquad (\mathbf{25})$$

where

$$\beta = \frac{S}{a}\sqrt{\frac{1}{C_d}} \qquad (\mathbf{26})$$

The terminal conditions are that $v = 0$ for $y = L_d$ and $v = v_0$ for $y = 0$ (See Fig. 9).

The final forms of the solution are then:

$$v = v_0\,\frac{\sin\beta\,(L_d - y)}{\sin\beta\,L_d} \qquad (\mathbf{27})$$

$$V = \frac{v_0}{\sqrt{C_d}}\,\frac{\cos\beta\,(L_d - y)}{\sin\beta\,L_d} \qquad (\mathbf{28})$$

$$\frac{p}{\gamma} = C_d\,\frac{V^2}{2\,g} = \frac{v_0^2}{2\,g}\,\frac{\mathrm{Cos}^2\,\beta\,(L_d - y)}{\sin^2\beta\,L_d} \qquad (\mathbf{29})$$

At the division lines, $y = 0$, and then

$$\frac{p_0}{\gamma} = \frac{v_0^2}{2\,g}\cot^2\beta\,L_d \qquad (\mathbf{30})$$

The total pressure in the intake chamber may now be determined readily by eliminating $\frac{p_0}{\gamma}$ between equations (**21**) and (**30**). Then:

$$\frac{P_i}{\gamma} = \frac{v_0^2}{2\,g}\left[\frac{1+K-K_L}{\sinh^2\,\alpha\,L_i} + (1+K) + \cot^2\beta\,L_d\right] \qquad (\mathbf{31})$$

If the density is taken as 0.074 lb. per cu. ft., the velocities are in ft. per min., and the head is in inches of water, this equation is:

$$P = \left(\frac{v_0}{4030}\right)^2\left[\frac{1+K-K_L}{\sinh^2\,\alpha\,L_i} + (1+K) + \cot^2\beta\,L_d\right] \qquad (\mathbf{32})$$

Considering now the air that enters directly from the end bell and is then discharged (Fig. 9), it will readily be seen that the same equations apply as before, except that now the loss at entrance must be considered, and one velocity head in addition must be added. The equation for end bell pressure then is

$$\frac{P_e}{\gamma} = \frac{v_e^2}{2\,g}\,(1 + k + \cot^2\beta\,L_d) \qquad (\mathbf{33})$$

Now

P_e = the end bell pressure.
v_e = velocity in the gap at the end bell entrance.
k = loss coefficient at entrance.

$\beta = \frac{S}{a}\sqrt{\frac{1}{C_d}}$ but C_d is generally different from that for the other case, as the velocity v_e is usually not the same as v_0.

L_d = length from end bell to the balance point where $v = 0$.

The question may arise why there is a velocity head term $\frac{v_e^2}{2\,g}$ in equation (**33**) and a corresponding term $\frac{v_0^2}{2\,g}$ does not appear in equation (**30**). The explanation is that it is necessary always to establish the velocity in the gap, but for the case when the air flows from the intake through the gap and then is discharged, the velocity head $\frac{v_0^2}{2\,g}$ is charged against the intake side, as the air is flowing at the velocity v_0 in the gap when it leaves the intake.

SUMMARY OF FINAL EQUATIONS

Intake	*Discharge*
$v = v_0 \frac{\sinh \alpha\,(L_i - x)}{\sinh \alpha\, L_i}$	$v = v_0 \frac{\sin \beta\,(L_d - y)}{\sin \beta\, L_d}$
$V = \alpha \frac{a}{S} v_0 \frac{\cosh \alpha\,(L_i - x)}{\sinh \alpha\, L_i}$	$V = \frac{v_0}{\sqrt{C_d}} \frac{\cos \beta\,(L_d - y)}{\sin \beta\, L_d}$

$$P_i = \gamma \frac{v_0^2}{2\,g}\left[\frac{1 + K - K_L}{\sinh^2\,\alpha\,L_i} + (1+K) + \cot^2 \beta\, L_d\right]$$

$$P_e = \gamma \frac{v_0^2}{2\,g}\,(1 + k + \cot^2 \beta\, L_e)$$

Note that to compute the velocities on the discharge side adjacent to the end bell, the above velocity equations may be used, but y and L_d are now measured from the end bell end instead of from the division point, v_e must be substituted for v_0, and the value of C_d must be used that corresponds to v_e instead of to v_0.

In using the equations it is perhaps a little more convenient to take S as the area of one circle of stator vents at the minimum section, and then L_d, L_e and L_i are in number of vents instead of in actual lengths.

In the above equations:

$$\alpha = \frac{S}{a}\sqrt{\frac{1 + K - K_L}{C_i}} \qquad \beta = \frac{S}{a}\sqrt{\frac{1}{C_d}}$$

a = cross-sectional area of the air gap.
C_d = discharge coefficient in the radial vent ducts.
C_i = intake coefficient in the radial vent ducts.
k = loss coefficient at entrance to gap, end bell end.
K = loss coefficient for sudden increase in velocity.
K_L = loss coefficient for turning a right angle.
L_d = distance from division point to balance point, discharge side.
L_e = distance from end bell to balance point.
L_i = distance from division point to balance point, intake side.
P_e = pressure in the end bell.
P_i = pressure in the intake chamber.
S = cross-sectional area of stator vents at the minimum section per unit length axially.
v_0 = velocity in gap at division points.
v_e = velocity in gap at end bell end.
V = velocity in stator vents at minimum section.
x = variable distance from division point, intake side.
y = variable distance from division point, discharge side.

DETERMINATION OF THE BALANCE POINTS

The equations which have been derived contain the values of L_d and L_i. These lengths are unknown since the points where the velocities in the gap are zero have not been determined. Those points, which we have called "balance points," are determined by the following conditions:

1. The pressure drops for all parallel paths must be the same.
2. The vent duct velocities at the balance point must be the same when computed for the two parallel circuits which meet at that point.

The transcendental forms of the equations make a simultaneous solution impossible, so a trial and error, or an approximate solution must be used. Those methods were followed by Bratt as given in his paper, but his solutions were not found to be suitable, and other lines of attack were followed.

An approximate solution was obtained by making the following additional assumptions:

1. The velocities in the vent ducts in a given belt are equal. This applies to the intake and discharge, but the two are not taken equal to each other.
2. The pressure drop in any circuit from intake chamber to outlet $= C_i \frac{V_i^2}{2\,g} + \frac{v_e^2}{2\,g} + C_d \frac{V_d^2}{2\,g}$.*
3. The pressure drop in the circuit from the end bell to outlet $= (1 + k)\frac{v_e^2}{2\,g} + C_d \frac{V_d^2}{2\,g}$.

The equations are then in simple algebraic form. A simultaneous solution is then possible regardless of the number of parallel paths, and the approximate values of L_d, L_e and L_i can be determined. The pressure drops in the various parallel paths and the vent duct velocities at the balance points are compared, using the values of the lengths as found in the transcendental equations.

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*The symbols are the same as before, but V_i and V_d represent respectively the intake and discharge vent duct velocities.

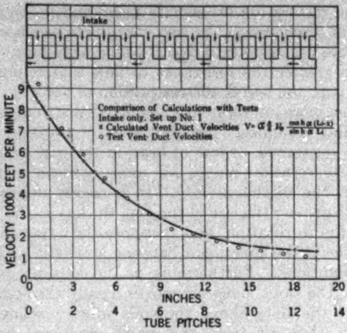

Fig. 10—Comparison of Calculations with Tests. Intake Side Only. Set-Up No. 1

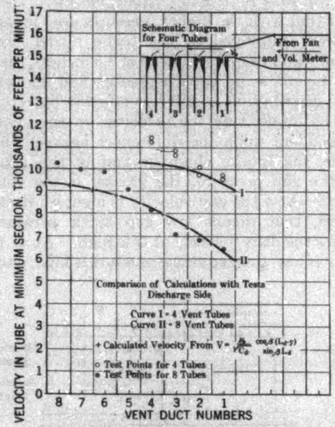

Fig 11—Comparison of Calculations with Tests. Discharge Side Only. Set-Up No. 2

Then, if the pressure drops for parallel circuits, and the vent duct velocities at the approximate balance points, do not compare favorably, the lengths may be changed, and the calculations repeated. In a number of cases tried, it was not found necessary to repeat the results more than once. If the lengths are not great, so that the velocity distribution in a belt is not far from uniform, the first approximation is generally sufficiently accurate.

Checks on the Accuracy of the Equations

Most of the experimental work was to determine the losses and to check them, as summarized in Tables I and II. Many tests were thus made for the intake and discharge sides. After having derived the equations, as given elsewhere in this paper, they were checked by comparing the calculated air velocities for a large number of conditions with test results. Only a few of those comparisons are given here.

In Fig. 10, the comparison is given for one condition, using the intake tubes only. In Fig. 11, outlet vents were used, there being four for one test and eight for the other. The solid lines are calculated curves. For the intakes, the agreement between tests and calculations is better than for the discharge. This is as might be expected, as the test data were known to be less accurate for the latter, and the assumption made in regard to values of the discharge coefficient C_d, and the losses were certain to introduce some inaccuracies.

The data plotted in Fig. 12 were taken with the set-up as shown in Fig. 6. The solid lines are calculated curves, and the circles are test points. It will be noted that the calculated curves for the intake do not quite meet at the balance point. It was considered unnecessary to make another attempt to calculate the velocities with the assumed balance point shifted slightly. Attention is called to the agreement between calculations and tests for the air-gap velocities at the division points. These velocities multiplied by the cross-sectional area give the volumes. The average of these calculated velocities differs only 2.5 per cent from the average by test.

In Fig. 13 will be found comparisons of calculations

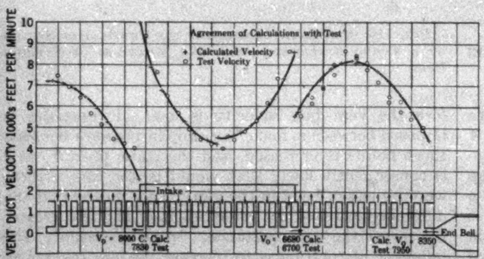

Fig. 12—Comparison of Calculations with Tests for Set-Up No. 3 (Fig. 6), Imitating One-Half of a Turbo Alternator.

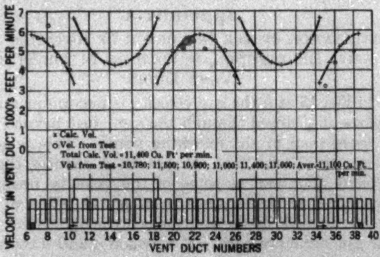

Fig. 13—Comparison of Calculations with Tests, in Original Turbo Model of 1923

and tests on the turbo model of 1923.[6] No air entered direct from the end bell, so that the balance points were always at the middle of a belt. The comparison for vent-duct velocities covers the outlet vent ducts only, as no tests were taken for the intake vents. The average of six volume meter readings was 11,100 cu. ft. per min. and the calculated was 11,400 cu. ft. per min. Those tests were with the rotor stationary.

In Fig. 14 will be found a comparison of tests and calculations on the machine for which the test data were given in Fig. 36, p. 496 of the 1924 A. I. E. E. Transactions. Here the calculated volume is 69,500 cu. ft. per min. and that by test 72,500 cu. ft. per min. The agreement between test and calculated velocities in the vents adjacent to the end bell is very good, but the test velocities are somewhat higher than calculated in the middle belt.

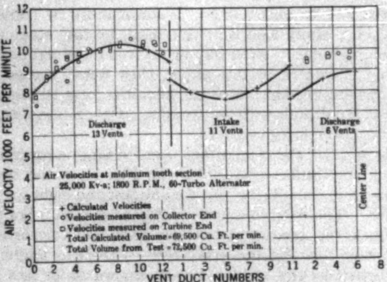

Fig. 14—Comparison of Calculations with Tests in a 25,000-Kv-a., 1800-Rev. per Min. Turbo Alternator

In general, it seems that the agreement of calculations with tests is as close as one might expect with so complicated a phenomenon as fluid flow. It is believed that, for the particular type of ventilating system, the volumes and their distributions can now be calculated with a reasonable degree of certainty, and rule of thumb methods need no longer be used.

Appendix A

Explanation of the Influence of Rotation upon Axial Distribution of Volumes in the Stator Vent Ducts

In Fig. 15 at *a* the gap velocity is maximum, and the volume discharged through the vent just beyond at *A* is lower than in the following vents. This is so because the volume that escapes at *A* reduces the gap velocity at *b*, and as some of the velocity head at *a* is restored to static pressure at *b*, the efflux at *B* is greater than *A*, as the vent velocity is dependent upon the gap static pressure. If between *a* and *b* there had been no restoration of velocity head, and if the coefficients of discharge in the two vent ducts had been the same, the velocities in *A* and *B* would have been equal. So, the greater the resistance to flow that is imposed in the gap, the less is the restoration of velocity head, and the more uniform are the axial distributions of vent duct volumes. One influence of rotation is to impose resistance, or loss of head in the gap, and consequently rotation makes for more uniform distribution on the discharge side. It is readily understood that the greater the size of the gap, the less is the resistance due to rotation, and therefore the larger the gap the closer is the agreement between standstill and running conditions. Furthermore, as the coils are usually sunk in the slots, some air moves axially through portions of the slots above the coils which considerably protects that air from the whirls produced by the rotor. (Compare Figs. 33 and 34 and 35, pp. 495 and 496 in the 1924 A. I. E. E. paper, loc. cit.) It is also evident that with the higher pressures used in machines than were employed in the turbo model tests, (10 inches of water or more, instead of 3.3 inches), the velocities in the vents and gap are greater, and the vent-duct volume distribution is still less affected by rotation.

On the intake side the velocities in the vent ducts are dependent upon the differences between the pressures in the intake chamber and in the gap. The lower the gap pressure, the greater is the vent duct velocity. At *q* the gap velocity is high; by Bernouilli's equation, that means low gap static pressure, and therefore the duct velocity at *Q* is high. At *m* the gap velocity is low, the gap static pressure is high, and therefore the vent-duct velocity is low. If resistance is imposed between *m* and *q*, the drop in pressure between those points is increased, and the static pressure at *q* is consequently lower than would be the case if there were no loss of head in the gap. That means that the difference in vent-duct velocities at *M* and *Q* is greater the larger the gap resistance. In all probability, the influence of rotation is to increase the gap resistance, and therefore augments the differences between maximum and minimum intake vent-duct volumes. However, with the large gaps, with the coils sunk in the slots,

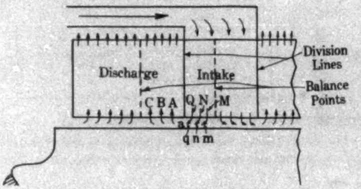

Fig. 15—Schematic Diagram of the Ventilating System of a Turbo Alternator, Showing Meanings of Symbols Used

and with the high pressures used, the influence of rotation upon the intake volumes and their distribution must be small, just as for the discharge vents.

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[6] In working up this test, the velocities in the individual ducts were obtained by comparing the average anemometer reading with the total volume by the volume meter.

Appendix B

NOTE ON THE "TOY BALLOON METHOD" FOR MEASURING DISCHARGE VOLUME DISTRIBUTION

On page 486 of the 1924 A. I. E. E. TRANSACTIONS a method for measuring pressures in the air gap is described which was called the "toy balloon method" by Mr. Knowlton when discussing the paper. The tests described in the present paper showed that the method is sound for measuring pressures. But the tests made for evaluating the vent-duct discharge coefficient, C_d, indicate that C_d is not constant, but is dependent upon the air-gap velocity. Consequently it is not correct to calculate the velocities in the vent ducts from the gap static pressures, and therefore the distribution of volumes in the discharge vent ducts of a machine can not be determined with accuracy by that method, unless suitable corrections are made. Such corrections consist in changing the velocity (or volume) in a vent inversely as the square root of the ratio of C_d corresponding to the approximate gap velocity at a particular position to the value of C_d, corresponding to zero gap velocity. The method is somewhat tedious, but is the best that we know of at present.

In Fig. 14 in the present paper (in which some of the data of Fig. 36, p. 496 of the 1924 A. I. E. E. paper are reproduced), no corrections were made for the changes in C_d. Had that been done the velocities in the end vents would have been slightly lower; that is, the test curves would have been a little steeper.

Discussion

S. L. Henderson: This paper covers the results of a considerable amount of experimental work done subsequent to the papers written in 1924 by Mr. Fechheimer and Mr. Bratt. As the later tests were made on stationary models, the constants could be determined more accurately than on a moving model, and the accuracy of the equations is demonstrated in the comparison between tests and calculations as given in Figs. 13 and 14. The results of this work are being used in the design of turbine generators with considerable practical benefit. On one design, in particular, it was possible to shorten the machine considerably through the use of these equations.

The choice of the number of vents and the division of these vents into intake and outlet zones is thus made relatively simple. The number of vents is determined by the velocity of air required in the radial vents to obtain satisfactory cooling on the sides of the vent spaces. The number of vents is also determined by the width of the iron package between vents which can be allowed without having an excessive temperature drop in the package. Several trials, as in most questions of design, must be made in order to obtain the best proportions. After the number of vents has been decided upon, the division of these vents into intake and outlet zones can be carried out through the use of the equations in this paper.

Development and Application of Loading
for Telephone Circuits

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Member, A. I. E. E.
and
WILLIAM FONDILLER[2]
Member, A. I. E. E.

***Synopsis.**—A review of the art of loading telephone circuits as practised in the United States. The introductory section briefly reviews the theory of coil loading, and summarizes the principal characteristics of the first commercial standard loading coils and loading systems, thereby serving as a background for the description of the various improvements of outstanding importance which have been made in the loading coils and loading systems during the past fifteen years to meet the new or changing requirements in the rapidly advancing communication art.*

These major improvements are described in detail under the appropriate headings (1) Phantom Group Loading, (2) Loading for Repeatered Circuits, (3) Incidental Cables in Open-Wire Lines, (4) Cross-talk, (5) Telegraphy over Loaded Telephone Circuits, (6) Loading for Exchange Area Cables, and (7) Submarine Cables. The discussion of these various developments sets forth the relations between the loading features and the associated phases of telephone development, such as the cables, repeaters, telegraph working, and carrier telephone and telegraph systems.

The concluding part of the paper gives some general statistics regarding the extent of the commercial application of loading in the United States, and a brief statement indicative of the large economic importance of loading to the telephone using public.

INTRODUCTION

THE year 1926 marks the fiftieth anniversary of the birth of the telephone, and the completion of the first 25 years of the commercial application of loading to telephone circuits by means of inductance coils inserted at periodic intervals. The present time is thus peculiarly appropriate for a survey of loading developments.

The purpose of this paper is to present a review of the art of loading telephone circuits, as practised in the United States. In a paper presented before the Institute in 1911[3] Mr. B. Gherardi described the developments in loading up to that time and gave a comprehensive statement of the results obtained. In the present paper, therefore, references to the early developments in loading may be confined to matters that are necessary to the treatment of the subsequent developments in the art.

During the period under consideration many improvements of outstanding importance have been made in the characteristics of the loading coils and in the loading systems, in order to meet new or changing requirements in the rapidly advancing communication art. The more important of these improvements are listed below and will be discussed in the sequence noted:

I. Phantom Group Loading
II. Loading for Repeatered Circuits
III. Incidental Cables in Open-Wire Lines
IV. Cross-Talk
V. Telegraphy over Loaded Telephone Circuits
VI. Loading for Exchange Area Cables
VII. Submarine Cables

As a basis for the discussion of the characteristics of commercial loading systems and the various developments which have been made, the elementary theory of loaded lines and a review of the first loading standards will be given. Those interested in the exact mathematical theory are referred to more complete discussions which may be found in the bibliography appended hereto.

Theory. It is convenient to discuss the coil loaded line in terms of its corresponding smooth line, a hypothetical line in which the constants of the inductance coils are assumed to be distributed uniformly along the line.

Table I gives simplified formulas which define the important line characteristics in terms of the primary line constants, the formulas being so arranged as to indicate directly the nature of the changes in line characteristics which occur when uniformly distributed inductance is added to a uniform line initially having zero inductance.

TABLE I
APPROXIMATE LINE FORMULAS

Line Characteristics	Uniform Line Having Zero Inductance	Uniform Line Having Distributed Inductance	
α, attenuation constant	$\sqrt{\frac{p\,R\,C}{2}}$	$\sqrt{\frac{R}{2\,L\,p}} \cdot \sqrt{\frac{p\,R\,C}{2}} = \frac{R}{2}\sqrt{\frac{C}{L}}$	(1)
W, velocity of wave propagation	$\sqrt{\frac{2\,p}{R\,C}}$	$\sqrt{\frac{R}{2\,L\,p}} \cdot \sqrt{\frac{2\,p}{R\,C}} = \sqrt{\frac{1}{C\,L}}$	(2)
Z_0, characteristic impedance	$\sqrt{\frac{R}{p\,C}}\ /\underline{45^\circ}$	$\sqrt{\frac{L\,p}{R}}\ /\underline{45^\circ} \cdot \sqrt{\frac{R}{p\,C}}\ /\underline{45^\circ} = \sqrt{\frac{L}{C}}$	(3)

In the above, α is the real part of the propagation constant; and $W = p/\beta$, in which $p = 2\,\pi\,f$ (f = frequency) and β is the wave length constant; *i. e.*, the imaginary part of the propagation constant. The formulas assume the leakage conductance G to be negligibly small; and in the case of the line with inductance, that R is small with reference to $p\,L$, R, L, and C being the line resistance, inductance, and capacitance per unit length.

Inspection of the formulas shows that the addition of distributed inductance:

(a) Reduces the attenuation constant and the velocity, provided that the ratio $R/2\,L$ is less than p.

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[1] American Telephone & Telegraph Co., New York, N. Y.

[2] Bell Telephone Laboratories, Inc., New York, N. Y.

[3] "Commercial Loading of Telephone Circuits in the Bell System," B. Gherardi, TRANS. A. I. E. E., Vol 30, 1911, p. 1743.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

(b) Increases the impedance, and improves the power factor.

(c) Makes the attenuation, velocity and impedance independent of frequency over the frequency range where R is small with reference to $p\ L$.

Investigating the question of concentrating the line inductance at uniformly spaced intervals, Professor Pupin gave his famous solution in a paper[4] presented before the Institute in May, 1900. Dr. G. A. Campbell in his paper[5] of March, 1903 also gave a mathematical development of the loading theory along somewhat different lines.

These early investigations showed that a coil loaded line should have several coils per wave length in order to simulate a uniform line. The more closely the coils are spaced the more exact is the degree of equivalence. When there are 10 coils per wave length the equivalence is very close. However, the cost of the loading increases as the spacing is shortened. Thus, from the standpoint of commercial application, the question "What is the smallest number of coils per wave length that will give satisfactory transmission?" is very important. In the investigation which was made to determine the magnitude of the changes in attenuation, velocity, and impedance, as the number of coils per wave length is reduced, abrupt changes in these characteristics were found to occur at the spacing of two coils per actual wave length; accordingly, the frequency at which this spacing condition applies in a loaded line became known as the critical frequency of the loading. The velocity of the coil loaded line at the critical frequency is lower than the velocity of the corresponding smooth line approximately in the ratio 2 to π; in other words, at the critical frequency there are (approximately) π coils per wave length, in terms of the velocity of the corresponding smooth line.

The following expression defines the critical frequency in a coil loaded line having zero distributed inductance:

$$f_c = \frac{1}{\pi \sqrt{L\,s\,C}} \qquad (4)$$

in which

f_c = critical frequency
L = coil inductance
s = coil spacing
C = line capacitance per unit length

[If the loaded line has distributed inductance, a correction is required in equation (**4**).]

At the critical frequency and higher frequencies, the attenuation loss is so extremely large that it amounts practically to a suppression, or "cut-off," effect. For this reason the term "cut-off frequency" has come into general use as a substitute for the somewhat less definitive term, "critical frequency."

The differences between the characteristics of a coil loaded line and its corresponding smooth line are sometimes designated "lumpiness" effects. They are due to internal reflection phenomena at the points of electrical discontinuity in the line caused by the insertion of the loading coils. The lumpiness effects are usually small for the frequencies below approximately 75 to 80 per cent of the cut-off frequency. As the frequency exceeds this value, however, the lumpiness effects increase at an accelerated rate.

Figs. 1, 2 and 3 illustrate the differences in the attenuation, velocity, and impedance characteristics of a typical telephone cable, with and without loading. The characteristics of the corresponding smooth loaded line are also indicated, to illustrate the theoretical differences between uniform loading and coil loading. Fig. 1 includes curves which give an analysis of the different types of line losses, (*a*) the "series" losses due to heat dissipation in the conductor and the loading coils which are proportional to the square of the line current, (*b*) the "shunt" losses due to heat losses in the dielectrics, which are proportional to the square of the line voltage, and (*c*) the lumpiness effects due to internal reflections. The sharp rise in attenuation with frequency as the cut-off frequency is approached is clearly indicated.

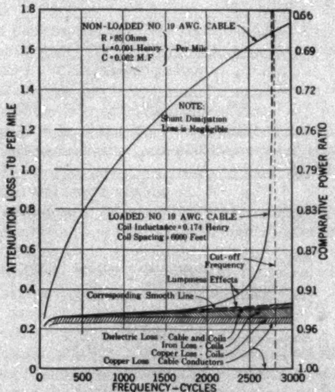

FIG. 1—ATTENUATION-FREQUENCY CHARACTERISTICS OF LOADED AND NON-LOADED NO. 19 A. W. G. CABLE

Fig. 3 illustrates the dependency of the characteristic impedance of a coil loaded line upon the terminal condition. The most frequently used loading terminations are "mid-section" and "mid-coil." In the mid-section termination, the first loading coil is located at a distance equivalent to one-half of a regular loading section from the beginning of the line. Mid-coil termination is obtained by installing at the beginning of the line, a coil having one half of the inductance of the regular coils, the first full coil being installed at the end of the first complete loading section. For mid-coil and mid-section terminations, the characteristic impedance is approximately a pure resistance, which varies with frequency as a complicated function of the ratio of the frequency to the cut-off frequency. With mid-coil termination the impedance-frequency characteristic droops with rising frequency, approaching zero at the cut-off frequency. On the other hand, the mid-section termination has a rising characteristic, approaching infinity at the cut-off frequency.

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[4] "Wave Transmission over Non-Uniform Cables and Long Distance Air Lines," M. I. Pupin, TRANS. A. I. E. E., Vol 17, 1900, p. 445. Refer also to Pupin, U. S. Patents Nos. 652, 230 and 652, 231, June 19, 1900.

[5] "On Loaded Lines in Telephone Transmission," G. A. Campbell, *Philosophical Magazine*, March, 1903.

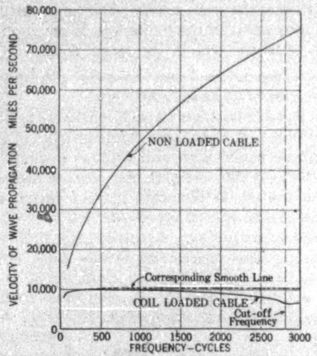

FIG. 2—VELOCITY-FREQUENCY CHARACTERISTICS OF LOADED AND NON-LOADED NO. 19 A. W. G. CABLES OF FIG. 1

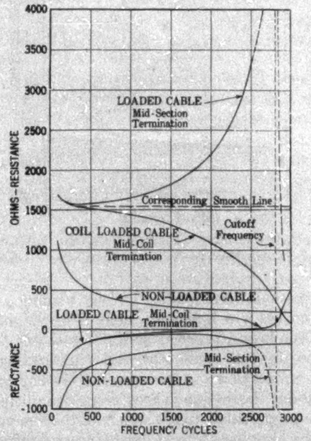

FIG. 3—IMPEDANCE-FREQUENCY CHARACTERISTICS OF LOADED AND NON-LOADED NO. 19 A. W. G. CABLES OF FIG. 1

Early Standard Loading Systems. One of the fundamental questions involved in the early commercial development work was that of determining what range of frequencies should be transmitted in order to furnish a satisfactory grade of speech transmission. The investigation of this point resulted in the adoption of a standard cut-off frequency of about 2300 cycles. Table II lists the other important transmission characteristics of the first loading systems standardized about 1904 for use on cables:—

TABLE II

FIRST STANDARD CABLE LOADING SYSTEMS

Loading Designation	Coil Inductance (Henrys)	Coil Spacing (Miles)	Inductance per Mile (Henrys)	Nominal Impedance (Ohms)	Attenuation Loss (TU per mile) 19 A.w.g.	16 A.w.g	14 A.w.g.
Heavy	0.250	1.25	0.200	1800	0.28	0.16	0.11
Medium	0.175	1.75	0.100	1300	0.39	0.21	0.14
Light	0.135	2.5	0.054	900	0.51	0.27	0.17
		(Non-loaded Cable)			1.05	0.74	0.59

NOTE. These data apply to cables having a mutual capacitance of 0.070 $\mu f.$ per mile and assume loading coils the electrical characteristics of which are given in Table IV. The nominal impedance is defined by the expression $\sqrt{L/C}$. The new unit of transmission loss (TU) is described in a recent Institute paper[6].

For open wire loading, only one loading system, known as "heavy" loading, was standardized. This involved the use of coils having an inductance of 0.265 henry at a spacing of approximately 8 miles. This loading had approximately the same cut-off frequency as the cable loading standards described in Table II. The other important line and transmission characteristics are summarized in Table III.

TABLE III

FIRST STANDARD OPEN-WIRE LOADING

Wire Diameter (In.)	Loading Condition	Constants per Loop Mile *R* Ohms	*L* Henrys	*C* Mf.	Nominal Impedance Ohms	Attenuation Loss TU per Mile
0.104	Non-loaded	10.4	0.0037	0.0084	660	0.075
0.104	Loaded	11.1	0.037	0.0086	2100	0.031
0.165	Non-loaded	4.14	0.0034	0.0091	610	0.033
0.165	Loaded	4.8	0.037	0.0094	2000	0.014

NOTE. Transmission efficiency figures assume dry weather insulation conditions, 5 megohm-miles, or better.

Loading Coils. The loading coils developed for use in the loading systems described in Tables II and III were of the toroidal type; i.e., they had ring-shaped cores formed by winding up a bundle of insulated fine wires on a suitably shaped spool. The core wire was 38 A. w. g., (0.004 in. diameter).

The wire used in the cable loading coil cores was a commercial grade of mild steel, hard drawn under conditions which gave it an initial permeability of 95. The

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6. "The Transmission Unit and Telephone Transmission Reference Systems," W. H. Martin, TRANS. A. I. E. E., Vol. 43, 1924, p. 797.

term "initial permeability" signifies the permeability at very weak magnetizing forces; i.e., below 0.1 gilbert per cm. The core wire used in the open wire loading coils was drawn from the same stock, but differences in the drawing and annealing treatments gave it an initial permeability of about 65. This core wire had lower eddy current and hysteresis losses than the 95-permeability wire. A black enamel insulation was used on the 95-permeability wire. A celluloid-shellac compound which could be applied at a lower temperature was used on the 65-permeability wire.

As illustrating the magnitudes involved, it may be noted that in order to meet the service requirements, the coils were designed so that for telephone currents of the order of 0.001 ampere, the magnetizing force H has a value of about 0.04 gilbert per cm., corresponding to a flux density of approximately $B = 2$ gauss.

The winding space on the cores was divided in half by means of fiber washers, and the winding was applied in two equal sections, one being located on each half of the core. In installing the coils, one of these windings was inserted in one line wire and the other winding in the other line wire, so connected that the mutual inductance between windings aided the self-inductance for current flowing around the circuit through both windings.

TABLE IV
FIRST STANDARD LOADING COILS

Type Loading	Coil Code No.	Inductance (Henrys)	Average Resistance		Overall Dimensions	
			D-C. (Ohms)	1000-Cycle (Ohms)	Diameter (In.)	Height (In.)
Open Wire	501	0.265	2.5	5.9	9	4
Cable	506	0.250	6.4	22.3	4⅛	3¼
"	508	0.175	4.2	13.0	"	"
"	507	0.135	3.2	9.1	"	"

NOTE. Effective resistance values apply for a line current of 0.002 ampere.

The high costs of the open wire lines warranted considerable refinement in the design of the open wire coils. They were, therefore, made much more efficient and correspondingly larger than the cable coils. They were wound with insulated stranded wire and had much lower core losses. Another important difference between the open wire and cable coils was the use of high dielectric strength insulation in the open wire coils. The coils were subjected to a breakdown test at 8000 volts (effective a-c.) and were protected in service by means of a special type of lightning arrester having non-arcing metal electrodes designed to operate at 3500 volts direct current.

Table IV lists the principal characteristics of the loading coils initially used in the standard loading systems listed in Tables II and III.

Loading Coil Cases. The cases used for potting the cable loading coils were designed so that they could be installed in underground manholes or on pole fixtures.

The general method of assembly is to dry the loading coils thoroughly and then impregnate them under vacuum with a moisture-proofing compound. The coils are then mounted on wooden spindles, adjacent coils being separated by iron washers. After carefully adjusting the individual coils to meet the electrical requirements, the spindles of coils are cabled to a short length of lead-covered cable which is referred to as a "stub" cable. Cast-iron cases with iron partitions were designed so as to provide a shielded compartment for each spindle of coils.

Commercially manufactured toroidal coils may have small irregularities in their windings resulting in a weak stray field which tends to cause cross-talk. The iron washers between coils and the partitions between spindle groups of coils provide effective cross-talk shields.

After placing the spindles of coils in the various compartments, the case is filled with a moisture-proofing compound. The lead-sheathed cable stub is brought through a brass nipple in the cast iron cover of the case, and the cover is then bolted to the case. By means of a special design of case and cover joint, a double seal is provided to prevent entrance of moisture at this point. A wiped joint is made between the lead sheath of the cable and the brass nipple.

The conductors in the stub cable have an appropriate color scheme in their insulation to identify the terminals of the loading coils, thus facilitating splicing of the coils into the line circuits. A series of multi-spindle cases was standardized, ranging in capacity from 21 to 98 coils. Smaller quantities of coils were potted in a single spindle pipe type case.

Generally similar assembly and potting methods were used for the open wire coils, the important differences being, first, that the open wire coils were always mounted in individual cases designed for mounting on pole fixtures, and second, that the coil terminals were brought out of the case in individual rubber-insulated leads.

I. PHANTOM GROUP LOADING

In Mr. Gherardi's paper,[7] already mentioned, reference was made to the development of means for (*a*) phantoming loaded circuits and (*b*) loading phantom circuits. The large plant economies made possible by these developments have resulted in extensive applications of these principles.

The following discussion will consider first the coil winding schemes, after which the transmission characteristics of the loading systems and the electrical characteristics of the loading coils will be briefly described.

Loading Methods. Fig. 4 schematically illustrates the standard method of phantom working and the

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[7] B. Gherardi, Loc. cit.

Bell System standard method for loading phantom circuits and side circuits of phantoms.[8]

The loading problem is to introduce the desired inductance into each of the three circuits of a phantom group without causing objectionable unbalances. The method illustrated in Fig. 4 involves individual loading coils for each circuit, the design being such that the side circuit coils are substantially non-inductive to the phantom circuit, while the phantom loading coil is substantially non-inductive to the side circuits. These desirable results require close magnetic coupling between the line windings in each coil. Consequently, in the side circuit coils each line winding, in effect, is distributed evenly about the entire core. The necessary high degree of symmetry required by balance considerations is obtained by dividing each line winding into two equal sections and interleaving them with the sections of the other line winding; thus each complete line winding consists of an inner section winding on one half of the core and an outer section winding on the opposite half core. Similar design principles are applied to the phantom loading coils, with added complications, however, arising from the increased number of line windings. Each of the four line windings consists of an inner section winding located on one core quadrant and an outer section winding located on the opposite core quadrant, the two line windings associated with a given side circuit being distributed about the same pair of opposite core quadrants. In arranging the windings on the core, precautions are taken to secure a symmetrical arrangement of the direct admittances among the line windings and from the line windings to the core and the case.

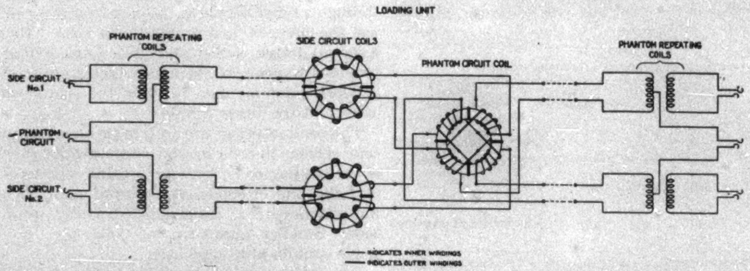

Fig. 4—Bell System Standard Method of Loading Phantom Circuits and Their Side Circuits

It is interesting to note that the three-coil loading scheme illustrated in Fig. 4 was employed in the Boston-Neponset cable, installed in 1910, which was the first successful installation of loaded phantom circuits in the world. Other schemes of phantom group loading using two-coil and four-coil arrangements have been developed here and abroad, but none of them is considered to be as satisfactory as the scheme illustrated in Fig. 4 from the standpoint of service and cost. These other schemes are described in a recent article[9] which compares them with the scheme above described.

Loading Systems. In adapting the circuits to phantom working, the electrical constants of the two-wire circuits were changed as little as possible in making them suitable for use as side circuits of phantoms. In the cables, two pairs having different lengths of twist were twisted into quad formation on a still different length of twist. The required balance was obtained on open wire lines by cutting in a large number of additional transpositions.

The construction methods chosen resulted in the phantom circuits having approximately 60 per cent greater distributed capacity than their side circuits, and a lower distributed inductance, approximately in inverse proportion. It was obviously desirable to install the phantom circuit coils at the same points as the side circuit coils; accordingly, in working to the same standard of cut-off frequency, the relative circuit constants summarized above resulted in the phantom loading systems having a nominal impedance approximately 60 per cent as high as their associated side circuit loading systems. The transmission efficiency of the phantom circuit was 20 to 25 per cent better than that of its associated side circuits, on which basis, the phantom circuits were suitable for use over somewhat longer distances than their side circuits.

Cable Loading. Data regarding the general characteristics of the first phantom group loading systems standardized for use on quadded telephone cables are given in the first four rows of Table V. These loading systems were used principally on interurban toll cables. Because of the extra cost of the terminal and signaling equipment, and other factors involved in phantom working, it was not economical to use phantom circuits in the shorter lengths of loaded cable ordinarily involved in exchange area connections.

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[8] U. S. Patents (No. 980,021 "Loaded Phantom Circuit," G. A. Campbell and T. Shaw. No. 981,015 "Phantomed Loaded Circuit," T. Shaw.

[9] "Commercial Loading of Telephone Cable," W. Fondiller, Electrical Communication, July 1925.

As soon as the development work on quadded toll cables and phantom group loading had progressed to a point where satisfactory commercial results were assured, active development work commenced on the Boston-New York-Washington cable project, involving the use of coarse gage quadded conductors and new types of high efficiency loading coils designed especially for use on the coarse gage wires. The Boston-Washington cable was the first link in a rapidly growing network of toll cables which now interconnects the large population centers of the Atlantic Seaboard and the upper Mississippi Valley region, providing increased reliability of service as compared with open-wire lines.

TABLE V

FIRST LOADING STANDARDS FOR QUADDED TOLL CABLES

Item	Loading Designation	Type Circuit	Coil Inductance (Henrys)	Coil Spacing (Miles)	Nominal Impedance (Ohms)	Attenuation Loss—TU per mile			
						19 A. w. g.	16 A. w. g.	13 A. w. g.	10 A. w. g.
1	Medium-Heavy	Side	0.210	1.4	1500			0.085	0.050
2	" "	Phantom	0.130	1.4	950			0.069	0.040
3	Heavy	Side	0.250	1.25	1850			0.081	0.050
4	"	Phantom	0.155	1.25	1150			0.066	0.042
5	Heavy	Side	0.250	1.25	1850	0.24	0.14		
6	"	Phantom	0.155	1.25	1150	0.20	0.12		
7	Medium	Side	0.175	1.75	1300	0.31	0.17		
8	"	Phantom	0.106	1.75	800	0.26	0.14		

Notes: A capacitance of 0.062 μf. per mile is assumed in side circuits and 0.100 μf. per mile in the phantom circuit. The pair capacitance value is smaller than that assumed in Table II, due to improvements in the cables.

All of the above loading systems have a cut-off frequency of about 2300 cycles.

It should be kept in mind that at the time under discussion (1910-1911) no commercially satisfactory type of telephone repeater was available. Accordingly, in order to assure satisfactory service between Boston, New York, Washington, and intermediate points, it was necessary to provide 10-A. w. g. and 13-A. w. g. conductors in the new cable. Cost studies showed it to be desirable to use a special weight of loading intermediate between the old heavy and medium loading systems, which was therefore designated "medium-heavy" loading. Information regarding this special loading is given in Items 1 and 2 of Table V. In Items 3 and 4, corresponding data are given on the "high-efficiency" heavy loading designed for coarse gage conductors. This heavy loading was used on certain sections of the Boston-Washington cable where plant construction reasons made it desirable to install the coils in existing loading manholes installed at heavy loading spacing.

From the last column of Table V it is seen that there is very little difference between the efficiencies of the heavy and the medium-heavy loading systems when used on 10-A. w. g. conductors. This explains the more general use of the medium-heavy loading, which was less expensive because of the greater distances between coils. The effects under discussion are due to the part played by the loading coil resistance. The loading coils themselves conformed as closely as practicable to the cost-equilibrium principle:—a condition of cost balance where a small improvement in transmission would require approximately equal expenditure whether by improving the loading or by adding copper to the cable conductors. On this basis, a somewhat less expensive grade of coil was used on the 13-A. w. g. wires than on the 10-A. w. g. wires. The grade of coils developed primarily for use on 16- and 19-A. w. g. cables, giving transmission results illustrated in Items 5-8 of Table V, was in turn less expensive than the "high-efficiency" coils. In each case, since the phantom circuits were somewhat more efficient than their associated side circuits, a somewhat higher grade coil was used in the phantom circuits than in the side circuits.

Open-Wire Phantom Loading. Phantom loading came into general use on open wire lines at about the same time as on quadded cables. In general, the methods used in applying phantom group loading to the open-wire lines were used for the cable systems. The line characteristics for the side circuits were practically the same as for the original non-phantomed circuits (Table III); the principal difference being that caused by the small resistance of the phantom loading coils. The important linear and transmission characteristics of the phantom circuits are given in Table VI.

TABLE VI

OPEN-WIRE PHANTOM LOADING

Wire Diameter (In.)	Loading Condition	Distributed Constants per Loop Mile at 1000 Cycles			Nominal Impedance (Ohms)	Attenuation Loss TU per Mile
		R (Ohms)	*L* (Henrys)	*C* (Mf.)		
0.104	Non-loaded	5.2	0.0022	0.0141	400	0.064
0.104	Loaded	5.8	0.023	0.0141	1300	0.027
0.165	Non-loaded	2.1	0.0021	0.0154	400	0.028
0.165	Loaded	2.6	0.023	0.0154	1200	0.012

The phantom loading coil had an inductance value of 0.163 henry.

Loading Coils. Table VII gives general information regarding the first standard side circuit and phantom loading coils used in the phantom group loading systems listed in Tables V and VI. The coils designed for open-wire lines and for 10-A.w.g. cable had 65-permeability wire cores and stranded copper windings. The coils designed for 13-A.w.g. cables had 65-permeability wire cores and non-stranded copper windings. The other coils had 95-permeability wire cores.

TABLE VII
FIRST STANDARD LOADING COILS FOR PHANTOM WORKING

Type Line	Inductance	Coil Code No.	Type Circuit	Average Resistance-Ohms		Overall Dimensions	
	(Henrys)			D-C.	1000 cycles	Diameter (In.)	Height (In.)
Open-Wire	0.265	512	Side	5.0	8.4	9.0	4.0
	0.163	511	Phantom	2.5	4.4	11.0	4.9
10-A. w. g. Cable	0.210	520	Side	3.8	6.6	8.5	3.5
	0.130	519	Phantom	1.9	3.4	10.4	4.0
	0.250	532	Side	4.1	7.8	8.5	3.5
	0.155	531	Phantom	2.1	3.9	10.4	4.0
13-A. w. g Cable	0.205	538	Side	6.0	9.2	5.7	2.5
	0.130	521	Phantom	3.0	4.5	7.9	3.0
	0.250	534	Side	6.6	10.7	5.7	2.5
	0.155	533	Phantom	3.3	5.3	7.9	3.0
16 and 19-A. w. g. Cable	0.250	515	Side	8.9	23.1	4.6	2.4
	0.155	530	Phantom	4.4	11.9	5.9	2.9
	0.175	514	Side	5.4	14.4	4.6	2.4
	0.106	513	Phantom	2.7	7.1	5.9	2.9

NOTE. The resistance data apply to circuits of a complete phantom group; *i. e.*, the side circuit data include effects of the phantom coils, and phantom circuit data include effects of the side circuit coils. Effective resistance values correspond to line current of 0.002 ampere.

Potting Features. The general practise for cable loading is to pot side circuit and phantom loading coils in the same case as phantom groups, since this has important installation and transmission advantages. The phantom coils, being considerably larger than the side circuit coils, are mounted in separate spindle compartments. The cross-connections between the side circuit and phantom coils are made within the case, in order to reduce the amount of splicing required in the field. Thus, the stub cable contains only the conductors to be spliced to the "east" and "west" conductors in the line cable. Quadded construction is used in the stub cable of all loading coil cases for phantom loading in order to avoid serious capacitance unbalances.

The multi-spindle cases used in potting the small size coils for 16 and 19-A.w.g. cables ranged in capacity from 12 to 24 phantom units. The larger size coils used on the coarser gage cables were potted in smaller complements. Occasionally it is desirable to install side circuit loading alone and to install the phantom loading at a later period. Accordingly, cable loading coil cases were designed to meet these conditions. The open wire coils were potted in individual cases.

II. LOADING FOR REPEATERED CIRCUITS

General. The development of telephone repeaters to the point where they could be used for commercial service in extending the range of telephone transmission was the beginning of a new era in the communication art. In this development work, the adaptation of the lines to the requirements of repeater operation was secondary in importance only to the development of satisfactory repeater elements and circuits for associating the repeater elements with the line. The reader is referred to an Institute paper by Messrs. B. Gherardi and F. B. Jewett[10] for general information regarding telephone repeaters and to a more recent Institute paper by Mr. A. B. Clark[11] for a general discussion of subsequent developments in the application of repeaters to long telephone circuits.

The early work on the line problem was primarily concerned with obtaining a sufficiently high degree of regularity in the line impedance-frequency characteristics, so that the requisite high degree of balance could be obtained and maintained between the line and the repeater balancing network. Later on, particularly in preparing for the application of telephone repeaters to long toll cables, such as the New York-Pittsburgh-Chicago cable, it became necessary to change the fundamental transmission characteristics of the loading.

Early Work—Reduction of Line Irregularities. Commercial telephony, requiring two-way transmission, imposes severe balance requirements on repeater circuits over the entire band of frequencies which the repeater is designed to transmit, in order to avoid singing or distortion due to near singing. Within certain limitations, the higher the degree of balance between the line and the balancing network circuit, the higher will be the permissible amplification gain of the repeater.

The practical solution of this fundamental repeater-line balance problem required (*a*) the construction of lines having extremely regular impedance characteristics over the frequency band which the repeater is designed to transmit and (*b*) the development of balancing networks[12] capable of accurately simulating the sending-end impedance characteristics of the improved lines throughout this frequency range. On account of the great difficulty of getting a high degree of balance at frequencies near the cut-off frequency of the loading, partly due to line irregularity effects and partly due to network design complications, it has been found desirable to use electric wave filters[13] in the repeater sets which cut off at a frequency below the cut-off frequency of the loading. This margin of cut-off effects is usually 200 cycles or more, depending upon the repeater design and the type of loading involved.

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[10] "Telephone Repeaters," B. Gherardi and F. B. Jewett, TRANS. A. I. E. E., Vol. 38, 1919, p. 1287.

[11] "Telephone Transmission over Long Cable Circuits," A. B. Clark, TRANS. A. I. E. E., Vol. 42, 1923, p. 86.

The "regular" line referred to in (*a*) is one which is free from impedance irregularities. In the case of loaded lines, the loading coils should have very closely the same inductance values, and the sections of line between loading coils should have closely the same value of capacitance. These uniformity features should be permanent, which requires that the coils should have a high degree of stability in their inductance characteristics; i. e., they should be capable of resisting the magnetizing effects of abnormal service conditions. Some of the older types of coils did not meet this requirement. The satisfactory way in which these fundamental coil requirements are fulfilled in the newer types of coils will be described in a subsequent section.

Uniformity in the loading section capacitance values involves uniformity in cable and line capacitance values as well as precision in the coil spacing. In toll cable loading the maximum deviations from the average spacing are kept below 2 per cent, and the average deviations are in the order of 0.5 per cent or less.

In exceptional cases where physical obstructions are encountered in reducing the spacing deviations to a sufficiently low value, use is made of "building-out condensers" or "building-out stub cables" to normalize the capacitance of loading sections.[14] Abnormally long loading sections can usually be split up into two sections, one or both of which may then be "built out" to the nominal standard capacitance values.

Transcontinental Lines—High Stability Loading Coils

The inauguration of commercial transcontinental telephone service over the New York-San Francisco line in January 1915 marked the first commercial application of these general improvements in regularity of line construction, including the use of an improved type of loading coil.

In the extensive field work which was done in preparing for transcontinental telephone service, it was found that the inductance values of a considerable percentage of the open-wire loading coils then in use (Nos. 511 and 512 types, Table VII) had changed appreciably from the nominal values to which they were adjusted at the factory prior to shipment, and that these changes were due to core magnetization caused by abnormal currents induced by lightning discharges. In some cases abnormal currents induced by power transmission lines or electric railway distribution systems were responsible for the loading coil magnetization.

The inductance changes were not sufficiently large to have serious reactions on transmission over non-repeatered circuits. Although individual coils varied in inductance from time to time, the general average of groups of coils was fairly constant. The effects of these individual variations on the impedance of the line, however, were too large to permit satisfactory operation with telephone repeaters. Some experiments made with improved lightning arresters, in an effort to reduce the coil magnetization trouble, were unsuccessful.

The solution of the problem of repeatering loaded open-wire circuits required the development of loading coils which would be stable magnetically when subjected to extreme conditions of magnetizing current in the windings. The requirement was laid down for these coils that the inductance to speech currents should not be affected more than about 2 per cent when a magnetizing current of two amperes was passed through either line winding. In view of the fact that the extreme residual magnetizing effect of this current on the No. 511 and No. 512 loading coils was approximately 30 per cent., it will be appreciated that this imposed a very severe stability requirement.

The design adopted involved the use of air-gaps in the cores of the iron wire core loading coils.[15] Two air-gaps were employed at opposite points in the cores and suitable clamping means were provided to hold the coil halves in proper alinement. The use of only two air-gaps in the cores of the phantom loading coil brought in unbalance tendencies not present in older designs, which were corrected by special refinements in the design.

The use of a magnetic circuit having "ends," while effective for producing self-demagnetization, brought in troublesome magnetic leakage which necessitated special potting methods. Because of the economy of cast-iron loading coil cases, it was decided to continue their use, but to increase their dimensions sufficiently to reduce eddy-current losses in the case to a tolerable point.

The air-gap type loading coils designed for the transcontinental circuits, coded Nos. 549 and 550 for the phantom and side circuits respectively, were more generally potted as phantom loading units than as individual coils, and in such instances the cross-connections between the phantom and side circuit coils were made inside the case. Important advantages of this arrangement were that the leakage losses during periods of low line insulation were greatly reduced as well as the liability of wrong connections of windings during the installation work. Fig. 5 is a photograph of an installation of open-wire loading coils illustrating both the individual coil and loading unit methods of potting.

Table VIII contains data on the air-gap coils stand-

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[12] R. S. Hoyt, "Impedance of Loaded Lines and Design of Simulating and Compensating Networks," *Bell System Technical Journal*, July 1924.

[13] U. S. Patents Nos. 1,227,113, and 1,227,114,—G. A. Campbell.

[14] U. S. Patent No. 1,219,760—John Mills and R. S. Hoyt.

[15] U. S. Patents Nos. 1,289,941 and 1,433,305—Shaw and Fondiller.

ardized for open-wire circuits. It will be noted that these coils are somewhat less efficient from the standpoint of effective resistance than the older type coils (Nos. 511 and 512) listed in Table VII, though having marked superiority over the latter with regard to magnetic stability. To assist in getting maximum line regularity, the Nos. 549 and 550 coils were adjusted in the factory to meet ± 1 per cent inductance precision limits. In the older types of coils ± 5 per cent deviations had been allowed. The nominal inductance values of the Nos. 549 and 550 coils are somewhat below those of the Nos. 511 and 512 coils, the inductance difference corresponding roughly to the average magnetization effect of normal service conditions on the older types of coils.

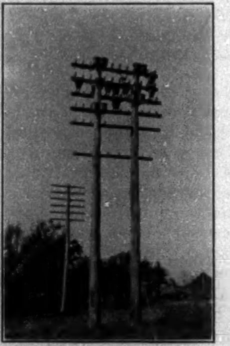

Fig. 5—Typical Open Wire Loading Installation

Showing four phantom group (3-coil) cases and nine individual coil cases

TABLE VIII
HIGH STABILITY COILS HAVING WIRE CORES WITH AIR GAPS

Type Loading	Coil Code No.	Type Circuit	Inductance henrys	Average Resistance Ohms		Overall Dimension Inches	
				D-C.	1000-Cycles	Diameter	Hgt.
Open Wire	550	Side	0.245	5.4	11.1	8.1	3.9
" "	549	Phantom	0.150	2.7	6.4	10.0	4.0
10 and 13	556	Side	0.255	7.0	14.0	5.6	2.9
A. w. g. Cable	555	Phantom	0.155	3.5	7.0	7.5	3.6
10 and 13	558	Side	0.200	6.2	10.9	5.6	2.9
A. w. g. Cable	557	Phantom	0.135	3.1	5.9	7.5	3.6

Notes. Open-wire coils used in Loading Systems, Tables III and VI. Cable coils used in Loading Systems, Table V.

Resistance data apply to side circuits and phantom circuits of complete phantom groups. Effective resistance values are for 0.002-ampere line current.

The solution of the transcontinental line problem involved improvements in the regularity of the coil spacing as well as improvements in the magnetic stability of the coils. The line "clearing up" work usually involved a great deal of retransposing, since cross-talk considerations made it necessary to have the coils placed at balanced or neutral points in the transposition layout.

In the case of coarse gage cable circuits, such as the Boston-Washington and other toll cables installed prior to the advent of repeaters, the new requirements were met by the design of an air-gap type of wire-core coil on which data are given in Table VIII. They were somewhat smaller and not quite so expensive as the improved open-wire coils.

Compressed Powdered Iron Core Loading Coils. It soon became evident that the economical extension of the toll plant would involve the general introduction of telephone repeaters in cable as well as open-wire circuits. The use of telephone repeaters made it possible to supersede the coarse gage conductors by 16- and 19-A. w. g. conductors for toll connections, and this greatly increased the need for an efficient and stable loading coil of lower cost than the air-gap wire-core coil.

As a result of investigations carried on over a period of several years, there was developed for commercial use early in 1916 a new magnetic material, compressed powdered iron, which has been of the utmost value in loading coil design.[16] This improved magnetic material is described in a paper presented before the Institute by B. Speed and G. W. Elmen[17] which also discusses the electrical and magnetic properties of this material.

Briefly, the method of production consists of grinding electrolytically deposited iron to the desired fineness, insulating the particles of iron, and finally compressing these insulated particles in steel dies at such very high pressures as to consolidate the mass into a ring, the specific gravity of which is substantially equal to that of solid iron. The rings are then stacked in a manner similar to laminations of sheet material to form a core of the desired dimensions. Though the separate rings are approximately 0.2 in. thick, the insulation between the individual particles is so effective that despite the use of molding pressures of 200,000 lb. per sq. in., the eddy current loss in a powdered iron core is less than that obtainable with 0.004-in. iron wire. Depending on the heat treatment and the amount of insulation, the initial permeability can be varied from approximately 25 to about 75. The specific resistance is about 20,000 times that of ordinary iron. The permeability can be controlled within comparatively narrow limits by the manufacturing processes, thus making for greater uniformity. The great advantage of this material for loading coils, however, lies in its self-demagnetizing property. The powdered iron core by virtue of its very numerous, though extremely small, distributed air-gaps, affords a means for constructing magnetically stable cores without the production of poles and their attendant magnetic leakage.

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[16] U. S. Patents No. 1,274,952, B. Speed; 1,286,965, G. W. Elmen; 1,292,206, J. C. Woodruff.

[17] "Magnetic Properties of Compressed Powdered Iron," B. Speed and G. W. Elmen, Trans. A. I. E. E., Vol. 40, 1921, p. 1321.

Fig. 6 gives photographs of (*a*) a standard compressed iron powder core ring such as is used in the cores of toll cable loading coils; (*b*) a completely assembled core with part of the core taping removed; (*c*) a completely wound coil of the side circuit type; and (*d*) a coil in cross-section.

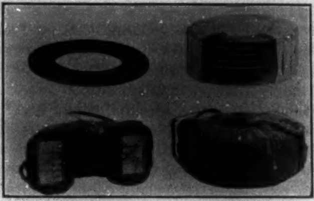

FIG. 6—COMPRESSED POWDERED IRON CORE LOADING COIL

The first application of powdered iron cores was to replace some of the 95-permeability wire core loading coils in 16- and 19-A. w. g. cable. The effective permeability of the 95-permeability wire cores, making correction for air spaces and insulation, was approximately 60, and accordingly, the replacing powdered iron cores were designed to have the same effective permeability.

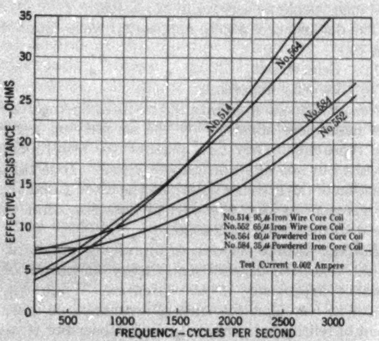

FIG. 7—EFFECTIVE RESISTANCE-FREQUENCY CHARACTERISTICS TOLL CABLE LOADING COILS

As a result of further developments in the direction of applying vacuum tube repeaters to loaded cable circuits, it became necessary with the extension of the length of these circuits to improve the characteristics of the loading coils. This led to the development of an improved grade of powdered iron core having an initial permeability of 35 which corresponds closely to the effective permeability of cores using iron wire having a permeability of 65. It was decided that for circuits such as interoffice trunks and short cables which would not be operated with superposed telegraph, the 60-permeability compressed iron core coils should be used; while for toll cable work involving repeatered composited circuits, 35-permeability cores should be employed. All of the compressed powder core coils intended for repeatered circuits were adjusted to meet ± 2 per cent inductance limits.

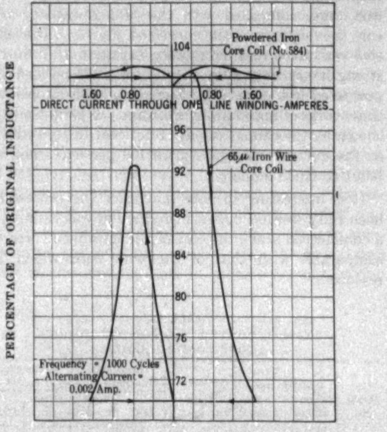

FIG. 8—RESIDUAL MAGNETIZATION CHARACTERISTICS OF COMPRESSED POWDERED IRON CORE AND IRON WIRE CORE LOADING COILS

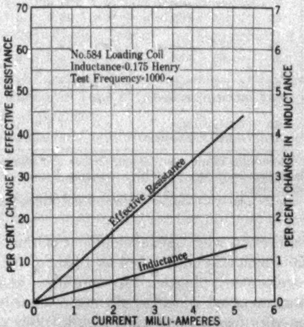

FIG. 9—VARIATION OF INDUCTANCE AND EFFECTIVE RESISTANCE

With line current in 35-permeability compressed powdered iron core loading coil.

The effective resistance-frequency characteristics of 95-permeability and 65-permeability wire core coils and 60-permeability and 35-permeability powdered iron core coils having the same inductance (0.175 henry) and the same over-all sizes are given in Fig. 7. The large improvement as to freedom from residual magnetization effects afforded by the 35-permeability powdered iron core, compared with the 65-permeability wire-core is evident from the curves of Fig. 8. The effective resistance and inductance variation with current strength are shown in Fig. 9 for a 35-permeability powdered iron core coil. The remarkable property of these cores of maintaining constancy of permeability is shown by the change of only 1 per cent in permeability as the current strength varies 400 per cent from, say, 0.001 to 0.005 ampere.

It is interesting to note that after the process had been fully worked out and production was running on a commercial scale, the cost of the improved cores was comparable with that of the wire cores which they replaced.

TABLE IX
TYPICAL COMPRESSED POWDERED IRON CORE LOADING COILS

Coil Code No.	Core Permeability	Inductance (henrys)	Type Circuit	Resistance Ohms		Dimensions Inches	
				D-C.	1000-Cycles	Diameter	Height
562	60	0.250	Side	11.4	25.8	4.5	2.1
561	60	0.155	Phantom	5.7	11.7	6.3	3.0
564	60	0.175	Side	6.6	15.4	4.5	2.1
563	60	0.106	Phantom	3.3	6.7	6.3	3.0
582	35	0.250	Side	15.9	21.8	4.7	2.4
581	35	0.155	Phantom	8.0	10.0	6.7	3.1
584	35	0.175	Side	10.8	14.1	4.7	2.4
583	35	0.106	Phantom	5.4	6.6	6.7	3.1
584	35	0.175	Side	12.1	15.3	4.7	2.4
587	35	0.063	Phantom	6.1	7.0	4.7	2.8
590	35	0.044	Side	4.0	4.6	4.7	2.4
591	35	0.025	Phantom	2.0	2.0	4.7	2.8

NOTE. Resistance values apply to side circuits and phantom circuits of complete phantom groups. Effective resistance corresponds to 0.002-ampere line current.

These coils are used in the loading systems listed in Tables V and X.

In connection with the development of the new core material which was undertaken as a part of the loading coil development program, an enormous amount of work was involved which would not ordinarily be associated with loading coil design work. For instance, there were undertaken chemical studies on electrodeposition of iron and methods of insulating the iron particles, metallurgical studies of the production of finely divided iron by various means, refinements in shielded electrical measuring equipment for accurate determination of small core losses at voice frequencies, development of special permeameters to make possible the rapid determination of the permeability of rings, the design of the steel moulding dies, selection of suitable grades of alloy steel to withstand the enormous pressures, and also various other special problems. These are mentioned here as illustrative of the scope of the problem of developing this new core material.

The value of the compressed powdered iron core loading coils in the modern loading art has been recognized to the extent of their adoption as an international standard for repeatered circuits[18].

New Requirements for Cable Loading Systems. In the first commercial applications of telephone repeaters, the new features in the loading were the improved types of coils already described and the precision of spacing the coils. No fundamental changes were made in the loading systems then standard.

The completion of the development of a satisfactory commercial type of telephone repeater marked the beginning of a long period of experimental work for the purpose of determining the commercial possibilities of the use of repeaters over long cable circuits. When loaded cables of improved impedance regularity became available, long circuits were built up for experimental purposes by looping back and forth. As the length of these circuits was increased, phenomena not previously observed in cable circuits became increasingly troublesome, and it became apparent that it would be necessary to develop new loading systems having improved velocity and higher cut-off frequency characteristics in order to realize the full possibilities of repeaters in extending the range and reducing the cost of long distance telephone service over cables.

The disturbances above mentioned were found to be due to

(*a*) Echo effects
(*b*) Velocity distortion

These phenomena originate in the lines themselves and are made more apparent by the amplifying action of the repeaters. They are present in non-repeatered circuits but not to a noticeable degree. It is the combination of the extreme length of the circuit and the use of repeaters to keep the over-all loss low that makes the disturbances troublesome.

Echoes. Echoes are due to unbalance currents; i.e., to the reflection of electrical energy at points of impedance irregularity in the circuits. When the circuit is so long that the time of transmission from the point of reflection to the disturbed subscriber is appreciable, there will be echo effects unless the losses in the circuit are so large as to cause the reflected energy to become inappreciably small. On such circuits it may be necessary to work the repeaters at gains well below those at which "singing" occurs or distortion due to "near singing" is experienced.

Since the time of transmission is such an important factor in echo phenomena, reductions in the harmful effects of these disturbances have been obtained in the improved loading systems which have been developed for use on long repeatered circuits, by substantially increasing the velocity of transmission. Recently there has become available commercially a device known as an "echo suppressor" which interrupts the path of the echoes without disturbing the main transmission. A description of the device and its field of application was given in a recent Institute paper.[19]

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[18] Minutes of Second Conference of Permanent Commission, Le Comité Consultatif Internationale de Communications Telephonique a Grande Distance, page 55—p. 119, English Version.

Velocity Distortion. In a coil-loaded line the steady state velocity of wave propagation varies with frequency. At the upper frequencies the velocity change is principally due to lumpiness effects of the loading and is, therefore, a function of the ratio of the frequency under consideration to the cut-off frequency. As illustrated in Fig. 2, the departure of the actual velocity from the nominal velocity of the corresponding smooth line ($\sqrt{1/CL}$) increases as the frequency is raised, the rate of change increasing rapidly as the cut-off frequency is approached. At frequencies below approximately 0.3 of the cut-off frequency the coil loaded line has substantially the same velocity characteristics as the corresponding smooth line; when the frequency is further reduced, the departure of the actual velocity from the nominal velocity increases as a function of the ratio of the line resistance to the inductive reactance per unit length.

As a result of these velocity-frequency relations, a long loaded repeatered circuit may have seriously objectionable quality, even when the attenuation-frequency distortion is made negligible by the use of special devices at the repeater stations for correcting the attenuation-frequency distortion effects.

The velocity distortion is particularly noticeable during the building-up and dying-down periods, when it manifests itself as transient distortion. The duration of transient distortion depends, among other factors, upon the length of the line, the nominal velocity, and the cut-off frequency of the loading. In the old standard loading systems the high frequency velocity distortion caused by the lumpiness effects of the loading was more serious than the low frequency velocity distortion. Accordingly, a substantial reduction in the transient distortion has been obtained in the new standard loading systems by raising the cut-off frequency of the loading.

For further discussion of velocity distortion reference should be made to Mr. A. B. Clark's paper[20], previously mentioned, which gives experimental results and to an earlier Institute paper by Mr. J. R. Carson[21] which gives the results of theoretical studies.

Characteristics of Improved Cable Loading Systems. The principal electrical features of the H-44-25 and H-174-63 phantom group loading systems which have been developed primarily for use on long repeatered cables are given in Table X. Corresponding details of the older standard loading system developed for non-repeatered cables are also included in this table. Typical attenuation-frequency curves of the old and new loading systems are given in Fig. 10.

In the following discussion of detailed characteristics, the various phantom group loading systems will be referred to in terms of recently standardized designations which include a letter to symbolize the coil spacing, in combination with two numbers which correspond with the effective coil inductance values in milhenrys, the first number referring to the side circuit coils and the second number to the phantom coils. The individual side circuit or phantom circuit loading systems have designations which include a letter to symbolize the coil spacing, coupled with the inductance value of the loading coils in milhenrys, and having a letter suffix "*S*" or "*P*" indicating the type of circuit, side or phantom.

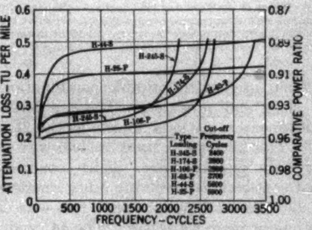

Fig. 10—Attenuation-Frequency Characteristics of Toll Cable Loading

The fundamental differences between the new and the old loading systems are with respect to velocity of wave propagation and cut-off frequency, these changes having been made in accordance with the preceding discussion primarily for the purpose of reducing echo effects and transient distortion. For reasons of plant economy and flexibility, the new loading systems all have the same coil spacing of 6000 feet.

The coil spacing being fixed, it necessarily follows that any reduction in coil inductance for the purpose of raising the cut-off frequency will also increase the transmission velocity. The attenuation improvement obtained by the loading decreases as the velocity is increased. High velocity loading is more expensive than low velocity loading, in the sense that more repeaters are required for the same over-all loss. Obviously, although high velocity loading could be used for short haul traffic, it would not be so economical as a low velocity loading. Commercial considerations thus justify a series of loading standards, graded to meet the requirements of the different lengths of circuits.

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[19] "Echo Suppressors for Long Telephone Circuits," A. B. Clark and R. C. Mathes, Jour. A. I. E. E., p. 618, June, 1925.

[20] Clark, Loc. Cit.

[21] "Theory of the Transient Oscillations of Electrical Networks and Transmission Systems", J. R. Carson, Trans. A. I. E. E., Vol. 38, 1919, p. 345.

TABLE X
LOADING SYSTEMS—SMALL GAGE REPEATERED TOLL CABLES

Item	(*a*) Loading System	Circuit	(*b*) Coil Code No.	Nominal Impedance (Ohms)	Nominal Cut-off Frequency (Cycles)	Transmission Velocity Miles-per Second	(*c*) Attenuation Loss TU per Mile at 1000 Cycles 19 A. w. g.	 16 A. w. g.	(*d*) Maximum Geographical Length (Miles)
(1)	H- 44-25	Side	590	800	5600	19000	0.48	0.25	5000
(2)	"	Phantom	591	450	5900	20000	0.40	0.21	5000
(3)	H-174-63	Side	584	1550	2800	10000	0.28	0.16	500
(4)	"	Phantom	587	750	3700	13000	0.28	0.16	1500
(5)	H-174-106	Side	584	1550	2800	10000	0.28	0.16	500
(6)	"	Phantom	583	950	2900	10000	0.22	0.13	500
(7)	H-245-155	Side	582	1850	2400	8000	0.25	0.16	250
(8)	"	Phantom	581	1150	2400	8000	0.20	0.12	250

Notes: (*a*) Nominal coil spacing is 6000 feet in cable having a capacitance of 0.062 μf per/mile in the side circuits and 0.100 μf per mile in the phantom circuit.

(*b*) The loading coil data are given in Table IX.

(*c*) These attenuation values apply at 55 deg. fahr. Under extreme temperature conditions, the actual attenuation may be approximately 12 per cent larger or smaller, due principally to changes in conductor resistance with temperature. In long repeatered cable circuits these variations of attenuation with temperature require special corrective treatment by means of automatic transmission regulators. (Reference No. 11)

(*d*) The length limitations are set by transient distortion effects; echo currents may limit circuit lengths to lower values, depending on the grade of balance of the lines and the permissible over-all loss.

At the present time the two phantom group loading standards H-44-25 and H-174-63 are sufficient to meet the graded requirements of commercial toll cable circuits, when used with suitable combinations of conductor sizes and repeaters. Three different general types of repeaters are used, known as the 21, 22, and 44 types.[22] The 21 type is used on two-wire circuits requiring only one repeater, under conditions where switched connections involving other repeaters are not involved. The 22 type is used on two-wire circuits requiring one or more repeaters. The 44 type is used on four-wire circuits, where one pair of wires is used for one-way transmission in one direction and the other pair of wires for transmission in the opposite direction.

TABLE XI
TYPES OF TOLL CABLE FACILITIES

Item No.	Length Circuit	Cable Gage	Type of Loading	Type Circuit	Type Repeater
(a)	(short)	19	H-174-63	2-wire	—
(b)		16	"	"	—
(c)		19	"	"	21
(d)		16	"	"	21
(e)		19	"	"	22
(f)		16	"	"	22
(g)		19	"	4-wire	44
)		16	H- 44-25	2-wire	22
(i)	(very long)	19	"	4-wire	44

When phantom circuits are worked on a four-wire basis, each one-way transmission path actually uses four wires.

Table XI lists the combinations of loading, conductor gage, and type of repeater circuit which are used in meeting the wide range of commercial requirements. The position of the facility item in the table indicates the sequence of transmission excellence, Item (i) being the highest grade facility in this respect. In general, the cost of these facilities is in reverse order to the sequence of electrical excellence.

The exact limits of the field of use of a given type of facility depend upon the magnitude of the permissible over-all transmission loss, and upon the grade of repeater balance obtainable. A discussion of these features would bring in complicated engineering questions beyond the scope of the present paper. So far as loading features are concerned, it is sufficient to state that H-44-25 loading is generally used on circuits of approximately 500 miles or more. On circuits intended for switched business, it is frequently necessary to use this type of loading for much shorter distances. For further discussion of the use of repeatered loaded lines reference is made to recent papers presented before the Institute by Mr. J. J. Pilliod[23] and Mr. H. S. Osborne.[24]

H-63-P versus H-106-P Loading. The standardization of the H-63-P loading to replace the H-106-P loading for association with H-174-S loading, is of particular interest in illustrating the reactions of repeater requirements on loading design. Phantom circuits necessarily have a lower attenuation constant than the associated side circuits, when the loading is designed to meet the same standard of cut-off frequency and the coils are spaced at the same loading points. When repeaters are used on such loaded phantom circuits, the net equivalent is practically no lower than the net equivalent of the associated side circuits, due principally to the fact that the loaded sides

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22. Gherardi—Jewett, Loc. cit.

23. "Philadelphia-Pittsburgh Section of New York-Chicago Cable," J. J. Pilliod, TRANS. A. I. E. E., Vol. 41, 1922, p. 446.

24. "Telephone Transmission over Long Distances," H. S. Osborne, TRANS. A. I. E. E., Vol. 42, 1923, p. 984.

and phantoms have practically the same velocity and cut-off frequency characteristics.

Under present operating conditions for short small gage-loaded circuits of such lengths that satisfactory transmission results can be obtained without using telephone repeaters, there is ordinarily no important advantage in having the phantom circuit more efficient than the side circuits. It is a distinct operating convenience, of course, to be able to use the phantom circuit and its associated side circuits indiscriminately for the same class of service.

Having the above situations in mind, it was decided to redesign the phantom loading so that it would have approximately the same attenuation constant at 1000 cycles as the associated H-174-S circuit loading. This resulted in the reduction of the phantom loading coil inductance to 63 milhenrys. On the basis of equal attenuation losses in the phantom circuit and its side circuits, the continued use of a higher grade coil in the phantom circuit was no longer justified from a cost standpoint. Accordingly, the new 63-milhenry phantom coil (Code No. 587, Table IX) was designed to have approximately the same d-c. resistance as the earlier standard 106-milhenry coil (Code No. 583), since this permitted a substantial reduction in the size of the loading coil and a consequent reduction in cost, without increasing the over-all losses in the associated side circuits. The design finally chosen resulted in the phantom coil having approximately the same over-all dimensions as the associated side circuit coils. This permitted the phantom coils to be mounted on the same spindles with the associated side circuit loading coils as phantom groups, thus reducing the amount of inside cabling. This gave improved electrical results, besides reducing the potting costs. The use of the smaller sized phantom coil, in combination with a larger sized case, made it practicable to pot a total of 45 phantom group combinations (135 coils) in a single case. Using the same sized case for potting phantom group combinations involving the older large sized phantom coils, the limit on the number of coils was 108 (36 phantom groups).

The reduction of the phantom coil inductance from 106 to 63 milhenrys made a substantial increase in the cut-off frequency and in the velocity of transmission, as noted in Table X. These improved characteristics made the H-63-P circuit much superior to the H-106-P circuit from the standpoint of echoes and velocity distortion characteristics. On this basis the H-63-P circuit is intermediate in transmission excellence between H-174-S and H-44-25 circuits.

It was found inadvisable to make a similar change in the H-44-25 loading system owing to cross-talk reactions following from the necessary use of higher repeater gains in the phantom circuit. These undesirable reactions, though present to a lesser degree in the case of the H-174-63 system, were offset by the factors already described. The size of the H-25-P coil was reduced, however, to conform to the potting method adopted for H-174-63 loading.

From the standpoint of repeater circuits the H-174-63 system is inherently better than the H-245-155 system because of its higher velocity and higher cut-off with resulting higher quality of transmission. Furthermore, as far as non-repeatered circuits are concerned, there is a negligibly small difference between the transmission performances, considering frequency distortion effects as well as volume efficiency effects. The standardization of the H-174-63 phantom group loading system, therefore, marked the abandonment of use in new facilities of the old standard H-245-155 phantom-group loading system.

Attenuation-Frequency Distortion. In addition to their improved velocity and cut-off frequency characteristics, the H-44-25 and H-174-63 loading systems have an important advantage from the standpoint of attenuation-frequency distortion effects, as is illustrated in Figs. 10 and 11. The frequency distortion effects illustrated in Fig. 10 may become very serious in very long lines. An indication of this is given in Fig. 11. The heavy line curves in this diagram illustrate the attenuation-frequency characteristics of a 500-mile 19 A.w.g. cable circuit involving the various types of loading noted, assuming that "perfect repeaters" are used in each case to reduce the total line loss to 10 TU at 1000 cycles. The foregoing "perfect repeater" is assumed to have the same amplification at all frequencies. Of course, in order to have the same over-all efficiency in the different types of circuits at 1000 cycles, it is necessary to assume different total amounts of repeater gain. The dotted lines in Fig. 11 illustrate corresponding frequency characteristics of short non-repeatered cables having the same types of loading as before; in each case the length of 19 A.w.g. cable circuit being chosen so that the non-repeatered circuits have the same loss (10 TU) at 1000 cycles. A visual inspection of the dotted and heavy line curves indicates how the line losses pile up in long connections. In the old standard low cut-off loading, the accumulated

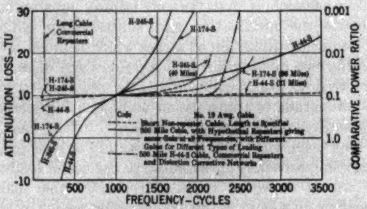

FIG. 11—ATTENUATION-FREQUENCY CHARACTERISTICS OF SHORT AND LONG LOADED TOLL CABLE CIRCUITS HAVING A NET ATTENUATION LOSS OF 10 *TU* AT 1000 CYCLES

losses in very long lines amount to a suppression effect for frequencies above 1600 cycles.

In very long lines having the newer grades of loading, the line losses are still sufficient to cause serious attenuation distortion effects if allowed to go uncorrected. The improved types of repeaters now used on long loaded circuits provide somewhat higher gains at the upper speech frequencies, thereby obtaining approximately a flat frequency characteristic over a wider frequency range. In repeaters used in conjunction with the H-44-25 loading, losses are introduced at the lower speech frequencies by auxiliaries to the repeater circuit, for the purpose of flattening the frequency characteristic at low frequencies. An indication of the improvement obtainable in the above ways is given by a dot-dash curve in Fig. 11, which illustrates the attenuation-frequency characteristic of a 500-mile H-44-S circuit having the best types of repeaters now commercially available.

In view of the difficulties brought into repeatered circuits by the use of loading, the question comes up: "Why not use more repeaters and do without the loading?" In the case of long cable circuits the answer to this question is that the coil loading substantially improves the attenuation and substantially reduces the frequency distortion at a cost which is much lower than the cost of the additional repeaters and distortion corrective networks which would be required to give the same grade of transmission without using loading.

Long Repeatered Open-Wire Lines. In the case of the long open wire lines, the present day answer to the foregoing question is unfavorable to the use of loading. The use of improved types of repeaters now makes it possible to secure better transmission results in long repeatered circuits without loading, than can be secured in loaded repeatered lines. In this connection it should be noted that in the case of non-loaded open-wire lines the distributed inductance is sufficiently large to keep the attenuation-frequency distortion low. Also the velocity of transmission is very high relative to that of a coil loaded line and there is no cut-off effect except that produced by the filters and other apparatus in the repeater sets.

These general transmission considerations are resulting in the removal of coil loading from high grade open wire lines. This dismantling work is being accelerated in order to adapt the open wire plant for a much more extensive application of carrier telephone and carrier telegraph systems.

The present expectations are that in the future new applications of open-wire loading will generally be limited to isolated cases of short lines where carrier telephone or telegraph systems are not contemplated and where the maintenance and operating conditions are unfavorable to the use of telephone repeaters.

Cable Loading Installation Features. Cost considerations make it desirable to use aerial cable in the long toll cable installations, so this type of construction is generally used in the open country. In the vicinity of large population centers, underground cable is used.

Typical aerial cable loading installations are illustrated in Figs. 12 and 13. On the main trunk cables two-pole *H* fixtures capable of supporting four or six large coil cases are usually required. Fig. 12 illustrates a fixture of this type designed for supporting four cases, three of which are already in place. On the smaller branch cables a single pole fixture such as illustrated in Fig. 13 is commonly used.

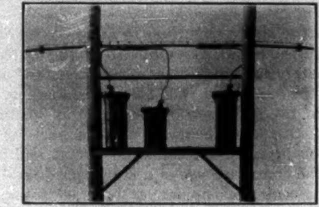

Fig. 12—Installation of Aerial Toll Cable Loading on 4-Case "H" Fixture

At the time a toll cable is installed, provision is made in the cable splices for the ultimate requirements as well as for the initial loading installation. Ordinary splices are made for the coils which are installed at the time the cable is placed, and "balloon" splices which provide the slack wire required for splicing are arranged for subsequent installations.

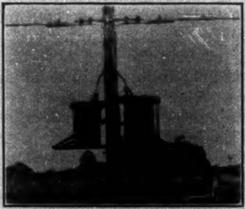

Fig. 13—Installation of Aerial Toll Cable Loading—Single Pole Fixture for Small Branch Cables

III. Loading for Incidental Cables in Open Wire Lines

In the loading applications discussed in the preceding sections, the primary purpose of the loading is to reduce line attenuation losses and frequency distortion effects. In the case of incidental pieces of cable in open wire lines, however, the primary function of the loading is to give the inserted cable approximately the same impedance characteristics as the open wire line, in order to minimize reflection effects at the junction of the cable and the open wire construction. An incidental cable occurring at a line terminal is ordinarily known as a toll entrance or a terminal cable; when occurring at an intermediate point, it is known as an intermediate cable.

The reduction of junction impedance irregularities has become especially important during recent years as a result of the rapidly increasing use of telephone repeaters, since in repeatered circuits, line impedance irregularities, by virtue of their effect upon the repeater circuit balance, may reduce the effective repeater gain and thereby impair transmission by an amount much larger than the ordinary reflection loss. Prior to the general use of telephone repeaters, satisfactory results were obtained by using some one of the standard heavy or medium weight cable loading systems on the entrance and intermediate cables associated with loaded open-wire lines, and a special weight of loading was used on the incidental cables in the non-loaded open-wire lines. In some cases ordinary medium loading was used, with suitable types of step-up or step-down transformers at the terminals of the inserted cable.

Incidental Cables in Loaded Open Wire Lines. In toll entrance and intermediate cables associated with loaded open wire lines, the primary requirements for matching impedance are that the nominal impedance and the cut-off frequency of the cable loading and of the loaded open wire line should be closely the same. To a first degree of approximation this means that the cable loading sections should have the same total mutual capacitance as the open wire loading sections, which, of course, requires a very much closer spacing. The cable loading system which was standardized for use in association with loaded open wire lines is designated "E-248-154". Its primary electrical characteristics are given in Table XII. Besides meeting the impedance requirements for use in association with repeatered open-wire lines, it is also very satisfactory with respect to attenuation characteristics. (In placing this loading, it is customary to locate the first loading point in the cable at such a distance from the last loading point in the open wire line that the total capacitance of the junction loading section is closely the same as that in the regular open-wire loading sections.)

Incidental Cables in Non-Loaded Open-Wire Lines. The problem of designing coil loading for incidental cables in non-loaded open wire lines is considerably more complicated than the case above considered, primarily because it involves an impedance match between a smooth line and a lumpy line. Broadly stated, the first part of the problem is to design a loaded cable of such characteristics that its corresponding smooth line is closely similar to the non-loaded open wire line. The second and more complicated part of the problem is to determine the coil spacing. This usually involves some degree of compromise, because of the dependence of the impedance of a loaded cable upon the loading termination.

TABLE XII

TYPICAL LOADING SYSTEMS FOR TOLL ENTRANCE AND INTERMEDIATE CABLES

Loading System Designation	Type Circuit	Coil Inductance Henrys	Coil Spacing Miles	Nominal Impedance Ohms	Cut-off Frequency Cycles	Attenuation Loss TU per Mile at 1000 Cycles	
E-28-16	Side	0.028	1.09	650	7200	0.15	(13 A.w.g.)
	Phantom	0.016	1.09	400	7800	0.13	
CE-4.1-12.8	Side	0.0041	0.176	600	45000	0.22	(13 A.w.g.)
	Phantom	0.0128	1.09	400	8500	0.19	
M-44-25	Side	0.044	1.66	650	4600	0.29	(16 A.w.g.)
	Phantom	0.025	1.66	400	4900	0.24	
E-248-154	Side	0.250	1.09	1950	2400	0.081	(13A.w.g)
	Phantom	0.155	1.09	1200	2500	0.070	

NOTE. Cable capacitance is assumed to be 0.062 μf per mile for side circuits, and 0.100 μf per mile for phantoms.

The first general requirement is that the ratio of inductance to capacitance to resistance per unit length in the loaded cable should be the same as the corresponding ratio in the non-loaded open wire line. Ordinarily, the loading coil resistance does not play an important part in the determination of the optimum resistance for the loaded cable; the choice of conductor gage being far more important. From this point of view, No. 13 A.w.g. is practically the best gage of conductor for entrance cable circuits connecting with 165-mil open wire lines. For the optimum impedance match on cables connecting with 104-mil open wire lines, it is necessary to use much higher resistance conductors, the choice between Nos. 16 and 19 A.w.g. conductors depending upon a number of factors which space limits do not allow to be discussed.

As noted in the discussion under "Theory" in the first part of the paper, the characteristic impedance of a uniform line is substantially a pure resistance, having the value $\sqrt{L/C}$ over the frequency range throughout which the inductive reactance per unit length is large with reference to the resistance. On the other hand, the characteristic impedance of a coil loaded cable varies over a wide range with frequency, depending upon the particular loading termination used.

Typical impedance-frequency curves for mid-coil and mid-section terminations are illustrated in Fig. 3. As will be seen from this diagram, the rising slope of the mid-section termination and the drooping slope of the mid-coil termination do not deviate greatly from a straight line relation for frequencies below approximately 0.5 of the cut-off frequency. The higher the cut-off frequency is, the more closely will the impedance-frequency characteristic of the loaded cable approach the flat characteristic of the non-loaded open-wire line over the range of frequencies involved in speech transmission. In this connection, it is to be noted that the repeaters now used on open wire lines are designed to transmit frequencies between approximately 200 and 2600 cycles. Of course, the higher the cut-off frequency, the more expensive will be the loading. Practical reasons make it desirable to space the loading coils on the cable circuits connecting with non-loaded open-wire lines at the same intervals as the coils which are used on the cable circuits connected with the loaded open-wire lines. This consideration in combination with the nominal impedance requirement previously mentioned, fixes the cut-off characteristics and, hence, the slope of the termination impedance-frequency characteristic.

These general considerations have led to the standardization of the E-28-16 loading system for use on entrance cable and intermediate cable conductors associated with non-loaded open wire lines. General data for this system are given in Table XII, and the computed impedance characteristics are illustrated in Fig. 14, which also gives the characteristic impedance curves for the non-loaded open wire line and the non-loaded cable. Since the E-28-16 loading system is a low impedance loading, the attenuation improvement is small relative to that of other types of loading system which are primarily installed for attenuation improvement.

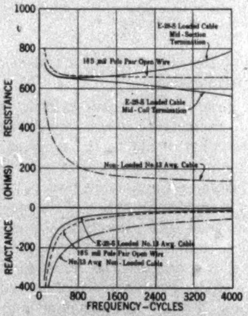

Fig. 14—Typical Impedance-Frequency Characteristics of Loaded and Non-Loaded Entrance Cable and Non-Loaded Open-Wire Line

Table XII also gives general data regarding the M-44-25 entrance cable loading system which has been used to some extent as a substitute for the E-28-16 loading system on cables connected to non-loaded open wire lines. The M-44-25 system used higher inductance loading coils and considerably longer spacing intervals than the E-28-16 system, and was consequently less expensive. The impedance characteristics, however, were not so satisfactory at the upper speech frequencies because of the greater slope of the impedance-frequency characteristic, due to the lower cut-off.

Carrier-Frequency Loading. Special types of entrance cable loading have beeen developed for use on incidental cables in open-wire lines on which carrier telephone or carrier telegraph systems are superposed. Loading system CE-4.1-12.8 listed in Table XII exemplifies this type of loading. The present standard carrier telephone systems operate up to frequencies of the order of 30,000 cycles.[25]

In order to get satisfactory impedance and attenuation characteristics in the loaded incidental cables, a cut-off frequency of approximately 45,000 cycles is used.

It will be noted from the foregoing that the highest working frequency in carrier loaded cables is approximately 0.75 of the cut-off frequency. The ordinary mid-coil and mid-section terminations do not give sufficiently close approximations to a flat impedance-frequency characteristic over this wide range of frequencies, so it has been necessary to use at the terminals of carrier loaded cables, a simple impedance corrective network.

Data regarding attenuation losses in a typical carrier-loaded cable are given in Table XIII. For purposes of comparison similar data are given on a corresponding non-loaded cable. Effective resistance values of the carrier loading coil are also included.

TABLE XIII
CARRIER FREQUENCY LOADING

Frequency Kilocycles	Attenuation Loss - TU per Mile (13 A. w. g. Cable) Non-Loaded	C-4.1 Loading	Resistance-Ohms per Carrier Loading Coil
1	0.49	0.23	1.5
5	0.78	0.27	1.6
10	0.90	0.33	1.9
20	1.14	0.52	4.1
30	1.37	0.90	8.1

The high frequency loading is used only on the side circuits, since at the present time it is not customary to operate carrier telephone systems over phantom circuits. The associated phantom circuit loading is designed for ordinary speech transmission. In order to transmit the high frequency carrier currents over the side circuits, it is necessary to have the side circuit loading coils spaced much more closely than for the ordinary voice frequency loading coils in the phantom circuit. On this account the theoretically best loading points for the carrier circuits frequently occur at places where it is inconvenient to locate the loading coils. The actual loading sections in such cases are made shorter than the theoretical lengths, and the deficiencies in loading section capacitance are remedied by adding lumped capacitances in the form of "building-out condensers." Recently, special types of stub cable designed specially for building-out purposes have come into use as substitutes for building-out condensers.

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[25] "Carrier Current Telephony and Telegraphy," E. H. Colpitts and O. B. Blackwell, Trans. A. I. E. E., Vol. 40, 1921, p. 205.

Loading Coils. The design of the coils used in the E-28-16 and M-44-25 loading systems is generally similar to the toll cable-loading coils having 35-permeability compressed powdered iron cores already described. The loading coils used in the E-248-154 loading system are larger coils of the air-gap type 65-permeability wire core construction listed in Table VII.

As regards the carrier loading system, CE-4.1-12.8, since this involves the transmission through the loading coils of frequencies up to 30,000 cycles or somewhat higher, special coil designs are required. The coil which loads the audio frequency phantom circuit, aside from being specially balanced for association with the side circuit coils, is generally similar in construction to the compressed powdered iron core phantom coil for toll cables.

The side circuit coil, however, is used for loading the high frequency circuit, and more severe requirements are, therefore, imposed on it owing to the multi-frequency transmission. Ordinarily the circuits are equipped to provide three or four carrier telephone channels or 10 carrier telegraph channels over a pair of wires, in addition to the ordinary audio frequency telephone and grounded telegraph channels. The primary added requirements as regards the loading coils are freedom from intermodulation between channels, and low energy losses at carrier frequencies. The most satisfactory solution as regards freedom from magnetic modulation is the avoidance of the use of ferro-magnetic core materials. The side circuit-loading coils were, accordingly, designed as toroidal wood core coils, with finely stranded copper windings in order to limit the eddy-current losses. Data regarding resistance-frequency characteristics are included in Table XIII.

The air core side circuit coils have a small leakage inductance which must be allowed for in determining the phantom coil inductance. For this reason the phantom coil inductance is lower than in the E-28-16 system (Table XII). In order to avoid impedance irregularity in the carrier circuits at the phantom loading points, it is necessary that the combination carrier-phantom loading units should have closely the same total inductance and shunt capacitance as the ordinary carrier loading coils. This requires the use of a different type of carrier loading coil at the phantom loading point, from that at the non-phantom loading points, having a lower inductance and capacitance corresponding to the leakage inductance and shunt capacitance of the associated phantom coil. Other refinements of design are involved in these combination loading units.[26]

IV. CROSS-TALK

One of the greatest practical difficulties which has been encountered in extending the commerical range of long distance telephone service is that of keeping at a tolerably low value, the speech overhearing effects known as cross-talk, which occur between adjacent telephone transmission circuits whenever there is an appreciable amount of electromagnetic or electrostatic coupling between them.

From the early days of telephony great care has been exercised in plant design and construction work to avoid circuit and apparatus unbalances, but as is to be expected from the nature of the problem, it is practically impossible to obtain and maintain absolutely perfect balance. In short telephone circuits, there is no particular difficulty in keeping the unbalance effects small enough so that the over-all cross-talk is not serious. As the length of the line increases, however, there are more and more opportunities for unbalances in the lines and in the associated apparatus in the lines and offices. In repeatered lines, moreover, the repeaters amplify the cross-talk as well as the speech transmission. Thus we have the cumulative effects of cross-talk from successive sections in the long repeatered lines. From the service standpoint, however, it is necessary that the cross-talk in the very long lines should be within the limits set for the shorter lines.

The problem of keeping cross-talk low between a phantom circuit and its associated side circuits, and between the two associated side circuits of a phantom group, is by far the most difficult phase of the general cross-talk problem in long repeatered cables. It is present in the cables, the loading coils, the terminating apparatus and the office cabling. Of these, the cable and associated loading coils are the major sources of unbalances.

The phantom-to-side and side-to-side cross-talk unbalances in the cable quads are reduced to small values by exercising great care both in the various manufacturing processes and in the selection of raw materials. When the cable is installed in the field, a large improvement in cross-talk conditions is secured by splicing adjacent lengths of cable together in such a way that the unbalances in one length of cable substantially neutralize the unbalances contributed by the adjacent length of cable. Usually, three such "capacity-unbalance test" splices are made at symmetrical points in each loading section, and as a result the average over-all capacity unbalance in a loading section is reduced to about one-tenth of the magnitude which would hold if these tests were not made.

In the design of the standard phantom circuit and side circuit loading coils, special care was taken to make them substantially free from inherent unbalances. Also in the manufacture of the coils, great care is exercised to realize the benefits of the inherent symmetry of the designs. In the early days before telephone repeaters came into general use on loaded lines, satisfactory results from the standpoint of self inductance and mutual inductance unbalances were obtained by adjusting the different windings to the nearest turn; *i. e.*, a condition of balance where either adding or subtracting one turn to one of the line windings would increase the cross-talk rather than reduce it.

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[26] U. S. Patents Nos. 1,501,959, Martin and Shaw; 1,501,926, Shaw.

Later when repeaters came into general use, it was found necessary to obtain much more refined adjustments. Further improvements have been worked out in manufacturing methods and processes which allow a greater degree of symmetry. As a result of these various improvements, the phantom-to-side cross-talk unbalances in the loading coils have been reduced approximately 75 per cent or more below the values obtained before repeaters came into general use on small gage toll cable. The coil cross-talk unbalances are now nearly as low as the cross-talk unbalances in the associated cable sections after the completion of the capacity unbalance test splicing.

The loading coils used in the very long circuits having H-44-25 loading obviously are more important from the standpoint of cross-talk limitations than the coils used in the shorter circuits having H-174-63 loading, and somewhat greater care is required in their manufacture. These coils are adjusted and tested in a factory test circuit which at the cross-talk test frequency simulates the service impedance conditions. In the phantom-to-side cross-talk test, the disturbing test current is superposed on the phantom circuit, and measurements of the cross-talk are made in the side circuits, the cross-talk being expressed in millionths of the current into a transformer connected to the phantom circuit and of such ratio as to make the impedance at its input equal to that of the side circuit. As a result of the improvements previously mentioned, the average cross-talk in the coils used for the H-44-25 loading is now about 20 millionths. This corresponds to an attenuation of about 95 TU.

To assist in visualizing the real achievement which this minute value of phantom-to-side cross-talk represents, Table XIV gives information regarding the cross-talk of different elementary types of unbalance in H-44-25 loading coils:

TABLE XIV
CROSS-TALK DUE TO UNBALANCE IN H-44-25 LOADING COILS

Type of Unbalance	Amount of Cross-talk
1 ohm resistance	400 millionths (68 TU)
1 micro-henry inductance	2.5 " (112 TU)
1 turn of winding	280 " (71 TU)
1 micro-microfarad capacitance	0.94 " (121 TU)

These values apply at 1000 cycles.

In the loading coils designed for H-174-63 loading, the cross-talk per unit of electromagnetic unbalance tends to be smaller and the cross-talk per unit of electrostatic unbalance larger, in rough proportion to the differences in line impedance between the H-44-25 and H-174-63 circuits.

Side-to-side cross-talk is uniformly lower than phantom-to-side cross-talk, as would be expected from the less intimate coupling between circuits. Accordingly, the special adjustments which are made are primarily for the purpose of reducing phantom-to-side cross-talk.

In the loading coils intended for H-44-25 circuits the special cross-talk adjustments are applied for minimizing "far-end" cross-talk or for minimizing "near-end" cross-talk, according as the coils are required for four-wire or two-wire repeatered circuits, respectively. The term "far-end" cross-talk applies to cross-talk heard at the distant end of the disturbed circuit, and correspondingly the term "near-end" cross-talk applies to the cross-talk heard at the end of the disturbed circuit near the talker.

Considering now the cross-talk between four-wire circuits in the same quad, it is to be noted that the directional effects of the telephone repeaters block the transmission of cross-talk in the one-way path back to the near end of the circuit, and consequently the special cross-talk adjustments on the coils for four-wire H-44-25 circuits are made primarily for reducing far-end cross-talk.

In two-wire circuits, near-end and far-end cross-talk both occur, and generally near-end cross-talk is much greater because its "average" cross-talk path has less attenuation than that of the far-end cross-talk. Consequently, the special cross-talk adjustments made in the two-wire circuit coils are for the purpose of reducing the near-end cross-talk to a minimum.

In the foregoing connection, it is to be noted that the cross-talk current caused by electromagnetic unbalances flows around the two ends of the disturbed circuit in series. On the other hand, the cross-talk current caused by electrostatic unbalances divides and flows from its point of origin in opposite directions around the two ends of the circuit in parallel. Consequently, when electrostatic and electromagnetic cross-talk currents are in phase at one end of the circuit, they will be practically in phase opposition at the other end of the circuit. The special cross-talk adjustments are made in such a way as to get the maximum benefit from the phase-opposition at the particular end of the circuit where the reduction is more important.

In the four-wire type of circuit used in very long cable circuits, relatively large amplification gains are possible in the repeaters because of the characteristic circuit feature which allows the repeaters to act as one-way amplifiers. As a result of these high amplifications, there are large differences in power level on the input and output sides of the repeaters. This fact has made it desirable for cross-talk reasons to segregate the oppositely transmitting branches of the four-wire circuits. In the cables, the "east-bound" and "west-bound" branches of the four-wire circuits are in different groups. This segregation is also carried out in the loading coil pots, and in the office cabling.[27]

With loading coils as manufactured at present, the cross-talk unbalances in the loaded cables are such that the resultant over-all cross-talk is expected to be tolerable for the longest circuits now definitely planned in cable. The margin below commercial limits is much less in two-wire circuits than in four-wire circuits. At present, there is a growing tendency to use two-wire circuits for longer distances than formerly, for reasons of plant economy. This trend thus increases the severity of the cross-talk requirements.

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[27] U. S. Patent No. 1,394,062—O. B. Blackwell.

Unbalances in loaded circuits which contribute to noise due to induction from power transmission and distribution circuits are similar in nature to those contributing to cross-talk. The precautions which are taken in the design, manufacture, and installation of loaded circuits to reduce unbalances have the effect, therefore, of reducing both cross-talk and noise.

V. TELEGRAPHY OVER LOADED TELEPHONE CIRCUITS

It had been the practise in the Bell System, before the advent of loading, to employ circuits for simultaneously transmitting telephone and telegraph currents. Two methods were in general use, (1) the composite system, in which each line wire of the telephone circuit provided a telegraph channel with ground return, and (2) the simplex system, in which the two conductors in parallel were used with a ground return.

It was very desirable to continue to superpose d-c. telegraph currents on telephone circuits after the introduction of loading. The possible detrimental effects of the superposed telegraph and telephone currents passing through the loading coils did not require serious consideration so long as the circuits were relatively short, since the magnetic modulation in the loading coil cores due to superposed hysteresis effects was sufficiently small to be negligible. As the length of the loaded circuit was increased, the interaction between the telegraph and telephone currents which has been designated in an Institute paper[28] as "flutter," was aggravated and serious distortion of speech resulted.

Measurements of flutter effects obtained with the two grades of core material then in use, viz., 65-permeability and 95-permeability wire, showed the lower permeability core material to be substantially better in this respect. This material had already been adopted for the high efficiency loading coils used on the large gage toll cable circuits, and for open wire lines used in spanning considerable distances.

In order to obtain improved transmission over composited and loaded Nos. 16 and 19 A. w. g. cables, the side circuit and phantom loading coils for this grade of service were redesigned in 1913 to employ 65-permeability cores working to the same over-all dimensions. In addition to improved flutter characteristics, the replacing loading coils had somewhat lower iron losses, and their cost was slightly higher.

A substantial reduction in flutter distortion effects was later obtained in superposed telegraphy over loaded open wire lines and long coarse gage cable circuits, with the adoption of the air-gap type of loading coils already described (Table VIII) as the latter had considerably better hysteresis characteristics than the corresponding types of continuous wire core coils which they superseded.

The operating requirements for the grounded telegraph systems referred to above, necessitated the use of telegraph currents of very large amplitude relative to the telephone currents; consequently on such circuits it was impracticable to realize the benefits of reduced flutter distortion which would have resulted from the use of small amplitude telegraph currents. These possibilities, however, have been fully realized by the development of a metallic polar duplex telegraph system[29] to meet the special requirements imposed by superposed telegraph operation over long small gage telephone circuits. In this system, the superposed telegraph current is of the same order of magnitude as the telephone current. Under these favorable operating conditions, the flutter distortion effects caused by modulation in the cores of the present standard 35-permeability compressed iron powder core loading coils, are within satisfactory limits on the longest circuits which are used simultaneously for telegraph and telephone service.

The recent development of a voice-frequency carrier telegraph system[30] providing 10 or more independent channels over a loaded four-wire cable circuit has made it economical to concentrate a large part of the telegraph service over the long repeatered cables on a special group of wires which are not used simultaneously for telephone purposes. This method of operation obviously eliminates all possibility of modulation effects between the carrier telegraph circuits and the speech transmission circuits. However, the possibility of intermodulation effects between the different superposed carrier telegraph channels involves the same fundamental requirements in the loading coils as when the telegraph circuits are superposed in telephone circuits. The requirements of these systems are satisfactorily met by the 35-permeability compressed iron powder core loading coils now standard for use in toll cable loading.

VI. RECENT IMPROVEMENTS IN LOADING FOR EXCHANGE AREA CABLES

The developments discussed in the preceding sections were directed to improving and extending the range of long distance telephone service. During the greater part of this period the loading standards for exchange area trunk cables remained fixed. The first important change occurred about 1916, when compressed powdered iron core coils came into general use in place of the old standard wire core coils.

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[28] "Hysteresis Effects with Varying Superposed Magnetizing Forces," W. Fondiller and W. H. Martin, TRANS. A. I. E. E., Vol. 40, 1921, p. 443.

[29] "Metallic Polar-Duplex Telegraph System for Cables," Messrs. Bell, Shanck and Branson, TRANS. A. I. E. E., 44: 327, 1925.

[30] "Voice-Frequency Carrier Telegraph Systems for Cables," Messrs. Hamilton, Nyquist, Long and Phelps, TRANS. A. I. E. E., Vol. 44, 1925, p. 316.

In the period 1922-4, the use of new types of fine wire cables had reached a point which required that certain changes be made in the old standard loading systems. Accordingly a new series of improved loading systems having a considerably higher cut off frequency than the original standard systems, described in Table II, were developed.

Cable Developments. Notable advances have been made in the art of cable manufacture during the last decade or so, including the standardization of 450-pair 19-A. w. g. cable, 900-pair 22-A. w. g. cable, and 1200-pair 24-A. w. g. cable, all contained in standard full sized sheaths (2-⅝ in. outside diameter). For each of the conductor gages involved, each of these new maximum sized cables has approximately 50 per cent more conductors than the previous maximum sized cable, typified by the old standard 300-pair No. 19-A. w. g. or 600-pair No. 22-A. w. g. cables.

The newer types of cables have a smaller amount of paper insulation on the individual conductors with a resultant increase in mutual capacitance.

About 1921 the methods of stranding the newer types of fine wire cables (No. 22 and 24 A. w. g.) were changed in order to improve their balance characteristics. These changes made the cables suitable for the application of loading.

The use of the old standard loading systems on the new types of cables would have resulted in an objectionable impairment of quality, due to the reduction of the cut-off frequency resulting from the increased cable capacitance. Also the types of coils available were more expensive than could be justified for permanent standards on the low cost fine wire cables. Accordingly the development of new loading systems and less expensive coils was undertaken.

Determination of New Cut-Off Frequency Standard. The coil design cost-balance study was taken up as one phase of a general transmission-cost study of exchange area transmission, which also included a theoretical investigation of cut-off frequency standards.

In this work use was made of recent investigations of the effect of variations in the frequency distortion and volume efficiency of a telephone circuit on the capability of the circuit to transmit and reproduce intelligible speech.

In the cost studies, allowance was made for the reduced costs of the new types of cable facilities, and the use of less expensive types of coils proportioned to be in approximate cost-equilibrium with these facilities. On the basis of these new cost relations, it was found that an increase in the cut-off frequency of exchange area loading could be justified. Further studies showed that if the cut-off frequency should be raised materially above 3000 cycles the increased costs would be large in proportion to the resultant improvement in transmission. On this basis, it was decided to adopt 2900 cycles as the cut-off frequency standard for the new loading, when used on higher capacitance cables. This corresponds to a cut-off frequency of approximately 3200 cycles on the older types of low capacitance cables.

New Standard Loading Systems. Having decided upon a new cut-off frequency standard, the next step was to choose coil spacings and inductance values. Obviously, in order to make full use of available loading manholes and vaults, it is desirable to adhere to the established spacing standards. Also, it is desirable to make as much use as practicable of the old standard loading coil inductance values, in order to minimize the expense of rearranging the existing loading to conform to the new standards. Furthermore, there are important advantages in having a graded series of standards. This avoids economic waste otherwise involved in the use of expensive loading on trunks where a less expensive loading is good enough.

The foregoing general considerations resulted in the standardization of certain new loading systems, the principal transmission features of which are summarized in Table XV.

TABLE XV
NEW LOADING STANDARDS FOR EXCHANGE AREA TRUNKS

Loading Designation	Coil Spacing Feet	Coil Code Nos.	Approx. Cut-off Frequency Cycles High Capacitance Cable	Approx. Cut-off Frequency Cycles Low Capacitance Cable
M-88	9000	602	2900	3200
H-135	6000	603	2800	3200
H-175	6000	574	—	2800
D-175	4500	574	2900	3200

NOTE. High capacitance cable has approximately 0.083 μ per mile. Low capacitance cable has approximately 0.066 μf per mile.

The M-88 system is especially suitable for the shorter lengths of fine wire trunk cables which constitute the predominating bulk of the exchange area trunk mileage. In longer trunks, the other more expensive loading systems find their field of service. The H-175 system is limited to low capacitance cables because of the lower cut-off effects on high capacitance cables, but has considerable commercial importance because of the large number of low capacitance cables now in the plant.

Table XVI gives general transmission data on typical exchange area trunks using the new loading systems, including also non-loaded trunks. Attenuation-frequency characteristics of some of these trunks are given in Fig. 15. A dotted line curve shows the characteristics of the old standard medium loading when used on 0.065 μf per mi. No. 19 A.w.g. cable.

The position of the different types of facility in Table XVI indicates in a general way the sequence in regard to costs of the different types of facility; *i. e.*, No. 24-A.w.g. non-loaded cable circuits are the least expensive of those listed and H-175 loaded No. 16-A.w.g. cable the most expensive, the intermediate facilities being correspondingly intermediate in cost.

TABLE XVI
TRANSMISSION CHARACTERISTICS OF TYPICAL EXCHANGE AREA TRUNKS

Cable Conductor A.w.g.	Capacitance μf./Mile	System	Coil Code No.	Cut-off Frequency Cycles	Circuit Impedance Ohms	Attenuation Loss TU per Mile
24	0.079	Non-loaded	—	—	740	2.2
22	0.083	" "	—	—	570	1.8
24	0.079	M-88	602	2900	900	1.48
22	0.083	M-88	602	2900	990	0.96
22	0.083	H-135	603	2800	1300	0.68
19	0.085	Non-loaded	—	—	400	1.27
22	0.083	D-175	574	2900	1690	0.53
19	0.083	M-88	602	2800	860	0.51
19	0.085	H-135	603	2800	1280	0.38
19	0.066	H-175	574	2800	1640	0.29
19	0.085	D-175	574	2800	1680	0.30
16	0.066	M-88	602	3200	960	0.24
16	0.066	H-135	603	3200	1420	0.20
16	0.066	H-175	574	2800	1640	0.17

NOTE. The impedance and attenuation figures hold at 1000 cycles. Impedance values for loaded circuits assume mid-section termination.

In general, it will be noted from Table XVI that the facility cost is in reverse order to the transmission efficiency. However, there are exceptions to this general tendency: for instance, non-loaded No. 19-A.w.g. cable is less efficient and more expensive than No. 22-A.w.g. cable with M-88 or H-135 loading. From this it will be apparent that non-loaded No. 19-A.w.g. cable has practically no economical field of service on a competitive basis with loaded No. 22-A.w.g. cable.

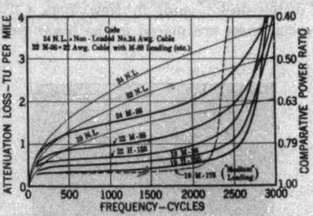

FIG. 15—ATTENUATION-FREQUENCY CHARACTERISTICS OF TYPICAL EXCHANGE AREA LOADED AND NON-LOADED CABLES

The problem of the plant engineer in laying out trunk cables is to determine for each trunk group the type of facility which will meet the established transmission standards with the lowest cost. The permissible loss in a given length of trunk in a particular area depends on the type of service involved; for instance, a larger loss is allowable in a direct interoffice trunk than in other types of trunks forming part of a toll connection.

The allowable loss for a given length and type of trunk varies widely among different metropolitan areas, due to local conditions. These limits are established by means of "loop-and-trunk" studies which determine for a particular area the most economical allocation of the permissible over-all loss between the subscriber loops and the interoffice trunks. On account of the wide range of local conditions the fields of service of the different types of loaded and non-loaded trunks cannot be sharply defined. An indication of the service uses is given in the upper part of Fig. 16, which illustrates the possible applications of the new standard types of facilities for direct interoffice trunk service, assuming maximum allowable losses of 11 and 15 TU, respectively. The diagram shows, for instance, that M-88 loading on No. 22-A.w.g. cable is the preferred construction for trunks from 7.5 to 11.5 miles long, when working to an 11-TU limit. The lower part of Fig. 16 indicates on a cumulative-percentage basis the distribution of direct interoffice trunk lengths in the Bell System.

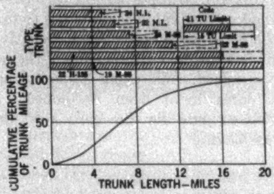

FIG. 16—DIRECT INTEROFFICE TRUNKS

Field of use of different types of loaded and non-loaded facilities, working to 11 to 15 *TU* limits on attenuation loss. Curve in lower part of diagram shows distribution of trunks with respect to trunk length

In the design of the exchange area trunk plant, it is necessary, of course, to consider the signaling characteristics of the facilities and associated equipment, as well as the transmission characteristics. After a certain length is reached in a given type of facility, it may become necessary to use relatively expensive signaling equipment for working longer distances. In some cases of this kind, the total facility cost may be reduced by using a more expensive grade of circuit which will allow less expensive signaling equipment.

In the application of the new standard loading systems, the same standards of over-all attenuation loss are adhered to, as in the older loading systems. In consequence, there is an appreciable improvement in the intelligibility of transmission, due to the ability of the new loading systems to transmit efficiently a range of high frequency voice over-tones which are suppressed by the old standard 2300-cycle cut-off.

Along with this improvement in service, the new loading systems substantially reduce the plant cost; partly due to the economies which result from the extension of the transmission range of the new types of fine wire cables, and partly because of the use of materially less expensive types of loading coils.

Loading Coils and Cases. As previously noted, the first important change in the coils used for exchange area loading from the early 95-permeability wire core type was the substitution of compressed powdered iron in place of wire for the cores. Initially, only coils having powdered iron cores with a permeability of 60 were designed, as this value corresponds to the effective value of the cores displaced. More recently, in order to better fit in with the requirements of the new cut-off frequency standard, coils using 35-permeability powdered iron cores have been developed. In Table XVII are listed data for the coils now used in exchange area loading.

TABLE XVII
COILS FOR LOADING EXCHANGE AREA CABLES

Coil Code No.	Inductance (Henrys)	Core Permeability	Resistance-Ohms		Over-all Dimensions Inches	
			D-C.	1000 Cycles	Diameter	Height
602	0.088	35	8.9	10.5	3.6	1.3
603	0.135	35	12.8	14.1	3.6	1.3
574	0.175	60	4.6	10.6	4.5	2.1

Effective resistance values are for a line current of 0.001 ampere.

The standardization of the small sized Nos. 602 and 603 loading coils has made it possible to design containing cases and assembly methods which permit much larger numbers of coils to be enclosed in cases conforming to the dimensional limitations set by existing vault conditions. A series of cases having capacities up to 300 coils have now been developed. The use of these large potting complements will be of considerable value in reducing the space congestion encountered in the "downtown" sections of the larger metropolitan areas.

In the 300-coil case, a total of 1200 soldered joints are required to connect the coil terminals to the stub cable conductors. It was accordingly very important that the assembly method should involve a minimum liability to open circuits, crosses, or grounds. To accomplish this, a method was devised whereby the various spindles of coils were assembled to a skeleton frame to which the cable stub containing the 600 terminal pairs is also attached. All splices to the outgoing conductors, are made immediately adjacent to the individual coil terminals, after which the skeleton unit consisting of the coils and stub cable with case cover attached, is picked up with suitable tackle, and the coil unit inserted in the case. Fig. 17 illustrates this stage of the assembly. The case is subsequently filled with moisture-proof compound and sealed in the usual manner.

Installation Features. In general, the exchange area cables on which loading is required are run in underground ducts and consequently the great bulk of the exchange area loading is installed in underground vaults. Fig. 18 shows a typical loading installation in a "double-deck" vault in New York City. The loading coil cases are placed in the lower part of the vault permitting the coil terminal stub cables to be brought up vertically behind the horizontal cable runs and spliced to the trunk cables in such a way as to minimize the difficulties of future work on the cables

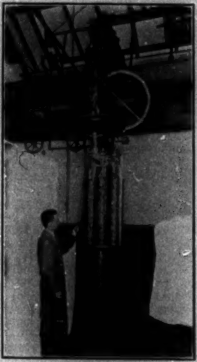

FIG. 17—ASSEMBLY OF 300-COIL CASE

Lowering loading coils into case after coil spindles have been mounted on frame and coil terminals spliced to stub cable conductors

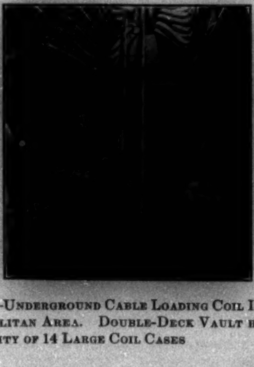

FIG. 18—UNDERGROUND CABLE LOADING COIL INSTALLATION IN METROPOLITAN AREA. DOUBLE-DECK VAULT HAVING ULTIMATE CAPACITY OF 14 LARGE COIL CASES

passing through the vault. The trunk cables enter the vault through ducts which may be seen at the top of the picture, and are supported on racks mounted on the upper side walls of the vault.

At present, a total of eight loading coil cases is installed in the vault illustrated in Fig. 17. Only five of these, however, appear in the picture. The cases now in place contain a total of 645 loading coils. The vault has space for six additional large cases, on which basis it is estimated that this vault will ultimately contain about 2400 coils. Some of the largest vaults are capable of accommodating a total of 30 large cases containing a total of 9000 coils.

VII. Loading for Submarine Cables

Coil Loading. The special problem of applying coil loading to submarine cables is a mechanical one, rather than one concerning the principles of loading. The situation in the United States is such that only a few coil-loaded submarine cables have been required; this, of course, does not refer to the considerable number of instances where the submarine cables are so short that ordinary types of coils installed at the terminals satisfy the transmission requirements.

To date there have been installed in the United States a total of five cables having submarine coil loading details of which are given in Table XVIII.

TABLE XVIII
COIL LOADED SUBMARINE CABLES

Location	Year of Installation	Length of Cable Miles	No. of Load Points	Number of Loaded Ccts.	Spacing of Coils-Miles	Coil Inductance Henrys
Chesapeake Bay No. 1	1910	4.5	2	17 pr. 13 A.w.g.	1.97	0.117
Chesapeake Bay No. 2	1916	4.0	1	12 qd. 13 A.w.g.	2.0	0.067-S 0.042-P.
Tarrytown-Nyack	1916	2.7	2	37 qd. 16 A.w.g.	0.89	0.250-S 0.155-P
Raritan Bay No. 1	1917	5.3	5	37 qd. 16 A.w.g.	0.91	0.250-S 0.155-P
Raritan Bay No. 2	1918	5.3	5	37 qd. 16 A.w.g.	0.89	0.250-S 0.155-P

The Raritan Bay cables each have 37 quads loaded at five points, 111 coils at each point, constituting the largest installation of submarine coil loading in the world. The depth of water in which these cables are installed is about 35 feet.

In each of the above instances, dry core paper cables were used. The design of the coils was made to fit the special designs required for the submarine loading pots. The case design was such as to furnish complete protection of the coils against moisture penetration and adequate mechanical strength for taking up the tension in the cable. In installing the cables, the procedure was the splice the loading coil cases into the cable while the cable was coiled on a barge, and then lay the cable and coils as a continuous operation. The service record of these loaded submarine cables is excellent, thus demonstrating a satisfactory solution of the many difficult mechanical problems involved.

Continuous Loading. Another form of submarine cable loading, first put into practise by the Danish engineer, C. E. Krarup,[31] is to wind an iron wire or tape spirally around the copper conductor. This gives a continuous loading which has found important applications in the case of telephone and telegraph cables laid in deep water. So far as land cables are concerned, it has been found that continuous loading is uneconomical in comparison with coil loading. The only instances of continuous loading in the plant of the Bell System are the Florida-Cuba cables,[32] connecting Key West and Havana, which are the longest and most deeply submerged cables in use for telephonic communication in the world.

VIII. Extent of Commercial Application

The following data will assist in visualizing the practical importance of the developments which have been described in this paper.

In 1911, when Mr. Gherardi addressed this Institute on the subject of loading practise in this country, there were about 125,000 loading coils in service which loaded about 85,000 miles of open-wire circuits and 170,000 miles of cable circuits. Although precise figures are not yet available regarding the number of loading coils in service in the Bell System as of January 1, 1926, conservative estimates set this total at about 1,250,000 coils. These coils load about 1,600,000 miles of cable circuits and 250,000 miles of open wire. In round numbers, 500,000 coils are installed on non-quadded local area trunk cables and 700,000 in toll and toll entrance cables (the bulk of these being quadded cables). Nearly two-thirds of the total number of coils have compressed iron powder cores, all of these being installed on cable circuits. About 4500 coils having wooden cores are installed on carrier-loaded entrance cables. The remainder have iron wire cores, approximately 60,000 being of the so-called "air-gap" types.

Prior to the development of satisfactory types of telephone repeaters, the principal use of loading coils was in exchange area trunk cables in large metropolitan areas such as New York, Chicago, Philadelphia, and Boston. The successful application of telephone repeaters to loaded small gage cables has greatly increased the use of loading in the telephone plant. As illustrating this trend, approximately 150,000 toll-cable coils were manufactured for the Bell System in 1925, and approximately 100,000 exchange area cable coils. Recent estimates of the loading coil requirements for the next five years indicate an annual demand at a rate which would double the total number of loading coils in service about 1930.

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[31] C. E. Krarup, Submarine Telephone Cables with Increased Self-Induction, *ETZ.*, 23:344, April 17, 1902.

[32] W. H. Martin, G. A. Anderegg, B. W. Kendall, "Key West-Havana Submarine Telephone Cable System," Trans. A. I. E. E., Vol. 41, 1922, p. 184.

As regards the field of application for cable loading in terms of cable lengths, the entrance and intermediate cables represent the minimum lengths; for instance, pieces as short as 500 feet when present in carrier telephone systems may require loading. In the local exchange areas, toll switching trunks as short as two miles may require loading. On the other hand, as illustrating the longest circuit now entirely in cable, a connection between Boston and Milwaukee—via New York, Pittsburgh, Cleveland and Chicago—typifies the possibilities in the existing repeatered loaded cable plant. The over-all length of such a circuit is approximately 1200 miles. There is no technical obstacle to the use of repeatered loaded cables for distances several times as great; i.e., in the present state of the art, this is primarily a question of economics rather than of development.

XI. CONCLUSION

It will be appreciated from the foregoing account that the invention of coil loading was the beginning of an era of intensive development which has been marked by enormous advances in the design of telephone transmission lines, and that there has been no slackening of the inventional or development activity devoted to this subject. It is significant that at present more engineers and physicists in the departments represented by the authors are engaged on loading development problems than at any previous time.

In this account of the progress of the loading art during the past quarter century, the authors have endeavored to point out the relation of the loading developments to other phases of telephone development such as cables, repeaters, telegraph working, and carrier telephone and telegraph systems. In the space that is available, it would be impracticable to assign full credit to the many individuals who have been engaged in the development work on loading and the related problems. The final accomplishments should be regarded as the result of well coordinated efforts along many lines.

In conclusion, it may be of interest to note what the development and use of loading has meant to the telephone using public from an economic standpoint. Leaving out of consideration altogether loading on long toll cables—where the interdependence of repeaters and loading is such that it is impracticable to assign to each its share of the savings—and taking into consideration only the loading of interoffice trunks and toll open-wire circuits, it has been estimated that the larger wires which would have been required to give the present grade of transmission if loading had not been available, taken together with the heavier pole lines and additional underground ducts, would have entailed an additional investment in Bell System telephone plant of over $100,000,000.

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Discussion

Bancroft Gherardi: The invention of loading and the development of telephone repeaters have, together, revolutionized the engineering of toll-line plants in the United States. For the invention of loading we are indebted to Dr. Pupin.

Before loading was invented, cable was to toll-telephone transmission much the same as strychnine is to the human system. Any great amount of cable produced definitely unfavorable reactions. The advent and application of loading produced a substantial change in this situation. Whereas, before loading was available, a toll-line cable of even twenty or thirty miles in length presented a serious problem which could only be solved by the generous use of copper, loading very much extended the range of cable so that practical toll cables of even 200 mi. length were available. With the advent of telephone repeaters and their use in combination with loading, toll cables 1000 mi. in length or even longer became practical and economical.

Today we are giving commercial telephone service regularly through cable between Boston and New York on the one hand and Chicago on the other. Before the end of 1926 New York and Boston will be connected by an all-cable route to St. Louis. From Boston to St. Louis it will be about 1400 mi. following the route of the cable.

Many other important cable routes are completed or now under way so that within a few years the principal toll routes in the northeastern section of the United States will be cable.

There are several advantages in this type of construction. On the more congested routes right-of-way problems would have been formidable without cable. Between New York and

Philadelphia, by the end of this year we would have required twenty 60-wire pole lines to take care in overhead construction of the circuits that we shall have in cable. The maintenance of these lines under unfavorable conditions would have been a formidable problem.

To some, it might appear that the loading coils and repeaters would necessarily be a somewhat incidental factor in the total amount of construction involved in the toll-line cable. This is not the case. Considering a cable between New York and Chicago, it is found that only about 50 per cent of the total cost of the cable is in the cable itself and the structure supporting it.

F. B. Jewett: In looking over the paper and in casting my mind back over the history of my connection with the telephone business (which is in point of time almost coincident with the span of time in which loading has been a factor) I was interested to recall the several steps through which this whole art of loading has gone from the days when it was first presented through Dr. Pupin's work. My mental review included the early stages of our attempts to use loading up to the present time when it is recognized as a factor of very great importance.

I chuckled to myself as I thought of a time, nearly twenty years ago, when we thought our development work as applied to loading was nearing completion. The word went out that we ought to hustle up the work we then had in hand and button up this loading development so that it would be standard for all time and we could take our forces off and go to other things. Well, just as in the case of all things of an engineering nature which have real continuing merit, so it is in the case of loading. Being a real thing, there is no end to the development work and the field of application continues to grow. As the paper states, there are more men engaged in loading development problems today than there ever were in the history of the work; also the problems which open up before us as possible of solution through loading and the combination of loading with repeaters, are more extensive now than they ever have been in the past.

As a result of the work to date, which has resulted not only in improvements in kind but in cheapening the forms of loading, we are continuing to reduce the size and length of circuits on which it is economical to apply loading. At any given stage of the development of a thing like the loading coil, there comes a time when although physically possible of use on certain kinds of circuits, it is economically not feasible to use it because it is cheaper to get the desired result in some other way. But, with the cheapening of loading coils and their improvement, the gage and length of circuits on which loading is commercially applicable and advantageous have been reduced until we are approaching the point where it would appear to be economical to use loading on wires as fine as it is mechanically possible for us to employ them.

As the paper indicates, the development work started by the requirements of loading per se, has ramified in all sorts of unexpected nooks and crannies. It has expanded to include electrochemical processes, matters of tool design for the utilization of those enormously powerful presses for forming the core material, special machinery for winding coils, the kinds of insulating material, a thousand and one things which you wouldn't ordinarily think of as being connected with loading-coil developments. Loading-coil developments have also had profound effect on the methods of line construction. The precision with which we have to space these coils, the accuracy with which we have to maintain the balances between the parts of a circuit and the circuits themselves—all of these things indicate what a vast engineering field has grown up as a result of this thing which appeared relatively so simple.

Just one point in closing: I have no statistics to give but will mention one interesting calculation to indicate what the traffic possibilities are in a group of circuits of modern structure, equipped with loading coils and with modern repeaters. Mention has been made of the New York-Chicago cable. That cable in the main, as far as I know, is a cable of standard size. That means it is a cable which has an outside diameter of 2⅝ in. The lead sheath is ⅛ in. thick and inside there is a core of paper-insulated wires. It is primarily a telephone cable. Just as an illustration, let us consider what the communication-carrying capacity of this telephone cable would be if it were used just as it stands for telegraph purposes instead of telephone. With the voice-frequency multiplex system, it is possible to obtain about ten telegraph channels per telephone pair. If each of these telegraph channels were equipped with multiplex printing telegraph apparatus commonly used by the Western Union Telegraph Company, we could operate about ten thousand printer circuits in each direction. Working simultaneously at their normal speeds, it would be possible with this set-up to transmit between New York and Chicago the whole contents of a New York daily newspaper, exclusive of advertising material, in less than two minutes.

M. I. Pupin: The authors mentioned that it is convenient to discuss the coil-loaded line in terms of its corresponding smooth line. I should like to add one word to that statement —that it is still convenient—because I discussed it that way twenty-six years ago, in my first paper before the American Institute of Electrical Engineers. They also say that "Professor Pupin gave his famous solution in a paper presented before the Institute in May, 1900." That is a very complimentary statement but I should like to see that compliment completed. I should add that in that paper was also given the fundamental mathematical method of treating networks, filters and balancing networks. Before that time no one knew anything about it. They call them filters in West and Bethune Streets. At Columbia University we call them pilot conductors. But they are more persistent than we are, and so they have their own way. But the theory of the filters was first given at Columbia University and not at West Street.

Now, having said all the disagreeable things that I can think of, I am going to add another one which is not so very disagreeable perhaps, or perhaps more so. They talk of the loading coil, the toroidal loading coil. There is a very, very important bit of history attached to that. When I came out with the toroidal form of the loading coil, the engineers of the American Telephone & Telegraph Company couldn't see any virtue in it at all. It took them some time to recognize its virtues but when they did, they recognized them very clearly. Since that time they have been eliminating all other forms of coils that they employed by the hundreds of thousands in their business.

Now I won't be disagreeable any more. The work that that company has done upon the development of the toroidal coil (invented at Columbia University and not in Bethune St.) is most remarkable. The development went on with a steady progress until today they have a coil that represents scientific research efforts perhaps unequalled by anything else that has happened in the telephone art during the last twenty-five years, since I made the invention.

It is a remarkable result and I think by next year they may have another announcement to make which will be even more remarkable than anything else they have announced so far, but I am not at liberty to talk about that.

It goes without saying that, in the course of the development of an invention, other things are brought in to supplement the invention and it speaks very highly for the loaded telephone cable that it is supplemented so beautifully by the telephone repeater of the vacuum-tube type. A fellow who can't associate with anybody else is not a very good fellow as a rule. An invention that can't stand the companionship of other inventions is probably a poor invention. The loaded-conductor invention takes the repeater invention into its arms and they hug each other and make a beautiful pair of congenial companions. That to me is more encouraging and more complimentary to the loaded-conductor invention than anything else that has happened. Attempts have been made to supplant the loading coil, to get along with repeaters only by adding attenuation equalizers, but they didn't prove practicable. This created the impression in places that the loaded-conductor invention was dead and a friend of mine came to me one day and repeated a sentence which I published in a book some time ago: "Inventions grow old and are superseded by other inventions."

Some one once said to me, "Isn't that a sad expression, Professor? Doesn't that indicate that your invention is dead?"

That doesn't refer to my invention at all. If it did I wouldn't have put it there. It would betray too much disappointment and I dislike exhibitions of disappointment. No, the invention is not dead; it is still alive and quite vigorous.

It is very true, as Mr. Shaw has said, that not even a very small fraction of the hundred million dollars is in my pocket. Anybody can see that; and it isn't in the pockets of the Telephone Company either, which is not quite as obvious. It is in the pockets of the people of the United States, where it belongs, and they will get a great deal more in their pockets in the course of time, because every year the combination of loading and telephone repeaters is saving an enormous amount of money to the people of the United States. And what is it producing? It is producing an effect for which Washington and Lincoln longed. Washington thought of nothing so much as of the consolidation of the American Union. Lincoln, much against his will, even went to war for the purpose of saving that Union and consolidating it.

Now when you hear of that beautiful telephone system, from Boston to New York, to St. Louis and to all the big centers in the United States, tell me of something else that has that power of consolidating the American Union! There is nothing else unless it be the radio broadcasting. That is another electrical art but that is not a part of the paper.

Who has produced this wonderful art which is producing this wonderful effect? I read another passage from my book: "It is not so much the occasional inventor who nurses a great art like telephony and makes it grow beyond all our expectations as it is the intelligence of a well organized and liberally supported research laboratory," like the Bell Telephone Laboratories at West and Bethune Streets, which I have been abusing so much.

The industries of this country have finally discovered the correct method of doing things. Their great research laboratories take the more or less crude ideas of individuals and develop them and put them together as the American Telephone and Telegraph Co. has put together the loaded conductor and the telephone repeater. This is creating one of the finest telephonic systems that the world has ever known, and has gone far beyond our expectations.

The paper testifies to that better than anything I have seen.

William Fondiller and Thomas Shaw: Dr. Jewett has referred to the early stages of loading development work, and recollected that it was thought at that time that loading was to be standardized for all time so that the engineers might give their attention to other problems. While such a consummation may have appeared very desirable to some at that time, new developments in materials and processes and in methods of operation continually open up new fields of investigation and design. Our paper has attempted to chronicle the major steps in this development.

In considering Professor Pupin's discussion, the authors very much regret that he appears to feel that they have not fully recognized his contributions to this art, and that as a result of this has made several remarks which he designates as "disagreeable things."

The paper refers to Professor Pupin's famous solution of the coil-loaded transmission line. It is so generally known that it hardly requires repetition that Professor Pupin obtained the fundamental patents on the coil-loaded line and the toroidal form of loading coil.

As stated at the beginning of the paper, this does not attempt to cover the origin of loading or the early developments of the art, as these have been treated quite fully in earlier papers, particularly Mr. Gherardi's 1911 paper before the Institute. The loading-coil invention, as was true of Bell's original invention and as is true of nearly all important advances ,was not without its controversies. This is similarly true of the developments in filters and balancing networks which Professor Pupin also mentions. Any reference to these past controversies is entirely aside from the purpose of the paper.

Now, about that part of Dr. Pupin's discussion relating to the beautiful companionship of the loading invention and the repeater invention, a casual reading of the paper may lead one to infer that the application of repeaters to loaded circuits was easily accomplished. Be it understood, however, that the paper is a statement of end results, rather than a detailed account of the difficulties which had to be overcome to reach these results; in fact a great deal of investigation was required, extending over a number of years, before it was commercially possible to combine phantoming, loading, and repeatering telephone circuits. A study of the line characteristics indicated that the failure of the early types of loading coil to maintain the necessary stability was principally responsible for the inability of the repeaters to get along with the loading coils. Development of new magnetic materials and improved construction methods were required before these obstacles were overcome.

Cipher Printing Telegraph Systems
For Secret Wire and Radio Telegraphic Communications

BY G. S. VERNAM[1]
Associate, A. I. R. E.

***Synopsis.**—This paper describes a printing telegraph cipher system developed during the World War for the use of the Signal Corps, U. S. Army. This system is so designed that the messages are in secret form from the time they leave the sender until they are deciphered automatically at the office of the addressee. If copied while en route, the messages cannot be deciphered by an enemy, even though he has full knowledge of the methods and apparatus used. The operation of the equipment is described, as well as the method of using it for sending messages by wire, mail or radio.*

The paper also discusses the practical impossibility of preventing the copying of messages, as by wire tapping, and the relative advantages of various codes and ciphers as regards speed, accuracy and the secrecy of their messages.

* * * * * *

INTRODUCTION

THE purpose of this paper is to discuss briefly certain methods for obtaining secrecy in connection with messages sent by wire or radio telegraphy, and to describe in particular printing telegraph cipher systems that were developed for this purpose during the World War.

The desirability of obtaining secrecy in telegraphic communications and the possible advantages of a system that would be capable of sending messages in such form as to be entirely secret, and which at the same time, would be more rapid and accurate than the codes and ciphers ordinarily used, were brought out in conversations with officers of the Signal Corps, U. S. Army. These discussions made it evident to the engineers of the Bell System that it would be very helpful if the well-known automatic features of the printing telegraph art could be made available for enciphering and deciphering telegraph messages, and could at the same time be made practical for use under service conditions.

The engineers recognized that printing telegraphs[2] were rapid and accurate, but were not secret except to the extent that their signals could not be read from a telegraph sounder. With the general requirements for secrecy systems in mind, studies were made of printing telegraph systems to determine how their messages could be made secret. The result of this work was the development of a cipher system that is capable of rendering messages entirely secret, is rapid and accurate, and is practical to use.

This "Cipher Printing Telegraph System" was called to the attention of the Signal Corps. The Signal Corps became very much interested, tested the secrecy of communications handled by the system and tried it out between New York and Washington. This trial proved that the system could be successfully used to send messages secretly and at a speed many times faster than by methods previously in use.[3]

Each message is automatically enciphered at the sending station and deciphered in the same manner at the receiving station. The method of ciphering will be described later in this paper and is such that under certain conditions of use, the messages are rendered entirely secret, and are impossible to analyze without the key, even if it is assumed that the enemy can capture a machine, learn its method of operation in all details, and intercept a large number of messages.

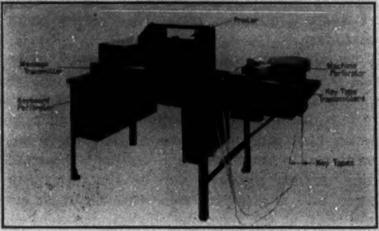

FIG. 1—CIPHER PRINTING TELEGRAPH MACHINE

FLEXIBILITY OF SYSTEM

This method of ciphering can be used with machines of various types. The electrically-driven machine shown in Fig. 1 was developed during the war particularly for the Signal Corps, U. S. Army. In order to save time in production, standard printing telegraph parts were used wherever possible with the result that this machine has the appearance of a "start-stop" printing telegraph set with some additional units mounted on a shelf at the right end of the table. This type of cipher set is particularly suitable for handling large amounts of traffic at high speed.

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[1] Engineer, Dept. Development and Research, Am. Tel. & Tel. Co.

[2] See John H. Bell, "Printing Telegraph Systems," TRANS. A. I. E. E. for 1920, Vol. XXXIX, Part 1, p. 167, and A. H. Reiber, "Printing Telegraph Systems Applied to Message Traffic Handling," TRANS. A. I. E. E. for 1922, Vol. XLI. p. 39.

[3] Note: See page 140, "Report of the Chief Signal Officer to the Secretary of War" for the year ending June 30, 1919.

Presented at the Midwinter Convention of the A. I. E. E., New York, Feb. 8-11, 1926.

If something smaller in size and portable is required, the machine shown in Fig. 2 may be used. This machine is light and strictly portable as no electric current is required for its operation. It is slower than the large machine and requires a knowledge of the standard "Baudot" printer code (see Fig. 8) on the part of the operator, but its messages are equally secret.

These machines are considered suitable for general use by government departments, business concerns, etc., for handling confidential messages rapidly and secretly. The method of using them can be varied to suit conditions and so as to make unauthorized decipherment as difficult as may be necessary up to the point where it becomes impossible even for an expert cryptanalyst.

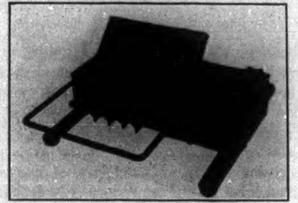

FIG. 2—PORTABLE CIPHER MACHINE

If an appreciable demand exists for machines of special sizes or having particular operating features for special uses, these can be built to employ the same secrecy principle. For example, the functions of ciphering and transmitting over a telegraph circuit can be combined in one machine, if desired, so that, at the sending station, messages can be simultaneously enciphered and transmitted over the telegraph circuit, and so that, at the receiving station, messages can be received, deciphered automatically and printed directly in plain text; thus avoiding the slight delay caused by separately enciphering and deciphering each message. This method is particularly suitable for cases where the cipher equipment can be directly connected to a telegraph line or to a radio transmitter and receiver and can be operated by the same personnel.

RUIYW TGCZG PIETY RJGUA ELKEJ EZIAO
ISCFE LCXHF CONEC XELVY DXJBT WFEJM
HLGDL DDPYD TPGVQ EZAYI LXSZX

FIG. 3—SAMPLE CIPHER MESSAGE IN PRINTED FORM

If the cipher messages are to be turned over to a telegraph or cable company to transmit, they should be in written or printed form. For this purpose, the cipher machine can be arranged to print the cipher messages in groups of five letters each, spaced to form "words." Fig. 3 is a copy of such a message shown exactly as it was prepared by the cipher set. Such messages can be printed by the machine directly on the telegram blank with the address and signature in plain English, and if desired, a carbon copy can be made at the same time for record purposes.

PREVENTING ACCESS TO MESSAGES

There appear to be two general methods for securing secrecy in connection with communications, namely, (1) by preventing or at least attempting to prevent access to the messages or to the lines of communication and, in the case of telegraphic communications by rendering the lines incapable of being tapped, and (2) by the use of codes and ciphers with key systems known only to the proper parties.

As regards wire tapping, sensitive alarm devices arranged to operate on small changes in the electrical constants of the line circuit are unsuccessful as a means of preventing unauthorized parties from obtaining access to the circuit. The electrical condition of a long telegraph circuit is continually changing as a result of variation in temperature and other weather conditions. This fact limits the useful sensitivity of any such alarm devices, whereas by using vacuum tube amplifiers, a record of the signals passing over a circuit can be obtained without appreciably disturbing the line circuit and even without actual contact with the wire.

Telegraph systems have been invented that will operate successfully on very small line currents, and which use coils and condensers to suppress the harmonics in the signal impulses, or in other words to avoid sudden changes in current value. The currents induced in neighboring circuits by such a system would be small, so that it would be rather difficult, if ordinary methods are used, to obtain a record of the signals by their inductive effect. This can be readily done, however, if modern vacuum tube amplifying equipment is used. It is also obvious that a record can be easily obtained if the wire is tapped.

Many attempts have been made to obtain secrecy during the actual transmission of telegraph messages by making them unintelligible. In one system of this sort, successive signal impulses are sent alternately over two line wires by means of a rotary switch which puts the sending key in connection first with one wire and then with the other at each movement of the key. At the receiving end, the impulses are combined through one relay. With this system, the messages may be readily copied if both wires are tapped, and it is quite possible to decipher the messages even if only one wire is tapped.

Proposals have also been made to use complex devices or methods, or so to mutilate the normal impulses that they become unintelligible to anyone tapping the line circuit or intercepting the signals if sent by radio. Any secrecy system of this general class can be readily "broken" by anyone having a knowledge of the methods used and the ability to assemble and operate the necessary apparatus.

Tapping Duplex and Multiplex Circuits

It has also been considered that a full duplex circuit or a multiplex printer circuit, in which messages are being transmitted simultaneously in opposite directions, could not be tapped and that circuits of this character insured secrecy to the communications thus being handled. This is not true, however, and means have been invented by which a message originating at one station of an ordinary duplex circuit can be tapped at any part of the circuit, even though a second message is also going over the same circuit simultaneously in the opposite direction. This means that a multiplex printer circuit, in which as many as eight messages, four in each direction, are handled simultaneously, may be tapped and a person who is familiar with the system can readily analyze the multiplex impulses to distinguish between adjacent channels and the letters of each message in each channel.

An arrangement for tapping a duplex circuit is shown in Fig. 4. A single sensitive polar relay may be used to receive the signals from either end of the circuit, or by using two such relays, the signals in both directions may be read simultaneously. Each relay may control a sounder or a suitable recording device. One winding of each relay is connected in series with the line, the other winding being connected in a circuit from line to ground through an "artificial line" composed of adjustable resistances and condensers. The line winding of each relay should have relatively few turns and should be of low resistance, the other winding having a large number of turns. Each artificial line should be adjusted to be substantially equivalent to the impedance of the corresponding section of line including that of the terminal station equipment multiplied by the ratio of turns of the relay windings.

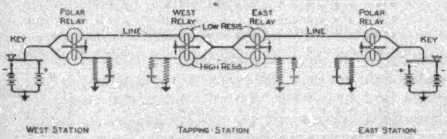

Fig. 4—Method of Tapping a Duplexed Line

Signals transmitted from the west station will pass through the line windings of both relays at the tapping station, a small part of the signal currents also going through the lower windings and artificial lines to ground. The signal currents pass through both windings of the west relay in series, the magnetic effects of the two windings aiding each other, so that the armature of this relay will follow the signals. The same signals pass through both windings of the east relay in parallel, the magnetic effects opposing and balancing so that this relay does not respond to signals from the west station.

In a similar manner, signals from the east terminal station will energize the east relay but not the west relay at the intermediate station, so that by using suitable recording devices associated with each relay, a copy of signals in both directions may be obtained.

Tapping a Multiplex Circuit

This method may be used to tap a multiplex printer circuit, in which case a tape record of the form shown in Fig. 5 will be obtained. If this is taken from a "double-duplex" circuit alternate letters must be read as indicated, to get the message from either channel. Every third or every fourth letter must be chosen if the circuit is operated by the "triple-duplex" or "quadruple-duplex" method. The individual letters are in the ordinary five-unit printer code, the polarities of alternate signals being reversed. To decipher such a tape, it should be divided into units of five dot lengths each. The correct starting point can be found in not more than five trials and can be recognized by the fact that the letters of each message then form sensible combinations.

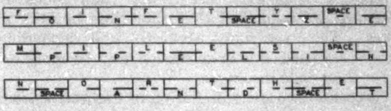
F O I N F E T SPACE Y Z SPACE E
M P I P L E E L S I SPACE N
N SPACE O A R N T D H SPACE E T

Fig. 5—Tape Record of Multiplex Printer Signals

Codes and Ciphers

Secrecy in connection with telegraphic communications is usually obtained at the present time by means of codes and ciphers, the term "code" being applied in cryptography to the method in which entire words or phrases of a message are replaced by arbitrary groups of letters or numbers usually printed in a code book, identical copies being kept by those using the code, "cipher" referring usually to a system in which the individual letters of a message undergo a change either in arrangement or nature.

It is obvious that the combinations of letters in a cipher message will not form pronounceable groups or genuine words except occasionally by accident, but "code" systems can be arranged to use pronounceable artificial "words" or actual dictionary words, if desired. This is usually done, as such "code words" are handled by the telegraph and cable companies at a cheaper rate than the unpronounceable so called cipher "words."

Each of these two general systems has advantages and disadvantages which cause them to be used for certain classes of work, depending upon the conditions. The code system has the outstanding advantage, especially for commercial work, of enabling messages to be shortened so that the tolls are reduced, and it is chiefly for this reason that commercial codes are used. Code is not very accurate, as a mistake in a single code group or even a single letter may change the meaning of an entire message, or necessitate its repetition. If secrecy is required, it is necessary to use carefully guarded private code books, the maintenance of secrecy and accuracy during the printing and distribution of which may cause great trouble. Such books must be carefully used to maintain secrecy, and must be immediately replaced, sometimes at great expense and inconvenience if they should become compromised.

Ciphers, in general, are slower than codes unless machines are used, but then they may be very much faster. They are more accurate, and depending on the system used, cipher messages may be more or less secret than code messages.

There are two general classes of ciphers, known respectively as transposition ciphers and substitution ciphers. In the first class, as the name suggests, the letters of the original message are rearranged, according to a definite system, and in the second class, substitutions for the original letters are made according to some prearranged key. In one, the relative positions of the letters are changed and in the other, the letters themselves.

TRANSPOSITION CIPHERS

A transposition cipher may be distinguished from a substitution cipher by a study of the frequency of occurrence of the letters of the message by comparison with a frequency table of the language of the original message. Studies which we have made of the frequency of the different letters of the English language as they occur in telegrams sent over our private wires, indicate that they are used about as shown in Fig. 6. It is apparent that some letters are used very frequently, the vowels a, e, i, o, u, forming approximately 40 per cent of the total, e being the most commonly used letter of all.[4] This chart is similar to those used by cipher experts.

In a transposition cipher the letters must be rearranged according to a definite system known to the receiving correspondent. Those who make a study of ciphers tell us that such systems are usually easy to discover, particularly if a considerable number of messages are intercepted including two or more of exactly the same length. Transposition ciphers are not suitable for use with machines.

SUBSTITUTION CIPHERS

In substitution ciphers the order of the letters remains unchanged, but for each letter is substituted its equivalent in one or more cipher alphabets. For example, using the table below, for each letter in the plain text alphabet we may substitute its equivalent in the cipher alphabet. To decipher, this process is reversed.

Plain Text–A B C D E F G H I J K L M N O P Q R S
T U V W X Y Z
Cipher –F Q R U K A H G Z S E M L Y P O B C J
V D T X W N I

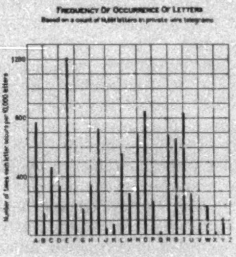

Fig. 6

If a chart is prepared from a frequency count of the letters in such a cipher message, it will have the general appearance of Fig. 6 but the crests will correspond to different letters. Messages of this type are readily deciphered by an expert even when a "mixed" alphabet is used, such as that illustrated above in which the letters of the cipher alphabet are not in the usual alphabetic order.

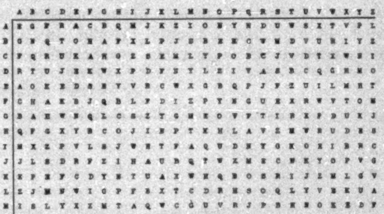

Fig. 7

By using more than one alphabet, the cipher may be made more difficult to "break." The method may be described by referring to the "cipher square" shown in Fig. 7. In this table, the top alphabet represents the plain text, while below it are shown 26 cipher alphabets, each designated by a "key" letter given in the left-hand column. Some form of key, usually a word, is used, the letters of this key word designating the alphabets and the order in which they are to be used. A different cipher alphabet is used in a repeating manner, with each successive letter of the message.

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[4] See "Manual for the Solution of Military Ciphers" by Lt. Col. Parker Hitt.

This type of cipher may be distinguished by the fact that the frequency chart is rather flat, the frequency of occurrence of all letters being roughly the same. Each cipher alphabet is used repeatedly at regular intervals. By first finding this interval and then studying each alphabet separately, messages of this type can be deciphered readily by an expert.

RUNNING KEY CIPHERS

If the key used with this type of cipher is made very long, so that it never repeats and if any portion of this key is never used for more than one message, the operation of "breaking" the cipher becomes very much more difficult. If, now, instead of using English words or sentences, we employ a key composed of letters selected absolutely at random, a cipher system is produced which is absolutely unbreakable.

This method, if carried out manually, is slow and laborious and liable to errors. If errors occur, such as the omission of one or more letters, the messages are difficult for the recipient to decipher. Certain difficulties would also be involved in preparing, copying and guarding the long random keys. The difficulties with this system are such as to make it unsuitable for general use, unless mechanical methods are used.

CIPHER PRINTING TELEGRAPH SYSTEM

By using machine methods, this type of cipher may be made practicable for use. Fig. 1 is an illustration of the cipher machine previously referred to, and which operates on this principle. As previously mentioned, this machine was developed during the recent war and adopted by the Signal Corps, U. S. Army.

Certain parts of this machine are the same as those used for ordinary printing telegraphs, such as those described in recent papers before the Institute. For this reason, it will not be necessary to describe in detail the parts which are commonly used in such systems, such as the keyboard perforator, the transmitters, and printer.

CIPHER MACHINE—METHOD OF OPERATION

The messages are first punched in a paper tape by means of the keyboard perforator. The code used is shown in Fig. 8. This is the well-known five-unit printing telegraph code. Each letter is represented by a small feed hole and one or more larger holes which may be punched in five different positions across the tape. Since in each of these five positions a hole may or may not be punched, there are $(2)^5$ or 32 possible different combinations in this code of which 26 are used to designate letters, the other 6 representing the so-called "stunts," which are the "space," "carriage return," "line feed," "figure shift," "letter shift," and the "blank" or "idle" signal.

The cipher "key" may take the form of another tape of similar form having characters punched in it at random and with every tenth character numbered, so that the tape may be set to any designated starting position. The key tapes are prepared in advance, the original key being perforated by hand, as by working the keyboard at random, additional copies being made automatically by the machine.

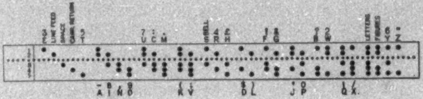

FIG. 8—TAPE SHOWING PRINTING TELEGRAPHIC CODE

The message tape is passed through a unit known as a transmitter, where the holes in the tape serve to control the positions of five contact levers, each of which makes contact with either of two bus-bars. The key tape controls the contacts of a second tape transmitter. The contacts of the two transmitters are connected to a set of five magnets or relays as shown in Fig. 9. Each magnet will be energized if the correspondingly numbered contacts of the two transmitters are against opposite bus-bars, but not if they are making contact with similar bus-bars. In the diagram, contacts 1 and 2 of the message transmitter are against the left or positive bus-bar, this setting representing the letter *A*. Contacts 1, 4 and 5 of the key transmitter are against the positive bus-bar, representing the letter *B* in the printer code. This will energize magnets 2, 4 and 5, which combination represents the letter *G*.

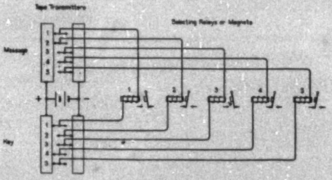

FIG. 9

All of the possible combinations resulting from various characters in the two tapes might be shown in a cipher square similar to that of Fig. 7 except that it would have 32 characters on a side instead of 26.

The characters of the cipher messages formed in this way may be recorded as perforations in a third tape. For this purpose a "machine perforator" is used. This device is similar in many respects to a keyboard perforator and is shown in Fig. 10. The tape, from a reel on the top of the machine, passes through the punch block at the front left corner of the machine. Here it passes under a die plate and over a group of six punches, which may be forced up through the tape by the action of an electromagnetic hammer. Five of these punches are too short to be acted on directly by the hammer and are pushed through the tape only when an individual "selecting finger" is interposed between the punch and hammer. The five selecting fingers are actuated by five magnets which may be controlled by the relays shown in Fig. 9. A ratchet-operated, star-wheel feeds the tape forward after each character has been punched.

The cipher message tape prepared in this way is unintelligible in form and may be sent to the receiving station by messenger or by mail, or if desired, it may be transmitted by wire or radio and reproduced by another machine perforator at the receiving point. The cipher tape is there run through the message transmitter, where its characters combine with those of a duplicate key tape to reproduce the original message, which will be printed out in page form and in "plain text."

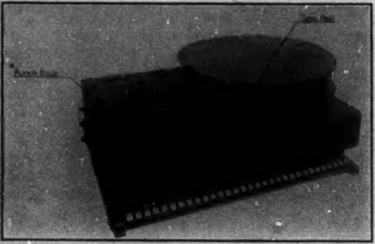

Fig. 10—Machine Perforator

Length of Key Tape

With the system as described above, the key tape must be at least as long as the sum of all the message tapes used with it, as the messages will lose their secrecy to some extent if the key tape is used repeatedly. The use of a short repeating key may give sufficient secrecy for some uses, however.

A roll of tape eight inches in diameter contains about 900 feet of tape and would serve to encipher about 18,000 words counting five printed characters and a space per word, without repeating the key. If sent at an average speed of 45 words per minute this number of words would require 400 minutes or nearly 7 hours to transmit.

In order to reduce the amount of key tape required for handling large amounts of traffic, the "double key" system was devised. In this system two key tapes are used, the ends of each tape being glued together to form a loop preferably about seven feet in circumference. The tapes should differ in length by one character or by some number which is not a factor of the number of characters in either tape. A separate transmitter is used for each tape, and the characters of the two key tapes are combined, by a method similar to that shown in Fig. 9, with those of the message tape to form the cipher message.

The result is the same as though the two key tapes were first combined to produce a long single non-repeating key, which was later combined with the message tape. This long, single key is not, strictly speaking, a purely random key throughout its length as it is made up of combinations of the two original and comparatively short key tapes. The characters in this key do not repeat in the same sequence at comparatively short regular intervals, however, as would be the case if only one key tape loop were used.

The number of characters in this equivalent single key is equal to the product of the number of characters in the two tape loops, and may easily exceed 600,000 before any part of the key begins to repeat. If proper care is taken to use the system so as to avoid giving information to the enemy regarding the lengths of the two key tape loops or their initial settings and to avoid the possibility of ever re-using any part of the resultant single key, this system is extremely difficult to break even by an expert cryptanalyst having a large number of messages and full knowledge of the construction of the machine and its method of operation.

Captain W. F. Friedman, Cryptanalyst of the Signal Corps, U. S. Army, has recently invented some modifications[5] of this system intended to eliminate the loss in secrecy that results from using the two more convenient comparatively short repeating key tapes instead of the single long non-repeating key tape. These modifications consist of changing at intervals the order of connection of the five contacts of one or more of the tape transmitters or of adding a third key tape and transmitter so arranged that the extra key tape does not step ahead in unison with the other two key tapes, but starts and stops at irregular intervals. Either of these methods, properly used, makes unauthorized decipherment practically impossible and at the same time does not unduly complicate the machine or its method of operation.

With the double key tape system, the handling of large volumes of traffic is greatly simplified. The tapes should be numbered so that the deciphering operator can set them at the correct starting point for each message, and rules should be adopted so that both key tapes will never be set twice at the same starting point. Information regarding the proper settings for the key tapes for deciphering each message must be sent to the deciphering operator. These settings may be prearranged or they may be selected arbitrarily by the sending operator. In the latter case the numbers representing the key tape settings should be prefixed to the message. These "key indicators" should preferably be enciphered by running them through the machine together with a special key tape which is used only for this purpose.

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[5] See Patents 1,522,775 of Jan. 13, 1925 and 1,516,180 of Nov. 18, 1924.

SPEED OF OPERATION

This type of machine was operated by the Signal Corps over its private wire telegraph circuits. In service tests made by the United States Army, each outgoing message was checked by running it again through the machine to decipher and print it, and the deciphered copy was then compared with the original message, so that each message tape was run through twice. A certain amount of time was lost, due to setting and resetting the key tapes, checking, etc., for each message, but an average enciphering speed of 10-15 words per minute was readily maintained. We understand this to be many times faster than those manual methods for enciphering or coding which are used where a high degree of secrecy is required. Incoming messages were deciphered at the rate of 30-40 words per minute.

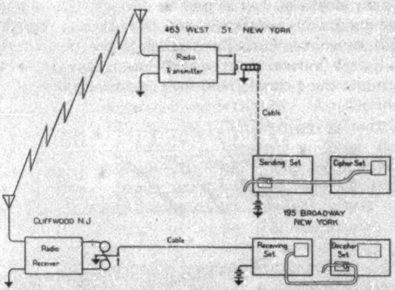

FIG. 11—CIRCUIT FOR RADIO CIPHER DEMONSTRATIONS

OPERATION BY RADIO

This cipher system was demonstrated before the delegates to the Preliminary International Communications Conference in October, 1920. During this demonstration, cipher messages were sent over a circuit containing a radio link, as illustrated in Fig. 11. The radio equipment was the same as that employed a year previous in tests on the operation of multiplex and start-stop printing telegraphs by radio and is described elsewhere.[6]

A considerable number of cipher messages was transmitted over this radio circuit during this demonstration, these messages being automatically enciphered before and deciphered after transmission, so that they were absolutely secret, even though transmitted by radio. No interference from atmospherics or from other radio stations was noticed, all messages being received without error.

In conclusion, we wish to express our appreciation of the assistance given us by the officers of the Signal Corps and the General Staff of the United States Army in making tests and trials of cipher printing telegraph systems; and we wish particularly to acknowledge our indebtedness to Lt.-Col. J. O Mauborgne, of the Signal Corps for his advice in connection with this development and for his assistance in arranging to have tests made to determine its secrecy and demonstrations and service trials to determine its practicability for Army use. We also wish to express our appreciation of the services rendered by the Cipher Department of the Riverbank Laboratories, Geneva, Illinois, and by Col. George Fabyan, the head of these laboratories, in making tests of the secrecy of messages enciphered in various ways with these machines.

Discussion

L. F. Morehouse: This work on the development of arrangements for secretly transmitting telegrams was done during the War under the direction of Mr. Gherardi. The problem was to see if a simple and effective means that would be entirely secret could be devised for handling telegraph business. The method used should be such that an enemy could not decipher the messages even if he could capture the mechanism used, thoroughly understand its operation, and obtain contact with the line circuits.

A large number of systems were studied. Of the many suggestions made, the most promising were set up and tested out in the laboratory. Various schemes for breaking up the dots and dashes were proposed, but in the last analysis all were found to be unsatisfactory. We found in studying this problem that many methods had been devised. Where mechanical devices were used, they involved, sooner or later, a repetition of the key. If there is such a repetition, the messages can be deciphered and the real secrecy has disappeared.

Printing telegraph methods were found to be more promising. The key in the form of a paper tape or resulting from the combination of two such tapes can be made as long as desired without repetitions with the result that the cipher becomes impossible to break. For ordinary business purposes, however, a simpler cipher using a key that might repeat at infrequent intervals would be entirely practical and sufficiently secret.

P. W. Evans: None of the present military codes or ciphers offers complete satisfaction in its security, speed or simplicity. The Signal Corps of the Army will appreciate any assistance or suggestions which may be offered on this important phase of our national defense problem.

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6. "Printing Telegraph by Radio" by R. A. Heising, *Journal of the Franklin Institute*, January 1922, pp. 97-101.

Refraction of Short Radio Waves
in the Upper Atmosphere

BY WILLIAM G. BAKER[1] and CHESTER W. RICE[2]
Non-member
Associate, A. I. E. E.

Synopsis.—*The paper shows that the striking phenomena of short-wave radio transmission (i. e., below 60 meters) can be quantitatively accounted for on a simple electron refraction theory in which the effect of the earth's magnetic field and electron collisions may be neglected as a first approximation. The distribution and number of electrons per unit volume in the upper atmosphere required on this theory to account for the meager experimental data appear to be in general accord with the values required in the explanation of the diurnal variations of the earth's magnetic field, aurorae and long-wave radio transmission.*

The paths taken by the waves from an antenna to distant points on the surface of the earth are calculated. The path calculations give a definite picture of the now familiar skip distance effects. Ideal signal intensity curves (i. e., neglecting absorption and scattering) are given, which show how the energy sent out by a transmitter is distributed over the surface of the earth. A focussing of energy just beyond the skip distance, and again just inside the point where the ray tangent to the ground at the transmitter comes back to earth, is clearly shown. The reflection of waves at the surface of the earth is also considered.

The results of these calculations make it possible to estimate the most suitable wave lengths for night and day communication between any two points on the earth's surface. It is also pointed out that there will be a minimum wave length, in the vicinity of 10 meters, below which long distance communication becomes impossible. It is shown that from the point of view of long distance communication low angle radiation is most effective. The ray paths and energy flux density in the wave front of the sky waves are independent of the plane of polarization of the transmitter. The effects of polarization on the reception problem are not discussed.

I. INTRODUCTION

SHORT-wave (*i. e.*, 60 to 15 meters) transmission experiments during the past two years have definitely brought to light many peculiarities which were entirely unexpected as extrapolations from our many years of long-wave experience. Until recently any announcement of long-distance short-wave transmission was put down as an unexplained freak by the average radio man, and dismissed from his mind. As the number of such reports increased, we could no longer be content to dismiss them as freaks and were forced to abandon our preconceived notions as to what normal short-wave transmission should be, and extend our theory in such a way as to give these remarkable results a definite place in the new scheme of things.

In Fig. 1 we have attempted to summarize the available data on short-wave transmission characteristics. Most of the data are from the valuable papers by Taylor[3] and Taylor and Hulburt[4] with a few check points kindly supplied by Young.[5] We have also obtained considerable help, in drawing the smooth curves through the few scattered points, from the valuable work published by many amateurs.[6] The curves assume that 5 kw. are being supplied to an average antenna and that the practical limit of reception is reached at 10 microvolts per meter. The curves further assume full daylight or night conditions at both the transmitter and receiver in order to eliminate sunset and sunrise effects as well as peculiarities which arise when one station is in darkness and the other in daylight. This, of course, limits the diagram to practically north and south transmission; nevertheless, it may be used to estimate the general trend for other conditions.

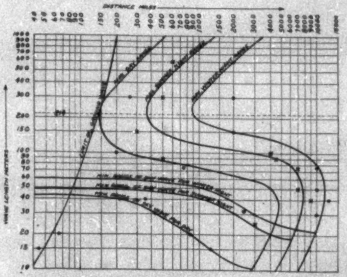

FIG. 1—APPROXIMATE TRANSMISSION CHARACTERISTICS MAINLY FROM DATA BY A. H. TAYLOR FOR 5 KW. IN ANTENNA AND LIMIT OF RECEPTION, 10 MICROVOLTS PER METER. NORTH AND SOUTH TRANSMISSION

As a typical example of the peculiarities of short-wave transmission, let us describe the experience obtained with a 5-kw., 30-meter transmitter. Here the signal rapidly decreases as we leave the transmitter and reaches the lower useful limit of 10 microvolts per meter at about 70 miles. This short range is what might be called the expected value as viewed from our long-wave experience and is represented in Fig. 1 by passing to the right of the line marked *Limit of Ground Wave.* If we now continue to greater distances the signal remains out until we reach approximately 450

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[1] W. and E. Hall Fellow of the University of Sydney, New South Wales, Australia. Research Laboratory, General Electric Co., Schenectady, N. Y.

[2] Research Laboratory, General Electric Co., Schenectady, N. Y.

[3] A. Hoyt Taylor, *Inst. Radio Eng.*, Vol. 13, p. 677, 1925.

[4] A. Hoyt Taylor and E. O. Hulburt, *Q. S. T.*, p. 12, Oct. 1925.

[5] C. J. Young, Unpublished Reports on Short-Wave Transmission Tests by the General Electric Co. at Schenectady, N. Y.

[6] See for example *Q. S. T.*, 1924 and 1925.

Presented at the Midwinter Convention of the A. I. E. E., at New York, N. Y., February 8-11, 1926.

miles where the day signal unexpectedly becomes strong again. This is represented in the figure by crossing to the right of the curve marked *Minimum Range of Day Sky Wave.* Continuing to greater distances we find the signal gradually falling off in intensity, reaching the useful limit of 10 microvolts per meter in the vicinity of 4500 miles by day. This is represented in Fig. 1 by passing to the right of the curve marked *Maximum Day Range.* On a summer night the signal does not reappear after the 70-mile extinction until we are approximately 2000 miles from the transmitter, after which the signal falls off gradually to a very low value at 7500 miles.

The present explanation of the above peculiar phenomena is as follows: Assume, for simplicity, that energy is radiated equally in all directions by the transmitter. As we go away from the source, the signal strength will decrease in the usual manner due to spreading and energy absorption by the ground, with the result that the 30-meter signal practically vanishes in the vicinity of 70 miles. In other words, the ground wave component of the 30-meter signal behaves as we should expect from our long-wave experience, *i. e.*, is rapidly attenuated. The unexpected thing happens when we go out to 450 miles and find that the signal by day has reappeared. This reappearance of signal is accounted for by electronic refraction of a portion of the energy which is radiated towards the sky. A reflection theory of this effect has been proposed by Reinartz.[7] More recently a refraction theory has been developed by Taylor and Hulburt.[4] The calculations in the present paper are based on the electron theory of optical dispersion in metallic media as developed by Lorentz,[8] Drude,[9] etc. We are also greatly indebted to Eccles,[10] Larmor,[11] Appleton[12] and Nichols and Schelleng[13] who have worked out many important conclusions which follow from an application of the optical theory to various phases of the radio transmission problem.

The present calculations show that by making certain reasonable assumptions as to the number and distribution of the free electrons in the upper atmosphere the main characteristics of the relatively meager experimental results can be accounted for.

The calculated paths for rays going out from the transmitter at different angles to the horizontal show the following general characteristics: A ray starting out at a low angle will be only slightly refracted and come to earth again at a great distance from the transmitter. For higher angles the rays will return to earth progressively nearer the transmitter. Finally a critical angle is reached for which the refracted ray comes down at the nearest distance to the transmitter. For higher angles the points of return recede from the transmitter until finally a second critical angle is reached where the ray does not return to earth, but instead goes out into space and is lost.

The distance from the transmitter to the nearest point at which the refracted sky wave returns to earth has been called the "skip distance." For a given wave length the skip distance is a minimum in the middle of the day and a maximum on a winter night, the summer night value being somewhat less than the winter night skip. This variation is theoretically accounted for by a change in the height, thickness and maximum value of the electron density in the upper atmosphere.

The experimental summary given in Fig. 1 shows that the skip distance for a given time of day or night decreases with increasing wave length. This observation is in agreement with the increase in refraction on the longer wave lengths. If we neglect the effect of collisions between the electrons and gas molecules which prevent the refractive index from going to zero, we obtain a sharp upper limit in wave length above which skip distances fall to zero. For the ionization values assumed in the paper, this occurs for a wave length of 60 meters on a winter night as shown in the calculated skip distance curves of Fig. 16.

If the effect of collision frequencies were taken into account, this sharp upper limit would disappear and we would obtain skip distances on wave lengths greater than 60 meters for the assumed winter night ionization values. The effect of the earth's magnetic field will also require consideration in the vicinity of the upper limiting wave length. The experimental determination of skip distances on the longer wave lengths will be difficult owing to the masking effect of a relatively strong ground wave.

On the present theory we may expect severe fading near the transmitter under certain circumstances. For example, consider the case of 60-meter transmission on a winter night. Here a refracted or sky wave will fall inside of the ground wave limit and at a certain distance may be approximately equal in magnitude to the ground wave value. Under these conditions severe interference effects between the two waves will result. If the ground absorption is high this effect may be found quite close to the transmitter. Over salt water the effect should occur at a greater distance.

On shorter waves where the skip distance is well beyond the ground wave limit, severe fading is expected in the region just beyond the skip distance, where the two sky waves of approximately equal intensity overlap.

Appleton[12] and a little later Nichols and Schelleng[13] independently pointed out that the earth's magnetic field should produce some very interesting effects on radio transmission, especially in the vicinity of 214 meters and above. It is interesting to note, in this connection, that Fig. 1 shows a marked absorption in the 214 meter region. A very interesting study of transmission phenomena in the broadcast wave length band has recently been reported by Bown, Martin and Potter[14]. The present paper is confined to the propagation phenomena on the short-wave side of 214 meters where the effective electron restoring force due to the earth's magnetic field will cease to be important compared with the electron inertia force, and may, therefore, be neglected as a first approximation.

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[7] John L. Reinartz, *Q. S. T.*, p. 9, April 1925.

[8] H. A. Lorentz, The Theory of Electrons, Teubner, 1909.

[9] Paul Drude, The Theory of Optics (Engl. Trans. by Mann and Millikan) Longmans, 1917.

[10] W. H. Eccles, *Proc. Roy. Soc.*, Lond., Vol. 87, p. 79, 1912.

[11] Joseph Larmor, *Phil. Mag.*, Vol. 48, p. 1025, 1924.

[12] E. V. Appleton, *Proc. Phy. Soc.*, Lond., Vol. 37, Part 2, p. 16D, 1925.

[13] W. H. Nichols and J. C. Schelleng, *The Bell System Technical Journal*, Vol. IV, p. 215, 1925.

II. REFRACTIVE INDEX OF A MEDIUM CONTAINING FREE ELECTRONS

When an electromagnetic wave passes through an electron atmosphere it sets the electrons in motion, with the result that the apparent or phase velocity of the wave is increased above that in free space. Following Larmor[11], let the electric field intensity of the wave be given by

$$E = E_m \sin \omega t \tag{1}$$

The force exerted on the electron of charge e is $e\,E$ and since for a free electron no elastic restoring or dissipative forces exist, the total applied force is consumed in accelerating the electron mass m, giving

$$m \frac{d v}{d t} = e\,E_m \sin \omega t \tag{2}$$

Integrating equation (**2**) and putting the constant of integration equal to zero, since we are not concerned with the random velocities, we obtain

$$v = -\frac{e}{m}\frac{E_m}{\omega} \cos \omega t \tag{3}$$

where v = electron velocity due to electric field.

If there are N of these electrons per cu. cm. moving with the velocity v they will constitute a current i_1 per square cm. equal to $N\,e\,v$. We thus have from (**3**)

$$i_1 = -\frac{N e^2}{m}\frac{E_m}{\omega} \cos \omega t \tag{4}$$

In addition to this current we have the ordinary condenser charging current flowing across the unit cube. The electric intensity or potential gradient in the wave is E and, therefore, the potential difference between two planes one cm. apart is E times one cm. The capacity of our imaginary condenser which has two plates of one sq. cm. area and one cm. apart is $C = 1/4\,\pi$ since the dielectric constant is unity. The charging current for this unit condenser is

$$i_2 = \frac{1}{4\,\pi}\frac{d E}{d t} \tag{5}$$

Differentiating equation (**1**) with respect to t we have

$$\frac{d E}{d t} = \omega\,E_m \cos \omega t \tag{6}$$

Substituting this relation in (**5**) we obtain for the charging current density

$$i_2 = \frac{1}{4\,\pi}\,\omega\,E_m \cos \omega t \tag{7}$$

The total current, i, is obviously the sum of the two currents i_1 and i_2.

$$i = \frac{1}{4\,\pi}\left\{1 - \frac{4\,\pi\,N\,e^2}{m\,\omega^2}\right\}\omega\,E_m \cos \omega t \tag{8}$$

By substituting equation (**6**) in (**8**) the total current density becomes

$$i = \frac{1}{4\,\pi}\left\{1 - \frac{4\,\pi\,N\,e^2}{m\,\omega^2}\right\}\frac{d E}{d t} \tag{9}$$

It will be observed that this becomes the ordinary expression for the charging current of our unit condenser if we let the bracketed quantity stand for the effective dielectric constant of the material between the plates. In other words, the effective dielectric constant of an electron atmosphere becomes

$$k = 1 - \frac{4\,\pi\,N\,e^2}{m\,\omega^2} \tag{10}$$

The quantities in the above expression are all in c. g. s. electrostatic units.

N = number of electrons per cu. cm.
e = electron charge = 4.77×10^{-10} e. s. u.
$m = 8.97 \times 10^{-28}$ grams
$\omega = 2\,\pi f$
f = frequency in cycles per second
$4\,\pi\,e^2/m = 3.2 \times 10^9$ c. g. s. (e. s. u.)

We see from equation (**10**) that the effect of electrons in reducing the apparent dielectric constant of space is, for a given frequency, proportional to the total number N present per cu. cm., to the square of the charge and therefore independent of the sign of the charges, and inversely proportional to their mass. The lightest gas ion is that of hydrogen having a mass of approximately 1800 times that of an electron. It is evident, therefore, that for anything like equal numbers the effect of gas ions on the effective dielectric constant can be neglected.

A comparison of the signs of equations (**4**) and (**7**) shows that the electron current is opposite in phase to the condenser charging current. In other words, the inertia due to the to-and-fro motion of the electrons is equivalent to shunting the capacity of each unit cube of space by an inductance equal to $m/N\,e^2$.

The velocity of an electromagnetic wave in a medium is inversely proportional to the square root of the permeability times the dielectric constant of the medium. For our case the permeability is unity and the effective dielectric constant that given by equation (**10**). We may, therefore, write for the velocity

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[14] Ralph Bown, De Loss K. Martin and Ralph K. Potter, *The Bell Syst. Tech. Jour.*, Vol. V, No. 1, p. 143, 1926.

$$c' = \frac{c}{\sqrt{k}} \text{ cm. per sec.} \tag{11}$$

where $c = 3 \times 10^{10}$ cm. per sec. velocity of light.

The refractive index μ of a medium is defined as the ratio of the velocity of light in a vacuum to that in the medium.

$$\mu = c/c' \tag{12}$$

From (**10**), (**11**) and (**12**) we obtain

$$\mu = \sqrt{1 - \frac{4\,\pi\,N\,e^2}{m\,\omega^2}} \tag{13}$$

as the refractive index of our electron atmosphere.

This expression for the refractive index depends upon the assumptions that the average time between the collision of an electron with molecules is sufficiently great to allow the full effect of the accelerating action of the electric field to be realized. When an electron collides with a massive particle, the directed velocity given to it by the wave is turned at random and cannot be returned to the wave. Thus the effect of a collision between an electron and a molecule is to convert the energy given to the electron by the wave into random motion or heat energy. The effect of a collision between two electrons does not withdraw energy from the wave, because a collision between two particles, of equal charge and mass, results in such an interchange of velocities that the total charge movement is unchanged by the collision.

Suppose, for example, that on the average an electron makes ten collisions with molecules during one cycle of the wave then it is evident that the wave would scarcely have begun to accelerate the electron before a collision would interfere with the process. It seems clear that where a great number of collisions occur per cycle of the wave the amount of energy which goes into electron acceleration during the short interval between collisions is relatively small, and consequently the energy abstracted from the wave is small. It is also evident that under these conditions the effect of the electrons in reducing the refractive index from unity is slight.

From a qualitative point of view, we can see that the maximum energy dissipation will occur when there are about four electron collisions per cycle of the wave since the full acceleration is obtained in a quarter cycle. Calculation shows that actually the maximum effect occurs for $1/2\,\pi$ collisions per cycle. This may readily be understood since the greater number of collisions will more than make up for the slightly lower velocity acquired by the electrons during the shorter interval of time. With fewer collisions per cycle, the energy loss will rapidly decrease and the full effect of the electrons on the refractive index will result.

A rigorous solution of the problem taking into account the average frequency of collision between an electron and a molecule is given in Appendix I.

III. GENERAL EQUATION FOR THE PATH OF A RAY IN A MEDIUM OF VARYING REFRACTIVE INDEX

The distribution of ionization in the upper atmosphere will, in general, increase with height, reaching a maximum, and will then decrease again to a small value at very great heights. We have seen that the apparent refractive index of the medium is reduced by the presence of the free electrons which are present in an ionized gas. This means that the energy, which is radiated up into this region from our antenna, will be bent around, and return to earth at some distant point, instead of vanishing into space. We picture the energy leaving the antenna as consisting of an infinite number of rays just as is commonly done when dealing with problems in light. We can then follow the paths of the various rays, which leave the antenna in different directions, up through the medium and back to earth again. In this way we obtain a complete picture of what happens.

In Fig. 2 we have drawn two rays at a distance $d\,n$ apart. Let the wave front be along $A\,B$ at a time t and along $C\,D$ at a time $t + d\,t$. It then takes the wave the same time to go from B to D as from A to C.

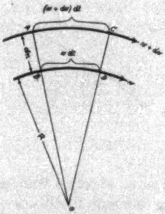

FIG. 2

The distance from B to D is $v\,d\,t$ and the distance from A to C is $(v + d\,v)\,d\,t$, where v and $(v + d\,v)$ are respectively the velocities along the lines $B\,D$ and $A\,C$.

If $O\,B$ is the radius of curvature of the ray $B\,D$ and is equal to R, we must have

$$A\,C/A\,O = B\,D/B\,O \tag{14}$$

since each is equal to the angle $B\,O\,D$ in radians. Thus

$$\frac{(v + d\,v)\,d\,t}{R + d\,n} = \frac{v\,d\,t}{R} \tag{15}$$

From this we obtain

$$\frac{v + d\,v}{v} = \frac{R + d\,n}{R} \tag{16}$$

and upon subtracting one from each side

$$d\,v/v = d\,n/R \tag{17}$$

It follows that

$$\frac{1}{R} = \frac{1}{v}\,\frac{d\,v}{d\,n} \tag{18}$$

But we have from the definition of the refractive index that $v = c/\mu$ where c is the velocity of light. On substituting this relation in (**18**) we obtain[15]

$$\frac{1}{R} = -\frac{1}{\mu}\,\frac{d\,\mu}{d\,n} \tag{19}$$

If we know the value of μ everywhere this equation enables the path to be calculated. The calculations are much simplified, however, by making the following transformations.

Let z denote the height above the ground, and let us assume that the electron density is a function of z only. In this case μ is a function of z alone since the collision frequency ν can also be considered as being a function of z. We may write

$$\frac{d\,\mu}{d\,n} = \frac{d\,\mu}{d\,z}\,\frac{d\,z}{d\,n} \tag{20}$$

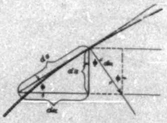

Fig. 3

To evaluate $d\,z/d\,n$, observe that we must take an element $d\,n$ of the normal to the ray path drawn inwards, and find the corresponding change $d\,z$ in z. Reference to Fig. 3 will bring out the construction. Since the inward normal has a downward component, it is evident that z will decrease and, therefore, $d\,z$ will be negative. It follows from Fig. 3 that

$$d\,z/d\,n = -\cos\phi \tag{21}$$

where ϕ is the acute angle between $d\,z$ and $d\,n$. But the angle ϕ is also the angle between the tangent to the path and the horizontal so that

$$d\,z/d\,x = \tan\phi = \sin\phi/\cos\phi \tag{22}$$

Furthermore

$$\sin\phi = d\,z/\sqrt{(d\,x)^2 + (d\,z)^2} \tag{23}$$

Combining (**22**) and (**23**) we find

$$\cos\phi = 1/\sqrt{1 + (d\,z/d\,x)^2} \tag{24}$$

Substituting (**24**) in (**21**) gives

$$d\,z/d\,n = -\,1/\sqrt{1 + (d\,z/d\,x)^2} \tag{25}$$

We now put (**25**) in (**20**) and obtain

$$\frac{d\,\mu}{d\,n} = -\frac{1}{\sqrt{1 + \left(\frac{d\,z}{d\,x}\right)^2}}\,\frac{d\,\mu}{d\,z} \tag{26}$$

Substituting (**26**) in (**19**) we have

$$\frac{1}{R} = \frac{1}{\mu}\,\frac{d\,\mu}{d\,z}\,\frac{1}{\sqrt{1 + \left(\frac{d\,z}{d\,x}\right)^2}} \tag{27}$$

It remains to express the value of $1/R$ in terms of z and x.

The required relation for the curvature in rectangular coordinates is given in any standard book on the calculus[16] as

$$\frac{1}{R} = \frac{\frac{d^2\,z}{d\,x^2}}{\left\{1 + \left(\frac{d\,z}{d\,x}\right)^2\right\}^{3/2}} \tag{28}$$

On equating the two expressions (**27**) and (**28**) for the curvature we obtain:

$$\frac{1}{\mu}\,\frac{d\,\mu}{d\,z} = \frac{\frac{d^2\,z}{d\,x^2}}{1 + \left(\frac{d\,z}{d\,x}\right)^2} \tag{29}$$

This is the required differential equation for the path of a ray since we have assumed that μ is a function of z only. It is convenient to obtain a first integral of this equation before expressing μ as a function of z. To do this we multiply (**29**) by $2\,\frac{d\,z}{d\,x}\,d\,x$ and reduce to

$$\frac{2\,d\left(\frac{d\,z}{d\,x}\right)^2}{1 + \left(\frac{d\,z}{d\,x}\right)^2} = 2\,\frac{d\,\mu}{\mu} \tag{30}$$

The two sides of the equation may now be integrated directly to

$$\log\left\{1 + \left(\frac{d\,z}{d\,x}\right)^2\right\} = 2\log\mu + C_1 \tag{31}$$

Taking the anti logarithms we have

$$1 + \left(\frac{d\,z}{d\,x}\right)^2 = \mu^2\,C_2 \tag{32}$$

In order to determine the value of C_2, observe that just before the ray enters the ionized medium we have $\mu = 1$, while the left hand side is the square of the secant of the angle between the ray and the horizontal.

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[15] We must be careful in which direction we draw the normal to perform the differentiation $d\,\mu/d\,n$. If the refractive index at A is less than that at B, so that the velocity at A is greater than that at B, the path going through A will bend over towards B. As drawn it is a convention that the radius of curvature shall be considered negative. Hence $d\,\mu/d\,n$ must be considered positive, and we must differentiate along the inward drawn normal to the path.

[16] For example, Granville, Differential and Integral Calculus, p. 161.

This can readily be seen by reference to Fig. 4 as follows:

$$dz/dx = \tan\beta$$

$$1 + (dz/dx)^2 = 1 + \tan^2\beta = \sec^2\beta$$

Therefore when $\mu = 1$, $C_2 = \sec^2\beta$. Substituting this value of C_2 in equation (**32**) and putting $1/\cos^2\beta$ for $\sec^2\beta$ we obtain

$$dx = \frac{\cos\beta\,dz}{\sqrt{\mu^2 - \cos^2\beta}} \qquad (\mathbf{33})$$

This is the general differential equation for the ray paths, applicable to any distribution of ionization, which

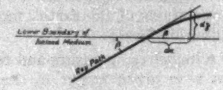

FIG. 4

allows us to express the resulting refractive index μ as a function of the height z alone. In order to plot the path of a ray we require the value of the horizontal distance x for any given value of the vertical height Z. This required relation between x and Z is obtained by performing the integration indicated below:

$$x = \int_{z=0}^{z=Z} \frac{\cos\beta\,dz}{\sqrt{\mu^2 - \cos^2\beta}} \qquad (\mathbf{34})$$

Here β is a constant for any given ray as will be seen from Fig. 4.

The above expression for the path of a ray is for a plane earth. The corresponding equation in polar coordinates which takes into account the curvature of the medium is given in Appendix II.

Before we can actually perform the integration indicated in (**34**) we must express μ as a function of z and substitute it into the equation. In the following section we shall review the present status of our knowledge concerning the constitution and probable state of ionization in the upper atmosphere, in order to select a reasonable, and at the same time analytically manageable, assumption as to the variation of refractive index with height above the earth.

IV. CONSTITUTION AND DISTRIBUTION OF IONIZATION IN THE UPPER ATMOSPHERE

A. *Constitution.* The earth's atmosphere is composed of two concentric spherical shells. The inner one is called the troposphere, and is distinguished by the existence of a temperature gradient, which decreases from the ground upwards, the mean gradient being in the neighborhood of 6 deg. cent. per kilometer. In this region, masses of air are continually being raised or lowered, and thus moved to places where the pressure is different.

In the absence of heat exchanges by conduction and radiation, the resulting temperature distribution would be that of an atmosphere in adiabatic equilibrium. In other words, the temperature distribution would be such that the adiabatic heating or cooling to which a mass of gas is subject when moved from one level to another would automatically bring its temperature to that of its surroundings. Actually, however, the heat exchanges by radiation and conduction reduce the observed gradient to approximately one-half of the adiabatic gradient. The height of the troposphere in different places may vary from 10 to 15 kilometers.

The outer shell is called the stratosphere and is distinguished by the absence of a temperature gradient and consequently any large scale mass motion. It is sometimes referred to as the isothermal layer. The boundary between the two layers is, of course, not sharp, and the thickness of the transition layer, in which the mass motion practically dies out, is not at present precisely known. We will assume, however, for the present discussion that convection ceases at a height of 20 kilometers.

In Fig. 5 we have reproduced the proportional composition of the atmosphere, by mass, as calculated by Chapman and Milne.[17] This is based on the assumptions that convection ceases at 20 kilometers, and that no hydrogen is present. The temperature of the stratosphere is taken at − 54 deg. cent. In Table I the pressure, number of molecules, approximate molecular mean free path, and the average collision frequency between an electron and gas molecule, are given for

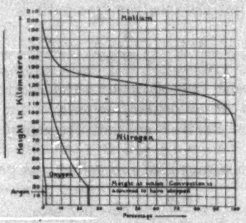

FIG. 5—PROPORTIONAL COMPOSITION OF AIR BY MASS

TABLE I

Height in kilometers	Pressure Dynes per sq. cm.	Number of molecules per cu. cm.	Molecular M. F. P. cm.	Electron Collision frequency per sec.
0	1.01×10^6	2.7×10^{19}	$9. \times 10^{-6}$	9.5×10^{11}
12	1.92×10^5	6.5×10^{18}	$4. \times 10^{-5}$	2.1×10^{11}
20	5.53×10^4	1.9×10^{18}	$1. \times 10^{-4}$	8.5×10^{10}
40	2.55×10^4	8.6×10^{16}	$3. \times 10^{-3}$	2.8×10^9
60	1.24×10^2	4.2×10^{15}	$6. \times 10^{-2}$	1.4×10^8
80	6.27	2.1×10^{14}	1.0	8.5×10^6
100	0.363	1.2×10^{13}	20.	4.3×10^5
150	1.49×10^{-2}	5.0×10^{11}	500.	1.7×10^4
200	5.62×10^{-3}	1.8×10^{11}	1000.	8.5×10^3
300	6.99×10^{-4}	2.4×10^{10}	1×10^4	8.5×10^2
400	1.05×10^{-4}	3.6×10^9	7×10^4	1.2×10^2
600	2.59×10^{-6}	8.8×10^7	3×10^6	2.8
800	7.97×10^{-8}	2.7×10^6	9×10^7	0.95
1000	2.92×10^{-9}	9.9×10^4	3×10^9	2.8×10^{-3}

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[17] S. Chapman and E. A. Milne: *Quarterly Jour. of the Roy Meteorological Soc.*, Vol. 46, p. 357, 1920. This is a very complete discussion of the present status of our knowledge of the upper atmosphere.

different heights. The approximate values of the molecular free paths were calculated by Chapman and Milne[16] from the relation

$$l = \frac{1}{\sqrt{2}\,\pi\, M \sigma^2} \text{ cm.} \qquad (35)$$

where

M = number of molecules per cu. cm.

σ = mean molecular diameter assumed 3×10^{-8} cm.

Thus no account is taken of the fact that the air is a mixture. The approximate values of the electron free paths are obtained from the relation

$$l' = \frac{1}{M \pi \sigma_1^2 \sqrt{1 + \dfrac{m'}{m}}} \text{ cm.} \qquad (36)$$

where

M = number of molecules per cu. cm. assumed large compared with the number of electrons.

$\sigma_1 = \dfrac{\sigma + \sigma'}{2}$ = average of molecule and electron diameters

m' = mass of electron

m = mass of molecule

For our purposes we may neglect the mass and diameter of the electron in comparison to those for the molecule so that the electron free path is

$$l' = \frac{4}{\pi M \sigma^2} \qquad (37)$$

Comparison with (**35**) shows that the electron free path is $4\sqrt{2}$ times the molecular free path.

The average electron collision frequency is

$$\nu = \bar{c}/l' \qquad (38)$$

where $\bar{c}$ is the average electron velocity as given by

$$\bar{c} = \sqrt{\frac{8\,k\,T}{\pi\, m'}} \text{ cm. per sec.} \qquad (39)$$

in which

$k = 1.37 \times 10^{-16}$ universal gas constant

T = electron temperature deg. K.

$m' = 9 \times 10^{-28}$ the electron mass in grams.

In making the calculations we have assumed an electron temperature of 6000 deg. K., *i. e.*, the electrons have not had sufficient time since entering the stratosphere to acquire the mean temperature of 219 deg. K. This gives

$$\bar{c} = 4.85 \times 10^7 \text{ cm. per sec.} \qquad (40)$$

and consequently

$$\nu = \frac{4.8 \times 10^7}{4\sqrt{2}\, l} = \frac{8.6 \times 10^6}{l} \qquad (41)$$

Inspection of the values in Table I shows that for a wave length of 60 meters we will obtain the maximum absorption at approximately 70 kilometers since at this point $\nu = \omega$. Consequently if we assume that the lower boundary of the ionized medium is considerably above 70 kilometers, the radio waves will not be subjected to large absorption.

Recent work by Lindemann and Dobson[18] based on the observation of falling meteors leads them to a higher estimate of the density above 60 kilometers than that corresponding to a temperature of 219 deg. K. for the isothermal layer. Their calculations indicate a value in the neighborhood of 300 deg. K. They have also suggested reasons why we may expect this higher temperature. It has occurred to the writers that the heating effect of the eddy currents induced in the ionized upper atmosphere required in the explanation of the diurnal variations of the earth's magnetic field, may be a contributing factor.

The effect of a higher temperature and consequently higher densities at a given height in the isothermal layer will be to increase the height at which the electron collisions cease to be important and will require us, therefore, to put the refracting medium at a greater height if we wish to avoid high absorption of the radio waves.

B. Ionization. The theories of the aurorae,[19] diurnal and semi-diurnal variations in the earth's magnetic field as well as magnetic storms[20] are based on the existance of ionization in the upper atmosphere. Again in the early days of radio, Kennelly[21] and independently Heaviside,[22] suggested the need of a conducting layer to account for the observed long distance transmission. Recently Watson[23] has shown that an equation of the Austin-Cohen type can be obtained theoretically if a conducting upper atmosphere is assumed.

The present views appear to favor the hypothesis that the ionization in the upper atmosphere is maintained principally by the action of high velocity electrons which reach the earth's atmosphere from the sun. When they approach the earth, the magnetic field will concentrate them at the poles and bend them around the earth into the dark hemisphere. On this view, the maximum ionization intensity will be expected in the polar regions and on the sunlit hemisphere. Rayleigh[24] has shown definitely that the green auroral line is always present in the night sky and in this way proved the existence of strong ionization on the dark side of the earth.

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18. F. A. Lindemann and G. M. B. Dobson, *Proc. Roy. Soc.* Lond., Vol. 102, p. 411, 1923.

19. For a summary of theories and references see W. J. Humphrey's Physics of the Air, p. 422, Lippincott, 1920.

20. See excellent summary by S. Chapman in Glazebrook, Dictionary of applied Physics. Vol. II, p. 543, MacMillan, 1922.

21. A. E. Kennelly: *Electrical World and Engineer*, March 15, 1902.

22. Oliver Heaviside: *Encyclopedia Britannica*, tenth edition, Vol. 33, Dec. 1902.

23. G. N. Watson: *Proc. Roy. Soc.* Lond., Vol. 95, p. 562, 1919.

24. Lord Rayleigh, *Proc. Roy. Soc.* Lond., Vol. 109, p. 428, 1925.

Chapman and Milne[18] have calculated the distribution of ionization which would result from the absorption of high velocity electrons in the earth's atmosphere. We have reproduced their curve in Fig. 6. From this we see that from the ground to nearly 40 kilometers height the relative ionization remains practically zero. It then begins to increase slowly at first, and then more and more rapidly with the height. As we approach the maximum, the rate of increase decreases again, and after passing the maximum at 54 kilometers, it falls off more gradually to a small value at 90 kilometers. Our present interest is mainly in the general shape of the curve, since for a different absorption coefficient the absolute values calculated would be changed, but the general shape characteristics would remain the same.

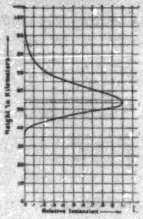

FIG. 6

In calculating the refraction of radio rays which rise into an ionized medium of this type and are refracted back to earth, we are only concerned with the shape of the curve from the ground to the maximum since any rays which rise above the maximum will turn out into space and be lost. With this in mind, it will be evident that a good approximation for our purposes may be had by writing

$$N = N_0 \sin^2 \frac{(z-b)\,\pi}{2\,(B-b)} \tag{42}$$

where

b = height of the lower boundary of the ionized medium above the earth.

B = height of the maximum ionization above the earth.

N_0 = maximum number of electrons per cu. cm.

N = number of electrons per cu. cm. at height z above the ground.

Here z is restricted to values between b and B. By varying the values N_0, b and B we can represent a thick or thin, high or low medium with any desired value of maximum ionization. The general shape of the curve is shown in Fig. 7.

V. CALCULATION OF THE PATH OF A RAY IN A MEDIUM IN WHICH THE ELECTRON DENSITY IS A SINE SQUARE FUNCTION OF THE HEIGHT

A. Case of Flat Earth. We have seen in the previous section that for short waves the effect of collisions between electrons and molecules on the refractive index of the ionized upper atmosphere can be neglected. We therefore substitute the expression for N given in equation (**42**) in equation (**13**) and obtain the following expression for the square of the refractive index as a function of the height z above the earth:

$$\mu^2 = 1 - \sigma \frac{N_0}{\omega^2} \sin^2 \frac{(z-b)\,\pi}{2\,(B-b)} \tag{43}$$

where $\sigma = \dfrac{4\,\pi\,e^2}{m}$

$\sigma = 3.2 \times 10^9$ c. g. s. (e. s. u.) for electrons.

This expression is only assumed to hold in the ionized part of the medium from $z = b$ to $z = B$. Below $z = b$ we assume that the refractive index remains equal to unity down to the surface of the ground, *i. e.*, $z = 0$. As previously pointed out, we are not concerned with the behavior of the refractive index after passing its minimum value at $z = B$. Above $z = B$ we merely assume that the refractive index returns gradually to unity at a great height.

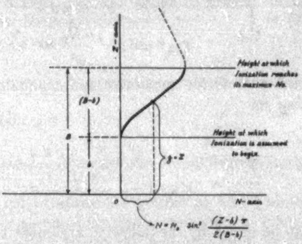
Z-axis
Height at which Ionization reaches its maximum No.
(B-b)
B
b
Height at which Ionization is assumed to begin.
z-Z
0
N-axis
N = N₀ sin² (Z-b)π / 2(B-b)

FIG. 7

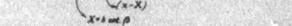
Z-axis
Height at which Ionization reaches its Maximum
(B-b)
x
B
b
β
Height at which Ionization is assumed to begin.
b
z
0
x-axis
(x-X)
X=b cot β

FIG. 8

Substituting the value for μ^2 given above by (**43**) in equation (**34**) we obtain

$$(x - X) = \int_{z=b}^{z=Z} \frac{\cos\beta\, d\,z}{\sqrt{1 - \sigma \dfrac{N_0}{\omega^2} \sin^2 \dfrac{(z-b)\,\pi}{2\,(B-b)} - \cos^2\beta}} \tag{44}$$

as the horizontal distance $(x - X)$ traveled by the ray in the ionized medium, while rising a distance $(Z - b)$ in the ionized medium. Inspection of Fig. 8 will make this clear.

The trick involved in the integration of this equation consists in finding substitutions which will transform it into a known form. In what follows it is shown that it can be reduced to an elliptic integral.

First of all let us put

$$u = \pi\,(z - b)/2\,(B - b) \tag{45}$$

and consequently

$$d\,u = \pi\,d\,z/2\,(B - b) \tag{46}$$

also

$$1 - \cos^2\beta = \sin^2\beta$$

in equation (**44**) and obtain

$$(x - X) = \int_{u=0}^{u=\frac{\pi(Z-b)}{2(B-b)}} \frac{2\,(B - b)\cos\beta\,d\,u}{\pi\sqrt{\sin^2\beta - \dfrac{\sigma N_0}{\omega^2}\sin^2 u}} \tag{47}$$

We will now divide numerator and denominator by $\sin\beta$, and get

$$(x - X) = \int_{u=0}^{u=\frac{\pi(Z-b)}{2(B-b)}} \frac{2\,(B - b)\cot\beta\,d\,u}{\pi\sqrt{1 - \dfrac{\sigma N_0}{\omega^2\,\sin^2\beta}\sin^2 u}} \tag{48}$$

It can be shown by the following argument that the expression $\sigma N_0/\omega^2\sin^2\beta$ under the radical sign is greater than unity. Going back to equation (**32**) we have

$$1 + (d\,z/d\,x)^2 = \mu^2/\cos^2\beta \tag{49}$$

and if the ray becomes horizontal and returns to earth, we must have $d\,z/d\,x = 0$ at the summit, and consequently

$$\mu^2 = \cos^2\beta \tag{50}$$

Subtracting each side of equation (**50**) from unity we have

$$1 - \mu^2 = \sin^2\beta \tag{51}$$

at the summit of the path.

But we have from equation (**13**)

$$1 - \mu^2 = \sigma N/\omega^2 \tag{52}$$

and N_0 is greater than N. Hence from equations (**51**) and (**52**) we have $\sigma N_0/\omega^2$ greater than or equal to $\sin^2\beta$ so that the expression $\sigma N_0/\omega^2\sin^2\beta$ is greater than or equal to unity. We may, therefore, put it equal to one over the square of the sine of some angle α since the cosecant square of an angle is always equal to or greater than one. Hence

$$(x - X) = \int_{u=0}^{u=\frac{\pi(Z-b)}{2(B-b)}} \frac{2\,(B - b)\cot\beta\,d\,u}{\pi\sqrt{1 - \dfrac{\sin^2 u}{\sin^2\alpha}}} \tag{53}$$

A further substitution will be required since the coefficient of $\sin^2 u$ is greater than one. If we put

$$\sin u = \sin\alpha\sin w \tag{54}$$

we have

$$\cos u\,d\,u = \sin\alpha\cos w\,d\,w \tag{55}$$

and the integral (**53**) becomes

$$(x - X) = \int_{w=0}^{w=\sin^{-1}\left\{\frac{\sin\frac{\pi(Z-b)}{2(B-b)}}{\sin\alpha}\right\}} \frac{2\,(B - b)\cot\beta\sin\alpha\cos w\,d\,w}{\pi\cos u\cos w} \tag{56}$$

Let us now define on angle ϕ such that

$$\sin\alpha\sin\phi = \sin\frac{\pi\,(Z - b)}{2\,(B - b)} \tag{57}$$

and substitute it in (**56**); we obtain

$$(x - X) = \int_{w=0}^{w=\phi} \frac{2\,(B - b)\cot\beta\sin\alpha\,d\,w}{\pi\cos u} \tag{58}$$

Squaring equation (**54**) and subtracting each side from unity we have

$$1 - \sin^2 u = 1 - \sin^2\alpha\sin^2 w \tag{59}$$

and hence

$$\cos^2 u = 1 - \sin^2\alpha\sin^2 w \tag{60}$$

Substituting equation (**60**) in (**58**) we obtain

$$(x - X) = \int_{w=0}^{w=\phi} \frac{2\,(B - b)\cot\beta\sin\alpha\,d\,w}{\pi\sqrt{1 - \sin^2\alpha\sin^2 w}} \tag{61}$$

If we now put the value of $\sin\alpha$ from the relation

$$\omega^2\sin^2\beta/\sigma N_0 = \sin^2\alpha \tag{62}$$

in the numerator of (**61**) we obtain

$$(x - X) = \frac{2\,(B - b)\;\omega\cos\beta}{\pi\sqrt{\sigma N_0}} \int_{w=0}^{w=\phi} \frac{d\,w}{\sqrt{1 - \sin^2\alpha\sin^2 w}} \tag{63}$$

The integral in the above equation is called the elliptic integral of the first kind. For brevity it is customary to write

$$\int_{w=0}^{w=\phi} \frac{d\,w}{\sqrt{1 - \sin^2\alpha\sin^2 w}} = F\,(\sin\alpha, \phi) \tag{64}$$

It is also customary to put $k = \sin\alpha$ and write it $F(k, \phi)$. The numerical value of the integral for any value of α and ϕ has been calculated and recorded in tables of elliptic integrals.[25] For example, we find from the tables that

$$F\,(\sin\alpha, \phi) = \int_{w=0}^{w=\phi} \frac{d\,w}{\sqrt{1 - \sin^2\alpha\sin^2 w}} = 0.4498 \tag{65}$$

when ϕ = 25 deg. and α = 75 deg.

The numerical values of ϕ and α for any value

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[25] For example, see A Short Table of Integrals, by B. O. Pierce, p. 118. A more complete table will be found in Jahnke und Emde, Funktionentafeln, p. 53-68.

$z = Z$ on a given ray may be obtained from (**62**)and (**57**), as follows:

$$\alpha = \sin^{-1} \{ \omega \sin \beta / \sqrt{\sigma N_0} \} \qquad (66)$$

$$\phi = \sin^{-1} \left\{ \frac{\sqrt{\sigma N_0} \sin \dfrac{\pi (Z - b)}{2 (B - b)}}{\omega \sin \beta} \right\} \qquad (67)$$

The corresponding numerical value of F (sin α, ϕ) is looked up in the tables and the numerical values ofthe horizontal component $(x - X)$, traveled by the ray in the ionized medium while rising a distance $(Z - b)$ in the ionized medium as

$$(x - X) = \frac{2 (B - b) \omega \cos \beta}{\pi \sqrt{\sigma N_0}} F (\sin \alpha, \phi) \qquad (68)$$

On the assumption of a flat earth, inspection of Fig. 8 shows that we must add the horizontal projection of the straight part of the path

$$X = b \cot \beta \qquad (69)$$

to equation (**68**) to obtain the total horizontal distance x to the point (x, Z) on the ray path. The complete expression for the case of a flat earth then becomes

$$x = b \cot \beta + \frac{2 (B - b) \omega \cos \beta}{\pi \sqrt{\sigma N_0}} F (\sin \alpha, \phi) \quad (70)$$

If we wish to know where the ray will return to earth we must take twice the value of x corresponding to the summit of the path since the path is clearly symmetrical. At the summit of the path we have by equations (**51**) and (**52**)

$$\sin^2 \beta = \sigma N / \omega^2 \qquad (71)$$

But

$$N = N_0 \sin^2 \frac{(Z - b) \pi}{2 (B - b)} \qquad (72)$$

from which

$$\sin^2 \beta = \frac{\sigma N_0}{\omega^2} \sin^2 \frac{(Z - b) \pi}{2 (B - b)} \qquad (73)$$

Substituting the value of sin β from (**73**) in equation (**67**) for ϕ we obtain

$$\phi = \sin^{-1} 1 = 90 \text{ deg.} = \frac{\pi}{2} \qquad (74)$$

at the summit of the path.

Substituting this value of ϕ in equation (**63**) we obtain

$$(x_s - X) = \frac{2 (B - b) \omega \cos \beta}{\pi \sqrt{\sigma N_0}} \int_{w=0}^{w=\frac{\pi}{2}} \frac{d w}{\sqrt{1 - \sin^2 \alpha \sin^2 w}} \qquad (75)$$

as the equation for the horizontal component $(x_s - X)$ traveled by the ray in the ionized medium while rising to the summit of the path in the ionized medium. Or, in other words, while rising a distance $(Z_s - b)$ in the ionized medium. In the above x_s is the horizontal distance from the transmitter to the foot of the perpendicular which passes through the summit of the path and Z_s is the height of the summit of the path above the ground.

The integral in equation (**75**) is called the Complete Elliptic Integral of the First Kind. For brevity it is customary to write it

$$\int_0^{\pi/2} \frac{d w}{\sqrt{1 - \sin^2 \alpha \sin^2 w}} = K (\sin \alpha) \quad (76)$$

It is also customary to put $k = \sin \alpha$ and write it $K (k)$.

Numerical values of this integral for any value of α will be found in the tables of elliptic integrals.[25]

We may now write the expression for the total horizontal distance between transmitter and the point at which a given ray will return to ground for the *Case* of a *Flat Earth* as follows:

$$2 x_s = 2 b \cot \beta + \frac{4 (B - b) \omega \cos \beta}{\pi \sqrt{\sigma N_0}} K (\sin \alpha) \text{ kilometers} \quad (77)$$

In which

$\alpha = \sin^{-1} \left\{ \dfrac{\omega}{\sqrt{\sigma N_0}} \sin \beta \right\}$ in degrees

b = height above the ground at which ionization is assumed to begin; measured in kilometers.

β = angle between ray and horizontal at the lower boundary of the medium in degrees.

B = heightabove the ground at which the ionization is assumed to reach its maximum value, in kilometers.

$\omega = 2 \pi f$ and f = frequency of the radiation in cycles per second.

$\sigma = 4 \pi e^2/m = 3.2 \times 10^9$ c. g. s. (e. s. u.) where e is the charge and m the mass of an electron.

N_0 = number of electrons per cu. cm. at the height B of maximum ionization.

$K (\sin \alpha) = K (k)$ = complete elliptic integral of the first kind to be looked up in tables. For example, when $\alpha = 10$ deg., $K(\sin \alpha) = 1.5828$.

B. Approximate Allowance Made for the Effect of the Curvature of the Earth on the Ray Paths. When considering long distance transmission, the effect of the earth's curvature cannot be entirely neglected. The analytical difficulties, however, become considerably increased when we attempt to take into account the curvature of the ionized medium and use equation (**15**) of Appendix II instead of equation (**34**). Happily the most important effect produced by the earth's curvature is to increase the angle β at which a ray of given initial inclination to the horizontal θ, strikes the lower boundary of the ionized medium. This effect can be readily taken into account as follows:

Let r_0 be the radius of the earth and let r_1 be the radius from the center of the earth to the lower boundary of the ionized medium. Then from Fig. 9 the angle $O\,A\,B$ is equal to (90 deg. + θ) while the angle $O\,B\,A$ is equal to (90 deg. − β). It follows that the angle $A\,O\,B$ is equal to ($\beta - \theta$) since the sum of the angles of a triangle is equal to 180 deg. From the sine rule for triangles we have

$$r_0/\sin\,(90\text{ deg.} - \beta) = r_1/\sin\,(90\text{ deg.} + \theta) \qquad (78)$$

from which

$$\cos\theta = (r_1/r_0)\cos\beta \qquad (79)$$

and since $r_1 = (r_0 + b)$ we can write (**79**)

$$\cos\beta = \cos\theta/(1 + b/r_0) \qquad (80)$$

From equation (**80**) we can calculate the value of β for any assigned value of initial angle θ and height of medium b. Knowing β we can calculate ($\beta - \theta$) and the distance $A\,C$ measured along the surface of the earth since the arc length

$$A\,C = \frac{2\,\pi\,r_0}{360}\,(\beta - \theta) \qquad (81)$$

where ($\beta - \theta$) is in degrees.

The method of approximating the portion of the path in the ionized medium for the case of a curved earth will be understood by reference to Fig. 10 in which the effects of curvature are greatly exaggerated for the sake of clearness. Assume a ray starting out from the transmitter at A which makes an angle θ with the horizon. This ray is assumed to take a straight path through the lower atmosphere and meet the lower boundary of the ionized medium, vertically over the point marked C. The angle between the ray and the tangent plane at the point of intersection being β. The value of β for any value of θ is obtained from equation (**80**).

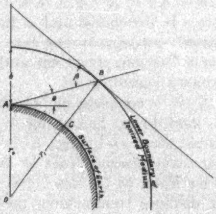

FIG. 9

The distance over the surface of the earth $A\,C$ is obtained from equation (**81**). We now calculate the path which a ray with this value of β would take in a Flat Medium by equation (**68**). This path is illustrated by the full line curve. The value of $(x - X)$ from 0 to 2 is indicated as $(x - X)_2$. Similarly the ordinate of the path at the point 2 is indicated as $(Z - b)_2$. The distance 0 to 6 is $2\,(x_s - X)$ as given by equation (**75**). The distances 0′ 1′, 0′ 2′, etc., equal to 0 1, 0 2, etc., are then laid off on the surface of the earth as shown. Thus the arc length 0′ 6′ is equal to the distance 0 6 or range of the ray on the assumption of a flat earth. Radii are then drawn from the center of the earth through the points 1′, 2′, 3′, 4′, 5′, and 6′ and continued above the lower boundary of the ionized medium. Distances equal to the ordinates at 1, 2, 3, etc., of the full line curve are then measured off on the corresponding radii 1′, 2′, 3′, etc., from the concentric circle representing the lower boundary of the ionized medium. These distances become the ordinates of the dotted curve which represents the approximate path of the ray in the ionized medium. At E the dotted ray emerges from the ionized medium and is assumed to take a straight path, E to D, back to the surface of the earth, the acute angle between the tangent plane at E and the ray being β.

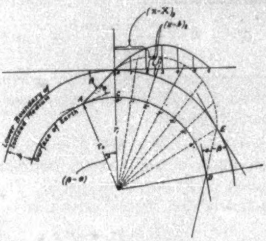

FIG. 10

It will be observed that the above method of constructing the approximate path for a curved medium from the calculated path for a flat medium increases the length of the transformed path. This effect is in the right direction since in the actual problem the medium would be bending around with the ray and consequently retain the ray in the medium for a greater distance.

On the above assumptions the total range of a ray for the case of a curved earth, *i. e.*, the arc length $A\,D$ in Fig. 10 is given by

$$\text{Range} = \frac{4\,\pi\,r_0}{360}\,(\beta - \theta) + \frac{4\,\omega\,(B - b)\cos\beta}{\pi\,\sqrt{\sigma\,N_0}}\,K\,(\sin\alpha) \qquad (82)$$

where

r_0 = 6300 kilometers, the approximate radius of the earth

θ = angle between ray and horizontal at the transmitter in degrees.

$$\beta = \cos^{-1}\left\{\frac{\cos\theta}{(1 + b/r_0)}\right\} = \text{acute angle between}$$

the ray and the tangent to the lower boundary of the ionized medium at the point where the ray of initial angle θ, enters the medium; measured in degrees.

The other factors are as given for the case of a flat earth under equation (**77**).

VI. TYPICAL PATH CALCULATIONS

In order to make clear the method used in calculating ray paths, we give an example which is representative of summer day conditions. The values used for the constants appearing in the formulas were selected with the idea of fitting the experimental data concerning skip distances given in Fig. 1. For summer day conditions we assume the maximum electron density to be $N_0 = 6 \times 10^5$ per cu. cm., the lower edge of the ionized medium at a height $b = 200$ km., and the distance from this lower edge to the height of maximum intensity, $(B - b) = 80$ km. Putting these values in the equation (**42**), we have

$$N = 6 \times 10^5 \sin^2 \left\{ \frac{9\,(z - 200)}{8} \right\}^{\circ} \qquad \textbf{(83)}$$

We shall calculate the paths for a wave length of 21 meters. On referring to the formula (**68**) developed in Section V we find that if $(x - X)$ denotes the distance to the right of the point where the ray enters the ionized medium,

$$(x - X) = \frac{2.12 \times 10^4}{\lambda \sqrt{N_0}} (B - b) \cos\beta\, F\,(\sin\alpha, \phi) \qquad \textbf{(84)}$$

and is therefore in this case given by

$$(x - X) = 104.25 \cos\beta\, F\,(\sin\alpha, \phi) \qquad \textbf{(85)}$$

The angle α is determined from the equation (**66**) which becomes

$$\sin\alpha = \frac{10^5}{3\,\lambda \sqrt{N_0}} \sin\beta \qquad \textbf{(86)}$$

and in this case reduces to

$$\sin\alpha = \frac{43}{21} \sin\beta \qquad \textbf{(87)}$$

The angle β is determined by the equation (**80**), which is

$$\cos\beta = \frac{r_0}{r_0 + b} \cos\theta \qquad \textbf{(88)}$$

in which $r_0 = 6300$ km. = earth's radius
and b = height of lower edge of medium.

The angle θ is the angle made by the ray with the horizontal at the transmitting station, β is the angle at which the ray cuts the ionized medium, and α is an angle which is required in the calculation of the path. The elliptic integral $F\,(\sin\alpha, \phi)$ is given in the tables[25] for various values of α and ϕ. The angle ϕ, called the amplitude of the integral, is determined from equation (**57**) which becomes (see also equation **45**)

$$\sin\phi \sin\alpha = \sin \left\{ \frac{9\,(z - 200)}{8} \right\}^{\circ} = \sin u \qquad \textbf{(89)}$$

and consequently varies along the path.

At the summit of the path ϕ becomes a right angle, and $F\,(\sin\alpha, \phi)$ becomes $K\,(\sin\alpha)$.

The path is symmetrical about a radius, (*i. e.*, vertical) drawn through its summit.

The distance X appearing in the expression is the horizontal distance to where the ray enters the ionized medium, and is given by $110\,(\beta - \theta)$, where $(\beta - \theta)$ is expressed in degrees.

All distances are expressed in kilometers.

The calculations are set out in the accompanying tables.

Table II gives the values of β, $(\beta - \theta)$ and X, for various values of θ. These values depend only on b.

TABLE II*

Line	Quantity	1	2	3	4	5	6	7	8
I	$\theta°$	0	5	10	15	20	23	25	25.7
II	$\beta°$	14.25	15.04	17.3	20.7	24.5	26.9	28.55	29.2
III	$(\beta - \theta)°$	14.25	10.04	7.3	5.7	4.5	3.9	3.55	3.5
IV	X, km.	1568	1104	803	627	495	429	390	385

*The values of β for different values of θ were unfortunately read from a curve instead of being directly calculated from equation (88). The parts of the paths in the ionized medium are calculated correctly for the values of β used, but some small errors made in reading the curve (β vs. θ) mean that the paths are for slightly different values of θ. These errors do not amount to more than a small fraction of a degree in θ, and therefore do not appreciably affect the results. In the same way small inaccuracies exist in the signal intensity calculations of Tables XI and XII.

Table III gives the calculations of α and $104.25 \cos\beta$, by means of $\sin\beta$, $\sin\alpha$ and $\cos\beta$. These quantities are constant for a given ray path.

Table IV gives the calculations for the points corresponding to $\phi = 18$ deg., 36 deg., 54 deg., 72 deg., and 90 deg. The quantities determined for each point are z and $(x - X)$.

Table V gives some additional points for the curve with $\theta = 25.7$ deg. This curve goes to infinity, and is the ray path which separates the rays which return to earth from those which go out into space.

Table VI gives the values of the co-ordinate x for the points calculated. The downward curve is filled in by symmetry, the corresponding points being tabulated against supplementary values of ϕ. The paths obtained from the calculations in the above tables are plotted in Fig. 11.

It is of some interest to consider the paths of those

TABLE III

Line	Quantity	1	2	3	4	5	6	7	8
I	$\theta°$	0	5	10	15	20	23	25	25.7
II	$\sin\beta$	0.246	0.2595	0.2974	0.3535	0.4147	0.4524	0.478	0.4879
III	$\sin\alpha$	0.503	0.531	0.608	0.724	0.849	0.927	0.979	1
IV	α	30.2	32.1	37.4	46.4	58.1	68	78.2	90°
V	$\cos\beta$	0.9692	0.9657	0.9548	0.9354	0.9100	0.8918	0.8784	0.8729
VI	104.25 cos. β km.	101	100.5	99.4	97.4	94.7	93	91.5	91

TABLE IV

Line	Quantity	ϕ	1	2	3	4	5	6	7	8
I	θ		0°	5°	10°	15°	20°	23°	25°	25.7°
II	$\sin\phi\sin\alpha$	18°	0.1565	0.164	0.188	0.2235	0.2625	0.2865	0.308	0.309
III	U		9°	9.44°	10.84°	12.9°	15.2°	16.6°	17.93°	18°
IV	z		208	208.4	209.6	211.5	213.5	214.8	215.9	216
V	$F(\sin\alpha, \phi)$		0.3154	0.3156	0.3161	0.3169	0.318	0.3187	0.3192	0.3195
VI	$(x - X)$		31.8	31.7	31.5	30.9	30.2	29.6	29.2	29.1
VII	$\sin\phi\sin\alpha$	36°	0.296	0.312	0.358	0.4255	0.499	0.5445	0.575	0.5878
VIII	U		17.3°	18.2°	21°	25.2°	29.9°	33°	35.1°	36°
IX	z		215.4	216.2	218.7	224	226.6	229.3	231.2	233
X	$F(\sin\alpha, \phi)$		0.6384	0.6397	0.6433	0.6502	0.6596	0.6666	0.6719	0.6743
XI	$(x - X)$		64.5	64.3	63.9	63.4	62.5	62	61.5	61.3
XII	$\sin\phi\sin\alpha$	54°	0.407	0.43	0.492	0.585	0.687	0.75	0.792	0.809
XIII	U		24°	25.5°	29.5°	35.8°	43.4°	48.6°	52.1°	54°
XIV	z		221.4	222.7	226.3	231.8	238.6	243.2	246.4	248
XV	$F(\sin\alpha, \phi)$		0.9746	0.9769	0.9946	1.017	1.054	1.086	1.111	1.124
XVI	$(x - X)$		98.5	97.5	94	98.5	100	101	101.5	102
XVII	$\sin\phi\sin\alpha$	72°	0.478	0.505	0.578	0.689	0.807	0.881	0.93	0.9511
XVIII	U		28.6°	30.3°	35.3°	43.6°	53.8°	61.8°	68.4°	72°
XIX	z		225.4	226.9	231.2	238.8	247.8	254.9	260.8	272
XX	$F(\sin\alpha, \phi)$		1.326	1.336	1.365	1.418	1.546	1.648	1.769	1.843
XXI	$(x - X)$		137	134	136	138	146.5	154	162	167.5
XXII	z	90°	228.4	228.5	233.3	241.3	251.7	260.4	269.5	280
XXIII	$K(\sin\alpha)$		1.687	1.704	1.757	1.875	2.117	2.419	2.995	∞
XXIV	$(x - X)$		170.5	171.5	174.5	183	200.5	225	274	∞

The values of z and $(x - X)$ are in kilometers.

rays which do not return to earth. We can only calculate these paths as far as the height of maximum electron density, because we have made no assumptions concerning the electron distribution at greater heights. For this purpose we require a different formula, since equation (**68**) breaks down when β exceeds the limiting value for return of the ray to earth.

TABLE V

When $\theta = 25.7°$ $\alpha = 90°$ and $\phi = U$

Line	Quantity	1	2	3	4	5	6
I	ϕ or U	75°	78°	81°	84°	87°	89°
II	z	267	269.3	272	274.7	277.3	279.1
III	$F(\sin\alpha, \phi)$	2.028	2.253	2.542	2.949	3.642	4.741
IV	$(x - X)$	184.5	205	231.5	268.5	331.5	432
V	x	570	590	617	654	716	817

TABLE VI

Line	ϕ	Values of x in k. m.								
I	–	θ	0°	5°	10°	15°	20°	23°	25°	25.7°
II	0	x	1568	1104	803	627	495	429	390	385
III	18		1600	1136	835	658	525	459	419	414
IV	36		1633	1168	867	690	558	491	460	475
V	54		1666	1202	897	725	595	530	492	487
VI	72		1705	1238	939	765	642	583	552	553
VII	90		1739	1276	978	810	695	654	664	∞
VIII	108		1773	1314	1017	855	748	725	776	
IX	126		1812	1350	1059	895	795	778	836	
X	144		1845	1384	1089	930	832	817	868	
XI	162		1878	1416	1121	962	865	849	909	
XII	180		1910	1448	1153	993	895	879	938	
XIII	Ground		3478	2552	1956	1620	1390	1308	1328	

If we go back a few steps in the derivation of the path formula to equation (**48**), we find

$$(x - X) = \int_{u=0}^{u=U} \frac{2\,(B - b)\cot\beta\, d\,u}{\pi\sqrt{1 - \dfrac{\sigma N_0}{\omega^2 \sin^2\beta}\sin^2 u}} \qquad (\mathbf{90})$$

The fact is that we have now so great a value of β that the expression $\dfrac{\sigma N_0}{\omega^2 \sin^2\beta}$ becomes less than unity. We proved this expression to be greater than unity for all rays that return to earth, but in the case of rays going into space we can prove that we must have $\dfrac{\sigma N_0}{\omega^2 \sin^2\beta}$ less than unity, by a somewhat similar argument to that used previously. We may then put

$$\sin^2\alpha' = \frac{\sigma N_0}{\omega^2 \sin^2\beta} \qquad (\mathbf{91})$$

so that

$$\sin\alpha' = \frac{3\,\lambda\sqrt{N_0}}{10^5 \sin\beta} \qquad (\mathbf{92})$$

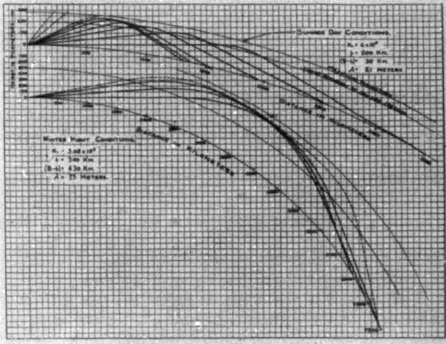

Fig. 11

The above integral then becomes an elliptic integral, and its value is

$$(x - X) = \frac{2\,(B - b)}{\pi} \cot\beta\, F(\sin\alpha', U) \qquad \textbf{(93)}$$

In the case considered, we have

$$\sin\alpha' = \frac{21}{43 \sin\beta} \qquad \textbf{(94)}$$

$$(x - X) = \frac{160}{\pi} \cot\beta\, F\,(\sin\alpha', U) \qquad \textbf{(95)}$$

In table VII are given calculations of paths which do not return to ground.

The curves given in Figs. 11 and 12 which correspond to the ionization conditions assumed for winter nights, were calculated in a similar manner, for a wave length of 25 meters. The assumed constants are as shown in the figure.

TABLE VII

Line	Quantity	ϕ or U	x	1	2
I	θ			40°	70°
II	β			42.07°	70.65°
III	$(\beta - \theta)$			2.07°	0.65°
IV	X		200	227	143
V	$\sin\beta$			0.6700	0.9435
VI	$\sin\alpha'$			0.73	0.519
VII	α'			46.9	31.3
VIII	$\cot\beta$			1.1076	0.3512
IX	$\frac{160}{\pi}\cot\beta$			56.3	17.9
X	$F\,(\sin\alpha', U)$	30°	226.7	0.5365	0.5299
XI	$(x - X)$			30.2	9.4
XII	x			257	152
XIII	$F\,(\sin\alpha', U)$	60°	253.3	1.151	1.096
XIV	$(x - X)$			64.9	19.6
XV	x			292	163
XVI	$K\,(\sin\alpha')$	90°	280	1.883	1.705
XVII	$(x - X)$			106	30.5
XVIII	x			333	174

It is seen from an inspection of these curves that we have a skip distance in both cases. As we increase θ from zero, the distance to the point where the ray returns to ground decreases at first, and afterwards increases, having passed through a minimum.

It is evidently not necessary to calculate the complete paths in order to determine the skip distances. For we are only interested in the points where the different rays reach the ground, and these are twice the distances to the summits of the various paths. We must, however, calculate the total range for several values of θ, and if we plot a curve giving the relation between this range and the angle θ, we shall be able to determine the skip distance by reading the minimum from the curve. These curves are given in Figs. 23 and 26 for the cases already calculated. We can, in this manner, make a special study of the skip distances apart from the paths.

We must admit the possiblity of a wave being reflected at the earth's surface when it comes down. We shall assume that the reflection is specular, that is to say, the reflected ray is in the plane of the incident ray, and the angle of incidence is equal to the angle of reflection. Clearly in this case the second path described will be a repetition of the first, provided that the reflection occurs at a horizontal plane, for example, at sea.

In Fig. 12 we show an example of some of these multiple paths, corresponding to the winter night curves given above in Fig. 11.

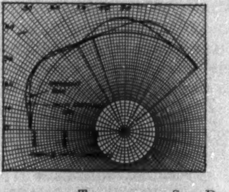

Fig. 12—Reflection of Tangent and Skip Distance Rays for Winter Night Conditions

VII. Discussion of Skip Distance Calculations

We have shown in the preceding section that under the conditions assumed we obtain a skip distance. Some experimental data are available concerning these, so that the parameters N_0, b and $(B - b)$, appearing in the path equations, can be chosen to fit the observed values for skip distances.

As a preliminary, it is necessary to find the effect of each of the three parameters. This is done by holding two constant, varying the third, and calculating the skip distances for various wave-lengths.

The effect of varying N_0, the maximum electron density, can easily be calculated. We have the fundamental equation

$$\mu^2 = 1 - \frac{\sigma N}{\omega^2} \qquad \textbf{(96)}$$

in which $\sigma = \frac{4\,\pi\, e^2}{m} = 3.2 \times 10^9$ for electrons. In the above expression collisions are entirely neglected.

Suppose now that we multiply N by 4, and ω by 2, or, what is the same thing, halve the wave length λ. On putting the new values in the above equation we obtain the same value for μ^2. But if we multiply N_0 by 4, we multiply the value of N at every point by 4 also. Hence if we obtain a skip distance S for a wave length λ with maximum electron density N_0 we shall obtain the same skip distances for a wave length $\lambda/2$ with maximum electron density $4\,N_0$.

In general, if we multiply N_0 by n^2 and divide λ by n, we shall not alter μ^2 at any point, and therefore we cannot alter the skip distance.

For convenience we show the relation between wavelength and skip distance by plotting log (skip distance) as abscissa and log (wave length) as ordinate. On such a plot, if we multiply N_0 by 4, we must slide the curve down so that every point will have for its ordinate log $\lambda/2$ instead of log λ. But $\log \lambda/2 = \log \lambda - \log 2$, hence we have moved every point of the curve through the same vertical distance, log 2, and the shape is unaltered. This rule applies also to any path at a given angle θ.

A family of curves for various values of N_0 is given in Fig. 13.

For the other parameters empirical results only can be obtained.

When we vary $(B - b)$, keeping N_0 and b constant, the result appears to be to move the curve towards the right by a constant amount all along the curve. Successive equal increments of $(B - b)$ appear to produce slightly diminishing displacements of the curve on the logarithmic plot. The shape is again unaltered. It must be noticed that at a certain wave length the skip distance tends to zero and the curve becomes asymptotic in the logarithmic plot. A displacement to the right can obviously have no effect on this asymptote. The limiting wave length is that which makes the value of μ^2 at the maximum ionization equal to zero, hence we have $\sigma N_0 = \omega^2$ from equation (**96**). A family of curves for various values of $(B - b)$ is given in Fig. 14.

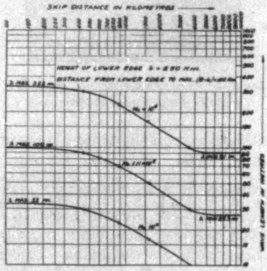

FIG. 13—EFFECT OF INTENSITY OF IONIZATION ON SKIP DISTANCE

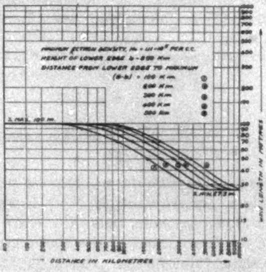

FIG. 14—EFFECT OF THICKNESS OF MEDIUM ON SKIP DISTANCE

When we vary b, keeping N_0 and $(B - b)$ constant, there appears to be practically no effect for wave lengths near that which makes the skip distance zero. The effect becomes larger as we move away from this wavelength, so that a family of curves for various values of b is spread out into a fan shape at the lower wave lengths. The effect of successive equal increments of b becomes less and less. A family of these curves is given in Fig. 15.

An interesting point is shown by all these curves. As we decrease the wave length, the maximum permissible value of β, in order that the ray may return, will decrease. It may happen that the ray tangent to the earth cuts the ionized medium at an angle greater than the critical angle for return. In this case no ray would be able to return at all.

When the ray tangent to the earth cuts the ionized medium at the critical angle, it is readily shown that the skip distance is infinite, since the ray at the critical angle goes off to infinity. The curves have, therefore, a horizontal asymptote, at the wave length for which the ray tangent to the earth cuts the medium at the critical angle.

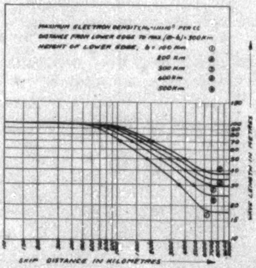

FIG. 15—EFFECT OF HEIGHT OF MEDIUM ON SKIP DISTANCE

When we vary $(B - b)$ we do not affect the critical angle, which depends only on N_0. The angle at which the ray tangent to the earth cuts the medium depends only on b. It is therefore evident that when we vary $(B - b)$, we do not alter the lower asymptotic wave length, but a variation of either N_0 or b will evidently change its value.

The critical angle is always given by

$$\sigma N_0 = \omega^2 \sin^2 \beta \qquad \textbf{(97)}$$

and $\cos \beta = \left(\dfrac{r_0}{r_0 + b}\right) \cos \theta$, so that the ray tangent to the earth will have $\cos \beta = \left(\dfrac{r_0}{r_0 + b}\right)$ and hence,

$$\sin^2 \beta = \frac{b^2 + 2\, r\, b_0}{(r_0 + b)^2} \qquad \textbf{(98)}$$

Substituting (**98**) in (**97**) we find that the greatest value of ω for which a ray can return to earth is given by

$$\sigma N_0 = \omega^2 \frac{b^2 + 2 r_0 b}{(r_0 + b)^2} \tag{99}$$

or

$$\omega = (r_0 + b) \sqrt{\frac{\sigma N_0}{2 r_0 b + b^2}} \tag{100}$$

which evidently depends on both N_0 and b.

Calculations show that there is a sudden bend in the curve very close to the lower asymptotic wave length.

Knowing the effect of varying each of the parameters, it is fairly easy to choose values to fit curves drawn to indicate the results of experimental observations. In this way we find that the assumed experimental values given in Fig. 1 and redrawn in Fig. 16, require:

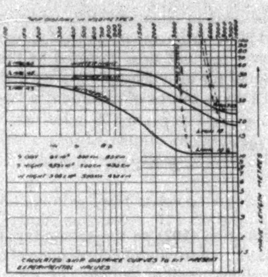

Fig. 16

For summer days $N_0 = 6 \times 10^5$ electrons per cu. cm., $b = 200$ km., $(B - b) = 80$ km. For summer nights $N_0 = 4.83 \times 10^5$ electrons per cu. cm., with $b = 500$ km. and $(B - b) = 430$ km. For winter nights the curve is nearly parallel to that for summer nights, so we take $N_0 = 3.08 \times 10^5$ electrons per cu. cm., $b = 500$ km., $(B - b) = 430$ km., and let the change in N_0 take up the whole of the difference in the curves. The data concerning skip distances, especially at night, are as yet very meager so that our assumed values may be far from accurate. As further data are obtained the curves may be moved in any desired direction by changing N_0 or $(B - b)$, and we may change their shape by changing b.

VIII. CALCULATION OF THE POWER RECEIVED AT THE SURFACE OF THE EARTH FROM A DISTANT SHORT WAVE TRANSMITTER, NEGLECTING ENERGY ABSORPTION

In the present discussion we will confine our attention to the power which comes down to the earth from the sky due to refraction in the upper atmosphere. The ground wave, which is only of importance near the transmitter, can be considered in the usual manner. We shall also neglect energy absorption in the sky wave path. We shall further assume that the energy radiated by the transmitter at an angle θ above the horizontal is the same in all directions, *i. e.*, the radiation curve is symmetrical about the vertical axis.

Suppose that a ray starting out from the transmitter at an angle θ above the horizontal comes down to earth again at a distance $x(\theta)$. We write this distance as $x(\theta)$ to keep before us the fact that this distance is a function of the initial angle θ at which the ray leaves the transmitter. An adjacent ray starting at the angle $(\theta + d\theta)$ will come down at a distance $x(\theta + d\theta)$ which may also be written $\left\{ x(\theta) + \frac{d x(\theta)}{d \theta} d\theta \right\}$. The power emitted by the transmitter between the angles θ and $(\theta + d\theta)$ is a function of the distribution curve of the antenna, which is here assumed to be a function of θ only. We will therefore write the power emitted by the transmitter through this zone as $p(\theta)\, d\theta$.

This power must come down distributed over the area lying between the two circles on the earth's surface at distances $x(\theta)$ and $\left\{ x(\theta) + \frac{d x(\theta)}{d \theta} d\theta \right\}$ from the transmitting station. This of course assumes that these distances are equal in all directions.

If r_0 be the radius of the earth, the angle $A\,O\,B$ in Fig. 17 will be $x(\theta)/r_0$ radians. The radius r of the zone will then be r_0 times the sine of $x(\theta)/r_0$ and hence the area will be

$$2 \pi r_0 \sin \frac{x(\theta)}{r_0} \, \frac{d x(\theta)}{d \theta} d\theta \tag{101}$$

Dividing the power sent out from the transmitter between the angles θ and $(\theta + d\theta)$ by the area of the zone on the earth's surface over which it is spread we obtain

$$w = \frac{p(\theta)}{2 \pi r_0 \dfrac{d x(\theta)}{d \theta} \sin \dfrac{x(\theta)}{r_0}} \tag{102}$$

for the power per unit area at the distance $x(\theta)$ from the transmitter.

For the *Case* of a *Flat Earth* the angle θ is equal to β and we therefore write

$$w = \frac{p(\beta)}{2 \pi x(\beta) \dfrac{d x(\beta)}{d \beta}} \tag{103}$$

for the power received per unit area at a distance $x(\beta)$ from the transmitting station. In this equation $x(\beta) = 2 x_s$ as given in equation (**77**).

It is customary to represent the field intensity of the wave radiated by an antenna in different directions by a polar diagram, in which the lengths of the radii drawn in various directions are made proportional to the field intensities emitted in the directions indicated by the radii.

Polar diagrams are also frequently drawn in which the lengths of the radii are made proportional to the power intensities emitted by the source in the different directions. The radii for a power intensity curve of a given source are then proportional to the squares of the radii for the corresponding field intensity curve.

If I be the amount of radiant power which falls on unit surface at unit distance from the antenna system in the direction θ, the *source* is said to have a power intensity I for the given direction. In other words the source radiates I watts per unit solid angle in the direction θ. Then the amount of power which will fall on unit surface at a distance r from the antenna system in the direction θ will be I/r^2, since for a source of given intensity I the radiation density in a given direction falls off inversely as the square of the distance.

The power radiated by the antenna through the zone lying between θ and $(\theta + d\,\theta)$ which we have called $p\,(\theta)\,d\,\theta$, will be equal to the area of the zone times the amount of radiation falling on the zone per unit area. From Fig. 18 this is seen to be

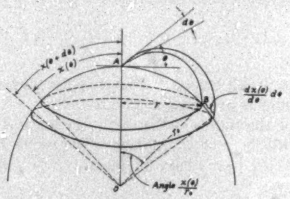
$x(\theta + d\theta)$
$x(\theta)$
A
$d\theta$
θ
$\frac{dx(\theta)}{d\theta}\,d\theta$
B
r
r_0
O
Angle $\frac{x(\theta)}{r_0}$

Fig. 17

$$2\,\pi\,r\cos\theta\,r\,d\,\theta \times I/r^2 = p\,(\theta)\,d\,\theta \qquad \textbf{(104)}$$

where $d\,\theta$ is in radians,

or

$$p\,(\theta) = 2\,\pi\,I\cos\theta \qquad \textbf{(105)}$$

The total power radiated by the antenna, assuming that it all goes out through the hemisphere above the surface of the earth, will be obtained by integrating (**104**) over the surface of the hemisphere.

This gives

$$P = \int_0^{\pi/2} 2\,\pi\,I\cos\theta\,d\,\theta \qquad \textbf{(106)}$$

For a simple vertical antenna in which the height is a small fraction of a quarter wave length we have the usual expression[26]

$$I = I_0\cos^2\theta \qquad \textbf{(107)}$$

where

I_0 = horizontal source intensity in watts per unit solid angle.

Substituting (**107**) in (**106**) and performing the integration we have

$$P = \frac{4\,\pi}{3}\,I_0 \qquad \textbf{(108)}$$

If we put the value of I_0 in terms of P in equation (**107**) and substitute the resulting value of I in (**105**) we obtain

$$p\,(\theta) = \frac{3}{2}\,P\cos^3\theta \qquad \textbf{(109)}$$

This may be used in equation (**102**) when calculating the received power from a short vertical antenna.

If the power per unit area radiated by the antenna system is equal for all values of θ, *i. e.*, if the system constituted a point source of radiation, we would have

$$I = I_0 \qquad \textbf{(110)}$$

and from (**106**)

$$P = 2\,\pi\,I_0 \qquad \textbf{(111)}$$

Therefore

$$p\,(\theta) = P\cos\theta \qquad \textbf{(112)}$$

for use in equation (**102**) on the assumption of a point source radiator. Actually of course none of our simple antenna systems radiate equally in all directions, nevertheless, an assumption of this kind is interesting when we wish to study the limitations imposed on short wave transmission by the properties and configuration of the medium, irrespective of the directive properties of our antenna system. When this ideal case has been studied, and the directions of useful radiation determined for the particular problem in hand, the most suitable type of antenna directive curve can be chosen readily.

Before we can calculate the radiation density at different distances from the transmitter from equation (**102**) we require the value of $d\,x\,(\theta)/d\,\theta$. This is obtained by differentiating equation (**82**) which expresses the range $x\,(\theta)$ of a ray as a function of the initial angle θ at which the ray left the transmitter. For the *Case* of a *Curved Earth* we have from equation (**82**) after expressing the angles in radians

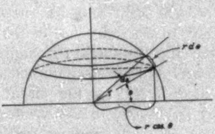

Fig. 18

$$x\,(\theta) = 2\,r_0\,(\beta - \theta) + \frac{4\,\omega\,(B - b)}{\pi\,\sqrt{\sigma\,N_0}}\cos\beta\,K\,(\sin\alpha) \qquad \textbf{(113)}$$

---

[26] Curves of this kind were first given by A. Blondel, French Association for the Advancement of Science at Angers in 1903. See J. A. Fleming, *Principles of Elect. Wave Tel. and Tel.*, Fourth Edition, 1919, p. 362. The directive curves for vertical antennas excited in various harmonics were first given by Balth. van der Pol, Jr., in *Proc. Phy. Soc.* Lond., Vol. 29, p. 269, 1917. Further reference may be made to Stuart Ballantine, *I. R. E.*, Vol. 12, No. 6, p. 823, 1924, and Balth. van der Pol, Jr., *I. R. E.*, Vol. 13, No. 2, p. 251, 1925.

in which β, the acute angle at which the ray enters the ionized medium, is given by

$$\cos\beta = \left\{ \frac{\cos\theta}{(1+b/r_0)} \right\} \qquad (\mathbf{114})$$

and $\sin\alpha$, which is called the modulus of the elliptic integral, $K\,(\sin\alpha)$, is given by

$$\sin\alpha = \frac{\omega}{\sqrt{\sigma N_0}} \sin\beta \qquad (\mathbf{115})$$

The differentiation of equation (**113**) involves a good many steps and is therefore set aside in Appendix III. The final value obtained in equation (**17**) of Appendix III gives for the *Case* of a *Curved Earth*,

$$\frac{d\,x\,(\theta)}{d\,\theta} = 2\,r_0\left(\frac{\tan\theta}{\tan\beta}-1\right) + \frac{4\,(B-b)}{\pi}\,\frac{\sin\alpha}{\tan\beta}\,\tan\theta \times \left[\frac{E\,(\sin\alpha)}{\tan^2\beta\cos^2\alpha} - \frac{K\,(\sin\alpha)}{\sin^2\beta}\right] \qquad (\mathbf{116})$$

The new expression $E\,(\sin\alpha)$ in the above equation denotes the complete elliptic integral of the second kind and its value for any value of α will be found in Tables.[24]

For the *Case* of a *Flat Earth* we require the value of $d\,x\,(\beta)/d\,\beta$ from equation[27] (**77**) for use in the power density equation (**103**). This gives

$$\frac{d\,x\,(\beta)}{d\,\beta} = -\,\frac{2\,b}{\sin^2\beta} + \frac{4\,(B-b)}{\pi}\,\sin\alpha \left[\frac{E\,(\sin\alpha)}{\tan^2\beta\cos^2\alpha} - \frac{K\,(\sin\alpha)}{\sin^2\beta}\right] \qquad (\mathbf{117})$$

We have now completed all of the expressions required by equation (**102**) for the calculation of the power per unit area which a ray leaving the antenna at an angle θ yields when it again reaches the earth, neglecting absorption in the medium.

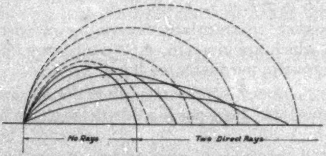

FIG. 19

For example let us assume that we have a short vertical antenna which radiates a total power of P watts into space, and desire to know the number of watts per square kilometer w received at a distance x kilometers from the transmitter, for the case of a curved earth.

We first have to calculate ranges of the rays leaving the transmitter at various values of the angle θ from equation (**82**). We have seen from the sample ray paths calculated in section VI and given in Figs. 11 and 12 and further illustrated in Figs. 19 and 20 that for the case of a flat earth there are in general two direct rays with two corresponding values of θ which traverse different paths but strike the earth beyond the skip distance at the same distance x from the transmitter. For the case of a curved earth illustrated in Fig. 20 we again have no sky rays until the skip distance has been passed. Beyond this there is a region of two direct rays, until we reach the point where the limiting ray, which starts tangent to the earth's surface at the transmitter, comes to earth. Outside of this region we again find only one direct ray. We shall postpone the discussion of reflected rays for the moment, and confine our attention to the direct rays.

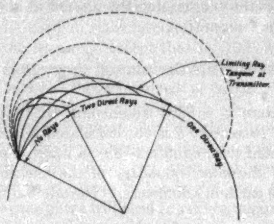

FIG. 20

Except near the skip distance we can usually neglect the ray which has the largest value of initial angle θ, since the power density which it produces at x is relatively small. Where both rays are important we add the two values of w to obtain the resultant power density at the point. This assumes that the relative phases of the two rays are distributed at random when they arrive at the ground.

If everything were constant in the medium traversed by the rays, there would of course be a definite phase relation between the two rays at the ground which would be a function of the time of propagation of the two rays over their respective paths and the wave length. The phenomena of fading indicate irregularities in the refracting medium, which means that at one instant of time the amplitudes of the two rays may add directly in phase and a moment later the phases will shift so that they will oppose each other. Under these conditions the resultant power density at the point, taken over a considerable length of time, is equal to the sum of the power densities of the two rays.

We shall now assume that we have determined the initial angles of the two rays which strike the ground at the required distance x from the transmitter, and that we are so far from the skip distance that we can neglect the power density contributed by the ray having the larger initial angle. Equation (**102**) then states that

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[27] For convenience we have put $2\,x_s = x\,(\beta)$ in this equation.

$$w = \frac{p\,(\theta)}{2\,\pi\,r_0\,\dfrac{d\,x\,(\theta)}{d\,\theta}\,\sin\dfrac{x\,(\theta)}{r_0}} \text{ watts per sq. km.} \qquad \textbf{(118)}$$

The value of $p\,(\theta)$ for our case of a short vertical antenna radiating P watts into space is given by equation (**109**) as

$$p\,(\theta) = \frac{3}{2}\,P\cos^2\theta \text{ watts} \qquad \textbf{(119)}$$

where θ is the initial angle which we have calculated for the important ray striking the ground at the distance x from the transmitter.

We next calculate the value of $\dfrac{d\,x\,(\theta)}{d\,\theta}$ from equation (**116**). Here $r_0 = 6300$ kilometers, the approximate radius of the earth; θ is our calculated initial angle; β is calculated from equation (**80**) in which b equals the height of the lower boundary of the ionized medium above the earth in kilometers. The angle α is given in equation (**66**) as

$$\sin\alpha = \frac{\omega}{\sqrt{\sigma\,N_0}}\sin\beta \qquad \textbf{(120)}$$

this may be written as

$$\sin\alpha = \frac{2\,\pi\,c}{\lambda\,\sqrt{\sigma\,N_0}}\sin\beta \qquad \textbf{(121)}$$

in which

c = velocity of light in meters per sec.

and

λ = wave length in meters

$\sigma = 4\,\pi\,e^2/m = 3.2 \times 10^9$ c. g. s. (e. s. u.) for electrons

N_0 = maximum number of electrons per cu. cm. which occurs at the height B above the ground.

Hence

$$\sin\alpha = \frac{10^5}{3\,\lambda\,\sqrt{N_0}}\sin\beta \qquad \textbf{(122)}$$

The value of $(B - b)$ is the thickness of the ionized medium from the lower boundary to the maximum in kilometers. The values of the complete elliptic integrals of the first and second kinds, $K\,(\sin\alpha)$ and $E\,(\sin\alpha)$, respectively, are found in tables of elliptic integrals for any value of α.

The final quantity required is the sine of the angle $\dfrac{x\,(\theta)}{r_0}$. Here $x\,(\theta)$ is the range of the ray which strikes the ground at the distance x kilometers from the transmitter, and r_0 is the radius of the earth in kilometers. This gives the angle in radians, and for looking up the sine we require the angle in degrees, which is $\dfrac{180}{\pi}\;\dfrac{x\,(\theta)}{r_0}$.

We have thus obtained the power density in watts per square kilometer of the earth's surface falling at the distance x from the transmitter. To find the power density on a surface normal to the ray we have to divide by $\sin\theta$, since the ray comes down at the angle θ to the horizontal

$$\text{Let } w_0 = w/\sin\theta \qquad \textbf{(123)}$$

We can calculate the received electric field intensity in microvolts per meter by means of Poynting's Theorem which states that the energy flux density w_0, in ergs per second per square cm. of an electromagnetic wave is equal to $\dfrac{c}{4\,\pi}$ times the product of the electric field intensity E in c. g. s. (e. s. u.) and the magnetic field intensity H in c. g. s. (e. m. u.) for the case of an isotropic medium of unit permeability and dielectric constant, that is

$$w_0 = \frac{c}{4\,\pi}\,[E\;H] \qquad \textbf{(124)}$$

where

$c = 3 \times 10^{10}$ cm. per sec. the velocity of light.

For an electromagnetic wave in space we have

E in c. g. s. (e. s. u.) = H in c. g. s. (e. m. u.)

This can be seen easily by comparing the expressions for the energy per unit volume stored in the electrostatic and magnetic fields, and remembering that both forms carry equal amounts of energy. We may therefore write

$$w_0 = \frac{c}{4\,\pi}\,E^2 \text{ ergs per sq. cm. per sec.}$$

where E is in c. g. s. (e. s. u.)

If we wish to translate our values of energy flux density, which are given in watts per square kilometer, into microvolts per meter we have

$$1 \text{ c. g. s. (e. s. u.)} = 300 \text{ volts}$$

$$1 \text{ erg per sec.} = 10^{-7} \text{ watts}$$

$$w_0 = \frac{E^2}{120\,\pi} \text{ watts per sq. cm.}$$

where E is in volts per cm.

Expressing E in microvolts per meter we have

$$E = 19.4 \times 10^8\,\sqrt{w_0}$$

where w_0 is still in watts per sq. cm.

If we express the power in watts per square kilometer we have

$$E = 19.4 \times 10^3\,\sqrt{w_0} \text{ microvolts per meter} \qquad \textbf{(125)}$$

Since in our equations the power is expressed in watts per square kilometer of earth surface we have

$$E = 19.4 \times 10^3\,\sqrt{w/\sin\theta} \text{ microvolts per meter} \qquad \textbf{(126)}$$

This is the total value of potential gradient and lies in the wave front. If it lies in a vertical plane, the resultant field obtained by combining with the reflected ray, assuming perfect metallic reflection at the surface of the earth, will be

$$2\,E \cos\theta \qquad \textbf{(127)}$$

If the plane containing the direction of the ray and the vector E makes an angle ψ with the vertical, the resultant field at the earth's surface will be

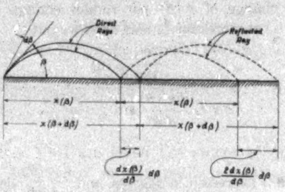

FIG. 21

$$2\,E\cos\theta\cos\psi = 38.8\times 10^3\cos\theta\cos\psi\sqrt{w/\sin\theta} \qquad \textbf{(128)}$$

In the above calculations we have determined the energy flux density received at the surface of the earth when a ray of given initial angle first strikes the ground. Obviously a portion of this incident energy will be reflected toward the sky and return to earth again at twice the initial distance from the transmitter. If we assume that the irregularities of the earth's surface, at the point of incidence, are small compared with the wave length, the angle of reflection will be equal to the angle of incidence and the path taken by the reflected ray will be a repetition of the first. The direct and reflected rays are illustrated in Fig. 21 for the *Case* of a *Flat Earth.* Here the power emitted by the transmitter between the angles β and $(\beta + d\,\beta)$ first strikes the earth in the ring between $x\,(\beta)$ and $x\,(\beta + d\,\beta)$, and the reflected portion next strikes the earth in the ring between $2\,x\,(\beta)$ and $2\,x\,(\beta + d\,\beta)$. This process of successive reflections will go on until all of the energy which was originally radiated from the antenna through the zone $d\,\beta$ is dissipated by absorption, scattering due to ground irregularities, etc. The power density received in the second zone from the power radiated by the antenna between the angles β and $(\beta + d\,\beta)$ will be

$$w_2 = \frac{\eta\,p\,(\beta)\,d\,\beta}{2\,\pi\times 2\,x\,(\beta)\times 2\,\dfrac{d\,x\,(\beta)}{d\,\beta}\,d\,\beta} \qquad \textbf{(129)}$$

or

$$w_2 = \frac{\eta\,p\,(\beta)}{8\,\pi\,x\,(\beta)\,\dfrac{d\,x\,(\beta)}{d\,\beta}} \qquad \textbf{(130)}$$

with similar expressions for the third, fourth, etc., zones. Here η is the fraction of the energy which is assumed to be specularly reflected.

Similarly for the *Case* of a *Curved Earth* we have for the second zone from Fig. 22

$$w_2 = \frac{\eta\,p\,(\theta)\,d\,\theta}{2\,\pi\times r_0\sin\dfrac{2\,x\,(\theta)}{r_0}\times 2\,\dfrac{d\,x\,(\theta)}{d\,\theta}\,d\,\theta} \qquad \textbf{(131)}$$

$$w_2 = \frac{\eta\,p\,(\theta)}{4\,\pi\,r_0\,\sin\dfrac{2\,x\,(\theta)}{r_0}\,\dfrac{d\,x\,(\theta)}{d\,\theta}} \qquad \textbf{(132)}$$

The quantities in the above expressions are the same as those already used in calculating the power density received in the first zone except that in (**132**) we have to look up the sine of the angle $2\,x\,(\theta)/r_0$ instead of $x\,(\theta)/r_0$. Thus the labor involved in taking account of repeated reflections is not serious.

To calculate the energy flux density at a point distant x from the transmitter we must take first that which comes by the direct ray, then that which has been reflected from rays at $x/2$, $x/3$, $x/4$, etc. Absorption in the atmosphere will tend to reduce the intensity of the direct and subsequently reflected rays, also the reflection efficiency factor η will be reintroduced for every reflection so that we may expect that most of the energy will come from paths with few reflections. It may of course happen that some or all of the multiple reflected rays will be absent at the distance x under consideration: For example we cannot have a ray coming down after two sky paths at less than twice the skip distance, etc.

IX. INTENSITY CALCULATIONS

To make clear the method of calculating the energy flux density at distant points, we give an example worked out in full. The case chosen is that for which the path calculations were given. The constants are $N_0 = 6\times 10^5$ electrons per cc. = maximum density; b = height of lower edge of medium above ground = 200 kilometers; $(B - b)$ = distance from lower edge to maximum = 80 km. The wave length chosen is 21 meters. If we consider only the end point of the path, and denote the distance x for this point by $x\,(\theta)$, we can draw a curve giving the relation between $x\,(\theta)$ and

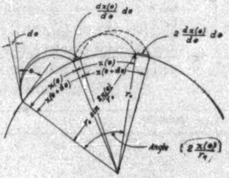

FIG. 22

the initial angle θ. This curve is given in Fig. 23.

If we wish to calculate the energy flux density at a distance x from the transmitting station, we must find the corresponding value of θ from this curve, since the formulas used give the energy flux density for rays starting out at the angle θ.

The energy flux per square kilometer per second is given by equation (**102**) which is

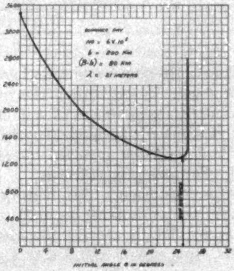

Fig. 23—Variation of Range with Initial Angle

$$w = \frac{p\,(\theta)}{2\,\pi\,r_0\,\dfrac{d\,x\,(\theta)}{d\,(\theta)}\,\sin\dfrac{x\,(\theta)}{r_0}} \qquad (\mathbf{133})$$

for a point source $p\,(\theta) = P \cos\theta$, where P is the power radiated into the sky. The quantity $\dfrac{d\,x\,(\theta)}{d\,\theta}$ is the change of $x\,(\theta)$ per radian change in θ. In order to get more convenient units we put

$$w = \frac{\dfrac{\pi}{180}\,p\,(\theta)}{2\,\pi\,r_0\,\dfrac{d\,x\,(\theta)}{d\,\theta^\circ}\,\sin\left\{\dfrac{x\,(\theta)}{110}\right\}^\circ} \qquad (\mathbf{134})$$

where θ is expressed in degrees, and $\dfrac{d\,x\,(\theta)}{d\,\theta^\circ}$ denotes the change in $x\,(\theta)$ per degree change in (θ). The angle $\dfrac{x\,(\theta)}{r_0}$ radians is also changed to its equivalent, $\dfrac{(x\theta)}{110}$

TABLE VIII

Line	Quantity	1	2	3	4	5	6	7
I	θ	0°	5°	10°	15°	20°	23°	25°
II	β	14.25°	15.04°	17.3°	20.7°	24.5°	26.9°	28.55°
III	α	30.2°	32.1°	37.4°	46.4°	58.1°	68°	78.2°
IV	$\sin\beta$	0.246	0.2595	0.2974	0.3535	0.4147	0.4524	0.478
V	$\sin\alpha$	0.503	0.531	0.608	0.724	0.849	0.927	0.979
VI	$x\,(\theta)$	3478	2552	1956	1620	1390	1308	1328
VII	$K\,(\sin\alpha)$	1.687	1.704	1.757	1.875	2.117	2.419	2.995

TABLE IX

Line	Quantity	1	2	3	4	5	6	7
I	θ	0°	5°	10°	15°	20°	23°	25°
II	$\tan\theta$	0	0.0875	0.1763	0.2679	0.3640	0.4245	0.4663
III	$\tan\beta$	0.254	0.2685	0.3115	0.3779	0.4557	0.5073	0.5441
IV	$\cos\alpha$	0.8643	0.8471	0.7944	0.6896	0.5284	0.3746	0.2045
V	$E\,(\sin\alpha)$	1.688	1.453	1.414	1.338	1.229	1.136	1.052

TABLE X

Line	Quantity	1	2	3	4	5	6	7
I	θ	0°	5°	10°	15°	20°	23°	25°
II	$\dfrac{\tan\theta}{\tan\beta}$	0	0.326	0.566	0.709	0.799	0.837	0.857
III	(II − 1)	−1	−0.674	−0.434	−.291	−0.201	−0.163	−0.143
IV	$\sin\alpha \times$ I	0	0.1731	0.3445	.513	0.688	0.775	0.839
V	$\dfrac{B-b}{45} \times$ IV	0	0.308	0.612	0.912	1.223	1.379	1.49
VI	$\tan^2\beta$		0.0721	0.0971	.1428	0.208	0.2575	0.296
VII	$\cos^2\alpha$		0.717	0.63	0.475	0.2795	0.1402	0.0417
VIII	VI × VII		0.0517	0.0612	0.0679	0.0581	0.0361	0.01236
IX	$E\,(\sin\alpha) \div$ VIII		28.1	23.1	19.7	21.12	31.45	85.2
X	$\sin^2\beta$		0.0674	0.0884	0.1136	0.172	0.205	0.2285
XI	$K\,(\sin\alpha) \div$ IX		25.3	19.87	16.5	12.29	11.79	13.1
XII	IX − XI		2.8	3.23	3.2	8.83	19.66	72.1
XIII	220 × III	−220	−148.2	−95.5	−64.	−44.2	−35.9	−31.5
XIV	XII × V	0	0.86	1.98	2.92	10.8	27.1	107.5
XV	$\dfrac{d\,x\,(\theta)}{d\,\theta^\circ}$	−220	−147.5	−93.5	−61.1	−33.4	− 8.8	+76

degrees. The value of $\frac{d\,x\,(\theta)}{d\,\theta}$ is given by

$$\frac{d\,x\,(\theta)}{d\,\theta^\circ} = 220\left(\frac{\tan\theta}{\tan\beta} - 1\right)$$

$$+\frac{(B-b)}{45}\,\frac{\sin\alpha\,\tan\,(\theta)}{\tan\beta}\left[\frac{E\,(\sin\alpha)}{\tan^2\beta\cos^2\alpha} - \frac{K\,(\sin\alpha)}{\sin^2\beta}\right]$$

$$\text{kilometers per degree} \qquad \textbf{(135)}$$

which is obtained from the formula (**116**) by multiplying by $\frac{\pi}{180}$ and putting in the value 6300 for r_0, the earth's radius.

In Table VIII are given the values of quantities already determined in the path calculations, viz., β, α, $\sin\beta$, $\sin\alpha$, $x\,(\theta)$, and $K\ (\sin\alpha)$.

In Table IX are given the values of $\tan\theta$, $\tan\beta$, $\cos\alpha$, and $E\,(\sin\alpha)$, as read from appropriate tables.

In Table X are given the calculations to determine $\frac{d\,x\,(\theta)}{d\,(\theta)}$.

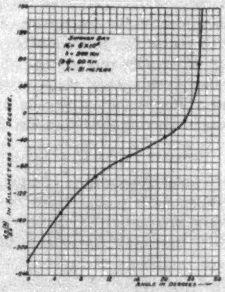

FIG. 24—RATE OF CHANGE OF RANGE WITH RESPECT TO INITIAL ANGLE

A curve is given in Fig. 24 showing the relation between $\frac{d\,x\,(\theta)}{d\,(\theta)}$ and (θ).

In the actual intensity calculation we are not concerned with the sign of $\frac{d\,x\,(\theta)}{d\,\theta}$, but only with its magnitude. This can be seen from the way that the expression is derived. We now have

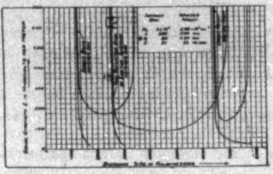

FIG. 25—CALCULATED INTENSITY OF SIGNAL

$$w = \frac{\frac{\pi}{180}\,.\,P\cos\theta}{2\,\pi\,r_0\,\frac{d\,x\,(\theta)}{d\,\theta^0}\sin\left\{\frac{x\,(\theta)}{110}\right\}^\circ} \qquad \textbf{(136)}$$

$$= \frac{\cos\theta}{2228\,\frac{d\,x\,(\theta)}{d\,\theta^0}\sin\left\{\frac{x\,(\theta)}{110}\right\}^\circ} \qquad \textbf{(137)}$$

on putting $P = 1000$ watts and $r_0 = 6300$ km.

The electric field strength is given by

$$E = 19.4 \times 10^3\sqrt{w\ /\ \sin\theta} \qquad \textbf{(138)}$$

in which E is in microvolts per meter and w is in watts per square kilometer.

In Table XI are given the calculations to determine E and w from the above formulas.

The values of E for various values of $x\,(\theta)$ are given in the form of a curve in Fig. 25.

TABLE XI*

Line	Quantity	1	2	3	4	5	6	7
I	θ	0°	5°	10°	15°	20°	23°	25°
II	$\frac{x\,(\theta)}{110}$	31.6°	23.2°	17.8°	14.7°	12.6°	11.9°	12.07°
III	$\sin\frac{x\,(\theta)}{110}$	0.524	0.3939	0.3957	0.2538	0.2181	0.2062	0.2091
IV	$\frac{d\,x\,(\theta)}{d\,\theta^\circ} \times \text{III}\ \frac{\text{km.}}{\text{deg.}}$	115.1	58.	28.6	15.5	7.3	1.81	15.9
V	$\cos\theta$	1	0.9962	.9848	0.9659	0.9397	0.9205	0.9063
VI	$\text{V} \div \text{IV}$	0.00868	0.01717	.0344	0.0623	0.1288	0.509	0.057
VII	$10^6 \times \text{VI} \div 2228 = 10^6\ w$	3.79	7.69	15.4	27.9	57.7	228.	25.6
VIII	$10^3\sqrt{w} \div \sqrt{\text{VII}}$	1.946	2.77	3.92	5.28	7.6	15.1	5.06
IX	$E = \frac{19.4 \times \text{VIII}}{\sqrt{\sin\theta}}$ μv per m	∞	182.4	182.4	221.2	252.3	469.5	151.4
X	$x\,(\theta)$	3478	2552	1956	1620	1390	1308	1328

*See foot note to Table II.

The calculations were also made for a wave length of 25 meters under winter night conditions, and the results are given in Figs. 25, 26 and 27. Fig. 26 gives the relation between $x\ (\theta)$ and θ; Fig. 27 gives the value of $\dfrac{d\,x\ (\theta)}{d\theta}$ for different values of θ, while in Fig. 25 are plotted the signal strength values against the distance $x\ (\theta)$.

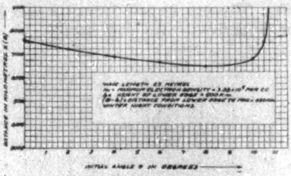

FIG. 26—RANGE OF RAYS WITH DIFFERENT INITIAL ANGLES

To show the effect of multiple reflections, we have calculated the signal strength of rays coming down a second time after being reflected once at the earth's surface. If the direct ray comes down at $x\ (\theta)$, the reflected ray comes down at $2\,x\ (\theta)$, and the value of $\dfrac{d\,x\ (\theta)}{d\,\theta}$ will also be doubled.

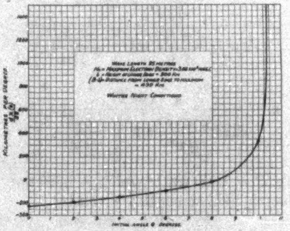

FIG. 27—CHANGE OF RANGE PER DEGREE CHANGE OF INITIAL ANGLE

The ratio of the two values of w is

$$\frac{w_1}{w_2} = \frac{\dfrac{d\{2\,x\ (\theta)\}}{d\,\theta}\ \sin\left\{\dfrac{2\,x\ (\theta)}{110}\right\}^{\circ}}{\dfrac{d\,x\ (\theta)}{d\,\theta}\ \sin\ \dfrac{x\ (\theta)}{110}} \qquad \textbf{(139)}$$

in which w_1 refers to the direct ray and w_2 to the reflected ray. This simplifies to

$$w_2 = \frac{w_1}{4\cos\dfrac{x\ (\theta)}{110}} \qquad \textbf{(140)}$$

so that the value of E_2 will be

$$E_2 = \frac{E_1}{2\sqrt{\cos\dfrac{x\ (\theta)}{110}}} \qquad \textbf{(141)}$$

The calculations for the intensity of the received signal after reflection are given in Table XII. When there are several rays coming down at a point, the most probable signal strength would be given by the square root of the sum of the squares of the separate values, since the energies may be assumed to be additive. Actually there may be fluctuation all the way from the sum to the difference of the intensities of the rays, so it seems best to leave the curves in the form of the separate signal strengths due to the different rays. We can also see in this way where severe fading is likely to occur. The signal strengths for the direct and once reflected rays are plotted in Fig. 25.

X. SUMMARY AND CONCLUSIONS

We have seen that the striking phenomena of short wave radio transmission (*i. e.*, below 60 meters) can be quantitatively accounted for on a simple electron refraction theory in which the effects of the earth's magnetic field and collisions of electrons with molecules may be neglected as a first approximation. The distribution and number of electrons per unit volume in the upper atmosphere required on this theory to account for the meager experimental data appear to be in general accord with the values required in the explanation of the diurnal variations of the earth's magnetic field, aurorae and long wave radio transmission[28].

TABLE XII*

Line	Quantity	1	2	3	4	5	6	7	8
I	$x\ (\theta)$	3478	2552	1956	1620	1390	1308	1328	1400
II	$E\ (x)$	∞	182.4	182.4	221.2	252.3	460.5	151.4	80
III	$\dfrac{x}{110}$	31.6	23.2	17.8	14.7	12.6	11.9	12.07	12.72
IV	$\cos\dfrac{x}{110}$	0.8517	0.9191	0.9521	0.9673	0.9759	0.9785	0.9779	0.9754
V	$\sqrt{\text{IV}}$	0.923	0.957	0.976	0.984	0.988	0.989	0.989	0.9875
VI	$E\ (2\,x)$	∞	95.2	93.5	113	128.	237	76.5	41
VII	$2\,x\ (\theta)$	6956	5104	3912	3240	2780	2616	2656	2800

*See foot note to Table II.

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[28] H. J. Round, T. L. Eckersley, K. Tremellen and F. C. Lunnon. *Journal. Inst. Elec. Engg*, Vol. 63, No. 346, p. 933, Oct. 1925.

Thus the large increase in the skip distance on a given wave length at night compared with that by day is a natural consequence of the greater ionization produced on the sunlit hemisphere by the streamers issuing from the sun. The ideal field intensity calculations given in Fig. 25 show that we may make an ample allowance for scattering and absorption and still account for the strong signals observed at great distances. The high field intensities indicated in the ideal curves of Fig. 25 at the skip distance and where the tangent ray strikes the earth are due to the focussing effect discussed in Appendix IV. The intensities at the two foci will, of course, be limited by the finite size of the source as well as by absorption and scattering.

Let us now discuss the problem of maintaining night and day communication between two points 5000 kilometers apart from the point of view of the present theory. For full winter night conditions (*i. e.*, night at both transmitter and receiver) Fig. 16 indicates that the selection of a 30-meter wave length would put the receiver right at the skip distance. Such a wave length selection would result in the arrival of two intense sky waves and consequently severe fading. Inspection of the field intensity calculations of Fig. 25 shows that the field intensit produced by the ray leaving the transmitter at the higher initial angle dies out very rapidly compared with that produced by the ray having the lower initial angle. Therefore, if we pick a somewhat longer wave length than 30 meters we will receive a reasonably strong signal from one ray only and consequently be less likely to find severe and rapid fading effects. We therefore would probably choose a wave length in the vicinity of 32 meters for winter night operation.

An alternative would be to select a wave length in the vicinity of 55 meters, with the idea of taking advantage of the focussing which occurs just inside of the distance at which the ray leaving the transmitter tangent to the ground comes back to earth. For full day conditions this wave length would be weak since 5000 kilometers is too far from the day skip distance. If we applied the above reasoning in selecting the best value for full day operation from Fig. 16 we would probably be led to select a wave length in the vicinity of 11 meters. This, however, would bring us down very close to the lower wave length limit which would mean a very weak signal. The reason for the weak signal near the low wave length limit will be readily appreciated when we remember that under these conditions we are working close to the value at which a ray leaving the transmitter tangent to the earth's surface strikes the lower boundary of the ionized medium at the second critical angle. Under these conditions there is only a small fraction of the emitted radiation which returns to earth.

It would be better, therefore, to select a wave length in the vicinity of 15 meters and operate on the ray which is reflected from 2500 kilometers to 5000 kilometers. Under these conditions we would, of course, face the possibility of considerable fading due to interference between the weak direct ray and the reflected ray. Another method would be to use a still longer wave length, for example around 25 meters, and depend upon the reflection of the energy falling at 2500 km. which is already beginning to pile up towards the focus at 3500 km.

In this connection it is interesting to note that the reflected 25-meter signal should be much stronger at 6400 km. than at 5000 km. since the focus is in the vicinity of 3200 km. In other words, it is apparently easier to get a good day signal at 6400 km. than at 5000 km. This point of view indicates that in short wave transmission problems, there will be certain favored distances. Another point of interest is that a 28-meter wave length should give good day and night communication between two points about 6400 km. apart due to the direct ray by night and the reflection from the tangent ray focus by day.

The numerical values deduced for this example are, of course, very uncertain since the ionization constants for the upper atmosphere which are required before a set of radio transmission characteristics like Fig. 16 can be calculated were figured backward from the very meager radio data. In other words, it is probably fair to say at the present time that short wave radio transmission experiments are the most direct method we have of estimating the ionization conditions in the upper atmosphere. We should not lose sight of the fact that the skip distances etc., which depend upon the ionization conditions in the upper atmosphere are probably not constant but will vary from year to year following the 11-year sun spot period, the last minimum of which occurred in 1922.

When both the transmitter and receiver are not in the sunlit or darkened hemisphere the ray paths will no longer be symmetrical about the middle point, and due allowance will have to be made for the variation of ionization conditions between the two stations. Sunset and sunrise effects will also require special treatment. It is also probable that transmission to or from the polar regions will require special study of this kind, due to the high concentration of ionization over the polar regions as compared with that over the middle belt of the earth. We can conclude in a general way that transmission from the sunlit into the darkened hemisphere will result in longer skip distances than would result if daylight extended over the whole path. For example, a ray entering the ionized medium from the sunlit side will at first meet the normal day refraction, which starts bending the ray back towards the earth and, as it moves into the darkened hemisphere, the bending by refraction will become less and less until normal night conditions exist. Thus the ray will strike the ground at a greater distance from the transmitter than it would have, if full day conditions extended over the entire path. Here the general conditions for reciprocity[29] are satisfied, so that if a ray were started back along the same path it would retrace the entire path back to the starting point.

Inspection of the day curve of Fig. 16 shows that it will be impossible to maintain communication between two distant stations on less than a 10-meter wave length. This limitation is due to the fact that the tangent ray will strike the lower boundary of the ionized medium at an angle greater than the second critical angle and will therefore not return to the earth, but be refracted out into empty space.

The precise value of this minimum wave length is, of course, not 10.6 meters, as indicated in Fig. 16, because of the meager data upon which the figure is based. There is a similar limit for full winter night conditions given as 22.5 meters in Fig. 16. Near sunset or sunrise, however, we can use a wave length less than the above full winter night limiting value, when transmitting from the dark into the light hemisphere, since the refraction is increasing in the direction of travel of the ray and may be sufficient to bend the ray back to earth.

It is now interesting to see what type of antenna directive curve will be most effective for long distance communication from the point of view of the present theory. We have seen that all of the energy which strikes the lower boundary of the ionized medium above the second critical angle is refracted out into space and is lost. The initial angle at the transmitter which corresponds to this condition is obtained from equations (**80**) and (**97**) as

$$\theta = \cos^{-1} \left\{ (1 + b/r_0) \sqrt{1 - \sigma N_0/\omega^2} \right\}$$

which may be written for convenience as

$$\theta = \cos^{-1} \left\{ (1 + b/6300) \sqrt{1 - 9 \times 10^{-9} N_0 \lambda^2} \right\}$$

The summer day and winter night conditions on long wave lengths yield the largest useful values of the initial angle. For our assumed ionization condition we obtain $\theta = 67.8$ deg. for 40 meters on a summer day and $\theta = 64.5$ deg. for 55 meters on a winter night. For shorter wave lengths the critical values for the initial angles will be much less as will be seen from Figs. 23 and 26. Here $\theta = 25.9$ deg. for 21 meters on a summer day and $\theta = 11.2$ deg. for 25 meters on a winter night. Thus we may conclude that on these short wave lengths all of the useful radiation is emitted between the horizontal and approximately 70 deg., and the greatest distances are reached by the low-angle radiation. We therefore conclude that for long-distance work, on short waves, maximum efficiency is obtained by low-angle radiation. This also means that nearby obstructions which cut off the low angle radiation will be detrimental to long distance working. It is therefore desirable to place the transmitter on a hill or mountain, so as to obtain an unobstructed path to the horizon in the desired direction. Raising the antenna system well above the ground will also assist by reducing ground losses and lowering the horizon.

For long-distance work the plane containing the electric vector at the transmitter may make any angle whatever with respect to the ground without appreciably affecting the ray paths or energy flux density in the wave front.

In any case the earth's magnetic field will produce enough rotation of the plane of polarization to make the angle of polarization of the received ray at the surface of the ground purely a question of chance. The best type and orientation of receiving system (loop or antenna) will depend upon the direction and polarization of the arriving wave as well as upon the conductivity of the ground and height of the receiving system above the earth. Some interesting work on determining the direction of arrival of signal waves has recently been done by Appleton and Barnett[30].

The best polarization of the transmitter can therefore be considered from the point of view of ground losses, mechanical construction and such questions as nearby interference, due to the ground wave, etc.

It should be pointed out in closing that electron collisions and the effect of the earth's magnetic field will modify the shape of the skip distance curves in the vicinity of the upper asymptotic wave length. Here absorption and double refraction (*i. e.*, splitting of a ray into two components having different velocities of propagation) will require consideration in a complete theory of short wave transmission.

The above theory is based on continuous wave theory and will not apply directly to very highly damped waves. In highly damped spark transmission we are dealing with a wide band of frequencies and therefore skip distance effects, etc., will be considerably blurred. Here methods similar to those used by Eckersley[31] in his very interesting explanation of the familiar "trolley car noise" often heard by radio men, would have to be applied.

The relatively small effect of molecular refraction, due to density and temperature gradients in the lower atmosphere, discussed by Fleming[32] and Larmor,[11] has been neglected.

It should also be kept in mind that only an approximate allowance has been made for the effect of the curvature of the ionized medium on the ray paths.

ACKNOWLEDGMENT

The writers owe a great deal to Mr. E. W. Kellogg for the many illuminating suggestions which he contributed during the preparation of the paper.

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[29] Cases where two-way communication on the same wave length will not hold, due to the effect of the earth's magnetic field, or because of an electron drift velocity, have been discussed respectively by E. V. Appleton, *Nature*, p. 382, March 7, 1925, T. L. Eckersley, *Nature*, p. 466, Sept. 26, 1925.

[30] E. V. Appleton and M. A. F. Barnett, *Proc. Roy. Soc. A*, Vol. 109, p. 621, Dec. 1925.

[31] T. L. Eckersley, A Note on Musical Atmospheric Disturbance, *Phil. Mag.*, Vol. 49, p. 1250, June, 1925.

[32] J. A. Fleming, *Proc. Phy. Soc. Lond.*, Vol. 26, p. 318, 1913-1914.

Appendix I

EFFECT OF ELECTRON COLLISIONS WITH MOLECULES ON THE REFRACTIVE INDEX OF AN IONIZED MEDIUM

Let N = number of electrons per c. c.
ν = collision frequency between an electron and molecule (per sec.)
v = average instantaneous velocity in the direction of the electric field.

The equation of motion is

$$\frac{d\,v}{d\,t} = \frac{e\,E}{m} \qquad (1)$$

where

e = electron charge
E = instantaneous value of electric field intensity in the wave
m = electron mass

If we multiply (1) by $N\,e$, we obtain the rate of change of current density in the absence of collisions

$$N\,e\,\frac{d\,v}{d\,t} = \frac{d\,i}{d\,t} = \frac{N\,e^2\,E}{m} \qquad (2)$$

The total number of collisions which occur per c. c. in the time interval $d\,t$ will be

$$N\,\nu\,d\,t \qquad (3)$$

and in the absence of an accelerating field these collisions would cause the current density to decrease in the interval $d\,t$ by the amount

$$d\,i = -\,N\,\nu\,d\,t\,e\,v \qquad (4)$$

or

$$\frac{d\,i}{d\,t} = -\,N\,\nu\,e\,v \qquad (5)$$

The net rate of change of current considering the effect of collisions will then be the sum of (2) and (5), that is

$$\frac{d\,i}{d\,t} = \frac{N\,e^2\,E}{m} - N\,\nu\,e\,v \qquad (6)$$

But the total charge movement or current density is always

$$i = N\,e\,v \qquad (7)$$

Substituting (7) in the second term on the right hand side of (6) we obtain

$$\frac{d\,i}{d\,t} = \frac{N\,e^2\,E}{m} - i\,\nu \qquad (8)$$

Transposing and rewriting (8) we obtain

$$E = \frac{m}{N\,e^2}\,\frac{d\,i}{d\,t} + \frac{m\,\nu}{N\,e^2}\,i \qquad (9)$$

If we put

$L_e = m/N\,e^2$ . . . . C. G. S. (E. S. U.) (10)
$R_e = m\,\nu/N\,e^2$ . . . . C. G. S. (E. S. U.) (11)

in (9) we have

$$E = L_e\,\frac{d\,i}{d\,t} + R_e\,i \qquad (12)$$

which is the familiar equation for a voltage impressed on an inductance and resistance in series.[33] Thus equations (10) and (11) give the effective inductance and resistance of a unit cube of space due to the presence of the electrons and molecules.

Let us now imagine an infinitely long square section of one sq. cm. area cut out of space through which a plane electromagnetic wave is being propagated.

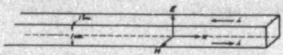

FIG. . 1

The length is taken in the direction of propagation and the sides of the square are taken parallel to the electric intensity E, and magnetic intensity H, respectively. The resulting slab or beam is pictured in Fig. 1. Here the electric intensity produces a potential difference between the top and bottom surfaces of the beam and we are therefore concerned with the capacity between these two faces per unit length of the beam. This will be $\frac{1}{4\,\pi}$ c. g. s. (e. s. u.) assuming unit dielectric constant.

If an imaginary current i, per cm. of width, flowed from right to left on the top of the slab and an equal and opposite current flowed along the bottom of the slab, we would obtain a magnetic field H in the direction shown which would be equal to $4\,\pi\,i$, if i is in c. g. s. (e. m. u.). The flux from a cm.-length of slab would be $4\,\pi\,i$ lines and therefore the inductance per unit length of slab will be $4\,\pi$ in c. g. s. (e. m. u.) since the inductance is defined as the flux linkage per unit current. The inductance per unit length in c. g. s. (e. s. u.) will then be $\frac{4\,\pi}{c^2}$, where c is the velocity of light in cm. per second.

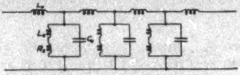

FIG. 2

We have already seen that the effect of the electron current is to shunt each cu. cm. of space by a resistance and inductance in series of the values given in equations (11) and (10). Hence we may represent the equivalent circuit of our wave beam in an ionized medium as a transmission line of the form shown in Fig. 2. The constants per unit length in c. g. s. (e. s. u.) are as follows:

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[33] The above simple deduction proposed by E. W. Kellogg replaces a rather intricate proof by the authors.

$$\text{Series Inductance } L_0 = 4\,\pi/c^2 \tag{13}$$

$$\text{Shunt Capacity } C_0 = 1/4\,\pi \tag{14}$$

$$\text{Shunt Inductance } L_e = \frac{m}{e^2\,N} \tag{15}$$

$$\text{Shunt Resistance } R_e = \frac{m\,\nu}{e^2\,N} \tag{16}$$

The propagation constant of such a line is[34]

$$P = \alpha + j\,\beta = \sqrt{\frac{\text{Series Impedance}}{\text{Shunt Impedance}}} \tag{17}$$

$$= \sqrt{\text{Series Impedance} \times \text{Shunt Admittance}} \tag{18}$$

For our line we have

$$P = \alpha + j\,\beta = \sqrt{j\,\omega\,L_0\left(\frac{1}{R_e + j\,\omega\,L_e} + j\,\omega\,C_0\right)} \tag{19}$$

We shall substitute the following expressions for convenience

$$x = L_0\,\omega \tag{20}$$

$$g = \frac{R_e}{R_e^2 + (L_e\,\omega)^2} = \text{Conductance} \tag{21}$$

$$b = \frac{L_e\,\omega}{R_e^2 + (L_e\,\omega)^2} = \text{Susceptance} \tag{22}$$

$$b_c = C_0\,\omega \qquad = \text{Susceptance} \tag{23}$$

Then

$$P = \alpha + j\,\beta = \sqrt{j\,x\,(g - j\,b + j\,b_c)} \tag{24}$$

Squaring (**24**) we have

$$\alpha^2 + 2\,j\,\alpha\,\beta - \beta^2 = j\,x\,g + x\,b - x\,b_c \tag{25}$$

Equating the real and imaginary parts we have

$$\left.\begin{aligned} \alpha^2 - \beta^2 &= x\,(b - b_c) \\ j\,2\,\alpha\,\beta &= j\,x\,g \end{aligned}\right\} \tag{26}$$

Equating the absolute value of these vectors we have

$$\sqrt{(\alpha^2 - \beta^2)^2 + (2\,\alpha\,\beta)^2} = \sqrt{x^2\,(b - b_c)^2 + x^2\,g^2} \tag{27}$$

from which

$$\alpha^2 + \beta^2 = x\,\sqrt{(b - b_c)^2 + g^2} \tag{28}$$

Adding (**26**) and (**28**) we obtain

$$\alpha = \sqrt{\frac{x}{2}\left\{\sqrt{(b - b_c)^2 + g^2} + (b - b_c)\right\}} \tag{29}$$

$$= \sqrt{\frac{x}{2}\left\{\sqrt{(b_c - b)^2 + g^2} - (b_c - b)\right\}} \tag{30}$$

Subtracting we have

$$\beta = \sqrt{\frac{x}{2}\left\{\sqrt{(b - b_c)^2 + g^2} - (b - b_c)\right\}} \tag{31}$$

$$= \sqrt{\frac{x}{2}\left\{\sqrt{(b_c - b)^2 + g^2} + (b_c - b)\right\}} \tag{32}$$

α is called the *attenuation constant per unit distance* and is the reciprocal of the distance in which the amplitude of the wave will decrease to $1/\epsilon$ of its value. β is called the *wave length constant* and is the phase change, measured in radians, in the wave per unit distance of travel. In other words, a wave length on the transmission line is $\frac{2\,\pi}{\beta}$ units long. If the frequency of the wave is f, the phase velocity or apparent velocity of propagation c' will be

$$c' = \lambda\,f = \frac{2\,\pi\,f}{\beta} = \omega/\beta \tag{33}$$

Hence the refractive index of optical theory for the ionized medium through which the light beam was drawn is

$$\mu = \frac{c}{c'} = \frac{c\,\beta}{\omega} \tag{34}$$

From equations (**20**), (**21**), (**22**), (**23**), and (**13**), (**14**), (**15**), (**16**) we have

$$x = \frac{4\,\pi\,\omega}{c^2}\ \text{c. g. s. (e. s. u.)} \tag{35}$$

$$g = \frac{e^2\,N}{m}\ \frac{\nu}{(\nu^2 + \omega^2)} \tag{36}$$

$$b = \frac{e^2\,N}{m}\ \frac{\omega}{(\nu^2 + \omega^2)} \tag{37}$$

$$b_c = \frac{\omega}{4\,\pi} \tag{38}$$

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[34] See for example J. A. Fleming, The Propagation of Electric Currents in Tel. and Tel. Conductors, D. Van Nostrand Co., or K. S. Johnson, Transmission Circuits for Telephonic Communication, D. Van Nostrand Co.

Substituting these values in (**30**) and (**32**) we obtain

$$\alpha = \frac{\omega}{c\sqrt{2}}\sqrt{\sqrt{\left[1-\frac{4\pi e^2 N}{m(\nu^2+\omega^2)}\right]^2+\left[\frac{4\pi e^2 N\nu}{m\,\omega(\nu^2+\omega^2)}\right]^2}-\left[1-\frac{4\pi e^2 N}{m(\nu^2+\omega^2)}\right]} \qquad \textbf{(39)}$$

$$\beta = \frac{\omega}{c\sqrt{2}}\sqrt{\sqrt{\left[1-\frac{4\pi e^2 N}{m(\nu^2+\omega^2)}\right]^2+\left[\frac{4\pi e^2 N\nu}{m\,\omega(\nu^2+\omega^2)}\right]^2}+\left[1-\frac{4\pi e^2 N}{m(\nu^2+\omega^2)}\right]} \qquad \textbf{(40)}$$

From equation (**34**) we obtain for the refractive index

$$\mu = \frac{1}{\sqrt{2}}\sqrt{\left[1-\frac{4\pi e^2 N}{m(\nu^2+\omega^2)}\right]+\sqrt{\left[1-\frac{4\pi e^2 N}{m(\nu^2+\omega^2)}\right]^2+\frac{\nu^2}{\omega^2}\left[\frac{4\pi e^2 N}{m(\nu^2+\omega^2)}\right]^2}} \qquad \textbf{(41)}$$

If we assume that ν is small compared with ω, we may neglect the second term under the inner radical sign of equation (**40**) and obtain the approximation

$$\beta = \frac{\omega}{c}\sqrt{1-\frac{4\pi e^2 N}{m(\nu^2+\omega^2)}} \qquad \textbf{(42)}$$

and by equation (**34**)

$$\mu = \sqrt{1-\frac{4\pi e^2 N}{m(\nu^2+\omega^2)}} \qquad \textbf{(43)}$$

For the same conditions an approximation for α may be obtained as follows,
Let

$$G = 4\pi e^2 N/m(\nu^2+\omega^2) \qquad \textbf{(44)}$$

Then

$$\alpha = \frac{\omega}{c\sqrt{2}}\sqrt{\sqrt{(1-G)^2+\left(\frac{\nu G}{\omega}\right)^2}-(1-G)} \qquad \textbf{(45)}$$

$$= \frac{\omega}{c\sqrt{2}}\sqrt{\sqrt{(1-G)^2\left\{1+\frac{\left(\frac{\nu G}{\omega}\right)^2}{(1-G)^2}\right\}}-(1-G)} \qquad \textbf{(46)}$$

$$= \frac{\omega}{c\sqrt{2}}\sqrt{(1-G)\sqrt{1+\frac{\left(\frac{\nu G}{\omega}\right)^2}{(1-G)^2}}-(1-G)} \qquad \textbf{(47)}$$

$$= \frac{\omega}{c\sqrt{2}}\sqrt{(1-G)\left\{1+\frac{1}{2}\,\frac{\left(\frac{\nu G}{\omega}\right)^2}{(1-G)^2}\right\}-(1-G)} \qquad \textbf{(48)}$$

$$\alpha = \frac{\nu G}{2c\sqrt{1-G}} \qquad \textbf{(49)}$$

Using equation (**43**) we may write it

$$\alpha = \frac{2\pi e^2 N\nu}{c\,\mu\, m(\nu^2+\omega^2)} \qquad \textbf{(50)}$$

The value given by Nichols and Schelleng,[13] page 229, for the absorption coefficient yields the following value of α, in our notation,

$$\alpha = \frac{2\pi e^2 N\nu\mu}{c\, m(\nu^2+\omega^2)} \qquad \textbf{(51)}$$

When μ is nearly equal to unity both expressions will give substantially the same result.

A further approximation may be obtained from equation (**49**) by expanding the denominator in a binomial series and taking the first term only. This gives

$$\alpha = \frac{\nu G}{2c}\left(1+\frac{1}{2}G\right) = \frac{\nu G}{2c}+\frac{\nu G^2}{4c} \qquad \textbf{(52)}$$

If now G is small compared with unity we can neglect its square and write the simpler formula

$$\alpha = \frac{\nu G}{2c} = \frac{2\pi e^2 N\nu}{c\, m(\nu^2+\omega^2)} \qquad \textbf{(53)}$$

Appendix II

GENERAL EQUATION FOR THE PATH OF A RAY IN A MEDIUM OF VARYING REFRACTIVE INDEX IN POLAR COORDINATES

Transforming equation (**27**) into the polar coordinates r and θ we write,

$$\frac{1}{R} = -\frac{1}{\mu}\frac{d\mu}{dr}\;\frac{r}{\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}} \qquad \textbf{(1)}$$

in which the sign has been reversed because of the difference in the respective conventions concerning the sign of the curvature when using rectangular and polar coordinates. For rectangular coordinates a positive curvature is such that the convexity is towards the X-axis while in polar coordinates a positive curvature has its concavity towards the origin. The general expression for the radius of curvature in polar coordinates is given in text books of the calculus[35], as

$$\frac{1}{R} = \frac{r^2+2\left(\frac{dr}{d\theta}\right)^2-r\frac{d^2r}{d\theta^2}}{\left\{r^2+\left(\frac{dr}{d\theta}\right)^2\right\}^{3/2}} \qquad \textbf{(2)}$$

Equating expressions (**1**) and (**2**) we have

$$\frac{r^2+2\left(\frac{dr}{d\theta}\right)^2-r\frac{d^2r}{d\theta^2}}{r^2+\left(\frac{dr}{d\theta}\right)^2} = -\frac{r}{\mu}\,\frac{d\mu}{dr} \qquad \textbf{(3)}$$

Multiplying both sides by $\frac{1}{r}\frac{dr}{d\theta}\,d\theta$ we obtain

$$\frac{\left(r\frac{dr}{d\theta}+\frac{2}{r}\left(\frac{dr}{d\theta}\right)^3-\frac{d^2r}{d\theta^2}\,\frac{dr}{d\theta}\right)d\theta}{r^2+\left(\frac{dr}{d\theta}\right)^2} = -\frac{d\mu}{\mu} \qquad \textbf{(4)}$$

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[35] See for example Granville: Differential and Integral Calculus, Ginn and Co., p. 162.

For purposes of integration we split up the left-hand side as follows:

$$\frac{\frac{1}{2}\left[2r\frac{dr}{d\theta}+2\frac{dr}{d\theta}\frac{d^2r}{d\theta^2}\right]d\theta}{r^2+\left(\frac{dr}{d\theta}\right)^2}$$

$$+\frac{\left[\frac{2}{r}\left(\frac{dr}{d\theta}\right)^3-2\frac{d^2r}{d\theta^2}\frac{dr}{d\theta}\right]d\theta}{r^2+\left(\frac{dr}{d\theta}\right)^2}=-\frac{d\mu}{\mu} \tag{5}$$

On dividing the numerator and denominator of the last term on the left-hand side by r^2 we have

$$\frac{\frac{1}{2}\left[2r\frac{dr}{d\theta}+2\frac{dr}{d\theta}\frac{d^2r}{d\theta^2}\right]d\theta}{r^2+\left(\frac{dr}{d\theta}\right)^2}$$

$$+\frac{\frac{2}{r^3}\left(\frac{dr}{d\theta}\right)^3 d\theta-\frac{2}{r^2}\frac{d^2r}{d\theta^2}\frac{dr}{d\theta}\,d\theta}{1+\frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2}=-\frac{d\mu}{\mu} \tag{6}$$

We may now integrate and obtain

$$\frac{1}{2}\log\left[r^2+\left(\frac{dr}{d\theta}\right)^2\right]-\log\left[1+\frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2\right]=C-\log\mu \tag{7}$$

which reduces to

$$\log r-\frac{1}{2}\log\left[1+\frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2\right]=C-\log\mu \tag{8}$$

or

$$\frac{1}{2}\log\frac{r^2}{1+\frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2}=C-\log\mu \tag{9}$$

The denominator on the left-hand side of this expression is the square of the cosecant of the angle which the ray path makes with the radius vector. At the lower boundary of the ionized medium, *i. e.*, $r = r_1$, we have $\mu = 1$ and the angle between the ray and the radius vector is $\left(\frac{\pi}{2}-\beta\right)$.

Hence

$$C=\frac{1}{2}\log\left(r_1^2\cos^2\beta\right) \tag{10}$$

$$=\log\left(r_1\cos\beta\right) \tag{11}$$

Substituting this value in (**9**) we obtain

$$\frac{r^2}{1+\frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2}=\frac{-r_1^2\cos^2\beta}{\mu^2} \tag{12}$$

Inverting and transposing we have

$$\left(\frac{dr}{d\theta}\right)^2=\frac{r^2\left[\mu^2r^2-r_1^2\cos^2\beta\right]}{r_1^2\cos^2\beta} \tag{13}$$

so that the equation for the path in polar coordinates becomes

$$d\theta=\frac{r_1\cos\beta\,dr}{r\sqrt{r^2\mu^2-r_1^2\cos^2\beta}} \tag{14}$$

and

$$\theta_2=\int_{r_1}^{r_2}\frac{r_1\cos\beta\,dr}{r\sqrt{r^2\mu^2-r_1^2\cos^2\beta}} \tag{15}$$

Appendix III

DIFFERENTIATION OF THE RANGE EQUATION WITH RESPECT TO θ FOR THE CASE OF A CURVED EARTH

The equation for the range of a ray for the case of a curved earth is given in equation (**113**) as

$$x(\theta)=2r_0(\beta-\theta)+\frac{4\omega(B-b)}{\pi\sqrt{\sigma N_0}}\cos\beta\,K(\sin\alpha) \tag{1}$$

in which

$$\cos\beta=\frac{\cos\theta}{\left(1+\frac{b}{r_0}\right)}=A\cos\theta \tag{2}$$

$$\sin\alpha=\frac{\omega}{\sqrt{\sigma N_0}}\sin\beta=C\sin\beta \tag{3}$$

Differentiating (**2**) we have

$$-\sin\beta\,d\beta=-A\sin\theta\,d\theta$$

so that

$$\frac{d\beta}{d\theta}=A\frac{\sin\theta}{\sin\beta} \tag{4}$$

Differentiating (**3**) we obtain

$$\frac{d\alpha}{d\beta}=C\frac{\cos\beta}{\cos\alpha} \tag{5}$$

Multiplying (**4**) by (**5**) we have

$$\frac{d\alpha}{d\theta}=\frac{d\beta}{d\theta}\cdot\frac{d\alpha}{d\beta}=AC\frac{\sin\theta}{\cos\alpha\tan\beta} \tag{6}$$

The differentiation of $K(\sin\alpha)$ is given by Whittaker and Watson, Modern Analysis, 3rd edition p. 521 in the form

$$\frac{dK(k)}{dk}=\frac{1}{k}\left\{\frac{E(k)}{1-k^2}-K(k)\right\} \tag{7}$$

in which $k = \sin\alpha$

from which

$$\frac{dK(\sin\alpha)}{d\alpha}=\frac{\cos\alpha}{\sin\alpha}\left\{\frac{E(\sin\alpha)}{1-\sin^2\alpha}-K(\sin\alpha)\right\}$$

$$=\cot\alpha\left\{\frac{E(\sin\alpha)}{\cos^2\alpha}-K(\sin\alpha)\right\} \tag{8}$$

In the above the new expression $E(\sin\alpha)$ denotes the complete elliptic integral of the second kind and its values will be found in Tables[36].

For convenience we shall simplify equation (**1**) by writing

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[36] For example, Jahnke and Emde, Funktionentafeln, p. 68, B. O. Pierce, Short Table of Integrals, p. 118.

$$D = 2\,r_0 \qquad (9)$$

and

$$F = \frac{4\,\omega\,(B-b)}{\pi\,\sqrt{\sigma N_0}} \qquad (10)$$

On differentiating (1) we then obtain

$$\frac{d\,x\,(\theta)}{d\,\theta} = D\left(\frac{d\,\beta}{d\,\theta} - 1\right) + F\cos\beta\,\frac{d\,K\,(\sin\alpha)}{d\,\alpha}\,\frac{d\,\alpha}{d\,\theta}$$

$$- F\sin\beta\,K\,(\sin\alpha)\,\frac{d\,\beta}{d\,\theta} \qquad (11)$$

On substituting the values found in (4), (8) and (6) we have

$$\frac{d\,x\,(\theta)}{d\,\theta} = D\left(\frac{A\sin\theta}{\sin\beta} - 1\right) + F\cos\beta\cot\alpha\,\times$$

$$\left\{\frac{E\,(\sin\alpha)}{\cos^2\alpha} - K\,(\sin\alpha)\right\}\left\{A\,C\,\frac{\sin\theta}{\cos\alpha\tan\beta}\right\}$$

$$- F\sin\beta\,K\,(\sin\alpha)\,A\,\frac{\sin\theta}{\sin\beta} \qquad (12)$$

We may simplify to

$$\frac{d\,x\,(\theta)}{d\,\theta} = D\left(\frac{A\sin\theta}{\sin\beta} - 1\right) + F\,A\,C\,\frac{\cos^2\beta\sin\theta}{\sin\alpha\sin\beta}$$

$$\left\{\frac{E\,(\sin\alpha)}{\cos^2\alpha} - K\,(\sin\alpha)\right\} - F\,A\sin\theta\,K\,(\sin\alpha) \qquad (13)$$

Substituting the value of C given by equation (3) we readily obtain

$$\frac{d\,x\,(\theta)}{d\,\theta} = D\left(\frac{A\sin\theta}{\sin\beta} - 1\right) + F\,A\sin\theta$$

$$\left[\cot^2\beta\left\{\frac{E\,(\sin\alpha)}{\cos^2\alpha} - K\,(\sin\alpha)\right\} - K\,(\sin\alpha)\right] \qquad (14)$$

$$= D\left(\frac{A\sin\theta}{\sin\beta} - 1\right) + F\,A\sin\theta$$

$$\left[\frac{E\,(\sin\alpha)}{\tan^2\beta\cos^2\alpha} - \frac{K\,(\sin\alpha)}{\sin^2\beta}\right] \qquad (15)$$

Putting back the values of A, D, and F we obtain

$$\frac{d\,x\,(\theta)}{d\,\theta} = 2\,r_0\left(\frac{\tan\theta}{\tan\beta} - 1\right) + \frac{4\,(B-b)}{\pi}\,\frac{\omega}{\sqrt{\sigma N_0}}\cos\beta\tan\theta$$

$$\left[\frac{E\,(\sin\alpha)}{\tan^2\beta\cos^2\alpha} - \frac{K\,(\sin\alpha)}{\sin^2\beta}\right] \qquad (16)$$

Putting in the value of $\frac{\omega}{\sqrt{\sigma N_0}}$ from (3) we have

$$\frac{d\,x\,(\theta)}{d\,\theta} = 2\,r_0\left(\frac{\tan\theta}{\tan\beta} - 1\right)$$

$$+ \frac{4}{\pi}(B-b)\sin\alpha\,\frac{\tan\theta}{\tan\beta}\left[\frac{E\,(\sin\alpha)}{\tan^2\beta\cos^2\alpha} - \frac{K\,(\sin\alpha)}{\sin^2\beta}\right] \qquad (17)$$

as the final expression for the rate of change of the range with respect to the initial angle θ.

Appendix IV

FOCUSING EFFECTS AT THE SKIP DISTANCE AND INSIDE OF THE TANGENT RAY

It will be observed from Fig. 25 that there is a focusing of the rays in the vicinity of the skip distance, and also where the ray which starts out tangent to the ground returns to the earth's surface. The reason for the focusing effect at the skip distance may be readily seen from the following consideration. As we increase the angle θ from zero to larger values, the range of the rays decreases rapidly at first and then more slowly until the minimum range (*i. e.*, skip distance) is reached for a certain value of θ. For higher values of θ, the range increases rapidly until the second critical angle is reached. Near the skip distance the rate of change of the range with respect to θ is very small and consequently the energy emitted between the angles θ and $(\theta + d\,\theta)$ must come down in a small zone, which means a high energy flux density. This effect is shown in Fig. 26 between the angles θ = 7° and θ = 9°.

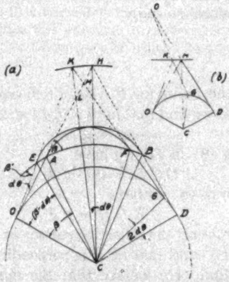

Fig. 1

The focusing effect where the tangent ray returns to earth will be made apparent from the following geometrical explanation. The tops of the ray paths are curved while below the ionized medium the paths are straight. If we produce the straight portions of any path they will meet above the summit of the path, and the result is as if sharp reflection had occurred at this point instead of a gradual bending at lower heights. Referring to Fig. 1A, let $O\,A\,B\,D$ be the actual path of a ray starting out tangent to the earth. Produce $O\,A$ and $D\,B$ to meet in H. Then H is the point at which sharp reflection may be imagined to occur. The ajdacent ray, $O\,E\,F\,G$, has the very small initial angle $d\,\theta$ with its sharp reflection point at K.

The arc length, $A\,B$, may be shown to be equal to the arc length, $E\,F$, as follows. The difference between $A\,B$ and $E\,F$ measured along the circle $E\,A\,F\,B$ is equal to $r_1\,d\,\theta/r_0$ times the second term on the right hand side of equation (**116**), and as the appropriate value of θ is zero, the difference vanishes.

Hence

$$\text{arc } A\,B = \text{arc } E\,F \tag{1}$$

and

$$\angle\, A\,C\,B = \angle\, E\,C\,F \tag{2}$$

We also have

$$\angle\, O\,C\,H = \angle\, O\,C\,A + \frac{1}{2}\,\angle\, A\,C\,B \tag{3}$$

and

$$\angle\, O\,C\,K = \angle\, O\,C\,E + \frac{1}{2}\,\angle\, E\,C\,F \tag{4}$$

Taking the difference we have

$$\angle\, K\,C\,H = \angle\, O\,C\,A - \angle\, O\,C\,E \tag{5}$$

But the angle $O\,C\,A$ is equal to the entrance angle, β, as may be readily seen by reference to Fig. 9 or 10.

We may, therefore, write (**5**) as

$$\angle\, K\,C\,H = \beta - \angle\, O\,C\,E \tag{6}$$

The angle $O\,C\,E$ is equal to

$$(\beta' - d\,\theta) \tag{7}$$

where β' is the entrance angle for the ray $O\,E\,F\,G$ and is, therefore, given by

$$\beta' = \beta + \frac{d\,\beta}{d\,\theta}\,d\,\theta \tag{8}$$

where β is the value for $\theta = 0$. From equation (**4**) of Appendix III we find that for $\theta = 0$, $d\,\beta / d\,\theta = 0$, so that

$$\beta' = \beta \tag{9}$$

Substituting (**9**) in (**7**) we have

$$\angle\, O\,C\,E = (\beta - d\,\theta) \tag{10}$$

Putting (**10**) in (**6**) we have

$$\angle\, K\,C\,H = d\,\theta \tag{11}$$

By construction the reflecting surface at H is perpendicular to $H\,C$ and that at K is perpendicular to $K\,C$. From equation (**11**) we see that the mirror at K is turned through an angle $d\,\theta$ with respect to H. If the mirror at K remained parallel to that at H as shown in Fig. 1B, a ray $O\,K$ inclined at an angle $d\,\theta$ to $O\,H$ would continue to diverge from $H\,D$ along $K\,G$ as illustrated. On the other hand, if the mirror is turned through an angle $d\,\theta$ the reflected ray will be turned through twice the angle $d\,\theta$, and therefore convert the divergence $d\,\theta$ into a convergence $d\,\theta$. Hence the ray $K\,G$ will intersect the ray $H\,D$ and therefore give a focusing effect.

The problem may be further analyzed as follows: Suppose that $K\,G$ meets $H\,D$ at D'. Let $H\,O$ meet $K\,C$ in L and $K\,G$ in M. Then in the triangles $O\,K\,L$ and $C\,H\,L$ we have the angles at 0 and C equal, also the angles at L equal, and consequently

$$\angle\, O\,K\,L = \angle\, C\,H\,L \tag{12}$$

and

$$\angle\, O\,K\,D' = \angle\, O\,H\,D' \tag{13}$$

In the triangles $O\,K\,M$ and $D'\,H\,M$ we have the angles at K and H equal and the angles at M equal, whence it follows that

$$\angle\, K\,D'\,H = \angle\, H\,O\,K \tag{14}$$

It is a well-known property of circles that the angle subtended by a given chord at any point on the circle, and on the same side of the chord, is constant. The converse is obviously true. Hence it follows that the five points K, H, D', C and O lie on one circle. Again, since the angles $K\,D\,C$ and $H\,O\,C$ are right angles, the points H, O, C and D lie on one circle. It follows that K, H, O, C, D and D' lie on one circle. The straight line $H\,D$ meets this circle in H, D and D' which can only be true if D' is the same point as D.

Thus D is a focus for rays leaving O at sufficiently small initial angles.

The point D may be regarded as the mirror image of O in a nearly spherical mirror with center at C and going through H. This image is, of course, real; that is, the rays would go through the point D if not intercepted by the earth. On the other hand, the mirror image O' in Fig. 1B is virtual. In this case the rays would diverge by the angle $d\,\theta$, appearing to come from the point O'. The focussing effect is due, therefore, to the curvature of the imaginary reflecting surface $H\,K$.

Discussion

A. E. Kennelly: The paper is very timely and interesting because so much attention has been drawn in recent months, not to say years, to the marvelous properties of very short waves. The paper makes a definite and very reasonable attempt to explain some of those properties. The direct wave dies out at a relatively short distance from the sending station and then nothing more is heard of it or received from it until it has traveled a relatively great distance. That phenomenon repeats itself at least once.

Here we are between certain rival theories of refraction and reflection from effects produced in the upper atmosphere at a distance at which we can only guess. It is very remarkable that we know so little. We are necessarily ill-informed concerning the conditions that exist in the atmosphere at a distance of, let us say, fifty kilometers above our heads, while fifty kilometers along the ground we can cover in an automobile in an hour or less.

The wonderful thing is that we are able already to form opinions, such as are expressed in the paper, as to what does take place in that region above our heads which is so near and yet so far. The promise is very brilliant that we shall be able to learn from radio observations, in the not far-distant future, the electrical properties of the atmosphere at distances from ten to 100 or more kilometers above the earth, and it will be very surprising if the information thereby gained does not have a marked influence upon weather forecasts at least in the region up to twenty or thirty kilometers above the surface.

Personally, I think it is early yet to form hard and fast opinions or to make very definite conclusions as to just what these phenomena are. We know that there must be refraction. We know that there must be rotary polarization and we also know that there must be reflection. Probably all three of these things occur simultaneously. I think we must therefore retain an open mind for the present—until more information can be secured by observation.

W. B. Kouwenhoven: The author's conclusions are similar to those of Dr. J. Zenneck who published a paper on this subject in "Elektrotechnik und Maschinenbau", Vol. 43, p. 593 and 612, 1925. Dr. Zenneck concludes from observations that radio waves enter the upper atmosphere and that these waves may come back to earth at some distant point because of refraction.

Radio transmission takes place by means of ground waves and by means of waves that pass through the upper atmosphere. In the case of short waves the ground radiation is rapidly absorbed as pointed out by the authors, while the absorption is much less for long-wave transmission.

The electric field produced by a radio wave in the upper atmosphere sets the ionized particles in motion. In the case of long waves the mean free path of the ionized particle is short compared to the wave length, and collisions occur. The energy is therefore absorbed and very little if any of the wave is refracted back to earth. In the case of short waves the absorption is much less in the upper atmosphere, and the wave is refracted and reaches the earth again at some point distant from the transmitter.

M. I. Pupin: Mr. Einstein would probably say that you are wrong when you say that the wave can be propagated by a velocity greater than the velocity of light wave. What would you answer if he did object?

W. G. Baker: I think it is one of the cases of distinction between the velocity of a group of waves and the phase velocity.

With reference to the question of reflection versus refraction raised by Dr. Kennelly, I believe that most of the people who talk about reflection from the Kennelly-Heaviside layer use it as an approximation; they do not wish to bother with the complexity introduced by refraction. Actually the transition from the neutral to the ionized medium must be gradual, and will therefore not be sufficiently abrupt to produce appreciable reflection except on the very long wave lengths. Unless the ionization changes by a large amount in a distance comparable with the wave length of the radio wave no appreciable reflection can occur, but electron refraction may easily bend the wave back to earth.

Rotary polarization and other effects due to the action of the earth's magnetic field on the electron motion will require consideration in a complete theory of the propagation of radio waves as Dr. Kennelly has pointed out. We have evaded that phase of the question as well as we could by dealing only with the case of short waves where the earth's magnetic field cannot have much effect. There is a certain resonance frequency of the electrons produced by the earth's magnetic field which corresponds to a wave length of about 214 meters. If we are working well above that frequency the effect of the earth's magnetic field on the motion of the electrons can be neglected without serious error.

We wish to thank Dr. Kouwenhoven for calling our attention to Dr. Zenneck's paper, and we are interested to hear that his conclusions concerning the propagation of short radio waves are similar to ours.

Recently there has been a good deal of surmising, on the part of radio men, as to the height of the ionized medium. Obviously we cannot speak of the height of the actual medium since we do not know where to say it begins, where the maximum is, or where it ends. The effective height of the layer as judged by a sharp reflection theory will vary, of course, with the wave length, distance between transmitter and receiver, etc. On short waves we require a relatively large amount of ionization to bend the ray back to earth. Therefore the ray will penetrate deep into the ionized medium before it returns to earth. Here the apparent height of the layer as judged on the reflection theory may be very great. On longer waves a smaller ionization density is sufficient to bend the same ray back to earth so that the depth of penetration will be less and the apparent height of the medium will be lower than that estimated from the short-wave experiments. A further difference is brought in by the effect of the earth's magnetic field.

Methods of High Quality Recording
and Reproducing of Music and Speech based on Telephone Research

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Member, A. I. E. E. Associate Member, A. I. E. E.

***Synopsis.**—The paper deals with an analysis of the general requirements of recording and reproducing sound, with the nature of the inherent limitations where mechanical records are used, and a detailed description of a solution involving, first, the use of electrical equipment for the purposes of recording and, second, the use of mechanical equipment based on electric transmission methods for reproducing.*

Probably the most useful feature of the paper is the complete description of the application of electrical transmission theory to mechanical transmission systems. A detailed analysis is made of the analogies between the electrical and the mechanical systems.

INTRODUCTION

THE problem with which this paper is concerned, in its broadest sense, may be stated as that of taking sound from the air, storing it in some permanent way and reproducing it again without appreciable distortion. It is immaterial from the general standpoint whether the means used are mechanical or electrical or a combination of the two. The choice of which method to use will depend largely upon the commercial requirements accompanying the specific purpose for which the reproduction is being made. For instance, it is quite probable that the means chosen for reproduction in residences would differ materially from those used in large ballrooms or in the presentation of synchronized motion pictures.

Before considering the methods and results referred to in the title of this paper, it may be well to make a rough division of the problem. The storing or recording of sound requires, first, a mechanical system which will respond faithfully to the sound waves which are to be recorded. Then there is required some material in or on which this sound may be recorded and an intervening system which permits the sound waves to make the record in this material. In the usual case, and in that with which we are particularly concerned here, there is a mechanical system which will vibrate in response to the sound which is to be recorded and directly through some mechanical linkage or less directly through an electrical linkage, drive a cutting mechanism which will impress a wax record.

The first consideration, therefore, is the character of the sound which is to be recorded including all of the effects of reverberation and the general questions of studio design. Next to be considered is the manner in which the cutting instrument shall impress this speech or musical record upon the constantly rotating wax disk, which disk is commonly called the wax master. In this connection, there will be discussed also the relative value of the electrical and mechanical linking of the cutting knife with the mechanism which receives the sound waves. Following the discussion of these problems and a brief reference to the state of the prior art, there remains to be considered the reproduction of the sound which is stored in the cuts or grooves of the wax record.

In the case of reproduction, also, there is required a mechanical system which will respond to these cuts in the wax and a system which will set up in the air sound waves essentially identical to those picked up by the first mechanism of the recording system. Between these two systems, a mechanical linkage intervenes in the case under discussion, but reference is made to the relative advantages of this system compared with the use of an electrical linkage.

First to be described is the character of the sound which is to be recorded and reproduced and the effects of reverberation and transients upon the listener's sensation of this sound.

STUDIO CHARACTERISTICS AND TRANSIENTS

Phonographic reproduction may be termed perfect when the components of the reproduced sound reaching the ears of the actual listener have the same relative intensity and phase relation as the sound reaching the ears of an imaginary listener to the original performance would have had. Obviously, it is very difficult, if not impossible, to fulfill all of these requirements with a single channel system, that is, with a system which does not have a separate path from each ear of the listener to the sound source.

The use of two ears, that is, two-channel listening, gives the listener a sense of direction for each of the various sources of sound to which at a given moment he may be listening, and, therefore, he apprehends them in their relative distribution in space. It has been found possible with a single channel system, however, by controlling the acoustic properties of the room in which the sound is being recorded, to simulate

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[1] Both members of the technical staff, Bell Telephone Laboratories.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

to a considerable degree in the reproduced music the effective space relationships of the original. In this case, with a one-channel system, the directional effect is, of course, entirely absent, and the spatial relationship which is apprehended is probably due to the increased apparent reverberation of the instruments situated at the far end of the room as compared with those in the near foreground.

In recording work, therefore, one of the important acoustic characteristics of a room is its time of reverberation. Although it is probable that this is the most comprehensive single factor, experiment has shown that the shape of the room and the distribution and character of the damping surfaces play a part in the excellence of music in such a room.

It has been shown by Sabine[2] that for piano music, studios should have a time of reverberation measured by his method of 1.08 seconds. Experience has indicated that this figure is also very closely correct for other types of music. This figure of Sabine's assumes binaural listening. With single-channel systems, such as most of the present reproduction systems, whether for radio or the phonograph, the ability of the listener to separate the reverberation from the direct music by means of the sense of direction is completely removed and there is thrust upon his attention an apparently excessive amount of room echo. Experiment has shown that a time of reverberation for the recording room ranging from slightly more than ½ to slightly less than ¾ of Sabine's figure affords in the reproduced music the effect of a room with proper acoustics. When this effect is accomplished, the person listening to the reproduced music has the consciousness of the music being played in a continuation of the same room in which he is listening and also has a sense of spatial depth.

Experiment has indicated further that any transients set up by the recording or reproducing system constitute a second cause of apparent increased reverberation. The data obtained thus far are insufficient to permit assignment of quantitative values to the importance of these two factors.

At the present state of the art, the most important requirement of a recording or reproducing system is its frequency characteristic. This involves two factors—intensity versus frequency, and phase distortion versus frequency. The effect of the second of these factors is not thoroughly understood but as it is closely related to the production of transients it has to be considered, as mentioned above. The system to be described is relatively free, however, from violent phase shifts within most of the range covered, but does have some undesirable phase-shift characteristics with small accompanying transients near its limiting cut-off frequencies.

FREQUENCY REQUIREMENTS

The frequency range which it would be desirable to cover, if it were possible, with relatively uniform intensity for the transmission of speech and all types of music including pipe organ is from about 16 cycles per second to approximately 10,000.

It may be interesting to examine the record requirements for a band of frequencies this great. For the purpose of this illustration, a lateral cut record will be assumed although in all the factors except the time which the record will run, the arguments apply in a similar manner to the hill-and-dale cut. Since, for mechanical reproduction, the sound at a given pitch is radiated by means of a fixed radiation resistance, it is necessary that the record must be cut with a device the square of whose velocity is proportional to the sound power. Under these conditions, it is seen that for a given intensity of sound the amplitude is inversely proportional to the frequency of the tone, and that a point will be reached somewhere at the low end of the sound spectrum where this amplitude will be great enough to cut from one groove into the adjacent groove, or in case of vertical cut, to cut so deeply that with present materials the wax will tear instead of cut away with a clean surface. This means that there is an inherent maximum amplitude beyond which it is not commercially feasible to go. Similarly the minimum radius of curvature of sine waves of various frequencies cut at constant velocity is inversely proportional to the frequency, so that as higher and higher frequencies are reached the radius of curvature becomes smaller and smaller until finally it becomes too small for the reproducing needle to follow. There is, therefore, an inherent limit at the upper end.

In order to extend these limits, it is necessary in the case of the low end to make the spiral coarser and in the case of the high end to run the record at a higher speed. Both of these changes tend to decrease the time which a record of a given size can be made to play. The only alternative of these methods is to cut the record less loud than is the present standard practise and make the reproducing equipment more sensitive. This could be done easily if it were not for the "record noise" or "surface noise," as it is commonly called. Since this surface noise is already loud enough in comparison with the reproduced music to be somewhat objectionable, no appreciable gain in this direction can be made until the technique of record manufacture has been distinctly improved.

In this connection, there is one other interesting point. It has been suggested that if electric reproduction were used, it would be possible to cut the record with a characteristic other than uniform velocity sensitiveness and correct for the error by an electrical system whose characteristic is the inverse of the characteristic of record. If the change which is made in the recording characteristic tends toward cutting at uniform acceleration sensitiveness, the amplitude varies inversely as the square of the frequency and hence the difficulties at the low end of the scale are greatly enhanced. Similarly, if the records are cut more nearly at constant amplitude, the radius of curvature of the sine waves decreases as the square of the frequency, hence the difficulties are placed at the upper end. In the process which is being described in this paper, these limitations have been met commercially by having a frequency characteristic of the uniform velocity type between the frequencies of 200 and approximately 4000 cycles per second. Below 200 it has been necessary to operate at approximately constant amplitude with a resulting loss in intensity which loss increases as the frequency decreases. Above 4000 it has been necessary to operate at approximately constant acceleration with its consequent slight loss in intensity at the very high overtones. With a characteristic of this type, a range of frequencies from 60 cycles to 6000 can be recorded with reasonable success although the very low and very high range are slightly deficient. (See Fig. 14.) With a record having such a frequency characteristic, the inherent limitations are divided between the two ends of the frequency band and where electrical reproduction methods are used, it is possible to employ a reproduction system whose frequency characteristic compensates for that of the record.

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[2] Collected papers of W. Sabine.

It should be pointed out that an attempt to record notes lower than the low cut-off of the above mentioned apparatus would result in recording only those harmonics of the notes which lie above the cut-off. This in no way prevents the listener from hearing the notes, reproduced by means of the harmonics only, as notes with the pitches of the missing fundamentals although it does somewhat change the quality of the tone.[3] If it were not for this ability of the ear to add the fundamental pitch of a note, of which only the harmonics are being reproduced, most of the older phonographs and loud speakers would have been totally useless for the reproduction of speech and music.

Mechanical Versus Electrical Recording

In attacking the recording part of the problem, two ways at once present themselves: first, the direct use of the power, of the sound being recorded, to operate the recording instrument; and second, the use of high quality electric apparatus with vacuum tube amplifiers in order to give more freedom to the artists and better control to the process. The amount of power available to operate the recorder directly from the sound in the recording room is so small as to make it extremely difficult to make records under natural conditions of speaking, singing, or instrumental playing. As the use of high quality electric apparatus with associated amplifiers has a very distinct advantage over the acoustic method, they have been adopted for the recording part of the process. Fig. 1A shows a picture of a group of artists recording by means of the sound power directly, while Fig. 1B shows a record being made by the same artists with the electric process.

It will be noticed in Fig. 1A that the artists are grouped very closely about the horn. In the case of the weaker instruments such as violins, it has been possible to use only two of standard construction. The rest of the violins are of the type known as the "Stroh" violin which is a device strung in the manner of a violin but so arranged that the bridge vibrates a diaphragm attached to a horn. This horn is directed toward the recording horn, as shown by the player in the foregound.

Fig. 1A—Picture of an Orchestra Recording for the Acoustic Process. This Picture was Furnished through the Courtesy of the Victor Talking Machine Company, Camden, New Jersey

Fig. 1B—Picture of the Same Orchestra Shown in Fig. 1A, but Recording for the Electric Process. This Picture was Furnished through the Courtesy of the Victor Talking Machine Company, Camden, New Jersey

With such an arrangement of musicians, it is very difficult to arouse the spontaneous enthusiasm which is necessary for the production of really artistic music. In Fig. 1B the musicians are sitting at ease more nearly in their usual arrangement and all are using the instruments which they would use were they playing at a concert. Furthermore, the microphone is now sufficiently far away from the orchestra to receive the sound in much the manner that the ears of a listener in the audience would receive it. In other words, it picks up the sound after it has been properly blended with the reflections from the walls of the room. It is in this way that the so-called "atmosphere" or "room-tone" has been obtained.

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[3] Physical Criterion for Determining the Pitch of a Musical Tone, H. Fletcher, *Phys. Rev.*, Vol. 23, No. 3, March, 1924.

In the old process, it sometimes happened that after the instruments had been arranged in such a manner that the relative loudness of the various parts had been balanced correctly, it was found that the whole selection was either too loud or too weak. This usually meant a complete rearrangement of the players. With the flexibility introduced by the use of electrical apparatus including amplifiers, the control of loudness is obtained by simple manipulation of the amplifier system and is in no way related to the difficulties of the relative loudness of one instrument to another. The only problem for the studio director in this case is to obtain the proper balance among the various musical instruments and artists. The advantages derived from this added ease of control are also made manifest in that it is much easier and less tiresome for the artists and it is usually possible to make more records in a given time.

MECHANICAL VERSUS ELECTRICAL REPRODUCING

Where the question of reproduction is concerned, the same two alternatives mentioned for recording present themselves, namely, direct use of power derived from the record itself versus the use of electromechanical equipment with an amplifier. In this case, however, the situation is a little different as the power which can be drawn directly from the record is more than sufficient for home use. Since any method of reproducing from mechanical records by electrical means involves the use of a mechanical device for transforming from mechanical to electrical power and a second such device for transforming from electrical back to mechanical power, that is, sound, it is necessary to use two mechanical systems, one at each end of an electrical system. Where the power which can be supplied by the record is sufficient to produce the necessary sound intensity, as in the case of home use, it is in general simpler to design one single mechanical transmission system than it is to add the unnecessary complications of amplifiers, power supply and associated circuits. In cases where music is to be reproduced in large auditoriums, the power which can be drawn from the record may be insufficient and some form of electric reproduction using amplifiers becomes necessary.

BRIEF DESCRIPTION OF RECORDING SYSTEM

The system used for recording consists of a condenser transmitter, a high quality vacuum tube amplifier and an electromagnetic recorder. Fig. 2 shows the calibration of the condenser transmitter and the associated amplifiers. The condenser transmitter and amplifiers are so designed that the current delivered to the recorder circuit is essentially proportional to the sound pressure at the transmitter diaphragm. The electromagnetic recorder, which will be described later, is designed to work with this type of system. With the exception of this electromagnetic recorder, apparatus of this type has already been described in the literature.[4] In addition to this equipment which might be called the recording amplifier system, there is a volume indicator for measuring the power which is being delivered to the recorder and also an audible monitoring system. The audible monitoring system consists of an amplifier whose input impedance is high compared with the recorder impedance and a suitable loud speaking receiver. The monitoring amplifier is bridged directly across the recorder and operates the loud speaking receiver so that the operator may listen to the record as it is being made.

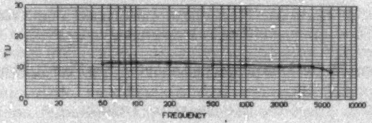

FIG. 2—CALIBRATION OF THE CONDENSER TRANSMITTER AND ASSOCIATED AMPLIFIERS

This curve shows merely the relative frequency sensitiveness of the system, the zero line having been chosen arbitrarily

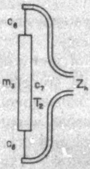

FIG. 3—SCHEMATIC MECHANICAL ARRANGEMENT OF DIAPHRAM AND AIR CHAMBER

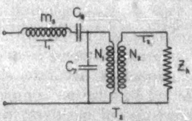

FIG 4.—ELECTRICAL EQUIVALENT OF MECHANICAL SYSTEM SHOWN IN FIG. 3

In the design of the recording and reproducing systems each part of the system has been made as nearly perfect as possible. Errors of one part have not been designed to compensate for inverse errors in another part. Although this method is the more difficult, its flexibility, particularly as regards the commercial possibilities of future improvements, justifies the extra effort.[5] There is, therefore, no distortion in the record whose purpose is to compensate for errors in the reproducing equipment; the only intended distortion in the record being that required by the inherent limitations mentioned above. See Figs. 2, 14 and 20.

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[4] Wente, E. C., "Condenser Transmitter as a Uniformly Sensitive Instrument for Measuring Sound Intensity," *Phys. Rev.*, Vol. 10, 1917.

Crandall, I. B., "Air-Damped Vibrating Systems," *Phys. Rev.*, Vol. 11, 1918.

Wente, E. C., "Electrostatic Transmitter," *Phys. Rev.*, Vol. 19, 1922.

Martin, W. H. and Fletcher, H., "High Quality Transmission and Reproduction of Speech and Music," TRANS. A. I. E. E., Vol. 43, 1924, p. 384.

Green, I. W. and Maxfield, J. P., "Public Address Systems," TRANS. A. I. E. E., Vol. 43, 1923, p. 64.

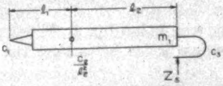

Fig. 5—Schematic Mechanical Arrangement of Needle Arm Transformer

General Basis of Design

An interesting feature of the development of the mechanical and electromechanical portions of the recording and reproducing system is their quantitative design as mechanical analogs of electric circuits. Both the recording and reproducing systems are good examples of the use of this type of analogy.

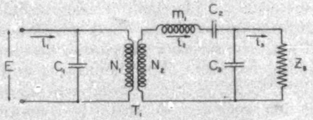

Fig. 6—Electrical Equivalent of System Shown in Fig. 5 with Its Termination

The economic need for the solution of many of the problems connected with electric wave transmission over long distances coupled with the consequent development of accurate electric measuring apparatus has led to a rather complete theoretical and practical knowledge of electrical wave transmission. The advance has been so great that the knowledge of electric systems has surpassed our previous engineering knowledge of mechanical wave transmission systems. The result is, therefore, that mechanical transmission systems can be designed more successfully if they are viewed as analogs of electric circuits.

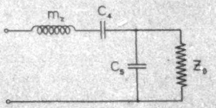

Fig. 7—Electrical Equivalent of the Spider Section

While there are mechanical analogs for nearly every form of electrical circuit imaginable, there is one particular class of electrical circuits whose study has led to ideas of the utmost value in guiding the course of the present development. This class of circuits consists of infinitely repeated similar sections of one or more lumped capacity and inductance elements in series and shunt and are commonly known as filters. The study of filters began with the work of Campbell[6] and a recognition of their importance as frequency selective systems in telephone repeaters, carrier systems, radio, signalling systems, etc., led to their intensive study. In the available literature is to be found a fairly complete statement of their properties and details of their design.[6]

It will be recalled in the case of the telephone circuit that the introduction of inductance coils at regular intervals in the circuit produced a remarkable change in the transmission characteristic. Over a broad band of frequencies the attenuation was reduced and made fairly uniform over that range while beyond a critical frequency called the cut-off frequency the attenuation became very high. In the ideal filters with zero dissipation the transmission characteristics are of the same nature but more clear cut. Structures of this type with infinitely repeated sections will have one or more transmission bands of zero attenuation and one or more bands having infinite attenuation. The impedance characteristics of such a structure measured from certain characteristic points will be pure resistance more or less uniform in the transmission bands, and pure reactance in the attenuation bands. These terminations are

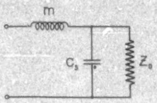

Fig. 8—Electrical Equivalent of Simple Low Pass Type of Network which Occurs Frequently in this Work

mid-series; that is, the entering element being a series one of half the normal series element; or mid-shunt; that is, the entering element being twice the impedance of the normal shunt element. The corresponding impedances are called the mid-series and mid-shunt characteristic or iterative impedances.

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[5] Green, I. W. and Maxfield, J. P., "Public Address Systems," Trans. A. I. E. E., Vol. 42, 1923, p. 64.

[6] Campbell, G. A., "On Loaded Lines in Telephonic Transmission," *Phil. Mag.*, March 1903.

Campbell, G. A., U. S. Patents 1,227,113; 1,227,114; "Physical Theory of the Electric Wave Filter," *Bell System Technical Journal*, November 1922.

Zobel, O. J., "Theory and Design of Uniform and Composite Electric Wave Filters," *Bell System Technical Journal*, January 1923.

Peters, L. J., "Theory of Electric Wave Filters Built up of Coupled Circuit Elements," Trans. A. I. E. E., May 1923.

Carson, J. R. and Zobel, O. J., "Transient Oscillations in Electric Wave Filters," *Bell System Technical Journal*, July 1923.

Zobel, O. J., "Transmission Characteristics of Electric Wave Filters," *Bell System Technical Journal*, October 1924.

Johnson, K. S., and Shea, T. E., "Mutual Inductance in Wave Filters with an Introduction on Filter Design," *Bell System Technical Journal*, January 1925.

Johnson, K. S., "Transmission Circuits for Telephonic Communication," D. Van Nostrand, 1925.

If we retain the first few sections of such a structure and terminate them with a resistance which is equal to the resistance impedance of the infinite line from which they were taken, the characteristics are substantially unchanged. It is understood, of course, that this resistance equals approximately the resistance impedance of the remainder of the infinite line at most of the frequencies in the transmission band in which we are interested.

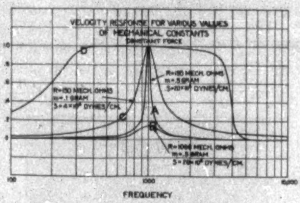

FIG. 9—VELOCITY RESPONSE FOR VARIOUS VALUES OF MECHANICAL CONSTANTS

The presence of small amounts of damping in the various elements also has but slight effect on the general characteristics. These results could in general be readily applied to the various telephone transmission problems because the source and load between which the filter system was inserted generally had or could be made to have a nearly resistance impedance equalling the mid-series or mid-shunt impedance of the filter within the transmission band. The filter and terminating impedances may then be said to be matched. Where adjacent sections in the filter have impedances similar in character but different in absolute magnitude they may be joined by a suitable transformer.

Many early attempts were made to design mechanical transmission systems having a wide frequency range in which highly damped single or multi-resonant systems were employed. In these attempts both of the obvious methods of increasing the damping were used, namely, that of adding a resistance to the system and that of increasing the value of the compliance and decreasing mass in such proportion as to maintain the same natural frequency. The former of these methods reduces the sensitivity of the system at the point where it is most efficient, (See Fig. 9), while the second method increases the response at the points where the system is less sensitive, namely, away from its resonance point. Fig. 9 shows four curves—first, a singly resonant system, Curve *A*; second, the same system with friction added, Curve *B*; third, the same system without the added friction but with an increase in compliance and a decrease in mass such that the natural period remains the same, Curve *C*; and fourth, a band pass type of circuit whose resistance impedance is the same as that of the system shown in Curve *A*. (See Curve *D*.)

The results of filter theory have shown how these resonances should be coordinated so that when a proper resistance termination is used high efficiency and equal sensitivity are obtained over a definite band of frequencies by elimination of response to all frequencies outside the band. With the electrical case of a repeated filter, each section considered by itself resonates at the same frequency but when combined into a short-circuited filter of *n* sections, there will be *n* natural frequencies. However, when such a system is terminated with a resistance which equals the nominal characteristic impedance in the transmission band, uniform response in the terminating resistance is obtained over the entire band.

DETAILED ANALYSIS OF MECHANICAL AND ELECTRICAL ANALOGS[7]

Before going on with a detailed treatment of the electrical analogues of the mechanical structures used in the problem of phonographic reproduction, a list of the corresponding quantities used in the two systems will be given, together with the symbols employed.

Mechanical		Electrical	
Force	= F (dynes)	Voltage	= E (volts)
Velocity	= v (cm./sec.)	Current	= i (amperes)
Displacement	= s (cm.)	Charge	= q (coulombs)
Impedance	= z (dyne sec./cm. or mechanical ohms)	Impedance	= Z (ohms)
Resistance	= r (dyne sec./cm.)	Resistance	= R (ohms)
Reactance	= x (dyne sec./cm.)	Reactance	= X (ohms)
Mass	= m (gms)	Inductance	= L (henries)
Compliance	= c (cm./dyne)[8]	Capacity	= C (farads)

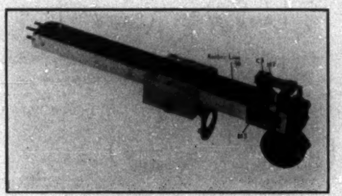

FIG. 10—THIS FIGURE SHOWS AN ELECTROMAGNETIC RECORDER COMPLETE EXCEPT FOR THE BOTTOM OF THE CASE

In addition to the above certain other quantities such as angular displacement, pressure and impedance per unit area, and a few others which have no direct electrical analog will be used. These quantities, however, are either standard in the literature or may always be reduced to those given above.

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[7] The authors wish to express their appreciation to Mr. E. L. Norton for his courtesy in working out the mathematics of the mechanical and electrical analogs which are shown in this paper.

[8] H. W. Nichols, "Theory of Variable Dynamical Electrical Systems," *Phys. Rev.* Vol. 10, 1917.

As illustrations of the general methods employed certain important portions of the reproducer will be considered in detail. Considering first the electrical analog of the air chamber[9] between the diaphragm and horn, we make use of the following list of Symbols (see Figs. 3, 4, 15 and 16).

m_3 = Effective mass of diaphragm in grams
A_1 = Equivalent area of diaphragm in cms^2
c_6 = Compliance of edge of diaphragm
A_2 = Area of throat of horn
z_h = Impedance of horn—Vector ratio of applied force at the throat of the horn to the resultant linear velocity of the air
s_1 = Displacement of diaphragm
v_1 = Velocity of diaphragm
s_2 = Displacement of air in throat of horn
v_2 = Velocity of air in throat of horn
P_0 and V_0 = Initial pressure and volume of air-chamber
F = Force applied to diaphragm
p = Small change of pressure in air-chamber.

For a small change p in the pressure within the air-chamber we have:

$$p = \frac{n\,(A_1 s_1 - A_2 s_2)\,P_0}{V_0} \qquad (1)$$

where $n = 1$ for an isothermal change and 1.4 for an adiabatic change. For the case under consideration $n = 1.4$ very nearly.

If the horn opening is closed, $S_2 = O$, and we get for the compliance of the air chamber as measured from the diaphragm

$$c_7 = \frac{S_1}{p\,A_1} = \frac{V_0}{n\,p_0\,A_1^2}.$$

We have the two force equations

$$m_3 \frac{d\,v_1}{d\,t} + \frac{s_1}{c_6} + p\,A_1 = F \qquad (2)$$

$$z_h\,v_2 - p\,A_2 = 0 \qquad (3)$$

or substituting the values of p and c_7

$$m_3 \frac{d\,v_1}{d\,t} + \frac{s_1}{c_6} + \frac{1}{c_7}\left[s_1 - \left(\frac{A_2}{A_1}\right)s_2\right] = F \qquad (4)$$

$$z_h\,v_2 + \frac{1}{c_7}\left[\left(\frac{A_2}{A_1}\right)^2 s_2 - \left(\frac{A_2}{A_1}\right)s_1\right] = 0 \qquad (5)$$

Considering now the analogous electrical circuit, and assuming the velocity, current, force and voltage to vary sinusoidally, we have the parallel relationship for the steady state conditions:

Mechanical case (Fig. 3)

$$m_3 \frac{d\,v_1}{d\,t} + \frac{s_1}{c_6} + \frac{1}{c_7}\left[s_1 - \left(\frac{A_2}{A_1}\right)s_2\right] = F$$

$$z_h\,v_2 + \frac{1}{c_7}\left[\left(\frac{A_2}{A_1}\right)^2 s_2 - \left(\frac{A_2}{A_1}\right)s_1\right] = 0$$

If $v_1 = j\,\omega\,s_1$, etc.
$z_1\,v_1 - z_m\,v_2 = F$
$z_2\,v_2 - z_m\,v_1 = 0$
where:

$$z_1 = j\left(\omega\,m_3 - \frac{1}{\omega\,c_6} - \frac{1}{\omega\,c_7}\right)$$

$$z_2 = \left[z_h - j\left(\frac{A_2}{A_1}\right)^2 \frac{1}{\omega\,c_7}\right]$$

$$z_m = -j\left(\frac{A_2}{A_1}\right)\frac{1}{\omega\,c_7}$$

Electrical case with ideal transformer of turns ratio $\frac{N_2}{N_1}$ (Fig. 4.)

$$L_3 \frac{d\,i_1}{d\,t} + \frac{q_1}{C_6} + \frac{1}{C_7}\left[q_1 - \left(\frac{N_2}{N_1}\right)q_2\right] = E$$

$$Z_h\,i_2 + \frac{1}{C_7}\left[\left(\frac{N_2}{N_1}\right)^2 q_2 - \left(\frac{N_2}{N_1}\right)q_1\right] = 0$$

If $i_1 = j\,\omega\,q_1$, etc.
$Z_1\,i_1 - Z_m\,i_2 = E$
$Z_2\,i_2 - Z_m\,i_1 = 0$
where:

$$Z_1 = j\left(\omega\,L_3 - \frac{1}{\omega\,C_6} - \frac{1}{\omega\,C_7}\right)$$

$$Z_2 = \left[Z_h - j\left(\frac{N_2}{N_1}\right)^2 \frac{1}{\omega\,C_7}\right]$$

$$Z_m = -j\left(\frac{N_2}{N_1}\right)\frac{1}{\omega\,C_7}$$

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[9] The use of the air chamber to increase the loading effect of the horn on the diaphragm has been appreciated for a number of years. It has been used in telephone receivers, phonographs, and loud speaking receivers since their earliest developments. A treatment of the force equations of the air-chamber was given by Hanna & Slepian, "The Function and Design of Horns for Loud Speakers," TRANS. A. I. E. E., 1924, p. 393. The equivalent structure, however, was analysed as a compliance and resistance in series instead of in shunt.

The last five equations in each case give the complete solution of the network. By analogy between the two sets of equations, therefore, the air-chamber corresponding to the shunt capacity C_7 is spoken of as a shunt compliance, $c_7 = \frac{V_0}{n\,P_0\,A_1^2}$, together with a transformer inserted before the horn, which has an equivalent turns ratio equal to the ratio of the areas of the diaphragm and horn openings.

Taking up now the somewhat different illustration of the needle arm, the following symbols are needed (Figs. 5, 6, 15, 16).

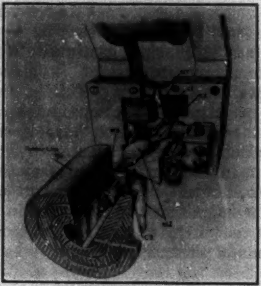

FIG. 11—DETAILED DRAWING OF THE MECHANICAL FILTER OF AN ELECTROMAGNETIC RECORDER. (LETTERING SAME AS IN FIG. 12)

List of Symbols:

l_1 = Distance from pivot point to end of needle
l_2 = Distance from pivot point to center of "spider" (Fig. 15)
I = Moment of inertia of needle arm
m_1 = Apparent or equivalent mass of arm as measured from the center of the spider $= \frac{I}{l_2^2}$
c_1 = Compliance of needle point
c_2 = Compliance of bearing to turning of the needle arm, as measured from end of arm at the spider
c_3 = Compliance of end of needle arm attached to spider
s_1 = Displacement of tip of needle
s_2 = Displacement of end of arm at the spider
s_3 = Displacement of spider
z_s = Mechanical impedance of spider and remainder of structure = Vector ratio of applied force to resultant velocity
θ = Angular displacement of needle arm
F = Applied force at needle point

We have the three force equations:

$$\frac{s_1 - l_1\,\theta}{c_1} = F \tag{6}$$

$$I\,\frac{d^2\theta}{d\,t^2} + \frac{(l_1\,\theta - s_1)\,l_1}{c_1} + \frac{\theta\,l_2^2}{c_2} + \frac{(l_2\,\theta - s_3)\,l_2}{c_3} = 0 \tag{7}$$

$$\frac{s_3 - l_2\,\theta}{c_3} + z_s\,\frac{d\,s_3}{d\,t} = 0 \tag{8}$$

Replacing θ by $\frac{s_2}{l_2}$ and I by $m_1\,l_2^2$ gives:

$$\frac{s_1 - \frac{l_1}{l_2}\,s_2}{c_1} = F \tag{9}$$

$$m_1\,\frac{d^2 s_2}{d\,t^2} + s_2\left[\left(\frac{l_1}{l_2}\right)^2 \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}\right] - \frac{l_1}{l_2}\,\frac{s_1}{c_1} - \frac{s_3}{c_3} = 0 \tag{10}$$

$$\frac{s_3 - s_2}{c_3} + z_s\,\frac{d\,s_3}{d\,t} = 0 \tag{11}$$

Considering now the analogous electrical circuit; assuming as before sine functions for v, i, F, and E, we have:

Mechanical case substituting $v_1 = j\,\omega\,s_1$, etc., in the last equations:

$$-j\,\frac{v_1}{\omega\,c_1} + j\,\frac{l_1}{l_2}\,\frac{v_2}{\omega\,c_1} = F$$

$$j\,v_2\left[\omega\,m_1 - \left(\frac{l_1}{l_2}\right)^2 \frac{1}{\omega\,c_1} - \frac{1}{\omega\,c_2} - \frac{1}{\omega\,c_3}\right]$$

$$+ j\,\frac{l_1}{l_2}\,\frac{v_1}{\omega\,c_1} + j\,\frac{v_3}{\omega\,c_3} = 0$$

$$+ j\,\frac{v_2}{\omega\,c_3} + v_3\left(z_s - j\,\frac{1}{\omega\,c_3}\right) = 0$$

Electrical case with ideal transformer of turns ratio $\frac{N_2}{N_1}$

$$-j\,\frac{i_1}{\omega\,C_1} + j\left(\frac{N_2}{N_1}\right)\frac{i_2}{\omega\,C_1} = E$$

$$j\,i_2\left[\omega\,L_1 - \left(\frac{N_2}{N_1}\right)^2 \frac{1}{\omega\,C_1} - \frac{1}{\omega\,C_2} - \frac{1}{\omega\,C_3}\right]$$

$$+ j\left(\frac{N_2}{N_1}\right)\frac{i_1}{\omega\,C_1} + j\,\frac{i_3}{\omega\,C_3} = 0$$

$$j\,\frac{i_2}{\omega\,C_3} + i_3\left(Z_s - j\,\frac{1}{\omega\,C_3}\right) = 0$$

The analogy between the two sets of equations is quite obvious. It will be noticed that the effect of the lever arm is to introduce an equivalent transformer of a turn ratio which is the reciprocal of the corresponding lengths of the arms.

The general method of deducing the equivalent electric circuits should be clear from the above illustrations of the air-chamber and of the needle arm. For example, in the spider section, Fig. 15, the mass is driven directly by the force from the needle-arm compliance, there being a small series compliance in the connection owing to bending of connecting rod. The diaphragm is connected through the compliance of the prongs of the spider. The equivalent circuits are shown in Figs. 7 and 16.

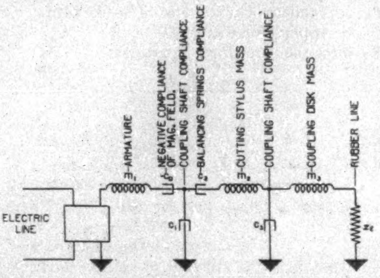
m_1-ARMATURE
$-c_0$-NEGATIVE COMPLIANCE OF MAG. FIELD.
COUPLING SHAFT COMPLIANCE
c_2-BALANCING SPRINGS COMPLIANCE
m_2-CUTTING STYLUS MASS
COUPLING SHAFT COMPLIANCE
m_3-COUPLING DISK MASS
RUBBER LINE
ELECTRIC LINE
c_1
c_3
z_t

Fig. 12—Equivalent Electric Circuit of the Electromagnetic Recorder

The equations of this network may be obtained from the equations for the needle arm by placing c_1 equal to zero, taking a unity ratio transformer, and substituting m_2 for m_1, c_4 for c_2, c_5 for c_3 and z_d for z_s.

Another type of network which occurs frequently in the building of mechanical vibrating systems is represented diagrammatically in Fig. 8. This is clearly a particular case of Fig. 7 with c_4 made infinite.

By combining Fig. 6 representing the needle arm, Fig. 7 representing the spider section and Fig. 4 representing the diaphragm, air-chamber and horn, the complete reproducer may be built up. The resultant network is shown in Fig. 16. Since methods are available in the theory of electric wave filters to determine the proper values of the elements of the complete network for a free transfer of energy throughout an assigned frequency band, the analogous mechanical elements may be determined in the same manner.

General Design of Mechanical Systems

In designing mechanical systems of the band pass type, the problem is three-fold—first, that of arranging the masses and compliances such that they form repeated filter sections; second, determining the magnitude of these quantities so that with or without transformers the separate sections all have the same cut-off frequencies* and characteristic impedances; third, to provide the proper resistance termination. Where the transmitted mechanical power has not been radiated as sound this third part has been one of the most difficult to fulfill.

In designing these systems, practical difficulties arose—first, the difficulty of insuring that the parts vibrated in the desired degrees of freedom only, and second, the difficulty of determining the magnitudes of the various effective masses, compliances and resistances. Before the work to be described could be carried out practically it became necessary to develop a method of measuring mechanical impedances[10]. Such a method has been developed which at the present time covers a range of frequencies from somewhere below 50 to about 4500 pps. Work is still being continued to extend the method to higher frequencies. This method of measurement has been very useful not only in determining the magnitudes of the impedances in the degrees of freedom in which it is desired that they shall operate, but in determining the impedances to motion of the various parts in directions in which they should not be permitted to vibrate. In connection with the measurement of the magnitudes of the parts in the desired degrees of freedom this method enables us to determine the constants of the mechanical networks under their conditions of operation. Experience so far has indicated that when all the degrees of freedom have been taken into account and when the dynamic axes of vibration have been properly chosen, the static and

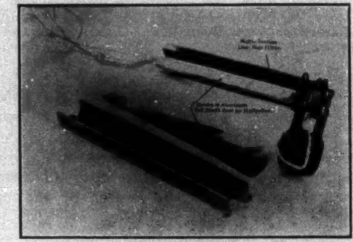

Fig. 13—Electromagnetic Recorder using Lumped Loaded Termination

The method of furnishing dissipation to the lumped loaded line is shown.

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*It is of course permissible to have a section having a higher cut-off than the others provided its characteristic impedance is the same as that of the others over the transmission band of those having the lower cut-off.

[10] Kennelly, A. E. and Affel, H. A., "The Mechanics of Telephone Receiver Diaphragms, as Derived from their Motional Impedance Circles," *Proc.* A. A. A. S., Vol. 51, No. 8, November, 1915.

Kennelly, A. E. and Pierce, G. W., "The Impedance of Telephone Receivers as Affected by the Motion of their Diaphragms," *Proc.* A. A. A. S., Vol. 48, No. 6, September, 1912.

dynamic constants of the parts are the same, and it is then possible to check the parts by simple static measurements. In the early attempts to build these systems very large discrepancies between the static and dynamic characteristics were found.

The Recorder

One of the early practical phonographic applications of electric filter design to mechanical problems was the development of an electromagnetic recorder. The instrument as finally constructed is essentially a properly terminated three-section mechanical filter in which the recording stylus and its holder constitute the series mass in the second section. Since a filter of this type appears at its input end as approximately a pure resistance within the transmission band, the current in the series inductances, that is, in the mechanical case, the velocity of the series masses is proportional to the driving force.

Figs. 10, 11 and 12 show respectively, a complete recorder, a drawing of the mechanical filter of such a recorder and a diagram of the equivalent electric circuit. The armature acts as the series mass m_1 in the first section; the magnetic field as the series negative compliance $-c_0$; the shaft between the armature and the stylus holder as the shunt compliance c_1; the balancing springs as the series compliance c_2; the stylus holder and the stylus as the series mass m_2; the shaft between the stylus holder and the disk, coupling the system to the terminating resistance, as the compliance c_3; the coupling disk as the series mass m_3 and the terminating line as approximately a mechanical resistance.

All of these equivalents are seen from the simple analogues previously outlined with the exception of the terminating resistance and the negative compliance $-c_0$. The terminating resistance was originally made up of a series of filter sections of lumped series masses and shunt compliances with a small amount of damping added to the motion of each of the series masses. Fig. 13 shows one of the early recorders equipped with this type of resistance termination. The reason for using such a complicated termination lies in the fact that most of the known mechanical resistances have values which are functions of frequency or of amplitude or both. Also in most cases, the mechanical resistance is accompanied by either a mass or compliance reactance. By using a multi-section filter which is sufficiently long so that a wave entering it will be essentially absorbed before it has reached the far end, been reflected and returned to the entering end, it has been possible to use imperfect types of damping for this line and still obtain over the desired band, an essentially pure resistance at the input end.

More recently a continuous line has been developed which is much easier of practical attainment than the complicated lump-loaded filter. The recorder shown in Fig. 10 is so equipped. Fig. 14 shows calibration curves of three types of recorders.

The compliance $-c_0$ is a mechanical quantity for which there is no simple electric analog. In a balanced armature type of structure such as that shown in Fig. 11, the action of the field on the armature, when it is at its center point, is balanced. If, however, the armature be deflected a small distance from this equilibrium, there is exerted by the magnetic field a torque tending to pull the armature further away from its center position. The value of this torque for small amplitudes is proportional to the angular displacement. It is seen therefore that this quantity is of the nature of a compliance but that the back force is in a reverse direction to that required for a positive compliance.

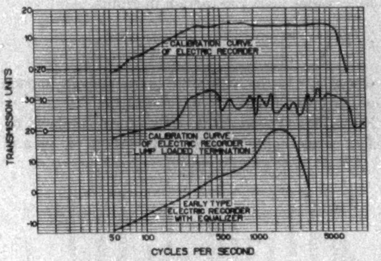

Fig. 14—Calibration Curve of Three Types of Electromagnetic Recorders

Design of the Reproducing Apparatus

As the analogy between the mechanical and electrical filter is more perfectly shown in the case of the reproducing equipment, the detailed quantitative description will be given in this connection. Figs. 15 and 16 show respectively a diagram of the reproducing system and its equivalent electric circuit. From these diagrams it is evident which units in the mechanical system correspond to the various electrical parts. As the series compliances c_2, c_4 and c_6 have been made so large that the low frequency cut-off caused by them lies well below the low frequency cut-off of the horn, an inappreciable error is introduced in using for design

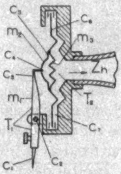

Fig. 15—Diagrammatic Sketch of the Mechanical System of the Phonograph

purposes formulas of low pass filters[11]. The two formulas which will be used are as follows:

$$f_c = \frac{1}{\pi}\sqrt{\frac{1}{m\,c}} \qquad (12)$$

where

f_c = cut-off frequency of a lumped transmission system in cycles per second

c = shunt compliance per section in centimeters per dynes

m = series mass per section in grams

$$z_0 = \sqrt{\frac{m}{c}} \qquad (13)$$

where z_0[12] is the value of characteristic impedance over the greater part of the band range.

Equations (**12**) and (**13**) which form the basis of the design work contain four variables, f_c, c, m and z_0. It is, therefore, necessary to determine two of them by the physical requirements of the problem after which the other two are determined. The upper cut-off frequency f_c was arbitrarily chosen at 5000 pps. as a compromise between the highest frequency occurring on the record and the increase in surface noise as the cut-off is raised. The choice of the other arbitrarily set variable came after considerable preliminary experimenting and was fixed by the difficulty of obtaining a diaphragm which is light enough and has a large enough area. Hence the effective mass of the diaphragm m_3, (Figs. 15-16) was fixed at 0.186 grams which value can be obtained by careful design. The effective area can be made as large as 13 square centimeters. For convenience let the arbitrary value chosen for $f_c = \bar{f}_c$ and the value of $m = \bar{m}_3$.

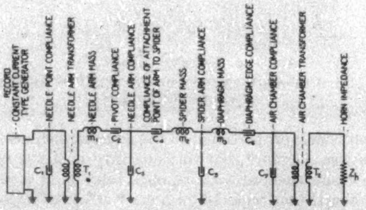

FIG. 16—ELECTRIC EQUIVALENT OF THE SYSTEM SHOWN IN FIG. 15

Solving Equations (**12**) and (**13**) for c and z_0, we get

$$c = \frac{1}{\pi^2\,\bar{f}_c^{\,2}\,\bar{m}_3} \qquad (14)$$

$$z_0 = \pi\,\bar{f}_c\,\bar{m}_3 \qquad (15)$$

also

$$z_0 = \frac{1}{\pi\,c\,\bar{f}_c} \qquad (16)$$

In order to obtain the low value of mass mentioned, with a large enough area, it was necessary to make the diaphragm of a very stiff light material. An aluminum alloy sheet 0.0017 in. thick was chosen and concentrically corrugated as shown in Figs. 17 and 18. These corrugations are spaced sufficiently close so that the natural periods of the flat surfaces are all above $\bar{f}_c$. To insure that this central stiffened portion should vibrate with approximate plunger action, which is more efficient than diaphragm action, it is driven at six points near its periphery.

Reference to Figs. 15 and 16 and Equation (**14**) shows that the compliance of the air chamber c_7, of the spider legs c_5 and shunt tip of the needle arm c_3 are determined. Also the mass of the spider m_2 and the effective mass of the needle arm m_1, as viewed at the point where it is attached to the spider, are determined.

The impedance looking into the system from the record is determined by the rate at which it is necessary to radiate energy in order that the reproduction may be loud enough. The power taken from the record is approximately $v^2\,z_0$ since z_0 is a resistance over most of the band. Experiment has shown this value of z_0 to be approximately 4500 mechanical ohms.

But substituting in Equation (**13**) the value of $\bar{m}_3$, and from Equation (**14**) the value of c_5, we find that the impedance is only 2920 mechanical ohms. It is, therefore, necessary to use a transformer whose impedance ratio is $\frac{4500}{2920}$. From this and a knowledge of filter structures the needle-point compliance can be determined. The value obtained is easily realized with commercial types of needle.

It will be noted that the record is shown in Fig. 16 as a constant current generator, *i. e.*, a generator whose impedance appears high as viewed from the needle point. That this is necessary is obvious when it is remembered that, if the impedance looking back into the record were to equal the impedance of the filter system, the walls of the record would have to yield an amount comparable with one-half the amplitude of the lateral cut. This would cause a breakdown of the record material with consequent damage.

The design of the system is complete, therefore, except for the resistance termination which is supplied

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[11] Campbell, G. A., "On Loaded Lines in Telephonic Transmission," *Phil. Mag.*, March, 1903.

[12] z_0 may be called nominal mid-shunt or mid-series impedance. Their actual values in the transmission band being at any frequency f.,

$$\text{mid-series} = z_0\sqrt{1-\left(\frac{f}{f}\right)^2}$$

$$\text{mid-shunt} \quad \frac{z_0}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

by the horn for all frequencies above its low frequency cut-off. The characteristics of the horn will be dealt with later. The resistance within the band looking in at the small end of the horn is $G\,A_2$ where G = mechanical ohms per square centimeter of an infinite cylindrical tube of the same area, and A_2 = area in square centimeters of the small end of the horn.

Let A_1 = the effective plunger area of the diaphragm (as previously mentioned this is 13 sq. cm.). The impedance looking back at the diaphragm is

$$z_0 = \pi \bar{f}_c \bar{m}_3 = 2920 \text{ mechanical ohms}$$

from Equation (**15**), and the impedance looking at a horn whose small end area equals A_2 is

$$z_h = r_0 = A_2\,G \tag{17}$$

Substituting

$A_2 = 13$ sq. cm.
$G = 41$ ohms per cm.^{2} we get
$z_h = r_0 = 533$ mechanical ohms

This is entirely insufficient so that the air-chamber transformer becomes necessary.

To calculate the necessary ratio of areas on the two sides of the air-chamber transformer, the following formula is needed. The formula assumes the chamber to be relatively small compared with all wave lengths of the sound to be transmitted, that is, the pressure changes throughout the chamber are substantially in phase.

$$\frac{z_0}{z_h} = \frac{A_1^2}{A_2^2} \tag{18}$$

where

z_0 = the impedance of the primary side of the transformer in mechanical ohms
z_h = the impedance on the secondary side of the transformer in mechanical ohms, *i. e.*, the horn impedance
v_1 = mechanical current, *i. e.*, velocity on the primary side of the transformer in centimeters per second
v_2 = mechanical current on the secondary side of the transformer in centimeters per second
F_1 = alternating force on primary side of air-chamber transformer in dynes
F_2 = alternating force on secondary side of air-chamber transformer in dynes
A_1 = effective area working into the primary side of the air-chamber in centimeters squared
A_2 = effective area working into the secondary side of the air-chamber in centimeters squared.

The characteristic impedance of the line on the diaphragm or primary side of the air-chamber as shown by equation (**15**) is

$$z_0 = \pi \bar{f}_c \bar{m}_3 \tag{19}$$

From Equation (**17**) the characteristic impedance on the horn or secondary side is

$$z_h = G\,A_2 \tag{20}$$

Therefore,

$$\left(\frac{A_2}{A_1}\right)^2 = \frac{z_h}{z_0} = \frac{G\,A_2}{\pi \bar{f}_c \bar{m}_3} \tag{21}$$

Solving this for A_2, we get

$$A_2 = \frac{G\,A_1^2}{\pi \bar{f}_c \bar{m}_3} \tag{22}$$

The equivalent of the air-chamber to a transformer shunted by a compliance is shown earlier in the paper.

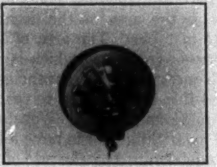

FIG. 17—MECHANICAL REPRODUCING SYSTEM WITHOUT THE HORN

In applying the foregoing method of design to a practical structure, a number of design problems had to be solved. The construction of the diaphragm and the method by which it is actuated have been already described, except for the tangential corrugations constituting the series compliance. The use of these corrugations results in the value of the series compliance being practically independent of the nature of the clamping, and has eliminated a tendency to "rattle" introduced by unevenness in the clamping surfaces.

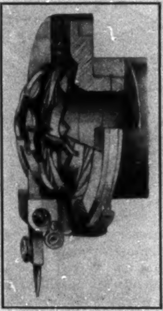

FIG. 18—SECTIONAL DRAWING SHOWING CONSTRUCTION OF THE SYSTEM SHOWN IN FIG. 17

Another feature in connection with the sound box is the needle-arm bearing shown in Figs. 17 and 18. Ordinary knife-edge bearings are not sufficiently rigid as fulcrums and the rotational reactance as well as the rotational resistance is undesirably large. A construction which has been found to meet the necessary requirements is the ball bearing type with the steel balls held in position by magnetic pull. By making the ball-containing case of soft steel and magnetizing the shaft, it has been possible to manufacture this bearing reliably and cheaply.

The horn which has been used as a terminating resistance to the mechanical filter structure is a logarithmic one. The general properties of logarithmic horns have been understood for some time.[13]

There are two fundamental constants of such a horn—the first is the area of the large end and the second the rate of taper. The area of the mouth determines the lowest frequency which is radiated satisfactorily. The energy of the frequencies below this is largely reflected if it is permitted to reach the mouth.

From the equations given by Webster,[13] it can be shown that all logarithmic horns have a low frequency cut-off which is determined by the rate of taper. If the rate of taper is so proportioned that its resulting cut-off prevents the lower frequencies from reaching the horn mouth, the horn will then radiate all frequencies reaching its mouth and very little reflection will result.[14] It is possible, therefore, to build a horn having no marked fundamental resonance.

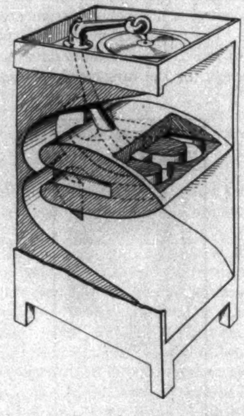

Fig. 19—Sectional View of the Folded Horn showing the Air Passage

Since the characteristics of the horn are determined by the area of its mouth and by its rate of taper the length of the horn is determined by the area of the small end. This area is determined in turn by the mechanical impedance and effective area of the system which it is terminating, as shown in Equation (22). It is seen, therefore, that the length of the horn should not be considered as a fundamental constant. A paper describing the design of horns based on these principles is being prepared.

An interesting feature of the horn which has been built commercially is its method of folding. The sketch in Fig. 19 shows a shadow picture of the horn. It will be noticed that the sound passage is folded only in its thin direction, which permits the radius of the turns to be small and thereby makes the folding compact.

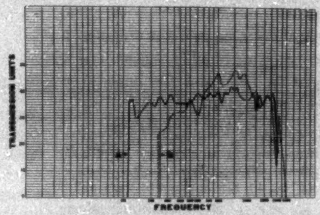

Fig. 20—Response Frequency Characteristic of Two Phonographs. Curve *A* Shows the Characteristic of the Band Pass Filter Type Described. Curve *B* Shows the Characteristic of One of the Best Commercial Machines Previously on the Market

Fig. 20 shows the frequency characteristic of a phonograph designed as shown above with a logarithmic horn whose rate of taper and area of mouth opening place the low cut-off at about 115 cycles. It also shows the characteristics of one of the best of the old style phonographs. Curve *A* represents the new machine, while Curve *B* represents the old style standard machine.

Discussion

C. R. Hanna: The following discussion applies particularly to that part of the paper dealing with the reproducing mechanism. The relative merits of the several improvements that were made are not clearly brought out in the paper and it is the purpose of the writer to compare the importance of the various developments.

In listening to reproduction from one of the new-type phonographs, the average person is impressed with just two things; first, the apparent greater volume of sound, and second, the great improvement in the response at low frequencies. The greater volume of sound is due partly to the fact that there are more low frequencies present, and perhaps, in a measure, to the fact that the diaphragm is one which acts like a piston, causing a greater volumetric rate of displacement of air into the horn for a given needle velocity than with the old type of flat diaphragm.

The improvement in the low-frequency characteristic of the reproducer, as described, could not have been obtained without the use of the slowly expanding logarithmic or exponential horn. The authors refer to the work of Arthur Gordon Webster in this connection: the general properties of the exponential horn were given in his National Academy of Science paper of 1919. Webster did not, however, carry his work sufficiently far to show the properties which the authors have stated in their paper; namely, that the exponential horn is a uniform radiator of sound down to a certain frequency, known as the cut-off frequency, which is determined by the rate of increase of section and the area of the large end of the horn.

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[13] Webster, A. G., "Acoustical Impedance and Theory of Horns and Phonograph," *Proc.* Nat. Acad. of Sci., 1919.

[14] The authors wish to express their appreciation in this connection of the work of Mr. P. B. Flanders who carried out the mathematical investigation of these relationships and to Mr. A. L. Thuras who checked experimentally the mathematical theory.

The authors cite some work (as yet unpublished), by Messrs. Flanders and Thuras, in which these properties are shown both theoretically and experimentally. I desire to call attention to the fact that the paper by Hanna and Slepian on "The Function and Design of Horns for Loud Speakers"[2] showed these same properties for the exponential horn. The equation for such a horn is

$$A = A_0 \epsilon^{Bx}$$

where

A = Area at any point

A_0 = Initial area

x = Distance from initial area, cm.

B = Constant which determines the rate of increase. It was demonstrated that the cut-off frequency is determined by the relation

$$\frac{2\pi f}{B} = \frac{a}{2}$$

where a = velocity of sound. From this it is seen that the smaller B is, the lower will be the cut-off.

The radiation characteristic of the infinite exponential horn for a fixed velocity of air in its throat was also shown in the paper by Hanna and Slepian. Fig. 1, herewith, shows this curve. The abscissas are $\frac{\omega}{B}$ and the ordinates give the comparison between the exponential horn and the infinite straight pipe which is a uniform radiator down to zero frequency. The cut-off point is seen to be as stated above.

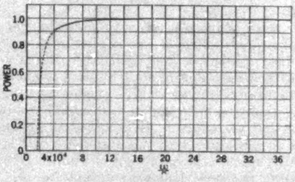

Fig. 1

It was clearly brought out in this paper that an exponential horn could be made with much smaller dimensions than any other shape of horn giving equal performance. A comparison was also shown between a particular exponential horn and a conical horn of equal length and terminal dimensions. This is given in Fig. 2, the superiority of the exponential horn being quite pronounced. Up to this time many persons had advocated the conical horn. It is believed that this paper was the first to show the superiority of the exponential horn.

Now, taking up the matter of the final or large area of the horn, as is pointed out by Maxfield and Harrison, if this area is large enough to prevent end reflections in the range of frequencies where the horn is a good radiator along its length, a horn will be secured which has very little resonance. The curves of Fig. 3 were presented by Hanna and Slepian to show the variation of reflection with frequency and area. The curves indicate that the smaller the area and the lower the frequency, the greater will be the reflection. It is seen, however, from the curve for the largest area, that the reflection becomes appreciable only in the range of frequencies where the horn ceases to be a good radiator along its length. Hence it follows that a horn of this shape can be designed with no marked fundamental resonance.

The degree of horn resonance and the position of the cut-off frequency as indicated by Fig. 20 of the Maxfield and Harrison paper agree very closely with values that can be predicted from the curves of Figs. 1 and 3 in this discussion.

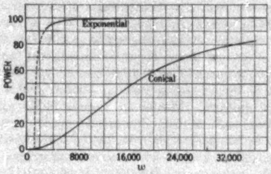

Fig. 2

The very careful proportioning of masses and compliances in the mechanical system of the reproducer has played only a minor part in the securing of a more uniform frequency-response characteristic than in the older types of phonographs. The slight extension of the upper frequency range may be attributed to the accurate design of mechanical parts. It should be pointed out, however, that since the phonograph record is a constant-current (or velocity) generator, the impedance of the mechanical system does not have to be uniform over a wide band of frequencies for it to be forced to vibrate in accordance with the vibrational velocity of the record. A departure from this fact, not apparent from the electrical analogy given by the authors in their paper, is the ability of the whole arm of the reproducer to vibrate in the low-frequency ranges instead of just the diaphragm mechanism. This may be overcome either by increasing the mass of the arm or, as the authors have done, by reducing the stiffness of the diaphragm.

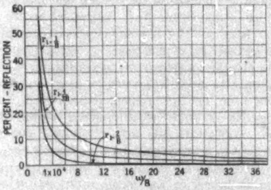

Fig. 3

Great credit is due the authors for the design of a mechanical system which is light and resilient, enabling the needle to track the record with small reaction force (and consequent decrease in wear) at the high frequencies where the accelerations are great, and at low frequencies where the deflections are great. The big improvement in the quality of reproduction, however, is due to the use of an exponential horn whose rate of increase of section is very small and whose final section is quite large.

E. W. Kellogg: I think most of us have thought of the rocking arm, which connects the needle with the diaphragm in a phonograph, as a simple lever, rigid enough so that when the needle moves one way, the diaphragm moves the opposite direction by a corresponding amount. If we could see what is really going on during a high-frequency vibration, we should probably find that the motion was more nearly like that of a snake. Messrs. Maxfield and Harrison and their associates have accepted the wave-motion picture and based their design upon it. The most striking resultant change in design is the interposition of a flexible link, or spring, between the end of the lever and the diaphragm. On first thought, it seems like deliberately throwing away some of the available motion, but the result is quite the opposite. I refer to the spider through which the diaphragm is driven.

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[2] A. I. E. E. Transactions, 1924, page 393.

If telephonic currents are to be transmitted without distortion over a high-efficiency line of length exceeding a sixth of a wavelength for the highest frequencies, the line must end in a non-inductive resistance of a definite value. A corresponding resistance is required in a mechanical system. In the case of the reproducing system the required resistance is obtained from the sound radiation of the diaphragm. But for the cutting tool, some other resistance must be found. In an electrical circuit nothing is easier to get than resistance, yet its mechanical counterpart is by no means easy to obtain. Sliding friction is not at all suitable. Motion in viscous fluids and electromagnetic drag, such as used in wattmeters, are true analogs. I wish to draw an illustration from the case of electromagnetic drag. An aluminum ring, weighing about four grams, surrounds a magnet pole, so that it is in a radial field of about 10,000 gausses. If one pushes it up and down, it feels as if it were in thick molasses. Under its own weight it settles about one millimeter per second. Yet if this ring is vibrated in an axial direction at 4000 cycles, its mass so predominates over the resistance that the power factor is only about 40 per cent. Mechanical hysteresis is another means for absorbing energy from vibrations. Rubber has long been used for such purposes. But rubber, so far from being pure in mechanical resistance, is a spring with a power factor of only about 10 per cent. I think the authors of the paper are to be complimented upon the ingenious device by which they obtain with the use of rubber a practically pure resistance with which to load the cutting tool. It should be borne in mind that the damping required for the cutting tool is of an altogether different order of magnitude from that which many of us have employed to take out the resonance peaks from loud-speaker diaphragms and similar applications.

The paper mentions methods of measuring mechanical impedances. I should be much interested to hear something further of the means used, for the problem presents many difficulties, and the results of such measurements would find many applications.

One statement in the paper causes considerable surprise. The knife edge was discarded because it has too great an elastic yield, and because it brings in too much rotational friction. The knife edge, of course, must work with an initial pressure exceeding the maximum force on the bearing, due to the vibrations, and is not well adapted to stand forces in more than one direction, but, in the case of the pivot for the reproducing lever, one would expect a well designed knife edge to work very satisfactorily.

L. T. Robinson: I am in agreement with the statement of the authors that "There is therefore no distortion in the record whose purpose is to compensate for errors in the reproducing equipment." In employing so many elements, some of which can be so readily modified in performance the temptation is very strong to look only at the final result and not be too critical as to where any corrective treatment is to be administered. I hope the stand taken by the authors will be firmly adhered by them and others who are working along similar lines. In this way, any progress that has been made, or will be made, becomes permanent.

Speaking of the electrically-cut record in general, we need not, for the moment, be concerned with minor details of the process. The results already obtained are so good that we may feel sure that the electrically cut record has come to stay and will place the phonographic art on an entirely new plane of excellence.

The mechanical reproducing system described by the authors is a distinct advance over former phonographs. However, I feel that full realization of the advantage of the electrically cut record will come through electrical reproduction.

One great advantage of the electrical method of reproducing is that the control of the sound volume is obtainable quite independent of the cut on the record and the cut on the record is controllable with consideration for the best conditions for the record alone. The advantages of such separation can be learned from the paper if it is read with this point in mind.

Volume of the sound reproduced is quite important and reproduction to be quite satisfactory must be about equal to the volume of the original sounds. A loud tone produced on a given musical instrument is quite different from a soft tone produced on the same instrument and reproduced with larger volume.

A. E. Kennelly: We have here presented to us the wonderful analogy which underlies mechanical and electrical phenomena, with mechanical phenomena interpreted in electrical terms.

We have long known that mechanical inertia was really electrical, and now we are finding that all these mechanical phenomena are primarily electrical quantities.

J. P. Maxfield: There are one or two technical questions brought out in Mr. Hanna's discussion which are of interest. The first deals with the statement that the new reproducing mechanism has a greater apparent volume of sound. In this connection, it is interesting to note that the response curves of the new and the old machines shown in Fig. 20 indicate that in the frequency region from around 800 to 2000 cycles per second, the old machine produced a louder sound for a given needle velocity. It will be seen, therefore, that this apparent increase in volume has been obtained by the widening of the band reproduced rather than by increasing the amount of energy radiated in that frequency band in which the old machine was most efficient.

The other point of interest refers to his statement that "The very careful proportioning of the masses and compliances in the mechanical system of the reproducer has played only a minor part in the securing of the more uniform frequency response characteristic than in the older type of phonograph." In view of the high quality which is obtained and of the commercial requirement that the wear on the record shall not be excessive, the authors do not agree with this statement. It is not necessarily true that because the record is a constant-current generator and, therefore, delivers constant current to the sound-box mechanism, that the diaphragm necessarily delivers constant current to the air. If a relatively stiff, heavy, vibrating system is used, it becomes exceedingly difficult with the constant-current type of generator to obtain good quality and if it is possible to obtain it, the wear on the record becomes excessive. A reference to Fig. 16 indicates that the first part of the system reached is the needle point which has a definite compliance. At the higher frequencies, if the impedance of the reproducing system is too high, the needle will bend instead of moving the rest of the system and the response will be reduced thereby. Similarly, at the low-frequency end, if the diaphragm-edge compliance is too small, that is, if the diaphragm is too stiff, the whole tone arm will vibrate and thereby reduce the motion of the diaphragm relative to its case. It is true that, so far as response is concerned, this effect can be corrected by increasing the moment of inertia of the tone arm—a method which is equivalent to increasing the mutual inductance of the transformer, T_1, (Fig. 16); but if the solution is thus obtained, the wear on the record becomes excessive and in some cases the force becomes so great that the needle will not track in the groove.

The solution presented in the paper is one in which the mechanical impedance has been made as nearly as possible independent of frequency and is of the nature of a pure mechanical resistance. The result of this type of solution is that a maximum of sound energy at all frequencies within the band is radiated with a minimum of wear on the record.

Parameters of Heating Curves of Electrical Machinery

BY VLADIMIR KARAPETOFF[1]

Fellow, A. I. E. E.

Synopsis.— *When a body is being heated by a uniform addition of a constant quantity of heat per unit time, its temperature above the ambient air (the latter being assumed to remain at a constant temperature) increases approximately according to an exponential law. The exponent is proportional to the ratio between the heat capacity of the body and the coefficient of thermal dissipation into the surrounding medium. In a paper read before the Institute's Midwinter Convention, 1925,* (JOURNAL, *Vol. 44, p. 142) Doctor A. E. Kennelly has proposed to include such a coefficient among other characteristics of an electrical machine. In the present paper it is pointed out that for thermal purposes an electrical machine cannot be considered as a single body, since the stator consists of two metal bodies (the winding and the core) between which there may be a considerable heat interchange, and that the rotor is also such a composite body. Differential equations of heat flow in a combinational body are established and solved. The stator winding is thermally determined by its heat capacity and its heat dissipation coefficient, and so is the stator core; further, there is a coefficient of mutual flow. The rotor also requires five similar coefficients. Thus, while an electrical machine could be defined by its thermal coefficients, and the temperature rise of the different parts predicted for a given operating regime, the number of required parameters is much larger than for a single body.*

* * * * *

WHEN an attempt is made to represent, analytically, the temperature rise with the time in an electrical machine at constant losses, the time-temperature curve is usually assumed to be exponential.[2] Recently, Dr. A. E. Kennelly has extended the treatment and has shown that the curve remains exponential even when the losses themselves are linear functions of the temperature.[3]

Actual heating curves sometimes differ materially from the simple exponential form. This can be shown by plotting the differences between the ultimate temperature and the instantaneous temperatures against time as abscissas, on semi-log paper. An exponential curve should give a straight line, and this is not always the case. The principal reason for this discrepancy is that the stator of a machine cannot be considered as one "chunk" of metal; it consists of two metal bodies, the winding and the core, between which there may be an appreciable difference of temperatures. The same is true of the rotor. In a transformer, three separate metal bodies at different temperatures may be distinguished.

It is the purpose of the following investigation to show that with two metal bodies at different temperatures, and with heat interchange between them through a layer of insulation, the heating curve for each consists of two exponential terms with different exponents. Thus, each part of a machine should be characterized by at least two composite thermal time constants, and these will represent an experimental heating curve much more closely than is possible with one thermal time constant and with a common curve for both the winding and the core.

Doctor Kennelly compares the transient period of temperature rise to a transient rise of current in a d-c. circuit containing a resistance r and an inductance L. In the latter case, the rise in current is also exponential and depends upon the time constant (L/r) of the circuit. However, a stator, or a rotor, is more nearly analogous to a system of two coupled electric circuits, in which the current rise is represented by two or more exponential terms, each with a different time constant[4].

With the notation given at the end of the paper[5],

$$p_1\,d\,t = \theta_1\,(s_1 - s_{12})\,d\,t + (\theta_1 - \theta_2)\,s_{12}\,d\,t + k_1\,d\,\theta_1 \qquad (1)$$

$$(\theta_1 - \theta_2)\,s_{12}\,d\,t + p_2\,d\,t = \theta_2\,(s_2 - s_{12})\,d\,t + k_2\,d\,\theta_2 \qquad (2)$$

These equations are similar to Doctor Kennelly's equation (**19**), and refer to the metal parts 1 and 2 of one of the principal members of the machine, respectively,— say the stator winding and the stator core. Equation (**1**) expresses the fact that the heat $p_1\,d\,t$, developed in the part 1 during an infinitesimal element of time, $d\,t$, is used up in three ways: The part $\theta_1\,(s_1 - s_{12})\,d\,t$ is communicated to the ambient air or other cooling medium; the part $(\theta_1 - \theta_2)\,s_{12}\,d\,t$ is communicated to the part 2 of the machine; and the remainder, $k_1\,d\,\theta_1$, raises the temperature of the part 1 by $d\,\theta_1$. Equation (**2**) has a similar meaning for part 2, with the term $(\theta_1 - \theta_2)\,s_{12}$ considered as part of the heat input.

Dividing throughout by $d\,t$, and introducing the "deficiencies", $\delta_1 = \theta_{10} - \theta_1$ and $\delta_2 = \theta_{20} - \theta_2$, in place of the temperatures themselves, equation (**1**) becomes,

$$p_1 = (\theta_{10} - \delta_1)\,(s_1 - s_{12}) + [(\theta_{10} - \delta_1) - (\theta_{20} - \delta_2)]\,s_{12} + k_1\,d\,(\theta_{10} - \delta_1)/d\,t \qquad (3)$$

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[2] For a theory of such simple heating curves see, for example, V. Karapetoff, *Experimental Electrical Engineering*, First Edition, 1909, p. 442.

[3] A. E. Kennelly, *The Thermal Time Constants of Dynamo-Electric Machines*, A. I. E. E. JOURNAL, 1925, Vol. 44, p. 142.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., Feb. 8-11, 1926.

[4] G. W. Pierce, Electric Oscillations and Electric Waves, 1920, Chap. 7.

[5] This notation is made to agree, as much as possible, with that in Dr. Kennelly's paper referred to above.

After the ultimate temperatures, θ_{10} and θ_{20}, have been reached,

we have, $\delta_1 = \delta_2 = 0$, and $d\,\delta_1/d\,t = 0$. Hence, for $t = \infty$, equation (3) becomes:

$$p_1 = \theta_{10}\,(s_1 - s_{12}) + (\theta_{10} - \theta_{20})\,s_{12} \qquad (4)$$

By analogy, equation (2) gives,

$$p_2 = \theta_{20}\,(s_2 - s_{12}) + (\theta_{20} - \theta_{10})\,s_{12} \qquad (5)$$

Substituting these values of p_1 and p_2 in equation (3) and in a similar equation obtained from equation (2), after reduction we get,

$$\delta_1\,s_1 - \delta_2\,s_{12} + k_1\,d\,\delta_1/d\,t = 0 \qquad (6)$$

$$\delta_2\,s_2 - \delta_1\,s_{12} + k_2\,d\,\delta_2/d\,t = 0 \qquad (7)$$

Equations (6) and (7) are simultaneous differential equations for δ_1 and δ_2, and their solution gives the desired expressions for the heating curves of the two metal parts of the stator or the rotor. To eliminate δ_2, differentiate equation (6) with respect to t. This gives,

$$s_1\,d\,\delta_1/d\,t - s_{12}\,d\,\delta_2/d\,t + k_1\,d^2\,\delta_1/d\,t^2 = 0 \qquad (8)$$

Multiply equation (6) by s_2, equation (7) by s_{12}, equation (8) by k_2, and add the three equations together. The variable δ_2 is then eliminated, and the result is

$$k_1\,k_2\,d^2\,\delta_1/d\,t^2 + (k_1\,s_2 + k_2\,s_1)\,d\,\delta_1/d\,t + (s_1\,s_2 - s_{12}^2)\,\delta_1 = 0 \qquad (9)$$

The solution of this equation is of the form

$$\delta_1 = A_1\,\epsilon^{-t/\sigma} + B_1\,\epsilon^{-t/\tau} \qquad (10)$$

where A_1 and B_1 are the constants of integration, and σ^{-1} and τ^{-1} are the roots of the "auxiliary" quadratic equation[6]

$$k_1\,k_2\,x^2 - (k_1\,s_2 + k_2\,s_1)\,x + (s_1\,s_2 - s_{12}^2) = 0 \qquad (11)$$

Solving this equation for x, gives

$$x = m \pm \sqrt{n^2 + q_1\,q_2} \qquad (12)$$

where

$$m = 0.5\,[(s_1/k_1) + (s_2/k_2)] \qquad (13)$$

$$n = 0.5\,[(s_1/k_1) - (s_2/k_2)] \qquad (14)$$

$$q_1 = s_{12}/k_1 \;;\; q_2 = s_{12}/k_2 \qquad (15)$$

For the sake of abbreviation, let,

$$r = \sqrt{n^2 + q_1\,q_2} \qquad (16)$$

so that

$$q_1\,q_2 = r^2 - n^2 \qquad (16a)$$

Since the thermal time constants, σ and τ, are the reciprocals of the two values of x in equation (12), we have,

$$\sigma^{-1} = m + r \qquad (17)$$

$$\tau^{-1} = m - r \qquad (18)$$

The subscripts of the coefficients in equation (9) are symmetrical with respect to 1 and 2. Hence, an identical equation may be written for δ_2, and the exponents σ and τ are the same for both δ_1 and δ_2. Thus, the general expression for δ_2 is:

$$\delta_2 = A_2\,\epsilon^{-t/\sigma} + B_2\,\epsilon^{-t/\tau} \qquad (19)$$

Substituting in equations (10) and (19) $t = 0$, gives

$$\theta_{10} = A_1 + B_1 \qquad (20)$$

$$\theta_{20} = A_2 + B_2 \qquad (21)$$

Substituting the values of δ_1 and δ_2 from equations (10) and (19) in equation (6), and equating separately the coefficients of $\epsilon^{-t/\sigma}$ and $\epsilon^{-t/\tau}$, gives the following two necessary relationships between A_1 and A_2, and between B_1 and B_2:

$$A_1\,(\sigma\,s_1 - k_1) = s_{12}\,\sigma\,A_2 \qquad (22)$$

$$B_1\,(\tau\,s_1 - k_1) = s_{12}\,\tau\,B_2 \qquad (23)$$

A substitution of the same values of δ_1 and δ_2 in equation (7) will give no new relationships between the above constants of integration. Solving equations (20) to (23) as simultaneous equations, and using equation (16a), we get:

$$A_1 = [(r + n)\,\theta_{10} - q_1\,\theta_{20}]/2\,r \qquad (24)$$

$$B_1 = [(r - n)\,\theta_{10} + q_1\,\theta_{20}]/2\,r \qquad (24)$$

$$A_2 = [(r - n)\,\theta_{20} - q_2\,\theta_{10}]/2\,r \qquad (26)$$

$$B_2 = [(r + n)\theta_{20} + q_2\,\theta_{10}]/2\,r \qquad (27)$$

These values are to be used in equations (10) and (19).

As a check on the foregoing expression, let, in a limiting case, the two hot bodies be entirely independent of each other: that is, put $s_{12} = 0$. Then $q_1 = q_2 = 0$; $r = n$; $\sigma^{-1} = s_1/k_1$; $\tau^{-1} = s_2/k_2$; $B_1 = A_2 = 0$; $A_1 = \theta_{10}$; $B_2 = \theta_{20}$.

Hence, equations (10) and (19) become:

$$\delta_1 = \theta_{10}\,\epsilon^{-t\,s_1/k_1};\; \delta_2 = \theta_{20}\,\epsilon^{-t\,s_2/k_2} \qquad (28)$$

which agrees with Doctor Kennelly's results for a single hot body.

If the losses themselves are functions of temperature, so that p_1, instead of being a constant, is, for example, a linear function of δ_1, the general form of equation (3) remains unchanged, although the coefficients will have a different meaning. The same is true of p_2. Hence, the general expressions for δ_1 and δ_2 will be of the same mathematical form as equations (10) and (19), only the coefficients and their interpretation will have to be deduced anew, following the general method used above.

PRACTICAL APPLICATION OF THE ABOVE FORMULAS

In order to apply equations (10) and (19) to a given machine, it is necessary to determine the thermal dissipation coefficients s_1 and s_2, the heat transmission coefficient s_{12}, and the thermal capacities k_1 and k_2. As a concrete example, consider the stator of a synchronous machine, and let the subscript 1 refer to the winding and the subscript 2 to the iron core. Let the machine be run at a certain load until the constant ultimate temperatures, θ_{10} and θ_{20}, have been reached and measured. Let the values of the copper loss, p_1, and of the core loss, p_2, be also known. Then, equations (**4**) and (**5**) contain three unknown quantities, s_1, s_2, and s_{12}. Let a heat run be made also at some different values of the losses, say p_1' and p_2', and let the final temperatures be θ_{10}' and θ_{20}'. Then two more equations, similar to equations (**4**) and (**5**), may be written, giving altogether four equations with three unknown quantities. If these quantities, determined from three of the equations, also satisfy the fourth, then all is well and an additional check has been obtained on both the theory and the measurements. In case of an unimportant discrepancy, an adjustment can be made of all or some of the quantities involved, to satisfy the four equations with a reasonable accuracy.

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[6] See any text-book on differential equations, chapter on linear equations with constant coefficients.

To determine k_1, equation (**6**) is applied to the beginning of the experimentally obtained heating curves of both the winding and the core. Namely, at $t = 0$, $\delta_1 = \theta_{10}$ and $\delta_2 = \theta_{20}$; $(d\,\delta_1/d\,t)_0$ is the slope of the curve (taken with the minus sign) of the lower portion of the heating curve where it is practically a straight line. Substituting these values in equation (**6**), we get:

$$k_1 = (\theta_{10}\,s_1 - \theta_{20}\,s_{12})/(-\,d\,\delta_1/d\,t)_0 \qquad \textbf{(29)}$$

A similar expression for k_2 may be written from equation (**7**).

Knowing the foregoing five constants of the machine, the auxiliary quantities $m, n, q_1, q_2, r, \sigma,$ and τ may be readily computed from the expressions given above. After this, the parameters A_1, B_1, A_2, B_2 may be evaluated for any desired values of ultimate temperature rise, θ_{10} and θ_{20}, and equations (**10**) and (**19**) used to predict the shapes of the heating curves of both the core and the winding for any desired interval of time.

The values of θ_{10} and θ_{20} depend upon the losses in the machine. Knowing the losses p_1 and p_2, and the coefficients s_1, s_2, s_{12}, the values of θ_{10} and θ_{20} may be determined by solving equations (**4**) and (**5**) as simultaneous equations. The result is

$$\theta_{10} = \frac{p_1\,s_2 + p_2\,s_{12}}{s_1\,s_2 - s_{12}^2} \qquad \textbf{(30)}$$

$$\theta_{20} = \frac{p_2\,s_1 + p_1\,s_{12}}{s_1\,s_2 - s_{12}^2} \qquad \textbf{(31)}$$

Thus, with the aid of the foregoing theory, knowing the ultimate temperature rise at two different loads, and the initial slope of the heating curves at one of these loads, it is possible to predict the complete shape of the heating curves under any load conditions for which the losses are known.

Instead of determining, experimentally, the ultimate temperature rise at two different loads, it is also possible to use only one set of heating curves (one for the core and one for the winding), even without reaching the ultimate temperatures. In this case the unknown thermal constants of the machine should be determined by the method of least squares, to satisfy equations (**10**) and (**19**)[7]. The computations will be considerably more involved, but a heat run is saved, and the only heat run to be performed need not be continued until the stationary conditions have been reached. With very large machines, these considerations may outweigh the tediousness of extra computations.

NOTATION

A	a constant, in deg. cent.
B	a constant, in deg. cent.
k	thermal capacity of a body, in kw-hrs. per deg. cent.
m	defined by equation (**13**), in (hours)$^{-1}$
n	defined by equation (**14**), in (hours)$^{-1}$
p	heat input, in kw.
q	defined by equation (**15**), in (hours)$^{-1}$
r	defined by equation (**16**), in (hours)$^{-1}$
s	thermal dissipation coefficient, in kw. per degree centigrade of temperature difference; this coefficient includes the heat loss to the ambient medium and that to the other part of the composite body.
t	time, in hours
x	auxiliary notation for σ^{-1} and τ^{-1}, equations (**11**) and (**12**)
δ	deficiency in temperature, that is, the difference $\theta_0 - \theta$.
θ	temperature rise, in degrees centigrade, above the ambient medium.
θ_0	ultimate temperature rise.
σ, τ	time constants of a combination of two bodies, in hours.

NOTE 1. Where the subscripts 1 and 2 are used in the text, 1 refers to a copper winding and 2 refers to the iron core separated from it by a layer of insulation. The subscript 12 refers to the heat conductance of this insulation.

NOTE 2. The fundamental units assumed in the notation are the kilowatt, the kw-hr., the hour, and the deg. cent. However, the formulas hold true with any units, provided that these are consistent among themselves; for example, the watt, the joule, the second, and a degree of any desired thermometer scale.

Some recent articles on heating of electrical machinery:

E. Hughes, (British) *Inst. El. Engrs. Journal*, 1924, Vol. 62, p. 628.

M. L. Keller, *Archiv f. Elektrot.*, 1924, Vol. 13, p. 292.

W. H. Cooney, JOUR., A. I. E. E., Vol. 44, p. 1342, Dec. 1925.

See also: R. Richter, *Elektrische Maschinen* (Berlin, 1924), Vol. 1, pp. 350 to 365.

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[7] V. Karapetoff, *Engineering Mathematics*, Vol. III, pp. 59-66.

Discussion

G. E. Luke: As mentioned in this paper, Professor Kennelly derived the heating characteristics of an electric machine by considering it as one "chunk" of metal[1]. This paper gives the thermal equations on the basis of two "chunks" of metal (core and windings). An approximate solution of this problem based on three "chunks" of metal (core, windings, and frame) was given in another previous Institute paper[2]. As a matter of fact, a correct solution of this problem would have to be based upon an infinite number of "chunks."

Professor Karapetoff assumed two thermally connected masses of metal (core and windings), each having a uniform temperature and loss throughout. In practise, the losses and temperatures are usually much higher in the teeth than in the main part of the core; hence, there will be a heat flow in that direction. In the windings, there may be a considerable heat flow from the embedded part to the ventilated end portion. Thus, the heating curve of the "hot spot" on the copper will depend not only upon the thermal constants of the embedded copper and core but also upon the thermal conditions of the end windings. Again the large variation in the temperature of the cooling fluid in its passage through the machine will tend to cause all parts of the machine to vary with it.

The assumptions which must be made in order to derive the approximate thermal characteristics of a machine are more closely approached in small machines with relatively little forced cooling. On the other hand, in large machines, where long heat and fluid flow paths are found, together with a high velocity of the cooling fluid, the derivation of the heating characteristics on the assumption of uniform core and winding temperatures is likely to be erroneous.

The method of solution of the problem as simplified by Professor Karapetoff interested me, as several years ago I had occasion to solve the same problem. In my solution, θ_1 was evaluated from eq. (2) (also $d\,\theta_1$) and substituted in eq. (1). This also gave a linear differential equation having constant coefficients, but the second member, instead of being zero, as in eq. (8), was a function of p_1, p_2, k_1, k_2, s_1, and α, where α is the temperature coefficient of resistance. The solution of this was similar to eq. (10) with a constant term added. Although the method of solution given in this paper seems to be indirect, yet the equations representing the arbitrary constants are more systematic than the method I used.

For the application of these equations, the author suggested that the many constants be calculated from experimentally obtained heating curves. The difficulty is to obtain accurate heating curves of the copper and iron temperatures especially on the rotating members.

I have applied these equations for predetermining the temperatures of machines on short-time overloads by calculating the constants from the known physical conditions. Thus, on such loads, most of the losses are stored and since the copper has such a high rate of loss, its temperature rises quickly. For example, the temperature rises of a 50-h. p., d-c. railway motor on a 150-per cent load, for one-half hour, as calculated (starting cold) were 51.5 deg. cent. armature core and 85.5 deg. armature copper, while the actual tested values were 48.0 and 87.5 deg. cent. respectively.

Another interesting application of these heating equations was to calculate the heating and cooling curve of a 600-volt, 31,000-cir. mil., rubber-insulated, stranded copper cable, with a load of 214 amperes for 2½ min. The calculated points were based only on the weight of copper and insulation, the conductivity of heat of the rubber, and the surface heat transfer from the cable. The solution is necessarily approximate since the insulation and copper were assumed to have a uniform temperature θ_2 and θ_1, respectively. The results are given in Fig. 1 herewith. Since the loss in the rubber was zero, its slope was zero at the start of the cycle.

Although the assumption of two thermal masses simplifies the problem considerably, the final solution is still long and complicated and therefore will be used seldom by the average engineer. This solution should prove of value to the specialist, however, when considering problems such as I have mentioned. This solution also will indicate the limitations and errors of the less approximate solution based on a single uniform mass of metal.

W. F. Dawson: I want to show some curves (Fig. 2 herewith) that follow the curves given in Goldschmidt's paper[3] and also by Dr. Kennelly.

These curves are the heating curve of a mass of copper imbedded in insulation and other surrounding material. To simplify the problem, let us assume that it is not armature copper in an armature, but a field coil where the only loss is the copper loss. The initial rate of temperature rise depends upon the specific heat of the material, the specific gravity, (density), the specific resistance, and the current density. It so happens that, at 1100 amperes per sq. in., where there are no parasitic or extra losses, and the mean temperature is about 60 deg. cent., the rate of temperature rise is almost exactly one deg. cent. per min. This current density also gives approximately one watt per cu. in. (0.95 watt).

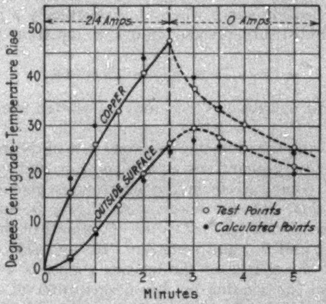

Fig. 1—Heating and Cooling Curve of a 600-Volt, 32,000-cm. Rubber-Insulated, Stranded Copper Cable

In practise, it is found that at 1100 amperes per sq. in., the initial rate of temperature increase is much less than one deg. cent per min., seldom more than one-half of a deg. cent. per min. and sometimes as low as one-third of a deg. cent. per min. The explanation lies in the fact that these windings are always surrounded with a considerable amount of insulation and frequently with masses of iron. Goldschmidt has suggested that the specific heat of cotton insulation is approximately six times that of copper; hence it is easy to explain that the added thermal capacity of the surrounding insulation and iron is responsible for the lowered rate of temperature rise. A divisor of two is usually quite safe, hence with a current density of 2200 amperes per sq. in., one can safely assume that the initial rate of temperature rise will not exceed two deg. cent. per min.

I have made several tests with temperature rise plotted against time and found that, when a proper factor was used for the thermal capacity, the test results compared very closely with curves calculated according to Goldschmidt's formula. In one, a strange "hump" was shown in the observed heating curve and it was referred back to the testing department with the comment that "the phase currents in the armature became temporarily unbalanced." This was found to be the case. The extra heating was due to double-frequency currents induced in the field because currents in the armature phases became unbalanced. Fig. 2 will be found useful in the study of short-time ratings of windings, and the effect of fractional loads and overloads. It should be considered as a suggestion rather than something to be followed under all circumstances. It is plotted on the assumption that 2200 amperes per sq. in. give an initial temperature rise of two deg. cent. per min. (this assumes that total thermal capacity equals twice that of the copper winding), that the final temperature rise will be 40 deg., and that the temperature rise varies as the square of the current load.

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[1] Temperature Rise of Electric Machines on Intermittent Duty, by G. E. Luke, *Electrical World*, May 27, 1922.

[2] *Heating of Railway Motors in Service and Test-Floor Runs*, by G. E. Luke, Journal A. I. E. E., (1922), p. 165..

[3] *Journal* I. E. E. (London) May 1905, Vol. 34, No. 172, p. 660.

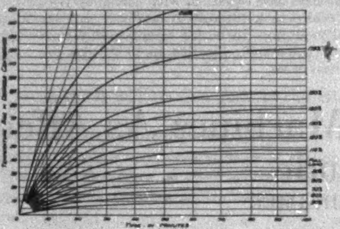

FIG. 2—TEMPERATURE GRADIENT CURVES BASED ON 2200 AMPERES PER SQ. IN., FULL LOAD AND TWO-DEG. RISE PER MIN.

A little study will suggest many uses for curves of this sort; for example, a short-time overload is required on a machine that has been operating at fractional load until it has reached steady temperature. From the steady temperature indicated by the curve for that fractional load, follow the abscissa line to the left until it intersects the curve corresponding with the overload and note the corresponding time. Add the time for the required overload, and the intersection of the corresponding ordinate with the overload curve will indicate the total temperature rise.

C. J. Fechheimer: A few more words may be added in regard to the longitudinal heat flow in the copper. In machines which have, let us say, over 50-in. core length, the influence of the longitudinal flow upon the copper temperature at the middle of the core may be neglected. But in short machines, that flow plays an important part in the heat dissipation at the center. In some short machines, (say eight in. of core length or less), most of the heat generated in the copper is dissipated from the end windings, and there may even be flow of heat from the iron to the embedded copper whence it flows longitudinally to the ends. Frequently, then, there is not a great difference in temperature between the embedded copper and the ends.

It is well known that the time constant in a simple exponential equation is proportional to the ratio of the storage of heat to the dissipation of heat. In general, that time constant is considerably higher for the embedded part than for the ends, and, since there is so large an interchange of heat between the two in the short machine, the influence of the longitudinal flow cannot be ignored. About eight years ago I tried to obtain a solution of the problem, taking into account time and the distance along the conductors as independent variables. I obtained the partial differential equations and was able to integrate them. But the equations became so unwieldy when the terminal conditions were substituted that I abandoned the solution. I believe I still have the papers, and should be glad to send to Professor Karapetoff the results as far as I went, if he cares to go over them.

Suppose the time-temperature curves for a short machine are illustrated graphically as in Fig. 3 herewith. Curve I shows the rate of heating of the embedded part, and Curve II that of the ends, assuming that neither is influenced by the other. The actual time-temperature curve of the copper at the middle of the machine is about as in Curve III. The ends attain their maximum temperature in considerably less time than the embedded parts, each acting alone, and the maximums are quite different. If the equation of Curve III can be taken as of simple exponential form $[\theta = \theta_e\,(1 - \epsilon^{-\frac{T}{t}})]$, and if the final temperature rise θ_e and the initial slope $\left(\frac{d\theta}{dt}\right)_{t=0}$ be known, the entire curve can be plotted.

In Dr. Kennelly's paper of last year and in this paper of Professor Karapetoff's, mention is made of the similarity between the time equations of the heat circuit and the electric current containing inductance and resistance. In the discussion of a paper by Mr. Luke in 1922, that similarity was noted.[3] Perhaps that was brought to the attention of the electrical engineers long ago.

A. E. Kennelly: In the paper which I presented last year, I was not interested primarily in the determination of the conditions of heating, but in the presentation of the statistical factors concerning heating. As has been pointed out by several speakers, and as pointed out in the discussion last year, there are a number of instances in which the heating curves of parts of machines, or of parts of windings, or of core bodies, follow exponential curves, and it is marvelous, I think, how common exponential curves, either of the rising type or of the falling type, are in natural phenomena.

We have been in the habit of referring the unit of time in which the exponential change occurs to a time constant in which the change is 63.2 per cent of the final ultimate change and in the next time constant 63.2 per cent of what again is left. That is a very awkward numerical calculation to present to practical men, so that, in order to avoid that unnecessary numerical and statistical combination, I ventured to point out that it is sufficient to take approximately 70 per cent of that exponential time constant (69.3 per cent theoretically) and to state that, as the binary time-constant or the time to come up to one-half; so that if in the first binary time constant of, let us say, twenty minutes, this element of the machine will come to half its final temperature elevation, then, in the next binary time constant of twenty minutes, it will come to half of what is left or three-quarters of the whole.

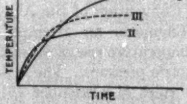

FIG. 3

As has been pointed out by Professor Karapetoff and others, probably no machine develops a strictly exponential time curve of heating. As engineers, we try to find reasonable practical applications and we say, "Well, granted that it is never quite true, are there not cases which present themselves where the deviations for ordinary practical purposes can be neglected?" It is my opinion, after having gone over a considerable number of such heating curves, that many cases can be treated upon the basis of an experimental time constant.

So I think the procedure we can agree upon is this: Let us find the cases which present themselves in engineering practise where the actual curves are nearly constant, so that we may say, for practical purposes, that this machine, under such a load, will come up to one-half its final temperature elevation in so many minutes. Then one knows just how the heating will be beyond that point. Then, let us tabulate the exceptions.

I think we may look upon this as a valuable tool for the future, but I want first to see a tabulation of what can be done with existing machinery of various classes, using the simple, single time-constant. From what I have been able to ascertain from actual engineers,—men who are using this,—there will be a considerable number of cases where that does apply reasonably well.

F. Fabinger (in writing[4]): It may be of interest to call attention to my article entitled "A Contribution to the Theory of Heating of Electrical Machinery" published in the May 1923 issue of the *Elektrotechnicky Obzor* which is the official organ of the Association of Electrical Engineers of Czechoslovakia. In this article, I consider a body which is a source of constant quantity of heat per unit time to be completely surrounded by another body of infinite conductivity, in contact with a cooling medium of constant temperature. The differential equations in this case lead to a solution similar to Karapetoff's eq. (**10**); that is, the instantaneous temperature is expressed by a sum of two exponential terms.

Vladimir Karapetoff: As Dr. Kennelly has pointed out, the next step is to apply the proposed theory to a few machines to see how handy (or unhandy) it will be. I also hope that someone will be spurred to improve the two-chunk theory, but let us hope that this will be not in the direction of a three-chunk theory, but in the direction of distributed flow of heat.

To me, the next step is this: Assume the heat flow in the stator copper to be parallel to the shaft of the machine and the heat flow in the stator core to be radial. The temperature at a point will then not only be a function of time, but of the coordinates of that point as well, so that, as Mr. Fechheimer has pointed out, we shall have equations with partial derivatives. It would be of considerable interest to establish such equations. Even should they prove to be too complicated for a straightforward solution, some approximation methods may be applicable in numerical cases.

The Ratings of Electrical Machines As Affected by Altitude

BY CARL J. FECHHEIMER[1]

Fellow, A. I. E. E.

Synopsis.—*The paper contains equations applicable to machines cooled by forced air convection currents. It is to be hoped that the A. I. E. E. Standards Committee will find some helpful suggestions in the paper.*

Ignoring differences in ambient temperature, the effect of altitude may be considered from two standpoints:

(*a*) *The change in temperature rise, the rating remaining the same; or* (*b*) *the change in rating, the temperature rise at a given altitude equaling that at sea-level. The applications of both are considered, and equations are solved in both ways. The difference in ambient temperature at sea-level and at altitude is also taken into consideration in other equations.*

The present A. I. E. E. rule is known to be faulty, applies only on the basis of temperature change, and starts at 1000 meters. These and other objections are believed to be met in this paper.

Several of the equations are plotted in families of curves, which should be of assistance to the reader. By assuming other values for some of the factors, other similar curves can be drawn.

In Appendix I, the derivations of equations are given. Appendixes II and III cover discussions of two of the factors used in the equations. In Appendix IV, some cases other than a slotted core are discussed.

A number of assumptions were made in the derivations of the equations: (*1*) *The temperature coefficient of resistance was neglected;* (*2*) *the heat that flows transversely through the insulating wall is the same percentage of the heat generated in the copper at altitude as at sea-level. The equations are intended to apply to a slotted core, although they may be used for other parts of an electrical machine. Some examples are given in Appendix IV.*

* * * * *

It has long been known that the density of a gaseous cooling medium influences the rate at which heat is dissipated from a heated surface, and at the February 1913 Convention of the American Institute of Electrical Engineers an attempt was made to correlate the temperature rise of an electrical machine with the barometric pressure. The revised A. I. E. E. standardization rules of December 1914 contained a statement for correcting temperature rise or rating to correspond to higher altitude, but the rule was intended only to be temporary, and it is now recognized as faulty. Several articles on this subject have recently appeared in the *Technical Press*[2] and in this article another method of attack is presented. It is to be hoped that the A. I. E. E. Standards Committee will find some helpful suggestions herein.

If the change in ambient temperature is ignored, it is evident that the effect of altitude may be considered from two standpoints: (a) The change in temperature rise, the rating remaining the same; or (b) the change in rating, the temperature rise at a given altitude equaling that at sea-level. Perhaps the most usual case, in which the purchaser of apparatus is interested, is how much less the temperature rise of the machine should be at sea-level than at a given altitude for the same load. The machine usually is tested near sea-level, and the customer wishes to know how much to decrease the test temperature rises to equal approximately those at the higher altitude. This is the most useful way for the correction to appear in the A. I. E. E. Standards. It is well, however, to include another method to cover the change in permissible output with

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[4] From Prague, Czechoslovakia.

[1] Research Engineer, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

[2] Particularly the paper by Doherty and Carter: *Effect of Altitude on Temperature Rise*, TRANS. A. I. E. E., 1924, p. 824.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

altitude for operating recommendations. Equations in this form should also be of value to the designing engineer, as with their use he can tell how much the rating of a machine should be altered if transferred from sea-level to a higher altitude.

In the 1925 A. I. E. E. Standards, the rule reads: "For apparatus intended for service at altitudes greater than 1000 meters, it is provisionally agreed that the permissible temperature rises (to be included in contracts and checked by test at low altitude) shall be less than specified in these standards by one per cent of the specified rise for each 100 meters of altitude in excess of 1000 meters." The "Usual Service Conditions" are given as "(a) When and where the temperature of the cooling medium does not exceed 40 deg. cent. (b) Where the altitude does not exceed 1000 meters."

From the above it will be seen that in the existing A. I. E. E. rules:

(1) The correction for altitude is only on the basis of temperature change, not on a change in rating.

(2) The correction for altitude starts at 1000 meters, there being no correction for altitudes lower than that elevation.

(3) The correction for altitude is in very simple form, and as such cannot possibly bring in the various factors upon which such correction depends.

(4) The correction for altitude is empirical, and is based upon a limited amount of data; it is consequently liable to be considerably in error.

It is believed that these objections are met in this article. The derivations of the equations are given in Appendix I. The final equations may be put into various forms, according to what the point of interest is, what items are to be considered and what ones are to be ignored. They may be classified according to whether the solution is for temperature or for rating. The solutions are applicable to forced convection conditions only. The equations are intended to apply chiefly to a slotted core, in which some of the losses are constant and some are proportional to the square of the load. The factor k is the fraction of loss in the core that is proportional to the square of the load at sea-level rating. (Thus, in estimating k, the numerator is the embedded copper loss, and the denominator the embedded copper loss plus the core loss). For the more usual applications of the equations 1 and 2 below, k fortunately does not appear. The equations can be applied to parts other than the slotted core, such as field coils in synchronous alternators, but then, although $k = 1$ and the equations are therefore simplified, account must be taken of the rate at which the field current changes with the load, and that is beyond the scope of this paper. In Appendixes II and IV, k and cases other than a slotted core are briefly discussed.

The final equations are as follows, the list of symbols following the equations:

(a) Solutions for temperatures:

I. Surface temperatures only considered:

$$\frac{\theta_{s1}}{\theta_{s2}} = \left(\frac{B_2}{B_1}\right)^m \tag{1}$$

II. Internal (copper) temperatures considered:

$$\frac{1}{b} = \frac{\theta_1}{\theta_2} = \frac{1}{a + (1-a)\left(\frac{B_1}{B_2}\right)^m} \tag{2}$$

(b) Solution for ratings:

I. Surface temperatures only considered:

1. Permissible temperature rise at a given altitude the same as at sea-level.

$$K = \sqrt{1 - \frac{1}{k}\left[1 - \left(\frac{B_2}{B_1}\right)^m\right]} \tag{3}$$

2. Permissible temperature rise at a given altitude different from that at sea-level:

$$K = \sqrt{1 - \frac{1}{k}\left[1 - b\left(\frac{B_2}{B_1}\right)^m\right]} \tag{4}$$

II. Surface drop and drop through insulation considered:

1. Permissible temperature rise at a given altitude the same as at sea-level.

$$K = \sqrt{\frac{1 - (1-k)(1-a)\left(\frac{B_1}{B_2}\right)^m}{a + k(1-a)\left(\frac{B_1}{B_2}\right)^m}} \tag{5}$$

2. Permissible temperature rise at a given altitude different from that at sea-level.

$$K = \sqrt{\frac{b - (1-k)(1-a)\left(\frac{B_1}{B_2}\right)^m}{a + k(1-a)\left(\frac{B_1}{B_2}\right)^m}} \tag{6}$$

In the above equations,

a = ratio of the thermal drop from the copper to the iron to the total temperature rise, at sea-level.

b = ratio of the total temperature rise at a given altitude to the total temperature rise, at sea-level.

B_1 = barometer reading at sea-level.

B_2 = barometer reading at a given altitude.

k = fraction of loss in the core that is proportional to the square of the load at sea-level rating.

K = fraction of sea-level rating that should apply at a given altitude.

m = exponent of barometric pressure ratio, and is the power to which that ratio should be raised to determine how much the surface temperature rise is altered by a change in density. Its value is between 0.75 and 1, and for most purposes 0.9 may be used.

θ_1 = total temperature rise at sea-level.

θ_2 = total temperature rise at a given altitude.

θ_{s1} = temperature rise of the cooling surface above the ingoing air, at sea-level.

θ_{s2} = same as θ_{s1}, but at a given altitude.

In Fig. 1 is plotted the ratio of the barometric pressure at sea-level to that at a given altitude. With these data, and that contained in equation (**1**), the curves in Fig. 2 have been plotted. They show what the ratio of the temperature rise at sea-level to that at altitude should be, outside surface temperatures only being considered. They are plotted for three different values of *m*. The corresponding ratio, using the provisional A. I. E. E. rules, is also plotted. If the assumption is made that the probable most usual value of *m*, is say, 0.9, data from the curve corresponding to that value could conveniently be incorporated in the rules. Or, for $m = 0.9$, the data may be quite accurately represented by the equation

$$\frac{\theta_{s1}}{\theta_{s2}} = 1 - 0.09 \times \frac{\text{alt.}}{1000} \text{ up to about 4000 meters.}$$

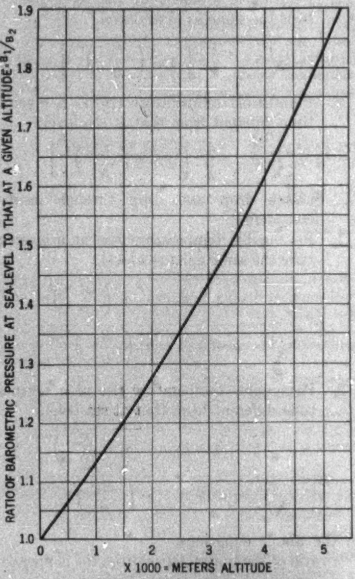

FIG. 1—CURVE OF BAROMETRIC PRESSURE

In those machines in which temperatures are measured by embedded temperature detectors, equation (**2**) may be used. The plots for a = 0.4, 0.5, and 0.6 are given in Fig. 3 for $m = 0.9$. Those three curves are so nearly straight lines that their approximate equations may be written as follows:

$$a = 0.4, \frac{1}{b} = 1 - 0.0604\left(\frac{\text{Alt.}}{1000}\right)$$

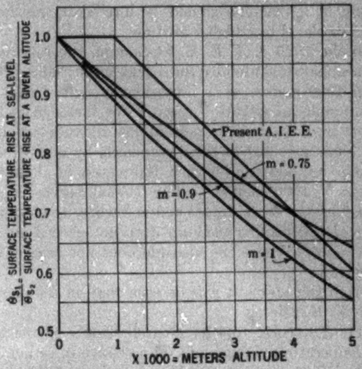

FIG. 2—RATIO OF SURFACE TEMPERATURE RISE AT SEA-LEVEL TO THAT AT A GIVEN ALTITUDE. "*m*" HAS A DIFFERENT VALUE FOR EACH CURVE.

$$\frac{\theta_{s1}}{\theta_{s2}} = \left(\frac{B_2}{B_1}\right)^m$$

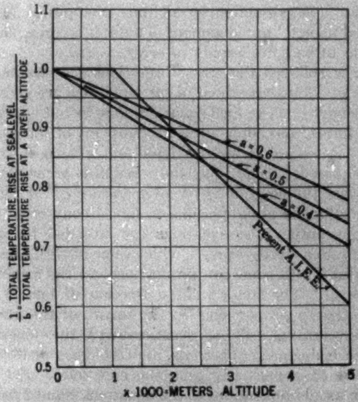

FIG. 3—RATIO OF INTERNAL TEMPERATURE RISE AT SEA-LEVEL TO THAT AT A GIVEN ALTITUDE. "*a*," THE RATIO OF DROP FROM COPPER TO IRON TO TOTAL TEMPERATURE RISE, HAS A DIFFERENT VALUE FOR EACH CURVE.

$$\frac{1}{b} = \frac{\theta_1}{\theta_2} = \frac{1}{a + (1 - a)\left(\frac{B_1}{B_2}\right)^m}$$

m = 0.9 FOR ALL CURVES

$$a = 0.5, \quad \frac{1}{b} = 1 - 0.0526\left(\frac{\text{Alt.}}{1000}\right)$$

$$a = 0.6, \quad \frac{1}{b} = 1 - 0.044\left(\frac{\text{Alt.}}{1000}\right)$$

Alt. = Altitude in meters.

The error for the high altitudes is not negligible when the present A. I. E. E. rule is used. Thus, for $a = 0.5$, the temperature rise at sea-level is 0.6, instead of 0.735, of the rise at 5000 meters.

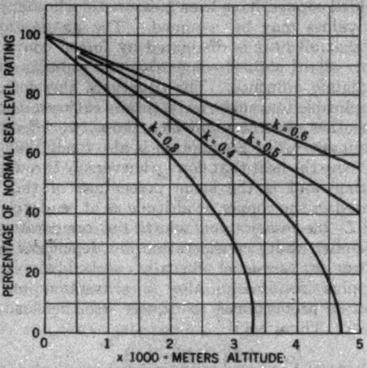

FIG. 4—RATINGS AS AFFECTED BY ALTITUDE. $m = 0.9$ FOR ALL CURVES, SURFACE TEMPERATURE ONLY CONSIDERED. k, THE FRACTION OF LOSS IN THE CORE THAT IS PROPORTIONAL TO THE SQUARE OF THE LOAD AT SEA-LEVEL, HAS A DIFFERENT VALUE FOR EACH CURVE

$$K = \sqrt{1 - \frac{1}{k}\left[1 - \left(\frac{B_2}{B_1}\right)^m\right]}$$

It will be noted that equations (**1**) and (**2**) do not contain k, the fraction of the loss in the core that is proportional to the square of the load. This follows because the temperature of the external surface of, say, an armature, is independent of the distribution of losses in the copper and iron. This is fortunate, as one less variable means simplification.

When the equations are solved for the rating ratio K, the term k is present. When so solved, the simplest solution, equation (**3**), applies when surface temperatures only are considered, and when the permissible temperature rise at a given altitude is the same at sea-level. Perhaps that is the form of "rating" equation that an engineer would use most frequently. The percentage of normal sea-level rating may then be read directly from a family of curves, such as are shown in Fig. 4. Those curves were calculated on the basis that the barometric pressure ratio = 0.9, and that the value of k, the fraction of total loss that, at sea-level, is proportional to the square of the load, is constant along any one curve. The value to assign to k is considered in Appendix II. If a fixed value is chosen for k, the equation may again be plotted in the form of a family of curves, the exponent m being fixed for each curve. The value of 0.4 has been chosen for k in the plot in Fig. 5. From data on various machines, it seems as though 0.4 is a fair average value. As stated elsewhere in this paper, a value of 0.9 for m is probably the one that may be generally adopted. It is, therefore, suggested that for machines on which surface temperatures only are measured, the data from either Fig. 4 or 5, for $m = 0.9$ or $k = 0.4$, be used.

If again surface temperatures only are considered, and values are assured for m and k, a family of curves may be plotted with the use of equation (**4**). As shown in Fig. 6, $m = 0.9$ and $k = 0.4$. Equations (**3**) and (**4**) are the same except that in (**3**) the temperature rise is taken to be the same at altitude as at sea-level, for a given rating $\left(b = \frac{\theta_2}{\theta_1} = 1\right)$. The ratio b is constant along any curve in the family in Fig. 6. It will be seen that it is quite within the range of possibilities for a machine to be good for a higher rating at a higher altitude than at sea-level. For example, if the air temperature is 40 deg. at sea-level, and the rise is 40 deg., then at a certain location at 3000 meters altitude, where the air temperature is 20 deg., the permissible rise is 40 + 40 − 20 = 60 deg. to secure the same total temperature. The rating may then be, from Fig. 6, 110.5 per cent of sea-level rating.

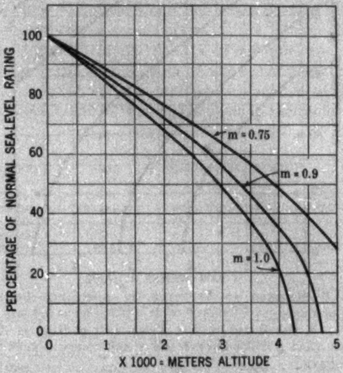

FIG. 5—RATINGS AS AFFECTED BY ALTITUDES. SURFACE TEMPERATURE ONLY CONSIDERED. "m" HAS A DIFFERENT VALUE FOR EACH CURVE. $k = 0.4$ FOR ALL CURVES.

$$K = \sqrt{1 - \frac{1}{k}\left[1 - \left(\frac{B_2}{B_1}\right)^m\right]}$$

Equations (**5**) and (**6**) are solved for the percentage of sea-level rating, account being taken of the drop through the insulation. Thus, the factor a is the ratio of the thermal drop from the copper to the iron to the total temperature rise, at sea-level. The assumption is made that the heat that flows transversely through the insulating wall is the same percentage of the heat generated in the copper at altitude as at sea-level. Evidently these two equations apply only when the temperatures are measured by embedded temperature detectors. Equation (**5**) is the same as (**6**), except that in (**5**) the temperature rise at altitude is taken to be the same as at sea-level.

With the number of factors as given in equations (**5**) and (**6**), it becomes more difficult to plot them in families of curves, and such curves are, therefore, not included in this paper. As those equations will probably be of value chiefly to the designer, and as it is necessary to evaluate such factors as k and a, with which frequently only the designer is familiar, he can readily substitute the numerical values in the equations and solve for K.

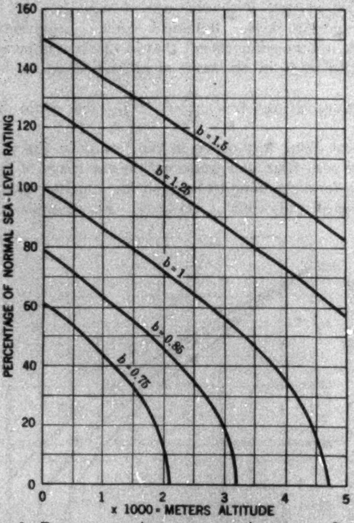

FIG. 6—RATINGS AS AFFECTED BY ALTITUDE. SURFACE TEMPERATURE ONLY CONSIDERED. $m = 0.9$ AND $k = 0.4$ FOR ALL CURVES. b, THE RATIO OF THE TEMPERATURE RISE AT ALTITUDE TO THAT AT SEA-LEVEL, HAS A DIFFERENT VALUE FOR EACH CURVE.

Appendix I

DERIVATION OF EQUATIONS

For the usual case, the slotted core construction is of greatest interest. While that construction is the one chiefly considered, the equations are applicable to other parts. Some cases are considered in Appendix IV.

The derived equations take into account the relative values of $I^2 R$ loss, the percentage thermal drop through the insulating wall, and the comparative allowable temperature rises at altitudes and at sea-level. The equations are simplified if any of these are neglected. Furthermore, the equations can be readily applied if tests have been made at the factory where such items as the barometric pressure, ingoing air temperature, and the temperature rise of the iron surface and of the embedded copper, have been measured or whose approximate values may be assumed. The assumption is made that all heat is dissipated by forced convection currents of air, natural convention and radiation being of negligible influence. The equations, therefore, are not applicable to transformers, totally enclosed machinery, or other apparatus in which those two effects are predominant. The assumption is also made that in a slotted core the heat that flows transversely through the insulating wall is the same percentage of the heat generated in the copper at altitude as at sea-level.

Call L_1 the losses upon which the temperature in the member under consideration are dependent when operating at sea-level (usually embedded copper losses plus iron loss). Also k = fraction of loss L_1 that is proportional to square of the load, at sea-level. Then:

$$L_1 = k\,L_1 + (1 - k)\,L_1 \qquad (7)$$

Thus, $k\,L_1$ = variable losses that are proportional to the square of the load, and the remainder, $(1 - k)\,L_1$ = constant losses. At a given altitude, the constant losses are still $(1 - k)\,L_1$, and the variable losses are as at sea-level multiplied by the square of the ratio of ratings. If L_2 is the loss at altitude,

$$L_2 = k\,K^2\,L_1 + (1 - k)\,L_1 \qquad (8)$$

The difference between the surface temperature and that of the cooling medium adjacent to the surface is proportional to the losses, and inversely proportional to a fractional power of the density of the medium. The fractional exponent is probably between 0.75 and 1, and its value is further discussed in Appendix III. The cooling medium, air, is itself heated by the absorption of losses up to the particular surface, and that air temperature rise must be added to the thermal drop from the surface to the adjacent air. The air rise is inversely proportional to the density of the cooling medium. The temperature rise of the surface above the air entering the machine is then made up of two parts, one of which is inversely proportional to the first power of the density ratio and the other is inversely proportional to a power of that ratio whose value is between 0.75 and 1. In the general case in which the temperature rise above the ingoing air is considered, the resultant exponent is probably not far from 0.9, but in these equations it is written as m. The equation coordinating losses and barometric pressures with surface temperature rise may then be written as:

$$\frac{\theta_{s2}}{\theta_{s1}} = \frac{L_2}{L_1}\left(\frac{B_1}{B_2}\right)^m \qquad (9)$$

Here θ_{s1} = temperature rise of cooling surface above the ingoing air, at sea-level. θ_{s2} = same as θ_{s1}, but at altitude. B_1 and B_2 are respectively the barometric pressure readings at sea-level and at altitude.

From equations (7), (8) and (9),

$$\frac{\theta_{s2}}{\theta_{s1}}\left(\frac{B_2}{B_1}\right)^m = \frac{L_2}{L_1} = k\,K^2 + (1-k) \qquad (10)$$

Whence, for $\theta_{s2} = \theta_{s1}$,

$$K = \sqrt{1 - \frac{1}{k}\left[1 - \left(\frac{B_2}{B_1}\right)^m\right]} \qquad (11)$$

Equations (11) and (3) in the text are identical. Assuming that the load at sea-level and at altitude are the same, $K = 1$, the equation may be written:

$$\frac{\theta_{s1}}{\theta_{s2}} = \left(\frac{B_2}{B_1}\right)^m \qquad (12)$$

This is the same as equation (1) in the text.

Consider next the transverse flow of heat through the insulation wall adjacent to the copper in the slot; the total temperature rise of the copper above the ingoing air θ_2 is the sum of the drop through the insulating wall θ_{i2} and the surface rise θ_{s2}. That is:

$$\theta_2 = \theta_{s2} + \theta_{i2} \qquad (13)$$

Similarly at sea-level, using subscripts 1 instead of 2,

$$\theta_1 = \theta_{s1} + \theta_{i1} \qquad (14)$$

The heat flows in part transversely through the insulation from the copper to the iron, and in part longitudinally and the relations are too complex to embody in these equations.[3] (In some cases the flow may be from the iron to the copper). It is believed, however, to be reasonable to assume that the percentage of the total heat generated in the copper that flows transversely is the same at a given altitude as at sea-level. The difference in temperature between the copper and the cooling surface may then be taken as proportional to the copper losses:

$$\frac{\theta_{i2}}{\theta_{i1}} = \frac{K^2\,k\,L_1}{k\,L_1} = K^2 \qquad (15)$$

Call a the ratio of the drop from the copper to the cooling surface to the total temperature rise at sea-level:

$$a = \frac{\theta_{i1}}{\theta_1} \qquad (16)$$

One other factor may enter, as the permissible temperature rise at a given altitude is frequently greater than at sea-level, due to the lower ambient temperature at the higher altitudes. The ratio of the permissible rise is:

$$b = \frac{\theta_2}{\theta_1} \qquad (17)$$

Equations (10), (13), (14), (15), (16) and (17) may readily be combined, and the solution may be written as:

$$K = \sqrt{\frac{b - (1-k)\,(1-a)\left(\dfrac{B_1}{B_2}\right)^m}{a + k\,(1-a)\left(\dfrac{B_1}{B_2}\right)^m}} \qquad (18)$$

This is the same as equation (6) in the text. By taking the ambient temperature at sea-level to be the same as at altitude, $b = 1$, equation (5) is obtained. If in (18), a, the ratio of the drop from the copper to the iron to the total temperature rise at sea-level, is taken as zero, equation (4) is obtained. Then, again, if in equation (4), $b = 1$, equation (3) follows. Also, if in equation (18) the ratio of ratings at altitude and at sea-level be taken as unity, equation (2) is obtained. Equation (1) may be obtained from (2) by placing $a = 0$. Then the surface rises (θ_{s1} and θ_{s2}) are taken to be equal to the total rises θ_1 and θ_2.

Appendix II

DISCUSSION OF VALUES OF *k*

The value of the fraction of the loss in the core that is proportional to the square of the load, k, is necessarily largely dependent upon the type of machine, upon the speed, upon the voltage, upon the choice of proportions by the designer, etc. A number of machines of three types were chosen at random, as given in the table.

SALIENT POLE ALTERNATORS

Rating at Sea Level

Kv-a.	Volts	Frequency	Rev. per min.	*k*
18750	12000	60	150	0.498
1250	13200	60	112	0.302
850	2200	60	100	0.730
12500	13200	60	720	0.370
10000	15000	50	600	0.590
				0.498 = Av. *k*

LARGE INDUCTION MOTORS

H. p.	Volts	Frequency	Syn. Rev. per min.	*k*
1200	2200	60	600	0.206
1200	6600	25	500	0.475
1400	2200	60	514	0.448
1500	6600	25	375	0.320
1200	2200	60	300	0.316
1500	2200	60	360	0.357
				0.353 = Av. *k*

D-C. GENERATORS AND MOTORS

Kw.	Volts	Rev. per min.	*k*
500	250	1200	0.216
1000	250	720	0.380
1500	250	514	0.420
300	250	150	0.407
600	250	100	0.450
1000	250	100	0.508
			0.397 = Av. *k*

Mean of the three values of *k* = .416

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[3] Those equations will be found in a paper by the author: "Longitudinal and Transverse Heat Flow in Slot Wound Armature Coils.", TRANS. A. I. E. E., 1921, p. 589.

From Fig. 4 it will be seen that the value of k has considerable influence upon the rating. For the general average case, the value of 0.4 is probably not far wrong. The curves in Figs. 5 and 6 were plotted for that value. Inasmuch as the equations involving k are of principal value to the designer, he can readily determine the proper value to assign to it, and estimate the rating at the higher altitude by substituting in the proper equation. For short machines the longitudinal flow affects the distribution, and for such machines the value of k should be reduced.

Appendix III

DISCUSSION OF VALUES OF m

The value to assign to m, the barometric pressure ratio exponent, could easily be made the subject of an entire paper. Only a brief outline of the subject and mention of several papers are given in this note. As stated in Appendix I, m is dependent upon two factors which are additive, (a) the rate at which heat is transferred from a heated surface to the moving fluid, and (b) the temperature increase of the cooling fluid from entrance to the machine up to the point under consideration. These two do not bear a fixed relation to each other; in one type of machine the relation may be quite different from another type. For example, in a d-c. armature, the temperature rise of the air up to the parts of the radial vents considered is probably quite small; on the other hand, in a high speed steam-turbine driven alternator the air rise is usually considerable before it reaches those parts of the vent ducts which are closest to the points where the temperatures are measured.

The volume of air per unit time which passes through a machine is independent of the barometric pressure. (This follows because the pressure generated by the fans and the pressure drop through the various paths of the machine are both proportional to the density, and the generated and consumed pressures are equal to each other). As the mass of air per unit of time is proportional to the density, the mass varies directly with the density or with the barometric pressure. As the temperature rise of the air is inversely as its mass, (assuming that the same heat is taken up by the air), the temperature rise is inversely as the barometric pressure.

In regard to the rate of transfer of heat from the surface to the cooling medium, there are a number of papers available. Perhaps the best work is that of Nusselt[4], and he found experimentally that the density ratio exponent is 0.786. Pohl[5] used that value in the derivation of equations applicable to machines using a closed circuit system of cooling. One of the most recent publications is that of Rice[6], who obtains his results largely theoretically, using the dimensional method, and in his final equations for heat transfer for the turbulent state of ideal gases, for smooth and for moderately rough surfaces, the exponent of density is given as unity. Rice's paper contains a large number of references, and any one interested can consult that paper. In the paper by Doherty and Carter[7], the exponent for forced convection was found to be 0.73, and to simplify calculations, they used 0.75. This value seems to be approximately correct for the machines for which they give data in their paper. As we understand their results, the exponent 0.75 takes account of the air rise as well as the surface transfer. It is felt, however, that their tests are too limited to warrant us to draw conclusions.

It is believed that since the total drop is made up of two items which are additive and since the exponent for one of them is probably not far from 0.80 and the other is unity, a mean of 0.9 may be chosen for the general case. It would be well to obtain further experimental checks. As may be seen from Figs. 2 and 5, if 0.9 be adopted, the error in temperature or in rating is not great for probable departures above or below this value.

Appendix IV

CASES OTHER THAN A SLOTTED CORE

The manner of treatment of a few cases other than a slotted core will be considered.

1. *Commutators.*

The losses are evidently the brush friction and $I^2 R$, the influence of windage loss upon temperature being negligible. The friction losses are evidently constant and the $I^2 R$ losses are proportional to the square of the load. Equations (1), (3) and (4) are applicable.

2. *Stationary Field Coils as in D-c. Machines.*

A. Shunt Coils. If the field current may be assumed to be the same at all loads, and surface temperatures are to be measured, equations (1), (3) or (4) may be used; otherwise, if the temperature rise is measured by resistance, equations (2), (5) or (6) may be used, taking $k = 1$. If the field current changes with the load, the temperature equations (1) or (2) may still be used. Unless the change in current with load can be incorporated in a simple equation which can be combined with other elementary equations, the rating may be approximated by a cut-and-try method.

B. Series or interpole coils, or compensating windings. The current is proportional to the load, and the losses to the square of the load. If the surface temperatures only are to be measured, equations (1), (3) or (4) may be used, taking $k = 1$. If these windings are insulated and temperatures are measured by resistance, equations (**2**), (**5**) and (**6**) may be employed.

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[4] Nusselt, "Heat Transmission in Conduits," *Zsch. d. V. D. I.*, 1909, p. 1808.

[5] Robert Pohl, "Fundamentals of Heating Calculations of Electric Machines, Especially Turbo Generators, Cooled by the Circular Process", *Arch. f. Elek.*, 1923, p. 361.

[6] C. W. Rice, "Forced Convection of Heat in Gases and Liquids", *Industrial and Engineering Chemistry*, May, 1924, p. 460.

[7] "Effect of Altitude on Temperature Rise", TRANS. A. I. E. E., 1924, p. 824.

It is recognized that, due to change in armature losses with load, the temperature of the cooling air changes with the load. That introduces a complication, and furthermore its influence is usually small, if consideration is given to the fact that at the higher altitude the rating and losses are reduced.

3. Revolving Field Alternator Field Coils.

A. Single layer edgewise winding. Temperatures measured by resistance. The temperature rise for a given load may be estimated by equation (**1**). The changed rating may be estimated by approximating the field current for various loads, and with the use of equation (**1**), calculate the temperature rise ratio. A curve may then be plotted coordinating temperatures with ratings.

B. Embedded windings, such as for turbo alternator rotors; or field coils for salient pole alternators with insulation. Temperature rise measured by resistance. Equation (**2**) may be used for temperature rise ratio. The changed rating may be estimated in the same manner as for single layer edgewise winding, except that equation (**2**) should be used instead of equation (**1**).

Discussion

R. E. Doherty: The accompanying Fig. 1 represents a generator. The cooling air enters at a temperature T_1. We are considering some spot, S, on the surface. There is a temperature rise of the cooling air which is occasioned by its passing over the heated surface before it reaches the spot in question. This brings the cooling air, the air to which the surface transfer is made at that spot, up to T_2. Then there is a rise of the surface temperature above that, which is T_3. There is a still further rise of the copper to T_4.

I wish to emphasize that Mr. Fechheimer's equations refer to some particular spot. T_3 is the surface temperature at that spot, and T_4 is the copper temperature. The temperature rise $(T_4 - T_3)$ is not a function of the barometric pressure. The temperature rise $(T_3 - T_2)$ is a function of the pressure, varying, let us say, as the 0.75 power. The rise $(T_2 - T_1)$ is also a function of the barometric pressure, but not the same function as for $(T_3 - T_2)$. It varies as the 1.0 power instead of 0.75.

The temperature rise of the surface spot, S, will depend upon the heat dissipated from it. Mr. Fechheimer's factor k, which corresponds to the factor a in the paper[1] presented by Mr. Carter and myself in 1924, is the fraction of the loss dissipated from the spot or surface which is due to $I^2 R$ loss. It should be observed that this is not the ratio of the copper loss in the machine to the total losses. There may be spots from which there is practically no heat due to $I^2 R$ loss, and others where all of it is due to $I^2 R$ loss. Hence, the table for k in Appendix II is without meaning until it is specified to what surface the values apply.

The author brings out the fact that there are two view-points from which this problem may be treated, which I should state as follows: One may take the temperature rise at sea-level for a given load, and calculate the increase in temperature rise *for the same load and losses* when the machine is operated at altitude. The other is to calculate what reduction in load, and, therefore, losses, is necessary to give the same temperature rise as at sea-level. The author takes the latter viewpoint; our paper in 1924 took the former. If correctly done, either gives adequate data for rules. I found the problem difficult enough without having to worry about the redistribution of losses and the necessary additional and very questionable assumptions required thereby. Since, however, the author's results check with those we obtained the other way, I naturally endorse it on the grounds of Dr. Kennelly's philosophy. If it is a convenient form of representation, and fits the facts, it is a perfectly good thing to do.

Now as to what Mr. Fechheimer says about my paper: "In the paper by Doherty and Carter, the exponent for forced convection was found to be 0.73 and to simplify calculations, they used 0.75. This seems to be correct for the machines for which they give data in their paper. As we understand their results, the exponent 0.75 takes account of the air rise as well as the surface transfer. It is felt, however, that their tests are too limited to warrant us to draw conclusions." He adds also that the results were of too complicated a nature to be used as a basis of engineering rules.

Our paper covered the general case. Convection, which is the only factor considered by Mr. Fechheimer, is not the total means by which heat is dissipated. Radiation may be a dominating factor. There are all grades between predominating radiation and predominating convection. Certain tests which I made, gave this figure 0.75, which checked with results on machines so far as they were available at the time. I still think it is about right. We took the rise $(T_3 - T_2)$ into account in a term which we called y. The rise $(T_2 - T_1)$ was accounted for in the term z. We properly combined the two results and called that x. We did not lump them, as Mr. Fechheimer has done, into one factor varying as, say, 0.9 power. It depends upon other factors.

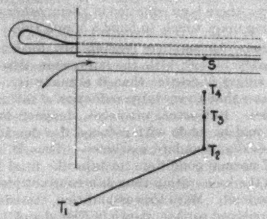

Fig. 1

Mr. Fechheimer says that our expressions are too complicated. The phenomenon itself is complicated of course, when the general case is considered. But after obtaining the equation for the general case, the specific data applying to a special case, such as Mr. Fechheimer has treated, may be substituted, and the resulting forms are extremely simple. Our paper gave these simple forms for various special cases, and the relation for the present case was a linear relation as simple as Ohm's law; and Mr. Fechheimer's numerical results, as shown in his Fig. 2, are in substantial agreement with ours.

So, his first point that the exponent 0.75 was taken as applying to both temperature rise, $(T_3 - T_2)$ and $(T_2 - T_1)$, isn't correct. They were not lumped together but were treated separately, and then logically combined.

Now, as to the point that results given were too limited for the drawing of conclusions: there may be justification of the statement that they are too limited to draw *broad* conclusions, but I question his justification in saying they are too limited to draw *any* conclusions. We set up a logical theory under definite assumptions and then justified the conclusions by experiment. That is sound engineering. If it satisfies all the facts as they are known, it is reasonable to take the results as applicable within the limit of those facts and assumptions. That is just what we did. A rational theory was set up, taking into account not merely the condition of forced convection, but also the general case where part is radiation and part convection which takes into account the other details which may be thrown out or left in, as conditions may demand.

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[1] *Effect of Altitude on Temperature Rise*, by R. E. Dougherty and E. S. Carter, Trans., A. I. E. E., 1924, p. 824.

Then we made certain tests which checked up points on which existing data were insufficient, and applied that theory to actual machines under the conditions of pressure below atmosphere. The results agreed. Other tests were made on an actual machine in the field, first at sea-level, then at high altitude. The data obtained on the machine were not complete, but such as were complete were in satisfactory agreement. So upon that basis I venture to suggest that our results are competent within the limits we specify.

I wish to say just a word about the several statements that equations are too complicated for A. I. E. E. rules. Of course they are. An investigation, which of course involves equations to determine a basis for framing simple rules, is one thing, but making those simple rules is quite another. The present paper, covering one case, and our paper in 1924, which covers many, including that one, are such investigations. Mr. Paxton's proposal represents such simple rules. Therefore don't throw out results because there is an equation in them; otherwise you won't have anything upon which to build a simple rule.

P. L. Alger: An amateur reading Mr. Fechheimer's paper would probably be left with the impression that the effect of altitude on rating is greater than it actually is. For instance, Fig. 6 indicates that a very large reduction of rating may occur at high altitudes. But actual machines, designed for use at high altitudes, would be made with reduced flux densities and lower no-load losses than standard machines. Thus, it is more representative of normal conditions to take the total losses as proportional to the kv-a. rating than as a constant plus K times the square of the load. With this assumption, the effect of altitude on rating is reduced to that shown in Figs. 2 and 3 of the paper.

In the second place, the normal ambient temperature falls about 5.5 deg. cent. for each thousand-meters increase in altitude, or by nearly the same amount as the increase in full-load temperature rise of a normal machine due to the reduction of atmospheric pressure for the same increase in altitude. Therefore, if the cooling air is taken from outside the station, there is practically no change in rating required by change in altitude in the average case. Only when open machines designed for use in heated stations are considered is it necessary to allow for any reduction of rating. And even in these cases the maximum ambient temperature to be expected is certainly less than the maximum ambient temperature that may be met in sea-level operation, so that here, too, the reduction in rating need not be as great as that indicated by Figs. 2 and 3.

E. B. Paxton: I think we should thank Mr. Fechheimer for pointing out quite clearly the different ways in which the altitude correction may be taken into account provided there is sufficient information to do so. As he has said, the present A. I. E. E. rule takes into account only the case where the temperature rise is limited in a test near sea-level by the amount of the correction for installation at a higher altitude. For this condition, the results that he obtains check, fairly closely, the results obtained previously by Mr. Doherty, as well as the present A. I. E. E. rule.

I think the present A. I. E. E. rule is faulty principally in that it takes care of that one condition only. It is my opinion that in a great many cases, a correction is made where it is unnecessary, and that, as Mr. Alger has pointed out, the ambient temperature will be sufficiently low to offset the correction necessary for the altitude.

This matter of correction for altitude would seem to be a relatively unimportant matter considering the many other conditions that affect the design and installation of a machine. For that very reason, we should be just as sure that any rule which is standardized does not require corrections unnecessarily as we are to cover the necessary cases.

Along this line, about a year and a half ago, I proposed a rule for inclusion in the A. I. E. E. Standards which is, I believe, an improvement on the present A. I. E. E. rule in that it does take advantage of a reduction in ambient temperature at the higher altitudes and covers one of the cases which Mr. Fechheimer mentions in his paper. I think we must regard two kinds of applications: first, that of a standard machine, and we should make rules which will allow its application under all conditions where it may safely be used; second, we must recognize that there are conditions where it is necessary to use a special machine. I should like to see the present A. I. E. E. rule extended to allow the use of machines of standard temperature rise at high altitudes, provided the ambient temperature is low enough to compensate for the increased temperature rise. I am convinced that there are many cases where standard machines may be used safely at the higher altitudes without any correction.

W. F. Dawson: First of all, I want to suggest the probability of the ambient temperatures at high altitudes not necessarily being lower than at sea-level. I have a record of certain tests on a 2550-h. p., 3600-rev. per. min., synchronous motor which I designed and tested some four years ago at our factory in Lynn, Mass., and repeat tests that were made in the Andes Mountains at an altitude of 14,000 ft. The ambient temperature during the mountain test in January was 21.5 deg. cent., which compares approximately with that in this vicinity. Of course, had that 14,000-ft. altitude been in the Canadian Rockies, the situation would have been different, but one must not forget that high altitudes exist also in tropical countries.

It seems to me that the crux of the situation is all ambient temperature. If the density of the cooling air, due to greater elevation, is lower than standard, its thermal capacity for a given volume is proportionately less. The average temperature difference between inlet and outlet air on self-ventilated turbine alternators at sea-level will be between 15 deg. cent. and 30 deg. cent.

At the end of the test of the machine to which I have just referred, inlet air was 28.7 deg. cent. and outlet air 45.8 deg. cent., a difference of 17.1 deg. cent.

According to the formula used in Mr. Fechheimer's paper, the reference-air density was about 0.6 atmosphere which would mean that, at the 14,000-ft. station, the temperature rise of the air after passing through the motor would be 17.1 deg. divided by 0.6, or 28.5 deg., an increase of 11.4 deg., and that is slightly more than the temperature differences we ordinarily expect. It is unfortunate that, while we used temperature indicators at the factory, they were not available at the high elevation; but we did get a very good check on the field in spite of the fact that it was run at the higher current.

In turbine alternators (and this synchronous motor is a turbine alternator fitted with a starting winding), one of the major losses of the machine is the windage loss. Windage loss will be from 0.3 to 0.5 of the total. In this particular case, at sea-level the total loss passed through the machine at its rating, that is, the loss handled by the air, was 57.3 kw. Of that amount, 19.8 kw. was windage. What happened at the higher altitude? The 19.8 must be multiplied by 0.6 because the volume is the same at higher altitude as at the lower altitude, and the windage loss is in proportion to the density of the air, so 8 kw. of that loss are saved at the higher altitude. That is one of the credits of the problem.

The heating by increase in resistance of that field with 93.5 amperes at sea level was 40.8 deg. cent., of which 12.7 deg. was due to windage loss alone, leaving 28.1 deg. attributable to $I^2 R$. The test at higher elevation was run at 100 amperes. If we multiply that 28.1 by $\left(\frac{100^2}{93.5}\right)$, we obtain 32.3 deg. That will be the temperature rise due to copper loss, but the total temperature rise must allow also for the air friction. This will be no longer 12.7 deg. but 0.6 of 12.7 deg. = 7.6 deg. Adding this to 32.3 gives 39.9 deg. as the estimated field rise for the higher altitude. The value reported from Peru was 40.3 deg. cent.

My feeling is that the problem does not involve much of mathematics, but judgment combined with experience, and that it probably reaches its greatest simplicity in turbine alternators and similar machines where the air paths are systematically directed. The principal difference in temperature rise will be due to the fact that the ventilating air, because of lower density, will have a higher average temperature at the air-gap and where the windings are to be cooled; hence their temperature will be correspondingly greater.

The fact that the low-density air has a correspondingly less windage loss must not be overlooked. The full-load efficiency of the motor I have just been discussing was 96.45 per cent at sea-level. About 8 kw. of windage loss was saved due to rarefied air at 14,000 ft. altitude, thus bringing the efficiency up to 96.84 per cent.

Sometimes it has been possible to apply the principal of the supercharger to machines which otherwise would have to be derated for operation at high altitudes. It is only necessary to provide special fans which increase the volume of air to correspond with its reduced density and thus insure the same temperature rise ordinarily obtained at sea level.

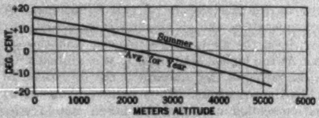

FIG. 2

H. M. Hobart: I am of the opinion that from a standardization standpoint this subject will be best straightened out on the basis that the rating will be the available capacity of the machine at sea-level. As Mr. Fechheimer suggested, sometimes you can take more out of the machine at higher altitudes, and sometimes less, according to the prevailing temperatures. That won't change the rating. The rating will be the available capacity at sea-level and 40 deg. cent. cooling-air temperature.

G. E. Luke: The present A. I. E. E. rules (1925) for rotating motors and generators, (excepting railway motors), cover altitude limitations as follows:

"Unusual Service Conditions: The use of machines in cooling mediums having temperatures higher than 40 deg. cent. or at altitudes greater than 1000 meters (3300 ft.) should be considered as special."[2]

A similar statement is found for air-blast transformers; however, for self-cooled oil insulated transformers, the correction is 4/10 per cent per 100 meters instead of one per cent. From the wording of the footnote ("it is provisionally agreed"), it is assumed that this correction was subject to doubt.

Both this paper by Mr. Fechheimer and a previous one by Messrs. Doherty and Carter[3] have brought out the fact that the altitude correction is considerably influenced by the ratio of copper-to-core loss and by the temperature gradient through the insulation. These factors are of considerable importance when temperatures are determined by embedded temperature detectors.

On the basis of certain assumptions, all of these factors can be combined into a group of equations; however, since some of the constants are factors of design, the equations are not in a suitable form for incorporating in the A. I. E. E. Standards.

In the discussion of the Doherty and Carter paper, Mr. E. B. Paxton suggested a plan whereby the maximum ambient temperature for various altitudes could be limited on the basis of a sea-level rating. Mr. F. D. Newbury's modification of this plan, I believe, offers a suitable solution for this unusual service condition. Naturally, such a simple solution is a compromise and is only approximate. It is warranted due to the very small per cent of machines operated at altitudes above 1000 meters (3300 ft.) and also to the average decreasing ambient temperature with increasing altitude. The accompanying Fig. 2 shows the average summer temperature at various altitudes taken near Paris, Brussels, Strassburg, and Munich, as given by the Smithsonian Physical Tables.

Where machines are to be operated indoors at high altitudes with ambient temperatures greater than those specified, the conditions should be regarded as special and the corresponding rating can be specified by the manufacturer as calculated by the method given by Mr. Fechheimer.

C. J. Fechheimer: It would seem that Mr. Doherty has understood me to be critical of the work which he has done. That is far from my purpose, for I appreciate the value of it. As he has pointed out, I have evidently mis-read one statement in his paper. Since he is considering the temperature rise of the surface above the air adjacent to it, and since his results and mine agree, there is no occasion for further discussion on this point; in fact, it is gratifying to note that we are in agreement.

With the simplified equation (36) in his paper, he takes account only of the surface drop and must then add the air temperature rise to it. In general, the air temperature rise is not known, particularly if it is taken from the entrance of the machine up to a particular point. Consequently, the purchaser of a machine is still considerably in doubt as to how to apply the equations. The method which I use, of taking a mean between the exponent, 1, for the air rise, and the exponent of about 0.75 to 0.80 for the surface rise, is not mathematically correct; nor with the possible variations in air temperature rise as compared with the surface drop, is it strictly correct to apply the method which I have used. My purpose was to place this complex subject on such a simple basis that the A. I. E. E. Standards Committee could embody it in its rules. Referring to Fig. 2 in my paper, it will be seen that the influence of the variation from an exponent of 0.75 to 1. is not very great; consequently, taking as an approximation a value of, say, 0.9, the error is not of great magnitude. After all, that is all with which we are concerned. In the present A. I. E. E. rule, as the total temperature rise is considered, it is not broken up into its two components, and in my opinion it will be necessary, in any revised rule, to follow this same practise.

Mr. Doherty refers to the constant k in my paper, which is the fraction of the loss in the core that is proportional to the square of the load at sea-level rating. In making up the tables in Appendix II, I considered that all the $I^2 R$ loss that was in the embedded part was liberated from the core, or from the straight parts of the coils, and that the end windings took care of themselves so far as dissipating their generated heat. That assumption may not be justified, and for accurate solutions, consideration should be given to the longitudinal heat flow along the copper and the transverse heat flow through the insulation. That, however, puts the problem entirely out of the range of a practical solution. Fortunately, this factor k disappears when only the temperature-rise ratios are considered. It appears in the equations when the ratio of ratings is solved for. After all, those solutions for ratings will be of value probably only to the designer and not to the purchaser of the machines, and the designer can approximate the percentage of heat generated in the embedded copper which is transmitted longitudinally or transversely. In the long-core machines, he will not introduce much of error if he considers all of the embedded $I^2 R$ loss to be liberated in the straight part. More of the error is introduced in the short-core machine.

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[2] "For apparatus intended for service at altitudes greater than 1000 meters (3300 ft.) it is provisionally agreed that the permissible temperature rises (to be included in contracts and checked by test at low altitude) shall be less than specified in these standards by 1 per cent of the specified rise for each 100 meters (330 ft.) of altitude in excess of 1000 meters (3300 ft.)."

I confess that I do not understand how Mr. Doherty has reached the conclusion that I considered only the reduction in load, and not the increase in temperature. Equations (1) and (2) are solutions on the basis of temperature. Equations (3) to (6) are on the basis of ratings.

It is true that I have considered forced convection only. In the majority of machines in which fans or their equivalent are used, radiation may be excluded, and even free convection is of little consequence. My paper is not intended to cover such apparatus as self-cooled transformers or motors which liberate a large percentage of their heat through the outside casing by radiation or free convection. The statement which I made in my paper, to which Mr. Doherty refers, *i. e.*, that the equations in his paper were complicated, was prompted by such equations as (**18**) and (**30**), the former covering the general case and the latter the case for forced convection. My whole purpose in writing the paper in the manner as presented was to place it in simple form before the members of the Institute.

Mr. Alger states that by reducing the flux densities, the ratings need not be reduced. That is undoubtedly so, but in many cases machines with standard windings will be applied at the higher altitudes, chiefly to save the expense of making special machines. The other point that Mr. Alger makes in regard to ambient temperature would, I think, bear further study. While it is true that we have, from the Bureau of Standards, data on ambient temperature, and, as Mr. Luke points out in his discussion, from parts of Europe, a more thorough study of this whole subject could be made by the Standards Committee in order to offer suitable recommendations. Undoubtedly, in many cases the lower ambient temperature will practically offset the loss in rating that would otherwise be occasioned by the lower density.

Mr. Paxton's comments are largely along the same line of ambient temperature. He previously contributed valuable material in his discussion of Mr. Doherty's paper, and his suggestions are worthy of study by those who will participate in the revision of the rules.

The point that Mr. Dawson makes, that the ambient temperature at 14,000-ft. elevation in January was 21.5 deg., is of considerable interest. However, January is summer in South America, and the particular locality was not far from the equator. The specific machine upon which he had tests made, *i. e.*, a turbo alternator for which the windage loss decreased considerably, is undoubtedly a special case and cannot be treated along with the general cases. While special fans may be added to increase the volume and thereby obtain lower temperature rises at the higher altitude, in general such a scheme is not to be adopted as it would make the machine so highly specialized that the manufacturers would seldom go to the expense, nor would the customer wish to pay for it.

Since writing the paper, my attention has been called to a rather unusual phenomenon. It is claimed by some of the railway engineers that in a-c. commutator motors, brush-friction loss decreases with increase in current. It is even claimed that some of these commutators run cooler at normal full-load than they do at no-load. I am offering no explanation for the cause of this phenomenon, but, assuming that the statement is correct, the correction for altitude as given for commutators in Appendix IV no longer holds. Not knowing the rate of decrease in brush-friction loss with increase in $I^2 R$ loss, it is impossible for me to suggest a solution to that part of the problem.

One other item of comparatively small importance was not considered in the derivation of the equations. With lower ambient temperature, the air density is increased; with increase in density, both the air-temperature rise and the surface drop decrease. At the higher altitudes, if the ambient temperature is lower, the rating of the machine is not decreased so much as indicated by the equations. It is believed, however, that this additional correction need not be given much consideration.

Motor Band Losses

BY T. SPOONER[1]
Member, A. I. E. E.

Synopsis.*—It is shown that railway motor band losses are of appreciable magnitude, sometimes sufficiently large to be detrimental to the cooling of the machine. By tests of a small machine checked against those of a large one, the band losses are found to vary according to the 1.7 power of the frequency and from the 1.35 to the 1.8 power of the induction, depending on the width and type of band. These losses are shown to be due chiefly to the change in the radial component of the flux as the band passes by the pole tip. For average bands, about 15 per cent of the losses (hysteresis and eddy) are due to the tangential flux in the bands. A typical set of curves is shown for calculating band losses.*

* * * * *

IN most types of rotating electrical apparatus, the rotor windings are held in position by some sort of slot wedge. However, in the case of d-c. railway motors, it is almost universal practise (in this country at least) to hold the rotor windings in the slot by means of wire bands. These bands, when over the core material, are placed in shallow slots in the core. They are from ½ in. to ¾ in. wide and are spaced approximately three in. apart. Much wider bands are often used over the end windings but in this paper we shall be concerned only with the bands over the core.

It is often assumed that the band losses are so small as to be negligible; and even if they are not negligible, the heat is readily dissipated, since the bands are on the surface of the rotor. As a matter of fact, band losses are sometimes of quite considerable magnitude and they occur adjacent to the windings, thus preventing loss of heat from the windings and teeth, even though they do not actually transfer heat to these parts. Moreover, the band losses may materially heat the cooling air in the air-gap, thus making it much less effective.

BAND CONSTRUCTION

There are many kinds of band construction and band materials in use, but here we shall deal with only three or four common types. The steel banding wire for the

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experiments to be described had a diameter of either 0.0453 in. or 0.0641 in., and a resistivity of about 18.5 microhm-centimeters. The bands were wound on a 0.0125-in. soft iron, tinned strip which, in turn, was insulated from the core by means of a strip of asbestos tape. In order to reduce the band losses, in some cases each band was divided into two sections, each section having its own strip and the two sections insulated from each other. The wires of each band or section were held together by clips of 0.0125-in. soft iron about ½ in. wide, spaced approximately every three inches. The wires were wound on the armature under considerable tension and the wires, strips and clips were soldered together. The solder was of pure tin, having a resistivity of about 13 microhm-centimeters.

FACTORS AFFECTING THE BAND LOSSES

The problem of band losses is much more complicated than is evident from a first inspection. So far as the phenomena have been analyzed, the following factors enter into the problem:

a. Width of bands
b. Width of teeth at air-gap
c. Size of band wires
d. Amount of solder used
e. Dimensions of tinned strips and clips
f. Normal resistivity of wire, solder and tinned strips
g. Temperature of bands
h. Frequency
i. Induction
    1. Tooth (radial)
    2. Band (tangential)
j. Field form
k. Air-gap
l. Depth of band groove

If the bands are not insulated from the core, the losses due to the bands may be anywhere from the insulated values to many hundred per cent higher, the values depending on the intimacy of contact between the bands and laminations. The following discussion assumes perfect insulation from the teeth.

There are several types of band loss for a given total armature flux which may be classified as follows:

1. Eddy-current losses in band wires, solder and tinned strip, due to radial flux.

2. Eddy-current losses in band wires, solder and tinned strip due to tangential flux.

3. Normal hysteresis loss in band wires and strip due to tangential flux.

4. Additional hysteresis losses due to displaced hysteresis loops.

5. Additional armature tooth loss (eddy-current) due to the damping out of flux by the bands in the region of the pole tips, thus producing flux across laminations which may result in extra eddy losses.

The eddy-current losses which occur in the bands themselves, due to item 1, (radial flux), are by far the most important, as can be shown by a few simple calculations from data available. The induced voltage in the bands has much the same wave form as given by the familiar case of an exploring coil surrounding a tooth and connected to an oscillograph. It has, in general, two large positive humps, followed by two large negative humps, with many intervening ripples. The large humps are produced when the tooth passes a pole tip and their shape and magnitude are a function of the shape of the pole tips.

The tangential fluxes passing from one pole tip to the next, through the bands, are usually of much higher flux density than the radial fluxes, but due to the small thickness of the bands, the eddy-current losses are small. However, the hysteresis losses correspond to the tangential flux, namely, the maximum induction.

The bands are subjected to an elliptical field and we shall assume that the hysteresis loss is the same as would be produced by the maximum tangential induction acting under alternating flux conditions. This tangential flux is a maximum in the position just before the band passes under the pole tip. It is lower at a position midway between the poles and zero at the center of the pole. This decrease in flux at the midpoint between the poles produces a minor hysteresis loop. Since this loop is greatly displaced from the normal position, there results appreciable increase in the hysteresis losses.

Due to the eddy currents in the bands, caused by the radial flux and the consequent damping out of the flux through the bands, the air-gap flux must tend to pass around the bands, giving decreased relative induction in that portion of the tooth under the bands. These fluxes tend to become uniform again under the bands, thus producing a flux component at right angles to the plane of laminations which may produce appreciable eddy-current losses.

TEST APPARATUS

The test results to be described were obtained chiefly on a typical small four-pole railway motor direct-coupled to a d-c. shunt motor. Two armatures were provided with the following dimensions:

	Armature A.	Armature B.
Diameter	9 in.	9 in.
Length	7 "	7 "
Number of slots (open)	31	16
Slot width at air-gap	0.370 in.	0.635 in.
Slot pitch at air-gap	0.911 "	1.77 "
Slot depth	1.00 "	1.50 "

The armatures had three band grooves 0.125 in. deep and 0.75 in. wide. The motor was provided with two sets of poles, one normal and the other chamfered 1/16 in. at each tip. The bands consisted of fourteen turns each of 0.0453-in. wire or eight turns each of 0.0641-in. wire. Both single and double bands were used for the larger size of wire. In the case of the double bands, each section had four turns.

TEST METHODS

Tests were made with three air-gaps; namely, 0.1 in., 0.2 in. and 0.3 in. and three speeds; 600, 1200 and 1800 rev. per min., corresponding to 20, 40 and 60 cycles respectively. Chamfered poles were used with 0.0452-in. and 0.0641-in. bands and 0.1-in. air-gap only.

The d-c. drive motor was supplied by a storage battery. No-load losses were determined with the bare armature for the various air-gaps and for various field excitations. The armature flux-per-pole was determined for various excitations by means of an exploring coil and d-c. voltmeter connected through a synchronous contactor. Corresponding maximum armature tooth inductions were determined ballistically by means of a tooth exploring coil and a ballistic galvanometer. Also the r. m. s. tooth voltages were determined by means of the tooth exploring coil and a Paul dynamometer type voltmeter which had a very small frequency error. From a previous investigation, data were available for the actual tooth-voltage wave forms as obtained by oscillograph.

In order to determine the tangential inductions in the bands the exploring coil was wound on the center band between two teeth. The motor field was reversed and the band inductions obtained ballistically. This was done for various field strengths for the position opposite the center point between the poles and for the position of maximum tangential induction, namely, with the exploring coil a little beyond the tip of the pole. Also a curve was obtained for one field strength and the various air-gaps between tangential band induction and armature position for a rotation of 90 electrical degrees.

TEST RESULTS

While the test results are referred to the average tooth-top induction, since this factor probably correlates best with the band losses, it should be remembered that the actual radial band inductions are considerably less and sometimes only about one-half the values corresponding to the average tooth top induction, due to the greater reluctance produced by the band grooves.

Fig. 1 shows some typical curves for tangential band inductions, which inductions are chiefly responsible for the hysteresis losses. The radial flux is, of course, approximately proportional to the tooth-top inductions and has nearly the shape of the field form. Fig. 2 shows some typical band-loss data for armature *A*, with the armature core, tooth and pole-face losses included for comparison. It will be noted that in one case the tinned strip was omitted from under the band wires.

Fig. 3, plotted on double log paper, shows band-loss calculation curves for a 0.0453-in. insulated band, eight wires wide. This method of plotting makes a very convenient form in which to have the results, since the losses vary approximately exponentially with frequency and induction. From theoretical and somewhat meager test considerations the band losses apparently vary about as the square of the band width. If desired, another curve could be added to Fig. 3, giving a factor to take care of the width.

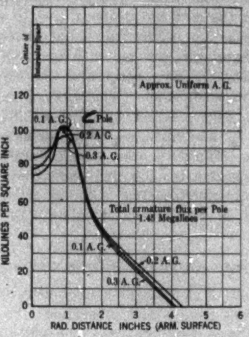

FIG. 1—TANGENTIAL BAND INDUCTIONS—0.0641 BANDS

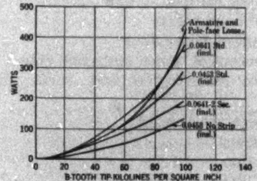

FIG. 2—BAND LOSSES FOR D-C. RAILWAY-MOTOR ARMATURE "A" 0.2 AIR-GAP—1200 REV. PER MIN.

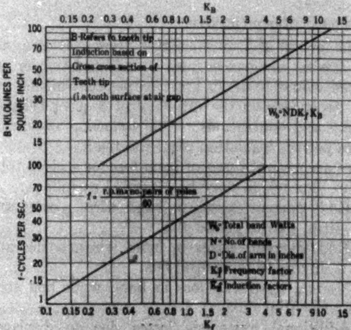

FIG. 3—BAND LOSSES FOR D-C. RAILWAY-MOTOR ARMATURE 8-0.0453-IN. DIAMETER WIRES— ONE-SEC. STRIP

It was found that the band losses varied only slightly with air-gap for a given armature flux, due probably to the fact that with larger air-gaps, though the radial flux increased, the rate of flux change became less, thus giving a relatively smaller induced band voltage. Also pole chamfering had only a small effect due to the fact that while the maximum individual voltages were less, the maximum tooth inductions were greater for the same total pole flux.

The following table gives the band loss coefficients, tooth-induction and frequency exponents for the *A* armature.

TABLE 1
BAND LOSS COEFFICIENTS AND EXPONENTS
Coefficients are for 1200 rev. per min. (40 cycles), 80 kilolines tooth tip induction, and 0.1-in. air-gap.

Band (Diameter of wires-in.)	Coefficient	Exponents		Remarks
		Frequency	Tooth Induction	
0.0453 in.	71	1.71	1.52	No strip
0.0453 in.	230	1.71	1.63	Strip
0.0641 in.	238	1.67	1.73	Strip—one sec.
0.0641 in.	188	1.85	1.87	Strip—ch. poles
0.0641 in.	180	1.68	1.34	Strip—two sec.

The coefficients are the total losses due to the presence of the bands for the specified conditions. There is some variation in exponents and coefficients for various air-gaps and frequencies. The values given are the mean results for the exponents. Tests on a much larger machine gave approximately the same results.

In order to show the relation between the various band losses, the hysteresis losses due to the tangential fluxes were calculated for certain conditions as follows: 0.0641-in. bands (one section), 60 cycles, 80 kilolines/sq. in. tooth-top induction and 0.2, in. air-gap.

Tangential eddy current loss = 43 watts
Hysteresis loss . . . . = 23 watts

Sum . . . . = 66 watts
Actual test loss . . . . = 470 watts

This indicates that for these conditions 85 per cent of the loss is due to the radial flux.

It would normally be assumed that wider teeth would produce higher band losses. Tests obtained with the *B* armature, however, having only 16 as against 31 teeth and correspondingly wider tooth tops, gave approximately the same band losses. By means of a ballistic exploration of the tooth-top inductions over the leading and trailing portions of the teeth as they passed by the pole tips, it was found that most of the change in flux as a point on the tooth top passed the pole tip occurred over a distance equal to less than one-half the tangential width of the tooth top for the *B* armature. This means that for these particular poles a tooth top width of from 0.5 to 1 inch or over will give approximately the same band losses. For still narrower teeth there might be considerable difference.

Some tests made on the experimental and other machines show that if the bands are not insulated from the core, very much larger losses than normal will occur. The armature laminations are short-circuited on the shaft and the bands touching the laminations at the periphery complete an electrical circuit, which may make a very good path for induced currents. It was found possible to produce losses of several kilowatts in the small 7 by 9 armature by this means. It is possible to apply uninsulated bands without producing these extra losses but only in the case that the slot windings are so thick radially that the bands do not touch the core. In practise, this procedure is not possible and there is a general tendency toward the use of insulating materials even though most of the band tension is carried by the coils.

DISCUSSION OF RESULTS AND CONCLUSIONS

The eddy currents in the bands due to the radial flux which are chiefly responsible for the band losses are undoubtedly of sufficient magnitude to appreciably hinder the change of flux as the band passes under the pole tip. This is analogous to skin effect. It will be noted that at low and moderate inductions, the two-section bands have nearly as much loss as the wide single-section bands, although theoretically the losses for the former should be ¼. This is probably due partly to the fact that the losses for the wide bands are reduced by skin effect at the lower inductions. At the higher inductions, the magnetizing forces become so great that the damping effect of the eddies becomes less important and relatively more radial flux passes through the wide bands, thus giving a higher induction exponent for the wide bands than for the narrow. Again due to these damping currents, flux may pass around the bands under certain conditions, thus altering the eddy-current losses in the teeth by producing flux components at right angles to the tooth laminations. This is purely speculative. These induction exponents are lower than would be expected. It may be that heating of the bands has much to do with it.

The frequency exponents are less than two, due, undoubtedly, partly at least, to the skin effect in the bands.

It has been impracticable so far to derive a theoretical mathematical formula for calculating the band losses caused by the radial component of flux. This is an interesting problem for anyone mathematically inclined. The chief difficulty, even without assuming any damping effects, is to determine the most probable path of the eddy currents in the bands.

Considerable reduction in band losses may be effected by omitting the metal strip under the wires, but this gives less satisfactory construction from a mechanical standpoint. The insulation under the bands can not be dispensed with safely in all cases.

Non-magnetic band wire may be used if desired, but due to its inferior mechanical properties and slight effect on the band losses, its use is probably not justified.

A band wire and a solder of high electrical resistance would be of considerable advantage if they were otherwise suitable.

In large railway motors, the band losses may, under certain conditions, amount to two or three kilowatts and are therefore not negligible. Band temperatures up to 200 deg. cent., or more, may occur. At very high inductions the band temperature may increase so rapidly that the rate of increase of band losses with induction will sometimes decrease very considerably due to the increased resistance of the bands.

Discussion

G. E. Luke: In particular I remember one experimental machine designed for high-speed railway work that gave considerable trouble due to the bands coming loose. Some attributed this to the high centrifugal stresses; however, later tests proved the trouble to be due to excessive band losses. Careful insulation of these bands reduced the stray losses and no more such trouble was experienced. After Mr. Spooner had completed his first tests on band losses I arranged tests on a 50-h. p. railway motor. Small thermocouples were soldered to the bands and others placed on the winding and core. These couples were brought out to special slip-rings so that the temperature could be determined with the motor running. In this way tests were made with insulated and uninsulated armature bands. An appreciable temperature difference was found between the two tests. In some cases the uninsulated armature had temperatures 10 deg. cent. higher than that armature with insulated bands.

Due to the many other losses and factors that must be controlled, work of this nature must be done with a high degree of accuracy and requires considerable patience on the part of the experimenter. Mr. Spooner has also put the data in such form that they can be used in determining the losses on other machines of similar design.

J. C. Lincoln: Some years ago, a case of trouble from bands throwing solder was solved in the following manner: In this particular case the machine was an eight-pole machine and the bands, as is customary, were soldered entirely around the armature. The trouble was corrected by placing four strips for holding the wire of the bands together at 90 deg. around the armature and soldering the bands only where these strips were placed. This left a north pole and a south pole between each of the strips and the bands were not soldered between these strips. The total magnetic induction between the bands was zero.

In other words, between each typing strip there was a north pole and south pole and therefore there was no voltage induced in the bands between each of the tie strips. As the result of this change, the machine which had been throwing solder around the bands and giving trouble was corrected entirely.

P. L. Alger: I should like to ask Mr. Spooner if he has made a study also of the losses in the binding bands on the end conductors of rotating armatures, and, if so, whether such losses are ever large.

Thomas Spooner: Replying to Mr. Alger's questions, we have never investigated systematically the losses occuring in bands which hold the end windings of machines in position. These losses are often appreciable but we have no quantitative data concerning them.

In the case of Fig. 2 of the paper, the losses there given are for the unwound armature. If the armature windings had been in position, as they were for certain later tests, the losses would have been higher, due to eddy currents in the copper.

Starting Characteristics and Control of Polyphase Squirrel-Cage Induction Motors

BY HORACE M. NORMAN[1]

Associate, A. I. E. E.

Synopsis.*—In the application of squirrel-cage induction motors to such severe service as frequent start-and-stop or frequent plugging operation, it is desirable to know how much loss the motor will be required to dissipate, plugging being understood as changing the motor from any speed in one direction to any speed in the opposite direction, by reversing the rotation of the field only.*

The subject of starting loss is also of interest from the control standpoint—such as the application of auto-transformers for starting purposes, or where an external resistance is inserted in the primary as is done in the control of elevator motors.

Consideration is given to the part that the primary and secondary resistance and the total reactance play in the determination and the manipulation of the starting losses; and, as it is true that in many applications the time spent in accelerating the rotor from rest is useless from the production standpoint, consideration is given to the value of secondary resistance that will give minimum starting time for a given field strength. This involves a method for the determination of the time taken to attain a given speed in general.

There are many cases where a motor has already been designed and tested, and is about to be applied on a given job where the cycle of operation is known and includes either starting and stopping or plugging. In such an event instead of working with the various test values to get the constants of the machine, it is much more convenient and accurate to work from such values as starting torque, maximum torque, slip at full load, locked current and primary resistance.

It will be shown how the preceding short-cut method, for most cases, can be made more simple without sacrificing the accuracy of the result to any appreciable extent.

When a motor is plugged the problem is complicated by the difficulty that eddy currents flow in the rotor which give a slight increased effect to the torque at negative speeds. Although this paper does not show how to predetermine its amount, it does show how to handle the losses and the time of reversal, if the test speed-torque values are known or can be estimated from another machine.

* * * * *

When an induction motor is started up in series with an auto-transformer, the ratio of the loss in the motor and auto-transformer at any instant is a constant. The ratio of total loss over the entire starting period must have this same value. As this value can easily be calculated by considering any particular instant, such as the very start, it is obvious that the total loss in the auto-transformer is known when that of the primary of the motor is known. From this it can be seen that the two problems are reduced to that of the motor only.

It is true that in the vast majority of cases the r. m. s. value of starting current is very large compared to the magnetizing current and that sufficient accuracy can be obtained by neglecting the magnetizing current except in so far as it affects the torque.

Assuming then that the magnetizing current can be neglected when finding the r. m. s. value of starting current, it follows that the secondary current is equal to the primary current, also neglecting the iron and friction and windage loss and provided, of course, that the secondary circuit is reduced to terms of the primary. Therefore, at any instant the secondary loss to the primary loss bears the ratio of the secondary resistance to the primary resistance. Since this ratio is practically constant for all positive speeds of the motor, then it can be assumed that

$$\frac{\text{Total prim. loss over starting period}}{\text{Total sec. loss over starting period}} = \frac{\text{Prim. Resistance}}{\text{Sec. Resistance}} = \frac{r_1}{r_2}$$

The primary resistance is taken to be per leg of the winding, and the secondary resistance in terms of the primary can be found by:

$$r_2 = \frac{T_{st} N}{7.04 \, \phi \, I_L^2} \qquad (1)$$

where

T_{st} = starting torque
N = synchronous speed in rev. per min.
ϕ = no. of phases
I_L = locked amperes per leg

The secondary loss during the starting period can be evaluated by the following:

Sec. loss (watts × secs. = joules) $= \phi \int I_2^2 \, r_2 \, dt$

where

ϕ = phases
I_2 = sec. amperes
r_2 = sec. res. per phase (ohms)
t = time (sec.)

Proceeding further, let

S = final slip
s = slip (in general)
T = torque (in general)
N = synchronous speed (rev. per min.)
$(M I)$ = total moment of inertia in poundals-feet reduced to terms of the rotor.

Then

$$I_2^2 = \frac{N s T}{7.04 \, \phi \, r_2}$$ for any two corresponding values of s and T

and

$$dt = - \frac{2 \pi}{60} \times \frac{(M I) N \, ds}{T}$$ where dt and ds are infinitely small changes of t and s.

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Therefore, secondary loss (joules)

$$= \phi \int_s^S \frac{N\,s\,T}{7.04\,\phi\,r_2} \times r_2 \times \frac{-2\,\pi}{60} \times \frac{(M\,I)\,N\,d\,s}{T}$$

$$= -\frac{(M\,I)\,N^2}{7.04} \times \frac{2\,\pi}{60} \int_s^S s\,d\,s$$

$$-\,(M\,I)\,N^2\,\frac{2\,\pi}{7.04 \times 60}\left[\frac{s^2}{2}\right]_S^s$$

$$= 0.00744\,(M\,I)\,N^2\,(s^2 - S^2)$$

Joules loss in rotor $= 0.00744\,(M\,I)\,N^2\,(1 - S^2)$

Starting from rest (slip = 1) to speed $N\,(1 - S)$ (2)

Joules loss in rotor $= 0.00744\,(M\,I)\,N^2\,(4 - S^2)$

When rotor is plugged (slip = 2) to speed $N\,(1 - S)$ (3)

Joules loss in rotor and stator $= 0.00744\,\frac{r_1 + r_2}{r_2}$

$(M\,I)\,N^2\,(1 - S^2)$ (4)

Starting from rest (slip = 1) to speed $N\,(1 - S)$

(inertia being the only resistance to motion).

The conclusion to be drawn from the above is that the secondary loss during acceleration or retardation depends only on the total inertia resisting the change of speed, and the initial and final speeds; and also that when a motor is plugged the secondary loss, and also primary loss, is approximately four times as much as when the motor is only started up from rest. Further, since the primary loss is to the secondary loss as r_1 is to r_2 and the secondary loss is independent of the secondary resistance, then it follows that the higher the secondary resistance the lower will be the primary loss during the starting or reversing period.

This indicates that if a motor be used for frequent start and stop, or reversing service, the full load running period being only a few seconds, then a high resistance rotor motor may have less average loss than a normal motor, in spite of the fact that during the running period the motor has a high slip and corresponding loss. This not only means a better average efficiency, but indicates that the temperature of the motor is lower with the high resistance rotor; and further, that it could, therefore, be built on a smaller frame with approximately the same rise of temperature as the normal motor.

At first sight it is rather hard to understand why the rotor loss is independent of the electrical characteristics of the motor, but it becomes obvious if the field be taken as the basis of rest instead of the stator. In this case the stator would have to be considered as revolving backwards at synchronous speed, and the rotor with it to correspond with the starting conditions of the motor; then when the line switch is thrown in, a stationary field is set up which acts as a brake on the revolving rotor and tends to bring it to rest. The only difference between a mechanical and an electrical brake is that the loss in one is in friction loss and the other in copper loss. Since it does not matter to what degree a mechanical brake is applied, the total loss is equal to the difference of the kinetic energy before and after, so in this case the loss is proportional to the square of the initial speed less the square of the final speed, the field being considered as stationary.

If the secondary winding of a motor be fixed and the primary winding changed so as to give various field strengths, and the primary resistance for each winding be assumed to vary directly as the effective turns squared, which is probable, then the primary loss will be the same over the entire starting period for all these windings. This is because the total impedance reduced to terms of the primary varies as the turns squared of the primary and the torque varies inversely in the same ratio. The time taken to change from any speed to any other speed varies inversely as the torque and, therefore, directly as the turns squared. This means that if the turns were increased by 10 per cent that the time taken to make any change of speed would increase by $(1.10)^2$, the primary resistance would increase by $(1.10)^2$ and the amperes by $\left(\frac{1}{1.10}\right)^2$. Since the loss is proportional to amperes squared and directly as the resistance, it follows that the watts loss goes down as $\left(\frac{1}{1.10}\right)^2$ but is used for a longer time by $(1.10)^2$, giving the same joules as before.

This can be even more readily seen by considering that when the secondary resistance is transferred to terms of the primary it is multiplied by the primary turns squared so that it varies in exactly the same way that the primary resistance was assumed to vary and since the ratio of primary to secondary loss is as r_1 to r_2 and also since the secondary loss is fixed then so also is the primary loss (in joules) fixed.

It might be thought that the starting loss could be changed by manipulating the impressed voltage (E). This, however, cannot be done because if the voltage were decreased by 20 per cent then the current would be 80 per cent of the full voltage locked amperes, assuming that the leakage paths are not saturated. This would make the loss at the moment of start only 64 per cent of the full voltage locked watts. However, if the rotor is considered as changing from slip of unity to slip of 0.95 then the increased time with the reduced voltage, compared with the time for full voltage, exactly compensates for the reduced watts because the time to make this change of speed is inversely as the torque, which itself varies as (E^2).

A similar argument applies for any other change of speed on the entire speed-torque curve. It, therefore, applies to the total loss during any change of speed, such as loss during plugging or for loss during starting from rest.

It is true that if the voltage be increased so that the leakage paths become saturated when the motor is starting, or reversing, the torques are increased by more than the voltage squared. This is because the current goes up slightly faster than the voltage and the torque is actually proportional to the current squared. However, the loss also goes up as the current squared so that any change of impedance due to saturation does not change the total joules loss during the starting period.

It can be seen then that the primary loss cannot be lowered by changing the primary winding, except that one coil might fill the slot with a greater total section of copper than another, or that the throw of one coil might be more effective compared with the total length of turn than another.

It is also true that any reactance whether in the primary or secondary has absolutely no effect on the loss in joules at start or when the motor is plugged. Any increase in reactance makes the time longer to attain a given speed.

It is, therefore, clear that the only way to decrease the starting loss of a squirrel-cage induction motor is to increase the secondray resistance. This, however, cannot be brought to the extreme because if enough resistance be added the accelerating torques will be reduced so much that the time taken to attain a given speed would be excessive and further the full-load slip would be too great, giving a very low running speed and poor efficiency.

The question then arises: What is the value of secondary resistance to best suit a given application?

There are two factors that enter into the determination of the answer. First, is the secondary resistance to be such that the losses in the motor during a complete cycle of operation are to be a minimum; and second, is it desirable to start up in the minimum time?

It is obvious that no mathematics can decide what compromise should be made between the two values of secondary resistance obtained from the two considerations given above; but even though neither ideal is finally selected it is well to know what they are, so that the secondary resistance can be made some intermediate value.

To find the total losses during a complete cycle of operation, it is only necessary to multiply the running losses by their duration in seconds and add the starting or plugging loss in joules. This gives the total joules loss over a complete cycle of operation and if divided by the total time for the complete cycle, it gives the average watts loss for continuous operation.

Before proceeding to get the value of secondary resistance which would give the least time of start, assuming the maximum torque fixed, it is necessary to derive a formula for the time of start for any value of secondary resistance.

It is true for a constant torque = T_c, a moment of inertia = $(M\,I)$, and a final speed of $N\,(1-S)$ that the time can be found by the equation

$$\text{time (seconds)} = \frac{2\,\pi}{60} \times \frac{(M\,I)\,N\,(1-S)}{T_c} \qquad (5)$$

(assuming that inertia is the only resistance to motion).

However, the torque of an induction motor is not constant, so the above formulas can only be used for an infinitely small change of speed. Then:

$$d\,t = -\frac{2\,\pi}{60} \times \frac{(M\,I)\,N}{T}\,d\,s \qquad (6)$$

and time

$$= t = -\frac{2\,\pi}{60}\int_1^s \frac{(M\,I)\,N\,d\,s}{T}$$

but

$$T = \frac{7.04\,E^2\,r_2\,\phi\,s}{N\left[r_2^2\left(\frac{E}{E_0}\right)^2 + 2\,r_1 r_2 s + (r_1^2 + X^2)\left(\frac{E}{E_0}\right)s^2\right]} \qquad (7)$$

$$\text{time (seconds)} = -\frac{2\,\pi}{60}\,(M\,I)\,N$$

$$\int_1^S \frac{N\left[r_2^2\left(\frac{E}{E_0}\right)^2 + 2\,r_1 r_2 s + (r_1^2+X^2)\left(\frac{E}{E_0}\right)_{s^2}\right]d\,s}{7.04\,E^2\,r_2\,\phi\,s}$$

$$= -\frac{2\,\pi}{60 \times 7.04} \times \frac{(M\,I)\,N^2}{E^2\,r_2\,\phi}$$

$$\int_1^s \left(\frac{r_2^2\,E^2}{s\,E_0^2} + 2\,r_1\,r_2 + \left(\frac{E}{E_0}\right)(r_1^2 + X^2)\,s\right)d\,s$$

time (seconds)

$$= 0.01487\,\frac{(M\,I)\,N^2}{E^2\,\phi}\left[r_2\left(\frac{E}{E_0}\right)^2 \log_e\left(\frac{1}{S}\right)\right.$$

$$\left. + 2\,r_1(1-S) + \left(\frac{E}{E_0}\right)\frac{(r_1^2+X^2)\,(1-S^2)}{2\,r_2}\right] \qquad (8)$$

where

$(M\,I)$ = moment of inertia of the whole system transferred to and including the rotor poundals ft.

N = synchronous speed (rev. per min.)

E = primary volts per leg

$E_0 = E - X_1\,I_m$

$X_1\,I_m$ = no-load reactance drop in primary.

ϕ = number of phases

r_1 = primary resistance per leg including external primary resistance if any.

r_2 = secondary resistance per leg in turns of primary

X = total reactance including external primary reactance if any.

S = normal or final slip

Use 0.0149 for constant.

Assuming that all the constants of the motor are fixed with the exception of the secondary resistance, then the minimum time to attain a given speed can be derived from the preceding equation, noting that it will then be of the form:

$$\text{time} = \text{constant}_1 \left[r_2 \,\text{constant}_2 + \text{constant}_3 + \frac{\text{constant}_4}{r_2} \right]$$

when

$$t = \text{minimum then } r_2 \,\text{constant}_2 = \frac{1}{r_2}\,\text{constant}_4$$

$$r_2^2 = \frac{\frac{1}{2}\left(\frac{E}{E_g}\right)(X^2 + r_1^2)(1 - S^2)}{\left(\frac{E}{E_g}\right)^2 \log_e \frac{1}{S}}$$

$$r_2 = \sqrt{\frac{(X^2 + r_1^2)(1 - S^2)}{4.6\left(\frac{E}{E_g}\right)\log_{10}\left(\frac{1}{S}\right)}} \qquad (9)$$

This formula gives the value of r_2 which would speed the motor from rest to $N(1 - S)$ revolutions per minute in the least time.

This, of course, assumes that the rotor has not an abnormal value of friction at start, which would cause a slight delay while the oil film in the bearings is being established. In cases where there is likely to be excessive friction at start, then some higher value of secondary resistance will give the minimum time.

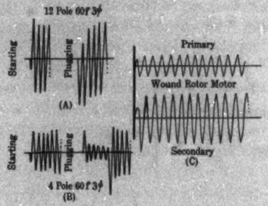

FIG. 1

Oscillograph records were taken of a squirrel-cage motor when starting up and when plugged in order to check the assumption that the dry bearings at start cause a delay which is small compared with the total time of start; they were also taken with a view to checking the time taken to establish the field.

Figs. 1 A and B show the current in the primary building up when the motor is started and plugged. It can be seen from A that the current is normal after about two cycles when the motor is started from rest and after four cycles when plugged. From B it can be seen that for starting the current has about 90 per cent of its full value for about four cycles, and is then normal; when plugged there is some irregularity but after seven cycles the current is stable. This irregularity is probably due to one of the poles of the switch making bad contact. Later tests with a wound rotor showed that one of the contacts was poor as the primary current in this particular phase did not start until about one-tenth of a second after the secondary current had started up. This indicates that a faulty switch can cause excess loss in the motor because as soon as two poles make contact the motor will draw single-phase current which causes a loss in the stator and rotor but gives zero torque. It is only when the third pole makes contact that the motor starts up.

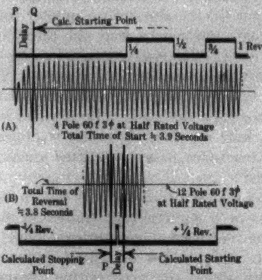

FIG. 2—(A) FOUR-POLE, 60-CYCLE, AT HALF RATED VOLTAGE, TOTAL TIME OF START = 3.9 SEC. (B) TWELVE-POLE, 60-CYCLE, AT HALF-RATED VOLTAGE, TOTAL TIME OF REVERSAL = 3.8 SEC.

Fig. 1 C shows the primary and secondary current of a wound rotor motor when started from rest. It can be seen that the secondary current builds up inside of 2 cycles.

From these tests it can be seen that it is not unreasonable to neglect the time taken to establish the field and the secondary current.

Fig. 2 A shows the primary current waves of a squirrel-cage motor when starting up; and above it a line which is displaced after the rotor turns one-quarter of a revolution and remains so for the next quarter revolution, etc. This line is obtained by mounting two contact strips diametrically opposite to each other on a paper pulley, each covering one-quarter of the circumference of the pulley.

The pulley is mounted on the motor shaft so that every time a contact piece passes a stationary carbon brush a current is established which actuates the oscillograph. Working back from slips of 10 per cent, 50 per cent and 75 per cent to get the moment of inertia of the rotor and pulley and then using the result to get the instant of start, assuming there is no delay, the point Q is reached. Since the switch was thrown in at the instant P then the difference represents the delay. Fig. 2 B represents the same thing when the motor is plugged. The delay in this case is for the excess friction when the rotor is changing direction of rotation. Both these delays are very small compared with the total time of start or plugging.

The usual value of friction and windage for a motor is such that it lowers the efficiency by only a few per cent so that it has the effect of reducing the torque exerted by the field on the rotor by only a few per cent at full load. For greater values of slip this reduction is less while the field torque is greater. This shows that it is justifiable to neglect it during the accelerating period, which has been done in the previous formulas.

Fig. 3 shows a series of curves each having the same maximum torque value, but which are for different values of secondary resistance. The various starting torques are obtained by using the value of secondary resistance obtained by formulas (**9**) when the slips corresponding to $A_1 B_1$ etc., are used. These curves then give an idea of what shape the speed torque curve should have for the minimum time of acceleration to a given speed (assuming that the only resistance to motion is in the form of inertia).

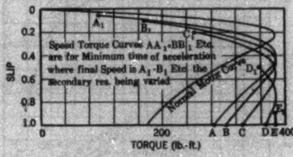

FIG. 3—SPEED TORQUE CURVES

The formulas (**8**) for time of start assume that the motor will have no cusps and will have a starting and maximum torque on test equal to the calculated torques.

In the event where there are cusps and the complete speed torque curve is taken on test then formulas (**6**) can be used by taking small changes in slip.

It should be noted that an erroneous conception is often held that the time of acceleration varies inversely as the average torque.

This would lead to the conclusion that a single-phase motor would start up since it has a definite average torque. The fact is that the time for any constant value of torque varies inversely as this torque, and this can be applied with reasonable accuracy for small changes in slip, but for the total accelerating period it should be noted that the time of acceleration varies as the average of the inverse of the torque, this average being obtained by taking the inverse of the torques for equal increments of speed. From this it can be seen that the time can be abnormal if the torque for any speed whatever approaches zero and would be infinite if it did drop to zero, which means that the motor would never speed up beyond this point.

There is another assumption, this time concerning the loss at start, which is often made. This is the shape of the current-time curve of the motor. It is sometimes assumed that if the value of locked amperes is used for zero time and full load amperes for the end of the accelerating period then the current curve could be assumed to be a straight line joining these two points. This is not necessarily true, but happens that for some high resistance rotor motors it is nearly so. However, it is far from true of the average motor where the starting current tends to hang on, and in Fig. 4 is shown to be as high as 93 per cent of locked value after half the starting time is passed. This is the upper curve for the low resistance rotor. The lower curve is for a rotor with four times as much resistance, and it can be seen that the current falls off nearly as a straight line.

The torque curve for this latter motor would have nearly maximum torque at start.

It will be noted from these two curves how much the starting current is reduced for the high resistance rotor compared with the normal rotor, and also that the average current is still a lower ratio. The r. m. s. value of the two curves should have a ratio of nearly two because the time of start for each case is practically the same. The r. m. s. value of each current curve when multiplied by the time of start should have a ratio of exactly two, which, when squared and multiplied by the ratio of resistances, gives unity, showing that the rotor loss is the same in either case, as previously shown.

Since the stator winding is the same for either case, and noting that there is little difference in the time of start, it can readily be seen that the stator loss is much lower for the high resistance rotor than for the normal rotor.

TIME OF ACCELERATION USING TEST VALUES OF TORQUES

Taking the case of a normal resistance rotor motor when the duty cycle is known, it is true that in the majority of cases the motor which is finally selected has already been built and tested, or it may be that the selected motor is so like another tested rating that the torques and other general characteristics can be estimated with greater precision than by calculation from the ground up.

In such a case it is quicker to use these estimated or tested torques, and there is also less likelihood of error than if a complete calculation were made of the machine.

The values required to find the time of acceleration from rest to a given speed are the slip at load torque equal to the starting torque, the maximum and starting torques, locked amperes, synchronous speed and moment of inertia of the rotor if starting light, and if loaded then the equivalent moment of inertia of the whole system, reduced to terms of the rotor.

Let

Maximum torque $= T_m$

Starting torque $= T_{st}$

Slip at load torque equal to $T_{st} = s_1$

Locked amperes $= I_L$ (per phase)

Synchronous speed $= N$

Moment of inertia (total) $= (M\,I)$

Slip at maximum torque $= S_m$

$$T = \frac{a\,s}{b + c\,s + s^2} \left(\text{refer to formula (7)}\right) \qquad (10)$$

where a, b and c are constants.

As a rule the test is worked up to show the slip s_1, but it is not a difficult matter to estimate it by producing the curve to the required point. The accuracy of the time from the resulting formula is not affected to any appreciable extent by the value of s_1 not being very accurately obtained.

Assume any two values of slip S_1 and S_2, such that they give the same value of torque; then from formula (**10**)

$$T = \frac{a\,S_1}{b + c\,S_1 + S_1^2} = \frac{a\,S_2}{b + c\,S_2 + S_2^2}$$

and therefore

$$b\,S_1 + c\,S_1\,S_2 + S_1\,S_2^2 = b\,S_2 + c\,S_1\,S_2 + S_1^2\,S_2$$

$$b\,(S_1 - S_2) = S_1\,S_2\,(S_1 - S_2)$$

$$b = S_1\,S_2$$

Therefore, for any two torques of equal value the product of the slips is a constant and, therefore, equal to the square of the slip at maximum torque.

Therefore, (a)

$$b = S_1\,S_2 = S_m^2 = s_1$$

Again

$$T_m = \frac{a\,S_m}{b + c\,S_m + S_m^2} = \frac{a}{2\,S_m + c}$$

and

$$T_{st} = \frac{a}{b + c + 1}$$

Combining these two equations

$$a = T_m\,(2\,S_m + c) = T_{st}\,(b + c + 1)$$

$$c\,(T_m - T_{st}) = T_{st}\,(1 + b) - 2\,S_m\,T_m$$

Substituting s_1 for b and $\sqrt{s_1}$ for S_m

(b) $$c = \frac{T_{st}\,(1 + s_1) - 2\,\sqrt{s_1}\,T_m}{T_m - T_{st}}$$

(c) $$a = T_{st}\,(b + c + 1)$$

From (a) (b) (c) the values of a, b and c can be found and can be substituted in a corresponding formula to (**8**) as follow:

Time of acceleration (seconds) =

$$\frac{\pi}{30}\,\frac{(M\,I)\,N}{a}\left[2.3\,b\log_{10}\left(\frac{1}{S}\right) + c\,(1 - S) + \frac{1 - S^2}{2}\right] \qquad (11)$$

from rest to slip of S.

N = synchronous speed (rev. per min.)

$(M\,I)$ = moment of inertia of whole system reduced to terms of the rotor.

S = final slip

$a = T_{st}\,(b + c + 1)$

$b = s_1$

$$c = \frac{T_{st}\,(1 + s_1) - 2\,\sqrt{s_1}\,T_m}{T_m - T_{st}}$$

T_m = maximum torque (lb.-ft.)

T_{st} = starting torque (lb.-ft.)

s_1 = slip at load torque equal to T_{st}

It can readily be seen from the above formula that the time to attain no-load speed is infinite. This is not false because as this speed is approached the available torque for acceleration is practically zero; and as the speed is infinitely closer to no-load speed the accelerating torque is infinitely small. However, when the speed reaches 90 per cent to 95 per cent of synchronous speed, it can be assumed that it is near enough to be counted up to speed; then if 90 per cent is taken the log to the base ten becomes unity and if 95 per cent it becomes 1.301. It makes only a slight difference in the time of start, whichever of the above final speeds is chosen.

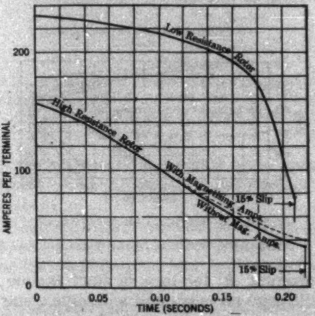

FIG. 4—AMPERE-TIME CURVES OF SIMILAR MOTORS

One having four times as much secondary resistance as the other

APPROXIMATE METHOD USING ONLY MAXIMUM AND STARTING TORQUES

The preceding method can be simplified by assuming that the torque curve has a formula

$$T = \frac{a\,s}{b + s^2} \qquad (12)$$

It should be noted that the value of a and b will be different in this formula from that in formula (**10**) so that the part that c plays is not entirely neglected; in fact the time as calculated by each is practically the same up to 90 per cent speed. Using only the maximum and starting torque, the value of a and b as given in formula (**12**) can be found so that the equation will give these same torques but will not give the correct slip for the load torques. However, as the time of starting is affected mostly by the torques for large values of slip so consequently this method gives very close results.

The values of a and b are obtained, as follows:

$$a = 2\,S_m\,T_m \text{ or } = T_{st}\,(1 + b)$$

$$b = S_m^2$$

where $$S_m = \frac{T_m}{T_{st}} - \sqrt{\left(\frac{T_m}{T_{st}}\right)^2 - 1}$$

The time equation then becomes:

time of start (seconds) =

$$\frac{\pi}{30}\,\frac{(M\,I)\,N}{a}\left[2.3\,b\log_{10}\left(\frac{1}{S}\right) + \frac{1 - S^2}{2}\right] \quad (\mathbf{13})$$

where N = synchronous speed (rev. per min.)

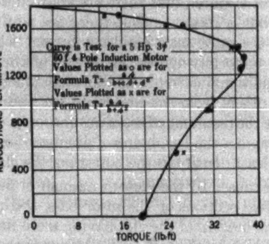

FIG. 5—TEST CURVE FOR 5-H. P., THREE-PHASE, 60-CYCLE, FOUR-POLE, INDUCTION MOTOR

Values plotted as O are for formula

$$T = \frac{a\,S}{b + c\,S + S^2}$$

Values plotted as X are for formula

$$T = \frac{a\,S}{b + S^2}$$

$(M\,I)$ = total moment of inertia reduced to terms of the rotor.

S = final slip

a = $2\,S_m\,T_m$, or $= T_{st}\,(b + 1)$

b = S_m^2

$$S_m = \left(\frac{T_m}{T_{st}}\right) - \sqrt{\left(\frac{T_m}{T_{st}}\right)^2 - 1}$$

Fig. 5 shows a test speed-torque curve and the calculated values for formula (**10**) plotted as (○) and those per formula (**12**) plotted as (×).

SPEED-TORQUE EQUATION FOR HIGH RESISTANCE ROTOR MOTORS

The two previous methods fall down when applied to the case of a speed-torque curve for a high resistance rotor motor.

In the first place, the starting torque is often nearly equal to the maximum torque so that the evaluation of a, b and c by the first method is more difficult besides the fact that the slip for a load torque equal to the starting torque is a doubtful quantity as the test values obtained for such a curve are not very accurate for large values of slip and are usually not taken for slips greater than about 50 per cent with the exception of the starting torque.

The second method falls down completely, because, as explained before, it does not give the correct values of slip for the load torques, by which was meant the torques on the upper part of the curve. But with a speed torque curve where the starting torque equals or nearly equals the maximum torque, the torques are nearly all on the upper part of the curve.

The value of a, b, and c for this high slip curve can be found by taking the angle which the curve makes at the point of slip = 0, torque = 0; the slip for some load torque (preferably the torque equal to two-thirds starting torque); and the starting torque.

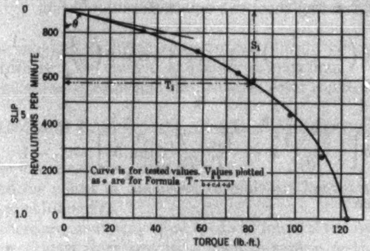

FIG. 6—CURVE FOR TESTED VALUES

Values plotted as O are for formula

$$T = \frac{a\,S}{b + c\,S + S^2}$$

Fig. 6 shows test speed torque curve for a motor which has a full-load slip of approximately 18 per cent. The tangent of the angle is taken as being the lb-ft. torque divided by the slip for a point on the curve infinitely close to the point of zero torque. These values are obtained by drawing a tangent to the curve at this point and taking the slip and torque for any point on this line.

The torque and slip for the load point are shown as T_1 and S_1. The torque T_1 is 82 which is two-thirds of T_{st}. This makes the final evaluation of the constants a, b, and c more simple.

Since

$$T = \frac{a\,s}{b + c\,s + s^2}$$

then

$$\frac{T}{s} = \frac{a}{b + c\,s + s^2}$$

and it is obvious that as T and s approach zero

$$\frac{T_0}{S_0} = \frac{a}{b},$$

therefore,

$$\tan\theta = \frac{a}{b} \text{ and } a = b\tan\theta,$$

then,

$$a\,(s) = T\,(b + c\,s + s^2) = b\,s\tan\theta$$

$$\text{when slip} = 1, \text{ then } (b + c + 1) = \frac{b\tan\theta}{T_{st}}$$

$$\text{when slip} = S_1, \text{ then } \left(\frac{b}{S_1} + c + S_1\right) = \frac{b\tan\theta}{T_1}$$

Subtracting these two equations:

$$b\left(1 - \frac{1}{S_1}\right) + 1 - S_1 = b\tan\theta\left(\frac{1}{T_{st}} - \frac{1}{T_1}\right)$$

and

$$b\left[\frac{S_1 - 1}{S_1} + \frac{T_{st} - T_1}{T_{st}\,T_1}\tan\theta\right] = S_1 - 1$$

then

$$b = \frac{1}{\dfrac{1}{S_1} - \dfrac{(T_{st} - T_1)\tan\theta}{T_{st}\,T_1\,(1 - S_1)}}$$

Since T_1 was selected as two-thirds of T_{st}, then the last expression for b can be simplified as follows:

$$\left.\begin{aligned} &\text{(a)} \quad b = \frac{1}{\dfrac{1}{S_1} - \dfrac{\tan\theta}{2\,(1 - S_1)\,T_{st}}} \\ &\text{(b)} \quad a = b\tan\theta \\ &\text{(c)} \quad c = \frac{a}{T_{st}} - b - 1 \\ &\text{and torque} = T = \frac{a\,s}{b + c\,s + s^2} \end{aligned}\right\} \quad (14)$$

If the torque curve be plotted which is obtained by finding a, b and c as described, then this curve should start from the point $T = 0$ and $s = 0$ with the same angle as the test curve; it should also pass through the point $T_1\,S_1$ and the starting torque point.

If, then, T_1 and S_1 are selected as previously suggested, and noting that the form of the equation is correct, it can be seen that the torques obtained from the equation will be extremely close to the test torques.

Fig. 6 shows the test speed-torque curve in full, and points calculated by this method plotted as ⊙. These points coincide with the test values except two which are slightly less. However, it should be remembered that this part of the curve is very difficult to obtain on test and that it may consequently be slightly off and not the calculated points.

PLUGGING SERVICE

When a motor is plugged there are eddy currents in the rotor which are very pronounced at negative speeds. These eddy currents cause an increase in the rotor loss over what it would be if measures were taken to prevent eddy currents. This increase in the rotor loss makes the torque larger and since the eddy currents increase as the slip increases it follows that the torques are affected more at negative speeds than at positive speeds. The increase in torque can be considerable and makes quite a difference in time of retardation, and also in the joules loss in the primary.

When it is stated that there are eddy currents in the rotor, what is meant is that there are additional currents in the rotor bars besides a current which is distributed uniformly across the section of the rotor bar. These additional currents add in some places and subtract in others from the uniformly distributed current and have the resultant effect of an unevenly distributed current across the bar, being of greatest density in the top and least density in the bottom of the slots.

This is caused by the varying amount of flux linkages for different sections of the rotor bars, the part of the bar in the bottom of the slot having more flux linkages than the part in the top. This causes more reactance per unit area for the bottom part than the top and since the resistance per unit area is the same for each, then the total impedance per unit area will be greater for the former. However, the $I\,Z$ drop for all parts of the bar must equal the voltage per bar which is a constant for a given value of slip. The current must, therefore, be greatest where the impedance is least which is at the top of the slot; and likewise is least at the bottom.

Since the loss is proportional to the density squared, it can be shown that the least loss for a given total current per bar is when the current is uniformly distributed.

The loss must, therefore, be greater for an uneven distribution of current and this condition can be replaced by a higher resistance rotor which would give the same loss if it had the same total current per bar with even distribution.

As the reactance in the rotor circuit is proportional to the slip, then it follows that the impedance bears a greater ratio to the resistance at high than at low values of slip, and since the resistance tends to make the current distribution uniform and the reactance tends to make it varied, then it follows that there will be more and more varied current distribution the higher the slip. This means that the effective secondary resistance does not bear a constant ratio to the primary resistance so that it is no longer true to say that the total secondary to the total primary loss during the reversing period bears the ratio of their respective resistances.

It is true that near standstill there is not a great deal of excess torque over what would be expected so that the equivalent secondary resistance is not very far from its normal value and it can be assumed, with reasonable accuracy, that the loss during the period from rest to full-load speed is divided in the primary and secondary as their resistance. This assumption was made for the derivation of formula (**4**).

However, when the loss is to be found in the primary and secondary for the period between the moment of plugging and of zero speed, then the changing ratio of primary and secondary resistance must be taken into account. Since the derivation of formula (**2**) shows that the secondary loss is independent of the secondary resistance, then it is obvious that it will also be independent of a varying value of secondary resistance.

This is the justification for using formula (**3**).

It will be noticed that the primary current for a motor which has a normal rotor winding is practically the same at the instant of plugging as at standstill so that if it be assumed that the current is equal to the plugging value during the entire period of deceleration, the loss so obtained is slightly in excess of the true value and is, therefore, on the safe side.

When a high resistance rotor is used, it is no longer true to say that the plugging current is practically equal to the starting current. It will probably be around 30 per cent greater for a motor that has sufficient secondary resistance to give maximum torque at start.

Therefore, the primary loss during the deceleration period for a high resistance motor should not be calculated in the same way as for a low resistance rotor motor.

It should be noted that the reason for not using formula (**3**), and multiplying it by $\frac{r_1 + r_2}{r_2}$ to get the total loss is because the primary loss is actually lower than this formula assumes. However, with a high resistance rotor the major loss is in the rotor, so that any difference there might be between what this formula gives and the actual total loss, is very small.

It is, therefore, reasonable to use the following formula for the total loss during the total plugging period, especially when it is remembered that it errs on the large side.

Joules loss in rotor and stator (when rotor is plugged)

$$= 0.00744 \frac{r_1 + r_2}{r_2} (M\,I)\, N^2 (4 - S^2) \qquad \textbf{(15)}$$

for high resistance rotors only.

For the time of reversal for such a motor there is very little mathematics that can be applied except, of course, the method given by formula (**6**). The following suggestions, however, will aid in making the problem shorter, but not so accurate. Formula (**8**) or (**11**) can be applied to the acceleration part of the speed torque curve and formula (**5**) to the retardation part, provided that the torque in the equation is the effective torque.

As the torque at negative speeds is very large, and the percentage variation small, this effective torque is extremely easy to estimate; provided that the test-speed torque curve is available at these speeds.

As there will be less eddy currents in a high resistance rotor motor than in a low resistance rotor motor it is very probable that sufficient accuracy can be obtained by a formula similar to (**8**) or (**11**) where the limits of the integral are 2 to S instead of 1 to S.

For a normal resistance rotor motor the method previously outlined should not be followed because the primary loss is not small compared with the secondary loss.

Then the total joules loss during the reversing period is given by the addition of formula (**2**) when multiplied by $\frac{r_1}{r_2}$ and formula (**3**) and $\phi\, I_p^2\, r_1\, t_p$ where t_p is the time required to bring rotor to rest during plugging and where I_p = current at instant of plugging.

Joules loss in rotor and stator (when rotor is plugged)

$$= 0.00744 (M\,I) N^2 \left[\frac{r_1}{r_2} (1 - S^2) + 4 - S^2 \right] + \phi I_p^2 r_1 t_p \qquad \textbf{(16)}$$

for low resistance rotors only.

All the values in the above formula are known with the exception of t_p.

There are many ways that t_p can be found if the speed torque curve from test be known but some methods lead to such complicated equations that their use is not justified when it is considered that, with the exception of the tedious method given by formula (**6**), they are evolved by some assumption that is only approximate.

As an example of this, take the most simple equation for the speed torque curve that shows excess torque at negative speeds and has the approximate shape for values between slip = 0 and 2.

$$\text{Torque} = T = \frac{a\,s}{b + s^2} + d\,(s)$$

The first part of this formula is the simplest expression that approximately follows the actual curve between slip = 0 and 1.

The second part makes the torques gradually increase as the slip increases and gives the excess torques which are noticeable at negative speeds. This equation can be made to follow very closely to the test curve, but when the time equation is evolved from it the complications and chance of error met with make it undesirable.

This equation is given for record only, as its use is not advocated.

$$t_p = -\frac{\pi}{30}(M\,I)\,N\left[\frac{b}{(a+d\,b)}\log_e\left(\frac{2}{S}\right)\right.$$

$$\left.+\frac{a}{2\,d\,(a+d\,b)}\log_e\left(\frac{a+d\,b+d\,4}{a+d\,b+d\,S^2}\right)\right]$$

It should also be remembered that the finding of a, b and d adds to the length of time needed to handle this equation.

One of the simplest methods that can be employed to get the value of t_p, and one which gives a result equally as accurate as the previous method, is found by the assumption that if the inverse of the torques between slip = 2 and 1 be plotted against the slip then a straight line will be the result.

This is not true but is approximately so. If there were no increased torque effect then the curve between slip = 2 and 1 is approximately a hyperbola and when the inverse of the torque is plotted it gives an approximate straight line, so that the assumption tends to be correct for this extreme.

The results of test indicate that where there is increased torque effect the assumption is nearly correct.

It may be mentioned here that there are nearly always some harmonics in the field form that show up on the speed torque curve. However, if these do not reduce the torque for any speed by more than, say, half, then it can be assumed that they cancel themeslves out over the entire period.

Since, as mentioned before, the time of change of speed varies as the average of the inverse of the torque, then it follows that if T_{st} = starting torque

and T_{plug} = torque at slip of two,

$$\frac{1}{\text{Effective torque}} = \frac{1}{2}\left(\frac{1}{T_{st}}+\frac{1}{T_{plug}}\right)$$

$$= \frac{T_{plug}+T_{st}}{2\,T_{plug}\,T_{st}}$$

$$\text{time (seconds)} = \frac{\pi}{30}(M\,I)\,\frac{N}{\text{Effective Torque}}$$

For retardation period:

(Slip = 2 to slip = 1)

$$t_p = \frac{\pi}{60}(M\,I)\,N\,\frac{T_{plug}+T_{st}}{T_{plug}\,T_{st}} \qquad \textbf{(17)}$$

where t_p = time in seconds,

for inertia loads only.

This also gives very nearly the correct result for cases where the torque for slip of two equals the starting torque, because in such a case the test values of torque are practically constant with the exception of cases where harmonics come into play; and as mentioned before these do not affect the time of retardation appreciably unless they reduce the torque to less than half its normal value.

To obtain the time to change from rest to, say, 10 per cent slip use either formula number **(11)** or **(13)**.

EQUIVALENT MOMENT OF INERTIA (IN TERMS OF THE ROTOR)

It will be noticed that every time and loss formula includes the term $(M\,I)$ which is the total inertia of the whole system reduced to terms of the rotor.

As this figure is the result of multiplying every small mass by its velocity squared and comparing it with that of the rotor, it can be seen that there are many cases where the rotor, due to its high speed, plays the major part in holding back the whole system from instantly coming up to speed.

The moment of inertia of the rotor should be known then if the preceding formulas are expected to give the correct losses and times. There are two accurate methods for finding this quantity; one is the well known method of swinging the rotor from some point of known distance from the center of gravity of the rotor, the axis around which it swings being parallel to the line through the center of the shaft.

The radius of gyration is given by

$$R = \sqrt{g\,l\left(\frac{P}{2\,\pi}\right)^2 - l^2}$$

where

R = radius of gyration (feet)

g = gravity

l = distance of axis from center of shaft

P = period (seconds) for a small amplitude of swing.

This formula gives the radius of gyration very accurately. However, it is difficult to fix the fulcrum so that it is absolutely rigid when the rotor is any size, besides the difficulty of measuring l accurately.

The other method lends itself to great accuracy due to the fact that the only measurement necessary is the period. The other dimensions used are the journal sizes, which are very accurately ground to size, and are both of equal size and known beforehand.

The equipment needed is two rails accurately machined to a very large radius. These are placed parallel to each other and the rotor is placed on them so that each journal rides on a rail.

As the rails are circular the rotor naturally will swing back and forth with a harmonic motion.

Fig. 7 shows the scheme suggested.

Let

R = radius of gyration of the rotor (ft.)

P = period of rotor swing (seconds)

r = radius of rotor bearings (ft.)

l = radius of the circular rails (ft.)

g = gravity

Then,

$$\text{radius of gyration} = R = r\sqrt{\frac{P^2\,g\,(l-r)}{4\,\pi^2\,l^2} - 1}$$

When this formula is used it will be noticed that the first term under the radical is very large compared with the second term, provided that l is very large. The accuracy with which R is obtained then approaches that of the period P.

The radius of the rails l should be made very large so that the period becomes very large and tends to eliminate windage, and also so that for a given amplitude of swing, the angle of the rails will be small and so prevent slipping of the journals on the rails.

The accuracy of R is unaffected by the fact that the rotor might be placed on the rails slightly skewed so that when at rest the bearings would not be at the lowest point of the rails.

In transferring the inertia of external moving parts to terms of the rotor the following rules are sufficient for most cases:

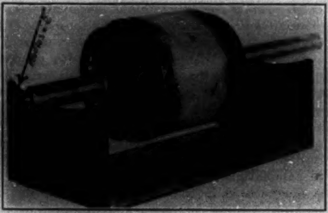

FIG. 7—SCHEME FOR FINDING RADIUS OF GYRATION OF ROTORS

1. All parts that have only linear motion should be transferred according to the following law.

$$(M\,I)\text{ equivalent} = \frac{W}{g}\left(\frac{30\,V}{\pi\,(\text{r. p. m.})}\right)^2$$

Where $(M\,I)$ equivalent = moment of inertia when transferred to the rotor

W = external weight in lb.

V = velocity of W in ft. per sec.

g = gravity

(r. p, m.) = revolutions per minute of rotor.

2. All parts that have a rotary motion only should be transferred by having their moment of inertia multiplied by the square of the ratio of their speed compared with the speed of the rotor.

All the previous formulas pertaining to loss or time have been evolved with the assumption that the only resistance to motion is inertia. This is approximately so in a great many cases, but if any particular application should have a large friction or hauling load to overcome then these formulas cannot be used. Any friction or hauling load should be applied as a reduction in torque and not as some supposedly equivalent inertia.

If the complete current torque curve be not available, then it can be built up from the speed-torque curve by the following:

$$I_2 = \sqrt{\frac{N\,s\,T}{7.04\,\phi\,r_2}}$$

where s and T are the values of slip and torque corresponding to the current.

The primary current is practically equal to this, the magnetizing amperes being the only difference; and for large slips the magnetizing current is much less than near synchronous speed.

Having the complete speed torque curve, the accelerating torque curve can be found by subtracting the friction or hauling torque.

The inverse of this torque is proportional to the time to change speed at any point. A current-time curve can then be built up similar to Fig. 4; or better still, a "current squared"-time curve can be built up without the intermediate step of plotting the current against time. The arithmetical average of the current squared curve over the entire starting period gives the average current squared. The square root of this value gives the r. m. s. value of current during the starting period.

CONCLUSION

For all cases worked out the primary loss has been found so that if some type of control be in the circuit, the loss can be found for it during a complete cycle of operation. The proper sized controller can then be applied which will not have an excessive factor of safety.

When a motor is to be used for either starting and stopping or reversing service, the total losses of the motor during a complete cycle can be found by the preceding formulas. Knowing from test the amount of loss that the motor can dissipate for a definite rise of temperature, and knowing the loss it will be required to dissipate, the temperature rise of the motor can be predicted.

This estimated rise indicates whether or not the proper sized motor has been selected.

Discussion

B. F. Bailey: I should like to point out the use of the following formula

$$\eta = \frac{0.142 \times \text{Lb-Ft.} \times \text{Sync. Speed}}{\text{Volt-Amperes}}$$

in which η is the volt-ampere torque efficiency. By substituting the input to the motor in watts for the volt-ampere input we have the watt–torque efficiency. This formula is of course correct for any speed of the motor, although it is most useful in computing the starting efficiency.

In the case of a squirrel-cage induction motor the starting efficiency will usually be about 20 per cent. The wound-rotor induction motor will show higher values and a high-resistance; squirrel-cage motor still higher values. Split-phase induction motors have starting efficiencies of about 10 per cent.

This formula gives us a ready means of comparing the starting performance of different types of motors.

It can readily be shown that the starting torque of a polyphase motor expressed in synchronous watts is equal to the input to the rotor, but the demonstration is based upon the supposition that we have a rotating field which does not change shape as it rotates. In practise, the field does change its shape somewhat and in fact there may be backwardly rotating components of the field which directly subtract from the torque. In addition, there is some loss of torque due to bearing friction. The result is that the torque as measured by a brake is always somewhat less than the torque as computed from the input to the rotor. My experience has been that usually the actual torque will be from 85 to 90 per cent of the theoretical torque, although instances are common where somewhat higher or lower torques are developed.

K. L. Hansen (communicated after adjournment): Aside from containing useful suggestions and formulas, Mr. Norman's paper assists materially in advancing problems pertaining to the transient phenomena of acceleration and retardation from the rough approximation stage on to a more sound, mathematical foundation.

Under the assumption of negligible resistance load, the author has found the secondary copper loss, when accelerating from slip s to slip S, to be

$$\text{Watt-seconds} = 0.00744\ (M\ I)\ N^2\ (s^2 - S^2)$$

and the primary copper loss to be the secondary loss multiplied by $\frac{r_1}{r_2}$.

In the October 18th, 1924 issue of the *Electrical World*, I published the following formula for the secondary copper loss: (on account of lack of space, derivation of formulas was not shown)

$$W_s = \frac{W\ G^2\ N^2\ (s^2 - S^2)}{4324} = 0.000231\ W\ G^2\ N^2\ (s^2 - S^2$$

where the moment of inertia $W\ G^2$ is expressed in lb.-ft. Changing to poundal-ft. by introducing gravity acceleration 32.2, it becomes

$$W_s = 0.00744\ (M\ I)\ N^2\ (s^2 - S^2)$$

which is identical with the one in the paper. An interesting point in this connection which Mr. Norman does not seem to mention in his paper is that when accelerating from standstill the secondary loss is equal to the energy stored in revolving parts.

In discussing motors in intermittent service, I made the following statement in my article:

"Comparing (3) with (1), it will be seen that when a motor accelerates from standstill to synchronism, the secondary energy loss is equal to the stored energy and the primary loss is the stored energy multiplied by $\frac{r_0}{r_1}$. As the secondary loss is independent of, and the primary loss inversely proportional to, the secondary resistance, it follows that the ratio of secondary to primary resistance should be as high as the rate of acceleration permits in order to reduce the energy loss of the system to a minimum."

It will be noticed that this paragraph covers substantially the discussion on the second and part of the third page in the paper.

It might also be of interest to know that I have used the author's speed-time formula and found it satisfactory in applications where the resistance load is small and the inertia load is high, as, for example, in centrifugal extractors. When there is an appreciable resistance load in addition to the inertia load, however, other formulas give better results.

P. L. Alger (communicated after adjournment): Mr. Norman has given us a complete synopsis of the starting behavior of an induction motor. As he points out, the object of studying starting conditions is to design the motor and the control so as to secure minimum losses at starting and a reasonable, but not excessive, factor of safety in design.

To bring out some of Mr. Norman's ideas more emphatically, there are two points which seem to be worthy of mention. Both of these points arise from the fact that a motor designed primarily for starting duty should have different characteristics from one designed for steady operation. In order to obtain the best starting torque per ampere, it is necessary to have a high rotor resistance, and, in order to obtain low starting losses, it is necessary to have a low primary resistance. These two things naturally result in the designer's making a starting motor with few primary turns and therefore with a very high magnetizing current.

A high magnetizing current results in an appreciable difference between the primary and secondary currents during the starting period. Though Mr. Norman suggests that this difference can be neglected, it is often quite appreciable. As Professor Bailey stated in his discussion of the paper, the torque efficiency, defined as the ratio of the starting torque as measured with a brake to that as calculated from the copper loss due to the measured primary current and the measured secondary resistance, is usually appreciably below unity. If the secondary resistance used in calculating the torque is the apparent value obtained by dividing the primary input at standstill, less the primary copper loss, by the square of the primary current, the torque efficiency is 100 per cent. But if the true value of secondary resistance is used, the calculated value of torque will be too high, due to the secondary current being smaller than the primary current.

Thus, in the attempt to make a motor with good starting characteristics, the magnetizing current is exaggerated, and so the accuracy of the simple theory which neglects magnetizing current may be said to be less, the better the motor is for starting duty. Mr. Norman also points out that the existence of eddy currents in the rotor during starting again makes the theory less accurate and generally requires some approximations to be employed in calculating the primary copper loss. These eddy currents are exaggerated in the attempt to obtain high starting resistance and low starting current without too seriously impairing the running efficiency by the increasing employment of the double squirrel-cage type of winding. However, if a double squirrel cage is so designed as to obtain the maximum possible starting resistance with a given running reactance, expressions for the secondary resistance referred to primary of the form $\frac{R}{S} + \frac{S\ X_0}{1 + S^2}$, and for the secondary reactance of the form $X + \frac{X_0}{1 + S^2}$, may be employed in place of the simpler ones, $\frac{R}{S}$ and X, and, with their aid, accurate formulas similar to Mr. Norman's may be derived for the time of starting and other characteristics.

H. M. Norman: In answer to Prof. Bailey's remarks concerning what he calls efficiency of the starting torque, I should say that there is nearly always a discrepancy between the measured starting torque by brake and that indicated by the watts loss in the rotor at start. The brake value is sometimes lower and sometimes higher than that given by the secondary loss. However, the average value obtained from a large number of different motors showed that there is only about a four-per cent difference.

It should also be remembered that there is difficulty in measuring the input watts correctly, and also in estimating the proper value of the primary resistance (variation due to temperature) to use for calculation of the watts in the primary so as to obtain the correct watts loss in the rotor. In view of these difficulties, the best guide is the average of a large number of motors.

I think this is a very important point because it touches on the accuracy of the loss formulas in the paper. If it were true that this starting efficiency were appreciably below 100 per cent then the starting losses would be greater than the formulas in the paper indicate.

If there is no unbalance of phases, however, then it is my opinion that any adjustment of the constant 0.00744 in the loss equations need not exceed the addition of the four per cent already mentioned.

In reply to Mr. K. L. Hansen's remarks, I think it unfortunate that the same formula should have to be worked out independently at two different times, but in this case it has the comforting feature that we are both in exact agreement.

Regarding the fact that when starting from rest and accelerating to synchronous speed the secondary loss is equal to the stored energy of the system (this includes the energy of all the moving parts). I should like to point out that I purposely avoided mentioning this fact as it is subject to misuse for the following reason. Suppose that for a given system on account of the nature of the application the rotor of the motor has high resistance so that the full-load speed is, say, 90 per cent of synchronous speed. Then if the secondary loss be taken as equal to the stored energy of the system, it would be found to be 81 per cent of what it would have been had the system been accelerated to synchronous speed. This is not true, however, as the loss in the rotor under these conditions is equal to 99 per cent of this value.

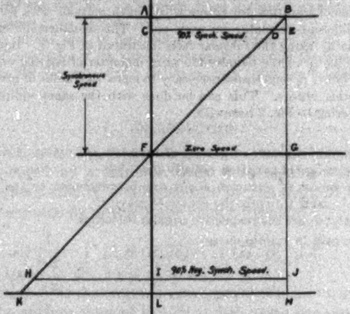

FIG. 1

This error is approximately equal to twice the full-load slip and even for low-resistance rotor motors is not altogether negligible.

Fig. 1 herewith gives a diagrammatic representation of the proportions of loss in the rotor of the motor and the stored energy of the moving parts. Consider a motor that has been accelerated from rest and has reached a speed proportional to $F\ C$ on this diagram where $F\ A$ represents synchronous speed. The area $F\ C\ D$ bears the same ratio to the stored energy of all the moving parts as the area $D\ E\ G\ F$ bears to the loss in the rotor. These areas become equal if the motor starts from rest and accelerates to synchronism so that for this particular case (*and this particular case only*) the loss in the rotor equals the stored energy of the moving parts.

This diagram also shows the ratio of lost and stored energy to the loss in the rotor when the rotor is plugged by extending the diagram into the negative-speed region. If a motor is running at, say, 90 per cent of synchronous speed and is plugged, the rotor reversing to 90 per cent synchronous speed in the reverse direction then, (referring to Fig. 1,) the energy lost in the moving parts by bringing them to rest is represented by area $H\ F\ I$, the energy gained by bringing the moving parts up to 90 per cent synchronous speed is represented by area $C\ D\ F$, and the energy lost in the rotor by area $D\ E\ J\ H$.

Referring to Mr. Hanson's statement that the short paragraph in his article covers substantially the discussion in part of my paper, I should point out that on the second page immediately following the loss formulas, I made a statement of approximately the same length and more or less equivalent to that in Mr. Hansen's article, and as I did not repeat myself in the subsequent paragraphs, I cannot see that his claim is well founded. Further, where there are only a few cycles of operation per minute I do not agree with Mr. Hansen's statement that the secondary resistance should be as high as the rate of acceleration will permit in order to reduce the energy loss to a minimum. This is neglecting entirely the losses during the running period. There are so many different cycles of operation that it is not desirable to compile an abundance of formulas to give the best value of secondary resistance for each, but as there is one cycle which is perhaps the most common, I might add the following formula:

$$r_2 = \sqrt{\frac{0.00744\ (M\ I)\ N^2\ (1 - S^2)\ r_1}{\phi\ I_2^2\ t\ K}}$$

(for inertia starting loads only)

where

I_2 = Secondary load current,

r_2 = Value of secondary resistance which gives the minimum losses over an entire cycle consisting of a start from rest to speed of $N\ (1 - S)$ and a full-load run for a time $= t$ (seconds) and a shut-down of any length of time. The constant is the ratio of effective resistance at start to that at full load and is slightly greater than unity.

The value of r_2 as obtained from this formula is for the load condition, $K\ r_2$ being the effective resistance at start.

Mr. Hansen seems to understand that I say that my speed-time formulas applied to all types of loading conditions. However, the method by which they were evolved, combined with the paragraph on the last page of my article drawing attention to friction or hauling loads, shows this impression to be incorrect.

In reply to Mr. Alger's statements, I should like to point out that the object of making the secondary resistance high for motors which are used for frequent starting is two-fold. It not only reduces the starting current, but also reduces the total losses during acceleration. The reduction of the primary resistance by means of reducing the primary turns has little or no affect on the starting losses, however, when measured in joules, and motors intended for frequent starting may sometimes be designed by changing the secondary winding only.

Motors to be used for special starting duty can be built with either a high rotor resistance, few primary turns to strengthen the field, or a combination of both, depending on the nature of the application.

If the starts are very frequent then the first method gives the best results as the starting losses are lowered and this results in a lowering of the total losses over a complete cycle of operation provided the rotor resistance is not increased excessively.

If the number of starts are very few and high starting torque is required on account of, say, excessive static friction, then the second method can be used provided the increased starting current is permitted and that low power factor is not of any great consequence.

When there are limits placed on the starting torque and current and the full-load power factor and efficiency, then the combination of more rotor resistance and fewer primary turns is used to meet these requirements. Again it may be used when the application requires high starting torque and a medium number of starts and stops.

From this it can be seen that the designer does not always use few primary turns.

Regarding Mr. Alger's statement that the use of few primary turns exaggerates the magnetizing current and so upsets the simple theory that during acceleration the primary and secondary current can be considered of equal value, I cannot agree. Neglecting saturation, the magnetizing current changes inversely as the turns squared. The starting current also changes in the same ratio. Therefore, the ratio of these currents is the same no matter what the primary turns may be. Should saturation be taken into account, then the consequent increase in current applies much more to the starting current than to the magnetizing current during acceleration because at start the main field is only one-half or one-third of its value at synchronous speed, while the leakage paths are saturated.

The difference between the primary and secondary current of the motor of average size and speed is about three per cent so that the value of the secondary resistance as given by formula (1) in the paper is about six per cent too small. Since the secondary loss (measured in joules) during acceleration is independent of the secondary resistance, then this error does not affect the rotor loss.

Now consider the primary loss during acceleration: the exact value could be obtained by multiplying the secondary loss in watts by the ratio of the primary resistance to the true value of secondary resistance and then by the mean square of the ratio of primary to secondary current over the starting period.

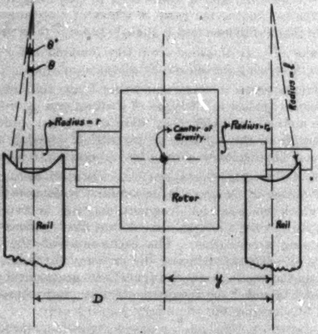

FIG. 2

It so happens that this ratio of the mean square over the entire starting period is very nearly equal to the ratio of the square of the primary to secondary current at the instant of starting which is also the ratio of the true value of secondary resistance to the value obtained by using formula (1) in the paper. Therefore, by using formula (1) to get the secondary resistance and assuming the primary and secondary currents equal, very little error is introduced.

To illustrate how negligible this error is, consider a 50-h. p., 6-pole, 60-cycle motor. The ratio of the mean squares of the primary and secondary currents up to 90-per cent synchronous speed is equal to 1.041. The ratio of the true value of secondary resistance to that calculated by formula (1) is equal to 1.031, so that there is only one per cent error in calculating the primary loss. Consider an extreme case of a 50-h. p., 20-pole, 60-cycle motor with only 4¾ times starting current and 68 per cent full-load power factor. The ratio of the mean currents to 90 per cent synchronous speed is equal to 11.37 and the ratio of true to calculated secondary resistances is equal to 11.17 which shows that there is less than two per cent error even in this extreme case. From this it is obvious that it is sufficiently accurate to use the method given in the paper.

Mr. Alger seems to be under the impression that the work on reversing motors, in which I made correction for eddy-currents in the rotor, had something to do with a difference in the primary and secondary currents. This is not so; the reason for finding the primary loss by the method worked out in the paper was on account of a varying ratio between primary and secondary resistance while retarding the speed.

Regarding the statement that formulas similar to those given in my paper could be worked out for double-cage windings, I think that these formulas would be very complicated and therefore would not be extensively used. Further, it is doubtful if such a motor would have any advantages over a high-resistance rotor motor as the higher full-load efficiency is misleading, the true guide being the total losses over a complete cycle of operation. Also there would have to be a decided advantage in favor of a double-cage winding to warrant its use as it is more complicated and expensive.

I should like to add the following formula which gives the radius of gyration for rotors which have journals with bearings of different diameter on each end. This measurment can be made by using the circular rails illustrated in Fig. 7. Squirrel-cage motors have usually the same diameter of bearing on each end but it is sometimes necessary to know the radius of gyration of other rotors. This can be done with the same equipment. Referring to Fig. 2 herewith,

let

v = vel. of center of gravity of rotor.

R = radius of gyration radially (feet).

R_1 = radius of gyration about axis perpendicular to the shaft and through the center of gravity.

ω and ω_1 are the respective angular velocities.

Then gain in kinetic energy

$$= \frac{1}{2} M v^2 + \frac{1}{2} M R^2 \omega^2 + \frac{1}{2} M R_1^2 \omega_1^2$$

Loss in potential energy

$$= M g (\theta'^2 - \theta^2)\left[\frac{1}{2}\left(\frac{r_0}{r}\right)^2 (l - r_0)\left(1 - \frac{y}{D}\right) + \frac{1}{2}(l - r)\frac{y}{D}\right]$$

Substituting $\omega_1 = \omega \dfrac{r_0 - r}{D}$

$$\omega = \frac{l}{r} \frac{d\theta}{dt}$$

$$v = \left(\frac{D - y}{D} r_0 + \frac{y}{D} r\right)^{\omega}$$

Equating these two quantities and integrating gives:

$$R = \sqrt{\left(\frac{P}{2\pi l}\right)^2 g\left[r_0^2(l - r_0)\left(1 - \frac{y}{D}\right) + r^2(l - r)\frac{y}{D}\right] - \left[r_0\left(1 - \frac{y}{D}\right) + r\frac{y}{D}\right]^2 - \left[R_1 \frac{r_0 - r}{D}\right]^2}$$

The term $\left[R_1 \dfrac{r_0 - r}{D}\right]^2$ can be neglected.

From this equation it can be seen that the short-cut method which suggests itself of taking the average of r and r_0 and then substituting in the formula given on the tenth page of my paper does not give sufficiently accurate results.

The Calculation of Magnetic Attraction
By the Aid of Magnetic Figures

BY TH. LEHMANN[1]

Non-member

Synopsis.—*The present paper treats of the following matters:*

1. Simplification of the physical formula for magnetic attraction,

2. Definition of magnetic reluctance when the magnetic field is bounded by non-equipotential surfaces,

3. Calculation of the virtual variation in magnetic reluctance due to any displacement of the limiting surfaces,

4. Demonstration of the theorem that the magnetic attraction between two ferromagnetic bodies depends only upon their common magnetic fluxes ϕ and upon the virtual gradient of its air-reluctances R_a, and that the formula

$$F_l = \frac{\phi^2}{8\pi} \frac{\delta R_a}{\delta_l}$$

which gives the attraction along any given direction l is as general and as exact as the formulas of physics,

5. Direct deduction of magnetic attraction from the lines of magnetic flux depicted by a magnetic figure without resorting to the components of the magnetic field.

The method used is developed by the application of the well-known principles of the potential energy function to the magnetic flux in the air-gap. Paths for the magnetic flux are established across the air-gap between the two ferromagnetic surfaces bounding the air-gap, by decomposing the magnetic flux into elemental tubes of magnetic force, the envelopes of which enclose spaces in which the flux is constant. The element of boundary-surface intersected by each elemental tube at the two boundary-surfaces encloses an equal number lines of magnetic flux, irrespective of the magnetic density at these points. By replacing the element of boundary-surface by its geometrical projection on a plane normal to the axis of the elemental tube, an equivalent equipotential surface is obtained at each end of the elemental tube. Any non-equipotential surface bounding an air-gap can thus be replaced by an equivalent equipotential surface composed of an aggregation of elemental equipotential surfaces which produce denticulations in the contour of the boundary-surface. It becomes possible, in this way, to evaluate magnetic reluctance and magnetic attraction by reference to summations of elemental magnetic tubes and without the necessity of considering directly the magnetic density of the magnetic field at any point. The potential energy of each elemental tube of magnetic force depends only upon the potentials at its ends, at the boundary-surfaces, it being entirely independent of the path followed by the tube in traversing the air-gap.

* * * * *

I. INTRODUCTION

THE resultant magnetic effort, F, which is experienced by a soft iron body in a medium whose permeability is independent of the field, H, can be calculated by means of the known formula

$$F = \frac{1}{8\pi} \int_s \{2H(B, N) - N(H, B)\}\, dS \qquad (1)$$

in which N is the external unit vector which is normal to the surface-element dS, H and B being the vectors of the magnetic field and induction at the same point.

By means of equation (1), the resultant force, F, can be calculated whenever the distribution of the magnetic field along the external surface of the magnetic body is known. When that body is surrounded by air or like medium, the formula (1) can be further simplified; but even in that case, there are no analytical solutions in the majority of the cases of magnetic field distribution met with in practise, so that, as a rule, all that can be done is to obtain the graphical integration of equation (1) by deducing the values of the magnetic field from a chart of its lines of magnetic flux. It is possible, now, to determine these values within one per cent[2].

This method of evaluation of the quantity indicated by equation (1) is not free from error, owing to the points of saturation which occur at the sharp angles and at the narrow portions of the ferromagnetic material enclosing the slots.

For these reasons it seems worth while to inquire whether magnetic attraction could not be deduced directly from the lines of force shown in a magnetic figure without the necessity of first obtaining the components of the magnetic field and then recombining them tensorially. It so happens that this is possible within the entire range of validity of equation (1).

In the present paper the only case considered will be that of the attraction between ferromagnetic bodies that are separated by a medium like air, this case being of more immediate interest.

II. TRANSFORMATION OF THE PHYSICAL FORMULA FOR ATTRACTION

In (1), let $\{2H(B,N) - N(H,B)\} = P$. This quantity is a linear vectorial function of the normal unit-vector N. By putting this expression in the form $aP + bH + cN = 0$, it is seen that P, H, and N, and also B when $B \parallel H$, are situated in the same plane. Moreover, the vector H bisects the angle formed between P and N (Fig. 1)[3] as is seen at once by taking the scalar product of P and H, and observing that $P^2 = B^2 H^2$. This allows P to be written in simpler form.

Let us use the term "*directed algebraical product*" of two vectors, H and B, to designate the vector the length of which is equal to the algebraical product of the scalar values of H and B, and the angle of which, with the

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[1] Consulting Engineer, Urmatt, France.

[2] See *Revue Générale de l'Electricité*, 1923, Vol. XIV, pp. 347 and 395.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

[3] See, for instance, A. Vaschy, "Théorie de l'Electricité," 1896, p. 64.

outward normal N, is equal to the sum of the angles, α, which H and B make with N; that is to say, $2\,\alpha$ when $B \parallel N$. This vector, which is parallel to the same plane as N, H and B, will be symbolized by $(\overline{B\,H})_{2\alpha}$, so that for an iron body surrounded by air, formula (1) may be expressed in the following form:

$$F = \frac{1}{8\,\pi} \int_S B^2_{2\alpha}\, d\,S \qquad (2)$$

Equation (2) may be considered a generalization of the ordinary formula of Maxwell, with which it becomes identical when $\alpha = 0$.

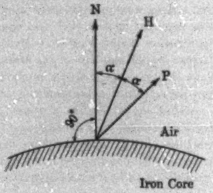

FIG. 1

It is seen at once that the algebraical value of the quantity to be integrated P, is always equivalent to B^2 whatever may be the angle of incidence, α, of the magnetic field. In air, the tensorial ellipsoid consequently becomes a sphere. For the condition $\alpha = 0$, the integration is performed along a level surface, and $P = \overline{B_0}^2$ has the direction of N. For an angle of incidence $\alpha = 45$ deg., the elemental effort $P = \overline{B_{\pi/2}}^2$ is tangential to the surface. Finally, if the integration is performed along a surface generated by lines of magnetic flux, and more especially along a line of plane flux, we have $2\,\alpha = \pi$, and $P = \overline{B}^2$ is now directed in a direction contrary to that of N. Since the direction of P is obtained by simply reflecting N with respect to B, the application of formula (2) is a very simple matter.

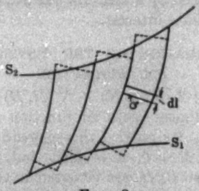

FIG. 2

III. DEFINITION OF THE RELUCTANCE BETWEEN TWO NON-EQUIPOTENTIAL SURFACES WHICH ARE SEPARATED BY AN AIR-SPACE

Let us subdivide the air-space between two non-equipotential surfaces, S_1 and S_2, (Fig. 2), into n very thin aliquot or subsidiary tubes having the reluctances $\rho_1, \rho_2, \rho_3, \ldots$, and let us replace the oblique bases of each tube by orthogonal bases having the same distance between centers. The elemental tubes being now limited by orthogonal bases, their reluctances can now be defined in the ordinary simple manner. It is known, moreover, that, in a medium of constant permeability, the magnetic energy of a Laplacian or vortex magnetic field is given by the volume-integral

$$W_m = \frac{1}{8\,\pi} \int B\,H\,d\,v$$

Integrating across the air-gap, along each tube, by tube-elements of sectional area σ and length $d\,l$, we will have

$$W_m = \frac{1}{8\,\pi} \sum_1^n \int B^2\,\sigma\,d\,l = \frac{1}{8\,\pi} \sum_1^n \phi_k^2 \int \frac{d\,l}{\sigma} = \frac{1}{8\,\pi} \sum_1^n \phi_k^2\,\rho_k$$

But since the flux, ϕ_k, of each aliquot tube is equal to the nth part of the total flux Φ, we have

$$W_m = \frac{1}{8\,\pi}\,(\Phi_n)^2 \sum_1^n \rho_k = \frac{\Phi^2}{8\,\pi}\,R_0 \qquad (3)$$

whence we deduce the total reluctance

$$R_0 = \frac{1}{n^2} \sum_1^n \rho_k \qquad (3')$$

or, more precisely,

$$R_0 = \underset{n\,=\,\infty}{\text{Limit}}\ \frac{1}{n^2} \sum_1^n \rho_k \qquad (3'')$$

It is evident that, by defining the reluctance ρ_k of each tube in accordance with (3″), formulas (3) and (3′) will remain rigorously accurate, whatever may be the magnitude of the elemental tubes.

In principle, the reluctance R_0 between two non-equipotential surfaces may be defined by the aid of any drawing of magnetic tubes. If the fractional coefficients of the tubes be designated by $m_1 = \frac{\Phi}{\phi_1}$, $m_2 = \frac{\Phi}{\phi_2}$, ... we obtain, for R_0, the following more general expression:

$$R_0 = \sum_1^n \frac{\rho_k}{m_k^2} \qquad (4)$$

The most frequent case is that where an arrangement of n tubes, conveying a total flux Φ, is composed of $n-1$ aliquot tubes, each containing the flux $\phi_k = \frac{\Phi}{m}$ and a residual flux $\phi_n = \frac{\Phi}{m_n}$. The total reluctance then becomes equal to

$$R_0 = \frac{1}{m^2} \sum_{1}^{n-1} \rho_k + \frac{\rho_n}{m_n^2} \tag{5}$$

IV. DEMONSTRATION OF THE EQUIVALENCE OF THE FORMULA $F_l = \frac{\Phi^2}{8\,\pi} \frac{\delta\,R_0}{\delta\,l}$ TO FORMULA (2)

Let us represent in cross-section (Fig. 3) the origin of a very thin tube, 2 – 3, and of the adjacent tubes, 1 – 2 and 3 – 4. Let us make the plane of the drawing parallel with the magnetic field in the air-gap B, and with the normal, N, with respect to the surface-element, S. The lines of magnetic flux being supposed to be very close to each other, no change will be caused in the reluctance of the tubes or in the external field if the oblique bases are replaced by orthogonal cross-sections through the same centers. Let us now displace the denticulated surface in a direction parallel to itself to the distance $d\,z$. All that need be considered is the projection $d\,x = d\,z \cos\eta$ of this displacement on the plane containing the vectors N and B, because any displacement of the surface-element S perpendicularly to that plane will have no influence on the reluctance of the tube under consideration. By taking any suitable unit of length for measuring the thickness of the tube in the direction perpendicular to the drawing, the width, σ, of the tube in the plane of the drawing can be made equal to its sectional area, and the intersections of its oblique base can be made equal to the surface-element, S.

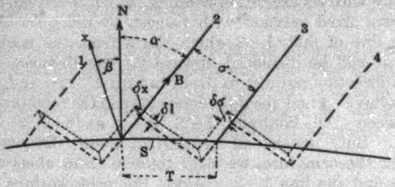

FIG. 3

Let (in Fig. 3) β designate the angle between $\delta\,x$ and the normal, N, and let α designate the angle of incidence of the vector, B. It will easily be seen that the displacement of the denticulated surface to the distance $d\,z$ causes in the tube of magnetic force a shortening equal to $d\,l = d\,x \cos(\alpha + \beta)$ in the direction of its length, and a decrease in its sectional area equal to $d\,\sigma = d\,x \sin(\alpha + \beta)$ over a length equal to $s \sin\alpha$. The result is that the reluctance of the portion of the tube thus modified is changed from $\rho = \frac{s \sin\alpha}{\sigma}$ to $\rho' = \frac{s \sin\alpha - \delta\,l}{\sigma - \delta\,\sigma}$, so that the variation in reluctance caused by the displacement of the surface-element, S, when $\sigma = s \cos\alpha$, and $s = S$, becomes equal to

$$\delta\,\rho = -\frac{\delta\,x}{\sigma^2} \cos(2\,\alpha + \beta)\,S$$

$$= -\frac{\delta\,z}{\sigma^2} \cos\eta \cos(2\,\alpha + \beta)\,S \tag{6}$$

Now, let $\frac{\Phi}{m_k} = B\,\sigma$ represent the magnetic flux in any tube. The summation of the values $\frac{\Phi^2}{8\,\pi} \frac{1}{m_k^2} \frac{\delta\,\rho_k}{\delta\,z} = -\frac{1}{2\,\pi} B^2 \cos(2\,\alpha + \beta)\,S$ along the whole surface will then, with the aid of (4), give us

$$\frac{\Phi^2}{8\,\pi} \sum \frac{1}{m_k^2} \frac{\delta\,\rho_k}{\delta\,z} = \frac{\Phi^2}{8\,\pi} \frac{\delta\,R_0}{\delta\,z},$$

from which, by passing to the limit, we obtain

$$\frac{\Phi^2}{8\,\pi} \frac{\delta\,R_0}{\delta\,z} = -\frac{1}{8\,\pi} \int_s B^2 \cos(2\,\alpha + \beta) \cos\eta\,d\,S \tag{7}$$

It now becomes immediately apparent that the quantity under the integral sign is nothing more than the projection of the vector, $P = \overline{B_{2\alpha}{}^2}$ upon the direction of $\delta\,z$. Therefore, the formulas (2) and (1) lead to the same effort in any given direction δz, as the formula $F_z = \frac{\Phi^2}{8\,\pi} \frac{\delta\,R_0}{\delta\,z}$, and the three formulas are consequently equivalent to each other. This result can also be expressed vectorially as follows:

$$-\frac{\Phi^2}{8\,\pi} \times (\text{Virtual gradient of } R_0) = \frac{1}{8\,\pi} \int_s \overline{B_{2\alpha}{}^2}\,d\,S.$$

We shall now show that the attraction between two bodies depends only upon the field that is common to both of them and that it can be evaluated without knowing the leakage-field.

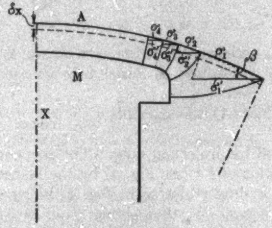

FIG. 4

V. CALCULATION OF MAGNETIC ATTRACTION BY MEANS OF THE COMMON FIELD ONLY

A. Unsaturated circuits. Let us consider the attraction in a radial direction between a field-magnet M

with salient poles and the armature (or stator) A (Fig. 4), on the assumption that $\mu = \infty$ in the iron. For this purpose, let us suppose a virtual decrease equal to δx in the air-gap, as if the armature were brought closer in the direction of the axis (X) of the pole, and let us, for a moment, assume as being true, the physical figment that the external field is not disturbed in consequence of the displacement in question (δx). Under such conditions, the length of each tube of magnetic force will be decreased by an amount equal to $\delta x \cos \beta$, where β designates the angle between the normal to the surface S at the point under consideration and the direction of δx. Therefore, if σ is the sectional area of the tube as it emerges from the surface of A, the variation of its reluctance, ρ, will be $\delta \rho = - \dfrac{\delta x \cos \beta}{\sigma}$. The portion of the virtual energy pertaining to this tube, when ϕ is the flux, consequently becomes equal to $\delta T = \dfrac{\phi^2}{8 \pi} \dfrac{\delta x \cos \beta}{\sigma} = \dfrac{B^2}{8 \pi} \delta x \cos \beta \, \sigma$, and the elemental force along δx will be equal to

$$f_x = \frac{B^2}{8 \pi} \sigma \cos \beta$$

which is evidently the projection upon δx of the force which is normal to σ, namely, $f = \dfrac{B^2 \sigma}{8 \pi}$, in perfect accordance with formulas (1) and (2).

Inasmuch as the common flux ϕ is generally the primary information available in practise, it is important to note that the total force of attraction F_x can be obtained without any integration, when the drawing shows m isometric tubes, by obtaining, along the armature, the value of the sum

$$F_x = - \frac{\Phi^2}{8 \pi} \frac{\delta R_0}{\delta x} = \frac{\Phi^2}{8 \pi} \frac{1}{m^2} \sum \frac{\cos \beta}{\sigma}$$

All that is necessary, therefore, is to obtain, at the place where the end-section, σ, of each tube emerges from the polar surface (A), the segments $\sigma' = \dfrac{\sigma}{\cos \beta}$, which are marked off by projection upon a line perpendicular to the direction of δx (see Fig. 4) by the normals to the chord of σ drawn (dotted in Fig. 4) between its two opposite edges. We thus obtain

$$F_x = \frac{\phi^2}{8 \pi} \frac{1}{m^2} \sum \frac{1}{\sigma'} \tag{8}$$

The degree of precision of F_x naturally depends upon the number of isometrical tubes into which the useful flux is subdivided. When the drawing comprises $n - 1$ tubes, each containing the flux $\phi = \dfrac{\Phi}{m}$, and a residual tube containing the flux $\phi' = \dfrac{\Phi}{m'}$, it is evident that we shall have

$$F_x = \frac{\Phi^2}{8 \pi} \left\{ \frac{1}{m^2} \sum_{1}^{n-1} \frac{1}{\sigma_h'} \; . \; . \; . \; . + \frac{1}{m'^2} \frac{1}{\sigma_n'} \right\} \tag{8'}$$

Let us now reverse the roles by bringing the field-magnet closer to the armature by the distance δx in a direction parallel to the axis of the pole. In order to make the physical figment plausible, we must now assume (Fig. 5) that the magnetic field is displaced bodily with its excitation-winding, which is assumed to have a fixed relation with respect to the pole (M). The line of flux $1 - 2$ which starts from the neutral point, will be displaced upward by the distance δx in a direction parallel to itself, toward $1' - 2'$; and we now have to find the difference between the reluctances between the contours $1 - 2 - 3 - 4$ and $1' - 2' - 3 - 4'$. But, instead of measuring these differences along the armature, we shall totalize them along the pole and the line $1 - 2$. Along the polar surface, as far as the line $1' - 2'$, we shall obtain, as before, the segments $\sigma' = \dfrac{\sigma}{\cos \beta}$, where σ designates the sectional area of the tubes measured where they emerge from the pole and β designates the angle between the normal N, to the chord of σ and the direction of δx. For the tube at the extreme right hand, there will be, in addition, a decrease in sectional area equal to $\delta x \cos \gamma$, where γ designates the angle between the direction of δx and the normal, N, to the line of flux at the point considered. The variation of reluctance per element of tube of length, λ, which results in the case of the last tube, in addition to the shortening, will be equal to

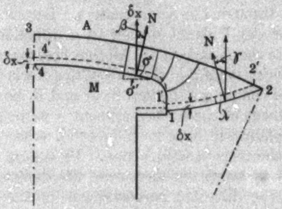

Fig. 5

$$\delta \rho' = \frac{\lambda}{\sigma - \delta x \cos \gamma} - \frac{\lambda}{\sigma} = \frac{\lambda}{\sigma^2} \delta x \cos \gamma,$$

and the corresponding decrease of attractive force will be equal to

$$f_x' = - \frac{\phi^2}{8 \pi} \frac{\delta \rho'}{\delta x} = \frac{B^2}{8 \pi} \lambda \cos \gamma,$$

which is the geometrical projection on the pole-axis of a force, $-\dfrac{B^2 \lambda'}{8\pi}$ which is normal to λ. The same term may also be obtained with equations (1) and (2), by making the surface-integral pass over the polar face and the line of flux 1 – 2 which ends at the neutral point.

The tube of magnetic force that is contiguous with the line 1 – 2 is composed (Fig. 6) of two squares, the mean sectional areas of which may be replaced approximately by the chords, λ_1 and λ_2, of their average lengths. The quantities $\lambda_1' = \dfrac{\lambda_1}{\cos\gamma_1}$ and $\lambda_2' = \dfrac{\lambda_2}{\cos\gamma_2}$ correspond to and can be obtained directly from segments cut off from a horizontal line (Fig. 6) by the perpendicular lines erected at their extremities in each case; and with the aid of these quantities, we finally have, in the case of m aliquot tubes of magnetic force,

$$F_x = -\frac{\phi^2}{8\pi}\,\frac{\delta R_0}{\delta x}$$

$$= \frac{\phi^2}{8\pi}\,\frac{1}{m^2}\left\{\sum\frac{1}{\sigma'} - \left(\frac{1}{\lambda_1'} + \frac{1}{\lambda_2'}\right)\right\} \qquad (9)$$

It is therefore possible to evaluate the virtual derivative, $\dfrac{\delta R_0}{\delta x}$*, from the original drawing of the magnetic field, without representing the displacement* δx *and without introducing the components of the magnetic field.*

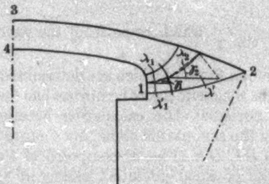

FIG. 6

It should be noted that the virtual work is equal to the energy contained in the zone involved only when the armature is displaced. If, instead, the magnetic field-pole and its winding are displaced, the magnetic energy contained in the area swept over by the polar base of the useful flux, 1 – 4 (Fig. 5) should be reduced by an amount equal to the potential energy contained in the zone swept over by the extreme lines 1 – 2 of the common flux.

Example. Let us take as an example the magnetic figure (Fig. 7) reproduced from Fig. 19 in the "*Revue Generale de l'Electricite,*" 1923, Vol. XIV, p. 397. After the fourth magnetic tube, counting from the neutral point, the lines of magnetic flux are radial, and they constitute, between that point and the polar axis, 6.4 unit tubes; so that the common flux, ϕ, per half pole consists of 10.4 unit-tubes. The air-gap being equal to 1.2 cm., and the polar half-pitch being equal to 9 cm., we have, for the sectional areas along the armature, reckoned from the neutral point:

2.85 cm.; 0.97 cm.; 0.71 cm.; 0.62 cm.; 6.4/0.6 cm.

The fractional coefficients are m = 10.4 for the first four tubes and m' = 10.4/6.4 for the last tube; so that, for the radial attraction per half polar pitch, we have:

$$F_x = \frac{\phi^2}{8\pi}\,\frac{1}{10.4^2}\left[\frac{1}{2.85} + \frac{1}{0.97} + \frac{1}{0.71} + \frac{1}{0.62} + \frac{6.4}{0.6}\right]$$

$$= \frac{\phi^2}{8\pi}\,\frac{1}{10.4^2}\,.\,15.06$$

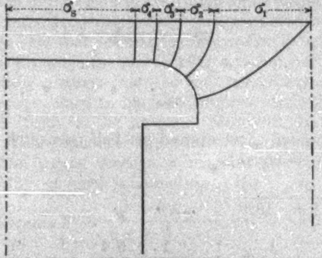

FIG. 7—DETERMINATION OF THE MAGNETIC ATTRACTION OF THE ARMATURE

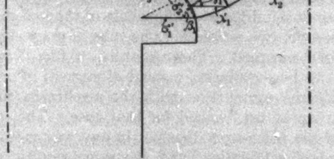

FIG. 7 A—DETERMINATION OF THE MAGNETIC ATTRACTION OF THE FIELD

Let us now start from the field-pole. The bases of the first three tubes that emerge from the polar surface are no longer horizontal, and it is necessary to begin by erecting perpendicular lines on the chords of σ and to obtain in a horizontal direction (see Fig. 7A) the values of $\sigma' = \dfrac{\sigma}{\cos\beta}$ which gives us the following values: σ_1' = 1.57 cm.; σ_2' = 0.70 cm.; σ_3' = 0.34 cm.; and we can then point off directly the values σ_4 = 0.55 cm. and σ_5 = 6.4/0.6 cm. We still have to obtain, on the line of median flux of the first tube (shown dotted), the segments $\lambda_1' = \frac{\lambda_1}{\cos\gamma_1}$ and $\lambda_2' = \frac{\lambda_2}{\cos\gamma_2}$, which may be cut off along any horizontal lines whatever by the lines perpendicular to the chords of the curvilinear elements λ_1 and λ_2 erected at their extremities. These values are found to be: $\lambda_1' = 1.10$ cm., and $\lambda_2' = 3.40$ cm.

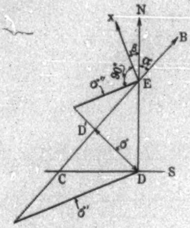

FIG. 8

The radial effort exerted per half pole pitch is thus found to be equal to

$$F_x = \frac{\phi^2}{8\pi}\,\frac{1}{10.4^2}$$

$$\left[\frac{1}{1.57}+\frac{1}{0.70}+\frac{1}{0.54}+\frac{1}{0.53}+\frac{6.4}{0.6}-\frac{1}{1.10}-\frac{1}{3.4}\right]$$

$$= \frac{\phi^2}{8\pi}\,\frac{1}{10.4^2}\,.\,15.20$$

The difference between this value and the preceding one is barely one per cent which seems satisfactory if we bear in mind that for the whole of the half distance between the two poles only one tube of magnetic force has been assumed. The armature in Fig. 7 was assumed to be rectilinear, instead of curved, in order to simplify the comparison with the algebraical formula which can be established for that case. The inclination of the polar axes does not in any way modify the method of calculation, and the same results should always be obtained whether we start from the armature or from the field-pole.

B. Saturated Circuits. The physical figment that a displacement does not disturb the external field leads us immediately, by the aid of (6), to expressions for an armature in which there is no current,

$$F_x = \frac{\phi^2}{8\pi}\sum\frac{\cos(2\alpha+\beta)}{m^2\,\sigma\cos\alpha}, \qquad (10)$$

and for a field-pole which is driving its field without alteration,

$$F_x = \frac{\phi^2}{8\pi}\left\{\sum\frac{\cos(2\alpha+\beta)}{m^2\,\sigma\cos\alpha}-\frac{1}{m_n^2}\left(\frac{\cos\gamma_1}{\lambda_1}+\frac{\cos\gamma_2}{\lambda_2}\right)\right\} \qquad (10')$$

Let us consider (Fig. 8) the base $C\,D$ of a tube having the thickness $D\,D' = \sigma$, which emerges from an element $C\,D$ of the surface, S, of the iron, at an angle α with respect to the normal $D\,E$ to $C\,D$. We can then obtain the projection of the superficial force f_n, corresponding to this tube, in a direction x, forming an angle β with $D\,E$, by constructing directly

$$f_x = -\frac{\phi^2}{8\pi}\,\frac{\delta\rho}{\delta x} = \frac{\phi^2}{8\pi}\,\frac{\cos(2\alpha+\beta)}{\sigma\cos\alpha}\,.$$

But it is often easier to decompose

$$\frac{\delta\rho}{\delta x} = \frac{1}{\sigma'} - \frac{1}{\sigma''}$$

$$= \frac{\cos(\alpha+\beta)}{\sigma} - \frac{\tan\alpha\sin(\alpha+\beta)}{\sigma}$$

and to obtain σ' and σ'' from Fig. 8 by cutting off segments on the perpendiculars to x emerging from D and E through the mean vector of the field B passing through C on the one hand and through the normal to the vector, passing through D, on the other hand. In the case of the field-pole it is still necessary to add the terms $\frac{\lambda}{\cos\gamma}$ which come from the extreme tubes of the common flux and which are determined as before.

When the magnetic figure comprises one or more lines of equal potential, the calculation becomes simpler by making the summation along any contour whatever, such as $A\,B\,C\,D\,E$ (Fig. 9) composed of lines of flux and of lines of equal potential, ending at the neutral points of the armature, or, in more general manner, ending at boundary points of the common field. The values of $\sigma' = \frac{\sigma}{\cos\beta}$ and of $\lambda' = \frac{\lambda}{\cos\gamma}$ may be obtained, as explained in paragraph A, from the sections σ on the equipotential lines and the segments λ on the isometric lines of flux, by noting that the terms

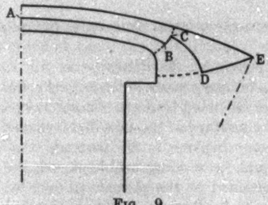

FIG. 9

$\frac{1}{\sigma}$ are of the same sign as the areas swept over, whereas the terms $\frac{1}{\lambda'}$ are of contrary sign.

The equivalence of the process can be easily demonstrated by transforming, by means of the generalized form of Green's theorem, the surface-integral (**1**) into a volume-integral:

$$\int_s P\,d\,S = \int \{2\,H \operatorname{div} B - H^2 \operatorname{grad} \mu - 2\,(B, \operatorname{rot} H)\}\,dv \qquad (\mathbf{11})$$

In the space in the air-gap which is traversed by the useful flux we have, inside any closed aerial surface, div $B = 0$; grad $\mu = 0$; rot $H = 0$; therefore, we will have

$$\int_S P\,d\,S = 0 \qquad (\mathbf{12})$$

Let us now suppose that the air-gap is swept over once by the surface of the armature, for a distance equal to one polar division $0_1\,I\,0_2$ (Fig. 10) and a second time by a string of any kind whatever, such as 0_1 II 0_2 which extends between the same points 0_1 and 0_2 of the armature. Multiplying (**12**) scalarly by the displacement $\overline{\delta\,x}$, and integrating along the closed contour 0_1 I 0_2 II 0_1, we have:

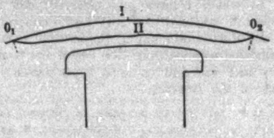

Fig. 10

$$\int_s (\overline{\delta\,x}, \overline{B_{2\alpha}{}^2})\,d\,S = 0; \text{ whence}$$

$$\int_{0_1 I\, 0_2} (\overline{\delta\,x}, \overline{B_{2\alpha}{}^2})\,d\,S = \int_{0_1 II\, 0_2} \overline{\delta\,x}, \overline{B_{2\alpha}{}^2})\,d\,S; \qquad (\mathbf{13})$$

or, on bringing (**13**) closer to the identity (**7**), we see that

$$(\delta\,R_0)_I = (\delta\,R_0)_{II}$$

It is, therefore, possible to determine the virtual variation of the reluctance, on the assumption that the air-gap is cut through by any contour whatever passing through the neutral points 0_1 *and* 0_2 *of the armature.* This manner of proceeding has advantages in the case of windings placed in slots (as in turbo generators).

The physical formulas for magnetic attraction are not much used in practise. Is this due to the hypotheses used in physics? It cannot be denied that, at first glance, our practical intuition, clarified by the idea of the magnetic circuit, finds it difficult to accept the figment which excludes any disturbance of the field during a virtual displacement. But this figment, although it may lead to a state of unbalance, is, in general, justified by observing that, in accordance with Thomson's theorem, one of the two neighboring states of distribution corresponds to a minimum of potential energy.

VI. APPLICABILITY OF THE DIFFERENT FORMULAS

Theoretically, formulas (**1**) and (**2**) enable the attraction to be evaluated whenever the magnetic field in the air-gap is known along the whole surface. But since no analytical solutions that are of interest in practise are available in the majority of cases the best we could hope to do would be to obtain the integration of these formulas graphically after the values of the magnetic flux have been deduced from a drawing of the lines of flux. And even this calculation becomes uncertain in the neighborhood of the slots for the windings, on account of the points of saturation found at the sharp corners and narrow parts of the cores that close around the slots. On that account the integration is rendered possible only when the surface of integration is situated at a suitable distance from the limiting surfaces. It is therefore necessary to have a complete survey of the magnetic field in the air-gap. Moreover, formulas (**1**) and (**2**) are no longer applicable when the surface of integration passes through a medium the magnetic permeability of which is a function of H.

The formula $F_l = \frac{\phi^2}{8\,\pi}\,\frac{\delta\,R_0}{\delta\,l}$ makes it possible to evaluate the magnetic attraction in the case of any ferromagnetic body surrounded by a medium of constant permeability (air) without the necessity of taking into account the components of the field. The permeability of the body may or may not depend upon the field. In the form in which the formula is completed by the saturation-term[4], it can also be used when the permeability of the external medium depends upon the field. It is then possible, moreover, to do without the magnetic figures of the lines of flux, provided the reluctance, R_0, of the air-gap is known as a function of the extent l, of the air-gap, It is also more easy, by means of this formula, to note critical values of the effort of attraction near points of contact, where the attraction may become more than double, even when the flux is assumed to be maintained constant by means of equipotential connections.

VII. CONCLUSIONS

The magnetic effort exerted upon an assemblage of ferromagnetic bodies and of currents, surrounded by air as a medium, is determined by the corresponding fluxes ϕ and by the virtual gradient of their air-reluctances, R_0.

The formula $F_l = \frac{\phi^2}{8\,\pi}\,\frac{\delta\,R_0}{\delta\,l}$ is as exact and as general as the formulas of physics. It has the advantage over them that besides giving an algebraical solution, it renders possible the evaluation of magnetic attraction by the direct utilization of the lines of magnetic flux of a magnetic figure, without the necessity of first determining the components of the field and then recombining them tensorially.

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[4] This case is treated in "*Revue Genérale de l'Electricité*," Vol. XV.

In a general way, the attraction between two bodies depends only upon the field common to them both. That is still true when the two fields have nothing more in common than their lines of separation (the case of the ideal short-circuit); and the mutual magnetic effort is then determined entirely by the value of the field along these lines of separation.

Discussion

C. O. Mailloux: This is another of the papers of the kind which, like one by Mr. Fortescue[1] published some years ago and one by Mr. Rice[2] written more recently, serves to advance our knowledge of ways and means of attacking problems that have baffled all others before these authors, and at the same time gives us very interesting evidence of the fact that the methods of mathematical treatment of previous generations—say of the days of Maxwell, Kelvin, Mascart, Helmholtz, etc.—are still capable of giving magnificent results. Mr. Fortescue, in an epoch-making paper, showed the valuable use that can be made of the principles of the *potential energy function* and of their application to the discussion of equipotential surfaces, etc., as a means of mapping out the field of electric force around insulators that are subjected to high electrostatic stresses. Those who may have had doubts at that time in regard to the utility of the study of the potential function as a preparation for the analysis of phenomena in fields of force, and were disposed to look more sympathetically upon the more "modern" methods devised or elaborated by Bjerknes, Lorenz, and others, found that their fears in regard to the "staleness" of the older methods were not wholly well founded. What Mr. Fortescue did was, in a sense, an extension of Maxwell's work, and his diagrams of lines of electric force in electrostatic fields show at least a family resemblance to some given in Maxwell's treatise. Mr. Rice's able paper furnished further valuable evidence of the great usefulness of this method of attack on seemingly difficult problems of like character.

In the present paper, we have the very interesting case in which the theory and the principles of the potential function enable the author to perform an entirely new *tour de force* that would have done credit to Maxwell himself, showing such a simple and practical way of "surveying" and *appraising* the magnetic force in an air-gap that we wonder how such a clever expedient has remained so long undiscovered. Every student of the theory of the potential function knows that the concept of the "tube" of magnetic force, even though it may be after all only a figment of the scientific imagination, does account, in a very simple way, for differences of magnetic density and distribution that are often impossible to describe by any other method, notably by the measurement or expression of variations in magnetic density in different parts of the magnetic field. It was an inspired idea of the author to decompose the magnetic field in the air-gap into *elemental tubes of magnetic force*, with boundaries (or envelopes) enclosing spaces in which the magnetic flux is constant. This amounts virtually to the same thing as finding paths across the air-gap where the magnetic density may be treated as if it were constant. In this way the author found a means of getting across the air-gap without having to stop midway to rearrange and readjust the magnetic density; and with the aid of the principle of "solenoidal distribution" and of equipotential surfaces, all of which is a part of the theory of the potential function, he had the absolute certainty of being able to get across safely before he started. The only problem that remained to be solved was that of making "landings" at both ends of the path followed in crossing the air-gap through a tube of magnetic force when that path is not exactly straight across, *i. e.*, when the tube of magnetic force is not "normal" to the surface of the poles or other ferromagnetic portions of the magnetic circuit on the two sides of the air-gap. The author solved that problem in the well-known, simple way, familiar to mathematicians, of replacing any contour line by an equivalent broken line, on the principle that a circle may be regarded as a polygon of an infinite number of sides. Now, the fundamental fact that makes this method rigorous as well as interesting and effective is the well-known principle that whenever a potential energy function *exists*, as is the case in an air-gap, the *initial* and the *final* stages of the potential developed (which in this instance is magnetic potential; that is to say, the conditions determining the *strength* of the physical *bond* between the two edges of the air-gap) are really independent of the locus of the path through which the tube of force is "laid." Indeed this far-reaching generalization, a great achievement and precious inheritance from the mathematical physics of previous generations, was the key that unlocked the secret of the wide power and range of the method and that inspired confidence in it by showing how far it can be relied upon.

The paper deserves to be regarded as a pathfinder, pointing the way to new and really wonderful applications of old but still "up-to-date" and most effectual methods of attacks on difficult problems. It is seen that the figments of the scientific imagination, to which I have just referred, can play a most important role and can lead to highly useful concrete results as they did in Mr. Fortescue's paper. As in that case (aside from the useful practical applications) the result is a contribution to general theory and will add value in a permanent way to the TRANSACTIONS of the Institute.

The paper may not prove as easy of perusal and comprehension as might be, because some of the ideas in it are presented somewhat tersely and are not, therefore, placed within as easy reach of the ordinary reader as might have been, perhaps, had the author realized the desirability of so doing. The mathematics are simple enough; in fact, they are elementary to those who have studied the theory of the potential energy function and mathematical physics to even a slight extent, although they may seem abstruse, imposing, and forbidding to those who have not done so. The paper really deserves to be made accessible to a very large constituency. This could have been done easily by taking a little more space for the presentation of the mathematics in slightly more "dilute" form. The objection most often made to papers of this type is, indeed, that their authors do not sufficiently recognize the importance of making them entirely clear to those who do not know or remember as much as they do themselves about the "short cuts" in mathematical transformations and demonstrations. A few notes given in an appendix, clarifying the more abstruse portions, would help greatly to remove the impression that the paper is merely a "high-brow" product which will, or can, interest only mathematical students and experts. In this respect, however, the paper will be found to be not nearly so formidable as it may look. Moreover, those who are interested in this general subject will find such additional information and elucidation in other articles by Dr. Lehmann which have appeared in the *Revue Générale de l'Electricité*; notably in the article published in the numbers of July 12 and 19, 1924. This was written at about the same time as the present paper and covers substantially the same ground, but does it in a more detailed manner, and also throws full light upon the theoretical considerations which the author kept in mind and which underlie the methods described in the paper.

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1. TRANSACTIONS A. I. E. E., 1913, p. 907.
2. TRANSACTIONS A. I. E. E., 1917, p. 905.

J. Slepian: It is usual to derive the forces acting on material bodies in a magnetic field by starting with the principle of conservation of energy, calculating the electrical energy input and the increase in magnetic energy, and assuming that the difference in these quantities gives the work done by magnetic forces on bodies which are displaced. This is the procedure followed by Doherty and Park in their paper.[4] Mr. Lehmann, however, starts with Maxwell's expression for the stresses in a medium in which a magnetic field exists.

In the time of Maxwell, the great program of physics was to explain all phenomena mechanically. Therefore it was very natural to try to explain the transmission of force over distance through the mediary of a magnetic field by a mechanism similar to that by which force is transmitted mechanically through material bodies, namely by a system of mechanical stresses. Maxwell was led to believe that the following system of stresses would account for the mechanical forces in a magnetic field: a tension equal to $H^2/8\pi$ per sq. cm. across any surface perpendicular to the lines of magnetic force, and a pressure equal to $H^2/8\pi$ across any surface parallel to the lines of magnetic force.

In applying these stresses to determining the forces on material bodies, however, a difficulty arises, for these stresses are supposed to exist in empty space or in the ether as well as in material bodies. And since the ether pervades all material bodies, we cannot tell how much of the stress at any point acts on a material body there, and how much is limited to the ether. Maxwell resolved this difficulty to some extent by showing for a stationary magnetic field that although these stresses do not give the actual ponderomotive force at each and every point of a material body, when integrated over a closed surface, they do give correctly the total ponderomotive force on all the material bodies enclosed in that surface.

Herein lies one of the difficulties which I have found in trying to follow Mr. Lehmann, for apparently he integrates the stresses over a surface which is not closed. Such a procedure if carried out, for example, over a portion of the surface of a wooden body, would give a resultant force, and yet we feel quite sure that no such mechanical force exists on any portion of such a body. If the integration is carried out over the whole surface of the body, the correct zero resultant is obtained, but not if the integration is limited to a part of the surface.

It is easy to see that if a body has infinite permeability, then integration over any portion of its surface will give correctly the mechanical force on that portion, and it so happens that the example which Mr. Lehmann has worked out is for a body with infinite permeability. But if the method is applicable to the case of finite permeability, it seems to me that some further justification is necessary. I would be very happy if Mr. Lehmann could clear up this point for me.

The formula which Mr. Lehmann has obtained is similar to that obtained by Doherty and Park by considering displacement of a magnetic body under constant flux, and I presume is intended to apply also for the case of saturation. However, Mr. Lehmann does not limit himself, as do Doherty and Park, to special magnetic circuits where the flux density is constant in the air-gap during the displacement, but suggests that the result is general. This is a question of very great importance and I therefore considered it worth while to try to derive this result from the principle of conservation of energy. In so doing I ran upon a difficulty which Mr. Lehmann hints at in his paper but which I have not been able to surmount. Perhaps Mr. Lehmann can show how to take care of it.

I used the following artifice: In the magnetic system under consideration, assume that all currents which are flowing are in resistanceless circuits so that no impressed voltages are required. Now assume that with the bodies in the position for which the force is to be calculated, all bodies which are subject to magnetic saturation are made infinitely conducting. No change in the magnetic force results from these hypotheses. Now let one of the saturating bodies be displaced a small amount, Δx. Then currents are induced in the various circuits and conducting bodies, the magnetic field changes, and work is received by the moving body. Now, since the electromotive forces are zero, there is no input of electrical energy. Hence the work received by the moving body must be equal to the decrease of magnetic energy. Since the saturating bodies are by hypothesis infinitely conducting, there will be no change in the magnetic field within them at any point, so that the magnetic energy in their interiors remains constant. Hence the whole change in magnetic energy is external to the saturating bodies. This change in the external magnetic energy may be resolved into two parts. First we may consider the change which would take place if the energy density at every point remained constant, so that the motion of the saturating body merely changed the boundaries of the external field. This change may be determined by multiplying the energy density at each point of the surface of the body by the normal component of the displacement of the surface and integrating over the whole surface. This change in magnetic energy considered by itself leads directly to Lehmann's formula. However, there is the second part of the change in magnetic energy which must be considered; namely, that due to the change in energy density at the various points of the field. If ΔW is the change in magnetic energy density at any point of the field, resulting from the displacement of the saturating body, then the volume integral,

$$\int\int\int \Delta W \, dx \, dy \, dz$$

taken over the whole external field gives the other part of the change in magnetic energy.

If Mr. Lehmann's result is correct, this integral must be equal to zero. However, I have not been able to establish the fact that this is necessarily the case and would like to ask Mr. Lehmann if he can suggest how this is to be done.

R. H. Park: Two methods of calculating the mechanical forces due to magnetic attractions have been presented at the 1926 Midwinter Convention; namely, a method of calculation based on analysis of flux through circuits, and a method based on the analysis of stresses acting at each point of a closed surface bounding the volume on which the force is to be calculated. Both methods have special fields of application in which they are of particular value. In the formula

$$F_l = \frac{\phi^2}{8\pi} \quad \frac{\delta R_o}{\delta l}$$

of the paper by Dr. Lehmann, he has given a useful simplification of the known methods of calculating force on the basis of stress acting on bounding surfaces. For this particular result and also for his introduction of the general method of calculating force from stresses on bounding surfaces to problems of practical character, Dr. Lehmann deserves much credit. Without wishing to detract in any way from the practical value of Dr. Lehmann's paper, I should like to mention a few considerations which may deserve attention.

In particular, the application of formula (1) of the paper to problems in which saturation exists both within and without the volume in which the force is to be calculated may be open to question. In his "Electrical Papers," Volume I, pages 542 to 553, and Volume II, pages 543 to 574, Oliver Heaviside shows definitely that formula (1) applies in the case of saturation when there is no saturation outside the region on which the force is to be calculated. He also gives a formula which applies in the more general case and in the interior of saturated regions. This result would indicate that formula (1) of the paper was correct in application in the case when there is iron both within and without the region on which the force is to be calculated, if the intersurface does not pass through saturated material. The arguments employed, however, in establishing these results are

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3. *Mechanical Force between Electric Circuits*, by R. E. Doherty and R. H. Park, A. I. E. E. Journal, March, 1926, page 231.

not entirely clear since no explanation is given that the assumptions on which the proof is based are in accord with experimental facts.

In regard to Dr. Lehmann's definition of his reluctance R_0, it may be pointed out that the determination of this reluctance requires a previous knowledge of the distribution of the field in the region under consideration. It therefore depends on the character of the medium at points outside of the particular region under consideration. Also there is a difference between the term $\frac{\delta R_0}{\delta l}$ and the ordinary total derivative of $R_0 \frac{d R_0}{d l}$. The derivative $\frac{\delta R_0}{\delta l}$ corresponds to a change in δl with the flux density and direction of the field fixed at the bounding surfaces.

C. O. Mailloux: With reference to the remark made by two of the discussers that the method is of direct application to questions where the permeability is independent of the field, that statement is brought out in the paper itself. The author concedes that point and there is no question about it. At the end he does refer to the fact that the formula can be completed by a saturation factor, but there is no detailed reference given to that, so we may assume that he didn't feel sufficient confidence in that new development to introduce it into the paper.

The inquiries made by one of the speakers are partly answered in the paper itself; notably where the author speaks of a complete integration around a circuit and counter-balancing effect produced by the integration of the second part of the circuit that is already shown diagrammatically in Fig. 10.

In regard to the detailed analysis of the formula, he preferred the purely practical method of finding what allowance, if any, could be made for the variation of magnetic field, when the length of the air-gap is increased or decreased, by calculating two cases where it has been done, proceeding with one from the inside and the other from the outside. He thus finds the difference between the two is within one per cent.

Now, for the cases with which he was interested in dealing, that error certainly is sufficiently small and, while the theory might be very interesting in a question of splitting hairs, it is not of immediate concern in connection with the present paper. I hope, however, that the author will be given an opportunity to explain himself to any such extent as he may deem necessary.

In the paper he does not mention the potential function, but anybody at all familiar with the subject knows that it is based upon that. One of the characteristics of potential function is that it goes into no speculation as to action at a distance. That may be the difficulty encountered by the gentleman. It is independent of all speculations as to the intervening medium or the actions at a distance. Had the author wished to go into it, he would have to begin by discussing the physics and mathematics of a tube of force, itself. He has assumed that as implicitly as one would assume the multiplication table in mathematics. Hence, if that is accepted as a fact and the explanation is sought for in some other books on mathematics, it seems to me a great many of the difficulties will disappear.

Th. Lehmann: I am indebted to Mr. R. H. Park and to Mr. J. Slepian for giving me, by their remarks, an opportunity to define further the scope of my paper. I also thank Dr. Mailloux for already having replied in part to their discussions.

My A. I. E. E. paper is a condensed summary of articles which have already appeared in the *Revue Générale de l'Electricité*, (Paris), July 12th and 19th, 1924, in which I made explanation (especially in paragraph VIII) of the fictitious character of Maxwell's stresses and pressures. In writing the A. I. E. E. paper it appeared to be desirable to leave out that paragraph and certain others to avoid lengthening it too much. In saying in the introduction to my paper, however, that I was taking into consideration only the resultant magnetic effort produced in a ferromagnetic body entirely surrounded by air, it seemed to me that there could be no doubt that the formulas (1) and (4) should be integrated along the *whole* of surface S in the body, especially since the letter S was added at the bottom of the integral sign to show that a *surface-integral* was intended.

Maxwell himself observed that the vectorial function, $P/8\pi$, which appears in the integral expression, is of fictitious character, inasmuch as it represents magnetic effort per unit of surface. The fact of having shown how this function may be evaluated and used in the determination of the true resultant does not at all imply that a real entity is attributed to the elemental effort per unit of surface, $P/8\pi$.

This point now having been made clear, I wish to assert that, for any closed aerial surface of integration, the formula

$$F = \frac{1}{8\pi} \int B_{1\alpha}^2 \, d S, \tag{4}$$

is equivalent to formula (1), the range of validity of which seems to have been determined previously by Oliver Heaviside. In any case, formula (4) gives exactly the *resultant* magnetic effort exerted upon a ferromagnetic body, C, having a surface, S, entirely surrounded by air, whether the permeability of the body be variable or not and even when the body, C, has hysteresis[4]. In the external medium, (the surrounding air), there may be other saturated ferromagnetic bodies and also some currents, provided the whole of the surface, S, of the body, C, be separated from the other ferromagnetic bodies by an air-zone.

Here I shall go no further than to state that I have given elsewhere (in the *Revue Générale de l'Electricité*, 1925, Vol. XVII, p. 167) the demonstration of the validity of formula (4) for a body, C, that is saturated but free from hysteresis, this being done by means of a schedule of the virtual potential energy involved, obtained by applying the theorem of W. Thomson. Another demonstration which does not use this theorem but which also includes the case where the body C is not free from hysteresis will appear shortly in the *Revue Générale de l'Electricité* (in the number for September 25, 1926, p. 427).

The same conditions, as regards validity hold also in the case of the formula

$$F_l = \frac{\phi_0^2}{8\pi} \frac{\delta R_0}{\delta l} \tag{7}$$

If, for the value of the flux, ϕ_0, we take the circumflux[5] and, for the reluctance, R_0, the value obtained by means of the formulas (3) and (3′), with that flux, then for example formula (7) will give the *true resultant* in the direction of δl, on the armature of a dynamo. Of course, $\delta R_0/\delta l$ is the *virtual* derivative of that reluctance, and, as pointed out correctly by Mr. Park, it should not be confused with the ordinary total derivative, $d R/d l$. Naturally the formula can be applied also to each pole or to each magnetic circuit in turn. The respective magnetic efforts thus obtained should then be considered as being component portions of the total magnetic effort, and their vectorial sum will give the *true resultant effort* rigorously, whether the magnetic circuits be saturated or not.

The effort actually exerted upon a pole or upon a tooth can be determined in the manner which I have already indicated in the *Revue Générale de l'Electricité*. When there is no deformation (*i. e.* when the magnetic effort produces no change in geometrical figure) in the pole and in the tooth, the method of procedure is as given on page 101 of the issue of July 19th, 1924, in paragraphs VI and VII; and when deformation is produced, the method corresponds to the formulas (**9**) and (**11**), as given in the April 4th, 1925 issue. If Mr. Slepian will kindly refer to these earlier papers, I dare say that he will recognize the fact that the fictitious character of Maxwell's stresses and pressures had not escaped me in the least. But all this was quite outside the scope of my A. I. E. E. paper; in that I intended to take into consideration only the effort exerted upon a ferromagnetic body that is entirely surrounded by air.

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[4] It should be stated that A. Liénard has already pointed out (in the *Revue Generale de l'Electricite* of October 20th 1923, p. 563) that the formula (1), when integrated on the *air* side along the surface of a ferromagnetic body with hysteresis, gives exactly the resultant magnetic effort. Moreover, A. Guilbert has given a very satisfactory experimental confirmation of formula (7) in his paper (Paris, 1926) which was published as a supplement to the Bulletin of the Société Française des Electriciens for January 1926.

[5] That is to say, the absolute sum of the fluxes per pole which are common to all the poles; this, in the case of an armature which is not eccentric in the magnetic field, is the same as the product of the number of poles and the common flux per pole.

The evaluation of the virtual derivative of the reluctance of the air, $\delta R_0/\delta l$, should be made by reference to Fig. 4 to 7A whenever the boundary air-surfaces may be considered equipotential, of which condition the reader is duly warned at the beginning of paragraph V (by the condition $\mu = \infty$ in the iron). In the case of superficial saturations of the pole-pieces of the armature, up to $B = 18{,}000$ c. g. s. (or $\mu > 100$), the departure of the lines of magnetic force passing through the air from the orthogonal direction with respect to the boundary surfaces, is still scarcely noticeable, and the methods described are still applicable with a degree of precision that is amply sufficient for practical purposes. The case given by way of example for a half-pole, should, naturally, be extended to all the poles when the magnetic field is excentric; this can be done by making δx coincide with the direction of the axis of excentricity. When the field or the armature is saturated to such a point that the lines of magnetic flux are no longer normal to the boundary surfaces, it becomes necessary to determine the segments σ' and σ'' along the whole of the air-surface for each tube of magnetic flux in accordance with the method indicated in Fig. 8; and, in that case, by following the external contour of the armature, we have

$$F_x = \frac{\phi_e^2}{8\pi} \sum \frac{1}{m^2} \left(\frac{1}{\sigma'} - \frac{1}{\sigma''} \right) \qquad \textbf{(10)}$$

In making the summation along a portion of the poles and of the outermost lines of force of the flux per pole that is common to both parts of the magnetic circuit, it is necessary to add, for each outer line of force, a term which corresponds to

$$\frac{\phi_e^2}{8\pi m_n^2} \left(\frac{1}{\lambda_1'} + \frac{1}{\lambda_2'} \right).$$

As I pointed out in the *Revue Générale de l'Electricité* of July 19th, 1924, at the end of paragraph VIII, the elemental contributions per tube should be considered fictitious for the same reason as in the case of Maxwell's effort per unit of surface, $P/8\pi$, in formulas (1) and (**4**).

As to the last question asked by Mr. Slepian, it will be found to be answered fully in the article already mentioned, which is to appear soon in the *Revue Générale de l'Electricité*, and in which the magnetic attraction between bodies that are saturated and that also have hysteresis is deduced from a schedule of the actual energy involved, in a virtual displacement, without abstractions or hypotheses.

For the convenience of those who do not have ready access to the files of the *Revue Générale de l'Electricité*, a brief reference to the article published in July, 1924, may be useful.

As stated in the preface of that article, the following points are considered in it: (1) simplification of the physical formula for attraction; (2) definition of magnetic reluctance when the field is bounded by non-equipotential surfaces; (3) calculation of the virtual variation of the reluctance due to any displacement whatever of the limiting surfaces; (4) demonstration that the magnetic effort between two bodies depends only upon their common fluxes, ϕ, and upon the virtual gradient of their air-reluctances, R_0, and that the formula

$$8\pi F_l = \phi_2\, \delta R_0/\delta l$$

which gives the attraction in any direction whatever, l, is as exact and general as the formulas of physics; (5) direct deduction of the attraction from the lines of flux of a magnetic figure without having to determine the components of the field; (6) evaluation of the local magnetic effort exerted upon a tooth or pole that is inserted in a groove, when the permeability is a function of the field.

The article contains nine sections, of which the last four include material that supplements the A. I. E. E. paper, which latter, as a matter of fact, was submitted to the Meetings and Papers Committee early in 1924, or before the publication of the articles in the *Revue Générale de l'Electricité*. In Sections VIII and IX, many points in regard to the validity and applicability of the formulas are made clear. Section VIII, perhaps, ought to have been included in the paper. It is reproduced here (translated) with equation (**13**), taken from another part of the paper which could not be understood unless reproduced.

I think that this extract from my other paper will cover sufficiently the point raised by Mr. Slepian.

I am grateful to Dr. Mailloux for his discussion which is a really elegant summary of the mathematical principles utilized in my paper and of their usefulness I am deeply touched by his high praise of my work.

(Extract from *Revue Générale de l'Electricité*) (July 19, 1924)

"VIII. DIGRESSION. The formulas of physics for magnetic attraction are not much use in practise. Are physical hypothesis at all responsible for this? It cannot be denied that at first glance our sense of practical intuition, refined by the concept of the magnetic circuit, finds it difficult to admit the figment that excludes all disturbance in the magnetic field during a virtual displacement. But that figment, although it may lead to a state of disequilibrium, can be justified by the theorem of Thomson applied to two neighboring states of distribution, one of which corresponds to a minimum of potential energy."

It may, perhaps, be better not to look so far but simply to attribute the above reflection to the dilemmas that one meets when Maxwell's stresses are applied to surfaces which are not closed. An example will make the difficulty more clearly apparent.

Let us suppose a mass of soft iron of constant permeability, μ, and free from all internal currents. The effort exerted upon such a body in an air medium of permeability, μ_0, may be evaluated according to formula (**13**) in either of two ways. We have

$$\frac{1}{8\pi} \int_s P\, d\, S = \frac{1}{8\pi} \int_v \{ 2\, H \operatorname{div} B - H^2 \operatorname{grad} \mu - 2\, [B, \operatorname{rot} H] \}\, d\, v \qquad \textbf{(13)}$$

The term on the left-hand side is a surface-integral which is equal to (**3**), or

$$F = \frac{1}{8\pi} \int_s \{ 2\, H\, (B, N) - N\, (H, B) \}\, d\, S \qquad \textbf{(3)}$$

The term on the right-hand side in (**13**) is a volume-integral which is obtained by the aid of the generalized theorem of Ostrogradsky (see Vaschy, *Théorie de l'Electricité*, 1896, p. 66).

The first of the two ways of evaluation is by means of the surface-integral,

$$8\pi F = \int_s P_0\, d\, S \qquad \textbf{(25)}$$

and the second (since div $B = 0$ and rot $H = 0$) is by means of the volume-integral

$$8\pi F = -\int_V H^2 \operatorname{grad} \mu\, d\, v \qquad \textbf{(26)}$$

Since the permeability, μ, has been assumed to be constant, we have, in the iron, grad $\mu = 0$, except at the surface S, where grad $\delta\, \mu = N\, (\mu - \mu_0)$. For each surface element, $d\, S$, the volume integral, it will be recalled, will receive the contribution

$$(P_0 - P)\, d\, S = N\, (H, H_0)\, (\mu - \mu_0)\, d\, S$$

where the subscript zero refers to the surrounding medium. But the Maxwellian elemental effort being given (outside of the factor $1/8\,\pi$) by the value of the quantity P_0 under the integral sign in equation (25), in air, it is clear that the quantities to be integrated in (25) and (26) cannot both give the true elemental effort. It is commonly admitted that the quantity under the integral sign in (26) corresponds to the true effort, so that the Maxwellian effort P_0, considered as an elemental effort, appears to be fictitious, and it leads merely to the same resultant effort as (26).

For the condition μ = constant, formula (26) becomes, in effect,

$$8\,\pi\,F = \int_s (P_0 - P)\,d\,S \qquad (26')$$

But, by virtue of theorem (13), when integrating along the *inside* contour (the iron side) of the *closed* surface S, (since grad μ then vanishes) we have,

$$\int_s P\,d\,S = 0,$$

and we thus see that (26′) actually becomes identical with (25) for any closed surface.

If we consider $-\dfrac{1}{8\,\pi}\,H^2$ grad μ as the true effort per unit of volume, we can then scarcely attribute any real meaning to the Maxwellian stresses except in so far as they can be applied in the case of a surface of integration that is entirely closed.

The same remarks apply to the efforts deduced from the variation in the reluctance, which do not prevent us, of course, from assembling the efforts represented by tubes when these are considered component parts of the total effort.

The Magnetic Hysteresis Curve

BY HANS LIPPELT[1]
Member, A. I. E. E.

Synopsis.—*An analysis of the phenomena of hysteresis is presented in the paper which introduces the conception of a reactive component and a dissipative component of the counteracting force, which appears when magnetizable material is subjected to a magnetizing force. Using this conception, equations and curves are developed for the hysteresis curve, for the various components and for the hysteresis loss. The loss is shown to depend directly on the dissipative component.*

I. Nature of the Problem

1. The purpose of this paper is an analytical study of the fundamental character of magnetic hysteresis with a view to formulating, if possible, reliable mathematical terms in convenient form.

Foremost among the historical works along this line is that by Hopkinson (giving a theory of the magnetic circuit), which was well augmented by Fröhlich as published in the *Elektrotechnische Zeitschrift* of 1881, pages 90 and 139; and a treatise by Professor Kennelly printed in the TRANSACTIONS of the A. I. E. E., 1891, page 485.

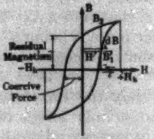

FIG. 1

The most familiar representation of magnetic hysteresis is the well-known loop, as shown in Fig. 1, in which the salient features are duly emphasized.

2. *Nature of Forces Involved in the Process of Magnetization.* As will be explained, the process of magnetization of the so-called magnetic metals and alloys seems to involve three forces.

(a). Magnetizing force H, which tends to produce magnetic induction in the material in the direction of H.

(*b*). Reactive force R, which tends to demagnetize the material, that is, to reduce the induction to zero. It seems to be a force of the nature of internal elastic stress, similar to a reactive tension.

(*c*). Dissipative force D, which opposes *changes* of magnetic induction. It seems to be a force of the nature of (molecular) friction and tends to maintain the material in the state of magnetization in which it happens to be at the moment.

The first force, H, is of external origin, being produced usually by an electric current.

When the magnetic circuit contains an iron core the total flux per sq. cm. of cross-section is

$$B = H + 4\,\pi\,I = H + \beta \qquad (1)$$

where $\beta = 4\,\pi\,I = B - H$ is called the intrinsic induction in the iron.

To better illustrate the relations, let us assume that a magnetic hysteresis curve $B_1\ B_2$ has been determined by experiment, Fig. 2. Its course is indicated by dotted lines, $B_1\ B_2$. True to definition, corresponding curves β_1 and β_2 have been produced by subtracting from the ordinates of $B_1\ B_2$ the corresponding abscissas, H.

3. *Manifestation and Character of Forces R and D.* That the external force H cannot be the only force entering into the process of magnetization may readily be understood from the axiom of physics, according to which any action entails a reaction. H represents the action in this case and two forces, R and D, the reaction. That becomes particularly evident from observations which are common to the well-known experiment of magnetizing iron by an electric current, and which it becomes necessary to again review.

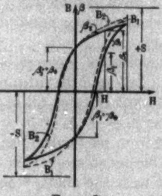

FIG. 2

(*a*). When an electric current flows through the spool, a magnetic field H is created within its cross-sectional area. This field H acts upon the iron core and causes it to become traversed by a strong magnetic flux β of the same direction as H. The iron will hold that flux as long as the magnetic field H exists. This indicates a tendency of H to produce (impress) induction in the iron in the direction in which it operates.

(*b*). Now if we interrupt the current, the magnetic field H will disappear and with it most of the magnetism β of the iron. This abatement of magnetism can be explained by the assumption of an internal

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force R, which tends to drive the magnetic flux β out of the iron, as an elastic reaction (tension) would do. Therefore, R is of opposite sign to H.

(*c*). If we put the current on again, there will reappear both the field H and the magnetism β. With the presence of R conceded under b, we will now have to broaden our conclusion (a) to the extent that the field H causes magnetism to be impressed upon the iron even against the opposing force R. Since the degree of magnetization is the same for the same recurring field H, it is likely that R depends on the degree of magnetization, the latter reaching the steady state when its sequel, the tension R, has likewise reached its original value. On the other hand, if R were not present, magnetization should be expected to rise to any abnormal value.

(*d*). If we increase (or decrease) the field H over (or under) the value obtained under c, it will be followed by a strengthening (or weakening) of β, β coming in each case to a steady state when its associate reaction R has adjusted itself to the new value of H. Therefore, R is surely a function of the quantity β

$$R = F(\beta)$$

If no other forces were contingent upon the process, there should be $R = -H$, because in physics the reaction is always equal and opposite to the action. This also fixes the dimension of R to be the same as that of H, namely $C^{-\frac{1}{2}}G^{\frac{1}{2}}S^{-1}$.

It will be shown, however, that these two forces R and H do not suffice to explain all the magnetic states, nor any one of them completely. A third force will exhibit itself when existing conditions are scrutinized more thoroughly.

(*e*). If the flow of current be interrupted altogether, it will be found that also most of the magnetism has been lost but not all of it. Residual magnetism, either $\beta_2 = +\beta_0$ or $\beta_1 = -\beta_0$, has been retained in the iron, particularly so when the material under test is hardened steel (Figs. 1 and 2).

Consistent with the above reasoning the presence of that residual magnetism β_0 ought to entail a residual reaction R_0. Since the field $H = 0$ (when no current flows), there would now be no force counteracting the reaction R_0 and all the magnetism should disappear from the iron. As it does not disappear, we are compelled to admit the existence of another, a second internal force. We recognize at once one of the characteristics of this new force; namely, it tends to *resist changes* in the magnetic state existing in the iron at the time, inasmuch as the residual magnetism continues to exist in a permanent magnet. The nature of that force, therefore, seems to be that of a friction. It will be designated by the letter D.

When the field $H = 0$, this friction has the special value D_0 and balances up against the internal tension R_0. Thus

$$D_0 + R_0 = 0$$

and the coincident state of magnetization is expressed by one of the two equations

$$\text{either } \beta_2 = +\beta_0 \text{ or } \beta_1 = -\beta_0$$

as per Fig. 2.

The fact that for any other value of H we observe likewise two values of β (Fig. 2) is proof enough of the magnetic friction D obtaining for all states of magnetization. D manifests itself particularly by the difference between β_1 and β_2 where the changes of β are large as referred to unit changes of H (*e. g.*, where the β curves are steep), and abates where they are small. Hence

$$D = F_1(\Delta\beta)$$

To amplify this reasoning, let us remember that for rapid changes of β (*e. g.*, $\Delta\beta$ = large), the reaction R also changes rapidly. Since H was assumed to change in unit steps only, D must necessarily make up for the rapid changes of R, which in turn conform to rapid changes of β, that is to $\Delta\beta$. This argument is also consistent with the force D being a "resisting" force and for this very reason D is negative, when $\Delta\beta$ is positive and vice versa.

It is obvious that the dimension of D is likewise $C^{-\frac{1}{2}}G^{\frac{1}{2}}S^{-1}$ because only forces of like character can enter into a play of action and reaction.

This force D will after further study reveal itself as the actual cause of hysteresis.

Having conceded three forces H, R and D to govern the process of magnetization, we were able to explain all and any conditions observed to exist in a magnetized material. To do that *all* of these *three forces* were *necessary*, but it is also evident that they are sufficient. The *magnetization* (β), however, when in the steady state depends on the equilibrium of those three forces. That is mathematically expressed in general by

$$H + D + R = 0 \tag{2}$$

and when applied to the ascending branch β_1 by

$$H_1 + D_1 + R_1 = 0 \tag{3}$$

and to the descending branch β_2 by

$$H_2 + D_2 + R_2 = 0 \tag{4}$$

Our problem was outlined in the first paragraph of Section I. It amounts, among others, to finding a formula $\beta = f(H)$, expressing in explicit terms the relation between the field H and the magnetization β. After what was said, this problem resolves itself now into two separate problems, *i. e.*, to determine primarily the equations for R and D as functions of H; and in conjunction therewith functions expressing R and D in terms of β.

II. Equation of Reactive Force $R = F(\beta)$

4. *Relation of R to β near Saturation.* It is known that the magnetization β of a magnetizable material can rise only to its saturation value S as a limiting value. When magnetization is approaching saturation, it will do so gradually and $\Delta\beta$ is small for large changes of H. Small variation of β, however, entails a negligible

force D and such a state of affairs in connection with equation (**2**) will be used as a guide for finding a mathematical relation between R and β.

5. *Law of Magnetization near Saturation.* As is well known, the curve of β plotted against the field intensity H as abscissa approaches, for large values of H, an asymptote, which runs parallel to the H-axis at a distance S = maximum value of β, Fig. 3.

That figure confirms the statement made in the foregoing paragraph (4) that β increases only little for large increments of H. In other words the difference

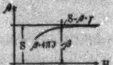


FIG. 3

$$S - \beta = \gamma$$

decreases rather slowly, because $\Delta\beta$ is small and so is the magnetic friction D.

Since the large external action H *must* meet a large internal reaction, it becomes necessary to ascribe a *large* value to the internal magnetic tension R. This quantitative relation becomes very plausible, when comparing the magnetism impressed upon the iron with an elastic medium being heavily compressed in a suitable container. (See Appendix I).

A law that is true in physics for compressed elastic mediums may apply also in our case of magnetism being impressed under similar symptoms upon suitable materials. That law applied will fix the internal reactive tension R as being inversely proportional to the capacity of the material for further magnetization. If $+S$ is the saturation value and β the amount of magnetism impressed by a positive field $+H$, the capacity for further magnetization is

$$\gamma = S - \beta$$

(See Fig. 3.) Under our assumption, the reactive component R would be expressed by

$$R_+ = -\frac{K}{S-\beta} = -\frac{K}{\gamma} \qquad (\mathbf{5})$$

where K is a factor of proportionality depending on the units selected for β and R. R_+ has been written to refer it to positive values of H and β.

In formula (**5**) the constant $-K$ is that reactive tension which exists when $\gamma = 1$.

We had to place the minus sign before the right hand side of formula (**5**) because R_+ is of a direction opposite to H (and also β), which we had assumed positive.

6. *Applying Formula* $R_+ = -\dfrac{K}{\gamma}$ *to the Whole Range of Magnetization.* This relation (**5**) applies, strictly speaking, only to the range near positive saturation. So long as the contrary is not proved, it is well worth while to try out its alleged validity for the whole range of magnetization and study the effect. Such an initial study of R, without paying attention to D, appears all the more justified, because R is a force depending on the degree of magnetization, while D depends primarily on the quality of the material.

In order to better understand this relation between β and R_+, a curve has been drawn up (Fig. 4) corresponding to formula 5. It proves to be an equilateral hyperbola, whose one axis coincides with the axis of abscissas β, while the other is parallel to the axis of ordinates at a distance equal to $+S$, *e g.*, the saturation value of β.

For values of β approaching $+S$, R_+ grows infinitely large and is of negative sign. For smaller values of β the numerical value of R_+ decreases first rapidly then slowly until it attains

$$\text{when } \beta = 0 \text{ a value } R_+ = -\frac{K}{S}$$

According to our curve, Fig. 4, R_+ continues to have negative values and does not become zero until

$$\beta = -\infty$$

Returning now for a moment to Fig. 2, we notice that for heavy negative values of field intensity H, the value of β approaches a negative maximum $-S$. Formula 5 and curve Fig. 4 both yield for this value of $\beta = -S$

$$R_+ = -\frac{K}{+S-(-S)} = -\frac{K}{2S}$$

while we should expect an infinitely large value for R in conformity with our previous reasoning.

7. *Adapting the Law of Reactive Tension R to Negative Magnetization.* This discrepancy of the quantitative relation between R and β, which became evident with negative magnetization, may easily be taken care of by repeating, for that condition, the application of the law of stressed elastic mediums.

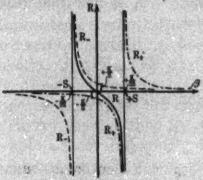


FIG. 4

Similar to the process carried out in chapter (**5**) we write now for strong negative magnetization

$$R_- = -\frac{K}{(-S)-\beta} = +\frac{K}{S+\beta} \qquad (\mathbf{6})$$

Formula (**6**) [as well as formula (**5**)] represents an equilateral hyperbola (Fig. 4), and in this curve the same discrepancy as that encountered above is here confronted in reversed form for $\beta = + S$.

8. *Modification of the Law of Inverse Proportionality.* The validity of each individual formula (**5**) and (**6**) to the limited zone of magnetization near positive or negative saturation respectively renders both of them unsuitable for general use. The situation is aggravated by the duplicity of value for R when $\beta = 0$, as given above $\left(R_+ = -\dfrac{K}{S} \text{ and } R_- = +\dfrac{K}{S} \right)$.

Even if it were conceded that formula (**5**) holds for positive magnetization and formula (**6**) for negative magnetization, the discontinuity and an abrupt change from $-\dfrac{K}{S}$ to $+\dfrac{K}{S}$ at zero magnetization is not compatible with the behavior of elastic mediums, which had been referred to for comparison (see Appendix I).

However, continuity is a necessary prerequisite and that can readily be obtained by admitting the simultaneous validity of both formulas (**5**) and (**6**). Such a step finds its mathematical expression in the following form:

$$R = R_+ + R_- = -\frac{K}{S-\beta} + \frac{K}{S+\beta}$$

or

$$R = -\frac{2K\beta}{S^2-\beta^2} \qquad (7)$$

Formula (**7**) is the sum of the two terms 5 and 6 and so is curve R in Fig. 4 the composite curve of the individual curves R_- and R_+.

The new curve has actually three branches, but the two external ones, which belong to abscissas whose numerical values are larger than S, are at present of no utility. Our present consideration shall be limited to the range between $+ S$ and $- S$.

With our fundamental law modified by formula (**7**), it is obvious that the constant K has now a different physical meaning. $- K$ is now that reactive tension (R) which exists when

$$\frac{2\beta}{S^2-\beta^2} = 1$$

A better definition is (Fig. 14)

$$K = \frac{1}{2} P_0 S^2$$

We have in Fig. 4 a continuous curve devoid of abrupt changes in values of R. Furthermore for

$$\begin{array}{ll} \beta = -S & R = +\infty \\ \beta = 0 & R = 0 \\ \beta = +S & R = -\infty \end{array}$$

That R becomes zero for $\beta = 0$ is particularly noteworthy, because this relationship affords a smooth reversal of the reactive tension R, when the magnetization β changes from positive to negative values. Besides, this point $\begin{cases} \beta = 0 \\ R = 0 \end{cases}$ is a point of symmetry for the curve.

Otherwise curve R in Fig. 4 and formula (**7**) answer well to the call for a reactive force, patterned after the tension in a stressed elastic medium. (See Appendix I).

For further reference, the first derivative of the reactive tension as referred to magnetization shall be given here.

$$\frac{dR}{d\beta} = -2K\frac{S^2+\beta^2}{(S^2-\beta^2)^2} = \frac{R}{\beta}\,\frac{S^2+\beta^2}{S^2-\beta^2}$$

$$= -\frac{R^2}{2K\beta^2}(S^2+\beta^2) = -\frac{R^2}{2K}\left(\frac{S}{\beta}\right)^2 - \frac{R^2}{2K} \qquad (8)$$

Of the unknown quantities contained in equations (**3**) and (**4**) two may now be considered known, namely

$$R_1 = -\frac{2K\beta_1}{S^2-\beta_1^2} \qquad (9)$$

$$R_2 = -\frac{2K\beta_2}{S^2-\beta_2^2} \qquad (10)$$

9. *Determination of Constants S and K.* One of these constants, S, depends surely on the magnetic quality of the material under consideration. Experiments have shown that.

Whether and to what extent K depends on the material is still to be proved by tests. Primarily K is a special value of magnetic tension.

From a few coordinated values of β near saturation, the saturation value S itself can be found by extrapolation with sufficient accuracy.

The same test readings of H and β will lend themselves to the determination of the constant K, when recourse is had to formula (**7**) and $- H$ is substituted for R.

Another method for determining K is explained below in connection with the study of force D.

10. *Conclusions in Regard to Reactive Tension R.* We have seen that: (*a*) The reactive component R of the internal counteracting force is a force of the nature of elastic recoil. (*b*) This force R is caused by and is a mathematical function of the magnetic induction β. (*c*). The relation of R to β may be expressed hypothetically by the equation $R = -\dfrac{2K\beta}{S^2-\beta^2}$ (*d*). Although the reliability of this equation has been demonstrated by experimental data pertaining to but one material, its plausibility can be enhanced by the agreement of its corollaries with known facts and theories. (See Appendix I.)

(*e*.) For values of β near saturation, the equation reduces itself to the form $R = -\dfrac{K}{\gamma}$ (Appendix I). Under these same limitations Fröhlich's law is reduced to the form $H = \dfrac{K}{\gamma}$. It follows that $R + H = 0$

and that magnetic friction D and hysteresis are negligible near saturation. This conclusion is in harmony with known facts.

(*f*). For small values of β the equation reduces itself to the form $R = -\frac{2K}{S^2} \times \beta$ which makes it very analogous to Hooke's Law for elastic bodies. (Appendix I.)

III. Equation of Dissipative Force D

11. *Review and Aspect.* The ultimate object before the author was the establishment of an equation which relates the magnetization β to the field H. Our basic equation (**2**) harbors the solution of the problem in so far as it ties into a law three magnetic forces which in turn are related to β and H.

The force R proved to be a force having the magnetization β as its cause and being quantitatively related thereto as per equation (**7**).

The dissipative force D, however, depends on the magnetic quality of the material magnetized. To study the character of this force D and its magnitude necessitates, therefore, a recourse to actual test of suitable materials.

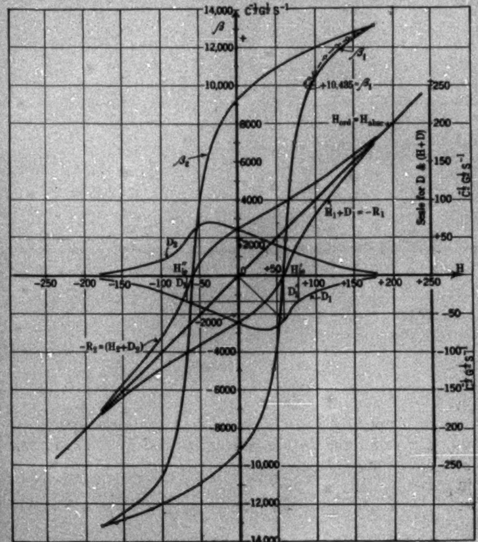


Fig. 5

Should we succeed in developing for this force D mathematical terms containing D as a function of H or β, the analytical representation of the magnetic hysteresis curve would then become possible.

12. *Exploitation of Experimental Data.* For the purpose of investigating to what laws the internal friction D (*i. e.*, D_1 and D_2) is subject, we employ a hysteresis curve that shows clearly the properties in question. By experiment the writer has determined such a curve for a material known as hardened tungsten steel. The β-curve for this material is shown in Fig. 5.

The method employed was the step-by-step method applied to Hopkinson's Divided Bar, using also a ballistic galvanometer.

The curve is composed of two branches, β_1 and β_2. Only the β_2-branch has been plotted from observed data, because the observed results for the β_1-branch may contain errors of observation. Ascertainment of degree of accuracy is not possible now. The β_1-branch shown is a copy of β_2-branch by virtue of symmetry.

13. *Numerical Values for K and S.* In order now to apply equations (**9**) and (**10**) to our curve, the values of S and K ought to be known.

From the curvature of β curve near its highest ordinates (Fig. 5), $S = 16000$ is estimated by graphical extrapolation.

To determine K, curves were plotted first taking K as an arbitrary constant which was later corrected as will be explained. For plotting these curves equations (**9**) and (**10**), $R = -\frac{2K\beta}{S^2 - \beta^2}$, were made use of. By their aid $+ R$ could be plotted as a function of β (similar to Fig. 4). However, it was found more useful to plot $- R$ as a function of H. The hysteresis curve, Fig. 5, gives values of H corresponding to different values of β and by use of these values, taken from the hysteresis curve, it was possible to plot $- R$ as a function of H.

The curves for $- R_1$ and $- R_2$ so plotted are shown in Fig. 5. This Fig. 5, however, shows the final corrected curves which were not obtained until the correct value of K had been determined.

As already mentioned, K was first given an arbitrary value and the curves for $- R_1$ and $- R_2$ were plotted against H. An inspection of these original curves showed that for large values of H they approached the straight line $H_{ordinate} = H_{abscissa}$. Knowing this it was possible to plot the curves to their true scale as shown in Fig. 5. Also the true value of K could be determined and it was found to be $K = 0.5682 \times 10^6$ for the sample tested.

14. *Curves for D_1 and D_2 as Depending on Field H. Permanent Magnets.* As $- R = (H + D)$ it is possible to get values of D from the $- R$ curves in Fig. 5. This was done and curves for D_1 and D_2 as functions of H were accordingly plotted in Fig. 5. $(- R - H = D)$.

Observe that the ordinates of D_1 are negative, while those of D_2 are positive.

Of special interest is the maximum value, which—referring to curve D_1—occurs for

H_1 = approximately $+ 36\, C^{-1/2}\, G^{+1/2}\, S^{-1}$

D_1 = approximately $- 70\, C^{-1/2}\, G^{+1/2}\, S^{-1}$

Furthermore for

$$H_1 = 0 \text{ we have } \left\{ \begin{aligned} D_1 &= -\ 61.5\, C^{-1/2}\, G^{+1/2}\, S^{-1} \\ \beta_1 &= -\ 9230\, C^{-1/2}\, G^{+1/2}\, S^{-1} \\ +\ R_1 &= +\ 61.5\, C^{-1/2}\, G^{+1/2}\, S^{-1} \end{aligned} \right.$$

When no external field H exists the two internal forces D_1 and R_1 (also D_2 and R_2) are numerically equal, but of opposite sign and therefore balance one another.

If we now go along curve β_1 (Fig. 5) from value $-$ 9230 units, where $H = 0$ to $-$ 8000, which represents a decrease in the magnetic flux, we get for this lower state of magnetization

$$\begin{aligned} R_1 &= +\ 47.4\ C^{-1/2}\, G^{1/2}\, S^{-1} \\ D_1 &= -\ 68.4\ C^{-1/2}\, G^{1/2}\, S^{-1} \\ H_1 &= +\ 21.\ \ C^{-1/2}\, G^{1/2}\, S^{-1} \end{aligned}$$

This means for this decrease in flux the internal friction D_1 has increased and is stronger than the internal tension R_1, which has decreased. Therefore, without the addition of $H_1 = + 21$ to force R_1, the magnetism within the iron could not diminish. This fact explains the phenomenon of permanent magnets. We may put it in this way.

"In permanent magnets magnetic friction and reactive tension are equal. For diminishing flux the friction increases, while the tension decreases. Therefore, the flux *cannot* decrease as a result of the reactive tension."

Also for other small and increasing values of H_1 the magnetic friction is both increasing and greater than the reactive magnetic tension, being smaller and diminishing. Such conditions will obtain until D_1 reaches its maximum value. After that, both D_1 and R_1 are decreasing, and, as a sequel, the magnetization β_1 of the steel does rapidly decline.

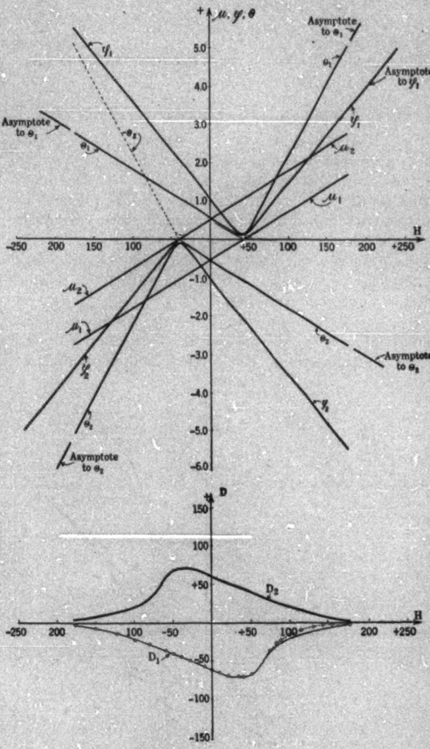
$$\text{For } \beta = 0 \text{, we have } \left\{ \begin{aligned} &- D_1 = + H_1 = \text{approx.} + 59 = \\ &\qquad \text{the coercive force} \\ &R_1 = 0. \end{aligned} \right.$$

For this condition magnetic friction and field intensity counteract and cancel one another, while the magnetic tension is zero.

For further rising field strength the magnetization β_1 increases rapidly at reversed (positive) values, owing to a rapid receding of D_1. The increase in flux, however, will soon cause R_1 to rise and the increments in magnetization, as referred to constant increments in field intensity, become smaller and smaller the more we approach the highest value of β_1. At the same time the magnetic friction D_1 converges towards the limit zero, while the reactive magnetic tension R_1 is striving towards an infinitely large value.

15. *Mathematical Relation of D and H.* After having analyzed in a general way the curves plotted from observed data, the problem remains to find mathematical terms expressing the intrinsic and mutual relations of all the forces involved and of the flux.

As regards the D curves discussed above it was possible to get a mathematical relation between D and H. The method of deriving this relation is given in Appendix II. It was found that the graphical curves D_1 and D_2 are closely satisfied by the following equations:



FIGS. 6-7

$$D_1 = -\frac{A}{\cosh \theta_1} \tag{11}$$

with

$$\left.\begin{aligned} \theta_1 &= u_1 + \varphi_1 \\ u_1 &= +c\,(H_1 - f) \\ \varphi_1 &= +q\sqrt{p + u_1^2} \end{aligned}\right\} \tag{12}$$

and

$$D_2 = +\frac{A}{\cosh \theta_2} \tag{13}$$

with

$$\left.\begin{aligned} \theta_2 &= u_2 + \varphi_2 \\ u_2 &= + c\,(H_2 + f) \\ \varphi_2 &= - q\,\sqrt{p + u_2^2} \end{aligned}\right\} \qquad (14)$$

For the sample tested the constants are as follows: $A = 70.061\ C^{-1/2}\ G^{1/2}\ S^{-1}$; $f = 42$; $c = 0.012545$; $p = 0.003255$; $q = 2$.

In connection with these equations the respective curves have been computed and drawn up in Figs. 6 and 7. Fig. 6 shows the graphs for u_1, φ_1, θ_1 and u_2, φ_2, θ_2, while in Fig. 7 a copy is given of the D_1 and D_2 curves as plotted from *observed* results.

In curve D_1 a few small circles indicate such points of D_1 curve as have been computed by formula (**11**).

It is seen from Fig. 7 that the agreement between the observed values of D_1 and those computed by formula (**11**) is rather good. For positive values of H_1 a deviation is noticeable. As will be seen later this is of little account.

IV. MAGNETIC FLUX

16. *Relation of β to R.* By solving equations (**9**) and (**10**) for β_1 and β_2 respectively, we get in general

$$\beta = + \frac{K}{R} - \frac{1}{R}\sqrt{K^2 + R^2 \times S^2} \qquad (15)$$

Values for β_1 and β_2 have likewise been computed. Between observed and computed values of β_1 and β_2 an agreement of even better degree than for D_1 (Fig. 7) is apparent in Fig. 5. And such better agreement is due to the fact that quantity $R = -(H + D)$ enters into formula (**15**). If D were not quite correct, the error would not manifest itself any more than D partakes in the sum $R = -(H + D)$.

Deviation between observed and computed values is indicated for the β_1 branch only by a dotted line (Fig. 5).

However, for $H_1 = + 93.4$ the *observed* value $\beta_1 = + 10435$ has been plotted and it falls very close to the computed (dotted) line. The above formulas may, therefore, be considered as being correct to satisfy any reasonable demands.

The term on the right hand side of formula (**15**) permits a nice transformation. Move the product $R^2\,S^2$ outside the square root and write:

$$\beta = S\left[\frac{K}{S\,R} - \sqrt{\left(\frac{K}{S \times R}\right)^2 + 1}\,\right]$$

By applying now the following substitution

$$\frac{K}{S\,R} = \sinh\psi \text{ or } \psi = \sinh^{-1}\frac{K}{S\,R} \qquad (16)$$

our formula will turn into

$$\beta = \pm\, S\,[\,\sinh\psi - \cosh\psi\,] = \mp\, S \times e^{-\psi} = \mp\, S\,\frac{1}{e^{+\psi}} \qquad (17)$$

For the exponential form we have

e = 2.7188, the base of hyperbolic logarithms

ψ = a positive hyperbolic angle

$-$ sign applies when R is positive (β = negative)

$+$ sign applies when R is negative (β = positive)

These relations are represented graphically by full line curve in Fig. 4.

By substituting in formula (**15**) for R the term $-(H + D)$, with D as per equations (**11**) and (**13**), β may be expressed in terms of H:

$$\beta = + \frac{K}{-\left(H + \dfrac{\mp A}{\cosh\ \theta}\right)} - \frac{1}{-\left(H + \dfrac{\mp A}{\cosh\ \theta}\right)} \times$$

$$\sqrt{K^2 + \left(H + \frac{\mp A}{\cosh\theta}\right)^2 \times S^2} \qquad (18)$$

and this is the equation of the hysteresis curve.

V. ENERGY LOSS AND OTHER RELATIONS

17. *Loss of Energy.* This problem of the hysteresis curve would not be completely answered without stating the so-called hysteresis loss.

As known, this loss in ergs is computed by (Fig. 1)

$$L = \frac{V}{4\,\pi}\;{}^{+H_h}\!\!\int_{-H_h}^{-H_h} H\,d\,B = 2\frac{V}{4\,\pi}\int_0^{-B_h} (H_1 - H_2)\,d\,B\ (C^2\,G\,S^{-2}) \qquad (19)$$

whereby V is the volume of the magnetized material in cubic centimeters and ${}^{+H_h}\!\!\int_{-H_h}^{-H_h}$ indicates that we have to integrate between the limits from $-H_h$ to $+H_h$ and back to $-H_h$. H_h is the highest field intensity attained.

Observing now equation (**1**) our integral dissolves into

$$L = \frac{V}{4\,\pi}\;{}^{+H_h}\!\!\int_{-H_h}^{-H_h} H\,d\,H + \frac{V}{4\,\pi}\;{}^{H_h}\!\!\int_{-H_h}^{-H_h} H\,d\,\beta$$

The value of the first of those two integrals is nil, leaving for the loss

$$L = \frac{V}{4\,\pi}\;{}^{+H_h}\!\!\int_{-H_h}^{-H_h} H\,d\,\beta = \frac{V}{4\,\pi} \times I \qquad (20)$$

$I = {}^{+H_h}\!\!\int_{-H_h}^{-H_h} H\,d\,\beta$ represents the area within the loop $\beta_1\,\beta_2$ of Fig. 2.

Designating the highest value that β attains by β_h and by further transformation we get

$$I = -\,2\,\{\int_{\beta_1 = 0}^{\beta_1 = \beta_h} \beta_1\,d\,H_1 - \int_{\beta_2 = 0}^{\beta_2 = \beta_h} \beta_2\,d\,H_2\,\} \qquad (21)$$

The following equations (**15**), (**9**), (**10**), (**3**), (**4**), made use of in the order named, will permit the expression of these integrals in terms of H, and eventually yield the loss thus:

$$L = -\frac{V}{2\pi}\left\{ + \int_{+H_0}^{+H_h} \left(-\frac{K \cosh \theta_1}{H_1 \cosh \theta_1 - A} + \frac{1}{H_1 \cosh \theta_1 - A} \times \right.\right.$$

$$\left. \sqrt{K^2 \cosh^2 \theta_1 + (H_1 \cosh \theta_1 - A)^2 S^2} \right) d H_1$$

$$- \int_{-H_0}^{+H_h} \left(-\frac{K \cosh \theta_2}{H_2 \cosh \theta_2 + A} + \frac{1}{H_2 \cosh \theta_2 + A} \times \right.$$

$$\left.\left. \sqrt{K^2 \cosh^2 \theta_2 + (H_2 \cosh \theta_2 + A)^2 \times S^2} \right) d H_2 \right\} \quad (22)$$

with θ_1 and θ_2 as per equations (**12**) and (**14**) and
$H = + H_0$ when $\beta_1 = 0$, $H = - H_0$ when $\beta_2 = 0$,
highest value attained by H is H_h.

Although equation (**22**) represents the answer to our call for the loss, its form is not well suited for the busy engineer. We will, therefore, prepare another solution.

$d H_1$ and $d H_2$ as they appear in equation (**21**) may be quickly derived from equations (**3**) and (**4**).

$$d H_1 = - d R_1 - d D_1$$
$$d H_2 = - d R_2 - d D_2$$

With the aid of these differentials, our integrals in equation (**21**) change to this new form:

$$I = -2\left\{ \int_0^{\beta_h} \beta_1 (- d R_1 - d D_1) - \int_0^{\beta_h} \beta_2 (- d R_2 - d D_2) \right\}$$

$$= - 2 \left\{ - \int_0^{\beta_h} \beta_1 \, d R_1 + \int_0^{\beta_h} \beta_2 \, d R_2 - \int_0^{\beta_h} \beta_1 \, d D_1 + \int_0^{\beta_h} \beta_2 \, d D_2 \right\}$$

The first two of these integrals are easily taken care of by solving general equation (**8**) for $d R$, $d R_1$ and $d R_2$ to go into our integrals with the limitation $d \beta_1 = d \beta_2$, resulting in

$$- \int_0^{\beta_h} \beta_1 \frac{R_1}{\beta_1} \times \frac{S^2 + \beta_1^2}{S^2 - \beta_1^2} d \beta_1 + \int_0^{\beta_h} \beta^2 \frac{R_2}{\beta_2} \times \frac{S^2 + \beta_2^2}{S^2 - \beta_2^2} d \beta_2 = 0$$

This result leaves us for I only the following term:

$$I = - 2 \left\{ - \int_0^{\beta_h} \beta_1 \, d D_1 + \int_0^{\beta_h} \beta_2 \, d D_2 \right\} \quad (23)$$

two other forms of which will prove of value for the purpose of integration. β_1 and β_2 may be considered the common abscissa.

$$\beta_1 = \beta_2 = \beta$$

Therefore

$$I = - 2 \int_0^{\beta_h} \beta (- d D_1 + d D_2) = - 2 \int_0^{\beta_h} \beta \, d(- D_1 + D_2) \quad (24)$$

and the loss

$$L = - \frac{V}{4\pi} 2 \int_0^{\beta_h} \beta \, d(- D_1 + D_2) = - \frac{V}{2\pi} \int_0^{\beta_h} \beta \, d(- D_1 + D_2) \quad (25)$$

Of this formula we will make use in a later paragraph. For the present we return to transforming equation (**24**) into

$$I = - 2 \left\{ \left[- \beta_1 D_1 + \beta_2 D_2 \right]_0^{\beta_h} + \int_0^{\beta_h} D_1 \, d\beta_1 - \int_0^{\beta_h} D_2 \, d \beta_2 \right\} \quad (26)$$

It would not require much explanation to show that

$$\left[- \beta_1 D_1 + \beta_2 D_2 \right]_0^{\beta_h} \equiv 0$$

which reduces equation (**26**) under the strength of previous argument to

$$I = - 2 \int_0^{\beta_h} (D_1 - D_2) \, d \beta = + 2 \int_0^{\beta_h} (- D_1 + D_2) \, d \beta \quad (27)$$

and the loss

$$L = + \frac{V}{2\pi} \int_0^{\beta_h} (- D_1 + D_2) \, d \beta \quad (28)$$

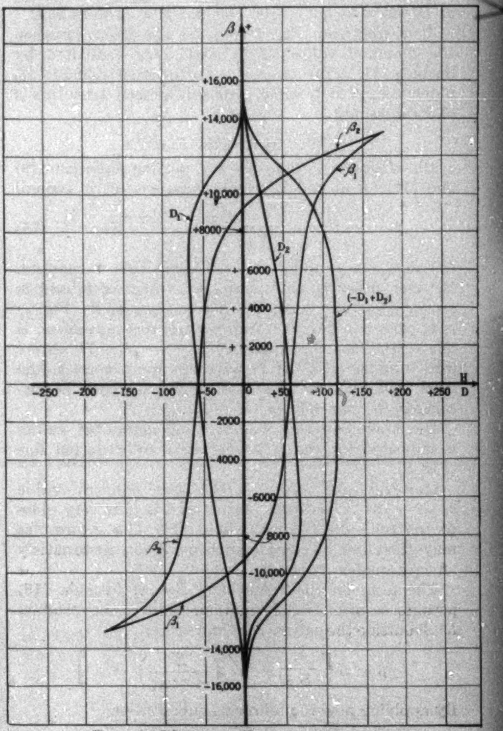


Fig. 8—Magnetic Hysteresis Curve

The solution of this latter integral requires D_1 and D_2 or $(- D_1 + D_2)$ as a function of β. That will be accomplished in the next chapter.

But it may be remarked right here, the differential of energy loss, according to formula (**28**), reveals itself as the product of the magnetic friction times the change in the flux, to wit:—"friction times magnetic current." Formula (**28**) in this form $\frac{dL}{dt} = \frac{V}{2\pi}$ $(-D_1 + D_2)\frac{\partial\beta}{\partial t}$ expresses the "Hysteresis Power Loss" as being proportional to the product "Force of Friction times Magnetic Current."

18. *Magnetic Friction D as a Function of Induced Flux β.* Fig. 8 shows the curves of magnetic friction D_1 and D_2 over the induced flux β as abscissas.

These curves D_1 and D_2 were derived by transposing the ordinates D_1 and D_2 from their H abscissas to such β abscissas as are coordinated to the H abscissas.

No matter between what limits the magnetic cycle occurs, for $H = \pm\infty$ the values of D_1 and D_2 are zero and those of β_1 and β_2 are $\pm S$.

Consequently our two curves D_1 and D_2 must have zero points for the abscissas $+S$ and $-S$. This enables us to consider D_1 and D_2 as two half waves of two combined sine and cosine series with higher harmonics.

Referring now to curve D_1 particularly, its equation has been determined on that basis, using a method developed and described by Prof. Runge,[2] namely

$$D_1 = 10^{-3}\, S \sum \left(b_n \sin n\frac{\pi}{2}\frac{\beta_1}{S} + a_n \cos n\frac{\pi}{2}\frac{\beta_1}{S} \right) \qquad (29)$$

and

$$D_2 = 10^{-3}\, S \sum \left(b_n \sin n\frac{\pi}{2}\frac{\beta_2}{S} - a_n \cos n\frac{\pi}{2}\frac{\beta_2}{S} \right) \qquad (30)$$

with

n =	1	3	5	7	9	11	13	15
b_n =	+0.612	+0.606	−0.2485	−0.702	+0.0976	−0.030	+0.0249	−0.0202
a_n =	−3.89	+0.014	+0.345	−0.195	+0.0528	+0.020	−0.015	−0.004

19. *The Dissipated Energy (Numerical).* By virtue of equations (**28**), (**29**) and (**30**), we have for the loss

$$L = \frac{V}{2\pi}\int_0^{\beta_h} (-D_1 + D_2)\, d\beta \qquad (31)$$

In forming the difference $(-D_1 + D_2)$ all sine members disappear and we get

$$\frac{L}{V} = -10^{-3} \times \frac{2}{\pi^2} \times S^2 \times \left[\sum \frac{\text{const}}{n} \sin n \frac{\pi}{2}\frac{\beta_h}{S} \right]_0^{\beta_h}$$

$$= -10^{-3} \times \frac{2}{\pi^2} S^2 \times \lambda \qquad (32)$$

wherein β_h = approx. 13183; S = 16000

$$n = 1, 2, 3 \;.\;.\;.\;.\; 15$$

$$\lambda = \left[\sum \frac{\text{const}}{n} \sin n \frac{\pi}{2}\frac{\beta}{S} \right]_0^{\beta_h} \qquad (33)$$

or in our case $\lambda = 3.7423$

Finally

$$\frac{L}{V} = -10^{-3} \times \frac{2}{\pi^2} 16000^2 \times 3.7423$$

$$\frac{L}{V} = -194100 \frac{C^2 G S^{-2}}{C^3} \text{ ergrs. per cu. cm.}$$

The only dimensional quantity on the right side of formula (**32**) is S^2 whose dimension in the $C\,G\,S-$ system is $(C^{-1/2} G^{1/2} S^{-1})^2 = C^{-1} G S^{-2}$ = ergrs. per cu. cm., what it ought to be. In that way we get a confirmation that our theory and results are right.

According to the well-known Steinmetz formula, the hysteresis loss is computed by

$$\eta \times \underset{max}{B}^{1.6} \text{ in ergs per cu. cm.} \qquad (34)$$

The writer does not know the *exact* value of η for hardened tungsten steel; and for that reason the check shall not be made on the loss of energy, but on the coefficient η, thus

$$194100 = \eta \underset{max}{B}^{1.6} = \eta\,(\beta_h + H)^{1.6} = \eta\,(13183 + 177)^{1.6}$$

$$= \eta \times 13360^{1.6}$$

hence

$$\eta = 0.04862$$

This figure seems to be quite reasonable, which means that the two formulas are equivalent.

If the magnetic cycle takes place between other limits, the "constants" in some or all of our formulas will change. But we have to conclude that for all cycles (for which the Steinmetz formula holds) λ should satisfy this equation.

$$10^{-3} \times \frac{2}{\pi^2} \times S^2 \times \lambda = \eta \times \underset{max}{\beta}^{1.6}$$

or

$$\lambda = \frac{\eta \times 10^3 \times \pi^2}{2} \times \frac{\beta_m^{1.6}}{S^2} = \frac{\eta \times 10^3 \times \pi^2}{2}\; \frac{(H_h + \beta_h)^{1.6}}{S^{1.06} \times S^{0.4}}$$

$$= \frac{\eta \times 10^3 \times \pi^2}{2} \times \frac{\left(\frac{H_h}{S} + \frac{\beta_h}{S} \right)^{1.6}}{S^{0.4}} \qquad (35)$$

or by neglecting $\frac{H_h}{S}$, which in our case is $\frac{177}{16000}$ = 0.011, we get approximately

$$\lambda = \frac{1000\,\eta \times \pi^2}{2} \left(\frac{\beta_h}{S} \right)^{1.6} \times S^{-2/5} \qquad (36)$$

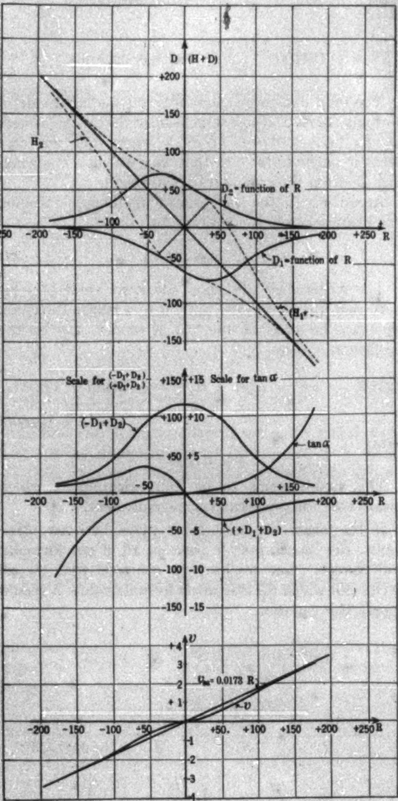
---

[2] See *Electrotechnische Zeitschrift*, 1905, p. 247. *C. Runge, "Theorie & Praxis der Reiben", Goshen at Leipzig, Germany.*

Further experiments should show to what extent this formula is correct.

Appendix III contains formulas which give the energy loss in explicit terms of the leading quantities.

20. *Magnetic Friction D as Function of Magnetic Tension R.* It is very difficult to determine analytically the value of magnetic friction D as a function of magnetic tension R. However, D_1 and D_2 may be plotted as functions of R_1 and R_2 respectively by taking values from the curves in Fig. 5 which shows $-R_1$ and $-R_2$ as functions of H. Fig. 9 shows the curves so plotted. It is possible to obtain from these curves formulas for D_1 and D_2 as functions of R_1 and R_2, respectively. Such formulas were obtained and it was found that the resulting values for D_1 and D_2 check very closely with the values of D_1 and D_2 in Fig. 5 and Fig. 8. For formulas see Appendix III.



FIGS. 9-10-11

21. *Loss of Energy in Terms of Flux β and Tension R.* Although energy loss is given in equations (**28**) and (**32**) in terms of β and of D, energy loss may also be computed in terms of β and R. It was computed in this manner and for the sample given was found to be 188,000 ergs per cu. cm. which checks well with the former value of 194,100 ergs per cu. cm.

Formulas and computations to augment the above statements will be found in Appendix III.

VI. ALTERNATING CURRENT EXCITATION

22. *Sine Current.* In the case where an alternating current is used to furnish the magnetomotive force, the intensity of the magnetic field will be expressed by

$$H = H_h \times \sin \frac{2\pi}{T} t = \text{const} \times I_0 \sin \frac{2\pi}{T} t \qquad \textbf{(37)}$$

where

t = time in seconds $\qquad H_h$ = max field intensity

$T = \dfrac{1}{\text{Frequency per second}}$ $\qquad I_0$ = max current

= Period

Such a condition will exist in a current transformer with its secondary windings open or removed and the primary traversed by a sine current.

What is the curve followed by magnetic flux β?

For our particular hysteresis curve (tungsten steel), we have

$$H_h = 177$$

and

$$H = 177 \sin \frac{2\pi}{T} t \qquad \textbf{(38)}$$

For values H thus computed the corresponding values of β may readily be obtained and plotted against the angle $\frac{2\pi}{T} t$ as abscissa, Fig. 12. The same method that was employed for the derivation of formulas (**29**) and (**30**) is here applicable to determine an expression for β by trigonometric functions. In the present case the process has been carried one step further by doing away with the cosines and introducing constants into arguments of the sine functions.

$$\beta = S \sum c_n \sin\left(\frac{2\pi}{T} t + \omega_n\right) \qquad \textbf{(39)}$$

with

n =	1	3	5	7	9	11
c_n =	+0.9482	+0.2324	+0.1212	+0.07891	+0.05684	+0.09989
ω_n =	−8° 26′	−61° 7′	−106°28.4′	−151°48.6′	+163° 3.6′	−102°23.6′

Observe that

$S = 16000 = 1.20755\,\beta_h$ and $\beta_h = 13250$ = highest magnetization attained.

The coefficients of higher harmonics bear the following relation to fundamental 3rd, 5th, 7th, 9th and 11th harmonic.

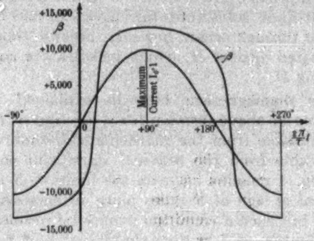


FIG. 12

coefficient =
0.245, 0.1279, 0.0831, 0.0539, 0.1053 × coeff. of fundamental approximately

$$\frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{12} \quad \frac{1}{19} \quad \frac{1}{9.5}$$

The Fourier series (**39**) has been derived from the graph, Fig. 12. If the analytical formulas for β be used as a basis it ought to be possible to show why the lower harmonics (3rd and 5th) are of comparatively large amplitude.

23. *Sine Voltage.* For the case of a sine voltage

$$E = E_0 \sin \frac{2\pi}{T} t$$

being applied to the windings around the magnetic material, the magnetic flux must change in such a manner that it induces a sine counter e. m. f. in opposition to impressed sine voltage. In other words (neglecting ohmic resistance)

$$-\frac{dB}{dt} = -\frac{d(H+\beta)}{dt} \times \text{const} = -E_0 \sin \frac{2\pi}{T} t$$

hence

$$B = (H + \beta) = \frac{E_0}{\text{const}} \left(-\cos \frac{2\pi}{T} t \right)$$

$$= \frac{E_0}{\text{const}} \times \sin \left(\frac{2\pi}{T} t - \frac{\pi}{2} \right)$$

or

$$B = B_h \times \sin \left(\frac{2\pi}{T} t - \frac{\pi}{2} \right) \qquad (\mathbf{40})$$

$B_h = H_h + \beta_h$ = highest value which B attains in cycle considered. In our case $B_h = 177 + 13250 = 13427$.

If now the field intensity and current are again proportional to one another, what is the curve of the current flowing?

Formula (**1**) together with (**40**) furnishes a suitable relation for H, namely

$$H = B_h \sin \left(\frac{2\pi}{T} t - \frac{\pi}{2} \right) - \beta$$

Plot B curve into Fig. 5 as per formula (**1**). Select angular step $\frac{2\pi}{T}$ and compute B-values as per formula (**40**). For such B-values take the corresponding values of β and H from Figure 5.

The following two formulas satisfy respectively the magnetization β and field intensity in terms of the time angle $\frac{2\pi}{T} t$.

$$\beta = S \sum C_n \sin \left(\frac{2\pi}{T} t + \Omega_n \right) \qquad (\mathbf{41})$$

with

n =	1	3	5	7	9	11
C_n =	0.8314	$+\frac{2.868}{1000}$	$+\frac{0.6956}{1000}$	$+\frac{0.069}{1000}$	$+\frac{0.08924}{1000}$	$+\frac{0.5953}{1000}$
Ω_n =	−90°14.8′	+82° 35′	+60°37.9′	+94° 56′	+176° 0′	−161°38.4′

and

$$H = 177 \sum h_n \sin \left(\frac{2\pi}{T} t + \nu_n \right) \qquad (\mathbf{42})$$

with

n =	1	3	5	7	9	11
h_n =	+0.7559	−0.2592	−0.0629	−0.00624	−0.00806	−0.00538
ν_n =	−64°43.7′	+82°35′	+60°37.9′	+94°56′	+176°0′	−161°38.4,

In the formula for β, the higher harmonics are of no account, while in the term for H only the 3rd and 5th harmonic are of appreciable value when compared with the fundamental wave. Fig. 13 has been drawn to show how such relations find their graphical expression.

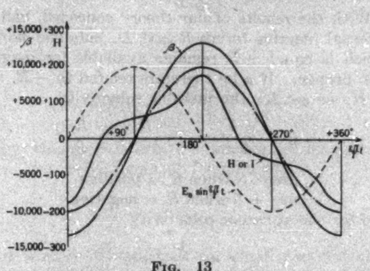


FIG. 13

As stated above, the current is proportional to the field intensity H. The dash-dotted line curve visible in Fig. 13 would be the curve of the current, if there were no steel present within the magnetizing winding. Said curve is drawn at random scale, but it serves nevertheless to emphasize the distortion suffered by the current I through magnetic hysteresis.

VII. APPLYING THE THEORY

To show the ability of the new theory we shall now apply it to the well-known theory of Hopkinson's Magnetic Circuit, in conjunction with Professor Kennelly's paper on Reluctivity (TRANSACTIONS of the A. I. E. E., 1891, page 485).

In said paper (Figs. 4 and 5) the rectilinear characteristic of the metallic reluctivity $\rho = \dfrac{1}{4\,\pi\,\kappa}$ when plotted against H is particularly stressed.

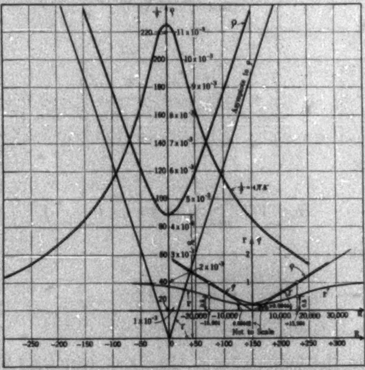


FIG. 14

With the results of our theory conceded, calling for internal reactive forces R and D, *only that part of* H which is equal $-R$ remains available for magnetization proper. If ρ be therefore related to $-R$, instead of H, we get for the metallic reluctivity

$$\rho = \frac{1}{4\,\pi\,\kappa} = \frac{-R}{\beta} = \frac{-R}{\mp S}e^{\psi}$$

use $-S$ when R = positive
use $+S$ when R = negative

and for the apparent reluctivity

$$r = \frac{1}{\mu} = \frac{\zeta}{1 + \zeta}$$

both of which are illustrated in Fig. 14 at two different scales for our sample of hardened tungsten steel.

We note particularly that for large values of $R-$ (which differ only slightly from corresponding values of H) ρ is very nearly a straight line whose asymptote runs through the origin of the system at an inclination

$$\tan\tau = \frac{1}{S}\text{, which in our case } \frac{1}{16000} = 0.0000625.$$

It is of the same order as the gradient of right hand end of curve for "Glass Hard Pianoforte Steel, Ewing 1890," given in Fig. 5 of Professor Kennelly's paper. The straight lines (substitutes for curves) in said Fig. 5 do not run through origin, however, because those curves are plotted against H, which includes the component opposing D.

Even diamagnetism may be explained now, by reversing in above formula for ρ the sign of R. That is permissible from the mathematical standpoint and conceivable from the physical standpoint so long as β in Fig. 4 remains between the limits $+S$ and $-S$. Reversal of sign of R takes place automatically when $\beta > S$, but such a condition cannot be reconciled with our definition of S as a maximum value of β.

NOTATIONS AND SYMBOLS USED IN THIS PAPER

B Induction or flux density
H Magnetizing force; also spatial induction
β Intrinsic induction (*i. e.*, $B - H$)
S Saturation value, that is the limiting value of β
γ Capacity for further induction (*i. e.*, $S - \beta$)
R Reactive component of internal force, opposing magnetization in either direction
D Dissipative component of the internal force, opposing all changes of induction β
M Maximum value of $(-D_1 + D_2)$
Subscript$_1$ indicates appertainment to ascending branch of hysteresis loop
Subscript$_2$ indicates appertainment to descending branch of hysteresis loop
Subscript$_h$ indicates highest value attained
Subscript$_0$ indicates that a related quantity has zero value
$\theta = u + \varphi$, a hyperbolic angle
$\alpha =$ a circular angle
$v =$ a hyperbolic angle

The unit for quantities B, H, β, S γ R, D, M in the $C\,G\,S$—system is $C^{-\frac{1}{2}}\,G^{\frac{1}{2}}\,S^{-1}$; that is, the gauss = number of lines of force per square centimeter.

L = Energy loss in ergs $(C^2\,G\,S^{-2})$.
1.356×10^7 ergs = 1 ft-lb.

For illustration of leading symbols see Figs. 1 to 4.

Appendix I

Magnetization and the Characteristic Equation for Gases. The symbols P, V_1, v, T, C, as used in this appendix, refer to gases.

In Section II under heading 5 the existence and magnitude of reactive tension R was explained by referring to the behavior of an elastic medium under stress. Perfect gases are elastic media and lend themselves particularly well for this analogy.

In thermodynamics the well-known characteristic equation of a perfect gas is:

$$\frac{\text{Pressure} \times \text{Volume}}{\text{Absolute Temperature}} = \text{Constant}$$

which when applied to the case of a gas enclosed in a cylinder under a piston (Fig. 15A) will read (symbols as per Fig. 15A and B)

$$\frac{P \times (V_1 - v)}{T} = C \text{ or } (V_1 - v) = \frac{1}{\frac{1}{C\ T} \times P}$$

For isothermic compression is $C\,T$ = constant and

$$P = \frac{\text{Const}}{V_1 - v}$$

This formula is identical with our formula (**5**), if we replace external pressure P by $H = (-R)$; V_1 by S; v by β and Const. by K. The analogy is more evident when a liquid is substituted for the moving piston (Fig. 15B). The liquid received in the cylinder bears a greater similarity to the magnetization imparted to the iron than does the so-called piston displacement.

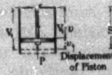


Fig. 15 A

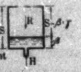


Fig. 15 B

The well-known formula for magnetization near saturation as given by Fröhlich[3] may also readily be derived from this example of analogy by mere transformation of the thermodynamic formula quoted. We start out by writing same thus

$$V_1 - v = \frac{1}{\frac{1}{C\ T} \times P}$$

The state of near-saturation is analogized by the liquid v (or β) almost completely filling up the whole volume V_1 (or S), Fig. 15A and B. This condition is mathematically expressed by

$$v \equiv V_1$$

By dividing these two equations, we get

$$\frac{V_1 - v}{v} = \frac{1}{\left(\frac{V_1}{C\ T}\right) \times P}$$

which, when solved for v, yields:

$$v = V_1 \frac{\frac{V_1}{C\ T} \times P}{1 + \left(\frac{V_1}{C\ T}\right) P}$$

or by virtue of analogy, Fig. 15A, B, writing "a" for $\frac{V_1}{C\,T}$; $\beta = S \frac{a\,H}{1 + a\,H}$ and that is Fröhlich's Law.

The analogy cited pertains to formula (**5**) which is correct only near saturation. For the whole range of magnetization formula (**7**) is to be used. The physical process of magnetization and its reactive tension R, as embodied in that formula (**7**), has a like analogy in thermodynamics. The case has been sketched out in Fig. 16.

A thin piston moves in a cylinder, which is closed at both ends. When the piston is in its middle position, the gas pressures on either side are the same. When the piston is moved out of its neutral position by an external force P, internal pressures P_1 and P_2 are being set up to which the following relations apply:

For upper chamber $\frac{-P_1(-V_1 - v)}{T} = C$ or

$$P_1 = -C\,T\frac{1}{V_1 - v}$$

For lower chamber $\frac{-P_2(-V_1 - v)}{T} = C$ or

$$P_2 = -C\,T\frac{1}{-V_1 - v}$$

Hence the total internal reactive pressure is

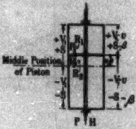


Fig. 16

$$(P_1 + P_2) = -\frac{2\,(C\,T) \times v}{V_1^2 - v^2}$$

and by changing over to notations of magnetizing process, we arrive again at formula (**7**).

$$R = -\frac{2\,K\,\beta}{S^2 - \beta^2}$$

It is worthy of note that for small magnetizations β^2 is negligible in comparison to S^2 and our formula changes thus into

$$R = -\frac{2\,K\,\beta}{S^2} = -\left(\frac{2\,K}{S^2}\right) \times \beta$$

which is the same as Hooke's Law, pertaining to elastic bodies under small stress.

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[3] *Elektrotechnische Zeitschrift*, 1881, pp. 90, 139. 1882, p. 71.

Appendix II

DERIVATION OF EQUATION FOR CURVE D

The Theory. The dissipative force, when it first occurred in the analysis of the hysteresis curve, appeared in the form of two graphs D_1 and D_2, having the field H for abscissas, Fig. 5, and the problem arose immediately to find a mathematical function which would satisfy these curves, D_1 and D_2.

With that problem in view we will limit ourselves for the present to curve D_2 only.

Although that curve is unsymmetric when referred to its maximum ordinate, it was assumed at the outset that a symmetrical curve, but otherwise of configuration similar to D_2, would hold the key to a mathematical equation for the unsymmetric curve D_2.

Instead of considering the distorted curve D_2 as the resultant curve of several superposed symmetrical curves[4], it was preferred to look upon curve D_2 as being intrinsically a symmetrical curve, which is satisfied by some explicit function whose argument is the abscissa. The distortion is considered as being due to the argument being a function of the abscissa.

Disregarding, with that object in view, the unsymmetry of our proposed curve D_2, its great similarity is conspicuous with the graph of the function

$$A \times \text{sech}\, u$$

when plotted against H as abscissa. (Fig. 17). The scale in Fig. 17 has been chosen such that the curve A sech u is generally of the same proportions as curve D_2 in Fig. 5. This limitation implies u to be linear function of H, whose zero point, 0, is to the left of origin O of the system of coordinates. Calling $O\,O_1 = -f$, the equation for u will be in general

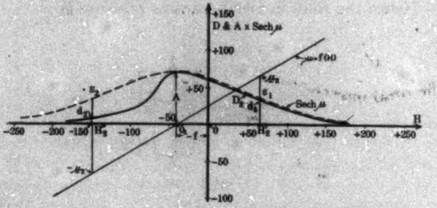

FIG. 17

$$u = c\,(H + f) = c\,H + c\,f$$

where c is the trigonometric tangent of the angle that u forms with the axis of abscissas.

Considering now two ordinates of the graph A sech u, which are located symmetrically to the maximum ordinate A, the abscissas are $O\,H_2'$ and $O\,H_2''$ (Fig. 16) with the limitation that $H_2''\,O_1 = O_1\,H_2'$. The ordinates are

$$H_2''\,s_2 = H_2'\,s_1 \text{ for the sech } u \text{ curve}$$

$$-\,u_2 \text{ and } +u_2 \text{ respectively for the } u\text{-line.}$$

The ordinates of the proposed curve D_2, however, which stand at the same abscissas are respectively

$$H_2''\,d_2 < H_2''\,s_2 \text{ and } H_2'\,d_1 \lessgtr H_2'\,s_1$$

If curve D_2 were to be represented mathematically by a sech-function, same would have to read for (compare Fig. 16):

ordinate $H_2''\,d_2 = A \text{ sech } (-\,\underline{u}_2 - \Delta_2\,u)$
ordinate $H_2'\,d_1 = A \text{ sech } (+\,\underline{u}_2 \pm \Delta_1\,u)$

Underscored letter indicates a definite fixed value in contradiction to a variable.

The values $\Delta_1\,u$ and $\Delta_2\,u$ may readily be considered values of a function φ_2, thus that

$$-\,\Delta\,u = \varphi_2 = -\,f_2\,(u_2)$$

For simplicity let us call $\Delta_1\,u$ equal $\Delta_2\,u$; *e. g.*, considering φ_2 a curve being symmetric to the ordinate $O_1\,A$ (Fig. 17).

The deviation of D_2 curve from the sech u curve is small along right hand side of the maximum ordinate and increases toward the left. Consequently φ_2 must attain larger values the more remote they are to the left from that maximum value. For the right hand side $(u_2 \pm \Delta\,u)$ must be numerically nearly equal u_2. These two requirements jointly do not tolerate the + sign in $(u_2 \pm \Delta\,u)$. The minus sign only is possible. A function which meets both requirements is

$$\varphi_2 = -\,q\,\sqrt{p + u_2^2}$$

where q and p are constants. With a hyperbolic angle θ_2 consisting of the basic element

$$u_2 = c\,(H_2 + f)$$

and the above corrective element φ_2, namely

$$\theta_2 = u_2 + \varphi_2 = u_2 - q\,\sqrt{p + u_2^2}$$

(curves for u, φ and θ for the sample tested are shown in Fig. 6), the function sech θ_2 should be flexible enough to cover the proposed curve D_2. Thus

$$D_2 = +A \times \text{sech}\,\theta_2 = +A \times \text{sech}\,[u_2 + \varphi_2] = \frac{+\,A}{\cosh\theta_2} \qquad \mathbf{(13)}$$

The five constants A, c, f, q and p should insure agreement between the graph D_2 and this formula at five suitable points anyway and near-agreement for the rest of the curve.

Similar arguments apply to curve D_1 and equations (**11**) and (**12**).

Numerical Evaluation of Curve D_2. Equation (**13**) and its companion equation (**11**), when duly transformed, will prove suitable means for determining the numerical values of the respective constants occurring in those equations.

Referring now to curve D_1 (Fig. 5), we rewrite its equation in this form

$$\theta_1 = u_1 + \varphi_1 = c\,(H_1 - f) + q\,\sqrt{p + c^2\,(H_1 - f)^2}$$

$$= +\cosh^{-1}\frac{-\,A}{D_1} \qquad \mathbf{(a)}$$

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[4] *Electrical World*, July 29, 1909, p. 262. col. 2.

We select now such pairs of abscissas H_{1e}' and H_{1e}'' (Fig. 5) whose ordinates D_1 are equal. For any such pair of ordinates equation (a) will read thus, when square root is avoided:

$$q^2\,[p + c^2\,(H_{1e}' - f)^2] = \left[\cosh^{-1}\frac{-A}{D_1} - c\,(H_{1e}' - f)\right]^2$$

$$q^2\,[p + c^2\,(H_{1e}'' - f)^2] = \left[\cosh^{-1}\frac{-A}{D_1} - c\,(H_{1e}'' - f)\right]^2$$

Carrying out the multiplications, rearranging terms and subtracting the lower equation from upper one produces

$$c\,(q^2 - 1)\,[H_{1e}'' + H_{1e}' - 2f] = -\,2\cosh^{-1}\frac{-A}{D_1} \qquad (\mathbf{b})$$

Taking now from Fig. 7 the following special values

$H_{1e}'' = O$	$-\,65.63$	$-\,109.8$
$H_{1e}' = +\,54.9$	$+\,77.98$	$+\,92.6$
$D_1 = -\,61.165$	$-\,34.0$	$-\,20.4$

and applying them to equation (**b**)

$$c\,(q^2 - 1)\,[O + 54.90 - 2f] = -\,2\cosh^{-1}\frac{-A}{-\,61.165}$$

$$c\,(q^2-1)\,[-65.63+77.98-2f] = -2\cosh^{-1}\frac{-A}{-\,34.0}$$

$$c\,(q^2-1)\,[-109.8+92.6\;-2f] = -2\cosh^{-1}\frac{-A}{-\,20.4}$$

After eliminating the constants c, q and f by due process of algebra an equation results of this form

$$1.445 = \frac{\cosh^{-1}\dfrac{-A}{-\,61.165} - \cosh^{-1}\dfrac{-A}{-\,34.0}}{\cosh^{-1}\dfrac{-A}{-\,34.0} - \cosh^{-1}\dfrac{-A}{-\,20.4}}$$

which is satisfied, within good enough accuracy, by a value

$$A = 70.061$$

With A thus determined, the above special values of $\cosh^{-1}\dfrac{-A}{D_1}$ may be considered known and the problem of finding the other constants is thereby reduced to solving simple algebraic equations.

Appendix III

Noteworthy Results

Special Values of the Leading Quantities. As the new theory has produced new forces, special values of the latter will be of interest. Numerical values given below refer to the sample of tungsten steel tested.

Associated with residual magnetism are residual friction for $H = O$

$$D_0 = \mp\frac{A}{\cosh\,[\pm cf \pm q\sqrt{p + c^2 f^2}]} = \mp 61.165\ C^{-\frac{1}{2}}\ G^{\frac{1}{2}}\ S^{-1}$$

Residual Tension:

$$R_0 = -\,D_0 = \pm\,61.165\,C^{-\frac{1}{2}}\,G^{\frac{1}{2}}\,S^{-1}$$

Residual Magnetization:

$$\beta_0 = +\frac{K}{\pm\,R_0} - \frac{1}{\pm\,R_0}\sqrt{K^2 + R_0^2\,S^2} = -9230\,C^{-\frac{1}{2}}\,G^{\frac{1}{2}}\,S^{-1}$$

Coercive Force H_c occurring when $R = O$ to be computed from

$$D = \frac{\mp\,A}{\cosh\ \theta} = \mp\,H_c$$

or

$$\cosh^{-1}\frac{\mp\,A}{\mp\,H_c} = c\,(H_c \mp f)\ \pm q\sqrt{c^2\,(H_c \mp f)^2 + p}$$

$$H_c = \pm\,58$$

The maximum value of D_1 occurs for

$H_1 = +\,39.38 \qquad D_{1max} = -\,69.721 \qquad R = +\,30.341$

$\beta = -\,5880\,C^{-\frac{1}{2}}\,G^{\frac{1}{2}}\,S^{-1}$

Equal numerical values of D_1 and D_2 for common value of H occur for three values of H.

$$\left.\begin{matrix} D_2 = -D_1 \\ H_2 = H_1 \end{matrix}\right\}\ \begin{matrix} & H & = +84 & 0 & -84 \\ \text{for} & \theta & = 1.586 & 0.533 & 1.586 \\ & D & = 27.8 & 61.5 & 27.8 \end{matrix}$$

In connection herewith check up in Fig. 6 the intersection of $-\ \theta_2$ with $+\ \theta_1$.

Equations Relating D to R. Fig. 9 shows curves typifying the relation between the magnetic friction D and the magnetic tension R. To render such relationship available for more exact analysis a few pertinent formulas will be given below.

$$(-\,D_1 + D_2) = M\cos\alpha = +\,116.479\cos\alpha \qquad (\mathbf{43})$$

$$\tan\alpha = 1.671686 \times 10^{-15}\,(R^5 + 13\times 10^5\,R^3 + 179058\times 10^5\,R)^{1.2132876} \qquad (\mathbf{44})$$

$$(D_1 + D_2) = \frac{a\,R + b\,R^3}{1 + r\,R^2 + s\,R^4} = -\,\frac{110.065\left(\dfrac{R}{100}\right) + 17.266\left(\dfrac{R}{100}\right)^3}{1 + 1.812\left(\dfrac{R}{100}\right)^2 + 2.513\left(\dfrac{R}{100}\right)^4}$$

Compare herewith Figs. 9 and 10.

By adding and subtracting formulas (**43**) and (**45**), terms will be arrived at which permit the calculation of D_1 and D_2 individually.

Analytical work is often simplified when the quantities under consideration are proportional to one another or nearly so. With such simplicity in view the following terms are mentioned here and are augmented by Fig. 11.

$$(-\,D_1 + D_2) = M\,\mathrm{sech}\,v = 116.479\,\mathrm{sech}\,v \qquad (\mathbf{46})$$

$$v = 0.0173\,R\left[1 + \left(\frac{1.5658^{21}}{R^2}\right)^{4\cdot354}\right]^{-1/21} \qquad (\mathbf{47})$$

$$R = 57.80\,v\,[1 + 63.10\,v^{-6\cdot80}]^{1/21} \qquad (\mathbf{48})$$

Energy Loss. By making use of the above relations, the energy loss may be calculated as follows. Recourse is made also to equation (**25**) on page 8.

$$\frac{L}{V} = -\frac{M}{2\pi}\Big[\beta\cos\alpha\Big]_0^{\beta_h} + \frac{M}{2\pi}\int_0^{\beta_h}\frac{1}{\sqrt{1+\tan^2\alpha}}\,d\beta$$

ergs. per cu. cm. (**49**)

Formula (**49**) is a sequal to formula (**43**), involving D, R, β. The actual integration is hard to execute.

The inherent simplicity mentioned above in connection with formula (**46**) manifests itself when employing that formula for computing the energy loss. The necessary integration can be carried out for one part exactly and for the remainder with sufficient approximation. Relying again on equation (**25**),

$$-\frac{L}{V} = \frac{1}{2\pi}\int_0^{\beta_h}\beta\,d\,M\,\text{sech}\,v$$

$$= \frac{1}{2\pi}\times\frac{-M}{2\times 0.0173\times K}$$

$$\left\{(S^2-\beta^2)\left[\frac{v}{\cosh\ v}+\frac{\pi}{2}-2\tan^{-1}e^v\right]\right\}_0^{\beta_h}$$

$$-\ 96.90\times 10^6$$

$$+\frac{1}{2\pi}\times\frac{-M}{2\times 0.0173\,K}\int_0^{\beta_h}\left[\frac{v}{\cosh\ v}\right]d\,\beta^2$$

$$+\ 23.40\times 10^6$$

$$+\frac{1}{2\pi}\times\frac{-M}{2\times 0.0173\,K}\int_0^{\beta_h}\left[+\frac{\pi}{2}-2\tan^{-1}e^v\right]d\,\beta^2$$

$$-\ 125.90\times 10^6$$

$$\text{Total}\ -\ 199.4\ \times 10^6$$

$$\frac{L}{V} = +\frac{1}{2\pi}\,\frac{+116.5}{2\times0.0176\times0.5682\times10^6}\times(-199.4\times10^6)$$

$$= -\ 188000 \text{ ergs per cu. cm.}$$

This is about 3 per cent less than the value given above. The difference, no doubt, is due to the fact that the corrective value of parenthesis in formula (**47**) has been disregarded, *e. g.*, substituting for the curve v (Fig. 11), its asymptote.

Discussion

S. L. Gokhale: I believe that Mr. Lippelt's analytical conception of reactive and dissipative components of magnetic forces within a magnetic material, if carried to its logical consequence, will eventually do for the magnetic circuit what the conception of "energy component" and "idle component" have done for the electric circuit. Mr. Lippelt also deserves credit for his ingenious method of evolving from an unsymmetrical hypothetic equation another of symmetric form. This method is likely to have a very wide and useful application in other fields.

As to his equation (**7**) which is one of the two important equations in the paper, I take the liberty of disagreeing with the author. His line of reasoning, as I understand it, is as follows:

a. Near saturation, hysteresis practically disappears, making $R = -H$.

b. Near saturation, the relation of H to β follows the law $H = +K/\gamma$.

c. Therefore, near saturation, the relation of R to β must also be represented by the law $R = -K/\gamma$.

Mr. Lippelt's final equation for the magnetic hysteresis curve depends entirely on proposition c, and therefore ultimately upon proposition b, which he accepts as an established and indisputable fact.

At this point I differ from Mr. Lippelt. I have in my hands a few hundred magnetization curves, to demonstrate that the relation of H to β cannot be represented correctly by the equation, $H = K/\gamma$; that relation can be represented much more closely by the equation $\log\gamma = f - g\,H$, or $H = a - b\log\gamma$. It is not possible to present the full evidence in a brief discussion like this, and I shall therefore limit myself to a single illustration for the present.

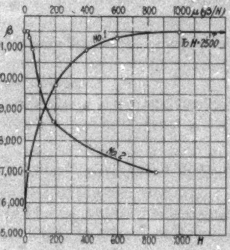

Fig. 1

Fig. 1 represents the curves for intrinsic induction (Curve No. 1) and permeability (Curve No. 2) for a sample of electrolytic iron. This sample was included in a paper by Dr. Yensen.[1] The sample was tested at the Bureau of Standards, Washington. The material as well as the test data are therefore of the highest possible reliability. A study of Curves 3 and 4 of Fig. 2 shows conclusively that the relation of H to β for this material does not follow the law $H = K/\gamma$, but instead, follows the law $\log\gamma = f - g\,H$. As this material has a remarkably low hysteresis, it follows that near saturation, the hysteresis should be practically negligible so as to suggest that the true law of reactivity should be $\log\gamma = f + g\,R$.

If, after a careful study of these and several other curves, Mr. Lippelt decides to revise his views as to the equation of reactivity near saturation, he might have to revise his equation for hysteresis also.

As to the equation for the dissipative component, *viz.*,

$$D = -A/\cosh\theta$$
$$= -A/\cosh\left\{(c\,H + f) + q\sqrt{h + (c\,H + c\,f)^2}\right\}$$

I notice that the term $\beta\,h$,—that is, the peak value of β for the hysteresis loop,—is conspicuously absent. We all know that hysteresis depends in some way on the peak value of β; any formula for hysteresis in which $\beta\,h$ is absent is, therefore, not likely to be the true formula, even approximately. On this point, I believe we need some explanation before we could accept Mr. Lippelt's equation.

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[1] The Magnetic Properties of the Ternary Alloys Fe-Si-C, by T. D. Yensen, TRANSACTIONS A. I. E. E., 1924, page 145.

In the course of the presentation of his paper, Mr. Lippelt claimed to have established a rational equation for hysteresis; I am unable to follow him at this point. If I understand his argument correctly, he has attempted to formulate a rational equation for R and an empirical equation for D.

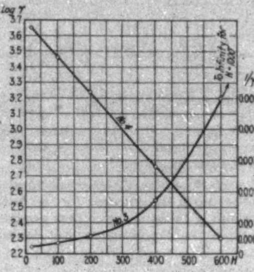

FIG. 2

TABLE I

MAGNETIZATION TESTS ON ELECTROLYTIC IRON

Test made at the Bureau of Standards—Test No. Tem. 41893. Data for H and B received through courtesy of Dr. T. D. Yensen. The other columns of the table are obtained by computation.

H	B	β	γ	μ $(= \beta/H)$	$1/\gamma$	Log γ
0.2	460					
0.4	4600					
0.5	7300					
1.0	11150					
2.0	14230					
4.0	15770					
20.0	17020	17000	4500	850	0.0002222	3.6530
100.0	18700	18600	2900	186	0.0003450	3.4625
200.0	19960	19760	1740	99	0.0005745	3.2405
400.0	21320	20920	580	52.3	0.001724	2.763
600.0	21900	21300	200	35.5	0.005000	2.310
1000.0	22500	21500	0	21.5	inf.	
1500.0	23000	21500	0	14.3	inf.	
2000.0	23500	21500	0	10.7	inf.	
2500.0	24000	21500	0	8.6	inf.	

Notation: In this table
$\beta = B - H$
$\gamma = S - \beta$ (Where $S = 21500$).
$\mu = \beta/H$ (Intrinsic permeability).

In Section VII (Applying the Theory), the author shows that Kennelly's law of reluctivity follows as a corollary from his theory.

Kennelly's law is only another form of Frolich's law. Mr. Lippelt's hypothesis is admittedly a form of Frolich's law, and must necessarily agree with all other forms of that law, including Kennelly's law. Such agreement implies nothing more than this—that any hypothesis in harmony with any other hypothesis will agree with that hypothesis in all its forms. Such an agreement has obviously no evidential value in support of the hypothesis in question.

Mr. Lippelt's hypothesis ($H = K/\gamma$) is analogous to Boyle's law for perfect gases. But in formulating the law on this basis he has to assume that magnetization consists of charging the molecules with something that is not in the molecules to start with, *viz.*, the magnetic induction β. Such an explanation of the equation would therefore be in agreement with Poisson's theory, but not in agreement with Weber's theory, which is, at present, the generally accepted theory.

Hans Lippelt: It is obvious that in presenting this paper I did not intend to advance a final and conclusive treatise on magnetism. Of the experimental data which I had on hand, only those appertaining to one magnetic cycle of tungsten steel appeared to be best suited for the study. With such fundamental limitations, it cannot be expected that the results obtained and conclusions drawn will cover all possible cases of magnetization. This state of affairs is reflected in Division 10, under "Conclusions," by the statement that Equation (7) is hypothetical. At the same time it should be noted that this equation is rather plausible and is also rooted in well-known laws of physics. Mr. Gokhale's apprehension that Equation (7) is "indisputable" does not seem to be claimed in the paper.

Mr. Gokhale expresses his disagreement in particular with Equation (7) and rests his point of view on research work of his own and on premises mentioned under a and b and on a conclusion, c.

In deriving Equation (7), use has been made of the relation

$R = -\dfrac{K}{\gamma}$, but it has been used merely as a stepping stone for

evolving Equation (7), which constitutes the improved form of the relation between R and γ.

The paper in exploiting the test data makes *no use* of Mr.

Gokhale's relation c: $R = -\dfrac{K}{\gamma}$, but employs Equation (7)

exclusively. The effect will be understood when, for example, both relations are applied to the case of $\beta_h = 13183$, the highest value of magnetization which the tungsten steel attains in its magnetic cycle.

$$\text{Mr. Gokhale's relation (c) } R = -\frac{K}{\gamma} = -\frac{K}{S-\beta}$$

$$= -\frac{0.5682 \times 10^6}{16000 - 13183} = -202 \text{ wrong value}$$

$$\text{As per Eq. (7) of paper, } R = -\frac{2 \times 0.5682 \times 10^6 \times 13183}{16000^2 - 13183^2}$$

$$= -182 \text{ correct value.}$$

The difference amounts to 11 per cent of the true value.

Referring now to Curves 1, 2, 3, and 4 submitted by Mr. Gokhale, essential characteristics of those curves, not mentioned yet, should be stated before making further use of them. The equation of Curve 4 is

$$\log \gamma = \log (S - \beta) = 3.6996 - 0.002331 \times H \qquad \textbf{(d)}$$

from which follows (see also Curves Nos. 4 and 1).

for $H = 0$ $\gamma = S - \beta = 5007$ and therefore $\beta = +\beta_0 = +16500$
for $H = 1588$ $\log \gamma = 0$; $\gamma = S - \beta = 1$ $\beta = 21500 - 1$

$$\text{simultaneously} \left.\begin{matrix} H = 0 \\ \beta = 0 \end{matrix}\right\} \text{would make } \gamma_\bullet = S \text{ and } \log \gamma_\bullet = 4.3324 \text{ which is not situated on the straight line No. 4.}$$

The fact that for $H = 0$ a residual magnetization $\beta_0 = +16500$ occurs, renders the cited curves unsuitable for comparison with the hysteresis curve of the paper. Curve 1 looks more like the descending branch of a hysteresis curve, while Table I showing test data indicates that the H-values progressed from $H = 0$ to 2500, characterizing it as an ascending branch.

TABLE II

Comparison of Equation (7) or rather (15) of the paper with Equation (d) of discussion

$- R = H \qquad K = 0.1205 \times 10^8 \qquad S = 21500$

$$= - \frac{K}{H} + \frac{1}{H} \sqrt{K^2 + S^2 \times H^2} = - \frac{0.1205 \times 10^8}{H} + \frac{1}{H} \sqrt{(0.1205 \times 10^8)^2 + 21500^2 \times H^2} \text{ corresponding to Equation (15)}$$

$H \quad -R$	20	60	100	200	400	600	1000	1588
β as per above Equation (15)	16303	19586	20331	20906	21200	21300	21380	21424
β as per Equation (d)	17000	17872	18573	19789	20925	21300	21477	21499
Difference	−697	+1714	+1758	+1117	+275	0	−97	−75

Due correction should be made, because "Weber's theory" also implies that the magnetizing effect of a field H depends on the configuration of molecules at the beginning of the process.

On the other hand, if the low hysteresis is to be neglected altogether, there should be $\beta = 0$ for $H = 0$.

Fig. 3 herewith has been drawn up to facilitate, as far as possible, comparison between Mr. Gokhale's results of research and the theory of the paper. Curve 1 has been copied from Mr. Gokhale's Fig. 1, and Curve 5 is a magnetization curve computed on the basis of Equation (7), or rather Equation (15), which is another form of Equation (7) of the paper. Numerical values are given in Table II. K as it appears there is a value adapted by mathematical trial to Curve 1.

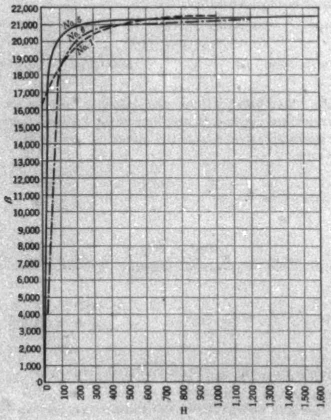

FIG. 3

Now inspecting Fig. 3, if, as called for, Curve 1 were duly corrected to start at the origin of the system, the two curves 1 and 5 would be close together for values of β from 0 to about 18,000 and from 21,000 to saturation. A difference, however, is conspicuous for values of β from 18,000 to 21,000. The agreement between the two curves would be better yet, if Curve 1 would start at a residual magnetization $\beta = -\beta_0$, *e.g.*, including that amount of hysteresis which exists in the electrolytic iron under test. In the latter case H would be replaced by $-R$ and a conclusive comparison of the two pertinent Equations (d) and (7) could be made. If, then, a need for improvement of Equation (7) should come into evidence, it might be taken care of by adding an exponent n, possibly thus

$$R_n = - \frac{2\,K\,\beta}{(S^2 - \beta^2)^n} = - 2 \left(\frac{k}{S^2 - \beta^2} \right)^n \times \beta \qquad \text{(e)}$$

In this form, the dimensional homogeneity of the equation would be retained. Curve 6 shows the corrective effect of a coefficient $n = 0.949$ upon Curve 5. Further research work along the lines of the new theory, however, should establish a term for R which covers all cases of magnetization.

Such a revision of Equation (7), or one similar, should have only a limited effect upon the equation for the dissipative component D, on account of the anticipated near agreement between the curve as per Equation (7) and the graph of the improved term.

Since, however, the molecular friction (and hysteresis) depends very much on the material under test, it is to be expected that for different materials the D-curves will have different configurations. The many constants in Equations (11) to (14) give quite a great latitude in this respect and should accord to Equations (11) and (14) more than transitional importance.

The quantity β_h does not occur in Equations (11) to (14), because they relate D to H. β is very much in evidence in Equations (33), (35), and (36), and in calculations of loss in Appendix III.

The empirical character of the equations for D is admitted, but the Curves D_1 and D_2 are rational inasmuch as they are the outgrowth of rational theory.

Section VII (Applying the Theory) was added to show first, that by reason of functional appertainment, the magnetization curve should be referred to R and not to H, and secondly, that

$\tan \tau = \dfrac{1}{S}$, which was not known before.

Regarding Mr. Gokhale's reference to Weber's theory, according to which the magnetism is resident in the molecules, the gist of that theory will not be much altered by assuming that the molecules are ordinarily devoid of magnetism, but in magnetizable materials they can be polarized in one direction only. When magnetized to the extent of $\beta = H$ (for those molecules the polar axes of which happen to be parallel to H) and to $\beta < H$ (for those molecules the polar axes of which are not parallel to H), they tend and begin to aline their axes parallel to the field H and thereby also deflect the molecules from their original position. In so doing they encounter the friction D. At the same time the flux within them increases in accordance with their "magnetic ability." Each molecule then represents only a link in one of the many parallel chains constituting the magnetic flux of the circuit.

A further refinement of the theory will have to settle the question as to whether the polar axes are rigidly fixed to their respective molecules or whether they permit a certain shift around them.

As the flux enters the molecules and alignment of the polar axes takes place, a reaction R is encountered. While mathematical terms are given in the paper for D and R, the writer believes himself justified in assigning the cause of those forces R and D to the thermodynamic and mechanical state of the material under test. Such conclusions are drawn from results of research published by J. A. Ewing.[2]

Fig. 4 (herewith) shows the development along its axis and a cross-section through a rod of magnetizable material. Certain notations are also given which are necessary for further explanations.

The outstanding feature of the aforesaid conclusions is that magnetic reaction R, magnetic friction D, and, as a sequel, β, depend very much on the relative distance between the molecules. There are various forces affecting that distance.

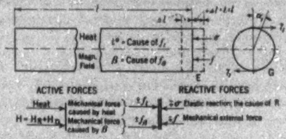

FIG. 4

Briefly stated

$$R = F_1(\epsilon, \alpha) \tag{f}$$

$$D = F_2(\epsilon, \alpha) \tag{g}$$

In general, *e. g.*, if not counteracted otherwise,

Separation of molecules ($\Delta l = +\epsilon l =$ positive) by expansion lowers D, β, R.

Mutual approach of molecules ($\Delta l = \epsilon l =$ negative) by compression increases D, β, R, each at its own rate.

If counteraction does take place, as, for instance, when mechanical pressure opposes thermal expansion, the effect upon β depends upon

$$\frac{dD}{d\epsilon} \gtreqless \frac{dR}{d\epsilon} \text{ or } \gtreqless \frac{d\beta}{d\epsilon} \tag{h}$$

$$\frac{dD}{d\alpha} \gtreqless \frac{dR}{d\alpha} \text{ or } \gtreqless \frac{d\beta}{d\alpha} \tag{j}$$

For ϵ, a critical maximum value ϵ_m exists. When ϵ_m is exceeded, the effect is similar to that of a large air-gap or a broken magnetic chain.

To augment the above equations (f and j), the following two well-known relations should be cited:

$$\epsilon = F_3(t_e, \beta, \sigma, f) \tag{k}$$

$$\alpha_1 = F_4(t_e, l_1) \tag{l}$$

Observing now that $\epsilon = \sigma E$ and $\alpha_1 = \tau_1 G$, it is obvious that R and D depend on the elastic quality of the material.

It is understood, of course, that the two fundamental laws of thermodynamics also apply. They must be referred to, for instance, when explaining the change of magnetization through heat in a permanent magnet. Here they force the conclusion that heat energy is transformed into magnetic energy.

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[2] "Conclusions drawn from references in Magnetic Induction in Iron and other Materials", by J. A. Ewing. Third Edition, 1900, D. Van Nostrand Co., New York.

EFFECT OF HEAT ON MAGNETICS

Page 171, Magnetization of Iron at Various Temperatures, Fig. 77 with $H = 0$ to 2.5. Fig. 78 $H = 0$ to 50, D decreases with increasing temperature; therefore $\frac{d\beta}{d\epsilon} > 0$ and $\frac{dR}{d\epsilon} \gtreqless 0$. For large H and β, ϵ_m is approached, hence saturation value goes down. Observe that β adds to the elongation when within a certain value. Compare Fig. 126. $H < 280$.

Page 172. Fig. 79, same case, but $H = 0.3 =$ constant: $t = 0$ to 800 deg. For low temperatures: $\frac{dD}{d\epsilon} < 0 =$ negative, but D remains too large. The weak $H = 0.3 < D$ cannot accomplish a parallel alignment of polar axes. R therefore remains near zero value. Above 700 deg. D is getting so small, ($D = 0$) that under the weak H, alinement of polar axes is accomplished, involving also $\frac{d\beta}{d\epsilon} > 0$ and $H = -R$, which brings about further elongation $\epsilon = F_3(\beta)$, carrying ϵ to critical value ϵ_m, thus reducing β. $\beta = H$.

Page 173. Fig. 80, $H = 4.0 =$ constant. Fig. 81, $H = 45 =$ const. Same process as in Fig. 79, but quantitatively different, inasmuch as the stronger field overcomes D at lower temperatures.

Page 176. Magnetization of nickel at various temperatures. Fig. 86, $H = 0$ to 80, t held constant at various values $t = 13$ deg., 245 deg., 284 deg., 298 deg., 308 deg.

D abates with rising temperature, which also entails increasing ϵ. As β grows larger due to the growing field intensity, D is falling off. $\epsilon = F_3(\beta)$ adds to the thermal elongation until the critical value is being approached. The greater the elongation by heat, the less elongation $\epsilon = F_3(\beta)$ is necessary to reach the critical value ϵ_m.

Page 177. Fig. 87, Field kept constant at various values $H = 2.5$, 10, 30, 50; while temperature varies from 0 to 310 deg. Explanations are similar to those for Fig. 79.

Page 183. Fig. 89. Bar magnet subjected to heat cycle 8-160-10 deg. Curve shows the abatement of β with separation of molecules by thermal expansion. Such separation also permits better alinement of axes of molecules, which is reflected in higher β values when the magnet cools off.

The recurrence of β and its stored magnetic energy leaves no other interpretation than that heat has been transformed into magnetism directly. This experiment should be repeated with a winding around the magnet and connected to a suitable galvanometer for checking the flow of coulombs in and out. In that form the outfit would be an apparatus adapted to change heat into electricity.

EFFECT OF MECHANICAL STRESS UPON MAGNETS

Page 204. Fig. 98, Nickel under compression. H varies from 0 to 160, while compressive stress is held constant at various values, *e. g.*, 0, 1.9, 3.5, 6.8, 10, 13.8, 19.8 kg/mm^2. Rate of magnetization increases with compression, *e. g.*, with mutual approachment of molecules.

$$\frac{d\beta}{d(-\epsilon)} > 0 \text{ and } \frac{H_R}{(-\epsilon)} \text{ numerically larger than } \frac{D}{(-\epsilon)}.$$

Residual magnetism $+\beta_0$ increases with pressure, therefore we have $\frac{dD}{d(-\epsilon)} > 0$.

Tension applied to nickel has the opposite effect ($\epsilon =$ positive). Compare Fig. 95 on page 200.

Pages 197, 224, Villary effect:—In iron, when subjected to a weak field, a mechanical tension will cause β to rise; when in a strong field and put under tension, β will decrease.

This Villari effect in iron seems to indicate that in iron a negative maximum for ϵ exists. The neutral point between $(-\epsilon_m)$ and $(+\epsilon_m)$ occurs when a certain external load is applied.

Page 243. Fig. 121, Nickel under tension and torsion. Tension alone (Curve *c c*) separates the molecules and therefore holds β low. Torsion added brings about lateral compression and a mutual approachment of molecules, and also an increase of D, the molecular friction. As soon as H has reached a value (12 in Fig. 121) to overcome this larger D, alinement of polar axes of molecules takes place "en masse," accompanied by a large increase in β. The molecules remain then in alinement for the descending branch, owing to the high value of D, and a high residual magnetism is the result.

Page 208. Fig. 102. Nickel in (successive) constant fields. $H = 6.9$, 21.8, 53.5, 118. Pull $= 0$ to 12×2.75 kg/mm^2. Separation of molecules lowers β and stored magnetic energy. Removal of tension brings back β and its energy. Case is similar to bar magnet under influence of heat, page 183.

Properties of the Single Conductor
New Fundamental Relations

BY CARL HERING[1]
Fellow, A. I. E. E.

Synopsis.—*The properties of a unit length of single, straight conductor, far removed from all other circuits, are investigated to endeavor to find whether such a unit is a basic, fundamental one, on which deductions and a method of mathematical treatment could be based, as was advocated by Ampere, to supplement (not to replace) the Maxwell system based on a complete circuit, and to test the correctness of some of the postulates now in common use, based on the latter system. By several new, simple, and direct proofs based only on a few well-established and accepted relations, chiefly the internal stresses in a conductor, but excluding infinities, self-inductances, induction, and any postulates, a constant is deduced for the energy stored by a current in such a unit length, which seems to be one of the most fundamental, basic constants in electrodynamics, from which many useful deductions can be made, some of which are given. This energy of the flowing current corresponds to the $m v^2/2$ energy of moving masses.*

Some of the results differ from those which have been in use; explanations are offered of the cause of these differences, and it is shown how the results may be brought into agreement, involving some changes in our previous conceptions. It is shown that with flux lines a distinction ought to be made which is analogous to that which distinguishes the wattless ampere from the one in phase, or between a true resistance and an impedance; it is shown why what might be called wattless flux, ought to be recognized. It is shown that self-inductance is used in two senses which may sometimes lead to different results, and that a distinction should therefore be made between them somewhat analogous to that between resistance and impedance or reactance. This, it is believed, would clear up the ambiguity now existing in that term. It is believed that some of these results could not have been obtained from the Maxwell complete circuit system, which rather leads one away from them.

* * * * *

INTRODUCTORY

In science, as in engineering and mathematics, it is always desirable to have two independent methods of getting a result; they not only afford a very desirable check on the result and on each other, but each one often has advantages over the other in the different fields for which it is best adapted. For the analytical treatment of electric currents and circuits, the "complete circuit" system of Maxwell has been taught universally and exclusively and for so many years that the strong though wrong belief has arisen among many that it is the only possible one, and that every case must be treated from that and only from that standpoint. In some cases this has not only misled us but has sometimes even led us away from useful facts and relations.

There is a second and older system, the one advocated by that great mathematical physicist Ampere, which is based on the single conductor, and which has many advantages over the other in specific fields for which it is better adapted. Neither system should be used to completely replace the other; they should be used to supplement each other, each in its own field. An electron may start from rest, move to another point and come to rest again there, as in a bolt of lightning; while moving, it is a current and generates a magnetic field in which energy is stored, quite analogous to the $m v^2/2$ energy stored in a moving body. The "complete-circuit" system is a misfit in such a case and involves complications which are burdensome and confusing to the student, while the single-conductor system applies directly.

On the other hand the complete-circuit system lends itself well to integrations around a completed path. It has served its purpose well and is very useful, and in most of the usual circuits it is quite reliable. But in a search for the true, basic fundamentals, there is no such a thing as a fundamental or unit or limiting circuit; the nearest approach to one is a circular circuit in the form of a tore or torus whose centerline radius is equal to the radius of the wire, like a doughnut with an infinitely small hole through it. This is the shortest circuit which can be made with a wire of given diameter, hence is at least a limit, but for various reasons this is not satisfactory as an ultimate fundamental.

The writer has therefore concluded that the only real fundamental is a unit length of a long straight, single conductor, far removed from all others, as this embodies absolute ultimata; it is the form which is most free from all external influences, and besides the current, it involves the least number of variables, in fact only one, the radius of the wire, and even that one falls out in some cases, as will be shown below, for relatively to the infinite free space around it for the magnetic flux, the small finite radius of the wire becomes negligible.

The purpose of the present paper is to examine analytically the properties of such a unit length of this fundamental conductor, without involving infinities, the limitations of complete circuit mathematics, self-inductances, induction, postulates, or any but well established relations, and to deduce from these properties any useful relations and constants that may exist.

To do so by considering a straight line as a special case of a circle of infinite radius, is unsatisfactory, as it introduces that dangerous quantity, infinity, with its serious pitfalls into which many otherwise able men

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have fallen[2]. In a French book by Fleury dealing with infinity he says: "Infinity has no other property than that of being impossible. Any calculation based upon absolute infinity, or upon any function whatever of absolute infinity, is itself absurd." The writer agrees with him in this, in part. When the results of two methods fail to agree, "look for the infinity" he says, and you will probably find the error.

When absolute infinity occurs, as it does, in mathematical deductions, there are two ways in which it can be safely applied in practise. The first one applies (as Fleury admits), when it is possible to assume that the quantity is simply so large (though finite) that others which depend on it may safely be taken as the same as they would be if it were infinitely large; this is often the case. For instance, in starting a current in circuits containing some small inductance, a few seconds is generally quite safely taken to be the equivalent of the eternity required theoretically for the current to reach its final value even in very accurate tests; the currents in even sensitive tests made in New York do not appreciably affect simultaneous researches made in Philadelphia, the 90-mile distance being practically infinite. Our atmosphere should extend theoretically to infinity, according to the laws of gases, yet beyond the relatively short distance of 50-60 miles there is not enough left to consider; the moon, sun, planets and stars encounter no friction in our atmosphere, even though it should extend theoretically to infinity.

The second one applies when some *property* of an infinite quantity can be found which is finite and therefore can be practically applied; thus the 760-mm. pressure of our atmosphere is finite, though according to the theory of gases, the atmosphere must extend to infinity; or similarly, the radial magnetic pressure on the surface of a single straight conductor, by the flux surrounding it, is known to be finite, although the flux, like the atmosphere, should extend theoretically to infinity. It will be shown later that under certain conditions the flux energy is also finite, though the flux is infinite.

One of the pitfalls in dealing with infinities obtained mathematically is that sometimes the real result is that the quantity is merely indeterminate and not infinite; this may arise when a factor has been suppressed or dropped because it is unity, but it is physically still present.

By treating the circle as a special case of a straight line, instead of the reverse, infinities and their pitfalls are avoided, which is another reason for preferring to start with the straight line as the fundamental.

THE SINGLE CONDUCTOR

In dealing with the single straight conductor as the fundamental, the real underlying condition (besides the straightness of a reasonable length) is not that it must be infinitely long, but merely that the unit length is assumed be so far removed from all other currents, or magnets, that they will have no appreciable effect on it; such currents include its own return circuit. Hence in a complete circuit in the form of a large square the sides may be made so large that a unit length *l*, Fig. 1, in the middle of a side, is no longer affected by the return current in the opposite side *L*.

The only three effects which neighboring currents can have on the unit length *l*, are attraction, repulsion and induction, which decrease as the square of the distance, and although they may also increase as the length *L* increases, yet the action of both combined is always a *decreasing* function, hence a distance may always be reached at which the effects are small enough to be neglected, as is always done when two or more independent tests with complete circuits are made at different places at the same time, even though with great accuracy.

Hence such a square can always be conceived in which the field at its center may safely be taken as zero, and therefore a unit length *l* in the middle of a side is practically as free from the effects of neighboring currents, as though it were a single conductor, and the flux and flux energy inherent with it are practically those in the disk-shaped space *D* between two parallel planes one unit apart, just as they are in a single conductor. Such a unit length *l* may therefore be taken to represent a unit length of a single conductor to any degree of accuracy desired by merely making the square large enough. Such a square circuit is therefore a connecting link between a unit length of a single, straight conductor and a complete circuit; and the condition of infinite length, so often used in order to degenerate the single conductor to the realm of impossibilities, is no longer justifiable; some of the pitfalls of infinity are thereby avoided.

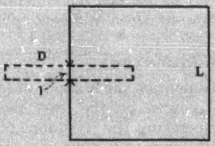

FIG. 1

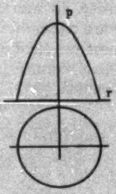

FIG. 2

Maxwell himself determined (Art. 478) the now classic law for the flux density around a single conductor, with such a closed circuit; Northrup's determination of the internal pressures in a single conductor produced by the flux, is not based on the condition of infinite length, but as he says, merely on freedom from the effects of neighboring currents, and the accuracy of his expression was checked experimentally in a closed circuit. Ampere's able analytical researches with the single conductor were no doubt based on the same conditions; so is the generally accepted $i^2/4$ energy of the flux in the interior of a conductor. Many other like cases could be cited, showing that the infinite length is not a necessary condition. The same condition of freedom from all external influences is assumed in all accurate electrical tests, hence it is not an unreasonable or impossible condition.

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[2] This was well brought out in a discussion by Dr. C. O. Mailloux, TRANS. A. I. E. E., Vol. 42, 1923, p. 328, col. 2, par. 4, to p. 329, col. 2, par. 2.

ENERGY

Energy is the one physical quantity which is common to all physical phenomena and processes, and its absolute unit, the erg, applies to them all; hence it is the best and in fact the only connecting link between all the different groups of physical phenomena. Forces may be electromotive, magnetomotive, or ponderomotive (tending to move masses), each of which is physically different from the other, and their units are different and are not directly convertible. Moreover some forces are vector quantities, have components and resultants, can be generated or annulled and follow different laws, while energy is always a constant quantity no matter what form it may be in and its laws are common to all its forms. Energy is therefore the best physical quantity to use as a fundamental and the best connecting link between different physical phenomena.

ZERO RESISTANCE

In searching for fundamentals the number of factors or variables should of course be the least possible. At absolute zero of temperature the resistance of pure metals is eliminated, as it is zero; a current once started, as by induction, will therefore continue (theoretically) to flow forever, just as a body set into motion will continue to move when it meets with no resistance. Zero resistance will be assumed in the present discussion.

In the electric case this stored energy, which was stored while the current was started, is represented entirely by what seem to be stresses or strains of the ambient ether, which are called magnetism, hence it is magnetic energy and seems to be very closely analogous to the mechanical stresses or strains in compressed or stretched elastic rubber, and is static or potential in its nature. This stored energy of a current is closely analogous to the $m\,v^2/2$ energy stored in a moving body. When any of this stored energy, at zero resistance, is released, as by doing external work, the current also decreases, as shown in a paper by the writer,[3] and when it has all been released the current will become zero.

Such a condition of zero resistance simplifies greatly the considerations and conception of some electrical phenomena involving flux and energy, as there is then no external source of energy to mask the conditions when some of the stored energy has been released, as is the case when a "constant current" is specified in stating the laws, as it usually is, in which case the released energy is at once restored again, and there is also a continuous flow of energy. At zero resistance and with no connected source, the variables are reduced to their minimum, hence they are the best conditions for studying the fundamentals. The permeability is, of course, assumed to be unity, in all such fundamentals.

UNIT RELATIONS

The very basis of the absolute or c.g.s. system is to make as many as possible of the numerical relations between different fundamental physical quantities, unit relations; that is, the coefficient is unity, in which, however, the factor 2 often resulting from integrations, and π which cannot be avoided, must be included. If, therefore, in the search for new fundamentals and relations between them, the latter turn out to be unit relations, the presumption is strong that they are correct, though it is not necessarily an absolute proof. A number of the relations deduced below will be seen to be unit relations.

THE FUNDAMENTAL CONSTANT

One of the most basic fundamental quantities in electrodynamics therefore seems to be the energy represented by, or inherent with, or stored in, a unit length of single, straight conductor by a given steady current, under the most basic conditions; if this quantity leads to unit relations its fundamental character is confirmed.

Determination of this Constant. The problem which the writer endeavored to solve therefore was to determine this basic fundamental constant and to do so by a method which does not involve any physical quantities other than energy, mechanical forces, and currents, hence excluding completely that ambiguous quantity called self-inductance, and the theory of linkages (perhaps still incompletely proved) in the case of self-produced flux, both of which are so often relied upon absolutely in such energy calculations; nor does it involve any infinities or "postulates"[4] and it is moreover very direct and simple; there should, therefore, be no difficulty in either confirming it or pointing out precisely the error in it, if there is one. This energy should of course be the same no matter by which process it has been determined; if not, an explanation must be looked for and it ought to be possible to find it, especially if it lies in this simple proof. Fleury's advice in such cases is to "look for the infinity." While this method is new and different from the orthodox ones, it involves no new or unproved laws.

The method is based on the recently discovered and now well-known fact that when a current passes through a conductor internal stresses and strains in the form of radial pressures and longitudinal tensile stresses are produced, quite analogous to those in a magnetic field;

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[3] Trans. A. I. E. E., Vol. 42, 1923, p. 325.

[4] A postulate is defined in the dictionary as "a proposition accepted *without proof;* something that must be *assumed* in order to account for something else." (Italics are the author's.)

but as the former act directly to tend to move the material of the conductor, they produce truly ponderomotive forces (tending to move masses), as is readily shown when the conductors are liquid and the current is large enough, as they increase with the square of the current, like most other electromagnetic forces. Being true mechanical forces they can be correctly specified and measured in dynes. Hence by letting these known mechanical forces act through known distances, the corresponding amounts of energy may be determined. Unfortunately neither Ampere nor Maxwell had the advantage of any knowledge of these internal stresses.

INTERNAL STRESSES

Many years ago the writer noticed these stresses when large currents were passed through liquid conductors in electric furnaces; the forces were sometimes strong enough to completely rupture the circuit by crushing it radially or tearing it longitudinally, and as this always occurred in one place by depressing the liquid where the cross-section of the open channel happened to be least, the phenomenon was colloquially called the "pinch effect" by which term it is now generally known. These forces are now used very effectively in hundreds of electric furnaces for lifting and circulating the liquid metal.

Later, Dr. E. F. Northrup, to whom the writer had described this peculiar phenomenon, developed the mathematical formula for the quantitative value of these pressures, in a very able paper.[5] He based it on the attraction of the filamentary conductors for each other, and also on the radial force acting on each filamentary conductor due to its being in the magnetic field inside of the conductor; both methods gave the same result. It is important to note that it was also confirmed by him later experimentally to a high degree of accuracy (using, of course, a complete circuit), showing that the pressures were true, mechanical ones, and that their units in the formula are in dynes per sq. cm This formula or law may therefore be safely accepted as quantitatively correct, reliable and accurate. The present writer showed later that in the c. g. s. system this law is a *unit relation* for the pressure at the center of a round conductor which, though it may not be a proof, is always strongly confirmatory of correctness. This easily remembered relation is $p = i^2/S$ in which S is the section.

Northrup's general law for the radial pressure p in dynes per square centimeter at any point in the interior of a round conductor of radius R at a radial distance r from the center, and carrying a current i, is $p = i^2 (R^2 - r^2) \div \pi R^4$, the now well-known "pinch" pressure formula; all the quantities are in c. g. s. units. This gives the *mechanical* stresses or strains in the form of radial pressures in any part of the cross-section of a round conductor, produced by a current flowing through it. The curve of these pressures is a parabola, Fig. 2; hence for the whole section the curved surface of the loci is a paraboloid of revolution. This mechanical pressure is zero at the circumference, a maximum at the center, and the mean over the whole section can readily be shown to be half of the maximum; the formula also shows that this radial pressure is independent of the length of the conductor. This formula is based on the condition that, as Northrup himself states it, the conductor is a part of a very long straight conductor of circular section "very far separated from its return conductor," hence is not limited to the impossible infinitely long conductor.

PROOFS

There are several proofs of this constant, $i^2/2$ ergs per cm. all of which lead to the same result and therefore confirm each other. They are rigid, being free from approximations, dropped factors, infinities, inductance, self-inductances, or mere postulates. One is based on the radial pressures, one on the longitudinal force, and other shorter ones based on what some may consider allowable assumptions. The energy referred to is that which is required to start a steady current; it is constant while the current is flowing, and is set free again when the current stops; it is quite independent of the energy which may be transmitted while the current is flowing; it is quite analogous to the vis viva or the $m\,v^2/2$ energy in a moving body, like that required to bring the cable itself up to its normal speed in a cable transmission. By the first method, based on the radial pressures, it is measured as it is set free, while the current decreases to zero; it involves only two of Maxwell's undisputed laws. By the second method, based on the longitudinal force, it is measured as it is being stored by a constant current in an increased length of the conductor; it involves only the Northrup and the so-called Kelvin laws. In both, the conductor is assumed to be a liquid, and the energy is measured by the mechanical work done by this energy.

Radial Pressure Method. A steady current is assumed to have been started by an external source, in a circuit of zero resistance, hence would continue to flow without a connected source until all its vis viva or stored energy has been consumed by transformation into some other form of energy. This stored energy is quite analogous to the $m\,v^2/2$ energy stored in a body moving at a constant velocity and without encountering any resistance.

An experimental proof that the energy stored in a body moving at a constant velocity is equal to that given by the well-known formula $m\,v^2/2$, might be obtained by opposing the motion of that body by a *constant* pressure or force until it comes to rest, when its energy is exhausted; then the product of this force and the distance over which it was applied before the body came to rest, would evidently be equal to this

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[5] *Physical Review*, June 1907, p. 474.

$m\,v^2/2$ energy, thereby proving that this expression gives the correct amount, or that this amount might be determined in that way if that formula were unknown.

The present method is quite similar. Assume any given length l of such a straight, single, liquid conductor of circular section having a radius R and a current I (c. g. s. units); or it may be the whole circuit of length l, if only it is large enough that each unit length is far enough from the return circuit not to be affected by it. It is well known and could easily be shown, that such a liquid conductor will tend to shrink radially, due to the pressures produced by this stored magnetic energy.

From Maxwell's $H = 2\,I/R$ for such single conductors, and his $H^2/8\pi$ pressure formula, this radial magnetic pressure on the outside surface is easily shown to be $I^2/2\pi\,R^2$ in dynes per sq. cm.; this is the total, *resultant* pressure; it will be explained below why this is the resultant and under what conditions Maxwell's $H^2/8\,\pi$ pressures are true mechanical pressures in dynes per sq. cm. Whatever may be the detailed explanation of the mechanism of this shrinkage, say through a radial distance d, it is true in any case that this pressure must have then acted through this distance d and has thereby done work. Let the outflow of this liquid due to this shrinkage be assumed to be opposed by a constant mechanical pressure on the liquid (analogously to the constant pressure referred to above opposing a moving body), as for instance by making the liquid which is ejected by this shrinkage raise a weight, as it does in hundreds of electrical furnaces in daily use.[6] This opposing constant outflow pressure is made equal to the above radial pressure.

In thus acting on the outside through the distance d this constant radial pressure has set free a known part of the energy originally stored in the flux; this is determined from the distance d and the known force equal to this constant pressure multiplied by the mean area. Hence there is then left less flux energy and therefore of course also less current. Let this shrinkage at constant outside pressure against an equal, constant, outflow pressure, continue until the conductor has shrunk to a line, that is, to zero.

At these radial and outflow pressures, assumed to be constant and equal, it can be shown that $I^2/i^2 = R^2/r^2$ in which i and r are the current and radius after a shrinkage; for the first pressure P is $I^2/2\,\pi\,R^2$ and the pressure p after this shrinkage to a radius r is $i^2/2\,\pi\,r^2$; when these are made equal to each other the above relation follows. Hence $I/i = R/r$, that is, the currents will diminish in proportion to the radius, and therefore the current, and with it of course the energy also, will become zero, when the radius has shrunk to zero. This shows that *all* the stored energy has thus been consumed in crushing the conductor to zero. The remaining energies are proportional to the squares of the currents or radii, but this is not an essential relation in this proof.

It also follows that for any given length l of the conductor the total radial *force*, as distinguished from pressure (force = pressure × area), diminishes in proportion to the radius, that is, $F/f = R/r$, and it is therefore also zero when the radius is zero. These simple relations hold only when the outflow pressure is made numerically equal to this constant radial pressure; this outflow pressure always exists at a distance from the center equal to the outside radius divided by the square root of 2, as shown by the Northrup formula, and his experimental demonstrations show that these forces are true, mechanical forces in dynes agreeing quantitatively with Maxwell's formulas.

As the original radial pressure is $P = I^2/2\,\pi\,R^2$ the force at first is $F = 2\,\pi\,R\,P\,l = l\,I^2/R$ in dynes, and as it diminishes in proportion to the radius, the work done by its acting radially to the center is the mean of the original and zero, hence is $l\,I^2/2\,R$, which acting through the distance R gives as the original stored energy $W = l\,I^2/2$ ergs or $I^2/2$ ergs per cm., *which is the basic, fundamental constant sought for.* This quantity was thought by our forefathers to be infinity, as deduced from the "complete circuit" system; the explanation why there is this disagreement will be given below. Attention is called to the fact that this total $i^2/2$ ergs, is just twice that long known to reside in the inside of the conductor.

The same result could be obtained by the calculus, and the liquid might then be assumed to be ejected continuously at the center where the pressure will vary from twice the radial pressure at the start, to zero at the end; the opposing outflow pressure must then be assumed to vary accordingly in order that at every moment there is equilibrium. The radial pressures will then no longer be constant but will decrease, and the current will decrease faster than in proportion to the radius. It is then analogous to stopping the movement of a body by means of a variable, instead of a constant, force.

Longitudinal Force Method. A steady current is assumed to be kept flowing in a liquid conductor of zero resistance, zero weight, and constant diameter, by a continuously applied source. Instead of allowing the mechanical stress to act radially to set free the stored energy as before, it is now allowed to act longitudinally to lengthen the conductor of constant diameter by a specified amount, whereby the source adds to the circuit the energy stored in this added length. This added energy is calculated by letting this known longitudinal force lift a known weight through a known height while lengthening the conductor by the latter distance; the source thereby does a known amount of external work, and according to the so-called Kelvin law it then simultanously adds to the stored energy of the circuit an amount equal to the external work done. Hence the energy stored in this added length is equal to this known external work done.

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[6] If necessary to picture the details (though they are immaterial and do not affect the theory involved) let the liquid be supposed to be ejected through a tube leading to a cylinder foreign to the conductor, having a piston which raises a constant weight; as the pressures in the interior of the conductor are known to be different at different distances from the center, this tube is assumed to be applied at that radial distance r from the center (namely when $r^2 = R^2/2$) at which this particular pressure exists and is constant during the shrinkage, hence it must be assumed to be moved toward the center as the conductor shrinks. When this is done, and only under those conditions, the quantitative mathematical relations become extremely simple, as will be shown. This shrinkage against an opposing pressure must of course be assumed to take place simultaneously throughout the whole of that circuit, though only a portion of it needs to be considered mathematically.

The force is calculated from the pressures given by the Northrup law, which is generally recognized as correct, both relatively and quantitatively, and the pressures have been shown experimentally to be true mechanical pressures in dynes per sq. cm.; it may now be safely classed under the "classic" laws. The Kelvin law, though not always referred to by this name, seems to be a universal law in dynamics and when properly worded, may also be safely classed under the "classic" laws. The conditions of the circuit are that only the additional length need be liquid, the parts near it must have the same diameter, the part that moves must be considered as weightless and of course it must be straight for a reasonable length and the return circuit must be far removed.

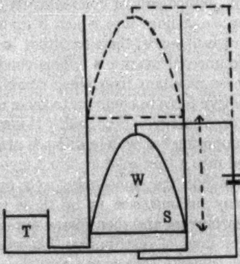

Fig. 3

In Fig. 3, let S be conceived to represent an infinitely thin cross-sectional layer of a circular, liquid conductor of radius R. Let W be a weight, which may be shaped as a paraboloid of revolution to correspond with the pressure loci in Fig. 2, though merely to simplify the conception, as a cylindrical body of half the maximum height and the same base would have the same weight or volume and would answer quite as well. Let a current i be conceived to be passed axially through this section, the rest of the circuit being shown diagrammatically only. Let the layer S be assumed to be connected at its circumference, where the pressure is zero, to a supply reservoir of more liquid diagrammatically represented by T; the liquid is assumed to be weightless and the supply from the outside is assumed to exert no pressure and to involve no energy.

The conductor S being a liquid, the radial pressures of the Northrup formula will act equally well axially, let it be said by hydraulic action, if that is preferred; hence they will raise the weight W, the additional weightless liquid necessary to provide for the lengthening of the conductor being supposed to be supplied from the outside under no pressure. The pressure formula will be seen to be independent of the axial length, hence these axial pressures will always remain the same as the weight is raised, that is, the total vertical lifting force is a constant, for a constant current. This axial lifting force is being used in hundreds of electrical furnaces in daily use lifting many tons per day, hence is well known and not a mere theoretical force on paper; its quantitative value is also well known.

Let the weight be lifted l cm. (Fig. 3). The surface of the loci of the pressures being a paraboloid of revolution, it can readily be shown[7] that the average pressure over the whole section is half the maximum at the axis. At the axis $r = 0$ (in the Northrup formula), hence the maximum pressure there is $p = i^2/\pi R^2$; multiplying half of this by the total area πR^2, gives $i^2/2$ as the force in dynes; that is, the total lifting force, and therefore the weight W which it will lift, is numerically equal to $i^2/2$ dynes. Having lifted it l cm. the external energy expended on the weight is: energy = force $\times$ distance $= l\,i^2/2$ or $i^2/2$ ergs per unit length. Hence according to the above mentioned Kelvin law, the stored magnetic energy of the conductor per centimeter of length, must be $i^2/2$ ergs also, the source having supplied double this energy. ***This is the basic fundamental constant sought for.***

Other proofs. A given radial pressure can always be replaced by its corresponding circumferential tension. If P is the radial pressure on the outside of a conductor as given above, it can be shown that the tensional force (not pressure) in a band encircling a unit length of the conductor and producing the same radial pressure, is $P\,R$ in dynes, in which R is the radius. If a stretched, elastic band having this tension be assumed to be developed into a straight line (equal to the circumference) and then allowed to shrink to zero length, following the law of the perfect spring, namely that the tension is proportional to its length, it can be shown that the energy set free thereby is again the same, $i^2/2$ ergs.

From the Northrup formula the pressure in dynes per sq. cm. at the center is just twice the $i^2/2\,\pi R^2$ pressure in the same units, at the periphery, from outside. As the outside pressure must also act at the center, it might be argued that the inside flux has added an equal amount, hence that the energies residing inside and outside are equal, therefore the total is double the well known inside energy $i^2/4$.

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[7] It is well known that the volume or weight of such a paraboloid of revolution is equal to that of a cylinder having the same base and half the height.

At the periphery the two mechanical forces balance, as there is no tendency to movement there, according to the Northrup formula; there is no resultant there. As the forces are equal the energies should be also, as both forces must be considered relatively to the same center, hence as acting through the same distance, the radius. Again the total is twice that inside.

The very basis of the c. g. s. system is that the fundamental relations are unit relations, at least as far as possible. If it may therefore be assumed that the self-inductance (in its true energy sense) of a unit length of the fundamental conductor, is unity, then if L in the usual expression $L\,i^2/2$ for this energy, is made unity, the energy becomes $i^2/2$. A self-inductance is stated to be physically a length, and in the c. g. s. system it is correctly measured in centimeters. Under fundamental conditions this length and the length of the conductor should be the same thing. This constant then follows directly.

Those who recognize the existence of the longitudinal force will find that in a fundamental conductor of uniform diameter it is numerically equal to $i^2/2$ dynes, from the Northrup formula, and that in this fundamental conductor it is independent of the diameter or the length. Hence when this force acts to stretch or lengthen its conductor without doing any external work (though generating a counter e. m. f.), the energy is being stored, just as it would be when the speed of a moving body is increased. If l is this added length, the added energy will of course be $l\,i^2/2$ ergs, which for a unit length again gives $i^2/2$.

Doubtless still more proofs could be found which are also free from the pitfalls of infinities, the ambiguous self-inductances, and the shortcomings of the complete circuit theories, which had misled us in the past.

DEDUCTIONS FROM THIS CONSTANT

It will be seen that this constant is independent of the diameter of the wire, and depends only on the current, both of which have long been known to be true of the flux energy residing in the interior of the conductor, $i^2/4$. For any length l the energy is $l\,i^2/2$, that is, it is directly proportional to the length. This $i^2/2$ ergs is apparently one of the most basic fundamental constants in electrodynamics, and from it interesting deductions follow.

This $i^2/2$ is the total stored energy, outside of the wire and inside. It has long been known and is easily proved, that the amount stored inside of the conductor is $i^2/4$ ergs per cm. It follows therefore that of the total, half of the energy resides in the inside and half on the outside, as one might expect nature to distribute it.

This constant also shows that the energy of the flux is finite, though the flux itself is of infinite extent, in the same sense that our atmosphere extends to infinity. It is one of those cases in which a *property* of an infinite quantity is finite and therefore affords a means of treating an infinite quantity mathematically without danger of falling into the pitfalls of mathematical infinity. Another finite property of this theoretically infinite flux is what might, in a certain sense, be called the resultant or equivalent flux, as will be described below.

Reduced to the units used in practise the general formula becomes $W = 0.00001524\,l\,I^2$ in which W is the stored energy in watt-seconds or joules, l is the length in 1000-ft. units, and I is the current in amperes. This shows how extremely small it is.

A very interesting and important deduction, showing another new unit relation, is that in the c. g. s. system each unit of current generates one unit of flux around such a conductor, in each unit of length, and independent of the diameter; it includes all the flux inside and outside. But this flux must be specifically defined, as it is a resultant, equivalent, or condensation of all the very large number of lines into which it divides itself as it spreads out into space; these resultant lines might be termed fundamental maxwells. This condensation is such that these resultant lines when combined with the magnetomotive force in common to them all, represent the true stored energy in that field. It is somewhat analogous to supposing our widely diffused atmosphere to be condensed into a thin solid or liquid layer around the earth, which has the same mass. Or these condensed lines may be imagined to be those originally generated by the current and then spread out into space according to the laws of distribution, *but without any change of energy contents*, just as the condensed atmosphere would spread out without change of mass. Such lines are a means of summing up an infinite quantity by one of its properties (energy in this case) to get something finite. They are useful in calculations of flux energy and they clear up some ambiguities, but they are sometimes distinctly different from those entering into calculations of the induction of e. m. fs. by cutting or linking. This will be further discussed below.

The proof of this equality is as follows. The energy residing in a complete circuit of flux is: ergs = maxwells × gilberts/8 π. The magnetomotive force in gilberts around a single conductor is $4\,\pi\,i$, and as the energy per unit of length is $i^2/2$, it follows that $i^2/2 = 4\,\pi\,i\,f/8\,\pi$ in which f is the number of lines in maxwells; hence $i = f$. This unit relation is a fundamental one and applies rigidly only to the fundamental conductor, and of course, not at all to bi-filar non-inductive circuits, nor when the permeability is not unity. The "complete circuit" mathematics leads us away from it rather than toward it. It seems to mean that in the c. g. s. system current and flux are merely different physical representations of the same quantity, as far as energy is concerned, though, of course, only when the resistance energy is zero; or that magnetism is merely an effect of current at a distance, the energy residing in the moving electrons.

For a unit length and unit radius the total radial force (not pressure) from the outside, is numerically equal to i^2 dynes, hence is another unit relation; for other radii it is inversely proportional to the radius. This force multiplied by half the radius (because it is radial) again gives the constant $i^2/2$.

The stored magnetic energy in any part of a circuit is generally calculated from the formula $L\,i^2/2$ in which L is the self-inductance of that part in centimeters. In the fundamental conductor this energy is $l\,i^2/2$ in which l is the length in centimeters. Hence in such conductors the self-inductance (in the energy sense of that term) is the same thing as the length of the circuit, that is, the distance over which the current flows; another interesting unit relation. This explains why a self-inductance is physically the same kind of a quantity as a length, a purely geometric quantity, at least under the most fundamental conditions; the permeability is of course taken to be unity in all fundamental cases. It also shows why in the c. g. s. system it is correctly expressed in centimeters, or in 10,000. kilometers for the henry. It also follows that under these fundamental conditions the self-inductance is independent of the diameter of the wire, which would seem to follow also from the long-known fact that the energy of the flux in the interior of the wire is independent of the diameter.

This stored electromagnetic energy, $l\,i^2/2$ of a current, is quite analogous to the visviva or stored mass energy $m\,v^2/2$ of a moving body. In the c. g. s. system both are equal to ½ erg when all the quantities are unity, and one may write $l\,i^2 = m\,v^2$, which means that if all the electromagnetic energy stored by a current i flowing for a distance l, in such a single conductor, be converted into moving mass energy, the relation of the mass to the velocity must be such that $m\,v^2 = l\,i^2$. Thus for say 1000 amperes flowing in such a conductor, the stored electromagnetic energy in every foot is the same as the mechanical energy stored in a weight of 0.723 or nearly ¾ lb. moving at 1 ft. per sec., or 0.181 lb. at 2 ft. per sec.; all are equal to 0.01124 foot-pounds or 152,400 ergs. This is the energy, per foot of conductor, set free when the current is stopped. For any other length of conductor this equivalent mass increases as this length.

It is of interest to note that for 1 ampere this energy stored per centimeter is equal to that of 5.10 millionths of a gram raised one centimeter, which is extremely small, and shows why the least resistance in such a conductor stops a current almost instantly after the e. m. f. ceases. Yet our forefathers claimed that this energy was infinitely large, which it seems is still being taught.

Another result which this constant, $i^2/2$, has led to, is a better understanding of the true nature of flux energy and calculations pertaining to it, as explained below.

DISAGREEMENT

When this stored magnetic energy per unit length of a single conductor is deduced by means of the mathematics of the complete circuit and by some of the older methods and postulates, or by means of self-inductance formulas (always only approximate), the result is that even for a very small current this stored energy per unit length is infinite, while $i^2/2$ is generally quite small. But as the above proofs are simple, brief, rigid, and involve nothing but well established laws, it does not seem possible to find any error in them in a long discussion since the writer's first publication of this result,[8] deduced by a similar though more involved process.

The discrepancy therefore must be looked for elsewhere. The mere fact that the result differs from our older views, cannot of course, be accepted as a proof of an error in it; if that were done in this and other cases, further progress in science would be checked.

The writer believes that this discrepancy can best be located and explained, and that the criticisms of the above proofs can best be answered, by first getting a clearer conception of the various factors and elements involved, and by endeavoring to show how and where some of our older conceptions had misled us, and which of them may be questioned and should be modified or revised if wrong. The new result, if correct, may well be used as a test of the correctness of some of the older laws and postulates.

THE SQUARE CIRCUIT

As explained above, a circuit in the form of a very large square (Fig. 4) is the best form of complete circuit for studying the properties of a unit length of a single conductor. A number of the criticisms involving the "rest of the circuit" are also answered by this form.

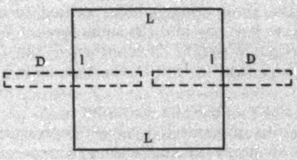

FIG. 4

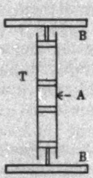

FIG. 5

If the lengthening of the circuit described in the second proof above, be assumed to take place simultaneously in the middle of two opposite sides, as in Fig. 4, and the material of the circuit be considered weightless, many of the criticisms made will be met. The only additional flux and flux energy will then be in the unit disks D and D; those in and around the rest of the circuit will not have been changed in the least.

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[8] "A Single Straight Conductor as a New Fundamental", *Jour. Frank. Inst.*, Feb. 1925, p. 235. In this article the first part of the paragraph forming the upper half of p. 243 contains an unfortunate arithmetical error and should be deleted.

It is of interest, and answers some criticisms, to suppose the reverse movement to be forced to take place, that is, that even when only a small steady current is flowing in such a circuit as Fig. 4, it be forced to be shortened by a unit length (half a unit on each of two sides) or even less. According to the older views that the stored energy inherent in a unit length of such a circuit (or even less) is infinite, it would have to follow that an infinite amount of energy had been set free by this small motion and small current, which of course would be absurd.

It has been claimed that the other sides L L repel each other, thereby tending to lengthen or stretch the others, and that this force *must be added* to that exerted in the unit length. The very premises are, however, that they are so far apart that their repulsion effect on each other along their whole length (which is due to their own flux) may be safely assumed to be zero. It is true, however, that no matter how long the other sides l, l are, the force which lengthened the unit length, caused by the mutual repulsion of the parallel disks of flux, exists along the whole length and therefore acts on L and L also, to repel them at their ends, as they are the abutments. But as this is the *same* force, it would be an evident error to add it to itself.

This may be illustrated by the following mechanical analogy. Let T, Fig. 5, be a tube of any length, even very long; it is conceived to be filled with some elastic material under pressure which is just resisted by pistons acting against the abutments B B which therefore seem to tend to repel each other. If now some more of the compressed material be introduced at A the abutments B and B will move apart, but the force that moved them is that applied at A, and must not, of course, be added to the prior force which seemed to repel the abutments, as it is one and the same force. When a rope supporting a weight is lengthened the force in it must not be added to that in the rest, as it is the same force.

FLUX AND FLUX ENERGY

A unit line of flux, a maxwell, by itself represents no specific amount of energy, any more than an ampere does (after it is started); it might therefore be "energyless" like a wattless ampere, though for different reasons. Amperes represent energy when combined with the electromotive force in phase with them; a maxwell represents stored magnetic energy when combined with its magnetomotive force in gilberts (or with ampere-turns). The energy in ergs is equal to the maxwells multiplied by the gilberts and divided by 8π. The magnetomotive force in gilberts in a complete magnetic circuit around a long straight wire, is numerically equal to $4\pi i$ (i is the c. g. s. unit of current equal to 10 amperes); hence one gilbert $= 10/4\pi$ ampere-turns.

But there is a very important difference in the analogy between the amperes in an electric circuit and the maxwells in a magnetic circuit. In an electric circuit the filamentary amperes, each multiplied by their voltage in common and in phase, may correctly be added together to get the total energy; but the writer maintains that this cannot always be done correctly with the maxwells.

The reason for this is best shown by analyzing the flux around and in a straight conductor. In Fig. 6 let C be the cross-section of a conductor; then according to the well known law, $H = 2\,i/r$, the ordinates of the Curve F give the flux densities H at any distance r from the center, outside of the wire, while the ordinates of the straight line f give the H inside of it. At the surface the two are, of course, exactly equal, namely h, as they are in equilibrium there.

The flux lines outside may be said to act by their contraction somewhat like layers of stretched rubber bands, the outer ones exerting a radial pressure on all those between them and the surface, just as the outer layers of our atmosphere exert a radial pressure on all those below them. Hence the magnetic pressures ($H^2/8\pi$ according to Maxwell) at the surface of the wire, are the *resultants* of all those outside of the wire, just as the atmospheric pressure of 760 mm. is the resultant of all the pressures above it. This radial pressure from outside contributes to the flux density H (Curve F) which is greatest at the surface. As H means maxwells per sq. cm. there are true maxwells in all this space around the wire, theoretically all the way to infinity.

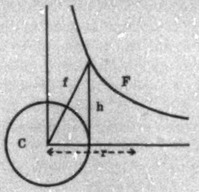

FIG. 6

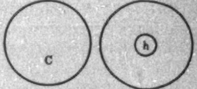

FIG. 7

But to combine all these maxwells with the gilberts and add all these energies together to get the total, as has been done by some (which of course gives infinite energy) is, in the opinion of the writer, neither justified nor correct. It is adding things again that have already been added. The magnetic pressure at the surface is already the sum (integration) of all those beyond, just as the pressure at the bottom of a wall of bricks is the sum or integration of all those at the individual layers above the bottom; it would surely not be correct to add together the total pressures at each layer of bricks to get the total at the bottom.

The stored energy is a function of this pressure; Maxwell's formulas give the pressure in dynes per square centimeter as $H^2/8\pi$ and the energy in ergs per cubic cm. as the same $H^2/8\pi$. Hence if it is wrong to add the pressures, it is equally wrong to add the energies deduced from them. If one calculates the energy from a *resultant*, one cannot again calculate it for all of the components which make up this resultant and then add them together (or integrate) to get the total. It is either one or the other but not both.

If, for instance, a rubber rod of a square centimeter section, were stretched say a centimeter, the stretching force would exist in all parts of its length, and energy would be stored in every centimeter of its length. But if it be then cut, and contracts for one cm., all its stored energy would be released in the cubic centimeter space in that cut, even though it had formerly been distributed along its whole length; it is the energy of the *resultant*; the cut could be made anywhere along its length with the same result. To then multiply the energy in that cubic centimeter by the length of the stretched rod to get the total stored energy while stretched, would evidently be wrong, as it would be far greater than the energy expended in stretching it. It would be adding again what had already been added in the resultant.

Maxwell's energy formula $H^2/8\pi$ per cu. cm., is, in the opinion of the writer, analogous to this case. It is not denied that this much energy could be released from a cubic cm., but the writer claims that it is energy based on a resultant, and if released, then other parts of the magnetic circuit that had contributed to it, would lose their energy also. What has misled us is that constant current has generally been assumed, in which case any energy taken out of a system is at once restored again by the source. In studying such fundamental cases one should conceive, as stated above, the conditions of zero resistance at which a current once started continues without a source of supply of new energy. If in such a case a part of the stored magnetic energy is set free, the current will fall as was described above. The squares of the currents will be proportional to the remaining energies.

The following analogy (though not quite perfect) will help to understand the physical nature of these magnetic stresses, and will perhaps help to clarify some disputed points. Let C Fig. 7 be the section of a long cylinder with a very thick wall and an infinitely small hole along its axis; let it be conceived to be made of something very elastic like rubber, in the sense that rubber is elastic. Let water be forced into the hole until it expands into the large hole h; the ends are closed, and may be neglected in considering a unit length in the middle.

The compressive and tensile strains or stresses in the wall around h will then be closely analogous to the magnetic strains or stresses in the ether around the outside of a straight conductor, h representing the conductor. They will be greatest at the surface of the column h, decreasing to practically zero on the outside surface of the rubber. If the mechanically elastic properties of the rubber were the same as what might be called the magnetically elastic properties of the ether, (Maxwell's statement to the effect that the tensile stress along lines of force is equal to the repulsion between like lines, seems to define these elastic properties), the analogy would be more perfect, as far as the stresses and energy are concerned; but it does not extend to the water, as the stresses in it are quite different from those in the inside of the conductor; nor does the conductor have to collapse to release the magnetic energy; it is the current in it which becomes zero when the field becomes zero (for zero resistance). A complete mechanical analogy does not seem to exist.

The energy stored in this stressed rubber per unit length is certainly *finite*, this cannot be questioned, and it is exactly equal to that which was required to force the water into the hole; it therefore cannot possibly be infinite. The radial pressure at the surface of the hole h is the *resultant* or summation of all the components beyond it; when it is allowed to act, as by letting it force out the water, the stresses in other parts disappear also, as they formed a part of the total. If the energy is set free by the action of the resultant of component forces, it evidently cannot then be set free again by the component forces also; either one or the other but not both, can act to set free the energy. To integrate these pressures in successive infinitely small, concentric layers would be summing up what had already been summed up before, hence would give erroneous results, far too great. Hence integrating the energies in such layers, based on the pressures existing there, would be equally absurd, yet this is what has been done in the electrical case as an alleged proof to show that the stored energy per unit length is infinite.

The above seems to show that, while there are lines of flux, maxwells, in the space surrounding a wire, it is not proper (as has been done) to combine all these maxwells with their magnetomotive forces, to get the energies represented by them, and then add all these energies to get the total. Hence it seems to be proper to apply to some of them the corresponding term used with amperes, and say they are *wattless* (to some extent) in the sense that they cannot all represent the energy attributed to them when considered individually. To do so would be to add to the energy of a resultant the energy represented by the components which make up this resultant. When a resultant exists, the energy in a system is that represented by this resultant, and no more.

Amperes are wattless when out of phase with their e. m. f. while maxwells are wattless when their energies are represented by, or attributed to, that of their resultant. The e. m. f. induced by the linkage or cutting of lines, is independent of whether they are wattless or not, as will be explained below.

There being no such thing as a magnetic insulator, the lateral spreading of flux due to the repulsion of like lines can be resisted only by other flux; the magnetic properties of iron are, of course, always excluded in a study of fundamental laws and properties. The radial magnetic pressure from the outside, at the surface of a round wire, must therefore be exactly balanced by an equal and opposite radial magnetic pressure from the inside, due to the lateral pressure of the inside flux. This is known to be the case and follows from what was shown in Fig. 6, in which the densities H just outside and inside of the surface, represented by the ordinate h, are equal, and therefore the pressures $H^2/8\,\pi$ in dynes per sq. cm. in opposite directions, are also equal and therefore in equilibrium. Hence during the steady state there is no resultant radial pressure between these two at this surface, though this surface, considered as a thin layer, may be said to be compressed magnetically as though between the jaws of a vise. This is confirmed by the Northrup formula in which the mechanical radial pressure at the surface, when $r = R$, is zero. This has been the subject of some discussion, as it was thought that the tensile stress of the outside lines ought to exert a radial mechanical pressure from the outside. When a weight is supported on a hand there is no resultant force which can do any work, the upward and downward forces being equal. But when the upward pressure is reduced by only the slightest amount, the weight will descend and do work equal to the weight multiplied by the distance. Similarly when the pressure in the inside of a conductor is slightly relieved, as for instance if it were a liquid, part of which was forced out, then the outside pressure will act. The conditions are somewhat analogous to those in a lever.

The inward pressures from the outside flux are balanced by the outward pressures of the inside flux; hence the former are represented by the latter, and are transferred to that flux, and it is in this interior flux that the conductors tend to move to the center, thereby exerting the pressures in the Northrup formula. It is for this reason that the mechanical pressures on the inside, and therefore the energy deduced from them, are based on the total pressures, inside and outside, and therefore represent the total energy.

Maxwell's $H^2/8\,\pi$ Pressures. Doubt has often been raised as to whether these $H^2/8\,\pi$ pressures and tensions are really in dynes per sq. cm. for if they are, they are real, ponderomotive or mechanical pressures, and not mere mysterious magnetic pressures. The fact that these magnetic pressures exist radially on the outside of a conductor though there does not seem to be any mechanical force there, under stable conditions, has been explained above. Northrup's experimental measurements of the actual mechanical forces in the interior of a liquid conductor, produced by these theoretical electromagnetic forces, demonstrated that the latter are correctly expressed in dynes per sq. cm. Hence these $i^2/2\,\pi\,R^2$ radial pressures from the outside are true mechanical pressures provided they are properly interpreted.

To be mechanical these $H^2/8\,\pi$ stresses must of course act on some mechanical abutment; the ether cannot act as one. The lack of proper consideration of such abutments no doubt has given rise to much confusion; they do not appear as factors in calculus calculations. The two unlike poles of nearby magnets are the abutments for the tension of the lines between them. The only abutment of the endless lines in space encircling a conductor is, of course, the surface of the material conductor against which they exert radial pressures, as a stretched rubber band would do.

In the writer's opinion this $H^2/8\,\pi$ expression can be applied correctly to parts of fields only when the density is uniform, which it can be only when the lines are straight and parallel; the pressures of none of them are then resultants of the pressures of others. Even then they cannot be added, as by integration, any more than one could add together the pressures at successive layers of a confined compressed gas. When the lines are curved, those on the inside of the curve support the pressures of those outside of it besides their own, as superimposed rubber bands would; hence to add them, or the energies deduced from them, by integration, would be adding again what had already been added. In differentially small parts of fields uniform density may no doubt be assumed. It seems that the $H^2/8\,\pi$ pressures and tensions in the differentially small layer next to the surface of the conductor, which forms their only abutment, can safely be taken as true mechanical stresses in dynes per sq. cm. but those radially beyond must not be added to them.

INDUCTION

Attention is called to the fact that the e. m. f. induced by cutting or linking a definite number of lines of flux per second is entirely independent of the energy inherent in that flux; hence conversely it is not proper to draw any conclusions as to the energy in that flux from this induced e. m. f.; yet this has been done.

Imagine a field between two parallel rectangular iron pole faces; let there be F lines in this field and let it be cut by or linked with, a moving circuit, in a given time. Now let the pole faces be doubled in length or width but contain the same number of lines F and be cut or linked in the same time as before. The induced e. m. f. will be exactly the same, yet these same lines in the first case will have four times the energy in them that they had in the second case, as their density H is twice as great. Yet in the case of self-inductance, from the counter e. m. f. induced by a rising current, very positive conclusions have been drawn concerning the energy inherent in the self-produced flux lines that have been cut or linked.

This, in the opinion of the writer, is not justified for the reasons just given, even though it is generally accepted and important conclusions are based on it. It seems that this is partly the explanation of the disagreement between the writer's results and those based on mere "postulates" and on some of our older theories.

It is true that in the case, for instance, of a large single-turn circuit, without iron, of course, the ampere-turns are the magnetomotive force of the flux circuits around the wire and linked with the loop; also that the energy of flux is proportional to the product of the maxwells and the ampere-turns; but in the opinion of the writer the latter is not true when the energy in some of the flux lines is the resultant of the energy of some of the others, that is, when the pressures of some lines are components of the pressures of others, hence some of them must be considered as wattless in the sense above described; if not, as explained above, it would be adding again what had already been added before.

In a very large single-turn circuit, for instance, the flux encircling a unit length of circuit would not differ greatly from that around a unit length of the fundamental straight wire, in that the flux distant from the wire exerts a pressure on that nearer to it, and that the pressures of those lines nearer the wire are therefore resultants of those beyond. The e. m. f. induced by the linkages or the cutting of such wattless lines is the same as with the others, but the energies represented by them are not. Hence it does not appear to be justifiable to draw such definite conclusions as have been drawn about the energy of the flux lines, from merely the e. m. f. induced by them, and the m. m. f.

SELF-INDUCTANCE

In the writer's opinion this term self-inductance has been used in two different senses, one when applied to energy and the other when applied to the induction of a counter e. m. f. in its own circuit when its own current varies.

When the energy per unit length of a circular circuit of, say, one turn, is calculated (as it generally and perhaps always has been) from the self-inductance as determined from the usual formulas, it becomes larger as the radius increases and is infinitely large for a straight conductor considered as a circle of infinite radius. But all self-inductance formulas for such circles are only approximate (some factors having been dropped as too small), and seem to involve empirical constants. To extrapolate an approximate formula to infinity is evidently not rational. Moreover in extrapolating even a rigidly exact expression to infinity the result may sometimes have been interpreted as infinity when it really is an indeterminate; a suppressed or dropped factor may make this difference.

Hence the explanation of the disagreement of the writer's result with that obtained in the older way from such formulas, is better looked for in the latter which involves infinities, as the method used in the former is mathematically rigid, and involves no infinities or indeterminates. These self-inductance formulas have no doubt been tested experimentally and have been found to be (or have been adjusted to be) correct to limited extents, but this applies only to the range of circuits tested, beyond which it is still an extrapolation, hence involves the pitfalls of that process. Within the ranges tested, the self-inductance per unit length no doubt increases when the diameter of the circuit increases, but the writer maintains that when the circles become very large it must start to decrease again reaching the above theoretical limit for a straight conductor. It is of interest that for the limit in the other direction, the smallest possible circuit (the torus referred to in the beginning of this paper), the usual formulas approach quite closely to the above theoretical limit.

If a flexible wire is curved, a current in it will tend to straighten it; this is due to the greater pressure of the disks of flux (due to a greater density H) against each other on the inner side of the curve; or to the longitudinal force which tends to stretch and therefore straighten it; Maxwell's pressure formula $H^2/8\,\pi$ shows this greater pressure, the abutments of the force being of course in the wire as the ether cannot act as an abutment. By this straightening it expends a part of its stored energy; hence it must have less when straight than when curved as claimed above, even though according to the approximate self-inductance formulas (though beyond their tested ranges) it ought to have more then. As this would take place with zero resistance, when a current once started will continue to flow without connection to any source, this straightening is done at the expense of its stored energy and it is therefore not a case under the Kelvin law, there being no source to supply additional energy.

Even for very large circuits the energy calculated from these approximate formulas is not enormously greater than the writer's small result and it is *nowhere near infinity*; hence it would take but a small change of curvature of the curve to make them correspond for a straight wire or a very large circle. In a small circuit the "return" part of the circuit tends by induction to *increase* the current in the other part; by the very definition of the fundamental conductor, this kind of induction no longer exists. As a circuit becomes larger this induction may become larger due to increasing length, but it diminishes as the square of distance, hence must *decrease* as the circuit becomes larger.

The error pointed out above, due to adding the pressures of components to those of their resultant which already contains them, seems to grow less as coils become smaller and especially when they are solenoids; it may, therefore, be small in the coils used in practise, which may explain why it was not noticed before. It may nearly disappear in the torus (a solenoid with a circular axis) in which the field is nearly uniform. When quite uniform this error naturally disappears.

Self-inductance is usually defined as flux per ampere, hence as a *derived* quantity and not as an independent one, which it seems to be in the energy sense. This definition makes it dependent on only the flux and the current, and nothing else. It is known, however, that in practise it is dependent also on such factors as frequency, shape, skin effect, phase difference, etc. From the physical dimensions it is true that flux divided by current is a length; and in the fundamental conductor the "resultant" flux (in the energy sense) is the product of the current and the length of its path in c. g. s. units.

A resistance (which is physically an independent quantity) was formerly considered to be defined correctly as a derived quantity, namely volts per ampere, but it is now known that although this is sometimes correct (for d. c.), it may be so very wrong (when the volts and the amperes are not in phase) that this derived quantity was given a special name, impedance. The wattless ampere resulted from this distinction. Today we can realize the confusion and misfits that must have existed before this distinction was recognized by Kennelly over 30 years ago. But the true quantity, called self-inductance, is a purely geometric one, a length, hence is really an independent quantity, like a resistance, independent of any flux or current. Why should it be defined then as *derived* from these? The only reason seems to be, that like in the case of resistance this quotient *sometimes* gives the correct result. But again, as in the case of resistance as a derived quantity may this not sometimes give incorrect results; may there not be a difference between two self-inductances analogous to that between resistance and impedance; and if so, would this not lead to wattless flux (as described above) just as the difference between resistances and impedance has led to the wattless ampere?

The counter e. m. f. in self-inductance induced by the self-generated flux of a varying current depends only on the number of lines cut or linked and is independent of whether they are wattless or not, as was explained above. The lines in the interior, however, can have but a fractional value of those outside, as they do not cut the entire conductor.

The writer believes that the disagreement between his result and that deduced by the older views, is due largely to the fact that self-inductance had been defined as a derived quantity and that a similar distinction should be made as was done in the case of resistance. That kind of self-inductance which it is always correct to use in energy calculations, should be distinguished from the kind which it is always correct to use in inductance and for the latter the term self-inductance might well be retained as it describes it correctly. Sometimes these two self-inductances may be equal, as in the case of resistance and impedance, but sometimes they may differ, and then some wattless factor is involved. There seems to be nothing unreasonable in this.

THE LONGITUDINAL FORCE

In the second proof above it was assumed that the radial forces, (which, being perpendicular to the conductor are in accordance with past teachings and therefore have been accepted), acted hydraulically to produce the axial forces which lifted the weight. For many years[9] the writer has maintained (as Ampere did) that there exists also a force in the direction of the axis, hence a longitudinal force, which, acting in both directions, he termed a "stretching" force tending to elongate the conductor. Maxwell in Art. 526 discusses it briefly and does not there deny its existence. The writer showed by experiments[10] that this force acted also in solids, hence can not be only hydraulic; also that theory demanded it; the newer electronic theories also seem to demand it. This force is due to the mutual repulsion of the lines of force encircling a conductor, which forces have their anchorage or abutments in the material of the conductor, the only place where they could be. The existence of this force, which cannot be denied, is now receiving more general acceptance.

In the opinion of the writer the recognized perpendicular force and this disputed longitudinal one, are in fact really the same force, as the ether seems to act like a liquid or gas, in the sense that a magnetic force exerted on it in any direction, acts in all directions, as a mechanical force does in liquids and gases, in which the parallelogram of forces no longer applies. Maxwell's conclusion that the tension along lines is numerically equal to the repulsion between them, seems to bear this out.

AMPERE'S METHOD REVIVED

That great mathematical physicist, Ampere, endeavored to develop a mathematical treatment of electric circuits based on straight elements of conductors. Maxwell (Art. 526) did not approve it and preferred his complete circuit system, which has been very useful; but it has unfortunately led us away from the Ampere system, the development of which has been neglected. Ampere had not the advantage of knowing anything about the more recently discovered internal stresses in a conductor (the so-called pinch effect), which have led to the above basic constant, and which, together with the Ampere system, leads to a possible mathematical treatment of electric circuits, which while not replacing the Maxwell system, should be developed as an alternative system to use when the Maxwell system is inadequate, or misleads (due sometimes to infinities) or fails to disclose important factors, such as this constant and the wattless elements. Many interesting and useful developments seem to be possible now in such a system of mathematical treatment in which the fundamental is the single conductor (Ampere's method), in place of the complete circuit of Maxwell, and some of the results in the former could not have been deduced from the complete circuit system.

CONCLUSIONS

In the opinion of the writer, the above proofs, deductions, and discussion, will show that a new system of treatment of electrical problems can be based on the single, straight conductor, as distinguished from that based on the Maxwell complete circuit; not to replace the latter system but to supplement it, and to test the correctness of parts of it. The single conductor system leads to some new and useful results and shows that we should modify some of our former conceptions; also that in flux lines there is an analogy to the wattless ampere which ought to be recognized. It also shows that the term self-inductance has been used in a dual sense and that a distinction should be made analogous to that between resistance and reactance. Some heretofore unknown and useful relations have been deduced from what is believed to be one of the most fundamental constants in electrodynamics, the value of which is determined by simple proofs. Some of the results deduced could not have been deduced from the complete circuit system, which has, in some cases, been misleading.

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[9] "The Stretching of a Conductor by Its Current." *Jour. Frank. Inst.*, Jan. 1911, p. 73. Also other later papers.

[10] *Electromagnetic Forces*, TRANS. A. I. E. E., Vol. 42, 1923, p. 311, see more particularly Figs. 6 and 9.

Discussion

S. L. Quimby: In the classical electromagnetic theory, self-inductance is specifically defined and invariably calculated as a property of a complete electric current circuit. No theory consistent with the facts revealed by experiment has yet been developed to make possible the calculation of the self-inductance of any portion of a circuit.

I now propose to analyze the two experiments described by Dr. Hering and to point out the way in which the energy relations derived by him differ from those which are in accord with the electromagnetic theory.

I shall first consider the second of these experiments. Dr. Hering bases his analysis of this upon a theorem ascribed to Lord Kelvin. This theorem states that if any system of current circuits in which the currents are kept constant by batteries is allowed to alter its configuration, then, of the work done by the batteries in keeping the currents constant, one-half will appear as increased magnetic energy of the system of currents and one-half as mechanical work.

Fixing our attention upon Dr. Hering's apparatus, we imagine that, initially, the weight is locked rigidly in place and furthermore that the whole conductor is perfectly rigid. A current flowing in this conductor now supplies us with a definite system of current circuits, namely, the very large number of filamentary conductors into which the finite conductor may be imagined to be divided. With the weight still locked in place, we now suppose that the conductor is no longer rigid, but elastic. Under the action of the pinch forces, it will shrink, radially. Its self-inductance will therefore increase by an amount ΔL, and the magnetic energy of the system will increase by an amount $1/2\ .\ \Delta L\ .\ i^2$. At the same time, a certain amount of mechanical energy is stored in the elastically strained conductor. Kelvin's theorem tells us that this energy of elastic strain is precisely equal to the increased magnetic energy. The radial displacement of the material of the conductor accompanying the shrink involves a cutting across the lines of magnetic force in the conductor, and the counter e. m. f. thereby developed accounts for the work done by the battery in supplying the elastic energy.

This is as far as Kelvin's theorem carries us in our analysis, for, if we now release the weight, it will move and this motion will cause the introduction of *new conducting material into the circuit.* We no longer have a definite system of conductors to consider but instead a system which is continually changing through the continual introduction of new conductors. Kelvin's theorem can no more be applied to an analysis of this machine than it could to an analysis of an ordinary d-c. motor, in which, barring resistance, all the energy supplied by the battery appears as mechanical work and the magnetic energy remains practically constant; and this for the same reason, since the commutator on a d-c. motor is merely a device for continually introducing new conducting circuits into the system.

The introduction of new mercury into the conducting system involves a radial flow across the lines of force in the conductor and it is easily shown that the work done by the battery against the counter e. m. f. thereby developed is precisely equal to the mechanical energy acquired by the raised weight.

Dr. Hering's analysis of his first experiment is based upon an assumption which, in the light of the electromagnetic theory, is false. He assumes that if the pinch pressure is allowed to annihilate a fluid-conducting circuit of the type described, then the mean pressure, *i. e.*, the pressure at a distance $R/\sqrt{2}$ from the axis of the conductor, will be constant. This, as the author shows, is equivalent to the assumption that the current will decrease linearly with the radius. Calculations based upon the electromagnetic theory show that under the conditions of the experiment the variation of the current with the radius of the conductor as it collapses would be given by the expression

$$I = [\log (2\,p\,l/R)]^{-1}$$

where

$$1/p = \log^{-1} (3/4),$$

and

l = the length of the conductor.

In conclusion it may be observed first, that the classical electromagnetic theory has never concerned itself with the calculation of the self-inductance of, nor the energy associated with, a unit length or any other portion of a current circuit, and second, that the argument by which the author arrives at his value of these quantities is not consistent with that theory.

A word should be said on behalf of the self-induction formulas supplied by the electromagnetic theory. Where these are calculated for the simple circuits, of whatever size, referred to by the author, they do not necessarily contain approximations, never involve empirical constants, and are never derived by extrapolation.

The author, if I understand him correctly, maintains that the process of evaluating the total magnetic energy in a certain space by integrating the Maxwellian energy density $H^2/8\pi$ throughout that space, is valid only when the magnetic field is uniform in direction. It seems to me that this criticism lacks weight. Consider, for example, a circuit consisting of a straight cylindrical conductor with a return through a concentric cylindrical conducting sheath. I doubt very much whether Dr. Hering will deny that the energy associated with such a current circuit can be obtained correctly by integrating $H^2/8\pi$ throughout the volume enclosed by the sheath. And yet this is precisely the type of field for which he denies the validity of this procedure. The argument which he uses in support of his contention is not, in my opinion, a good one. To adopt one of his own analogies, it is quite true that the pressure at the bottom of a column of bricks cannot be obtained by adding together the pressures at the bottoms of successive strata, but the total potential energy of the column most certainly can be calculated by summing up the separate potential energies of the different strata.

B. A. Behrend (communicated after adjournment): Dr. Hering's paper is a critical review of fundamental electrical principles and an attempt is made to call attention to, and suggest a different attitude toward, certain phenomena and experiments which in the conventional treatment appear distinctly awkward.

It has been known, of course, to the earnest and profound student, that unipolar or homopolar induction and allied phenomena strain somewhat the fundamental simplicity of Faraday's law of induction as given to us in its mathematical form by Maxwell. But Maxwell and his interpreters were aware of this, as was pointed out forty years ago by Oliver Lodge in his "Modern Views of Electricity." I discussed some of these difficulties with Prof. Lodge about 1890 and retain his pencil memoranda on the subject.

Attempts to compare the fundamental phenomena of electromagnetism to related phenomena in hydrodynamics and the theory of elasticity have proved useful. Temperature and electric and magnetic potential and the velocity potential appear as complete mathematical parallels. Lord Kelvin and Professor P. G. Tait have established the identity of mathematical conditions in St. Venant's torsion problem and a hydrokinetic problem.[1] Professor A. E. H. Love has also worked out the hydrodynamical analogy in which the problem of torsion of a twisted prism is compared with that of a frictionless liquid in a rotating cylindrical vessel or with liquid circulating with uniform spin in a fixed cylindrical vessel coinciding with the surface of the twisted prism.[2]

What I propose to point out here is that a similar complete analogy exists between the electromagnetic phenomena cited by Dr. Hering and the phenomena of torsion of prisms as treated by de St. Venant. The value of such analogies lies in creating mental pictures and substituting for unfamiliar problems those to which we have grown accustomed.

So far as I understand the Faraday-Maxwell electromagnetic theory and the ideas recently propounded in several papers before the Franklin Institute and this Institute by Dr. Hering, I daresay that Dr. Hering's views, far from being contradictory of the conventional theory, give an additional physical interpretation and expound some of the obscurities and difficulties of the traditional theory. Guided by his own views, Dr. Hering has made some valuable inventions which in themselves speak well for his point of view, and it seems as though both Bourbon and tyro of our art might well lend an open mind to these papers and to Dr. Hering's interesting and thoughtful experiments.

R. P. Jackson (communicated after adjournment): I was much interested in the statements of Dr. Hering in the February JOURNAL, page 123, concerning comparisons of the inductance of a circular circuit with that of a straight conductor of equal length. Some years ago I made an investigation of similar character and came to approximately the same conclusion.

At that time, I was investigating the requirements of conductors to and from lightning arresters. A great deal has been said about the increased impedance of bends and angles in such conductors and of the necessity of maintaining straight lines or easy curves and avoiding sharp bends. A little investigation demonstrated that as soon as you begin to curve a conductor away from a straight line you begin to reduce its inductance per unit length and that a single-turn loop could not possibly have as much inductance as the same conductor run in a straight line, far from any return circuit.

This conclusion, of course, does not entirely remove the objection to curves and bends in lightning conductors for the simple reason that a straight line is the shortest distance between two points. For that reason, being the shortest conductor, one following a straight line will probably have less inductance than any other conceivable path would afford.

The presence of necessary angles and bends is harmful only in so far as the total length of the conductor is increased and their harmful effect may be much less than ordinarily assumed, providing, however, the conductor itself is of proper section and shape.

It is apparently more important to avoid single, small cylindrical conductors than it is to fear convenient and necessary curves and bends in connection with high frequencies. When there is more than one turn, however, the condition is entirely changed as the same flux threads through the various turns and there is the well known approximation, depending on the relative position of the turns, for the inductance to increase with the square of the number of turns.

(EDITOR'S NOTE: Following the presentation of the paper, M. F. Skinker read from an article by Leigh Page, published in the *Journal* of the Franklin Institute for February, 1926, page 245, which article discussed a former article by Dr. Hering in the same *Journal*. Dr. Hering's reply to this is published in the April issue of that *Journal*, page 497.)

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[1] Treatise of Natural Philosophy, by Kelvin and Tait, part II, p. 242.

[2] A. E. H. Love, Treatise on the Theory of Elasticity, Vol. I, p. 158 et. seq.

Heaviside's Proof of His Expansion Theorem

BY M. S. VALLARTA[1]

Associate, A. I. E. E.

Synopsis.—*Heaviside's proof of his celebrated Expansion Theorem, found scattered in his "Electrical Papers," is reconstructed. It is based upon his so-called "conjugate theorem" also discovered independently by Routh, which establishes a relation between any two normal modes of oscillation of a dynamical system. Heaviside's argument applies to systems having finite number of degrees of freedom and no repeated or null roots of the determinantal equation of the system. The relation between Heaviside's, Carson's and Wagner's proofs is also pointed out.*

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LET an arbitrary electric network be given and suppose a unit constant voltage is impressed at a certain instant. Then the determination of the d-c. transient and of the steady state which follows consists in finding an integral of a linear differential equation of the nth order, with constant coefficients and satisfying n initial conditions. By ordinary methods the determination of the arbitrary integration constants using the initial conditions is, as a rule, extremely laborious, except perhaps in the simplest type of circuits; therefore, a formula giving directly the transient current, without requiring the determination of integration constants, would mean a considerable advance in circuit theory. If, further, it is realized that when the d-c. transient and the corresponding steady state are both known, the behavior of the network under all common types of impressed voltages is also known[2], the importance of such a formula becomes at once evident.

In his celebrated Expansion Theorem, Oliver Heaviside gave just such a formula. In accordance with his somewhat eccentric habits of thought, he merely wrote down his final result on page 127 of the second volume of his "Electromagnetic Theory," without giving a proof, even without the slightest hint to his previous investigations leading to this result. This seems to have eventually given rise to the impression that Heaviside did not prove his Expansion Theorem, but rather found it in some obscure fashion, peculiar to him, by considerations which might not be of the nature of a logical proof. Thus J. R. Carson in his latest paper on circuit theory[3] writes that "the Expansion Theorem was stated by Heaviside without proof; how he arrived at it will probably always remain a matter of conjecture," while K. W. Wagner[4] writes, "Heaviside gibt die Formel ohne Beweis, ja selbst, ohne einen solchen anzudeuten" (Heaviside gives the formula without proof; what is more, without giving an indication of one) and exactly the same view is expressed in a recent paper by L. Casper[5]. Now, although no proof of the Expansion Theorem, and even no indication of such, is given in the "Electromagnetic Theory," numerous scattered investigations which might lead up to this remarkable formula are found in Heaviside's "Electrical Papers" and also in many of his published writings; therefore, the possibility that he might have given a proof of his Expansion Theorem was recognized by some writers, among them V. Bush[6], L. F. Woodruff[7] and the present author[8]. It was not until quite recently, however, that a reconstruction of Heaviside's own proof of his Expansion Theorem was possible.

The first to examine Heaviside's investigations leading to the Expansion Theorem and to recognize their interest, extreme generality and correctness, was T. J. Bromwich[9], who, in an exhaustive paper published in 1916, but read in 1914, not only gave an independent derivation of the Expansion Theorem, similar to Wagner's[10], but also showed with extreme clearness the connection between Heaviside's argument and those methods of derivation which, like Bromwich's and Wagner's, make use of contour integrals to solve the dynamical equations.

Heaviside's derivation of his Expansion Theorem is based on the so-called "conjugate theorem" which we now proceed to establish. The conjugate theorem was probably first discovered by Routh[11], but Heaviside was undoubtedly unaware of Routh's results, his methods are completely independent and, further, his results are easier to state than those of Routh.

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[1] Massachusetts Institute of Technology, Cambridge, Mass.

[2] J. R. Carson, *Theory of the Transient Oscillation of Electrical Networks, etc.* TRANSACTIONS A. I. E. E., p. 346, 1919.

[3] "Electric Circuit Theory and Operational Calculus." *Bell System Technical Journal*, Vol. IV, p. 685, Oct. 1925; see p. 713.

[4] "Uber eine Formel von Heaviside zur Berechnung von Einschaltvorgänge," *Archiv für Elektrotechnik*, Vol. 4, p. 159, 1916.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

[5] "Zur Formel von Heaviside für Einschaltvorgänge," *Archiv für Elektrotechnik*, Vol. 15, p. 95, 1925.

[6] "Heaviside's Operational Calculus," Mimeographed notes for use of students at Massachusetts Institute of Technology, p. 37, July, 1925.

[7] "Principles of Electric Power Transmission and Distribution," p. 237, footnote, Wiley, New York, 1925.

[8] "Notes on Heaviside's Operational Method," Mimeographed notes for use of students at Massachusetts Institute of Technology, p. 17, March, 1923.

[9] "Normal Coordinates in Dynamical Systems," *Proceedings of the London Mathematical Society*, Vol. 15, p. 401, 1916. See pp. 415-420.

[10] Wagner's proof of the Heaviside theorem appeared as a part of his paper already quoted (l. c. footnote 4) on March 21, 1916. Bromwich's paper was read on March 12, 1914, received April 22, 1916, published in 1916.

[11] "Rigid Dynamics," Vol. 2, Articles 383, 384; 1892.

Consider an electric network of n degrees of freedom (n currents required to specify the circuit completely at any one instant). It is well known that such a network is the exact analogue of a dynamical system of n degrees of freedom in which the forces of reaction are proportional to the displacements from equilibrium or zero configuration (*i. e.*, elastic forces of Hooke type) and the forces of resistance are proportional to the velocities. In this electromechanical analogy, inductance is equivalent to coefficient of inertia, the reciprocal of capacity to the elastic coefficient and resistance to the coefficient of dissipation or frictional coefficient; electromotive force corresponds to applied force, current to velocity and charge to displacement. This fact has been already utilized in the analysis of circuit problems by mechanical models and conversely by Doherty and many others.

The equations of motion of a dynamical system of the type specified above are well known to be[12]:

$$\left.\begin{array}{l} \left(C_{11}\dfrac{d^2}{d\,t^2} + B_{11}\dfrac{d}{d\,t} + A_{11} \right) x_1 + \;.\;.\;.\;. \\ \qquad + \left(C_{1n}\dfrac{d^2}{d\,t^2} + B_{1n}\dfrac{d}{d\,t} + A_{1n} \right) x_n = F_1 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \left(C_{n1}\dfrac{d^2}{d\,t^2} + B_{n1}\dfrac{d}{d\,t} + A_{n1} \right) X_1 + \;.\;.\;. \\ \qquad + \left(C_{nn}\dfrac{d^2}{d\,t^2} + B_{nn}\dfrac{d}{d\,t} + A_{nn} \right) x_n = F_n \end{array}\right\} \quad (1)$$

In this system of linear differential equations, $C_{\kappa\kappa}$, $B_{\kappa\kappa}$ and $A_{\kappa\kappa}$ are the coefficients of inertia, dissipation and elasticity of the kth branch, x_κ the displacement of the kth branch and $C_{\kappa j}$, $B_{\kappa j}$, $A_{\kappa j}$ the coefficients of inertia, dissipation and elasticity between the kth and the jth branches. $F_1, F_2 \;.\;.\;.\;. F_n$ are the external forces acting on the corresponding branches.

Equations (**1**) form the starting point of Heaviside's argument[13]. On account of the precise analogy between the electric network and the mechanical system, it is immaterial whether use is made of electrical or mechanical entities and it is permissible to change from the network to the mechanical system if only the right equivalents are used. Let Q be the rate at which energy is dissipated in the system, due to the presence of frictional forces in it, U the potential energy of the elastic forces and T the kinetic energy. The power transferred to the system by the applied forces is $\sum\limits_{k=1}^{n} F_\kappa\, \dot{x}_\kappa$ ($\dot{x}_\kappa = d\,x_\kappa/d\,t$), so the conservation of energy gives

$$\sum_{k=1}^{n} F_\kappa\, \dot{x}_\kappa = Q + \dot{U} + \dot{T} \qquad (2)$$

which says that the power delivered by the applied forces is equal to the energy dissipated, plus the rate of increase of the kinetic energy of the system, plus the rate of increase of the stored elastic energy. Q, U and T are defined in terms of the variables $x_1 \;.\;.\;.\;. x_n$ which determine the state of the system at a given instant by the expressions,

$$\left.\begin{array}{lll} 2\,T &= \sum\limits_{i,j=1}^{n} C_{ij}\,\dot{x}_i\,\dot{x}_j & (C_{ij} = C_{ji}) \\ Q &= \sum\limits_{i,j=1}^{n} B_{ij}\,\dot{x}_i\,\dot{x}_j & (B_{ij} = B_{ji}) \\ 2\,U &= \sum\limits_{i,j=1}^{n} A_{ij}\,x_i\,x_j & (A_{ij} = A_{ji}) \end{array}\right\} \quad (3)$$

The symmetrical conditions imposed on the coefficients $A_{ij} = A_{ji}$, $B_{ij} = B_{ji}$, $C_{ij} = C_{ji}$ mean that there are no forces of gyrostatic origin, also no forces not derivable from a potential-energy function. Such relations are also assumed to be satisfied in the corresponding electric network, thus excluding thermionic devices, etc.

It is well known that the reduced system of differential equations obtained from equations (**1**), *i. e.*, the system obtained by putting all the impressed forces equal to zero, admits an exponential solution of the type $x_\kappa = X_\kappa\, e^{pt}$, X_κ and p being real, imaginary or complex constants to be determined. Such solutions are called the *normal* solutions. The substitution $x_\kappa = X_\kappa\, e^{pt}$ transforms (**1**) from a system of linear differential equations with constant coefficients, with time as independent variable, to a system of algebraic equations with p as unknown. It is further known that the condition that the system in question have a solution is that the determinant of the coefficients shall vanish. This determinant is defined by

$$D(p) = \begin{vmatrix} A_{11}+B_{11}\,p+C_{11}\,p^2 \;.\;.\;. & A_{1n}+B_{1n}\,p+C_{1n}\,p^2 \\ \cdots\cdots\cdots & \cdots\cdots\cdots \\ A_{n1}+B_{n1}\,p+C_{n1}\,p^2 \;.\;.\;. & A_{nn}+B_{nn}\,p+C_{nn}\,p^2 \end{vmatrix} \qquad (4)$$

$D\,(p) = 0$ is then known as the determinantal equation of the system. If p_κ is a root of $D\,(p) = 0$, p_κ also satisfies the original equations (**1**). For each root $p = p_\kappa$ of the determinantal equation, suppose we find a solution, $l_1, l_2, l_3 \;.\;.\;. l_n$ for the equations,

$$\left.\begin{array}{l} (A_{11} + B_{11}\,p_\kappa + C_{11}\,p_\kappa^2)\,l_1 + \;.\;.\;. \\ \qquad + (A_{1n} + B_{1n}\,p_\kappa + C_{1n}\,p_\kappa^2)\,l_n = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ (A_{n1} + B_{n1}\,p_\kappa + C_{n1}\,p_\kappa^2)\,l_1 + \;.\;.\;. \\ \qquad + (A_{nn} + B_{nn}\,p_\kappa + C_{nn}\,p_\kappa^2) = 0 \end{array}\right\} \quad (5)$$

If then the initial displacements u_κ and the initial velocities v_κ are adjusted so that

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[12] For a full discussion of these equations and their derivation see for example Whittaker's "Analytical Dynamics," p. 177, Cambridge, 1917.

[13] "Electrical Papers," Vol. 2, p. 202. First published in his paper "On the Self-Induction of Wires," Parts 3 and 4, Philosophical Magazine, Vol. 22, pp. 332-352 and pp. 419-442, October and November, 1886; see p. 335 and p. 426.

$$\left.\begin{aligned} \frac{u_1}{l_1} = \frac{u_2}{l_2} = \ldots = \frac{u_n}{l_n} = A \\ \frac{v_1}{l_1} = \frac{v_2}{l_2} = \ldots = \frac{v_n}{l_n} = A\,p_K \end{aligned}\right\} \quad (6)$$

a possible solution of the system (**1**) is given by,

$$x_1 = A\,l_1\,e^{pKt}\ x_2 = A\,l_2\,e^{pKt}\ x_n = A\,l_n\,e^{pKt} \quad (7)$$

that is, by placing,

$$X_1 = A\,l_1, \quad X_2 = A\,l_2 \quad X_n = A\,l_n \quad (8)$$

The constant A therefore fixes the amplitude of the displacement. Further, on account of the linearity of the equations, $x_r = \sum_K A\,l_K\,e^{pKt}$ is also a solution, the summation being over the roots of $D(p) = 0$. But it is by no means evident that the $2\,n$ equations (**6**), which determine the amplitude of the normal solutions, are themselves algebraically capable of solution; in fact examples can be given where the normal solutions are not algebraically independent. The most direct proof that the constants A can be found is to express them in terms of the initial values of displacements and velocities. This is done by the Heaviside-Routh conjugate theorem.

Coming back now to Heaviside's argument, suppose that all the impressed forces vanish, so that no energy can be transferred to the system, while the stored energy, due to the elastic connections of the system, is dissipated irreversibly through frictional forces. Let p_1 and p_2 be any two different roots of $D(p) = 0$, satisfying equations (**1**), so that $x_1 = X_1\,e^{p_1 t}$, $x_2 = X_2\,e^{p_1 t}$ are solutions of (**1**). Let further Q_1, U_1, T_1 be the dissipation, kinetic and potential-energy functions corresponding to the normal solution p_1 *i. e.*, to $x_1 = X_1\,e^{p_1 t}$ . . . . . . $x_n = X_n\,e^{p_1 t}$. Then the conservation of energy gives

$$Q_1 + \dot{U}_1 + \dot{T}_1 = 0 \quad (9)$$

but since

$$\dot{U}_1 = \frac{1}{2}\,p_1 \sum A_{ij}\,x_i\,x_j + \frac{1}{2}\,p_1 \sum A_{ij}\,x_i\,x_j = 2\,p_1\,U_1 \quad (10)$$

and, likewise,

$$\dot{T}_1 = 2\,p_1\,T_1 \quad (11)$$

therefore,

$$Q_1 + 2\,p_1\,(U_1 + T_1) = 0 \quad (12)$$

In the same way, let Q_2, T_2, U_2 be the dissipation, kinetic and potential-energy functions corresponding to the normal solution p_2, *i. e.*, to $x_1' = X_1\,e^{p_2 t}$ . . . . $x_n' = X_n\,e^{p_2 t}$. We have, precisely as above,

$$Q_2 + 2\,p_2\,(U_2 + T_2) = 0 \quad (13)$$

Now, since p_1 is a root of the determinantal equation, and p_2 is also a root of the determinantal equation, therefore x_1, x_2 . . . x_n and x_1', x_2', ... x_n' are both solutions of the original system of equations (**1**). If now, following a nomenclature suggested by Bromwich, we define the relative kinetic energy, the relative potential energy and the relative dissipation of the two normal modes of oscillation p_1, p_2, by means of the expressions

$$\left.\begin{aligned} \frac{1}{2}\,Q_{12} &= \sum_{i,j=1}^{n} B_{ij}\,\dot{x}_i\,\dot{x}_j' \\ U_{12} &= \sum_{i,j=1}^{n} A_{ij}\,x_i\,x_j' \\ T_{12} &= \sum_{i,j=1}^{n} C_{ij}\,\dot{x}_i\,\dot{x}_j' \end{aligned}\right\} \quad (14)$$

we find again from the conservation of energy that

$$\frac{1}{2}\,Q_{12} + p_2\,U_{12} + p_1\,T_{12} = 0,\ \frac{1}{2}\,Q_{12} + p_1\,U_{12} + p_2\,T_{12} = 0 \quad (15)$$

whence, by subtraction

$$(p_1 - p_2)\,(U_{12} - T_{12}) = 0 \quad (16)$$

and therefore, $U_{12} = T_{12}$, which is the so-called conjugate theorem. It says in words that the relative kinetic energy of two normal solutions (two normal modes of motion) is equal to the relative potential energy of these two normal solutions. It is to be noted quite carefully that the above reasoning holds only for the case of different roots, $p_1 \neq p_2$.

Heaviside also shows that the conjugate theorem holds for the electromagnetic system expressed by Maxwell's equations. It is to be observed in this connection that the equations (**1**) which correspond, as already explained, to the electric network, are consistent with the Maxwell circuital laws provided only that the displacement current is negligible compared to the conduction current[14], therefore any argument based on these equations applies to the dynamical system under consideration, but not conversely. The proof is quite simple, but will not be given here[15].

Now let T_{r0}, U_{r0}, be the relative kinetic energy of the normal mode corresponding to the root p_r with respect to the *initial* velocities and displacements, T_{rr}, U_{rr}, the kinetic energy of the normal mode under consideration with respect to itself. T_{r0}, U_{r0}, T_{rr}, U_{rr}, are defined by the expressions,

$$\left.\begin{aligned} T_{r0} &= \sum_{i,j=1}^{n} C_{ij}\,p_r\,l_i\,v_j \quad & T_{rr} &= \sum_{i,j=1}^{n} C_{ij}\,(p_r\,l_i)\,(p_r\,l_j) \\ U_{r0} &= \sum_{i,j=1}^{n} B_{ij}\,l_i\,u_j \quad & U_{rr} &= \sum_{i,j=1}^{n} B_{ij}\,l_i\,l_j \end{aligned}\right\} \quad (17)$$

whence it follows that $U_{rr} = 2\,U$, $T_{rr} = 2\,T$ (cf. equations (**3**)). Now if coefficients $B_K = A\,l_K$ are found as outlined above, so that,

$$\left.\begin{aligned} U_{r0} &= \sum_{k=1}^{n} B_K\,U_{K,r} \\ T_{r0} &= \sum_{k=1}^{n} B_K\,T_{K,r} \end{aligned}\right\} \quad (18)$$

then

$$B_r = \frac{U_{r0} - T_{r0}}{U_{rr} - T_{rr}} \text{ and } A = \frac{T_{10} - U_{10}}{T_{11} - U_{11}} \quad (19)$$

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[14] See for example M. Abraham, "Theorie der Elektrizität," Vol. 1, pp. 227-260, Leipzig. 1920.

[15] Heaviside, "Electrical Papers," Vol. 2, p. 203-204.

because, by the conjugate theorem, all the differences $U_{rs} - T_{rs}$ vanish.[16]

Let us now suppose that all the applied forces save one, F_1, are zero. On account of the linearity of the equations, this is just as general as inserting forces $F_1, F_2, \ldots F_n$ in each one of the n branches. Let F be an exponential, real, imaginary or complex function of time. Then, for any x,

$$x_K = \frac{F_1}{Z(p)} \tag{20}$$

$Z(p)$ being a function of p obtained from equations (**1**) by elimination of all the x's except the one desired. It is in general given by the ratio of two determinants. Now,

$$F_1 \frac{d\,x_K}{d\,p} - x_K \frac{d\,F_1}{d\,p} = -x_K^2 \frac{d}{d\,p}\,\frac{F_1}{x_K} = -x_K^2 \frac{d\,Z}{d\,p} \tag{21}$$

and, by the conjugate theorem,

$$2\,(T - U) = x_K^2 \frac{d\,Z}{d\,p} \tag{22}$$

Heaviside[17] now directs his attention to the electric network corresponding to the mechanical system so far considered and reasons in terms of voltage and current instead of force and velocity or displacement. It has been already shown that the internal connections of the system, *i. e.*, the coefficients A, B, C, determine how the variables chosen to fix the system should vary in order that the resultant system be normal and that the amplitude of the normal modes of oscillation is in turn determined by the conjugate theorem. The actual current and the actual voltage are then represented by sums of normal solutions (cf. equations (**7**) and (**8**))

$$V_K = \sum_j A_j\,u_j\,e^{p_j t} \qquad I_K = \sum_j A_j\,w_j\,e^{p_j} \tag{23}$$

the summations being over the roots of the determinantal equation $Z(p) = 0$. The coefficients are, as stated, determined by the conjugate theorem.

In order to find the current due to a constant applied voltage, Heaviside makes use of the following argument: Suppose that instead of inserting a constant voltage e, a condenser of capacity C is inserted in the circuit at the same point; by allowing C to increase indefinitely the effect of a constant applied voltage is obtained. This is an essential point, as it reduces the problem on hand to a subsidence problem, with no external forces. Therefore, the current can still be obtained as a sum of normal solutions. For suppose that the condenser C is inserted in the network at a given point and at a certain instant, there being no current and no voltage in any of its branches at that instant. The problem is then to find the subsidence to equilibrium of a system under its own internal stored energy. The conjugate theorem therefore holds. Let the algebraic function expressing the relation between a voltage F_K in branch K and the current I_K in the same branch be $Z(p)$ (see Equation (**20**)); suppose now that the condenser C is inserted in the same branch K and let the corresponding function be $Z_1(p)$. Remove the voltage F_K, charge the condenser and switch it on branch K. We have now the subsidence problem, to be solved through the same process as before, by making use of the known properties of normal functions and the conjugate theorem.

Let w_j, u_j, be the normal mode of oscillation of current and of corresponding voltage in the branch under consideration where the condenser is inserted. Since the current is equal to the rate of decrease of charge in the condenser, therefore

$$w_j = -C\,\dot{u}_j = -C\,p_j\,u_j \tag{24}$$

because u_j is a normal coordinate. Initially, that is, at the instant the condenser is switched on its initial voltage V is $V = \sum_j A_j\,u_j$ and $\sum A_j\,w_j = 0$, as there is no current[18]. The electric (potential) energy stored in the condenser is $\frac{1}{2}\,C\,V\,u_j$, so by the conjugate theorem

$$C\,V\,u_j = 2\,(U - T)\,A_j \tag{25}$$

U, T being the electric and magnetic energies of the normal mode. But we have also, by the conjugate theorem,

$$2\,(U - T) = -\,w_j^2 \left.\frac{d\,Z_1}{d\,p}\right|_{p\,=\,p_j} \tag{26}$$

Therefore by (**25**) and (**26**)

$$A_j = -\,\frac{C\,V\,u_j}{w_j^2 \left.\dfrac{d\,Z_1}{d\,p}\right|_{p\,=\,p_j}} \tag{27}$$

Now substitute u_j from (**24**) in (**27**); the result is

$$w_j\,A_j = V \left(p_j \left.\frac{d\,Z_1}{d\,p}\right|_{p\,=\,p_j} \right)^{-1} \tag{28}$$

and substituting in the second of (**23**):

$$I_K = \sum_j \frac{V\,e^{p_j t}}{p_j\,d\,Z_1/d\,p \mid p = p_j} \tag{29}$$

which is the subsidence current. Now to get the effect of a constant applied voltage, let C be increased indefinitely, keeping V constant. This is the same as though the circuit were connected to a source of infinite energy. Now the structure of $Z_1(p)$ may be readily seen to be such that

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[16] See Heaviside's "Electrical Papers," Vol. 1, p. 523, or "The Electrician," November 27, 1885, p. 46; also Routh, l. c. footnote (11).

[17] Electrical Papers, Vol. 2, p. 372; see also his paper "On Resistance and Conductance Operators, etc," in Philosophical Magazine, Vol. 24, pp. 479-502, 1887. See p. 501.

[18] The initial current is zero because A_{ij}, B_{ij}, C_{ij} are all finite.

$$\lim_{C \to \infty} Z_1(p) = Z(p) \tag{30}$$

and when C increases indefinitely one root of $Z_1(p) = 0$ approaches zero, because the condenser in the branch K introduces in the determinantal equation a term which varies inversely with C. It follows that when C increases indefinitely

$$p_j \frac{d\,Z_1}{d\,p}\bigg|_{p=p_j} = p_j \frac{d\,Z}{d\,p}\bigg|_{p=p_j} \quad \text{for all roots except } p = 0,$$

and for $p = 0$ (31)

$$\lim_{C \to \infty} \left(p \frac{d\,Z}{d\,p} \right)_{p=0} = Z(0) \tag{32}$$

therefore, finally

$$I_K = \frac{V}{Z(0)} + \sum_j \frac{V\,e^{p_j t}}{p_j \dfrac{d\,Z}{d\,p}\bigg|_{p=p_j}} \tag{33}$$

which is the Heaviside Expansion Formula.

Two points are to be noted in connection with Heaviside's derivation. The first is that his method of reasoning applies only to the case where the determinantal equation has no null and no repeated roots; otherwise, the conjugate theorem breaks down and therefore also the expansion theorem. The second is that the proof as it stands applies only to a system with finite number of degrees of freedom.

Heaviside also gave another alternative proof of his Expansion Formula, the essential point of which is the splitting of the quotient $F_{rs}(p)/D(p)$, where $F_{rs}(p)$ is the cofactor of the element containing the coefficients A_{rs}, B_{rs}, C_{rs} in the determinant $D(p)$, into a sum of partial fractions[19]. This is also the essential point in Carson's first proof[20].

Heaviside has also tried to generalize his proof so as to apply to a system of infinite number of degrees of freedom, but his argument, while quite convincing from a physical standpoint, is not mathematical. It may be formulated analytically, but the proof of the theorem for infinite roots leads to formidable difficulties, some of which have already been examined by Bromwich[21] (l. c. footnote 9). The relation between the conjugate theorem and Wagner's method of attack has been fully treated by Bromwich and will not be taken up here.

It is almost certain that few have realized the extraordinary completeness and generality of some of Heaviside's investigations. It is in the hope of calling attention to the unexplored parts of his work, also as an effort to secure a better appreciation of his labors, that this paper has been written.

Discussion

J. J. Smith: I should like to refer to the paragraph third from the end of Mr. Vallarta's paper in which he states that Heaviside gave another alternative proof of his Expansion Formula. This is given in Heaviside's Electrical Papers, Vol. II, p. 226, in a footnote. As this seems to be another very important way of proving the expansion theorem, I shall take the liberty of quoting it in full, as follows:

Let
$$C = \frac{f(p_0)}{\phi(p_0)} E \tag{1}$$

be the differential equation connecting C with E where p_0 stands for $d/d\,t$ and $\phi(p) = 0$ is the determinantal equation of the system, that is, $\phi(p)$ may be either the characteristic function in fully developed form, or the same multiplied by any function which does not conflict with its use in the determinantal equation. Then we have by the algebraical theorem:

$$\frac{1}{\phi(p_0)} = \sum \frac{1}{(p_0 - p)\,\phi'} \tag{2}$$

where ϕ' means $d\,\phi/d\,p$, and the summation includes all the roots of $\phi(p) = 0$. Therefore, by (1), using (2) and integrating:

$$C = f(p_0) \sum \frac{E}{(p_0 - p)\,\phi'} = E\,f(p_0) \sum \frac{e^{pt} - 1}{p\,\phi'} \tag{3}$$

E being zero before and constant after $t = 0$. But also by (2):

$$\frac{1}{\phi_0} = \sum \frac{1}{-p\;\phi'} \tag{4}$$

where ϕ_0 means ϕ with $p_0 = 0$, so that (3) becomes:

$$C = E\,f(p_0)\,\frac{1}{\phi_0} + E\,f(p_0) \sum \frac{e^{pt}}{p\,\phi'} \tag{5}$$

Now, perform the operations indicated by $f(p_0)$ and we get:

$$C = E\,\frac{f_0}{\phi_0} + E \sum \frac{f(p)}{p\,\phi'}\,e^{pt} \tag{6}$$

where f_0 means f with $p_0 = 0$. (See also the investigation at the end of the (later) paper on "Resistance and Conductance Operators.")

(6) is the form of the expansion theorem given by Heaviside in his Electromagnetic Theory, Vol. II, p. 135, a modified form of which is given c .

On going through the second volume of the Electromagnetic Theory, it is, I think, fairly evident that Heaviside used the above conception of the expansion theorem quite freely. For instance, on p. 88 he solves the problem of voltage applied at one end ($x = 0$) to a cable of length l which is grounded at the other ($x = l$) and for which the well-known formula

$$V = \frac{\sinh q\,(l - x)}{\sinh q\,l} E$$

By expanding

$$\frac{q\,l}{\sinh q\,l} = 1 - \frac{2}{1 + \left(\dfrac{\pi}{q\,l}\right)^2} + \frac{2}{1 + \left(\dfrac{2\,\pi}{q\,l}\right)^2} - \frac{2}{1 + \left(\dfrac{3\,\pi}{q\,l}\right)^2} \cdots$$

where

$$q = \sqrt{C\,R\,\frac{d}{d\,t}}$$

and then identifying

$$\frac{1}{1 + \dfrac{n^2\,\pi^2}{R\,C\,l^2\,\dfrac{d}{d\,t}}} \text{ with } e^{-\frac{n^2 \pi^2 t}{RSl^2}}$$

the solution of the problem is readily obtained.

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[19] "Electrical Papers," Vol. 2, p. 226, where the footnote supersedes the text.

[20] "On a General Expansion Theorem, etc.," *Physical Review*, Vol. 10, p. 217, 1917, also l. c. footnote (3).

[21] Wagner's proof is not applicable to infinite roots as it stands, because Cauchy's residual theorem, on which his proof is based, does not apply to infinite number of poles without special investigation of the behavior of the function under the integral sign.

If his work in this example is carefully followed out it will be found that he is doing nothing but developing the various steps in the expansion theorem given in the footnote in his Electrical Papers, p. 226.

There are two fundamental concepts back of this derivation which it may be interesting to examine. Writing (1) in the form:

$$\left(A_n \frac{d^n}{d\,t^n} + A_{n-1} \frac{d^{n-1}}{d\,t^{n-1}} + \ldots\ldots + A_1 \frac{d}{d\,t} + A_0\right) C$$
$$= f\,(p_0)\,E \tag{7}$$

where the actual value of $\phi\,(p_0)$ is inserted, it may be seen that this is a linear differential equation of the nth order for C. Now it is known that the general solution of this equation is:

$$C = B_1\, e^{p_1 t} + B_2\, e^{p_2 t} + \ldots\ldots + B_n\, e^{p_n t} + \frac{f\,(p_0)_{\;p_0 = 0}\, E}{A_0} \tag{8}$$

where $p_1, p_2 \ldots\ldots p_n$ are the roots of the equation

$$A_n\, p^n + A_{n-1}\, p^{n-1} + \ldots\ldots + A_1\, p + A_0 = 0$$

Each term $e^{p_r t}$ is a solution of the equation

$$\left(\frac{d}{d\,t} - p_r\right) y = 0$$

We are thus lead to expect that the solution of a linear differential equation of the nth order can be made to depend upon the solution of n linear equations of the first order, and if a comparison is made of equations (1) and (3) it will be seen that the equation of the nth order in (1) has been transformed into n equations of the first order in (3) by means of a partial fraction expansion.

The second point to notice is that nowhere in the above proof does Heaviside use the property $C = 0$ when $t = 0$. What he does postulate is that E is zero before and constant after $t = 0$. It is under this hypothesis that in order to find the value of $\frac{1}{p_0 - p}$ he assumes it to be, say, y, then $(p_0 - p)\; y = E$

Or

$$\left(\frac{d}{d\,t} - p\right) y = E$$

giving

$$y = i_0\, e^{+p(t - t_0)} + \frac{E\,(e^{pt} - 1)}{p}$$

Now in all practical problems the real part of p is negative, corresponding to circuits with resistance in which energy is dissipated; hence, whatever current may have existed at the instant $t_0 < 0$, where the actual value of t_0 may be made as large as we please, it will have become negligible by the time $t = 0$. Hence, the effect produced by the applied voltage E is contained wholly in the second term, giving

$$y = 0 \qquad t < 0$$

$$y = \frac{E\,(e^{pt} - 1)}{p} \qquad t > 0$$

Or, if we wish to put it another way, the y caused by E applied at $t = 0$ is given by this term, and the $i_0\, e^{p(t - t_0)}$ part is due to another cause. This is not the way in which Heaviside makes this particular step but the results are the same.

Whatever way we chose to look at it, the facts are that by using this substitution in (3) as shown, and completing the solution as in (6), we get the solution of (1), corresponding to the system (assumed to be a dissipative one) represented by (1) being at rest, and having been at rest for a very long time prior to $t = 0$. We have thus used the one condition

$$E = 0 \qquad t < 0$$
$$E = E \text{ constant} \qquad t > 0$$

to supplant the n terminal conditions which would be normally required to determine the constants in the solution of (1) or (7). The economy in such a reduction of terminal conditions should be apparent. The economy becomes more apparent when applied to problems in long transmission lines where an infinite number of terminal conditions arise in certain methods of obtaining the solution.

I presume the reason Mr. Vallarta did not discuss Heaviside's second derivation referred to above is that it involves so-called operational mathematics, and operational methods have been looked upon with disfavor by mathematicians for many years as lacking in rigor, which may or may not be true. I like to think, however, that between the operational mathematics and the conventional pure mathematics there must be some common ground on which they can both meet together by a little give-and-take on both sides. I have published two papers in the *Journal* of the Franklin Institute with this end in view, one in June, 1923, and the other as a serial in October, November and December, 1925. In these papers the ideas given above are developed more fully. The interesting part, however, is that although Heaviside, as far as I am aware, uses his expansion theorem only in continuous systems for one coordinate in addition to the time, such an attempted development of a common ground between pure and operational mathematics has led to a method of solution for two- and three-dimensional problems such as occur in the flow of heat, and in addition to the solution of potential problems in electrostatics in which no time factor appears. These results come from simple extensions of the development of the expansion theorem given above, which then becomes a particular expansion theorem in a great class of expansion theorems. Reference must be made to my paper for details. However, with Mr Vallarta I join in a plea for greater study of the works of Heaviside and for the development of both his ideas and the concepts of pure mathematics to the point where they will both merge into one as I have not the slightest doubt they should.

M. S. Vallarta: In his valuable discussion Mr. J. J. Smith has already given Heaviside's second proof of the Expansion Theorem, to be found in the second volume of his "Electrical Papers" (l. c. footnote 19 of the writer's paper.) The writer did not feel that a complete account of this second method of proof was necessary partly because it has already been discussed rather thoroughly elsewhere (see for example reference 9 in the writer's paper), but mostly because it is not quite so illuminating, or important from a physical standpoint, as the first method of proof based on the conjugate theorem.

Mr. Smith raises an interesting question when he speaks of operational calculus and its relation to the Expansion Theorem. As shown in the paper, operational methods are not required to establish the Expansion Theorem in the case of no null and no repeated roots of the determinantal equation, but the use of such methods really simplifies matters considerably in this exceptional case. For purposes of technical applications, however, the writer feels that both may properly be kept separate. That operational calculus can be rigorized if the proper tools and methods of attack are used has been shown by Wiener ("The Operational Calculus" forthcoming in the *Mathematische Annalen*) and Carson (see the Bell Technical *Journal* for 1925 and 1926). Once operational calculus is rightly understood, it may be applied confidently not only to engineering problems but also to problems in pure mathematical physics where less powerful methods break down completely. As a most advanced example of this type of problems, reference is made to a paper by Born and Wiener, (*Journal* of Mathematics and Physics of M. I. T., Vol. 5, p. 84, Feb. 1926) to quote but a single example.

The great importance of the conjugate theorem has been very recently recognized by K. W. Wagner ("Der Satz von der wechselseitigen Energie" *Elektrische Nachrichten-Technik*, Vol. 2, p. 376, Nov. 1925) who also gave in this article Heaviside's first method of proof of the Expansion Theorem. This and the writer's paper were written approximately at the same time (Wagner's paper was received for publication on Nov. 9, 1925; the author's paper on Dec. 1, 1925). The present writer is glad to add this interesting reference to the list already given in the paper.

A New Wave-Shape Factor and Meter

BY L. A. DOGGETT, J. W. HEIM and M. W. WHITE*
Member, A. I. E. E. Non-Members

***Synopsis.**—A star-connected circuit consisting of two voltmeters and one variable condenser has certain properties upon which a wave-shape factor may be based. When the voltmeter resistances are equal to each other and equal to the condenser reactance, the ratio of the voltmeter readings will always be $(2 + \sqrt{3})$ for an alternator producing sine waves and always less than $(2 + \sqrt{3})$ for all other wave shapes. From any measured ratio of voltmeter readings, the purity of the voltage wave may be determined; furthermore, the maximum possible percentage of any single harmonic present in the wave can be immediately obtained (See Table III). For the experimental application of this method a wave shape meter is proposed for practical application which consists essentially of two voltmeters and a variable condenser. Such a meter has advantages over the method of analysis based on oscillograms, namely: (1) cost, (2) portability, (3) ease of experimental procedure and (4) rapidity of arriving at results. This method of attack is not intended to supplant the oscillograph but rather to supplement it. The method has been checked with the aid of a harmonic alternator and has been applied to the local power system and to various alternators available in the Electrical Engineering laboratories of the Pennsylvania State College.*

* * * * *

INTRODUCTION

FOR a long time the wave-shape standard of the A. I. E. E. has been based on oscillograms. Recently† copies of seven oscillograms were sent to ten electrical manufacturing and power companies for measurement of deviation factor. The results obtained showed very considerable variations. For one oscillogram, the values found for this factor ran from 1 to 4.2 per cent, with an average value of 2.25 per cent. In other words, the maximum value was in excess of the average by 86½ per cent of the average value, while the spread between the maximum and minimum values was 3.2 or 142 per cent of the average value. For the other six curves the spread was less, but still considerable. The excess of the maximum values over the average values for each of the six other cases was 57 per cent, 54 per cent, 52 per cent, 47 per cent, 31 per cent, and 14 per cent. Quoting from the reference given, "The variations between results cannot be attributed to the processes of calculation employed, for neither the maximum nor the minimum spread occurred with the same party. The variations must, therefore, arise from difficulties inherent in the method such as difficulty in evaluating exactly the ordinates of the oscillogram, errors in evaluating the area included by the squares of the ordinates in finding the effective value, errors in measuring the maximum differences between the ordinates of the oscillogram and the equivalent sine wave, especially when these differences occur in the steep part of the curve."

Another test was made to determine the accuracy attainable in the determination of the amplitudes of harmonics by analysis of oscillograms. Three waves were prepared by the synthesis of known harmonics and fundamental, the waves traced, and the results turned over to various parties to be analyzed. The results indicated that for harmonics below the 15th, average variations from the correct values of amplitudes of harmonics of ± 5 per cent might be expected.

The usual alternative to the use of the oscillograph is some sort of a circuit method of which many have been proposed but so far no one has been permanently adopted. Here still another circuit method is proposed, differing primarily from other circuit methods in that it is based on a three-phase circuit.

DESCRIPTION OF THE METHOD

Two voltmeters, V_2 and V_3 (Fig. 1), having equal resistances and a variable condenser C (whose range of reactance can be varied above and below the resistance of the voltmeters), are connected in star to the three-phase source whose wave shape is to be investigated. The value of C is adjusted until the ratio of the larger to the smaller voltmeter readings is maximum. That this ratio, R, is $(2 + \sqrt{3})$ for a pure sine wave is proved in the Appendix. The greater the deviation from a pure sine wave, the lower this ratio will become. It is the dependence of this ratio upon the harmonics present in the wave which makes it possible to use such a voltage ratio as a measure of the purity of the wave.

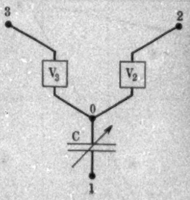

FIG. 1 A—A NEW WAVE-SHAPE FACTOR

FIG. 1 B—WAVE-SHAPE FACTOR AND METER

The vector diagram of Fig. 2 gives an idea of the displacement of the neutral as the wave shape departs from a true sine wave.

By the use of the following formula, also proved in the

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*All of Pennsylvania State College, State College, Pa.

†*Revue Generale de L'Electricite*, January 10, 1925, pp. 43-46.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

Appendix, the voltage ratio for any combination of harmonics may be calculated,

$$R = (2 + \sqrt{3}) \left[\frac{100 + a_2 (\% H_2)^2 + a_3 (\% H_3)^2 + \; . \; . \; . \; a_n (\% H_n)^2}{100 + b_2 (\% H_2)^2 + b_3 (\% H_3)^2 + \; . \; . \; . \; b_n (\% H_n)^2} \right]^{1/2} \tag{1}$$

Where $\% H_2$ is the per cent second harmonic.
$\% H_3$ " " " " third "
$\% H_n$ " " " " n^{th} "

The a and b constants for the various harmonics are given in Table I.

TABLE 1

Harmonic	a	b
2	0.0026	0.1967
4	0.0160	0.0939
5	0.0080	0.2230
7	0.0157	0.1330
8	0.0101	0.2165
10	0.0153	0.1500
11	0.0110	0.2106
13	0.0149	0.1590
14	0.0115	0.2066
16	0.0147	0.1640
17	0.0119	0.2036
19	0.0145	0.1680
20	0.0122	0.2014
22	0.0144	0.1710
23	0.0124	0.1993
25	0.0142	0.1730
∞	0.0134	0.1866

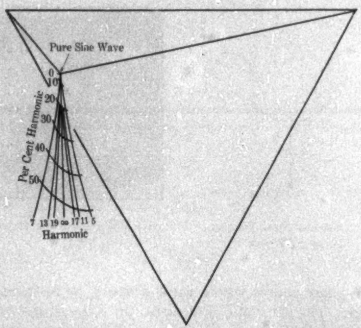

FIG. 2—LOCI OF FLOATING NEUTRAL FOR VARIOUS PERCENTAGES OF HARMONICS. (LINE VOLTAGE CONSTANT)

Since in the experimental application of this method, two voltmeters having equal resistances are required and as one meter must have a range approximately three times that of the other, some difficulty was experienced in selecting suitable meters from standard models, those of the dynamometer type being desired.

The meters first selected were a Weston instrument with a resistance of 301 ohms and a range of 30 volts, and a General Electric instrument having a resistance of 920 ohms and a range of 150 volts. Each voltmeter was equipped with a multiplier so as to arrange the resistance of V_2 in Fig. 1 equal to that of V_3. The voltmeters were then calibrated. Three one-μf mica and one eight-μf paper condenser were used. The capacities of these condensers were also checked.

The final arrangement, as shown in the photograph, Fig. 1, consists of two Weston dynamometer type voltmeters of 40- and 120-volts range respectively and of 1000.7- and 1000.0- ohms resistance respectively. Mounted in the same box are a one-μf. mica and two one-μf. paper condensers. The mica condenser is a Leeds and Northrup instrument, variable by steps of one-twentieth μf. from 0 to one μf. The small switch in the lower left hand corner controls the paper condensers, giving zero, one, or two μf.

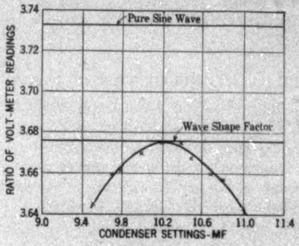

FIG. 3

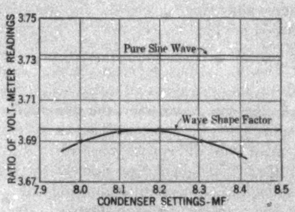

FIG. 4

SOME APPLICATIONS OF THE METHOD

With the apparatus as originally set up, tests were made on waves obtained from the Keystone Power Corporation System, from the Pennsylvania State College Power Plant (300 kv-a. turbo alternators), and various laboratory alternators. In the curves of Figs. 3 and 4 are plotted the data for two representative cases.

To obtain data for the curves of Fig. 5 the final arrangement shown in the photograph of Fig. 1 was used. If the following technique is employed, and if the frequency remains constant for about five minutes, the results obtained are independent of voltage, capacity and magnitude of line frequency. The technique employed in obtaining the five dotted curves of Fig. 5 was to vary rapidly the capacity from 2.0 to 3.0 μf. by 0.1 μf. steps, reading the two voltmeters at each step. The maximum ordinate of the plot of the ratio of these observations is the wave-shape factor.

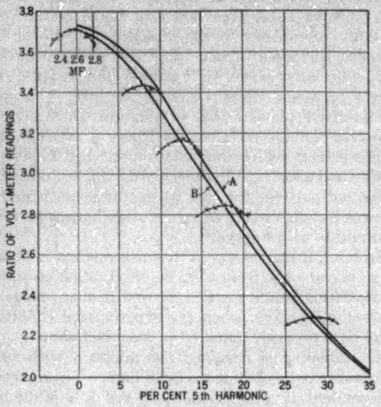

FIG. 5

CHECK OF OBSERVATIONS AGAINST CALCULATIONS

In order to check this method the following test was carried out. The fundamental and fifth harmonic members of a three-phase harmonic alternator were used. A description of this machine is given in the Appendix. The phases of the fundamental machine were star-connected, and each phase joined in series with the proper coil of the fifth-harmonic machine. This three-phase supply was connected to the wave shape meter (Fig. 1). Ratio curves, such as shown in Fig. 5, were obtained for 0 per cent, 8.0 per cent, 12.8 per cent, 17.5 per cent, and 28.0 per cent fifth harmonics. Through the maximums of the five dotted curves is drawn a curve, *B*, which should check with curve *A*, the latter being calculated from formula (**1**). Discrepancies between curves *A* and *B* are chargeable to deviation from a pure sine wave on the part of the fundamental machine and to slight inequalities in the magnitudes of the three-phase voltages of the fifth harmonic machine. In searching for various factors to account for this discrepancy it was proved by test that the wave-shape factor is independent of the phase position of the harmonics.

CONCLUSIONS

1. *Value of Voltage Ratio to Be Met in Practise.* In addition to the values of wave shape factors given above for systems and dynamos tested in our laboratories, there is given in Table II a compilation of wave-shape factors for some waves whose analyses into component harmonics have been found in the literature. It will be seen that wave-shape factors varying from 3.448 to 3.730 appear in practise. These factors were obtained by substituting the given percentage harmonics in formula (**1**).

TABLE II

Wave-Shape Factor	Per Cent Harmonic									Reference
	3	5	7	9	11	13	15	17	19	
3.712	0	2	1.44							1
3.605	0	5.3	2.46	0	1.04	1.2				2
3.715	0	1.77	1.22	0	0.48	0	0	0.395	0.294	2
3.633	13¼	5	1.2	0.9						2
3.676	0	3.5	1.6							3
3.493	3.67	8.05	1.63	0.058						4
3.520	0.98	7.6								4
3.448	7	9								5
3.670	3	4								5
3.730	0.46	0.724								6
3.724	5.5	1.25	0.49	0.55	0.37					6
3.591	2.92	5.2	0.66	1.07	0.62	3.77	4.6	0.48		6
3.555	2.83	5.43	5.51	0.26	1.28	0.51	0.23	0.09		6
3.624	1.0	2.73	2.2	0.41	4.4	0.79	0.23	0.09		6

1. "Specification and Design of Dynamo-Electric Machinery," Miles Walker, p. 332.
2. TRANS. A. I. E. E., Vol. 32, 1913, p. 781.
3. TRANS. A. I. E. E.., Vol. 38, 1919, p. 1185.
4. TRANS. A. I. E. E., Vol. 23, 1904, p. 408.
5. *Electrician*, Aug. 6, 1909.
6. Bureau of Standards, Vol. 9, 1913, p. 567.

Oscillograms were obtained from 19 machines of Table I of the 1919 report of the A. I. E. E. Subcommittee on Wave Shape Standards[13]. Based on analyses of these waves, the following wave-shape factors were calculated:

3.717, 3.706, 3.717, 3.719, 3.698, 3.723, 3.715, 3.724, 2.746, 3.700, 3.700, 3.696, 3.698, 3.702, 3.700, 3.708, 3.702, 3.553, 3.349; *i. e.*, a range of 3.724 to 2.746.

2. *Interpretation of Voltage Ratios.* As previously stated, a ratio of voltmeter readings of $(2 + \sqrt{3})$ or

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[13] Osborne

3.732 corresponds to a pure sine wave. Consequently the purity of a voltage wave may be immediately determined by noting whether or not the observed voltage ratio is equal to 3.732.

In case the ratio is found to be less than $(2 + \sqrt{3})$ it follows that the wave is not a pure sine wave. We may assume the absence of third harmonics or any multiple thereof in the ordinary commercial three-phase circuits. Furthermore, no second or fourth harmonics will be found in the waves from commercial alternators.

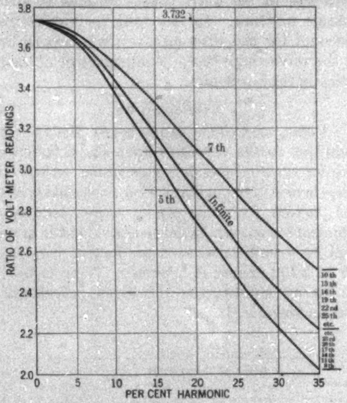

Fig. 6

The fifth and seventh harmonics are usually the most pronounced of all the harmonics, and Fig. 6 shows the ratio curves for these harmonics. If only a seventh harmonic is present, reference to Table III will immediately give the percentage harmonic corresponding to any observed voltage ratio.

TABLE III

7th Harmonic

Per Cent Harmonic	Voltage Ratio
0	3.7320
1	3.7298
2	3.7232
3	3.7123
4	3.6974
5	3.6753
6	3.6530
7	3.6248
8	3.5999
9	3.5685
10	3.5298
11	3.4962
12	3.4572
13	3.4163
14	3.3742
15	3.3312

Reference to the curves of Fig. 6 shows that a given voltage ratio corresponds to a smaller percentage harmonic when this is a harmonic of an order other than the seventh. Consequently the *maximum* possible percentage of any single harmonic present may be immediately obtained from the observed voltage ratio by reference to Table III. For example, a voltage ratio of 3.33 corresponds to the presence of a 15-per cent seventh harmonic (From Table III) and inasmuch as this ratio corresponds to a smaller percentage of any single harmonic other than the seventh, it is quite conservative to state that in a circuit which gives an observed voltage ratio of 3.33, there is no single harmonic present which amounts to more than 15 per cent of the fundamental. In other words, this last deduction holds not only for the case where the seventh alone is present but also where there are other harmonics found in addition to the seventh.

In brief, it can be stated that this method is capable of giving at once from a single observation of voltage ratio, the maximum possible percentage of any harmonic present in a wave, when the experimental conditions have been properly adjusted as described above.

3. *Accuracy of Results.* The accuracy with which the data may be obtained is quite high. The method is independent of voltage, since only the ratio of the two-phase voltages is required and this ratio is independent of the actual voltage used. The frequency need not be measured, provided means of checking its constancy are available; this can be conveniently done by any frequency meter. If good mica condensers are used, the accuracy with which they are guaranteed by the manufacturers (say ¼ per cent) is more than sufficient for our purpose, since the actual value of the capacitance does not enter directly into the computations. Consequently, the main factor which limits the precision of the measurements is the accuracy of the calibration of the voltmeters and our ability to read the same. The probable error of the voltage ratio, including instrumental and observational errors, will not exceed 0.2 per cent.

Our experiments have indicated that the use of the iron-vane type of voltmeter is not very satisfactory for this purpose.

Appendix I

Proof that $\frac{V_2}{V_1} = R = 2 + \sqrt{3}.$ for a pure sine wave: The following formula has been derived in a previous paper* for calculating the voltage to neutral for any branch of the star-connected circuit of Fig. 1.

$$I_p Z_p = E\left[a^{p-1} - \frac{a^0/Z_1 + a^1/Z_2 + \;.\;.\;.\;.\; a^{n-1}/Z_n}{1/Z_1 + 1/Z_2 + \;.\;.\;.\;.\; 1/Z_n}\right] \qquad \text{(a)}$$

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*Doggett. "Floating Neutral n-Phase Systems", Trans. A. I. E. E., Vol. 42, 1923, p. 860.

where

E is the numerical value of the impressed voltage to geometrical neutral.

a = cos $2\pi/\phi + j \sin 2\pi/\phi$ for counter clockwise rotation.

a = cos $2\pi/\phi - j \sin 2\pi/\phi$ for clockwise rotation.

ϕ = number of branches or phases.

Z_1 = impedance between 0 and 1 (see Fig. 1).

Z_2 = impedance between 0 and 2 (Fig. 1).

Z_n = impedance between 0 and n (Fig. 1).

p = 1, 2 or 3 for a three-phase system.

Assuming that the impedance of each meter is 1000 ohms resistance (inductance negligible†) and that of the condenser is also 1000 ohms (capacitive reactance), the reading of the voltmeter V_2 according to equation (**a**) is,

$$I_2 Z_2 = E \left[(-1/2 - j\sqrt{3}/2) - \frac{\dfrac{1}{-j\,1000} + \dfrac{-1/2 - j\sqrt{3}/2}{1000} + \dfrac{-1/2 + j\sqrt{3}/2}{1000}}{\dfrac{1}{-j\,1000} + \dfrac{1}{1000} + \dfrac{1}{1000}} \right]$$

$$= E\left[-1/2 - j\sqrt{3}/2 - \frac{j-1}{j+2} \right]$$

Rationalizing the last term, reducing and collecting,

$$I_2 Z_2 = \frac{E\,[-3 - j\,(6 + 5\sqrt{3})\,]}{10} = V_2$$

Proceeding in a similar manner, the reading of V_3 is

$$I_3 Z_3 = \frac{E\,[-3 - j\,(6 - 5\sqrt{3})\,]}{10} = V_3$$

Then the ratio R

$$= \frac{V_2}{V_3} = \frac{-3 - j\,(6 + 5\sqrt{3})}{-3 - j\,(6 - 5\sqrt{3})}$$

Combining real and j terms of both numerator and denominator, and rationalizing,

$$R = 2 + \sqrt{3} = 3.732.$$

(Q. E. D.)

Appendix II

Proof of Equation (1). (General formula for calculating R for any number of harmonics of known percentage).

Notation.

E_1' = Fundamental component of voltage across V_2.

E_2' = 2^d harmonic component of voltage across V_2.

E_3' = 3^d harmonic component of voltage across V_2.

E_n' = n^{th} harmonic component of voltage across V_2.

E_1'' = Fundamental component of voltage across V_3.

E_2'' = 2^d harmonic component of voltage across V_3.

E_3'' = 3^d harmonic component of voltage across V_3.

E_n'' = n^{th} harmonic component of voltage across V_3.

Then,

$$R = \left[\frac{(100\,E_1')^2 + (\%\,2^d\,E_2')^2 + (\%\,3^d\,E_3')^2 + \ldots (\%\,n^{th}\,E_n')^2}{(100\,E_1'')^2 + (\%\,2^d\,E_2'')^2 + (\%\,3^{d}\,E_3'')^2 + \ldots (\%\,n^{h}\,E_n'')^2} \right]^{1/2} \qquad (\mathbf{b})$$

Constants are worked out for the fundamental and fifth harmonic, in the following sample calculations: Using formula (**a**),

$$E_1' = -0.5 - j\,0.866 - \frac{-1+j}{2+j} \quad \text{(calculated in Appendix I).}$$

Rationalizing the last term and collecting,

$E_1' = -0.3 + j\,1.466$

Combining real and j terms and squaring,

$(E_1')^2 = 2.24.$

Similarly,

$$E_1'' = -0.5 + j\,0.866 - \frac{1+j}{2+j}$$

$(E_1'')^2 = 0.161.$

Similarly, remembering that the phase rotation of the fifth harmonic is opposite to that of the fundamental*,

$$E_5' = -0.5 + j\,0.866 - \frac{-1+j\,5}{2+j\,5}$$

$(E_5')^2 = 1.794.$

$$E_5'' = -0.5 - j\,0.866 - \frac{-1+j\,5}{2+j\,5}$$

$$= -1.293 - j\,1.383$$

$(E_5'')^2 = 3.585$

Equation (**b**) can be transformed into equation (**1**) by factoring out the ratio for the fundamental sine wave, $(2 + \sqrt{3}.)$ The b constants are derived from the E'' constants in equation (**b**):

$$b_5 = (E_5'')^2 \times \frac{1}{0.161} = \frac{3.585}{0.161} = 22.3$$

The a constants of equation (1) are derived from the E' constants of equation (**b**):

$$a_5 = (E_5')^2 \times \frac{1}{0.161} \times \frac{1}{(2+\sqrt{3})^2}$$

$$= 1.794 \times \frac{1}{0.161} \times \frac{1}{13.92} = 0.80$$

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†The inductance of the meters was found to be too small to affect these calculations appreciably.

*Fortescue, TRANS. A. I. E. E., Vol. 37, 1918, p. 1027.

Appendix III

The Harmonic Alternator. The various harmonic voltages were obtained by means of a special harmonic alternator. This machine consists of three three-phase a-c. generators of the revolving field type, one of which is a 15-kw., 60-cycle machine; the second has a capacity of 7½ kw. at 180 cycles, and the third, 3 kw. at 300 cycles. All three machines are mounted on the same bed-plate and have a common frame while the field coils of all machines are mounted on the same shaft. The armature of the 60-cycle machine is stationary, but the armature of the 180-cycle machine may be moved by means of a hand wheel through an angle corresponding to 360 electrical degrees, and the armature of the 300-cycle machine by similar means can be moved through an angle corresponding to 600 electrical degrees. The six terminals of the armature winding of each machine are connected to a switchboard so that the three-phase windings may be connected either in star or delta. The field control of each generator is independent of the other two fields. The machine is driven by a d-c. shunt motor and the field winding is separately excited.

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Discussion

J. J. Smith: When considering the effect of harmonics, it is perhaps well to divide them into different classes: (1) those which may give rise to resonant conditions or large circulating currents on the power circuit, (2) those which may give rise to telephone interference, (3) those which may give rise to radio interference, (4) those which may give rise to any other type of trouble that may be discovered. Let us consider, for instance, the case of telephone interference, as somewhat similar remarks will apply to other types of interference. In the telephone-interference-factor meter an attempt was made to weight the various harmonics in accordance with the experimental data available as to the interfering effect of each harmonic. A special type of network was designed to do this, and the impedance characteristics of such a network, which is used in the telephone-interference-factor meter, can be varied, within certain limits at least, to correspond with any new data which may show that it is desirable to weight the harmonics differently. In the method suggested by Messrs. Doggett, Heim, and White, no such adjustment is possible.

The method proposed may also be compared with a method such as the use of the telephone-interference-factor meter by noting the ratio between the maximum and minimum values obtained on machines in actual operation. The T. I. Fs. given on the machines of Table I of the 1919 Report of the A. I. E. E. Subcommittee on Wave Shape Standards vary from 11 to 550, while the ratios given by Messrs. Doggett, Heim and White vary from 2.746 to 3.724, giving a ratio of 50 to 1 with one method and 1.35 to 1 with the other.

It is well to remember that direct-current and other types of machines produce harmonics, and that it is undesirable to have a wave-shape meter which will apply to three-phase systems only. Also, in certain cases of telephone interference which have occurred on systems with the neutral grounded, it is the wave shape from line to neutral and not from line to line which is of interest.

I would like to inquire if the authors found any difficulty in making measurements on systems on which there were large variations in load. In making measurements of harmonics on such systems we have found that the magnitude of the harmonics varied with the load over short periods of time (in the order of one or two seconds). I wonder if fluctuations of this nature would not make it impossible to get a consistent curve from which to pick the maximum as in Fig. 5.

It may be well to consider the first paragraph of the paper by briefly considering what accuracy may be obtained in analyzing a wave obtained with the oscillograph. Let us assume an oscillogram has a maximum ordinate of 2 in. and that the line is of average thickness, which, in order to get a good film, may be 1/20 in. or less. Using a wave micrometer ruled in fortieths of an inch there is no reason why the values of the ordinates should not be measured to within half of one division or 1/80 of an inch. This is an error of 1 in 160 or let us say 0.7 per cent. As the probability is that all these errors will not be in the same direction, it would appear more proper to take, perhaps, half of this, or say 0.35 per cent, as the probable error in measuring the oscillograms. Using this figure the maximum deviation in the first case quoted by the authors from the *Revue Générale de l' Electricité* should have been between the limits 2.6 per cent and 1.9 per cent or the maximum value is in excess of the average by about 15 per cent of the average value, and correspondingly less in the other case.

The large discrepancies quoted by the authors may be due to the inherent difficulty in superposing the equivalent sine wave of equal length in such a manner as to give the least difference, as required by A. I. E. E. Standards, 1922, No. 3274. It would be impossible, however, to explain them without making a detailed study of the calculations which were made in each case. Also, a comparison of the T. I. Fs. calculated from oscillograms and those taken direct with the T. I. F. given in the 1919 Report of the Subcommittee on Wave Shape of the A. I. E. E. Standards Committee, will show that the large discrepancies quoted by the authors cannot be the general rule. Furthermore, it should be remembered that by using a filter or some such device for stopping off the fundamental, a very much higher degree of accuracy in measuring harmonics may be obtained with the oscillograph.

F. K. Brainard: It is generally recognized that sine waves in a-c. apparatus are desirable and since commercial alternators frequently have voltage waves which differ considerably from sines, it would be highly desirable to have a wave-form factor which could be easily determined and which would be a measure of the detrimental effect resulting from a distorted wave. Various factors have been suggested including the following: (a) form factor, (b) peak factor, (c) telephone-interference factor, (d) differential distortion factor, (e) integral distortion factor, (f) curve factor, and (g) harmonic factor. None of these appears to be entirely satisfactory, partly because the detrimental effect of a distorted wave depends upon the trouble under discussion and partly because some of the above factors are not readily measured. If telephone interference is being considered, the telephone-interference factor is undoubtedly the proper criterion to apply, but if the core loss of transformers is under consideration or the dielectric strength of insulating material is being measured, it is quite obvious that some other factor should be used.

While this new factor gives the greatest weight to the seventh harmonic, there is not an exceedingly great variation in weight given between any of the harmonics, and in this respect it resembles the "harmonic factor" which is defined by Bedell as the ratio of the effective value of the harmonics to the effective value of the fundamental, but is superior to it in that it is very easily measured, although it is applicable to three-phase circuits only. Possibly an Institute rule using both the telephone-interference factor and this new factor specifying limiting values of both, would be desirable. In that case, the telephone-interference factor would limit possible telephone interference and this new factor would limit other troubles due to the presence of the lower harmonics.

The authors are to be complimented for the development of a wave-shape factor which gives promise of being a valuable one. The question now seems to be the determination of the value of this factor as a criterion for the comparison of voltage waves.

C. W. Bates: The paper contains a satisfactory analysis of the proposed method which should give reasonably correct values for the harmonics present, when the conditions are ideal. The paper does not, however, contain any investigation of the errors which may arise from such sources as the use of voltmeters containing some inductance, condensers whose loss is not negligible, and unbalanced line voltages. The entire theory of the proposed method rests on the assumption that when no harmonics are present the potential of the neutral point of the circuit used is displaced to such a point that the voltmeter readings have the ratio of 3.732. This is true only when both voltmeters are absolutely non-inductive, the condensers are absolutely without loss, and the voltages of the three phases are absolutely equal.

If it is assumed that each voltmeter has an inductance L in addition to its resistance R and that the condenser has a loss represented by its equivalent series resistance r in addition to its capacity c the ratio of the voltages indicated with exactly balanced three-phase line voltages will be approximately equal to $3.732\left[1+\sqrt{3}\left(\frac{\omega L}{R}-r\,\omega\,c\right)\right]$ when $R=\frac{1}{\omega c}$. The fractional error is then equal to $\sqrt{3}\left(\frac{\omega L}{R}-r\,\omega\,c\right)$ or practically equal to the difference of the angles of defect (from the ideal) multiplied by $\sqrt{3}$. The approximation in this result is due only to considering that the angles of defect are equal to their respective sines and tangents and that the cosines are equal to unity. These approximations are entirely justifiable for any voltmeters or condensers whose use could be considered.

The importance of these errors may be judged by considering two examples:

Voltmeters: $R = 1000$ ohms, $L = 53$ millihenries

$$\frac{\omega L}{R} = .02 \text{ at 60 cycles,}$$

Condenser: $\frac{1}{\omega c}$ 1000 ohms, $r = 50$ ohms

$r\,\omega\,c = 0.05$ (3 deg. approximately)

The fractional error in the ratio will be equal to $\sqrt{3}$ (0.02 − 0.05) = − 0.052 or − 5.2 per cent. (By the exact expression this is 4.9 per cent, thus justifying the approximate expression given for the error). If the voltmeter inductance is 5.3 millihenries as may occur in the best modern dynamometer voltmeters and the condenser phase angle is 17 min. as might be found in a good mica condenser, each angle of defect is one-tenth the former value and the error is 0.52 per cent. Even this error is quite appreciable in the calculation of the method shown in the paper. Fortunately the errors due to inductance and to condenser loss are opposite in sign and therefore, by a suitable adjustment, they may be made to neutralize each other.

The error due to unbalanced three-phase voltage may be much greater and moreover varies continually with line-voltage fluctuations. This error may be readily analyzed by the use of the symetrical coordinates developed by C. L. Fortescue. By this method any unbalanced system of three-phase voltages is resolved into two balanced systems of opposite phase rotation. Since the direction of shift of the potential of the neutral point is determined by the phase sequence, it may be readily appreciated that voltages of opposite phase sequence simultaneously impressed will not result in the ideal ratio of voltmeters of 3.732, even if no harmonics are present.

The analysis of this error is too complicated to be presented in a brief discussion but the results of the analysis may be illustrated numerically. Let m be the degree of unbalance expressed by the ratio of the negative-sequence voltage to the positive-sequence voltage. This is roughly equal to the greatest deviation of any one of the three voltages from the mean. The error due to unbalance will depend not only on this ratio but on the phase angle between the two oppositely rotating components noted with reference to any instant. Accordingly the limits of the error are given in the first four lines of the tabulated illustrations below. Since the authors state that any small error due to voltage unbalance may be eliminated by stepping the line terminals around the circuit terminals and averaging the results obtained by the use of each of the three connections, a comparison of the error of this average is given also, in the last four lines.

Two numerical examples are given in the table, one corresponding to a 10 per cent unbalance such as would result from voltages of 90, 105, and 105, or to 91.3, 100, and 108.7, each of these groups being approximately respective phase voltages which would result in a 10 per cent negative-sequence voltage, the difference in line voltages resulting in a different phase angle. The other example is based on a 1 per cent unbalance due for example to voltages of 99, 100.5 and 100.5. In general the unbalance will be between these limits, as the first corresponds to very poor service voltage while the second will be difficult to maintain even under laboratory conditions. From the figures given in the table it is seen that the results of a single pair of readings are entirely untrustworthy even with very well balanced voltages, and that good balance is necessary in order to secure reliable results, even if the terminals are stepped around. It may be noted that the average ratio is always high.

Negative-sequence Voltage m	10 Per cent	1 Per cent
Maximum ratio of readings	6.04.	3.882
Error of maximum, per cent	62	4.0
Minimum ratio of readings	2.68	3.593
Error of minimum, per cent	29	3.7
Maximum average ratio	4.022	3.7335
Error of maximum average ratio, per cent	7.8	0.043
Minimum average ratio	3.755	3.7333
Error of minimum average, per cent	0.6	0.033

The formula from which these results were calculated is included without derivation, for the sake of completeness.

Ratio of voltmeter readings

$$= \sqrt{\frac{A + \frac{m^2}{A} + 2\,m \cos(60^\circ + \alpha)}{\frac{1}{A} + A\,m^2 + 2\,m \cos(60^\circ - \alpha)}}$$

$$= A \sqrt{\frac{1 + \frac{m^2}{A^2} + \frac{2\,m}{A} \cos(60^\circ + \alpha)}{1 + A^2\,m^2 + 2\,A\,m \cos(60^\circ - \alpha)}}$$

where

$A = 3.732$,

m = ratio of negative - to positive-sequence components,

α = angle between components in the reference phase which is taken to be that across which the voltmeters are connected.

L. A. Doggett: We have been very pleased to have Mr. Bates attack this problem from his own angle of approach and not only check our (2 plus $\sqrt{3}$) but also show us the method of eliminating the errors due to the inductance of the voltmeters and the resistance of the condenser.

So far as the effect of unbalance is concerned we would like to submit for comparison with Mr. Bates' figures the data for one test. With balanced voltages the wave-shape factor for a certain wave was 3.693. With the same wave and with voltages of 100, 97, and 97.5, the wave-shape factors, as the terminals of the wave-shape meter were stepped around, were 3.54, 3.95, and 3.60, averaging 3.697. Experimental study of the effect of unbalanced voltages indicated that useful results were obtainable up to 2 per cent unbalance.

Mr. Smith has brought out some of the contrasts between the telephone-interference-factor meter and the present wave-shape meter. He has also covered the question of the accuracy obtainable from oscillograms. As the wave-shape meter is not a telephone-interference-factor meter, no discussion is devoted to their comparisons. Mr. Smith has described what might be called the utmost accuracy obtainable from oscillograms. The results cited in our paper are considered fair samples of everyday accuracy obtainable. As to the effect of load pulsations, we find no difficulty in getting results while undergraduate laboratory work is in full swing.

In concluding the discussion, it should be pointed out that the instrument, made accurate in the manner described by Mr. Bates, is particularly well adapted to shop testing of three-phase alternators, which have three exactly balanced voltages. Like the telephone-interference-factor meter, this meter has its distinctive characteristic. While the T. I. F. meter penalizes the 17th and 19th harmonics for 60-cycle alternators, the present wave-shape meter penalizes harmonics of phase rotation opposite to that of the fundamental.

Lastly, we find this meter admirably suited for showing and recording changes in alternator wave shape as the character of the load changes. Functioning thus, the wave-shape meter has brought to our attention some rather remarkable facts.

The Use of Vibration Instruments on Electrical Machinery

BY J. ORMONDROYD[1]

Non-member

Synopsis.—*Several mechanical vibration instruments are described and actual problems which these instruments have helped to solve are mentioned. The theory of the seismographic instruments is developed to show the relation between the record or indication and the motion being measured. The limitations in the accuracy of amplitudes recorded or indicated are brought out.*

INTRODUCTION

VIBRATION problems in the shop have frequently been left to the care of "penny balancers," men whose practical judgment was not tempered by any accurate knowledge of the vibration properties of bodies or the possible causes of vibration. Their opinions were based on data gathered by the unaided senses of touch, sight and sound. The effects of vibration on the senses of touch and hearing depend on amplitude and frequency together; and it is well known that the eye exaggerates any vibratory motion it observes. Subjective data almost always fail in accuracy. Knowledge is useful only when it becomes quantitative. The question "How much?" must be answered explicitly or implicitly before any action can be undertaken.

In any vibration problem there are five things which should be known:

1. Frequency of vibration.
2. Amplitude of vibration.
3. Type of vibration—simple-harmonic or complex.
4. The elastic properties and mass distribution of the vibrating body.
5. A general knowledge of the possible mechanical and electrical forces acting on the body.

The first three must be gotten by quantitative measurement. Observations of these three without measurement is usually valueless. The fourth is gotten from a knowledge of the materials and dimensions of the body. The fifth involves a clear understanding of the mechanical and electrical functioning of the body. This usually must go beyond the knowledge used to design the body since that knowledge, generally speaking, does not take into account the possibilities of vibration. No instrument can supply four and five. Trained human intelligence is necessary here.

It is proposed to show that this problem can be studied quantitatively in the shop and field. The theory and use of a few instruments suitable for this study will be taken up. Only mechanical instruments will be discussed. While electrical methods for measuring vibrations are well known, the inconveniences attending the use of the oscillograph have always discouraged any extended application of these methods on machines in production and in operation. Mechanical instruments have the following advantages over electrical vibration measuring devices which depend on the use of an oscillograph:

1. They are easily portable.
2. They can be applied to the job quickly, easily and cheaply.
3. The record or indication is available for immediate use.
4. The record or indication can be taken over a long period of time.
5. No complicated electric circuits are needed.

It should be stated, however, that electrical circuits can be devised to give space-, velocity- and acceleration-time effects; while mechanical instruments are limited to space-time effects. Very high frequencies demand the use of electrical methods.

The theory and practise of five instruments will be given.

Indicating Instruments.

1. The vibrating reed.
2. The vibration amplitude indicator.

Recording Instruments.

3. The directly connected vibrograph.
4. The vibrograph.
5. The torsiograph.

The vibration amplitude meter as described in the paper was developed by engineers of the Westinghouse Company although forms of this meter can be bought on the market. The other devices are commercial instruments widely used in Europe and slowly coming to the attention of American engineers. The use of each instrument is illustrated in the following pages by concrete examples.

The theory of the instruments is given in the appendixes. Mechanical vibration-measuring instruments as they now stand cannot be used safely unless their limitations are clearly appreciated. The relationship between the record or indication and the actual motion is not a simple one. Knowledge of the theory of this relationship must be considered a necessary tool to the man who uses the instruments.

LINEAR VIBRATIONS

Most of the vibrations in electrical machinery are linear in nature. Four of the five instruments dis-

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[1] Motor Engineering Department, Westinghouse Electric & Manufacturing Company.

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cussed are for the study of linear vibrations. In general, the fields of application of these four instruments are:

1. The vibrating reed—for measuring frequency only.

2. The amplitude meter—for measuring amplitudes only.

3. The seismic vibrograph—used where a complete analysis of the motion is desired. Applicable where the instrument cannot be attached to a fixed reference point.

4. The directly connected vibrograph—used where a complete analysis is wanted. Applicable where the instrument can be attached to some fixed reference point.

MEASURING FREQUENCIES—THE VIBRATING REED

The properties of resonance in a thin cantilever beam have long been used in turbo generator tachometers and electrical frequency meters. Fig. 1 shows a reed which can be varied in length and which can be attached to any vibrating body. The body should have a weight considerably larger than the weight of the instrument, say ten times as great, to preclude the possibility of the instrument affecting the motion of the vibrating body.

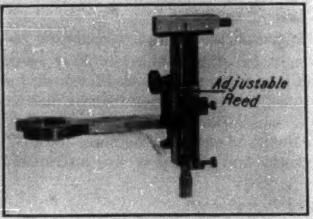

FIG. 1—REED VIBROMETER

The instrument shown in Fig. 1 consists of a claw to be clamped under a bolt head, two joints rotatable at right angles to each other, a main frame bearing a reed length scale on the side, an amplitude scale across the top and a long screw. A clamp carriage rides on the main frame; its position being adjusted by the screw. The reed is held tightly in a fixed clamp at the bottom of the frame and its free length is varied by the position of the movable clamp on the carriage.

The instrument is bolted to the vibrating machine and the free length of the reed is adjusted until the largest amplitude of motion is obtained at the end of the reed. This is read on the transverse scale. The instrument then is in resonance with the impressed frequency. This frequency can be determined by measuring the free length of the reed.

This device is so highly selective (damping forces extremely small) that it can be used only on vibrations with almost absolutely constant frequency. The least variation in frequency near the resonant point will give very large fluctuation in amplitude. This limits the instrument to uses on turbo generators and to cases where the cause of the vibration lies in magnetic pulls or in torque pulsations. These latter have frequencies depending on the voltage frequency of the line which varies slightly.

Given the same frequency, the amplitudes can be compared. Turbo generator balancing can be controlled by the use of a reed under these conditions.

A knowledge of the frequency of a vibration will often point to its cause. Unfortunately, the vibrating reed is too sensitive to be generally used for this purpose since in most cases the frequencies vary enough to spoil the accurate setting of the reed.

THE MEASUREMENT OF AMPLITUDES ONLY—AMPLITUDE METER

In many cases the frequency of vibration is known and the magnitude of its amplitude is of importance. Here the amplitude meter is a handy tool. This instrument consists of a mass suspended from a frame by a spring. (See Fig. 12.) The frame is held against or attached to the vibrating body. The relative motion between the spring-suspended mass and the frame is equal to the total range of the vibratory motion when the frequency of the motion is above a certain value. This relative motion is transmitted through levers to an indicator which makes a band image over a calibrated scale.

The most important use for the instrument is in field balancing of turbo generators. The instrument is attached to the bearing pedestal and four amplitude readings are taken—one with the original unbalance; the other three with an arbitrary unbalance applied successively in three different points of the balancing plane. The amplitude readings determine a system of vectors which give the amount and direction of the original unbalance. This rational procedure saves time and is more accurate than the old cut-and-try methods.

In cases where periodic forces have set stators of electrical machines in vibration, the amplitude meter has been used to determine the nodes or points at which the stator motion amplitude was zero. A check on the number and position of the nodes, along with a measurement of the frequency, indicates the curative measures to be taken. It also indicates the accuracy of design calculations.

The comparative roughness of commutators can be gaged with a small amplitude meter by holding it on the brushes. Where a definite minimum roughness is permissible on a given line of motors, this is a simple means of inspection.

Where large numbers of any one motor have to be made with a given minimum of vibration, the amplitude meter again serves as a means of objective inspection.

APPLICATION OF THE SEISMIC VIBROGRAPH

The seismic vibrograph is shown in Fig. 2. It consists of a heavy pendulum suspended between two springs. (See Fig. 13.) The pendulum is placed at right angles to the direction of the motion to be measured. The frame of the instrument is attached to the vibrating body and moves with it. The spring-borne pendulum stands still in space when the frequency of motion is above a certain value and the relative motion between the frame and the pendulum bob is transmitted through a system of levers to a recording pen. A strip of paper actuated by clockwork moves under the pen and a record of the motion shows as a wave on the paper. A timing device records equal time intervals on the same strip. The record can be analyzed for frequency, amplitude and type of motion.

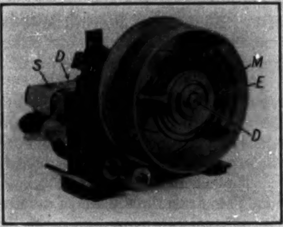

FIG. 2—SEISMIC VIBROGRAPH

The seismic vibrograph finds its chief application in the measurement of floor vibrations in buildings and electric locomotives. The absence of any fixed reference point in cases like these necessitates the use of a seismic instrument.

Although the cause of troublesome vibrations on factory floors or office buildings usually can be traced to its source without the help of instruments, there are cases where the cause is entirely elusive until vibration measurements are made. This is true where many pieces of rotating apparatus exist near together. A vibrograph record will show the frequency of the strongest vibration and this knowledge will usually point to the cause.

A common cure for floor vibration is mounting the machine which causes the disturbance on cork pads or springs. The design of a flexible mounting is determined by the weight of the machine and the frequency of the disturbance which it causes.[2] It is entirely possible to design a flexible mounting in such a manner that the transmitted vibrations become worse instead of better. In a rational design of flexible mounting the vibrograph supplies the necessary data on frequencies.

In gas or Diesel electric locomotives the floor vibrations may come from many sources. Unbalance in the reciprocating parts of the prime mover or in the rotating parts of the engine, generator and numerous auxiliaries—any one of these may be causing the objectionable disturbance. Usually they must all run simultaneously so no successive elimination of causes is possible. Another source of noticeable linear vibrations may be the periodic twist in the common foundation of the generator and engine due to the inherently varying torque of the prime mover. A vibrograph record usually shows up the troublesome frequency and leads to the proper remedy. This remedy may consist in the correcting of a bad unbalance, the changing of frame work structure to avoid local resonance, or the avoidance of resonance in the entire cab-spring system.

APPLICATION OF THE DIRECTLY CONNECTED VIBROGRAPH

Fig. 3 shows a side view of this instrument and Fig. 16 shows a top view. The instrument is clamped tightly to a body assumed to be absolutely stationary. A light rod transmits the motion from the vibrating body (See Fig. 16) to a bell crank which magnifies the motion and actuates the pen-push rod. The pen system, paper drive and timing device are all the same as on the seismic vibrograph (See Fig. 16).

The entire region close to a vibrating machine is often vibrating with the machine. In cases where this is so bad that the assumption of a stationary reference point cannot be approached even approximately, special mountings for the instrument must be used. In power houses a heavy weight—a ton or more—is suspended from the crane and the instrument is mounted on the weight (See Fig. 4). On the test floor a heavy mass mounted on springs has been used. In both cases a seismic system is formed having a low natural frequency which does not respond to the small high frequency forces transmitted through the bell crank. Usually the floor vibrations are small enough so that any object near the vibrating machine may be considered stationary enough to serve as a mounting for the instrument.

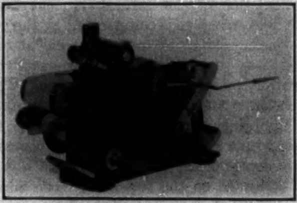

FIG. 3—DIRECTLY CONNECTED VIBROGRAPH

THE STUDY OF CRITICAL SPEEDS

In rotating apparatus the critical speeds are the most important objects of vibration study. Existing methods of balancing rotors are accurate enough to eliminate violent vibrations at any speeds except those in the region of the critical speeds. The directly connected vibrograph finds its most extensive use in the study of the critical speeds of structures and machines.

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[2] See C. R. Soderberg, *Elec. Journal*, Vol. XXI, 1924, p. 160.

EXPERIMENTAL DETERMINATION OF NATURAL FREQUENCIES

Any structure which is deformed by the application of external forces will return to its natural conformation when these forces are removed, assuming deformations within the elastic limit of the structure. If the applied force is in the form of a blow the return to the position of static equilibrium is made through damped oscillations having the natural frequency of the structure. This fact is used in determining the natural frequencies of bodies too complicated in shape to be calculated. The body is clamped in its natural position, a blow is struck and the vibrograph records the ensuing damped oscillation.

FIG. 4—VIBROGRAPH FOR POWERHOUSE

The calculation of the natural frequency of turbo generator field structure is complicated by the presence of field winding slots. A method of calculating this frequency was devised analytically and checked experimentally by means of the vibrograph. The mass of the generator rotor is too large to be affected by a blow; so the principle of resonance was used. A small d-c. motor with a badly unbalanced rotor was mounted on the top center of the large rotor which was held in pedestals mounted on the test floor. The vibrograph was mounted below the center of the field. The motor speed was varied until the amplitudes of vibration of the whole field became a maximum. A curve of the amplitudes read from the records indicated the critical speed very distinctly and a check between theory and practise was possible.

EFFECTS OF STRUCTURAL PARTS ON CRITICAL SPEEDS

Perhaps the most valuable service which the directly connected vibrograph has performed has been in the emphasizing of the tremendous differences which can exist between the critical speed in rotating apparatus, as calculated on the rotor alone and the actual critical speed of the completed and installed machine. When a draftsman in the ordinary course of design calculates the critical speed of his rotor he tacitly assumes that the bearings in which it will rotate are held rigidly fixed in space and that his calculated critical speed will be the critical speed of the completed machine. But the action of the bearing under the influence of unbalanced forces depends on the flexibilities of the pedestals, bed-plate and ultimate foundation. These additional flexibilities, usually neglected, all tend to make the actual critical speed lower than the calculated speed.[3] This fact is of importance since all small and medium sized machines are usually built to run below their calculated critical speeds.

The effects of parts of the structure external to the rotor shaft may range all the way from complete control of the actual critical speed (as in the case of all modern balancing machines, where the flexibility connected to the vibrating bed determines the frequency of the system) to the case where the bearing is in extremely rigid pedestals attached to an extremely rigid foundation. In this latter case the critical speed as calculated would be the actual critical speed in service.

An extensive series of tests was run by Dr. S. Timoshenko and Mr. L. S. Jacobsen on an experimental synchronous condensor to prove that the effects of structural flexibilities could be predetermined accurately. The machine was rated 5000 kv-a. at 750 rev. per min. They tried three shafts of different diameters and two different bed-plates—all other conditions remaining constant. Bed-Plate B was 40 per cent heavier and twice as stiff as Bed-Plate A (See Table I). The results are shown in Table I.

TABLE I

			Bed-plate A		Bed-plate B	
Direction of Vibration	Diameter of Shaft	Critical Rev. per min. Calculated on Rotor Alone	Actual Critical Rev. per min. No Field Excitation	Per Cent Lower	Actual Critical Rev. per min. No Field Excitation	Per Cent Lower
Horizontal	10.5 in.	1270	1050	17%	1110	12.5%
	12.5 in.	1470	1180	20%	1270	13.5%
	15.5 in.	1820	1325	27%	1460	20.5%
Vertical	10.5 in.	1270	1010	21%	1110	12.5%
	12.5 in.	1470	1130	23%	1280	13. %
	15.5 in.	1820	1220	33%	1410	22.5%

Another interesting case was an experimental elevator motor generator set consisting of two standard d-c. machines, the rotors of which were on a common shaft and the stators being mounted separately on a common bed plate. The critical speed calculated on the rotor alone was about 3200 rev. per min. The actual critical speed was found to be at 1850 rev. per min. due to the combined additional flexibility of the feet and bed-plate. The system in reality was one with several degrees of freedom instead of one. By tying the two stators together with a brace, the feet flexibilities were eliminated and the critical speed became 2800 rev. per min. (See Fig. 5.) When the inboard holding down bolts of one of the machines in the unbraced set was loosened only half of the foot material of that machine was acting as a restraint on its motion. This increase of foot flexibility lowered the critical speed to 950 rev. per min.

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[3] See C. R. Soderberg, *Elec. Journal*, Vol. XXI, No. 12, p. 579.

It can be seen that errors of critical speed calculation in the order of 40 per cent are possible in the ordinary course of design. This becomes important in case the designer thinks that he is running not far below the critical speed as he calculates it in the ordinary way.

Effects of Foundation. Synchronous machines have been tested on the test floor and in the field which showed a difference of 12 per cent in the critical speed on account of the differences in foundations under the bed-plate. And in turbo generators, mounted as they are in power houses, several critical speeds are passed through in going up to the running speed some of which are due to flexibilities in the foundation itself. This problem is one of extreme difficulty and it is being studied by means of the vibrograph.

Effect of Magnetic Pull. Unbalanced magnetic pull also lowers the critical speed.[4] A mechanical spring always tends to return to its position of equilibrium when stretched or compressed. A body in between the poles of a magnet is pulled away from its equilibrium position more and more the farther it gets from the position of equilibrium. An unbalanced magnetic pull has a negative spring effect which decreases the total spring constant of the whole system.[5] For a given machine this can be calculated. For a certain field excitation, it reaches a maximum effect; for stronger fields its effect falls off again. In the case of the synchronous condenser mentioned before the maximum effect was at 30 amperes, field current. Table II shows the effects at this maximum taken from vibrograph records.

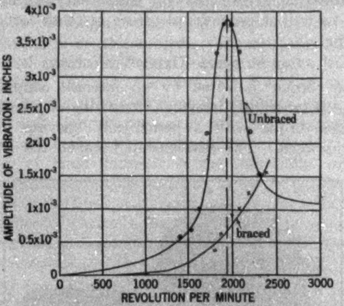

FIG. 5—AMPLITUDE CURVE—REV. PER MIN. M. G. SET VIBRATION OF EXPERIMENTAL M. G. SET WITH AND WITHOUT BRACE

TABLE II

Direction of Vibration	Shaft Dia.	Bed-Plate A			Bed-Plate B		
		Critical R.P.M		Per Cent Lower	Critical R.P.M.		Per Cent Lower
		No. Field	30 Amperes		No. Field	30 Amperes	
Horizontal	10.5	1050	930	11.5	1110	994	10.5
	12.5	1180	1070	9.0	1270	1165	8.0
	15.5	1325	1230	7.0	1460	1375	6.0
Vertical	10.5	1010	883	12.5	1110	994	10.5
	12.5	1130	1015	10.0	1280	1175	8.0
	15.5	1220	1120	8.0	1410	1320	6.0

VIBRATIONS NOT DUE TO CRITICAL SPEEDS

Noisy and violent vibrations cannot always be attributed to critical speeds. The case of an induction regulator standard for single phase, 60 cycles, but applied and rated for 25 cycles shows this very clearly. The machine was noisy on the test floor, due to impact between the segment teeth and the worm. The diagnosis of critical speed in resonance with the single-phase torque was made and the usual remedy of changing the shaft diameter was tried, but without diminishing the noise. The vibrograph was mounted to measure the tangential motion of the regulator gear segment. The supply frequency was varied and the kv-a. was kept constant. The resultant torsional vibration amplitude plotted from the records is shown in Fig. 6. Instead of resonance at 25-cycle voltage (50-cycle torque variation) the amplitudes tend to increase toward infinity at zero frequency—or to some extremely low frequency resonance amplitude due to the clearances in the worm and segment system.

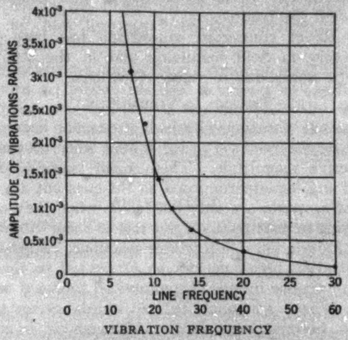

FIG. 6—AMPLITUDE-FREQUENCY CURVE—INDUCTION REGULATOR VIBRATION FREQUENCY CYCLES PER SECOND

The rotor was acting very similarly to a rotor free to rotate in bearings acted upon by a periodic torque.

The amplitude of torsional vibration of such a body is in general

$$\theta_0 = - \frac{\phi_0}{I\,\omega^2}$$

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[4] Rosenberg, TRANS. A. I. E. E., 1918, p. 1425.

[5] C. R. Soderberg, *Elec. Jour.*, Vol. XXI, No. 12, p. 582.

θ_0 = amplitude of torsional motion
ϕ_0 = amplitude of applied torque (proportional to the kv-a. in this case.)
I = the moment of inertia of rotor
ω = 4 π × line voltage frequency

The only fundamental way to limit the oscillation was shown to be by the application of a fly-wheel to increase the inertia of the rotor. Since this could not be done gracefully or conveniently (and the idea of a fly-wheel on an induction regulator being revolutionary to say the least) the noise was cut out by putting longitudinal flexibility in the worm and making the segment from a non-metallic material. This decreased the retardation value of the impact and completely eliminated the trouble.

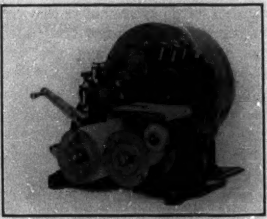

FIG. 7—TORSIOGRAPH

The direct connected vibrograph has been used extensively in field balancing and in the study of magnetic noises due to stator vibrations.

TORSIONAL VIBRATIONS

Torsional vibrations although of common occurrence cannot be observed very readily even when they are of destructive magnitude. These small periodic angles of lead and lag superimposed on the constant advance of rotating parts are only noticeable when they cause chattering in gear teeth or give rise to linear vibrations on the foundation due to stator reactions. Most shaft and coupling breakage can be attributed to the dynamical stresses induced by unnoticed torsional vibrations. Where torsional critical speeds occur these stresses become very large. Although it is much easier to calculate torsional critical speeds than linear critical speeds, very few designers check this important point. Torsional vibrations being hidden are ignored.

The torsiograph has an important educational mission in emphasizing the existence of these oscillations in a concrete way. Its chief uses are in the study of torsional critical speeds, in the location of the causes of vibration and in the checking of design theories.

The torsiograph consists of a light pulley and a fly-wheel mounted concentrically on the same axis. (See Figs. 7 and 8.) The fly-wheel is held in a definite static relationship to the pulley by means of two opposing springs. (See Fig. 14.) Relative motion between the pulley and the fly-wheel about this position of equilibrium is possible. The relative motion actuates a system of bell cranks which move a recording pen push rod. The recording-paper drive and timing device are exactly the same as for the vibrograph. In addition to the timer there is a pen actuated by a solenoid which is excited by the making of a contact once every revolution of the body being tested. This indicates the instantaneous angular position of the body.

The pulley is belted to the shaft to be tested and follows its motions. The fly-wheel tends to rotate at a uniform average speed for frequencies above a certain value. The relative motion between the irregularly moving pulley and the steadily moving fly-wheel produces the record.

TORSIONAL VIBRATIONS DUE TO MECHANICAL CAUSES

Gear vibrations have been studied with the torsiograph—the most interesting conclusion drawn being that torsional vibration with a steady tooth contact frequency only occurs when the gears are heavily loaded. This points to the fact that any dynamical theory of gearing must take into account the effects of deflections in the gear structure (the only thing that changes with load) as well as geometrical errors in tooth form and spacing.

GAS ELECTRIC DRIVE VIBRATIONS

The torque delivered by an internal combustion engine is inherently irregular, consisting of a constant average torque with a periodically varying torque superimposed. The relationship between the magnitudes of these two torques depends on the number of cylinders and the number of strokes per combustion cycle. The fundamental of the periodic torque has the firing frequency and all other higher harmonics also exist. The mechanical system consisting of generator rotor, fly-wheel and crankshaft with its attached reciprocating masses, has many natural frequencies. Theoretically the combination of the harmonics of the variable torque and the many natural frequencies of the rotational system lead to an infinite number of possible critical rev. per min. of the generating unit. Practically resonance with the fundamental and perhaps the second harmonic have given the im-

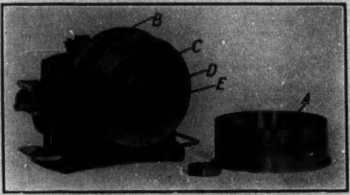

FIG. 8—TORSIOGRAPH

portant critical speeds. The other harmonics are usually too small to produce noticeable effects.

Several gas electric generating units have been investigated by means of the torsiograph—the critical speeds determined and operating speed limits have been established. The theories for precalculating the critical speeds have been checked experimentally by means of the instrument.

Where flexible couplings have been used on gas electric units records on both sides of the coupling have shown how the coupling acts as a wave filter, only the lowest harmonic of the variable torque being prominent on the generator side of the coupling and at speeds far above the lowest critical speed of the rotational system even the fundamental gets very small.

TORSIONAL OSCILLATIONS IN SIDE-ROD ELECTRIC LOCOMOTIVES

The torsional oscillation of the rotating system of a side rod electric locomotive is an important problem where locomotive frames, rods and cab structure are very light as in European practise. In the heavier American designs this problem loses much of its importance; but the careful designer attempts to make the possibilities of destructive torsional vibrations remote. The torsiograph has been used extensively in Europe in checking the numerous theories which have been developed on this subject. Theoretically, errors in side rod length (or "tram") errors in crank length and quarter, and pin clearances, can give rise to oscillation having driver rotational frequency, and the second and fourth harmonic of this frequency. Torsiograph records show that these exist; but the presence of large damping forces (especially where flexible gears and pinions are used) keeps the amplitude of these oscillations within safe limits even at the critical speeds. An experimental check can be made on design theories devised to lessen the possibilities of destructive action.

Where gears are interposed between the motors and drivers, resonance with the tooth frequency vibrations must be avoided on all parts of the motors and locomotive frame near the motor bearings. The torsiograph records show what conditions of load are necessary to bring about large amplitude of these gear frequency oscillations.

TORSIONAL VIBRATION FROM ELECTRICAL CAUSES

It is a well-known fact that single-phase rotating machines vibrate torsionally with double line-voltage frequency[6] and amplitudes proportional to the kv-a. The reaction of these single-phase torques through the stators of large machines have made it advisable in some cases to go to special spring mounting to protect the foundation of the machine from the double-line frequency forces.[7] The torsiograph makes a very convenient means of studying these oscillations.

It is generally assumed that polyphase motors rotate uniformly under all conditions. Torsiograph records in many instances show double line frequency torsional oscillation just as in the case of single-phase machines—the amplitudes being much smaller. Experimentally this was shown to be due to unbalance in the primary winding—unbalanced voltage or unbalanced impedance being equally effective. It can be shown theoretically by the use of symmetrical coordinates,[8] that the positive and negative sequence currents and voltage due to electrical unbalance produce a small torque of double line frequency superimposed on the average constant torque expected. The amplitude of the resultant vibrations is proportional to the amount of unbalance. Similarly it has been shown theoretically and experimentally that unbalanced resistances in the secondary of a wound rotor motor can produce destructive torsional vibrations on the rotor[9].

Unbalance in the secondary currents which have a fundamental frequency equal to the slip frequency will induce currents in the primary circuit having a frequency of $f - (n + 1)\, s$ where

f = line frequency
n = number of unbalanced secondary harmonic
s = slip frequency

Torques produced by a current having a frequency $f - (n + 1)\, s$ and a flux with frequency f will be of two frequencies, $2f - (n + 1)\, s$ and $(n + 1)\, s$.

This theory was developed by C. R. Soderberg and tested by him by means of the torsiograph and oscillograph.

In tying any a-c. motor onto a common shaft bearing other rotating masses the designer must be careful to avoid resonance with double the line frequency of the a-c. supply. This is especially necessary if the motor is single-phase but should not be neglected even in the case of polyphase motors.

Appendix I

GENERAL THEORY OF THE SEISMOGRAPH

All of the instruments discussed, except the directly connected vibrograph, are built on the same principles as the seismograph or earthquake recorder. In developing the theory of these seismic instruments the vibratory motion considered will be simple harmonic motion. Any complex periodic motion consists of a sum of simple harmonic motions, so that the problem is completely solved when the form of one term of the series can be determined. The instruments will be considered as systems with one degree of freedom. Deviations from this assumption will be taken up in Appendix III.

Fig. 9 represents the simplest seismographic system for measuring vertical vibrations. The mass M is suspended from the frame B by means of the spring which has the characteristics K (force per unit extension); B is rigidly attached to the body A which has the vertical motion.

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[6] A. L. Kimball and P. L. Alger, TRANS. A. I. E. E., 1924, p. 730.

[7] C. R. Soderberg, *Elec. Journ.*, Vol. XXI, No. 8, p. 383.

[8] C. L. Fortescue, TRANS. A. I. E. E., 1918, p. 1027.

[9] A. F. Kenyon, *Elec. Journ.*, Vol. XXII, No. 9, p. 435.

$$Y = Y_0 \sin \omega t \qquad (1)$$

It is tacitly assumed that the total mass of the instrument is small compared with the mass of the body to which it is attached. For engineering purposes the mass of the seismograph should not be more than about one-tenth the mass of the vibrating body.

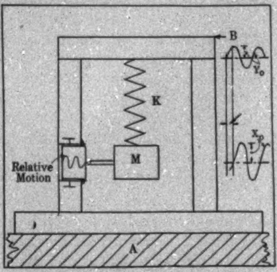

FIG. 9—SIMPLE SEISMIC SYSTEM RECORDING SEISMOGRAPH FOR VERTICAL MOTION

THE ABSOLUTE MOTION OF THE MASS

The motion of A is transmitted by the rigid frame B to the point of suspension of the spring-mass system. The mass has some motion x, as yet undetermined. Motions x and Y are both absolute motions taken positive in the same direction and referred to the same system of coordinates fixed in space. The center of gravity of M under static conditions will be taken as the origin of the coordinate system. The forces acting on M under the given conditions of motion are:

1. An inertia force $- M \dfrac{d^2 x}{d t^2}$
2. A damping force $- c \dfrac{d x}{d t}$
3. A spring force $- K (x - Y)$

c is a force per unit velocity. The damping factor c cannot be predetermined readily—depending as it does on friction, windage and internal mechanical hysteresis of the spring material. That the damping is proportional to the velocity of motion is a pure assumption made for simplicity and justified by experimental results. No other forces act on M and, since these must be in balance at all times, we have

$$M \frac{d^2 x}{d t^2} + c \frac{d x}{d t} + K (x - Y) = 0 \qquad (2)$$

Substituting the value of Y given in equation (1) we get

$$M \frac{d^2 x}{d t^2} + c \frac{d x}{d t} + K x = K Y_0 \sin \omega t \qquad (3)$$

The general solution of (3) is

$$x = e^{-\frac{c}{2M}t} \left[C_1 \cos \sqrt{\frac{K}{M} - \frac{c^2}{4 M^2}}\, t \right.$$

$$\left. + C_2 \sin \sqrt{\frac{K}{M} - \frac{c^2}{4 M^2}}\, t \right] \qquad (4)$$

$$+ \frac{K}{\sqrt{c^2 \omega^2 + (K - M \omega^2)^2}} Y_0 \sin \left(\omega t - \tan^{-1} \frac{c \omega}{K - M \omega^2} \right)$$

The complementary solution contains C_1 and C_2 arbitrary constants which must be evaluated by means of two initial conditions. But this term dies away in time and in the measurement of machinery vibrations we are not interested in this free vibration of the instrument system itself; but rather we are interested in the particular integral which gives us the law of the forced vibrations which we are measuring.

The forced motion of M, or the particular integral of (3) can be written:

$$x = \frac{1}{\sqrt{\left(\frac{c}{M \omega_c}\right)^2 \frac{\omega^2}{\omega_c^2} + \left(1 - \frac{\omega^2}{\omega_c^2}\right)^2}}$$

$$Y_0 \sin \left(\omega t - \tan^{-1} \frac{c \omega}{K - M \omega^2} \right) \qquad (5)$$

where ω_c is defined by equation (7) below.

From (5) it can be seen that M has the same type of motion as the impressed motion, simple harmonic; it has the same frequency and its amplitude is always proportional to the amplitude of the impressed motion. The motion of M lags behind the impressed motion. The angle of lag and the absolute value of the amplitude of x are functions of the impressed frequency and the constants of the seismic system.

At a certain frequency of the impressed motion the movement of M will become very large—it will reach a maximum value. This is called the critical speed of the instrument, and the impressed frequency is in resonance with the natural frequency of the instrument.

The natural frequency of the instrument can be seen to be (see equation (4))

$$\omega_c = \sqrt{\frac{K}{M} - \left(\frac{c}{2 M}\right)^2} \qquad (6)$$

but $\left(\dfrac{c}{2 M}\right)^2$ is usually such a small quantity that it can be neglected and the natural frequency is closely equal to

$$\omega_c = \sqrt{\frac{K}{M}} \qquad (7)$$

This occurs when the frequency is such that

$$K - M\,\omega^2 = 0 \tag{8}$$

At this frequency the amplitude of the motion x, x_0, is only limited by the damping factor.

$$x_{0(\omega=\omega_c)} = \frac{K}{c\,\omega_c}\,Y_0 \tag{9}$$

and

$$\tan^{-1}\frac{c\,\omega_c}{K - M\,\omega_c^2} = \frac{\pi}{2} \text{ or } 90^\circ \tag{10}$$

Fig. 10a showing the amplitude of the motion x and Fig. 10b showing the angle of lag are plotted on the following data:

Y_0 = 1 in. (constant for all frequencies)
M = 0.20 lb. mass
K = 10.00 pounds per inch
c = 0.00, 0.066 and 2.83 pounds per inch per second

$c = 2.83$ is the value at which the instrument is a periodic, that is where

$$\frac{K}{M} - \frac{c^2}{4\,M^2} = 0 \tag{11}$$

From Fig. 10a and Fig. 10b it can be seen that when $\frac{\omega}{\omega_c} > 3$, the mass M undergoes only a fraction of the impressed motion and that it lags behind the impressed motion almost 180 deg.

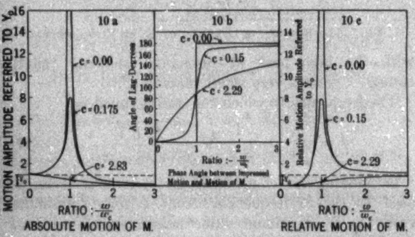

FIG. 10—AMPLITUDE-FREQUENCY CURVES, SIMPLE SEISMIC SYSTEM

Now, suppose a pencil were attached to M extending perpendicular to its direction of motion and scribing on a strip of paper moved in a direction perpendicular to the plane of the pencil motion by a mechanism attached to the Frame B (See Fig. 9). At and near $\frac{\omega}{\omega_c} = 0$ B and M are moving the same amount and in phase. There will be no record on the paper. At $\frac{\omega}{\omega_c} = 1$ the record will be much larger than the actual motion. At $\frac{\omega}{\omega_c} = \infty$ $\left(\text{practically for } \frac{\omega}{\omega_c} \geqq 4\right)$, B will move with amplitude Y_0, M will stand still in space and the record will have the same amplitude as the impressed motion, but it will be 180 deg. out of phase—when the motion is up, the record will be down.

Evidently, in interpreting the record, we are interested in the relative motion between the suspended mass and the frame of the instrument.

THE MOTION OF MASS M RELATIVE TO THE FRAME B

The motion of B is assumed again as an absolute motion

$$Y = Y_0 \sin \omega\,t \tag{12}$$

The *mass* M has a motion x_1 relative to the *frame* B. The force equation under these conditions becomes:

$$M\frac{d^2}{d\,t^2}(Y + x_1) + c_1\frac{d\,x_1}{d\,t} + K\,x_1 = 0 \tag{13}$$

or substituting the values of Y

$$M\frac{d^2\,x_1}{d\,t^2} + c_1\frac{d\,x_1}{d\,t} + K\,x_1 = M\,\omega^2\,Y_0 \sin \omega\,t \tag{14}$$

The inertia force on M depends on the total motion of M, and c_1 differs from c in equation (2) in that the atmosphere is here assumed to be moving with the instrument. Practically $c_1 = c$.

The particular integral of (14) can be written:

$$x_1 = \frac{\dfrac{\omega^2}{\omega_c^2}}{\sqrt{\left(\dfrac{c_1}{M\,\omega_c}\right)^2\dfrac{\omega^2}{\omega_c^2} + \left(1 - \dfrac{\omega^2}{\omega_c^2}\right)^2}}\; Y_0 \sin\left(\omega\,t - \tan^{-1}\frac{c_1\,\omega}{K - M\,\omega^2}\right) \tag{15}$$

where ω_c is defined by equation (7).

Using the same system constants as before, Fig. 10c shows the amplitude of X_1, or the record, as a function of $\frac{\omega}{\omega_c}$. The angle of lag is exactly like the angle of lag in the case of absolute motion as shown in Fig. 10B.

From Fig. 10C it can be seen that the amplitude of the record is very closely equal to the amplitude of the impressed motion when $\frac{\omega}{\omega_c} \geqq 4$. The error at $\frac{\omega}{\omega_c} = 4$ is about 6 per cent. It also shows that the value of the damping factor does not materially affect the results given above $\frac{\omega}{\omega_c} > 4$.

This brings out the fundamental limitation of seismic instruments. To get accurate records the vibration measured must have a frequency at least four times greater than the natural frequency of the instrument. For frequencies below this the instrument must be calibrated and corrections must be made on the records. What is more, the damping factor c changes with the age of the instrument, due to wear in the parts and accumulations of dirt and also with the lubricant used in any pivots or bearings. In the region above $\frac{\omega}{\omega_c} = 4$ this change can be very great without affecting the results. But when used below this range the instrument must be calibrated frequently.

Appendix II

THE THEORY OF THE ACTUAL INSTRUMENTS

The relative motion equations for actual instruments are of the same form as developed in Appendix I. The mechanical systems are usually more complicated and, as a result of this, the constants are different from those in the simple systems.

The assumptions made and units used will be:

TABLE III

	Linear Systems	Rotational Systems
Number of degrees of freedom	one	one
Impressed motion	$Y = Y_0 \sin \omega t$ Y_0 is small	$\phi = \phi_0 \sin \omega t$ ϕ_0 is small
Relative motion	x	θ
Coefficient inertia term	lb. mass	lb. mass in.^{2}
Coefficient of damping term	Lb. per in. per sec.	Lb-in. per radians per sec.
Coefficient of spring term	Lb. per in.	Lb-in. per radians
Right hand member-external action term	Lb.	Lb-in.

Since the damping factor is not predeterminable and small it will be designated simply by c in each case.

The Vibrating Reed. The simplest vibration instrument is the vibrating reed. It consists of a thin cantilever beam bearing a small mass on its free end. Fig. 11a shows such a system mounted for vertical vibration measurement.

This instrument is an indicating device—the eye seeing the relative motion between the mass M and the base B. The force equation is

$$\left(M + \frac{m}{4}\right)\frac{d^2 x}{d t^2} + c\,\frac{d x}{d t} + \frac{3 E I}{l^3}\,x = \left(M + \frac{m}{4}\right)\omega^2 Y_0 \sin \omega t \qquad (16)$$

where m is the mass of the cantilever beam and $\frac{3 E I}{l^3}$ represents the spring constant for a perpendicular force acting at the free end of the beam (Fig. 11b).

This instrument is usually used for detecting frequencies only and depends on its high selectivity or "sharp tuning" for this result. The value of c is usually extremely small which results in very large amplitudes of motion at and very near to the natural frequency of the reed and small effects in all other regions. Fig. 11c shows the effect of the magnitude of c on the "tuning." The curve of amplitudes against $\frac{\omega}{\omega_c}$ with the low peak is for a large value of c. The curve with the high peak at resonance—showing "sharp tuning"—is for a very small value of c.

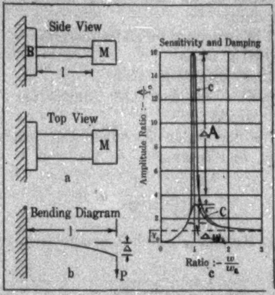

FIG. 11—SCHEMATIC DIAGRAM OF REED VIBROMETER

The frequency to which the reed is tuned is

$$f = \frac{\omega_c}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{12 E I}{l^3\,(4 M + m)}} \qquad (17)$$

By making the length l variable, a given reed can be used to detect frequencies from a given minimum value up to a maximum value determined by practical considerations.

THE VIBRATION AMPLITUDE METER

Fig. 12a shows a schematic arrangement of this instrument.

The relative motion between M and B is magnified by the indicator system and the maximum value of the total range is shown by the width of the band image of the indicator point vibrating over the calibrated scale. The magnification μ is

$$\mu = \frac{a}{b} \qquad (18)$$

The equation of motion is

$$\left(M + \frac{I}{b^2}\right)\frac{d^2 x}{d t^2} + c\,\frac{d x}{d t} + \left[K_1 + \left(\frac{c}{b}\right)^2 K_2\right]x = \left(M + \frac{I}{b^2}\right)\omega^2 Y_0 \sin \omega t \qquad (19)$$

where I is the moment of inertia of the indicator about its axis of rotation and all the other quantities are indicated in Fig. 12a. A typical curve of the indicated amplitudes in relationship to the impressed or measured amplitude is shown in Fig. 10c. In order to get an accurate indication the frequency measured should be at four times the natural frequency of the instrument.

The natural frequency of the instrument is

$$f_n = \frac{\omega_c}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K_1 + \left(\frac{c}{b}\right)^2 K_2}{M + \frac{I}{b^2}}} \qquad (20)$$

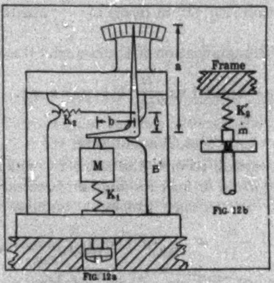

FIG. 12—SCHEMATIC DIAGRAM OF AMPLITUDE METER

UPPER LIMIT TO ACCURACY OF INDICATOR

The question of indication accuracy above $\frac{\omega}{\omega_c} = 4$ depends on the exact following of the motion of M by the indicator. This condition is expressed mathematically by saying that the force between the indicator and the mass M at their point of contact must never equal zero. An initial contact pressure is developed by putting an initial tension in K_2 and a compression in K_1. The amount of initial set necessary to get accurate readings under a given set of conditions can be calculated. Considering Fig. 12B, we have a schematic view of the indicator system where

$$m = \frac{I}{b^2} \qquad (21)$$

$$K_2' = \left(\frac{c}{b}\right)^2 K_2 \qquad (22)$$

and the mass M has the motion.

$$Y = Y_0 \sin \omega t \qquad (23)$$

The spring K_2' has an initial compression x_0 which produces a static contact force between M and m. When the position of M is positive, both the inertia reaction of m and the spring force in K_2' increase this contact force; but when the position is negative the inertia reaction is upward and the total spring force is diminished. Both these effects reach their extreme values at the instant the mass M reaches its lowest position. Let the contact force be F. At the lowest position in the limiting case we have

$$F = K_2' (x_0 - Y_0) - m\,\omega^2 Y_0 = 0 \qquad (24)$$

The value of ω^2 at this point is

$$\omega^2 = \omega_c^2 \left(\frac{x_0}{Y_0} - 1\right) \qquad (25)$$

where

$$\omega_c^2 = \frac{K_2'}{m} \qquad (26)$$

For $\frac{x_0}{Y_0} \leqq 1$ the indicator will not follow the motion of M accurately at any frequency since the limiting frequency becomes zero or imaginary. For $\frac{x_0}{Y_0} > 1$ the frequency which can be followed successfully depends entirely on the stiffness of the indicator spring and the inertia of the indicator. When the ratio $\frac{K_2'}{m}$ is large, high frequencies can be measured accurately.

THE SEISMIC VIBROGRAPH

The seismic vibrograph is shown in Fig. 2 and schematically in Fig. 13a, which shows the recording system as well as the spring mass system. Although the instrument measures linear motions it has a rotational mechanical system. The mass M becomes a pendulum suspended between the clock spring noticeable in Fig. 2 and the recording system spring. This is shown more clearly in Fig. 13b which further simplifies the system. In Fig. 13b, K_1' represents the clock spring reduced to an equivalent coil spring at the point where the recording system touches the pendulum and K_2' is the pen spring reduced to an equivalent spring acting at the same place. Fig. 13c shows

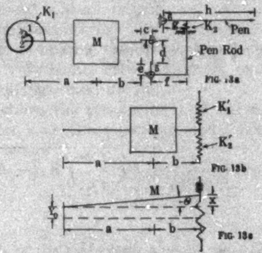

FIG. 13—SCHEMATIC DIAGRAM OF SEISMIC VIBROGRAPH

schematically the motions encountered by this system. The whole instrument undergoes the linear motion.

$$Y = Y_0 \sin \omega t \qquad (27)$$

and the pendulum also rotates through an angle θ with respect to the frame. Taking moments about the pendulum axis of rotation, the moment equation becomes

$$M \rho^2 \frac{d^2 \theta}{d t^2} + c \frac{d \theta}{d t} + \left[\frac{1}{(a+b)^2} K_1 + \left(\frac{f d}{e c} \right)^2 K_2 \right] (a+b)^2 \, \theta = a M \omega^2 Y_0 \sin \omega t \qquad (28)$$

where

ρ = the radius of gyration of pendulum about its axis of rotation.

a = the distance between the center of gravity of the pendulum and the axis of rotation.

$a+b$ = the distance between the pendulum axis of rotation and the point at which recording system is in contact with the pendulum.

$\frac{K_1}{(a+b)^2}$ and $\left(\frac{f d}{e c} \right)^2 K_2$ are reduced values of the spring constants K_1 (# in./radian) and K_2 (#/in.)

The particular integral of (**28**) gives us the relationship between the relative rotation θ and the impressed motion. Since θ is small it can be reduced to a linear relative motion at the recording system point of contact. The relative linear motion there will be

$$x = (a + b)\, \theta \qquad (29)$$

and

$$x = \frac{a + b}{\left(\frac{\rho^2}{a} \right)} \frac{\frac{\omega^2}{\omega_c^2}}{\sqrt{\left(\frac{c}{M \rho^2 \omega_c} \right)^2 \frac{\omega^2}{\omega_c^2} + \left(1 - \frac{\omega^2}{\omega_c^2} \right)^2}} \, Y_0 \sin (\omega t - \phi) \qquad (30)$$

where

$$\phi = \tan^{-1} \frac{c \omega}{(a+b)^2 \left[\frac{1}{(a+b)^2} K_1 + \left(\frac{f d}{e c} \right)^2 K_2 \right] - M \rho^2 \omega^2} \qquad (31)$$

and

$$\omega_c^2 = \frac{\left[\frac{1}{(a+b)^2} K_1 + \left(\frac{f d}{e c} \right)^2 K_2 \right] (a + b)^2}{M \rho^2} \qquad (32)$$

The record x_1 will be x multiplied by the factor μ due to the pen lever system.

$$\mu = \frac{d f h}{c e g} \qquad (33)$$

Equation (**30**) shows that the final magnification of the record not only depends on the frequency of impressed motion, and the system of levers in the recording system, but also on the distribution of mass in the pendulum. $\frac{\rho^2}{a}$ is the distance from the center of percussion to the axis of rotation of the pendulum. If, for any reason, the shape or mass of the pendulum is changed the record must be understood to have a new magnification constant. The total kinematic magnification of the record is

$$\mu = \frac{a + b}{\left(\frac{\rho^2}{a} \right)} \frac{d f h}{c e g} \qquad (34)$$

The fraction (Eq. 30) in terms of $\frac{\omega}{\omega_c}$ might be called the dynamic magnification of the record. It approaches the asymtotic value 1 as $\frac{\omega}{\omega_c}$ becomes large.

THE TORSIOGRAPH

The torsiograph records the angle by which the rotating body leads or lags its angular position in space referred to an average constant angular velocity.

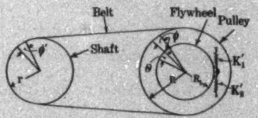

FIG. 14—SCHEMATIC VIEW OF TORSIOGRAPH ATTACHED TO SHAFT

The instrument records an angle-time curve. When the body tested rotates uniformly it records a straight line. There is no angle of lead or lag in this case.

The torsiograph is shown in Fig. 3 and Fig. 4; schematically it is shown in Fig. 14.

The fly-wheel with moment of inertia I is mounted by means of ball bearings on a hollow axis which is part of the pulley and acts as the pulley's bearing on the stationary axis attached to the instrument frame. The fly-wheel I can rotate relatively to the pulley B but it is held in a static position by means of two opposing springs, the clock spring and the pen spring, exactly in the same manner as in the case of the seismic vibrograph.

$$K_1' = \frac{K_1}{R_1^2} \text{ (K_1 is in # in./radian)} \qquad (35)$$

$$K_2' = \left(\frac{f d}{e c} \right)^2 K_2 \qquad (36)$$

(K_2 is in #/in. and the same type recording system is assumed as in the seismic vibrograph.)

We consider an oscillation superimposed on a steady rotation or existing by itself. The analysis is the same in either case. Taking moments about the center of rotation the moment equation becomes:

$$I\frac{d^2\theta}{dt^2}+c\frac{d\theta}{dt}+\left[\frac{1}{R_1^2}K_1+\left(\frac{fd}{ec}\right)^2K_2\right]R_1^2\,\theta$$

$$= I\,\omega^2\,\psi_0 \sin \omega t \qquad (37)$$

R_1 is the radius to the point of contact between the fly-wheel and the recording system. θ and ψ_0 are defined in Table III. The particular integral is

$$\theta=\frac{I\,\omega^2}{\sqrt{c^2\,\omega^2+(K-I\,\omega^2)^2}}\,\psi_0 \sin\left(\omega t-\tan^{-1}\frac{c\,\omega}{K-I\,\omega^2}\right) \qquad (38)$$

where

$$K=\left[\frac{1}{R_1^2}K_1+\left(\frac{fd}{ec}\right)^2K_2\right]R_1^2 \qquad (39)$$

INTERPRETATION OF TORSIOGRAPH RECORD

θ is relative angular motion between the pulley and fly-wheel; but the recording system transforms this to linear motion at the record and magnifies it.

Assuming an inextensible belt and an impressed frequency very large compared with the natural frequency of the instrument $\left(\text{defined by } \omega_c^2=\frac{K}{I}\right)$ an oscillation ϕ' on the tested shaft (see Fig. 14) will give a linear motion x_1 on the torsiograph pulley and this in turn will give a linear motion x_2 at the point of contact between the recording system and the fly-wheel such that

$$x_2=\frac{R_1}{R}\,\phi'\,r \text{ (quantities defined in Fig. 14)} \qquad (40)$$

This in turn is magnified to x_3 at the record where

$$x_3=\frac{dfh}{ceg}\,\frac{R_1}{R}\,\phi'\,r \qquad (41)$$

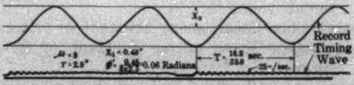

FIG. 15—INTERPRETATION OF TORSIOGRAPH RECORD

The recording lever multiplication is for a system as shown in Fig. 13A.

The kinematic magnification factor is

$$\mu=\frac{dfh}{ceg}\,\frac{R_1}{R} \qquad (42)$$

and

$$\phi'=\frac{x_3}{\mu\,r} \qquad (43)$$

Fig. 15 represents this interpretation of a torsiograph record. If the ratio $\frac{R_1}{R}$ is changed for any reason, the magnification is changed. This is important, since it indicates that any change in the size of pulley changes the magnification, a large pulley giving small magnification. Also for a given angular motion the magnitude of the record x_3 will be proportional to the radius of the tested shaft. Extremely small motion should be recorded from a shaft with as large a diameter possible. Large motions should be measured on small diameter shafts to keep the record within the limits of the instrument.

THE DIRECTLY CONNECTED VIBROGRAPH

This instrument merely transmits and magnifies the vibratory motion. Its use assumes that the frame of the instrument is absolutely at rest with respect to the earth. This assumption can be realized to a very high degree of approximation by mounting the instrument on a very heavy mass suspended on a long cable or on soft springs.

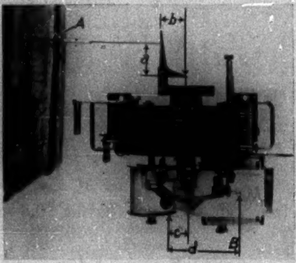

FIG. 16—DIRECTLY CONNECTED VIBROGRAPH (TOP VIEW)

Fig. 16 shows the arrangement of the instrument.

A motion x at the vibrating body becomes x_2 on the record where

$$x_2=\frac{bd}{ac}\,x \qquad (44)$$

Here

a = length of the bell crank arm parallel to the vibrating body.
b = length of other bell crank arm.
c = distance from pivot of pen to pen rod.
d = total length of pen.

TIMING DEVICE ON RECORDING INSTRUMENTS

The time measurement is made on the recording instruments by means of the free vibrations of a loaded Cantilever beam. The beam is mounted vertically above the record paper and bears a mass M near its lower free end. On the lower end, a pen is attached which traces the beam oscillation on the record strip.

The mass M is so large compared with the mass of the beam of length l that the mass of the beam can be neglected. The equation of forces is:

$$M \frac{d^2 x}{d t^2} + c \frac{d x}{d t} + \frac{3 E I}{l^3} x = 0 \qquad (45)$$

The initial conditions are determined by the fact that the beam is displaced from its position of equilibrium and released producing damped free vibrations. The conditions are

$$x = x_0 \text{ when } t = 0 \qquad (46)$$

$$\frac{d x}{d t} = 0 \text{ when } t = 0 \qquad (47)$$

The solution is

$$x = x_0 e^{-\frac{c}{2M} t} \left[\cos \sqrt{\frac{3 E I}{l^3 M} - \frac{c^2}{4 M^2}}\, t \right.$$
$$\left. + \frac{c}{2 M} \sin \sqrt{\frac{3 E I}{l^3 M} - \frac{c^2}{4 M^2}}\, t \right] \qquad (48)$$

Since $\frac{c}{2 M}$ is very small this becomes approximately

$$x = x_0 e^{-\frac{c}{2M} t} \cos \sqrt{\frac{3 E I}{l^3 M}}\, t \qquad (49)$$

This traces an oscillation having the constant frequency

$$f = \frac{1}{2 \pi} \sqrt{\frac{3 E I}{l^3 M}} \qquad (50)$$

which gives a convenient measure of the time.

Appendix III

LIMITATIONS OF THE INSTRUMENTS

The instruments have been treated as systems with one degree of freedom. This leads to results important for the correct interpretation of records or indications; but it does not explain certain anomalous actions which occur under certain conditions.

Under the assumption of one degree of freedom, accurate records should be possible from $\frac{\omega}{\omega_c} = 4$ to $\frac{\omega}{\omega_c} = \infty$ but under certain conditions records can be obtained which cannot be explained by any possible physical data of the machine tested. Taking the torsiograph as an example, two conditions may arise in practise:

1—Oscillations so large may be recorded that the forces necessary to produce them in the machine tested cannot possibly exist.

2—Small high frequency vibrations which record at the lowest magnification may disappear from the record when higher magnifications are used.

The only explanation for these anomalous actions lies in the assumption of several degrees of freedom in the instrument. When the torsiograph is examined in detail this seems to be justified. Instead of an inextensible belt turning an inertialess pulley as tacitly assumed in Appendix II, we have a belt material which is elastic and a pulley which has a small, but appreciable inertia. Similarly, instead of a weightless pen attached rigidly to the pen push rod, we have a pen which has weight gripped by pivots mounted on a springy fork. The torsiograph is in reality a system with three degrees of freedom.

For convenience in analysis the system will be shown as an equivalent linear system. This system in turn can be broken into two separate systems since the mass of the pen is so small that it does affect the motion of the fly-wheel even when it may serve to distort the record. The damping factor will be considered equal to zero. This will give us infinite values of amplitude at certain points and will give the curve of amplitudes positive and negative branches. By plotting all branches on the positive side of the coordinate system there will be a curve which will differ from the facts only in the narrow regions around the critical points.

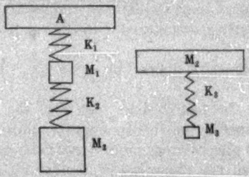

FIG. 17—EQUIVALENT LINEAR SYSTEMS FOR TORSIOGRAPH

Fig. 17 shows the systems assumed. In Fig. 17

A = body tested—moment of inertia large compared with moment of inertia of torsiograph rotational system.
K_1 = equivalent linear spring constant of belt.
K_2 = equivalent linear spring constant of fly-wheel springs.
K_3 = equivalent linear spring constant of pen rod pivot fork.
M_1 = equivalent mass of pulley.
M_2 = equivalent mass of fly-wheel.
M_3 = equivalent mass of pen.

$$K_1 = \frac{C}{L} \qquad (51)$$

where C depends on the material and cross-section of the belt and L is the distance between centers of torsiograph pulley and tested shaft.

$$M_2 = \frac{I}{r^2} \qquad (52)$$

where I is the moment of inertia of the pen about its axis of rotation and r is the distance from that axis to the pen rod fork pivots. This distance is inversely proportional to the magnification used.

The body A has the motion.

$$Y = Y_0 \sin \omega t \qquad (53)$$

M_1 has a motion x_1 relative to A. M_2 has a motion x_2 relative to M_1. M_3 is actuated only by the relative motion between M_1 and M_2 and in the pen system x_2 is considered as an absolute motion; M_3 has an absolute motion x_3 which traces the record.

The force equations under these conditions become

$$M_1 \frac{d^2 x_1}{d t^2} + K_1 x_1 - K_2 x_2 = M_1 \omega^2 Y_0 \sin \omega t \qquad (54)$$

$$M_2 \frac{d^2 x_1}{d t^2} + M_2 \frac{d^2 x_2}{d t^2} + K_2 x_2 = M_2 \omega^2 Y_0 \sin \omega t \qquad (55)$$

$$M_3 \frac{d^2 x_3}{d t^2} + K_3 x_3 - K_3 x_2 = O \qquad (56)$$

The solution for x_3 is

$$x_3 = \frac{K_3}{K_3 - M_3 \omega^2} \cdot \frac{\begin{vmatrix} K_1 - M_1 \omega^2 & M_1 \omega^2 Y_0 \\ - M_2 \omega^2 & M_2 \omega^2 Y_0 \end{vmatrix}}{\begin{vmatrix} K_1 - M_1 \omega^2 & - K_2 \\ - M_2 \omega^2 & K_2 - M_2 \omega^2 \end{vmatrix}} \sin \omega t \qquad (57)$$

Expanding the determinants

$$x_3 = - \frac{\omega_1^2 \omega_3^2 \omega^2}{\omega^6 - (\omega_1^2 + \omega_2^2 + \omega_3^2 + n^2) \omega^4 + (\omega_1^2 \omega_2^2 + \omega_1^2 \omega_3^2 + \omega_2^2 \omega_3^2 + n^2 \omega_3^2) \omega^2 - \omega_1^2 \omega_2^2 \omega_3^2} Y_0 \sin \omega t \qquad (58)$$

where

$$\omega_1^2 = \frac{K_1}{M_1} \qquad (59)$$

$$\omega_2^2 = \frac{K_2}{M_2} \qquad (60)$$

$$\omega_3^2 = \frac{K_3}{M_3} \qquad (61)$$

$$n^2 = \frac{K_2}{M_1} \qquad (62)$$

Taking values

$K_1 = 26750.00$ #/in. $M_1 = 0.0210$ lb. mass

$K_2 = 10.75$ #/in. $M_2 = 0.1180$ lb. mass

$K_3 = 100.00$ #/in. $M_3 = 0.0002$ lb. mass

the amplitude of equation (58) has been plotted against the impressed frequency (Fig. 18).

When the constants have the value assumed, the record is accurate only between four cycles per second and 40 cycles per second. From 40 cycles to 210 cycles the record is too large. Beyond 210 cycles the record becomes less than the impressed motion and at 300 cycles it practically disappears. It is entirely possible to get this situation with instruments now on the market. Fortunately, it is also possible to modify the belting used and pen rod fork structure so that the two higher frequencies will be pushed up into a much higher range thereby extending the accurate range.

Conclusion to the Appendixes

Seismographic vibration instruments can be the most convenient and valuable tools for every day investigations in the shop and field, if their limitations are clearly realized by the man who interprets the records. Several important points have been brought out in the appendixes.

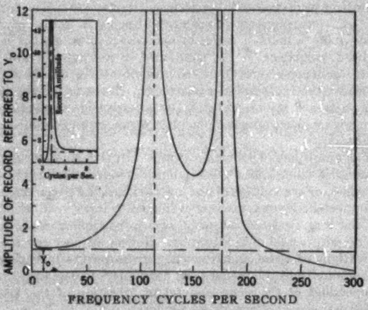

Fig. 18—Amplitude-Frequency Curves of Torsiograph Recorder

1. The record will always show the frequency of the motion being measured.

This is a fortunate circumstance since a knowledge of the frequency is the most important thing in determining the cause of the vibration.

2. The accuracy of the recorded or indicated amplitude depends on several factors:

   (a) It has a lower limit determined by the main spring—mass system of the instrument.

   (b) It has an upper limit determined by the masses and flexibilities of other parts of the instrument.

   (c) When parts of the indicating or recording system are held in contact by spring forces it is determined by the ability of these parts to remain in contact under the action of inertia forces produced by the motion of the parts.

A non-critical interpretation of the records may lead to large errors in the determination of amplitudes.

3. The kinematic magnification is determined by the design of the instrument.

(a) In the seismic vibrograph it depends on the mass distribution in the pendulum.

(b) In the torsiograph it depends on the ratio between the pulley diameter and the diameter at which the pen system is actuated.

In using the seismic vibrograph and the torsiograph it is often desirable to change the factors mentioned here. When they are changed it should be remembered that the record has a new magnitude relationship to the actual motion.

Discussion

W. B. Creagmile: Mr. Ormondroyd has mentioned a vibrating-reed type of instrument with one reed. The common use of vibrating-reed instruments having a series of tuned reeds along a marked scale, to measure revolutions per minute and a-c. frequency, is not mentioned in the paper. The reeds of the tachometer pick up mechanical vibration from the machine to which the instrument is attached. In the frequency meter, the reeds are set in vibration by alternating current flowing through an electromagnet placed in series with resistance across the mains.

The amplitude of vibration of any reed is dependent upon the amount of vibration of the machine to which the tachometer is attached, or the voltage of the line to which the frequency meter is connected. However, speed or frequency is not read by observing the amplitude of vibration of any particular reed, but by noting which reed is in vibration. Hence, we may say that the indications of speed and frequency given by the vibrating-reed tachometer and vibrating-reed frequency meter are entirely independent of the exact amount of vibration of the machine under test or of voltage of the a-c. line.

J. Ormondroyd: On the second page of the paper, the first sentence in the section on "The Vibrating Reed" makes brief mention of the two common uses of "tuned" reeds. Since the reed tachometers and electrical frequency meters are not used primarily to study vibration, no further mention is made of them.

The amplitude of motion at the end of a reed in a Frahm tachometer can be represented by equation (15) if

$$M = M + \frac{m}{4}$$

as in equation (16). This equation shows that the amplitude on any reed will be proportional to the impressed amplitude. But it also depends on the dynamic magnification which is a function of the frequency. Fig. 11c shows the relationship between reed-vibration amplitude and impressed vibration.

The reed which happens to be at or near resonance with the impressed frequency will have its motion magnified beyond all proportion to the magnification which the other reeds experience. The magnification at resonance is inversely proportional to the damping factor C as can be seen in Fig. 11c where the effect of two different values of C is shown. Putting $\frac{\omega}{\omega_c} = 1$, in equation (15) this is shown very clearly.

The speed or frequency is read by observing which reed has the greatest amplitude. The neighboring reeds usually show some smaller amplitude while the more distant reeds have no apparent motion.

In a sense the indications of speed are independent of the exact amount of the impressed vibration. This statement breaks down at the limit where the exact amount becomes zero. If turbine manufacturers could reach their ideal of perfectly balanced spindles and field-rotors the reed tachometers would become useless.

A High-Frequency Voltage Test for Insulation of Rotating Electrical Apparatus

BY J. L. RYLANDER[1]
Member, A. I. E. E.

Synopsis.—For many years the need for a higher test voltage between turns of individual coils or complete windings has been recognized. To obtain this higher test voltage, high frequency has been introduced. The high frequency used for this test is produced by damped oscillating discharge from a condenser and applied directly to the leads of the coil or winding. Practically any desired voltage can be applied to a coil or winding provided a sufficiently high frequency is used.

As a shop method for checking defects in material, or poor workmanship, the high-frequency test method has been found very effective in those classes of windings so far tried.

The method used for detecting short circuits on the principle of the radio receiving set are described.

INSULATION is used on rotating electrical apparatus to insulate between its various parts which have a difference of potential and also to insulate all parts of the circuit to "ground." The "ground" insulation is thoroughly tested by the present method of applying a high potential, usually at normal frequency, between the winding and the core or frame at various stages of construction. The present methods of testing the insulation of various parts of the same circuit such as from one turn to other turns, one layer of wire to other layers of wire, are far from adequate. It is the purpose of this paper to present a method of making a thorough and practical test between various parts of a coil or winding.

PRESENT METHOD OF TEST

The present method of testing between turns and layers of coils consists of placing the coil over a pole piece magnetized by a primary coil which induces a voltage in this coil as the secondary coil of a transformer. The maximum voltage that can be induced by this method will not exceed 10 or 20 volts per turn in most coils as the voltage is limited by the size of pole piece that can be inserted into the opening of the coil and to the frequency of the circuit. Usually 133 or 500 cycles per second are used. This test discovers short circuits where there is actual copper to copper contact. However, it does not discover any weak or damaged insulation where there is not actual copper contact. Therefore many coils are given no insulation test.

The windings of d-c. armatures are tested by this same induction method by revolving the armature slowly and testing each coil separately. This test discovers any short circuits caused by actual copper to copper contact and also discovers open circuits. As this type of testing apparatus is only adaptable for rotating windings, induction motor primary windings are not given this test.

The completed motor or generator is given a running test with rated voltage for one-half hour to several days depending on the test data needed. This test is applied to the motor when new and operating at normal voltage and under normal conditions of operation. It makes no allowance for future deterioration of the insulation or for any abnormal conditions of operation.

In order to obtain a higher test voltage between turns of coils both before and after winding the coils in the machine, that would be considered a real insulation test, high-frequency voltage is applied directly to the terminals of the coil or windings.

TYPES OF HIGH FREQUENCY

With high frequency, advantage can be taken of the inductance of the coil or winding and thus obtain almost any voltage desired.

The use of high frequency for various test purposes is not new but as far as the author is aware it has not been applied to the commercial testing of individual coils and wound apparatus.

In general, there are two types of high-frequency voltage: (1) undamped oscillations and (2) damped oscillations. The first type includes the high frequency voltage produced by high frequency generators which is, comparatively speaking, limited as regards the range of frequency. This class may also include the high frequency produced by vacuum tubes, such as used in radio work and which has a very high range of frequency compared to any generator.

The second type is produced by the oscillating discharge of a condenser through an inductance with a comparatively low resistance.

The production and control of both types of high frequency have been known practically and theoretically for many years and is described in many text books on radio and physics so no detailed description need be given.

The oscillating discharge high frequency was chosen for commercial test purposes because of its comparative ease of production and control.

It may be of interest to enumerate briefly a few of the experiments which were made with other types of high frequency. The reason for these experiments was mainly that when the damped oscillating high frequency

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Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

was first put in use the spark-gap was the only practical means of measuring the voltage. As is well known, the measurement of voltage with the sphere spark-gap is not very accurate under 2000 or 3000 volts. Inasmuch as there were many cases where a lower high frequency test voltage was desired attempts were made to measure the damped oscillating voltage by other means than spark-gap. Ordinary voltage measuring instruments were entirely unsuitable and it was also found, as was anticipated even before trial, that the electrostatic voltmeter would not give the correct measurements.

At this point, therefore, attempts were made to use the undamped high frequency obtained from vacuum tubes. However, the necessary frequency from this source was so high that it was difficult to screen the indicating instruments from the indirect effect, so this also was discarded for commercial reasons.

The high-frequency voltage obtained from a high-frequency generator was next resorted to but the current required up to 5000 cycles was so high that serious heating occurred in the bands of banded armatures in the case of direct-current machines.

In the meantime methods of detecting short circuits on wound apparatus, including commutating machines, were developed. This development permitted the high frequency test voltage to be applied across the normal span of the brushholders on a commutating machine, so the necessity for very low voltage at high frequency was largely removed. Later, also a method of instrument measurement of damped high-frequency voltage was developed, and for this reason all efforts were finally concentrated on the damped high-frequency test voltage.

THE PRINCIPLE OF OPERATION OF THE DAMPED HIGH-FREQUENCY SET

A very high voltage can be applied to any winding if the frequency is sufficiently high. The voltage drop due to the inductive reactance is $E = 2\pi f L I$ where f is the frequency per second and L is the inductance in henries and I is the current in amperes. For example, a 15-turn circular coil with a 12-in. diameter having a d-c. resistance of 0.1 ohm and an inductance of 0.001 henry would have approximately the following voltages across its terminals with 10 amperes in the circuit: one volt d-c.; 4 volts, 60 cycles per second, and 4000 volts with 60,000 cycles per second. It is thus seen that practically any desired voltage can be placed across a coil or a winding if sufficiently high frequency is applied. The voltage is applied by connecting the two leads from the high frequency apparatus directly to the terminals of the coil or winding either partial or completely finished.

When high frequency is applied to the apparatus under test, it sets up an alternating electromagnetic field of a particular frequency in a manner similar to a radio transmitter. If a short circuit occurs in the winding, the wave length and frequency will be changed accordingly, and the strength of the outgoing signals is reduced. A wave meter is used to measure the outgoing waves and thereby determine whether any insulation failure has occurred.

The wave meter consists of a set of inductance coils, variable condenser with a vernier attachment, two variable resistances and a low reading ammeter of the thermo-element type, all connected in series with one of the inductance coils. The inductance coils are such that by changing the setting of the variable condenser, resonance may be obtained for the frequency generated by the coil or winding under test. The condenser, the ammeter and the variable resistances are mounted on a panel and a tuning coil is placed near the apparatus being tested. The tuning coils have low resistance and a minimum capacity for a given inductance. Coils with taps are unsatisfactory on account of the end-turn loss.

The wave meter is tuned to the frequency of the waves omitted by the apparatus under test and it acts in a manner similar to a radio receiving set. A current flows in the wave meter circuit which is measured by the ammeter. The variable condenser is adjusted until a maximum current is shown on the meter which thereby indicates the condition of resonance and also the frequency of the circuit. The meter reading alone does not mean anything in particular; it is the relative meter readings used in connection with the condenser readings and the distance from the tuning coil to the apparatus under test which tells the story.

The principle by which high frequency is produced by this outfit is that a condenser placed across the secondary terminals of the transformer is charged during each alternation of the 60-cycle current. The condenser is automatically discharged through rotating disks when the voltage reaches a predetermined value on each alternation. The discharge of the condenser through the apparatus under test produces a high-frequency current whose frequency is determined by the apparatus under test and the capacity of the condenser.

DESCRIPTION OF HIGH-FREQUENCY TESTING APPARATUS FOR COMMERCIAL USE

The high-frequency testing apparatus consists essentially of (1) equipment for generating the high frequency and applying it to the apparatus under test, and (2) a means of detecting any short circuits or failures in the insulation of the apparatus under test while the high frequency is being applied. A photograph of the apparatus is shown in Fig. 1. A schematic diagram of this apparatus in shown in Figs. 2A and 2B, Fig. 2A showing the generation of the high frequency and its application to the coil or winding and Fig. 2B showing the apparatus used in the detection of an insulation failure.

The power is furnished by a transformer of 10-kw. capacity with 7500-15,000 and 30,000 volts on the secondary. A 70-per cent reactance limits the current to one and a third times full-load current with the short-circuit current in the secondary. The condensers have a capacity of 0.05 microfarads at 30,000 volts. The air-gap for discharging the condenser is formed between two motor-driven rotating brass alloy disks so that the arc across the gap will not burn the metal. The sphere-gap limits and measures the voltage across the apparatus under test. A resistance of 15,000 ohms is placed in series with the sphere-gap to prevent the spheres from being burned by the arc. For voltages of 5000 and higher, five-cm. spheres are used, and for lower voltages one-cm. spheres are used, also instrument measurement. A panel contains switches for the circuit breaker and plugs for tapping in different values of reactance. A hand-wheel is used for controlling the spacing of the gap of the rotating disks. Voltages ranging from 2000 to 30,000 volts can be obtained with this particular apparatus. By a combination of series and parallel connections the transformer will give 7500, 15,000 or 30,000 volts and intermediate voltages are obtained by adjustment of the rotating gap.

FIG. 1

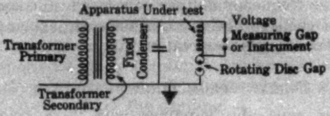


FIG. 2A

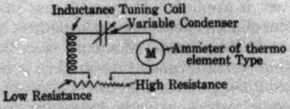


FIG. 2B

The manipulation of the test apparatus consists of connecting the leads to the coil or winding under test and setting the sphere-gap for the value corresponding to the desired test; then closing the circuit breaker and opening up the rotating disk gap until the desired voltage is obtained. This is indicated by a spark across the measuring gap. The voltages are generally applied for from 10 to 15 seconds. The frequency is determined by the apparatus under test usually within the limits of 10,000 to 200,000 cycles per sec. for individual coils and 5000 to 100,000 cycles per sec. for wound apparatus.

THE HIGH-FREQUENCY VOLTAGE

The damped high-frequency voltage as produced by this set gives a somewhat more severe test on the insulation than a corresponding voltage at normal frequency. Fortunately such a test can be used to discover faults without damage to the normal insulation.

In general the voltage builds up somewhat on the end coils. At first thought this may seem to be a disadvantage, but in actual service the end coils of a group are subjected to greater voltage in the case of surges than are the other coils. The voltage on the inner coils is sufficiently high, however, to give an adequate test for defects in material or workmanship.

VALUE OF TEST VOLTAGES USED

The high-frequency test voltage applied to the terminals of wound apparatus corresponds closely to the regular ground test as given by the A. I. E. E. standards. This allows a liberal margin of safety over the strain due to operation at its normal voltage, and also to the momentary high voltage, high frequency surges that may be impressed in service.

The voltage which it is desirable to apply to individual coils before winding into a machine is largely a question of judgment based on experience. We have used successfully on induction regulator coils a test voltage on single coils somewhat higher than the operating voltage of the machine for voltages lower than 2500 volts while for voltages over 2500, a test voltage somewhat lower than the operating voltage, has been used.

METHODS USED IN TESTING WITH HIGH FREQUENCY

The methods used for various types of apparatus are somewhat different, depending on various conditions. The method of testing a single coil is shown in Fig. 1.

The method of testing the windings of a two-pole, single-phase, induction regulator is shown diagrammatically in Fig. 3 for the stator and Fig. 4 for the rotor. The two leads from the high frequency set are connected to leads *A* and *B* of one pole and the tuning coil of the wave meter is placed alongside the frame as shown at *C*. The voltage is applied for 10 to 15 seconds. The two high frequency leads are then connected to the leads *E F* of the other pole of the winding and the tuning coil placed at *G*.

The high-frequency leads can be connected to three-phase machines by either of the methods shown in Figs. 5 to 8 inclusive.

The high-frequency leads may be connected either across each pole-phase group as shown in Fig. 5, or from each lead to the star connection, as illustrated in Fig. 6 or across the leads as in Figs. 7 and 8. Each of these connections has merits for certain conditions as determined by the number of poles, the voltage of the machine, the number of turns in the winding, the number of parallel connections and the size of the machine.

On a d-c. armature the voltage is applied at two points on the commutator corresponding to the location of the brushes, and the armature is then rotated.

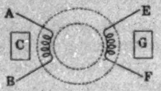


Fig. 3—Two-Pole Single-Phase Stator Winding

The tuning coil of the wave meter is placed in the most advantageous position which usually is close to the outside circumference of rotors and either inside the bore or outside the frame of stators.

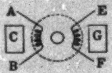


Fig. 4—Two-Pole Single-Phase Rotor Winding

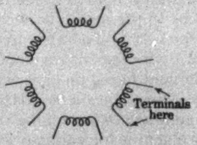


Fig. 5—Voltage Applied to each Group Separately

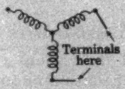


Fig. 6—Voltage Applied to Each Phase Separately

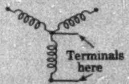


Fig. 7—Voltage Applied Across Two Terminals of a Star-Connected Winding

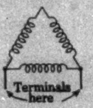


Fig. 8—Voltage Applied Across Two Terminals of a Delta-Connected Winding

For each coil and winding the wave meter settings are recorded. These settings show both the frequency and the strength of signal transmitted from any particular apparatus when the winding is normal and free from faults. As each coil or winding is tested, the readings will correspond to the standard readings if the winding is free from faults, but if short-circuited, different readings will be obtained. It is thus easy to check any apparatus that has been previously tested. If new, the approximate readings can usually be estimated with sufficient accuracy by comparing the general characteristics of the windings with other known windings, and checking one part of the winding with another is an additional check. The fact that a short circuit has a deadening effect on the radiations transmitted as well as changing the wavelength and frequency facilitates the discovery of any short circuits.

Scope of Application

This method of insulation testing was first applied commercially in the testing of induction regulator coils. The results were so satisfactory that the test was extended to the wound regulators, both stators and rotors of single-phase and three-phase regulators. Thousands of induction regulator coils for machines of all voltages up to 14,000 volts and hundreds of wound primaries and wound secondaries are now being tested each month with high frequency. The development has now reached a point where completely wound d-c. armatures, a-c. generator coils, turbo-generator coils and wound induction motors either partly or completely connected have been tested experimentally.

Considerable experimental work must be done before this method can be applied safely as a commercial shop test to complete windings of large alternators or induction motors. For example, sufficient information as to satisfactory test values is not yet available to apply high frequency voltage as a test to the complete windings which will be a proper measure of their ability to withstand surges in operation.

Benefits Derived from High Frequency Testing

Testing coils and windings with high frequency has had an effect on the insulation of the apparatus to which this test is applied. It showed that insufficient insulation for reliable operation was used in some cases, that different kinds of material should have been used in others, that certain insulating materials had been badly crushed during one of the coil forming operations, that the vacuum impregnation had not fully penetrated to all parts of the coil and that the moisture had collected into these spots and that the insulation used on certain designs was too liberal. It also furnished a good idea as to whether the insulation was sufficiently balanced between the various turns and layers to withstand the natural uneven voltage distribution that occurs in coils and windings whenever the windings are subjected to surges and line disturbances. It is thus seen that high frequency testing has enabled the improvement of the insulation of the windings by eliminating weak or defective coils, which might otherwise have been used. High frequency also checks the ability of the insulation to withstand the occasional severe dielectric strains that will occur in service and to which at present so many of the otherwise unaccountable insulation failures are attributed.

TEST RESULTS

As induction regulator coils have been tested regularly for over four years some of the results of these tests will be of interest. During the first two years 5 per cent of all coils tested failed to stand the prescribed test but the weekly percentage of failure varied from 1 per cent to 30 per cent. During the past year the percentage failures varied from 0 to 2 per cent weekly with an average of 1 per cent. This great reduction in the number of coils that failed to stand the test was accomplished by correcting the cause of the trouble which can usually be determined by a thorough examination. However, if these coils had not been rejected on account of the high frequency test, all of the coils would otherwise have been used, as none of the coils had any defect that could have been discovered by any other method than high frequency testing.

A record of the complete windings that have been tested shows that out of 100 machines of a certain particular type there were no failures, but on other types there were as high as 10 per cent and an average of all the types of 1 per cent. It is not to be inferred that the faults detected by this test would all have resulted in service failures. The applied test has purposely been made severe in order to certainly weed out all weaknesses of this class.

There has been a distinct improvement in these machines from the service standpoint. Whereas there was a considerable number of failures before high-frequency testing was started on these coils, there has not been reported a single insulation breakdown in service of any machine on which high-frequency test had been applied to the coils or the winding.

HIGH-FREQUENCY TESTING MEETS A GREAT NEED

High-frequency testing shows up poor workmanship, checks the insulation design and the processes of manufacture. It insures that proper and adequate insulating materials are in their proper place and not missing or damaged and that no harmful foreign materials are present. It checks up such features as the elimination of moisture and the thoroughness of impregnation, undue mechanical pressure at any point and the ability of the materials to stand the mechanical operations of coil construction, and whether the various materials are sufficiently uniform.

All of the above conditions may exist but they may not be detected or discovered without the aid of high-frequency testing as they are hidden from view and visual inspection is useless.

One real merit of high-frequency testing is that it invariably shows the cause of the failure, as the arc is of only such intensity as to burn out the weak insulation without burning the copper. When a machine fails in service the opposite nearly always occurs, as the copper is usually so badly burned as to destroy all evidence of the cause of the insulation failure.

High-frequency testing has now reached a stage of development where it is used as a shop test on some classes of apparatus as a check on manufacturing processes. The author does not wish to convey the idea that a point has been reached where the method may be applied as a universal commercial test.

Discussion

C. T. Weller: About ten years ago we started to investigate the effect of high-frequency discharges through the windings of standard types of current transformers. This investigation led to the development of the by-pass protector. The outfit developed at that time has proved to be very satisfactory for work of this nature. Referring to Fig. 2A in Mr. Rylander's paper, we found the quench-gap to be the most suitable for our purpose, the setting being obtained by varying the number of units in series. We utilize an adjustable air-core reactance in series with the test device to vary the frequency in the oscillation circuit and the voltage across the test device, the total voltage being fixed by the setting of the quench gap. We utilize two crest voltmeters, each consisting of a kenotron and an electrostatic voltmeter for measuring voltage and detecting breakdown. One crest voltmeter is connected across the series reactance and the other across the test device, the test voltage being held on the latter. In case of breakdown, the voltage across the series reactance rises while that across the test device falls. The changes in voltage are accompanied by a change in frequency. The approximate frequency is determined by means of a wave meter coupled inductively to the series reactance. Some difficulty has been encountered in arriving at this determination, however, due to harmonics in the oscillation circuit. However, the frequency is not so important with this outfit since the main reliance is placed on the crest voltmeter. The crest voltmeter has an advantage over a spark-gap in that it will indicate the crest value of the voltage at all times during the test. The outfit described may be used for testing the insulation of the windings of a wide variety of apparatus at high frequency.

V. M. Montsinger: The testing of coils between turns by means of voltage at high frequencies has been practised for years by the company with which the speaker is connected. We have used for that purpose damped oscillations and for detectors we have used a telephone receiver and an ammeter. A diagram of the arrangement for testing with damped oscillations of 20,000 cycles per second is shown in the accompanying illustration.

The charging circuit of the condenser C consists of a resistance R, an inductance L, and a step-up transformer T. The transformer T is excited by means of an induction regulator so as to obtain a voltage control of $E = 0$ to 440 volts from a 220-volt, 60-cycle supply.

The discharge circuit of the condenser C consists of a spark-gap G and an inductance L_1 which also serves the purpose of a linking coil, or excitation coil, of the testing yoke, Y. The discharge circuit of the condenser has a natural frequency of about 20,000 cycles per sec.

The coil X to be tested is placed on the table t in such a manner that the coil encircles both the testing yoke Y and the detector yoke y. Both of these yokes are of split construction so that the coil may be inserted. Yoke y carries a coil n of only a few turns connected to a phone receiver P and to a hot-wire ammeter A.

When the spark-gap, G, is arcing, the condenser voltage is discharged across the linking coil L_1 and an oscillating voltage at a frequency of about 20,000 cycles is induced in the coil X under test. When the coil under test is perfect, or when it withstands the induced voltage test, the phone receiver will be silent and the ammeter will indicate zero.

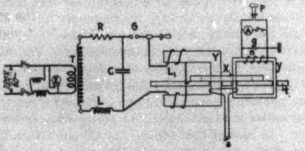
However, the phone receiver will be noisy and the ammeter will indicate a deflection when the coil under test is defective or when it breaks down during test, because then a short-circuit current in the defective coil will magnetize the detector yoke y and energize its coil n.

A small gap, g, with one terminal grounded is provided for the protection of the indicating instruments.

One auxiliary turn mounted under the table, encircling the testing yoke Y and the detector yoke y, may be short-circuited by the switch s at any time to show whether or not the equipment is operating satisfactorily.

The voltage per turn induced in the coil under test is directly proportional to the voltage E impressed on the transformer, provided that the setting of the spark-gap G is maximum for the discharge voltage of the condenser.

This testing method offers high induced test voltages per turn at the high frequency of about 20,000 cycles per sec. This frequency does not overheat the insulation because each train of damped oscillations, which starts at every half-cycle of the supply frequency ($2 \times 60 = 120$ times per sec.), lasts a very short time only and is followed by a comparatively long period of rest.



ARRANGEMENT FOR TESTING COILS WITH HIGH-FREQUENCY VOLTAGE

Experience has demonstrated that this outfit is a practical shop method for inducing high voltage between turns. It does not require an experienced person to operate it and will not only detect weak points of insulation but will give evidence of corona discharges.

E. S. Lee: Mr. Rylander has described a somewhat modified form of a method which, as has been indicated by other speakers, has been variously used during the past eight or ten years. Where it is desired particularly to obtain a high enough voltage between the individual turns of a coil to enable weak spots in the insulation to be broken down, the use of high-frequency voltage is advantageous, as higher voltages are thereby obtainable between turns than with the usual methods employing low-frequency voltage.

It is my understanding that this becomes a comparison method; that is, a coil is selected which is considered a good one and if the reading on the instrument for other coils approximates the reading as given by the good coil, then these coils may be assumed to be suitable and fit for use. If, on the other hand, the reading on the instrument does not closely approach the established reading, there is something wrong with the coil and if the voltage is high enough to cause a breakdown of the insulation, it will be easily detected.

I should like to ask Mr. Rylander how important it is to maintain the relative positions of the coil under test and the tuning coil. If we use this test for more complicated structures, as suggested by Mr. Rylander, how great a factor will be introduced because of the relative difficulty of placing the detector coil in about the same relative position with respect to the coils to be tested, since these may be assembled into stators of different sizes and forms? I believe this condition will have to be taken into account.

Mr. Rylander's paper quite clearly brings out the many defects which may occur in the manufacture of the insulation. If it is found that quite a bit of trouble is arising from these factors, then such a test as has been described is probably necessary. If, however, there are no troubles from these, it is a question whether or not such a test ought to be applied. This is true particularly because it is not altogether simple to determine the exact voltage distribution in the different turns of the coil. Mr. Rylander speaks of this in the paragraph in which he says, "In general, the voltage builds up somewhat on the end coils." Also, in so far as we do not know definitely the conditions that exist in the coils due to surges, we are not absolutely sure that by applying a high-frequency test, we are applying a test that is comparable to service conditions. It may not necessarily have the same characteristics, and we must be sure to have the same characteristics.

I think that this test must be put in the same category with any other test. In the section entitled *Benefits Derived from High-Frequency Testing*, Mr. Rylander states many defects which this test has shown to exist in the coils which he has tested. I think that Mr. Rylander wishes to bring out that this form of testing will show that some defect is present, or, if breakdown occurs, will allow such evidence to remain that the cause of the failure may be ascertained by visual inspection. Although where required, the test as indicated is of value, still of itself it does not show the cause of failure.

G. E. Luke: The A. I. E. E. standardization rules give limits for test voltages between circuits and grounds, but nothing is said concerning test voltages between turns. The reason for this omission is that there has been no standard or established method for conducting this inter-turn dielectric test. This paper and the discussions show that the manufacturing companies recognize the importance of this insulation and are developing means for testing its factor of safety.

In my experience on dielectric tests of windings there have been some puzzling breakdowns between turns; especially do I remember one series of armatures that gave a high percentage of failures between end connectors. Although the clearance was small, the insulation should have withstood a potential of 1000 volts or more, but some failed on a low voltage of from 50 to 100. In this particular case, the trouble was due to insufficient drying of the impregnating varnish or compound. A high-frequency test, such as described, would reveal such a weakness, whereas, without such a test, the machine would be passed as correct. Anything we can do to improve the methods for testing insulation, particularly that between turns, will be of great value to the electrical industry.

J. L. Rylander: Mr. Montsinger has described a method of testing individual coils with induction and the use of high frequency. I wish to point out that that method of test has no similarity in principle to the test described in my paper either as to method of applying the voltage or to the method of detecting failures; and furthermore, it is very limited in its application, as will be explained later. I understand that the test referred to by Mr. Weller and Mr. Lee is the same as that described by Mr. Montsinger as regards the method of applying the voltage; and for detecting breakdown, they use a crest voltmeter instead of the telephone receiver used by Mr. Montsinger.

With the method described by Mr. Montsinger, the voltage is applied to the coil under test by "induction," the coil under test acting as the secondary winding of a transformer. With the method described in my paper the high-frequency high voltage is applied *directly* to the coils or completed windings or other apparatus under test, as shown in Figs. 1 to 8 of the paper.

For the method of detection, Messrs. Weller, Montsinger and Lee use telephone receivers or crest voltmeters instead of the wavemeter shown in Fig. 2B of the paper.

The merits of the directly applied, high-voltage high frequency as compared with the induction method are as follows:

1. It can be applied to completed or partially wound machines as well as to coils of any type or shape, large or small, armature coils, field coils, magnet and other types of coils. The test shown in Mr. Montsinger's diagram is not and cannot be applied to completed a-c. windings; nor can coils be satisfactorily tested if the coils have a very small or a very large opening into which the testing yoke and the detector yoke must pass.

2. Invariably, the cause of any failure can be ascertained. The reason for this is that when a defect occurs, the insulation is burned away only at a small spot between the turns at the defect. With the induction method of test, the result of a defect is that the short-circuited turns are heated to a very high temperature the full length of the turns, so that the insulation is charred the full length of these turns as well as on the adjacent turns; and, as a result, the exact location of the defect cannot be determined; and usually the evidence of the cause is destroyed.

3. This directly-applied test corresponds to service conditions. Current flows in the coils or windings and also voltage is impressed between the turns, and the high-frequency oscillations correspond to the service surge effects upon any particular winding.

4. The wavemeter method of detecting failure, as used with the directly-applied, high-voltage high frequency, is an adequate and most effective method of detecting short circuits and insulation breakdown.

Mr. Lee asked about the importance of the location of the wavemeter tuning coil in respect to the coil or more complicated structure under test. This is a very important feature. Our wavemeter readings consist of the frequency as determined by the inductance and the condenser setting at resonance, and to the intensity and direction of the transmitted electromagnetic field as shown by the ammeter reading and the distance from the wavemeter tuning coil to the apparatus under test. Therefore, the reading for a defective piece of apparatus is different from the reading for the same apparatus without any defects. However, the point should be brought out that even when there is only one piece of apparatus to test, it can be tested and the readings will indicate whether the winding is satisfactory or whether defective. There is no difficulty in placing the tuning coil in the same relative position with respect to the apparatus under test. Mr. Lee stated that my paper "brings out clearly the many defects that may occur in the windings." We have found since we started this method of test that every one of the defects mentioned did occur, although we did not know it before.

Mr. Lee pointed out that it is difficult to determine the exact voltage distribution in the different turns of the coils. When we test a particular type of winding for the first time, we place a crest voltmeter across the end coils or others and measure the voltage on them while the voltage is applied to the leads. We can now estimate very well how it does divide on apparatus which is similar to something we have already tested. It should be pointed out, however, that a knowledge of the approximate voltage distribution in a winding (easily obtainable by this test method) is significant from an operating standpoint rather than from a testing point of view.

In conclusion, it may be stated that, by means of this directly applied high-frequency high-voltage test, the "internal" parts of windings can now be tested as thoroughly as the "external" insulation is tested by the "ground" test.

The Cross-Field Theory of Alternating-Current Machines

BY H. R. WEST[1]
Associate, A. I. E. E.

Synopsis.—*It is the purpose of this paper to show how analysis by the cross-field theory may be used to obtain accurate, purely numerical methods of calculating performance characteristics of alternating-current machines. Methods of calculation are derived and sample calculations are given for the single-phase induction motor and the repulsion motor.*

ALTERNATING-current machines may be analyzed according to either the revolving-field or the cross-field theory. Each of these theories has its own individual merits, depending more or less on the type of machine under consideration. Some phenomena are more easily understood by a study following the revolving-field theory, and other phenomena are made more clear by use of the cross-field theory.

For routine calculation of performance characteristics of alternating-current machines, many graphical and analytical methods have been developed. These different methods also have their own individual merits and the choice of a method of calculation should depend partly on what is most desired; *e. g.*, speed, accuracy, or aid to visualization.

Therefore, we may say that, in general, neither the revolving-field nor the cross-field method of analysis, and no one method of calculation, graphical or analytical, should be used exclusively. Although this paper deals only with a general analytical method of studying some types of a-c. machines using the cross-field theory, it is not by any means intended as a plea for the exclusive use of this general method, for it is recognized that whatever usefulness it may have will be found in rather limited fields.

In the following, the attempt is made to show how a general method of analysis, following the cross-field theory, may be applied to alternating-current motors to obtain simple and accurate, purely numerical methods for routine calculations of performance characteristics. The fundamental principles of the analysis of motors by the cross-field method are, of course, very well known. The general method given below is fundamentally the same as that outlined by Steinmetz in his "Theory and Calculation of Electrical Apparatus," but differs from it in the treatment of the leakage reactance, and in the arrangement of the results.

Briefly stated, the general method as applied to a motor is as follows: Kirchoff's voltage equations are set up for the different circuits of the motor and are solved to obtain equations for the currents in each of the circuits in terms of the applied voltage, the design constants of the motor and the speed. From these equations, other equations for the fluxes linking or cut by the rotor conductors are obtained. The torque corresponding to any one of the rotor circuits is obtained by multiplying the in-phase components of the current and the fluxes cut by the conductors of that circuit. Adding the components of torque corresponding to each of the rotor circuits, we obtain an expression for the total torque developed, which, multiplied by the speed, gives the power generated. Subtracting from this the friction losses gives the power output. The power input is, of course, given by the product of the applied voltage and the in-phase component of the line current.

In this paper, the angle of hysteretic lag between flux and m. m. f. is neglected in the analytical solutions, and correction for core loss is made in the numerical calculations by treating the core loss the same as if it were a friction loss. If it should be desirable, the angle of hysteretic lag can be taken into account by using the complex quantity $Z_m = R_m + j X_m$ for the magnetizing impedance in place of the pure reactance $j X_m$ which is used in the equations in the following part of this paper. This would complicate matters slightly by adding to the lengths of the equations, and would yield but a very slight increase in accuracy. In almost all cases, the slight increase in accuracy would not justify the extra labor and chances for numerical errors.

Sine wave distributions of m. m. f. and uniform air-gap permeance are assumed in all cases.

The details and application of the method can best be shown by means of examples, as follows:

The Single-Phase Induction Motor

According to the cross-field theory, the components of the main flux of the motor and the rotor currents are considered separately in two axes at right angles to each other. The axis of the stator winding may be called the transformer axis, and the axis at right angles to it the field axis. A squirrel cage is considered as equivalent to a commutated winding with brushes bearing on the commutator short-circuited on themselves in the transformer and field axes. The motor can then be represented diagrammatically as in Fig. 1.

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The following symbols will apply:

E = applied voltage
I_1 = line current
I_{1a} = power component of line current
I_{1b} = reactive component of line current
I_{2t} = rotor current in the transformer axis
I_{2f} = rotor current in the field axis
r_1 = resistance of the stator winding
r_2 = resistance of each of the rotor circuits
X_m = mutual inductive reactance of the stator and rotor windings
x_1 = leakage reactance of the stator winding
x_2 = leakage reactance of each of the rotor circuits
N = effective number of turns in each of the circuits
f = frequency of applied voltage
S = speed as a fraction of synchronism

The symbols for voltage and current all represent r. m. s. values of time vector quantities. The positive senses of the currents are indicated by the arrows in Fig. 2.

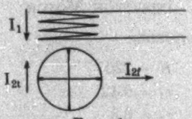

FIG. 1

The motor flux is resolved into the following four components: (1) the transformer flux Φ_{mt} which is the flux that is mutual to the stator winding and the rotor circuit in the transformer axis; (2) the field flux Φ_f which is the flux produced by the m. m. f. of the current I_{2f} in the field axis of the rotor; (3) the leakage flux φ_1 of the primary; and (4) the leakage flux φ_{2t} of the rotor circuit in the transformer axis.

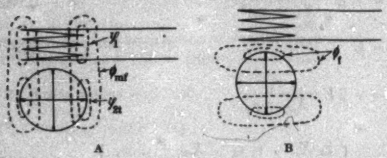

FIG. 2—FLUX COMPONENT IN SINGLE-PHASE INDUCTION MOTOR

(a) In transformer axis
(b) In field axis

In terms of the currents and the motor reactances, the equations for the above flux components are:

$$\Phi_{mt} = \frac{X_m\,(I_1 - I_{2t})}{2\,\pi\,f\,N} \tag{1}$$

$$\Phi_f = \frac{(X_m + x_2)\,I_{2f}}{2\,\pi\,f\,N} \tag{2}$$

$$\varphi_1 = \frac{x_1\,I_1}{2\,\pi\,f\,N} \tag{3}$$

$$\varphi_{2t} = \frac{x_2\,I_{2t}}{2\,\pi\,f\,N} \tag{4}$$

Voltage applied to the stator terminals must overcome the resistance drop and the mutual and leakage reactance drops due to alternation of Φ_{mt} and φ_1. The equation is

$$E = r_1\,I_1 + j\,x_1\,I_1 + j\,X_m\,(I_1 - I_{2t}) \tag{5}$$

In the transformer axis of the rotor, the sum of the voltages induced by transformer action of Φ_{mt} and φ_{2t}, and by rotation through the flux φ_f plus the resistance drop $r_2\,I_{2t}$ must equal zero. The equation is, for counter clockwise rotation of the rotor,

$$0 = -j\,X_m\,(I_1 - I_{2t}) - S\,(X_m + x_2)\,I_{2f} + r_2\,I_{2t} + j\,x_2\,I_{2t} \tag{6}$$

Similarly for the field axis of the rotor,

$$0 = j\,(X_m + x_2)\,I_{2f} - S\,X_m\,(I_1 - I_{2t}) + S\,x_2\,I_{2t} + r_2\,I_{2f} \tag{7}$$

Solving these three voltage equations for I_1, I_{2t}, and I_{2f} we get

$$I_1 = E\,\frac{[-r_2^2 + (1 - S^2)(X_m + x_2)^2 - j\,2\,r_2(X_m + x_2)]}{U_1' + j\,W_1'} \tag{8}$$

$$I_{2t} = E\,X_m\,\frac{(1 - S^2)\,(X_m + x_2) - j\,r_2}{U_1' + j\,W_1'} \tag{9}$$

$$I_{2f} = -\,\frac{S\,E\,X_m\,r_2}{U_1' + j\,W_1'} \tag{10}$$

where

$$U_1' = -\,r_1\,r_2^2 + 2\,r_2\,x_1\,(X_m + x_2) + r_2\,X_m\,(X_m + 2\,x_2) + (1 - S^2)\,r_1\,(X_m + x_2)^2 \tag{11}$$

$$W_1' = -\,r_2^2\,x_1 - 2\,r_1\,r_2\,(X_m + x_2) - r_2^2\,X_m + (1 - S^2)\,[x_1\,(X_m + x_2)^2 + x_2\,X_m\,(X_m + x_2)] \tag{12}$$

Substituting the above values of I_1, I_{2t}, and I_{2f} in equations (**1**), (**2**) and (**4**), we get

$$\Phi_{mt} + \varphi_{2t} = \frac{-\,E\,X_m\,[r_2^2 + j\,r_2\,(X_m + x_2)]}{2\,\pi\,f\,N\,(U_1' + j\,W_1')} \tag{13}$$

and

$$\Phi_f = -\,\frac{S\,E\,X_m\,(X_m + x_2)\,r_2}{2\,\pi\,f\,N\,(U_1' + j\,W_1')} \tag{14}$$

The torque developed by the motor consists of two components, one component T_1 due to the interaction of the current I_{2t} and the flux Φ_f, and another component T_2 due to the interaction of I_{2f} and the flux $\Phi_t = \Phi_{mt} - \varphi_{2t}$. These torque components in synchronous watts are equal to the products of the in-phase components of the currents and the fluxes with which they interact multiplied by $2\,\pi\,f\,N$; that is, from equations (**9**) and (**14**)

$$T_1 = \frac{E^2\,X_m^2\,(X + x_2)^2\,r_2\,S\,(1 - S^2)}{U_1^{1\,2} + W_1^{1\,2}} \tag{15}$$

and from equations (10) and (13)

$$T_2 = \frac{-E^2 X_m^2 r_2^3 S}{U_1^{1\,2} + W_1^{1\,2}} \tag{16}$$

and the total torque developed by the motor is

$$T = T_1 + T_2 = \frac{E^2 X_m^2 r_2 S\,[(1-S^2)(X_m + x_2)^2 - r_2^2]}{U_1^{1\,2} + W_1^{1\,2}} \tag{17}$$

This equation shows that the torque developed by the motor is zero when

$$1 - S^2 = \frac{r_2^2}{(X_m + x_2)^2}$$

or the ideal no-load speed is

$$S_0 = \sqrt{1 - \frac{r_2^2}{(X_m + x_2)^2}} \tag{18}$$

Substituting the above value for the no-load speed in equation (7), we obtain for the no-load current,

$$I_0 = \frac{2\,E\,(X_m + x_2)}{2\,r_1(X_m + x_2) + \dfrac{r_2 X_m^2}{X_m + x_2} + j\,X_m\,[X_m + 2\,(x_1 + x_2)]}$$

For all usual relative values of the design constants, this is very closely

$$I_0 \simeq -j\,\frac{2\,E}{X_m}\left[\frac{X_m + x_2}{X_m + 2\,(x_1 + x_2)}\right]$$

$$\simeq -j\,\frac{2\,E}{X_m}\left[1 - \frac{2\,x_1 + x_2}{X_m + 2\,(x_1 + x_2)}\right] \tag{19}$$

It is obvious that by means of this equation, the value of the magnetizing reactance X_m can be calculated with any desired degree of accuracy from the values of the short-circuit and no-load currents.

The performance characteristics of a motor of given design constants can be calculated completely by means of equations (8) and (17). The solution of these equations can be reduced to a simple matter of arithmetic by means of a suitable printed form, arranged for carrying out the calculations for a number of different speeds simultaneously. The use of very large numbers will be avoided and the calculations for motors of different sizes will be more nearly alike if the numerators and denominators are divided by $(X_m + x_2)^2$ and $(X_m + x_2)^4$ in equations (8) and (17) respectively. This gives

$$I_1 = \frac{M_1 + j\,N_1}{U_1 + j\,W_1} \tag{8'}$$

$$T = \frac{S\,F_8 + S\,(1 - S^2)\,F_9}{U_1^2 + W_1^2} \tag{17'}$$

where

$M_1 = F_1 + (1 - S^2)\,F_2$
$N_1 = F_3$
$U_1 = F_4 + (1 - S^2)\,F_5$
$W_1 = F_6 + (1 - S^2)\,F_7$

The complete expressions for F_1 to F_9 inclusive are given in the following sample calculation.

Performance Calculation of Single-Phase Induction Motor

Motor design constants:

$E = 220$
$r_1 = 0.12$
$r_2 = 0.3$
$x_1 = 0.4$
$x_2 = 0.4$
$X_m = 15$

Core loss and friction = 600

$$F_1 = -E\,\frac{r_2^2}{(X_m + x_2)^2} = -\,0.0835$$

$$F_2 = E = 220$$

$$F_3 = -\,2\,E\,\frac{r_2}{X_m + x_2} = -\,8.57$$

$$F_4 = r_2\left[\frac{x_2 X_m - r_1 r_2}{(X_m + x_2)^2} + \frac{X_m + 2\,x_1}{X_m + x_2}\right] = 0.315$$

$$F_5 = r_1 = 0.12$$

$$F_6 = -\,r_2\left[\frac{r_2\,(X_m + x_1)}{(X_m + x_2)^2} + \frac{2\,r_1}{X_m + x_2}\right] = -\,0.0105$$

$$F_7 = x_1 + x_2\,\frac{X_m}{X_m + x_2} = 0.789$$

$$F_8 = -\,E^2 r_2^3\,\frac{X_m^2}{(X_m + x_2)^4} = -\,5.23$$

$$F_9 = E^2 r_2\,\frac{X_m^2}{(X_m + x_2)^2} = 13{,}800.$$

	S	1.00	0.98	0.95	0.90	0.80
	$1 - S^2$	0	0.0396	0.0975	0.19	0.36
	$S(1 - S^2)$	0	0.0388	0.0927	0.171	0.288
(1)	F_1	−0.0835	−.08	−0.08	−.08	−0.08
(2)	$(1 - S^2) F_2$	0	8.70	21.4	41.7	79.2
(3)	$M_1 = (1) + (2)$	−0.0835	8.62	21.3	41.6	79.1
(4)	$N_1 = F$	−8.57	−8.57	−8.57	−8.57	−8.57
(5)	F_4	0.315	0.315	0.315	0.315	0.315
(6)	$(1 - S^2) F_5$	0	0.005	0.012	0.023	0.043
(7)	$U_1 = (5) + (6)$	0.315	0.320	0.327	0.338	0.358
(8)	F_6	−0.0105	−0.010	−0.010	−0.010	−0.010
(9)	$(1 - S^2) F_7$	0	0.031	0.077	0.150	0.284
(10)	$W_1 = (8) + (9)$	−0.0105	0.021	0.067	0.140	0.274
(11)	$M_1 U_1$	−0.0263	2.76	6.98	14.05	28.3
(12)	$N_1 W_1$	0.09	−0.18	−0.57	−1.20	−2.35
(13)	$M_1 U_1 + N_1 W_1$	0.0637	2.58	6.41	12.85	25.95
(14)	$N_1 U_1$	−2.70	−2.74	−2.80	−2.90	−3.06
(15)	$- M_1 W_1$	−0.0009	−0.181	−1.43	−5.82	−21.66
(16)	$N_1 U_1 - M_1 W_1$	−2.70	−2.92	−4.23	−8.72	−24.72
(17)	U_1^2	0.0992	0.1025	0.107	0.1143	0.1282
(18)	W_1^2	0.0001	0.0004	0.0045	0.0196	0.0751
(19)	$U_1^2 + W_1^2$	0.0993	0.1029	0.1115	0.1339	0.2033
(20)	$I_{1a} = (13)/(19)$	0.642	25.1	57.5	95.8	127.5
(21)	$I_{1b} = (16)/(19)$	−27.2	−28.4	−38.0	65.2	121.5
(22)	$I_1 = \sqrt{I_{1a}^2 + I_{1b}^2}$	27.3	38.0	69.4	116	176
(23)	Power factor = (20)/(22)		0.66	0.83	0.825	0.725
(24)	Power input = E . (20)		5510	12660	21100	28000
(25)	$S F_8$	−5.23	−5.1	−5	−5	−4
(26)	$S(1 - S^2) F_9$	0	535	1280	2360	3970
(27)	(25) + (26)	−5.23	530	1275	2355	3966
(28)	$T = (27)/(19)$	−52.8	5150	11450	17600	19450
(29)	Core loss and friction	600	600	600	600	600
(30)	Net torque = (28) − (29)	−653	4550	10850	17000	18850
(31)	Power output = S . (30)	−653	4460	10300	15300	14700
(32)	Efficiency = (31)/(24)		0.808	0.812	0.725	0.532

It will be noted that all the steps are indicated in the above. It is obvious that if a printed calculation form arranged as the above is used, the calculation of the complet performance characteristics of the single-phase induction motor is made comparable to a simple problem of bookkeeping, and can be done by a person without any technical training.

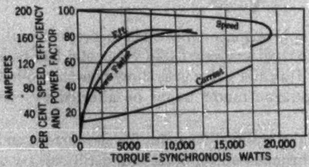

FIG. 3—PERFORMANCE CHARACTERISTICS OF SINGLE-PHASE INDUCTION MOTOR

The performance curves plotted from the above calculated values for current, torque, power factor, and efficiency are shown in Fig. 3.

As a further illustration, the method will be applied to the repulsion motor.

THE REPULSION MOTOR

As usually built, the stator of a repulsion motor has a simple single-phase winding. This stator winding may be considered as made up of two windings, one which may be called the transformer winding and which is in inductive relation to the commutated winding, and the other which may be called the field winding, which is at right angles to the axis of the commutated winding. According to the assumption of sine wave distributions of m. m. f., which is approximately realized in most cases, the transformer winding component will have $N \cos A$ effective turns, and the field winding component will have N sine A effective turns, where A is the angle in electrical degrees between the axis of the commutated winding as determined by the brush position and the transformer axis. The motor can be represented diagrammatically as in Fig. 4.

The same symbols that were used for the single-phase induction motor will apply for the repulsion motor and in addition we have

I_3 = local short-circuit current in the coils short-circuited by the brushes,

r_3 = resistance of the local short circuits formed by the coils short-circuited by the brushes and the brush contacts.

The leakage reactance of the local short circuits is assumed the same as that of the rotor winding as a whole, viz., x_2.

We have, therefore, three circuits to consider in the repulsion motor, and three corresponding voltage equations. For the stator winding we have

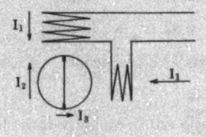

FIG. 4

$$E = j X_m (I_1 \cos^2 A - I_2 \cos A) + j X_m (I_1 \sin^2 A - I_3 \sin A) + (r_1 + j x_1) I_1 \qquad (20)$$

For the rotor winding proper,

$$O = - j X_m (I_1 \cos A - I_2) + (r_2 + j x_2) I_2 + S X_m I_1 \sin A - S (X_m + x_2) I_3 \qquad (21)$$

For the coils short-circuited by the brushes,

$$O = - j X_m (I_1 \sin A - I_3) - S X_m (I_1 \cos A - I_2) + S x_2 I_2 + (r_3 + j x_2) I_3 \qquad (22)$$

Solving for I_1, I_2 and I_3, we get the equations,

$$I_1 = E \frac{[r_2 r_3 - (1-S^2)(X_m + x_2)^2 + j(r_2 + r_3)(X_m + x_2)]}{U_2' + j W_2'} \qquad (23)$$

$$I_2 = E X_m \frac{[-S r_3 \sin A - (1-S^2)(X_m + x_2) \cos A + j r_3 \cos A]}{U_2' + j W_2'} \qquad (24)$$

$$I_3 = E X_m \frac{[S r_2 \cos A - (1-S^2)(X_m + x_2) \sin A + j r_2 \sin A]}{U_2' + j W_2'} \qquad (25)$$

where

$$U_2 = - r_2 X_m^2 \cos^2 A - r_3 X_m^2 \sin^2 A - (r_2 + r_3)(x_1 + x_2) X_m - (r_2 + r_3) x_1 x_2 + r_1 r_2 r_3 - (1-S^2) r_1 (X_m + x_2)^2$$

$$W_2' = r_1 (r_2 + r_3)(X_m + x_2) + r_2 r_3 (X_m + x_1) + S(r_3 - r_2) X_m^2 \sin A \cos A - (1-S^2)(X_m + x_2) [X_m (x_1 + x_2) + x_1 x_2]$$

The components of the motor flux which react with the currents I_2 and I_3 to produce torque are the field flux Φ_f and the transformer flux Φ_t,

where

$$\Phi_f = \frac{X_m I_1 \sin A - (X_m + x_2) I_3}{2 \pi f N} \qquad (26)$$

and

$$\Phi_t = \frac{X_m I_1 \cos A - (X_m + x_2) I_2}{2 \pi f N} \qquad (27)$$

The torque developed by the motor at any speed is, in synchronous watts,

$$T = 2 \pi f N [(I_2 . \Phi_f) - (I_3 . \Phi_t)]$$

where $(I_2 . \Phi_f)$ and $(I_3 . \Phi_t)$ represent the products of the in-phase components of I_2 and Φ_f, and I_3 and Φ_t, respectively. Substituting for the currents and fluxes their values from equations (23) to (27) and multiplying out, we get

$$T = \frac{E^2 X_m^2}{U_2'^2 + W_2'^2} \left\{ \begin{array}{l} (r_3^2 - r_2^2)(X_m + x_2) \sin A \cos A \\ + (r_3 \sin^2 A + r_2 \cos^2 A) \\ [-S \ r_2 \ r_3 + S(1-S^2)(X_m + x_2)^2] \end{array} \right\} \qquad (28)$$

The no-load speed is obtained by solving the cubic equation that is obtained by putting $T = 0$ in the above. An approximate solution is

$$S_0 \simeq \sqrt[3]{1.5 + \frac{r_3}{X_m \left(\tan A + \frac{r_2}{r_3} \cot A\right)}}$$

This approximate equation is theoretically much more accurate than any calculated value of r_3 can be expected to be. Its chief value is to show how the no-load speed of a repulsion motor might be affected by changes in design.

Special simplified formulas for the current and torque at synchronous speed or at standstill can be obtained by putting $S = 1$ or $S = 0$ in equations (23) and (28). Slight approximations can then be made by which the equations are reduced to rather simple formulas which need not be given here.

The performance characteristics of the motor can be calculated completely from equations (**23**) and (**28**). For calculation purposes, it is advisable to divide the numerators and denominators of these equations by $(X_m + x_2)^2$ and $(X_m + x_2)^4$ as in the case of the single-phase induction motor. This gives

$$I_1 = \frac{M_2 + j\,N_2}{U_2 + j\,W_2}$$

$$T = E^2\,\frac{G_9 + S\,G_{10} + (1 - S^2)\,G_{11}}{U_2^2 + W_2^2}$$

where

$$M_2 = G_1 + (1 - S^2)\,G_2$$
$$N_2 = G_3$$
$$U_2 = G_4 + (1 - S^2)\,G_5$$
$$W_2 = G_6 + S\,G_7 + (1 - S^2)\,G_8$$

The complete expressions for G_1 to G_{11} inclusive are given in the following sample calculation.

Performance Calculation of Repulsion Motor

Motor design constants:

$r_1 = 1.1$
$r_2 = 3.5$
$r_3 = 100$
$x_1 = 2.8$
$x_2 = 2.8$
$X_m = 60$
$A = 15$ deg.

Core loss and friction = 110 watts

$$G_1 = E\,\frac{r_2\,r_3}{(X_m + x_2)^2} = 19.5$$

$$G_2 = -E = -220$$

$$G_3 = E\,\frac{r_2 + r_3}{X_m + x_2} = 363$$

$$G_4 = -(r_3 \sin^2 A + r_2 \cos^2 A)\left(\frac{X_m}{X_m + x_2}\right)^2$$

$$-\left(\frac{r_2 + r_3}{X_m + x_2}\right)\left(x_1 + \frac{x_2\,X_m}{X_m + x_2}\right) + \frac{r_1\,r_2\,r_3}{(X_m + x_2)^2} = -17.98$$

$$G_5 = -r_1 = -1.1$$

$$G_6 = \frac{1}{X_m + x_2}$$

$$\left(r_1\,r_2 + r_1\,r_3 + r_2\,r_3\,\frac{X_m + x_1}{X_m + x_2}\right) = 7.39$$

$$G_7 = (r_3 - r_2)\left(\frac{X_m}{X_m + x_2}\right)^2 \sin A \cos A = 22.0$$

$$G_8 = -\left(x_1 + \frac{x_2\,X_m}{X_m + x_2}\right) = -5.47$$

$$G_9 = +\frac{X_m^2\,(r_3^2 - r_2^2)\sin A \cos A}{(X_m + x_2)^3} = 36.3$$

$$G_{10} = -\frac{X_m^2\,r_2\,r_3}{(X_m + x_2)^4}$$

$$(r_3 \sin^2 A + r_2 \cos^2 A) = -0.80$$

$$G_{11} = \left(\frac{X_m}{X_m\,x_2}\right)^2$$

$$(r_3 \sin^2 A + r_2 \cos^2 A) = 9.05$$

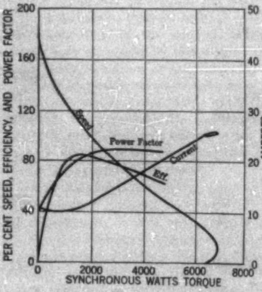

Fig. 5—Performance Characteristics of Repulsion Motor

The performance curves corresponding to the following calculated values are shown in Fig. 5.

It will be noted that this calculation form is, in general outline, the same as that used for the single-phase induction motor, practically the only differences being in the expressions for the constants F_1, F_2, etc., and G_1, G_2, etc.

The single-phase induction motor and the repulsion motor have been chosen to illustrate the general method of analysis by the cross-field theory on account of their relative simplicity. The same general method might be applied to machines of more complicated types, such as those having four or more circuits to consider, but the method of procedure would be essentially the same, and the results would merely be somewhat more complicated. The analytical solution of such a motor by the same general method has been given in a previous paper. (Trans. A. I. E. E., 1924, p. 1048.)

PERFORMANCE CALCULATION OF REPULSION MOTOR—(*Continued*)

	S	0	0.75	1.00	1.25	1.80
	$1 - S^2$	1	0.4375	0	−0.5625	−2.24
	$S(1 - S^2)$	0	0.328	0	−0.703	−4.03
(1)	G_1	19.5	19.5	19.5	19.5	19.5
(2)	$(1 - S^2)\ G_2$	−220	−96.3	0	123.8	493.
(3)	$M_2 = (1) + (2)$	−200.5	−76.8	19.5	143.3	512.5
(4)	$N_2 = G_3$	363	363	363	363	363
(5)	G_4	−17.98	−17.98	−17.98	−17.98	−17.98
(6)	$(1 - S^2)\ G_5$	−1.1	−0.48	0	0.62	2.46
(7)	$U_2 = (5) + (6)$	−19.08	−18.46	−17.98	−17.36	−15.52
(8)	G_6	7.39	7.39	7.39	7.39	7.39
(9)	$S\ G_7$	0	16.50	22.0	27.50	39.6
(10)	$(1 - S^2)\ G_8$	−5.47	−2.39	0	3.08	12.26
(11)	$W_2 = (8) + (9) + (10)$	1.92	21.50	29.39	37.97	59.25
(12)	$M_2\ U_2$	3820	1420	−350	−2490	−7950
(13)	$N_2\ W_2$	696	7800	10680	13750	21500
(14)	(12) + (13)	4516	9220	10330	11260	13550
(15)	$N_2\ U_2$	−6930	−6700	−6520	−6300	−5640
(16)	$-\ M_2\ W_2$	380	1655	−570	−5450	−30400
(17)	= (15) + (16)	−6550	−5045	−7090	−11750	−36040
(18)	U_2^2	364	341	323	302	241
(19)	W_2^2	4	462	863	1440	3510
(20)	= (18) + (19)	368	803	1186	1742	3750
(21)	$I_{1a} = (14)/(20)$	12.28	11.5	8.75	6.45	3.62
(22)	$I_{1b} = (17)/(20)$	−17.8	−6.3	−5.98	−6.75	−9.6
(23)	$I_1 = \sqrt{I_{1a}^2\ I_{1b}^2}$	21.6	13.15	10.55	9.35	10.25
(24)	Power factor = (21)/(23)		0.875	0.83	0.69	0.35
(25)	Power input = E . (21)		2530	1920	1420	796
(26)	G_9	36.3	36.3	36.3	36.3	36.3
(27)	$S\ G_{10}$	0	−0.6	−0.8	−1.0	−1.4
(28)	$S(1 - S^2)\ G_{11}$	0	3.0	0	−6.3	−36.4
(29)	= (26) + (27) + (28)	36.3	38.7	35.5	29	−1.5
(30)	$T = E^2 . (29)/(20)$	4790	2340	1450	805	−19
(31)	Core loss and friction	110	110	110	110	110
(32)	Net torque = (30) − (31)	4680	2230	1340	695	−129
(33)	Power output = S . (32)		1674	1340	868	−232
(34)	Efficiency = (33)/(25)		0 66	0.70	0 61	−0.29

Discussion

P. L. Alger: Mr. West has presented in his paper a businesslike and accurate way of obtaining the characteristics of an a-c. motor. The method is particularly adapted, and, in fact, was especially intended, for use by engineers having to do with large numbers of motors. Probably we all agree that the circle diagram, or some similar graphical method, is most suitable for college instruction, and for use by persons only dealing with motors occasionally. When large quantities of motors are to be designed, however, it is necessary to take into account many of the minor features of the theory, and, at the same time, to make the method extremely systematic. Thus, I am heartily in sympathy with the point of view which Mr. West expounds so thoroughly.

While the title of the paper emphasizes the cross-field theory, the same method could be used equally well with the rotating-field theory, for most machines. It is my opinion that both the rotating-field and the cross-field theories have advantages in particular cases, and that a complete understanding of design can be obtained only by a mastery of both theories. So far as I know, there is only one respect in which the two theories give different results.

This one exception occurs when the secondary winding of a single-phase machine has sufficient skin effect to make its effective resistance and reactance at line frequency appreciably different from their values with direct current. Since the actual current in any one secondary conductor consists of components of two different frequencies, the two components encounter different impedances, and so the effective voltage across the conductor is not equal to the product of the total current by any definite value of impedance. The cross-field theory treats the entire current in one axis as being of line frequency, and so does not take into account the variation of impedance with frequency. On this account, the rotating-field theory gives more accurate results for machines of this type.

I believe that the obvious objection to Mr. West's method, that it is very complicated, can be largely overcome by a study of the formulas, and their rearrangement into a form consisting of a principal term and various corrective terms. For example, if the magnetizing reactance is large by comparison with the leakage reactance, many of the formulas can be simplified by expressing them in power series in terms of $\frac{x}{X_m}$, and then neglecting higher powers than the first of this ratio. In some cases, the most complicated formulas can be put into a simple and usable form by assuming average values for certain of the less important constants, and preparing charts showing the effects of variations in the major constants. Then, correction formulas can be made up, showing the effects upon the result of changes in the minor variables made independently, and these auxiliary formulas can be used whenever especial accuracy is desired.

I believe that the ultimate design method is one which takes into account all of the variables, but only carries the result to the desired accuracy, and no further.

K. L. Hansen: In his analysis of the single-phase, induction motor, Mr. West has followed the usual and much discussed method of the cross-field theory. He has, apparently, introduced a slight modification by adding the secondary leakage flux to the mutual flux in the expressions for some of the induced voltages. The introduction of this refinement is, however, incidental and is obviously not the main object of the paper.

In the opening paragraph the author states that some phenomena are more easily understood by a study following the revolving-field theory, and other phenomena are made more clear by use of the cross-field theory. This statement, in connection with the title of the paper, would naturally lead one to believe that some distinct advantage of the cross-field theory over the revolving-field theory would be pointed out, at least in the specific instances which the author has chosen for illustration. The paper does not fulfill this expectation.

According to the author, the choice of methods should depend on what is most desired, *e. g.*, speed, accuracy, or aid to visualization. From the standpoint of speed and accuracy the cross-field theory obviously offers no advantage over the rotating-field theory. From the standpoint of visualization or physical conception the cross-field theory appears to be rather inferior to the rotating-field theory. Indeed, at the very outset an artifice is resorted to of considering the squirrel cage as the equivalent to a commutated winding with brushes bearing on the commutator short-circuited on themselves in the transformer and field axes.

There can be no objection to replacing the actual circuits we are dealing with by artificial equivalent ones when something is gained from the standpoint of clearness or shortening of the analytical work; but in this instance nothing is apparently gained thereby. In the revolving-field theory the squirrel cage is considered to be exactly what it is—a system of polyphase circuits. It seems that the currents flowing in the artificial secondary circuits employed by the author are of constant frequency and that therefore the secondary reactance x_2 is a constant quantity independent of the slip. Have the calculations in the paper been made on that assumption, and if so, do they lead to correct results?

A few years ago a writer in the JOURNAL argued that when several theories are advanced to explain the same phenomena, economy of thought demands that all but one of these theories be abandoned, unless each of the several theories possesses distinct advantages not possessed by the others. He then proceeded to make a misapplication of the revolving-field theory, and as his deductions naturally were in error he concluded that the revolving field theory is more liable than the cross-field theory to lead to erroneous results. He also was laboring under the delusion that the revolving-field theory is not applicable to commutator motors, and therefore advocated that this theory be abandoned in favor of the cross-field theory.

The present writer's opinion is that the point of retaining, in the interest of economy of thought, only one theory to explain a certain set of phenomena, is well taken; but I can not agree that in this case the revolving-field theory should be abandoned. My preference for the revolving-field theory is that is appears to be the more general one of the two. If the two components of the revolving field are equal, we have a single-phase system; if they are unequal, we have an unbalanced polyphase system. If one component vanishes, we have a balanced polyphase system, and if the angular velocity of the rotating field is equal to zero, we have a direct-current system. In other words, all phenomena of rotating electrical machines can be explained as special cases of the general theory based on two oppositely rotating magnetic fields.

H. R. West: I agree with Mr. Alger that the best method of calculation is that which gives results of the necessary accuracy, and no more, with the least amount of labor. On the other hand, I cannot accept the statement that the methods of calculation that I have given are very complicated. The greater part of the work of calculating a motor is contained in the tabulated part. Examination of this tabulated part of the calculation will show that nearly all of the rows of numbers are obtained with only one setting of the slide rule, or merely by addition. With very little practise, one will find that the calculation can be carried through very easily and quickly. The method given for the single-phase induction motor has been compared in actual trial with the most up-to-date methods of accurately calculating the polyphase induction-motor characteristics, and although it has been found to be somewhat more laborious, as obviously it must, since the operation of the single-phase, induction motor is very complicated in comparison to that of the polyphase motor—still it is found to be comparable in the amount of work and time required. Furthermore, anyone who can use a slide rule can carry out the calculation completely. It is not necessary for him to know anything about motors or electric circuits in order to carry out a complete calculation.

There are some very obvious approximations that can be made in the calculations, particularly in calculating the constants F_n and G_n. These approximations would result in very little more than an apparent reduction in the labor of calculating, and therefore the expressions were given in full for the sake of completeness.

As Mr. Alger suggests, a systematically arranged calculation form would probably be found equally helpful in calculating the characteristics of a motor from equations that might be derived by the revolving-field method. However, I do not believe that the revolving-field method of analysis will lead as directly to equations or formulas that are most useful for calculation purposes.

Mr. Hansen calls attention to the terms in my equations that represent the speed voltages corresponding to the rotor leakage fluxes. The inclusion of these terms is not new[1] with the author, although they have been neglected by some writers, generally with only very slight effect on the accuracy of the results.

The correctness of the results that have been obtained by the cross-field method of analysis has been questioned by Mr. Hansen. The exact equivalence of the results obtained by the revolving-field and cross-field methods was demonstrated several years ago[2]. Furthermore, examination of the analytical expression which Mr. Hansen derived for the line current taken by the single-phase induction motor in his very instructive paper[3] on the rotating-field theory will show that his equation is exactly equivalent to the corresponding one of my paper.

Reply to the remaining portions of Mr. Hansen's discussion must consist largely of repetition of the ideas expressed in the opening paragraphs of my paper. Both methods have their own advantages, depending partly on the types of motors. For instance, it is obvious that the revolving-field method should be used for the balanced polyphase induction motor, and the cross-field method should be used for the a-c. series motor. At the same time, I believe that a study of both of these motors from both points of view will often be found worth while. In the case of the single-phase induction motor, which seems to offer the most debatable ground in regard to the cross-field and revolving-field methods, it seems that the revolving-field method is much superior for the purpose of giving a mental picture of how the motor works. At the same time, I have found that the cross-field theory adds to and clarifies that picture. On the other hand, I am convinced that the cross-field method of analysis is more useful for exact analysis, since it leads directly to results that are immediately useful for exact numerical calculation without the necessity of making two separate calculations for a polyphase motor and then combining them.

As Mr. Alger has pointed out, there seems to be only one case in which the revolving-field method can be used and where the cross-field method cannot be used, *i. e.*, where the rotor inductances depend upon the frequencies. Otherwise, one method seems to be as general as the other.

The paper was not intended as an attempt to prove the general superiority of one method over another for any particular motor, but to demonstrate the possibility of using the cross-field method of analysis to obtain readily usable, accurate, numerical methods of calculation of motor characteristics. My experience leads me to believe that such methods of calculation can be most readily derived by the use of the cross-field method, in the case of many kinds of motors, particularly single-phase motors.

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[1] See, for instance, Arnold, Wechselstromtechnik. V 1.

[2] Arnold, Wechselstromtechnik, and Karapetoff, JOURNAL A. I. E. E., Aug. 1921.

[3] JOURNAL A. I. E. E., February, 1925.

Rating of Heating Elements for Electric Furnaces

Test Data and Integration of Interference Between Resistors

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Associate, A. I. E. E. Associate, A. I. E. E.

***Synopsis.**—A study of the practical considerations involved in assigning kilowatt ratings to radiant heating elements in furnace chambers. Constants for radiation and absorption are given. Consideration is given to shape and spacing of elements with respect to shielding due to adjacent elements. A method of determining the equivalent unshielded element is worked out to provide a basis for comparing dissimilar shapes of heaters.*

* * * * *

THE failure of an electric-furnace heating element is usually due to the chaotic methods used in establishing its ratings. Frequently heating elements are given arbitrary ratings, with no analysis of the conditions under which they are to operate. Some designers have developed rule-of-thumb methods of rating elements, based on current density, or on energy dissipated per unit area. Many of these methods are further elaborated to allow for increased ratings at low chamber temperatures. These methods must be used with considerable caution as they are almost invariably based on experience with only one type of heating element, used under restricted conditions. The determination of the general constants involved has been hampered by the cost of making scientific measurements at high temperatures. A general formula must provide for the following variables:

1. The physical characteristics of the heating surfaces.
2. The relative spacing and location of the component parts of the heating surfaces.
3. The allowable difference in temperature between the element surface and the surrounding atmosphere.

Data and mathematical analysis, made available within the last year, give these constants and make it possible to determine factors of safety of various shapes and arrangements of heating elements, and so make direct comparisons. This will permit the heating engineer to select his units with greater freedom and predict the temperatures at which they will operate. The calculation is analogous to the calculation of the mechanical engineer, who determines the unit stress on the extreme fiber of the beam he is loading with a fairly accurate knowledge of the stress at which it may be expected to fail.

A conductor supported in air and carrying electric current will increase in temperature until the heat removed by conduction, convection, and radiation equals the heat generated within the conductor by the passage of the current. This is true whether the conductor is inclosed in a furnace chamber to act as a source of heat, or is a part of a transmission line where heat is an undesirable by-product. The relative amounts of heat removed by the three different means of transmission change rapidly with the operating temperature of the conductor.

The convection loss from a surface increases as the 1¼ power of the difference in temperature between the surface and the surrounding room, while the radiation loss increases as the fourth power. At a surface temperature of 1000 deg. cent. (1832 deg. fahr.), the loss from a conductor in free air is roughly 90 per cent by radiation and 10 per cent by convection. (Conduction through air is so small as to be negligible and will be disregarded throughout the remainder of this article.) At this temperature, it is evident that an air space is a very inefficient heat insulator, since 90 per cent of the energy travels freely and almost without loss between the enclosing surfaces.

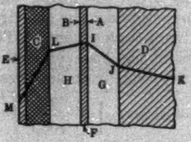

Fig. 1

The simplest heating element might be a sheet of resistance material covering an area of furnace wall and parallel to the wall, the arrangement being as indicated in Fig. 1; *F* is the plate in which heat is generated with surface *A* toward, and parallel to, the charge *D* which is to be heated, and surface *B* toward, and parallel to, wall *C* which is a partial heat insulator. We will consider *F* a large thin plate so the losses from the edges may be eliminated as negligible. Heat generated in the plate is conducted to the surfaces *A* and *B*. We will assume that wall *C* is of such material and thickness that 20 per cent of the heat generated in the plate escapes to the outside. Surface *B* can then dispose of only 20 per cent of the energy, generated so surface *A* must attain such a temperature as to dispose of the remaining 80 per cent of the energy. Surfaces *A* and *B* are opposite sides of a thin plate and so are at approximately the same temperature. When a state of equilib-

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[1] Both of the Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 8-11, 1926.

rium of heat-flow is reached, the temperature-gradients will be approximately as represented by lines *I–J–K* and *I–L–M*, the temperature at the surface of the wall *L* will be considerably higher than the temperature *J* at the surface of the charge because the difference in temperature between *I* and *J* is forcing four times as much heat across the space *G* as the difference in temperature between *I* and *L* is forcing across the space *H*.

It is particularly noticeable in this example that all surfaces of an element are not equally effective in disposing of the generated heat. If we consider surface *A* as 100 per cent effective, we can then consider the surface *B* only 25 per cent effective because under operating conditions it disposes of only one-fourth as much of the generated heat. The surfaces of plate *F* as located are then only 62.5 per cent as effective in disposing of heat as they would be if charge *D* were disposed on both sides of plate *F*. The factor 62.5 per cent (in this case) may be called the "surface efficiency" since it is the factor by which the total surface area of the resistor must be multiplied to obtain the area of an equivalent resistor whose surface is 100 per cent effective.

THEORY AND EXPERIMENTAL DATA

(*a*) *Single Linear Heater.* Due to the relatively poor thermal conductivity of the supporting refractories, and to the small area of contact, very little heat is conducted away from the heater elements. Hence radiation and convection are the principal agents in the transfer of heat from the elements. The fundamental equation, expressing the rate of heat dissipated by radiation and convection from surfaces, is:

$$W = a(T_2 - T_0)^{1.25} + 36.9\,e\left[\left\{\frac{T_2}{1000}\right\}^4 - \left\{\frac{T_1}{1000}\right\}^4\right]$$

where

W = Watts dissipated per sq. in. of surface

T_2 = Absolute temperature deg. cent. of heater surface

T_0 = Absolute temperature deg. cent. of room or furnace air

T_1 = Absolute temperature deg. cent. of surrounding surfaces

a = Convection constant

e = Coefficient of emissivity (1.00 for a black body)

In the above equation, the convection loss is shown as proportional to the 5/4th power of the temperature rise of the heater above the ambient air. The radiation loss, however, increases at a much greater rate, since it is a function of the fourth power of the absolute temperatures. The constants (*a*) and (*e*) vary slightly with the temperatures; however, the variation is so small that it can be neglected for practical purposes. The values of these constants were determined from experimental tests. The single linear heater element was suspended in free air. The input was measured electrically and the final temperatures were determined by the use of thermocouples. An optical pyrometer was also used to check the heater temperatures. The thermocouples were 0.008-in. diameter platinum and platinum-rhodium spot welded to the conductor.

From the large number of tests made, average values of the constants were obtained as follows:

Heater Element	Constant (*a*)	Constant (*e*)
¼-in. diameter nickel chromium..........	0.0032	0.91
¾-in. wide strip nickel chromium..........	0.0013	0.90

Using these constants in the equation previously given, the curves shown, Fig. 2, were plotted. Experimental tests on this curve covered the range of 100 to 1100 deg. cent. The elements were tested both vertically and horizontally and very little difference was found, especially at commercial temperatures where most of the heat is transmitted by radiation.

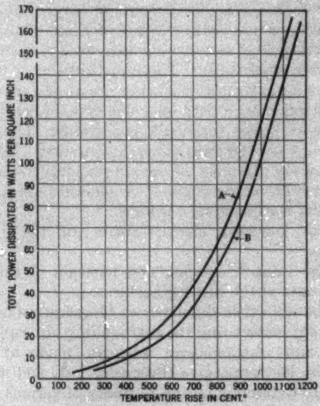

FIG. 2

A = ¼-inch rod
B = 1½-inch wide strip
Ambient temperature 27 deg. cent.

(*b*) *Parallel Heater Elements—Single Row.* In a commercial furnace, the heater consists of a large number of parallel ribbons or rods. It is evident that the rating of such elements will not be as great as a single element, due to interference of the adjacent elements. For such parallel heater elements arranged in a row, the relative heat loss can be calculated on the basis of the angle subtended by the opposite interfering resistor. Thus, in Fig. 3 is shown a row of parallel strip resistors. The approximate radiation from any small element (dx) will be proportional to the ratio of (180 deg. $- \alpha$) per 180 deg. An integration of these elements, either mathematically or graphically, over the total surface will give the effective radiation. Results of such calculations are given in Figs. 5 and 6 for round rods and strips respectively. The curves are plotted in terms of ratios in order to be applicable to any particular arrangement and size of heating element. While these curves as calculated refer only to radiated energy, tests show that they also represent very closely the effectiveness of the surface for both radiation and convection. The reason for this primarily is due to the relatively small amount of heat lost by convection —although it is a fact that by the grouping of the elements the convection and radiation losses are reduced to about the same degree. These curves were experimentally checked throughout the range shown, both round rods and strips being used. In making these tests, three equally spaced parallel elements were used. Thermocouples were placed on all three. Means were provided for independently varying the current through the middle heater and the two outside heaters. In this way the current was adjusted so as to give equal temperatures for all three heaters. The loss from the middle conductor was measured.

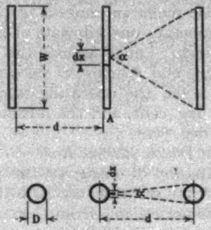

FIG. 3

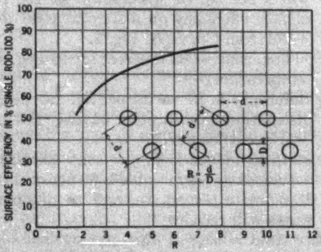

FIG. 4—CURVE OF SURFACE EFFICIENCY FOR DOUBLE BANKED RODS

(c) *Parallel Heater Elements—Double Row.* In some furnaces it is desirable to double bank the heating elements. The two rows of elements will interfere to some extent so that the rating will be reduced. The "surface efficiency" of a double bank resistor composed of round parallel rods arranged equidistant from each other, was calculated in the same manner as the single bank. The calculated curve is shown in Fig. 4.

The curves, Figs. 4, 5, and 6, show values of surface efficiencies plotted against the ratios of spacings to widths and diameters respectively. The curves are not directly comparable as the abscissas are not expressed as ratios of similar dimensions.

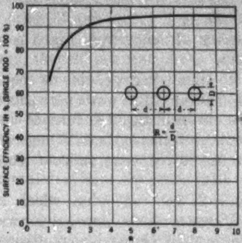

FIG. 5—CURVE OF SURFACE EFFICIENCIES FOR RODS

It is interesting to analyze commercial heating elements on the basis of the above data.

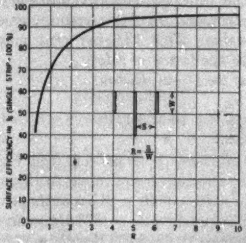

FIG. 6—CURVE OF SURFACE EFFICIENCIES FOR STRIPS

Let us assume the following conditions:

1½-in. wide strip 1/10-in. thick spaced, 1½-in. centers arranged as in Fig. 3, 0.289-in. diameter rod 1-in. centers arranged in two layers, (double banked) as shown in Fig. 7, and 0.289-in. rod ¾-in. centers single layer (single banked) also shown in Fig. 3. In each case, power equivalent to 15 watts per square inch of radiating surface, is generated in the element. The furnace chamber temperature is fixed at 900 deg. cent. (1652 deg. fahr.), room temperature 27 deg. cent.

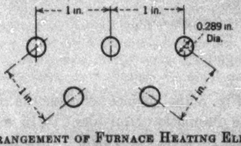

FIG. 7—ARRANGEMENT OF FURNACE HEATING ELEMENT RODS

We will assume that the resistor enters the danger zone, in which oxidation becomes progressive and, therefore, destructive at 1100 deg. cent. (2012 deg. fahr.).

For the Case of the Double-Banked Rod:

d = 1-in.
D = 0.289
R = 03.46

The surface efficiency from Fig. 4 is then 70 per cent.

Then 15 watts per sq. in. is equivalent to $15 \div 0.70 = 21.43$ watts per sq. in. dissipation.

To use the curves in Fig. 2 at furnace chamber temperatures, we must transpose from 27 deg. cent. ambient temperature to 900 deg. cent. ambient temperature as follows:

900 deg. − 27 deg. = 873 deg. which corresponds to 78 watts on the Curve *A* for rods (Fig. 2).

Then 78 + 21.43 (the equivalent dissipation determined above) = 99.43 watts which corresponds to a temperature rise of 950 deg. cent. (Curve *A*, Fig. 2). The operating temperature of the rod is then 950 deg. cent. + 27 deg. cent. = 977 deg. cent.

If 1100 deg. cent. marks the entrance to the danger zone, a 12.6 per cent rise in temperature will be required to bring the temperature of this heating element up to a dangerous value. This 12.6 per cent may then be considered as the factor of safety.

In the Case of the 1½-in. Ribbon:

W = 1½
S = 1½ − 1/10 = 1 4/10
R = 0.933

Surface efficiency = 69 per cent from Fig. 6.

Then the dissipation equivalent to 15 watts per sq. in. = $15 \div 0.69 = 21.74$ watts per sq. in.

900 deg. cent. minus 27 = 873 deg. cent. which corresponds to 65 watts dissipation on Curve *B*, Fig. 2.

65 watts + 21.74 = 86.74 watts per sq. in. This is equivalent to a 960 deg. cent rise. The operating temperature is then 960 deg. cent. + 27 deg. cent. = 987 deg. cent.

This gives a factor of safety as above of 11½ per cent.

For the Single Banked Rod:

d = 0.75
D = 0.289
R = 2.6

From Fig. 5 the surface efficiency is 89.5. Then the dissipation equivalent to 15 watts per sq. in. is $15 \div 89.5 = 16.65$.

900 − 27 deg. cent. = 873 deg. cent. which corresponds to 78 watts on Curve *A*, Fig. 2. Then 78 + 16.65 = 94.65 watts per sq. in. which corresponds to a temperature rise of 930 deg. cent. The operating temperature of the rod is then 930 deg. + 27 deg. = 957 deg. cent. This gives a factor of safety of 14.94 per cent.

It should be noted that these factors apply only to the physical conditions specified. A narrower ribbon or smaller rod would have a higher factor of safety for this spacing. A wider strip would have a considerably lower factor of safety as we are working on the steep part of the surface efficiency curve, Fig. 6.

We may also make an analysis of the cost of metal required to generate one kilowatt of heat energy in a furnace chamber. Let us assume the same conditions of physical size and spacing as were used in the preceding illustration, the operating temperature of the resistor 1000 deg. cent. and the furnace chamber temperature 900 deg. cent.

Considering the Double Banked Rod:

The dissipation of energy at 100 per cent surface efficiency on a rod at 1000 deg. cent. is 106 watts per sq. in. (from Curve *A*, Fig. 2). The dissipation at 900 deg. cent. (chamber temperature) = 79 watts per sq. in. correcting for 27 deg. ambient temperature in each case.

The difference is 27 watts per sq. in. or the permissible dissipation from a rod having 100 per cent "surface efficiency."

The double banked rod as before is 70 per cent efficient so the actual watts permissible dissipation is 70 per cent of 27 = 18.9 watts per sq. in.

To dissipate 1000 watts will require $1000 \div 18.9 = 52.91$ sq. in. of rod surface.

The area of one foot of 0.289-in. diameter rod is 10.9 sq. in.

$52.91 \div 10.9 = 4.854$ linear feet of rod required weighing 0.239 lb. per foot = 1.16 lb.

At a list price of $4.05 per pound this is $4.80 for the cost of double banked rod to dissipate one kilowatt.

For the Strip:

The watts per sq. in. dissipation at 100 per cent surface efficiency on a strip at 1000 deg. cent. = 92 from Curve *B*, Fig. 2.

At 900 deg. cent. = 65 (correct for 27 deg. ambient temperature)

The difference is 27 watts per sq. in. permissible dissipation from a strip having 100 per cent "surface efficiency."

The strip as before is 69 per cent efficient so the actual watts permissible is 69 per cent of 27 = 18.6 watts per sq. in. and to dissipate 1000 watts will require

$1000 \div 18.6 = 53.8$ sq. in.

The area of one linear foot of strip is 38.4 sq. in.

$53.8 \div 38.4 = 1.4$ linear feet of strip weighing 0.81 lbs. per foot = 0.782 lbs. at $5.10 per pound = $3.99 for the cost of strip to dissipate one kilowatt.

In the Case of the Single Banked Rod:

The dissipation from a 100 per cent surface efficiency rod as in the case of the double banked rod is 27 watts per sq. in.

The efficiency as in the earlier single banked rod example is 89.5 per cent so the actual watts permissible is 89.5 per cent of 27 = 24.16.

To dissipate 1000 watts will require 1000 ÷ 24.16 = 41.35 sq. in. of heater area.

The area of a linear foot of 0.289-inch diameter rod is 10.9 sq. in.

41.35 ÷ 10.9 = 3.793 feet of rod at 0.239 pounds per foot this weighs 0.9065 lb. at \$4.05 per pound = \$3.67 for the cost of single banked rod to radiate one kilowatt.

In computing the above costs prices have been listed for nickel chromium as quoted by a well-known manufacturer. As in the case of efficiencies, other physical dimensions will materially change the results. The dimensions used are those of heating elements in common use.

For convenience, the results of the above problems are tabulated below:

Shape of element	Arrangement of element	Factor of safety at 15 watts per sq. in. dissipation	Cost of metal per kw. radiated at constant factor of safety
Strip 1½ in. wide	Single banked Fig. 3	11¼ per cent	\$3.99
Rod 0.289 in. dia.	Single banked Fig. 3	14.94 per cent	\$3.67
Rod 0.289 in. dia.	Double banked Fig. 7	12.6 per cent	\$4.80

The above problems serve to illustrate several important points in design.

1. A resistor element of any shape may be supported in a furnace and so rated as to give a suitable factor of safety and reasonable service.

2. The shape of the element does not inherently insure a lower first cost of material. (Low cost frequently sacrifices a part of the factor of safety).

3. The selection of the elements for any furnace is an engineering problem requiring proper analysis. It should be approached with the idea of applying a suitable heating equipment, not with the idea of adopting some specific type of element to the installation.

In the foregoing calculations, we have disregarded the effect of adjacent walls. This should not introduce a serious error where the element itself is not so located as to act as a screen preventing reradiation and reflection of heat energy from the wall. The wall would perhaps have considerable effect on convection currents, but convection plays a relatively small part in the dissipation of heat at these temperatures.

Heat radiated to the wall is either reflected or is absorbed by the wall. That absorbed is conducted away from the surface or re-radiated into the furnace chamber. For the relative values of these various heat transfers, we must again refer to test data available. Fire brick at a temperature of 1000 deg. cent. reflects 40 per cent of the radiant heat falling on its surface and absorbs the other 60 per cent. If 50 per cent of the heat generated finds its way to the wall surfaces as radiant energy, then 20 per cent of the heat generated in the furnace is reflected from the walls much as light is reflected from a mirror. Thirty per cent is absorbed by the walls. If one-half of this heat absorbed is conducted to the external surface and lost, the remainder, 15 per cent of the heat generated in the heating elements, is re-radiated from the walls into the furnace chamber.

Thus, it can be seen that with the above data, the furnace engineer can calculate with reasonable accuracy the temperatures attained by resistor elements in service and can determine the factors of safety. He can also anticipate the manner in which heat will be transmitted to the charge. These calculations are of utmost importance to the furnace buyer. The fact that the conditions in a projected furnace can be determined by competent engineers, much as the loading of a bridge is determined, is insurance against loss through furnace failure.

Discussion

G. E. Luke: The equations for heat loss as given in this paper are only approximate. The heat liberated by convection varies to some extent with the position of the resistor. Its variation with temperature rise to the 1.20 power is only empirically true, but it seems to hold over a rather wide range of size and temperature. Fortunately, the rate of loss by convection is a small percentage of the total so that it does not have to be so accurately known. The accuracy of the Stefan-Boltzmann equation for radiation depends upon the correctness of the coefficient of emissivity (e). This value (e) is not constant but changes somewhat with temperature. For practical purposes it can be considered as a constant without great error.

Although considerable data are published regarding the rate of heat transfer from high-temperature conductors, yet few are available as to the reduction in this loss due to the proximity of other parallel conductors. At first the method of calculation as shown on Fig. 3 was used. It was recognized that this method was not rigidly correct since the intensity of the heat radiation from a point on a plane surface in any direction is proportional to the cosine of the angle that direction makes with the perpendicular. Nevertheless the approximate method gave results within 2 to 3 per cent of the true loss as determined experimentally.

It is hoped that this paper will show the fallacy of designing resistors according to the constant-current-density method or the constant-surface-loss method. Figs. 5 and 6 show that the possible rating depends largely upon the resistor arrangement.

Electrification of Paper-Making Machines

BY STEPHEN A. STAEGE[1]

Fellow, A. I. E. E.

Synopsis.*—The ever increasing speeds of paper-making machines has made necessary the replacement of the inadequate mechanical drive with electrical sectional drive, and the perfection acquired in modern electrical sectional paper-machine drive has made possible unlimited paper speed in so far as the drive and control is concerned.*

Not only is electrical sectional drive necessary for high speed machines but it is highly economical for even the slowest speed machines on account of the saving in power over mechanical drive, the great saving in maintenance, and the greater precision with which the section speeds can be maintained thereby greatly reducing the number of paper breaks and increasing production.

This paper outlines the several trends of development in an historical sketch of progress in the art up to the present time, devoting special attention to a unique system of direct current drive in which the section driving motors through a differential electrical field control means, cause the several section driving motors to operate in synchronous relation at any desired relative speed values, which speed relations are at the same time adjustable at the will of the operator.

INTRODUCTION

THE production of a continuous sheet of paper 12 to 18 ft. wide and 250 to 1000 mi. long without a break on the machine—a paper speed of 1000 ft. per min. or more—has been made possible by electrical sectional paper-machine drive and control.

The frail web of paper, about three mils thick, passes without support from section to section of the machine, passing between heavy press rolls, tons in weight; threading around 30 or 40 steam-heated dryer rolls; through the eight- or ten-roll calender stack weighing a hundred tons or more; and onto the reel.

In the travel of the sheet through the paper machine from the "wire" where it is formed to the calender where it is finished, the sheet usually increases in length from five to ten per cent. Therefore, the sheet is actually traveling faster when it leaves the machine than at the beginning of its travel. In fact, there is usually a progressive increase in the speed of travel of the sheet as it goes from section to section, on account of its stretch.

There are two principal types of paper-making machines, known as cylinder machines and Fourdrinier machines. The fundamental differences are in the wet or forming-end of the machine.

The Fourdrinier machine is the type always used for high speed, in making newsprint and kraft paper and in the manufacture of book, writing and the higher grade papers.

The cylinder machine is inherently a slower speed type and is used chiefly in the manufacture of box and container board, roofing felt, and the heavier sheets.

The paper-making machine in its usual form consists of a number of sections independently driven but which must be maintained in operation at the proper speed relations to each other. Not only must the proper speed relation be maintained between sections, but these speed relations must be adjustable at the will of the operator so that they can be adapted to the requirements of the sheet of paper. These may change slightly in elongation or shrinkage on account of variations in the condition of the stock, temperature, atmospheric conditions and a number of other factors; the stretch or shrinkage of the sheet depending largely upon the thickness, quality, condition of stock and amount of pressure used in removing the water.

In addition to the requirement of adjustability of speed between sections to take care of variations in elongation and shrinkage, it is essential that the entire machine be adjustable in speed over a wide range, without disturbing the speed relation of the several sections.

The speed range of a newsprint machine may be 600 to 1200 ft. per min.; a kraft machine, 200 to 800 ft. a book machine, 100 to 600 ft.; a specialty Fourdrinier machine, perhaps 100 to 800 ft.; while a cylinder machine sometimes has a speed range as great as 35 to 350 ft. although a more common range for this type of machine is 50 to 200 ft. of paper per min.

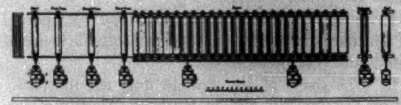

FIG. 1—TYPICAL SECTIONAL PAPER-MACHINE DRIVE

A typical high-speed Fourdrinier machine (Fig. 1) usually consists of eight sections; a "wire" or couch section, three press sections, two dryer sections, one calender section and a reel section. Fig. 2 shows schematically a side elevation of such a machine.

The first section of the Fourdrinier machine, known as the "Fourdrinier," "wire", or "couch" section, has a revolving wire screen of fine mesh. This Fourdrinier wire, in the form of a wide belt the width of the machine, operates around two rolls, the one at the extreme end of the machine called the breast roll and the one at the other end of the "wire," the "couch" roll. This is the driven member of the section, the breast roll being rotated as a pulley by the wire. The wire may be from

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50 ft. to more than 100 ft. long, depending upon the size and speed of the machine; and the span of the wire is kept from sagging by a large number of small "table" rolls located close together under the top pass of the wire, which carries the stock.

The press sections consist of pairs of heavy rolls, weighted with compounding levers, and between the upper and lower rolls of which the sheet, on its felt conveyor, passes. The lower roll is driven and the upper roll rotates by contact traction with the lower roll.

In the dryer sections some 15 or 20 dryer rolls or drums, four, five or six feet in diameter are nested in a group and geared together by intermeshing spur gears. There are usually two dryer sections which may be independently driven and regulated or may be geared together.

The calender stack consists usually of eight or ten chilled-steel rolls accurately ground and polished and driven by the bottom roll, all of the upper rolls being rotated by surface contact friction.

A "uniform speed" reel is commonly used, the spool and roll of paper being rotated by the traction of surface contact with the reel drum.

From the "couch" the sheet, though very wet and frail, spans a distance of a foot or more until it reaches a felt conveyor by means of which it is carried through the first press, then to the second and third presses, on independent felts and across another unsupported span to the dryer sections. Dryer felts hold the sheet in contact with the hot dryer drums as it is threaded through the nest of dryers. From the dryers, the sheet passes across a span of more than five ft. to the calender stack and thence to the reel.

Passing from the "wire" at the "couch," additional water is removed by pressure in going through the presses, and most of the remaining water is removed by heat in the steam-heated dryer sections.

The function of the paper machine is to reduce the stock mixture, evenly distributing the fibers in proper relation, fabricating them into a continuous web, removing the excess water by filtration, pressure and heat and giving the final sheet of paper the required finish, and to wind it into rolls of convenient size.

In putting the sheet on the machine, it is put over at the full normal operating speed by means of air jets, in the case of high speed machines at least, by means of which a leader strip is detached from the "wire" at the couch and blown across the gap to the felt conveyor of the first press. At each section the air jet is used to transfer the narrow leader strip to the succeeding section. As soon as the leader strip has been started the sheet is widened by tapering out at the "wire" until it is the full width of the machine.

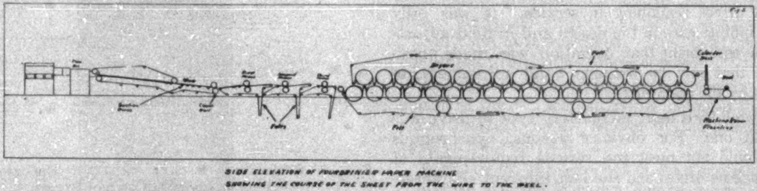
Fig. 2
Wire
Suction Boxes
Couch Roll
Felts
Dryers
Felt
Calender Stack
Reel
Machine Room Floor Line
SIDE ELEVATION OF FOURDRINIER PAPER MACHINE
SHOWING THE COURSE OF THE SHEET FROM THE WIRE TO THE REEL.

FIG. 2—SIDE ELEVATION OF FOURDRINIER PAPER MACHINE SHOWING THE COURSE OF THE SHEET FROM THE WIRE TO THE REEL

A very large percentage of the paper of today is made from wood pulp, the individual fibers having been reduced or separated by mechanical or chemical processes and mixed in a suitable proportion of water.

In the manufacture of paper, the raw material passes through several processes and refinements before it goes to the paper-making machine in a fluid mass consisting of approximately ½ of 1 per cent stock and 99½ per cent water.

The Fourdrinier section of the machine receives the stock from a flow box, located above the breast roll, from which it flows by gravity onto the revolving fine wire screen from a thin submerged orifice extending the width of the machine and located just above the "wire."

When the stock flows onto the "wire," much of the water immediately passes through by gravity, and further along its travel, a considerable part of the remaining water is drawn through the wire from the stock film on its surface by a number of flat suction boxes located below it in which 10 to 15 inches of vacuum is maintained.

In cylinder machines, the design and functioning of the various parts is much the same as that of the Fourdrinier machine, except that the forming of the sheet is done on cylinder moulds instead of on a Fourdrinier wire. A machine may have several cylinder moulds consisting of skeleton drums covered with fine wire screen. These revolve slowly in vats containing paper stock and each cylinder mould contributes a thin layer of stock to the final heavy sheet. A revolving felt conveyor picks up the stock from the cylinder mould drums, carries the laminated stock film between small presses where it is pressed into a homogeneous fabric and part of the water is removed, and thence carries it through the first press from which the operation is similar to that of a Fourdrinier machine.

EARLY METHODS OF DRIVE

For many years, the paper-making machines were mechanically driven by variable speed engines or motors or other variable speed means to take care of the speed range required, and the individual sections were driven from a line shaft through gears and belt or rope drive with cone pulleys, so that by shifting the belt on the cone pulleys, the required relative speed between sections could be obtained. Such systems were unusually subject to belt slippage on account of the poor belt contact on the cone pulley surfaces and on account of the vertical-belt drives which were usually necessary; they required mechanical clutches or belt slackening and tightening devices so that any section could be stopped and started independently of the rest of the machine; and they suffered from more than the usual troubles and limitations to which mechanical drives are subject.

As early as 1905 to 1910 several installations of electrical sectional paper-machine drives, but without automatic speed regulation, were made in an effort to overcome the serious shortcomings of mechanical drive, but most of these early installations because of poor speed regulation were unsuccessful—only two of these early sectional drives remaining in service. It was only through infinite care in the design and in field adjusting of the equipment that operation was made possible, and that only by frequent hand adjustments of rheostats by the operators.

The search for a more satisfactory type of drive was a difficult one. For obvious reasons, synchronous motors could not meet the various requirements of paper-machine drives and the best inherent characteristics of d-c. motors did not provide speed regulation of sufficient precision over the speed range required, and were furthermore subject to the effects of changes in resistance due to temperature changes resulting in variations in speed. An automatic speed regulator was sought, therefore, to control the speed of the d-c. driving motors for each of the sections. It was immediately apparent that none of the usual types of speed governors or regulators would suffice, inasmuch as speed control within limits of the order of $^1/_{10}$ of 1 per cent or closer was necessary for satisfactory performance. The differential principle was, therefore, finally made use of as this offered possibilities of practically infinite precision of speed control of integrated values. To correct for a tendency of the motor to change in speed, a small change in angular displacement of the motor armature with respect to a time cycle is necessary, but the motor can be made to operate for an indefinite period without gaining or losing in revolutions per minute.

Paper machines sectionally driven by d-c. motors and controlled by regulators of the electrical differential type have now been in operation in this country and Canada for some six years. The details of the mechanism by means of which differential speed regulation with field control is obtained, has gone through a process of evolution whereby the apparatus required has been greatly reduced and simplified and improved in quickness of response.

FIRST AUTOMATICALLY REGULATED ELECTRICAL SECTIONAL DRIVE

In the first electrical sectionally-driven paper machine in this country to be automatically controlled by the electrical differential regulator principle, a master unit was provided to which the speed of the sections was referred (Fig. 3). The master consisted of a motor-driven a-c. generator, and from each section of the paper machine there was also driven, through a control speed changer, a small a-c. generator. Connected electrically between the master generator and the section generator was a wound rotor induction motor, the stator being connected to the master generator and the rotor to the section a-c. generator. When connected in the proper phase relation so that the magnetic field in the rotor would rotate in the same direction as the magnetic field in the stator and at the same speed, no rotational movement of the rotor would take place. If, however, the d-c. section driving motor for any reason should slow down or speed up, this would, of course, cause a corresponding change in the frequency of the generator driven from the section motor, from that of the master, and the rotor of the induction motor would then rotate in the one direction or the other, depending upon the higher frequency and at a rate of speed corresponding to the difference in frequency of the two generators between which it was connected. This constituted an electrical differential.

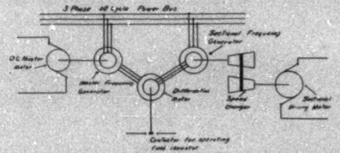
3 Phase 60 Cycle Power Bus
Sectional Frequency Generator
D.C. Master Motor
Master Frequency Generator
Differential Motor
Speed Changer
Sectional Driving Motor
Contactor for operating field rheostat

FIG. 3—SCHEMATIC DIAGRAM OF FIRST ELECTRICAL DIFFERENTIAL SPEED CONTROL SYSTEM

A motor-operated field rheostat was provided in the shunt-field circuit of each of the section driving motors (Fig. 4) and through movement of the differential element, the wound rotor induction motor, contact means were provided so that a rotational movement of a very small fraction of a revolution in either direction would serve to make a contact thereby operating the rheostat in one direction or the other as required to correct the speed of the section-driving motor. This relay method of operating the field rheostat was used instead of coupling the rheostat directly to the shaft of the differential motor so as to relieve the differential control system of the necessity of developing sufficient torque to turn the rheostat which would detract from its sensitivity; and also to obtain characteristics whereby the travel of the rheostat arm was not a direct function of the movement of the rotor of the differentially connected induction motor. Reaction magnets were provided to prevent overtravel of the rheostat and hunting.

This system was quite effective but had its limitations particularly in range of control, speed of regulation and tendency to oscillate on account of the definite step characteristic of the rheostat which obviously could not be made to meet the exact requirements of the speed necessary for the section motor.

The master and section a-c. generators used were of the induction frequency changer type which appeared to be the most desirable for this service. Standard wound rotor induction motors were used and were driven at a speed usually about 50 per cent below synchronous speed so that when excited from a 60-cycle source, the output frequency of these frequency changer generators would be about 30 cycles at the maximum paper speed and this frequency would increase as the paper speed decreased. The use of this type of generator avoided the necessity of d-c. excitation and provided all of the characteristics required and any variations in the frequency of the exciting source would have no effect upon the regulation as they would take place equally on both sides of the differential.

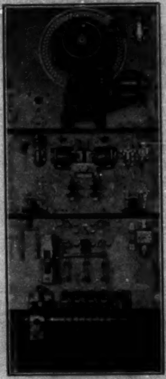

Fig. 4—Differential Relay and Motor-Operated Field Rheostat of First Electrical Differential Speed Control System of Sectional Drive

With this arrangement it will be seen that when the speed of the section motor is correct, the secondary frequency of the master and section frequency changer or generator is the same and no rotation of the induction motor connected between the two frequency changers takes place. If, however, a change in angular displacement of the section frequency changer takes place with respect to the master, a similar movement takes place in the rotor of the wound rotor motor and through the rheostat indirectly actuated by this movement resistance is cut in or out of the shunt-field circuit of the section driving motor, thereby tending to maintain its speed at the required value.

Following shortly after the installation in 1919 of the first control system of the type described, two other systems of sectional paper-machine drive and control also came into use; one known as a "synchronous motor tie-in" system, and the other as an "interlock" system.

Synchronous Motor Tie-In System

The "synchronous motor tie-in" system (Fig. 5), instead of automatically regulating the d-c. section motor field strength to maintain the proper speed relation between sections, employed a synchronous motor for each paper-machine section, of about one-fifth the capacity of the main d-c. section driving motor. The synchronous motors were driven from the main d-c. section driving motors through a spur-gear reduction and a pair of cone pulleys with belt drive. The synchronous motors were d-c. excited and had their three-phase armature circuits connected to a common or "dead" bus, as it was called. This "dead" bus was not connected to any other source of power.

The object of the synchronous motors was to hold the speeds of the d-c. section driving motors at their proper speed relation, as determined by the position of the belts on the cone pulleys.

In the event of a tendency of the d-c. section driving motor to change in speed, due to a load change or temperature change, the synchronous motor was called upon to carry the increase or decrease in the load of the section, or the load thrown off or taken on by the d-c. section motor due to a temperature change and corresponding change in field resistance. This was necessary in order that the synchronous motor might remain in synchronism with the "dead" bus and the other synchronous motors.

It will be observed that with this arrangement, any load taken by the section synchronous motor as a motor must be furnished by the other synchronous motors connected to the "dead" bus, as generators; and any load furnished by the section synchronous motor in question, as a generator, must be absorbed by the other synchronous motors. Since any change in load, or tendency to change in speed, on the part of a d-c. section driving motor creates a call indirectly upon each of the other section motors to carry or absorb the change in the section in question, the entire paper machine would tend to change in speed, to a degree depending upon the droop of the load speed characteristic curves of the d-c. section driving motors and in an amount, compared to the section tending to change, approximately inversely proportional to the number of sections.

It will be observed, also, that the d-c. section driving motor in question could only change in speed with respect to the frequency of the "dead" bus, or the equivalent approximate mean paper speed, an amount permitted by the change in belt stretch, creep and slip on the cone pulleys, resulting from the change in load being transmitted, with the exception of the transient speed change coincident with the angular displacement and change in phase position of the synchronous motor accompanying its change in load, the changes in load being transmitted by the belt drive on the cone pulleys being subject to any amount within the capacity of the synchronous machine and possibly passing through zero torque on the synchronous machine in going from motor to generator or generator to motor (Fig. 6).

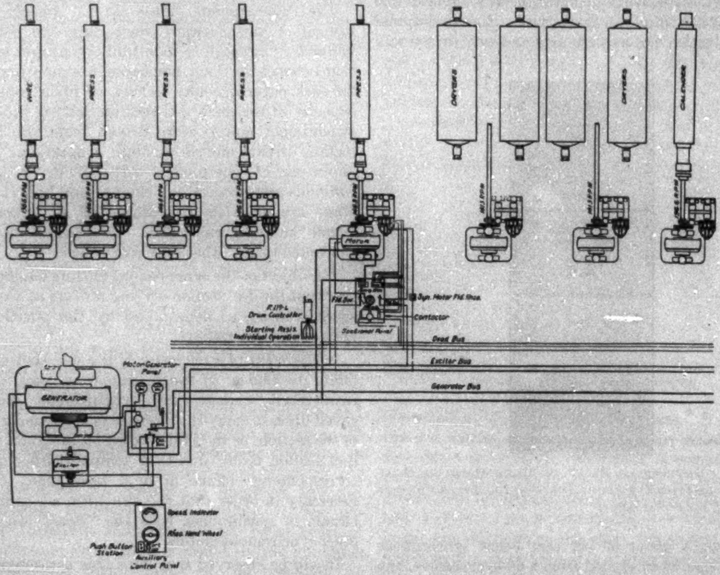
WIRE
PRESS
PRESS
PRESS
PRESS
DRYERS
DRYERS
CALENDER
Motor
Syn. Motor Fld. Rhse.
Fld. Sw.
R 119-L
Drum Controller
Starting Resis.
Individual Operation
Contactor
Sectional Panel
Dead Bus
Exciter Bus
Generator Bus
Motor Generator Panel
GENERATOR
Exciter
Speed Indicator
Rheo. Hand Wheel
Push Button Station
Auxiliary Control Panel

FIG. 5—SCHEMATIC DIAGRAM OF SYNCHRONOUS MOTOR TIE-IN SYSTEM OF SECTIONAL DRIVE WITH BELTS AND CONE PULLEYS

In order to maintain the synchronous motor at approximately no-load, so as to be in a position to compensate for any change in either direction, it was, of course, necessary to readjust the shunt-field strength of the d-c. section driving motors at intervals, by suitable hand rheostats, a double indicating instrument being provided for each synchronous motor to indicate the approximate load as motor or generator.

This system of speed control was soon superseded by a modification wherein the belt and cone pulleys for the transmission of power from the synchronous motors was eliminated and the synchronous motor rotor was geared direct to the d-c. section driving motor and the synchronous motor stator was made to rotate at a slow speed by a small variable speed d-c. motor which was geared to the stator member of the synchronous motor. Additional bearings and slip rings were provided to permit the rotation of the stator.

In order to adjust "draw" of the sheet, or the relative speeds of the d-c. section driving motors, it was now only necessary to adjust the speed of the small d-c. motor driving the stator of the synchronous motor, by means of a hand rheostat in its field circuit; and then to balance the load by adjusting the rheostat in the field of the d-c. section driving motors.

This arrangement eliminated the belt slippage factor incident to the cone-pulley belt drive, and except for variations in speed of the small d-c. motors, which were under no automatic control, the major possibilities of speed variation of the d-c. section driving motors had been eliminated. In this connection it will be seen that, while the small d-c. motors driving the stators of the synchronous motors were subject to load changes from zero to maximum and possibly even from motor to generator, if the worm gear efficiencies were high enough, considering the ratios used, the effect of the resulting speed variations of these small motors on the speed of the d-c. section driving motors was only approximately inversely proportional to the ratio of the worm gear used in driving the stators of the synchronous motors.

"INTERLOCK" SYSTEM

The other or "interlock" system of electrical sectional paper-machine drive referred to, employed a mechanical differential gear system for the operation of face-plate rheostats in the shunt-field circuits of the several d-c. section driving motors (Fig. 7).

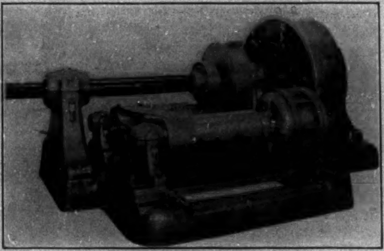

FIG. 6—SECTIONAL DRIVE OF SYNCHRONOUS MOTOR TIE-IN SYSTEM

A master shaft ran the entire length of the paper machine and adjacent to each of the paper machine sections, a shaft at right angles was taken off from the master shaft by means of a bevel gear. One end of the right angle shaft engaged with one member of a differential gear and the other end member of the differential gear engaged with a shaft driven by the d-c. section driving motor, through a small speed changer, consisting of belt and cone pulleys or expanding V pulleys. The middle element of the differential gear was geared to the face-plate rheostat in the shunt-field circuit of the d-c. section driving motor. The master shaft was driven by the dryer section of the paper machine, which, therefore, became the master speed reference, and was not under regulator control.

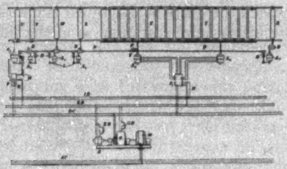

FIG. 7—"INTERLOCK" SPEED CONTROL SYSTEM OF SECTIONAL DRIVE

Speed regulation of the individual d-c. section motors by this system is a function of the speed of the differential gear members, the ratio of the middle member to the rheostat arm shaft, the total resistance in the rheostat and the number of resistance steps; as well as the inherent regulation of the motors themselves and the electrical and mechanical inertia of the system. The regulation is also adversely affected by any back-lash or lost motion in the bevel, differential and rheostat gears and by the torsional elasticity of the master shaft; in fact, by any deformation or departure from synchronism or exact phase position of one part of the speed regulating system to another.

To overcome the disadvantages of the speed regulating rheostat with a rather limited number of resistance steps and of fairly high total resistance, no individual step of which is likely to give exactly the right speed, an arrangement was employed wherein the rheostat contact shoe overlaps one resistance step or button so that a very small movement of the rheostat arm will serve to transfer contact to any one of three resistance steps of the rheostat. To maintain a correct mean resistance value, it is necessary that the rheostat arm oscillate back and forth from one contact point to another, remaining in contact with either at each oscillation such a percentage of time that the mean resistance is of the required value to maintain the d-c. section driving motor at the correct average speed. Ostensibly, the only force available to cause oscillation of the rheostat arm is the d-c. section driving motor which must oscillate at the same frequency as the rheostat arm. The frequency of the oscillation is, of course, determined by the reactance of the electrical system involved, the motor field flux changes, the corresponding changes in motor torque, the inertia of the mechanical parts involved and the lost motion and torsional deformation of the gear train and shafts in the control system.

In each of the systems of electrical sectional paper-machine drive and control described, the usual method of obtaining desired changes in paper speed is by adjustment of the voltage of the d-c. generator supplying current to the armatures of each of the d-c. section driving motors.

ELECTRICAL DIFFERENTIAL REGULATOR

The second step in the development of the electrical differential type of field control speed regulator system was the elimination of the a-c. system entirely and the substitution of master and section rotary contactors. These consisted of commutators or ring segments, the master and section rotary contactor being in series and serving to periodically short-circuit resistors in the shunt-field circuit of the section motor under control as illustrated schematically in Fig. 8. Both master and section rotary contactor drums or commutators operate normally at the same speed and the angular displacement of the one to the other determines the percentage of time the resistance is cut in or out of the field circuit.

The operation, as will be seen, is somewhat similar to that of a Tirrill regulator, but does not have vibrating contacts.

Differing from the Tirrill regulator, there are a multiple of resistors in series which instead of being short-circuited at the same time, are short-circuited alternately in such a way as to tend to produce the least change in voltage periodically across the motor field. To handle four resistors in series, six rings on the section rotary contactor and four rings on the master rotary contactor are required.

With this arrangement, when the section rotary contactor is exactly in phase with the master rotary contactor and running at the same speed, the regulating resistance in the shunt field circuit of the d-c. section motor is short-circuited and, therefore, cut out 100 per cent of the time. When the section rotary contactor is 180 deg. out of phase, lagging behind the master, then the regulating resistance is all cut in all of the time.

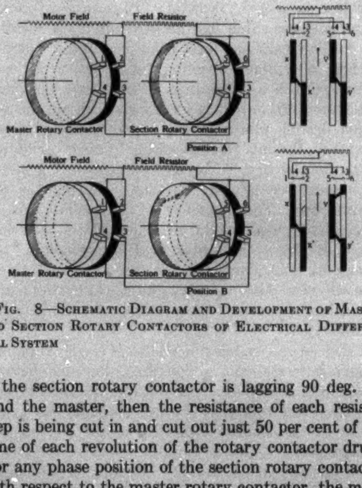

FIG. 8—SCHEMATIC DIAGRAM AND DEVELOPMENT OF MASTER AND SECTION ROTARY CONTACTORS OF ELECTRICAL DIFFERENTIAL SYSTEM

If the section rotary contactor is lagging 90 deg. behind the master, then the resistance of each resistor step is being cut in and cut out just 50 per cent of the time of each revolution of the rotary contactor drum. For any phase position of the section rotary contactor with respect to the master rotary contactor, the mean effective regulating resistance in the shunt-field circuit of the d-c. section driving motor is a direct function of the angular displacement of the section rotary contactor with respect to the master. With more than four resistance steps and a corresponding increase in the number of commutator rings, the wiring becomes very complex and difficult to trace and offers various objections for this reason.

LATEST TYPE ROTARY CONTACTOR ELECTRICAL DIFFERENTIAL SPEED REGULATOR

The third and latest development in improving and simplifying the electrical type of differential speed regulator by field control is a change in the rotary contactor design whereby it becomes in effect a continuous rheostat automatically controlled. The rotary contactor drum consists of an annular ring segment tapered at one end, forming a conducting segment and a complementary insulating segment, the two constituting the entire surface of the drum or cylinder, (see Fig. 9).

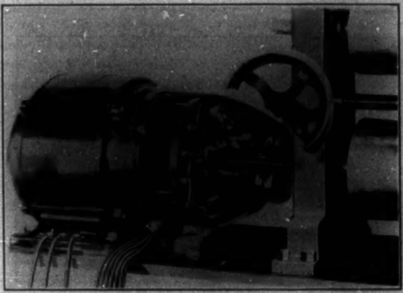

FIG. 9—LATEST SECTION ROTARY CONTACTOR UNIT OF ELECTRICAL TYPE DIFFERENTIAL SPEED REGULATOR SYSTEMS

Twelve brushes disposed radially around the periphery of the drum form a single spiral helix. Resistance units are connected between the brushes from one end of the spiral to the other in series with the shunt-field circuit of the motor. The conducting segment of the drum serves to short-circuit as many steps of resistance as there are brushes in engagement with the conducting segment at any time.

The drum is assembled on a sleeve which is mounted on the shaft of the section master synchronous motor, and is movable longitudinally on the sleeve but prevented from rotating thereon by a spline key, and the sleeve on which the drum is assembled is pinned to the shaft. The brushes are stationary and the full length of the taper of the conducting segment of the drum is equal to the "pitch" from one brush to the next in a direction parallel to the axis of the drum, plus a small overlapping allowance.

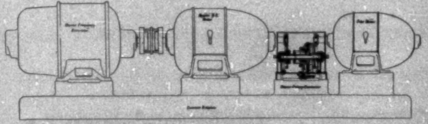

FIG. 10—MASTER SET OF LATEST ELECTRICAL TYPE DIFFERENTIAL SPEED REGULATOR SYSTEM

The speed reference in this arrangement is a master set (Fig. 10) consisting of a master frequency generator driven by a d-c. master motor and controlled with great precision in speed by a d-c. pilot motor and master rotary contactor. It is from this master frequency generator that each of the section master synchronous motors of the section rotary contactor units operates. Both the d-c. master motor and the pilot motor of the master set operate in parallel with the d-c. section driving motors and the master rotary contactor resistors are in the shunt field circuit of the d-c. master motor. The pilot motor is the final speed reference and as it operates with substantially no load changes and is usually located where there is little temperature change, its speed is very constant for any desired paper speed; voltage regulators being used to maintain the exciter voltage uniform and the main d-c. generator voltage constant at any desired voltage within the speed range of the paper machine.

A control speed changer, consisting of a pair of cone pulleys and belt, is provided for each section of the machine, one cone pulley being driven from the d-c. section motor to be controlled, by a chain drive; and the other cone pulley shaft coupled to the screw member of the section rotary contactor; a nut member being fixed in the outer end of the rotary contactor drum with which the screw engages. This screw member is assembled on the same sleeve with the drum to assure perfect alinement and is rotatable thereon but is restrained from moving endwise by a shoulder in the sleeve at one end of the screw and a collar at the other end on the end of the master synchronous-motor shaft. From this definite longitudinal location of the screw on the sleeve of the section rotary contactor—the spline key in the sleeve and drum preventing rotational movement of the drum on the sleeve; and considering the engagement of the screw within the nut member of the drum—it is apparent that any difference in speed between the screw and the nut will cause a longitudinal movement of the drum beneath the brushes.

In normal operation, the screw member driven by the d-c. section driving motor through the control speed changer operates at the same speed as the nut member, drum and master; and certain resistance steps are continuously short-circuited corresponding to the brushes engaging with the continuous portion of the conducting segment of the drum. One step is being cut out periodically once every revolution, the percentage of time it is cut out corresponding with the percentage of time the brush of this step is in engagement with the tapered portion of the conducting segment which depends upon the longitudinal position of the drum beneath the brushes. The remaining resistance steps are cut in continuously, as the brushes are only in engagement with the non-conducting segment of the drum. As a result, the motor operates at a speed resulting from the mean effective resistance valve of the rotary contactor resistor—an automatic rheostat—with an infinite number of steps of infinitely small increments of effective resistance.

The extreme sensitivity of this regulating system will be appreciated by the fact that any change in angular position or phase relation of the lower cone pulley shaft of the control speed changer with respect to the master section synchronous motor, through the screw action, causes a longitudinal movement of the drum beneath the brushes and a longitudinal movement of only a few thousandths of an inch is sufficient to correct for most of the tendencies on the part of the d-c. section driving motor to change in speed. A travel of the drum of approximately three-fourths of an inch suffices to cover the range from resistance all-out to resistance all-in; and sufficient resistance is frequently used to produce a speed increase of as much as 50 per cent or more when all cut in to the shunt field circuit of the motor.

Another very important characteristic of such a differential type of regulator is the fact that any variation in speed, no matter how small, produces a positive corrective effect; and the corrective effect is cumulative and continually increases in value until the correct speed has been restored. A change in angular or phase position only, of the section with respect to the master serves to cut in or cut out sufficient resistance in the shunt field circuit of the d-c. section driving motor to maintain its speed at the same rate as before although the load may have changed.

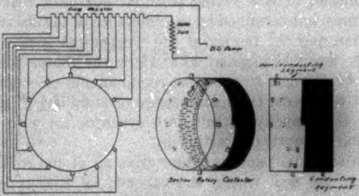

FIG. 11—SCHEMATIC DIAGRAM AND DEVELOPMENT OF SECTIONAL ROTARY CONTACTOR DRUM, BRUSHES AND WIRING CONNECTIONS OF LATEST TYPE ELECTRICAL DIFFERENTIAL SPEED REGULATOR SYSTEMS

In this respect it is interesting to observe that the d-c. motor is made to function much like a synchronous motor in so far as maintenance of speed is concerned. With the synchronous motor when a load is thrown on, the rotor falls back in phase position with respect to the generator or the frequency source and the new load is carried without more than a transient change in speed and at the same rev. per min. as before; sufficient torque having been developed in the phase displacement to carry its load. In the case of the d-c. motor with electrical speed differential field control, when a load is thrown on, the rotor falls back in phase position with respect to the master and the new load is carried without more than a transient change in speed, at the same rev. per min. as before; sufficient torque having been developed in the phase displacement and corresponding longitudinal movement of the rotary contactor drum beneath the brushes, thereby changing the motor field resistance, so as to carry the new load without change in rev. per min. When the load is thrown off, the rotor will move forward again in a manner similar to that of the synchronous motor. The number of mechanical degrees through which the d-c. motor must fall back in phase displacement in order to carry a load of given value, at the same speed as before, depends upon the droop of the load speed characteristic curve of the motor, the resistance in the rotary contactor regulating circuit, and the ratio of longitudinal drum movement to the differential angular displacement of the rotor of the motor.

This electrical speed differential field regulator in its simplest form is not fundamentally an especially high speed regulator, nor is it especially adapted to correct with exceptional quickness for extremely heavy load changes and corresponding tendencies to speed change, although with suitable auxiliary reaction features, not necessary in paper machine drive, such conditions can be taken care of. The upper limit of speed of regulation of this system without auxiliary reaction features, is determined by the reactance of the electrical system and the inertia of the mechanical system; in other words, the speed with which the motor torque can be changed to meet the requirements of constant relative speeds, the maintenance of stability of speed and the capacity to pull into synchronism with the master promptly on starting up.

In sectional paper-machine drive all of the requirements are well within the capacity of this type of electrical speed differential regulator system and the relative speeds of the section driving motors are held constant with such precision that no variation in the "draws" of the sheet can be observed even though the paper may be traveling through the machine at the rate of 1000 ft. per min. or more which speeds are now common in modern high speed paper-making machines.

The purpose of the control speed changer between the d-c. section driving motor and the screw member of the section rotary contactor is to make possible adjustments of the speed of the d-c. section driving motors and at the same time maintaining the speed of the screw member of the rotary contactor at the same speed as the section master synchronous motor. Coincident with any shifting of the belt on the control speed changer an angular displacement of the screw with respect to the nut takes place, moving the drum longitudinally, changing the regulating resistance so as to give the d-c. section motor its new speed corresponding with the new belt position on the cone pulleys.

The coning of the pulleys of the speed changer is so small that even though a change in section "draw," or relative speeds of 25 per cent to 30 per cent can be obtained, good belt contact is always secured; and as no load is transmitted by the belt no slippage or creep that can in any way be detected takes place.

Of much value in sectional paper-machine drive and fundamental in this electrical speed differential field regulator system, is the fact that the section motors do not have to oscillate in speed to maintain the proper mean regulating resistance values; nor does any other part of this control system oscillate. The motors operate at a constant uniform speed and the exact mean effective regulating resistance values required for uniform motor speeds at any desired value are obtained by the constantly rotating tapered conducting segment of the drum of the section rotary contactor.

With the twelve steps of regulating resistance normally employed in the design of the rotary contactor, resistance values sufficiently high to meet every requirement of electrical sectional paper-machine drive can be used with practically no visible sparking at the brushes and with negligible wear of brushes or drum. The mechanical movement between the screw and nut of the rotary contactor is so small that there is practically no wear of those parts.

Fig. 12—Sectional Paper-Machine Drive Installation of Electrical Differential Type, Field Control, Speed Regulator Systems

In order that the screw member may readily re-engage with the nut member of the section rotary contactor, should it run out of engagement in shutting down or starting up, small helical springs are so disposed within the drum that they come into engagement and begin to compress just before the screw leaves the nut so that there is a sufficient force to cause the re-engagement of the screw and nut as soon as the speeds of the respective members are such as to permit.

Full automatic push-button control of starting and stopping operation, "inching" and changing of "draw," as well as the adjustments of the speed of the machine as a whole for any paper speed within its range, adds greatly to the flexibility and makes the equipment practically fool-proof.

Rumors have been heard of the development in Europe of an a-c. type of electrical sectional paper-machine drive wherein commutator-type variable speed a-c. motors are understood to be used for driving the paper-machine sections with a differential mechanism for automatically shifting the motor brushes to control the relative speeds of the section driving motors and by changing the master speed to effect speed adjustment over the entire range of the paper machine. Information is not at hand to confirm the successful installation and performance of this type of drive and control.

The use of electrical sectional paper-machine drive has made possible the operation of paper-making machines economically at higher speeds than was possible with existing mechanical systems of drive. This was largely due to the rather limited amount of power that can be transmitted by means of cone pulleys and belt drive. With electrical sectional paper-machine drive a very considerable saving in power is accomplished and maintenance is reduced to a minimum as compared to mechanical drive. On account of the elimination of belt slippage and sectional speed variations incident thereto, breaks in the paper are greatly reduced and the operating efficiency materially increased. More exact speed regulation throughout is made possible and many other distinct advantages in specific cases and kinds of paper machines and types of product have added greatly to the general efficiency, flexibility and operating technique in the modern paper-making machines.

The object of this paper has been to chronicle the advance in the art of electrical sectional paper-machine drive and control, to indicate the several more or less distinct lines of development, to outline the outstanding characteristics of each of the several types and to point out novel and rather unique methods, so far as the author knew, for obtaining speed regulation of great precision and for the making of d-c. motors to operate at synchronous relative speeds.

The author wishes to express his appreciation of the privilege of using in this paper illustrations kindly furnished by the following: Vocational & Educational Committee of the Paper Industry; McGraw-Hill Book Company, Publishers of "Manufacture of Pulp and Paper"; The Paper Mill and Wood Pulp News; The General Electric Company; Westinghouse Electric & Manufacturing Company.

The Development of the Sectional Paper-Machine Drive

BY H. W. ROGERS[1]

Associate, A. I. E. E.

Synopsis.—*To an increasing degree, large machines designed for quantity production are being subdivided into their elements, each section being driven by a separate motor as contrasted with former practise of driving these machines, as a whole, from a single unit. With the paper machine, this is called the sectional-electric drive.*

In this paper, the author has confined himself largely to the history and development of the several types of the sectional drive and the relative merits of each particular type.

An attempt has been made to set forth clearly the advantages of the sectional drive and to show that its field of application should increase with a fuller understanding of its advantages as to increased production, improved product and lower operating cost.

* * * * *

THE sectional drive, as it is commonly called, consists of a number of individual motors driving the various sections of a single machine with provision for maintaining a definite speed relationship between the sections, and while it is not limited in its application to paper machines, the use of it in this industry has been so extensive that it is impossible to mention sectional drives without immediately thinking of the paper industry.

The sectional drive is an extremely interesting engineering problem and is very important commercially, especially to the pulp and paper industry. Of approximately 1600 paper machines in use at the present time, probably 80 per cent are still being driven by means of the old type mechanical drive and steam engine, which would indicate that, in spite of the progress already made, the field of application for the electric drive has barely been entered.

The advantages of the electric motor are recognized, as practically all new mills are 100 per cent electrically equipped and most old mills use motors to a limited extent on pumps, beaters, jordans, screens, etc. However, the paper machine, the most important machine in the mill and the heart of a billion dollar industry, has been neglected. A vast amount of time and thought, research and development work, has been devoted to this problem by the engineers of some of the larger electrical manufacturers and, like all other new applications, developments, changes and refinements have taken place.

In a sense, these changes have come as the result of a broader knowledge of the problem and an honest endeavor to meet all of the requirements, but it is equally true that the paper manufacturers have gradually become more exacting in their requirements. In any event, change and development go hand in hand

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with progress; it is a natural process and should not be held out as a criticism.

To appreciate fully the problem which confronted the electrical manufacturer in 1919, it is necessary to consider the details of the paper machine and its requirements.

There are certain parts of every paper machine which run at a constant speed, regardless of the speed at which paper is being produced, and these parts, consisting of screens, pumps, suction and agitators, constitute what is commonly called the constant-speed end of the paper machine. The driving of this end of the machine presents no difficulty whatever in its application and requires very little attention.

The variable-speed end of the machine consists of a number of separate sections, which have been in the past and are very largely at the present time driven from a single line shaft through a system of cone pulleys and bevel gears with friction clutches for starting and stopping each individual section separately if occasion requires.

The quality of the product depends very much upon the type of drive selected and the proper application of it with respect to the machine. Speed regulation, flexibility of control and uninterrupted service are all important factors and must be given careful consideration. For this reason it is desirable to have the paper machine and its drive a complete unit in itself, independent of the rest of the mill, so that trouble in the rest of the mill will not result in any interruption in the actual production of paper.

The line drawing, Fig. 1, illustrates the more common form of paper machine, consisting of the following sections:

1. Couch (either suction or standard)
2. First press (either suction or standard)
3. Second press
4. Third press
5. First dryer
6. Second dryer
7. Calender
8. Reel.

A fourth, and even a fifth press, is sometimes encountered, and it is not uncommon to find a smoothing press with one or two breaker stacks and two or three calendar stacks, making a total of twelve or thirteen sections on one single machine. These sections, however, are practically independent of each other so that they are readily controlled, but on machines which involve felts common to two or more sections, the problem becomes complicated and both the paper manufacturer and the electrical manufacturer begin to court trouble.

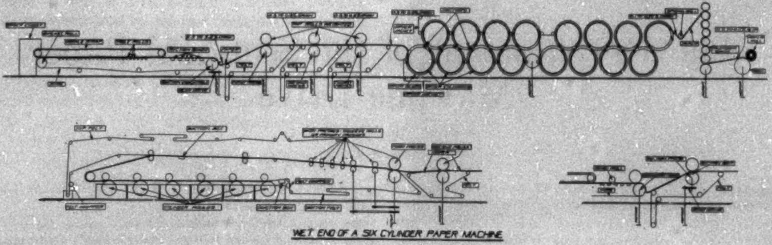

FIG. 1—FOURDRINIER PAPER MACHINE

The stock enters the machine at the wet end on the wire and consists of 99½ per cent water and ½ per cent stock. Here the sheet is formed and, in three seconds, 99 per cent of the water is removed after which the sheet passes through one or more press rolls and over the dryers to the calenders, reel and rewinder as finished paper with about 8 per cent moisture, the whole operation requiring probably 40 seconds.

A continuous sheet must, therefore, be maintained throughout the machine and owing to its condition, there is a slight difference in speed between each section which must be maintained absolutely. This difference in speed is called "draw" and varies from time to time with the condition of the stock and the grade of paper and must, therefore, be capable of adjustment.

The drive must not only permit a very close speed regulation, but must also permit adjusting the "draw" between sections and at the same time absolutely maintain the relative speeds of the various sections.

The "draws" or slight variations in speed between consecutive sections are not fixed but are subject to modification due to the condition of the stock, kind of paper and suction; consequently, provision must be made to vary the speed of each individual section within certain limits (approximately 20 per cent) and still maintain a definite speed relationship between all of the sections. The approximate draws are indicated in Fig. 1.

The driving of a single machine with a multiplicity of motors, or a sectional drive as it has come to be known, is not a new idea, although the demand for it is relatively recent, beginning in 1919.

In 1909 the first real sectional drive was installed at the Chisholm Falls plant of the International Paper Company, consisting of d-c. motors with quarter-phase collector rings and mechanical speed changers for "draw" adjustment and, although it operated successfully for months, it was evidently in advance of the times and was eventually taken out.

In 1919 the enormous demand for paper, which had been aggravated by the curtailment of paper machine production during the war, started a controversy regarding the relative merits of wide, slow-speed machines and narrow, high-speed machines. Needless to say, the high-speed machine won out, although there is now a mild tendency to reverse that decision, and the demand for sectional drives was born at a time when most of the electrical manufacturers were not fully prepared to receive it.

It was frankly admitted among machine builders and operators that while the mechanical drive was a possibility on high speed machines, it was not at all practical on account of the high maintenance and this condition magnified the necessity for a solution to the problem.

There were no reliable power data available, since it was practically impossible to obtain any with mechanical drives; nor were there any accurate data on "draws" for the same reason. However, the need was urgent and the first so-called synchronous tie-in type of paper-machine drive, in spite of adverse criticisms and prophesies to the contrary at the Crown-Willamette Paper Company's Mill, was built and put into successful operation. This drive was really the forerunner of the ever increasing demand that has spread over the entire country, including Canada, and its success has had a marked bearing upon the development of the industry and the attitude of the trade toward sectional drives in general. A failure at that time would have been a reflection upon the electrical industry as a whole and undoubtedly would have been a serious set-back to the sectional drive as such.

In theory this type of drive was all that could be desired; in practise it fulfilled all expectations and its success was duplicated in eleven other orders during the same year. These drives made it possible to secure a vast amount of valuable data heretofore unavailable, the careful analysis of which has had a great deal to do with the developments that have followed since their creation.

During this period drives of an entirely different character were being exploited, although not in such large numbers.

Strictly speaking, there are but two types of sectional drive on the market today:

I. The synchronous tie-in type of drive, wherein there is an actual interchange of energy between the various machine sections and where the relative speeds are held constant by synchronous motors. This, truly, is preventative rather than corrective.

II. The regulator type of drive, wherein the speed of the various sections is maintained by shunt field adjustment on the motors. It will operate on an angular displacement similar to Type I, if the regulator element is of the synchronous type, but there is no actual restraining power to hold the motor in place and the success or failure of this type is determined by the amount of angular displacement which causes the regulator to function and by the time element of the motor field.

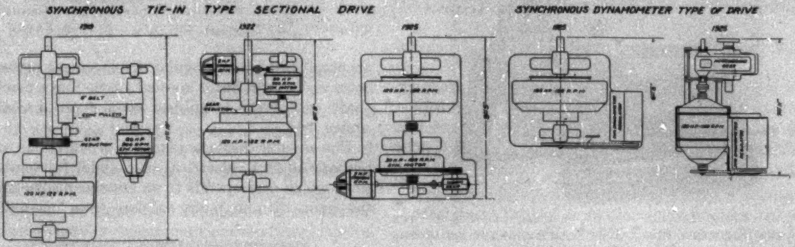

Fig. 2

The real difference between these two types of drive can readily be appreciated when the peculiar characteristics of the d-c. motor, operating at reduced voltages and speeds, are considered. The speed regulation at full voltage and full speed may be three or four per cent or even less, but, owing to the armature drop, it becomes extremely poor when operating at low speeds on low armature voltage. To maintain a constant speed under these conditions with a varying load requires a very wide change in the field strength which detracts from the motor torque at low speeds. In other words, a d-c. motor will not deliver as much torque under reduced armature voltages and this demands the use of over-capacity motors where the regulator type of control is used.

With the synchronous tie-in type of drive, the motor capacity need not be increased because the synchronous motors absorb the change in load and the d-c. motors may be operated at a fixed field. This type of drive, therefore, while higher in cost and space requirements, has the advantage of smaller capacity motors and will hold the motor speed within extremely close limits even where instantaneous load changes are encountered, whereas the regulator type of drive requires a certain time element between the change in load and the correction. However, the regulator type of drive, which will be fully treated later in this paper, has been improved to a point where it meets the most exacting requirements of commercial operation and, despite the larger capacity driving motors which it requires for wide range machines, offers advantages as to first cost and space requirements.

Coincident with the development of the regulator type of drive, the synchronous tie-in drive, while remaining fundamentally the same, has passed through several stages of improvement.

I. The original type consisted of slow-speed d-c. motors with high-speed synchronous tie-in motors connected to the d-c. motor through a pair of cone pulleys and a gear reduction.

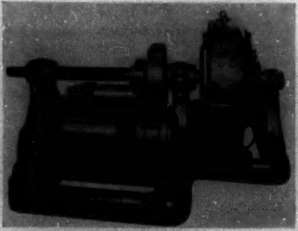

Fig. 3—First Development of Synchronous Tie-in Type of Drive Showing Moderate-Speed Synchronous Motor Connected to Main Motor Through Gear Reduction and Cone Pulleys

II. In the second development the cone pulleys and belts were eliminated and a high-speed synchronous motor with revolving stator frame was used, the stator frame being driven by a small adjustable-speed d-c. motor. This change reduced the space requirements and overcame the objection to belts.

III. The final development of this type of drive utilizes a slow-speed synchronous motor mounted on the d-c. motor shaft, thus eliminating the gear reduction.

On all three of these drives the synchronous motors were 20 per cent of the capacity of the main driving unit with 200 per cent pull-out torque either as a motor or as a generator; consequently, they could take care of a maximum load change of 40 per cent without exceeding their normal rating and still hold the speed within very close limits. The development of this type of drive is shown in Fig. 2.

It is a well-known fact that the synchronous motor operates at a fixed speed which is dependent upon the frequency of the system to which it is connected and the speed cannot be changed so long as it operates within its capacity. The use of a multiplicity of synchronous motors connected to the same source of supply or interconnected between each other is analogous to a mechanical line shaft and gears connecting the various sections of a machine together.

It is true that the loading of a synchronous motor is accomplished by an angular displacement between the rotor and stator, but it is equally true that a mechanical shaft with gears is subject to angular displacement under load.

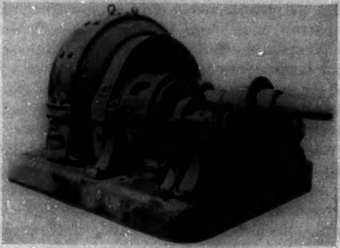

Fig. 4—Second Development of Synchronous Tie-In Type of Drive Showing Moderate-Speed-Geared, Synchronous Motor with Revolving Stator Frame

The angular displacement in a synchronous motor is perhaps 10 or 12 electrical deg. under full-load conditions which corresponds to an angular displacement on the main driving motor of less than one-half of a mechanical degree. This displacement affects the sheet of paper that is passing over the roll at the time the load change occurs, and is present in all types of drive whether mechanical or electrical. It, however, is a negligible quantity and has no bearing upon the operation.

Fig. 5—Third Development of Synchronous Tie-in Type of Drive Showing Slow-Speed, Direct-Connected Synchronous Motor with Revolving Stator Frame

The original type of drive which has been so successful from the beginning has been installed on twelve paper machines, although one of them has since been modified to the regulator type for the sake of uniformity in equipment. The use of cone pulleys and belts between the main driving d-c. motors and the synchronous tie-in motors has been questioned in a few instances on the basis that belts in ordinary commercial service operate with a slip of two or three per cent. However, the belts used on these drives are very much over capacity and under actual test show a maximum slip of $\frac{30}{100}$ of one per cent. The load on a paper machine is very constant and under normal operating conditions will not vary more than 10 per cent. Consequently, the speed is maintained with $\frac{15}{100}$ of one per cent or less, whereas the same load change on a mechanical drive would be twice this amount or even more, depending upon the condition of the belts. Each synchronous motor is free to act as a motor or a generator with an actual transfer of energy between it and the balance of the synchronous motors. A load change on one section, therefore, may affect the speed of that section $\frac{15}{100}$ of one per cent, but the effect on the remaining sections, which must of necessity supply or absorb the energy, is only $\frac{2}{100}$ of one per cent on an eight-section machine.

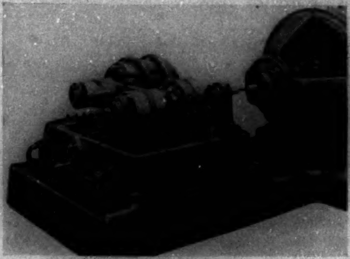

FIG. 6—DISK-TYPE SPEED REGULATOR
As installed on sectional paper-machine drives

The second development of synchronous tie-in drive utilized a slow-speed d-c. motor to which a moderate-speed synchronous motor was connected through a gear reduction. The "draw" adjustments or speed changes, were obtained by rotating the stator frame of the synchronous motor by means of a small adjustable speed d-c. motor. This change eliminated the cone pulleys and belts and had the advantages of smaller space requirements and "draw" adjustment through the medium of a field rheostat rather than the shifting of a belt. This type has been in successful operation since 1922 and has also been furnished on two other machines.

The third and final development of the synchronous tie-in drive utilizes a slow-speed d-c. motor and a slow-speed synchronous motor mounted on the same shaft. The "draw" adjustment is obtained by rotating the stator frame of the synchronous motor by means of a small adjustable speed d-c. motor. In this drive the cone pulleys, belt and the gear reduction are eliminated. This drive has been furnished for four paper machines which are motored for 1440 ft. per min., a speed far in excess of anything that has ever been attempted in the past on any machine.

In both of these developments where revolving-frame synchronous motors are used, the stator frame revolves at an extremely low speed compared with the rotor and consequently a speed change on the adjustable speed motors, which drive the stator frame, has little effect upon the speed of the section. In reality the regulation of the small d-c. motor only affects the speed of the stator which is but a small percentage of the synchronous speed. Consequently, the synchronous speed or the speed of the main driving motor will be maintained within $\frac{15}{100}$ of one per cent or less for the load changes ordinarily encountered on a paper machine.

Fifteen of these drives are now in successful operation and most of them have been operating for five years with practically no maintenance.

THE SYNCHRONOUS DYNAMOMETER (REGULATOR) TYPE

The possibilities of a speed regulator which operates on the shunt field of a d-c. motor have been fully appreciated and as early as 1920 a synchronous dynamometer type of speed regulator was built by the company with which the writer is connected.

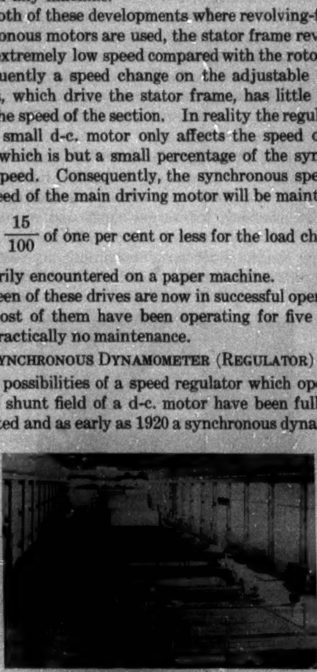

FIG. 7—SYNCHRONOUS TIE-IN TYPE OF DRIVE WITH CONE PULLEYS AND BELTS
As installed on a 164-in., 1200-ft.-per-min. paper machine

This regulator consisted of a small synchronous motor, the rotor of which was driven from the motor on which the speed was to be held constant through a pair of small cone pulleys and a small belt. The stator was excited from a master generator in such a way that it remained stationary unless there was a tendency for the controlled motor to change in speed, thereby causing the stator to rotate and alter its field to maintain the speed constant. It operated on an angular displacement and handled the field current direct through a commutator type of rheostat, the brush mechanism of which was connected direct to the synchronous motor stator.

In 1921-1922, the disk-type regulator with synchronous motor actuating element followed closely on this development operating on the principle of a voltage regulator, except that the contacts were revolving instead of stationary. In its developmental stage the disks were modified to give a total angular displacement of 180 degrees and were so constructed as to cut the motor field resistance in and out of the field circuit twice every revolution or twenty times a second, the effective field resistance being determined by the angular displacement between the disks, one of which was driven by the motor to be controlled and the other by the synchronous motor. Subsequently, the brush mechanism was altered to cut the resistance in and out in three steps rather than one.

During 1922-23 this final form was furnished with ten paper machine drives, making newsprint, book, kraft, tissue, crepe and glassine; on high-speed machines and low-speed machines, for narrow-range and wide-range speeds. The regulator was subject to limitations in its operating range, under some conditions necessitating adjustments in an auxiliary field rheostat. This feature, however, only applied to wide-range machines and the endeavor to overcome it and make the adjustment of the auxiliary rheostat automatic resulted in one more development in which a mechanical differential was added to the disks to automatically adjust the external field rheostat. It was built and tested under actual operating conditions but has never been furnished on any drive.

FIG. 8—REGULATOR TYPE OF SECTIONAL DRIVE

As installed on a 170-in., 700-ft-per-min. machine

As a result of past experience with all kinds of paper machines making all grades of paper, it is believed that the speed regulator which will best satisfy the exacting conditions of operation and appearance is the development and refinement of the synchronous dynamometer regulator built and tested in 1920 and known as a synchronous dynamometer. It consists of a small synchronous motor, the rotor of which is driven from the motor to be controlled through a pair of cone pulleys and a belt which gives 20 per cent "draw" adjustment. The stator frame, mounted on ball bearings, is excited from a master generator and is free to rotate. It actuates the brush mechanism of a commutator type rheostat which has a total movement of 180 deg. The rheostat has both coarse and fine steps and gives the equivalent of 450 operating points.

FIG. 9—CONTROL SWITCHBOARD FOR SECTIONAL PAPER-MACHINE DRIVE

This regulator combines a number of features not found in any other type of regulator:

1. Is of the synchronous type
2. It operates on an angular displacement corresponding to approximately 0.05 per cent change in speed of the controlled motor
3. Has 450 operating points
4. Has anti-hunting features
5. Has an absolute minimum of moving parts as the rotor of the synchronous motor, which is mounted on ball bearings, is the only moving part
6. The rheostat mechanism is stationary except when load changes demand action. It is not of the make and break contact type
7. Remains stationary, maintaining its operating position when the master generator is shut down or when the synchronous motor is disconnected from its power supply, thus overcoming any possibility of a "wild" machine
8. Has a wide operating range
9. Low maintenance
10. Minimum space requirements
11. Is totally enclosed
12. Is mounted direct on the motor base
13. Presents a neat compact appearance
14. Is readily accessible
15. Is sturdy in design
16. Has motor-operated "draw" adjustment.

It would seem that there has been an unusual number of modifications in the sectional drive but a careful analysis of the situation will reveal that there have been from the very beginning but two distinct types. The changes that have taken place are a natural process of development and do not affect the fundamental principle of operation.

Thus far only the development of the two types of drive and their characteristics as separate units have been given consideration, without any particular reference to the machine as a whole.

The sectional drive usually consists of the various parts as enumerated below.

I. A Turbo Generator Set.

(a) This has a separately excited shunt-wound d-c. variable voltage generator and a non-condensing steam turbine with a direct-connected exciter of sufficient capacity to excite the generator and all of the sectional motor units.

Exhaust steam is utilized to dry the paper and the turbine is designed to operate at back pressures to meet the required conditions. In some instance an a-c. generator is also furnished as a part of the turbine unit to furnish power to the constant speed end of the paper machine or to float on the mill system and thereby insure a perfect steam balance under all conditions of operation.

Synchronous motor-generator sets are sometimes used as prime movers and in such cases either live steam or exhaust steam from some other source is used to dry the paper.

(b) A generator-control panel

(c) A voltage-regulator panel with

1. A voltage regulator for the main generator
2. A voltage regulator for the exciter

(d) An auxiliary control panel for controlling the generator voltage and the speed of the machine as a unit.

II. Motor Equipment.

(a) An adjustable speed, d-c. motor for each section of the paper machine. This may be of the slow speed direct-connected type or of the moderate speed geared type. Both the open type motor and the enclosed, externally ventilated type motors have been furnished, but this is entirely a matter of preference with the paper manufacturer.

(b) A regulator.

III. Mechanical Parts.

All motors are furnished with base, shaft, bearings, couplings, shaft extensions, and all necessary equipment not included as a part of the paper machine.

IV. Control.

(a) A five-point, full automatic-contactor control panel with overload and under-voltage protection and indicating meters for each sectional motor unit.

(b) A continuous duty starting and regulating rheostat designed for 75 per cent speed reduction for each sectional motor unit.

V. Master Generator Set.

The master generator set which supplies a-c. power to the speed regulators consists of a 10-kw., 1200-rev. per min., three-phase, 60-cycle generator, direct connected to an over capacity d-c. motor which is controlled from the main generator panel and automatically comes up to speed when the voltage is brought up on the generator.

It controls the speed of the paper machine as a unit and its speed is adjustable within certain limits although it depends primarily upon the voltage at which the main generator is operating.

When the synchronous tie-in type of drive is used, the master generator is not necessary because any one or more sections which happen to be running constitute a master for controlling the speed of the other sections.

VI. Draw Adjustment.

Provision is made on each sectional unit for a sufficient "draw" adjustment to take care of all operating conditions such as variations in stock, grade of paper, grinding press rolls and the like.

The "draw" adjustment is motor operated and may be made from the front side of the machine.

VII. Temperature.

All equipment is based on 40 deg. cent. rise under normal load.

VIII. Location of Equipment.

The power unit should be located preferably in the basement under the paper machine and in close proximity to this unit the generator control and all of the motor control units may be lined up as a single switchboard. This arrangement leaves nothing in the machine room except the motor equipment and the control buttons.

IX. Operation.

In starting up the paper machine it is to be assumed that the motor-generator set or turbo generator set is running and that there is a proper voltage supply available. Each section of the paper machine may be started up in succession, starting with couch, by merely pressing a button after which the motor automatically comes up to its full speed or to a speed corresponding to the supply voltage and the speed regulator is automatically connected to the master generator bus and begins to function immediately.

Provision is made to operate the motors at slow speeds for washing felts, changing wires or spearing broke.

The individual control of the motors permits operating any section of the paper machine independent of any other section and at the same time assures an absolute synchronous tie-in between the sections when all motors are operating at their full speed.

SLOW-SPEED MOTORS

It is not difficult to design for this service either slow-speed direct-connected motors or moderate speed motors with herring-bone gear reductions. Either type of motor is satisfactory and it is simply a matter of capitalizing the higher maintenance and increased space requirements of the moderate-speed geared motor as against the slightly higher first cost of the slow-speed direct-connected motor.

Sectional Electric Drive for Paper Machines

BY R. N. NORRIS[1]

Non-Member

***Synopsis.**—This paper describes a system of electric control for sectionalized electric drive which makes use of a mechanical differential. Further, it discusses the drive of the constant-speed end of the paper machine and the general advantages of electrical sectional drive for paper mills. Some figures are given on the power consumption of several classes of mills. The type of drive described has been used for some time in Europe and Canada and it has now been successfully introduced in the United States.*

IN this paper will be described a system of sectional electrical drive which utilizes a mechanical differential as the basis of its control. First, this particular type of system, called the Interlock System, will be described and this will be followed by comments on the drive of the constant-speed end of the paper machine, with statement of the advantages of sectional driving in general.

Like all other sectional electric drives the Interlock installation consists essentially of a d-c., compound-wound, interpole generator with an exciter, which exciter supplies the excitation current to the field circuit of the generator and also the field circuits of the several section motors.

It is essential that the generator and exciter be designed to give good regulation under varying load conditions and that voltage regulators be provided to maintain a steady voltage in order to compensate for variations in load or speed of the prime mover.

Fig. 1 is a schematic diagram and layout of a typical installation showing the section motors at the couch, press, dryers and calender sections of a newsprint machine. This illustration does not show the reel or winder sections, but the reel and additional calender sections or smoothing roll sections, etc., can be driven in a similar manner, and the sketch serves to show the arrangement of the drive in general.

Each section of the paper machine is driven by a compound-wound interpole d-c. motor, which may be either directly coupled to the paper machine in-driving shafts or coupled thereto through some form of gear unit—double helical, worm, chain, etc. Sometimes a very convenient arrangement is to use direct-coupled motors for the heavier powers at couch, dryer and calender sections, and geared units at the press sections.

Fig. 2 shows direct-coupled motors driving a 234-in., 1200-ft. per min. newsprint machine. The sectional drives are housed in a separate room, the in-driving shafts of the paper machine passing through the walls into the machine room proper.

Fig. 3 shows an all-geared arrangement of Interlock drive, and Fig. 4 shows the semi-geared arrangement referred to above. Fig. 5 is a line drawing of a direct-coupled motor and also shows control unit.

In large newsprint machines the dryer sections, consisting as a rule of 40 to 46 cast-iron cylinders, five or six feet in diameter and up to 240 in. in face, are usually geared together and driven by motors driving through two pinions on to the same load.

However, the arrangement of the machine, whether direct-coupled, gear-driven, or, as here termed, semi-geared, is a matter of detail which may vary with the particular case in question.

As illustrated by Fig. 1, a d-c., adjustable-voltage generator, *G*, supplies the necessary power to the section motors, *A* 1 to *A* 7. This generator is separately excited, and the exciter also supplies the excitation current to the field circuit of the several section motors.

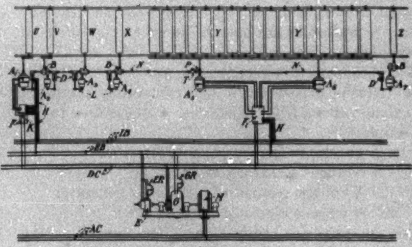

FIG. 1—SCHEMATIC DIAGRAM AND LAYOUT OF TYPICAL INSTALLATION

A.	Section motors
B.	Speed reduction gear
D.	Differential regulator
E.	Exciter
F.	Motor starting panel
F.	Dryer starting panel
G.	D-c. generator
H.	Contact and field connections
K.	Armature starting resistance
L.	Cone pulleys and belt for adjusting draw
M.	Main synchronous motor
N.	Master shaft
P.	Chain drive
T.	Bevel gears
U.	Couch press
V.	1st press
W.	2nd press
X.	3rd press
Y.	Dryer section
Z.	Calenders
DC.	Main d-c. bus, 500 or 250 volts
EB.	Exciter bus, 250 volts
ER.	Exciter field rheostat
GR.	Generator field rheostat
IB.	Interlock bus, 250 or 125 volts
AC.	Main a-c. bus

*A*1 to *A*7 represent the various section motors which, on the couch and the two dryer sections, are direct coupled to the intake shafts. On the three press sections and the calender section, a single reduction herringbone gear is employed. For purposes of simplicity no main switchboard is shown in this illustration; however, a main switchboard is provided between the generator and exciter and the main busbars, *D C*, and exciter busbars, *E B*. On the couch section is shown one of the motor starting panels, *F*, with the necessary connections from the field and armature circuits of the motor, and the differential regulator *D*; *F*1 is the starting panel for the dryer motors.

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Speed adjustment within the desired range of the paper machine is obtained, in the case of large newsprint machines, entirely by variation of the generator voltage which is applied to the section motor armatures. In book-paper machines, or other machines requiring large speed ranges, this speed adjustment is obtained by a combination of generator voltage regulation as above and variation of the field strength of the section motors. The whole of this speed adjustment is obtained from a combined regulator which is operated by one handwheel or by remote-control, push-button operation.

Fig. 2—Direct-Coupled Motors Driving a 234-in.,1200-Ft. per Min. Newsprint Machine

The arrangement of the Interlock control system shown in Fig. 1 consists of a master shaft *N*, located in any convenient place, and extending the entire length of the variable speed end of the paper machine. This master shaft transmits practically no power, as it is only a governor shaft. The power required to drive the master shaft is negligible, and the whole shaft, with the differential regulators but without the draw control belts in position, can very readily be turned by hand from any point of the shaft. At each section, the differential control gear, *D*, is inserted.

Fig. 3—Interlock Drive in Operation on Third Press Section of 234-in., Newsprint Machine, 600/1000 ft. per Min

In Fig. 1 the shaft is shown passing underneath the reduction gear units, but it, of course, has no connection with these units and is simply shown in this position as being a convenient position for it.

It can be placed in any convenient position—under the floor if desired, with the regulator equipment suspended from the ceiling—and if the master shaft is an insuperable difficulty either in the layout of the plant at the back of the paper machine or in reality in the mind of the prospective user, then it can be replaced by a synchronous generator and synchronous motor at each section; but in the author's opinion the shaft is the better drive from a point of simplicity and reliability and provides a more rigid and positive drive for the control units.

Fig. 4—Semi-Geared Arrangement of Interlock Drive

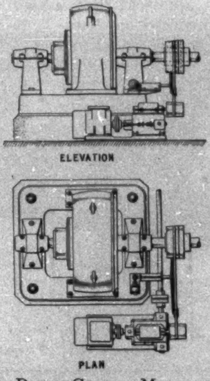

Fig. 5—Showing Direct-Coupled Motor and Control Unit

The master shaft can be driven by either a small, separate electric master motor or from one of the sections of the paper machine, preferably the dryer section, as shown in Fig. 1. This master shaft when driven from the dryer sections is driven by means of a small chain drive or a small geared drive, whichever happens to be the more suitable for the layout in question.

A diagrammatic illustration of the differential-control apparatus, geared to the master shaft at each of the sections to be controlled, is given in Fig. 6. This differential control consists of bevel gear, *P*, free to revolve on cone pulley shaft, *S*, and a bevel pinion, *Q*, which is mounted on the master shaft, *R*. Attached to the bevel wheel, *P*, are three planetory pinions, *L*, which mesh with the sun wheel, *K*, the last named being keyed to the cone-pulley shaft, *S*; the pinions, *L*, also mesh with the internal teeth of the annular ring, *N*, which turns freely on shaft, *S*. Around the outside of the annular ring, *N*, are teeth that engage with another spur wheel, *O*, mounted on shaft, *T*, this shaft being connected through a coupling to a shaft carrying the brush arm, *D*, in the automatic differential regulator *C*.

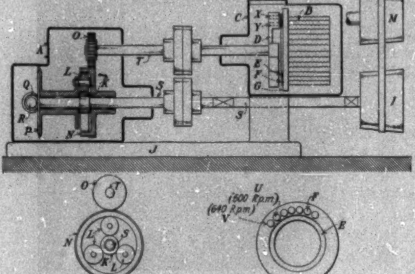

FIG. 6—DIAGRAMMATIC ILLUSTRATION OF DIFFERENTIAL CONTROL APPARATUS GEARED TO MASTER SHAFT AT EACH SECTION TO BE CONTROLLED

If the shaft, *S*, carrying the cone pulley, *I*, which is driven by the section motor, runs (in the opposite direction) at the same speed as the bevel gear, *Q*, located on the master shaft, then there will be no resultant movement of the annular ring, *N*; but however slightly these two may vary in angular position relative to each other, a correct and instantaneous response is given on the annular ring, *N*, and coincidentally on the wheel, *O*, which is driven by the annular ring. The bevel pinion, *Q*, is driven by the master shaft in each case, and the sun wheel, *K*, is driven from the cone pulley through the section motor itself, which must be controlled in each case. Therefore, any tendency of difference in angular relation between the master shaft and the motor to be controlled is instantly reflected on the annular ring.

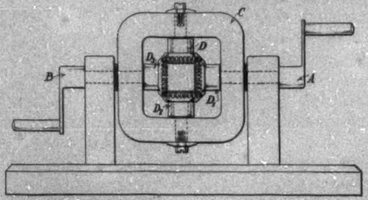

FIG. 7—BEVEL-GEAR DIFFERENTIAL

An epicylic train of wheels is an elusive creature when it is desired to know exactly what takes place, and to more readily grasp the simple and effective nature of the control it is easy to look upon it as a bevel gear differential as Fig. 7. This consists of a small frame, *C*, with four small bevel gears, *D*, *D*1, *D*2, *D*3, all of the same size and having the same number of teeth. Gears *D* and *D*2 turn on journals that are attached to the frame *C*; gear *D*1 is keyed to the shaft, at the other end of which is keyed the handle *B*; and gear *D*3 is keyed to a shaft, at the other end of which is keyed the handle, *B*; the frame *C* is free to turn on these two shafts. Suppose the frame to be held stationary and handle *A* to be turned clockwise; then, gears *D* and *D*2 will turn in opposite directions, and will cause gear *D*3 to turn in a direction opposite to that of gear *D*1, which is keyed to the same shaft as the handle *A*. The two handles *A* and *B* will, therefore, turn in opposite directions at the same speed in revolutions per minute.

Now if the frame, *C*, be free to turn and handle *A* and *B*, together with gears *D*1 and *D*3, are turned in opposite directions at the same speed, the frame *C* will remain stationary, the conditions then in so far as the revolutions of the gears are concerned, being exactly the same as they were when the frame was held stationary. If, on the contrary, while *A* makes 100 turns, *B* makes one turn more or less than this, *i. e.*, 99 or 101 turns, the frame, *C*, must make half a turn forwards or backwards, the same result that would be had if *D*1 were stationary and handle *B* made one turn.

FIG. 8—EPICYCLIC INTERLOCK DIFFERENTIAL WITH COVER REMOVED

The angular displacement has nothing to do with time; it depends only on the difference between the number of turns made by the gears *D*1 and *D*3 regardless of whether this takes one minute or one year. Therefore, if in *any* period of time, gear *D*1 has an angular velocity of 1 deg. greater or less than gear *D*3, the frame *C* must be displaced ½ deg. from its initial position.

The Interlock system of control is merely the application of this principle to the regulation of the speed of the electric motor connecting a single field rheostat to the third member of the differential, which, in the epicyclic differential Fig. 6, is in the ring, *N*, and is so connected by gear *O*, shaft *T*, and coupling *X*.

Fig. 8 shows the epicyclic Interlock differential with the cover removed, and Fig. 9 closed, and it gives a good idea of the robust construction. The gears all run in oil, and being enormously large in comparison to the power required, they will run indefinitely without wear. In fact, after four years' operation at 24 hours per day on a large Canadian newsprint installation, the gear teeth still show the signs of the hobbing cutters which milled them out of the solid.

Fig. 9—Epicyclic Interlock Differential with Cover Closed

The field regulator portion of the control contains a large number of resistance units which are cut in and out of circuit of the motor field, the circuit being completed through the brushes *X* and *Y* which travel over a number of contacts *F* and form a complete circuit through the brass ring contact *E*. In practise, the brush arm, *D*, quietly breathes between the two contacts, of which there are 80 to 100 on the regulator.

The accuracy with which the desired speed is required is so great that if there were a million contacts instead of 80, they would not suffice to have one for every flicker in speed; the brush would steadily breathe but the movement would still be between the contacts, one on each side of the desired speed. Referring to the lower right hand part of Fig. 6, suppose that one of these contacts *U* represents 600 rev. per min., on the motor, that the next contact, *V*, represent 640 rev. per min. and that a speed of exactly 620 rev. per min. on the motor were required; then the brush arm would be quietly breathing between the two contacts *U* and *V*, spending sufficient time on either contact to produce an average field that will give a perfectly steady motor speed of 620 rev. per min., the speed that was desired for the moment. Should this desired speed be 635 rev. per min., say, then the action is exactly the same as before, except that the arm spends a fraction more time on contact *V* than on contact *U*.

If the load varies on any motor, it will immediately tend to speed up or slow down, which will produce a tendency to change any angular variation between the two halves of the differential. The annular ring, *N*, will move and the rheostat arm, *D*, will take up this new position to suit the load in question.

The control is absolutely automatic and instantaneous for all conditions of load which may occur, and no hand adjustment of the motor field strength is required for any load change which does occur such as happens at the press sections, for instance, when the weights are changed, or at the couch section when the vacuum is altered.

If the machine operator desires to change the draw between sections of the paper machine, he has only to move the belt on the cone pulley, *I*, and cone pulley, *M*, Fig. 6, attached to the motor shaft; the motor is then trying to drive, *K*, at a speed that is different. This immediately sets in motion the annular ring *N*, which comes to rest again in a moment, after moving through a few degrees of arc, enough to alter the resistance in *C*; harmony is then once more established, and the motor, running at a different speed, as desired, is still driving the sun wheel, *K*, at exactly the same speed as that of the bevel pinion. The 1½-in. belt may be shifted on the cone pulleys by a hand-wheel or by remote control of a fractional h. p. motor that operates the belt-adjustment gear.

Starting up of the respective section motors is obtained usually by means of automatic push-button-operated contactor motor-starting panels. The push buttons can be located anywhere and are usually on the front of the paper machine at the respective sections, the panels themselves being arranged usually as a switchboard in a substation preferably alongside the generator and main switchboard; Fig. 10 shows a typical switchboard of this nature.

Fig. 10—Typical Switchboard for Automatic Push-Button Operated Motor Starting Panel

The starting panels are provided with resistance units capable of giving very slow crawling speeds which are required when putting on new Fourdrinier wires or felts at the press sections or the dryer sections, and also for the purpose of inching round the various sections for inspection purposes.

Each section can be started, run at slow speeds, or stopped, independent of the other sections, without interfering with any of the other sections. They will all continue to run at the predetermined speed. Should the master section shut down—whether it is the dryer sections, or a small master motor—all the other sections automatically still continue to run at the speed at which they were first operating.

Of course, for the time being, while the master section is standing, the Interlock is not in operation, but immediately the master section has again been brought to full speed the other sections automatically go into interlock.

These conditions are all made by the means of simple automatic devices.

MOTORS

All motors, whether for direct-coupled machines or for geared installations, are totally enclosed and supplied with forced ventilation. This is a distinct advantage at the back of the paper machine, as behind the dryer sections high temperatures are met, and behind the wet end of the paper machine at the couch and press sections a great amount of splashing of water mixed with paper stock occurs which is not good for the windings of any electric motor. Furthermore, at times careless use of a hose by some operator may ruin a motor. All this is guarded against by the use of totally enclosed machines.

This Interlock sectional electric drive which was developed by the Harland Engineering Company has been in successful use since 1914 when it was first installed in Scotland on a small machine having a wide speed range of ten-to-one. During the four or five years previous to this installation there were two sectional electric drives in operation in Great Britain. One of these was not altogether successful and was replaced by mechanical drive. The other was converted to the Interlock differential-control system and is the drive in Scotland referred to above. This equipment is still in operation. Since the installation of this equipment, constructional improvements in mechanical and electrical details have been made as a result of experience, but the principles of control and operation have not been changed.

Before passing from sectional electric drive reference will be made to the driving of the constant-speed end of the paper machine, which seems liable to suffer from neglect owing perhaps to the tremendous interest that has been focused on its sister part.

The paper-machine drive has always been separated into two groups,—the constant speed part, and the variable speed end. The development of the sectional electric drive has provided for the latter, but there appears to be little movement as yet to study and change driving arrangements of the constant speed end. Where a separate driving unit is provided it is quite usual to have two generators and exciter, all driven together, consisting of a d-c. generator, its exciter (which also excites the sectional motor fields) for the variable speed end, and an alternator to provide a-c., three-phase supply to operate the constant speed end. This constant speed end may be driven by one motor, or it may be divided into groups, but no attempt seems to be made to adjust the running speed of its elements to suit the changing speed of the paper machine.

A little progress has already been made in one or two mills in so far that in a few cases the pumps, etc., are being driven by separate motors, thus eliminating belts as far as possible. For this purpose the d-c. motor has been employed, which has wherever possible been direct-coupled to pumps, and which allows a moderate amount of speed control by shunt regulation, thus enabling the speed of the pump to be adjusted to suit the operating speed of the paper machine itself. For such a purpose, direct current clearly has an advantage, and the author believes will gradually be more widely adopted.

It is true that some portions on the constant speed part have been initially conceived and developed with a drive from a steam engine or a counter shaft in mind, and perhaps do not seem to be specially suited for direct coupling to a motor, but doubtless with the development and more extended use of an individual drive on the constant speed end, designs which the business seems to demand will gradually be evolved. In such matters engineers do not usually lag far behind. With such an arrangement the speeds of the various pumps can be adjusted to suit the output required of them, which to a moderate extent varies with the speed of the paper machine. In this way economies in power can be attained and it is believed other advantages also will result. It will be evident that the control of this part of the machine will be more complete than is otherwise possible.

The following table gives averages of some hundreds of power readings taken from time to time on various machines. The first row covers readings on 14 machines of similar size in operation, and the second row, 6 machines. The others are averages on selected individual machines:

From several readings obtained from machines having mechanical drives with single motor the average e. h. p.

Width & Type of *M/c.*	Average Speed Ft.per min.	Electric H. P. Input to Motor per inch width of Machine per 100 Ft. per Min. Paper Speed									
		Couch	1st. Press	2nd. Press	3rd. Press	4th. Press	Dryers	1st. Cal.	2nd. Cal.	Exciter.	Total
234-in. News	875	0.039	0.016	0.023	0.022	..	0.060	0.036		0.0057	0.2017
166-in. News	750	0.054	0.029	0.027	0.0	..	0.674	0.650		0.009	0.243
168-in. Kraft	675	0.039	0.027	0.020	0.017	..	0.041	0.036		0.008	0.188
148-in. Book	530	0.046	0.044	0.026	0.044	..	0.035	0.031	0.068	0.008	0.300
148-in. Tissue	230	0.017	0.041	0.017		..	0.047	0.043		0.006	0.171

per inch of width for 100 ft. per min. is 0.35 e. h. p.

Then there is the very interesting side of the question relating to the effect of sectional driving on the thermodynamic efficiency (a) of the paper machine itself—(b) of the paper mill as a whole,—as a result of the saving in power effected by sectional driving.

From the power table just given, it will be seen that a 234-in. paper machine, operating at 1000 ft. per min. would require 472 e. h. p. A mechanically operated machine operating at the same speed would require about 819 e. h. p.

Assuming that the average output of paper from the machine is 125 tons per twenty-four hours, and that the average steam requirements for drying the paper are 3.75 lb. of steam per pound of paper, then we get a steam requirement in the drying cylinders of the paper machine, 38,900 lb. of steam per hour.

A steam turbine operating under normal conditions usually met with in a paper mill would have a water-rate of about 38 lb. of steam per e. h. p.-hr. allowing for generator efficiencies, so that by utilizing to the fullest extent the power in the steam required in reducing it from the normal pressure to the pressure required for drying in the paper machine cylinders we can obtain approximately 1000 e. h. p.

Now, if the paper machine is driven mechanically, the balance of horse power so obtainable (181. e. h. p.) would hardly justify the installation of an additional generator to supply the excess power to the constant-speed end of the machine, but the balance of power obtainable by using sectional drive, 528 e. h. p., is sufficient to justify this and is usually enough to drive the whole of the constant speed end of the paper machine. This renders the paper machine an individual unit, which has many advantages.

Sectional driving of paper machines has also had its effect upon building design by reducing the cost of basements and saving space.

The foregoing does not pretend to deal with the general advantages to be obtained in the use of sectional electric drive as compared with mechanical drive. These are becoming so generally well-known that it is only necessary to outline briefly a few of them—such as the elimination of belts and ropes, and their consequent very high up-keep costs; the saving of power as is shown on the power figures given above; the very much steadier speed control between sections of the paper machine which result in a better quality and production of paper from a given machine with a given quality of stock supplied to the machine.

Each section of the paper machine is made a separate unit and with electrical indicating instruments it is possible to detect faults which may arise on the paper machine itself which allows investigation and probably rectification of the trouble before it becomes serious, whereas with mechanical drive very probably the first indication of trouble is when the trouble has become serious enough to necessitate expensive repairs.

The greater ease in handling the paper machine by the paper mill operators is very marked and needs to be experienced to be realized. The absence of ropes and belts on the back side of the machine makes the machine more accessible, which all helps in the upkeep and maintenance of the plant and reducing life hazard.

Due to the very even starting torque exerted by electric motors, particularly of the d-c. type, dryer gear life is increased as compared with the mechanical drive where power application of the dryer gears is usually by clutch or snatching of cone pulleys which puts sudden stress on the gears.

These notes have been put together on the assumption that they will be read to those who have some knowledge of paper machines and are interested in present and future operation and development. There is to my mind a great field in the United States for conversion of old mechanically driven machines to sectional drives and although I expect there are many paper makers who will disagree with me, I maintain that there are very many advantages to be obtained in doing this.

One, of course, still meets the man who says the mechanical drive is good enough and he is clearly right from the standard of his own point of view. Everybody knows that mechanical drives have made and will still make good paper. In the beginning it was the large powers and high speeds required by the large newsprint machines which forced forward this subject of sectional driving, although its actual development was accomplished before the advent of the large high-speed machine.

The steady increase in the size of newsprint machines finally made sectional driving a necessity.

When its reliability became established, it made wide machines and high-speed machines more readily available than formerly. Its advent also has apparently had an influence on the design of the paper machine itself as regards the widths and speeds at which it can be built.

The reliable nature of the drive in operation on newsprint machines has been proved and has demonstrated also that for the small book machines, etc., the same good results can be obtained: in fact, it is becoming more general, and nearly all new book machines are fitted with sectional drive. As stated before there is a big future for the conversion of existing machines to sectional drive.

In this connection it is interesting to consider that some form of sectional drive has been constantly the subject of research by independent peoples in different parts of the world; contemporary with the work which we were doing in Great Britain, other people were carrying on similar experiments on this continent and in Europe. All these experiments, the author believes, were made quite independently of each other showing a widespread belief in the necessity and usefulness of sectional drive.

Discussion

PAPERS ON PAPER-MACHINE DRIVE

(Staege, Rogers and Norris)

Cleveland, Ohio, March 18, 1926

T. D. Montgomery: Mr. Staege says that a speed adjustment of from 0.1 to one per cent is required. Does this mean that some classes of paper require 0.1 per cent while others require one per cent, or that regulation anywhere between 0.1 per cent and one per cent meets all conditions of paper making?

Reference is made to differentials, either electrical or mechanical, and Mr. Staege refers to the accumulative lag in drive and master shaft and field clutch. Would there not be the equivalent lag with either the mechanical or electrical differential, in so far as affecting the regulation?

L. W. W. Morrow: The object of all of these drives as outlined in the papers seems to be to answer the speed-regulation requirements in the production of paper. I should like to see something that would show the response of these drives to the actual requirements; in other words, if there is a change in load in any one section, what is the time interval until adjustment to that situation by the drive? Each drive has been described, but we haven't definite time data as to the results accomplished by the drive.

F. C. Bowler: We have heard a great deal about the exact regulation which will be produced by these various types of drives and emphasis has been put on the need of exact regulation. I presume that the authors have made some experiments tending to show what the actual regulation has been with the various types of mechanical drive which we have had in the past. I wish that might be brought out. In other words, it is a question in my mind whether we aren't putting emphasis on something that is more refined than we need. We have made paper a good many years with belts and gears, and personally I doubt if we have ever had such regulation with mechanical drives as any of these new sectional drives will give us.

N. D. Paine: I would like to agree with Mr. Bowler and say that we are inclined, perhaps, in the sectional drive, to attain too exact speed requirements. As he said, we have made paper for years with belts and pulleys, and now, with the electrical drive, it is more a question, especially on the large 234-in. machines, of starting the dryers than of the actual speed control.

We do get exceptionally good speed control with the mechanical interlock, of which our company has four on the 234-in. machines.

The papers lay quite a bit of stress on the fact that there is a large amount of stored energy in some types of motors. With regard to the mechanical interlock drive, that stored energy is used to very good purpose. The master shaft being driven from the dryer section of 34 6-ft. dryers, weighing 12 tons each, naturally maintains an extremely steady speed, as the energy stored in this section smooths out any slight speed variation which might occur due to small voltage fluctuations.

Coming down to the various section motors, I can say, especially on the couch—which we have watched more particularly—that speed control, where the paper is the weakest, with our Harland or mechanical interlock, is maintained with a small field variation, from 0.05 to 0.10 ampere, in the shunt field, which is with the machine running at 750 to 800 ft. per minute.

H. L. Sanborn: There seems to be a question as to exactly what basic principles the electrical manufacturers are endeavoring to correct with their electric drive.

Mr. Staege refers to a steel mill as having a very heavy load, which would not exactly apply to the electric drive. There is little comparison between the steel-mill and the paper-machine variable loads, yet a plugged calender will subject its motor to two and one-half times its starting torque, probably as great a variation as the steel mill when the ratio of power in use is considered. This and other inherent conditions peculiar to paper-machine operation produce a system of variables making load changes one of the big factors the electric drive is called upon to correct.

These load variables may be caused by mechanical conditions, vacuum changes on the wire, poor bearings, sections out of alignment, stock changes, clothing, etc.

We have in the Abitibi Mill at Iroquois Falls the Westinghouse electric sectional drive. We find it more reliable than our mechanical types after four years of operation. The whole machine has greater flexibility. The operating efficiency has increased and as a whole the close regulation tends to produce greater tonnage over the corresponding machines of the same size having mechanical drive. The breaks on this particular machine have decreased and there has been a reduction in repair labor and material.

In the main, however, I think, we have perhaps forced upon the manufacturer of electrical material a very severe problem, but you must endeavor to bear this in mind:—The paper mill has been running for years and years, as Mr. Bowler states, and the electric drive is endeavoring to take up these old-time conditions and produce results superior to the mechanical type of drive, without redesigning the paper machine to fit the electric drive.

Our sectional drive has accomplished this feat at the Abitibi mill and the average operating efficiency on our paper machine equipped with this drive, conditions being equal, has not been exceeded, and on the average has not been approached by either the mechanical or non-sectional electric-driven machines in our plant.

J. F. Rhodes: I think my two friends who have just spoken have brought up very important arguments. I believe the Mead Pulp & Paper Company put in two of the first drives.

I think the main object in the machine drive is to keep the sections together, that is, synchronized. With the old machines we had an engine that required steam pressure to be constant all the time. Then we depended upon the governor to maintain the speed. After the paper maker got his paper over the machine, he had nothing more to worry about unless his thickness changed or something caused a break.

As we get into the electrical drives the main thing seems to be to get away from the engine and the only problem left is to supply the steam for drying. However, paper makers feel that if anything breaks down it will do so at the weakest point. If these things become so complicated that a brush gives away or a contact goes, we have to hunt the trouble, and in the meantime we may have lost three or four hours of production. The old type of machine was so simple that almost any paper maker could very quickly find out what the trouble was and get started quickly.

In building a new paper mill I do not think there would be any doubt that he would adopt the extraction type turbine and put in the late type of electric drive, getting the one that was the simplest, and required the least amount of space, and of course the financial end would come in.

I think we should aim towards simplicity in choosing drives.

I do not believe there is any change in the load of the machine after it is started and the paper is going over the machine perfectly. If there should be a break at the calenders there probably would be a change in load there for a while which would react back to the motor-generator set and that would in turn slow down, if it was not properly regulated, reacting back into the machine again, causing another break.

R. T. Kintzing: These papers are so broad in scope that they necessarily have devoted but a limited space to electric control used on sectional drives. Control designers should be credited with solution of the very difficult electrical problem of precise motor-speed regulation. I should like to point out a few of the other control features involved in this application, and also to supplement the statements made in some of the papers concerning the various types of speed regulators.

Starting, stopping and inching are the usual operations required in controlling the individual section motors. Certain sections are occasionally reversed, others are temporarily slowed down, and other motors start and stop simultaneously. Running-load conditions are usually constant. Starting-load conditions may be those of a friction load or those of a load having high inertia. The latter are the most difficult to regulate. Starting-load conditions on the dryer and calender section are unusually severe.

Individual resistor-type automatic starters having positive time-limit acceleration are best on account of starting from a bus, the voltage of which may vary over a broad range to provide for different machine speeds. Push-button-operated starters with motor-driven cam-operated accelerating contactors have proved very satisfactory. Time-limit overload and low-voltage protection is necessary. Starting motors at full field strength is desirable to obtain high starting torque with minimum starting currents.

The armatures of the section motors are connected to an adjustable-voltage generator while their fields and the control circuits obtain power from a constant-voltage exciter generator. All circuit breakers, disconnect switches, etc., should be connected to interlock the two sources of d-c. power and prevent damage from incorrect sequence of operation. It should be impossible to energize the control circuit of any section motor unless all of the switches furnishing power to the motor circuits are closed. Opening any switch should immediately de-energize the control circuits of all sections depending upon power supply from that switch.

Auxiliary motors driving exciters or blowers for forced motor ventilation should have their control apparatus interlocked and connected to insure their operation when the power-supplying generator is started and to cut off the power supply in case of overload on these auxiliary motors.

The many panels necessary for controlling and regulating the required apparatus should be assembled into one switchboard which can be located remote from the heat, water and dust of the machine room and under the charge of authorized workmen.

In addition to the usual push-button stations for starting, stopping and inching the section motors and for changing the speed of the entire machine, there is furnished with the equipment described in Mr. Staege's paper a "safe-run" station for each section which permits the section control to be made safe when it is necessary to work around dangerous parts of the machine and which cannot be restored to the running position until unlocked.

Heavy dryer sections and calenders require starting torques many times in excess of the running torque, while presses and other sections usually are started at reduced load and do not need much more than normal torque to start. If the range of operating speed and voltage is broad, special provision must be made to change the value of starting resistors as the voltage changes.

When starting currents become large enough to overload the generator, and cause undesirable voltage variations, means must be provided to obtain the necessary torque without these objectionable features. Separate low-voltage generators for starting purposes, separate starting motors, and series-parallel controls are methods in common use. In the case of dryer sections geared together, satisfactory stable division of the load on the driving motors can be obtained by a proper design of the motor. Motors having good speed regulation are necessary, but these unfortunately will not ordinarily parallel with equal division of load. It has been possible by means of special windings to design motors which have excellent speed-regulation characteristics and at the same time divide their loads equally when geared together.

A successful speed-regulating system must have ability to hold constant motor speed in spite of the various causes tending to change it, must respond quickly enough to avoid damage to the partly finished sheet of paper, and must operate with a minimum of attention. If the regulator can make a change in field strength of sufficient magnitude, can make it fast enough, and can stop when exactly the right amount of change has been made, successful operation is assured. Sturdiness and serviceability must be obtained in the design to make it a commercial success after these fundamental requirements have been secured. The most important feature of the regulator described by Mr. Staege is its ability to produce a perfectly graduated change in effective value of the field resistor by the simple method of intermittently short-circuiting resistor steps by means of a conductor having a variable width. Enough resistor steps can be used to give all of the regulating capacity ever needed. Unlike former systems, the amount of resistance is not limited by the master speed. Those sections having high inertia and requiring more gradual changes in resistance can be equipped with smaller-pitch screws. As a result it is not necessary to sacrifice regulating capacity on the high-inertia sections and the lighter sections can be made to respond more quickly. An infinite number of resistance values between maximum and minimum may be secured. The regulator is truly differential in action and must produce 100 per cent speed correction before its action stops. Mechanically the apparatus employed is simple and substantial. It is evident that this system possesses to a very marked degree all three of the fundamental requirements: capacity, quick response and smooth regulation.

E. F. Bearce: The sectional electric drive for paper machines has accomplished some very definite results for the industry.

In the manufacture of news print, this drive is especially advantageous since it permits the high paper speeds with positive regulation. For the higher-grade paper manufacturer the drive has provided regulation of speed which is one of the important features required to get a sheet of uniform weight and caliber. These points of advantage are in comparison with the older methods of drives using a variable-speed back line in which the speed may be limited by belt or pulley design and the use of a variable-speed steam-engine drive having a regulation not less than three per cent.

It is also interesting to compare the cost of the sectional drive with the variable-speed drive using a back line, both as to first cost and maintenance. It has been determined that the first cost of the sectional electric drive is a little greater than the back-line drive, but is considerably more economical as regards maintenance.

The papers have brought out some very interesting points as to the details of construction and arrangement of the three different drives, each one having distinct points of advantage, and all three in successful operation in different parts of the country.

The manufacturers of this equipment are to be congratulated on the development of this drive as one of their greatest contributions to the advancement of the paper industry, and I am sure every paper maker is ready to cooperate with them in the continuance of their good work.

E. B. Wright: We operate seven machines; three of them are driven electrically by d-c. variable-speed motors through a line shaft with cone pulleys and friction clutches.

This type of drive works out very nicely, the regulation being good if the exciter voltage remains constant. The least variation in exciter voltage causes a change in speed of the d-c. motors.

A motor-generator set, with exciter direct-connected, is used to drive the variable-speed lines. This set takes its power from an extraction turbine, which when operating at maximum rating and extracting the maximum amount of steam, at times falls below normal speed, which affects the exciter voltage causing a variation at the paper machine.

I should like to ask the speakers how the cost of installing the individual drive compares with line-shaft drive in new installations, also how maintenance costs compare.

R. S. White: Mr. Wright mentioned a variation in speed of the prime mover causing a speed change on the paper machine. My experience has been the same.

We have a synchronous motor-generator set with exciter direct-connected. This exciter supplies the excitation to the d-c. generator as well as the synchronous motor. Therefore any slight change in speed causes a voltage variation in the d-c. generator due to the exciter voltage varying.

When first starting, we had considerable trouble with the governor of our turbine and consequently had all sorts of speed variations. Now if the paper-machine head box were set for, say, 25-lb. paper at 300 ft. per min. and then the machine suddenly changed to 325 ft., it would cause the paper to drop in weight and change the dryness at the same time.

All paper-machine drives, in my estimation, should have voltage regulators to take care of just such trouble as we experienced. I understand that all Harland drives are now furnished with voltage regulators.

The Harland drive for our No. 2 machine has been in operation four months and the No. 1 machine two months. Outside of the trouble mentioned we have not any fault to find with the performance of the drives.

When we started the No. 2 machine we had a suction press roll 22 in. in diameter. Soon this was removed and a 26-in. rubber press roll installed. Since this roll was 4 in. larger, it had to run slower. Now the field rheostat or regulator did not have a range wide enough to cover this, so we had to change the gears and get a larger ratio.

It seems to me that these regulators should be designed to cover a much larger range than they do in our machines, as paper mills change managers and each one wants different equipment. And in order to drive this equipment extra gears must be kept in stock.

Mr. Norris mentioned something that seems to me is more important than close regulation between sections, and that is the control of the supply to the machines. Why not have the pumps interlocked so as to give uniform supply? If the supply to the machine is not fed on the wire at a uniform rate, the paper cannot be of good quality. There should be an automatic arrangement so that in case the machine changed speed, the opening from the head box (if a Fourdrinier machine) would change in direct proportion to the speed.

C. A. Farrell: We have all three drives, single-motor, sectional and mechanical. I guess the proof of the pudding is in the eating, and all of our machine men are unanimous for the electric drive. We find that they like the sectional drive the best, although I can't see but that we get almost as good regulation with the single motor. Of course, we have belt trouble. I notice that one of the manufacturers has a provision also, I think, in the face of the motor for cutting in some series windings. I would like to ask whether that is necessary on all of the sections or just a few, and whether it is for starting purposes only.

Mr. Norris says that he uses compound-wound motors, and the other manufacturers use shunt motors. I wonder if there is a reason for that.

I think our sectional drive was one of the first. Our motors are larger than necessary and we don't have any trouble from temperature, but I wonder if the motor is enclosed only to keep the water out of the machine or if they depend upon the forced air to keep down the temperature and therefore use a smaller motor.

I would like to know what sizes of motors are used in the drives today compared with ours. We have 35- to 50-h. p. motors, I believe.

We are particularly interested from the maintenance standpoint in the temperature of the motors. As large as our motors are, in the heat of summer they frequently run temperatures that are pretty high.

Tom Harvey: I look at this from an owner's standpoint. I am wondering if these people who put in these very expensive electric drives are getting a return on the money invested.

We have four machines making box board. I was very much disappointed that none of the speakers said a word on box board. Our tonnage amounts to the total tonnage of all of the news paper, all of the book paper and all of the writing paper made in the United States, and I think it should be recognized.

One of the authors shows a drawing of a cylinder machine and does not say a word about it.

We have one old mill that has been running about 25 years, and because of the crowding of the mill we decided some few years ago to electrify it. We have spent considerable money in that mill and I suppose it is electrified as well as any other box-board mill with the exception perhaps of the mill at Ritman, Ohio.

We find that we don't get the return on the mill with the large investment that we do from the mill with the smaller investment. We also get much larger tonnage from the mill that is what I call directly driven from the steam engine with the rope drive. We have very little electricity in that mill.

I can readily see that the electric drive is a great advantage to the people who are making news paper and are running their machines up to a 1000 ft. a minute, but I have yet to be shown where a manufacturer is justified in spending the amount of money that he has to spend to put in the electric driven machine. I doubt very much if the stockholders would be satisfied if they could see the results from the machines that are not electrified.

J. H. Crossley: We have only just started up our first two sectional-drive machines, which are the General Electric Company's drive.

I should like to ask Mr. Norris what he means when he says that when their interlock is out of action, the wet end of the machine does not run wild? I am curious to know what would happen to our drive if our regulator should go out of action.

A. O. Spierling: I was glad to hear Mr. Harvey refer to the financial studies which must be made when considering the addition of electric drives to paper machines.

At the present time we, at Hammermill, are using the mechanical type of drive on each of our five paper machines. About four years ago we investigated the electric drive and have been giving considerable attention to it since. We credit the electrical manufacturers with doing a splendid piece of engineering work in perfecting the electric drive. Any of the three types of electric drive described, we believe, will work very well but their installation involves considerable expense which is sometimes very hard to justify.

We have thus far been unable to find how we could produce paper any cheaper by replacing our present drive with the electric type of drive.

There is one point, however, which has been brought out here at this meeting which if shown to be true would immediately touch any paper maker's heart and that is the statement that with the electric drive a better quality of paper might be produced. If this is true, the paper maker would be immediately much in favor of this type of drive because if he can produce a better quality of paper he can get more money for it and therefore justify the expenditure.

Of course, the thermodynamics of the matter enter very much into it, and it is a heat problem from start to finish. We charge against our paper machines the heat units that are used not only for the engine drive but also for the air that must be supplied to carry away the evaporated moisture.

Everything of that nature has to be taken into consideration before a person can say whether or not he is justified in putting in such a drive. It may, however, work out better with a new installation or with larger machines than it does with an existing installation of moderate-sized machines. Our machines are comparatively small compared to 300-in. machines. We manufacture a high-grade bond paper and run at speeds anywhere from about 100 ft. a minute up to 500 ft. a minute.

W. W. Spratt: One of the items touched upon is the heat balance of the paper mill. I believe it is an important subject to which we should give serious thought.

Mr. Norris, in his paper, mentioned that the 125-ton machine requires, roughly, 39,000 lb. of steam an hr. This, he estimates, is equivalent to 1000 e. h. p.

Let us assume that for the class of mills considered, power is worth from ½ to one cent a kw-hr. Mills having adequate hydroelectric development, principally in Canada, may, of course, get power for less than a ½ cent. Depending on the cost of competing power and cost of coal in the district analyzed, it is estimated that it may be worth from $5000 to $25,000 a year as an incentive for the mill to obtain by-product power from this quantity of steam. Taking an average condition of $15,000, the average mill would hardly, under these conditions, utilize live steam through reducing valves to do the work in the dryers, which points out the importance of consideration of some sort of non-condensing or by-product prime mover as a source of power for the paper machine.

This is the question which I know Mr. Harvey, in his discussion, was referring to, especially in relation to board mills. I have seen many cases in board mills which did not have the best type of heat balance. Unfortunately, in the case of mills that do not have the heat balance worked out well, and are electrified, the story gets around that it is uneconomical to drive the mill electrically, while, with a much better heat balance, taking full advantage of the possibilities of generating by-product power, such a situation would not be so marked and the advantages of electric drive could well gap the difference of some of the losses on the pure question of steam utilization, as referred to the engine drive which unfortunately still predominates in board mills.

I believe the greatest advantage of a sectional drive is in a new mill where advantage can be taken of the smaller space and the basement can be utilized for other purposes. I know of one mill which installed sectional drive merely on the basis of lower building cost and other items which have not been discussed. They operate a board machine from 50 to 200 ft. per minute. Fortunately, they happen to be located where they can buy power from Niagara and they have a cheap power rate which assists them. Even in their case, however, there would be some advantage in using a non-condensing unit to drive the d-c. generator.

We have heard a very interesting discussion touching on the relative merits of moderate-speed motors versus direct-connected, slower-speed motors. The discussion took us into the question of stored energy of the different motors. We are chiefly interested, however, in maintaining the speed of the motors, whether we do it by means of an interlock system, synchronous-motor tie-in system, or regulator system. In this connection, I believe we all agree that the system with the maximum stored energy is less susceptible to speed changes. The stored energy tends to maintain the speed at the proper value. Our problem is one of maintaining the desired speed and a large stored energy assists us. It is only when we are away from the desired speed that a large stored energy would be a disadvantage.

I should like to ask two questions; one is with relation to the system utilizing the dryer sections as a master. I should like to ask why that is done,rather than having a regulator which is entirely independent of any one section of the machine.

I should like to ask Mr. Rogers how long it takes the vernier brush arm of the synchronous-dynamometer regulator to travel from one extreme to the other.

N. D. Paine: May I answer one of the last questions,—as to why the dryer section is used as a master section? I mentioned before that the immense amount of inertia in the dryers naturally tends to keep them at a very steady speed. As a matter of fact, using a Bristol tachometer which is extremely sensitive, we have found that when we had that tachometer on the master shaft, it would draw a straighter line than you could with a ruler for possibly four or five hours on end. It also obviates the necessity of using section regulators. You cut down your initial cost in that way.

It has been mentioned that when the dryer section is shut down the master shaft is stopped and the section motors are running wild. This is not correct. When the dryers are shut down, the section motors automatically go to the non-interlock point of the field regulator, which is only one or two commutator or face-plate segments away from the operating position of the regulator brush arm, and this position is less than 0.25 ampere in field-strength value. I happen to know that our own paper makers hardly realize that there is any difference in speed once we have our regulators set when the dryers are shut down. In fact, once the dryers are shut down, you are not making paper; you can't handle it the full width of the sheet; you can only take your lead strip up to the third press at most, so why worry?

L. E. Markle: It is common practise in paper mills to have the machine operating for 24 hours a day, over a period of six days in the week. In view of this continuous service, the mechanical duty required of the rheostat mechanism described by Mr. Norris as a breathing process must be rather severe. We should like to ask whether special precaution is taken in the selection of materials and the construction of the face plate and brush mechanism?

Several of the paper-mill representatives as well as the authors of all of the papers have emphasized the necessity for close speed regulation at all times. Since the permissible changes in speed are so small, it is certainly true that any system which will correct to a finer degree than other systems is desirable. All systems require some speed reference as a master. This unit may be a section of the paper machine such as the dryers, or it may be a unit driven by a separate motor. Since any change in the speed of the master unit affects the speed of the entire paper machine, causing it to speed up or slow down as the master speed changes, it would seem that an installation as described by Mr. Staege, where the speed of the master set is regulated just the same as the speed of any of the main driving motors, would naturally give a closer speed regulation.

I feel this point vital because as mentioned before the limits of permissible variation are small, and any system which can regulate better than another, no matter how small the amount, is preferable.

S. A. Staege: Mr. Rogers calls attention to the controversy on the relative merits of high-speed, medium-width machines as compared with medium-speed, very wide machines, and indicates that there is now a tendency to favor the former. A more complete statement would be to say that the high-speed, medium-width machine first reached its supremacy, but is now being followed by the high-speed, very wide machine, as there does not appear to be any fundamental reason why the very wide machines should not operate at equally as high a speed as the medium-width machine. Present practise indicates wider machines than ever built before combined with a maximum contemplated speed so far unapproached in actual commercial practise by several hundred feet per minute. The real limitations of the paper machine both as to speed and to width are determined by such factors as the maximum speed at which the sheet can be formed properly and by economic factors pertaining to the machine clothing, initial cost, etc., and not at all by the drive.

All available data confirm the indication that the power required by the several sections of the paper machine varies directly as the width of the machine and directly as the speed of the machine so long as the other relevant factors remain substantially constant.

I do not agree with Mr. Rogers' statement that the synchronous-motor tie-in system is a preventive and that the regulator type of control is a corrective system, nor am I able to agree that in the regulator type of control there is no restraining power to hold the motors in place. In fact, in the case of sectional drive, the d-c. motor has available a restraining torque equivalent to far more than the full-load torque of the motor, whereas, in the case of the synchronous, tie-in system, the full-load torque of the synchronous motor is far less than the normal torque of the d-c. driving motor. It is quite true, however, as Mr. Rogers has said, that the success of the regulator type of control depends upon the amount of angular displacement which causes the regulator to function and by the time element of the motor field. This means that there should be absolutely no lost motion and that resistance be cut in or out of the motor field circuit with the smallest possible change in angular displacement of the motor. It also means that the time element of the motor field circuit should be as short as possible, which is inevitably associated with relatively high-speed motors.

I should like to ask Mr. Rogers if he does not feel that a speed change of a section of the machine of 0.15 per cent which he has indicated may result from a load change, in the case of the synchronous-motor, tie-in system, is not too much for satisfactory operation, particularly at the dry end of the machine. Personally, I feel that such an amount of speed change between sections is far in excess of safe or desirable limits.

In the case of the synchronous-motor, tie-in system, I should like to ask Mr. Rogers at what speed the frame of the synchronous motor is made to rotate by the small variable-speed, d-c. motor. It would appear that if the speed at which the frame rotates is a very small percentage of the rotor speed, then the change in speed of the small, d-c. motor might have to be several hundred per cent to accomplish a change in speed of the d-c., section-driving motor of, say, 10 per cent to compensate for possible changes in diameter of the roll, change in draw, etc.

I would also like to ask whether the paragraph in his paper immediately preceding the description of the synchronous-dynamometer type applies to the paragraph immediately preceding it.

From Mr. Rogers' conclusions, it appears that the three authors are now unanimously in agreement that regulator control best meets the requirements of sectional paper-machine drive.

It is stated by Mr. Rogers that the synchronous dynamometer regulator operates on an angular displacement corresponding to approximately 0.05 per cent change in speed of the controlled motor. Obviously, if there is a change in speed, the angular displacement will continue to increase until the speed is corrected and it would be desirable to know how long this 0.05 per cent change in speed must continue to obtain a sufficient angular displacement to operate the regulator.

It would be interesting to know through how many mechanical degrees the stator of the synchronous dynamometer would have to rotate to move the commutator rheostat brushes through 450 operating points. I should also like to ask what means are employed to actuate the brush on the large-step portion of the commutator rheostat. I should like to ask Mr. Rogers of what the anti-hunting features consist in the regulator equipment described by him.

Mr. Norris in describing the interlock regulating system states that in practise the brush arm quietly breathes between two contacts. I should like to ask the order of the frequency of oscillation for the various sections of the paper machine and the actual amount of angular displacement through which the d-c., section-driving motor oscillates in response to the resistance changes.

A statement is made by Mr. Norris that should the master section shut down, whether it is the dryer section or a small master motor, all the other sections automatically still continue to run at the speed at which they were first operating. I should like to ask through what means this is accomplished.

H. W. Rogers: In connection with Mr. Staege's paper, I would like to say that the General Electric Company installed and put into successful operation in 1909 a sectional paper-machine drive with a definite tie-in between the sections.

Mr. Staege describes the synchronous tie-in type of drive and states that any change in load on the part of one section will affect the speed of the entire paper machine. In this connection, I wish to state that the use of synchronous motors on each section of the paper machine is analogous to a positive mechanical tie-in between the sections, and the machine operates as though driven by a single motor. Therefore, the effect on the total load of a change in load on one section is a matter of two or three per cent and the effect on the speed of the machine as a whole for such a change in load is less than 0.1 per cent.

After describing the improvements which have been made in the synchronous tie-in type of drive, Mr. Staege states that the effect of speed variation in the small d-c. motors which rotate the synchronous-motor stator frame on the main driving motor is approximately inversely proportional to the ratio of the worm gear used in driving the stator. This statement is incorrect inasmuch as the gear ratio has nothing whatever to do with the speed regulation of the d-c. motor. A small d-c. motor drives the synchronous-motor stator at a speed which permits proper draw adjustment, this speed being only a small percentage of the speed at which the main motor is operated. Therefore, any variation in speed of the small motor directly affects the speed of the stator, but the stator speed being a small percentage of the rotor speed, its effect on the main driving unit is inappreciable.

In my experience I have found that the control of the high-speed machine is the simplest problem with which we have to contend and that the low-speed and wide machines present the greatest difficulty.

I am interested to know just where the third type of regulator mentioned by Mr. Staege is in operation and on how many machines. I should also like to ask what provision is made to eliminate a so-called wild machine when the master set is shut down.

The European drive to which Mr. Staege refers on the ninth page consists of a-c. commutator motors and has been in successful operation at the Empire Mills, Ltd., for practically two years and is now being followed up by a second installation in another mill.

S. A. Staege: I think there is some misapprehension as to the effect of inertia in the moving system with respect to the regulation of the paper machine. As Mr. Spratt indicated, where there is considerable flywheel effect, regulation is frequently easier than where there is very little. I might say that in respect to the dryer sections where there is a very great amount of flywheel effect, the rotary-contactor regulator which has been described is able to take care of the regulation just as completely as on a press section where the inertia is almost negligible. In fact, I am unable to see that the amount of flywheel effect should seriously affect the regulation of the paper machine. The amount of stored energy in the motor is so small compared with the inertia in the dryers that I look upon it as a very small factor. It is perfectly true that if there is lost motion then a large amount of flywheel effect is undesirable; in fact, it is difficult if not impossible to regulate if you have lost motion in the regulating system. It is of course true that the high-speed motor does have more flywheel effect than a low-speed motor.

The speed of the motor field is the next controlling factor providing the regulator works instantaneously. The rotary-contactor regulator's movement is absolutely synchronous with the change in angular displacement. The only possible chance for any delay in the corrective effect is in the time element of the motor field, and with the high-speed motor the field is very quick as compared with the field of the large slow-speed motor where there is much more iron and turns and the inductance is a great deal more. The response of the armature current will follow with practically no lag as soon as the field is changed.

I am not able to follow Mr. Rogers in his conclusions that the field is not a controlling factor in the time element of the motor.

The magnetic contactor is a very reliable piece of apparatus. When the contacts are made they rub and roll together, presenting changing surfaces, and the contacts never blister nor accumulate dirt.

Mr. Montgomery inquired as to the degree of regulation required, mentioning 0.1 per cent, and asking whether that was necessary. On the wet end of the machine, between the couch and the first press, the machine can stand considerably more than 0.1 per cent variation without breaking the sheet or causing any very serious consequences. I feel that any change in speed, however, tends to weaken the sheet although it may not be very serious. Farther on toward the dry end of the machine, a variation of 0.1 per cent would cause very serious straining of the sheet and between the dryer and the calender an elongation of the sheet of 0.1 per cent would almost certainly break the sheet. There are some grades of paper, of course, that have more elasticity than others.

The differential accumulative effect referred to which applies to both mechanical and electrical differential devices is accumulative in characteristics. If there is a change in speed, no matter how small, as long as it is allowed to continue the angular displacement keeps accumulating. And the accumulated value of departure, if it is attached to something which is supposed to regulate, will become greater and greater until the required action has taken place. The rapidity with which the corrective effect will be brought about is dependent upon the relations between the various parts, gear ratios if it is gears, or some other similar factors in determining the speed at which the secondary movement takes place.

Mr. Bowler called attention to the degree of regulation as compared with that of the mechanical drive. As brought out in these papers, mechanical drives are subject to belt slippage, and it is very well known that belts are liable to slip anywhere from a small fraction of a per cent to two or three per cent, depending upon the loading or condition of the belt, the arc of contact with the pulleys, the tension and numerous other factors. Of course, the amount of belt slippage, other things being equal, is nearly a straight-line function of the load transmitted, and if the total slippage, for instance, is two per cent under full load, a change in load of 10 per cent could be expected to produce a variation in that slippage of 0.2 per cent. However, a little moisture on the belt is likely to cause variations in the slippage of considerably larger values. It is the variation in slippage which affects the draw rather than the total or base slippage. If the drives all slipped exactly the same amount and never changed, you would have just as good a draw as though they did not slip at all.

Mr. Paine speaks of starting the dryers and comments on the stored energy. It is true, as I have indicated, that there is a great deal of stored energy in the dryers and they have to be started more slowly than the other sections of the machine. But if the proper time element is given they should be brought to full speed and synchronized with the control system without any difficulty. And where the regulator is designed in such a way that it starts up with full field on the motors and then after the motor gets up to nearly the operating speed the regulator gradually cuts in the field resistance, bringing it up to regulating values, there is no shock brought about and nothing to start oscillations. Of course, if you cut in the regulating resistance suddenly in the full amount, it is possible, if it is too much, to start oscillations of the dryers which might not easily be damped out, but when the resistance values are of the correct amount and the speed at which it is cut in properly controlled, there is no difficulty.

However, it is true that on the dryer sections with large inertia, the steepness of the regulation curve should be less than on sections with less inertia, because they cannot respond so quickly. You have there not only the lag of the motor field, but you have the inertia lag of the rotating mass, and the speed of cutting in the resistance must be adapted to the torque the motor is able to develop and to the inertia of the system.

Mr. Sanborn speaks of sudden variations and overloading conditions. The rotary-contactor regulator, which has been discussed by me, is designed to take care of any changes in load from no load to 50 or 100 per cent overload and still maintain the draw, and it does it. It has enough resistance and cuts it in at a sufficient rate to prevent more than a transient change in speed. That differs from the earlier types of regulators in which there was not a sufficient amount of regulating resistance to take care of such wide changes and in which it couldn't be cut in fast enough to overcome the initial tendency to drop off in speed faster than it could correct it.

Mr. Rodes mentioned the possibility of the brushes breaking on the regulator. If a brush should break or a wire should break in the rotary-contactor control, which is nothing more than a rheostat in which every step is divided into an infinite number of small increments of resistance, all that would happen would be that the drum would move longitudinally on the shaft a small fraction of an inch. In fact, if you were to lift out one brush while running the drum would move up about 1/16 in. and present the next step, and you could take out a number of steps of resistance or cut the wires, and it would simply move up far enough to compensate for it. You would hardly see any change in the draw at all; probably you wouldn't see any.

The simplicity of the sectional drive is not fully appreciated by most paper-mill people. Electrical apparatus has been so standardized and used so long that there is no uncertainty about its performance. It will probably stand up and give good service as long as any kind of mechanical device with similar treatment. And the records show that the maintenance is extremely low, something that cannot be met or even approached by most mechanical drives. As an illustration, one drive, of which a record was sent to me recently, has been in operation just one year, and had a total maintenance expense of $21.

Mr. Bearch questions whether there is too much refinement. I believe as long as we can get refinement without complication we ought to have it.

Mr. Harvey has raised a very interesting point in regard to cylinder machines. We have sectional drives on two-cylinder machines and they are giving a very good account of themselves. The maintenance is negligibly small and the power used is about 50 per cent less than where mechanical drive is used.

The draw can be controlled with greater precision with sectional drives than with any possible mechanical drive. And it can be held where wanted.

H. W. Rogers: In answer to Mr. Staege's reference to my statement that "the speed regulator which will best satisfy the exacting conditions of operation and appearance is the synchronous dynamometer regulator," I might say that this refers only to the regulator type of drive and its latest development. The statement has no reference to the synchronous tie-in type of drive which is being furnished and will continue to be furnished for some applications.

These two types of drive are fundamentally different; they each possess certain advantages and are both successful drives and yet they cannot be considered in the same class.

The question of torque available in the motors for maintaining constant speed under varying load conditions has been brought up. It is evident that the main driving motors have an abundance of it, whether slow-speed or moderate-speed motors are used. With the synchronous tie-in type of drive the restraining power is in the synchronous motors which have 20 per cent of the main motor capacity and have a pull-out torque of 200 per cent; consequently, they will hold the speed constant within a 40 per cent change in load either as a motor or as a generator, which is well within the load changes encountered on a paper machine. With the regulator type of drive the restraining power must come from the main driving motor and it is simply a question of whether it is instantaneously available for maintaining constant speed.

Both slow-speed direct-connected motors and moderate-speed geared motors have been successfully used on paper machines, but in my experience the purchaser has shown a marked preference for the slow-speed direct-connected motor. No difficulty has been experienced in designing either type to meet the requirements and it has been largely a matter of balancing the lower maintenance of one against the lower first cost of the other.

The moderate-speed motor will not respond more readily to changes in field strength than the slow-speed motor nor will it give a greater amount of torque for starting heavy loads. Slow-speed motors with good regulation can be made.

The slow-speed motor has a heavy armature with comparatively large WR^2, while the moderate-speed motor is of small diameter with a low WR^2, and while this might appear as an advantage to the moderate-speed motor, the truth is that we are not particularly concerned with the WR^2.

The stored energy in the armature is, however, of vital importance and is a direct indication of the ease and rapidity with which the motor will respond to the regulator and to changes in field. Since the stored energy in the armature is proportional to the square of the speed, it increases very rapidly in the higher speed motor, in spite of the smaller WR^2, and is many times larger in the moderate-speed motor than in the slow-speed motor.

As a further comparison the torque required to bring the motors to full speed in a given time may be of interest. Here again is a direct comparison of the responsiveness of the slow-speed motor and the moderate-speed motor, and while such wide changes in speed are not to be encountered, the comparison holds true for any percentage change in speed.

The moderate-speed motor has fewer armature conductors and a lower armature reaction; it also has a much lower inductance in the field than the slow-speed motor. The field, therefore, should and does respond more quickly to changes in current but that question is beside the point. What we are primarily interested in is the rapidity with which the armature responds to field changes and that is an entirely different question. The advantages are all with the slow-speed motor, and in the nature of things the field of the moderate-speed motor must of necessity respond about six times as rapidly as the slow-speed motor field to be on the same basis. This is not the case, but if it were possible the moderate-speed motor would still require many times greater change in armature current to produce the same responsiveness in speed, as the slow-speed motor.

It is well known that where rapid cycles and quick reversals are required, as in rolling mills and some machine-tool applications, the moderate-speed motor is always abandoned in favor of the slow-speed motor. The most important example of this application is the high-speed elevator which is driven by the slowest speed motor (65 rev. per min.) to secure sensitive control and quick response.

As regards regulation, it should be sufficient to state that hundreds of motors have been built at speeds from 38 to 100 rev. per min. with practically flat speed curves.

The slow-speed motor not only has as much torque as it is possible to obtain from any moderate-speed motor, but it has more active material and a much greater heat-storage capacity. It will stand greater overloads and more punishment without ill effect than the moderate-speed motor.

In first cost the advantage is with the moderate-speed motor, but in all other respects the advantages are with the slow-speed motor which may be summed up as follows:

1. The slow-speed motor has a much lower stored energy.
2. It requires less torque to produce a given change in speed.
3. The armature will respond more quickly to field changes.
4. It has a greater heat-storage capacity.
5. It will stand heavier overloads and more punishment with no ill effects.

In his remarks justifying the use of moderate-speed motors, Mr. Spratt has stated that heavy inertia or large stored energy is an advantage in maintaining speed, whereas Mr. Staege has admitted that the dryers, which have a large inertia, are more difficult to control since they do not respond quickly to changes.

The stored energy of a couch or press section may approximate 29 or 30 kilowatt-seconds, whereas the stored energy of a slow-speed motor armature is probably not more than one-quarter of it. The stored energy of the modertate-speed motor armature is probably two and one-half times that of the couch or press section and constitutes the bulk of the stored energy involved.

Mr. Paine has suggested the use of the dryers as a master. This is a practise that has been followed extensively on news machines and other machines with a narrow speed range and I can see no objection to it. It simplifies the control somewhat and does not affect the speed of the other sections when shut down as the regulators retain their operating position.

On wide-range machines, however, making book kraft or heavy papers, there is a tendency for the calenders to pull the dryers ahead and change the speed of the whole machine. Consequently a master set is a decided advantage in this case.

Several discussors have intimated that perhaps the electrical manufacturers are striving for unnecessary perfection and Mr. Montgomery has asked what regulation is actually required. In actual tests on mechanical drives I have found the regulation between sections to be as high as 1.2 per cent without affecting the operation of the machine or causing any complaint, and while I do not think that this should be taken as any criterion, it does indicate that exact speeds are not absolutely essential. Any of the sectional drives thus far developed will maintain speeds far beyond the possibilities of any mechanical drive.

All of the regulators described are synchronous in type and operate on the principle of an angular displacement between the two elements, the successful operation depending upon the magnitude of the displacement which causes the regulator to function. With the synchronous dynamometer I have described, a displacement of one-quarter of a mechanical degree or less between the stator and rotor will cause the stator to move and the movement ceases when the angular displacement disappears. Under these conditions there is no cumulative effect as is common with a mechanical drive. We have made no oscillograms of the synchronous dynamometer regulator, but from close observations I have made it seems to respond instantly to changes in load such as are normally encountered on the calender. Under normal conditions of operation the vernier brush on the upper half of the commutator rheostat will have sufficient range to meet all requirements but for extreme load changes the lower brush, which operates on coarser resistance steps, comes into play. This lower brush is carried on a yoked arm between the yokes of which the vernier brush moves and until the vernier brush has made its complete travel, the lower brush does not move.

All of the regulators described operate on the same fundamental principle of shunt-field control; they all operate on the basis of an angular displacement between the two elements and it is simply a question of which one operates on the smallest displacement, and which one is the simplest mechanically and electrically and has the lowest maintenance.

Mr. Harvey has requested information regarding the cost of sectional drives and I might answer him by stating that, where a new machine is involved, it is always possible to justify the sectional electric drive over a mechanical drive with either single motor or steam engine.

The use of auxiliary series fields on the dryer and calender motors was practised to a certain extent on some of the earlier drives, but at present we use shunt characteristics on both motors and generators with saturated fields on the dryer and calender motors for starting.

The use of enclosed motors is dependent upon the wish of the purchaser, but in no case do the enclosing features affect the temperature rating nor are such features ever used to justify smaller frames.

R. N. Norris: Mr. Montgomery asked a question on the degree of control—was 0.1 per cent sufficient? I think that a little better than that is necessary. So far as I can see from measurements taken personally, the average we have got can be taken at 0.03 per cent. That is from very careful measurements taken over a period of years.

I don't think there is anything in the accumulative effect. I agree with Mr. Rogers in that.

Mr. Bowler referred to the exactness of regulation on mechanical drives. I think Mr. Staege replied to that in what he said about belt slip. I don't think I need to say anything more about it, except to say that Mr. Bowler is quite right when he says that for years paper makers have made paper on mechanically driven machines satisfactorily. That is perfectly true, but then, of course, we reached a stage in paper machines where they became bigger and they have to be operated at much higher speeds, and the increased speed necessarily led to the development of these sectional drives which, once developed, proved to be far superior to mechanical drives in several respects.

Mr. Paine referred to the question of using the dryer section as the master section, and Mr. Staege also made a comment on this point. It is of course perfectly easy for dryer sections to be controlled. It is not at all a question of not being able to control them. It is purely a question of convenient arrangement of the plant.

It is often of considerable advantage to use the dryer sections, which are heavy and have considerable momentum, as a means of giving steady speed to the master shaft. If it is desired to control the dryer section it can be easily done. But why do it if it is not necessary? Why not simplify the equipment? That is one of the things at which we have aimed, namely, simplification in construction and installation.

If, however, it is required to control the dryer sections, as is advisable on book machines, kraft machines, tissue machines, etc., etc., then all that is necessary is to drive the master shaft with a small master motor and interlock the dryer sections to that. As a matter of fact there are in operation in Canada three interlock drives so driven and two in England, the two in England being big newsprint machines and the three in Canada being smaller book machines.

I agree with Mr. Rogers that for the high-speed news machine the drive of the master section by the dryers is all right, but for book machines, kraft machines, and tissue machines, it is questionable, and my inclination is to have the master motor for driving the master shaft, and not to drive from the dryer section.

Mr. Sanborn asked what the basic principle is that we are endeavoring to correct. We are of course endeavoring to correct the degree of variation in angular movement of paper-machine rolls, and the amount of load variation we are endeavoring to take account of is the normal load variations which occur on a paper machine.

I agree with Mr. Staege that the average conditions of load on a paper machine do not vary very considerably and are nothing like as heavy as the load variation that will occur on, say, a heavy rolling-mill plant. I have seen, however, fairly large load variations take place over a period of time. I have seen a couch motor, for instance, taking as a normal load 125 h.p., go up quite unexpectedly to 160 and 170 h.p. without any apparent reason. The reason of course was something mechanically wrong in the bearings or the Fourdrinier part of the paper machine, or possibly increased suction, but with the interlock system the design of the equipment is such that we provide sufficient control and resistance to deal with very heavy overloads and to prevent suctions dropping out of step. In fact we can comfortably handle overloads as high as 200 per cent if desired. If you put into your control the extra ability to compensate for these big changes in load then it is an additionally desirable condition to have.

One discussor referred to the question of starting motors on full field. That of course we always do; in fact the motors are started on what we call "super-field," as they have the full normal shunt excitation and they have the additional heavy series turns in operation at the same time.

Another gentleman referred to the question of maintenance costs on the regulator faceplates and brush arms. All I can say in reply is that we have over 30 machines in operation in Canada alone, and we have not yet been called upon to supply any spare contacts or brush arms for these regulators. We have supplied, I think, probably along with the installations as spares, some new faceplates, and probably clients have put them into operation, but we have not known of it, nor have we ever been called upon to replace them. In fact, the amount of wear on the faceplates is practically nil. When we first went into this question in 1912 and 1913, this question of wear troubled us also, but the large equipment we installed in 1913 is still running with the original faceplates, and this question of wear on faceplates need not be taken into account.

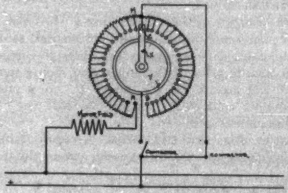

Fig. 1

In relation to the question of the use of direct-coupled motors of slow speed as against higher-speed motors geared to the sections, I can say that we are perfectly content to use either, and have used both with perfectly good results. All our motors are compound-wound, and the results obtained prove that the compound motors are quite capable of giving the service, and of dealing with the heavy starting torques without the necessity of inching motors. As a matter of fact I think it is one attribute to the interlock system that it will control large slow-speed compound-wound motors very satisfactorily.

Mr. White referred to an installation that had no voltage regulator. I agree with him that one is essential, and on this particular installation, I had recommended that this voltage regulator be installed. On the whole I believe voltage regulators do improve operation, but we have had some extraordinarily good results without voltage regulators. The Laurentide machines, which have operated over 1000 ft. per min. since August 1921, have no voltage regulators and I say without exception that the machines are among the safest in existence today.

The question has been asked as to the amount of draw control that can be allowed at each section to compensate for the change in roll diameter from, say, 22 in. to 26 in. This is entirely one of previous decision in the design of the equipment; usually we allow 20 per cent, but if somebody wants 50 per cent it can be given. If they want to make the equipment so that they can change rolls with ease, it can be given. The average paper machine does not need such a condition as this; in fact, I think it is the first time that I have ever heard of such a condition as this being required.

Mr. Harvey referred to the question of finance and the application of the drive to the cylinder machine, and made the statement that he did not see where the increased cost of the sectional drive warranted its installation on book machines, or how its installation was warranted in taking out an old machine and replacing it with a sectional type.

I think the question of the cost of the drive is bound up with the results one can get. We have installed drives on book machines which have given so much satisfaction to clients that they say the installation is more than warranted by the results they have obtained.

Of course, where the book machines are going to be operated at higher speeds, sectional drive is the only thing. We have book machines on order today that go up to 850 ft. per min. I received an order recently for one book machine which is to have a speed variation of from 150 ft. to 850 ft. That is to be 170 in. wide, which is a pretty big book machine.

Mr. Crossly asked what I meant by saying that the machine was not "wild" when a certain master section was shut down.

This can be explained by reference to the accompanying diagram, Fig 1. *A* is the full field position; *B* is the weak field position with all resistance of the regulator in circuit. *C* is the normal working position of the arm. The arm *X* makes contact from *Y* ring to contact *C* through the carbon brush. Possibly *C* may vary one contact or two contacts, as the case may be, but it is only a very tiny bit and gives only a very tiny difference in field current.

When the master section shuts down the arm, *X* travels around, but the connection from the + busbar to *Y* is broken by an automatic contactor operated when the dryer motor is shut down, and, in place of the connection being made through the ring *Y* and arm *X* to the point *C*, connection is made through another contactor to what we call the "mid-point" *M* on the resistance itself.

As a matter of fact the two contactors are combined in one, one of which opens, and the other closes, when the dryer motors shut down.

The resistance connection *M* is adjustable in the right working position for the paper machine, and it is found that this always remains in about the same position, no matter in what position the paper machine is working, as the equipment is designed to operate at the same field current on high-speed machines all the time. Where variation of field current is required for wide-range machines the motors are designed so that, by the alteration of one combined rheostat, all field currents are operated at the same time in the same degree, and the arm remains in the same position.

I think the above remarks also apply to Mr. Staege's question as to what happens when the master shaft shuts down.

One gentleman referred to the question of thermal conditions in the mill. This opens up a very interesting phase of the whole subject of sectional electric drive, and one to which I have given a little thought, of late. It is a very interesting study, and I say that by the use of sectional electric drive it is possible to obtain a better heat balance than with any form of mechanical drive. My reasons for this statement are too long to enumerate here, but it is a subject which deserves considerable investigation.

One gentleman raises the question of carrying the paper through to the third press if the dryers are shut down. This is not an important matter in spite of what some paper makers may say, because, if you shut down the dryer sections for anything at all, it means a stoppage of several minutes. During this time, much the best place to let the paper go to is the pit at the couch, and, as a matter of fact, there has lately been installed in the mills of the International Paper Company at Three Rivers a device that, when the paper breaks at any section of the machine, will automatically break the paper at the couch. I think this is good practise.

Mr. Staege asked the order of frequency of oscillation for the various sections of the paper machine. The frequency of breathing of the regulator arm is an absolutely mechanical reflection of what is happening in the paper machine room, and it is therefore possible to count the oscillations and the time period in which they take place with the interlock equipment.

A feature which is of considerable assistance to the paper maker is an indication that the sectional drive is working satisfactorily. I would not call this oscillation; in fact there is no oscillation. The arm breathes quietly—and I emphasize "quietly"—from contact to contact, often taking a very considerable period of time to do it; in fact, I have personally sat and watched the arm work without seeing it move an amount appreciable to the eye, for over a quarter of an hour.

The average speed of movement over the working range of the arm, under normal operating conditions, varies a little, but when it is calculated into items of percentage of the variation in feet per minute of the paper, it works out, as I previously said in reply to Mr. Montgomery, to the order of 0.033 per cent.

Mr. Staege, in his description of the interlock drive, I think, has quite inadvertently used the past tense in reference to many of his verbs, when he says that the dryer shaft "was" driven, etc., etc.; and all I can say is that the present tense is more applicable, as the drives not only "were" working satisfactorily, but "are" working satisfactorily, and "will" in future work satisfactorily.

Then there is the question of this "backlash" which he visualizes, and this elastic master shaft, and with all this "backlash" and "elasticity," it is a wonder that the interlock drive has been able to do anything at all. But facts show that it *does* do it, and does it with remarkable success, and today holds unbeaten records for output in the way of tonnage on the machines, one machine having produced as much as 127 tons of newsprint in 24 hours, and having given an average, over a period of months, of 114 tons per day.

Furthermore, the average highest operating speed is held by the interlock, since August 1921, on the Laurentide machines.

Mr. Staege, in his description of the new system, uses the word "automatic" regulation. I do not quite see how he claims that this is automatic. The regulator in itself cannot change the field, and the multiplicity of contact which he refers to is an endeavor to get slide-wire effect, but in my opinion is not necessary.

It is that which is at the back of the regulator which does the work, that is, the screw in the worm, which is in the nature of a mechanical differential, and I do not quite see how Mr. Staege can claimthat this entirely is an electrical differential.

I refer again to the question of having something in our equipment which the naked eye can see is happening, which is an instant reflection of what is happening on the paper machine roll, and that is the movement of the regulator arm. It doesn't matter whether you have 10 ohms in between stops, or 1000 ohms,—as long as the arm is only moving a short distance, it is an inner mechanical connection back to the regulator, unless the belt is slipping, and if the belt slips you couldn't make paper. There is something which is a mechanical indication to the eye independent of time lags of field, independent of back lash, independent of elastic shafts.

The question of the vernier on a rheostat is a matter of opinion, I think. Mr. Rogers described the interesting vernier they have employed on their regulator type. I think it is quite good. As a matter of fact we had also a vernier once upon a time, but we didn't carry it any further. We thought it was an additional complication, and we left it out.

Mr. Rogers referred to the question of the master shaft driving the high-speed machine. He says he has no objection to that. I quite agree with him. I think it is quite a sound scheme. If anybody insists on regulating it, it can be regulated.

On book-paper machines and kraft-paper machines I agree with Mr. Rogers that the calender can pull round the dryers through the paper alone. I have one kraft machine in mind which has a 75-h.p. motor on the calender and two 75-h.p. motors on the dryer. I have seen the calender pull 125 per cent full load, and the dryers practically no load.

We had another very interesting study once when we tried to drive a Harper machine. First, it gave us a lot of trouble simply because we did not appreciate (not being papermaking engineers) the conditions of a Harper-machine wet end. This machine has interconnecting felts between the couch and the first press, and the first press and the second press. The felts actually acted as belts and one would find that first the press motor was pulling the couch motor round as a generator; and then the couch motor would pull the press motors round as generators, depending upon the tension of the felts. Having discovered the difficulty, alterations were made within twenty-four hours which enabled the machine to perform under all conditions required of it, and it is today running with absolute satisfaction.

A reference has been made by both Mr. Staege and Mr. Rogers to a new drive at the Empire Paper Mills in England, using a-c. commutating motors. This drive works, but it is very complicated and occupies a large amount of space.

Rural Electrification

BY G. C. NEFF[1]
Member, A. I. E. E.

Synopsis.—*This is a paper on rural electrification, discussing some of the problems connected with this important development. The paper points out the effect of power and machinery on the living conditions of the American farm and the part electricity may play. It discusses the things being done by the agricultural and electrical industries to direct and bring about a proper development of this service.*

According to a report of the United States Department of Agriculture, in the July 1925 issue of *Crops and Markets*, the total gross income of the United States farmers for the year ending June 30, 1925, was approximately twelve billion dollars. This figure includes the value of food and fuel products produced and consumed on these farms. A 1924 report of the Department of Agriculture shows that there are thirty billion man hours of human energy used annually on the farms of the United States and this figure does not include any time allowance for the work done by the women in the house. If the entire gross income of the farms of the United States of twelve billion dollars is used to pay for these thirty billion man hours the hourly wage would be 40 cents. From these earnings the farmers must support and provide for their families, must pay all of the operating expenses in connection with their farms, must pay taxes on property which has a value of approximately sixty billion dollars, and must provide interest on this huge amount of money which is tied up in farm lands and improvements thereon.

The above statement shows that if the farmers of the United States are to be put in a prosperous condition, the hourly wage per worker must be raised. This can be done by either increasing the income or by maintaining the present income with fewer workers. I believe that the latter can be accomplished through a proper and more liberal use of power and machinery. Due largely to wide distribution of electric power by public utility companies, and to the ease with which this power can be automatically controlled, it will play a big part in reducing the number of necessary workers and in increasing general farm prosperity.

A study of general conditions shows that a very close relationship exists between the income of each productive worker and the average amount of mechanical power available to the workers. In China, the mechanical power per worker is very low and the wages are also very low. The average amount of power available to the factory worker of the United States is higher than in any other country in the world and likewise the average wage is higher.

The same relationship holds true in different parts of our country. Again, the United States Agricultural Department Survey shows that the use of power per farm worker in certain southern states is much lower than in certain northern states, and the income per worker is much less in these southern states than in the northern states referred to above. It seems, therefore, that if the average income of the worker is to be raised, it must be through increased production per worker and this can only be done with the help of machinery and this calls for power. Therefore, proper electric power supply on the farm is something which may have much to do with farm prosperity itself.

While many farms are now and for some years will be supplied with electric service from the individual farm lighting units, general and permanent rural electrification will be brought about through a further expansion of our electric public utilities which now so efficiently serve our cities and villages. In fact, this expansion has begun and about four per cent of the United States farmers are now enjoying central station service. It might appear from this that practically nothing has been done by the utility companies in extending electric service to the farms. To correct this impression, please consider the following statements. In that part of the United States lying between the Alleghany Mountains on the east and the Rocky Mountains on the west are approximately 100,000 farms receiving central station service.

On the average, three farms can be served from one mile of rural line. Therefore, there are located in this middle section of the United States approximately 33,000 miles of rural lines. If these rural lines were put into long straight lines, they would extend from Chicago to Tokio on the west, to Constantinople on the east, to the southern tip of South America on the south, and to the North Pole on the north; and there would be enough line miles left over to build a line from New York to San Francisco and back again.

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Presented at the Regional Meeting of District No. 5 of the A. I. E. E., Madison, Wis., May 6-7, 1926.

In the more thickly-settled region of the Atlantic Coast and in the region along the Pacific Coast, where electric service is used on farms for irrigation purposes, the number of farmers receiving electric service from central station lines is about equal to the number served in the great middle part of this country. The miles of line on the coast regions which are devoted to rural electric service add greatly to the mileage just described. This shows that an exceedingly large amount of rural lines have been built, that huge amounts of money have been spent, and that an appreciable start has been made in this greatly needed development. The fact that less than four per cent of the farms in this country are served by such a tremendous mileage of distribution lines clearly demonstrates that to extend adequate electric service to a large portion of the farms in this country is a task involving huge sums of money. This fact is fully appreciated and it is with the complete knowledge that the adoption of a wrong policy of development would result in losses very detrimental both to agriculture and the utility industry, and which would also seriously affect other industries, that a very extensive plan of study and investigation has been made in which every angle of the extension of service to the farm and the use of such service is being carefully studied and analyzed. This work is being carried on by representatives of agricultural organizations and agricultural colleges, by agencies of the National and State Governments which have to do with agricultural problems, and by the utility industry and the manufacturers of electrical and agricultural machines and appliances.

This cooperative work is guided by a committee known as the Committee on the Relation of Electricity to Agriculture and through its efforts and the excellent cooperation of various state organizations, there have been established 17 experimental rural lines in 17 different states. These experimental lines really form field laboratories in which to carry on the study of rural electric service. By a joint study of this kind it is believed many mistakes will be avoided which would otherwise be made and, while some mistakes will probably be made, even with the precautions taken, it is believed the major mistakes will be prevented. In the meantime, rural electric service development is going ahead as illustrated by the statistics given above. Thousands of farms are being connected every month and the rate of increase in connection is growing each year.

If rural electric service is to be built on a safe and permanent basis, it must return a profit to the farmer who makes use of the service, and also make a proper return to the electric utility which supplies the service. While there is much to be learned about this new development, those who have studied it most are convinced that both of these two conditions can be met if the development is intelligently directed. By intelligent direction is meant:

(a) Creation of a new department in electric utility organizations, devoted entirely to the proper handling and solving of all rural electric service problems, and made responsible for the direction of the development.

(b) Placing at the head of this new department a man who has had training in agricultural problems and who is sympathetic toward agriculture.

(c) The development of a fair, simple, workable and complete plan under which lines will be extended and service supplied.

(d) The development of a rate schedule which will give the utility proper compensation but which is so made as to encourage increased use by the farmer.

(e) The making of surveys and maps and the collection of such information that the main trunk lines may be intelligently laid out to most efficiently serve the entire rural territory before construction work starts. (This will eliminate much rebuilding, overloaded lines, etc.)

(f) Keeping the rural department fully informed on all experimental and research work now being conducted in agricultural colleges and on the rural experimental lines in many states. This will make possible the most efficient and fullest use of electric service on the farm.

(g) Close cooperation between this rural department and farm customers.

(h) A desire on the part of the utility to supply good and adequate service at lowest cost, and a desire on the part of the farm customer to make fullest profitable use of such service.

In the following "Program for Developing Rural Service" is given a complete plan of operation of a rural department in one of the large utility companies of Wisconsin. This particular department is headed by an agricultural engineer, a graduate of the Wisconsin University, who has had about two years' experience in giving intensive study to electric service on the farm.

PROGRAM FOR DEVELOPING RURAL SERVICE

Successful development depends upon at least three essential factors:

A. A sound policy of building and financing rural extensions.

B. A rate schedule which fosters large use of current by the farmers.

C. A complete understanding of the rural program and of the reasons underlying its adoption, by every employee who has any direct or indirect contact with rural customers.

I. Planning the Extensions

1. A study of the location of all existing substations at which rural service lines may originate.

2. Decision as to the location of additional substations strategically placed so that the entire territory may be served to the best advantage.

3. The division of the entire territory into definite rural service areas, each to be served by its own substation. The size and shape of these areas will depend on the local conditions.

4. A basic plan or map to be followed in the building of rural service trunk lines. The first of these lines will normally extend through the most thickly settled areas. These trunk lines should be planned with a view to serving the entire area ultimately, and all building should follow these plans.

Feeders to and stubs from these trunk lines may be built as development progresses.

II. New Rural Customers

1. First, campaign for new rural customers to be made along existing rural lines, in an effort to bring the average of customers per mile as high as possible.

2. Second, effort to be directed along existing pole lines on which it is practicable to string wires for rural service.

3. An intensive canvass of the more promising territories to be traversed by the definitely planned rural service trunk lines mentioned in Section I.

4. Definitely bringing our new rural policy to the attention of each existing rural customer, individually, and providing him with a comparison of rates based on his conditions. This work is to go on at the same time as that mentioned above.

III. Procedure

1. In securing new customers along existing lines, the rural service salesman will proceed as follows:

A. Call on prospect; explain rates and advantages of new rural policy; sign contract in duplicate.

B. Leave one copy of contract with prospect and send other to district office.

C. Make "Rural Prospect Report" in duplicate, sending one copy to local office and other to General office. This will be done whether prospect signs contract or not. Data for this report will be brought out during sales talk.

D. Make daily report showing calls made, contracts signed, miles of line covered, etc.

2. While canvassing the line for new customers, the rural service salesman will also call on each old customer and give him the opportunity to change from old to new rates. The procedure will be:

A, b, c, and d as above.

E. When canvass of line is finished, report length of line, number of old customers, number of new customers, and number of transfers from old to new rate.

3. District and local managers and all others working on rural service extension, but not specifically assigned to that work, will follow the same procedure, omitting (d), the daily report.

4. In extending rural service along existing transmission lines, the following procedure will apply:

A. Decision by engineers as to whether or not it is practicable to carry a rural distribution circuit on the same poles.

B. Estimate of costs by Engineering Department. Since the number of prospects who will become customers is not known, this estimate should show as separate item:

a. Cost of the line itself without transformers, services, or meters.

b. The cost of supplying service to a customer at some specified distance from the line, and

c. The amount to be added or subtracted for longer or shorter service lines.

C. Canvass of prospects by rural service salesmen, district or local managers, or others to create interest, and to discover how many customers may be expected. This makes possible an estimate of the cost per prospective customer.

D. Contracts and reports as detailed above.

5. The location of rural service trunk lines will be made by the Engineering Department in cooperation with the local representative and with the Rural Service Department. The location will depend upon such factors as straightness of roads, tree conditions, villages to be served, lakes, marshes, or rivers to be crossed, density of rural population, soil fertility, farm prosperity, demand for service, etc. When the location of the proposed trunk line has been fixed, procedure will be as shown above in 1 and 4.

IV. Reports to Rural Service Department

1. Daily report of rural service salesman.

2. Rural prospect report to be made in duplicate by any employee who learns of a prospect, one copy to local office, other to general office, Rural Service Department.

3. Completed canvass of rural line showing changes made.

4. Record of proposed location of rural trunk lines with cost estimate.

5. Route all work orders in any way affecting rural customers through Rural Service Department so that needed data may be taken off. Work order to show names of rural customers to be connected or disconnected, phase or service to each, and size of transformer and meter of each. Also estimated contribution to be paid by customer.

6. A complete report by each district manager of current used and bills paid each month by each rural customer.

V. Better Service

1. The closest possible cooperation between Rural Service Department and the Ripon Experimental Line.

2. Study by Rural Service Department of the reports of all electric farm projects and related articles.

3. Securing by Rural Service Department of all possible information relating to the different types and makes of all machines which are likely to be operated electrically.

4. Digests of this literature and of manufacturers' data to be supplied by Rural Service Department to district and local managers; also tables of electric consumption by different machines under various conditions.

5. A careful study of the relation between the appliances used and the resulting current consumption on the average farm. Data for this will come from the rural prospect reports and meter cards. This study will be made by the Rural Service Department and the results will be supplied to all local representatives.

6. The employment of at least one first-class electrical repair man who has a thorough knowledge of really good farming practise; this man to be furnished with a light truck and to travel the rural lines, calling on every customer. He will acquaint himself with the needs of the customers, and will seek the best solutions of the electrical problems of each. He will service all faulty equipment, explain ways of making fuller use of electric service, and will carry bulbs and some of the most commonly used standard appliances to supply customers who want them. He must remember that he is a service man, not a salesman, and that he only carries merchandise for the convenience of the customers.

VI. Publicity

1. The discovery and fostering of outstanding examples of the best uses of rural service in each community. Where good farmers can be found to cooperate, each local manager should try to develop electric farms that can be used as object lessons. But this development must not be subsidized.

2. Local managers will place news items in local papers. These will report the addition of new farm customers with brief

statements of how each expects to use service, or tell of the introduction of some novel application, or give definite figures on the performance of some machines, etc.

3. A definite and unified campaign of advertising in farm papers, supplemented by a series of articles explaining the company's policy and the results that may be expected from the use of electricity.

4. Statements of bills as computed under new rates, to be sent with regular monthly bills to old rate customers who would profit by changing to the new rate. Interest on the customers' investment in the line should be included in showing this advantage.

A carefully prepared contract form has been drawn up by this company which gives definite rules under which service will be extended to new farm customers and other customers living in the country. Under these rules, the utility companies finance the rural lines up to an average cost of $400.00 per farm customer whenever the customers agree to comply with certain conditions described in rules.

This contract gives the schedule of rates used in billing as shown in the accompanying table.

RATES FOR FARM CUSTOMERS

(a) Service Charge

Transformer Capacity Required in Kv-a.	Monthly Service Charge
Not more than 1½	$5.50
Over 1½ and not more than 3	6.00
Over 3 and not more than 5	6.60
Over 5 and not more than 7½	7.25
Over 7½ and not more than 10	8.00

If more than two rural customers are served from one transformer, each of them will be allowed a discount on his service charge as follows:

Customers Served	Discount on Service Charge, Per Cent
3	20
4	40
5	50

(b) Energy Charge

For the first 30 kw-hr. used per month	5½ cents per kw-hr.
For all in excess of 30 kw-hr. used per month	3½ cents per kw-hr.

A discount of ½ cent per kw-hr. will be allowed on any bill paid within fourteen days from the date rendered. The rate is made up of a service charge sufficient to cover the fixed costs on the rural extension and an energy rate covering only generating and certain transmission costs. The energy rate is comparatively low and if the farm customer increases the use of energy very materially the addition to the monthly bill will be relatively small. This rate encourages new uses of electric service by the farmers and brings about increased use with decrease in cost per kw-hr. to the farmer, all of which is very desirable to both the farmer and the utility and brings about general satisfaction with the service.

The service charge in this schedule deals almost entirely with the distribution line costs and is based upon an average investment per distribution line of $400.00 per customer. The service charge is intended to cover return on the investment, retirement expenses, taxes, operation, and maintenance expenses. These expenses may be considered as more or less fixed expenses and do not depend upon the amount of energy consumed by the customer. Fixed costs on the investment in power plant and transmission line could be included in the service charge, but in this contract all of these fixed expenses have been included in the energy charge.

The "energy charge" is low and is based upon the production cost plus a certain portion of the fixed charges in transmission lines and generating stations. In arriving at this cost the rural customer has been given the benefit of the system load factory rather than the load factor of the farm electric load. This results in lower costs of energy but it is believed that it is the fair way to handle the situation.

The accompanying table shows the kilowatt-hours used by 1226 farmers served by a Wisconsin company. These farms are largely dairy farms. This table shows that full use is not being made of electric service on the farms listed. A number of the farms shown in the table each use more than 100 kw-hrs. per month and this demonstrates in a practical way that a large use of electric service is possible.

CLASSIFICATION OF RURAL CUSTOMERS BY AVERAGE MONTHLY USE OF ENERGY

Kw-hr. Consumed per Month	Number of Customers	Per Cent of Total Number
Less than 10	123	10
10-19	390	32
20-29	275	22
30-39	158	13
40-49	75	6
50-100	131	11
Over 100	74	6
Total	1226	100

On the various experimental lines previously referred to, the farmers are using from 100 to 500 kw-hrs. per month and the average is continually increasing. It is believed that farm customers, in three or four years, will use on the average 150 kw-hrs. per month and if this can be brought about, those who have been making a study of electric service believe that such service, sold at any fair rate, will be very profitable to the farm customers and at the same time will be a load desired by the electric public utility companies. When this condition has been brought about rural electric service will be established on a sound and profitable basis, and electric service in the country will be taken as a matter of course, much as it is now in the city.

Important Features of a Successful Plan for Rural Electrification

BY G. G. POST[1]
Associate, A. I. E. E.

***Synopsis.**—Electric service in rural districts is considered to be a necessity, and a plan for bringing this about in southeastern Wisconsin is outlined in this paper. The main features discussed are: (1) organization for the work, (2) rural rates and financing of line extensions, and (3) advertising and education. A type of economical line construction which will render reliable service is described. The outstanding feature of this line construction is the use of 300-ft. spans. Interesting data, obtained in a survey of a number of rural lines, are presented. The paper shows that in the territory served there were 2740 farms receiving service under the rural rate on January 1, 1926.*

It also shows that the general plan, which has been followed since 1920, encourages the extension of lines and extensive use of the service after farms have been connected.

GENERAL

THE necessity for rural electrification and its economic importance have been widely discussed and there is general agreement that electric service should be made available to our rural population as rapidly as possible. To accomplish results in this direction requires that definite and effective plans be developed and adopted by those electric service supply companies which are in position to serve rural territory.

Plans for bringing about rural electrification vary considerably. Utilities in quite similar territories often differ on important points. Then, there are differences inherent in the territory to be served, such as the density of population and the farming resources, depending in large measure upon productivity of the soil and general knowledge of farming methods. It is not the writer's purpose to discuss or compare various plans but rather to outline a certain plan which has been worked out on a fairly extensive scale and has proved to be effective.

The territory in which this plan is in operation includes 11 counties or portions of counties in eastern and southeastern Wisconsin served by The Milwaukee Electric Railway & Light Company and its associated companies—the Wisconsin Gas and Electric Company and the Badger Public Service Company. Fig. 1 is a general map of the lines of these companies. There is a total of 21,000 farms in the territory.

ORGANIZATION FOR THE WORK

The most essential thing in a rural electrification plan is a proper organization to carry on the work. No company would consider the prosecution of a new and important undertaking without putting some one in charge and making that person responsible for the results.

Rural electrification is a new and important undertaking. It covers a field differing from any that has ever been covered before by electric utilities. If rural electrification is to succeed as fully as it should, the utilities must give it the same careful thought and effort they have given other important activities that have confronted them in the past and that have been successfully handled. The only way to do this is to establish a rural service commercial division, or department composed of a rural agent and such assistants or rural service salesmen as may be required, to adequately promote the use of electric service and electric equipment by the farmers of the territory the utility serves. The Milwaukee Electric Railway & Light Company employs a rural agent and from four to six rural service salesmen. These men have worked on farms, know the language of the farmer, and at the same time are ex-

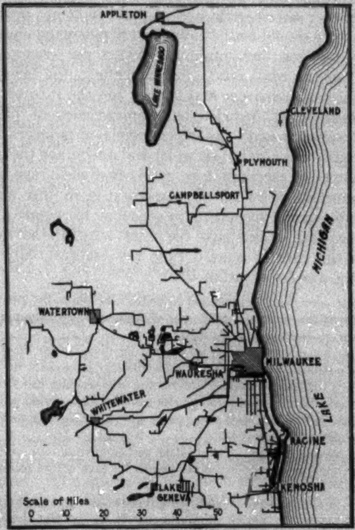

FIG. 1—DISTRIBUTION LINES OF THE MILWAUKEE ELECTRIC RAILWAY AND LIGHT COMPANY AND ASSOCIATED COMPANIES

perienced commercial engineers. They study all phases of the rural electrification problem, take such action as may be necessary to follow up advertising, keep prospective consumers interested in the service, carry on negotiations for line extensions, introduce new uses for the service, and do any other work which the proper conduct of the business requires.

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Presented at the Regional Meeting of District No. 5 of the A. I. E. E., Madison, Wis., May 6-7, 1926.

RURAL RATES AND THE FINANCING OF LINE EXTENSIONS

Before any organization can be effective, a system of rates and a method of financing line extensions, consumers wiring, and equipment must be developed.

It is a sound principle that country dwellers are not much different in their fundamental nature and aspirations from those residing in urban districts and that in establishing rates and line extension policies the same general principles should govern in rural as in urban districts. If it is right and equitable to have the equivalent of a demand charge (active rooms) in residence rates, and if a demand charge in commercial and power rates is right and equitable is it not proper that there should be the equivalent of a demand charge (active rooms and minimum charge per connected h. p.) in the rural rate? Fundamentally, the farm is the farmer's residence and factory and the rate to be equitable must include the characteristics of plain residence rates and those of commercial or power rates. The rate must also be one which will discourage the use of motors of excessive h-p. rating and encourage the consumption of kw-hr. To express it in another way, the rates should encourage the consumer to operate at a high load factor and make the fullest use of the service. With this idea in mind a rural rate, substantially as follows, was developed in 1920:

Rural consumers are defined as consumers generally located outside of the limits of cities and villages who are engaged in farming, stock raising, or dairying.

Rural consumers located upon existing distributing lines or who contribute to the cost of constructing extensions in accordance with the Company's rules and who execute the Company's standard form of rural service contract for a period of not less than two years may purchase single-phase electric service at the following rates.

(*a*) *Service Charge.* Two dollars per month for four or less active rooms plus forty cents per month for each active room in excess of the first four.

Service charge shall include energy consumption equivalent to five kw-hr. per active room per month or not less than 20 kw-hr., in case the active rooms are less than four.

(*b*) *Energy Charge.* Three and one-half cents per kw-hr. for all energy consumed in excess of five kw-hr. per active room, or in excess of 20 kw-hr. in case the active rooms are less than four.

The number of active rooms shall be computed on the following basis:

(1) All rooms contained in the residence shall be counted as active, except bathrooms, basements, garrets, closets, halls, pantry, unwired storage rooms, and three rooms used as bedrooms or sleeping quarters but not used for other purposes.

(2) Rural installations having barns, poultry houses, or greenhouses, but not wired, shall have the residence count increased by one active room.

(3) Rural installations having barns, poultry houses, or greenhouses wired, shall have their counts increased by one active room for each separate horse barn, dairy barn, poultry house, and greenhouse actually connected to the distribution system and receiving energy.

(4) Sheds or open barns used to shelter live stock, grains, feeds, or machinery (and not regularly used as horse barns or dairy barns), garages, and miscellaneous buildings, even though wired, shall be exempted from the active room count, excepting such garages, blacksmith shops, and other buildings that may be conducted for and open to the general public as repair or service shops. Each building so conducted shall be counted as an active room.

(5) A barn used as a combination horse and dairy barn shall be counted only as one active room.

(6) Where rural consumers, as herein defined, are located within the limits of cities and villages, such consumers may obtain service on either the rural rate or residence rate applicable in such localities.

The foregoing rate shall apply to all energy used for household purposes, including cooking, and for miscellaneous power purposes, providing the total nominal rated capacity of motors does not exceed three h. p. and providing the motors are not operated between sundown and 11:00 p. m. Service for additional motors will be supplied at the foregoing rates plus an additional charge of fifty cents per month for each h. p. or fraction thereof in excess of three h. p.

The foregoing rate does not include renewals of any incandescent lamps, but is subject to the Company's standard prompt payment discount of five per cent on the first $25.00 and one per cent on amounts in excess of $25.00 on monthly bills paid on or before the last discount date. Bills will be issued as far as practicable 10 days prior to the last discount date.

The gross bills under this schedule may be calculated from the following table.

MONTHLY BILLS FOR FARM CUSTOMERS

The minimum charge in this table allows (a) three h. p. in connected motor load and (b) energy consumption of five kw-hr. per active room, or 20 kw-hr. total, for less than four rooms.

For connected motor load in excess of three h. p., add 50 cents per month for each h. p. or fraction thereof in excess of three h. p.

For energy consumption, add three and one-half cents for every kw-hr. in excess of five kw-hr. per active room (or 20 kw-hr. total for less than four rooms).

No. of Active Rooms	Kw-Hr. Consumption Allowed on Minimum Charge	Minimum Monthly Charge
4 or less	20	$2.00
5	25	2.40
6	30	2.80
7	35	3.20
8	40	3.60
9	45	4.00
10	50	4.40

As far as the supply company is concerned, there is only one essential difference between rural and urban service, *i. e.*, the greater distances between consumers in the country than in urban territory. This necessitates more line investment per consumer in rural than in urban territory, making it necessary that some means be found for providing for the excess investment.

Before rural service was seriously considered, utility companies found it necessary to devise a plan under which lines could be extended to supply isolated consumers in the fringe surrounding urban territory where the extensions were so long that the revenue would not justify the investment. It became necessary, under such circumstances, for prospective consumers to assist in financing the lines. It was only logical that this principle should be applied in extending lines in rural communities. Consequently the plan was adopted of requiring the prospective consumers to contribute toward the cost of extending the service to them an amount equivalent to the difference between the cost of the extension and the estimated three years' gross revenue. At the same time provision was made that if other consumers were connected to the extension and it was not necessary for them to contribute toward the cost of their extensions, half of their net bills would be used as a refund to those contributing to the cost of the original extension. Such refunds are discontinued when the original extension has been in service three years, and the total refunds must not exceed the total of the original contributions.

It is usually found that some farmers desire the service more than others, or at least are willing to invest more money in getting it if necessary. Some farmers have been willing to use the service but have been unwilling to contribute anything toward the cost of the line. Others not only are willing to agree to use the service on as extensive a basis as possible, but are willing to contribute a reasonable amount where necessary in order to get the line constructed and the service furnished. It is not possible in these cases to get the farmer consumers on a rural line to bear an equal portion of the investment the company has to make in excess of the amount warranted by the revenue. It has been found necessary to lay the amount to be advanced and the complete proposition before the group and have them decide among themselves how they will finance the excess cost. One of the farmers, agreeable to both the group and the company, is designated as the official representative of the group and he executes the line extension agreement and turns over to the company the part of the investment required to be advanced by the farmers under the line extension agreement. He may have paid it all himself or he may have collected various parts of it from different farmers. Along with every line-extension agreement there is a blue-print showing the line as constructed and the services connected thereto. This record is carefully preserved and in the future, if there are refunds to the group to be made by the company due to additional consumers coming on the line, they are made to the official representative of the group who executed the line extension agreement and he is responsible for distributing these refunds to the farmers in a manner mutually satisfactory or in accordance with a basis previously agreed upon among themselves.

Although the ownership of lines partially financed by the consumers rests entirely in the utility, the amount contributed is not included in the utility's valuation for rate-making purposes. No special fixed charge is, therefore, included in the rates to cover any extra investment in rural lines and for this reason the monthly charge per kw-hr. is considerably lower than it would be if the utility carried all of the investment.

It must be borne in mind that, ultimately, there will be a vast network of distribution lines serving cities, villages and rural districts alike, and if rural consumers are to remain satisfied there must not be a special fixed charge in their rate to make their cost per kw-hr. higher than that of the urban consumers who may live next to them along the border lines of cities and villages.

Recognizing that it requires a considerable amount of money to contribute toward the cost of lines, to wire farms, and to purchase equipment, a plan has been devised under which the utility advances the money for the purchase of appliances or other equipment and wiring and the rural consumer pays it back on the installment plan in accordance with the terms of certain agreements.

Under the general plan for financing line extensions, farmers are encouraged to install as much equipment as possible so as to increase the gross revenue from the line and cut down the amount which must be contributed toward line extensions. In a good many cases it has been found that this results in the installation of electric ranges or other equipment which would probably not be otherwise installed.

The farmer is thus enabled to make immediate and full use of the service and derive the benefits which electric service always affords.

ADVERTISING AND EDUCATION

Advertising is just as important in the extension of electric service to the farmers as in any other line of endeavor. The farmer cannot be expected to want it if he is not told about its many benefits. Personal work with the farmers is most effective. The rural agent and his rural service salesmen are most valuable in establishing personal contacts with farmers and acquainting them with all known phases of rural electrification. The efforts of the rural service division are supplemented by the knowledge and experience of the company's regular advertising forces which are especially helpful in advertising in rural papers, in preparing special posters, talking at booster meetings in rural territory, etc. The specific plan adopted is to inform the farmer by all feasible means available as to the special applications and advantages of electric service to each individual farmer. Instead of waiting for the farmers to apply for electric service, the electric service company endeavors to develop the interest of all the farmers in the use of electric service and especially cooperate with those who already have the desire for the service in getting other farmers along the proposed route of the line interested, so that the line may be constructed and the service furnished at the lowest possible cost to any of the farmers. In connection with this work, farmers are being circularized and furnished booklets which show what electric service means to them. Advertisements are being inserted in local newspapers, and demonstrations are being held at various points where an audience of sufficient size to warrant a demonstration and exhibit can be got together. At these demonstration meetings well-informed representatives of the company are present, and an effort is made to furnish to the farmers any information desired regarding electric service and electric equipment and how they may be obtained.

The rural service salesmen always stress the point that the utility is not in the business of selling line extensions but rather the service and that it is desirable for the consumer to spend as much as he can afford on equipment which will enable him to make the most use of the service not only in the farm operations but in his home where the convenience of the service is so much appreciated and enjoyed by the housewife.

A roadside sign set up on a farm where a rural line extension is being installed is very helpful in attracting new customers. In some cases, as at the beginning of a line extension where service is available, these signs are illuminated. They are very effective, however, without illumination.

Much effective work is also done by establishing rural electrification booths at state and county fairs. These are strictly educational and informational displays to emphasize to farmers the benefits of the service.

LINE CONSTRUCTION

Because of the small number of consumers per mi. of line it is important that the cost of line construction be kept down to the minimum, consistent with reasonable service. This keeps fixed charges within reason and reduces to a minimum the amount which prospective consumers must contribute toward the cost of line extensions.

Utilities should not come to the conclusion that service to farmers can be less reliable than that to the ordinary small city or village. When a farmer installs a range upon which he depends for the cooking of his meals, when he installs a milking machine with which he expects to milk the cows, when he installs a heating plant which depends upon electric service for its operation, when he installs a motor-driven pump to furnish water to his stock or for the house, and when he installs other electrically driven equipment upon which his farm processes and operations depend, it is a serious matter if reasonably continuous service is not given and the farmer cannot be expected to electrify his farm if the service is not reliable.

The line construction employed, although it must of necessity be reasonable in cost, must embody all of the features which contribute to make possible safe and continuous operation.

The principal features of the line construction which has been adopted as most reasonable in cost and which at the same time meets the requirements for good service are as follows:

Size of poles	30 ft., 7 in. top, (western red cedar)
Spacing of poles	300 ft.
Size and kind of conductors	No. 2 A. W. G., A. C. S. R.
Size and kind of crossarms	3¼ by 4¼ by 5 ft. 7 in. fir
Size and kind of pins	No. 581 malleable clamp pin (St. Louis Malleable Casting Company)
Insulators	No. 9404 Ohio Brass (or equivalent)
Operating voltage	5000 primary, 120/240 secondary

Every tenth pole an anchor structure.

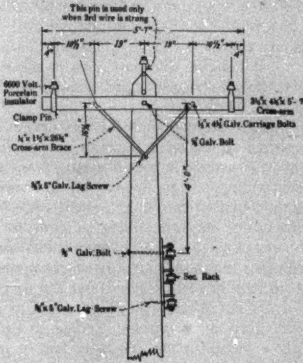
This pin is used only when 3rd wire is strung
5'-7"
10½"
19"
19"
10½"
4"
4"
6600 Volt. Porcelain Insulator
Clamp Pin
¼" x 1½" x 26¾" Cross-arm Brace
15¾"
⅝" x 5" Galv. Lag Screw
3¼" x 4¼" x 5'-7" Cross-arm
½" x 4½" Galv. Carriage Bolts
⅝" Galv. Bolt
4'-0"
⅝" Galv. Bolt
Sec. Rack
⅝" x 5" Galv. Lag Screw

FIG. 2—ARRANGEMENT OF POLE TOP

The details of the construction are shown in Figs. 2, 3, 4, 5, 6, and 7. The cost of a single-phase primary line, exclusive of transformers, lightning arresters, secondary mains, and services is approximately $750 per mi. in country where there is little trouble with trees and where there is neither rock nor too much gravel to interfere with pole setting. If tree conditions are bad or if rock or gravel is encountered, the cost will of necessity be increased. The details of cost are shown in Table I.

That the construction is safe is shown by the following:

Conductor Stresses:	
Elastic limit	1860 lb.
Ultimate strength	2660 lb.
Stress at 0 deg. fahr. with ½-in. ice and 8-lb. wind pressure	1860 lb.
Conductor Clearance from Ground:	
Clearance at 0 deg. fahr. with ½-in. ice	19.7 ft.
Clearance required by National Electrical Safety Code	18.0 ft.

Resisting Moments of Poles:	
Safe loading of 30-ft., 7-in. top western cedar, 30-in. circumference at ground line	17,800 ft-lb.
Required by loading ½-in. ice and 8-lb. wind	14,870 ft-lb.
Conductor Spacing:	
Provided by this construction (three-phase)	29½ in.
Required by National Electrical Safety Code for 300-ft. spans	18¾ in.
Pins and Insulators:	
Lateral stress with ½-in. ice and 8-lb. wind pressure	263 lb.
Stress in direction of line (conductors broken)	1860 lb.
Stress which two pins on anchor structure will safely withstand	1950 lb.
Stress which insulators will withstand	4000 lb.
Crossarms:	
Stress in direction of line which single arms will safely withstand	1880 lb.
Stress in direction of line (maximum loading) one conductor broken	1860 lb.

TABLE I

ESTIMATED COST PER MILE FOR THE EXTENSION OF TWO NO. 2 A. C. S. R. USING 300-FOOT SPANS

Material	Unit Cost	Quantity	Amount
No. 2 A. C. S. R	$0.0208	10,700 ft.	$222.56
5/16 in. guy wire	0.018	440 ft.	7.92
30 ft. western cedar poles	7.14	18	128.52
35 ft. western cedar poles	11.92	2	23.84
Everstick Anchors	1.52	4	6.08
No. 4 X insulators	0.27	8	2.16
Pole shims (large)	0.12	16	1.92
2 bolt guy clamps	0.42	32	13.44
5 ft. 7 in. crossarms	0.69	20	13.80
Crossarm braces	0.24	32	7.68
No. 581 clamp pins	0.457	40	18.28
5/8 by 12 in. galvanized machine bolts	0.083	16	1.33
5/8 by 16 in. galvanized machine bolts	0.105	2	0.21
5/8 by 5 in. galvanized lag bolts	0.0411	16	0.66
1 1/2 by 4 1/2 in. galvanized carriage bolts	0.0315	32	1.01
6600 volt pin type insulators	0.14	40	5.60
Aluminum sleeves (2 sleeves per joint)	0.16	4	0.64
Aluminum armor for insulator ties	0.35	2.9 lb.	1.02
No. 6 tie wire (annealed aluminum)	0.33	3.0 lb.	.99
Total Material			457.66
Labor			
Pole crew	5.94	12 hrs.	71.42
Line crew	3.75	16 hrs.	60.04
Supervision and engineering			19.72
			151.18
Sundries			
Truck	0.65	28 hrs.	18.20
Tools and injuries—5 per cent of labor			7.43
Right-of-way			54.08
			79.71
Contingencies			58.35
Grand Total			$746.90

Note: Pole Work—1 foreman, 1 driver and 6 polemen can set 12 poles per day of 8 hr.

Line Work—1 foreman, 2 linemen and 2 groundmen can install crossarms, string wire and tie in 1 mi. in 2 days of 8 hr. each.

Where secondary mains are necessary, 30-ft., 6-in., top poles spaced 150 ft. apart are used except for transformer poles which must have not less than 7-in. tops. This permits the use of No. 6 or larger, hard-drawn copper secondary mains installed on racks. All transformer connections are of copper and the copper conductors are connected to the aluminum conductors with approved clamps.

RESULTS ACHIEVED

During the fall and winter of 1925-1926 some interesting data were collected on a few rural lines.

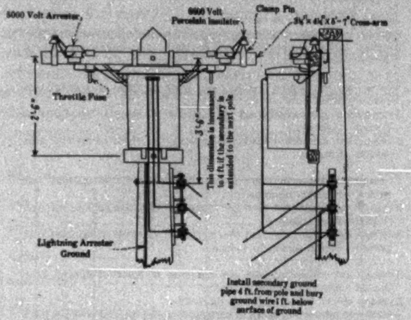
5000 Volt Arrester
6600 Volt Porcelain Insulator
Clamp Pin
3⅝"x4⅝"x5'-7" Cross-arm
Throttle Fuse
2'-6"
3'-6"
This dimension is increased to 4 ft. if the secondary is extended to the next pole
Lightning Arrester Ground
Install secondary ground pipe 4 ft. from pole and bury ground wire 1 ft. below surface of ground

FIG. 3—TRANSFORMER INSTALLATION SUPPLYING SERVICE DROP

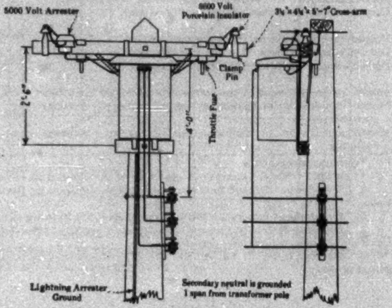
5000 Volt Arrester
6600 Volt Porcelain Insulator
3⅝"x4⅝"x5'-7" Cross-arm
Clamp Pin
2'-6"
4'-0"
Throttle Fuse
Lightning Arrester Ground
Secondary neutral is grounded 1 span from transformer pole

FIG. 4—TRANSFORMER INSTALLATION SUPPLYING SECONDARY MAIN

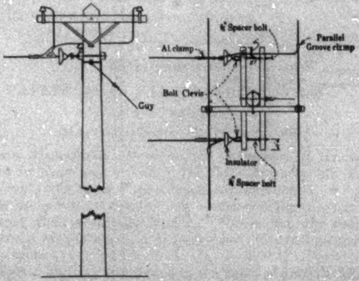
⅝" Spacer bolt
Parallel Groove clamp
Al. clamp
Bolt Clevis
Guy
Insulator
⅝" Spacer bolt

FIG. 5—METHOD OF MAKING DEAD-ENDS AND BRANCH-LINE TAPS

TABLE II—RURAL LINE SURVEY—AVERAGE LINE, TRANSFORMER AND CUSTOMER DATA

	Line No. 1	Line No. 2	Line No. 3	Line No. 4	Line No. 5	Average
Date of special test period	10/23/25 to 10/30/25	1/8/26 to 1/14/26	1/7/26 to 1/13/26	12/17/25 to 12/24/25	1/18/26 to 1/26/26	Winter
Number of customers per line	6	6	13	23	25	14.6
Length of line in mi	0.57	0.75	1.73	4.93	3.70	2.34
Average number of customers per mi	10.5	8.	7.5	4.66	6.75	6.*
Average size of farm served—acres	74.	63.	43.	88.	59.	66.*
Average load applied for at time customers requested service—kw	9.25	4.54	3.34	6.15	5.63	5.59*
Average initial connected load at time customers were given service—kw	7.75	5.68	3.70	6.51	3.81	5.11*
Average kw. connected load per customer at time of special test period—Total	9.69	7.296	4.613	8.684	5.65	6.88*
Average kw. connected load per customer at time of special test period—Lighting	0.877	0.708	0.552	0.875	0.845	0.80*
Average kw. connected load per customer at time of special test period—Ranges	7.08	3.785	2.691	5.350	3.318	4.20*
Average kw. connected load per customer at time of special test period—Laundry equipment	0.708	0.466	0.695	0.834	0.554	0.667*
Average kw. connected load per customer at time of special test period—Pump motors	0.685	2.090	0.59	0.557	0.746	0.761*
Average kw. connected load per customer at time of special test period—Cream separator motors	0.12	0.15	0.044	..	..	0.031*
Average kw. connected load per customer at time of special test period—Miscellaneous equipment	0.22	0.097	0.041	1.068	0.172	0.421*
Average size of transformers on line—kv-a	7.5	4.5	2.6	6.6	4.5	5.24*
Average 15 min. kv-a. demand on transformers during test period	3.60	2.64	1.61	3.11	2.19	2.63
Average transformer demand factor during test period (1)	0.19	0.298	0.32	0.273	0.333	0.28
Maximum 15 min. kw. demand of line during test period	5.54	6.6	8.45	15.4	12.	9.60
Line power factor at time of maximum demand	94%	89%	97%	96%	93.8%	94%
Demand factor of line during test period (2)	0.095	0.151	0.141	0.077	0.085	0.109
Diversity factor between line and transformer demands (3)	0.514	0.654	0.57	0.344	0.342	0.48
Per cent line, transformer and meter losses (4)	19.5%	24.4%	37.4%	32.8%	28.9%	28.6%
Annual kw-hr. delivered to customers—Average per customer	1694.	826.	640.	1401.	†	1046*
Month and kw-hr. of maximum use of energy—Average per customer	239 July	160 Aug.	80 Oct.	124 Sept.	†	121 Jul.*
Month and kw-hr. of minimum use of energy—Average per customer	91 Mar.	32 Sept.	28 Apr.	78 Mar.	†	69 Mar.*

*Weighted averages.

†New line. Kw-hr. figures not available or considered in determining weighted averages.

1. Transformer demand factors are the ratios obtained by dividing the maximum 15-min. transformer demands by the total transformer connected loads.

2. Line demand factors are the ratios obtained by dividing the maximum 15-min. line demands by the total connected loads on the line.

3. Diversity factors of line and transformers are the ratios obtained by dividing the maximum 15-min. line demand by the sum of the individual maximum 15-min. transformer demands.

4. Per cent losses are based upon the difference between measured kw-hr. line input and kw-hr. recorded on customers meters over an extended period of time.

These data are shown in Table II. It is interesting to observe that in general more load was found connected than the farmers agreed to connect when contracting to take the service. While the average contracted for was 5.59 kw., the average found connected was 6.88 kw.

Another fact of interest is that the transformer loading was found to be low. It is known from individual consumers' records that the maximum consumption is very apt not to occur in the winter, when these data were taken. This may be partly due to the fact that in the summer the farmer uses his electric range while in the winter he uses a wood range. He also does more pumping in the summer and carries on other operations more extensively. Comparable data taken during the summer months will show whether transformers larger than necessary have been furnished or not.

The power factor at time of maximum load was found to vary from 89 per cent to 97 per cent on the various lines tested. Annual kw-hr. per consumer on the vari-

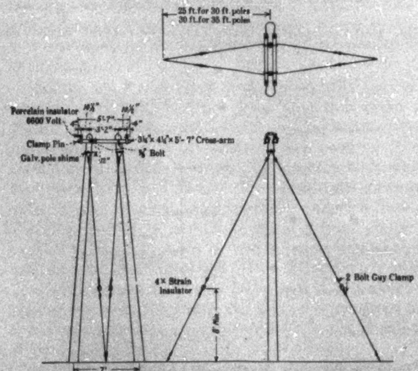

Fig. 6—Anchor Structure Used Every Tenth Span

ous lines averaged from 640 to 1694. Line, transformer, and meter losses were found to vary from 19.5 per cent to 37.4 per cent.

Fig. 8 shows the number of consumers served under the rural rate by years from 1920, when the rate was adopted, to January 1, 1926, when there were 2740 rural consumers connected.

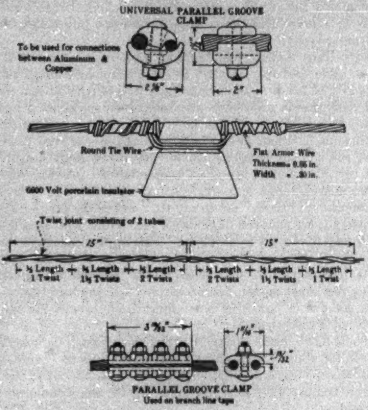
UNIVERSAL PARALLEL GROOVE CLAMP
To be used for connections between Aluminum & Copper
2 3/8"
2"
Round Tie Wire
Flat Armor Wire Thickness = 0.85 in. Width = .30 in.
6600 Volt porcelain insulator
Twist joint consisting of 2 tubes
15"
15"
1/3 Length 1 Twist
1/3 Length 1 1/2 Twists
1/3 Length 2 Twists
1/3 Length 2 Twists
1/3 Length 1 1/2 Twists
1/3 Length 1 Twist
3 7/32"
1 11/16"
11/32"
PARALLEL GROOVE CLAMP
Used on branch line taps

Fig. 7—Clamps, Splice and Tie

Conclusion

Utility companies must view the rural business broadmindedly and provide a reasonable line extension plan and rate schedule which will encourage the use of the service. A rate schedule designed to pay full returns on investment from the very start, by using burdensome minimum charges or high rates per kw-hr., will produce the opposite result.

A financing arrangement has been found to be necessary, because a rural line extension project involves a number of farmers, all of whom must arrive at the buying point at one time. Where the farmers' activities are somewhat varied, all may not have the funds available at the time the project is developed to the final stage. If it cannot be concluded then but has to be postponed because some do not have the funds available, when it is reopened, some of the others may not have the funds available, and often a project is delayed for an indefinite period of perhaps a year or two. A farmer who can afford to go into such a project at all will have the funds available at sometime within a year. A plan whereby his financial obligations can be taken care of satisfactorily until such time as he has the funds available leaves no obstacle in the way of concluding the negotiations when all agree upon terms.

Experience has demonstrated that the all important thing in general rural electrification is to make the service available to the farmers. The records show that as soon as the service is available the farmers will find new and increasing ways of utilizing it as industries in the urban centers have done and are continuing to do. Every farm can be made a laboratory in a greater or lesser degree for carrying on experimental work, and the greater the number of farms carrying on such work, the greater will be the increase of new applications for the service. It is important, therefore, for utilities to cover their rural territories as quickly as possible with suitable supply lines. This can be done with reasonable assurance that load not contemplated now will develop as new uses for the service are found and as the people come to realize, as they can only through use and reasonable encouragement on the part of the utility, its many and varied advantages.

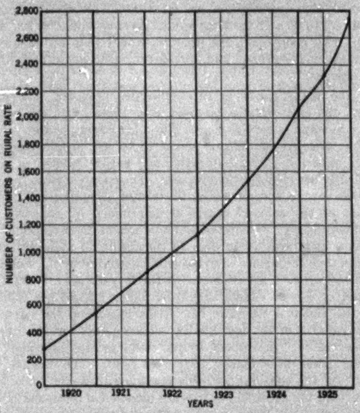

Fig. 8—Number of Rural Customers on Lines of the Milwaukee Electric Railway and Light Company and Associated Companies by Years

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Discussion

PAPERS ON RURAL ELECTRIFICATION

(Neff and Post)

Madison, Wisconsin, May 6, 1926

Eugene Holcomb: I notice the difference in the rate schedules in these two papers. The rates shown by Mr. Post seem undoubtedly too low for average rural territory.

It has been our experience that copper-weld lines are more satisfactory than aluminum lines and I believe that with proper sizes selected for the work to be performed, this will be found generally true.

The elastic limit as mentioned in the paper as 1860 lb. might also be checked. The sum of the strands in aluminum cable gives an elastic limit of 1593 lb., whereas this is stated in the table as 1860 lb.

The clearances given in the paper are mentioned as 18 ft. I believe 18 ft. in the code is for a 150-ft. span and for a 300-ft. span it is 19.3 ft. However, the clearances mentioned in the paper may be sufficient.

Mr. Neff has given a vivid illustration of the magnitude of rural electrification. Notwithstanding that only a very small dent in the total has thus far been made, the 33,000 mi. of lines mentioned in the inter-mountain section of the country would reach once entirely around the earth and one-third of the way on the second lap. Suppose we consider the possibilities of completely serving one township which is six miles square. A line on each one-mile road would total 72 mi. of line, counting the border lines as being half in the adjoining townships. A cost of $1200 per mile gives a total investment of $86,400 for lines. With an average of three customers per mile there would be 216 customers per township. Using the rates given in the paper and assuming the lowest monthly service charge at the stated average consumption, the monthly bill for each customer would be $10.60 and the total annual revenue would be $27,475 which, with good operating practise, would yield an adequate return on the investment and provide for replacements and other fixed charges.

Of the many forms of rate schedules proposed for rural service, the most practicable seem to be those which are based upon a demand or service charge plus a low energy charge. The simplest measure of demand seems to be the installed transformer capacity. This is tangible and easily understood and sufficiently accurate for the purpose. The energy charge should be made low to encourage the liberal use of equipment. The amount of the demand charge is determined by capacity or investment costs. Where the customer makes the investment in the lines and turns them over to the utility to maintain and operate, then the service charge should be reduced by the amount of interest on such investment. If the investment is $400, the demand or service charge should be reduced $2.66 per month from the amounts stated in Mr. Neff's paper and for a transformer capacity of not over 1½ kw., the amount of service charge becomes $2.84. In respect to the costs to the customer, it matters little who makes the investment. The matter of permitting the consumption of a few kilowatt-hours to be included with the minimum charge is a question of costs and if we are to have the large monthly consumptions desired, the difference would be very slight and by that time undoubtedly most of the rates now in effect will be materially modified.

To electrify all the 6,500,000 farms in America will require some $6,500,000,000 of capital and years of time and probably some remote parts never will be served. As stated in the paper, however, progress is being made and as equipment is developed and experience gained from the experimental lines, the work will proceed more rapidly. Above all things in order to avoid costly mistakes, this work should be thoroughly mapped out and handled by men who are devoted to rural service.

Unquestionably our industry will be able presently to show how electricity can be utilized by the farmer so that it will be profitable to him and not only that, but will help do things for him and the people in the farm home that will make life there very much more attractive and satisfactory.

C. B. Hayden: There are a few points which I shall mention as important from the standpoint of the Railroad Commission in particular. The question of rural electrification is unquestionably important and the activity of the utility organization in cooperating with the farm bureaus and college experimental departments in developing a workable plan to bring about a complete consideration of the problems involved is highly commendable. The response of the utilities has been quite unanimous and I do not believe that it can be said that, in their enthusiasm to provide this service, their interests have been completely submerged. And this is as it should be because in some instances in the past the rural policy, at least in the early years following the construction, would not provide returns sufficient to carry the investment and operating expenses.

I believe that the general policy shown by these papers, in that it tends to provide rural service for the larger consumer at very low service rates, is decidedly good. In this connection, however, we must not lose sight of the impression to be created in the mind of the average urban consumer when he realizes that, under the average community rate, he cannot purchase energy as advantageously.

I realize fully that in order to make the service of value to the farmer, aside, perhaps, for the use of lighting and small power appliances, he must be able to purchase in quantity at low rates, and I think that in a few years the energy will be so used by the average farmer. For a number of years, however, and until the experimental lines and special installations have proved that the farmer can actually make money by the use of large quantities of energy on the farm, we must remember to provide for the smaller user on terms that will not be prohibitive to him. In this connection, I should like to say that where the farmer wants the energy it has not been difficult for him to see that it is necessary to pay the cost of securing it. This applies to the average well-to-do farmer and does not apply to the smaller farmer who is operating on 15 to 20 acres where he does not figure on making very much money, but on making a living and that is all. There the use of power in large quantities, except for special uses, is out of the question.

I should like to make this point; that the average farmer must be given full consideration during this development period, and also it must be borne in mind that the rates must not favor the farmer as compared to the urban consumer for the same quantity consumption.

E. W. Lehmann: I should like to outline briefly what I think is the purpose of this rural electrification program. From the standpoint of the farmer, I believe it is first to provide service so that he can improve his efficiency in production; second, to improve the economy or lower the cost of production; third, if possible to increase or improve the quality of produce; fourth, to make the job easier and more pleasant; and fifth, to improve home life. From the standpoint of the public service company, however, I believe that the problem is to extend service to the farmer and to build up a load which will pay for the service rendered. The real job will be to sell the idea of the value of electricity and in that way sell more kilowatt-hours.

It is difficult, I think, to estimate the part that electricity will play in agricultural production in the future. A look into the past, however, seeing what machinery and equipment have done and analyzing our present problems, will give a fair basis for an estimate.

There is no question but that machinery and equipment have played a larger part during the past 75 years in increasing the productive capacity of the individual farmer than any other factor. While about 90 per cent of our population were farming 75 years ago, only 26 per cent are doing the job at the present time.

In regard to the development of the dairy industry, which is so important in Wisconsin, in a statement before an agricultural policy committee in Illinois a few years ago, Dr. M. W. Hepburn, an authority on dairying, said, "In a general way we may say that four factors have been largely responsible for the development of dairy production. They are:

"1. The introduction and utilization of the silo.

"2. The centrifugal cream separator.

"3. The discovery of the simple test for fat in milk,—the Babcock test, discovered at the University of Wisconsin.

"4. Better and more rapid transportation, together with the development of refrigeration."

All recognize these factors as being mechanical and their effectiveness is dependent on the application of power. Even the filling of the silo is a problem and there are other problems of the same kind.

In the whole field of crop production, equipment and machinery have played a large part; the production of wheat per person employed increased eighteen times from 1850 to the beginning of the twentieth century. With the advent of the gas tractor and the combine harvester-thresher, the efficiency per individual has been still further increased. A local farmer in Illinois who purchased a combine harvester-thresher rather recently told me that he harvested his grain at about one-third the former labor cost.

In discussing horticultural production, Dr. J. C. Blair, head of the Department of Horticulture in the University of Illinois, stated, "Without the protection afforded by spraying, insect and fungus attack will no doubt make apple-growing in Illinois an impossibility." This would apply also to other states. In certain sections of the country the application of electricity to the spray problem is being investigated with special attention to the stationary spray outfits. We should not overlook the fact that electrically driven equipment now plays an important part in the processing and manufacture of all sorts of fruits and garden produce. In addition to processing and canning, fruits are graded with electrically-driven graders and in that way made ready for market. In fact, in every phase of agriculture, mechanical equipment has played and still is playing an important part.

In considering the farmer's immediate problem we must recognize the fact that he is interested most in increasing his net income. There are three ways in which this may be done: First, by getting a better price for his produce; second, by lowering the cost of production; third, by producing more per worker. I believe the application of electricity will help in all of these methods.

In considering the application of electricity to the farm, remember that there is more than the mere matter of equipment and power involved in the farmer's problems. It is safe to say that four types of problems will be met in a rural electrification program; namely, (1) economic, (2) agricultural, (3) educational, (4) engineering.

I do not want to deprecate the part machinery has played in what I shall now say. From an economical standpoint, there are three equally important factors in agricultural production—land, labor and capital, or equipment. In addition to this, we, of course, have the item of managerial ability; in other words, the farmer himself. I believe he is as important a factor as any of the other three. Every manufacturer recognizes that there must be a proper balance between the three economic factors mentioned for economic and efficient production.

Where there is an abundance of good land, the tendency is toward the bonanza type of farming, rightfully called "agricultural exploitation." The recent rapid agricultural development in the Argentine has been due to this type of farming. The primary need there was machinery. That was formerly true in the Middle West. We cannot put too much emphasis now on the value of the soil; in fact, the life of our nation depends upon it. Much of our soil is already becoming depleted of its fertility. For efficient, economic production, our land must be well-drained, protected from erosion and must be used in accordance with the best farming practises to maintain a high state of fertility. The first step in economizing on land is a more intensive type of farming and that is a thing farmers will do when there isn't a chance for expansion.

In countries where really intensive agriculture is practised, labor is cheap and a low standard of living is the rule. We do not want this situation in America. Under our condition, we must economize on labor as well as on land. With the apparently high wages paid in the industries competing with the farm, labor on the farm must not be wasted by the use of poor and inadequate equipment. Farm operations and organization must be more carefully studied so that the distribution of labor will tend toward more efficiency and greater economy in production.

Whether the farmer receives a living wage from his produce will depend on his ability as a manager as well as a worker. The average of forty cents an hour for the farmer as his wage is really a misnomer because he does not receive a wage until his expenses are paid and his products are sold. So the farmer is not receiving forty cents an hour as might be inferred from the remarks of the previous speaker.

We must also remember that the problems of the individual farmer will not be solved by greater production in the aggregate, but greater production per man and at lower cost. It is true that labor is one of the big items of farm production cost and we can well economize on this point. The farmer has many ways to economize on labor. His whole system of farming should be planned with this idea in mind. In agriculture, the rotation of crops, methods of handling, all affect the item of labor. One of the methods of harvesting submitted by our Farm Management Department is to use hogs. Another is to use sheep,—in other words, no equipment at all,—practically eliminating the entire item of labor and producing a finished product without the use of equipment as far as the harvesting of the crop is concerned.

It remains true, however, in production that ordinarily when we economize on labor we must spend more for equipment. We recognize that the farmer's equipment must be adequate for the particular problems on the farm. I am emphasizing this because I believe the average engineer, or the man who goes out on rural electrification development work, must have a true appreciation and a true understanding of the farmer's problems, for unless he does have this true understanding he is likely to make mistakes concerning the application and the substituting of power in farming operations.

As I said before, the farmer's equipment must be adequate. But what was adequate twenty years ago might not be now. At the present time the cost of equipment on the farm is quite an item. If we study any problem in agriculture in the aggregate, however, it will assume large proportions. For example the valuation of farm buildings in Illinois is around $750,000,000. On the basis of depreciation, interest on investment and upkeep at 10 per cent, the farm buildings in Illinois are costing each year $75,000,000. The cost of electrification from the standpoint of the farmer, getting his wiring done, lines connected and equipment installed might represent a little more than his building cost for a two-year period. Fifty dollars a year, if set aside on each farm, would electrify all of our farms in less than twenty years. This amount is about one-third of what it costs per year to own a low-priced car.

However, we cannot expect all our farms to be electrified. I think it is a wrong idea to believe that every section is going to be influenced the same way. The man who farms the poor land and has a small income cannot have the same type of equipment and the same buildings as the man who farms the most fertile lands.

There are situations where there is so much labor in a particular farm family that even if it costs only a few cents to operate a motor, it will be next to impossible to have it adopted.

From an agricultural standpoint, the individual farmer and the individual farm must be considered. There are a number of factors that would affect the amount and kind of equipment that can be economically used on a farm; but I will not discuss this phase of the subject. The present tendency is toward larger and fewer farms, better equipped. As you may know, there are fewer and larger farms now than ten years ago. On the larger farms the tendency is to displace labor with equipment, and this will continue as long as equipment costs less than labor. When one farmer drops out and sells his land to his neighbor, that neighbor expands and uses different methods.

There are agricultural as well as economic problems involved. In attempting to electrify farms the principle should be recognized that while the type of farming may be changed successfully by a few individuals on applying electricity, this practise cannot be generally followed. In other words, because electricity plays a large part in poultry and dairy farming, we cannot expect every farmer who has his farm electrified to buy cows and chickens and build new barns and poultry houses. We must electrify grain farms and cotton farms as well as dairy farms.

Then there is the big problem of education. The fourth factor in production which I mentioned, managerial ability, will be greatly influenced by education. The trained farmer is interested in the largest possible net income, but with all factors of production considered. His labor income should not be produced at the expense of his land.

One particular point that I wish to mention is the question of seeing that the buildings are properly wired and that they have the proper number of outlets. If the wiring is left to the farmer and he hires a contractor, it is likely that a lot of needed outlets will be left out. My suggestion would be to have the rural service man make a definite plan and get this plan approved by the farmer when he is interested and knows just what he expects to install and then have the plan carried out by a reliable contractor. I believe every man from the utility company who comes in contact with the farmer should be a salesman, not necessarily a salesman of merchandise but one who can sell the idea of electric service. Regarding rates, I believe that we should have a rate that will encourage the use of electricity, and to do that you don't want to put into the rate schedule something that is going to penalize the farmer who is going to use this service. We should have a simple rate that is easily understood.

The question of financing the line has been discussed. The possibility of having the company finance the line and eliminate all refunds and special records, and the possibility of eliminating misunderstanding and suspicion on the part of the farmer as to whether he is getting back all that he should are matters that should be considered.

Mr. Post's schedule of rates may be adequate, but I really doubt it. I wonder if it is taking care of the situation. It seems to me that any rate schedule that makes it necessary to charge a man when he installs additional outlets, penalizes him and makes it harder to build up a load. I don't believe that the farmer wants to have, as would be indicated by this particular paper, one rate when service is started and a different rate later. I think the farmer should be sold on the basis of what it is going to cost him. Farmers are not looking for charity or any special privileges. To avoid misunderstandings and dissatisfaction, the rate should be simple and easily understood; this is quite important.

It does not necessarily hold true that rural electric service is going to cost the farmer either more or less than the urban customer; it might be more or it might be less. A farmer might be classed as a manufacturer, and there would be no question so far as the people in town criticizing are concerned because of the fact that he was getting a lower rate. The farmer expects to pay for what he gets whether used in his home or in his production work.

F. W. Duffee: I wish to speak of the matter of rate, because I believe that the matter of rate is one of the most important things to be taken up and settled first, as we believe that the rate can either kill or make the proposition.

There are just a few things we might mention as having been discovered about electric service and rates. The first is that the farmers all want electric service and they want it very much. They want light probably more than anything else, and very frequently that is about all they think of until they find out through education and experience the things they can do with electricity.

The next thing they want is a low monthly bill, $1, $2 or $3 would suit them fine, but less than that would suit them still better. When you start talking about $5 or $6 as a minimum monthly bill it hurts and it hurts bad for a while until they are shown that they can actually get out of it $5, $6, $7 or $8 worth of value. That is a point we have to educate them up to, because probably the minimum monthly bill is going to be somewhere around $5 or $6.

Then, of course, we must show them the advantage of using equipment; and the rate should be such as to encourage the use of equipment, because there is no question in any one's mind, I believe, that a low rate per kw-hr. can only be secured by a large consumption and the rate should be such as to favor that to a very great extent. A great deal of effort should be directed toward educating the people to the fact that a large current consumption will bring a low rate. Some of the rates in effect now in Wisconsin are such that, with a consumption of around 200 kw-hr. per month, the rate per kw-hr. is actually less than in some of the fairly good-sized cities.

As to our line at Ripon which has been built and upon which we have been working, it might be interesting to say in connection with the point that Mr. Neff brought out regarding the amount of power which each worker uses, that on some of these farms we have from 10 to 12 h. p. of connected load around the farmstead at the present time. That includes all motors and all household equipment. I don't believe that we are beginning as yet to get the maximum results from that connected load by any manner of means. I believe we can do a great deal toward increasing the use of that equipment, thereby decreasing the amount of work which the man and woman have to do around the barn and around the farmstead, and as you reduce the time that he must spend around his barn, you can thereby increase the amount of time he can spend in his field. That is a way to make more money; it is an indirect way, but a very real way just the same.

Regarding the matter of rate, I have one very interesting example. Up near Ripon a farmer had built for himself a line. He was a rather well-to-do farmer. This line cost him about $1500. He was the only one on it. It was built when the prices were high, but nevertheless it was quite long, I think a mile. He had installed and connected an electric range, refrigerator, motor-operated milking machine, a water pump and a large number of household appliances. His total consumption had been running less than 100 kw-hr. for all of that equipment. When asked why he didn't operate it after he had spent all of his money for this line and equipment, he said it would break him to operate it because the rate was constructed in such a way that it was prohibitive to use any large quantity of current. He has gone onto a different rate recently and I haven't heard what his consumption is at the present time.

As to the particular projects, some things which we consider of the greatest importance, I shall just mention:

1. Grain elevating and handling.
2. Grinding feed.
3. Grinding bones for chickens.
4. General utility motor.
5. Hoisting hay.
6. Individual cooking units.

Some of the above applications are fairly successful now, but it seems as if there was room for development in order to obtain the maximum benefits. That will be discussed later.

The following equipment is mechanically satisfactory if one can afford the initial expense of the equipment or the high cost of operation:

1. Electric ranges.
2. Electric refrigerators.
3. Electric ironers.
4. Water heaters for household use.

Electric ranges, surprising as it may seem, have been found rather economical in certain conditions where there is no natural fuel on the farm. We found, in one case, that they could operate an electric range for just about what they would pay for coal. It has been shown by monthly readings that they must *earn* to use a range and they must use certain types of utensils with it to get the best results. For example, one farmer bought his wife a pressure cooker. The current consumption dropped off about 20 or 25 per cent immediately afterward.

The handling of grain by elevators is a thing that can be done very readily by electric power and is something that is not done generally at the present time. At threshing time it usually takes anywhere from two to four extra hands to handle the grain. That can be cut down very readily to about one hand and a small motor and little equipment.

We have found that a small feed mill operated by a 3-h. p. to 5-h. p. motor can be equipped with an overhead self-feed hopper and a bin underneath for ground feed or an elevator to take it away. Two such mills are in operation and another could be operated with self-feed, except that it makes a good job for an 11-year old boy.

During the six-months' grinding season, these mills have averaged, monthly, from 12 to 35 kw-hr., grinding from 150 to 300 lb. daily. On the average these mills increase the monthly bill of each farmer 63 cents, grinding about 3.45 tons.

The local mills charge about $2.00 to $2.50 per ton. Even after adding interest, depreciation and repair on the motor and mill, the actual cash saving will almost pay the total monthly bill for electricity during the six months of use, to say nothing of the time saved in bagging up the grain and hauling it to town and back.

In connection with this it is important to note that two of the mills are not troubleproof. In one case, a mill clogged and a belt was burned in two. The third mill is priced so high as to put it out of the reach of the average farmer. We firmly believe, however, that developments will soon be made overcoming these objections. All of the mills will produce a satisfactory product when grinding small grains, and one of them will handle corn on the cob with a little special feeding mechanism.

Hoisting hay is a comparatively short job in the year, but a good job, providing we can operate a hoist satisfactorily with a motor. From a mechanical standpoint there is a tremendous overload at the beginning of the operation in breaking the bunch of hay away from the load, and you need either some kind of gear mechanism for quickly changing from a low speed to a higher speed or a large motor to handle the load at that time.

The ordinary household appliances are, of course, just as satisfactory on the farm as anywhere else, and some more so. The washing machine, for example, is worth more to the farmer's wife than to the city man's wife, I believe, for the reason that she has a great deal of hard work and many chores to do.

Incubation and brooding is a matter to which we have given considerable attention. Last year electric incubators did not work as well as the oil incubators, probably partly due to the fact that the operators were unfamiliar with the electric incubators. This year results show that they now compare favorably. In the meantime we have done some work in trying to perfect some of the electric controls because it has seemed that they were not sufficiently sensitive.

The problem of operating a milking machine is one to which electricity adapts itself most admirably. Electrical milkers are being used more all the time.

In this connection it is interesting to note that just recently the tariff has been raised on butter from eight to twelve cents, to permit farmers in this country to compete with the Danish farmer and the New Zealand farmer. The New Zealand farmer does not put butter on the market at a low price altogether because of the climatic conditions. It is true they have pasture practically all the year round, but at the same time they do use all the latest and best labor-saving equipment. I should like to repeat a remark that was made to me recently by an engineer. He said he thought there wasn't a milking machine on the American market that was the equal of the average milking machine sold in New Zealand.

Another job that has proved very successful is water heating for the dairy. That has been a difficult problem. The electric water heater solved it very, very successfully and at a moderate cost, the average being 57 kw-hr. per month in one case. By turning on the switch when you start milking, the water will be hot by the time you get around to washing your utensils.

Water pumping is another job which can be very successfully handled by electricity. There are a few features about that which we have found are not taken care of in the ordinary system. The current will occasionally be disconnected and the stock absolutely must be watered. We must have some kind of standby power to be able to pump water while the current is off or else we must have storage. Most of the automatic systems are so arranged with the electric motor built into the system that it is impossible to operate them by hand or in any other way. This means you either have to have a big storage system, another well, or two pumps in one well so that you can operate one by hand or by some other means when the current is off. This is a rather important problem.

E. A. Stewart: The papers by Mr. Post and Mr. Neff are evidences of the fact that rural electrification is soon to be put on a sound scientific basis. The methods used in the past, whereby each public utility company blindly and arbitrarily developed some haphazard method of carrying out rural extensions, are now being superseded by orderly methods. The results secured by experimental work, such as we are doing at Red Wing, Minnesota, and in nearly a score of other states, will be of no avail to our farmers unless the electric utilities adopt some such plans as are proposed in these two papers. The steps taken by these two companies in adopting a uniform method of developing rural service in each of their territories is a wonderful step in the right direction. This movement should be enlarged so as to continue throughout the state and even across the state boundaries. I mean that these companies and other companies operating in this state should get together and agree upon certain fundamental factors so that the rural service will be put on a uniform basis throughout the state and eventually throughout all contiguous, comparable territory.

You will note that the methods of financing rural extensions and the fundamental methods of rate making as proposed by these two executives are radically different. One proposes that the company finance the major cost of the line, the other proposes the farmers finance the major part of the costs. Some companies propose to finance the entire cost of high line, transformers, and secondaries, while others propose to have the farmers finance the entire cost. Urban utility business is now financed largely on the same basis throughout all contiguous territory. It is essential that farm extensions be treated in a similar way.

On account of the difference in rate structures proposed by these two companies, at the fringes of their territories where the two services meet there will be a misunderstanding of these rates by the farmers and dissatisfaction produced. One proposes a rate based on a fixed or service charge, which we are inclined to call a delivery charge, and a low energy rate. The other proposes a rate using a minimum charge based upon energy consumption and type of installation. The former proposal is like the rate that was put into effect at Red Wing, Minn., in 1923, after a rate study extending over four years had been made. So far as I know, this was the first place in the U. S. A. to use this type of rate for farm service. This type of rate is being used now by companies in at least eight different states. Our Red Wing rate, based on three customers per mile of line with all 3-kv-a. transformer installations, is a delivery charge of $6.90 per month per customer, and the first 30 kw-hr. at 5 cents per kw-hr., and all excess at 3 cents per kw-hr. It may be necessary to modify this rate for consumptions beyond 500 kw-hr. per month, and for water-heating loads. There are about ten different types of rates in use throughout this middle-west territory. These rates must be unified and coordinated to allay distrust and to make some of them more equitable and applicable.

A movement has been started in Minnesota to unify methods of operating rural extensions. A committee is working on coordination of methods in making rural extensions, particularly as regards financing, rate structure, organization, line construction and utilization in rural service. We believe that it is essential for all states to agree on some comprehensive, workable plan that will unify methods of farm electric service.

The rate structure proposed by Mr. Post has some features which may be disadvantageous and complicate the explanations that must be given to the farmer. It makes no difference to a power company whether a man has ten rooms or three rooms, if his maximum demand at time of system demand is the same in both cases. Is not the installed transformer capacity necessary to carry the load a more probable indicator of demand? The active-room basis may apply to farm service as it has been, but not as it should be, and, may I predict, as it will be. Minimum-bill type of rates has influenced thousands of farmers to keep energy consumptions down to minimum-bill size by the psychological effect. High energy charges on the minimum amounts is bad propaganda and customers forget the cheap energy on the excess rate.

Methods of financing the farm lines are important. If the farmer is to make his electric service earn money and if he is to be a satisfied customer with such high basic costs as are necessary for farm service, he must utilize electricity for many uses. To use electricity in many operations requires adequate wiring and considerable equipment. Wiring and equipment will cost from $500 to $1500. The farmer needs his money and credit to carry out this program. It is obvious that the power company should carry the investment in the high line, and even services to the farm house, as is done in urban service.

In reference to disparity of average kilowatt-hour charges on farms and in adjacent towns and cities, I cannot see how they can be the same and be equitable. Are costs for taxes, foodstuffs, fuels, and transportation the same? Obviously they cannot be. The average kilowatt-hour rate on the Red Wing farms is now below six cents per kw-hr. Some farmers have as high as nine cents, and why should they be the same or the same as in urban service.

I want to commend the suggestion that the rural salesman sell service. Also I want to say "More power to you," when you suggest that farm service must be as reliable as city service. One hundred and eighty-five acetylene lighting plants were recently installed in a district having rural service. Many electric installations were removed. The farmers were right. The electric service was execrable. Electric incubators can be bought for a song in this territory. By its type of service this company has blasted the possibilities of electric service to hundreds of farmers.

I wish to commend Mr. Neff's suggestions on the formation of a department of rural service; also his suggestion as to the desirability of a proper type of rate. The manner of converting present customers over to the new rate structure must be carefully worked out. Some plan will be proposed for Minnesota very shortly. It may involve refunds to purchase equipment.

I am glad to hear the suggestion in regard to a rural service man. The farmer should have service. Many companies that are now selling farm equipment are already putting in service departments. Two large nation-wide organizations have started such service as suggested by Mr. Neff during the past year. The electric utility companies must adopt some such plan.

I wish to close this paper with a plea for unified action, not only within the state, but in adjoining states on rural service programs.

K. A. Pauly: I was particularly impressed with the convincing demonstration by Mr. Neff that the farmer could never hope to be prosperous so long as his production is so largely dependent upon manual labor. This is a fact which all of us, who are connected with the electrical industry, fully appreciate, but I am afraid that too few workers realize the part which machinery plays in the prosperity of the wage earner.

Mr. Neff confirms the statement which I have frequently made myself and have heard others make, to the effect that it is not an unhealthy sign to see farmers' sons leaving the farm and going to the city. This merely indicates that improved methods in agriculture have made it possible for the ever increasing needs of the cities to be met without a corresponding increase in farm labor, a distinctly healthy, rather than unhealthy, symptom.

Without electric power, it would have been impossible to place at the disposal of the industrial worker the power which he now directs and which is the secret of his prosperity and I believe that the opportunities for the application of electric power in agriculture are just as great as they were in the industrial field. The tremendous advances in industries have been due largely to new methods made possible by electric power. So we must look for modifications in agricultural methods for our greatest gains, which methods will be, as was the case with the industries, built up around the convenience and flexibility of electric service. Just as the developments in the industries have been gradual and the outgrowth of experience with electric power, so, I believe, they will be in agriculture and the new ways of doing things will come step by step.

While I heartily advocate a thorough study of all the problems involved to avoid unnecessary and expensive errors, I am not optimistic to the degree of believing that we shall ever approach the ideal without putting into practise improved methods as they appear, and confidently expecting in the future to make still further improvements. Experience thus gained may be of material assistance in hastening the further development or may even point the way to entirely different lines of study which otherwise would not have been suggested. Advance in the industries has not been haphazard. On the contrary, much study and experimentation has been at the bottom of most of the essential advances.

A. H. Ford: In connection with the question of financing farm lines I wonder how many public utility men have considered that the farmer turns his capital very much slower than does the public utility? As the result of this, the public utility should finance the farm lines rather than leave it to the farmer; because it is too hard for the farmer to get the necessary capital.

Practically all the discussion concerning electric service on the farm has been based on the premises that the farm is a factory and everyone has been waiting for the time to come when the farm factory can use electric service to advantage. It will now be demonstrated that this is not necessary. The farm is a home as well as a factory. If the interest and taxes on the farm home and on a city home of the same grade are computed, each at the common rate, it will be found that the difference amounts to about $100 per year. The fixed charges on a farm line will be less than this amount per customer. A farm with electric service can have the town advantages of water supply, electric light, refrigeration and electric cooking at a cost not greater than the same services would cost in town. It seems unnecessary therefore to wait until economical methods are developed for the use of electric service on the farm. Electric service can be sold to him at once for use in his home and at a price which he can afford to pay rather than move to town in order to get the use of the town utilities.

A couple of speakers have mentioned selling kilowatt-hours. I wish that we could forget that expression. We are not selling kilowatt-hours but electric power service. However, those who are engaged in the electric power business have thought in terms of kilowatt-hours so long that they seem unable to get away from it. We are selling electric power service which involves a demand cost as well as an energy cost and if we talk about selling kilowatt-hours we are apt to forget the demand cost and the corresponding charge which should be made. We are also prone to talk kilowatt-hours to our customers who know nothing of the term. What the customer wants is service and he is not interested in kilowatt-hours. The sooner we stop talking kilowatt-hours and talk service, the better off we will be.

G. G. Post: Mr. Holcomb questions the elastic limit of the aluminum conductor mentioned in my paper. Since the meeting, the elastic limit of 1860 lb. given in the paper was checked with data furnished by the Aluminum Company of America and was found to be correct.

Mr. Holcomb made the statement that there was little difference as to who makes the investment in the line, whether it is the farmer or the utility. Of course in the long run the farmer pays for all that he gets. There is this to be borne in mind, however; where the farmer finances a portion of the line cost, he pays his money at the beginning; where the utility finances the extension work, the utility reminds the farmer every month forever thereafter that he has paid something toward the cost of the line. People who contribute toward the cost of extensions have the faculty, you know, of forgetting in time that they have paid, and it seems to me that in the long run the man who pays the smallest monthly bill is the one who is going to be permanently satisfied.

Two of the discussors stated that the rates to the small consumer under the two plans given are about the same. Consider the case of the farmer who uses perhaps 20 kw-hr. in a month. Under one rate he would pay $2.00 and under the other he would pay $6.60.

Mr. Lehman apparently is under a misapprehension concerning some of the provisions of the rate outlined in my paper. He spoke of the effect of changing the number of outlets from time to time. The number of outlets does not affect the rates. The amount that the farmer pays depends upon the active rooms which he has in all of his buildings and upon the h. p. of connected motor load in excess of 3 h. p.

There also seems to be a misapprehension concerning the meaning of the statement, "a rate schedule designed to pay full returns on investment from the very start, by using burdensome minimum charges or high rates per kw-kr., will produce the opposite result." It is not the intention to change the rate in order to increase the return; the rate remains the same. What is meant is that a line which may not be productive of adequate return in the beginning will give an adequate return later when the use of the service on the line has grown sufficiently.

I think there was also some question as to the active-room basis taking care of motors. It is true it does not take care of the motors. Motors are taken care of in another way by adding fifty cents to the minimum charge for each h. p. connected above three.

The Quality Rating of High-Tension Cable
With Impregnated-Paper Insulation

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***Synopsis.**—During the past five or six years, practically all of the American cable manufacturers have made some changes in their impregnating compound so as to produce cable having a low dielectric loss. In a number of cases, these compounds were not well chosen. During the same period, some of the operating companies have reduced the thickness of their insulation, resulting in increased dielectric stresses.*

In a number of cases, the compounds and perhaps also the paper that was used developed faults that were more serious than high dielectric loss. Some cables have been very deficient in dielectric strength and others have developed a new type of trouble which has been termed "ionization" and, after a comparatively short time, the operating voltage of such cable had to be materially reduced in order to keep the lines in service or the cable had to be replaced.

The cable manufactured during this period has generally passed, with a wide margin, the high voltage tests required by the specifications. Occasionally a lot of cable which passes these tests by a narrow margin will be shipped by a manufacturer and, in such cases, the service record of the cable is unsatisfactory.

The investigations which form the basis of this paper consist of:

(a) Laboratory tests to determine what high-voltage tests cable known to be of good quality will withstand as well as the similar tests on other grades of cable.

(b) A careful examination and dissection of samples of the cable as received from the factory and samples removed from lines which have failed in service or which have been in service for a number of years without failure.

(c) A comparison of the operating records of the cable with the results of the dissection and examination of the samples and with the factory inspection and tests made on the cable before shipment.

(d) Accelerated life tests on long samples of cable having a wide range in quality.

The results of these investigations have been correlated in a manner which permits the determination of the relative quality of several lots of cable purchased, as well as changes in the specifications which must be made in order to secure satisfactory high tension cables with impregnated-paper insulation.

INTRODUCTION

IN the earliest days of underground transmission by lead-covered cables, rubber insulation was used. A little later, it was found that unimpregnated-paper tapes could be used to advantage in place of rubber. Still later, it was found that the paper could be greatly improved by impregnating it with various compounds, and this type of insulation came into general use. After a few years' experience, a solution of rosin in rosin oil was thought to be most suitable for impregnating compound, and for a number of years this type of compound was used by practically all the American manufacturers excepting one or two who used a solution of rosin in mineral oil instead. For the lower operating voltages which were then in use, this type of cable was found to be quite satisfactory; but when such cables were operated at voltages above 7500, it was found that the cables would not carry as much current without failure as the low tension cables of the same size. Rayner,[1] in 1912, first noted that this difficulty was due to the loss in the dielectric. In 1917, Clark and Shanklin[2] stated that the best method of comparing this particular quality of different cable was by determining the power factor of the dielectric, at the same time stating that there was only one instrument in this country suitable for making such measurements. In 1917, Bang and Louis[3] reported the results of tests on cables installed in special conduit, which indicated the limitations of cables with high dielectric loss. Farmer,[4] in 1918, gave details of laboratory methods of such measurements and outlined some of the laws of dielectric loss. In 1922, one of the authors[5] presented a method for determining the current limits of high voltage cables from the laboratory measurements of dielectric loss.

With the load limits of such cables well determined, the operating companies, by observing the prescribed limitations, were able to operate such cables in a very satisfactory manner and with a small percentage of failures. The load limitations in some cases were very troublesome and expensive as, in the larger cities, they required a considerable expense for cables for underground transmission to substations. Upon this point being brought to the attention of the cable manufacturers, they made changes in their insulation, principally in their impregnating compound, so that, by 1923, the insulation was improved to such an extent that for cables up to 15 kv., the dielectric loss did not materially limit the carrying capacity. Even for cables up to about 27 kv., which was the maximum voltage then in use, the reduction in carrying capacity due to the dielectric loss was very small.

In the earlier days of impregnated-paper insulation, the cable manufacturers used about the same thickness of insulation that they had previously used with rubber insulation. When troubles developed in service or on test, it was customary to add a little more insulation rather than to attempt to locate and remove the cause of the trouble. The result was that in a few years the cables were generally furnished with heavier insulation than the tests on the cables indicated as necessary. In recent years some of the larger operating companies purchasing a large amount of cable each year have made some reductions in the thicknesses of the insulation,

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*Both of the Street Department, Commonwealth Edison Company, Chicago, Ill.

1. See Bibliography for references.

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for the purpose of reducing the cost. In other cases, reductions were found expedient in order to increase the amount of copper and, therefore, the amount of power which might be carried over one cable with a given over-all diameter. In other cases, the operating voltage was increased without a corresponding increase in the thickness of insulation. These changes resulted in an increase in the dielectric stresses.

The change in the type of impregnating compound and the increase in the dielectric stresses developed a large amount of operating troubles on the newer cables. In 1924, Lamb[6] stated that such troubles with high-voltage cables in England and in other countries had made it necessary to replace these cables in some cases and temporarily reduce the voltage in others.

Up to the time that troubles of this kind developed, it had been customary in this country for the cable manufacturers to supply cable to the larger operating companies under a five-year guarantee. The expense to the manufacturers on account of this guarantee was very slight, as most of the claims were due to defective workmanship in the factory or mechanical damage during shipment. The claims for dielectric loss failures were practically eliminated by the A. I. E. E. rule for maximum operating temperature of 85 deg. cent. minus E, where E is rated voltage in kv. This rule, according to experience previous to that time, served to determine the maximum temperature below which failures from this cause would not occur. About two years ago the American manufacturers reduced their five-year guarantee to a maximum of two years.

As a matter of fact, the principal safeguard of the purchaser rested in the five-year guarantee, as, for cables above 7.5 kv. normal operating voltage, the factory tests were quite perfunctory; they served merely to eliminate sections with defective workmanship, but gave no clue to the quality of the insulation unless it was very bad indeed.

The situation today is not greatly different. High voltage cable which will meet the requirements of all specifications which it has been possible to obtain and which are published in this or any European country, may prove an utter failure within less than one year of operation at its rated voltage. Troubles of this character have not been confined to any one country, to any one manufacturer or to any one operating company; several cases of such troubles have occurred within the year immediately preceding the presentation of this paper. High-voltage cables that have proved successful in service will pass all requirements of the most recent specifications by margins of the order of 30 to 100 per cent.

As some of the troubles may require time for development ranging from a few months to a few years, it is entirely possible for the guarantee period to expire before the trouble has become apparent, and then the operating companies are confronted with the proposition of replacing the cable at their own expense or operating it at a greatly reduced voltage. Therefore, a reduction in the guarantee period threw upon the operating companies, with the aid of the manufacturers, the responsibility of devising tests, to be made upon the cable at the factory before shipment, which will enable them to determine whether the cable will prove satisfactory under their operating conditions.

Some of the manufacturers stated that the troubles were due to lack of thorough impregnation and suggested an ionization test for the purpose of determining which cables were satisfactory. A clause prescribing such a test has been used in recent specifications. A further attempt was made to improve the specifications by making moderate increases in the test voltages, but in spite of these slight changes in the specifications, the troubles continue.

This paper describes the methods which have been used in correlating operating experience on about 4,000,000 ft. of high-voltage, three-conductor cable purchased in the past six years from ten manufacturers, with the specification requirements and test results so as to determine what changes and additions should be made in the present requirements.

The object of the investigation upon which this paper is based was to determine just what properties of the insulation were responsible for the failures of the cable in service and to indicate the changes that should be made in the cable specifications in order to materially reduce the number of service failures.

OUTLINE OF METHODS

With such a complex material as impregnated-paper insulation, having a number of widely different and independent qualities or characteristics, it is not possible to compare the products of different manufacturers or different lots from the same manufacturer by any single measurement. The only feasible method of making a reasonably accurate comparison is to adopt the scheme which for many years has been used by engineers with other types of material; that is, to assign weights to the various qualities in the light of past experience and determine from test or inspection data the percentage of the maximum weight for each item which should be allotted to each manufacturer or lot of material.

Quality rating tables have been used by the authors for a number of years for the purpose of comparing the quality of cable indicated by the proposals of the several manufacturers submitted in accordance with specifications. Later, as data accumulated from the results of tests, inspection, and operation, other quality rating tables were made to compare the cable actually shipped. At first these ratings were made purely relative, by selecting the best performance for each of the items compared, and rating the product of the other manufacturers by simple arithmetical calculations. Such a rating table for 13-kv. cables was given by one of the discussors[7] of the paper by E. S. Lee on "Testing High

Tension Impregnated-Paper Insulated, Lead-Covered Cables," presented at the A. I. E. E. Convention in February, 1925. While this method gave a very good idea of the relative standing of the manufacturers at one time, it did not permit a determination of the amount of improvement which the manufacturers make in the quality of their insulation from year to year. To secure this result, the basis was later modified so as to make all of the comparisons on a fixed basis and this meant the adoption of a standard for each of the items in the specifications. Table I gives such a quality rating for 13-kv. cables, and Table II for 35-kv. cables.

TABLE I

QUALITY RATING OF 500,000-CIR. MIL, THREE-CONDUCTOR, 13,000-VOLT CABLE

Item No.		Base Weight	Limits between which values were proportioned		Values for Various Manufacturers				
			Zero Weight	Full Weight					
	Manufacturer Year Made				*A* 1925	*B* 1925	*C* 1925	B^1 1922	*D* 1925-6
	A—Results of dissections and visual examinations								
1.	Workmanship on copper	4	Serious imperfections	No imperfections	2.0	3.0	2.5	3.0	4.0
2.	Workmanship on lead	2	Serious imperfections	No imperfections	2.0	2.0	2.0	2.0	2.0
3.	Workmanship on insulation and fillers	8	Serious imperfections	No imperfections	7.5	6.5	6.0	6.0	5.0
4.	Thoroughness of imprgenation	8	Poor throughout	Perfect throughout	7.0	7.0	4.0	1.0	7.0
5.	Tearing in bending tests	4	2 adjacent tapes at one point; 3 torn tapes per foot of cable	No tears	3.6	3.9	4.0	2.0	1.0
	B—Results of factory and laboratory tests on insulation								
6.	Ratio of max. to min. insulation resistance—for lots —for sections	6	$\begin{Bmatrix}3.0\\1.5\end{Bmatrix}$	$\begin{Bmatrix}1.3\\1.075\end{Bmatrix}$	4.6	4.3	− 1.0	− 8.4	4.4
7.	Dielectric loss at 80 deg. cent, watts per foot	4	0.75*	0.075*	3.6	2.4	3.2	2.2	1.3
8.	Increase in power factor at room temperature from 20 to 100 volts per mil of insulation	8	0.02	0.00	3.2	0.6	0.2	0.0	2.2
9.	Puncture voltage on straight samples, kv	17	74.5	119.0	8.2	9.0	4.0	− 1.7	15.9
10.	Puncture voltage on cold bent samples, kv	18	65.0	104.0	21.2	22.6	8.3	− 8.8	26.3
11.	Ratio of item 10 to item 9	8	0.75	1.0	8.0	8.0	7.4	− 2.9	8.0
	C—General								
12.	Uniformity of insulation	13	Serious variations in quality of insulation	Reasonably uniform quality	11.0	11.0	6.0	7.0	11.0
	Total, Examination	26			22.1	22.4	18.5	14.0	19.0
	Total, Tests	61			48.8	46.9	22.1	−19.6	58.1
	Total, General	13			11.0	11.0	6.0	7.0	11.0
	Grand Total	100			81.9	80.3	46.6	1.4	88.1

*Corresponding power factors of dielectric are 10 per cent and 1 per cent at 60 cycles.

EXPLANATION

Items 1, 2, 3, 4 and 12, in the absence of definite limitations in the specifications, are based on arbitrary standards established for the guidance of the inspectors. The standards for zero weights would generally have caused the rejection of the cable had they been noted at the factory.

The figures for zero weight for Items 5, 6, 8, 9, and 10 are the requirements of the latest specifications under which the cable was purchased. Full weight is given for Items 6, 8, 9 and 10 for values which would give a large factor of assurance for these qualities. All of these values appear possible of attainment in the present state of the art.

Zero weight and full weight for Items 7 and 11 are about the extreme limits of current American practise. The reasons for Item 11 are given at some length in the paper.

In Item 6 the variation in insulation resistance is based on the largest ratios found for the insulation resistances in the lots of cable as submitted for inspection and tests by the manufacturer and also on the largest ratio of the insulation resistances of the several conductors of any given section in each lot. The weight for this item is divided equally between the values for the lots of cable and for the individual sections. The latter part would disappear for single-conductor cable.

The values for Items 1, 2, 3 and 4 were determined by the inspection at the factory plus information obtained during and after installation. The values for Items 6 to 10, inclusive, were based on the averages of the poorest 20 per cent of the test results. In Items 1, 2, 3, 4, 5, 8 and 12 the values given to each manufacturer were proportioned between zero and base weight and no negative values or bonuses were given on these items. In Items 6, 9 and 10 the values were proportioned arithmetically between zero and full weight and negative values were given if the results were poorer than the values for zero weight, and extra weight was given when the results were better than required for full weight. In Items 7 and 11, negative values were given if the results were poorer than required for zero weight, and no extra value was given if the test results were better than required for full weight.

All values are based on tests and inspection of cable received and accepted by the purchaser.

Note: At least 9 mi. of cable are represented in each of the above ratings.

During this period, it was the practise to make a careful examination of the cable immediately adjacent to each fault and also at the manhole ends of the same section of cable. A detailed study of the results of these inspections as the data accumulated and a careful comparison with the test and inspection data permitted the primary cause of the troubles to be determined. A reference to the corresponding test and inspection data then furnished a basis for determining the base weights for each item, a part of the limits between which the values were proportioned, and the values assigned to the several manufacturers for the various items in the tables.

TABLE II

QUALITY RATING OF 350,000-CIR. MIL, THREE-CONDUCTOR, 35,000-VOLT CABLES

Item No.		Base Weight	Limits between which values were proportioned		Values for Various Manufacturers					
			Zero Weight	Full Weight						
	Manufacturer Year made				*M* 1925	*N* 1924	*O* 1924	*P* 1924	*Q* 1924	M^1 1923
	A—Results of dissections and visual examinations									
1.	Workmanship on copper	3	Serious imperfections	No imperfections	2.7	3.0	2.3	2.6	2.5	2.0
2.	Workmanship on lead	2	Serious imperfections	No imperfections	2.0	2.0	2.0	2.0	2.0	2.0
3.	Workmanship on insulation and fillers	8	Serious imperfections	No imperfections	7.0	7.5	3.5	3.0	5.0	1.5
4.	Thoroughness of impregnation	8	Poor throughout	Perfect throughout	7.0	7.0	7.0	5.0	5.0	3.0
5.	Tearing in bending tests	3	2 adjacent tapes at one point; 3 torn tapes per foot of cable	No tears	2.9	0.5	0.5	2.0	3.0	2.5
	B—Results of factory and laboratory tests on insulation									
6.	Ratio of max. to min. insulation resistance—for lots —for sections	6	3.0 1.5	1.30 1.075	6.0	4.8	5.9	5.3	0.3	−4.8
7.	Dielectric loss at 80 deg. cent.-watts per foot	7	2.1*	0.35*	3.7	2.0	0.0	6.0	6.2	0.1
8.	Increase in power factor at room temperature from voltages of 20 to 100 volts per mil of insulation	8	0.02	0.00	3.3	0.4	0.0	0.4	0.0	0.0
9.	Puncture voltage on straight samples, kv	17	160	256	17.7	16.3	7.1	2.3	−1.8	−1.8
10.	Puncture voltage on cold bent samples, kv	17	140	224	20.2	12.1	12.1	10.1	−2.0	−3.4
11.	Ratio of item 10 to item 9	8	0.75	1.0	5.6	1.4	8.0	8.0	3.8	2.2
	C—General									
12.	Uniformity of insulation	13	Serious variations in quality of insulation	Reasonably uniform quality	11.0	12.0	12.0	6.0	10.0	5.0
	Total, Examination	24			21.6	20.0	15.3	14.6	17.5	11.0
	Total, Tests	63			56.5	37.0	33.1	32.1	6.5	−7.7
	Total, General	13			11.0	12.0	12.0	6.0	10.0	5.0
	Grand Total	100			89.1	69.0	60.4	52.7	34.0	8.3

*Corresponding power factors of dielectric are 6 and 1 per cent at 60 cycles.

The same letters are used in Table II and Fig. 12.

The data were obtained from orders of several thousand feet of cable for columns *M* and *Q*; all other ratings represent several miles of cables.

The explanation under Table I also applies to Table II.

A perfect method of rating the quality of cables would accurately predict their operating performance. To be useful, such a method must be approximately accurate over a wide range of quality of insulation and be equally applicable to the widely varying types of insulation now on the market. In the past few years there have been received several lots of cable of inferior quality, and also a larger amount of cable of a quality ranging upward to excellent. The system of the Commonwealth Edison Company includes over 1300 mi. of 9-, 13-, 20-, 22- and 35-kv., three-conductor cable. The service records for all of this cable and the test and inspection data on the 800 mi., purchased from ten manufacturers during the past six years, were at hand. All of these data were drawn upon in attempts to devise a quality rating method. Each new constructive suggestion was tested by calculating a new rating table with the same original data in the endeavor to secure results which would at least rate the cables in their proper order of merit.

No great difficulty was found when the study was confined to 13-kv., three-conductor cables, all of which were insulated with manila paper and impregnated with compounds that were very much alike. But when the method was extended to 35-kv., three-conductor cables, insulated with a variety of paper ranging from 100 per cent manila to 100 per cent wood pulp and impregnated with compounds ranging from heavy grease to a medium heavy oil, serious difficulties were encountered. By carefully going over the operating records and comparing them with the test and inspection data and the results of examination of the cable near failures, the rating tables were altered and adjusted so as to bring the final results in accord with the operating experience.

While these studies were in progress, a series of tests had been undertaken to determine what high-voltage tests successful cable should withstand. After some data along this line had been obtained, it appeared perfectly feasible to devise an accelerated life test so that the approximate life of the cable would be indicated by the time that the cables withstood the test. This resulted in a second method of quality rating entirely independent of the first. A comparison was then made of the two sets of results on the assumption that if the two methods were reasonably accurate, some simple relation must exist between them.

HIGH-VOLTAGE TESTS

For many years it had been noted that the high-voltage tests which the cables actually withstood when tested at the factory were very much higher than the specification requirements. It had also been noted in a few lots of cable which had proved to be of inferior quality, the high-voltage test results were only slightly higher than the specification requirements. A series of high-voltage tests were, therefore, undertaken to determine what changes should be made in the specification requirements in order to eliminate such inferior lots of cable and, at the same time, allow the manufacturer a reasonable margin. As these high-voltage tests form a large portion of the investigations on which this paper is based, the tests will be described in detail.

During the past few years, a number of papers have been published describing attempts to determine by laboratory tests how dielectric failures occur. It had been the hope that the solution of this problem would afford great assistance not only in improving the insulation by altering the manufacturing methods or materials, but also in indicating the tests that should be applied to the insulation to determine its quality. Although it has not been possible to determine how dielectric failures occur, it still appeared possible to make some considerable improvements in the quality of cable installed, by approaching the problem from an entirely different angle and determining the limiting conditions under which the cable would not fail.

The previous experience with high-voltage tests on short samples at the factory had shown a wide difference in the results of the tests on samples cut from different sections of the same lot of cable. In making the high-voltage tests, it was desirable to determine, if possible, what length of sample should be used so that the sample would indicate with reasonable accuracy the quality of the lot of cable which it represented.

When the tests were started, the sections of cable used were about 200 ft. long for 13-kv., and 100 ft. long for 35-kv., three-conductor cable. These sections were as long as could be conveniently handled in the testing room and, further, the length of the 35-kv. cable was also limited by the testing transformers.

The cable was supported on small wooden blocks on the testing room floor and arranged in one or two long loops with large radius bends so as to avoid injuring the insulation. The lead sheath was very thoroughly and carefully grounded, so that the test operators could freely handle the cable with their bare hands and determine the location of hot spots. Previous to applying the voltage, thermometers were fastened to the lead sheath at intervals of about 25 ft. Additional thermometers were suspended in the vicinity to record room temperature. The power was supplied by a three-phase, 20-kw., 60-cycle rotary converter through three transformers with delta-connected primary and Y-connected secondary, having the neutral point grounded.

The initial pressure applied to the cables was approximately rated voltage. The pressure was then raised at a rate of about one kilovolt per second for 13-kv. cable, and 3 kv. per second for 35-kv. cable until the desired test pressure was reached. During the test, and with the voltage on the cable, the lead sheath was carefully examined by the test operators every five or ten min. during the first hour of the test in order to detect points where the sheath temperature was higher than the remaining portions. At each of these warmer places, which are called "hot spots," an additional thermometer was fastened to the sheath. After the first hour, the examinations were made at longer intervals.

If the failure occurred at or near the terminal bell, a new bell was made and the test continued. If the failure occurred in the central portion of the length of cable, a section about 5 ft. long, including the fault, was removed and the remaining portions of the cable were connected by a joint. Pronounced hot spots were removed at the same time. The tests were continued until the portion remaining became too short to be a fair, representative sample. From three to seven tests usually were obtained on each original section of cable. Marks were made 10 ft. apart on the original length of cable and the locations of the hot spots were recorded so as to determine whether they would recur at the same location following an interruption of the test. The cable at and near the points of failure and the hot spots was carefully dissected and examined and the results recorded.

At the start and occasionally during the progress of the tests, when changes in the procedure appeared desirable, special tests were made to determine the details.

In the earlier tests, the procedure was to apply the full reel test voltage (2.5 times normal) for 60 min., followed by 15 min. at no voltage. Then the voltage was increased in steps of 20 per cent, the voltage being held constant for 15 min., followed by a rest period of the same length. Generally the tests were continued in this manner until failure occurred. After some preliminary information had been obtained, the procedure was changed so as to make the test at 3.6 times normal voltage, the voltage being maintained constant until failure occurred. While this plan was quite successful for the cables of low quality, it made the test unduly long when testing sections of high-grade 13-kv. cable. To reduce the expense of such tests, the voltage was increased 20 per cent after eight hours and held at the new voltage for a second eight-hour period, after which a second 20-per cent increase was made. A study of these test data indicated that one hour at 3.6 times normal voltage would, very roughly, for low grade cable, correspond to about one year in service at the rated voltage, and this test voltage was, therefore, adopted for the accelerated life tests. Later, in order to obtain a voltage time curve that would show the time that the cables withstand various voltages, tests were made on samples at a number of higher voltages, maintained constant until failure occurred.

About 8000 ft. of cable have been used in making about 250 such tests. The cable was about equally divided between new and second-hand cable. In selecting the sections of cable for these tests, samples were obtained from the cable that had been shipped by the several manufacturers, and in this manner information was secured concerning the quality of each lot or type of cable shipped. In the same way, the sections of second-hand cable were selected so as to obtain similar data for cables of a wide range of quality whose service records were definitely known. The sections of cable used for this purpose were removed on account of failure, external damage, or changes in the transmission system.

Discussion of Quality Ratings

A. WORKMANSHIP

The first five items in Tables I and II can be considered under the general heading of workmanship. The information for calculating the values for these items was secured from the factory inspection reports and by examination of samples of cable taken from the ends of sections as received or from the middle of the sections whenever possible. Further information was obtained from time to time by the examination of samples of cable taken from sections removed on account of failures or other causes. Some of the defects of workmanship that have been noted upon examination of such samples are as follows:

Defective Conductors. Sharp corners made by the die in shaping the sector conductors, buckled and protruding strands, defective brazing of strands, excessive twisting of sector conductors, omitted strands.

Defective Lead. Spots of dross, laminated lead, longitudinal and transverse splits, holes in lead.

Defective Insulation. Loose, misplaced or knotted fillers, creases in individual tapes, wrinkles in tapes extending through many layers, registered tapes, omitted tapes, tapes folded and mishandled, damage to conductor insulation during the cabling of the three conductors.

Defective Impregnation. Insufficient or unsuitable impregnating compound, as shown by occasional air pockets or waxy flakes or black spots or tree designs distributed through the insulation; a marked difference in the impregnation of the insulation of several conductors at one location; a marked difference in the impregnation of the samples from different portions of one section or of one lot of cable or of different lots of cable from the same factory.

The values assigned to each of the several manufacturers for the first four items in the rating tables were determined by the men who had charge of the examinations of the sections of cable as above described, and the values were assigned on the basis of the influence of similar defects found at or near the points of failures which had occurred on test or in service. The values assigned for Item 5 were determined from the results of factory inspection as later described.

Impregnation. In order that the impregnation of paper-insulated cable may be satisfactory, it is necessary that:

1. The impregnating compound be suitable for the purpose, that is, it must be stable under the electrical stresses at all temperatures that will be experienced in normal operation.
2. The impregnating process be performed so as to completely remove the air and moisture and thoroughly impregnate the insulation, and eliminate voids.

Several tests have been evolved by the cable manufacturers and the Electrical Testing Laboratories to determine whether the compound will be stable, but such a test was not included in the specifications and tests on the cable covered by this paper. Therefore, this characteristic is not considered in the tables, and Item 4 refers only to the completeness of the impregnation.

The men who examined the samples for Item 4 determined the variation in the impregnation by visual inspection. They did this with considerable accuracy as they correlated their experience gained from similar examinations on samples of cable, taken from sections on which laboratory tests had been made for ionization and chemical tests to determine the thoroughness of impregnation.

Several cable manufacturers were furnished with copies of the earlier rating tables with a request for their comments, and they contributed more discussion on this item of impregnation than on all of the other items combined. Apparently all of this discussion was entirely warranted by the importance of the subject. No satisfactory explanation has been obtained for the marked differences in the impregnation on the three conductors at one place in the cable nor for the large differences in impregnation throughout the length of one section.

This difference in the impregnation of the three conductors in a section is shown by measurements of the insulation resistance and power factor. When the insulation is examined near a point where a failure due to ionization has occurred, a pronounced difference in the impregnation is frequently found as, for example, the insulation of one conductor will be filled with the evidences of ionization, while the insulation of the other two conductors will be entirely free from such evidence. In other cases, many evidences of ionization will be found in the insulation of all three conductors near the point of failure, but will be entirely lacking in the manhole ends of the same section of cable.

Tearing of Paper Tapes in Bending Tests. The routine tests on the cable at the factory include a bending test on an occasional short sample that has been cooled to the minimum operating temperature which, for Chicago, is about minus 5 or 10 deg. cent. This is followed by a puncture voltage test and later by an examination of the paper tapes in a three-ft. section of the cable, cut from the middle of the bent portion. The number of edge tears and longitudinal tears of the paper tapes are noted and recorded. In the table, the number of torn tapes and the number of tears are considered, no tearing receiving full weight.

If the tapes are not too wide, longitudinal tears occur only when the tapes are applied with a lap, that is, when each turn of tape overlaps a portion of the preceding turn. Edge tears will not occur if the paper tapes are applied perfectly flat without lap and if the impregnating compound does not become too viscous or stiff at the low temperature.

The tearing of the paper tapes in the bending test has been the subject of many extended and animated discussions with the cable manufacturers for a number of years, but the improvements in the machines for applying the paper tapes within the past few years have about eliminated this prolific cause of controversy. The leading manufacturers are now furnishing cable which gives a practically perfect performance in this test. Incidentally, some of the manufacturers state that the new taping machines not only make better cable, but the cost of the cable is reduced, as the machines are run continuously until the rolls of tape are exhausted instead of being frequently stopped to repair broken tapes.

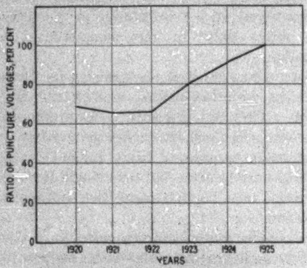

Fig. 1—Change with Time of Ratio

Puncture voltage on cold bent sample
Puncture voltage on straight sample

The evidences of defective workmanship were found upon examination of some of the earliest failures due to the recent troubles, but it did not appear, at that time, that the defects found were sufficient to cause the failures. Apparently this view was correct, as later, other failures occurred where there was no such evidence of poor workmanship, indicating that the quality of the insulation was poor. This experience indicates that if the insulation has a very small factor of safety, the first failures will probably occur at points where the dielectric strength of the insulation has been reduced by mechanical defects.

During the past few years, the leading American manufacturers have made some marked improvements in the workmanship of the insulation as shown by the paper tapes being applied more smoothly and evenly, the number of torn tapes in the bending test reduced, and the impregnation improved. The extent to which these improvements in workmanship have improved the quality of the insulation is indicated by the reduction in the effect of the bending test on the dielectric strength of the insulation. The extent of this improvement is shown in Fig. 1 which shows the ratio of the puncture voltage, obtained on the samples that have been subjected to the cold bending test, to the puncture voltage obtained on the straight samples. During the past few years this ratio has risen from about 65 per cent to 100 per cent. It therefore appears that this ratio can be used as an excellent test for workmanship.

B. Tests on Insulation

Insulation Resistance. The insulation resistance is one of the most sensitive tests that are made on each section of cable at the factory. At the present time, the greatest use for the results of this test is as an indication of the uniformity of the manufacturing processes. In Fig. 2 is shown the variation in insulation resistance of 125 consecutive reels as submitted by two manufacturers. Figs. 3 and 4 show the variation of the power factor of the dielectric with the insulation resistance as obtained from 30 consecutive reels submitted by two manufacturers. Fig. 5 shows similar data from two different lots of cable submitted, a few months apart, by another manufacturer; from such data and from other tests showing that no change in the paper was made during the same period, it is evident that the cable manufacturer changed the impregnating compound, a conclusion which has been verified by the manufacturer.

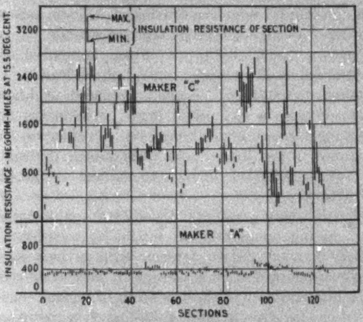

Fig. 2—Insulation Resistance vs. Order of Presentation of Sections

500,000-cir. mil, three-conductor, 13-kv. cable made in 1925
Note: The letters used designate the same manufacturer throughout the paper except for Table II and Fig. 13

Dielectric Loss at 80 Deg. Cent. For uniformity, the dielectric loss at 80 deg. cent. is used for comparison, although for 35-kv. cable it is realized that this is above the standard maximum operating temperature. Operating conditions arise infrequently, however, which render it expedient to carry an abnormal load on a cable for a short time rather than open the line and cause interruption of service. The dielectric loss tests at 80 deg. cent. permit the heating of the cable under such abnormal conditions to be predetermined.

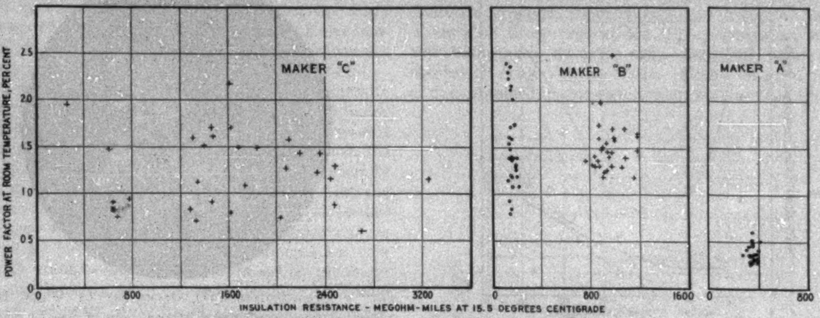

Fig. 3 Fig. 4 Fig. 5

Figs. 3-4-5—Power Factor vs. Insulation Resistance

For 500,000-cir. mil, three-conductor, 13-kv. cable, made in 1925 by three manufacturers.

Cables have 18/64 in. insulation between conductors and 14/64 in. insulation between each conductor and sheath

Power factor of dielectric is value obtained at room temperature on each section of cable at 100 volts per mil of insulation thickness. Insulation resistance is average for value for three-conductors obtained by the standard method with direct current

Data are shown for 30 consecutive sections of cable from makers *C* and *A* and for two lots of 30 consecutive sections, each presented several months apart, by maker B.

The manufacturers are now furnishing cable for voltages up to about 20 kv. with insulation with such low dielectric loss that this loss does not reduce the carrying capacity of the cable more than one or two per cent. For practical purposes, this can be considered inappreciable. A general idea of the effect of the power factor of the insulation on the permissible copper losses in the cable (and the sheath losses also in the case of single-conductor cable) is indicated in Table III. From the table it follows directly that, for commercial consideration, as the operating voltage increases, the maximum allowable power factor of the dielectric should decrease. Experience has shown that a dielectric loss of 1.75 watts per ft. of 35-kv., three-conductor cable does not result in service failures due to cumulative heating. The recent product of some of the leading manufacturers has shown losses materially below this figure, thus allowing an increased load to be carried on the cable without exceeding the maximum permissible operating temperature.

TABLE III

RELATION OF POWER FACTOR OF INSULATION TO OPERATING VOLTAGE

Cable*	Power factor of dielectric in %	Losses at max. operating temperature — watts per foot			Ratio
		Dielectric A	Total per cable B	Permissible copper losses† $B-A$	$\frac{A}{B}$
500,000-cir. mil 3-cond, 13 kv.	5	0.37	11	10.63	0.034
350,000-cir. mil, 3-cond, 35 kv.	5	1.75	8	6.25	0.22
600,000-cir. mil, 1-cond, 132 kv.	5	8	8	0	0.00

*Sizes used by the Commonwealth Edison Co.

†Including sheath losses in single-conductor cable.

"Ionization" Test. A test made by measuring the increase in power factor over a certain specified range in voltage, say, from 20 to 100 volts per mil of insulation thickness, is called an ionization test. In the later specifications under which this cable was purchased, there was a further provision that the upper voltage should be at least 150 per cent of the rated voltage of the cable. Several recent specifications require such a test on each section of cable.

Smit Kleine, Proos and Van Staveren[8] contend that there should be no rise in the power factor for about the same change in voltage, and this opinion is privately confirmed by other European manufacturers. Another manufacturer states that according to his experience a rise of two per cent for three-conductor cables, or one per cent for single-conductor cables, insures satisfactory impregnation. Probably each statement is correct as applied to the particular combination of paper and impregnating compound in the cable from which the experience was obtained, but this change in power factor over the specified range in voltage is very different for the different types of thoroughly impregnated insulation as made by the several manufacturers. Fig. 6 shows a number of such curves taken from actual tests on samples of commercial cables made by manufacturers that are considered among the leading manufacturers in the world. Some of these cables having the greatest increase in power factor over the specified range in voltage have given the best service, while other cables, which have a very small increase in power factor, have been found unsatisfactory for service at their rated voltage. In several cases, subsequent tests have developed that the impregnating compound was unstable, and this means that a test on the impregnating compound itself is necessary, in addition to the ionization test, in order to insure satisfactory cable.

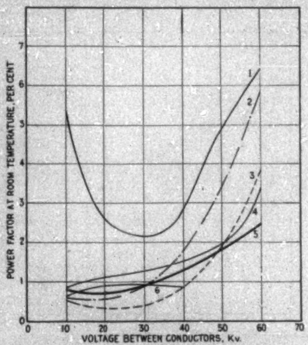

FIG. 6—POWER FACTOR VS. VOLTAGE

For 350,000-cir. mil, three-conductor, 35-kv. cables from various manufacturers

The fact still remains, however, that the experience to date shows that cables can be entirely satisfactory and still have a wide range in the increase in power factor with a given increase in voltage. The insulation in these cables also varies widely in their composition, that is, they contain varying proportions of wood pulp and manila paper and entirely different impregnating compounds. These facts indicate the necessity of considerably more information before the proper limits for the increase in power factor can be fixed for the various types of insulation, so that all types will be equally satisfactory.

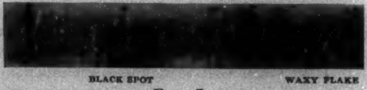

FIG. 7

Waxy flakes and black spots on center filler of three-conductor, 35-kv. cable, impregnated with a heavy grease. Cable was removed after being in service.

The several manufacturers making satisfactory cable use different methods of measuring the power factor, some tests being three-phase and others single-phase. In addition to this, the ratio of the thickness of the insulation on the conductor to the outer belt insulation varies. Apparently the ionization test in its present form cannot be considered satisfactory until we also have information regarding the factors to be used in converting the results of the single-phase tests so as to secure results strictly comparable with the results obtained from three-phase tests.

The visible evidences of high-voltage discharges or of ionization which are found upon dissecting the affected insulation, in the order in which they usually appear to develop in service are:

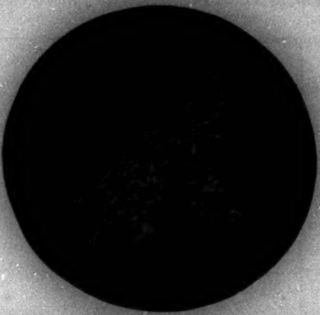

FIG. 8—MAGNIFICATION (7) DIAMETERS OF WAXY FLAKE

Removed from space between adjacent turns of tapes of conductor insulation of same cable as in Fig. 7

STRIP OF WAX
PAPER TAPE

FIG. 9—PHOTOMICROGRAPHIC CUT OF (7) DIAMETER MAGNIFICATION SHOWING STRIP OF WAX

Along edge of a tape of the conductor insulation of cable impregnated with a heavy oil

FIG. 10—TWO-DIAMETER MAGNIFICATION OF TREE DESIGN

Developed in and on tape of conductor insulation of three-conductor, 35-kv. cable, during test at 127 kv.

a. Waxy flakes and black spots in the compound
b. Black or brown spots in the paper
c. Fern-leaf or tree designs in the paper, and pin holes through the paper.

Figs. 7, 8, 9 and 10 show reproductions of photographs of these evidences.

The waxy flakes and black spots in the compound can be readily removed with the compound, but the other evidences of ionization will remain in the paper after the compound has been removed.

The waxy flakes and black spots in the compound, under a low power magnifying glass, appear similar in structure. The impregnating compound, whether a grease or an oil, apparently first turns to a jelly and, later, develops a granular structure. The appearance of various waxy flakes and black spots varies considerably as the appearance of honey turning to sugar will vary in different portions. If the impregnating compound is a grease similar to petrolatum, the waxy flakes are of a conspicuous yellow color, but if the compound is a heavy oil, the flakes are quite inconspicuous, as their color is not very different from that of the oil.

The brown or black spots in the paper generally appear in the middle or outer portion of the conductor insulation, sometimes accompanied by tiny punctures in the layers near the conductor and tree designs in the other layers. The evidences of ionization are generally found in the largest quantity where the stresses are the greatest, but their distribution throughout the insulation varies widely in different cases and appears to be affected by local variations in the impregnation. In the high voltage tests on samples of cable, the waxy flakes develop only when the tests continue for a long time, of the order of 40 hr. or longer; but the other evidences are developed by higher voltages in a much shorter time.

Dielectric-Strength Tests. High voltage tests on each section of cable have always been made on rubber insulated cables and this practise has been continued on paper-insulated cables. The test, however, was never intended or considered as a test of the quality of the insulation, but was made for the purpose of eliminating sections having serious defects in workmanship.

About six or seven years ago a dielectric-strength test on a short sample was first required by American specifications. This test was intended to insure insulation of the proper quality. At that time, a test of five times normal operating voltage for 30 sec. was considered sufficient for cables up to 15 kv., and four times normal for 30 sec. for cables above that voltage. The figures given were determined at a conference of interested parties, and, based upon general knowledge and experience, it was hoped that they would be sufficient. It was thought that the impregnated-paper insulation of the quality then being made would meet these requirements without difficulty, but in the discussion no test data were introduced or considered which would give definite information as to the dielectric strength of cables known to be satisfactory. The test voltage has been gradually increased in small steps until at the present time it is about 25 per cent higher and the time has been increased to five min.

The specifications require that after the sample has withstood the specified test voltage for five min., the voltage shall be increased in 10 per cent steps, held constant at each step for one minute, and the test continued in this manner until failure occurs. By carefully comparing service records with the records of tests made at the factory on the lots of cable which have failed in service, it has been noted that 13-kv., three-conductor cables which have met all the requirements of the specifications, but which failed on the dielectric-strength test at a voltage below seven times normal, have in general been quite unsatisfactory.

In order to secure definite information on the proper dielectric strength requirements, an extended series of tests on 13-kv. and 35-kv., three-conductor cables was undertaken as previously described under the subject on High-Voltage Tests. From the great mass of data obtained in this way, the curves shown in Fig. 11 have been prepared. It appears quite certain from the data obtained to date that if cable will pass the voltage tests corresponding to two or three well-separated points on line *S R*, in Fig. 11, it will have sufficient dielectric strength to give satisfactory service. The cable, however, may be deficient in other respects, as, for example, in ionization or in dielectric loss. If the insulation is quite uniform in quality, it may be possible to reduce the voltage tests, as represented by this line, but at the present time it appears to be perfectly feasible to secure cable from several manufac-

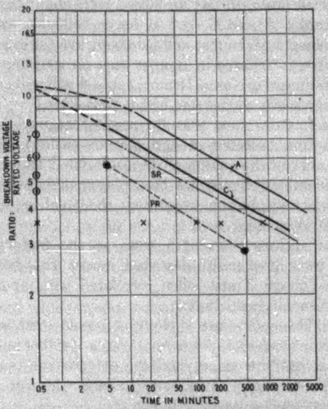

FIG. 11—VOLTAGE-TIME CURVES FOR 500,000-CIR. MIL, THREE-CONDUCTOR, 13-KV. CABLES

Curves *A* and *C* represent results from tests on 20-ft. samples. Cable *A* appears quite satisfactory; cable *C*, inferior and non-uniform, as shown by wide variation in test results.

S R—Suggested test requirements

P R—Present test requirements. ●Present specified tests

Typical test results on samples of cables that have proved unsatisfactory in service:

⊙Dielectric-strength tests at factory

×High-voltage tests in Chicago

turers which will average at least 30 per cent higher than the tests so specified, and at the same time leave the manufacturer a reasonable margin. It therefore seems inexpedient to consider accepting cables with an insulation of a lower quality than is represented by this line.

Lines *A* and *C* show the results of high voltage tests made on the recent products of two manufacturers. Tests on the products of two additional manufacturers give results similar to those shown by line *A*. Some months ago, lengths of about 300 ft. each of cables *A* and *B* were installed on 25-cycle lines operating at 20 kv., in order to extend the voltage-time lines to 1.5 times normal voltage.

As will be noted from Fig. 11, the test results for the dielectric-strength test for 30 sec. were about the same for cables *A* and *C*, but at lower voltages, cable *A* withstood a given test voltage about five times as long as cable *C*. Stated in another way, for a test to last a given time in excess of 10 min., cable *A* will withstand a voltage nearly 30 per cent higher than cable *C*.

These data show that the results obtained from a short-time, dielectric-strength test are misleading as an indication of the quality of the insulation. In the past, considerable reliance has been placed upon such misleading information regarding the quality of the insulation.

The dielectric strength tests at the factory and the tests previously described under the heading, High-Voltage Tests, when correlated with operating experience indicate that:

1. The short-time dielectric-strength test should be made at seven times normal voltage for five min., or, if this test introduces pothead or terminal troubles, it will be equally satisfactory to make the test at six times normal voltage for 16 min.
2. Satisfactory results can be secured on the short-time test with a sample about 15 ft. long under the lead.
3. A long-time test should be made on samples of the cable, and the requirements for this test should be four times normal voltage for six hours.
4. In order to secure reasonably accurate results on the long-time tests in which the failure may be caused by irregularities that require time to develop, the sample should be about 75 ft. long.

All of the points above mentioned are on line *S R*, Fig. 11, and if for practical reasons, a slight modification is desired in some of these tests, they should be made in accordance with the line.

On Fig. 11, some typical results are shown for dielectric strength tests at the factory and accelerated life tests in Chicago obtained on cable that has proved unsatisfactory in service.

During the last few months, the long-time tests at the factory have been made so as to secure additional information regarding the quality of the cable. The test requirements provide that the cable shall withstand 3.2 times normal voltage for eight hr., and the procedure has been that following this test, the voltage has been raised to 5.7 times normal voltage and continued at this voltage until the cable failed. The samples tested to date have withstood the higher test voltage from two to twelve hours. These data indicate that satisfactory cables which are now being made for 13-kv. service will readily withstand the proposed tests of four times normal for six hrs., as well as the test of six times normal for 16 min.

The data for lines *A* and *C* were obtained from tests on cables in which the insulation was impregnated with a grease or a heavy oil and their use should be limited to such insulation. The lines indicate that the voltage varies inversely as the seventh root of the time. It is entirely possible that a different line would be obtained for cable in which the insulation was impregnated with a materially different compound.

The fact that, in order to be satisfactory, the cables must withstand voltages of several times normal for many hours as a laboratory test, indicates that, in service, the cables occasionally must be subjected to voltages above normal. It is known from tests not here recorded that the number of transient voltages to which any one line is subjected is greater if the lines are interconnected. As these transient voltages are incidental to the normal operation of a large system or to a small system if supplied from overhead transmission lines, it appears that the cable must be designed to withstand such transients. There are in the records of the larger operating companies, a number of cases of cable failures which were simultaneous with other troubles on the system and definitely prove that these transient voltages occasionally cause cable failures.

As an example of the effect of transient voltages on underground cables, there may be cited the experience of a small suburban town adjacent to one of the larger Eastern cities. For a number of years, service was furnished from separate generators in a small steam plant in this town. The distribution feeders were single-phase and were supplied from a polyphase generator. Some time later, this lighting plant became a part of the system in the adjacent large city. The small generators were shut down and the distribution system was supplied by a transmission line from a generating station in the city. This underground system had been giving excellent service when supplied by the local generators, but when the system was supplied by a transmission line from the large generating station, the cable failures became so frequent that practically the entire system of primary feeders had to be replaced.

In view of the tendency to shut down the small plants in all parts of the country and to supply the smaller communities by transmission lines from large generating stations, it appears that similar experiences may occur in any small city if cables are installed with a smaller factor of safety than is found necessary in the large cities with interconnected transmission lines.

Cold Bending Test. Bending-test clauses have been a part of standard specifications in this and other countries for a number of years. The test is intended to insure that the cable will not be damaged by the bending incident to its installation in the conduit and training in the manhole. In Chicago, conditions require that cable installation shall proceed throughout the entire year, except the very cold days during the winter months and, for this reason, the clause in the specifications under which this cable has been purchased calls for the bending test to be made at a temperature of minus 5 deg. cent. or minus 10 deg. cent., depending upon the class of cable. The necessity of such a clause was shown by an investigation made several years ago in which it was found that the cable failures occurring in the three per cent of the cable which is in the manholes amounted to over 25 per cent of the total failures. An investigation made this year shows that this figure has recently dropped to about 18 per cent. The improvements in the behavior of American cable under this test are discussed elsewhere in the paper.

Ratio of Puncture Voltage Tests. It has been mentioned under the heading, *Workmanship*, that the ratio between the puncture voltage obtained on the sample subjected to the cold bending test and the puncture voltage obtained on the straight sample, appeared to be an excellent test of workmanship. While the ratio of these two voltages *averages* about 100 per cent, there is still quite a variation in the results of the tests on individual samples; *e. g.*, the puncture voltage on the cold bent sample is sometimes 10 or 15 per cent higher than that on the straight sample. A study of recent test data shows that, if the ratio were required to exceed 75 per cent, it would necessitate good workmanship and still allow an ample margin for the variation between samples, due to the ordinary variations in quality of the insulation.

C. UNIFORMITY OF INSULATION

This item is a recent addition to the rating table. Studies made during the process of modifying the rating table so as to bring it in accord with operating experience, plainly indicated that some of the properties of the insulation were not covered by the specifications and, further, that some of these neglected properties would probably have variations just as great as occur in those qualities which we know how to observe and measure.

Occasionally, deficiencies and irregularities are noted in the product of all manufacturers. Some of these deficiencies can be eliminated only by conscientious workmen and efficient inspectors at the factory. Irregularities in the impregnation is one of the greatest causes of variation in the quality of the insulation, and this is apparently one of the most difficult items for the manufacturers to control in the factory. Many examples of insulation, showing a marked deficiency in the impregnation, have been noted in the examinations of samples of cable which have been removed on account of failure or other causes, and an examination of the test records for those particular lengths has shown that such irregularities occurring throughout a small portion of a section of cable are not indicated by our present tests. It would be of great assistance in eliminating sections of cable having such deficiencies and in reducing the number of failures which result, if some test could be devised and applied to each section of cable at the factory, which would be a measure of the minimum quality of the insulation in any section tested.

During the many high-voltage tests which were made on three-conductor cable, it was noted that if the voltage was of the order of five or six times normal, a buzzing or crackling noise from the electrical discharges within the insulation could be distinctly heard. At such voltages, the life of the cable is comparatively short, but if the test voltage is reduced, the time to failure rapidly increases. At some voltage not well defined, the internal noise becomes inaudible and, at a voltage somewhat lower, the life of the cable becomes indefinitely long. From this it appears that a relation exists between the internal discharges and the life of the cable. The indications are that the failures of the insulation which occurred on high-voltage tests were brought about by the effect of these internal discharges upon the insulation and that failure of the insulation is inevitable, if the voltage is sufficiently high to cause these internal discharges. If, then, a test could be devised which would indicate the voltage at which the internal discharges begin, this would be an indication of the limiting voltage at the weakest point in the insulation throughout the section of cable tested and, by comparing the results of such measurements with similar data on cable known to be satisfactory, the necessary factor of safety to insure satisfactory operation could be determined. Such a test applied to each section of cable would be an invaluable addition to the tests now available, as it would give a direct indication of the minimum quality of any section of cable, whereas, the best that can be accomplished with the present ionization test is to determine the average quality, and it is the minimum quality rather than the average quality which determines whether or not the cable will fail.

The ionization test discussed in detail elsewhere gives an indication of the average impregnation of a section of cable, but neither the ionization test nor any other test now known gives any indication of the minimum impregnation at any part of the section. This variation in impregnation, plus local defects in workmanship and the resulting variations in the quality of the insulation throughout a section of cable, have been the cause of many failures of cable that have occurred in service in the last few years in cable that had passed with a wide margin all of the requirements of the specifications. These non-uniformities probably account for the fact that in some cases the remainder of a section of cable which has failed in service, upon being tested at the factory, will pass all of the requirements of the specifications.

An example of non-uniform quality of insulation is shown by the results of tests on a portion of a section of three-conductor, 13-kv. cable which failed after about three months' service. Upon examining and dissecting the cable, no apparent cause was found for the failure. About 210 ft. of the remaining section was tested and withstood 46 kv. for 33 min., after which failure occurred in the cable one in. from the end of the lead sheath. The pothead was replaced, the 193-ft. piece remaining was tested at 46 kv., and failure occurred in the cable after five hr. The 140-ft. piece remaining was tested and withstood 46 kv. for eight hr., followed by 55 kv. without interruption of the voltage for 8 hr., and followed by 65 kv. until failure occurred in the cable after 55 min. The sections of the cable containing the failure and the maximum hot spot were removed for examination. This left some shorter pieces and one, 69 ft. long, was tested and withstood 46 kv. for eight hr., followed by 55 kv. for eight hr. and 65 kv. for 16 min., after which failure occurred in the cable. In a series of similar tests at 127 kv., made on a section of 35-kv. cable, different portions of this section gave results which, according to Fig. 11, indicate a two-to-one range in quality.

Further data, including unusual examples of non-uniformity in insulation, are given in Table IV in the appendix.

Hot Spots. Further evidence of the non-uniformity of the insulation is contained in the record of the hot spots on the lead sheath in the high-voltage tests. These hot spots were distributed with great irregularity along most of the sections of cable tested. Generally the hot spots extended only a short distance along the cable. The temperature of the lead sheath at these hot spots ranged from 2 or 3 deg. cent. up to 74 deg. cent. above adjacent portions of the sheath.

In tests on one section of cable, sudden temperature rises of about 8 deg. cent. developed in less than 10 min. in two hot spots, accompanied by loud crackling and buzzing noises, followed by a disappearance of the noises, and a decrease of about eight deg. cent. in the temperatures. Then the temperatures increased five deg. cent. slowly to the end of the test, when failure occurred elsewhere. A broken filler was found at one of these hot spots.

As the location of the hot spots was recorded by reference to marks made on the original section of cable, it was possible to determine whether the hot spots would reappear at the same locations after an interruption of the test. Sixty per cent of the hot spots which developed in the first tests on 13-kv., three-conductor cable reappeared on subsequent tests. For the 33-kv., three-conductor cable, the corresponding figure was 84 per cent. In those tests where failure was in the cable and the time of the test was sufficient to result in hot spots of 2 deg. cent. or more above the minimum sheath temperature, 74 and 53 per cent of the failures occurred in hot spots for, respectively, the 35- and 13-kv. cables; 41 and 27 per cent, respectively, of the failures were at the maximum hot spots.

The study of all of the records shows that, in general, the cable manufacturers who have the best control of their materials and manufacturing processes, as demonstrated by the uniformity of test results and inspection data, make the best cable.

STABILITY OF IMPREGNATING COMPOUND

It has recently been discovered that certain previously used impregnating compounds are entirely unsuitable for use in high-voltage cables—that is, cables with average stresses exceeding about 40 volts per mil—and that when such compounds are used, the most perfect process of drying, vacuum treatment and impregnation will not prevent rapid deterioration of the insulation. F. M. Farmer[9] has devised a test which will eliminate unsuitable compounds, and this test will be in a paper presented by Mr. Farmer at the same meeting as this paper. On the basis of this information presented by Mr. Farmer, it appears perfectly feasible to devise a clause for insertion in the specifications which will effectively eliminate unsuitable compounds. A large number of failures of the 35-kv., three-conductor cable can be directly traced to this cause as well as a smaller number of failures occurring on 22-kv. and 13-kv., three-conductor cables.

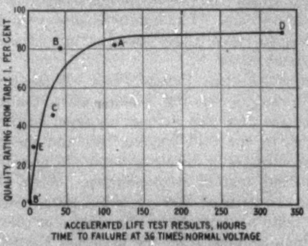

FIG. 12—RELATION OF QUALITY RATINGS WITH ACCELERATED LIFE TEST RESULTS, FOR 500,000-CIR. MIL, THREE-CONDUCTOR, 13-KV. CABLES

Tests which were at 46 kv. and higher were evaluated for 46 kv., which results in a maximum dielectric stress of about 72 kv. per cm.

COMPARISON OF QUALITY RATINGS WITH ACCELERATED LIFE TEST RESULTS

The relation between the quality ratings given in Table I and the results of the accelerated life tests made upon sections of the same cable is shown in Fig. 12. The variations of the individual points from the curve as drawn are no greater than might be expected from the variations in the quality of the insulation. As the accelerated life test for those cables having a life of only a few years has been found to be a fairly accurate indication of the service record of those cables, it appears that the curve in Fig. 12 verifies the results of the quality rating table, as a prediction of the service records of the cables of widely varying quality.

As the zero base weights in Table I practically correspond with specification requirements, Fig. 12 indicates that if the cable passes the specifications with a narrow margin, it will be unsatisfactory in service, and this indication is confirmed by experience. The cable shown by *B′* passed the dielectric strength tests at the factory by a narrow margin; there was an unusually high percentage of failures in the high-voltage test on each section at the factory; there were several failures on the test after installation before placing the line in service, and several failures in service within a few months after the line was placed in regular operation. Another lot of cable obtained from another manufacturer at about the same time, and having about the same quality, gave very similar results. A study of the rating table clearly indicates the changes in the specifications that must be made in order to eliminate such cable by means of the tests at the factory.

Cable *E* was of slightly higher quality and gave somewhat better results in the factory tests and in service. Cable *C* was of still better quality, but the quality as indicated by inspection and all of the tests was very irregular. About 10 mi. of this cable was installed and placed in service last fall, and one service failure, which was due to no apparent cause, occurred within two months after the cable was placed in service.

Cables shown by *A* and *B*, and installed last fall, have had no failures in service. The quality of these cables appeared so high that lengths of about 300 ft. each were installed on 25-cycle, 20-kv. lines last November and September, respectively, in order to verify the quality rating as determined from the table. No failures on these test lengths have occurred to date. If the voltage-time curves obtained in the laboratory on these cables correctly represent them, then these test lengths should last for several years at 1.5 times their normal voltage.

A further study of Table I will indicate the changes that must be made in order to insure cable of the desired quality.

Table II is a rating similar to Table I for typical examples of several different grades of 35-kv. cable which have been received to date. Fig. 13 shows the relation between these quality ratings and the accelerated life test results. Several of the cables represented have proved unsatisfactory for operation at 35 kv.; they developed the signs of ionization previously described and tests on their impregnating compounds show that they would have been rejected by Mr. Farmer's test for stability. The curve in Fig. 13 cannot be drawn with as much accuracy as the curve in Fig. 12, but Table II and Fig. 13 appear to indicate necessary changes in the specifications to insure satisfactory 35-kv., three-conductor cable.

It is recognized that Tables I and II, in their present form, are not perfect. They can never be made more accurate than the data from which they are developed, and these data will always vary in accordance with the non-uniformity of the insulation. Changes in the quality rating tables will be required from time to time as additional information is secured and especially to permit the tables to be properly applied to some of the new types of insulation which are now being developed by the cable manufacturers. However, there appears to be no reasonable doubt as to the feasibility of using the quality rating methods outlined in this paper as a basis for modifying the specifications in order to secure cable of the desired quality.

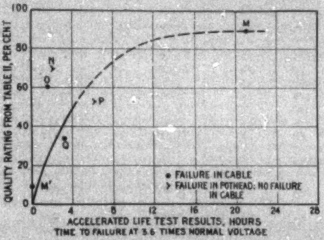

Fig. 13—Relation of Quality Ratings with Accelerated Life Test Results, for 500,000-Cir. Mil, Three-Conductor, 35-Kv. Cables

Tests were at 127 kv., which results in a maximum dielectric stress of approximately 125 kv. per cm.

Conclusions

1. All high-voltage tests on samples should be continued until failure occurs.

2. Long-time tests at several times normal voltage should be made on samples of the cable at the factory in order to obtain definite information as to their dielectric strength.

3. A test requirement should be inserted in high-voltage cable specifications to prevent the use of unstable impregnating compounds.

4. The ratio of the puncture voltage obtained on the cold bent sample to the puncture voltage obtained on the straight sample appears to be an excellent test of workmanship.

5. A test is needed to indicate the minimum impregnation in a section of cable instead of the average impregnation as determined by the present ionization test. Until such a test has been devised, only careful supervision of the impregnating processes will eliminate the variations in quality due to irregular impregnation.

6. The best cable is, in general, made by the manufacturers that have the best control of their processes,

TABLE IV—TYPICAL ACCELERATED LIFE TESTS

Test No.	Feet Tested between Potheads	Service Before test-months	Month and year made	Test volt. kv.	Failure		Temp. Rises of Sheath above Min. Sheath Temeprature—Deg. cent.		Remarks
					Time mins.	Location	At Failure*	Max. hot spot	
					500,000-cir. mil, 3-conductor, 13-kv. cables				
14	206	2.5	10–23	46	33	Cable	0	2.6	Failed from outer corner of 1 conductor, 1 in. below end of cable lead in pothead. Tape was wound with a negative lap and failure was along the edge of 4 superimposed layers.
14*A*†	193	2.5	10–23	46	300	Cable	5.8	5.8	Failed from outer corner of 1 conductor to sheath.
14*AA*	140	2.5	10–23	46	480				
				55	480				
				65	55	Cable	6.8	16.9	Failed between parallel faces of 2 conductors.
14*AAA*	67	2.5	10–23	46	480				
				55	480				
				65	16	Cable	4.9	4.9	Failed between parallel faces of 2 conductors.
17‡	213	12	11–22	46	96	Cable	0	3.2	Failed between 3 conductors. Blackened compound was found 2 ft. from the failure in the vicinity of a string wrapped around the belt.
17*A*	164	12	11–22	46	112	Cable	3.3	3.3	Failed under pothead wipe from the corner of 1 conductor to the sheath.
17*AA*	151	12	11–22	46	178	Cable	0	2.4	Failed between 3 conductors.
17*AAA*	73	12	11–22	46	267	Cable	0	4.5	Failed between 3 conductors.
28	200	11	3–23	46	0	Cable			Failed between 2 conductors at the corners of the sectors.
28*A*	148	11	3–23	46	0.75	Cable			Failed between parallel faces of 2 conductors.
28*AA*	140	11	3–23	46	3	Cable			Failed from cornor of 1 conductor to sheath.
28*AAA*	113	11	3–23	46	155	Cable	1.0	2.6	Failed between 3 conductors.
28*A*–4	93	11	3–23	46	341	Cable	0	2.8	Failed between parallel faces of 2 conductors, and from corner of 1 conductor to sheath.
34	213	13	9–22	46	480				
				55	109	Cable	48.5	48.5	Failed between parallel faces of 2 conductors.
34*A*	102	13	9–22	46	480				
				55	78	Cable	56.0	56.0	Failed between 3 conductors at center of cable.
34*AA*	95	13	9–22	46	480				
				55	164	Cable	74.0	74.0	Failed between 3 conductors at center of cable at a point where the center filler had wandered between conductors. These are the highest hot-spot temperatures developed in these tests. All three failures occurred at the max. hot spots for this section of cable.
64	22	0	8–24	50	3724	Cable	10.3	10.3	Failed between 3 conductors at the center of the cable. Black spots and waxy flakes were found in compound near failure.
79	97	0	12–25	62.5	1314	Pothead	0	7.2	Failed between parallel faces of 2 conductors at the end of lead in the pothead.
79*A*	86	0	12–25	62.5	850	Pothead	0	14.8	Failed between 3 conductors just below end of lead in pothead. One hot spot developed suddenly just previous to failure and its sheath temperature rose from 39 deg. cent to 69 deg. cent. in the last 10 minutes of test. One outer filler was found to be broken at this point.
79*AA*	68	0	12–25	62.5	270	Cable	0	26.1	Failed between 2 conductors at center of the cable. A broken outer filler was found 1 ft. from the failure. 2 hot spots developed sudden rises in temperature after 90 and 100 minutes, accompanied with a loud intermittent "ripping" and "cracking" sound which lasted about 20 sec. There were no unusual noises in either hot spot afterwards. In one, the temperature increased from 43 deg. cent. to 50 deg. cent. in 10 min., decreased to 41 deg. cent. in one hour and then rose to 46 deg. cent. by the end of test; a broken outer filler was found in this section. The other hot spot increased in temperature from 42 deg. cent. to 51 deg. cent. in 10 min., decreased to 43 deg. cent. in 40 min., and rose slowly to 48 deg. cent. at time of failure.
Total time to first cable failure					2434				
					350,000-cir. mil, 3-conductor, 35-kv. cables				
4	102	0	8–24	127	49	Pothead	5.8	6.0	Failed between 2 conductors in crotch.
4*A*	90	0	8–24	127	41	Pothead	4.7	4.7	Failed between 2 conductors in crotch.
4*AA*	83	0	8–24	127	19	Pothead			Failed between 2 conductors in crotch.
4*AAA*	69	0	8–24	127	28	Pothead	4.3	4.3	Failed between 2 conductors in crotch. In each test one portion about 20 ft. long remained about 5 deg. cent. cooler than the rest of the section.
				Total time	137				
26	96	0	5–23	127	0.25	Cable			Failed from outer corners of 2 conductors to sheath.
26*A*	82	0	5–23	127	32	Cable	4.0	4.0	Failed from outer corner of 1 conductor to sheath.
26*AA*	68	0	5–23	127	43	Cable	2.0	4.1	Failed from outer corner of 1 conductor to sheath.
36	98	0	8–25	127	408	Pothead	20.5	20.5	Failed between 2 conductors in crotch.
36*A*	82	0	8–25	127	585	Pothead	0.5	2.5	Failed between 2 conductors in crotch.
36*AA*	68	0	8–25	127	269	Cable	2.5	2.5	Failed between 2 conductors and to the sheath at a point where two strands of the outer filler wandered between the conductor and belt insulation.
Total time to first cable failure					1262				
42	98	0	8–24	127	33	Cable	0	2.0	Failed from outer corners of 2 conductors to sheath.
42*A*	80	0	8–24	127	172	Cable	18.0	18.0	Failed between 3 conductors at center of cable.

*Last reading taken before failure occurred. ‡14, 17, 28, etc. are different sections of cable.

†14–*A* is the portion remaining after the piece including the fault in Test 14 had been removed; 14*AA* is the portion remaining from Test 14*A*; etc.

TABLE V

RELATION BETWEEN LOCATIONS OF HOT SPOTS AND FAILURES IN VOLTAGE TESTS ON 500,000-CIR. MIL, 3-CONDUCTOR, 13-KV. CABLE*

Item No.	Time to failure in minutes	No. of Failures 0 to 5	6 to 20	21 to 120	121 and over	Total	Per Cent of Item No. 5
1.	Failures in max. hot spots	0	0	0	8	8	26.7
2.	Failures in other hot spots	0	0	2	6	8	26.7
3.	Total failures in hot spots. (1) + (2)	0	0	2	14	16	53.4
4.	Failures in "cold" portions after hot spot developed in same section of cable	0	0	4	10	14	46.6
5.	Total failures occurring in sections of cable in which hot spots developed, (3) + (4)	0	0	6	24	30	100.0
6.	Failures in sections in which no hot spots developed	12	4	1	1	18	
7.	Grand Total, (5) + (6)	12	4	7	25	48†	

*Only those tests were considered where failures were in cable proper, that is, not less than three in. from wipes of potheads or joints connected.

†22 samples of cable were used.

Note: The time to develop 2-deg. cent. hot spots in the High-Voltage Tests was found to be as follows:

Average.......................... 24 min.
Minimum.......................... 10 min.
Maximum..........................130 min.

TABLE VI

RELATION BETWEEN LOCATIONS OF HOT SPOTS AND FAILURES IN VOLTAGE TESTS ON 350,000-CIR. MIL, 3-CONDUCTOR, 35-KV. CABLE*

Item No.	Time to failure in minutes	No. of Failures 0 to 5	6 to 20	21 to 120	121 and over	Total	Per Cent of Item No. 5
1.	Failures in max. hot spots	0	1	9	1	11	40.7
2.	Failures in other hot spots	0	1	4	4	9	33.3
3.	Total failures in hot spots, (1) + (2)	0	2	13	5	20	74.0
4.	Failures in "cold" portions after hot spot developed in same section of cable	0	2	2	3	7	26.0
5.	Total failures occurring in sections of cable in which hot spots developed, (3) + (4)	0	4	15	8	27	100
6.	Failures in sections in which no hot spots developed	21	0	2	0	23	
7.	Grand Total, (5) + (6)	21	4	17	8	50†	

*Only those tests were considered where failures were in cable proper, that is, not less than three in. from wipes of potheads or joints connected.

†23 samples of cable were used.

Note: The time to develop 2-deg. cent. hot spots in the High-Voltage Tests was found to be as follows:

Average..........................16.5 min.
Minimum.......................... 5 min.
Maximum..........................70 min.

TABLE VII

REAPPEARANCE OF HOT SPOTS

Item No.		500,000-cir. mil, 3-conductor, 13-kv. Cable Number	% of Item 3	350,000 cir. mil, 3-conductor, 35-Kv. Cable Number	% of Item 3
1.	Samples tested	10	..	20	..
2.	Tests on samples*	36	..	62	..
3.	Hot spots which could reappear†	56	100	73	100
4.	Hot spots reappearing in second test	31	55	53	72.5
5.	Hot spots reappearing in third test only	3	5	8	11.0
6.	Total reappearance of hot spots	34	60	61	83.5
7.	Hot spots known definitely not to reappear	17	30	11	15.0
8.	Balance (unknown or doubtful)	5	10	1	1.5

*In the earlier tests, the results were not recorded in sufficient detail for this analysis. In a number of cases, only one test was made on a sample. In some tests, the duration was too short to develop 2-deg. cent. hot spots.

†Only those hot spots were considered which could reappear under the following conditions for subsequent tests:

(1) Common section of cable in tests considered.

(2) Sufficient duration of subsequent test to develop a hot spot (not less than the average time for 2-deg. cent. hot spot, that is, 16.5 minutes for 35-kv. cable and 24 minutes for 13-kv. cable). Subsequent tests of shorter duration were disregarded, if hot spot did not reappear. However, if hot spot actually reappeared in shorter time, it was counted as reappearing spot in spite of short time.

as indicated by the uniformity of test results and inspection data.

7. If tests on three-conductor, high-voltage cables are of sufficient duration (average time is about 20 min.) to develop hot spots,—that is, portions of the sheath that are two deg. cent. or higher above the minimum sheath temperature,—it has been found that:

a. A majority of the cable failures will occur in hot spots and about one-third of these failures will be in the maximum hot spots.

b. A majority of the hot spots will reappear in subsequent tests on a given section of cable.

8. Higher quality of the insulation is necessary as the operating voltage increases.

9. If the compound is stable, and test and inspection data are properly correlated with operating experience, then quality rating tables can be used to predetermine with reasonable accuracy:

a. The relative merits of several different types of insulation or lots of cable.

b. The service record of any particular lot of cable.

The authors gratefully acknowledge the efficient assistance they have received from C. E. Betzer, K. W. Miller, Karl Horine and L. B. Schofield in securing and correlating the information on which this paper is based.

Appendix

Tables IV, V, VI and VII give data extracted from the log of the tests made in Chicago as outlined under the heading, High-Voltage Tests, in the paper.

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Discussion

W. S. Clark: One very important feature in the paper of Messrs. Roper and Halperin is the correlation of tests and service, and nobody could do that except an operating man.

Now as to the rating of cable, I think it is most important to get into it a figure based on the so-called accelerated life test, as we have not yet found in Schenectady any test which would give us as correct a measure of the value of the cable as that. I should suggest that the tests on the cold and hot samples might have their ratings reduced to ten each, and the value of the added accelerated life test be put in at twenty, possibly reducing some of the observation tests because, of course, those are very undesirable on account of the personal element.

I should like to second Mr. Atkinson's suggestion that instead of testing to destruction by increasing the pressure 10 per cent every 30 sec. or every minute, the increase be at the rate of 15 per cent and at 5-min. intervals. I believe this would give more reliable data. The volts per mil, quoted in Mr. Farmer's paper, for instance, represent probably less than the one-minute strength of the cable.

Referring to Fig. 11 of Mr. Roper's paper, the slope of the curve in the type of cable on which it is based would be modified by temperature. It has been our experience with semi-solid compounds and on single conductors that the average life of a very large number of samples with insulation 0.28 in. thick, tested at 44 kv., or about 156 volts per mil, with a 2/0 conductor, would run around 40 hr. On the same samples, tested at 85 deg. the average life was in the nature of 1000 hr., the reason being that voids were not present in the second case in the same degree as in the first. You cannot prevent some voids in cable due to the process of manufacture unless you go back to the old Siemens process and apply the lead cold, because the lead goes onto the cable at a temperature around 200 deg. cent. and it is evident that this must warm up the cable as a whole, so that when it cools to room temperature there is some shrinkage and some voids are created. It is for this reason, I believe, that in the past cables which were operated at average voltage stresses of less than 50 volts per mil have not given as much trouble as cables operated above 50 volts per mil average stress. This is due to the fact that at atmospheric pressure you get ionization effects around 50 volts per mil.

To illustrate how the curve in Fig. 11 may be varied with different types of impregnating compound, I have checked up and found that there the voltage ordinarily varies inversely as the seventh root of the time. We have this check on tests running up to 2800 hours and the results agree quite well with the formula.

On oil-filled cables of the Pirelli type, however, we have found that the exponent, instead of being seven, should be something in the neighborhood of nineteen, indicating a very much flatter curve.

There is one piece of information involving very long and expensive testing with reference to the effect of surges on cable on which I have data and I am giving a brief summary of results below:

Two samples of single-conductor 2/0 cable with insulation 0.28 in. thick, 10 ft. under the lead.

Sample No. 1 was continuously subjected to 13.2 kv. at 60 cycles at room temperature. Once a day a 50-kv., 10,000-cycle surge of approximately 300 microseconds was applied. Total number of surges applied up to the termination of the test, 357.

Sample No. 2 was subjected to an impulse voltage of approximately 300 microseconds duration from the lightning generator (the sphere-gap being set for 75 kv.) once a day. This sample was, of course, also under continuous stress of 13.2 kv. Total number of applications, 333.

Total length of time involved in the test on each sample—9933 hr.

The initial power factor at 28 kv., at room temperature, was 0.5 per cent. The final power factors were as follows:

At 5 kv.—0.6 per cent

At 15 kv.—0.5 per cent

At 28 kv.—0.6 per cent (an extremely small increase in the power factor.)

The above tests were on sample No. 1.

Sample No. 2 at the end of the test at 5 kv. showed 0.45 per cent; at 15 kv., 0.5 per cent, and at 28 kv., 0.6 per cent.

After the completion of the tests, samples were put under continuous stress at 48 kv. a-c. and they stood in excess of 36 hr.

R. W. Atkinson: Mr. Roper and his aids are to be commended for the large measure of accomplishment they have made toward the solution of the complex problem. They have shown that cable having the insulation of their 13-kv. system will not be free from operating difficulties unless it can meet certain test standards and that when it does meet these standards continued satisfactory operation seems assured, and, indeed, so far as their considerable experience goes, is assured by a large margin.

I endorse most of Mr. Roper's conclusions but wish to discuss two of them. The fourth one, stating that the ratio of puncture voltage obtained on cold bent samples to the puncture voltage obtained on the straight sample appears to be an excellent test of workmanship, contains a qualification which I wish to emphasize. Mr. Roper uses the words "that this ratio *appears* to be an excellent test of workmanship." Undoubtedly, in many cases it is a test of workmanship. In general, American cables made at this time show approximately the same dielectric strength on the two tests, that is, this ratio is in the neighborhood of unity and without doubt the increase in the ratio from former times has resulted in no small measure from improvement in general excellence of workmanship. However, I do not believe that there is a fundamental relationship between the ratio and the quality of workmanship such that this condition will always remain true and I believe that the individual tests themselves and other tests and actual direct determinations of the quality of workmanship will be found to be much more important than any chance ratio which may seem to exist between these two tests. If proper weight is given to these other things, then the ratio between the two tests can be omitted from consideration.

I agree with conclusion 9, that quality rating tables can be used with reasonable accuracy to determine relative merits of different lots of cable, but if this comparison is to be more than a very rough approximation, some additional types of measurements must be made in order to find better means than are now available to cover the subject of uniformity.

There cannot be any doubt of the value of the so-called life tests, and the suggestion of these authors to change the eight-hour test in the Edison specification on reels to be used afterward to destruction tests on shorter lengths marks a distinct advance. For many years, we have been carrying on life tests in our laboratories, making these tests at relatively lower voltages and carrying them for many hundreds or thousands of hours. It is interesting to note that very much the same sort of data is obtained in the relatively shorter period, and of course tests for the shorter period have the very practical advantage that far more data can be obtained with a given space and equipment. I agree with these authors in placing this test as doubtlessly the most important single test. It is interesting therefore to note curves in Figs. 12 and 13, from which the authors conclude that this test gives substantially the same result as the rating table and that both are in general agreement with operating experience. The data given, however, do not justify this as a *general* conclusion.

Actually, the life test depends so largely upon the same things upon which the operating experience may be expected to depend that ordinarily there will be a very close relationship between them. It is easy, however, to cite cases where this will not be true. The cable might be made with a compound of such characteristics that it would not well withstand a cold bending test or handling while cold; then be installed in extremely cold weather and give a very bad service record in spite, perhaps, of being able to pass an extremely severe life test.

If, when a group of cables is rated in the order of relative merit, (as determined by the best information available) any characteristic is found to fall in the same order as the rating of the cables, on first thought this may be assumed to indicate that this characteristic has fundamental importance in regard to the rating, or that it varies directly with some other very important property. I have already shown that even such an important test as the life test may vary in a way different from the serviceableness of the cable. It is evident, then, that relatively minor properties may vary, in some cases, as the relative value of the cable, and in other cases, in an entirely contrary direction. To establish the importance of any particular test or item in the rating table, the individual item must be studied and its importance studied separately.

It follows, therefore, that we should be able to take a complete group of mechanical characteristics and get the same comparison as for a correlated and complete group of electrical characteristics. If these results do not coincide, there is no doubt that one or the other is not complete or is not properly evaluated. Each individual item that is given weight should be studied analytically and the weight determined in that way. Evaluation of these different properties by mere comparison with operating experience, without the initial analytical comparison, is likely to be misleading.

These authors have followed the analytical method to some extent; for instance, in the case of the cold bending test. They have pointed out installation conditions and have shown the relatively excessive number of failures which once occurred in the manholes. They have ascribed the large reduction in such failures to the use of cable better able to withstand the bending test. I should ask, at this point, if the better results may not be partly a result of closer supervision of installation and greater assurance that cables are not subjected to conditions from which they can be protected or for which they were not designed. The general analytical treatment given the bending test justifies that it be considered of important weight.

I have already mentioned that there is no justification for the use in rating table of the ratio between breakdown of the bent and unbent samples. Substantially the same arithmetical result can be obtained by changing the weights and some changes in the limits. It may be asked, if there is no final arithmetical change in the rating due to making the change here proposed, what is the use of making it? In the first place, logic demands it; secondly, there is a very important practical reason. If the test on the bent sample is three times as important as the other, it warrants a great deal more relative attention. If it is three times as important, there should be three times as many tests made of this kind as of the other. Half of the present number of samples now intended for the straight test should be added to those tested after bending.

Uniformity of insulation resistance is given considerable weight in the rating table because it is assumed to be an index of the uniformity of the cable. It is, however, a measure of uniformity of only one property—insulation resistance. If insulation resistance measurements can be used to indicate uniformity in properties important in themselves, those advocating it as important for that purpose should suggest what properties it indicates and should use the insulation resistence measurements to aid in picking out non-uniformities in the important property.

There is, however, another aspect of the matter. Conditions in Chicago are known to be peculiar in one respect that has undoubtedly greatly influenced the whole handling of the cable problem. Street conditions have dictated a limitation of diameter to three inches and power requirements have been for very large capacities at high voltages. No doubt the standard necessarily set for 33-kv. cable had a bearing on a choice in Chicago of insulation thickness less than for usual American practise. Mr. Roper's experience has demonstrated that the problem of suitable specifications for cables with such insulation thickness is vastly more complex than where thicker insulation is used.

I want to express, too, my pleasure in hearing Mr. Roper mention two things which I have been preaching for quite a number of years. One is that low dielectric loss can be overemphasized, and the other is that the cable manufacturers should not be limited to a particular kind of compound. Mr. Roper spoke of resin and its value. Some years ago the cable manufacturers had considerable difficulty in preventing operating people from insisting on putting into specifications the stipulation that a compound should be a mineral-base compound.

D. M. Simons: In general I agree with most of the conclusions of the authors, but I feel that real consideration should be given to the matter of whether or not some of the specific recommendations for tests are of general application or of general necessity. When the authors speak in general of a 13,000-volt cable, they have in mind a three-conductor, 500,000-cir. mil sector cable, insulated with 9/64-in. paper on the conductor, and 5/64-in. belt insulation. The mental picture of a 13,000-volt cable in the mind of a manufacturer is very different. He will have in mind various sizes of conductor with conductor insulation thicknesses varying from 9/64 in. up to the more usual thicknesses of 12/64 to 16/64 in., and of various belt thicknesses. The manufacturer therefore has a tendency to wonder if the tests and data are of sufficient generality, large as they may be in extent, to justify general conclusions?

I should like to mention one other point. The authors are known to advocate very severe tests and also extremely thin insulation. I believe, however, that most companies go rather slowly in reducing thicknesses. It is of course possible to go to thinner insulation than the average used in the country for a given voltage, but, apparently, in order to do so, the severity and number of tests must be greatly increased so that at least part of the small saving in cable cost, due to thinner insulation, may be lost due to the increased cost of testing to find out if the cable is adequate. The authors state that *some* of the large operating companies have made reduction in the thicknesses of insulation. I do not find that this is by any means the general tendency; in fact out of a list of some fifty of the larger central stations I was able to note only three or four such cases in the entire country.

Percy Dunsheath: For twelve months I have known Mr. Roper has been working on this point of rating and I have not agreed with him. I have always felt that if you have a dozen different factors you can't add them together; any one of them, no matter how good the others are, may cause the cable to fail. For instance, if you have the papers on the cable very well registered, that counts for nothing if there is no compound in the cable. I have always criticized this method of Mr. Roper's on that score, but now I see this curve and I think I am converted. The curve does demonstrate pretty definitely that the testing of a cable by Mr. Roper's method comes very near the truth.

R. J. Wiseman: Mr. Roper's quality rating is most valuable to manufacturers. By using the weighted values, each of us can rate our own cables. Some of us may not agree entirely with the various weights of the separate factors, but as a whole the result should be a guide. As we get to know the various influencing factors which will cause a cable to fail in service, we can assign more weight to them. Is tearing in bending tests equal to half the weight of thoroughness in impregnation? I don't believe so. Today practically all manufacturers butt the tapes or use a slightly open wrap. There is not much difference between a tear and an open wrap. To be sure, too many openings at one point are undesirable. Registration of tapes is more serious than tears.

We still must consider that the dielectric strength of a cable is a big factor. I am glad to see Mr. Roper give most weight to items No. 9 and 10, dealing with voltage on the hot sample and cold bent sample. I believe that practically all manufacturers are getting nearly the same breakdown voltage for hot and cold samples. Therefore, it is possible to reduce the weight value of the ratio of the two and add a new factor, the long-time voltage test on a 75-ft. length. Some of its weight can be taken from item No. 11 and some from item No. 8. I think the weight of item No. 8 is too high. It is definitely known that power factor varies with voltage. Therefore, full weight should be for an actual change in power factor, say 0.5 per cent.

Uniformity of impregnation is the great need today. As the authors state in their paper, cable showing uniform tests gives the best service. Insulation resistance is one of the best ways to determine uniformity but sometimes even this will not show up a defective reel of cable. If used in conjunction with the a-c. specific inductive capacity, however, it is a good guide to the uniformity of the cable. I am glad the authors recognize that it is the variation in insulation resistance and not the actual value that is important. It appears that there may be a relationship between insulation resistance and dielectric loss, but in each case it must be for each individual manufacturer.

There are some who do not believe the ionization test is of value. Here again, it is the individual manufacture as shown in Fig. 6 of the paper. Each one of us has his own curve, depending on the type of materials he uses and his own manufacturing methods. Let the user obtain the curve for each manufacturer and then compare results with this curve.

The new test for stability of compound needs to be perfected before we can use it as an accepted one. The idea is good but until we are able to prepare samples without entrapping air, we shall have to postpone using the test as a criterion for good compound.

The section of the paper dealing with the relationship between voltage and time of application is most valuable. Although others have referred to it and based their ideas upon short sample tests, the amount of data collected here give weight to the curves obtained. I think it would have been interesting if the authors plotted as ordinates, the logarithm of the maximum stress at the conductor instead of the ratio of breakdown voltage to rated voltage. Although some may question the accuracy of the formulas for calculating maximum stresses at the conductor for multiple-conductor cables, the work done by Atkinson and Simons has cleared up pretty well the discrepancies in the exact formulas. As it is now, a new set of curves must be drawn for each voltage. It is the maximum stress that the material can stand which is important. Low-voltage cables are being made today the maximum stress of which at breakdown is as high as for super-tension cables. Therefore, to expect a 33-kv. cable to withstand, say, seven times rated voltage for the same length of time as a 5-kv. or 13-kv. cable is asking too much.

There is a problem in connection with high-voltage testing which needs to be solved. We all know that we get different results if we apply a constant voltage until failure, noting the time. If we build up to this same voltage in steps, holding at each step, say one, five or ten minutes, and finally noting the time at the last voltage, the total time will be less, and it should be. On the way up to the last voltage the dielectric is being stressed; therefore, it is going through the fatigue pertaining to the breakdown phenomenon. The ultimate effect is the same failure. I believe the dielectric has been subject to just the same total amount of strain as in the first case. Although it did not stand up as long at the final voltage, it may be equally as good; in fact, it may be better. I have tried to evaluate the step test and the single voltage test but so far I have not been successful.

We must not misinterpret the formation of hot spots in a cable while subject to high voltages. If only one or two occur, it is an indication of possible weak spots but where there are many and uniformly distributed over the sample, I believe it is an indication of a uniformly manufactured cable, and although it may not stand as high a voltage ultimately as one with less hot spots, it may be a preferable cable. The whole length is failing at the same time and not here and there.

H. G. Burd: Many cable failures are charged too readily to defective cable. I should like to see considerable more study of operating conditions. A 25 per cent improvement of cable quality (as shown by recent tests) should be paralleled by a corresponding improvement in treatment of cable during and after installation. Quite radically different operating results in different cities on the same quality of cable present most convincing argument that many failures have a very close relation to operating problems and that even the best of cable as now made wouldn't overcome some operating handicaps.

F. A. Farmer: Some of us have been spending much time during the past two years trying to arrive at some method of taking all of the more or less intangible things which go to make up "quality" and combine them in such a way as to get a quantitative figure which we can call "quality." But even after we have done this, the proposed method must be tested finally by comparison with actual performance, and that is what Mr. Roper has attempted to do. I think he is going to get a satisfactory answer before very long. That, to my mind, will be one of the most important contributions that has been made in many years to our discussion of this subject.

E. S. Lee: If you study the tables in the paper carefully you will find many interesting things. Take, for example, in Table I, the cable as represented by manufacturer *D*. This cable has the lowest rating for tearing in the bending tests and it has the lowest rating for dielectric power loss at 80 deg. cent. It has a relatively low rating on workmanship, insulation and fillers, and it has a relatively low rating on the ionization test. In spite of these deficiencies, the relatively high value of the puncture voltage on straight, and particularly on cold bent samples, earned this cable the highest total quality rating. Nor does this honor seem to be misplaced, for from Fig. 12 we see that samples from cable *D* outlived all others by several fold. Evidently tearing in bending tests, dielectric power loss, workmanship on insulation and fillers, and ionization test values, may have very little weight when the insulation has high dielectric strength.

Those interested will certainly eagerly await the results of this cable in actual operation. In other words, if we had had these figures from some cable that we were trying to rate, wouldn't the tendency have been for some to say, "Well, it has low tearing; it has relatively high dielectric power loss; it has a low rating in the ionization test. Even though it has high dielectric strength, will not these other factors possibly react to give a lower endurance of the cable?" Evidently in this case they have not, but perhaps in some other case they might.

Equally interesting is the fact that cables *A* and *B* rate practically equal in all respects, the total rating being 81.9 and 80.3; yet from Fig. 12 we learn that the life test of cable *B* was 45 hr. as opposed to a life of 115 hr. for the sample of cable *A*, or an increase in life of 156 per cent for the same quality rating. Perhaps this may be attributed to non-uniformity of insulation, though both cables are rated practically perfect as regards uniformity of insulation.

Now on the other hand, cable *C*, which rates inferior to cable *B* in practically every respect, having a total quality rating of only 46 as opposed to 80 for cable *B*, has a life of 35 hr. compared with life of only 45 hr. for cable *B*, or an increase in life for cable *B* of only 29 per cent for a quality rating 72 per cent greater.

I bring these to your attention because I should like to have explained to me how one can use a quality rating of this kind with these irregularities. I recognize that we have wide variations in all of this work, and to me, it seems that the wide variations are still present in this particular quality rating table.

I make an appeal, as has been done by several others, for the inclusion in this quality rating of two other tests, (1) the short-time breakdown test, with five-minute steps rather than one-minute steps, and (2) an electrical endurance test. Increasing the time length of steps in the short-time breakdown test will not increase the difficulties of the test but it will bring into account more prominently the effect of electrical endurance which I know both Mr. Farmer and Mr. Roper advocate. An electrical endurance test where some voltage such as Mr. Roper suggested as 3.6 times normal or some other value times normal for different ratings of cable, (because it will probably have to be different for different ratings of cable) are applied continuously, giving results as in Fig. 12, will serve to take into account all the factors in the quality rating which we can otherwise accomplish only imperfectly.

In Table II, cables *P* and *Q*, with total quality ratings much below those for cables *N* and *O*, have longer life, as shown by Fig. 13. A possible explanation is the fact that cables *P* and *Q* have low dielectric power losses, while cables *N* and *O* have high dielectric power losses. Apparently this factor obtains more prominence in 35-kv. cables than in 13-kv. cables, and is an added argument for including breakdown and electrical endurance tests in the quality ratings as suggested above.

I do not present these points to disparage the idea of quality ratings; I advocate them, and I hope that we may be able to perfect them so that they will be of greater use; but there are still uncertainties in these particular quality ratings, and I hope that they may either be explained so that we won't have to worry about them or that we can improve them so that they can be made of greater use to us.

W. A. Del Mar: I should like to mention something in relation to the point brought up by Mr. Wallace Clark; namely, that some of his associates, some years ago, showed that a determining critical stress in a cable is a stress of about 50 volts per mil or 18-kv. per centimeter, in which a very thin film of air ionizes at atmospheric pressure.

It is interesting to classify cables in accordance with the average stress at which they operate. I believe the average stress is of more significance than the maximum stress in this connection, and I therefore divided the cables into three groups.

The diameter of a cable must not exceed from 3 to 3½ in. but conductor sizes have been going up from the A. W. G. sizes to 500,000 cir. mils and larger and in the last few years working voltages have been rising rapidly. The net result has been a great decrease in the insulation thickness per volt; or in other words, a great increase in the electric stress. Confining our discussion to triplex cables, this increase may be seen from Table I:

TABLE I*

13-Kv. Cable

Insulation Thickness			Average Stress	
64ths on each cond.	Mils between cond.	Cm. between cond.	Volts per mil	Kv./Cm.
16				
14	437	1.111	29.7	11.7
12	375	0.953	34.6	13.6
11	244	0.874	37.8	14.9
10	312	0.794	41.6	16.4
9	281	0.714	46.3	18.2
(a)*				
8	250	0.635	52.0	20.4
7	219	0.556	59.4	23.4

27-Kv. Cable

Insulation Thickness			Average Stress	
64ths on each cond.	Mils between cond.	Cm. between cond.	Volts per mil	Kv./Cm.
23	719	1.825	37.5	14.8
22	688	1.746	39.2	15.5
20	625	1.588	43.2	17.0
19	594	1.508	45.5	17.9
18	563	1.429	48.0	18.9
(a)*				
16	500	1.270	54.0	21.2
15	469	1.191	57.7	22.6
14	438	1.111	61.6	24.3

35-Kv. Cable

Insulation Thickness			Average Stress	
64ths on each cond.	Mils between cond.	Cm. between cond.	Volts per mil	Kv./Cm.
30	938	2.38	37.3	14.7
23	719	1.83	48.7	19.1
22	687	1.75	50.9	20.0
21	656	1.67	63.4	21.0
20	625	1.59	56.0	22.0
19	594	1.51	59.0	23.2
(a)*				
18	563	1.43	62.1	24.5

*The lines (a) represent 1926 insulation minima.

Considering cables on a basis of approximate equality of stress, we have the groups shown in Table II.

TABLE II

	Group I		Group II		Group III	
	About 38 V./Mil or 15 Kv./Cm.		About 48 V./Mil or 19 Kv./Cm.		About 58 V./Mil or 23 Kv./Cm.	
Rated Kv.	64ths each cond.	Mils between cond.	64ths each cond.	Mils between cond.	64ths each cond.	Mils between cond.
13	11	344	9	281	7	219
27	23	719	18	563	15	469
35	30	938	23	719	19	594

The cables in Group I have, as a rule, given practically no trouble in operation; those in Group II have given some trouble, but those in Group III have very generally been failures both in this country and abroad.

The reason is not far to seek if we remember the classic paper presented to the A. I. E. E. in 1919 by G. B. Shanklin and J. J. Matson[1]. These authors showed that at a stress of about 19 kv. per cm., (48 volts per mil), ionization of thin films of entrapped air occurs, and it has since been shown that such ionization is destructive due to its promotion of internal surface discharges and chemical deterioration of the compound. Hence it is not surprising to find Group I, without ionization, giving perfect service, while Group II, on the verge of ionization, is in the balance, and Group III, with decided ionization, is almost invariably in trouble[2].

Mr. Roper's paper brings into prominence the significance of the time–breakdown voltage characteristic of a cable as a measure of its probable life in service. In this respect the paper deals practically exclusively with Group II and attacks the problem with rare insight and rationality. Mr. Roper shows that in order that a Group II cable may operate continuously, (*i. e.*, at a stress of 48 volts per mil or 19.0 kv. per cm.), it must have a time–voltage characteristic at least equal to that represented by the stresses shown in Table III.

TABLE III

Minutes	Factor = times the rated voltage	Average Stress	
		Volts/Mil	Kv./Cm.
5	7	323	128
16	6	278	110
65	5	231	91
360	4	186	73
1000	3½	162	64

He has shown also that certain manufacturers have solved the problem of making Group II cables which are entirely satisfactory and he knows exactly how to obtain these cables and exclude those which are of poor or uncertain quality. This is a big forward step which marks an epoch in the cable industry. It should be noted that as thick insulation is weaker for unit thickness than thin insulation, the higher voltage cables having heavier insulation will be somewhat more severely tested than those for the lower voltages.

Mr. Roper now takes a step which, in the light of events, may or may not prove to be justifiable, for he does not furnish his justification. I refer to the extension of his conclusions to cover all cables, including those in Group III. It should be noted that his time–voltage curves and tables are not expressed in terms of stresses but of factors of so many times the rated working voltage. When applied to cables in any one of my groups, this is equivalent to specifying stresses but if applied to another group, the factors do not correspond to the stresses, but to either lower ones in Group I or higher ones in Group III.

The question arises, which is the correct procedure to use, certain stresses or factors based on the working voltage? Let us see the voltages obtained by these two systems, as shown on Table IV, considering for brevity, only the 5- and 1000-min. tests.

TABLE IV

Group I

Rated Kv.	64ths each cond.	Kilovolts		
		Min.	by Stress	by Factor
13	11	5	111	91
	11	1000	55.5	45.5
27	23	5	232	189
		1000	116	94.5
35	30	5	304	245
		1000	132	122.5

Group II

Rated Kv.	64ths each cond.	Kilovolts		
		Min.	by Stress	by Factor
13	9	5	91	91
		1000	45.5	45.5
27	18	5	182	189
		1000	95.5	94.5
35	23	5	232	245
		1000	116	122.5

Group III

Rated Kv.	64ths each cond.	Kilovolts		
		Min.	by Stress	by Factor
13	7	5	71	91
		1000	35.5	45.5
27	15	5	152	189
		1000	76	94.5
35	19	5	192	245
		1000	96	122.5

A study of Table IV shows that if uniform quality of insulation is used in the three groups, so that the cables will stand the voltages calculated from the stresses in Table III, the test voltages for Group I will be far above those calculated by Mr. Roper's factors, whereas those for Group III will be equally below. Conversely, if the cables are to stand tests based on Mr. Roper's factors, the insulation of Group I may be of inferior quality to that of Group II, whereas the insulation of Group III must be of superior quality to that of Group II.

Group I cables, however, are used in order to obtain a greater safety factor, and it would be unfair for a manufacturer to give an inferior quality, as this would result in Group I cable users, who are paying for heavier insulation, being given no greater security than users of Group II. The obviously correct procedure for Group I cable users, is to rate their cables as of Group II in their specifications.

Experience tells us that 35-kv. Group III cables which pass the test voltages calculated on the stress basis, are not necessarily good cables. Cables impregnated with what would now be considered highly unstable oils, will pass both the 5-min. and the 1000-min. tests. These cables would not have passed the tests based on the working voltage factors. Unfortunately the 5-min. test calculated by the factors cannot be tried for the 35-kv. cables, as no known type of terminal will consistently permit 245 kv. to be maintained for five minutes.

Whether the tests calculated from Mr. Roper's factors will insure the permanence of Group III cables, in my opinion, is yet a matter of conjecture. If this opinion is correct, prudence would suggest that Group II design be adhered to in our general practise until Mr. Roper can do for Group III what he has so ably done for Group II.

Present indications point both to the necessity of departure from the present design of triplex cables, if Group III stresses are to become practicable, and to the exercise of special care in operation to insure the maintenance of saturation and suitable internal pressure, so that the original quality which is insured by factory tests, will not be destroyed by chemical deterioration and temperature variations in operation.

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[1] *Ionization of Occluded Gases in High-Tension Insulation*, TRANS. A. I. E. E., 1919, p. 489.

[2] The Shanklin and Matson paper deals with maximum stresses, but experience has since shown that the average stress is more significant in respect to ionization and breakdown. (See also the paper by P. L. Hoover, *The Mechanism of Breakdown of Dielectrics*, JOURNAL, A. I. E. E., September, 1926, p. 824.)

Departure from the present standard design of triplex cables will be necessary for Group III cables, because the crude, twisted, paper fillers constitute nuclei of ionization and consequent surface discharges which must eventually lead to failure. It is somewhat early to prophesy the designs of the future, but necessity will doubtless lead to their early perfection.

F. A. Brownell: These authors' idea of a quality rating seems to be the best that has been advanced for the rating of cables.

The engineers of our company were very much impressed with this idea and arrangements have been made with our inspection department to gather the necessary data while inspecting cable so that this method can be followed.

At the bottom of the fourth page there is a clause which implies that all the cable rated at 13 kv. in Table I, is made up with manila paper. I should like to ask the authors if this is a fact, or if any of the cable in this list is made up with wood-pulp paper?

When we stop to realize that the cable tested at the factory and the cable after it has been in operation should have different characteristics, it does not seem so strange that we are unable to devise factory tests that will eliminate failures after the cable has been installed. We have set up a recording expansion meter on our cables at different times and have found that the cable is constantly in motion either expanding or contracting and, due to the different coefficients of expansion of lead, oil, paper and copper, these several parts are moving at different rates.

S. J. Rosch: Mr. Roper's company is probably the only one that has attempted to shed a little light upon what transpires after a cable has been installed; but after all, the results obtained are peculiar mainly to this particular system, because for any given voltage service, their thickness of insulation is lighter than others and therefore the factors of safety in operation are not comparable; and yet it is the work of the Commonwealth Edison Co. that has supplied the proper impetus necessary for further progress in the cable field.

The statement that a cable in the factory is not the same after it has been installed is very true, since the different methods of pulling cables into a duct are bound to change the physical structure of the cable more or less and consequently change some of the electrical characteristics as well.

I believe in placing greater reliance in the testing of cables than of samples, because the results obtained on a sample may not be indicative of a weak portion which may actually exist in the full length from which it is taken. This was very clearly borne out in Mr. Lee's paper in 1925 in which he pointed out that eight sections of cable made under identical conditions gave entirely different results. These are facts which cannot be denied, and in my opinion the only solution is a continuation of the 15-min. voltage test on all reels of cable for operation at voltages above 15 kv.

The quality rating table is a very desirable criterion of cable quality; nevertheless the points just brought out by Mr. Lee are very pertinent. For example, cable *D* had practically the poorest rating on all counts save one and yet the weight attached to this particular phase caused it to have the highest rating of all. It seems that the method of evaluation must be in error when poor workmanship on insulation and filler, excessive tearing on bending test samples, and high dielectric loss can be outweighed by high dielectric strength tests on samples of cable. If this method of evaluation is correct, then we ought to recognize this fact by changing our standards on those counts which are apparently unnecessary. This may change the entire method of manufacturing cables in this country.

But has cable *D* had an opportunity to demonstrate its superior operating characteristics in the duct system? Will the tears produced in the bending test repeat themselves with tears produced while pulling in the cables into the ducts, and will these tears impair the life of cable *D*? Not until these questions have been answered, shall we know whether the present method of quality rating of cables is the correct one.

J. L. R. Hayden (by letter): The curve in Fig. 1 herewith showing disruptive voltage vs. time of application for breakdown appears to be representative of all solid insulations, and various experiments indicate that some such relation may also hold for gases and liquids. As to the relative values of voltage, time and curve slope, these may vary over a wide range, depending upon type of insulation under consideration. The time element over which this characteristic has been generally determined is from a few seconds to an hour.

The accompanying Fig. 2 showing current through the insulation plotted against time at constant voltage illustrates a characteristic which has been frequently obtained on solid in-

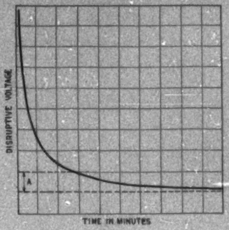

Fig. 1

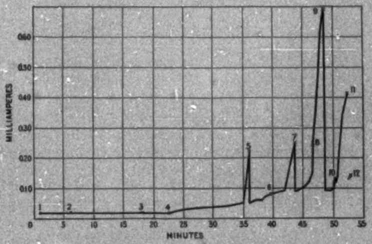

Fig. 2—Current-Time Curves, Continuous Voltage 10.9 Kv.—12 Electrodes—Total Current

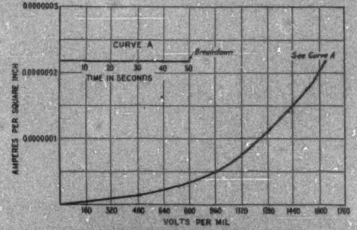

Fig. 3—Volt-Ampere Deterioration Characteristic of Black Varnished Cambric (0.012 in. Thick) at 25 Deg. Cent.

sulations of rather low specific resistance. Obviously, such a characteristic is obtained only when the voltage applied is in the range marked *A*, in Fig. 1. Such leakage currents are of the order of a few milliamperes at the highest, and it is observed that the current begins to increase rapidly (run away) at very low values of current. Because of these low current values and the rapidity with which the current runs away previous to failure, it is often found difficult to obtain this characteristic, especially on insulations of high specific resistance. For example, the magnetic oscillograph, when applied to the determination of this current before failure, will not usually indicate it, because the current is increasing at an enormous rate when it gets within the range of sensitivity of the oscillograph.

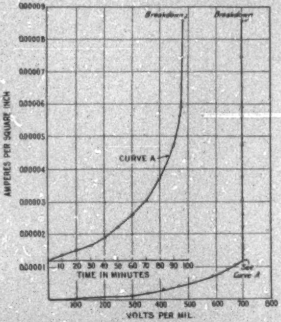

FIG. 4—VOLT-AMPERE DETERIORATION CHARACTERISTIC OF BLACK VARNISHED CAMBRIC (0.048 IN. THICK) AT 100 DEG. CENT.

The question of whether or not all failures are preceded by a rise in current is, of course, an unanswered one. Examination of Fig. 2 will reveal that disruption of the insulation occurred under several electrodes without a detectable rise in current preceding the failure. This may mean that no gradual rise of current took place, or that the values of current were too low to be indicated. Also, comparing Figs. 3 and 4 herewith, taken on varnished cambric, it is noted that, in one instance, a decided increase in current was measured before failure, while in the other the breakdown seemed to be instantaneous. In similar tests on impregnated paper cable insulation no slow current rises previous to disruption of the insulation were noted at 25 deg. nor 100 deg. cent. The results obtained are shown on the curves in Figs. 5 and 6, of this discussion.

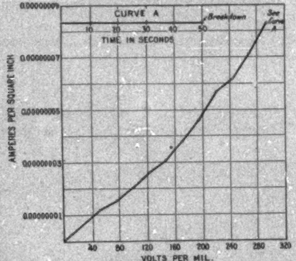

FIG. 5—VOLT-AMPERE DETERIORATION CHARACTERISTIC OF PETROLATUM IMPREGNATED PAPER CABLE INSULATION (0.115 IN. THICK) AT 100 DEG. CENT.

Attempts to substantiate the pyroelectric theory of failure by limiting this run-away current with a series resistance have usually been unsuccessful, because the current at the critical spot is a small proportion of the total leakage through the sample[3]. We succeeded in overcoming this difficulty in an electrode of small area, as the Nernst filament, and in another way with a large number of electrodes in parallel. Dr. Wagner's[4] demonstrations of the same characteristic involved the use of the wooden block electrode.

While various experimenters have obtained results which seem to indicate that the mechanism of failure may be something other than a heat phenomenon, it is important that due recognition be given to the difficulties which had to be overcome by Messrs. Hayden, Steinmetz and Wagner in their efforts to demonstrate its nature. Nor is it to be presumed that all insulation failures have this physical nature.

Very few data are available on insulation failures caused by high voltage transients of very short time duration, or upon the chemical or physical changes which may accompany long time applications of low voltages.

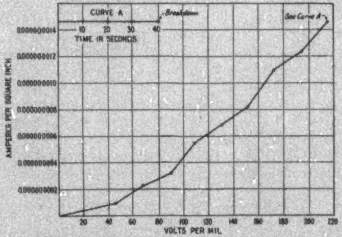

FIG. 6—VOLT-AMPERE DETERIORATION CHARACTERISTIC OF PETROLATUM IMPREGNATED PAPER CABLE INSULATION (0.130 IN. THICK) AT 25 DEG. CENT.

It appears that the cathode-ray oscillograph, because of its great speed of operation, may be of considerable value in obtaining information on the run-away currents and the mechanism of failure with high voltages of short time duration.

D. W. Roper: I wish to compliment Mr. Lee upon the very careful study he has given the paper; otherwise, he would not have discovered the weak point in the quality rating table, the weak points being that we have not quite discovered how to make a quality rating and an accelerated life test that would be equally fair to cables of low and of high dielectric loss. There must be some change apparently both in the quality rating tables and in the accelerated life test, that is, the voltage at which the test is made in order to do that. We have not yet quite discovered how it should be done. That accounts for some of the variations to which Mr. Lee called attention.

Mr. Brownell inquired for some information regarding the cables which contained wood pulp paper and manila paper. The information was purposely omitted. The object of the paper is to get better cable by cooperation with the manufacturers and not to boost or discredit any manufacturer.

Several of the speakers have mentioned the plan of incorporating the results of accelerated life test in the quality rating tables. To me it doesn't seem desirable to do that but it seems preferable to use the accelerated life test as a parallel method of quality rating rather than to attempt incorporating the results of the accelerated life test in the quality rating table. Apparently we shall do better if we carry along the two methods of quality rating independently and see that we get results which are in accord rather than to put the one result in the other table.

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[3] Insulation Failure—A Pyroelectric Effect: Hayden and Steinmetz, *Electrical World*, October 21, 1922.

[4] *Physical Nature of Electric Breakdown of Solid Insulations*: Dr. K. W. Wagner, A. I. E. E. TRANS., 1922, p. 288.

Mr. Del Mar referred to the subject of high-voltage cables and stated that for the very high-voltage cables we must either use a different quality of insulation or a different design. I think he is quite correct in that point. In my opinion, for each quality of insulation there is a maximum operating voltage for which that quality should be used. If we wish to go to higher operating voltages, we cannot do so by merely increasing the thickness of the insulation, but we must improve the quality.

One of the speakers mentioned the point that in making the test on the full reels, we may cause incipient troubles in the cables which will later develop in service. Our tests indicate that with the full-reel test we are now requiring, high-voltage test of each reel at the factory, and with the proposed requirements which are given in the paper, the full-reel test as now being made will take about 2 per cent of the life of the cable if the cable is exactly on the line of the suggested requirements, while it will actually take only about 1 per cent of the life of the average quality that we are getting. We are getting above the suggested requirement. I think the manufacturing companies are willing to sacrifice 1 per cent of the life in order to assure themselves that the cable is of proper quality. Mr. Rosch, I believe, mentioned the fact, however, that we should test the cable itself rather than samples, and in the paper is mentioned a proposed test, or a desired test should be made of each reel, if we could devise the test, so that we could from the test determine the minimum quality of the insulation of each section of cable as accepted by the purchaser at the factory.

Mr. Rosch commented upon cable *D* and mentioned its long life in spite of some deficiencies in workmanship as recorded in the early portion of Table I. It may not be proper to talk about the results of tests on the poorer grades of cable, but certainly no one can object to our giving a few details about the best cable.

This cable *D*, on the accelerated life test, showed a number of hot spots and we were somewhat curious to discover the causes of those hot spots. We opened up a number of them and what did we find? We found broken fillers, wandering fillers, missing fillers, and all of those other points that are given in the early part of Table I. My reaction is quite different from Mr. Rosch's on the point. To me it indicates that when this manufacturer *D* has been able to eliminate those defects in workmanship, which he undoubtedly will do, then what a remarkably high quality of cable he will secure!

Herman Halperin: The data presented by Mr. Clark regarding the slope of the voltage–time curves in Fig. 11 agree with our data in that the voltage varied inversely with about the seventh root of the time to failure for cables impregnated with a grease or a heavy oil and tested at room temperature. When an oil is used for the impregnating compound, the slope of the curve is different, according to Mr. Clark, and it becomes necessary to change the Curve *S R*.

Regarding conclusion No. 9, Mr. Atkinson's point that the rating tables can be used only as "a very rough approximation" until better tests are available for covering the subject of uniformity, this does not agree with our experience, since the rating tables have been more consistent with our operating records than Mr. Atkinson's statement indicates as possible. An attempt has been made to cover the subject of uniformity in accordance with the best practise of today; and as has been indicated in conclusions Nos. 5 and 6, uniformity is very essential to obtaining the best cable.

Cable specifications allow cable to be installed with a minimum radius equal to six times the diameter of the cable, and it is the practise of this company to make this radius in installation eight to twelve times the diameter of the cable.

We agree with Mr. Atkinson that the accelerated life test alone cannot be used as an indication of the quality of cable, but it has been our experience that the rating table and life test together give a fairly accurate prediction of the quality.

In regard to the point by Messrs. Atkinson and Simons, that with thicker insulations the specifications need not be so stringent, it might be mentioned that considerable trouble has been experienced in this country with the so-called thick insulations. The Commonwealth Edison Company, which has reduced the thicknesses of insulation considerably during the past seven years, has found that the quality of the newer cable with thinner insulation is such that for a given voltage cable, the breakdown voltages obtained on samples have increased from 60 per cent to 120 per cent. Another way of stating the improvement in quality is that on 5000-volt, four-conductor cable, with 5/64-in. insulation around each conductor and 3/64-in. belt insulation, the voltage tests are higher now than were considered sufficient a few years ago for 10-kv cable.

If we were to continue using thick insulations in this country there would be little progress towards obtaining satisfactory extra high-tension cable, such as 35-kv., three-conductor, and 75-kv. and higher, single-conductor cable. The important thing for high-tension cable is to have high quality insulation. Then there will be considerably less liability of local spots of poor quality which contain germs of failures to take effect almost regardless of the thickness. With good insulation, it is an economic waste to have the extra great thicknesses that have been and still are used by some companies.

With reference to Mr. Simons' point that manufacturers have to deal with a large number of sizes while our paper in arriving at certain test requirements presented data mainly for one size, our procedure is a common one in electrical practise. The knowledge of the variation of the requirements due to different sizes of conductor is entirely too indefinite now to arrive at any scale of test values which might be arranged according to the size of the conductor. From a practical standpoint the test on 500,000-cir. mil cable seems to be quite representative of the various sizes of commonly used conductors for transmission cables; that is, sizes from No. 0 to 750,000 cir. mils.

In regard to Mr. Dunsheath's statement that one type of defect in the cable may be so bad as to cause the cable to fail, it should be pointed out that the rating tables are for cable that has passed the specifications and has been shipped. Therefore, the cable would not be very likely to have a bad defect, such as a large number of registered tapes mentioned by him, as the specified tests at the factory would very probably cause such cable to be rejected. Furthermore, it has been our experience that when the cable of a certain manufacturer has more registered tapes than allowed by the specifications, this deficiency is found in several lengths and is revealed by poor results in several tests. In such a case, the poor quality of the cable would be indicated on the items concerning workmanship of the insulation, increase of power factor with voltage and puncture-voltage tests.

Dr. Wiseman's point that the tearing in the bending tests is given as much as one-half of the weight for thoroughness of impregnation, (for 13-kv. cable), appears to be in error. In item 8, eight points are given for the increase in power-factor test (ionization), which test is generally considered the best electrical test known for thoroughness of impregnation. This weight of eight points and the weight of eight points for the visual test for thoroughness of impregnation make a total of 16 points, which is four times the points for tearing.

His point that the power factor always varies with voltage is not borne out for us by commercial tests made on several makes of high-tension cable.

Dr. Wiseman's statement that the stability test for compound cannot be used until samples can be prepared with no air present does not seem to be warranted by Mr. Farmer's test; but be that as it may, a stability test for compound in high-tension cable is necessary, and no doubt a test satisfactory to all parties concerned will be evolved shortly.

Dr. Wiseman and Mr. Del Mar have referred to dielectric stresses in judging cables. One of them prefers to use maximum dielectric stress in discussing the stress in the cable, while the other prefers average stress, thus indicating the indefiniteness of judging the stress in the cable. From the standpoint of operation, what one wishes to know is the factor of assurance for the cable for a given service. For example, how many times the normal operating voltage will a sample of the cable stand for eight hours? The operators see no reason why extra high-tension cable should be purchased with a considerably lower factor of assurance than is obtained for cable of the moderate voltages, such as 15 kv. As has been pointed out, the quality of the insulation for 35-kv. and 66-kv. cable and even higher voltages should be such that it will give a proper factor of assurance. Two of the discussors have stated that potheads are not available for testing 35-kv. cable at seven times its rated voltage for 5 min. If this is true, then the test can be made at a lower voltage for which potheads are available; for instance, at six times the rated voltage for 16 min., as was suggested in the paper.

The curve *SR* indicates the minimum test requirements for satisfactory cable, and if a purchaser adds extra insulation for 13-kv. cable, (such as indicated in Group I of Table II in Mr. Del Mar's discussion), then the test requirements should be increased above the line *SR* in order to get the extra factor of assurance that is desired.

Mr. Del Mar's statement that cables in Group III of Table II "have been very generally failures abroad" is not warranted by our information concerning 35-kv. cables which have 19/64-in. insulation or less on each conductor. There are many cases, in England, France, and elsewhere, of these cables having been in successful operation from a few years up to ten.

It is agreed, with Dr. Wiseman, that the evaluation of a step test to an equivalent test at one voltage is not easy, but it appears possible to approximate the step test. In order to do this for any make of cable, it is necessary to know the nature of its voltage-time curve. After this curve is determined then the step test can be drawn in on the curve sheet on log-log paper by the following method:

Corresponding to the first step, draw a horizontal line at the ordinate of the length representing the duration of this step. At the end of this line, draw another line parallel to the voltage-time curve and as high as the voltage of the next step. From the end of this diagonal line, draw a third line representing, in length, the duration of the second step. This process is continued until the last step is drawn and the result is the equivalent test given for the voltage of the last step. The abscissa for the resulting point at the end of the last line represents the equivalent time. If this test is to be evaluated for a test at another voltage, then it can be done by drawing a line through this last point parallel to the voltage-time curve, and reading the values from this new voltage-time curve for this particular test.

As to Dr. Wiseman's discussion of the general distribution of hot spots, in the high-voltage tests mentioned in the paper, the cable usually had a few pronounced local hot spots which were from about 5 to 74 deg. cent. above the temperature of the adjacent cable sheath.

Mr. Burd's statement that different operating results have been obtained with cable of the same quality when installed in different operating systems presupposes that the insulation thicknesses were the same for given voltage cables installed in these different cable systems. From general information this does not appear to be correct. No doubt, the treatment given the cable during installation and operation will have some effect on the number of failures of the cable. It has been the aim of the company with which we are associated to install and operate the cable in accordance with the best practise. This kind of practise appears to be followed by most companies, especially the relatively few large operating companies, which are the ones that use most of the underground cable. In our experience of the cable failures, it has been frequently found that if the insulation, for instance, had been thoroughly impregnated, the failure would not have occurred.

In regard to Fig. 12, Mr. Lee has pointed out that the points for cables *A*, *B*, and *C* did not come as close to the curve as they might. The durations of the accelerated life tests on these three makes of cable were in consistent relative order as compared with the ratings in the table. The ratings are based on the large number of various tests covering all of the 50,000 ft. or more of the cable for each manufacturer, while the accelerated life test results are based on tests on a few samples. It has been usually found in dielectric-strength tests that there will be fairly large variations in the results and obtaining a life of 115 hr. for cable *A*, which was rated at 81.9, and only obtaining a life of 45 hr. for cable *B*, which was rated at 80.3, would not usually be considered a great discrepancy for such long tests. The voltage-time curve for these tests indicates a difference in voltage rating of only about 12 per cent.

Regarding the effect of dielectric loss on the life of the cable, the 35-kv. cables with high power factors of the dielectric will have shorter lives than would cables with low power factors, but with the same qualities in other respects. On the 13-kv. cables the makes that had the highest dielectric loss were the ones that had the longest lives, indicating high qualities in other respects.

Several discussors have made unfavorable comments about taking the variation of insulation resistance as an indication of uniformity. To begin with, this item has been given a maximum weight of only six points. The experience has been that the makers who have the greatest variation of insulation resistance also have other troubles. Manufacturers *B* and *C* in Table I, and manufacturer *Q* in Table II, had the highest variations, respectively, in the two tables, and these manufacturers are the ones who had the most trouble at the factory and the lowest quality insulation in other respects.

Mr. Hayden's information regarding the mechanism of failure of insulation is interesting, and it is agreed that more such information based on testing is needed.

Tests of Paper-Insulated High-Tension Cable

BY F. M. FARMER[1]
Fellow, A. I. E. E.

***Synopsis.**—The paper deals with those tests which are involved in specifications. The discussion is from the standpoint of the purpose and significance of the various standard tests which are made rather than that of the technique of the details of testing. A considerable number of data are given.*

The subjects treated are purpose and importance of tests of cable; insulation resistance test; high-voltage test; dielectric loss and power-factor test; "ionization" test; bending test; accelerated life test; preparation of samples for high-voltage tests and tests of components.

The possible significance of wide variation in some of the properties of cable is discussed. The relation found between time and voltage based on a large number of tests of both three-conductor and single-conductor cable is discussed at considerable length. A test for stability of the impregnating compound is described as is also a proposed standard load for dielectric-loss testing. Some data are given showing the effect of repeated bending on lead and how that effect is influenced by the tensile stress in the lead.

The paper concludes with a list of the ways by which, in the author's opinion, progress can be made in the improvement of the quality and the design of paper-insulated high-tension cable.

* * * * *

INTRODUCTION

TESTS which are made on paper-insulated high-tension cable may be divided into the three following general classes:

a. Research tests for the ultimate purpose of advancing this branch of the electrical industry by means of improvements in quality of materials and manufacturing processes, or improvements in cable design.

b. Tests on the commercial product at the factory to determine compliance with the manufacturer's standards or the purchaser's specifications.

c. Tests on cable after it is installed, including acceptance tests on new cable, periodic tests and fault location tests.

It is not feasible to cover even superficially all of these tests within the limits of one paper. It is, therefore, proposed principally to discuss those standard tests which are involved with specifications and certain other tests which, while not yet in specifications, aid in determining the quality of cable and the improvements which have been made therein. Furthermore, it is not proposed to discuss to any great extent the details of the technique of testing cable, but rather the purpose and significance of tests.

PURPOSE AND IMPORTANCE OF TESTS OF CABLE

Ostensibly tests in connection with the purchase and acceptance of cable are made for the purpose of determining whether or not the material complies with the specified requirements. The basic object, however, is to determine the suitability of the cable for the service to which it is to be applied. The specifications are, in effect, simply the engineer's method of describing cable which it is believed will be suitable for that service.

In no class of electrical apparatus does more reliance have to be placed on tests to determine suitability than in the case of cables. Insulation is a more or less important feature in all electrical apparatus but in high-tension cables it is *the* factor. The insulation in a high-tension transformer for example is, of course, of prime importance but the amount involved in the largest unit is insignificant compared to that in a cable line of average length. Consider, for instance, the 66-kv. line of the Cleveland Electric Illuminating Company which consists of two three-phase circuits of three, single-conductor cables each. This line is about eight miles long and contains, therefore, about 50 miles of cable. The mean area of the insulation of this cable is about 123,000 sq. ft. or a little over 2.8 acres. While a few hundred sq. ft. of insulation are continuously under stress in the transformers at each end of this line there are nearly three acres of insulation buried in the ground, every square inch of which is expected to withstand continuously all day and every day a potential of 38 kilovolts.

Unfortunately we know far less about insulation than about any of the other essential classes of materials in electrical apparatus. In the case of cables, we impose particularly severe conditions because we effectually cover the insulation with a sheathing of lead so that no visual inspection whatever is possible and the sins of omission and commission are thoroughly buried. We must, therefore, determine the suitability of this important part of a cable very largely by tests. Perhaps a better way of putting it would be to say that the best we can do is to make certain tests with the hope that, in the light of our present knowledge, they will eliminate cable which would be unsuitable for the intended service as prescribed by the specifications.

Acceptance tests of cable, together with the associated inspection, can be made to serve another important purpose and that is improving quality. By making tests and reporting results completely and in a uniform, systematic manner, a mass of data are available which can be pooled. Summaries of a large number of test results which are thus combined will have a maximum of reliability, and deductions therefrom should be of great value to the cable engineer in the exercise of one of his important functions, *i. e.*, the raising of standards of quality through revision of specifications.

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INSULATION RESISTANCE TEST

The test for insulation resistance is one of the earliest made on electrical conductors. In the early days of the art when only low voltages were involved it served very satisfactorily as a means of detecting sections of wire or cable which would be likely to prove unsatisfactory in service. At such low voltages the inherent quality of the insulation did not have to be of a very high order and the principal things to be guarded against were mechanical defects and accidental injuries. The insulation resistance test served very well in weeding out wire or cable defective from these causes.

With the advent of higher voltages it was found that factors other than mechanical defects and injuries have to be guarded against to avoid insulation which would soon prove unsatisfactory in service. It was found that insulation resistance tests were of very limited usefulness in determining the inherent value of the insulation and that other tests were much more important in insuring a satisfactory product. It is now generally recognized that the number of megohm miles of a cable has little significance and no one can predict that a cable with high insulation resistance will give better service than one with low resistance or vice versa.

While the actual value of the insulation resistance in megohm miles may not be significant, the *variation* in the megohm miles between different sections of the same cable line, which are supposed to be identical in quality, may be very significant. The variation in the quality of high tension cable is one of the greatest problems in the cable art for it is the weakest link in the chain that determines its strength and the problem of producing cable which is uniform from section to section or even foot to foot has yet to be solved. Obviously, cable having the least variation in quality is better than one the quality of which, although it may have a higher average value, varies over a wider range.

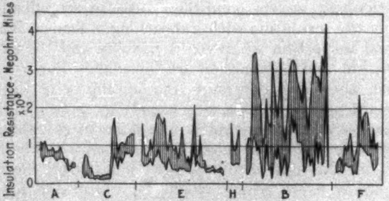

FIG. 1—INSULATION RESISTANCE OF 13-15-KV. CABLE INSPECTED DURING 1925

Upper line, maximum insulation resistance in each lot inspected
Lower line, corresponding minimum insulation resistance

The value of the resistance of a given section of cable depends upon the kind of paper, the kind of compound and the physical condition of each—particularly in regard to moisture—in which the drying and saturating processes have left it. It is only reasonable to suppose, therefore, that marked variations in the resistance between different sections means a difference due to some cause in the manufacturing process which may have a marked influence on the quality and therefore the serviceability of the cable.

Fig. 1 has been prepared from insulation resistance measurements made in the course of the routine inspection of high tension cable at six factories during 1925. The upper heavy line connects the values of maximum resistance found in each lot of cable and the lower heavy line connects minimum resistances. Each light vertical line connects the maximum and minimum values of resistance found in each lot submitted. The shorter the light vertical line is, the smaller is the variation in a single lot. The smoother the heavy lines are, the smaller is the variation between lots. It is evident that make "B" takes first place for variation while make "A" is conspicuously uniform, with "C" a close second.

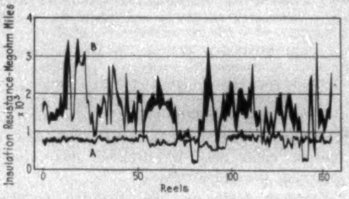

FIG. 2—[1]INSULATION RESISTANCE OF 15-KV. CABLE

B. Maximum and minimum insulation resistance of 150 consecutive reels presented by manufacturers *B*
A. Corresponding values for manufacturer *A*

Fig. 2 shows the difference in uniformity of insulation resistance in a still more pronounced manner. One hundred and fifty-five consecutive sections of cable submitted for inspection by makers "A" and "B" respectively have been plotted—the upper boundary of the black area being the maximums of the three conductors in three-conductor cable and the lower boundary the minimums.

Just what the practical significance of these differences in uniformity of insulation resistance is, we do not know. With respect to quality of the cable, these differences may mean much or they may mean little but it is submitted that until we have some means of measuring the quality of insulation more directly, we are justified in concluding that other things being equal, the cable with the smaller variation is the better.

HIGH-VOLTAGE TESTS

High-voltage tests of various kinds have long been considered the most important of all that are made on high-tension cable. They may be classified into (a) fixed or stated voltage tests, (b) momentary breakdown voltage tests and (c) time breakdown voltage tests.

(*a*) *Fixed-Voltage Tests.* A fixed-voltage test consists in the application of a prescribed voltage for a prescribed length of time to each reel of cable in the case of acceptance tests at the factory, or to an entire line of cable in the case of installed cable.

Volumes on the discussion of this subject of fixed-voltage tests of insulation for all purposes, and cable in particular, have been printed. How high should the test voltage be and how long should it be applied to insure that the cable meets the desired standard without injuring the insulation? Which is the more effective, high voltage for a short time or a lower voltage for a longer time? These are questions which are still answered by opinions to a large extent. We do not as yet have all the facts definitely established which are essential to the formulation of the true answer.

What is the purpose of this particular voltage test? We encounter two serious obstacles when arriving at the suitability of a piece of cable. In the first place, the insulation is buried out of sight and in the second place, it is far from homogeneous. A member in a steel structure can be put in place with almost complete confidence that it has 100 per cent reliability if it passes a visual inspection. Furthermore, we know as a matter of established fact that the strength of the member in question is uniform throughout, that a single test specimen will adequately represent it and that we can depend on that member having a certain factor of safety without actually loading it. But in the case of a length of cable we cannot judge by inspection whether it will serve the purpose intended nor can we depend absolutely upon a test to destruction of a specimen because of the lack of uniformity. We must "load" every foot of cable, that is, subject it to a voltage well above the rated voltage—if we are to be assured of a reasonable "factor of safety."

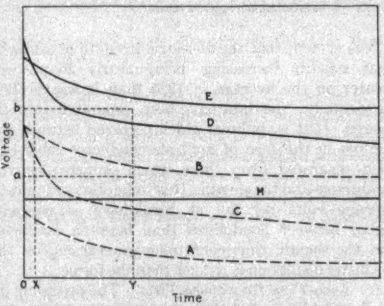

Fig. 3

The test should preferably accomplish two things,—eliminate cable the insulation of which is electrically weak because of accidental defects, and eliminate cable which is electrically weak because of the inherent low quality of insulation.

One way of visualizing the conditions with which we have to contend is indicated in Fig. 3. It is well established that the voltage which a given piece of insulation will withstand is greatly affected by the test conditions, one of the most important being the time of application of the voltage. Each portion of insulation has its own voltage-time characteristic which can be indicated in the general manner shown by the curves in the diagram.

Now consider a section of cable with an accidental defect of a mechanical nature such as external damage to the insulation, registration of a large number of tapes, cracked or torn insulation, etc. The breakdown voltage of the section of cable will, of course, be that of the insulation at this point, and if the injury is serious, the breakdown voltage will be much below that of the rest of the section. Nevertheless, that spot has a time-voltage characteristic which we can indicate by curve *A*. But a moderate test voltage, *a*, even if applied only momentarily, will cause puncture and therefore eliminate that section. If the defect is less extensive so that a lesser thickness of the insulation is affected, the curve may be something like *B* in which case, a test at *a* volts even if applied for *y* time, will not eliminate it. Nevertheless, its so-called permanent breakdown voltage, *i. e.*, the maximum voltage which it will withstand indefinitely is not sufficiently above the operating voltage, *M*, to provide the necessary factor of assurance. On the other hand, a test at *b* volts, even if only momentarily applied, would eliminate it.

Next, assume a spot which is inherently electrically weak due, for example, to incomplete saturation or overheating at a lead press mark. The insulation at that point may have a high momentary dielectric strength without a correspondingly high endurance strength due to high dielectric loss which causes failure in accordance with the thermal theory of Steinmetz and Wagner. The characteristic curve at this point might be of the shape indicated by curve *C*, in which case a test at *a* volts for a time *x* would not eliminate the section but *a* volts for *y* time would.

Normal cable may have a characteristic like curve *E* or curve *D*. The former is obviously the preferable type of curve. Both curves are higher than those for the other cases, consequently the test voltage should be adjusted to the normal cable and not only to the abnormal cable. The problem is then to determine the voltage which should be applied to insure as high a curve *E* as possible, and for as long a time as practicable in order to eliminate cable with too much of the characteristic indicated by *D*. This would be accomplished by applying *b* bolts for *y* time.

The practical question is then, what should the voltage be and for how long should it be applied? This question has been with us since the beginning of the industry and is likely to remain with us for some time to come. What we would like to have is *definite knowledge* of the excess over the rated voltage which a cable will withstand indefinitely. But it takes hours to reach the flat part of the time-dielectric strength curve, so that such a test on every section of cable is quite impracticable. On the other hand, too high a voltage may more or less damage perfectly good cable so that its endurance or permanent dielectric strength is reduced. We are accumulating knowledge of the relation between dielectric strength and time so that some day we may be able to set up an equivalent short time test based on facts, but at present our practise is based very largely on the coordinated opinions of engineers dealing with these matters.

It is to be noted, however, that the trend in fixed voltage tests is very definitely upward both as to voltage and time. For instance, the A. I. E. E. rules adopted in 1911 prescribe double normal rated voltage for one minute. The standards of 1922 prescribe two and one-half times operating voltage for five minutes while those adopted August 6, 1925 prescribe three times the rated voltage plus 2 kv. for five minutes. The most recent specifications in wide use, namely, those of the Association of Edison Illuminating Companies, prescribe test voltages substantially in accord with the latest rules of the A. I. E. E. but the test period is fifteen minutes instead of five minutes. So, despite our lack of facts and figures, general knowledge and experience has gradually pushed up the test voltage and lengthened the time of application.

(*b*) *Short-Time Breakdown Voltage Test.* The fixed-voltage test will not of course tell us what factor of assurance we are getting. We try to get some light on that question by means of short-time breakdown voltage tests, *i. e.*, by testing short samples to destruction. The light we get, however, is rather feeble for after we have obtained the so-called momentary puncture voltage, we cannot coordinate it with the rated or operating voltage of the cable. This is due not only to the dielectric strength-time relation referred to in the preceding discussion, but to the variation in the material. The latter point could be taken care of by testing enough samples but any such procedure sufficiently extensive to truly represent the product would be quite impracticable. Some day perhaps manufacturers will be able to produce cable which will be absolutely uniform from foot to foot. Then one sample will be as good as an infinite number and we will have only the task of correlating the result of the short-time test (which for obvious reasons is the most practical one to make) with an infinitely long time test corresponding to the desired operating requirement, namely, practically infinite life at operating voltage.

In order to eliminate partially the indefiniteness of a "rapidly applied" voltage test—the time effect being most marked at the higher voltages obtained in such a test—it is customary to use a step procedure, *i. e.*, increase the voltage by ten per cent steps, holding each for a prescribed period such as thirty seconds or one minute. While we may not be able as yet to state from such tests made on samples just what margin of assurance we may have in the cable from which the samples were cut, they do give us data which indicate the *trend* at least of this very important factor.

Table I shows a summary of a large number of tests made in 1923, 1924 and 1925 respectively on cable manufactured under the N. E. L. A. specifications. The results are reported in average volts per mil. Cables of all sizes and voltages are included, so that some common basis must be used. Whether or not the average stress at breakdown is a true measure of the dielectric strength of the insulation need not be discussed here. For *general comparisons* it serves the purpose and has the merit of simplicity and convenience.

TABLE I
DIELECTRIC STRENGTH OF CABLE

Year	No. of Tests	Average volts per mil at breakdown			Per Cent inc. in Average over 1923
		C-C	*C-S*	Average	
1923	260	291	283	287	..
1924	290	307	284	306	6.6
1925	106	354	357	355	23.7

"*C-C*" – between conductors.
"*C-S*" – between conductors and sheath.

It is evident that the dielectric strength of cables has been steadily increasing—being nearly 25 per cent greater on the average in 1925 than it was in 1923. Incidentally, the data have been separated into two groups—that is, failures which occurred between conductors in the case of multiple conductor cable have been separated from those which occurred between conductors and the sheath. It is to be observed that the average volts per mil at breakdown is somewhat higher between conductors than between conductors and the sheath, thus confirming general opinion that the latter insulation is weaker than the former.

(*c*) *Long-Time Breakdown Tests.* The preceding discussion has indicated that one of the most important things which we would like to know is the *maximum* voltage which each length of cable will withstand indefinitely. The only way to get this information is by a series of tests to destruction by subjecting several samples of the same cable to different voltages continuously until failure occurs. Sufficient tests of this kind will give data from which a reasonable reliable answer to the above question can be made. Such tests are, however, very expensive as much time is required and also a considerable amount of costly cable. However, the significance and importance of such information is being recognized and it is felt that we have reached a step in the development of the art where this knowledge is essential to further intelligent progress. The cost of acquiring the necessary data is secondary in view of the importance of cables in the electrical industry and the large amounts of money being invested in them.

A considerable amount of this kind of testing is being done—some by the manufacturers and some by the utility companies, particularly the Commonwealth Edison Company of Chicago. Much expensive research work along these and other lines dealing with cables is being done by this company under the able direction of D. W. Roper and, in accordance with the broad-gage policy of this company in such matters, the results are being made available to the industry.

Electrical Testing Laboratories are also doing a considerable amount of this class of cable testing and some of the data shown in the accompanying diagrams are made available through the courtesy of its clients, particularly the High-Tension Cable Committee of the Association of Edison Illuminating Companies under whose direction a series of tests is being made looking to the establishment of a tentative standard accelerated life test for specification purposes—a subject which is discussed in another part of this paper. The tests to which Figs. 5, 6 and 7 apply were made on three-conductor cables of various sizes and voltages and all but seven of the results shown were obtained with three-phase potential applied to the sample continuously except an hour or so a day when power-factor measurements were being made, and also Sundays and holidays. In the case of the seven exceptions, the sample was also loaded, *i. e.*, three-phase current was circulated through the conductors a portion of each day.

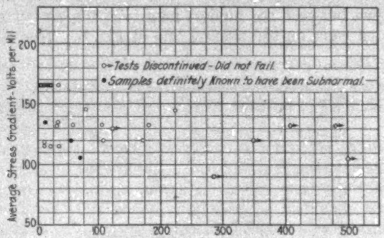

FIG. 4—CABLE LIFE TESTS (TIME-STRESS DIAGRAM) THREE-PHASE TESTS, THREE-CONDUCTOR CABLES

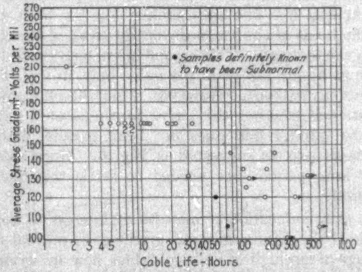

FIG. 5—CABLE LIFE TESTS (TIME-STRESS DIAGRAM) THREE-PHASE TESTS ONLY

The length of the sample in these tests varied from about 12 ft. to about 20 ft. in active length, *i. e.*, end to end of the sheath. The test was made by connecting a sample to a bank of three 20-kv-a., 100-kv. transformers connected in *Y* with the neutral grounded. Where current was circulated, this was provided by a current transformer in each phase insulated from the ground.

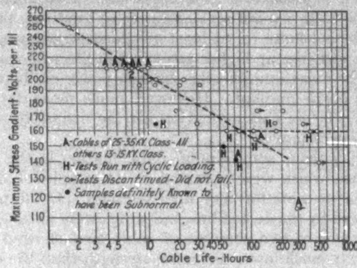

FIG. 6—CABLE TESTS (TIME-STRESS DIAGRAM) THREE-PHASE TESTS, THREE-CONDUCTOR CABLE

Graphic methods are, of course, the most convenient means of arriving at conclusions in tests of this kind but where a number of sizes and thicknesses of insulation are involved the selection of the best graphical method is not easy. Fig. 4 is a plot of thirty time tests using rectangular coordinates—*average* stress in volts per mil between conductors against time in hours. It is evident that the points are so scattered that no definite trend can be determined. Fig. 5 shows the same data plotted on logarithmic coordinates. Here the points are still rather badly scattered but, while it is difficult to determine the law, a straight line probably could be drawn which would reasonably represent the mean.

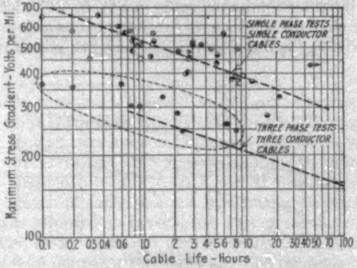

FIG. 7—CABLE LIFE TESTS (TIME-STRESS DIAGRAM)

Note: Points show single-conductor tests only. For three-phase test points see Fig. 6. Designations identify makes of cable tested

Going one step further and plotting the maximum stress gradient at the conductor surface instead of average stress gradient a more definite trend is indicated as shown in Fig. 6. Here it will be noted that while the data are quite insufficient to justify anything like a final conclusion, the data that we do have suggest a more or less critical point in the life curve beyond which the cable will have indefinite life. That is, the broken line which is a reasonable mean of all of the points, suggests that at a stress lower than about 150 or 160 volts per mil maximum stress gradient, the life will be indefinite (or at least will decrease at a very low rate) and that at higher stresses the life will vary inversely something like the seventh power of the maximum stress gradient. However, much more data are needed before this indication can be definitely established or disproved.

Fig. 7 shows some similar data, plotted in the same manner, which were obtained on 44 samples of single-conductor cable for very high voltages. If the points enclosed by the dotted line and which were obtained on specimens of two samples of cable that showed consistently much lower dielectric strength than the other samples, are omitted, a fairly definite straight line can be drawn, as indicated by the broken line. Seven makes of cable are represented in these tests with two different sizes of conductor and five thicknesses of insulation.

Whether or not a critical point exists here also is not determinable because we have not as yet enough data, specially long-time data. However, it will be observed that the slope of the curve as plotted is the same as that for the three-conductor cable in Fig. 6. The left-hand portion of the latter curve extended backwards is shown in Fig. 7. In other words, the seventh power relation between the maximum stress gradients is again suggested.

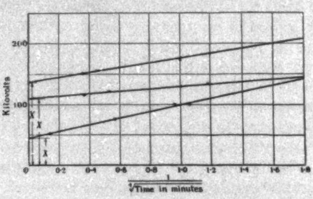

FIG. 8—TIME-DIELECTRIC STRENGTH RELATION

Dunsheath, in a recent paper,[2] expresses the breakdown voltage versus time relation found in some tests of cable in the manner employed by Peek,[3] in his experimental work on impregnated paper between concentric cylinders, but Dunsheath indicates an advantage of that method which is shown in Fig. 8, reproduced from his paper. The breakdown voltage is plotted against the inverse fourth root of the time and prolonging the curve to the infinity ordinate gives the apparent voltage which the particular cable will withstand indefinitely.

Unfortunately, the cable data are not given in Dunsheath's paper so that comparison cannot be made with the data given in the preceding discussion. The data for the three-conductor cable, Fig. 6, have, however, been replotted on this basis and are shown in Fig. 9 where the ordinates are maximum stress gradients on an equal-part scale and abscissas are hours on an inverse fourth-root scale. The heavy broken line is drawn between the highest point and the lowest point which were on the sloping line of Fig. 6. This curve gives a maximum gradient for indefinite life of only about 105 volts per mil. But it is at once evident that the curve does not fit the points on this plot as well as it did in Fig. 6. The light broken line has been drawn on the basis of the mean deviation of all of the results from the average. This second curve cuts the ordinate scale at about 115 volts per mil,—only 10 volts above the other intercept. However, it may be very properly questioned whether the curve continues on a straight line to infinity and does not bend parallel with the abscissas axis as suggested in Fig. 6. More data for very long periods are needed to settle this question definitely.

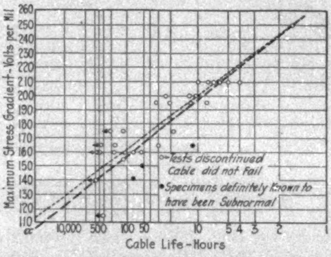

FIG. 9—CABLE LIFE TESTS (TIME-STRESS DIAGRAM) THREE-PHASE TESTS, THREE-CONDUCTOR CABLE

In this connection it may be of interest to note in Table II the maximum stress gradient at normal rated voltage of our highest voltage cables now in service or being manufactured. These are all single conductor cables and the figures are therefore comparable with Fig. 7.

TABLE II

City	Rated Kv. between phases	Thickness of insulation, inch	Stress Gradient volts per mil		Remarks
			Average	Maximum	
Cleveland	66	30/32	41	79	In service
Philadelphia	75	30/32	46	82	Being installed
Philadelphia	75	26/32	53	89	On order
New York	132	23/32	106	160	Being manufactured
Chicago	132	23/32	106	160	Being manufactured

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[2] "Dielectric Problems in High Voltage Cables," P. Dunsheath, *Journ.* I. E. E., Jan., 1926, p. 97.

[3] "Dielectric Phenomena in High-Voltage Engineering" F. W. Peek, Jr., p. 179.

DIELECTRIC LOSS AND POWER-FACTOR TEST

The practical significance of the dielectric losses in the insulation of cable was not fully appreciated until six or seven years ago when an epidemic of service failures led to the recognition of the dielectric loss type of failure. Bang and Louis[4] and other contributors to the TRANSACTIONS of the Institute have discussed the influence of dielectric loss on the carrying capacity of cables. Roper[5] has given concrete evidences of cable failures due to excessive dielectric loss. However, these losses have been so much reduced that they are no longer a serious element in cable operation. The pronounced decrease in dielectric losses which has taken place in the past four years is indicated in Fig. 10 showing typical curves for two classes of cable in 1921 and 1925, respectively.

For example, at 80 deg. cent., the loss for 30-kv. cable dropped from about 1.6 to 0.6 watt per foot and for 13-kv. cable from about 0.9 to 0.2 watt per foot. Nevertheless, this property has to be watched, particularly as voltages are increased, because for the same kind of insulation, it is greater for the high-voltage cable. Furthermore, the annual cost of the energy dissipated in the insulation of cables, although a very small per cent of the total loss is, nevertheless, a waste, and unnecessary waste is what the engineer is expected to eliminate. For example, in a cable 20,000 ft. long which is in service 90 per cent of the time, the value of the excess energy waste for every extra 0.1 watt per foot would be about $95.00 per year assuming an energy value of 0.5 cent. This is, of course, accompanied by a reduction in carrying capacity for the same copper temperature limit and this means less earning capacity per dollar invested. It is evident, therefore, that the dielectric loss and power factor of high-tension cables should be kept as low as possible, provided this is not accomplished at the expense of other important characteristics.

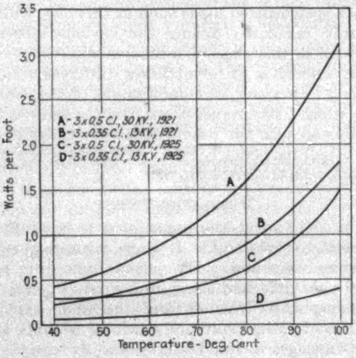

FIG. 10—TYPICAL DIELECTRIC-LOSS CURVES, 1921 AND 1925

Power-factor measurements also provide another means of studying the problem of uniformity of cable to which reference has been made. Everyone familiar with high-tension cable testing knows that the dielectric loss and the power factor vary from section to section in a given lot of cable although it was all manufactured at the same time and subjected to the same treatment. In fact, these quantities may not be constant throughout one section, for very considerable differences between a sample taken from a given reel and the whole reel are frequently found. In a recent investigation where a length of cable was cut into pieces, an opportunity was afforded to investigate this particular feature. The entire length of about 350 ft. had a power factor of 0.26 per cent; the first half, 0.38 per cent; the second half, 0.15 per cent and a piece about 20 ft. long from the first half had a power factor of 0.77 per cent.

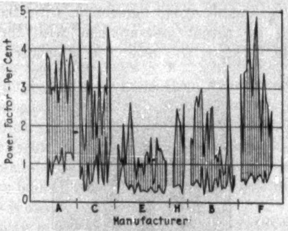

FIG. 11—POWER FACTOR OF 13-, 15-KV. CABLE, INSPECTED DURING 1925

Upper line—Power factor at rated voltage at 80 deg. cent.
Lower line—Power factor at rated voltage at 20 deg. cent.

Fig. 11 shows some power-factor data for 13- to 15-kv., three-conductor cable made by various manufacturers during 1925, the upper line being values obtained at 80 deg. cent. and the lower line values at 20 deg. cent. It is evident the cable of manufacturer *A* was much more uniform in this respect at least than that of the others, particularly *C* and *F*. However, make *E*, while not quite so uniform, was consistently much lower, with a minimum change between 80 deg. and 20 deg. The ideal lines would, of course, be straight and perhaps slightly sloping to the right, thus indicating absolute uniformity.

The methods in general use for measuring dielectric loss and power factor of high-tension cables have been described,[6] and it is not necessary to discuss them here. Suffice it to say that two general classes of methods are used, namely wattmeter methods and bridge methods. The former measure the active power and the reactive power dissipated in the cable,—the measurement being made with a wattmeter, either electrodynamic or electrostatic. This type of method is particularly applicable to measurements on three-conductor cable with three-phase potential. Bridge methods determine the active and reactive components of the impedance of the cable as a condenser and, of course, can only be used with single-phase potential applied to the cable. However, experience has shown that in three-conductor cables the dielectric loss and power factor calculated from measurements made with single-phase potential are in reasonably close agreement with the results obtained by actual measurement with three-phase potential. This is specially true of modern low-loss cables. The author advisedly says "reasonably close agreement" for there is no need for high precision in tests of a property which may vary in the manner indicated in the preceding discussion.

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[4] "Influence of Dielectric Losses in the Rating of High Tension Underground Cables," A. F. Bang and H. C. Louis, TRANS. A. I. E. E., Vol. 36, 1917, p. 341.

[5] "Dielectric Losses and Stresses in Relation to Cable Failures," D. W. Roper, TRANS. A. I. E. E., Vol. 41, 1922, p. 547.

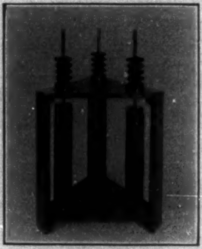

Fig. 12—Proposed Standard Load for Dielectric-Loss Tests

The suggestion has been made that one method of making tests for dielectric losses and power factors should be standardized in order to eliminate some of the differences between different laboratories, but it is not believed that this is necessary or even desirable. These quantities are definite physical quantities for which we have established units, and all methods which will measure such quantities accurately, whether they are in cable or anything else, should give the same result. What we need is not a standardization of one or more methods of making the measurement itself but a convenient *standard* load comparable in magnitude to the dielectric of a piece of cable. With such a standard having all of the characteristics which a standard should have, namely, permanence and minimum change with temperature and which can be easily and accurately calibrated, these various methods can be made to agree.

Fig. 12 is an illustration of such a standard which gives promise of serving the purpose very well and is in addition portable. It consists merely of three Leyden jars made of special glass, metal coated inside by silvering plus copper plating and outside by the spraying process. The tubes are about two in. outside diameter, about 16 in. long and the glass is about $^3/_{32}$ in. thick. They are filled with oil and will withstand the application of 25 kv. continuously. With three-phase potential applied to a set of three tubes, the three-phase power factor at 25 deg. cent. of the particular set in the illustration is 0.22 per cent at 10 kv., 0.27 per cent at 25 kv. and 0.32 per cent at 50 kv. At 35 deg. cent., the corresponding power factor values are 0.24, 0.30 and 0.36 per cent.

With standardized loads such as this one, which can be easily circulated among the various laboratories interested in this class of measurements, there should be no difficulty in eliminating differences due to methods. However, standardization of the *conditions* under which measurements are made in cable is highly desirable because the results obtained depend upon the conditions, particularly the temperature and the previous temperature history.

"IONIZATION" TEST

As is well known, the paper after it is wrapped on the conductor in a cable is then saturated with an insulating compound. The general procedure is first to dry the cable and then impregnate it, the initial step being under vacuum and the final step under pressure. In other words, as much as possible of the air or other gas in the voids of the structure—paper and stranded conductor—is removed and replaced by an insulating compound.

The work of many investigators in the field of insulation research has demonstrated that the less gas there is in the insulation the better it is as an insulator.

The so-called "ionization" test, a comparatively new one in the cable art, is intended to be a test of thoroughness of impregnation. We know that in a solid dielectric free from gas the loss will vary substantially as the square of the voltage and the power factor will be constant until the voltage approaches the rupturing value—barring, of course, temperature effects due to long application of the voltage. The theory of the "ionization" test is that, if air or gas is present, it will become ionized at a relatively low stress and the power factor will depart from a constant value as the voltage is increased, at a lower value than it would otherwise.

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[6] "Compensated Dynamometer Wattmeter Method of Measuring Dielectric Energy Loss," G.B. Shanklin, *G. E. Review*, October 1916, p. 842. "The Quadrant Electrometer for the Measurement of Dielectric Loss," D. M. Simons and W. S. Brown, TRANS. A. I. E. E., Vol. 43, 1924, p. 311. "A High-Tension Bridge for the Measurement of Dielectric Losses in Cables," R. W. Atkinson, *Electric Journal*, February 1925, p. 57. "Power Measurements at Very Low Power Factors," A. Barbagelata and L. Emanueli, *L'Elettrotecnica*, August 5, 1922, p. 477. "Measurement of Dielectric Losses at High Voltages—A New Type of Alternating-Current Bridge," B. Hogue, *World Power*, August 1925, p. 81. "Methods of Measuring Dielectric Losses in High Tension Cables," P. Capdeville and R. Laroche, *Revue General de L'Electricite*, Dec. 6, 1924. "Measurement of Power Loss in Dielectrics of Three-Conductor, High-Tension Cables," F. M. Farmer, TRANS. A. I. E. E., Vol. 38, 1918, p. 221.

That there is a marked difference in this characteristic in cables is indicated in Fig. 13. One curve shows the relation between power factor and average stress at room temperature in a piece of three-conductor cable of 1922 vintage and the other shows the same characteristic for a similar piece of cable of 1925 manufacture. There is no doubt that the thoroughness of impregnation has much to do with this change in power factor because a section of cable which shows a relatively large change in power factor with change in the voltage applied, will, if reimpregnated, show a much smaller change.

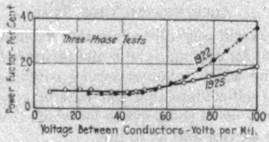

FIG. 13—IONIZATION CURVES 1922 AND 1925

Rated voltage: 13,000
Insulation: Paper, 9 x 7/32 in., *B C* and *B C L*
Conductors: 3-500,000-cir. mil sector

The "ionization" test, which is the present standard for specification purposes, is the determination of the change in power factor between an average stress of twenty volts per mil and one hundred volts per mil, respectively. The specifications issued by Association of Edison Illuminating Companies in November, 1924, prescribe that this change shall not exceed two per cent for a three-conductor cable or one per cent for a single-conductor cable. These limits were set very largely on the basis of European experience where this test had been in use for some time. However, experience of over a year shows that American manufacturers have no difficulty in meeting this requirement. In fact, a gradual reduction is indicated in Fig. 14 for it is evident that an average of the lines representing maximum and minimum values respectively, of successive lots of cable inspected, would, in most cases, have a gradual slope downward from left to right. Incidentally, this figure shows how this test also can be used to study uniformity. While there is not much difference between the manufacturers in this respect, the product of *E* shows a steady decrease in the amount of variation of this quantity.

The important question in connection with this test is, what is its real value? Will cable having a power-factor difference of two per cent for example, be better than one with a difference of four per cent and if so, in what respect and how much better will it be? Dr. Whitehead has shown[7] that insulating materials which were known to be poorly saturated had a high "ionization" test result and deteriorated rapidly under the application of voltage. However, we have no definite facts in the way of test data to show how much real gain is obtained in cables when this power-factor difference is two per cent instead of four per cent or one per cent instead of two per cent. Before reducing the permissible variation we should have definite knowledge on this point if such reduction involves increased manufacturing costs.

Referring to the test itself, its application to single-conductor cable is simple but in the case of three-conductor cable several methods may be used which will give different results. In one, the three-phase power factor is measured, using three-phase potential of the two prescribed values and noting the change in power factor. In another method, the change is noted between the three-phase power factor at each of the two voltages as calculated from measurements made with single-phase voltage between various conductor and sheath combinations. In a third method, the power factor at each of the two voltages is measured at each of the four combinations and the greatest change found in any one of the four sets of measurements is used.

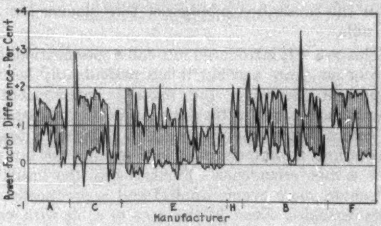

FIG. 14—IONIZATION TESTS OF 13-, 15-KV. CABLE INSPECTED DURING 1925

Upper line—Maximum power-factor difference in each lot inspected
Lower line—Corresponding minimum power-factor difference

Now, it is evident that the intent of this test is best fulfilled by the latter method as what we are interested in is the degree of saturation in that portion of the insulation where it is the least saturated. Of course, we cannot make a separation longitudinally and we must be content with an average over the entire length of cable but we can make a partial separation by making measurements between the conductors and between the conductors connected together and the lead. The highest difference in four such sets of measurements will generally be greater than that for three-phase measurements as would naturally be expected since the latter is an average value. However, this is not always the case, for frequently the two results will be found to be the same and occasionally the three-phase value will apparently be higher than the highest of the single-phase values.

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[7] "Gaseous Ionization in Built-up Insulation," J. B. Whitehead, TRANS. A. I. E. E., Vol. 42, 1923, p. 921, and Vol. 43, 1924, p. 116.

BENDING TEST

One of the most important tests in American cable practise is the bending test. In Europe most of the cable is laid either directly in the ground or in troughs, while in this country all high-tension cable is installed in conduits. This, of course, involves much more bending of the cable during installation than in European practise but as it is an unavoidable condition the cable must withstand it. Hence, the generally accepted test given below is used to see what the cable will do under this practise.

The sample, after subjection for two hours to a prescribed temperature (frequently −5 or −10 deg. cent.), is bent 180 deg. around a cylindrical form having a diameter twelve times the outside diameter of the cable. The sample is then straightened out and bent again 180 deg. in the opposite direction around the same or a similar form. It is again straightened and the cycle of operation is repeated making a total of four bending operations. The test is considered to have been successfully passed if the sample of cable withstands the application of a prescribed voltage for a prescribed length of time and shows, upon dissection, not more than the prescribed number of torn papers per foot of cable.

This test was introduced into cable specifications only five or six years ago but it has undoubtedly been an important factor in the marked improvement in workmanship and in the fabrication of the insulation of cables which has taken place. Excessive tearing was at first a common result in a bending test but now it is a rare occurrence. Furthermore, the insulation is tighter, more evenly applied and is practically free from wrinkles. Even in the case of cable with wood-pulp paper insulation, while there has been some improvement in the mechanical strength properties of wood-pulp paper, the improvements in fabrication processes have enabled such cable to pass this test—something that could not have been done two years ago.

Table III shows in a striking manner the improvement that has been made in the ability of Manila paper-insulated cable to withstand this bending test, as measured by puncture tests. The figures given are grand averages of tests made on samples of cable of various sizes and voltages.

TABLE III
DIELECTRIC STRENGTH, BEFORE AND AFTER BENDING

Year	No. of Tests		Dielectric Strength, Average Volts per Mil		
	Before Bending	After Bending	Before Bending	After Bending	Decrease per cent.
1923	260	288	287	225	21.6
1924	290	301	306	266	13.1
1925(*A*)	106	102	355	336	5.3
1925(*B*)	40	36	369	369	0.0

The last column shows what marked improvement has been made in the past three years—the effect of bending having dropped from 21 per cent to practically zero. In other words, present-day cable will withstand this severe bending test without any appreciable effect on the dielectric strength.

ACCELERATED LIFE TEST

Engineers will generally agree that the most valuable type of test for any material is the performance type—that is, one which involves service conditions and in which it is self-evident that the material passing it will satisfactorily serve the purpose for which it is intended both as to service and as to life.

In the case of high-tension cable, time is the important element and the problem is to produce, in a sufficiently short time to be practical, those conditions which develop in a cable in normal service that produce deterioration and ultimate failure. This is a very large problem which will require much experimentation and study to solve. The best that we can do at the present time is to subject a sample of the cable to those conditions which we know will shorten its life—such as excessive voltage and high current—and assume that the ratio of the life so obtained to normal life is a constant although unknown value—at least for all cable of the same type. In other words, such a test implies that where one make of cable or one kind of cable lasted half as long as another, the life of the second cable in normal operation would be at least twice that of the first cable.

An extensive series of tests is under way at the Electrical Testing Laboratories for the purpose of accumulating data upon which to base recommendations for a standard accelerated life test. While, as stated, any such test may not enable us to predict the life of cable as yet, it does seem highly probable that if a standardized accelerated life test were adopted, data could be accumulated which in the course of time could be correlated with the performance of cable in service so that ultimately we would be able to predict the performance of cable in service from the results of such a test.

It was hoped that periodic measurements of power factor during an accelerated life test of a piece of cable would show progressive deterioration and foretell approaching failure but some studies along this line have been disappointing. Figs. 15, 16 and 17 show such measurements made during the test of a piece of three-conductor, 15-kv. cable. Fig. 15 shows the power factor between one conductor and the other two at a stress of 20 and 100 volts per mil (average) respectively. It will be seen that at 20 volts per mil a gradual but steady rise took place until puncture occurred without any pronounced change immediately preceding failure. At 100 volts per mil the power factor rose relatively rapidly during the first 200 hours and rapid deterioration was suspected but it then went down as rapidly as it had gone up and continued to do so until failure occurred, even going below the initial value despite the fact that the test voltage had been raised twice in the meantime. The relatively small but sudden changes in the curve are due to changes in room temperature.

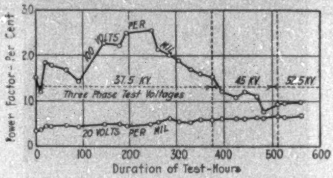


Fig. 15—Power Factor Variation with Time Between Conductors

Combination No. 1. Rated voltage: 15,000
Insulation: paper, 14 x 10/32 in. *B C* and *B C L*
Conductors: Three 350,000-cir. mil sector

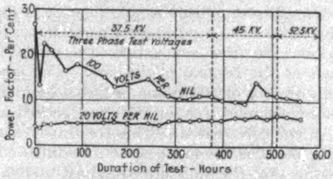


Fig. 16—Power Factor Variation with Time, Between Conductors

Combination No. 2. Rated voltage: 15,000
Insulation: paper, 14 x 10/32 in. *B C* and *B C L*
Conductors: Three 350,000-cir. mil sector

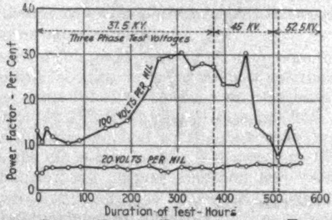


Fig. 17—Power Factor Variation with Time—All Conductors to the Sheath

Rated voltage: 15,000
Insulation: paper, 14 x 10/32 in. *B C* and *B C L*
Conductors: Three 350,000 sector

Fig. 16 shows similar data for another combination of conductors but here the power factor at 100 volts per mil decreased practically from the beginning with no preliminary signs of approaching failure before puncture occurred. The curve for the measurements at 20 volts per mil is similar to that in the preceding figure.

Fig. 17 is for the threec onductors against lead and here the results are of the same character as in the curves in the first diagram, Fig. 15.

Figs. 18, 19 and 20 show similar data for a sample of 35-kv. cable. In this test one combination of conductors, Fig. 18, shows a rise in power factor almost from the beginning which continued quite rapidly but at a constant rate until puncture occurred. Another combination, Fig. 19, showed even more pronounced results in that the curve rose more rapidly as failure was approached. On the other hand, the power factor between conductors and sheath, Fig. 20, was not only much lower but showed practically no change whatever during the test.

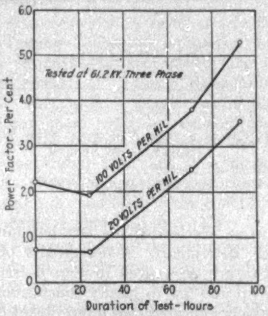


Fig. 18—Power Factor Variation With Time—Between Conductors

Combination No. 1. Rated voltage: 36,000
Insulation: paper, 33 x 25/64 in. *B C* and *B C L*
Conductors: Three 350,000 round

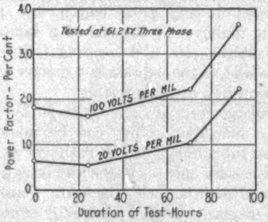


Fig. 19—Power Factor Variation With Time—Between Conductors

Combination No. 2. Rated voltage: 36,000
Insulation: paper, 33 x 25/64 in. *B C* and *B C L*
Conductors: Three 350,000 round

The results in this case were consistent with the conditions found upon examination of the sample after the conclusion of the test because failure had occurred between conductors due to leakage currents produced by tangential stresses so that deterioration was taking place between conductors from the beginning. While in this instance the indications that the power-factor test would be of value were promising, they were contradicted by the results obtained in the first test. Several other trials gave the same negative results so that it is concluded that life performance cannot be predicted from the power-factor change.

Whether or not intermittent loading has an appreciable influence on the life of cable and should be included in an accelerated life test has not yet been conclusively determined. A few results obtained up to this date are designated in Fig. 6 by the letter *H*. They are too few to justify conclusions although it is tentatively indicated that intermittent heating has no effect.

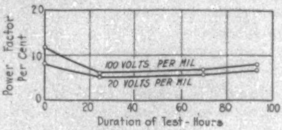


Fig. 20—Power Factor Variation with Time—All Conductors to Sheath

Rated voltage: 36,000
Insulation: paper, 33 x 25/64 in. *B C* and *B C L*
Conductors: Three 350,000 round

Preparation of Samples for High-Voltage Tests

Although it was indicated in the introduction of this paper that details of technique in tests of cable would not be discussed at any great length, the problem of preventing flashover at the end of a sample of cable in high-voltage tests and insuring failure occurring under the lead, is too important a matter to pass over in any discussion of cable testing.

High-voltage tests of cable up to 60 kv. involve no particular difficulties. They can be made with the ends of the cable in air without flashover at the ends—it being necessary only to cut the lead back a reasonable distance and slightly "bell" the lead. Care, of course, is always necessary to keep the freshly exposed surface of the insulation in its original condition. In the case of three-conductor cable some separation of the ends of the conductors is obviously necessary and greater care in bending the conductors has to be taken to avoid crotch failure by making the curvature a minimum at the crotch and putting the maximum curvature at the ends.

As the test voltages are increased, the difficulties multiply rapidly, particularly in time tests. In the acceptance testing of three-conductor cable rated at about 30 kv., it requires the application of about 75 to 90 kv. for 5 to 15 minutes and failure occurs at the crotch if the test is made in air. It is necessary to attach funnels so that the conductors at the crotch may be surrounded with a liquid or semi-liquid insulating material and then a test for periods of the order of 5 to 15 minutes can be carried up to about 125 to 150 kv. Short-time or momentary breakdown tests of samples of cable requiring as high as 200 kv. may be made in a similar manner with longer exposed conductor ends, greater clearances between ends and longer funnels.

Fig. 21—Testing Samples of Cable with Ends in Jars of Oil

A specially convenient method of disposing of the end problem is shown in Fig. 21. The sample is bent into the shape of an inverted *U* and is supported over two jars of transformer oil into which the ends of the cable, with the ends of the conductors separated, are lowered. Connections to the conductors are made through terminals at the bottom of the jars. The jars are of earthenware, are of 200-gallon capacity each and have been dried, vacuum treated and impregnated just as a section of cable is treated. This treatment was found necessary because the material in its normal condition is so porous that the moisture contained therein makes it quite valueless as an insulator. With this method, a reasonably high percentage of satisfactory test results (that is, punctures well under the lead) is obtained in time tests up to 100 kv. and in short-time tests up to the order of 150 or 175 kv.

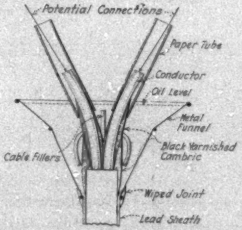


Fig. 22—Test End for Three-Conductor Cable

Note: For simplicity in drawing, only two conductors are shown

When long-time tests are to be made (that is, several hours or days) even at only moderately high voltages, none of these comparatively simple methods of taking care of the ends will always be satisfactory—crotch failure or burning of the conductor insulation due to leakage currents along the surface, frequently resulting. Even long-time tests of the order of 60 kv. should be made with specially prepared ends to insure ultimate failure under the lead. Fig. 22 shows a method of preparing the ends of three-conductor cables for long-time high-voltage tests which has been found satisfactory up to 100 kv., the highest voltage at which it has been used so far. The general procedure in preparing the cable end is first to cut back the belt insulation to within two inches of the lead and then wrap black varnished cambric tape on the conductors with cable compound between layers, care being taken to pull it tight and smooth. After building up the conductor insulation as shown to a maximum thickness of about 100 mils, the belt insulation is extended and reinforced in a similar manner so as to bring the "crotch" at the reinforced part of the conductors. When the wrapping is completed, all fillers are left intact and finally tied back as shown. Particular care is taken not to bend the conductors or disturb them any more than is absolutely necessary until after the reinforcing has been completed. A test sample of cable can be prepared for test with this type of end arrangement in two to three hours by an experienced man.

FIG. 23—STANDARD GENERAL ELECTRIC COMPANY CABLE TERMINALS

Schemes of this general character have been found by most testing laboratories to be the best solution of this problem. One difficulty, however, that has been experienced, has yet to be overcome. In tests where current is intermittently applied to simulate operating conditions, the oil in the funnels is drawn into the cable thus changing the normal condition and perhaps in some cases improving the saturation. This difficulty would be eliminated by a flexible barrier or dam at the crotch.

Where the test voltage exceeds about 150 kv., standard cable terminals have to be used. Fig. 23 shows a sample of 76-kv., single-conductor cable being tested with standard oil-filled General Electric Company cable terminals, rated at 350 kv. (dry flashover). Testing with these terminals is very expensive because the preparation of a sample alone requires about three men-days. A much quicker and less expensive method of preparing samples is needed, for, if the progress required by the art is to be made, much time-breakdown voltage data must be accumulated which means the testing of a great many samples of cable.

The simple expedient of cutting back more lead and testing in air does not, of course, remove the difficulty because the stress gradient along the surface is not uniform. The problem is to make it uniform. One simple remedy that is sometimes helpful is the application of a single layer of untreated cable paper wrapped over the insulation. The slight leakage current that results tends to equalize the potential gradients along the conductor surface.

Another scheme which is being developed at the Electrical Testing Laboratories promises to be very useful. It employs the principle of the condenser bushing and consists of a stack of disks strung on the cable ends. Each disk is simply an annular ring of corrugated cardboard which has been covered on one side with a thin coating of sprayed lead and then thoroughly saturated with oil. The diameters are computed to give a uniform stress gradient from the top disk, which is connected to the conductor, to the lower disk which is connected to the lead.

Fig. 24 is a sketch of one of these terminals on the end of a sample of 500,000-cm., single-conductor cable with 660 mils of insulation. The lead is cut back only 6¼ in. and over the exposed insulation are strung about 30 disks—the top one being about five in. in diameter and the bottom one about nineteen in. Flashover occurred at 110 kv. in air and 140 kv. in oil. With the terminal immersed in oil, 130 kv. was applied continuously for nineteen hours when the test was discontinued. No sign of overstressing at any point was observed during the test or upon examination at its conclusion.

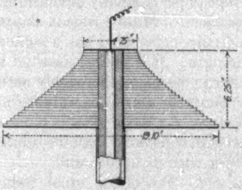


FIG. 24

TESTS OF COMPONENTS

Most cable specifications include some requirements, either general or specific, which necessitate certain tests to be made on most of the component parts of the cable. In addition, others not specifically required are frequently made for the purpose of obtaining information.

(*a*) *Paper*. Tests of paper removed from samples of cable are tested for composition of stock, tensile strength, tearing strength, folding endurance, dielectric strength and sometimes porosity or air resistance.

Occasionally, also the dielectric constant is determined. Methods which are more or less standardized are usually employed for these tests but they will not be discussed here. It may be of interest, however, to note the general average value of some of the properties during the past three years, as shown in Table IV.

TABLE IV
PROPERTIES OF CABLE PAPER (MANILA)

	Tensile Strength, lb. sq. in.*		Folding Endurance, No. double folds*		Tearing Strength, grams†		Dielectric Strength			
							Individual Tapes, Volts per mil		Average Stress Gradient, Volts per mil‡	
Year	No. of Tests	Avg.	No. of Tests	Avg.	No. of Tests	Avg.	No. of Tests	Avg.	No. of Tests	Avg.
1923	150	7230	150	1010	150	180	..	..	260	287
1924	825	7350	825	724	825	179	56	589	290	306
1925(*A*)	273	8435	273	881	273	163	43	638	106	355
1925(*B*)	263	9250	263	620	263	170	78	649	40	369

*Parallel to strips.

†Across strips.

‡Repeated from Table III to show, as a matter of general interest, the relation between the dielectric strength of single tapes and the dielectric strength of the total insulation (expressed in the same manner) as obtained in breakdown tests of samples of cable (before bending).

Manila rope paper exclusively has been used for high tension cables in this country up to very recently because wood pulp paper would not withstand the bending manipulation incident to American practise, although it is the standard type of cable insulation in Europe. However, recent improvements, particularly in the methods of applying the tapes, have made it possible for cable so insulated to pass the standard bending test.

This development is an advantageous one for the industry because the source of supply of raw material for Manila rope paper is rapidly diminishing on account of the replacement of Manila rope in shipping by steel rope. Wood-pulp paper also has a somewhat higher dielectric strength. It is further asserted by its advocates that it can be more thoroughly saturated than Manila rope paper for the reason, it is claimed, that the cellular structure of a wood fiber is more easily dried out and filled with compound than the tubular structure of a hemp fiber with its harder wall.

(*b*) *Compound*. Few of the usual tests which are normally made on insulating materials can be applied to the compound in cables by the purchaser because the only way he can obtain sufficient quantities of a compound from the sample of cable is by extraction with benzol or other solvents. The subsequent recovery of the compound by distilling off the solvent changes the viscosity and particularly the electrical characteristics, so that test results are likely to be quite misleading. Either some of the higher constituents of the compound are carried off with the benzol or some of the benzol remains in the compound.

A test that is frequently made is the determination of the approximate proportions of saponifiable and unsaponifiable constituents which are usually vegetable base and mineral base materials, respectively. It has been previously indicated that mineral base compounds have in general a much lower dielectric loss than vegetable base compounds but the latter has desirable viscosity and dielectric strength characteristics. A knowledge of the general composition of a compound is therefore useful as a check on the test results obtained on the cable itself.

A new test has recently been developed as the result of the unfortunate results which followed the change from vegetable base compounds to mineral base compounds in order to reduce dielectric losses which had been responsible for failures of the former type of cable.

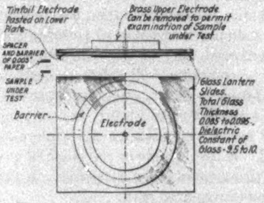

Tinfoil Electrode Pasted on Lower Plate
Brass Upper Electrode Can be removed to permit examination of Sample under Test
SPACER AND BARRIER OF 0.003" PAPER
SAMPLE UNDER TEST
Barrier
Electrode
Glass Lantern Slides. Total Glass Thickness 0.085 to 0.095. Dielectric Constant of Glass - 9.5 to 10.

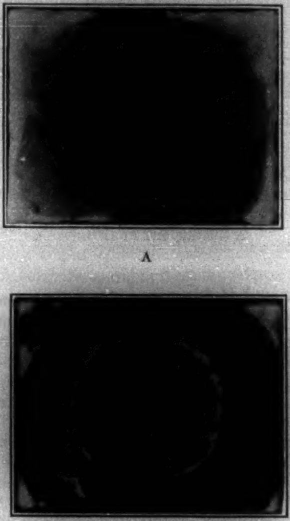
FIG. 25—E. T. L. "WAX" TEST

Equipment:
1. Test condensers
2. A simple rack to hold condensers
3. Transformer equipment to apply 15 kv. to test condensers continuously

Test:
1. 72 hr. at approximately 600 volts per mil on the compound, (15 kv. on the test condenser)
2. Wax visible through glass and marked by change of color and texture. Wax insoluble in oil or grease solvents
3. For tests on oils, the test space may be sealed with sealing wax

Long experience with mineral oils in various applications seemed to justify the general belief that a mineral base compound would remain perfectly stable indefinitely. However, a few months' service experience with the first high-voltage cables impregnated with mineral base compounds showed that such faith was not justified. An unknown, solid material, cellular in structure and extremely light, formed in the interstices of the insulation and failures too frequently followed in the course of time. It was evident, as is so often the case, that the elimination of one defect was being accompanied by the introduction of another.

In order to form this unknown material in any practical length of time, the compound must be subjected to stresses which are above the breakdown value. This is best done by placing the compound between concentric glass cylinders or between parallel glass plates. In the latter method as used at the Electrical Testing Laboratories, (Fig. 25) a film of the compound about three mils thick is placed between two lantern slide glasses on either side of which is a flat electrode to which the potential is applied. The upper electrode is removed at intervals and any change in the character of the film is noted.


A

B

Fig. 26—"Wax" Test

a. A specimen of petrolatum prepared for test

b. The same specimen after 48 hours at 600 volts per mil

Fig. 26 is a photograph of a test specimen before and after subjection to this test. Fig. 27 is a curve obtained with petrolatum which shows the time at which this unknown material, *X*, forms at various stresses—the stress being calculated from the measured dielectric constant of the glasses and the compound, the thickness of the glasses and the film, and the applied voltage. At the highest stress, 600 volts per mil, *X* formed very rapidly, and in a relatively short time all of the petrolatum would be entirely converted. The rate at which it forms falls off rapidly as the stress is reduced and at the lower voltages, the time is very uncertain so that many tests for very long periods would be necessary to establish the lower part of the curve definitely. It seems reasonably safe to assume from the trend of the curve that in a sufficiently long time more or less *X* could be expected to form even at stresses existing in high tension cables in normal operation, that is, of the order of fifty to seventy-five volts per mil.

It may properly be asked, how certain are we that the material formed in this test is the same as that developed in cables in service? Not only have the two products been carefully compared by visual and chemical examination but compound which developed *X* in a life test of a sample of cable in the laboratory, also developed the same identical material in this test.

While the absence of any signs of disintegration in a compound when subjected to this test is not a guarantee that it will remain unchanged indefinitely in a cable, it seems reasonable to assume that if 600 volts per mil for seventy-two hours at either room temperature or at 75 deg. cent. does not produce any change, the chances of disintegration taking place in service are rather remote.

(*c*) *Copper and Lead.* Copper is now so uniform in quality that tests of it for physical and electrical properties are rarely necessary. A measurement of the resistance of the conductors should be made if for no other reason than the fact that conductivity is, after all, the thing that is being purchased and some test should be made on each section. As a matter of fact, this test has quite often proved to be very much worth while through the detection of errors in length. Excess in length is a loss to the manufacturer, but a shortage may be very troublesome and the cause of considerable expense to the purchaser if it is not discovered until the cable is drawn into the duct.

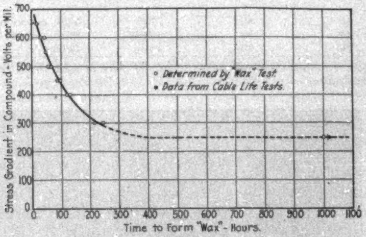


Fig. 27—"Wax" Tests—Time-Stress Diagram

It is not customary regularly to make tests of the lead. Most specifications prescribe commercially pure lead and any marked departure from the commercially pure state so seriously affects the flexibility that it is likely to be detected by an inspector accustomed to noting this feature. For instance, normal lead will have a tensile strength of the order of 2500 to 3000 lb. per sq. in., with an elongation of 30 to 35 per cent in two in., but in a recent investigation of obviously stiff lead which cracked during installation, the following impurities and physical properties were found:

Tin	0.24 per cent
Antimony	0.70 per cent
Copper	0.11 per cent
Tensile Strength	4950 lb. per sq. in.
Elongation	12 to 16 per cent in 2 in.
Hardness	double normal

In other words, the lead was abnormally hard due to impurities.

It may be of interest to note another investigation of a case of cracked lead. Chemical examination and mechanical tests showed it to be normal but it was learned that the cable had been subjected to a considerable amount of repeated bending during installation, possibly at the points of failure. Was this the cause of the cracking? A series of tests was made by subjecting specimens of lead sheath 1/8 by 1 in. by 9 in. long to a repeated but very slight bending between one-in. cylinders, the total angle being about 12 degrees. Tests were made with various weights on the end of the strip of lead with the results shown in Table V.

TABLE V

Load lb.	Stress lb. per sq. in.	No. cycles of bending	Remarks
0	0	8710	First cracks noted
5	40	3400	First cracks noted
10	80	1900	First cracks noted
15	120	1000	First cracks noted
0	0	8100	First cracks noted
0	0	11000	Badly cracked
0	0	12000	Ready to break
10	80	2700	First cracks noted
10	80	3600	Badly cracked
10	80	4040	Broke

Note: First group of specimens from one piece of lead, other two from another piece.

FIG. 28—FIRST THREE TEST SPECIMENS AT A MAGNIFICATION OF SIXTEEN DIAMETERS

a. First cracking, zero stress 8710 cycles
b. First cracking, 40 lb. sq. in., 3400 cycles
c. First cracking, 80 lb. sq. in., 1900 cycles

It is evident that lead will begin to crack after being bent back and forth a relatively small number of times—a number of cycles which was quite probable in the case in question. Furthermore, the more stress there is on the lead, the sooner will cracking start. Fig. 28 shows a magnification of sixteen diameters of a portion of the first three specimens in the table and Fig. 29 is an illustration of a portion of the fifth, sixth and seventh specimens, respectively.

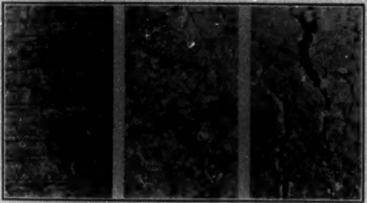


FIG. 29—TEST PORTION OF FIFTH, SIXTH AND SEVENTH SPECIMENS

a. First cracking. zero stress 8100 cycles
b. Badly cracked. zero stress 11,000 cycles
c. Ready to break. zero stress 12,000 cycles

CONCLUSIONS

The problem confronting the engineer responsible for an underground cable installation is the transmission of the most energy per dollar of annual cost, *i. e.*, per dollar of investment and per dollar expended in maintenance. The investment cost will depend not only upon the voltage employed, which presumably will have been selected after proper consideration of the economics of the situation and the local conditions, but also to some extent on the quality of the cable, for, the better the insulation, the less of it has to be used and the lower will be the cost. The maintenance cost, however, is largely a matter of quality although the cost of repairs due to inferior workmanship on splices, careless handling of cable during installation and other avoidable causes is not an insignificant item.

The last report of the Underground Systems Committee of the National Electric Light Association (June 1925) shows that the number of failures during 1924 in 7321 mi. of high-tension cable was 716 or practically ten failures per 100 mi. In addition, there were 256 failures of joints during the same period, making the total number of failures 13.3 per 100 mi. of cable.

Every failure in a cable costs money in direct outlay for replacement of cable and for withdrawing the old cable and installing the new, in addition to splicing and other incidental expenses. There is the further effect of cable failures that cannot be evaluated, namely, the disturbance to the service, the continuity and dependability of which is such an important element in the "good will" of an electric public utility.

The record of cable failures quoted above might be considered good and perhaps it is in comparison with the records of other electrical apparatus, particularly when the comparison is on the basis of the area of insulation involved. But it is the function of the engineer to make the record better and thereby reduce the maintenance cost in which cable failures form so large a part.

It is obvious that the greatest reduction in the number of cable failures is to be effected through improvement of the quality and through improvements in the design. Progress in both of these directions can be made in the following ways, all of which involve comprehensive and systematic testing of the kinds discussed in this paper:

(*1*) *Research by both manufacturers and users.* It is the manufacturers' problem to improve the quality of the materials which make up a cable. It is also their problem to improve manufacturing processes in order to produce cable which is not only better cable but which is more uniform. It is the users' problem to carry on investigations which will provide information that will permit more efficient operation based on facts rather than opinions and which will indicate the direction in which improvements are needed to meet the present and future requirements of the industry.

(*2*) *Thorough and systematic inspection and tests of cable when purchased.* While the immediate, primary purpose is to insure compliance with the contract, the greatest value of inspection and tests is likely to be the fact that it provides the means of obtaining definite, quantitative knowledge of the quality of cable. Such knowledge must be available before a reference point can be established and progress accurately measured.

(*3*) *Evaluation of cable.* The development of a satisfactory method of quantitatively evaluating cable through suitable weighing and combining the results of inspections and tests would provide a very useful incentive to progress. Such a method is now in the experimental stage of development but it will take time to demonstrate its reliability because that can only be measured by comparison with the performance of the cable after it goes into service. This involves keeping accurate, systematic records of all inspected cable after it goes into service and a record of the detailed examination of every failure.

(*4*) *Specifications.* Progress can be made through improvements based on *real knowledge* of the properties and characteristics of cable being purchased and through the developments of new tests which will enable us to predetermine the serviceability of cable in so far as possible. Cable specifications must, of course, be based on current practise and consequently revisions of existing requirements must follow progress in the art. However, keeping specifications up to date so that they include advances as they are made does, in itself, contribute to the general progress through setting a high standard for *all* manufacturers and for *all* users rather than the *few*.

(*5*) *Periodic examination of cable in service.* Improvement in cables would unquestionably be greatly aided if we had more knowledge of what takes place in a cable which causes it to deteriorate and ultimately fail. Turbines are carefully watched and periodically inspected and high tension transformers, switches and other important apparatus is periodically inspected and overhauled. Apparently all of the equipment on the system is given a more or less systematic inspection except the cable which is buried in the ground and forgotten until trouble develops. Samples should be removed from time to time (or at least when a convenient opportunity is afforded such as a joint failure) and examined and studied so that we can learn, if possible, what it is that determines the life of the cable and thus aid progress by indicating the direction of improvement both in the cable and in the specifications for the cable.

It is hoped that this paper will have been successful in presenting parts, at least, of a picture of the status of the high-tension cable art. It has been shown by means of reliable data that definite improvement has been made in several respects but that, on the other hand, new problems have developed which warn us that the millenium for the operating engineer who has a cable system in his charge is yet some distance in the future. Also an attempt has been made to indicate some of the channels through which further improvements can be effected or at least stimulated.

It is also hoped that the importance of cooperation in the industry has been indicated—cooperation among the users, among the manufacturers and between the two groups. Progress will be greatly facilitated when the spirit, "all for one object and one object for all", prevails throughout the entire industry, that object being to make American cable perfect cable and the best in the whole world.

Discussion

W. S. Clark: In connection with the figures given in Mr. Farmer's paper, one should not lose sight of the fact that there is no insulation of which we know in which the strength increases in direct proportion to the increased thickness of insulation.

The company by which I am employed makes small static condensers or capacitors for power-factor correction. These are operated with a stress of about 300 volts per mil, the total thickness of insulation being about 4 mils. We have found that if we desire to increase the voltage from 1200 to 2400 volts on these condensers, to secure the same factor of safety and freedom from breakdown it is necessary to more than double the thickness of the insulation.

In a general way the strength of saturated paper insulation, assuming uniform quality, appears to increase only about as the 0.8 power of the thickness and not directly as the first power.

With reference to the test referred to in Mr. Farmer's paper, regarding formation of wax or *X* compound in oil, the test as at present specified apparently cannot be duplicated in different laboratories with the same material. When the test is brought to a condition in detail where it can be duplicated in different laboratories with the same results, then I think it will be good.

E. C. Willman: We noticed that the *X* compound was greatest at the surface and in the layers of paper nearest the conductor; that is, at the point of highest stress. We therefore attempted to form the compound by stressing petrolatum placed between variously shaped electrodes, but without success. With the thought in mind that occluded air might be responsible for the formation, we tried bubbling ozone through melted petrolatum. This did not produce any *X*.

Later we dissected a piece of badly wrinkled cable which had a deposit of *X* in the wrinkles in the lead,—the point of least stress,—and had only traces at the conductor. This led us to believe that the formation was due to electric stress in combination with rarified atmosphere. We then made up a Geissler tube of glass 8 in. long and ¾ in. in diameter. An electrode was sealed into one end and the other end was provided with a rubber stopper through which an electrode and a smaller glass tube were inserted. Some petrolatum was placed in the tube, which was then exhausted to 28 in. of mercury and potential from a one-in. spark coil was applied to the terminals of the tube. In a short time the peculiar granular structure of the *X* compound became apparent on the surface of the petrolatum. Its identity was established by chemical and microscopic tests.

D. M. Simons: To my mind, the outstanding point of this paper is Fig. 7. If the breakdown strength of three-conductor cables under long-time application of voltage is only half that of single-conductor cables, or even if it is considerably less than single-conductors while not being as low as this figure suggests, then this is a fact of outstanding importance in the cable industry. I am interested in it not only for its general bearing, but also because it is a point that the engineers of the company with which I am associated have been claiming for many years, especially in connection with the use of three-conductor Type H cable in place of the usual belted construction for the higher voltages, say, 22,000 volts and higher. This form of cable is of course equivalent in its electric field to three single-conductor cables under one lead sheath, and our claim has been that this form of cable would avoid many of the troubles that have been observed in the outer layers of conductor insulation and especially in the filler spaces of the usual form of three-conductor cable and joints when used in the upper range of voltage. It is very significant to have this effect confirmed in such a striking manner by Mr. Farmer.

Percy Dunsheath: Mr. Farmer referred to the relative importance of the cable and the other part of the installation. The cable is, after all, from a financial point of view, *the* part of an installation to watch for possible improvement with a view to cutting down the costs of development work.

One point I should like to make is that the first thing to do is to settle this question of acceptance tests. If we can arrive at a test that will prove quality, then I think quality will follow. If there has been any delay in improving quality, it has been because we have not known what we wanted and have not been able to recognize quality when it already existed. We are still using the standard pressure test in spite of the fact that after a cable has been pressure-tested in a factory, if it isn't a good length of cable, the pressure test may leave it on the point of breakdown and it is sent out of the factory in this condition.

Unfortunately, after the completion of these three papers we are not left with a clause which we can put into a specification,—an acceptance clause; but I think we are all agreed that the work, particularly Mr. Farmer's analysis of the time–voltage curve, does bring us very much nearer arriving at a clause of this kind. I don't know whether it has been done before (to me it is quite new) but I see that Mr. Farmer has analyzed the time–voltage curve into a number of component time–voltage curves, some for the bad parts of the cable and some for the good. I think that that is a very important observation to make because if a cable consists of a number of different parts, some with a low time-voltage curve and some with a high time–voltage curve, then a short-time, high-pressure test is a means of finding out weakness in the cable. If there is only one time–voltage curve for the whole cable, then a short-time high-pressure test can be carried out but it may be destructive.

Page 8 of Mr. Farmer's paper advocates adopting a standard load. Generally speaking, I think that that is a good thing. If we could have a standard load which we could put onto any method of measuring losses,—dynamometer, bridge, or any other means,—and so arrive at a distinct, definite calibration, it would be a considerable step forward. I should suggest, however, that instead of the type of load which he proposes and which has to be measured, itself, we adopt a load which can be built up to a known power factor. I should suggest a condenser with negligible loss combined with a definite pure resistance.

On the eleventh page of Mr. Farmer's paper, those curves showing the variation of rising power factor with time are important and interesting and I am wondering whether the rise which he sometimes gets and sometimes does not get is due to a very slow increase of hydrostatic pressure. If so, possibly we can follow out that line with a view to getting an acceptance test, a cable which didn't give that original rise being a better quality than one which did.

Mr. Farmer describes the end which he uses for a three-conductor cable. I should just like to set forth a suggestion which I found useful in taking off ends for that type of test. To avoid the cost of a wiped joint, I have used ebonite cones about ½ in. thick, with a taped joint. Over the thickened dielectric on each core, I have carried tin-foil clear up to the crotch, with the result that no breakdowns can take place in the crotch. It is quite easy to bring an end of this kind to the highest voltage required.

I agree that the preparation of these samples is important. It is really astonishing the differences one can get on time-voltage tests on the same make of cable by first inverting the cable and putting the ends in oil, then turning the ends upward and putting on oil pockets. If anything should be standardized, it should be the method of making the ends. If an end is made which prevents the escape of any gases generated, up goes the time–voltage curve.

R. J. Wiseman: Referring to Fig. 1 of the paper, we find existing a rather interesting property aside from the wide variation in insulation resistance. The variation is very much greater for the compounds giving high values of insulation resistance.

Fig. 4, showing the distribution of failures for five-minute tests, bears out what some of us believe; namely, that failure on a reel test is due to mechanical defects and will be detected with a reasonably high voltage. Too few tests are reported for 15 min. to be able to draw any conclusions. It is quite likely that Mr. Farmer is right, that, due to pre-testing by the manufacturer, failures on short time are eliminated before inspection.

Table I shows an improvement in breakdown voltage for each year; but there is still room for improvement. When the maximum dielectric strength at the conductor approximates the dielectric strength of a sample between flat plates we shall have reached the limit.

Figs. 5 to 10 inclusive are of interest in suggesting the limiting stress we can put on a cable indefinitely. At the present time, for methods of manufacture I believe 125 volts per mil is nearer the safety limit of operation than 150 volts per mil. This will give a factor of safety of about four or five on strength of material.

Mr. Farmer's suggestion of a standard load for checking dielectric-loss testing equipment is a very good one; besides the load he describes, I should like to advise his considering a load giving larger charging currents and higher power factors. I have found this important in making check tests on equipment.

A study of the change in power factor with time for different stress values is one of the best means to determine the aging quality of a cable. We would expect a slight increase in power factor for about 50 hours and then either constant or decreasing values.

I hope Mr. Farmer will be able to solve for us the proper manner of preparing samples of cable for high-voltage test. At present we do not get the real short-time breakdown voltage due to flashovers. Potheads are of great help but still not sufficient. The time consumed in preparing ends for test is enormous. Any means of reducing it and still permitting higher voltages is welcome.

I should suggest that Mr. Farmer replot Fig. 30, using the maximum voltage stress at the conductor. Even though he may not think it accurate, he will get a better comparison of the test on cables and condensers for wax formation.

W. A. Del Mar: Unfortunately, the test for X which is described in Mr. Farmer's paper is open to the serious objection of giving discordant results. A case which came up quite recently was in relation to a barrel of compound which we had had in our laboratory for some weeks. We took out a sample which we designated A, divided it in two, sent half to the Electrical Testing Laboratories and kept the remainder ourselves. The Electrical Testing Laboratories sent back a report, "Abundant wax formation." Our laboratory, using apparatus as described in Mr. Farmer's paper, reported "none"; so we thought we would make a check test. We took another sample B out of the same barrel, again divided it in two, sending one part to the Electrical Testing Laboratories and keeping the remainder ourselves. They reported no wax and our laboratory reported considerable wax.

That bears out the general condition we have in our own laboratory; *i. e.*, we cannot obtain consistent results. We get X in a given compound perhaps in three or four tests in succession, and then, for some unknown reason, without any apparent change, the opposite result is obtained.

Even if the test can be perfected so as to be susceptible of unfailing check, insufficient evidence has been offered to connect the test with operating results.

Another interesting feature of Mr. Farmer's paper is the apparently greater dielectric strength of insulation on single-conductor cables as compared with triplex cables, as shown in his Fig. 7. This diagram, however, shows maximum stresses, as computed from the ordinary logarithmic formula. Recent researches by P. L. Hoover[1] have shown that the maximum stresses at failure cannot be derived from that formula, and that average stresses are a better criterion. As the average stress in a single-conductor cable is a much smaller fraction of the conventional maximum than in a triplex cable, the discrepancy which is made to appear in Fig. 7 disappears.

F. A. Brownell: For testing purposes Mr. Farmer has pointed out the need of an end bell that could be constructed or connected in a minimum of time. This is one of our chief difficulties at the factories today and it seems that more time and thought should be given to this subject. We have used, with some modification, the end bell shown in Fig. 22 of Mr. Farmer's paper and have had very satisfactory results, the only objection being the time required for wrapping the conductors with varnish cambric. I believe this can be overcome by the use of the roll of tapered paper that the Pirelli Company used in making up the joints on the 45-kv. line of the United Electric Light and Power Company of New York City. This paper could be made up into rolls and placed over each of the three conductors and tightened. This would give a snug-fitting insulation and the crotch would be entirely filled. It may be that the taper of the paper would have to be changed somewhat to give the desired fitting.

F. M. Farmer: In the captions for Figs. 25, 26 and 27 the word "wax" was used. This is unfortunate, since the product produced is not wax in the sense that the term is used in industry. In fact it is quite different from any known substance familiar to oil technologists and has none of the characteristics of any variety of wax. Furthermore, it is not soluble in any of the ordinary solvents. Mr. Del Mar has suggested the expression X. In view of our lack of knowledge of this substance, this designation seems particularly appropriate.

From the tests which he describes, Mr. Willman implies that this deterioration is due to the action of ultra-violet light resulting from a sufficiently high electric stress across microscopic air voids in the compound. Whether this X formation is due directly to the application of stress or due to the action suggested by Mr. Willman, which, in turn, requires the presence of stress, is, from a practical standpoint, immaterial to the user. He is interested in knowing whether or not the compound in the cable remains stable under the conditions of use which involve only two variables, namely, stress and temperature.

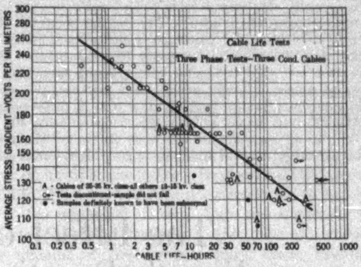


FIG. 6A—CABLE LIFE THREE-PHASE TESTS—THREE-CONDUCTOR CABLES

Mr. Del Mar has indicated that the test proposed for X formation is not dependable. It is probably true that it is not very sensitive where the compound forms X with difficulty so that X is formed at one time and not at another. Just why this variation occurs we do not know as yet but it is believed that the test being so simply and easily made justifies its use as a rough indicator of the probable performance of the compound.

Mr. Del Mar also raises the point that it has not been definitely demonstrated that this kind of deterioration in compound in a cable necessarily results in ultimate failure. This is true, but while we haven't positive proof that X in cable is objectionable, there is altogether too much circumstantial evidence available to justify engineers assuming that X is *not* objectionable. Since compounds can apparently be developed which do not form X and which are otherwise satisfactory, it seems only good engineering to avoid the use of X-forming compounds, even though we are not positive that they are objectionable.

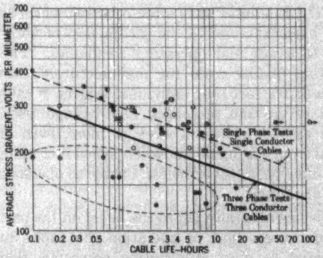


FIG. 7A—CABLE LIFE TESTS—SINGLE-CONDUCTOR CABLE

Referring to Figs. 6 and 7, some additional data have been obtained since these diagrams were made and new diagrams are given herewith,—Fig. 6A corresponding to Fig. 6 and Fig. 7A corresponding to Fig. 7. It will be observed that the slope is the same so that we are quite convinced that the seventh-power relation is approximately correct for at least the types of cable involved in these tests. Of course, if there is a pronounced change in the method of construction or in the kind of compound used in high-tension cables, the slope of these curves will undoubtedly be affected.

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[1] *The Mechanism of Breakdown of Dielectrics*, by P. L. Hoover, A. I. E. E. JOURNAL, September 1926, p. 824.

The point mentioned by Mr. Simons,—namely, the apparent superior showing of single-conductor cable over three-conductor cable,—is obviously of great importance; so much so, in fact, that a rather elaborate systematic investigation is being planned with specially constructed cable to determine the correctness or incorrectness of this indication. Mr. Del Mar points out that if average stress instead of maximum stress had been used in preparing these diagrams, the apparent difference between single-conductor and three-conductor cables would have been less. This, of course, is quite correct. For instance, the average *maximum*-stress gradient to give a life of 10 hr. in three-conductor cable is 215 volts per mil and in single-conductor cable, 400 volts per mil, or an apparent increase of 86 per cent. The average *average*-stress gradient to give a life of 10 hr. is 175 volts per mil in three-conductor cable and 220 volts per mil in single-conductor cable or an apparent increase of 26 per cent. But which of these two bases is the true measure of dielectric strength of the cable as a whole? As we all know, there has been much discussion of this point and papers have been presented before this Institute giving evidence which apparently favors both of these gradients as well as intermediate ones. Eventually we shall have the true answer; and while it is highly probable that it will not be the maximum-stress gradient, it is also probable that it will be something higher than the average gradient. It seems reasonable to conclude, therefore, from the evidence presented here, that the inherent dielectric strength of single-conductor cable is on the average substantially higher than that of three-conductor cable and that the difference may be of the order of 40 or 50 per cent. However, as previously stated, this apparently large difference must be confirmed by further investigation before a final conclusion is justified.

Referring to Capt. Dunsheath's suggestion in connection with the proposed standard load for dielectric loss testing that various power factors be obtained by the use of series resistance, I should say that we have had this point in mind.

The Effect of Internal Vacua

Upon the Operation of High-Voltage Cables

BY WILLIAM A. DEL MAR[1]

Fellow, A. I. E. E.

Synopsis.—*It has been known for some time that the internal mechanical pressure of cables is a factor in the quality of the insulation, and that the oil used in impregnating paper for cable insulation has a high thermal expansion which may alter the internal pressure of cables with varying temperature.*

This paper considers the rate of pressure equalization in cables and the relative effects of slow equalization upon long and short cables, indicating that there may be considerable difference in dielectric properties depending on the length and past history in regard to temperature cycles.

Means of overcoming these differences are suggested.

The paper is suggestive rather than demonstrative and it is proposed to publish a sequel giving experimental proofs of the facts stated.

CABLE failures have often occurred which have not been explained to the satisfaction of both manufacturing and operating companies. Their outstanding feature from the manufacturers' standpoint has been the fact that individual lengths of cable withstand severe and prolonged high-voltage tests sometimes extending over a week or more, even after removal from the ducts, but when joined into a long cable, they have been, nevertheless, unable to operate at their normal voltage.

This has naturally led to the belief on the part of the manufacturers that in operation the cables must be subjected to more destructive potentials than they experience in the tests, or in other words, that the trouble is due to transient high voltages.

The only alternative is that, in some way, the joining together of a number of dielectrically strong lengths can result in a dielectrically weak cable.

Until recently, this latter alternative seemed scarcely worth consideration since it was outside the range of reasonable probability, whereas the theory of transients seemed to have some foundation in actual circumstances such as the concentration of failures at certain points, regardless of time or place of manufacture of the cable.[2]

The point of view of the user usually seems to have been that the actual evidences of severe over-voltage are negative, whatever the theoretical evidences in their favor might be. They have discounted the proofs of excellence of the individual lengths of cable, and explained the failures on the assumption that the cables are either impregnated with chemically unstable oil or that the cable is incompletely impregnated, the evidences of which are likely to be found in the cables that fail. This theory has not satisfied the manufacturers because cables impregnated with identical compound have been known to operate satisfactorily at higher electric stresses than those in the cables that failed.

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[1] Chief Engineer, Habirshaw Cable and Wire Corporation.

Presented at the Regional Meeting of District No. 5, of the A. I. E. E., Madison, Wis., May 6-7, 1926.

[2] There is no doubt that transients have been a contributory cause of trouble, especially on large systems.

The manufacturers agree that practically perfect impregnation is essential to obtaining high dielectric strength but have not considered deficiencies in the impregnation or unstable oil to have been always the root of the trouble.

New information has now come to light, however, which explains how a number of good lengths of cable can be joined together into a bad cable, and the evidence is stronger than that which has been adduced in favor of the theories of abnormal voltage, or of defective cable.

The difference between the short length and the continuous cable, as manufactured in the past, is that the former has its ends open to the atmosphere so that the inside of the cable is at atmospheric pressure, whereas the latter, except near its ends, has an internal pressure independent of that of the atmosphere, by virtue of the continuity of the sheaths and joint sleeves, and the viscosity of the compound.[3]

On this difference, in our opinion, hangs the explanation of many mysterious cable failures.

CAUSES OF VACUA IN CABLES

Oils, such as are used for impregnation, have an extraordinarily high coefficient of thermal expansion, usually of the order of 0.09 to 0.1 per cent by volume, per deg. cent. This is about ten times that of the metals used in a cable. In order to visualize the effect of this, we may assume that all of this change in volume is concentrated at one end of the cable, and consider what happens when a length of, say, 400 ft. is cooled. If the cable is completely filled when it leaves the factory, which may be assumed to be at about 25 deg. cent., and is then put in a duct at 0 deg. cent., the volume of oil will have shrunk 2.5 per cent so that 10 ft. of the cable will be completely deprived of oil.

In actual fact, this shrinkage will not be concentrated at one place but it will be none the less real and, in the aggregate, will be equivalent to the 10 ft. of cable. Where the distance from the center of a cable to its open ends is several miles the time required for internal readjustment of pressure, with heavy petrolatum compound, may be several years, especially if the cable is lightly loaded. It is thus a matter of vital importance to load high-voltage cables, when impregnated with stiff compound, either prior to or upon the application of the voltage, and to maintain a load sufficient to liquefy the compound until internal pressures have been adjusted.

The contraction of compound in a cable is very beautifully shown by impregnating a piece of cable in a light-colored oil, and transferring it while hot to a dark-colored oil in which it is allowed to cool. The outside of the cable will be the first to lose its heat and the oil will, therefore, solidify there first. Hence, as the oil contracts, it will draw away from the center.[4] The dark grease used for cooling will, however, be sucked into the hollow so that the cable in cross-section will present the appearance of a hollow ring of light color with a dark core. A small amount of dark compound will also be found in the outer layers due to diffusion. If the cable were allowed to cool in the open air, instead of in dark oil, the core would be free of oil except for that held in the paper by capillarity. Other vacuous spots will form in the compound as it contracts, either as bubbles or streaks.

Lead, which is used universally for cable sheaths, is peculiar in being almost entirely lacking in elasticity; that is to say, when it is stretched, it does not return upon removal of the stretching force. When a cable is put on a reel at the factory, its sheath is stretched on one side, and when the cable is straightened, to go into a duct, the stretched sheath does not contract, but wrinkles, thereby increasing the internal volume. The amount of this increase may, under favorable conditions, be as high as ½ per cent, so that in a case where the oil constitutes ⅓ the cubic contents, the increased volume would take oil from about 1½ per cent of the cable, an amount of similar order of magnitude to the disappearance of oil due to contraction.

Thus, there are two causes of vacua in cables which are of somewhat similar order of magnitude, both operating simultaneously and of such magnitude they might desaturate a 400-ft. length of cable to the extent of leaving the equivalent of 15 ft. of cable without oil and with a vacuum in its place. Thus, a petrolatum-impregnated cable having atmospheric pressure at 25 deg. cent. when reduced to 0 deg. cent. shows an internal vacuum of 22 or 23 in., as measured by a pressure gage attached to one end. In this condition, ionization will occur at about ⅓ the voltage which would be required at atmospheric pressure.

It is not only in theory or laboratory tests that this phenomenon occurs. The lead sleeves of a large number of splices on high-tension lines have been known to collapse, proving the existence of very low pressure within the cable. Vacua of 15 to 20 in. have been noted in splices of installed cables, and in such vacua, ionization starts at 40 to 60 per cent of the voltage required at atmospheric pressure.

When high-tension cables are installed, it has been frequently observed that air is sucked in when the caps are broken. This absorption of air continues for hours at a diminishing rate. If the joints are applied before the completion of this action, the cable will necessarily start its working career with a partial vacuum.[5]

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[3] Cable compounds being very viscous, flow slowly, so that in the case of a petrolatum-impregnated cable, the readjustment of internal pressure is necessarily a very slow process, varying from practically zero speed with hard compounds and solid conductors to practically infinite speed with thin liquid compounds and hollow conductors.

[4] The same phenomenon may be observed by heating a small beaker of grease to the melting point and allowing it to congeal. When solid, the surface will be funnel-shaped instead of practically flat as when water congeals.

HARMFUL EFFECT OF VACUA

Having shown that vacua should exist in cables and then that they do exist, the next question is, do they do any harm?

Again the answer is in the affirmative.

Partial vacua ionize at much lower voltages than air at atmospheric pressure. Hence, a cable, having a partial vacuum within it, will show evidences of ionization at abnormally low voltages, so that ionization may occur even at working voltage or less.

The effect is not a mere corona glow, but a series of active streamer discharges, such as are familiar to us in vacuum tube lamps. These streamers emit rays, probably the ultra-violet, which have the effect of converting petrolatum compound into a solid, flaky substance, often known as *wax* or *X*.[6] Recent analyses have indicated that this substance has the approximate chemical composition,

$$(C_{12}H_{28}O)_n$$

It is apparently highly polymerized but of unknown degree of polymerization.[7] It is not a wax.

If petrolatum be placed in a vacuum tube so as to be out of the way of the terminals, but yet impinged upon by the discharge, the surface will be converted into *X*. This interesting laboratory experiment, due to Mr. E. C. Willman, of Cleveland, enables one to see, in process, the phenomenon which is known in cables only by its results.[8] It has been persistently denied by the manufacturers that the presence of this *X* is necessarily an indication of imminent failure of a cable and they are supported in this contention by tests on short lengths which continue in operation at very high stresses even after the practically complete conversion of the oil into *X*, yet cable users have known that, in some way, *X* formation and cable failure go together. These views are reconciled by the fact that short lengths of cable, such as are tested in laboratories by the manufacturers, will not fail due to *X* because their internal pressure quickly becomes atmospheric whereas long (installed) lengths are more liable to failure because, with stiff compounds, low pressures persist due to the slowness of pressure equalization. Furthermore, *X* formation increases vacua because *X* is less bulky than oil, and as mixed with the residual oil it forms a paste which is stiff and resistant to pressure equalization. If conditions are such as to create *X*, but yet maintain atmospheric pressure, the cable often will not fail.

A number of short lengths (10 ft. under lead) of cable were submitted to an accelerated aging test, one-half the cables being sealed hot, so as to have partial vacua within them when cool, and the other half allowed to cool with the compound at atmospheric pressure. The preliminary results show the vacuous cables have a much shorter life than those at atmospheric pressure, but difficulties in maintaining the seals have vitiated all experiments as far as quantitative results are concerned. The experiments are, therefore, being continued and Mr. C. F. Hanson expects to report them at a later date.

THE WAY TO AVOID HARMFUL VACUA

We have seen the cause of vacua in cable, and their baneful results. The next question is, naturally, how can they be avoided?

Before answering this, it should be borne in mind that an installed cable ordinarily has an internal pressure entirely independent of the atmosphere because its sheath and the sleeves of its joints are sufficiently thick and rigid to resist considerable pressure.[9] In order to maintain the inside of the cable throughout at atmospheric pressure, there must be the equivalent of openings to the atmosphere, compounds sufficiently fluid to transmit the pressure from these openings at a speed comparable with that at which thermal contraction occurs and longitudinal oil ducts of sufficient cross-sectional area to assist this action. In most cases the normal spaces between strands will suffice, but for very high voltages, a hollow core or special duct is desirable.

In order to provide the equivalent of openings in the cable or joint these must either be open to the air, or provided with a closed but collapsible part. The reservoir of liquid or semi-liquid oil applied to splices in an invention first used at Cleveland meets the latter requirements,[10] while the liquid oil reservoir system, developed in Italy, meets the former, although neither appear to have been intended for the specific purpose described above.

It should be remembered that pressure equalization is very slow with viscous oils, and differences of pressure should not be allowed to accumulate, but rather should be given an opportunity to equalize as they are forming.

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[5] A collapsible cap filled with compound may be placed over the ends of a cable when the seals are broken, which will prevent access of air and fill the voids with compound. Such a cap should be left on for several hours, preferably overnight. The use of such caps both in factory and field is the subject of a patent application.

[6] Ultra-violet rays have a similar effect upon most oils. In the case of some vegetable oils, the effect is so pronounced that it has been made the basis of a commercial lacquering process.

[7] Some experimenters report a very small proportion of nitrogen, but we regard this as too minute to be more than an impurity.

[8] It was formerly thought that *X* was due directly to the electric stress, but it has not been found possible to produce it by stress alone, in the absence of electric discharges.

[9] A pressure of 35 to 45 lb. per sq. in. will start expansion of 8/64 to 10/64 in. pure lead cable sheath.

[10] The Cleveland reservoirs were designed for the purpose of exerting atmospheric pressure "on the mobile compound to prevent the formation of voids during temperature changes," (Report of Underground Systems Committee, N. E. L. A., 1924) but as only the joint compound was mobile, the cables being impregnated with petrolatum jelly compounds, the reservoir appears to have been intended primarily as a protection for the joint rather than for the cable.

In conclusion, we summarize the following characteristics as among the most essential for the successful performance of high-voltage cables:

1. The compound should be sufficiently fluid or soft at all operating temperatures to permit readjustment of internal hydrostatic pressure at a speed comparable with that at which this pressure is disturbed by temperature variations.
2. Access to the atmosphere or to atmospheric pressure should be provided at points sufficiently close to ensure the communication of such pressure to all of the compound in the cable.
3. Oil ducts (special or incidental) must be provided to assist the transmission of this pressure along the cable.
4. The saturation of the cable should be as perfect as possible so as to provide a minimum of oxygen to be absorbed by the compound.
5. The compound should be as immune as possible from deterioration by the rays generated by electrical discharges.

In addition to these characteristics, which are related to the subject matter of this paper, the cable should have, of course, suitable dielectric strength, dielectric loss and flexibility.

Discussion

Wallace S. Clark: In connection with Mr. Del Mar's paper it should be remembered that the Pirelli idea is to keep in the cable, at all times, a pressure in excess of atmospheric pressure.

Percy Dunsheath: I think Mr. Del Mar has gone a little bit too far although probably not intentionally. I am rather afraid the impression that his conclusions will give is that all cables for high voltages should be supplied with oil ducts. I don't think he intended that and certainly I should resent any such suggestion, because I am sure that our experience with 33,000-volt cables, unhappy as it was years ago, today proves definitely and quite conclusively that a 33,000-volt cable can be made to give satisfactory service without oil ducts, whatever conditions of high voltages are called for.

R. J. Wiseman: Mr. Del Mar has presented to the public the views some of us have had for some time as to the effect of voids in cables. However, I think it would have been better if he had eliminated reference to atmospheric pressure. Voids are created in a cable due to the oil contracting as it cools and endeavoring at the same time to maintain a balanced pressure. This means that if a cable is sealed and the oil cools, it is below atmospheric pressure. A cable in operation with sealed ends is rarely at atmospheric pressure, usually above, and below only when cooled.

The size of voids in a cable will depend a great deal upon the type of oil and the freedom of motion. The size of the voids increases as we lower the temperature. This results from the capillary attraction of the oil to the paper. A tacky oil holds best and is likely to produce small voids rather than large ones, the latter being more objectionable.

Fortunately, shortly after a cable is installed at 0 deg. cent., it assumes the temperature of the ground and therefore heats up, eliminating most of the voids produced at low temperature. It is most desirable to load a cable up for a few days after installing it in order that the oil may become stabilized locally before making any voltage tests. Here the purchaser could help very well by actually putting a cable into service for a week and then making the acceptance test. This should be done on all high-voltage cables.

Take the case of the cables when they cool down after the load is removed, but the voltage is still on. Here it is advantageous that the dielectric loss be sufficient to heat the cable a few degrees, thus preventing the formation of voids. This brings out an important point. A reasonably low dielectric loss at low temperatures is preferable to a very low loss.

Creation of voids in joints is most probably due to the draining of the oil into the cable. It is not a case of stabilizing pressure, but rather improvement in cable impregnation. In such a case, the heating of the cable with load does not help the joint. The joint is on the way to failure unless refilled.

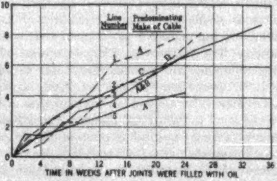

FIG. 1—CURVE SHOWING ADDITION OF OIL TO 22-KV. AND 33-KV., THREE-CONDUCTOR OIL-FILLED JOINTS WITH INSULATION FORMED

Herman Halperin: The impression is given in the paper that transient voltages cause many cable failures. In meetings of the A. E. I. C. Subcommittee on High-Voltage Transients on Underground Cable Systems, no information was given to show that transients cause any deterioration of the insulation. As expressed at a meeting of the subcommittee a few weeks ago, the opinion seems to be that the transients cause failures at unusually weak spots in the insulation; that is, the cable has usually been found obviously deficient at the point of failure, which cannot be said about cable failures in general.

The failures due to transients have generally been distributed over all parts of the underground systems, and rarely have they been located at any particular point, as may be inferred from the paper. This experience has been reported by several companies and is checked by the experience of the Commonwealth Edison Company.

In recent klydonograph investigations of surges on eight large systems, it was found that 85 per cent of the surges were less than twice normal voltage. About 1 per cent of the surges exceeded four times normal voltage, and it appears that surges must reach this magnitude to be disturbing. The latter surges may be slightly in excess of the full-reel test voltage at the factory. On the other hand, samples of high-tension cable will usually withstand test voltages of 7 to 12 times rated voltage of the cable for about one minute. The duration of the transient voltage is only a fraction of a second and Peek and others have

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NOTE: Emanueli, *Transactions* of the First World Power Conference, 1924, Vol. III, p. 1269, states: "The possibility of manufacturing cables destined to work at such high pressures (130 kv.) depends on careful reduction to a minimum of the gas which is occluded between successive layers of the insulating material," and *Jour.* I. E. E., Jan. 1926, p. 127, referring to the 130-kv. cable in Italy, "Its construction differs from that of an ordinary cable only in the fact that air or gas bubbles are entirely eliminated from the insulation."

presented data to show that the dielectric strength of insulation increases very rapidly as the time is reduced to a fraction of a second. One manufacturer has reported a case where the insulation withstood transient voltages of about three times the breakdown voltage he had been obtaining on dielectric-strength tests on samples. In regard to the decrease in dielectric strength on account of the higher frequency of the surges, it appears that this is more than counterbalanced by the increase in dielectric strength due to the shortness of the application of the transient.

In general, with cable of high quality, such as indicated as being desirable in the paper on the Quality Rating of High Tension Cable by Mr. Roper and myself, evidence seems to show definitely that transients will cause no failures and have no deleterious effects on the insulation.

In regard to the point in the first paragraph of the paper, that sections upon removal from ducts will withstand severe high-voltage tests even though they have failed in service, the failure occurred at the spot of lowest quality. These weak and irregular spots have been found repeatedly in accelerated life tests in Chicago, and the experience has been that if a portion a few feet long were removed from a section of cable, the voltage rating of the section would then be increased from 15 per cent to over 100 per cent. In the last part of the paper, there are listed the essential characteristics for successful high-tension cable, but the item of uniformity is omitted, although experience shows that it is of paramount importance.

In connection with expansion and contraction incidental to the operation of cables and the consequent formation of voids, the Commonwealth Edison Company has had an interesting experience with oil-filled joints on several lines of 22- and 33-kv., three-conductor, impregnated paper-insulated cables with fair impregnation. The joints were filled with a switch oil having a viscosity of about 200 seconds (Saybolt) at 25 deg. cent. Periodically the joints were checked as to the level of the oil in them, and sufficient oil was added (or removed in a few cases) to restore the level to its normal plane. Fig. 1 herewith shows the cumulative average number of pints of oil per joint added to these joints during the period of 5 to 8 months. In the average case, a total of about one gallon has been added in six months to each joint. This means that each length of cable had sufficient void space for one gallon of compound, which is sufficient compound to impregnate completely about 9 ft. of cable.

F. A. Brownell: Apparently Mr. Delmar's paper explains why so many of our cable failures occur during the off-peak when the cable is cooling or cool.

In 1923 we had 107 failures in our 13-kv. cables. Sixty-one of them could not be classified as to cause. Forty-two of this group failed during the early morning hours while the cables were carrying practically no load. Other years have shown similar results. If it is possible for vacua to form in our present cables at a temperature they would attain on an off-peak load, it would seem that this condition could be overcome by a rapid equalization of oil under pressure, the use of a less viscous oil than is used at present, and the installation of a reservoir on joints filled with the same oil that is used for impregnating the cables.

We have found the formation of X in cables operating at 26 kv., while in the laboratory it has required a potential of 100 kv., on the same make and type of cable before formation occurred. In cables impregnated with a rosin-content compound we were unable to produce it at potentials up to 210 kv. We have had cables that have failed in service operating at 26 kv. and have shown signs of ionization. This would indicate that the pressure in the cable must have been below atmospheric to have had ionization at this voltage.

D. W. Roper: Mr. Del Mar's theory seems to be borne out in a number of instances in our experience. He refers to the contraction of the compound in the cable; we have had a similar experience with compound in the joints. In some of our 33-kv. joints, we use a compound similar to that used in the cable; that is, a petrolatum. This particular line was operated practically without load for weeks at a time during the winter months. It was a tie line between generating stations and it was largely a reserve line on the generating capacity in the two stations.

After some weeks of operation during the winter season, we began to notice a bulging of the joints. We tried to discover the cause of this bulging, and as nearly as we could determine, it was due to the formation of small vacua or voids in the compound in the joint. Had this cable been operated every day at a load which would warm the compound to the melting point, the voids, when they reappeared, would probably not have reappeared at the same location. You can find these voids if you will take a mass of compound, either in a can or a glass vessel, and expose it to low temperature; you will find small voids perceptible to the naked eye, distributed throughout the entire mass of compound. If you take a different kind of a compound which has more cohesion between the particles,—a more viscous compound,—then when the compound cools there will be more of a settling of the horizontal surface so that there will be a serious depression, but with a compound of this kind, there is no great amount of settling of the surface; the cooling occurs by the formation of these minute voids throughout the mass of the compound.

It appears that these small voids will occur now and then adjacent to the conductor insulation which is in the joint, and when these occur, the ionization, due to the discharge in the voids, will occur, and in the course of time these voids will enlarge. By careful examination of the joints when opened, we actually found some voids of approximately the size of the joint of the thumb, and adjacent to such voids, we found the evidences of ionization in the factory-applied insulation in these joints.

There was undoubtedly some pressure within these joints as shown by the bulging of the lead sleeve at the joint, although these lead sleeves were covered with broken cement, so that it not only stretched the lead sheaths but cracked the cement. The trouble has been cured by removing those joints so as to remove this petrolatum and refill with a thin oil. This is the oil referred to by Mr. Halperin in his discussion and diagram. No such trouble occurred when the joints were filled with oil; in fact, trouble could not occur in a thin oil which was so thin as to be fluid at even the minimum operating temperature of the cable. The nature of the compound used in the joint, as nearly as our experience shows, must either be fluid or be of a character which will not permit the formation of these internal voids throughout the mass of the compound.

T. F. Peterson: One must recognize that the burden of failures resulting from conditions characteristic of long jointed lines, must be borne by operating companies. Failure due to vacua in installed cable falls into this class. The following is submitted because it is thought that too much importance has been attached to this condition.

Spaces or air pockets in insulation may be due to:

1. Contraction of oil after impregnation
2. Expansion of lead due to bending
3. Contraction of compound due to cheese formation.

Mr. Del Mar makes no attempt to show the harmful effects of pockets containing gases at atmospheric pressure. Indeed he attaches no importance to these. Failures are considered to be due to the fact that *after* cables are installed, vacua are invariably produced, ionization takes place and ultimately failure results. This reasoning tends to exonerate the manufacture and shift cause of failure to a rather unfortunate condition with which operating companies must cope.

It is my contention that air spaces, even at atmospheric pressure, constitute a very important cause of failure (the action being somewhat slower than for vacua, nevertheless very pronounced) and since, in a large measure, they are due to causes 1 and 3, which are of manufacturers' concern, it follows that the latter must assume some part of the responsibility for failure.

To substantiate this claim, consider the following:—Assume an air pocket at atmospheric pressure in a valley between turns of 5-mil paper; breakdown of this film, 0.0127 cm. thick, occurring at 68.3 kv. per cm. If the dielectric constant is assumed to be 3.5 the potential gradient in paper will be $\frac{68.3}{3.5} = 19.5$ kv. per cm. or 49.5 volt per mil. This is not an uncommon value for average gradient under operating conditions and so it appears that harmful results may easily result.

It remains, then, to explain the differences observed by the author between breakdown results on long and short lengths of cable. A partial explanation is given; namely, that of vacua. There is, however, another factor which tends to account for the results in so far as it assists the case of the short length. The latter is usually tested under oil where chances for reimpregnation are especially good; hence better test results.

The last paragraph of the paper gives ways of avoiding vacua. In the light of this discussion, it would seem that ways of diminishing possibilities of forming air pockets would have been a more appropriate subject for consideration.

W. F. Davidson (by letter): I am glad to note that Mr. Del Mar has attacked the problem of high-voltage cables from an angle which seems to be more in the direction of a real solution than many of our discussions of the "transient bugaboo." After all is said and done, our cables must be able to withstand such normal and transient voltages as do exist.

There is one phase of the paper which I wish to discuss; namely, the formula for this *X* or "cheese." When we have once definitely ascertained this factor we shall have made a long stride toward determining the true cause of its formation and definitely stating the effects of internal vacua, etc.

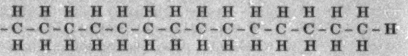
In the first place, I am surprised that it has been found possible to determine a chemical formula for *X*, because the original oils are known to be highly complex mixtures of several compounds and it is difficult to understand how we may be assured that the resulting *X* is really one chemical individual. Until we have this assurance, a chemical formula has no significance. Possibly Mr. Del Mar can give his data.

In the second place, as I understand the processes of analysis used in such cases—assuming that we have proved the existence of a single chemical individual—we would obtain the oxygen ratio by determining what was left after the hydrogen and carbon have been determined. This should give a definite value for n.

Finally, we have no evidence presented to substantiate the statement that "it is apparently highly polymerized." Here it is necessary to assume again that the substance is a chemical individual, for it is usual to ascertain the degree of polymerization by determining the molecular weight; and a mixture of several chemical substances cannot have a single molecular weight. But even so, the determination of molecular weight seems almost impossible in the case of a solid substance which is non-volatile and almost insoluble.

J. A. Duncan (by letter): Mr. Del Mar's formula $(C_{12} H_{28} O)_n$ is of special interest to those of us who have been studying the deterioration of impregnating compounds and it is this formula which I wish to discuss. In order to include all possible interpretations of such a formula, one must discuss all possible values of n. This means all values of n from plus one to plus infinity, since zero or negative values have no meaning.

Let us begin with the case n equal to unity. The formula is then $(C_{12} H_{28} O)$, which is very remarkable, because 26 atoms of hydrogen alone completely satisfy all the available valency bonds in a single molecule of 12 carbon atoms and it is difficult, if not impossible, to imagine how the other two hydrogens and one oxygen are attached to the molecule. The straight chain structure in normal paraffins contains the maximum number of hydrogens possible if we attribute to carbon the valency four which, I believe, has always been found to be its maximum value. It is seen from the diagram of a paraffin (dodecane for example)



$$\begin{array}{ccccccccccccccccccccccccccccccc} & & H & & H & & H & & H & & H & & H & & H & & H & & H & & H & & H & & H & & H & & H & & \\ H & - & C & - & C & - & C & - & C & - & C & - & C & - & C & - & C & - & C & - & C & - & C & - & C & - & C & - & C & - & H \\ & & H & & H & & H & & H & & H & & H & & H & & H & & H & & H & & H & & H & & H & & H & & \end{array}$$

that the maximum number of hydrogens per molecule is two plus twice the number of carbons. No carbon except the two at the ends of the chain can hold more than two hydrogens because two of its four bonds are occupied in holding the chain together. Each of the two end atoms holds three hydrogens.

The only way for a paraffin molecule to increase its oxygen content is for an oxygen atom to replace two hydrogen atoms or for a hydroxyl group O H to replace one hydrogen atom. In the former case, the hydrogen content is decreased and in the latter, it remains the same. If one O H group replaces a hydrogen in $(C_{12} H_{26})$ we should have $(C_{12} H_{26} O)$.

If one oxygen atom alone is placed within the molecule, it must replace two hydrogen atoms in order to find bonds for its two valencies. This would give us two hydrogens less than we had to begin with and the formula would be $(C_{12} H_{24} O)$.

It is thus highly improbable that one can have a compound with the formula $(C_{12} H_{28} O)$.

If n had any value greater than unity, the case would be even worse because this would mean a molecule consisting of two or more groups of $(C_{12} H_{28} O)$ and one valence bond of each group would be occupied in holding the two together. The possible number of hydrogens too would be still further decreased. One of the first two terms of the formula must be incorrect or Mr. Del Mar has a new arrangement of the oxygen atom with valency greater than two or a carbon atom with valency greater than four. The only case I've ever heard of before where oxygen has a valency greater than two is in the so-called oxonium compounds; and I think it is true that the science has never known a case of carbon with valency greater than four.

Of course, Mr. Del Mar may not mean his formula as that of a chemical individual, but merely gives it to indicate the proportions of carbon, hydrogen and oxygen in a mixture of substances which he found in a cable. If we allow mixtures, we can mix the original oils in the cable in many ways which would give us any specified ratio of carbon to hydrogen from the ratio one-to-four to the ratio infinity; and we could add air, moisture, or moisture and air, to take care of any amount of oxygen from zero to whatever number of oxygen molecules there are in a cable.

It is fairly well known that mineral oils usually used in cables consist of mixtures of several hydrocarbons of various carbon contents.

For example, suppose we had any number of molecules of normal hexane $C_6 H_{11}$ and half of the same number of oxygen molecules in a mixture which we shall call A. The proportions of carbon, hydrogen and oxygen would be 12 to 28 to 1 as Mr. Del Mar indicates.

Or suppose we had any number of molecules of normal dodecane $C_{12} H_{26}$ mixed with an equal number of water molecules $H_2 O$. Let us call the mixture B. The proportions of carbon, hydrogen and oxygen would again be 12 to 28 to 1.

Any mixture consisting of any amount of A with any amount of B would obviously still have carbon, hydrogen and oxygen in the proportions 12 to 28 to 1. Such a mixture we should properly call *X*, following Mr. Del Mar's happily selected notation.

Let us assume a cable oil with 0.1 per cent each of hexane and dodecane. One gram of this oil would contain 0.001 gram of each substance. Let p be the actual number of molecules of hexane and q the number of molecules of dodecane in 0.001 gram. Then

$$p = \frac{N}{86.112 \times 1000}$$

and $$q = \frac{N}{170.208 \times 1000}$$

where N is Avogadro's number, Millikan's value of which is $(6.062 \pm 0.006) \times 10^{23}$ and 86.112 and 170.208 are the molecular weights of the two substances. One can have then a mixture of any number of molecules from (1 to p) of A with any number from (1 to q) of B and still have the proportions of carbon, hydrogen and oxygen indicated by Mr. Del Mar. The number of possible different kinds of X, no two of which are identical and each of which would fit Mr. Del Mar's formula is

$$p \times q = \frac{N^2}{86.112 \times 170.208 \times 1000 \times 1000}$$

or $$p\,q = 2.5 \times 10^{37}.$$

The formula indicates no way except by guessing to tell which of the possible mixtures it represents. If each of the one hundred and ten million people in the United States would make a thousand guesses a day for fifty years this would be 2×10^{15} guesses. The chance of one of these guesses coinciding with Mr. Del Mar's X is a little less than one in 1.25×10^{22} or in other words one in twelve and half thousand billion billion and the guessing would be further complicated by the 99.8 per cent of the oil which we did not consider specifically in the above p and q.

C. F. Hanson (communicated after adjournment): In some localities, high-voltage cables operate at relatively low temperatures. From the point of view of dielectric loss and the possibilities of cumulative heating, these low temperatures of operation are considered favorable. However, from the point of view of internal vacua, these low temperatures are detrimental, if the cable is not permitted to "breathe."

The conditions upon which I base my calculations in arriving at the foregoing conclusions are as follows:

1. The maximum operating temperature of the cable insulation during summer months is 40 deg. cent. (104 deg. fahr.)

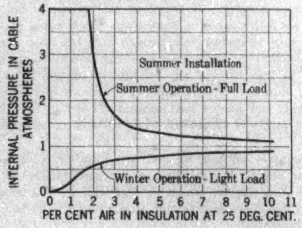


FIG. 2

2. The minimum operating temperature of the cable insulation during winter months is 20 deg. cent. (68 deg. fahr.)

3. The temperature of the cable insulation at the time of installation during summer months is 30 deg. cent. (86 deg. fahr.)

4. The temperature of the cable insulation at the time of installation during winter months is 15 deg. cent. (59 deg. fahr.)

5. It is assumed that an internal pressure of one atmosphere is established throughout a length of cable while the ends of the cable are open for splicing purposes.

6. The impregnation of the cable is expressed in terms of per cent of air by volume based upon the total air in the cable prior to impregnation. The air content is considered to be that which prevails at 25 deg. cent. (77 deg. fahr.)

7. The temperature coefficient of expansion of the impregnating compound is considered to be 0.1 per cent per deg. cent.

8. The internal pressure in the cable which will cause a permanent stretch in the lead sheath is considered to be 4 atmospheres. (Approximately 44 lb. per sq. in. gage pressure.)

9. The impregnating compound and the fibers of the paper are considered incompressible.

10. The temperature coefficient of expansion of the lead and of the copper is considered negligible, particularly in view of the fact that an expansion in the lead is partly counteracted by a simultaneous expansion in the copper conductors.

11. A length of cable is considered sealed at each end by virtue of a non-flowing splicing compound.

12. The volume of air varies according to Boyle's law in regard to pressure.

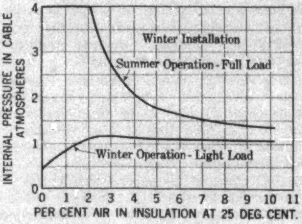


FIG. 3

13. The volume of air varies according to Gay-Lussac's law in regard to temperature.

Within the foregoing conditions, I have calculated for various percentages of air, the internal pressures which will prevail in a cable during maximum load in the summer and during minimum load in the winter when the cable is installed during summer months. The results of these calculations are shown in Fig. 2 herewith. I have also made corresponding calculations for a cable which is installed during winter months, and show the results in Fig. 3.

In Fig. 2, the lower curve shows that the internal pressure of the cable is always less than atmospheric during light loads in winter. If the ionization voltage of air is proportional to the pressure, then it is evident from this curve that a poorly impregnated cable is superior to a well impregnated cable. For example, if, in a cable of a particular wall and voltage, the ionization pressure is 0.8 atmosphere, then the cable having an air content of five per cent or more would operate without ionization. On the other hand, the cable having an air content of less than five per cent would operate during winter months with ionization which will produce carbonization.

If cables are installed during winter months, then operating conditions are more favorable in regard to internal pressure. A cable would have to be well impregnated before its internal pressure would be less than one atmosphere during winter operation. For example, only those cables having an air content of less than 1.5 per cent would have an internal pressure of less than one atmosphere during light load in winter. The reduced pressure in this case is brought about by a permanent stretch in the lead sheath. The permanent stretch is produced during heavy-load periods in summer.

A comparison of Figs. 2 and 3 indicates that when non-flowing splicing compounds are used, winter installation is more desirable than summer installation. Furthermore, the curves in these illustrations indicate that cables deteriorate mostly in winter, particularly if the cables are installed in summer.

It is not advocated that cables should always be installed during winter months. Winter installation is only a means for partly alleviating the unfavorable conditions caused by the use of non-flowing splicing compounds. The proper installation is that which will permit a cable to "breathe" as the temperature of the cable changes. Such installation is exemplified in the present-day use of very liquid splicing compounds and the application of syphons.

W. A. Del Mar: The principal discussion of my paper has taken place in the personal meetings and correspondence before and since the Madison meeting, and I have been surprised and gratified by the great interest manifested.

As I intimated in my paper and as repeated by Mr. W. S. Clark, there is nothing new in the basic ideas presented, but the implications which bear on ordinary cables seem to have been ignored almost completely by operating men and manufacturers alike. If any manufacturer had these ideas clearly in mind, his duty to the industry was to pass them on without delay to the cable users. So far as I know, this was done for the first time when I gave copies of a memorandum on this subject to several members of the Underground Systems Committee of the N. E. L. A., nearly a year ago.

We all recall the classic 1919 papers by Shanklin and Matson,[2] and Dubsky,[3] wherein it is shown that films of air entrapped in a cable ionize at an insulation stress of about 19 kv. per cm., but perhaps we neglected to note, as the three authors named above were silent on the subject, that this stress applies to air at atmospheric pressure only.

TABLE I

Air pressure, atmospheres	Relative ionization voltage as fraction of that at 1 atmosphere (Peek)	Ionization stress taking 19 kv./cm. for 1 atmosphere
0.1	0.2	3.8
0.2	0.3	5.7
0.5	0.53	10.0
0.7	0.70	13.3
1.0	1.00	19.0
1.25	1.22	23.2
1.50	1.44	27.4
2.0	1.90	36.1
2.5	2.35	44.7

We recall, moreover, the work of such experimenters as Peek, proving that the ionization stress of air is approximately proportional to the air pressure.

Combining these researches, we obtain Table I, showing the relation between ionization stress in cable insulation, and air pressure.[4]

We are now equipped with an important tool which, in our hands three years ago, might have saved the electric power industry and the cable manufacturers a considerable amount of worry. In order to understand this, consider the matter in Table II, which shows the maximum stresses for several typical triplex cables and the air pressures required to prevent ionization at these stresses, taken from Table I. The table may be summarized in a few words:

Cables of the 13,000-volt class do not ionize even though the pressure may be somewhat below atmospheric.

Cables for 22,000 volts have an ionization pressure of about one atmosphere.

Cables for all higher voltages as listed require an internal pressure of between one and two atmospheres. This is a conclusion of great importance which seems to have escaped the attention of the industry. It is well known that cables for over 22,000 volts belong in a different class, from an operating standpoint, from those for lower voltages. The former have had either excessive failures or at least show deterioration of the insulation, except in a small minority of cases. The latter give little or no trouble.

Similar information for single-conductor cables is given in Table III, from which it appears that cables for 132-kv. circuits with 60/64-in. insulation require about 3½ atmospheres, which is near the limit of strength of an ordinary lead sheath.

It is also a curious fact that the trouble seems to increase the more carefully the cable is made. This is because of the difference in impregnation; the more thorough this is, the greater the vacua due to contraction. This has been very clearly brought out by Mr. Hanson's discussion. For this reason, some of the cables of five years ago seem to be more reliable than those of today.

TABLE II

TRIPLEX 350,000-CIR. MIL CABLES

Working voltage between phases, Kv.	Insulation each conductor, 64ths in.	Maximum stress, kv./cm.	Internal pressure required to prevent ionization, atmospheres
13	12	16	0.8
13	9	19	1.0
22	19	20	1.0
22	18	21	1.1
24	18	23	1.25
26.4	18	25	1.35
27.6	19	25	1.35
33	19	30	1.65
33	23	26	1.40

The effects of ionization in the order of their occurrence are formation of *X*, carbonization of oil, formation of dendritic patterns, and eventually failure. The action may, however, stop at any point, as cables are in operation which have been full of *X* for years and carbonized oil has been known to exist for years in cables without further deterioration, probably because of increased internal pressure.

TABLE III

SINGLE-CONDUCTOR, 500,000-CIR. MIL CABLES

Working voltage to ground, Kv.	Insulation, 64ths in.	Maximum stress, kv./cm.	Internal pressure required to prevent ionization, atmospheres
23	40	24	1.3
26	40	27	1.5
43	52	38	2.1
43	60	35	1.9
76	60	62	3.4

Let us consider what has been done by operating engineers to influence the internal pressure of high-voltage cables.

First, they have written specifications which compel the furnishing of "over-saturated" cables, *i. e.*, cables in which high vacua are formed if operated as they have been in the past.

Second, they have done most of their installation work in summer, thereby insuring the occurrence of vacua in winter.

Third, until recently they have used splicing compounds which seal each length of cable and make pressure equalization impossible.

Fourth, they have given no consideration to the installation and operating conditions which influence internal pressure, thereby leaving the pressure to take a chance value which might or might not permit their cable to survive.

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[2] TRANSACTIONS A. I. E. E., 1919, p. 489.

[3] TRANSACTIONS A. I. E. E., p. 537.

[4] If the air films are very thin, this relation does not hold; but experience shows that it is approximately correct for most of the films of the dimensions found in cables.

It seems necessary to put a definite predetermined pressure on high-voltage cable insulation and maintain it with scrupulous care. Forward-looking practise points to the use of spindle oil splices and syphons. I believe that by this means some cables which have had their operating voltages lowered due to continual failures will be successfully restored to their rated voltages.

When all high-tension cables are equipped with pressure gages and pressure-maintaining devices, the guarantee clauses in cable specifications will assume a legitimate significance. Mr. Peterson sensed this point very clearly in his discussion.

I agree with Mr. Dunsheath that all high-tension cables do not have to be supplied with special oil ducts, as it is possible to maintain adequate pressure in most cases by relying upon natural interstices in the cable.

My paper supports Mr. Halperin in his views about transient voltages, by furnishing an explanation of certain failures which, in the past, have been blamed on transients.

Mr. Peterson's explanation of the higher breakdown voltage of short lengths as compared with complete feeders,—namely, that oil from the terminal tanks is sucked into the cable,—cannot be correct, both because the opposite action occurs due to the heating of the cable, (*i. e.*, the cable oil is expanded and driven out), and because the same relation holds if the short lengths are tested without oil terminals or tanks.

Mr. Brownell has brought out an interesting point in mentioning that X forms more readily at 26 kv. in service than at 100 kv. in the laboratory.

Mr. Davidson seems to have taken my formula for X more seriously than I had intended. I used a formula only because the results of analysis as percentages are hard to visualize. The evidences of polymerization are to be found in high specific gravity, solidity, insolubility, and infusibility considered in connection with the known origin of the material.

Mr. Duncan's discussion of my formula for X is very much to the point. He shows clearly that the suggested formula is untenable, and I am rather inclined to believe that the explanation is that the oxygen atom is superflous. We can now form X in such vacua that it seems that no atmospheric element can enter into its composition. The chemists obtained the oxygen by difference and not by any positive test, so that its existence is at least open to doubt. The formation of X is preceded by an apparent increase of surface tension, *i. e.*, the oil acts as if its surface tension had increased very greatly, causing it to form into drops. Perhaps the polymerization which characterizes X formation is a result of this surface tension; *i. e.*, it may be nothing but a further drawing together of the molecules.

Mr. Roper's and Mr. Halperin's discussions add important material to our general store of knowledge on this subject. There are more of this kind of data which bear on pressures and vacua in cables to be gathered from the experience of operating engineers. For instance, it has been found that splices which are equipped with unweighted syphons have a lower breakdown voltage than identical splices without the syphons, and some people would interpret that as indicating that the syphons might be an objectionable feature. The explanation as furnished by one of the earliest users of syphons is that splices tested with them are at atmospheric pressure inside, whereas those tested without them, being heated by the dielectric losses, have an internal pressure which may be very considerable. The internal pressure naturally increases the dielectric strength and raises the breakdown voltage at the splice. The obverse of this is seen in the trouble experienced with both cables and joints where vacuum joints have been used.

A full realization of the significance of cable vacua and pressures should materially assist in the design of cables for much higher voltages than now considered practicable, without departing from the diameters which ducts and handling difficulties now impose.

Behavior of Radio Receiving Systems to Signals and to Interference

BY LEO JAMES PETERS[1]
Associate, A. I. E. E.

***Synopsis.**—This paper develops a point of view and method by means of which it is possible to arrive at many of the transient effects occurring in radio systems by a consideration of steady-state properties alone. The scheme is to replace the voltages in radio receiving systems due to interference and signals by a group of generators having the correct voltages and frequencies. These generators can be thought of as having been in the circuit for an indefinitely long time so that only the steady-state response of the system need be considered. The generators which replace the voltages induced in an antenna by interrupted continuous wave stations, by spark telegraph stations, by telephone stations and by static are worked out. The desirable properties of radio receiving systems for receiving various types of signals through interference are arrived at and an ideal system is described which may be used as a standard of reference for judging the merits of any actual frequency selecting system. It is shown that this ideal system reduces the interference from all sources to the smallest possible value which can be obtained in a system which makes use of frequency selection to reduce interference. The paper thus arrives at the degree to which interference can be mitigated by frequency selection methods. In order to illustrate the method of treating actual systems calculations are given for a simple series receiver. The interference, caused by transmitting stations of various types and by static, is discussed and the factors determining such interference are pointed out.*

Part I. Character of the Signal Voltages Induced in Antennas and the Desirable Properties of Frequency Selection Systems for Receiving the Signals

1. Introduction and Purpose

THE behavior of a radio receiving system, both to signals and to interference, depends upon the properties of the system in the transient state. It is very often a difficult problem to arrive at the transient-state properties of a system whereas the formulation of the steady-state properties is a relatively easy matter. It is the purpose of this paper to develop a very simple but effective way of answering some of the questions which arise in dealing with the effects of both signals and interference upon radio receiving systems when the steady-state properties of the system are known or of arriving at the best steady-state properties of a system for receiving a given signal through interference. The primary purpose of the paper is to establish a viewpoint from which to judge the merits and the limitations of receiving systems, and also to arrive at the interference produced by transmitting systems of various types. Since the primary purpose of the paper is to establish a point of view and a method, only a few problems illustrating the method are discussed.

The solution for the transient properties of a circuit network can be made to depend upon the steady-state solution if the impressed voltage can be represented from $t = -\infty$ to $t = +\infty$ as a Fourier integral. This method has been used by Carson, Fry, and the author for observing the general behavior of circuits in the transient state. Carson has used this method to formulate the response to static impulses of those radio receiving systems in which the principle of frequency selection is used to reduce interference. Milner has used the Fourier series expansion to calculate the arrival current in submarine cables. These general methods, however, lead into fields of mathematics which are unfamiliar to many engineers, but they suggest a very simple and powerful but less general method of handling many problems that arise in radio communication. This paper has a twofold purpose; first of pointing out these simple methods with the hope that it may aid in giving the method the general use its power warrants, and second, to arrive at some interesting and useful conclusions as to the effects of signals and interference upon radio receiving systems.

2. Introduction to the Method of Treatment

Fig. 1 shows an alternator delivering a pure sine wave feeding a transformer on open circuit through a long line, *A*. Parallel to this line there is another line, *B*, which may be a telephone circuit. We wish to find the effect of line *A* upon line *B*. As is generally known, the current in line *A* has the form shown by Fig. 2. This current can be broken up into the two sine waves shown by Fig. 3. One of these sine waves has a frequency of 60 cycles per second and an amplitude *I*. The other has a frequency of 180 cycles per second and an amplitude of $\frac{1}{4} I$. If the mutual inductance between the lines is *M*, then the voltage induced in the line *B* is composed of two parts; one part has a frequency of 60 cycles per second and an amplitude of $2 \pi 60 I M$, while the other part has a frequency of 180 cycles per

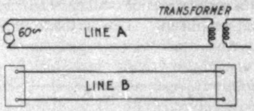

Fig. 1

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[1] Assistant Professor of Electrical Engineering, the University of Wisconsin.

Presented at the Regional Meeting of District No. 5 of the A. I. E. E., Madison, Wis., May 6-7, 1926.

second and an amplitude of $2\pi\, 180 \frac{I}{4} M$. If the amplitude of the 60-cycle voltage is represented by E then the amplitude of the 180-cycle voltage is $3/4\ (E)$. Then we can forget all about line A if, in line B, we introduce two alternators, one having a frequency of 60 cycles per second and a voltage of E and the other having a frequency of 180 cycles per second and a voltage of $3/4\ (E)$; that is, we can calculate the effect of line A upon line B by considering only the system shown by Fig. 4.

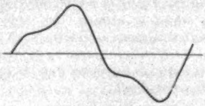

FIG. 2

The scheme used above for finding the effect of line A on line B is the one used throughout this paper for finding the effect of any voltage induced in an antenna upon the receiving system; for example, the voltage is replaced by a group of alternators having the correct frequencies and the correct voltages. The receiving system will then be represented schematically, as shown by Fig. 5. The generators are assumed to have no impedance and serve only as a device for fixing the attention on a sine wave of voltage having a given frequency and amplitude. Use is made of Fourier's expansion for obtaining the voltage and frequency of each alternator. Since each alternator is assumed to have been in the circuit for an indefinitely long time, only the steady-state properties of the receiving system need be considered.

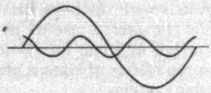

FIG. 3

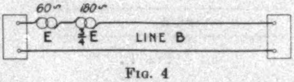

FIG. 4

3. INTERRUPTED CONTINUOUS WAVE TELEGRAPHY

The voltage induced in a receiving antenna by an interrupted continuous wave transmitter is assumed to have the form shown schematically by Fig. 6. In this figure, $2\,T$ represents the total time interval of a signal and the succeeding space, $2\,q\,T$, represents the signal time interval, and $2\,p\,T$ represents the space time interval. In order to simplify the calculations, it will be assumed that there is a complete number of cycles of the operating frequency in the time intervals $q\,T$ and $p\,T$. Under these conditions, if the operating frequency is $f_0 = \frac{\omega}{2\pi}$, the voltage is represented from $t = -\infty$ to $t = +\infty$ by the series:

$$e(t) = \sum_{m=1}^{\infty} a_m \sin \frac{m\pi t}{T}{}^{*} \qquad (1)$$

In which

$$a_m = \frac{E}{T}(\sin mp\pi)\left[\frac{2\omega}{\omega^2 - \frac{m^2\pi^2}{T^2}}\right]^{*} \text{ for } \frac{m\pi}{T} \neq \omega \qquad (2)$$

$$= E\,q \text{ for } \frac{m\pi}{T} = \omega \qquad (3)$$

That is, the voltage of the mth generator is given by equations (2) and (3). The generator having the largest voltage is the one having a frequency the same as the operating frequency (f_0) of the transmitting station. The frequency of the mth generator is, from equation (1),

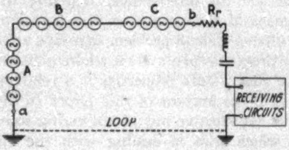

FIG. 5

$$f_m = \frac{1}{2\pi}\left(\frac{m\pi}{T}\right) = \frac{m}{2\,T} \qquad (4)$$

That is, the receiving circuit generators which replace the induced voltage, are spaced $\frac{1}{2\,T}$ cycles apart. The value of T depends upon the speed of signalling. At 30 words per minute $T = 0.05$ sec. At 150 words per minute $T = 0.01$ sec. Thus the speed of signal transmission determines the frequency spacing of the generators.

Let m_0 represent the m which gives the operating frequency, f_0.

Then

$$m_0 = 2\,T\,f_0 \qquad (5)$$

Let

$$m = m_0 + n = 2\,T\,f_0 + n \qquad (6)$$

$$n = 0, \pm 1, \pm 2, \pm 3 \ldots {}^{+\infty}_{-(2\,T\,f_0 - 1)}$$

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*For the derivation of these equations see Appendix, Part I.

If the signal sent out is a morse dot and the interval between signals is the morse dot interval, then

$$p = q = 0.5 \tag{7}$$

In equation (**2**)

$$\sin m\,p\,\pi = \sin m\frac{\pi}{2} = 0 \quad \text{if } m \text{ is even}$$

$$= \pm 1 \text{ if } m \text{ is odd}$$

Upon substituting these results and equation (**6**) in (**2**) and remembering that $\omega = 2\,\pi\,f_0$ we obtain:

$$a_{m_0} = \frac{E}{2} \tag{8}$$

$$|\,a_{m_0+n}\,| = \frac{E}{n\;\pi}\left[\frac{1}{1+\dfrac{n}{4\,T\,f_0}}\right] \tag{9}$$

From equation (**5**) it is evident that m_0 will always be an even number. It follows then, since $m_0 + n$ must be odd, that n will be an odd number, so that we write:

$$n = 0.\ \pm 1, \pm 3, \pm 5 \;\ldots\; \overset{+\infty}{\underset{-(2\,T\,f_o - 1)}{\ldots}} \tag{10}$$

For all of the important frequencies[2] $\dfrac{n}{4\,T\,f_0}$ is small compared to unity and (**9**) may be simplified to:

$$|\,a_{m_0+n}\,| = \frac{E}{n\,\pi} \tag{11}$$

We have now arrived at the following information relative to the generators which replace the induced voltage in the receiving antenna. The generator having the highest voltage is the one having a frequency equal to the operating frequency, f_0. The voltage of this generator is $\dfrac{E}{2}$. As we pass to generators having a frequency less than f_0, the first one we come to has a frequency $f_0 - \dfrac{1}{2\,T}$. This generator has a voltage of $\dfrac{E}{\pi}$. The next one has a frequency of $f_0 - \dfrac{3}{2\,T}$ and has a voltage of $\dfrac{E}{3\,\pi}$ and so on down the line. The first generator having a frequency higher than f_0 is the one which has a frequency $f_0 + \dfrac{1}{2\,T}$. The voltage of this generator is also $\dfrac{E}{\pi}$. The next one, up the frequency scale, has a frequency equal to $f_0 + \dfrac{3}{2\,T}$ and has a voltage equal to $\dfrac{E}{3\,\pi}$, and so on. These facts are brought out in a striking manner by the curve of Fig. 7. This curve is plotted with the ratios of the absolute value of the generator voltage to E as ordinates and with values of n as abscissa. Values of n are used as abscissa in order to make the curve hold for all speeds of transmission. To convert the abscissa scale to generator frequencies use is made of the relation:

$$(\text{Gen. freq.}) = f_0 + \frac{n}{2\,T} \tag{12}$$

At 30 words per minute this relation becomes:

$$(\text{Gen. freq.}) = f_0 + 10\,n \tag{13}$$

At 150 words per minute it becomes:

$$(\text{Gen. freq.}) = f_0 + 50\,n. \tag{14}$$

Thus a generator having a given ratio of voltage to E is five times as far removed in frequency from the operating frequency at a transmitting speed of 150 words per minute as at a transmitting speed of 30 words per minute. This fact, as we shall see later, has an important bearing upon the design of the receiving system and also upon the amount of interference created by the transmitting station.

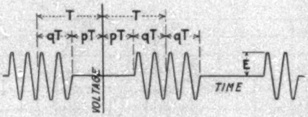

Fig. 6

The desirable frequency response characteristics of the interrupted continuous wave receiving system can now be obtained. If the system passes currents having the operating frequency f_0, and eliminates currents of all other frequencies, a continuous tone would be heard in the receivers and the dots and spaces could not be distinguished from each other; that is, no signals would be received. If the system passes currents of all frequencies with the same ease, the high frequency output would have the same wave form as the induced voltage, Fig. 6. This latter condition would lead to the distinguishing of the signals, but the system would have no selectivity against interference. The best circuit then would be one which passes just enough frequencies to make the signals distinguishable and eliminates all others. Let us assume that, in order to make the signals distinguishable, the receiving system must respond freely to all generators having a voltage greater than or equal to r decimal parts of the voltage of the generator the frequency of which is f_0; that is, the system must respond freely to all generators having a voltage equal to or greater than $\frac{r\,E}{2}$. Now the generator with a frequency of $f_0 \pm \frac{n}{2\,T}$ has a voltage equal to $\frac{E}{n\,\pi}$. We therefore write:

---

[2] See Appendix, Part I.

$$\left(\frac{E}{n\,\pi}\right)\frac{2}{E} = r$$

$$n_r \text{ is the odd number closest to } \frac{2}{r\,\pi} \qquad \textbf{(15)}$$

The receiving system therefore must pass a band of frequencies $\frac{n_r}{T}$ wide centered on the frequency f_0.

Let us define an ideal receiving system as one which has the following properties:

1. A radiation resistance R_r
2. No wasteful resistance
3. It acts as a pure resistance of magnitude $2\,R_r$ to frequencies lying in the range $f_0 \pm \frac{n_r}{2\,T}$.
4. Currents having frequencies lying in the range given by 3 shall be passed on to the detector either with a uniform amplification or without attenuation.
5. Currents of all other frequencies shall be eliminated before reaching the detector.

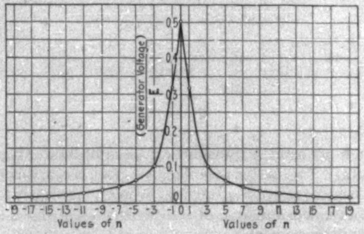

FIG. 7—INTERRUPTED C. W. TELEGRAPHY—VARIATION OF GENERATOR VOLTAGE WITH FREQUENCY—GENERATOR FREQUENCY

$= f_0 + \frac{n}{2\,T}$

$n = 0, \pm 1, \pm 3, \pm 5$, etc.
$f_0 = 70$ at 30 words per min.
$f_0 = 350$ at 150 words per min.

A system having the above properties is called an ideal system because it represents the best possible frequency selection system for receiving I. C. W. signals through interference. This ideal system can be only approximated, more or less imperfectly, in practise; but any actual receiving system, basing its selectivity upon frequency selection, should be made to fulfill the conditions stated as closely as possible.

The band of frequencies which the receiving system must pass freely is $\frac{n_r}{T}$. Now as both n_r and T are independent of f_0, so the band width is independent of the operating wave length. n_r is also independent of the speed of signal transmission, while T varies inversely as the speed of transmission. Therefore, the frequency band which the receiving system must transmit freely varies directly with the sending speed. Thus, the transmitted band width at 150 words per minute would be five times as great as at 30 words per minute.

If the receiving system acted as a pure resistance of magnitude, $2\,R_r$, to all frequencies, then during the dot interval the r. m. s. current would be

$$I_d = \frac{E}{2\,\sqrt{2}\,R_r} \qquad \textbf{(16)}$$

and the average rate of useful power delivery would be

$$P_d = \frac{E^2\,R_r}{8\,R_r^2} = \frac{E^2}{8\,R_r} \qquad \textbf{(17)}$$

During the space interval, the power delivery would be zero. Since we have taken the space interval equal to the dot interval, the average useful power delivery will be one-half of that given by (17) or

$$P_a = \frac{E^2}{16\,R_r} \qquad \textbf{(18)}$$

In the ideal receiving system, the mean square current in the frequencies which are freely transmitted is:

$$I^2 = \frac{E^2}{2}\,\frac{1}{4\,R_r^2}\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_s \frac{1}{s^2}\right] \qquad \textbf{(19)}$$

$$s = 1, 3, 5, \ldots n_r$$

The average useful power delivered by the generators in the band of frequencies $f_0 \pm \frac{n_r}{2\,T}$ is

$$P = I^2\,R_r = \frac{E^2}{8\,R_r}\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_s \frac{1}{s^2}\right] \qquad \textbf{(20)}$$

The ratio of the power actually utilized by the ideal receiver to the power available in the waves is, therefore:

$$\frac{P}{P_a} = 2\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_s \frac{1}{s^2}\right] \qquad \textbf{(21)}$$

Calculations indicate that if $n_r = 3$, the intervals and dots will be easily distinguishable. Under these conditions, the ratio of the power utilized in the ideal circuit to the power available becomes:

$$\frac{P}{P_a} = 2\left[\frac{1}{4} + \frac{2}{\pi^2}\left(1 + \frac{1}{9}\right)\right] = 0.9 \qquad \textbf{(22)}$$

With the above value of n_r, the ideal system would pass freely a band of frequencies lying between $f_0 - 30$

cycles per second and $f_0 + 30$ cycles per second at 30 words per minute. At 150 words per minute the band passed would be between $f_0 - 150$ cycles per second and $f_0 + 150$ cycles per second.

Let us now consider the simple receiving circuit shown in Fig. 8. This circuit consists of an elevated capacity network, A, a tuning inductance, L, and a detector which has a resistance R_d. The capacitance of the network to ground is represented by C. In addition to the detector resistance the circuit has a wasteful resistance, R_w, and a radiation resistance, R_r. The induced voltage is represented by the group of generators, G. If this station is to receive I. C. W. signals, the characteristics of the generators have already been found. We wish to design the circuit so that it will approach as nearly as possible the ideal receiver described above.

It is, of course, impossible to adjust this circuit so that it will act as a pure resistance to all of the generators lying in the band $n = 0$ to $n = \pm n_r$ and to eliminate the currents due to all of the other generators. We will therefore tune it to the frequency f_0. Let the r. m. s. value of the voltage of the nth generator $\left(\text{frequency } f_0 + \frac{n}{2T}\right)$ be designated by E_n and let the r. m. s. value of the current due to this generator be I_n. Since the circuit is tuned to the frequency f_0 we may write

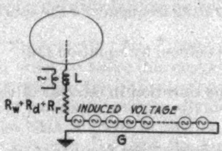

FIG. 8

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

Let the frequency change from f_0 to $f_0 + a$. The net reactance then is:

$$x = 2\pi L\,(f_0 + a) - \frac{1}{2\pi f_0 C\left(1 + \frac{a}{f_0}\right)}$$

$$= 2\pi L\,(f_0 + a) - 2\pi f_0 L\left(\frac{1}{1 + \frac{a}{f_0}}\right)$$

$$\left(1 + \frac{a}{f_0}\right)^{-1} = 1 - \frac{a}{f_0} \text{ very closely if } \frac{a}{f_0} < 0.05$$

Therefore

$$x = 2\,(2\pi L a) \qquad \textbf{(23)}$$

Now the number of cycles by which the nth generator is removed from f_0, is $\frac{n}{2T}$. It can be shown that the net reactance of the circuit to the nth generator is:

$$x_n = 2\pi L \frac{n}{T} \qquad \textbf{(23a)}$$

Let us now assume that, in order to have the circuit respond to as few frequencies as possible and still have the signals discernible, the reactance to the n_r generator should be equal to βR_t. From (**23a**) we arrive at

$$\frac{L}{R_t} = \frac{T_c}{2} = \frac{\beta T}{2\pi n_r} \qquad \textbf{(24)}$$

T_c is the time constant of the antenna circuit.

$$x_n = \beta R_t \frac{n}{n_r} \qquad \textbf{(25)}$$

The average useful power delivered to the circuit by the nth generator then is:

$$P_n = \frac{E_n^2 R_d}{R_t^2\left(1 + \beta^2 \frac{n^2}{n_r^2}\right)} \qquad \textbf{(26)}$$

But

$$E_n^2 = \frac{E^2}{8} \text{ for } n = 0$$

$$= \frac{E^2}{2 n^2 \pi^2} \text{ for all other } n\text{ s}$$

The total average power supplied to the detector then is:

$$P = \frac{E^2 R_d}{2 R_t^2}\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_n \frac{1}{n^2\left(1 + \beta^2 \frac{n^2}{n_r^2}\right)}\right] \qquad \textbf{(27)}$$

$$n = 1, 3, 5, \text{etc.}$$

From equation (**27**) it is seen that the best value of R_d is one-half of R_t. That is,

$$R_d \text{ should equal } (R_r + R_w) \qquad \textbf{(28)}$$

Now since $R_t = R_r + R_d + R_w$ must satisfy equation (**24**) it may not always be possible to satisfy (**28**). We therefore write:

$$R_d = \rho\,(R_r + R_w) \qquad \textbf{(29)}$$

Also let

$$k = \frac{R_r + R_w}{R_r} \qquad \textbf{(30)}$$

In terms of these symbols the total power delivered to the detector becomes:

$$P = \frac{E^2 \rho}{2 k R_r (1 + \rho)^2}\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_n \frac{1}{n^2\left(1+\beta^2 \frac{n^2}{n_r^2}\right)}\right] \qquad \textbf{(31)}$$

If this circuit is used with the same antenna as the ideal circuit, the power available is again given by equation (**18**). The ratio of the power utilized to the power available for this circuit is then:

$$\frac{P}{P_a} = \frac{8\rho}{k(1+\rho)^2}\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_n \frac{1}{n^2\left(1 + \beta^2 \frac{n^2}{n_r^2}\right)}\right] \qquad (32)$$

$$n = 1, 3, 5, 7, \text{etc.}$$

The summation term in (**32**) converges very rapidly because of the presence of n^4 in the denominator. Estimations based upon the shape of the antenna current wave and also upon the time constant of the circuit show that when $n_r = 3$, β should equal about 2.

If these values are assigned to n_r and β equation (**32**) becomes:

$$\frac{P}{P_a} = \frac{8\rho}{k(1+\rho)^2}[0.25 + 0.14 + 0.0045 + 0.0007 \; . \; . \; .]$$

$$= [3.2]\frac{\rho}{k\,(1+\rho)^2} \qquad (33)$$

If ρ is assigned the best value, namely unity, the ratio of the power utilized by the simple series receiver to the power available is:

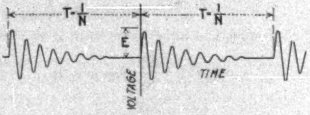

FIG. 9

$$\frac{P}{P_a} = \frac{0.8}{k} \qquad (34)$$

When equation (**24**) is satisfied then the product

$\frac{\rho}{k\,(1+\rho)^2}$ is the figure of merit of the simple series receiving circuit when the circuit has the minimum allowable transmitted band width. It will be shown in the section on interference that this product is the figure of merit of the simple series receiving circuit when receiving I. C. W. signals through interference.

4. *Spark Telegraphy.* The voltage induced in a receiving antenna by a spark transmitting station is assumed to have the form shown, schematically, in Fig. 9. The operating frequency is

$f_0 = \frac{\omega}{2\pi}$. In most spark systems the damping is such that the voltage dies very nearly to zero in the interval of time, T, ($\alpha T > 3$). Under these conditions the voltage is represented as a function of time during the interval $t = -T$ to $t = 0$ by the equation

$$e\,(t) = E\,\epsilon^{-\alpha(t+T)}\sin[\omega\,(t+T)] \qquad (35)$$

and in the interval $t = 0$ to $t = +T$ by

$$e\,(t) = E\,\epsilon^{-\alpha t}\sin\omega t \qquad (36)$$

The voltage induced in the antenna is represented from $t = -\infty$ to $t = +\infty$ by:

$$e\,(t) = A_0 + \sum_{m=1}^{\infty} B_m \cos\left[\frac{m\pi t}{T} + \theta_m\right] \qquad (37)$$

In our scheme of replacing the induced voltage by alternators we find from equation (**37**) that the voltage of the alternator with a frequency of $\frac{m}{2T}$ is equal to B_m.

It can be shown that B_n is stated very closely by the equation:

$$B_n = \frac{2E}{\omega\,T}\,\frac{1}{\sqrt{\frac{4\,\alpha^2}{\omega^2} + \left[\frac{2n}{f_0\,T} + \frac{n^2}{f_0^2\,T^2}\right]^2}} \qquad (38)$$

$$n = 0, \pm 1, \pm 2 \; . \; . \; . \; {}^{+\infty}_{-f_0 T}$$

In this equation the symbols have the following meaning:

B_n is the voltage of the generator whose frequency differs from f_0 by $\frac{n}{T}$ cycles per sec.

f_0 is the operating frequency of the spark-transmitting station.

E and α are defined by equation (**36**).

$\omega = 2\pi f_0$

T is the time duration in seconds of each spark.

Let

$N = \frac{1}{T}$ represent the number of sparks per second,

$\delta = \frac{\alpha}{f_0}$ represent the logarithmic decrement.

In terms of these symbols (**38**) becomes

$$B_n = \frac{2NE}{\omega}\,\frac{1}{\sqrt{\frac{\delta^2}{\pi^2} + \left[\frac{2nN}{f_0} + \frac{n^2N^2}{f_0^2}\right]^2}} \qquad (39)$$

The voltage of the generator whose frequency is the same as the operating frequency f_0 is:

$$B_0 = \frac{E}{\alpha\,T} = \frac{NE}{\alpha} = \frac{NE}{\delta f_0} \qquad (40)$$

In order to present at a glance the way in which the generator voltage varies with the frequency in the vicinity of the operating frequency, f_0, and the effect of the logarithmic decrement, δ, the number of sparks per second, N and the operating frequency on this variation, the curves of Fig. 10 have been drawn. The abscissa for all curves are values of the difference between the generator frequency and the operating frequency ($= nN$). Generator frequencies are located only at integral values of $n\,N$ divided by N. Thus at 1000 sparks per second, one generator has the frequency f_0 and the generators are located on a frequency scale every 1000 cycles per second above or below f_0. At 120 sparks per second generators are located on the frequency scale at f_0 and every 120 cycles per second above or below f_0. For curve (1) $f_0 = 10^6$ cycles per second, $\delta = 0.01$, $\alpha = \delta\,f_0 = 10^4$, $N = 1000$. The ordinates for this curve are values of the ratio of the generator voltage to the undamped peak voltage, E_1, induced in the antenna. $\left(\text{Ordinates are values of } \frac{B_n}{E_1}\right)$. The voltage assigned to the generators falls off at a fairly fast rate as the operating frequency is departed from. The generator, having a frequency which differs by 16,000 cycles per second from the operating frequency, has a voltage 10 per cent as great as the voltage of the generator with a frequency equal to the operating frequency. That is, the energy associated with a frequency removed from the operating frequency by 16,000 cycles per second is one per cent as great as the energy associated with the operating frequency. This curve also holds good for the conditions $f_0 = 10^5$, $N = 1000$, $\delta = 0.1$, $\alpha = \delta\,f_0 = 10^4$. This fact has an important bearing on the factors which determine the frequency-energy distribution as will be shown a little later.

For curve (2), $f_0 = 10$, $N = 1000$, $\delta = 0.1$, $\alpha = \delta\,f_0 = 10^5$. In order to make this curve directly comparable with curve (1), the ordinates have been taken as values of the generator voltage divided by the undamped voltage peak induced in the antenna by case (1) when both sets of waves have the same energy per wave train or per spark. That is, the ordinates in this case are values of $\frac{B_n}{E_2}$ multiplied by $\sqrt{\frac{\alpha.}{\alpha_1}}$ *. This curve is much flatter than curve (1) and the generator with frequency removed from f_0 by 16,000 cycles per second, has a voltage 70 per cent as great as the voltage of the generator whose frequency is f_0. This is a striking contrast to curve (1) for which $\alpha = 10^4$.

For curve (3), $f_0 = 10^6 \sim$, $N_2 = 120$, $\delta = 0.01$, $\alpha = 10^4$. The ordinates for this curve are values of $(B_n/E_2)\sqrt{\frac{N_1}{N_2}}$ †. This factor is used in order to compare this case with case (1) when both waves have the same energy associated with them. The rate at which the generator voltage falls off in this case with the frequency is about the same as for case 1. This curve is also valid for the conditions $f_0 = 10^5 \sim$, $N = 120$, $\delta = 0.1$, $\alpha = 10^4$.

These curves lead to the conclusion that the factor which determines the rate at which the generator voltages fall off with the frequency in the vicinity of f_0, is $\alpha = \delta\,f_0$. The other factors have but little influence upon the width of the band of frequencies over which most of the energy is spread. This is apparent upon examining equations (**38**) or (**39**).

When n is small, the second term in the bracket under the radical is small compared to the first and may be dropped without serious error. We then have for small values of n,

$$B_n = \frac{E\,N}{\sqrt{\alpha^2 + 4\,\pi^2\,n^2\,N^2}} \qquad \textbf{(41)}$$

This equation shows clearly the dependance of the band width upon α, because the larger the value of α the larger must be the $n\,N$ product before B_n differs much from B_0.

The ideal system for receiving spark signals has the same properties as the ideal system for receiving I. C. W. signals except that the transmitted band width will be different. In so far as obtaining a good tone in the receivers is concerned it would suffice to pass only the currents of three generators; however, this would result in the utilizing of only a small portion of the available energy. Let us therefore make the band wide enough to pass the currents of all generators with voltage equal to or greater than r decimal parts of the voltage of the generator the frequency of which is f_0. From equations (**40**) and (**41**) we then have

$$\frac{B_{nr}}{B_0} = \frac{\alpha}{\sqrt{\alpha^2 + 4\,\pi^2\,n_r^2\,N^2}} = r \qquad \textbf{(42)}$$

$$n_r\,N = f_c = \frac{\alpha}{2\,\pi}\sqrt{\frac{1}{r^2} - 1} \qquad \textbf{(43)}$$

Since n_r must be an integer it is taken as the integer which comes the closest to satisfying (**43**). $n_r\,N = f_c$ is then one-half of the transmitted band width.

The mean square voltage induced in the receiving antenna by the impinging waves is from equation (**36**).

$$E_a^2 = \frac{E^2}{T}\int_0^T \left[\epsilon^{-\alpha t}\sin\omega t\right]^2 d\,t$$

$$= \frac{E^2}{T}\left\{\frac{\epsilon^{-2\alpha t}}{4\,\omega^2 + 4\,\alpha^2}\right.$$

$$\left.\left[(\sin\omega t)(-\,2\,\alpha\sin\omega t - 2\,\omega\cos\omega t) - \frac{\omega^2}{\alpha}\right]\right\}_0^T$$

Since $\epsilon^{-2\alpha t}$ is vanishingly small when $t = T$ we have:

$$E_a^2 = \frac{E^2}{T}\left(\frac{1}{4\,\omega^2 + 4\,\alpha^2}\right)\frac{\omega^2}{\alpha} \qquad \textbf{(44)}$$

If α^2 is small compared to ω^2 this reduces to:

$$E_a^2 = \frac{E^2}{4\,\alpha\,T} = \frac{N\,E^2}{4\,\alpha} \qquad \textbf{(45)}$$

If the receiving system acted as a pure resistance of magnitude $2\,R_r$ to all frequencies, the mean square current would be:

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*See the discussion of energy relations which follows shortly.

†See discussion of energy relations which follows shortly.

$$I_a^2 = \frac{E_a^2}{4\,R_r^2} = \frac{N\,E^2}{4\,R_r^2\,4\,\alpha}$$

It can be shown that the power available in the damped waves is:

$$P_a = R_r\,I_a^2 = \frac{N\,E^2}{16\,\alpha\,R_r} \tag{46}$$

P_a as given by (**46**) represents the power available in the waves. Since this power varies directly as E^2, directly as N and inversely as α, it is evident that the correction factors applied to the curves of Fig. 10 are the proper ones.

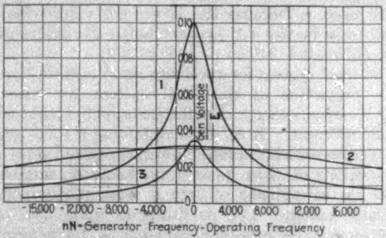

FIG. 10—SPARK TELEGRAPHY—GENERATOR FREQUENCY VERSUS GENERATOR VOLTAGE—GENERATOR FREQUENCIES LOCATED AT INTEGRAL VALUES OF $\frac{n\,N}{N}$.

Curve 1.
Operating frequency, $f_0 = 10^6 \sim$
Sparks per second, $N_1 = 1000$
Log. Dec., $\delta_1 = 0.01$; $\alpha\,1 = \delta_1 f_0 = 10^4$
Initial undamped voltage peak A, induced in antenna, $= E$
Also valid for $f_0 = 10^5$, $N_1 = 1000$, $\delta_2 = 0.1$, $\alpha = 10^4$ $A = E$
Curve 2.
$f_0 = 10^6 \sim$, $N_1 = 1000$, $\delta_2 = 0.1$,
$\alpha\,2 = 10^5$, $A_2 = \sqrt{\frac{\delta_2}{\delta_1}}\,E = 3.16\,E$
Curve 3.
$f_0 = 10^6 \sim$, $N_2 = 120$, $\delta_1 = 0.1$,
$\alpha_1 = 10_1^4$ $A_3 = \sqrt{\frac{N_1}{N_2}}\,E = 2.88\,E$
Also valid for
$f_0 = 10^5$, $N_2 = 120$, $\delta_2 = 0.1$,
$= 10^4$, $A = 2.88\,E$

If E_n represents the r. m. s. value of the voltage of the generator whose frequency differs from f_0 by $n\,N$ cycles per second, then the useful power delivered by this generator to the ideal receiver is 0 for n greater than n_r and equals:

$$P_n = \frac{E_n^2}{4\,R_r} \text{ for } n \leqq n_r \tag{47}$$

The total useful power delivered to the ideal circuit by the impinging waves is:

$$P = \sum_n P_n = \frac{E_0^2}{4\,R_r} + 2\sum_{n=1}^{n_r}\frac{E_n^2}{4\,R_r} \tag{48}$$

The factor 2 enters the summation of (**48**) because there are two generators having the voltage E_n. One has a frequency of $f_0 + n\,N$; the other has a frequency of $f_0 - n\,N$. Since the maximum value of n is n_r we may use equation (**41**) for finding the value of E_n^2. Since B_n is the peak value of the generator voltage $E_n^2 = \frac{1}{2} + B_n^2$ and (**48**) becomes:

$$P = \frac{E^2\,N^2}{8\,R_r}\left[\frac{1}{\alpha^2} + 2\sum_{n=1}^{n_r}\frac{1}{\alpha^2 + 4\,\pi^2\,n^2\,N^2}\right] \tag{49}$$

The ratio of the power utilized by the ideal receiver to the power available is:

$$\frac{P}{P_a} = 2\,\alpha\,N\left[\frac{1}{\alpha^2} + 2\sum_{n=1}^{n_r}\frac{1}{\alpha^2 + 4\,\pi^2\,n^2\,N^2}\right] \tag{50}$$

If the damping exponent, α, of the waves has a value of 10^4 and if r is taken to be 0.3, then the half band width as given by (**43**) is:

$$f_c = n_r\,N = \frac{10^4}{2\,\pi}\sqrt{11.1 - 1} = 5000 \sim \tag{51}$$

If the number of sparks per second N is 1000, $n_r = 5$ and the ratio of the power utilized by the ideal receiver to the power available is 0.82.

$$\frac{P}{P_a}$$

$$= (2)10^7\left[\frac{1}{10^8}(1 + 1.44 + 0.775 + 0.44 + 0.274 + 0.184)\right]$$

$$= 0.82 \tag{52}$$

If the receiving system consists of a simple series circuit, the general discussion given under the I. C. W. case still applies. In the present case, the number of cycles by which any generator frequency differs from the resonant frequency of the system is $n\,N$ and the net reactance to this frequency is from equation (**23**)

$$x_n = 4\,\pi\,L\,n\,N \tag{53}$$

If the reactance of the circuit to the frequency $n_r\,N$ is to equal $\beta\,R_t$ we have:

$$\frac{L}{R_t} = \frac{T_c}{2} = \frac{\beta}{4\,\pi\,n_r\,N} \tag{54}$$

Equation (**54**) is analogous to equation (**24**) of the I. C. W. case and fixes the freely transmitted band. The net reactance now becomes:

$$x_n = \beta\,R_t\,\frac{n}{n_r} \tag{55}$$

The average useful power delivered to the circuit by the nth generator then is given by the relation:

$$P_n = \frac{E_n^2\,R_d}{R_t^2\left(1 + \beta^2\,\frac{n^2}{n_r^2}\right)} \tag{56}$$

And the average power supplied to the detector is:

$$P = P_0 + 2\,\Sigma\, P_n$$

$$= \frac{E^2 N^2 R_d}{2\,R_t^2}$$

$$\left[\frac{1}{\alpha^2} + 2\sum_n \frac{1}{(\alpha^2 + 4\,\pi^2\,n^2\,N^2)\left(1 + \beta^2\,\frac{n^2}{n_r^2}\right)}\right] \qquad (57)$$

If the relations given by equations (**29**) and (**30**) are used here (**57**) becomes:

$$P = \frac{E^2 N^2 \rho}{2\,k\,R_r\,(1+\rho)^2}\left[\frac{1}{\alpha^2}\right.$$

$$\left.+ 2\sum_n \frac{1}{(\alpha^2 + 4\,\pi^2\,n^2\,N^2)\left(1 + \beta^2\,\frac{n^2}{n_r^2}\right)}\right] \qquad (58)$$

The ratio of the power utilized to the power available is then found from (**46**) and (**58**):

$$\frac{P}{P_a} = \frac{8\,\alpha\,\rho\,N}{k\,(1+\rho)^2}\left[\frac{1}{\alpha^2}\right.$$

$$\left.+ 2\sum_n \frac{1}{(\alpha^2 + 4\,\pi^2\,n^2\,N^2)\left(1 + \beta^2\,\frac{n^2}{n_r^2}\right)}\right] \qquad (59)$$

If there are 1000 sparks per second and $\alpha = 10^4$ and if $n_r = 5$ and $\beta = 2$, then the ratio of the power utilized to the power available is:

$$\frac{P}{P_a} = \frac{(8)\ 10^7\ \rho}{k(1+\rho)^2}\,[10^{-8}\,(1 + 1.24 + 0.47 + 0.18$$

$$+ 0.077 + 0.037 + 0.0195 + \ .\ .)] = \frac{\rho}{k\ (1+\rho)^2}\,[2.42] \qquad (60)$$

The best value for ρ is unity. If ρ has this value then (**60**) becomes

$$\frac{P}{P_a} = \frac{0.6}{k} \qquad (61)$$

Equations (**50**) and (**59**) show that for a given transmitted band width, the ratio of the power utilized to the power available becomes smaller as the damping exponent, α, is increased. This is because the greater the value of α the wider is the frequency band over which the energy is distributed.

5. TELEPHONY

The voltage of the generators which may be inserted in the receiving system to replace the voltage induced in the antenna by a radio telephone transmitter can not be written down in a general equation because it depends upon the character of the speech or music which is being sent out. The theory of modulation, however, shows that the voltage induced in the antenna of the receiving system has a group of frequencies consisting of the carrier frequency, an upper side band, and a lower side band. The carrier frequency is the operating frequency and determines the wave length of the transmitting station. The upper side band consists of a group of frequencies having values equal to the carrier frequency plus the frequencies of the voice or musical notes. The lower side band consists of a group of frequencies having values equal to the carrier frequency minus the frequencies of the voice or musical notes. If the highest musical note of importance is represented by f_c, then, in radio telephony, most of the energy is associated with a band of frequencies $2\,f_c$ cycles wide, centered on the carrier or operating frequency, f_0. If only a small portion of the energy is associated with frequencies outside this band, then we may replace the voltage induced in the receiving system by a group of generators having frequencies ranging from $f_0 - f_c$ cycles per second to $f_0 + f_c$ cycles per second. The voltage assigned to a generator having a given frequency will depend upon the nature of the speech or the music which is being received.

The ideal frequency selection system for receiving telephony will have the same properties as the ideal system described for the reception of I. C. W. signals with the exception that it must transmit freely the currents due to all generators having frequencies in the band $f_0 \pm f_c$ cycles per second. This ideal system would utilize practically all of the available energy and would be distortionless. The highest violin note of much importance has a frequency of about 5000 cycles per second. Therefore, for broadcast reception, it suffices to take f_c equal to 5000 cycles per second and the ideal receiver must transmit freely a band of frequencies 10,000 cycles wide centered on the operating frequency f_0.

It is impossible to design the simple series receiving circuit so that it will have no distortion and maintain good selectivity against interference. In the amplifying systems and in the loud speakers used for broadcast reception there is a good deal of distortion and the simple series circuit need not be freer from distortion than the rest of the system. The general discussion of the simple series receiver given under the I. C. W. case and the spark case applies here. If the system is tuned to the carrier frequency, then, from equations (**23**) and (**53**), the reactance of the system to the frequency $f_0 \pm p$ is given by the relation:

$$x_n = 4\,\pi\,L\,p \qquad (62)$$

If the reactance of the system to the generator whose frequency is $f_0 \pm f_c$ is $\beta\,R_t$ then (**62**) may be written as:

$$x_n = \beta\,R_t\,\frac{p}{f_c} \qquad (63)$$

If the r. m. s. value of the voltage of the generator whose frequency is $f_0 \pm p$ is E_p, the power delivered to the detector by the telephonic waves is:

$$P_p = \frac{E_p^2 R_d}{R_t^2 \left(1 + \beta^2 \frac{p^2}{f_c^2}\right)} \qquad (64)$$

The average power supplied to the detector is

$$P = P_0 + 2 \Sigma P_p \qquad (65)$$

In this equation, the summation is carried over all of the voice or music frequencies p which are present. If the relations given by equations (**24**) and (**3**) are used and if (**64**) is substituted in (**65**) we obtain for the average power supplied to the detector

$$P = \frac{\rho}{k R_r (1 + \rho)^2} \left[E_0^2 + 2 \sum_p \frac{E_p^2}{1 + \beta^2 \frac{p^2}{f_c^2}} \right] \qquad (66)$$

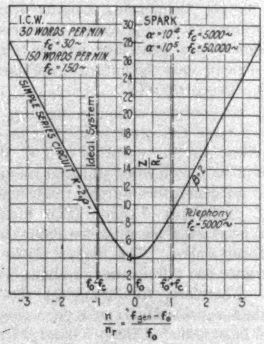

FIG. 11—TRANSMISSION CHARACTERISTICS

The average power received from the generator with a frequency of $f_0 \pm f_c$ is:

$$P_c = \frac{E_c^2 R_d}{R_t^2 (1 + \beta^2)} \qquad (67)$$

For distortionless reception this power should be:

$$P_{c1} = \frac{E_c^2 R_d}{R_t^2} \qquad (68)$$

We may then take as a measure of the distortion the ratio of P_c to P_{c1}. This ratio is:

$$D = \frac{P_c}{P_{c1}} = \frac{1}{1 + \beta^2} \qquad (69)$$

If β is taken equal to 2, the highest important voice or musical note will have one-fifth the energy it should have for distortionless reception. With the distortion as great as it is in the rest of the receiving equipment, this amount will probably not be excessive.

Since the ideal receiving system and the simple series receiving system are the same with the exception of the width of the transmitted band for all the three types of signals considered, it is possible to plot a single transmission characteristic for each one, to hold for all three types of signals. Such curves are plotted in Fig. 11. The abscissa scale is the ratio of the difference in generator frequency and the operating frequency to the cut off frequency f_c. The ordinate scale is the ratio of Z to R_r. Z is the ratio of the generator voltage to the detector current which has the same frequency as the generator voltage under consideration. The transmission characteristic of the simple series circuit is plotted for a value of β equal to 2 and for the best possible values of k and ρ. These curves show at a glance, the degree to which the simple series receiver falls short of the ideal receiver. These curves will be discussed more fully in the sections dealing with the reception through interference.

Part II.—Reception Through Interference

6. VOLTAGE INDUCED IN RECEIVING ANTENNA BY STATIC

The voltage induced in a receiving station by an I. C. W., a spark, or a telephone transmitter may be a source of interference as well as of signals. The frequency and voltage of the generator which may be used to replace these voltages in the receiving system have already been worked out. This section will be devoted to a discussion of the generators which represent the voltage induced in the receiving system by atmospheric strays or static. There is not very much information available on the wave form of the voltages induced in antennas by static. Watt and Appleton have published some observed static wave forms in the Proceedings of the Royal Society, 1923. These wave forms were sketched from visual observations made with a Braun tube oscillograph. The majority of the impulses were unidirectional in character and the time of rise and fall was about the same. The time duration of the majority of the impulses was of the order of one thousandth of a second. It is reasonable to expect then that some indication of the voltage and the frequency of the generators which replace the static voltage in the receiving antenna will be obtained if the wave form of the voltage induced by static is assumed as shown by Fig. 12. It is shown in the Appendix, Part III, that this voltage can be represented from $t = -\infty$ to $t = +\infty$ by the series:

$$e(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos \frac{m \pi t}{T} \qquad (70)$$

where

$$a_0 = \rho T \left[\frac{1}{2} - p + \frac{1}{2} p^2 \right] \qquad (71)$$

$$a_m = \frac{2 \rho T}{m^2 \pi^2} \left[\cos m \pi - \cos m \pi p \right] m \neq 0 \qquad (72)$$

Equation (**70**) shows that the generators which replace the voltage of Fig. 12 have the frequency $\frac{m}{2T}$ and voltages given by (**71**) and (**72**). Now the time duration of the impulses was around 10^{-3} seconds in the observations mentioned above. If the interval between impulses is assumed to be nine times as long we have the relations:

$$2qT = 10^{-3}, 2pT = 9 \times 10^{-3}$$
$$T(2q + 2p) = 2T = 10^{-2}$$
$$T = 5 \times 10^{-3} \text{ seconds}$$

The frequency spacing of the generators then is $\frac{1}{2T} = 100$ cycles. Equation (**72**) shows that the generators which have the high voltages have frequencies lying in the region from 100 to 1000 cycles per second. That is, the high-voltage generators have frequencies far lower than the operating frequencies of any radio system. Since m enters the denominator of the expression for the generator voltages, and since the m for a generator with frequency at $2f$ is twice as great as for a generator with frequency of f, it is evident that, neglecting the periodic variation caused by the term in the brackets of (**72**), the voltage assigned to a generator is inversely proportional to the frequency assigned to it. If we neglect the term in brackets, then, the voltage of all of the generators which have frequencies lying in a narrow band in the radio range is the same because the m for the generator at one end of the band differs only by a small percentage from the m for the generator at the other end of the band.

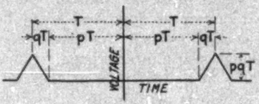

FIG. 12

It is not safe to follow too closely the generator frequencies and voltages given by the wave form of Fig. 12 because of the meager information upon which it is based. The consideration of this wave form, however, enables us to draw certain general conclusions as to the frequency and voltage of the generators which replace static in the receiving antenna. These inferences are:

1. The voltage assigned to a generator having a high frequency is less than the voltage assigned to a generator having a lower frequency.
2. The voltage assigned to all of the generators having frequencies which lie in a narrow band in the radio range of frequencies will be about the same.

In regard to assumption No. 1, it may be stated that the wave form assumed indicated that the voltage assigned to a generator was inversely proportional to the square of the frequency assigned to it.

If a voltage wave form having an abrupt rise, a flat top and an abrupt fall had been assumed, then the voltage assigned to a generator would be inversely proportional to the first power of the assigned frequency. Therefore, if the indications of the Watt and Appleton observations on the time duration of the impulses are anywhere near correct the voltage assigned to a generator will vary inversely as a power of the frequency which lies between 1 and 2.

For the wave form assumed, assumption No. 2 holds good, provided the periodic variation caused by the bracketed term of equation (**72**) is neglected. The justification for neglecting this term in drawing general conclusions arises from the fact that in the actual wave form of static the time duration of the impulses and the time interval between them vary at random. This random variation would smooth out the periodic variation indicated by the bracketed term of equation (**72**).

7. THE RECEPTION OF I. C. W. SIGNALS THROUGH STATIC INTERFERENCE

The receiving system is represented schematically as shown in Fig. 5, and we now have to consider the power received from two groups of alternators. One group of alternators replaces the voltage induced in the antenna by the I. C. W. transmitting station. The other group replaces the voltage induced in the receiving antenna by static. Since the voltage assigned to all the static generators whose frequencies lie in a narrow band is the same we will let E_s be the peak value of the voltage of each static generator. The power delivered to the ideal receiving system by each generator whose frequency lies within the transmitted band is:

$$P_{si} = \frac{E_s^2}{8 R_r} \tag{73}$$

All other static generators deliver no power to the detector. If the transmitted band of frequencies extends from $f_0 - f_c$ to $f_0 + f_c$ and if the static generators are spaced p cycles apart on a frequency scale, and if f_0 is taken as the location of one of the generators, then the number of generators in the freely transmitted band is $\frac{2f_c}{p} + 1$. The power delivered by static to the ideal receiving system is:

$$P_s = \left(\frac{2f_c}{p} + 1\right)\frac{E_s^2}{8 R_r} = \left(\frac{n_r}{pT} + 1\right)\frac{E_s^2}{8 R_r} \tag{74}$$

The power received from the I. C. W. transmitting station is given by equation (**20**). The ratio of signal power to static power is:

$$\frac{P}{P_s} = \frac{E^2}{E_s^2\left(\frac{n_r}{pT} + 1\right)}\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_s \frac{1}{s^2}\right] \tag{75}$$

$$s = 1, 3, 5, \ldots n_r$$

Now, because of the random nature of static, one frequency is as likely to be the location of a generator as any other; that is, p will be small compared to $\frac{n_r}{T}$. Under these conditions, the ratio of signal power to static power as given by (**75**) will be a maximum when the smallest possible value is assigned to n_r. In the ideal system, n_r was taken just large enough to permit the distinguishing of the signals. Therefore, the ideal system already described is the best possible frequency selecting system for receiving I. C. W. signals through static interference. Since p is assumed small compared to $\frac{n_r}{T}$, the 1 may be dropped in (**74**) and (**75**) compared to $\frac{n_r}{p\,T}$ and (**75**) may be written as:

$$\frac{P_{i.c.w.}}{P_s} = \frac{E^2\,p\,T}{E_s^2}\,\frac{1}{n_r}\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_s \frac{1}{s^2}\right]$$

$$= \frac{E^2\,p}{2\,E_s^2\,f_c}\left[\frac{1}{4} + \frac{2}{\pi^2}\sum_s \frac{1}{s^2}\right] \qquad (\mathbf{76})$$

$s = 1, 3, 5, \ldots\ldots\ldots\ldots n_r$

In equation (**76**), $\frac{E_s^2}{p}$ characterizes the static energy level associated with frequencies in the vicinity of the operating frequency f_0 of the I. C. W. station. This term varies from time to time and decreases as f_0 increases. $E^2\left[\frac{1}{4} + \frac{2}{\pi^2}\sum \frac{1}{s^2}\right]$ characterizes the I. C. W. signal energy level and its distribution over frequencies in the vicinity of f_0. The bracketed term is the same for all I. C. W. stations but E is dependent upon the transmitting station and the transmission efficiency. T characterizes the speed of signaling, its value decreasing directly with an increase in signaling speed. Thus the high signaling speeds are more subject to static interference than the low ones. n_r alone characterizes the receiving system. For the ideal receiver n_r has been assigned the minimum value which will permit the distinguishing of the signals. Thus the ideal receiver reduces the static interference to the lowest possible value that can be obtained with a frequency selecting system.

If the speed of transmission is 30 words per minute and if $n_r = 3$ we have:

$$\frac{P_{i.c.w.}}{P_s} = \frac{E^2\,p}{E_s^2}\,(0.008) \qquad (\mathbf{77})$$

At 150 words per minute, the ratio will be one-fifth as large as at 30 words per minute.

The mathematical formulation of the interference in actual receiving systems is left for a future paper, but it can be pointed out that, for the simple series receiver, the best signal-power, static-power ratio is obtained when f_c is assigned as small a value and β as large a value as possible and yet have the signals distinguishable. This follows because decreasing β or increasing f_c permits the generators removed from f_0 to furnish more power to the circuit. But as shown by the curve of Fig. 7, the voltage of the signal generators falls off rapidly as their frequency departs from the operating frequency whereas it has been shown that the voltage of the static generators remains about the same at all of the frequencies. Therefore, increasing f_c or decreasing β, that is decreasing the time constant of the circuit, increases the static power faster than the interferent power. From this it is apparent that the simple series receiver described is the best possible one for receiving through static.

8. THE RECEPTION OF RADIO TELEPHONE SIGNALS THROUGH STATIC INTERFERENCE

In the section on telephone reception it was shown that for distortionless reception the ideal receiving system must pass a band of frequencies running from $f_0 - f_c$ cycles per second to $f_0 + f_c$ cycles per second where f_c was about 5000 cycles. Under these conditions the static power picked up by the system is given by equation (**74**). Since the ideal system picks up energy only from generators which have frequencies lying in the band which must be transmitted, it is evident that the ideal system as described is the best possible frequency-selection system for receiving telephone messages through static interference. Since f_c for telephony is $5000 \div 30 = 167$ times as large as for I. C. W. reception at 30 words per minute, it is evident that 167 times as much static energy must be picked up in a telephone receiver as in an I. C. W. receiver.

The extent to which static interference can be reduced by the simple series circuit depends upon the allowable distortion of speech or music. In plotting the transmission characteristic of the simple series circuit given by Fig. 11, it was assumed that the energy in the 5000-cycle voice or musical note could be reduced to one-fifth of the value which it should have for distortionless reception. ($\beta = 2, f_c = 5000 \sim$)

9. THE I. C. W. TRANSMITTER AND THE SPARK TRANSMITTER AS SOURCES OF INTERFERENCE

It has been shown that the voltage induced in a receiving antenna by an I. C. W. transmitter could be replaced by a group of generators having the correct frequencies and correct voltages. Any I. C. W. transmitting station has generators with frequencies located in all frequency bands. It has also been shown that all receiving systems must transmit a band of frequencies centered upon the operating frequency of the station whose signals it is desired to receive. The receiving system must necessarily therefore pick up energy from all I. C. W. stations which induce a voltage in the receiving antenna. With a given receiving system the energy picked up from an interferent I. C. W. station depends upon the difference between the operating frequency of the interferent station and the operating frequency of the correspondent station and the manner in which the voltage assigned to any generator which replaces the interferent voltage in the receiving antenna vary with the frequency assigned to it. The curve of Fig. 7 gives a graphical picture of the dependance of the voltage assigned to any generator upon the frequency assigned to the generator. From equation (**22**) we draw the conclusion that 90 per cent of the energy available in the waves is associated with a band of frequencies $\frac{3}{T}$ cycles wide and centered on the operating frequency. At 30 words per minute, $\frac{3}{T}$ has a value of about 60 cycles, and at 150 words per minute $\frac{3}{T}$ has a value of about 300 cycles. We thus come to the conclusions that the energy associated with the waves of an I. C. W. transmitter is confined to a narrow band of frequencies and the width of this band varies directly with the signal speed. The above statement holds true if the I. C. W. transmitter has no harmonics. If harmonics are present in the waves sent out by the transmitter, the voltages assigned to generators having frequencies in the vicinity of the harmonic will vary as shown by Fig. 7 for the fundamental frequency and any receiving station having a transmitted band in the vicinity of one of the harmonics will pick up an appreciable amount of power from the transmitter.

The spark transmitter causes much more interference than the I. C. W. transmitter. This fact is brought out in a striking manner if the curves of Fig. 10 are compared with those of Fig. 7. The voltage assigned to a generator falls off slowly as the frequency departs from the operating frequency. From an examination of the curves of Fig. 10 and from the discussion of section (4) it is evident that the interference caused by a spark station is dependent upon the logarithmic decrement times the frequency rather than upon the logarithmic decrement. If $\alpha = \delta f_0 = 10^4$, then equations (**51**) and (**52**) show that 82 per cent of the energy in the waves is associated with a band of frequencies 10,000 cycles wide, centered on the operating frequency f_0. If $\alpha = \delta f_0 = 10^5$, then 82 per cent of the energy is associated with a band of frequencies approximately 100,000 cycles wide, centered on the operating frequency. This is in striking contrast to the I. C. W. case where most of the energy is associated with a band of frequencies 60 to 300 cycles wide. From the above discussion it is evident that a station operating at a frequency of 10^6 cycles per second, and having a logarithmic decrement of 0.01, has its energy spread over the same band width as a station operating at 10^5 cycles per second and having a logarithmic decrement equal to 0.1.

10. GENERAL CONCLUSIONS

With regard to the extent to which interference can be mitigated by frequency selecting systems all sources of interference to radio reception and all sources of signals have a definite frequency spectra and for convenience we can replace the voltages induced in an antenna by a group of generators having the correct voltages and frequencies. In order to receive signals the receiving system must pass freely the currents due to all generators having frequencies in a given band. This band is centered on the operating frequency of the station from which it is desired to receive signals. The width of the band is determined by the class of signals which it is desired to receive. If the interferent voltages have generators with frequencies in this transmitted band, the frequency selection system cannot eliminate the currents due to these generators. Since the ideal receiver as specified in this paper utilizes the maximum possible power from generators lying within the band of frequencies which must be passed and utilizes no power from generators which have frequencies outside this band, it is evident that the interference obtained in the ideal receiver is the minimum which can be obtained with frequency selecting systems. From this it is evident that the minimum interference which is obtainable depends upon the ratio of the voltage of the signal generators to the voltage of the interferent generators which lie in the transmitted band and upon the width of the band of frequencies which it is necessary to transmit. The width of the band of frequencies which must be transmitted depends upon the class of signals which are to be received. Thus while the necessary transmitted band width for I. C. W. telegraphy at 30 words per minute is only 60 cycles, at 150 words per minute it is 300 cycles. The necessary transmitted band width for telephony is about 10,000 cycles and the necessary transmitted band width for spark telegraphy varies from 10,000 cycles to 100,000 cycles depending upon the product of the logarithmic decrement times the operating frequency.

The ideal receiving system thus furnishes a criterion by which we can judge the merits of any actual frequency selection system for receiving through interference. The writer hopes in a future paper to show how near actual receiving systems of various types approximate the ideal receiver.

Appendix

Part I

(A) VOLTAGE AND FREQUENCY OF THE GENERATORS REPLACING VOLTAGE INDUCED IN A RECEIVING ANTENNA BY AN INTERRUPTED CONTINUOUS WAVE TRANSMITTER

The schematic diagram of the voltage assumed in this case is given by Fig. 6. The voltage is taken as a series of dots and spaces. The time duration of each space is represented by $2\,p\,T$ and of each dot by $2\,q\,T$.

In order to simplify the calculations the wave is taken as symmetrical with the origin and the time intervals $p\,T$ and $q\,T$ are assumed to be exactly divisible by the period of the operating frequency. Under these conditions the voltage induced in the antenna is expansible from $t = -\infty$ to $t = +\infty$ in the form:

$$e\,(t) = \sum_{m=1}^{\infty} a_m \sin \frac{m\,\pi\,t}{T} \tag{1}$$

where

$$a_m = \frac{2}{T} \int_{o}^{T} e\,(\lambda) \sin \frac{m\,\pi\,\lambda}{T}\,d\,\lambda \tag{2}$$

But $e\,(t) = 0$ from $t = 0$ to $t = p\,T$ (3)

$= E \sin [\,\omega\,(t - p\,T)]$ from $t = p\,T$ to T (4)

Upon substituting (**3**) and (**4**) in (**2**) we obtain:

$$a_m = \frac{2\,E}{T} \int_{pT}^{T} \sin \Big[\,\omega\,(\lambda - p\,T)\,\Big] \sin \frac{m\,\pi\,\lambda}{T}\,d\,\lambda \tag{5}$$

$$= \frac{E}{T} \int_{Tp}^{T} \left\{ \cos \left[\left(\omega - \frac{m\,\pi}{T} \right) \lambda - \omega\,p\,T \right] - \cos \left[\left(\omega + \frac{m\,\pi}{T} \right) \lambda - \omega\,p\,T \right] \right\} d\,\lambda \tag{6}$$

If $\omega \neq \frac{m\,\pi}{T}$ this becomes:

$$a_m = \frac{E}{T} \left[\frac{\sin [\omega\,q\,T - m\,\pi]}{\omega - \frac{m\,\pi}{T}} - \frac{\sin [\omega\,q\,T + m\,\pi]}{\omega + \frac{m\,\pi}{T}} \right] + \frac{E}{T}\,(\sin m\,p\,\pi) \left[\frac{1}{\omega + \frac{m\,\pi}{T}} + \frac{1}{\omega - \frac{m\,\pi}{T}} \right] \tag{6}$$

Because of the conditions on $p\,T$ and $q\,T$ the first term is zero and we therefore arrive at:

$$a_m = \frac{E}{T}\,(\sin m\,p\,\pi) \left[\frac{2\,\omega}{\omega^2 - \frac{m^2\,\pi^2}{T^2}} \right] \tag{7}$$

for $\omega \neq \frac{m\,\pi}{T}$

If $\omega = \frac{m\,\pi}{T}$ equation (**5**) becomes

$$a_{\frac{\omega T}{\pi}} = \frac{E}{T} \int_{pT}^{T} [1 - \cos\,(2\,\omega\,\lambda - \omega\,p\,T)\,]\,d\,\lambda$$

$$= E\,q - \frac{E}{T} \left[\frac{\sin\,(2\,\omega\,\lambda - \omega\,p\,T)}{2\,\omega} \right]_{pT}^{T}$$

$$= E\,q - \frac{E}{2\,\omega\,T}\,[\sin\,[\omega\,T\,(2 - p)\,] - \sin\,\omega\,p\,T]$$

Both sine terms vanish in the last equation because of the conditions on p, q, and T. We therefore have:

$$a_{\frac{\omega T}{\pi}} = E\,q \tag{8}$$

(B) THE APPROXIMATION IN DERIVING EQUATION (11) FROM EQUATION (9)

The approximation involved in this step entered because $1 + \frac{n}{4\,f_0\,T}$ was taken equal to unity. At a transmitting speed of 30 words per minute $T = 0.05$ seconds. If $f_0 = 10^6$ cycles per second the value of the term dropped is equal to $(n)\,5 \times 10^{-6}$. That is, the term for which $n = 2000$ is in error by only one per cent; all terms for which n is less are in error by less than one per cent. If $f_0 = 20{,}000$, (wavelength 15,000 meters) the value of the term dropped is $2.5\,n \times 10^{-4}$ and n may have a value of 40 before the error reaches one per cent; at 150 words per minute the value of n for which the error becomes one per cent is one-fifth of those given above, namely 400 at a frequency of one million cycles per second and 8 at a frequency of twenty thousand cycles per second.

Part II

(C) VOLTAGE AND FREQUENCY OF THE GENERATORS REPLACING THE VOLTAGE INDUCED IN A RECEIVING ANTENNA BY A SPARK STATION

For the purposes of this paper the voltage induced in a receiving antenna is taken to have the wave form shown schematically by Fig. 9. The voltage is assumed to have a peak value equal to E, a frequency $f_0 = \frac{\omega}{2\,\pi}$, and a logarithmic decrement equal to $\frac{\alpha}{f_0}$. The decrement is large enough so that the voltage due to one spark falls substantially to zero before the next spark takes place. Under these conditions the voltage can be represented as a function of time from $t = -\infty$ to $t = +\infty$ by

$$e\,(t) = \sum_{m=-\infty}^{m=+\infty} A_m\,\epsilon^{j\frac{m\pi t}{T}} \tag{9}$$

where

$$A_m = \frac{1}{2\,T} \int_{-T}^{T} e\,(\lambda)\,\epsilon^{-j\frac{m\pi\lambda}{T}}\,d\,\lambda \tag{10}$$

The expansion is used in the above form to facilitate the evaluation of the integral (**10**). As we have taken the wave form

$$e\,(t) = E\,\epsilon^{-\alpha\,(t+T)} \sin [\,\omega\,(t + T)\,]\,\Big|_{t=-T}^{t=o} \tag{11}$$

$$= E\,\epsilon^{-\alpha t} \sin \omega\,t\,\Big|_{t=o}^{t=T} \tag{12}$$

Substituting these values in (**10**), there results:

$$A_m = \frac{E}{2\,T} \left\{ \int_{-T}^{o} \epsilon^{-\alpha T}\,\epsilon^{-\left(\alpha + \frac{jm\pi}{T}\right)\lambda} \sin [\omega\,(\lambda + T)]\,d\,\lambda \right.$$

$$+\int_0^T \epsilon^{-\left((\alpha+\frac{jm\pi}{T}\right)\lambda} \sin\omega\lambda\, d\lambda \Bigg\} \qquad (13)$$

In the first integral put $r = \lambda + T$ and in the second put $r = \lambda$. (**13**) then becomes

$$A_m = \frac{E}{2T}(1+\epsilon^{jm\pi}) \int_0^T \epsilon^{-\left(\alpha+\frac{jm\pi}{T}\right)r} \sin\omega r\, dr \qquad (14)$$

$$= \frac{E}{2T}(1+\epsilon^{jm\pi}) \left[\left(\epsilon^{-\left(\alpha+\frac{jm\pi}{T}\right)r}\right) \right.$$

$$\left. \frac{-\left(\alpha+\frac{jm\pi}{T}\right)\sin\omega r - \omega\cos\omega r}{\left(\alpha+\frac{jm\pi}{T}\right)^2+\omega^2} \right]_{r=o}^{r=T} \qquad (15)$$

Since $\epsilon^{-\alpha r}$ is nearly zero, when $r = T$, (**15**), vanishes at the upper limit and we have:

$$A_m = \frac{E}{2T}\left[1+\epsilon^{jm\pi}\right] \frac{\omega}{\left(\alpha+\frac{jm\pi}{T}\right)^2+\omega^2} \qquad (16)$$

$$[1+\epsilon^{jm\pi}] = [1+\cos m\pi + j\sin m\pi]$$
$$= 0 \text{ if } m \text{ is odd; } = 2 \text{ if } m \text{ is even}$$

$$A_m = \frac{E}{T}\frac{\omega}{\left(\alpha+\frac{jm\pi}{T}\right)^2+\omega^2} \text{ for } m \text{ even} \qquad (17)$$

$$= 0 \text{ for } m \text{ odd} \qquad (18)$$

Equation (**9**) may be written in the form

$$e(t) = A_0 + \sum_{m=1}^{\infty} \left\{ (A_{+m}+A_{-m})\cos\frac{m\pi t}{T} + j(A_{+m}-A_{-m})\sin\frac{m\pi t}{T} \right\} \qquad (19)$$

Now A_{+m} and A_{-m} are conjugate complex numbers. Let

$$A_{+m} = a_m + j\,b_m \text{ and } A_{-m} = a_m - j\,b_m \qquad (20)$$

Then (**19**) becomes:

$$e(t) = A_0 + \sum_{m=1}^{\infty}\left[2a_m\cos\frac{m\pi t}{T} - 2b_m\sin\frac{m\pi t}{T}\right] \qquad (21)$$

$$= A_0 + \sum_{m=1}^{\infty} B_m\cos\left(\frac{m\pi t}{T}+\theta_m\right) \qquad (22)$$

where

$$B_m = 2\sqrt{a_m^2+b_m^2} = 2|A_m| \qquad (23)$$

$$\theta_m = \tan^{-1}\frac{b_m}{a_m} \qquad (24)$$

From (**17**), (**18**) and (**23**) we write:

$$B_m = 0 \text{ if } m \text{ is odd} \qquad (25)$$

$$= \frac{2E}{T}\frac{\omega}{\sqrt{\left[\alpha^2+\omega^2-\frac{m^2\pi^2}{T^2}\right]^2+\frac{4\alpha^2m^2\pi^2}{T^2}}} \qquad (26)$$

$$= \frac{2E}{\omega T}\frac{1}{\sqrt{\frac{\alpha^4}{\omega^4}+2\left[1+\left(\frac{m\pi}{\omega T}\right)^2\right]\frac{\alpha^2}{\omega^2}+\left[1-\left(\frac{m\pi}{\omega T}\right)^2\right]^2}} \qquad (27)$$

In any spark transmitter $\frac{\alpha^4}{\omega^4}$ is very small compared to the other terms under the radical and may therefore be dropped without appreciable error. The value of m which gives the operating frequency is $m = 2f_0T$ if f_0T is an integer. Take f_0T as an integer and since m must be a positive even integer take

$$\left.\begin{array}{l} m = 2f_0T + 2n \\ \qquad n = 0, \pm 1, \pm 2 \text{ etc.}\end{array}\right\} \qquad (28)$$

From (**27**) and (**28**) we find that the generator whose frequency differs from the operating frequency by $\frac{n}{T}$ cycles per second has a voltage given by:

$$B_n = \frac{2E}{\omega T}\frac{1}{\sqrt{2\left[2+\frac{2n}{f_0T}+\frac{n^2}{f_0^2T^2}\right]\frac{\alpha^2}{\omega^2}+\left[\frac{2n}{f_0T}+\frac{n^2}{f_0^2T^2}\right]}} \qquad (29)$$

For the poorest transmitting stations, (Log. dec. $=0.2$), $\frac{\alpha^2}{\omega^2}$ does not exceed 10^{-3}. Therefore the term under the radical of (**29**) has the form $2[2+A]10^{-3}+A^2$. If A is small compared to 2 the A in the first term may be dropped. If A is of the same order of magnitude as 2 or greater, the whole first term may be dropped. Therefore in all cases B_n is given very closely by:

$$B_n = \frac{2E}{\omega T}\frac{1}{\sqrt{\frac{4\alpha^2}{\omega^2}+\left[\frac{2n}{f_0T}+\frac{n^2}{f_0^2T^2}\right]^2}} \qquad (30)$$

Part III

(D) Voltage and Frequency of the Generators Replacing the Voltage Induced in a Receiving Antenna by Static

For the purposes of this paper, the wave form of the voltage induced in a receiving station by static is assumed to be as given by Fig. 12. The time duration of the impulse is $2qT$ seconds and the time between impulses is $2pT$ seconds.

The voltage as a function of time in the interval $t = -T$ to $t = +T$ is then given by

$$\left.\begin{aligned} e\,(t) &= -\,\rho\,(t + p\,T)\,\Big|_{t=-T}^{t=-pT} \\ &= 0\,\Big|_{t=-pT}^{t=+pT} \\ &= \rho\,(t - p\,T)\,\Big|_{t=pT}^{t=T} \end{aligned}\right\} \tag{31}$$

Since the voltage wave is symmetrical with respect to the voltage axis, it may be represented from $t = -\infty$ to $t = +\infty$ by the cosine series:

$$e\,(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} a_m \cos \frac{m\,\pi\,t}{T} \tag{32}$$

$$a_m = \frac{2}{T} \int_0^T e\,(\lambda) \cos \frac{m\,\pi\,\lambda}{T}\, d\,\lambda \tag{33}$$

Upon substituting (**31**) in (**33**) we obtain:

$$\begin{aligned} a_m &= \frac{2\,\rho}{T} \int_{pT}^{T} (\lambda - p\,T) \cos \frac{m\,\pi\,\lambda}{T}\, d\,\lambda \\ &= \frac{2\,\rho\,T}{m^2\,\pi^2}\,(\cos m\,\pi - \cos m\,p\,\pi) \text{ if } m \neq 0 \end{aligned} \tag{34}$$

$$\begin{aligned} a_0 &= \frac{2\,\rho}{T} \int_{pT}^{T} (\lambda - p\,T)\, d\,\lambda = \frac{2\,\rho}{T} \left[\frac{\lambda^2}{2} - p\,T\,\lambda \right]_{pT}^{T} \\ &= 2\,\rho\,T \left[\frac{1}{2} + \frac{1}{2}\,p^2 - p \right] \end{aligned} \tag{35}$$

Discussion

Edward Bennett: In order to appreciate the significance of the point of view which has been developed in Professor Peters' paper, it may be helpful to review the sequence in which the knowledge of the properties of circuits has developed. Practising engineers were first confronted with direct-current problems, and quickly obtained a very good understanding of the properties of complex networks or combinations of circuits, provided the problems related only to the steady flow of continuous currents in the networks.

Somewhat later came the realization of the possibilities of alternating current in the distribution of power. Engineers then became concerned with the performance of circuits when carrying alternating current. That situation was cleared by the adoption (largely through the writings of Steinmetz) of the method of the complex algebra for the calculation of the properties of alternating-current circuits.

In recent years, with the development of radio telegraphy and telephony, and with the development of extensive high-voltage transmission systems, engineers have become concerned, not so much with the performance of networks when carrying alternating currents in an undisturbed manner, but with the performance of these networks when they are subject to switching operations or when they are subject to disturbing electromotive forces,—the static impulses and so on.

Mr. Peters has developed and illustrated the application of the point of view that when we want to determine the effect of a transient disturbing electromotive force upon a receiving network, we may do this by replacing the transient electromotive force with the electromotive forces of suitably selected alternators which are conceived to remain in the circuit from the beginning of time to the end of time, and by calculating then the power delivery of these alternators to the receiving apparatus. The problem in the calculation of transient currents is thus reduced to the problem of calculating the steady-state values of alternating currents,—a calculation which is widely understood. It seems to me that the spread of this point of view will do for transient-current calculations what the spread of the method and notation of complex algebra did for the calculation of the steady-state values of alternating currents.

Can the University Aid Industry?

BY BENJAMIN F. BAILEY[1]

Fellow, A. I. E. E.

At first the author was inclined to reverse the title and make it read "Can Industry Aid the University?" There is no doubt in his mind that the engineering professor should have active and continuous contacts with industry. It is generally agreed that it is impossible for a school to teach engineering applications in detail. The field is altogether too broad and the variety of positions which students fill after graduation is so great that any attempt to fit a man to fill a particular position upon graduation must result in failure. The university can and should attempt to give a thorough foundation in the fundamentals underlying engineering and if this is well done it will require easily all the available time.

From this it might seem that if a teacher is well grounded in the fundamentals, he has all the necessary qualifications. To keep the interest of the student, however, and prepare him for his life work, the application of these principles to specific problems must be continually emphasized. If a teacher lacks definite personal knowledge of at least one field of engineering practise, he is seriously handicapped in his attempts to show how the fundamentals of physics, mathematics and economics may be applied to engineering practise. Moreover, the interest and ambition of the student is more easily aroused if he feels that his instructor has personal knowledge of the things about which he is talking. If the proper attitude on the part of the student is secured, the other problems of engineering education will largely take care of themselves.

Another reason for encouraging the teacher to make practical use of his knowledge and experience is that he may increase his income. Everyone knows that the salaries paid university professors are, in many cases, low as compared with the incomes that the same men could earn in industry. Under present circumstances in the colleges if the best teachers are to be secured and held there must be some way for them to earn something outside of their university salaries. Those who have been responsible for the conduct of departments of instruction have many times had the experience of losing their best men when they were offered twice as much salary by industry.

The author believes that there is a growing spirit of cooperation between industry and the universities. This is due to a number of causes. Doubtless the work of college professors during the war had a great deal to do with it. There are numerous instances of supposedly impractical professors who turned out to have the finest kind of executive ability and who were able to apply their theoretical knowledge to practical affairs upon short notice.

The decrease in the relative number of consulting engineers may have helped to foster this cooperation. The fact that the customer can obtain excellent engineering advice free from most of the large electrical companies has tended to deter men from becoming consulting engineers. Many problems, however, come up which cannot well be handled by reference to some manufacturer and if consulting engineers are not available, the work is likely to be turned over to a teacher of engineering.

Relatively, at least, the importance of the individual inventor seems to have decreased. It is coming to be more and more apparent that many problems are of the type best handled by scientists rather than by inventors and again the work is likely to be turned over to a teacher of engineering.

A third reason is the notable work accomplished by the research laboratories established by many of the larger electrical companies. The pronounced commercial success of many of the devices developed in such laboratories has shown the value of scientific services to the smaller manufacturers.

Whatever may be the cause, it seems to be a fact that so much work of a scientific nature is brought to the universities that it may not be long before the handling of this work will constitute a real problem.

For a great many years past, the Engineering Department of the University of Michigan, under the leadership of Dean Cooley, has encouraged the faculty to make contacts with industry. The question as to how far this sort of work should be encouraged was once raised and the regents of the university passed resolutions intended to clarify the situation. These were to the effect that the faculty of the Department of Engineering was encouraged to accept work of the proper character to as great an extent as possible without interfering with their regular work of instruction. They were given permission to use their offices and the laboratory facilities of the university for the carrying on of such work. It was clearly indicated that the work undertaken must be of more than routine character and that the amount and type of such work must meet with the approval of the Dean of the Department.

The work offered to the university usually falls in one of three classes.

a. Routine tests of materials and apparatus
b. Engineering advice
c. Research problems

A. Work of the first class has been discouraged, although not entirely prohibited. It frequently happens that a manufacturer wishes a particular test made and no other agency is prepared to do the work as quickly or as economically as the university. Not infrequently the university is asked to undertake work of this character because an absolutely impartial test is desired. With these facts in view, it does not always seem wise to refuse work of this character. When it is possible, however, it is turned over to the younger men to whom the experience is probably of some value.

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B. The type of work called *engineering advice* requires a distinctly higher grade of ability. A number of professors have performed notable services in connection with appraisals of railroads, telephone companies and power plants, by acting as expert witnesses in patent suits, by advising with municipalities regarding important bridges, roads and other public works, and by acting as consulting engineers to various manufacturing plants, as well as in numerous other ways. Many of these contacts have resulted in men being enticed away from the university by the offer of large salaries. In some cases these men have been lost permanently to the teaching profession; in other cases "leave of absence" for one or two years has been granted and the man has willingly returned to resume his university duties.

C. A considerable amount of research work has been carried out also; sometimes this is done for pay at the request of some manufacturer, but more often is undertaken merely on account of real interest in the work or with the hope that a good piece of work will result in promotion. Naturally most of us tend to concentrate on work which is paid for in cash as well as reputation, rather than on that which is paid for in reputation only. Nevertheless, some notable work has been done and if time were available, it would be interesting to list some of the papers published, dealing with research work accomplished at the University of Michigan.

Interest in research, particularly in applied research, has been stimulated by the establishment of the Department of Engineering Research in 1920. Some three or four years prior to this time, a group of engineering alumni proposed that the university establish such a department, and this was brought to a head a little later by a request from the Michigan Manufacturers Association. The manufacturers felt very strongly that such a department would be of distinct assistance to the industries of Michigan and other states, and that, in turn, industry might help the university. These concerns pointed to the large sums of money spent in agricultural research and, while admitting that every penny of this was well spent, insisted that money might be spent to equally good advantage to aid industry. The department has had a steady growth and the amount of work done has increased, until this year it is expected that approximately $100,000 will be paid into this department by industry for services rendered.

In establishing this department, a large number of perplexing problems had to be met. One of the first referred to the fees which should be charged for work done. It was universally agreed that the manufacturers requesting services should pay for them. The person doing the work receives a compensation approximately equal to that which he would receive for the same work done for the university. In addition an overhead charge of from 20 to 50 per cent is made to cover, at least partially, the use of laboratories and equipment and the cost of shop service, electricity, gas, air or other sources of energy.

The question of the character of work to be undertaken also demanded serious consideration. The policy of the department is to discourage routine tests as much as possible. If requests for such work come in, the inquirer is advised of the existence of commercial laboratories capable of doing the work requested. If it appears, however, that, for some reason or other, the work cannot be so satisfactorily accomplished elsewhere, the Department occasionally accepts such work. The percentage of tests in which no critical judgment is required, however, is very small.

On the other hand, the department fosters work of a research character and work in which judgment and engineering experience is definitely required.

The question of patents has proved to be perhaps the most perplexing of all the problems confronting the department, and the policy to be adopted has not yet been entirely settled. One proposal made was that if, in the course of a research, patentable inventions were made, the patent or patents were to be taken out at the expense of the university and thrown open to the use of the public without charge. To most of the members of the department, this did not appear to be a wise policy. It seemed that the manufacturer who supplied the funds for the investigation and the man who made the invention both deserved some reward. Since it furnished the facilities, the university was also entitled to consideration. Moreover, if a patent were thrown open to anyone, it is quite likely that no one would use it. The reason, as most engineers know, is that when an invention has been made, the trouble has just started. Almost invariably it is necessary to invest a large sum of money in developing the invention, putting it in form for economical manufacture and educating the public to use the new product. Since proportions and designs cannot be protected by patents (to say nothing of the desire aroused by advertising to possess the new article) a manufacturer might expend large sums in developing an invention and then be at the mercy of anyone who wished to steal the results of his efforts. Under such conditions a meritorious invention might never be put to practical use.

It was considered inadvisable also to allow patents taken out in the course of an investigation to be assigned exclusively to the manufacturer financing the work. An invention made in the course of research would be in the nature of a byproduct and, in general, would not be expected when the work was started. It was thought that if some invention of great value should sometime appear, the university might be severely criticized for allowing it to fall into the hands of a single individual or corporation.

The plan tentatively adopted provides that patents may be taken out by the inventor with the consent of the Board of Regents and at the expense of the person financing the investigation. All inventions are to be assigned to the university with the understanding that the manufacturer involved will have first consideration in the case of the sale of the patent or the granting of licenses under it. It is also understood that in case an invention proves valuable, the inventor may expect to profit to some extent. A number of patents have been applied for but in no case has the point been reached where this problem has had to be frankly faced. Under the present scheme the manufacturer is obliged to pay the expenses of taking out the patent and has no definite assurance as to what interest he will have under it. Neither has the inventor any assurance as to what returns he may expect.

The actual work of investigation under the Department of Engineering Research is carried out very largely by the teaching staff of the Engineering Department. There are, however, a few men who devote their entire time to this work. These men hold similar rank to men of the same attainments in the engineering college. The titles used are investigator, associate investigator, assistant investigator, research assistant, and assistant.

One of the first questions of policy to be settled when the Department was organized was whether or not a manufacturer might obtain data for his exclusive benefit. It was definitely determined from the first that the results of all investigations were to be made available ultimately, but publication is necessarily deferred for some time. Within the past year, the department has begun the publication of a bulletin in which the results of investigations are available to anyone interested.

The department is now undertaking the direction of a Meter Testing Laboratory in the Department of Electrical Engineering. This is somewhat of an extension of the original scope of the work of the department since the work of checking meters and instruments can hardly be considered as research. This laboratory was established at the request of the Detroit Edison Company and the Consumers Power Company as a convenience to the public utilities of the state. It was thought that the actual administration could most readily be carried out through the Department of Engineering Research. When this laboratory is in operation, the Public Utility Companies will be able to obtain prompt tests upon the accuracy of voltmeters, ammeters, wattmeters, current and potential transformers, and rotating standard watthour meters.

It is believed that the future work of the Department of Engineering Research will lie largely in cooperative research. In Michigan, there are a number of industries each carried on by a large number of manufacturers. Familiar examples are the furniture industry, the beet-sugar industry, the automobile industry, the paper industry and many others. In some of these, very little is done in the way of research, and it would seem that the manufacturers could well afford to cooperate to the extent of financing extensive research work. Some progress has been made in this direction, but naturally it will require time to interest enough manufacturers in each line to make the scheme a success.

The department also plans, as far as its finances will permit, to carry out researches in both pure and applied science upon its own initiative. A number of problems immediately suggest themselves which might well be the basis of extensive work and which promise ample returns for the money invested. In the case of some of these problems no one manufacturer is sufficiently interested to finance the work with the understanding that all results will be public property. Before very much can be done along these lines the department will have to secure more adequate financial support. At present, the state of Michigan spends, yearly, about $250,000 upon agricultural research. Every penny of this is undoubtedly justified. The value of the agricultural products in the state of Michigan is approximately $600,000,000 per year, but the value of the manufactured products is $3,500,000,000, or nearly six times as much. If engineering research received financial support on the same basis as agricultural research may, it be seen that there would be approximately $1,500,000 available per year. It is to be hoped that in the near future this work will come somewhere near to securing the support it deserves.

Some mention of several notable investigations already undertaken by the department may be of interest.

Natural Lighting. Work has been going on for about two years upon the subject of day lighting. Apparently very little had previously been done along this line and the importance of the work is obvious.

Natural Ventilation. A similar investigation was carried on to determine the extent to which ventilation occurs naturally in factory buildings. This is another subject upon which little or no data were available.

Bearing and Gear Noises. Some very interesting work has been done upon the sources of noises in machinery and the methods of measuring noises. A very ingenious device has been developed by which quantitative readings of noise may be made and some advance has been made in locating the source of noises in roller bearings and gears.

Single-Phase Motors. Work has been carried on for about three years upon the subject of single-phase motors, and considerable progress has been made in developing a new type of single-phase motor.

Cutting of Metals. A fundamental research is in progress upon the art of cutting metals with particular reference to milling cutters. A method has been developed for determining accurately the forces involved in action of a single tooth of such a cutter.

Industrial Waste Disposal. This work was undertaken at the request of the city of Flint, where the subject of waste disposal has assumed great importance.

SUMMARY

The writer believes that a distinct advance has been made within the last few years in the relations between the university and industry. It is hoped and believed that this tendency to cooperate will continue and will be of great advantage to both parties to the transaction.

Discussion

For discussion of this paper see page 604.

Research Relations Between Engineering Colleges and Industry

BY W. E. WICKENDEN[*]
Member, A. I. E. E.

IN our present easy-going attitude toward language, we are tempted to use the word "research" to mean so many things that we have no word left to mean research. The effort to sell a research program to industry has induced us to translate the word into such unmistakably practical terms as "in a word it is invention," to quote a recent official pamphlet.

In the present discussion it is probably safe to assume that we are dealing only incidentally with disinterested pioneer work in the realm of pure knowledge, which is the scientist's calling. Engineering is always utilitarian and concrete, and most engineers abandoned the alternative of becoming physicists, chemists or mathematicians in order to become engineers. This observation holds for professors as well as practitioners. Every professor of engineering worthy of the title, however, is qualified to investigate engineering problems involving elements of novelty which call for extended observation, refined technique, and a resort to fundamental principles. The indications are that both the men and facilities available for these purposes in the engineering colleges are now being utilized very inadequately, to the detriment of both industry and education.

A fairly complete survey of the engineering colleges in the United States shows that not less than 58 have organized arrangements for research, at least on paper. Included in this number are 41 institutions which have organized plans through which the services of the engineering staff are made available in consulting capacities to industries, public service utilities and others. If we include those institutions in which research is fostered on a purely individual basis, the total rises to 110.

The total expenditures of the organized research departments are not less than $1,250,000 per annum and may reach as high as $1,500,000. Of such expenditures in the academic year 1924-'25, approximately 11 per cent represented direct appropriations for research by the several states, 40 per cent funds allotted by the colleges from their general funds, and 49 per cent funds derived from outside sources, principally from public, professional and industrial organizations.

At least 34 institutions have full-time research staffs, with between 250 and 300 men so engaged. To this number may be added about 350 more who render part-time research service for definite compensation and about 200 who give part-time service in organized departments without special compensation. The number engaged on a purely individual basis is difficult to estimate.

Impressive as these totals are, it must be remembered that there are two corporations in the electrical industry which spend on research more than double the total outlay of the engineering colleges and that at least two in other fields exceed the total of the colleges. Organizations at this end of the industrial scale are unlikely to turn to the engineering colleges to get their problems solved. Big business can probably be induced to give support to organized investigation in engineering colleges only on the plea that such activity is necessary in order to maintain the proper setting and staff for the training of the grade of engineers big business requires. Pure research rather than engineering investigation seems likely to get the bulk of the financial assistance from this source, and it seems probable that it will be disbursed through some intermediary or clearing house, instead of passing directly to individual colleges or professors. The present campaign of the National Academy of Sciences for a national research endowment, under the leadership of Mr. Herbert Hoover, is working successfully on this principle.

At the other end of the scale are innumerable, small units of industry which live from balance sheet to balance sheet, with no consciousness of research needs and little margin for the support of research activities. Such organizations occasionally meet acute problems outside the scope of their normal routine and staff. When these emergencies call for investigation, a college professor or laboratory staff may fit the need admirably, but is this not first aid rather than research?

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Many of these smaller units of industry are concerned with the traditional arts rather than with the applications of modern science. They are likely to remain deaf to any appeal to support research for the sake of repaying their debt to science or for the sake of maintaining a wide margin between pure knowledge and practical invention. It is just these industries, however, which are most apt to be handicapped by the decline of sporadic invention and likely to profit most relatively, from an adequate program of research. Support for such investigation is not to be expected from the single units of such industries, but their state and national trade associations are the natural agencies to foster such activities. There are attractive possibilities of dividends in the form of better methods, economies and more exact control in the present state of the art. Research, looking in advance of the present art, is a form of mutual insurance covering the risks of supersession or revolutionary changes in the art.

A recent report of the Babson Statistical Organization announced that an English factory was testing out the production of a flexible, colorless, resilient, noninflammable glass of organic origin. It went on to say:

> "The point of this letter, however, is not only about glass. This is but one of numerous far-reaching discoveries which have recently been reported. We believe that business is entering an era of the most rapid and revolutionary changes in the chemistry and physics of manufacturing. To those who are quickest in taking advantage of such discoveries, they present unlimited opportunities, but the manufacturer, merchant or investor who is asleep to these changes will be hurt.
>
> "The time has passed when advertising alone will get sales. The two best salesmen today are 'a better product' and 'a cheaper way of making it.' Research opens the way to both. Furthermore, since the most deadly competition is not between concerns, but between industries, we urge clients to combine their energies with others in the same line of business. This saves duplication of efforts, leads to maximum results and keeps one best informed about all impending developments. Make use of the help which the United States Bureau of Standards at Washington stands ready to give you, and also that of other technical organizations of high standing. Cease worrying about gaining the temporary advantage of exclusive patents, and combine your resources in the way that will do most to permanently help your industry."

It is significant that the occasion which brought the above report to the writer's desk was a request from its authors for a complete list of the engineering colleges with facilities for organized research in cooperation with industry. The writer is strongly of the opinion that trade and industrial associations offer by far the most promising agencies through which to promote industrial cooperation with the colleges in engineering research. The support of such activities by individual corporations is apt to be sporadic while trade associations can effectively maintain a continuing relation. The results of work supported by individual corporations are apt not to be widely disseminated or used. A good deal of such support is frankly on a courtesy basis, without a vital interest in its effectiveness or utilization, and with little genuine collaboration between the college and the industry.

A trade association, administering its funds in trust for its constituency and genuinely concerned to show a good return for the investment, provides an excellent medium for the joint shaping of research projects and a widely ramified channel through which to disseminate results. It would seem that the most fruitful result may be expected when a trade association concentrates its cooperative program at a single college, or at a small group of institutions at most. Such work ought not to be located on a courtesy basis or split up on a political basis, but should seek to capitalize distinctive advantages of men, plant and environment. There is a definite loss when such resources are spread over many institutions as a fairly ineffectual mist, when they might be concentrated into a fairly powerful stream. In this respect the European situation is far better ordered than our own. The policy of concentrating resources abroad has led to the upbuilding of a group of notable and distinctive research centers which are the principal factor in attracting and holding to educational and research work the outstanding authorities in the several fields. The result has been to give the several institutions an individuality quite unknown among us.

The writer does not feel that the colleges can rest their case for industrial backing for their research programs on the needs of the colleges or on an assumed sense of obligation on the part of industry to see that the colleges obtain the men, money and facilities needed to do their best work. The industries recognize these obligations rather vaguely, and when confronted by the competing pleas of 100 to 150 institutions are justified in feeling rather helpless. The Babson report, however, speaks in the tongue which industry understands and to which it responds.

The work being done at Cincinnati in cooperation with the Tanners' and the Lithographers' national organizations, the power brake studies at Purdue in cooperation with the American Railway Association, the studies in warm air heating and ventilating at Illinois, and the great project of the Portland Cement Association in association with Lewis Institute are worthy examples of adequately supported, nationally backed and effectively concentrated research programs, involving real collaboration between colleges and industries and leading to results of great significance and value.

Discussion

For discussion of this paper see page 604.

Seminars for Practising Engineers:

A Challenge to Engineering Teachers

BY EDWARD BENNETT[1]
Fellow, A. I. E. E.

STATED in broad and comprehensive terms, the *primary responsibility* of the engineering colleges is for the development of four groups of men for engineering work; namely,

a. Undergraduate students
b. Graduate students
c. Practising engineers
d. Engineering teachers.

The responsibility of the colleges of engineering for the first two groups is evident and is not discussed in this paper. On the other hand, the possibilities of educational activities with the third group are just beginning to appear on the educational horizon, and the question naturally arises as to the grounds for listing such activities as a responsibility of the colleges.

Two considerations seem to warrant our regarding educational activities with practising engineers as responsibilities, or at least as promising fields of service for the colleges of engineering.

First, an appreciable percentage of practising engineers and of engineering teachers contend that an adequate foundation for an engineering career cannot be obtained in the four-year course. They advocate the plan of keeping the engineering students within the college walls for an additional year or two. If this contention is well founded, but if, for one reason or another, it is not deemed best to require engineering students to remain on the campus for a fifth year, it seems logical to consider the extent to which it may be feasible to carry the college to the young engineer in practise.

Second, it is a commonplace that most engineers (and this applies equally to all of the professions including the teaching) are unable to keep abreast of the rapid scientific developments in their field, and as a result are not in a position to do clean-cut effective work. There would seem to be a distinct opportunity for the colleges to be of service to engineers whose training along scientific lines may have been quite adequate at the time they entered practise but who have come to feel the need of a fundamental re-examination of certain fields.

In view of these considerations and others to be presented later, it would seem that the engineering colleges should, by trial, determine the feasibility and desirability of conducting in nearby engineering centers *seminars* or *conferences* of one or more of the following types for practising engineers.

Type A. Advanced studies of the kind given in residence in postgraduate work; studies in mathematics, physics, engineering subjects, etc.

Type B. Seminars dealing with recent developments, designed to enable the older graduates to keep abreast of the scientific advances.

Type C. Seminars for the discussion, in the light of fundamental theory, of

a. Allied or common research problems
b. Allied or common design problems
c. Allied or common operating or manufacturing problems.

A brief sketch of two seminars for practising engineers sponsored by the University of Wisconsin may be of interest.

Two years ago a group of technical graduates of ten or more years' experience, and all in responsible positions in the metallurgical industries, requested the opportunity to work for an advanced degree and to carry on this work in Milwaukee, 85 miles east of Madison, the seat of the University. Since an experimental research in some technical subject would be a necessary part of the proposed graduate work, the proposal was welcomed because it held promise of opening the way to a very desirable type of cooperative research between the engineering industries and the college. I refer to the type of research in which problems are not brought to the college laboratory to be solved, but in which the facilities and views of the college are carried to the industrial laboratory or the engineering office, and in which the primary function of the college in research—namely, the training of men in engineering research—is emphasized and is given wider scope. The positions held by the eight men in this group are as follows:

Works Manager of the Milwaukee Steel Foundry Company,
Vice-president of the Badger Malleable Company,
Works manager of the Globe Electric Company,
General superintendent of the Federal Malleable Company,
Metallurgist of the Federal Malleable Company,
Metallurgist of the Glancy Malleable Corporation,
Metallurgist of the Vilter Manufacturing Company.

The *significant* thing about the list is that these men are in competing industries and are cooperating in a highly effective manner in the solution of their common problems.

During the past two school years this group has met in Milwaukee each Friday evening for a conference extending from 7:30 to 10:30, or later, with Professor R. S. McCaffery or other members of the Mining

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[1] Professor of Electrical Engineering, University of Wisconsin, Madison, Wis.

Presented at the A. I. E. E. Regional Meeting of District No. 5, Madison, Wis., May 6-7, 1926.

or Metallurgical Department.[2] On Saturday, the professor visits one or possibly two of the members of the seminar at his plant or laboratory and discusses the research project which is under way. The research projects, which were selected by the men themselves, are as follows: A study of the reactions which take place in a basic steel furnace; an electrical method of rapidly determining the quality of the molten metal in malleable air-furnaces; the determination and comparison of the heat balances of hand-fired, oil-fired and pulverized-coal-fired malleable furnaces; the effect of silicon and manganese on the properties of malleable iron; and the study of the reactions of combustion in a malleable furnace for the purpose of obtaining greater accuracy of control of the finished product.

In the conduct of a seminar for the discussion, in a fundamental way, of allied questions, the first and most difficult problem which presents itself is that of getting the members of the group to talk the same technical language. This requires a review of the experiments, concepts, postulates, definitions, sequences and principles which underlie the branch of science with which the group is concerned. Accordingly, the Friday evening conferences during the first five months were devoted mainly to a review of the fundamentals of physical chemistry applied to metallurgy. This involved a study of equilibriums in chemical and metallurgical reactions and of equilibrium diagrams, the application of the phase rule, mass law, atomic structure and the velocity of reactions.

The impression made by this work is indicated first, by the fact that some of the concerns listed above have established fellowships at the university in order that the holder of the fellowship might collaborate with the member of the seminar by carrying on supplementary researches at the university, and second, by the fact that some 26 grey iron foundaries operating in the Fox River Valley have formed a local association to carry on and finance cooperative research at their own plants and in Madison; a similar group of about 20 grey iron foundries in Milwaukee is organizing to carry out a program of the same kind.

The second seminar was organized at the solicitation of electrical engineers in Milwaukee. The men in this group, which started with 18 men and ended with 13, are electrical engineering graduates with from two to six years of experience. During the past year this group met in Milwaukee each Thursday evening with a professor from the university for a seminar of the A type,—a seminar for the discussion of *transient phenomena and waves in electric circuits*. This course was largely a mathematical development of circuit theory, with, of course, constant illustrations of the application of the theory to transformers, generators and power and communication systems. As a result of the experience of the men with this seminar, courses of a similar nature have been solicited and will be given during the coming year. At least one industrial organization has under consideration plans for meeting all or part of the tuition expense of its employees who successfully complete courses of this kind.

THE SIGNIFICANCE OF THE SEMINARS TO INDUSTRY

It may be well to list the significant possibilities to industry of seminars conducted by the engineering colleges for men engaged in engineering practise. They are as follows:

Such seminars, if available, will enable some graduates of four-year engineering courses to obtain a more thorough scientific training for engineering work.

Seminars of the B type might be used to serve as the line of communication between the advance scientific workers and the main body of engineers, or to enable engineers who, in the press of work, have lost touch with certain allied fields, to "come back" in these fields.

During the apprenticeship period of the first few years after graduation, many engineering graduates slip in their grasp of mathematical methods and analysis. A course of the A type, if pursued during this period, should be of value in more firmly fixing these methods and in warding off the mental slump which frequently occurs during a depressing apprenticeship period.

That both industry and engineering education have much to gain by closer cooperation in engineering work, particularly of the research type, needs no argument. The kind of cooperation which has received more attention in the past is that in which certain research problems have been taken to the college laboratory for solution. By seminars of the C type, the views and the methods of the college are carried to the industrial laboratory, and the primary function of the college in research—namely, the training of its own staff and of its students in research—is made more apparent and is given wider scope. More research for engineering teachers both in the college and in industry is badly needed.

Engineering seminars sponsored by associations formed by the smaller industrial enterprises for the fundamental examination of their common technical problems may be a means by which the smaller enterprises with their desirable social characteristics may retain a place in the sun by the side of the modern corporation.

One of the barriers in the way of more effective cooperation between the industries and the state colleges results from a difference in the social philosophies in the two fields of action, namely, in the industrial field and in the professional field in general, or the teaching field in particular. In the educational and scientific field, the rewards and increased opportunities come from the sharing and free disclosure of all achievements which result in the advancement of knowledge. In the industrial field, the careful guarding of trade secrets is still regarded in many industries as highly essential, and the obtaining of exclusive rights through patent control is frequently the main consideration which leads to the support of research work. These conflicting demands—for the unreserved disclosure and for the exclusive use of knowledge—are the greatest barrier in the development of cooperative relations in those fields in which important improvements and inventions are likely to be made.

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[2] For a more detailed statement of this work, see the paper by Professor R. S. McCaffery entitled "Research Cooperation between University and Industry," Canadian Inst. of Min. & Met., Vol. XXIX, 1926.

The most promising sphere for the organization of seminars for the consideration of allied or common engineering problems is with associations of manufactures or of public utilities. In this case it would seem quite possible to formulate a policy with reference to the granting of preferential but not exclusive patent rights which would harmonize the views and rights of all parties in the enterprise. The essential features of such a policy are outlined in Sec. VIII and IX of Circular No. 9 of the Engineering Experiment Station of the University of Illinois, entitled "The Functions of the Engineering Experiment Station of the University of Illinois," by C. R. Richards.

Not the least among the possibilities for good of these seminars is the possibility that the advantages of cooperation may be shown to be so great that it may become obvious that the thing to do is to modify those features of the patent law which make the existing law a barrier to cooperation.

The type of seminar which holds the greatest promise of achievement is a seminar of experienced engineers from the same or allied fields who have come together to conduct a critical analysis of certain cases or certain lines of engineering practise. It may seem like presumption to suppose that men from the colleges can be of much service to such a group. The presumption is tempered somewhat by the consideration that the introduction of a foreign body into saturated solutions frequently initiates crystalization. In a group of experienced engineers, the man from the college may at least play a role not unlike that of the foreign body in the chemical solution. The achievements should come mainly from the contributions of the practitioners to the conference. Judgment should not be passed as to the relative contributions to be expected, however, until we have considered the significance of these seminars to the engineering teachers.

SIGNIFICANCE OF THE SEMINARS TO ENGINEERING EDUCATION

The fourth responsibility of the engineering college is for the development of engineering teachers. One of the strongest arguments for sponsoring seminars for practising engineers is the part which these seminars will play in the development of engineering teachers.

The seminars will compel and will reward a broader and a more thorough training than is common in the teaching ranks today.

They will afford greater opportunities to determine the adequacy, the relative importance, and the real significance of the principles and the methods taught to undergraduates.

They will make teaching attractive to a wider range of engineers by making it possible for men in teaching to have closer contact with engineering practise.

They will help to supply one of the greatest needs of the engineering colleges of the country; namely, the atmosphere, the spirit and the prestige which will accompany more examples of engineering work in progress or carried to a successful conclusion in the colleges or by the engineering teachers.

The opportunity to develop these seminars comes to the teaching profession as a challenge; a challenge because the task is no light one but one beset with difficulties and even with an element of danger to undergraduate instruction; but above all, a challenge because the acceptance of the opportunity means the acceptance of a goad and the acquirement of an incentive which will bring the work of the engineer and of the engineering teacher to a higher plane.

Discussion

COOPERATION BETWEEN THE COLLEGES AND THE INDUSTRIES IN RESEARCH

(WICKENDEN, BAILEY, BENNETT)

MADISON, WISCONSIN, MAY, 7, 1926

E. B. Paine: The Engineering Experiment Station of the University of Illinois was established December 8, 1903. The object of this experiment station was stated to be the encouragement of training in engineering and the study of problems of special importance to professional engineers and to engineering industries.

The control of the Engineering Experiment Station is vested in the executive staff, composed of the director, the heads of the nine departments in the College of Engineering, and the professor of Industrial Chemistry. The research work of the Station is conducted by a corps consisting of 32 full-time investigators, 14 research graduate assistants who devote one-half of their time to research and the other half to graduate study, and 50 members of the teaching staff of the college who devote part of their time to research investigations. For the year 1925-26, approximately $95,000 has been appropriated for this work from state funds and over $60,000 has been contributed from outside sources for carrying on cooperative investigations.

The present list of research investigations includes 88 titles distributed among the engineering departments as follows:—Ceramic Engineering, 10; Civil Engineering, 11; Electrical Engineering, 8; Mechanical Engineering, 14; Mining Engineering, 4; Theoretical and Applied Mechanics, 13; Physics, 8; Railway Engineering, 6; Industrial Chemistry, 14. Some of these investigations are carried on in part by funds from the state and in part by funds from industrial organizations. Work is now in progress on 25 cooperative investigations.

The Engineering Experiment Station, supported chiefly by public funds, cannot be employed in the exploitation of inventions or processes or in the conduct of scientific work the results of which are to be held from the public. Cooperative research is undertaken only in those instances where the chief purpose is to establish fundamental principles and physical laws which have a wide practical application.

The standard agreement for a cooperative investigation at the University of Illinois provides that the University contribute the use of its facilities. It will also assume the general administration of the investigation and publish in bulletins or circulars of the Engineering Experiment Station the results of the investigation. The cooperating agency furnishes the funds which are necessary to pay the salaries of special investigators, to purchase materials and special apparatus needed for the work, and other necessary expenses of the investigation. The university retains the ownership of all data secured. The research program is outlined by an advisory committee representing the cooperative agency and the executive staff of the Engineering Experiment Station.

The Engineering Experiment Station will not undertake commercial tests except under unusual circumstances. In no case will it undertake such a test if the results are to be used for advertising purposes.

Whenever discoveries and inventions result from research investigations, whether conducted with funds from the state or with funds furnished by a cooperating agency, the member of the University staff who made the discovery may be required to obtain a patent at university expense and assign the patent to the University.

John Mills: Professor Bennett's well ordered paper is both diagnostic and specific of a functional disorder in our educational system. Financial considerations and a natural impatience to get to work will probably always act to limit the number of students who can pursue continuously more than the usual four years of an engineering course. Financial progress and increased scales of personal expenses will also act to prevent any large number of practising engineers from returning to resident graduate study. Since, in general, the student will neither remain nor return, the university must go to him; and Professor Bennett's suggested method seems worthy of wide trial and very definite support.

It presupposes on the part of the practising engineers an interest in further study which is found as a rule only in those engaged in investigative work as distinct from work in commercial relations with the public or in the industrial management of routine operations. It is immediately evident that it is not for those who brag that they have never had occasion in their engineering careers to consult the theoretical books or to use the calculus tables of their student days.

Of the three types of seminar which are proposed, the first two deserve the emphasis in the judgment of the present writer who believes that a combination of the two would be most satisfactory. In such a combination the consideration of recent advances would accompany and serve to motivate reviews and extensions of theoretical considerations.

This is the type of post-graduate study in industry with which the writer is most familiar. Within Bell Telephone Laboratories there was evolved a scheme for such study which for some years has been organized by George B. Thomas, its Personnel Director. There the group of engineers and scientists is large enough to carry on such work as a self-contained group without assistance from educational institutions. Courses of graduate grade of difficulty, highly analytical in character, but illustrated by current problems and recent developments, are offered to the members of its technical staff. These are given by other members of the staff who are experts in the particular subjects. Each year there are several hundred registrations for these courses. Both instruction and attendance and study are on the time of the individual and are entirely optional. The preparation of the text material, however, is carried out as part of the regular company duties as are all the other matters with the exception of instruction and attendance. Incidentally there are neither tuition fees nor special payments to instructors, for the immediate rewards of instruction are non-financial although ultimately there may be financial return.

Some of the text material has attracted interest in academic circles and one of the texts, namely on telephonic transmission, was published some time ago. Another on the subject of sound is at present on the press, and the text material of certain other courses, in mimeograph form, is being revised for publication. Such a cooperative scheme of advances in education, however, is probably unique and will always be limited to large groups, homogeneous in training and interests. To provide for similar groups representing members of different corporations some scheme like Professor Bennett's must ultimately be adopted. That advanced education of the character described is valuable both to the individual and to industry has been proved in the eight years' experience with the so-called "Out-of-Hour Courses" in Bell Telephone Laboratories. That it should be available more widely and could be accomplished under the proposed plans appears to the writer of these remarks to be so self-evident that Professor Bennett's program should meet with immediate acceptance and trial in several localities.

S. H. Mortensen: Speaking as a practising engineer, I feel that seminars conducted along the lines suggested in Professor Bennett's paper would be of great value to the experienced engineer as well as to the young graduates.

Practising engineers frequently encounter problems the successful solution of which could be quickly facilitated by means of a discussion with university professors and research workers as well as with engineers active in the same or allied fields.

Certain of these problems would be suitable for investigation in a research department of the universities and others for factory or field tests. Cooperation in this work between the universities and the industries would lead to improved measures and a broader interpretation of the results obtained.

In arranging a program for practising engineers it might be well to bear in mind that they are called away frequently on business trips. For this reason it would be desirable if each meeting would take care of one subject in such a manner that it would be more or less independent of the preceding and the following meetings, thereby making continuity of attendance less imperative.

There is little doubt in my mind that the proposed seminars will benefit universities and industries alike. The university research department and teaching staff will be in touch with the problems of the day and the practising engineers will gain added experience and broadened vision.

J. S. Coldwell: Professor Bennett's paper appeals to me very much and I believe if any university would plan several such seminars and properly approach practising engineers and industrial concerns, they would be well attended.

Cooperation between the universities and industry would be mutually beneficial. The industrial organization would keep pace with the advance of science and would have available a very high type of research organization. On the other hand, the arrangement would broaden the university professors, keep them in touch with industrial conditions, make their teaching more effective and probably serve as a means for increased remuneration. Of the two I should say that the universities and the professors would profit the most and it is significant to note that the five papers presented were given either by a commissioner of education or by college deans. I get the impression of an effort to sell the research ability of the university to industry.

In the first place, this cooperation can be achieved only when both the university and industry have the same point of view. At present the universities' point of view is generally the advancement of science whereas industry's point of view is profit.

In order to serve industry properly and to be sought for service by industry, it will be necessary for the universities to get the profit point of view and sell their services to the industry as anything else is sold to industry. Any time the university can show industry anything which will lead to a profit in dollars and cents, it will be taken up immediately by industry.

At present it is doubtful that industry can refer immediate and concrete problems to the university for research on account of the need for familiarity with the many commercial, manufacturing, service, and other factors, also the need for quick action and furthermore on account of patent complications and competitive situations.

It is significant that in the ten years I have been with the Cutler Hammer Mfg. Company, I have seen only two university professors who visited our department. The universities and the professors should interest themselves in industry, call upon industry with much the same general point of view as a salesman, get acquainted with industrial problems and bring themselves to the attention of industry. If I were a university professor, I should visit various industrial plants, making myself acquainted with their problems and their personnel, at the same time letting them know of my familiarity with certain particular problems which they are working upon and that in a very short time the industry would be naturally turning to me as a source of information on their particular problems.

The university professors would be very welcome at our plant or other plants, and we would be glad to discuss many interesting problems with them, even pointing out the road for many important investigations of a general nature which we know have a commercial demand but which we either cannot afford to work upon or which do not tie in with our particular problems occupying most of our time.

Furthermore, if the universities want to serve industry, and at the same time maintain their organization and personnel, they will have to cut loose from traditional restrictions and make it possible for their professors to sell their services and their patents at such a price that their income would be sufficient to keep them with the university.

Some years ago when I was with a concern in the East, we had an instructor from the Cornell Metallurgical Laboratory working as a draftsman during the summer. While he was there, a rather difficult heat-treating problem came up and in the course of time it occurred to some one that he might be able to help us. The minute we put the problem up to him he gave us a general answer, and after very little checking he gave a specific answer. Immediately all the other heat-treating problems which arose were referred to him, and he continued to take care of their metallurgical problems after he went back to Cornell.

F. E. Turneaure: The question of research in the engineering colleges has been of much interest to me for many years,—in fact, ever since I began teaching at Washington University, St. Louis, where the late Dean J. B. Johnson was operating a materials-testing laboratory. In the early days of college research the subject of probably the most general interest was that of strength of materials. It is a research subject comparatively easy to develop in a college laboratory, and many of the engineering constants and empirical formulas used in civil-engineering design have been determined in the college research laboratory. Later on, many other lines of research have been developed, as have been explained before this meeting, and concerning which most of you are well informed. Along with this development of college research work has come a much closer cooperation between engineers and teachers. I suppose this is because the practitioner is becoming more scientific and the college professor more practical. This is well illustrated in the organization of the research committees of our national engineering societies, made up as they generally are of a combination of engineers and professors. The college professor is relied upon generally to conduct the research, and the practitioner offers practical suggestions relative to his experience. It is interesting to note how harmoniously they now cooperate, and at their meetings it is often difficult to distinguish between the professor and the practitioner. It is a good illustration of the growing scientific basis of engineering practise; and in electrical engineering this condition has, of course, obtained for many years.

Considering the trend of things, it would seem certain that there will be a considerable further extension of cooperative research between engineers and colleges or between industries and colleges. I believe that such cooperation is one of the very best aids in the development of the right kind of engineering instruction. Perhaps the most valuable feature is the effect that it has upon the teachers themselves, and through them, upon the students.

C. F. Harding: President Frank of the University of Wisconsin has emphasized the importance and necessity of making practical the pure research which has been developed during the last ten years. It seems to me the principal object of this discussion is the converse of that proposition; namely, the development of research in the colleges and universities resulting from practical problems. Many of these have been listed in the different institutions of the country.

At Purdue University we have been particularly interested in high-voltage developments. The great educational value of our high-voltage laboratory and the research carried on therein has resulted very largely from what may be considered commercial tests undertaken primarily as tests of a practical engineering nature of particular interest to the public utilities of the State. Like the tests at Illinois, as outlined by Professor Paine, no commercial tests are undertaken unless they have some development or research value, but, for example, if competitive high-voltage insulator tests are made, in practically every case a long series of researches has resulted which has been valuable, I believe, to the profession as a whole. The same thing is true of corona tests between wires, started in the first place as a purely commercial investigation and developed in a number of years into a fundamental research in connection with corona losses. A number of other similar cases might be listed as illustrations.

In addition to this, the value of senior and graduate theses and investigations carried on partly as a result of cooperation on the part of utilities and manufacturers has enhanced very greatly the educational work of the institution.

Commenting upon Professor Bennett's excellent suggestion, it seems to me that the conferences which have been held at different technical universities are bordering very closely upon the *B* and *C* types of seminars which he outlined. Most of you are familiar with the conferences for electric meter men that are held at the different educational institutions. Following that precedent, conferences for superintendents of distribution, particularly in the State of Indiana, and quite recently the innovation of industrial electric heating conferences, one of which was just concluded last month, have in all cases developed a fine cooperation between the university, manufacturers and public utilities of the state and have resulted in continued research and development problems of value.

So it seems to me that the work outlined in these papers is perhaps on a par with the application of pure research to industry as outlined by President Frank.

Edward Bennett: In his discussion, Mr. Coldwell makes one statement which calls for comment; namely, the statement that "if the universities want to serve industry, and at the same time maintain their organization and personnel, they will have to cut loose from traditional restrictions and make it possible for their professors to sell their services and their patents at such a price that their income would be sufficient to keep them with the university."

In other words, university professors, whose primary loyalty should be to the ideals and objectives of the educational world, are to find it feasible to remain in educational work because of the compensation received from industry for services rendered directly to industry!

When the proposal is stated in these terms it should require but slight reflection to see that in the conduct of its own affairs, industry does not subscribe to such a doctrine of divided loyalty. The old statement still holds that "No man can serve two masters: for either he will hate the one, and love the other; or else he will hold to the one and despise the other."

I cannot emphasize too strongly my view that a policy of meeting the situation or the competition pictured in Mr. Coldwell's statement simply by permitting and encouraging educators (engineering, medical, or otherwise) to supplement an inadequate salary by private practise is a short-sighted policy for which society is paying dear. The results obtained under such a program, as contrasted with those obtained under a program displaying educational statesmanship, are well presented in an article entitled "The Extension of the Full-Time Plan of Teaching to Clinical Medicine", appearing in *Science* for Aug. 11, 1922.

As stated in my paper, it seems to many educators that the essential features of a policy under which a state university and an industrial enterprise can effectively cooperate in industrial research are set forth in the circular of the Engineering Experiment Station of the University of Illinois, entitled "The Functions of the Engineering Experiment Station of the University of Illinois." In his discussion, Professor Paine has outlined this policy.

There is, however, an element of truth in Mr. Coldwell's statement, and it seems to me to lie in his definite recognition of the fact that no engineering college can hope to enter upon the worth-while and greatly desired program of cooperative research with industry so long as it pays inadequate salaries. One of the essential first steps in the advancement of such a program is the adoption of an adequate scale of salaries.

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Symposium on Dielectrics and Power-Factor Measurements*

I—Phase Difference in Dielectrics

BY J. B. WHITEHEAD[1]
Fellow, A. I. E. E.

Synopsis.—*A brief description of the origins and causes of phase difference in dielectrics.*

HIGH power factor is earnestly sought after in transmission, distribution, and all station loading. Low power factor is just as earnestly sought in the case of insulation, for the higher its value, the greater the internal loss, the higher the temperature, the lower the current capacity, the shorter the life. The term "power factor" in this case describes a property of the material, and as such is of quite different character and significance from its older connotation. It is no doubt largely for this reason that the custom has arisen of describing this property of dielectrics, not as the cosine of the angle of advance of the charging current over the applied voltage, but as the sine of the difference between that angle and 90 electrical deg. This angular difference is known as the "phase difference" or "phase defect" and its usage has the added advantage that up to about 2 deg., the phase difference, itself, in radians, its sine, its tangent, and obviously the power factor in its usual sense, all have the same value within a very small fraction of a per cent. This makes it possible to use these several quantities indiscriminately, thereby greatly simplifying many computations. The use of the tangent of the phase difference is especially convenient, as the ratio of the in-phase to the wattless components of the current.

The importance of phase difference in dielectrics was first appreciated in its influence on the performance of telegraph and telephone cables. Attenuation, distortion, and internal loss are all increased thereby, thus greatly restricting distances of communication. In the field of power transmission and utilization, attention was first focussed on dielectric-phase difference by the observations of Siemens in 1864 on the heating of condensers. This is an early date in the history of electrical engineering, and for many years thereafter the losses in dielectrics under alternating stress received the attention of physicists rather than that of engineers. It was in this early period also that residual charge, discovered in the Leyden jar in 1746, was still further stimulating the interest of physicists in dielectric phenomena. This interest showed its first fruits in the brilliant experiments of Hopkinson and in the unassailable theory of Maxwell of the phenomenon of dielectric absorption. It is not generally realized that since Maxwell's time it has been recognized by engineer-physicists, such as Rowland, Hess and a few others, that dielectric absorption necessarily causes alternating-dielectric loss; *i. e.*, dielectric-phase difference. Engineers became immediately concerned in the value of dielectric-phase difference with the first upward step in transmission voltages in the early '90s, and with the use of cables for transmission. The problem of the limitation of dielectric-phase difference thereafter was clear cut and has been with us ever since. Obviously the applications in which it assumes its greatest importance are the high-voltage cable and the commercial power

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*The following eight papers constitute a symposium arranged by and presented under the auspices of the Committee on Instruments and Measurements.

[1] School of Engineering, Johns Hopkins University, Baltimore, Md.

Presented at the Regional Meeting of District No. 1, of the A. I. E. E., Niagara Falls, N. Y., May 26-27, 1926.

condenser, but its presence and behavior must always be borne in mind in connection with all high-voltage insulation, especially that of composite or flexible character.

Although, as stated above, phase difference is a property of the material of the dielectric, and although this has been recognized for many years, and although an abundant literature is replete with experimental observations, it is astonishing to find that our knowledge even of the values to be assigned to particular materials is extremely indefinite, and that little has been accomplished towards a systematic understanding and regulation of the factors which control the values of phase difference. This is largely due to the fact that there are at least four and possibly more different classes of phenomena in dielectrics, any one of which will cause an angle of phase difference and to the further fact that probably all these causes follow different laws under the influence of varying temperature, frequency, and electric intensity. Each of the following well-recognized properties or conditions of insulation, if present, will cause a dielectric-phase difference: (1) normal conductivity, (2) dielectric absorption, (3) anomalous conductivity, (4) absorbed moisture, (5) dielectric hysteresis and (6) gaseous ionization.

Normal Conductivity. Under the original theory of Faraday and Maxwell, still regarded by physicists as fundamental in spite of obvious insufficiency, dielectrics are treated as being perfect of their kind, *i. e.*, as possessing specific inductive capacity only. Maxwell, however, recognized that no such perfect solid dielectric exists, and he treated at considerable length the properties of dielectrics which also possess conductivity. We have then in conductivity one of the fundamental causes of dielectric-phase difference. It is usually assumed that conductivity contributes a negligible proportion of the losses in dielectrics. This is undoubtedly true for most pure and simple materials, especially at ordinary temperatures. It should be noted, however, that the values of conductivity of different dielectrics extend over a wide range and in many cases may contribute a considerable phase difference. The conductivity of most dielectrics increases rapidly with increasing temperature. The rapidly rising phase difference of such a composite material as impregnated paper, for example, under increasing temperature, may be accounted for in very large measure in some instances by the increase in conductivity alone. The important influence of water on conductivity is described below.

Dielectric Absorption. Probably the most important of all causes of dielectric-phase difference is dielectric absorption, *i. e.*, the phenomenon of after-charge and residual charge. It is obvious that if under applied continuous voltage, current continues to flow over a period of time into or through a dielectric, then under alternating voltage there will flow a component of current in phase with the voltage, which, of course, means a definite angle of phase difference. Absorbent dielectrics, in effect, have for a short interval after the application of voltage a greatly increased value of apparent conductivity. On short circuit a charge continues to flow out long after the electrostatic charge has disappeared, and on reversal of the applied voltage the high initial conductivity appears again. Thus, for the rapid reversals of alternating voltage, the dielectric behaves as though this increased conductivity were continuously present.

Absorption usually occurs whenever a dielectric is composed of two or more different materials. Further, it appears that a very small proportion of a foreign material may cause a large absorption effect. Thus, the absorption often observed in many supposedly pure, simple materials is usually attributed to impurities. This is one of the obscure questions in our imperfect knowledge of dielectric absorption. Although it has been known for years that absorption is one of the most important causes of phase difference, very little attention has been given to the problem of controlling its value, and to the study of the behavior of various dielectric materials singly and in combination. That such study would prove profitable is indicated by the fact that a few composite dielectrics have very low values of phase difference and of absorption. These materials apparently are few in number, and their properties obtained only by cut-and-try and by great care in preparation. However, there is no reason why a further study and control of composite materials should not lead to dielectrics having not only low phase difference, but also other desirable thermal and mechanical properties.

Moisture. Many insulating materials, particularly those of porous and fibrous character, absorb moisture readily from the air. Their conductivity is very often greatly increased thereby. In many cases the moisture is taken up quite rapidly and is completely driven off again only with considerable difficulty. As a consequence, exact statements as to the influence of moisture on conductivity and power factor are not possible. Several definite qualitative relations stand out, however, and the behavior of cable paper in this regard may be taken as generally characteristic. Cable paper when standing in the open will absorb from 10 to 15 per cent of weight of moisture. If continuous voltage is applied to the paper in this state, the current, which in this case is pure conduction current, will increase slowly with time, tending to a steady value. If the voltage is increased, the final current will increase roughly as the square root of the voltage. A theory for this behavior has been proposed by Evershed and is based on the assumption that the variation in conductivity is traced to changes in the water films around air bubbles in the capillary fibers of the material. As the paper is heated above ordinary temperatures, the conductivity increases and the general behavior outlined above continues up to about 50 deg. or 60 deg. cent. Above this point traces of the normal residual charge and discharge of dielectrics begin to appear, although the conductivity is still very high. If allowed to stand at about 70 deg. for some hours, large quantities of the moisture are driven off, the Evershed effect has disappeared, and the paper begins to show the qualities of a good dielectric and insulator, having, however, pronounced absorption and conductivity. With further increase in temperature both absorption and conductivity are decreased further, and on standing for long periods at temperatures from 95 deg. to 115 deg. cent. there is little change, and the paper appears to be in more or less steady condition, still showing pronounced absorption but with relatively low value of final conductivity.

Thus, it will be readily seen that the influence of moisture on the phase difference on insulating materials is very complex. The variation with both voltage and time is often to be found in the alternating case. Increase in the amount of absorbed moisture shows itself almost immediately in increased values of phase difference and loss. As a consequence every effort is made in the manufacture of commercial insulation to exclude moisture as completely as possible and to prevent its subsequent absorption. It is safe to say, however, that in few cases is it possible to completely eliminate moisture and that observed values of phase difference and loss are always in some measure increased by residual moisture.

Anomalous Conduction. Nearly all liquid dielectrics show some conductivity. Moreover, this conductivity generally varies with both time and voltage. On the application of continuous voltage the resulting current decreases, approaching a constant value. With increasing voltage the final constant values generally show a departure from Ohm's law. Many attempts have been made to coordinate the results of investigation of this property but without great success. This conductivity is undoubtedly ionic in character and in some cases follows closely the known laws of the ionic conductivity of gases. Values have been obtained for the mobilities of both positive and negative ions for particular materials, but in general the results of such investigations are far from definite in character and this is attributed to the presence of ions or molecular aggregates of different size. Phenomena of this character seem to be particularly susceptible to the presence of impurities in small amounts, to traces of water, etc.

Electrolytic dissociation and resulting conductivity is known to exist in some complex insulating materials. Glass is a remarkable example of this. The metallic constituents of some glasses may be separated out under continuous voltage and deposited on electrodes. Here is an instance of a rigid insulating material through which it is known beyond a doubt that electrolytic ions pass from one electrode to another.

As regards the influence of these two types of conductivity on the phase difference of commercial insulation, it may be said that very little is definitely known. There would appear to be good possibility of the presence of the former type, and it is probably true that the residual conductivities of composite dielectrics made up originally with liquid binders, is in considerable proportion due to the motion of ions of the type usually to be found in liquids.

Hysteresis. Since the days of Siemens, who first noted the heating of impregnated paper under alternating stress, it has been customary to attribute the losses in dielectrics to some form of molecular friction, apparently arising in the same types of cause pertaining to the case of magnetic hysteresis. There are many differences between the two phenomena which indicate that they are of essentially different character, and it is undesirable, therefore, to use the word hysteresis in connection with dielectric behavior. In addition to the differences mentioned it is to be noted that in the case of dielectrics there are many other factors, as enumerated above, that appear to be quite sufficient to account for all of the losses which are observed. It is, of course, possible that in addition to these well-recognized causes for loss there may be some residual type of loss arising in the orientation or deformation of molecules and atoms. In fact, there have been numerous suggestions that the phenomenon of absorption itself arises in frictional deformation or the motion of electrons within the atom itself. With due weight given to these considerations it may still be said that the evidence that the nature of the losses in dielectrics is of the character usually understood by the word hysteresis is so small as to make it appear very unlikely.

Gaseous Ionization. Many forms of commercial insulation are composite in character and built up in layers. Conspicuous examples are the insulation of all electromagnetic machinery and of many types of high-voltage underground cable. The assembly and application of this type of insulation invariably provides opportunity for the enclosure or entrapping of certain amounts of air, which is never completely removed by such processes as evacuation and subsequent pressure impregnation. This air, when voltage is applied to the insulation, breaks down electrically under the process known as ionization. It is also known that in some cases the constituents of impregnating materials break down under stress, with the generation of gases and the further increase in the size of the voids in the body of the insulation. These gases are also subject to ionization. The products of this gaseous ionization are usually highly active oxygen and ozone. Not only is the ionized gas a good conductor but the products of the ionization attack the surrounding material rendering it conducting, thus further increasing the conductivity.

It will be seen, therefore, that this type of conductivity is not necessarily inherent in the material itself, but is a result of imperfect methods of assembling and applying the material. However, it is probable that even at very low stresses this type of conductivity is present to some extent in all insulations in the class mentioned. Standard specifications for cables, for example, include clauses giving tests for increase in power factor due to this cause, and limiting values for such increase, thereby recognizing its necessary presence. Power-factor curves of this type of insulation only show marked increase above certain values of stress, and such increase is in all probability rightly attributed to this type of ionization.

It can not be emphasized too strongly, however, that internal ionization is highly destructive to the structure of composite insulation, and that if such insulation is operated above a stress at which ionization is known to occur, the days or months of the life of this insulation are already numbered.

Bearing in mind these various causes giving rise to a phase difference in dielectrics, it is not to be wondered at that there is much conflict of evidence as to the behavior under different conditions of insulation of any particular type. Of the several causes enumerated, the laws of only two of them may be said to be well known, namely, conduction and absorption. Inherent conductivity of the type found in metals leads to a value of losses readily calculable. Dielectric absorption has been carefully studied and it is known that in insulation of comparatively simple structure absorption accounts for most of the loss. In the uncertain laws controlling the losses of other character there is plenty of explanation of the erratic behavior often reported. It may be pointed out, however, that these uncertain phenomena are usually of a type which seems to yield to care and control in preparation of materials and methods of assembly. Moisture may be largely driven out, anomalous conductivity reduced to extremely small proportions, and it is possible to reduce gaseous ionization to negligible proportions. With care of this character, therefore, it is legitimate to consider the behavior of dielectrics from the standpoint of conductivity and absorption alone. It has been shown that the extension of Maxwell's theory of absorption to the alternating case will account for the outstanding features of the behavior of composite insulation. For example, the loss is in proportion to the square of the voltage, and to the frequency. Moreover, it is shown that the variation of power factor with the frequency shows a maximum value, which may lie at a low or very high value of frequency, depending upon the constituents of the material. Further, and due to the same causes, variations of temperature within the usual range may cause either sharp increase or sharp decrease of power factor. The fact that variations of this character are in accord with the simple theory of absorption as the principal source of loss is extremely encouraging and a great step in advance.

In closing this brief discussion of the origin of phase difference in dielectrics, it may be pointed out that a moderate value of phase difference or power factor appears to be practically a necessary feature for composite insulation. On the other hand, there is no objection to a phase difference of moderate amount provided that it remains constant. If the phase difference of a given dielectric increases with the electric stress its cause should be carefully investigated and the insulation always operated below the stress at which such increase in phase difference begins. The increase due to temperature is more difficult to control, but there is reason to suppose that it has a definite value in any particular case, and that it is therefore subject to control. If these considerations are correct, there is good reason to hope that a well considered program of experimental research will result in the placing of the design of insulation on an engineering basis comparable to that pertaining to the other elements of circuits and machinery. The essentials of such a program would be principally the checking of the laws which now appear most probable, and the study of the properties of insulating materials, singly and in combination. The preparation and coordination of a program to meet this need is already under way in the work of the Committee of Insulation of the Division of Engineering and Industrial Research of the National Research Council.

Discussion

For discussion of this paper see p. 658.

II—Standards for Measuring the Power Factor of Dielectrics at High Voltage and Low Frequency[1]

BY HARVEY L. CURTIS[2]
Fellow, A. I. E. E.

***Synopsis.**—This paper points out the need which exists in the electrical industry, particularly in connection with the testing of high-voltage cables, for convenient standards for use in the measurement of dielectric loss. At present most laboratories make use of air condensers. In the paper these are classified and certain sources of error which must be guarded against are mentioned. Condensers with solid dielectrics would be much more convenient, more portable, and cheaper, but so far none have been produced which have satisfactory constancy for use as standards at high voltage.*

In any kind of measurement the need of standards is not apparent until the importance of this kind of measurement becomes evident. For example, standards of resistance were not needed so long as men were interested only in electrostatics. It was only when current electricity became of importance that resistance standards were developed. Likewise, the recent realization by industry of the importance of measuring the power factor of dielectrics has raised the question as to whether or not suitable standards can be prepared and maintained.

At the present time practically all laboratories which make measurements of the power factor at high voltages use an air or gas condenser as a standard. It is assumed that there is a negligible loss in such a condenser. Descriptions of a number of these condensers have appeared in print.[3] Also a number of concerns, in response to a circular letter, have furnished the author with descriptions of the condensers which they are using.[4] Space does not permit a detailed description of all of these condensers. However, it is possible to classify them under certain general headings and thus indicate the important features of the condensers which are now being used.

In all of these condensers, great care has been exercised to so design the condensers that no corona will occur at the highest voltages at which they are to be used. This requires that the spacing between the plates shall be adequate and that the edges of the plates shall be finished by a curved surface having at all points a large radius of curvature. If a large room is available, these conditions are not difficult to meet.

The condensers may be classified either in regard to the kind of dielectric or in regard to the shape of the electrodes. When classified in regard to the kind of dielectric, the condensers naturally fall into two classes, (1) condensers using air at normal pressure as a dielectric and (2) condensers using a compressed gas as a dielectric. As the dielectric strength of a gas increases approximately in the same ratio as the pressure of the gas, the plates of a compressed gas condenser can be placed relatively close together, decreasing greatly the size required for a given capacitance. With a pressure of only 10 atmospheres, the plate separation for any given voltage need be no greater than with an oil condenser for that voltage. Moreover, there are no data to show that there is any dielectric loss in a compressed gas. The advantages and disadvantages of a compressed gas condenser in comparison with one using air at atmospheric pressure is shown in the following statement furnished by Mr. R. W. Atkinson:

Advantages	Disadvantages
Compactness	Lack of adjustability
Portability	Necessity of maintaining pressure
Cheapness	Inaccessibility of electrodes
Constancy	Difficulty of designing a gas-tight, high-tension bushing
Complete shielding	

Air condensers for high voltage may also be put in two classes as regards the shape of electrodes, *i. e.*, those using flat plates and those using coaxial cylinders. Those using flat plates may again be divided into those having a single high-voltage plate and those having several high-voltage plates. When there is a single high-voltage plate, most laboratories use two low-voltage plates, one on either side of the high-voltage plate. This doubles the capacitance with an increase in the cost of only 50 per cent. Many laboratories use a guard plate to shield each of the low-voltage plates of the condenser.

In the condensers made of coaxial cylinders, two types are used; those having the high voltage on the inner cylinder and those having the high voltage on the outer cylinder. In either case guard rings are generally used at the ends of the low-voltage cylinder. If the low-

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[1] Approved by the Director of the Bureau of Standards of the U. S. Department of Commerce.

[2] Senior Physicist, Bureau of Standards.

[3] Shanklin, *G. E. Review*, Vol. 19, 1916, p. 844. Whitehead & Isshiki, TRANS. A. I. E. E., Vol. 30, 1920, p. 1076. Atkinson, *Electric Jour.*, Vol. 22, 1925, p. 62. Rayner, *Jour. Sc. Instruments*, Vol. 3, 1925, p. 33. Dawes & Hoover, JOUR. A. I. E. E., April, 1926, p. 337.

[4] The following firms have furnished unpublished descriptions:

- American Steel & Wire Company.
- Electrical Testing Laboratories.
- Habirshaw Cable & Wire Corporation.
- Safety Cable Company.
- Simplex Wire & Cable Company.
- Westinghouse Electric & Manufacturing Company.

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voltage cylinder is outside, it is customary to surround the low-voltage plate by a metallic screen which is connected to the guard rings at the end, thus completely shielding the low-voltage plate. If the low-voltage plate is on the inside cylinder, the guards at the end of the cylinder form a complete shield for this plate.

Certain difficulties arise in the use of air condensers as standards for power-factor measurements. There is often some uncertainty concerning the loss in the leads which go to the air condenser as well as those which go to the specimen. In case a substitution method is used, care must be exercised to see that the loss in the leads is the same when the specimen is being measured as when the air condenser is substituted.

If the condenser does not have any guard plates, then it is important that all the lines of electrostatic force passing from one plate to the other shall go through air. Such condensers should be kept well away from the walls of the room, and the solid insulating material used to support the low-voltage plate must be so arranged that the loss in it is negligible.

In condensers using a guard, there is frequently a large capacitance between the guard and the shielded plate. If the guard plate is at every instant kept at the same potential as the shielded plate, then this large capacitance has no deleterious effect. However, in very few of the published methods is any provision made for keeping the voltage of the guard plate the same as that of the shielded plate.

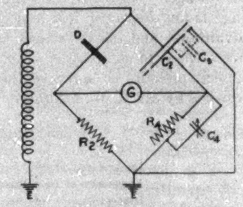

FIG. 1

The effect of a slight difference of potential between the guard plate and low-voltage plate can easily be illustrated by reference to the Schering bridge.[5] This bridge, arranged for use with a guarded condenser, is shown in Fig. 1. The capacitance C_s between the shielded plate and the guard plate is in parallel with the measuring capacitance C_4. The power factor of the specimen D is given by the equation

$$\text{Power Factor} = 2\,\pi\,f\,R_4\,(C_4 + C_s)$$

Hence, if the value of C_s is omitted in computing the power factor, an error is introduced the magnitude of which will depend on the relative values of C_4 and C_s. For a given sample, the error will increase as R_4 is increased. A rough estimate of the value of the capacitance C_s that may be expected in types of condensers now in use indicates that if R_4 is as much as 1000 ohms, the error would be of the order of 0.01 per cent in the power factor, whereas if R_4 is 10000 ohms the error would be of the order of 0.1 per cent.

All other methods of measuring power factor with a guarded condenser are subject to errors of the same magnitude as those indicated for the Schering bridge. The current that flows through the capacitance from the shielded electrode to earth decreases the current through the measuring instrument. Unless special provision is made to insure that the guard electrode is at the same potential as the shielded electrode, the errors that may result in the power factor of the specimen under test may be quite as large with a guarded condenser as with an unguarded condenser.

The problem of making power-factor measurements would be considerably simplified if suitable condensers with solid dielectric were available. These condensers could be certified for power factor in a standardizing laboratory. In order to be suitable, such condensers must be small enough to be portable and should have a low power factor which is stable with time and which is nearly independent of voltage, frequency, and temperature. No condenser which fulfills these requirements at high voltages has been produced. A mica condenser serves admirably at low voltage, but as yet it has not been adapted to high voltage.

Paper condensers for use on voltages up to 3000 volts are now a commercial article. Some unpublished experiments performed at the Bureau of Standards indicate that their power factor may change with time. Hence such condensers cannot at present be considered suitable for standards.

Glass condensers have been used in some cases. While very stable, the loss with most kinds of glass is high and there is a large change with temperature. Such condensers are not suitable as standards. There are unpublished data indicating that glass having a much lower loss than ordinary glasses will soon be available. This offers interesting possibilities for use in standard condensers.

Clear-fused quartz, often called silica glass, has been suggested as a suitable dielectric for high-voltage condensers. It answers the requirements of stability and of low dielectric loss. However, the effect on the power factor of changes in voltage, frequency, and temperature has not yet been determined. Until a condenser of this type has been given a complete test, there will be no certainty that fused-quartz condensers will make satisfactory standards for power-factor measurements.

The situation today in regard to power-factor standards at high voltage in many ways resembles the voltage situation forty years ago. Then if one wished to measure a voltage he must himself set up a Daniell cell which he could use as a standard. Now, practically every voltage measurement depends directly or indirectly on values maintained at a standardizing laboratory. The change from the first condition to the second has taken place gradually as new methods of maintaining standards have been developed, and as standardizing laboratories have been able to convince industry that the standards which they furnish are reliable.

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[5] See description by Everett S. Lee, JOUR. A. I. E. E., Feb. 1925, p. 160.

Now it is practically necessary for every laboratory that wishes to measure power factor at high voltage either to build an air condenser which is supposed to have zero power factor or to resort to difficult absolute measurements. However, if there is sufficient demand, standards will be developed and standardizing laboratories will put themselves in a position to certify to the value of the power factor under working conditions.

Discussion

For discussion of this paper see p. 658.

III—Accuracy Required in the Measurement of Dielectric Power Factor of Impregnated Paper-Insulated Cables

BY C. F. HANSON[1]

Member, A. I. E. E.

Synopsis.—*This paper deals with the effect of errors in the measurement of power factor upon the usefulness of impregnated paper-insulated cables. It points to error in the knowledge of the thermal properties of the cable and the cable duct. The latter error affects the usefulness of a cable to such an extent as to permit a limited error in the power factor without materially reducing the efficiency of the cable. This limited error defines the required power-factor accuracy.*

The required power-factor accuracy in general is found to vary directly with the frequency and the specific inductive capacity of the insulation, and to increase with the number of cables in a duct bank and the ratio of E^2/G, where E is the operating voltage of the cable in kilovolts and G is the geometric factor. For very high-voltage single-conductor cables the power-factor accuracy should be within the order of 0.002.

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INTRODUCTION

THE significance of an error in the measurement of one property of a commodity depends upon the limitations which that error imposes upon the usefulness of the commodity. At first hand, it would seem that this error should be reduced to as low a value as possible in order that the commodity may have a maximum usefulness. However, in the measurements of other properties of the commodity, errors exist which also impose limitations. An error in the measurement of the property, therefore, need not be made as small as possible, but need be reduced only to a value which will impose very little additional limitations upon the usefulness of the commodity.

The purpose of this paper is to determine the significance of an error in the measurement of the dielectric power factor of impregnated-paper insulated cables. The method of attack lies in an endeavor to compare the power-factor error with the error existing in the knowledge of the thermal properties of the cable. This comparison is based upon the effect of each of these errors on cable efficiency. The efficiency of a cable is measured by its capacity to carry current.

No economic gain of any consequence can be realized by reducing power-factor error below a certain limit as long as errors exist in the thermal data of the cable and the cable duct. In other words, for a given accuracy in the thermal data, an appreciable gain in cable efficiency can be realized by increasing power-factor accuracy only up to a certain limit. This limit of power-factor accuracy can be defined, for the purpose of this paper, as the required power-factor accuracy.

If the accuracy of the thermal data should increase, then the required power-factor accuracy would likewise increase. To provide for this situation, it is shown how the latter varies with respect to the former. With this information available, the required power-factor accuracy can be determined and then compared with the estimated power-factor accuracy obtained in the measurements. If the estimated accuracy is less than the required accuracy, then the methods for measuring power factor should be improved. On the other hand, if the estimated power-factor accuracy is far in excess of the required, then the accuracy of the thermal data should be increased.

THREE-CONDUCTOR CABLE

The following formula gives the current-carrying capacity of a three-conductor cable:

$$I = \frac{18.08}{\sqrt{R}} \sqrt{\frac{T_0 - T_G}{R_{th}} - W_{DL}} \qquad (1)$$

This formula is practically the same as formula (**24**) given by Simons[2]. The term W_{DL} is the dielectric power loss in watts per foot of cable at 60 cycles, and is assumed, for the sake of simplicity, to originate on the surface of the conductors.

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[1] Habirshaw Cable and Wire Corporation, Yonkers, New York.

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[2] "Calculation of the Electrical Problems of Transmission by Underground Cables," Donald M. Simons, The *Electric Journal*, August, 1925, p. 375. To avoid confusion the author has used the same symbols as those used by Simons. The first published formula to include W_{DL} was given by Wm. A. Del Mar in Harold Pender's "Handbook for Electrical Engineers," 1922 edition, p. 2021.

W_{DL} is proportional to the power factor $\cos \theta$.

R = the resistance per conductor in ohms per 1000 ft. at the allowable temperature, T_0, in deg. cent. (two per cent is added for cabling).

T_G = the base temperature of the earth in deg. cent. used as 20 deg. cent.

R_{th} = thermal resistance between conductors and base in thermal ohms per foot of cable.

I = current per conductor at the generator end of the cable in amperes.

The value of the current I is known with an accuracy of one per cent or better. The temperature, T_0, can be established under laboratory conditions with considerable accuracy. When T_0 is established, the value R becomes known to an accuracy within 2 per cent, the allowance required for manufacturing variation. The accuracy of the terms T_G and R_{th} is low. These two terms need be studied, therefore, in considerable detail in trying to assign the accuracy required in power-factor measurements.

Simons' equation (**20**) gives the following expression for R_{th}:

$$R_{th} = \frac{0.00522\, \rho\, G_1}{n} + \frac{0.00411\, B}{D} + N\, H \qquad (2)$$

ρ = thermal resistivity of insulation in watt-cm. units

G_1 = geometric factor of cable, all three conductors against sheath

n = number of conductors in the cable

B = surface thermal resistivity of lead sheath in watt-cm. units

D = outside diameter of the lead sheath in inches

H = heating constant of duct in thermal ohms per foot

N = number of similar cables in duct bank

The value of the fraction $\dfrac{T_0 - T_G}{R_{th}}$ can be varied either by assigning different values to T_G or to R_{th}. Any error existing in R_{th} can therefore, in equivalent terms, be added to the error existing in T_G. The foregoing fraction can therefore be rewritten into the following form in which ΔT_G is an equivalent error which combines the actual errors in T_G and R_{th}.

$$\frac{T_0 - (T_G \pm T_G)}{R_{th}}$$

The term W_{DL} contains an error of ΔW_{DL} and can therefore be written in the form $W_{DL} \pm \Delta W_{DL}$. Now, a certain equivalent error of ΔT_G in the base temperature of the earth will affect the value of I by the same amount as a certain error of ΔW_{DL}. Accordingly, the following equation can be written:

$$\Delta W_{DL} = \frac{\Delta T_G}{R_{th}} \qquad (3)$$

From Simon's formula (**17**) in the foregoing citation:

$$\cos \theta = \left(\frac{3144}{f\, k\, E^2/G_2} \right) W_{DL} \qquad (4)$$

E = Phase voltage in kilovolts

G_2 = Simon's geometric factor for three-phase operation

k = The specific inductive capacity and is taken to be 3.4

f = Frequency, and equals 60.

The power-factor error may be represented by $\Delta \cos \theta$ which corresponds to the dielectric power-loss error, ΔW_{DL}. Using these symbols in equation (**4**), the following relation is obtained:

$$\Delta \cos \theta = \left(\frac{15.4}{E^2/G_2} \right) \frac{\Delta T_G}{R_{th}} \qquad (5)$$

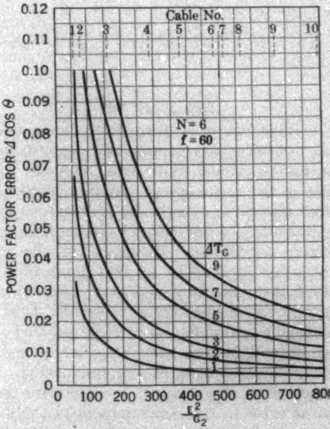

FIG. 1—POWER-FACTOR ERROR IN TERMS OF EQUIVALENT BASE TEMPERATURE ERROR Δ T_G FOR VARIOUS THREE-CONDUCTOR PAPER-INSULATED CABLES

In calculating the value for R_{th}, the author has used 500 for the thermal resistivity of paper insulation in watt-cm. units. The surface thermal resistivity of lead sheath B in watts per cm. units, is 1200, the same as that used by Simons. The heating constant H of the duct in thermal ohms per foot is taken as unity in accordance with Simons and Atkinson.

Table I gives the power factor error, $\Delta \cos \theta$, caused by an equivalent error of 1 deg. cent. in the base temperature T_G for ten cables. $\Delta T_G = 1$ in formula (**5**). In this table, T is the conductor insulation thickness and t is the belt insulation thickness with both expressed in

64ths of an inch. The column of cos θ gives what may be considered as a basic value of the power factor corresponding to the temperature T_0. To be sure, cables are manufactured having power factors less than those listed here, but on the other hand, cables are also made having higher power factors. In regard to operating voltage E, the table indicates an optimistic outlook on the performance of cables. The reason for this optimism is that the purpose of this survey is to consider power-factor measurements not only for cables being manufactured now, but also for possible cables which may be manufactured during the next decade.

The power factor error $\Delta \cos \theta$, is independent of the basic power-factor $\cos \theta$ according to formula (5). In other words, had other values been chosen for the basic power factor, $\cos \theta$, than those listed in the table, the power-factor errors, $\Delta \cos \theta$, would remain the same. This observation leads to an important conclusion. If the allowable power-factor error is limited only by errors existing in the base temperature T_G and the thermal resistance R_{th}, then the power-factor error must be expressed, not in terms of percentage, but in terms of a constant. For example, a power factor of 0.05 is known to be correct within the limits ± 0.005 and not ± 10 per cent.

The relation of the power factor error $\Delta \cos \theta$, and E^2/G_2 is shown in Fig. 1 for various values of ΔT_G. When the power-factor error has been calculated for an equivalent temperature error of 1 deg. cent. ($\Delta T_G = 1$), it is very easily obtained for other temperature errors. The power-factor error increases in the same proportion as the temperature error. For example, if $\Delta \cos \theta = 0.0331$ when $\Delta T_G = 1$, then $\Delta \cos \theta = 0.0662$ when $\Delta T_G = 2$.

The curves in Fig. 1 show that the required accuracy of power-factor measurements increases as the value of E^2/G_2 increases. For the sake of illustration, an equivalent error of 3 deg. cent. may be assumed in the base temperature. Under this condition the power factor of cable No. 9 should be measured with an error not to exceed 0.008. On the other hand, cable No. 2 may have a power-factor error not greater than 0.07.

A conversion of the term ΔT_G into its corresponding effect on the current carrying capacity of the cable may be helpful. The following formula provides the solution:

$$\frac{I_s}{I} = \sqrt{\frac{\cos \theta_0 - \cos \theta - \Delta \cos \theta}{\cos \theta_0 - \cos \theta}} \qquad (6)$$

I_s is the current capacity of the cable when its power factor is $(\cos \theta + \Delta \cos \theta)$.

I is the current capacity of the cable when its power factor is $\cos \theta$.

Fig. 1 supplies values of $\Delta \cos \theta$ for various cables and for various values of ΔT_G. Table I contains values of $\cos \theta_0$ and $\cos \theta$. Fig. 2 shows the effect of different values of ΔT_G upon the current capacity of the ten cables listed in Table I, when N equals six and when the basic power factor is that listed in the table.

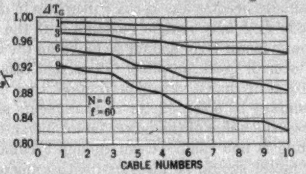

Fig. 2—The Effect of a Change in the Base Temperature Upon Current Capacity of Three-Conductor Paper-Insulated Cables

When N Equals Any Number

The subject thus far has been confined to the condition of six similar cables in a duct bank. The question naturally arises as to how the desired power-factor accuracy would change if some other number of similar cables in a duct bank were chosen. The change can easily be shown by introducing a factor Q.

$$\Delta \cos \theta = Q \, \Delta \cos \theta \, (N = 6) \qquad (7)$$

$$Q = \frac{R_{th} \, (N = 6)}{R_{th}} \qquad (8)$$

TABLE I

THREE-CONDUCTOR CABLE

$T_0 = 90 - E$ (minimum = 60) $T_G = 20$ $N = 6$ $\Delta T_G = 1$ $f = 60$

Cable No.	Cir. Mils X 10 −3	T	t	K. V. E	°C T_0	Cos θ	G_1	G_2	Cos θ_0	R_{th}	E^2/G_2	Δ Cos θ
1	500	8	6	7.6	82	0.053	0.70	1.04	2.05*	8.60	55.7	0.0331
2	212	16	4	13.2	77	0.046	1.29	2.31	1.31*	9.15	75.4	0.0231
3	500	9	5	13.2	77	0.046	0.71	1.12	0.678	8.58	156	0.0119
4	350	19	7	25	65	0.033	1.29	2.18	0.286	8.74	286	0.0064
5	500	9	5	20	70	0.038	0.71	1.06	0.246	8.58	376	0.0049
6	350	23	9	35	60	0.029	1.49	2.57	0.156	8.75	478	0.0038
7	350	20	10	35	60	0.029	1.45	2.41	0.142	8.79	509	0.0035
8	350	19	7	35	60	0.029	1.29	2.18	0.130	8.74	561	0.0032
9	500	18	8	35	60	0.029	1.16	1.86	0.113	8.53	658	0.0028
10	350	23	9	45	60	0.029	1.49	2.57	.092	8.75	790	0.0023

*The column of Cos θ_0 is, on first consideration, more ridiculous than interesting. Some of the values are imaginary. This column of figures gives the power factor which each cable would have if it had zero current carrying capacity. Those cables for which the figure is greater than unity will carry current regardless how great the basic power factor cos θ may be (cos θ, of course, will never be greater than unity). On the other hand, when the figure becomes rather low, as for example Cable No. 9, it has a significant meaning. This cable will not carry current at 60 deg. cent. if its power factor is equal to 0.113 at 60 deg. cent.

By trial calculation on the ten cables listed in Table I, Q was found to be constant, within an average deviation of 1.3 per cent. The following values of Q can be safely used for the purpose of this paper:

N	2	4	6	8	10	12
Q	1.85	1.30	1	0.81	0.69	0.59

The use of Q can easily be seen by referring to Fig. 1. The values of $\Delta \cos \theta$ for a condition of $N = 4$, may be obtained upon multiplying the values taken from Fig. 1 by 1.30. Likewise, the values of $\Delta \cos \theta$ for a condition of $N = 12$, may be obtained upon multiplying by 0.59. Because Q becomes smaller as N increases, the conclusion follows that power-factor measurements should be obtained with greater accuracy as the number of similar cables in a duct bank increases.

According to formula (**6**), the power-factor error $\cos \theta$ should become less as $\cos \theta$ increases in order that the ratio I_s/I may remain constant. If the limits of the power-factor error should be based on the condition that the error must not affect the current capacity of a cable by more than a certain percentage, then evidently, the accuracy of power-factor measurements should increase as the value of the power factor increases. This reasoning is not valid. Even though a cable of high power factor should be measured with a higher degree of accuracy, nothing would be gained in current capacity because the equivalent error ΔT_G would not permit a gain. The conclusion therefore follows that the required power-factor accuracy cannot be based directly on a limited change in current capacity.

SINGLE-CONDUCTOR CABLES

A slight modification of Simons' equation (**26**) for current capacity yields formula (**9**):

$$I = \frac{31.6}{\sqrt{R \dfrac{R_{ac}}{R_{dc}}}} \sqrt{\frac{T_0 - T_G - R_{th'} W_{DL}}{R_{th}}} \qquad (9)$$

The ratio $\dfrac{R_{ac}}{R_{dc}}$ is introduced because the skin effect in large conductors becomes appreciable. The term $R_{th'}$ is somewhat different from R_{th} because in the case of single-conductor cables the dielectric power loss is not assumed to originate on the surface of the conductors but actually originates in such a location as to cause a temperature drop from conductor to sheath equal to one half the drop it would have if the dielectric power loss had originated on the surface of the conductor. The value of R_{th} must be calculated in such a manner as to take into account sheath losses which are of relatively large magnitude in single-conductor but negligible in three-conductor cables. The other terms in formula (**9**) are the same as those in formula (**1**).

The ratio of R_{ac}/R_{dc} is taken from Simons' paper[3], p. 369. R_{th} and $R_{th'}$ were calculated according to his method. However, 500-watts per cm. units were used, in place of his 850-watts per cm. units, as the heat resistivity of impregnated paper.

By introducing the terms ΔW_{DL} and ΔT_G in formula (**9**), the following formula is obtained:

$$\Delta W_{DL} = \frac{\Delta T_G}{R_{th'}} \qquad (10)$$

Stating ΔW_{DL} in terms of $\Delta \cos \theta$ and $\dfrac{e^2}{G}$ formula (**10**) becomes:

$$\Delta \cos \theta = \frac{46.5}{e^2/G} \frac{\Delta T_G}{R_{th'}} \qquad (11)$$

e = voltage to neutral in kilovolts. In single-conductor cable it is the voltage from conductor to sheath.

G = Geometric factor

k = 3.4 and f = 60.

The power-factor error $\Delta \cos \theta$, is given in Table II when $\Delta T_G = 1$, for six different cables when N equals 6. The center of each of the three cables on a three-phase circuit was assumed to be at the apex of an equilateral triangle, the sides of which were taken to be 12 in. long ($S = 12$). Cables No. 14, No. 15 and No. 16 each has a hollow core of ¾ in. diameter in the conductor. All the cables are assumed to have a single lead sheath the thickness of which is given as U in 64ths of an inch. The thickness of the insulation in 64ths of an inch is given as T.

In Fig. 3 the relation of power-factor error, $\Delta \cos \theta$, to the ratio e^2/G is shown for various values of ΔT_G. As in three-conductor cables, the power-factor error is easily obtained for other values of ΔT_G when it is known for one value of ΔT_G. If $\Delta \cos \theta$ equals 0.005 when ΔT_G equals unity, then $\Delta \cos \theta$ equals 0.01 when

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3. *Loc. Cit.*

TABLE II
SINGLE-CONDUCTOR CABLE

$T_G = 20$ $N = 6$ $\Delta T_G = 1$ $f = 60$

Cable No.	Cir. Mils X 10 – 3	T	U	E	T_0	Cos θ	G	R_{th}	Cos θ_0	$R_{th'}$	e^2/G	Δ Cos θ
11	500	40	8	40	67	0.035	0.93	16.8	0.409	9.4	569	0.0087
12	500	60	9	66	60	0.029	1.19	17.2	0.167	9.2	1213	0.00417
13	750	52	10	75	60	0.029	0.97	20.4	0.114	8.9	1830	0.00285
14	600	46	10	100	60	0.029	0.81	17.9	0.0515	8.7	4130	0.00129
15	600	46	10	132	60	0.020	0.81	17.9	0.0296	8.7	7180	0.00074
16	600	46	10	132	60	0.010	0.81	17.9	0.0296	8.7	7180	0.00074

ΔT_G equals 2. The curves show that for high-voltage single-conductor cables, the permissible power-factor error is much less than it was for the three-conductor cables considered in Fig. 1. If the equivalent error ΔT_G, in the base temperature is assumed to be 3 deg. cent., then the power-factor error, $\Delta \cos \theta$, should not exceed 0.0022. This accuracy would be required regardless of what the actual power factor of the cable may be.

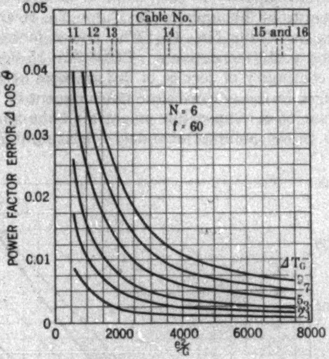

Fig. 3—Power-Factor Error in Terms of Equivalent Base Temperature Error ΔT_G for Various Single-Conductor Paper-Insulated Cables

The frequency used in the calculations is 60 cycles. If some other frequency were used, the permissible power-factor error would change. Although the exact variation of dielectric power loss with frequency is not known, the dielectric power loss may be considered to be proportional to the frequency within the limits of commercial power frequencies. On this basis, the permissible power-factor error, both for single-conductor and three-conductor cables, varies inversely as the frequency. In other words, the greater the frequency, the greater should be the power-factor accuracy.

The effect of ΔT_G upon the current capacity of single-conductor cables can be calculated by formula (**6**). This effect is shown in Fig. 4, when N equals 6 and when the basic power factor is that given in Table II.

If N equals some other number, the power-factor error $\Delta \cos \theta$ may be obtained by formula (**7**). In this case

$$Q = \frac{R_{tN'}\,(N = 6)}{R_{tN'}} \qquad \textbf{(12)}$$

The values of Q may be considered as follows:

$N = 3$	6	9
$Q = 1.50$	1	0.75

132-Kv. Cable

Fig. 4 brings out some rather interesting information. Each curve has a sudden break at cable No. 16; consequently cables No. 14 and No. 15 have some property which is abnormal relative to the other cables. This property is the power factor $\cos \theta$. Even though cable No. 15 has a power factor which is only 0.02 at 60 deg. cent., this power factor value is relatively much higher than the power factor 0.029 at 60 deg. cent. is for cable No. 12. Cable No. 15 should have a power factor of the order of 0.01 at 60 deg. cent. in order that that cable may perform in accordance with experience with other cables. The equivalent base temperature error ΔT_G, should be low and also the power-factor error $\Delta \cos \theta$. Furthermore, the power factor of the cable and the heat resistance of the cable and the duct line must run very uniform throughout its entire length for satisfactory and economical service.

Fig. 5 shows the power factor-temperature characteristic $A\,B$ which cable No. 15 may be expected to have, and characteristic $C\,D$ for Cable No. 16. Each of these characteristics was obtained in actual power-factor measurements of finished cables. The dotted line $E\,F$ is the unstable power factor-temperature characteristic for Cable No. 15 and likewise the dotted line $G\,H$ is the unstable characteristic for Cable No. 16.

The current capacity of Cable No. 15 is calculated to be 187 amperes when it is operated with a conductor temperature of 60 deg. cent. and a power factor of 0.020 when N equals 6. If this cable had $E\,F$ in Fig. 5, as its power-factor characteristic, it would carry 187 amperes at any temperature from 40 to 90 deg. cent. or more. Nobody knows what temperature the cable would choose.

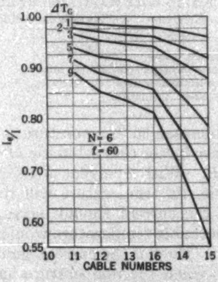

Fig. 4—The Effect of a Change in the Base Temperature Upon Current Capacity of Single-Conductor Paper-Insulated Cables

The unstable characteristic $E\,F$ is obtained with the use of formula (**11**). Instead of using $\Delta \cos \theta$ as a power-factor error, it is used as a power-factor increment. For the term ΔT_G, ΔT_0 is used because an increment change in T_0 produces the same effect as an increment change in T_0 according to formula (**9**). **Formula** (**11**) then becomes:

$$\frac{\Delta \cos \theta}{\Delta T_0} = \frac{46.5}{e^2/G} \frac{1}{R_{1N}} \qquad (13)$$

Formula (**13**) provides the slope of the characteristic *E F*. The point, $\cos \theta = 0.02$ and $T_0 = 60$, on the characteristic is already known. The characteristic is, therefore, established by a point and the slope. In the derivation of the slope the change in the resistance *R* due to an increase in temperature is disregarded. The actual power-factor characteristic of Cable No. 15 is the curve *A B*. With a current of 187 amperes flowing in this cable, its temperature will rise to 60 deg. cent. Now, if the current should increase slightly to 190 amperes, the cable temperature would immediately rise indefinitely. Even if the current should drop back to 187 amperes when the temperature has reached 70 deg. cent., the temperature would continue to rise anyway because the cable temperature has passed the critical point of 68 deg. cent. This temperature is generally known as the critical temperature of cumulative heating. Cable No. 15 is not suited for 187 amperes, because a small error in the thermal data or power factor of the cable would cause cumulative heating.

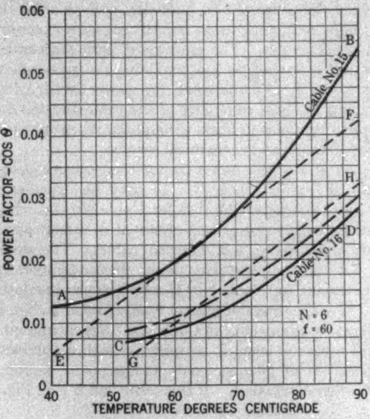

FIG. 5—CABLE NO. 15 WILL HAVE A SHORT LIFE]REGARDLESS OF POWER-FACTOR ERRORS. AN ERROR OF 0.002 MAY JEOPARDIZE THE LIFE OF CABLE NO. 16

132-kv. cable
One-conductor, 600,000 cir mils
3/4-in. hollow core
46/64-in. insulation

The current capacity of Cable No. 16 is calculated to be 268 amperes at 60 deg. cent. with a power factor of 0.01 when *N* equals 6. With a current of 268 amperes flowing through it, this cable with its power-factor characteristic *C D*, in Fig. 5, would come up to a temperature of 58 deg. cent. provided no power-factor error exists and that all other cable and duct properties are known exactly. If, however, a power-factor error of 0.002 or an equivalent base temperature error of 3 deg. cent. exists, the cable would come up to a temperature of 63 deg. cent. and stay there as long as nothing changes. A little overload for a few hours might, however, seriously jeopardize the life of the cable.

Fig. 6 tells a similar story except that *N* equals three. The current capacity of each cable at 60 deg. cent. is calculated to be 303 amperes for Cable No. 15 and 358 amperes for Cable No. 16. The power factors are 0.020 and 0.010 respectively. Cable No. 15 does not offer a continuity of service of 303 amperes because a small error in the power-factor measurement or in the equivalent base temperature might send the temperature up to the point of cumulative heating at 90 deg. cent. Cable No. 16 looks promising for 358 amperes even though a power-factor error of 0.002 might exist. If in addition to a power-factor error of 0.002, an equivalent base temperature error of 4 deg. cent. exists the probable error of the combination would be of the order of 0.005. Even with this equivalent error of 0.005 in the power factor, the cable would probably continue to serve with 358 amperes but no overload should be permitted. Under this condition it would operate at 65 deg. cent.

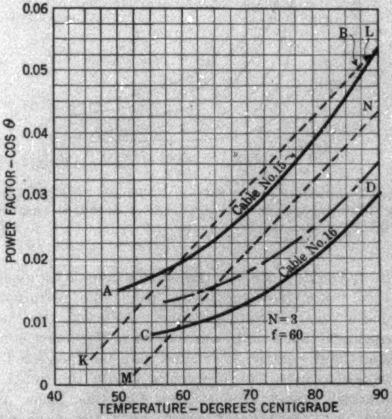

FIG. 6—ELIMINATION OF POWER-FACTOR ERROR AND EQUIVALENT BASE TEMPERATURE ERROR MIGHT SAVE CABLE NO. 15. AN EQUIVALENT POWER-FACTOR ERROR OF 0.005 MAY NOT SERIOUSLY IMPAIR THE USE OF CABLE NO. 16

132-kv. cable
One-conductor, 600,000 cir. mils
3/4-in. hollow core
46/64-in. insulation

For the 132-kv. cable, for operation in a three-cable duct bank, the power-factor error evidently should not exceed 0.002 and the equivalent error in the base temperature should not exceed four deg. cent. in order to obtain a cable service similar to that obtained with cables of lower operating voltage.

CONCLUSIONS

A survey of the factors which have a bearing upon the accuracy required in power-factor measurements shows that:

1. Power-factor accuracy should be stated as a constant and not as a percentage. For example, a power factor of 0.05 is accurate within ± 0.005 and not ± 10 per cent.
2. The power-factor accuracy required increases in direct proportion to the frequency, and also to the specific inductive capacity of the insulation.
3. The power-factor accuracy required increases as the number of similar cables in a duct bank increases.
4. The required accuracy increases as the ratio E^2/G increases, where E is the operating voltage in kv. and G is the geometric factor of the cable.
5. The required power-factor accuracy cannot be defined by percentage limits in the current-carrying capacity of a cable.
6. The required accuracy is relative to the accuracy obtained in the measurement of the heat resistivity of the insulation, the surface heat resistivity of the lead sheath, the "heating constant" of the cable duct and the base temperature of the earth.
7. For 132-kv. cable the power-factor accuracy should be of the order 0.002.

Discussion

For discussion of this paper see p. 658.

IV—The Use of the Dynamometer Wattmeter for Measuring the Dielectric Power Loss and Power Factor of the Insulation of High-Tension Lead-Covered Cables

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Associate, A. I. E. E.

Synopsis.—*The use of the dynamometer wattmeter for measuring the dielectric power loss and power factor of cable and capacitor insulation is not new, but dates from about 1890.*

Dynamometer wattmeters as available today are suitable for making these measurements. Care and attention must be given to their application.

The usual methods of application are:

1. Compensated dynamometer wattmeter method, with air capacitor,

2. Inductance variation method (phase-defect compensation method), with air capacitor,

3. Series resistor and wattmeter method,

4. Resonance wattmeter method.

Comparative measurements of dielectric power loss and power factor of cable samples indicate that results are being obtained with the dynamometer wattmeter wherein the probable departure from the true value is within from 10 to 20 per cent.

There is need for an effective means of standardizing any measuring equipment. Study of the calorimeter method for this purpose seems desirable.

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INTRODUCTION

PRESENT-day purchase specifications for high-tension lead-covered cables contain clauses limiting the values of the dielectric power loss and power factor of the insulation; these properties must therefore be measured either on the factory product in reel lengths, or on samples cut therefrom, or on both as may be specified. The dielectric power loss is the total power consumed in the insulation, and is expressed in watts. The power factor of the insulation is the ratio of the dielectric power loss to the product of the voltage across the insulation in volts and the resulting total current in amperes. The measurement of the dielectric power loss and power factor of the insulation therefore involves either (1) the measurement of the power, voltage, and current from which the power factor may be calculated; (2) the measurement of the power factor, voltage, and current from which the power may be calculated; or (3) the measurement of the power and reactive volt-amperes from which the power factor may be calculated.

The range of the values to be measured for present-day, single-conductor cables up to 43-kv. rating, conductor to sheath, (75-kv. between lines, three-phase), and three-conductor cables up to 33-kv. rating between treated paper-insulated conductors, is shown in the following:

Values for samples, 10 ft. of lead

Range of Impressed Voltage, 5 to 100 kv.
Range of Temperature, 20 to 100 deg. cent.
Dielectric Power Loss, 0.05 to 25 watts
Leading Current Power Factor, 0.002 to 0.10
Charging Current, 1.5 to 20 mil-amperes

For factory lengths which frequently extend up to 750 ft., the values of dielectric power loss and charging current are proportionately greater for the same voltage. The values of power factor remain the same.

The measurement of such low values of power and and power factor over a relatively wide range of voltages extending to high values is not simple, but requires special equipment. The instruments available for such measurements are the dynamometer wattmeter, the electrostatic wattmeter, and a-c. bridges. It is the object of this paper to discuss particularly the use of the dynamometer wattmeter in making these measurements.

THE DYNAMOMETER WATTMETER

A dynamometer wattmeter is one which depends for its action upon the electromagnetic interaction between sets of fixed and moving coils, one set being traversed by the circuit current (or a current proportional to the circuit current) and the other set by a current proportional to the circuit potential. The external resistor is usually considered as an integral part of the instrument potential circuit. Most present-day wattmeters for switchboard and portable use are of the dynamometer type, the moving coils being supported between jeweled bearings. For measurements of dielectric power loss in cable insulation where the power to be measured is of very low value, a sensitive dynamometer wattmeter is used in which the moving coils are delicately suspended by a fine filament. The deflections are read from a scale set at some distance from the instrument and traversed by a light beam reflected from a small mirror carried on the moving system. There are usually two complete elements connected in series, one element being mounted above the other, the upper and lower elements being of opposite polarity thus making the

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instrument astatic so as to eliminate errors due to external magnetic fields. Such an instrument is called an astatic reflecting electrodynamometer wattmeter, but it is usually referred to in connection with cable testing as a dynamometer wattmeter. A commercial form of an astatic reflecting electrodynamometer wattmeter is shown in Figs. 1 and 2.

USE OF THE DYNAMOMETER WATTMETER

The use of the dynamometer wattmeter is not new, but on the contrary is quite old. Its introduction is generally attributed to Professors Ayrton and Perry in 1881. In 1899, Rosa and Smith[1] published results of their measurements of dielectric power loss and power factor of capacitors using a resonance wattmeter method and using a calorimeter method, and following these in 1904 Dr. Rosa[2] published his further work in measurements made upon capacitors and circuits of low power factor using a dynamometer wattmeter with different methods of application, some of which are employed today in measuring dielectric power loss and power factor of cables. In 1901, Dr. C. V. Drysdale[3] described a dynamometer wattmeter of his own design, and at that time wrote the theory of the a-c. wattmeter which is still in use today. In the same year Dr. Drysdale[4] described the results of his measurements, using his dynamometer wattmeter to determine the dielectric power losses in capacitors and cables. At about this same time, instruments such as shown in Figs. 1 and 2 were developed and have been improved with the progress of the art.

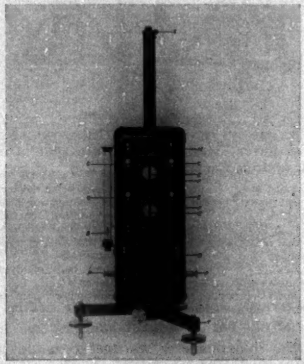

FIG. 1—ASTATIC REFLECTING ELECTRODYNAMOMETER WATTMETER

B Plumb bob
C, C, C, C Current terminals
D Damping chamber
E, E Moving coils
F, F Fixed coils (Field coils)
M Mirror
P, P Potential terminals
T Tripod
Z Zero Setter

From these early days to the present it is possible to trace the use of the dynamometer wattmeter for measuring dielectric power losses in capacitor and cable insulation through the writings of such men as Heinke[5], Potts[6], Mordey[7], Apt and Mauritius[8], Semenza[9], Humann[10], Hochstaedter[11], Shanklin[12],[13], Farmer[14], Renneson[15], Frigon[16], Barbagelata and Emanuele[17], Granier[18], Marbury[20], and Bruckman[21], and still this list is quite incomplete. The work by Carrol[19] and others in measuring corona losses in high-voltage transmission lines deserves mention in this regard.

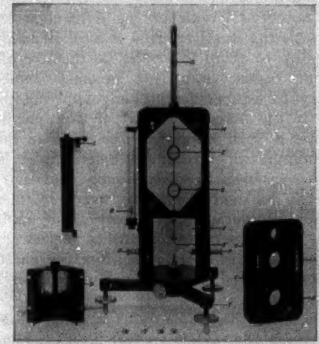

FIG. 2—PARTS OF ASTATIC REFLECTING ELECTRODYNAMOMETER WATTMETER

B Plumb bob
C, C Current terminals
D Damping chamber
E, E Moving coils
F, F Fixed coils (Field coils)
M Mirror
P, P Potential terminals
S Suspension
S' Spiral
T Tripod
V Damping vanes
Z Zero setter

The use of the dynamometer wattmeter for these measurements is thus seen to be quite firmly established. The theory of the instrument has been adequately stated by several writers[27],[28],[29]. Instruments are commercially available in which the inherent errors have been reduced to the minimum consistent with good instrument design. The problem in connection with measurements of dielectric power loss and power factor of cable insulation becomes, therefore, mainly a problem of correct application.

APPLICATION OF THE DYNAMOMETER WATTMETER

In applying the dynamometer wattmeter to the measurement of dielectric power loss and power factor of cable insulation, means must be provided for supplying current to the current circuit of the dynamometer and potential to the potential circuit of the dynamometer, so that these will represent the cable-circuit current and potential in known proportionate value and in phase. Dynamometer wattmeters are usually supplied with a current circuit of suitable rating for direct connection into the cable circuit. If this condition is

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1. See Bibliography attached hereto.

not satisfied and a shunt is required, then compensation must be made for any difference that may exist between the phase angles of the shunt and the current circuit of the dynamometer in addition to the usual compensation for differences in temperature coefficients. On the basis that it is desirable to eliminate cause for compensation wherever possible, it would appear that the use of a directly-connected current circuit was preferable.

The potential circuit of the dynamometer wattmeter with the usual external resistor is adapted for voltages up to about 150 volts, so that for measurements at higher voltage it becomes immediately necessary to arrange for properly applying the high voltage to the potential circuit of the dynamometer wattmeter in such a way that the power in the high-voltage circuit can be accurately determined. Though this same problem is solved simply and universally in general instrument practise today by means of instrument potential transformers[22], the application of these alone to measurements of dielectric power loss is not so satisfactory because of the phase-angle correction involved at the low power factor of the measurement[23].

In this connection Table I is of interest, for it shows the maximum allowable departure in phase angle from the true value for different percentage errors for different values of power factor. Since values of phase angle for instrument potential transformers are usually certified to as being correct within from 3 to 5 minutes, it is reasonable to assume that a departure in phase angle of at least 2 minutes from the true value may exist. For this condition it is seen that measurements to within an error of one per cent are not obtainable practically, to within five per cent are not obtainable for values of power factor 0.01 and less, and to within 10 per cent are not obtainable for values of power factor 0.005 and less. It follows, therefore, from the standpoint of phase angle alone that for the majority of these measurements the least error that can be reasonably expected will be in the order of 10 per cent.

TABLE I

Power Factor cos θ	θ	Maximum Allowable Departure in Phase Angle for an Error of					
		1%	5%	10%	25%	50%	100%
		Minutes					
0.002	89°53′	0	0.3	0.6	1.6	3.3	7
0.005	89°43′	0	1	2	4	9	17
0.01	89°26′	0	2	4	8	17	35
0.02	88°51′	0.5	4	8	17	35	70
0.03	88°17′	1	5	11			
0.05	87° 8′	2	8				
0.10	84°16′	4	19				

An additional correction must be made for the phase angle of the potential circuit of the dynamometer wattmeter. Unless the potential circuit is accurately shielded, the phase angle is difficult to calculate and may introduce an additional error in excess of those shown in Table I. For these reasons, the practise of correcting by calculation has been generally abandoned.

USE OF THE AIR CAPACITOR COMPENSATED DYNAMOMETER WATTMETER METHOD

The errors resulting from calculating corrections and from those causes not readily susceptible to calculation can be almost entirely eliminated by the use of a high-voltage capacitor so designed as to have a negligible loss[12, 24, 25, 31], and across which the high voltage of the circuit may be directly impressed. Such a capacitor is usually one with air as a dielectric though capacitors with CO$_2$ gas under pressure have been used. If the loss in the capacitor is zero, then when the current circuit of the dynamometer wattmeter is connected into the capacitor circuit and voltage is impressed across the potential circuit of the wattmeter such as by instrument potential transformer, the dynamometer wattmeter should indicate zero assuming no loss in the current circuit. It probably will not do so because of the combined phase angle of the instrument potential transformer and the potential circuit of the dynamometer wattmeter, but it may be made to do so by changing the reactance of the potential circuit until such a result is obtained. The dynamometer wattmeter is thus correctly compensated for existing phase-angle differences. The capacitor may now be replaced with the cable sample of the same capacitance, and the resulting deflection read on the scale of the dynamometer wattmeter. The capacitor and the cable sample are connected to the circuit at all times. The power in watts is then directly calculated from the deflection and the watt-constant as obtained with direct current. This method is usually referred to as the Compensated Dyna-

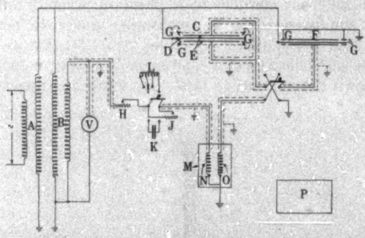

FIG. 3—DIAGRAM OF CONNECTIONS FOR COMPENSATED DYNAMOMETER WATTMETER METHOD OF MEASURING DIELECTRIC POWER LOSSES IN CABLE SAMPLES

A	Power transformer
B	Instrument potential transformer
C	Air capacitor
D	High-voltage plate of air capacitor
E	Low-voltage plates of air capacitor
F	Cable sample
G	Guard rings
H	External resistor
J	Auxiliary series resistor
K	Compensating capacitor
L	Compensating inductor
M	Dynamometer wattmeter
N	Potential coils of dynamometer wattmeter
O	Current coils of dynamometer wattmeter
P	Potentiometer for calibrating dynamometer wattmeter

Dotted lines indicate grounded shielding mometer Wattmeter Method, and since first described by Shanklin[12] in 1916 has been found to give reliable results.[26] The circuit diagram for this method is shown in Fig. 3, in which diagram are shown the shields and ground connections necessary to be provided to eliminate errors due to electrostatic and electromagnetic induction.

By this method the manipulation and corrections are reduced to simple form. The dynamometer wattmeter can be calibrated in position at any time by taking reversed readings on direct current, reading the values with a potentiometer. This assures accuracy of watt calibration. Losses in the current circuit can be calculated and corrected for if necessary. All phase-angle differences are corrected for by the compensation of the potential circuit.

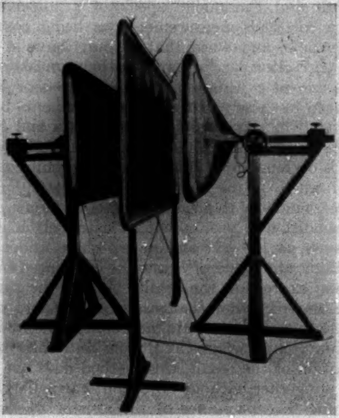

FIG. 4—AIR CAPACITOR. THREE-PLATE VARIABLE-CAPACITANCE TYPE

The middle plate is the high-voltage plate and is 9 ft. long and 5 ft. wide. The low-voltage plates are each 7 ½ ft. long and 3 ½ ft. wide. The capacitance range is from 0.0001 to 0.002 microfarad. The maximum voltage is 100 kv.

The use of the air capacitor is quite prevalent in the art today and its use greatly facilitates measurements made with the dynamometer wattmeter. Two commercial forms of such capacitors are shown in Figs. 4 and 5, being of the variable and fixed-capacitance types, respectively. The suitability of any given capacitor of this type can be determined from a consideration of the design and from tests such as those described[12]. Though the loss is not absolutely zero, it is possible to determine its negligibility within satisfactory limits. A value of phase angle of 0.5 minute is considered possible of attainment, and the design must be such as to provide this attainment if the advantages of the method are to be realized. The error introduced by phase angle of the air capacitor is to make the final reading of power to be lower than the true value.

For best results the capacitance of the air capacitor should be equal to that of the sample being measured so that circuit conditions are maintained as nearly identical as possible during compensation and during measurements. Such a condition can be obtained without undue difficulty for 10-ft.-of-lead samples of present-day cables. For long factory lengths, however, this condition is practically prohibitive. It is therefore necessary either to compensate on a current coil of low-current rating and make the measurement on a current coil of high rating, or use a shunt. Either of these methods introduces corrections which may be uncertain. Bridge methods are apparently more susceptible to accurate results for such measurements as these[26].

The sensitivity of the dynamometer wattmeter varies with the design, and is in the order of from 10^{-4} to 10^{-5} watts per millimeter deflection at 110 volts when the scale is set at the conventional distance of one meter from the instrument. This is also the order of the lowest values of dielectric power loss to be measured (at the wattmeter terminals) in cable samples (10 ft. of lead) so that for such values the reading error alone may be in the order of 20 per cent since the deflection may be only a few millimeters accurately read to from 0.3 to 0.5 millimeter. For higher values of power the reading error expressed as a percentage value becomes proportionately less.

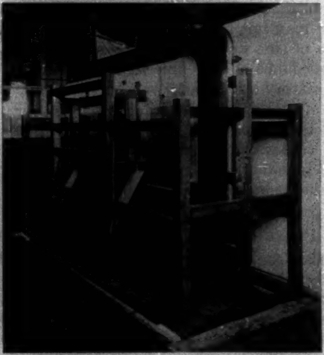

FIG. 5—AIR CAPACITOR. THREE-PLATE FIXED-CAPACITANCE TYPE

The plates are the same size as those shown in Fig. 4. Capacitance 0.0004 microfarad. Maximum voltage 100 kv. This type of capacitor used especially in high-voltage bridge.

The choice of means of voltage transformation is pertinent. A set of instrument potential transformers for different voltage ranges is undoubtedly the best, and these are commercially available up to 132 kilovolts. Ratio transformations obtained with these are independent of the current in the high-voltage circuit. The use of transformer voltmeter coil, though convenient, requires compensation for every measurement, since the transformation is affected by the current in the high-voltage circuit. The same applies to the practise of taking the potential from the low-voltage side of the testing transformer.

The effect of the earth's field is eliminated by making the dynamometer wattmeter astatic, although this is hardly necessary when measuring alternating power. Strong external alternating fields such as from high-current circuits or reactors nearby may, however, seriously affect the dynamometer wattmeter of the reflecting astatic type. Hence, it is quite essential to see that such external fields are eliminated when the dynamometer wattmeter is in use.

The dynamometer wattmeter accurately summates the power due to the like frequencies in voltage and current, and therefore should give the true power regardless of wave form. For cable testing, however, where the current circuit of the dynamometer wattmeter is in the cable circuit, and where the potential circuit of the dynamometer is supplied by means which may or may not be affected by the cable capacitance, the use of sine wave is highly desirable because of the low impedance of the cable to higher harmonics which act to distort the current wave. Also, from the standpoint of voltage measurement, the applied voltage should be of sine-wave form. The prevalence of the use of sine-wave generators today for cable testing is largely eliminating the need for worry regarding errors due to wave form[26].

INDUCTANCE VARIATION METHOD
PHASE-DEFECT COMPENSATION METHOD

Another somewhat similar application of the dynamometer wattmeter and air capacitor is used whereby with the capacitor in the circuit the dynamometer reading is brought to zero by means of inductance in the potential circuit. The cable sample of same capacitance is then substituted in the dynamometer wattmeter current circuit for the capacitor and the dynamometer reading is again brought to zero by means of the inductance in the potential circuit. From the change in inductance and the resistance of the potential circuit, the power factor is calculated.[2] This is frequently referred to as the inductance variation method, or the phase-defect compensation method.

SERIES RESISTOR AND WATTMETER METHOD

A series resistor may be used with the dynamometer wattmeter in either of two ways. First, it may be used as a substitute for the instrument potential transformer, the air capacitor still being retained for use in compensating the dynamometer wattmeter. In this case the conditions of use and possible results would be somewhat similar to those when the instrument potential transformer is used. A series resistor has also been used instead of the instrument potential transformer and air capacitor, connecting the resistor directly across the high-voltage circuit with the potential circuit of the wattmeter connected either in series with it, or in parallel with a portion of it. Such a resistor must be of fixed value, wherein the leakage to ground and the reactance are so extremely small as to be negligible. For the higher voltages such a resistor is necessarily large and quite expensive, and the problem of shielding is quite complicated. Barbagelata and Emanueli[17] speak of a suitable series resistor for such measurements as being costly and cumbersome, and state that it may have a phase-angle correction larger than those being measured.

The use of a water-column series resistor is being employed in these and similar measurements by some observers[17, 19]. As used by Barbagelata and Emanueli[17] the power factor of the cable insulation being measured is obtained by calculation from circuit constants, one of which is the resistance of the water column. A measurement of this value is therefore required under each condition of use, except at extremely high voltages such as 100 kv. and more, where, under certain conditions, the value of R may be eliminated from the equation expressing the result. Where the resistance of the water column changes frequently and requires frequent measurement as in most of the work in our country today, and where circuit constants enter into the calculation of the result, this method does not appear to be altogether without objection, particularly if the error due to capacitance of the resistor to ground is considerable, as it may be. The authors, however, describe a form of water-column resistor which is claimed to be free from capacitance error.

In this particular regard the work being done under the direction of Professor Ryan[19] in measuring corona losses at high voltages merits consideration, in that a method is given for shielding the water-column resistor and adjusting it to eliminate errors due to capacitance. This result is obtained by varying the resistance of the water-column resistor by varying the water flow and thus its temperature, the capacitance remaining constant. By taking readings of the wattmeter for different resistances, the shielding can be adjusted to give a zero error.

Some comparative measurements have been made with this method and with the compensated dynamometer wattmeter method, the results of which are shown in Figs. 6, 7, 8 and 9. The dynamometer wattmeter with water-column resistor was set up as first described[19] by Clark and Miller although the instrument was connected into the ground side of the circuit and the pail was replaced with a continuous hose connection from the point of high potential down nearly to the drain. This hose was banded at intervals and the bands connected to similar bands on the resistor hose to distribute the potential properly.

Under these conditions agreement was obtained with measurements of power factor made with the compensated dynamometer method to within 0.004 on the cable sample, Figs. 6 and 7, and within 0.005 on the experimental capacitor, Figs. 8 and 9, except at the lowest voltage. These departures represent differences from the results obtained with a compensated dynamometer wattmeter method in the order of from 20 per cent to 80 per cent.

The later work as reported by Carrol[19] wherein a salt solution under controlled circulation is used to obtain the necessary control of the resistance of the potential circuit would also have to be utilized if this method were used for measuring dielectric power losses in cable insulation. In making these tests the need for greater control of the resistance of the potential circuit was evident. Short-circuiting one turn of the water-column resistor was tried to obtain the desired range, but the resulting values obtained as shown in Figs. 8 and 9 show that error was introduced thereby.

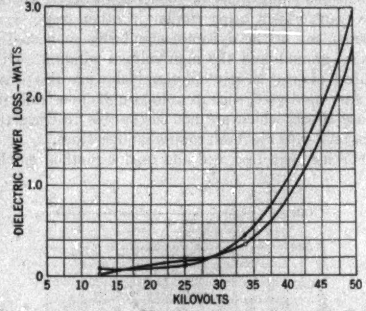

Fig. 6—Comparative Measurements of Dielectric Power Loss in Length of Experimental Three-Conductor Cable, Measuring from Single-Phase Supply with One Conductor "High" and the Other Two Conductors Grounded to Sheath

* By shielded water column series resistor and dynamometer wattmeter method

⊙ By compensated dynamometer wattmeter method

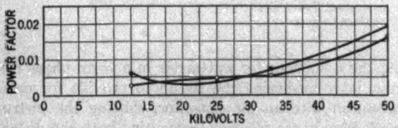

Fig. 7—Comparative Measurements of Power Factor of Length of Experimental Three-Conductor Cable, Measuring Dielectric Power Losses from Single-Phase Supply with One Conductor "High" and the Other Two Conductors Grounded to Sheath. Same Cable as in Fig. 6

* By shielded water column series resistor and dynamometer wattmeter method

⊙ By compensated dynamometer wattmeter method

While the use of the water-column resistor in measurements of dielectric power loss and power factor of cables doubtless has merit and warrants further investigation, it does not appear to be inherently more accurate than the present established methods, such as the use of the instrument potential transformer and air capacitor.

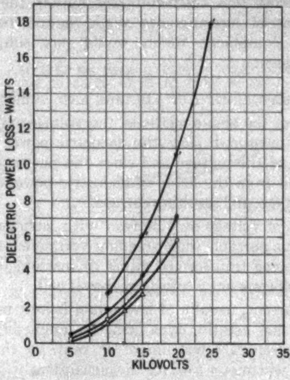

Fig. 8—Comparative Measurements of Dielectric Power Loss in an Experimental Capacitor

○ By compensated dynamometer wattmeter method

△ By Shering bridge method

* By shielded water column series resistor and dynamometer wattmeter method

*′ By shielded water column series resistor and dynamometer wattmeter method, with one turn of series resistor short-circuited

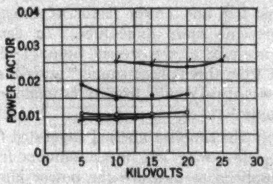

Fig. 9—Comparative Measurements of Power Factor of an Experimental Capacitor. Same Capacitor as in Fig. 8

⊙ By compensated dynamometer wattmeter method

△ By Shering bridge method

• By shielded water column series resistor and dynamometer wattmeter method

*′ By shielded water column series resistor and dynamometer method, with one turn of series resistor short-circuited

RESONANCE WATTMETER METHOD

The resonance wattmeter method as early described by Rosa[1] and as used recently by Marbury[20] for measuring dielectric power losses and power factor in capacitors has merit, but would doubtless be somewhat difficult of application to cables because of the wide range of capacitance of samples of different rating and cables of different lengths. The accurate determination of the losses in the inductance to be subtracted from the wattmeter reading for the various conditions of measurement would entail the use of detailed equipment which would not simplify the measurements.

MEASUREMENT OF VOLTAGE AND CURRENT

In all of the above methods, the voltage and current must be measured, since the dielectric power loss is a function of the voltage and since the power factor is calculated from the value of dielectric power loss, applied voltage, and resulting current.

Values of voltage may be read with sufficient accuracy with a portable direct-reading voltmeter, the high voltage being transformed either by instrument potential transformer or by transformer-volt coil.

The current, usually of low value, is best read with a reflecting astatic dynamometer ammeter, which may be a separate instrument or may be the dynamometer wattmeter used for the measurement of dielectric power loss, connecting the potential circuit, with a suitable shunt, in series with the current circuit.

MEASUREMENTS ON THREE-CONDUCTOR CABLES

The discussion thus far has been confined to the dynamometer wattmeter itself and to its application in a single-phase circuit. The measurement of dielectric power loss and power factor of three-conductor cables presents more difficulties. In general, two methods are used at the present time: (1) direct measurement from three-phase supply[14] and (2) calculation of the three-phase results from tests made from single-phase supply[13, 26].

Probably the simplest method suggested for making the dielectric power loss measurements from three-phase supply is to measure the power input to the transformers from the low-voltage side with the cable sample connected to the high-voltage side and then disconnected therefrom, the difference of these readings giving the value of dielectric power loss in the cable insulation. Unfortunately, the latter is too small to be accurately measured by this method except at the higher temperatures. Even under these conditions, the final result is the difference of two comparatively large values differing by only a small amount which is not highly accurate.

It thus becomes necessary to make the measurements on the high-voltage side. Although the principle of compensating the dynamometer wattmeter for circuit effects is still applicable, as in the single-phase circuit, it cannot be applied directly to each phase because the current circuit of the dynamometer wattmeter must be connected into the cable circuit between the grounded neutral point and the high-voltage winding of each transformer separately. Thus, the current due to capacitance to ground of the high-voltage winding, and between high-voltage and low-voltage windings, traverses the current coil of the dynamometer wattmeter but not the cable sample or the air capacitor. In a small transformer, error due to this cause may not be large. In a large transformer, shielding is absolutely necessary between the high-voltage and low-voltage winding. Fig. 10 shows the difference in power factor of a cable as measured with a shielded and an unshielded transformer of 100-kv-a. rating at 100 kv., the transformers being similar in other respects, the current coil of the dynamometer being connected between ground and the high-voltage winding. Other sources of error largely indeterminable are also present to varying degree, such as voltage unbalance and interphase effects which are not susceptible to correction, thus rendering the result somewhat uncertain.

Farmer[14] has described and used a method employing the dynamometer wattmeter in a three-phase circuit, later modifying the set-up to provide for compensation with an air capacitor, shifting the instrument from phase to phase, measuring both the dielectric power loss and the reactive volt amperes, the power factor being determined from the ratio of the two.

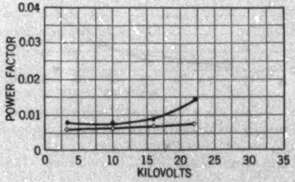

FIG. 10—COMPARISON OF VALUES OF POWER FACTOR OF A THREE-CONDUCTOR CABLE SAMPLE WHEN MEASURED WITH CURRENT COIL OF DYNAMOMETER WATTMETER CONNECTED BETWEEN "GROUND" AND HIGH-VOLTAGE WINDING OF TRANSFORMER, AS IN A THREE-PHASE CIRCUIT

Rating of transformers 100 kv-a. at 100 kv. One transformer shielded between high- and low-voltage winding; the other transformer unshielded. Transformers identical in other respects. Note that measurements are at comparatively low voltage, and that differences are increasing considerably with voltage increase.

• Unshielded transformer

○ Shielded transformer

The method of calculating the dielectric power loss from measurements made from single-phase supply[13, 26] has the advantage of employing the simpler single-phase measurement which is subject to greater accuracy and certainty in final result. It has the disadvantage of the uncertainty of the calculations embodying the resulting phenomena in the cable exactly. Check measurements between those made from three-phase supply and single-phase supply in general show good agreement, neither method giving consistently lower or high values. Farmer[14] states that the measured value is frequently considerably lower than the calculated value. Figs. 11 and 12 show comparative results in this regard, obtained in two different laboratories on two different samples of cables cut from the same length. Agreement of power-factor values is within from 0.002 to 0.004. The per cent of difference from the mean is 20 per cent over the entire range of values. Figs. 13 and 14 show results of similar measurements made on the same sample of cable in the same two laboratories. Agreement of power-factor values is within 0.002. The maximum per cent difference from the mean is 20 per cent; remaining values show closer agreement, and at some points actual agreement. On the assumption that the different methods give values subject to approximately the same errors, these results indicate the departure from the true result for both methods to be in the order of from 10 to 20 per cent.

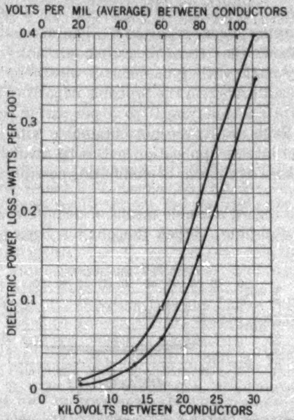

FIG. 11—COMPARATIVE MEASUREMENTS IN TWO DIFFERENT LABORATORIES OF DIELECTRIC POWER LOSS IN TWO DIFFERENT CABLE SAMPLES CUT FROM THE SAME LENGTH OF CABLE

Cable rated three-conductor, 500,000-cm., 9/64-in. x 5/64-in. treated-paper insulation, lead-covered

- • Laboratory A. Sample No. 1. Results calculated from measurements made from single-phase power supply by compensated dynamometer wattmeter method
- ○ Laboratory B. Sample No. 2. Results obtained by measurement with dynamometer wattmeter in a three-phase circuit

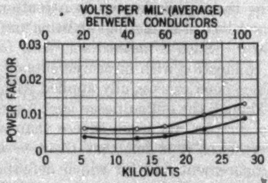

FIG. 12—COMPARATIVE MEASUREMENTS IN TWO DIFFERENT LABORATORIES OF POWER FACTOR OF TWO DIFFERENT CABLE SAMPLES CUT FROM THE SAME LENGTH OF CABLE—SAME CABLE AS IN FIG. 11

- • Laboratory A. Sample No. 1. Results calculated from measurements made from single-phase power supply by compensated dynamometer wattmeter method
- ○ Laboratory B. Sample No. 2. Results obtained by measurement with dynamometer wattmeter in a three-phase circuit

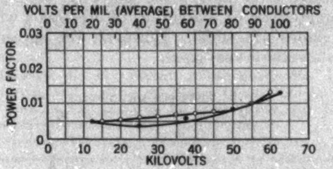

FIG. 13—COMPARATIVE MEASUREMETTS IN TWO DIFFERENT LABORATORIES OF DIELECTRIC POWER LOSS IN THE SAME CABLE SAMPLE

Cable rated three-conductor, 350,000-cm., sector, 10/32-in. x 5/32-in. treated-paper insulation, lead-covered

- • Laboratory A. Results calculated from measurements made from single-phase power supply by compensated dynamometer wattmeter method
- ○ Laboratory B. Results obtained by measurement with dynamometer wattmeter in a three-phase circuit

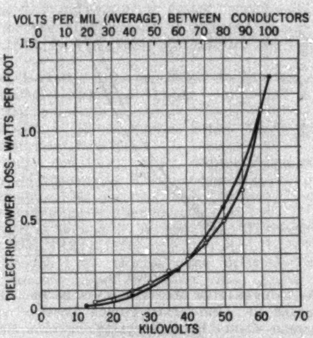

FIG. 14—COMPARATIVE MEASUREMENTS IN TWO DIFFERENT LABORATORIES OF POWER FACTOR OF THE SAME CABLE SAMPLE—SAME CABLE AS IN FIG. 13

- • Laboratory A. Results calculated from measurements made from single-phase power supply by compensated dynamometer wattmeter method
- ○ Laboratory B. Results obtained by measurement with dynamometer wattmeter in a three-phase circuit

ACCURACY OF MEASUREMENT

In the absence of known means for definite comparison, the accuracy of these measurements must be determined in other ways. Comparison of measurements made by independent methods is helpful.

It has been shown[26] that for single-conductor cables measured by compensated dynamometer wattmeter method and by Schering Bridge method that agreement in power factor has been obtained within 0.002. This is at a value of power factor in the order of 0.005, thus the resulting departure from the average value by the two methods is about 20 per cent. Note that the percentage error may seem somewhat high, but this is characteristic of this means of expression where the base values are small. However, the agreement in absolute value is very good.

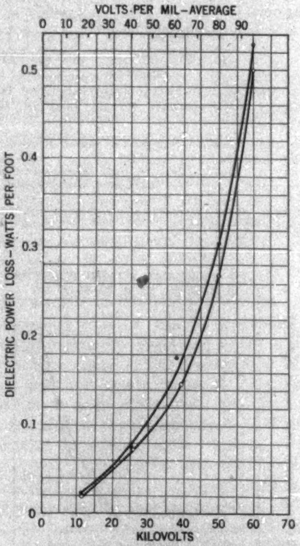

FIG. 15—COMPARATIVE MEASUREMENTS IN TWO DIFFERENT LABORATORIES OF DIELECTRIC POWER LOSS IN THE SAME CABLE SAMPLE—CABLE RATED SINGLE-CONDUCTOR. 500,000-CM., ROUND, 5/8-IN. TREATED-PAPER INSULATION, LEAD-COVERED

● Laboratory A. Compensated dynamometer wattmeter method
○ Laboratory B. Phase-defect compensation dynamometer wattmeter method

Figs. 15 and 16 show results of measurements made on the same sample of single-conductor cable in two different laboratories using the compensated dynamometer wattmeter method and the inductance variation dynamometer wattmeter method (phase-defect compensation method) respectively. Agreement is within 0.02 watt in 0.1 watt for dielectric power loss, and within a power factor of 0.001 at a value of power factor in the order of 0.005. This indicates departure from the average value for the two measurements to be in the order of 10 per cent. The curves of Figs. 11 to 14 show similar results for two methods of measuring three-conductor cables where the departure from the average value for the two methods is in the order of from 10 to 20 per cent.

CONCLUSION

An appeal has been made for greater attention to the question of making these measurements, large differences—as much as 100 per cent—on a percentage basis having been reported.[30] The discussion given above shows that these measurements are now being regularly made with dynamometer wattmeter methods giving results wherein the probable departure from the true value is within from 10 to 20 per cent in percentage value. Such results are not obtainable with ease, but only after intense application and using available intercomparisons as a means of eliminating possible unknown errors.

In this regard a usable means of definite comparison is quite desirable. Much thought has been placed on procuring a suitable comparator which could be used for standardizing any measuring outfit for dielectric power loss and power factor. No satisfactory means has yet been made applicable.

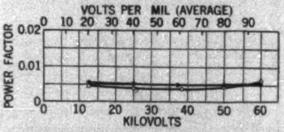

FIG. 16—COMPARATIVE MEASUREMENTS IN TWO DIFFERENT LABORATORIES OF POWER FACTOR OF THE SAME CABLE SAMPLE—SAME CABLE AS IN FIG. 15

● Laboratory A. Compensated dynamometer wattmeter method
○ Laboratory B. Phase-defect compensation dynamometer wattmeter method

As far as the author knows no attention has been paid to the calorimeter method as being useful for such a comparator. Such a method was used early in the the art in connection with measurements on capacitors and has since been successfully applied with satisfying results. Though the difficulties of such a measurement with cable samples are more severe than with capacitors, nevertheless it is felt that there is a field for such a measurement and for which definite specifications could be drawn up as a comparator, which would enable different measuring equipments to be adequately standardized. The possibilities of development work in this regard are quite worth the effort.

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Discussion

For discussion of this paper see p. 658.

V—Three Methods of Measuring Dielectric Power Loss and Power Factor

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***Synopsis.**—The paper presents a brief description of the methods of measuring dielectric power loss and power factor in commercial use at the Electrical Testing Laboratories.*

The several methods are handled individually and their advantages and disadvantages listed.

Appendixes are included on the use of the shunted electrodynamometer as an ammeter, determination of compensation, effect of incorrect compensation in the wattmeter shunt, effect of slightly unbalanced voltages on three-phase measurements, a three-phase wattmeter switch and shielding, grounding, etc.

INTRODUCTION

THE very fact that a symposium on methods of measuring dielectric power loss and power factor has been arranged is an indication that such measurements offer considerable difficulties, particularly when applied to present-day dielectrics. The purpose of this paper is to describe and discuss the methods now employed at the Electrical Testing Laboratories for such measurements.

Originality is not claimed for the methods described, but they have been improved and adapted as need for them has arisen. Experience has shown each to be satisfactory for the class of work for which it has been developed.

GENERAL

The three general methods to be described and the measurements for which they are adapted are as follows:

1. Compensated Wattmeter Method
   a. For single-phase measurements on cables
   b. For three-phase measurements on cables
2. Phase-Defect Compensation Method
   a. For high-voltage single-phase measurements on cables
   b. For single-phase measurements on low-voltage condensers of high capacitance
3. Transformer Bridge
   a. For single-phase measurements on very low capacitances (short pieces of cable, oils, compounds, sheet materials, etc.)

1a. The Compensated Wattmeter for Single-Phase Measurements.

This method consists in a direct measurement of the dielectric power loss in watts when an alternating potential is applied to the cable under test. As there is always a certain amount of "leakage" power in the test circuits and instruments, a measurement is first made of this "leakage" before the cable is connected for test. This "leakage" loss is then subtracted from the total loss with the cable or other dielectric in circuit in order to obtain the "net" or actual loss in the material under test.

This direct measurement is made using a shunted reflecting electrodynamometer wattmeter in connection with a potential transformer. The phase angle of the potential transformer and the self inductance of the potential coil of the wattmeter are compensated for by the use of a shunted condenser of the proper value in the potential circuit of the wattmeter. The self inductance of the current element of the wattmeter is compensated for by the use of a shunted condenser in the wattmeter shunt. A diagram of the complete circuit used is shown in Fig. 1.

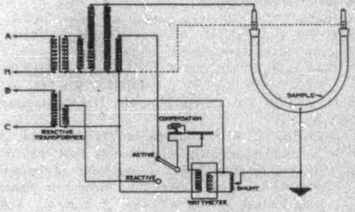

FIG. 1

The equipment used in making such measurements is as follows:

1 Insulating transformer
1 Testing transformer
1 Voltage transformer
1 Reflecting electrodynamometer wattmeter
1 Compensated wattmeter shunt
1 Voltage-circuit resistance with compensation
1 Calibrating load
1 Lamp and scale
1 High-resistance voltmeter
1 Frequency meter
1 "Reactive" insulating transformer

When using this method the wattmeter is calibrated on low voltage against a resistance load. In this operation the voltage circuit resistance is adjusted to give some even scale constant such as 0.001-watt loss in the high-voltage circuit per millimeter scale deflection.

After the calibration is completed the resistance in the compensation circuit is set for its correct value for the first test voltage. Since the burden on the potential

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1. Both of Electrical Testing Laboratories, New York City.

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transformer is practically constant from day to day, regardless of the sample under test, the value of compensating resistance necessary for various test voltages can be determined in advance by the methods described in Appendix II.

For speed in testing it is the usual practise to take a complete set of "leakage" readings covering the full range of test points and follow this immediately by "cable on" readings made at the same test voltages. These readings are referred to as "active" readings as distinguished from the "reactive" readings referred to below.

In order to determine power factor it is then necessary to measure either the current, Appendix I, or the apparent power, reactive volt-amperes. It is the practise at these Laboratories to determine the reactive volt-amperes in this case rather than the current. With a three-phase, *Y*-connected supply and the test set operated on one phase, a transformer with a ratio of $\sqrt{3}$ to one connected across the remaining two phases furnishes a potential supply which makes it possible to read the reactive volt-amperes directly. In this case, as well as in the power readings, "leakage" readings and "cable on" readings are made for all test points. These readings are referred to as "reactive" readings.

A typical set of data is as follows:

	Active	Reactive
Voltmeter indication	20	
True voltage—kv	10	
Compensating resistance	249	
"Leakage" Measurements		
Dynamometer indication	−33	−67
Shunt	1	1
Total deflection	−33	−67
"Cable On" Measurements		
Dynamometer indication	112	168
Shunt	1	50
Total deflection	112	8400
Leakage deflection	−33	−67
Net deflection	145	8467
Net volt-amperes	0.145	8.467
Cotangent	0.0171	
Power factor, per cent	1.71	

The time required for an operator to take complete data on a test requiring six test voltages is about one-half hour. During this time he can make all computations as indicated in the sample data, these computations being made while waiting for the wattmeter to assume a steady deflection.

The sensitivity of this equipment is, of course, fixed in the calibration and remains constant throughout the test range.

The accuracy depends chiefly on the accuracy of compensation. With the present methods of determining compensation it is believed that an accuracy of ± 0.05 per cent in power factor should be obtained.

This type of equipment is sensitive to external conditions. The loss read by the wattmeter varies as the square of the applied voltage, so that small voltage variations multiply in their effect on the loss reading. The reactive volt-amperes vary directly as the frequency and as the voltage, hence these conditions must be constant and correct.

Dielectric power loss and power-factor measurements should be made using sine-wave potential. With this method, however, the power loss is measured for the wave used, whatever its form may be.

Advantages:
1. Rapid operation—five minutes per test point including time for computations
2. Simple operation
3. Not affected by wave form of supply
4. High sensitivity

Disadvantages:
1. Accuracy of method depends on accuracy of compensation
2. Particularly susceptible to variations in supply voltage

1b. The Compensated Wattmeter for Three-Phase Measurements.

This method consists in a direct measurement of the dielectric loss in watts when a three-phase alternating voltage is applied to the sample of cable under investigation.

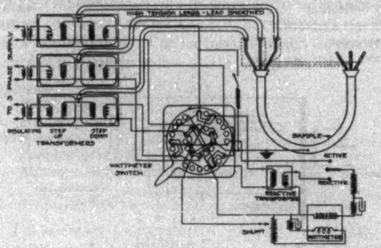

Fig. 2

The method is essentially the three-wattmeter method for the determination of power in a three-phase, four-wire circuit. This method has been simplified by the use of one wattmeter in connection with a specially designed switch for connecting the wattmeter in each of the three circuits as desired. A complete diagram of this circuit is shown in Fig. 2.

The operation of this equipment is essentially the same as that described in Method 1a. Since it is very difficult to obtain balanced three-phase voltages, it has been found that best results are secured by holding the voltage constant on one phase, rather than trying to adjust to the correct voltage on each phase. The reasoning back of this is to maintain the conditions constant for each test point. When working on an unbalanced system this would not be true if the voltage were adjusted on each phase when the wattmeter is read.

The equipment used in making these tests is the same as that described in Method 1 *a*, except that three insulating, three testing and three potential transformers are required together with the special wattmeter switch, Appendix V.

A typical set of data from one of these tests is as follows:

	Active		Reactive
Voltmeter indication		29	
True line-kv		10	
Compensating resistance		279	
"Leakage" Measurements			
Dynamometer Indication Ph. I	−24		96
" " " II	0		141
" " " III	−52		120
" " Sum	−76		357
Shunt	1		1
Total deflection	−76		357
"Cable On" Measurements			
Dynamometer Indication Ph. I	−36		87
" " " II	107		81
" " " III	−42		79
" " Sum	29		247
Shunt	10		200
Total deflection	290		49400
Leakage deflection	−76		+357
Net deflection	366		49043
Net volt-amperes	0.366		49.043
Cotangent		0.0075	
Power factor, per cent		0.75	

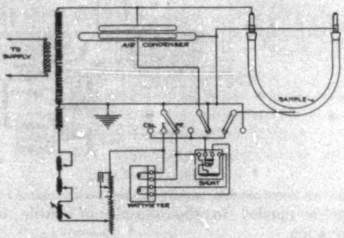

FIG. 3

The advantages and disadvantages of this method are essentially the same as those described in Method 1a.

2a. *Phase-defect Compensation Method for High-Voltage Measurements.*

This method consists in a determination of the power factor or phase angle of the sample under test by a direct comparison of its apparent phase defect angle with that of a "no-loss" air condenser.

In this method a reflecting type electrodynamometer wattmeter, connected as for measuring watts loss, is made to read zero by changing the phase of the current in the potential circuit by inserting inductance of known value. Such a balance is obtained, first, against a "no-loss" air condenser, and then against the sample. From the difference in shift necessary the power factor or phase angle of the sample is obtained.

Complete connections used in this test are shown in Fig. 3.

The equipment used is as follows:

1 Testing transformer
1 Reflecting electrodynamometer wattmeter
1 Dynamometer shunt
1 Voltage circuit resistor with compensation
1 50- to 250-milhenry constant resistance tapped inductor
1 6- to 50-milhenry variable inductor
1 Lamp and scale
1 High-resistance voltmeter
1 Frequency meter
1 High-voltage variable air condenser, "no-loss"

In using this method the sample and the "no-loss" air condenser are kept connected to the transformer at all times in order to maintain the relative phase positions. For convenience in computing, the potential circuit resistance is kept constant at a value in ohms equal to 6 π times the frequency in cycles per second.

In order to determine the dielectric power loss the charging current is measured, using the electrodynamometer as an ammeter, Appendix I. Then from the charging current, the power factor and the test voltage, the dielectric power loss is computed.

A typical set of test data and computations is as follows:

Voltmeter indication	20
Test kv.	
Cable (Condenser Connected Directly to Ground)	10
Potential circuit resistance	1131
Observed Inductance—Direct	44.0
" " —Reverse	43.8
" " —Average	43.9
Ammeter shunt	5
Ammeter reading	452
True milliamperes	2.74
Condenser (Cable connected directly to ground)	
Observed inductance—Direct	35.0
" " —Reverse	35.6
" " —Average	35.3
Difference in inductance	8.6
Power factor of sample, per cent	0.29
Dielectric power loss, watts	0.079

To determine the constant of the ammeter the electrodynamometer is calibrated against a resistance load. This connection is obtained with the selector switch on "calibrate."

Fig. 4 is the vector diagram for this method where

E = Test voltage
I_a = Current to air condenser
I_s = Current to sample
E_p = Voltage applied to potential circuit
I_1 = Current in potential circuit when balanced against air condenser
I_2 = Current in potential circuit when balanced against sample
ϕ_1 = Apparent phase defect angle of air condenser
ϕ_2 = Apparent phase defect angle of sample

The computation of the power factor is based on the following theory. If L_1 is the inductance setting for zero reading on the air condenser, the phase shift of the current in the potential circuit in order to obtain the balance is

$$\tan \phi_1 = \frac{\omega L_1}{R} \tag{1}$$

Likewise, if L_2 is the inductance setting for balance

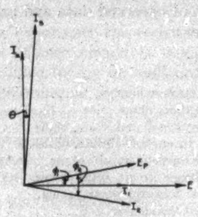

FIG. 4

against the sample, the phase shift of the current in the potential circuit is

$$\tan \phi_2 = \frac{\omega L_2}{R} \tag{2}$$

Since these are all small angles, the tangent is numerically approximately equal to the sine of the angle. Then, if θ is the phase-defect angle of the sample (difference between the actual angle and 90 deg.)

$$\sin \theta = \tan \phi_2 - \tan \phi_1 = \frac{\omega}{R} (L_2 - L_1) \tag{3}$$

or

$$\text{Power factor} = 100 \sin \theta = \frac{100\ \omega}{R} (L_2 - L_1) \tag{4}$$

But R has been made equal to 3 ω

Therefore,

$$\text{Power factor} = \frac{L_2 - L_1}{30} \tag{5}$$

when L_2 and L_1 are read in millihenries.

The computation of the power factor is so simple that it is readily made while waiting for the ammeter to assume a steady deflection.

Computation of the charging current depends upon the ammeter connection used. See Appendix I for methods of using the electrodynamometer as an ammeter.

The time to take a set of readings covering six-test voltages is approximately one-half hour, during which time the power-factor values are also computed. The remainder of the computations may take as much as an additional 20 min.

The sensitivity of this equipment, when used for power-factor measurements, varies with the test voltage and the charging current to the sample under test. It is naturally more sensitive under higher currents and voltages.

When making the power-factor balance, voltage variations cause changing sensitivity, although at balance voltage variations have no effect (assuming no change in power factor with voltage). Likewise, frequency variations affect the sensitivity on power-factor measurements as well as change the apparent defect angle. On current measurements any variable causing a varying charging current (voltage or frequency) causes a variable reading, which is particularly objectionable since the deflection varies approximately as the square of the current.

The accuracy of this method is dependent chiefly upon the operator and upon the "no-loss" condenser. A careful operator should be able to make measurements to within ± 0.05 per cent power factor. The losses as computed from the current and power factor should be well within ± 5 per cent accuracy.

In this equipment errors in wave form introduce an error in the power factor since it is computed on the basis of a sine wave. For harmonics which do not run above five per cent in value, this is negligible.

Advantages:

1. Rapid operation
2. Simple operation

Disadvantages:

1. Variable sensitivity
2. Accuracy dependent on operator and on "no-loss" condenser
3. Frequency and voltage must be maintained constant

2b. *Phase-Defect Compensation Method for Measurements on Low-Voltage Condensers of High Capacitance.*

The measurement of losses in low-voltage condensers of high capacitance (such as are used for power-factor correction) may be accomplished by a modification of Method 2a. The method then consists in a determination of the power factor of the test condenser by a comparison of the apparent phase-defect angles with different known values of resistance in series with it.

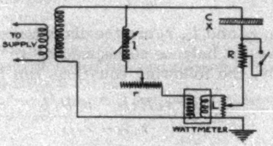

FIG. 5

The diagram of connections is shown in Fig. 5. Here again a reflecting type electrodynamometer wattmeter is used, connected as for power measurement, and made to read zero by changing the phase of the current in the potential circuit by inserting inductance of known value. Such a balance is obtained for each of two or more known values of resistance in series with the test condenser.

In employing this method the following equipment is used:

1 Resistor having two or three steps
1 Reflecting electrodynamometer wattmeter
1 Lamp and scale
1 Variable inductometer
1 Voltage circuit resistor
1 Voltmeter
1 Frequency meter

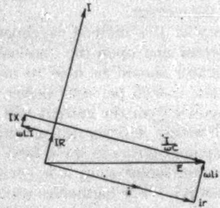

FIG. 6

Then, if

X = Series resistance of test condenser
C = Capacitance of test condenser
R = Resistance in condenser circuit (not including X)
L = Inductance of current circuit (usually that of current coil of wattmeter)
l = Total inductance of voltage circuit
r = Total resistance in voltage circuit
$\omega = 2\pi f$,

Fig. 6 shows that at balance

$$\frac{\omega l}{r} = \frac{X + R}{\dfrac{l}{\omega C} - \omega L} \tag{6}$$

When l_1, r_1 and l_2, r_2 are the inductance and resistance values at balance corresponding to R_1 and R_2, respectively, the following equations can be deduced.

$$X = \frac{r_2 R_2 l_1 - r_1 R_1 l_2}{r_1 l_2 - r_2 l_1} \tag{7}$$

$$C = \frac{l_1}{r_1 (R_1 + X) + \omega^2 L l_1} \text{ or } \frac{l_2}{r_2 (R_2 + X) + \omega^2 L l_2} \tag{8}$$

If r_1 can be kept equal to r_2, equation (7) reduces to

$$X = \frac{R_2 l_1 - R_1 l_2}{l_2 - l_1} \tag{9}$$

On the other hand, if a variable inductor is not available and r must be varied, equation (7) reduces to

$$X = \frac{r_2 R_2 - r_1 R_1}{r_1 - r_2} \tag{10}$$

This latter arrangement produces one peculiar condition in that the sensitivity of the meter may change more rapidly than the phase angle of the potential circuit thus causing the electrodynamometer indication to become greater for a time as the balance point is approached.

A typical set of observed data and calculated results are shown below:

Test voltage, volts		440	
Frequency, cycles per sec		60	
Current-coil inductance, henrys		0.008	
Voltage-coil resistance, ohms		8000	
Resistance of current coil and leads, ohms		18	
"Test" resistance in current circuit, ohms	0		10
Total resistance in current circuit, ohms	18		28
Inductance of voltage circuit, millihenrys	400		588
Series resistance of test condenser, ohms		3.3	
Capacitance of test condenser, μf		22.6	
Power-factor test condenser, per cent		2.81	
Loss in test condenser, watts		46.3	

Advantages:

1. No standard condenser needed
2. Settings only indirectly affected by frequency

Disadvantages:

1. Sensitivity variable
2. Constants of circuits must be known accurately

3a. The Transformer Bridge.

By this method the capacitance and equivalent shunting resistance are determined by comparison of the test sample with a "no-loss" air condenser. Each is inserted separately in one circuit of the bridge while the other circuit of the bridge containing known capacitances and resistances is adjusted until the currents in the two circuits are equal in magnitude and opposite in phase. From these values of capacitance and resistance the power factor and dielectric loss are computed.

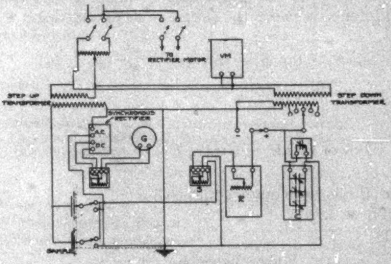

FIG. 7

A diagram of the circuit is shown in Fig. 7. The detector used consists of a synchronous commutator and a d-c. galvanometer. The rectifier is provided with two sets of brushes spaced 90 electrical degrees apart so that when properly set the bridge may be balanced for either component (active or reactive)—each practically independently of the other.

The balance circuit is unique in the use of a four-dial 100,000-ohm resistance box in connection with a universal shunt in order to simulate a continuously variable resistance of 0- to 500-megohm range. This is obtained by connecting the shunt across the detector and using the movable point as a connection for the resistance circuit.

Under certain conditions the resistance component required of the balance circuit is negative. Such a condition is provided for by an additional winding on the balance transformer as shown in Fig. 7.

The equipment used in this bridge consists of

1 D-c. reflecting galvanometer
1 Lamp and scale
1 Synchronous commutator
1 10,000-ohm galvanometer shunt
1 2000-ohm galvanometer shunt
1 4-dial, 100,000-ohm resistor
1 3-dial, 1 micro-microfarad mica condenser
1 1000-micro-microfarad variable air condenser
1 High-resistance voltmeter
1 Voltage regulator
1 Small step-down transformer
1 Testing transformer
1 High-voltage variable air condenser

In this method, also, both the sample and the air condenser are kept connected to the testing transformer, each being connected directly to ground when not connected into the test circuit. This maintains circuit conditions constant at all times.

A set of typical test data is as follows:

	Air Condenser	Sample
Capacitance reading	0.2048	0.4378
Resistance reading	84000	42100
Shunt used	20	5
Tan ϕ	0.0077	0.0288
Power factor of sample, per cent		2.11

The tangent of the *apparent* defect angle, ϕ, is computed as for a parallel circuit

$$\tan \phi = \frac{1}{\omega C R S} \qquad (11)$$

where

$\omega = 2 \pi f$
C = Capacitance read
R = Resistance read
S = Multiplying power of shunt

The defect angle of the sample is then the difference of these two angles, and since the angles are small,

Power factor $= \tan \phi_2 - \tan \phi_1$

where

ϕ_1 = the apparent defect angle of the air condenser and

ϕ_2 = the apparent defect angle of the sample

The capacitance of the test sample is obtained from the capacitance read by

$$C_s = C \times r \qquad (12)$$

where

C_s = Capacitance of sample
C = Capacitance read
r = Transformer ratio between test and balance circuits.

The time required for readings is about three minutes per test point. This includes the time for balances against both the air condenser and the sample.

Computations are not readily made during the test so that the time required for complete results on a test of six test points would be about one-half hour.

The sensitivity of this equipment is a function of the current in the test circuit and the galvanometer used. Since a d-c. galvanometer of medium high sensitivity (500 meg.) is used, the resultant sensitivity is high.

Accuracy is such that capacitances of the order of 50 micro-microfarads can be measured to within better than one per cent, while the power factor can be determined within ± 0.05 per cent (absolute value).

The bridge is extremely sensitive to leakages, so that all high-potential parts must be completely shielded and all low-potential wiring metallically sheathed.

Frequency and voltage change affect the results only in so far as they change the properties of the test sample.

Potential waves of other than sine characteristics affect both the test and balance circuits alike and cause error only if neglected in the computations—the bridge determines the capacitance and resistance components for the wave applied.

Advantages:
1. Rapid operation
2. Simple operation
3. Very high sensitivity
4. Measurement range
    Capacitance 1-1000 micro-microfarads
    Power factor 0-100 per cent
5. Independent of frequency and voltage variations
6. Accuracy high

Disadvantages:
1. Very sensitive to leakages

Appendix I

THE USE OF THE REFLECTING ELECTRODYNAMOMETER AS A UNIVERSAL SHUNTED AMMETER

There are two methods of using the reflecting type electrodynamometer as a universal shunted ammeter. The first is that shown in Fig. 8. In this case only one coil is shunted and, providing the dynamometer has a uniform watt scale, the instrument follows the law,

$$\Delta S = K^2 I^2 \text{ or } I = \frac{\sqrt{\Delta S}}{K} \qquad (13)$$

where

Δ = the deflection in millimeters
S = the multiplying factor of the shunt
K = the ammeter constant
I = the current

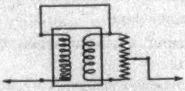

FIG. 8

It is preferable under some circumstances to shunt both coils as shown in Fig. 9. The instrument then follows the law,

$$\Delta S^2 = K^2 I^2 \text{ or } I = \frac{S\sqrt{\Delta}}{K} \qquad (14)$$

In this latter case, if the instrument can be calibrated to be direct reading on one shunt the product of the reading on this scale and the shunting value gives the current on any shunt. In the first case it is necessary to compute the constant over the range used and then the current which is a longer process.

Appendix II

DETERMINATION OF COMPENSATION

a. Using a "no-loss" Air Condenser.

To determine the correct wattmeter compensation for a given voltage, set air-condenser plates as close together as safe operation permits and read the wattmeter deflection for a series of values of "compensation resistance." These results when plotted, with wattmeter reading as abscissas and the square of the "compensation resistance" as ordinates, will give a straight line. Similar sets of data are then taken for a wider setting of the air condenser. These straight lines should then all cross at a point, corresponding to the correct value of compensation, at which point the wattmeter reading should be a constant regardless of the condenser spacing.

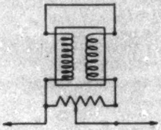

FIG. 9

While this method is described as for a "no-loss" air condenser, it applies equally well for a "constant-loss" variable condenser. Leakages play no part, providing they are constant for each voltage.

This procedure, while applying to single-phase equipments in particular, is readily adapted to three-phase equipments. In this case, each phase is handled separately as for single-phase determinations and then the average of the three values used as the correct three-phase compensation.

b. Using a Built-up Three-Phase Load.

A second scheme, sometimes more convenient for determining three-phase compensation, is to take three single-phase loads of approximately the same capacitance and loss, say three pieces of single-conductor cable, on which determinations can first be made by single-phase test methods. These loads can then be combined into a three-phase load and the "compensation resistance" adjusted until the loss measured under three-phase potential equals the sum of the losses measured at a corresponding single-phase potential.

Appendix III

EFFECT OF INCORRECT COMPENSATION IN WATTMETER SHUNT

Incorrect compensation in the wattmeter shunt will introduce no error if the phase defect compensation method is used. Any such phase error will affect both balances of the wattmeter alike and hence be eliminated when the difference is taken.

In the straight wattmeter methods, however, incorrect compensation in the wattmeter shunt will cause an error unless the potential circuit compensation values are obtained as in Appendix II. Mathematical analysis and actual tests have shown that any error caused by incorrect compensation in the wattmeter shunt appears as a constant angular error independent of shunt position.

Appendix IV

EFFECT OF SLIGHTLY UNBALANCED VOLTAGES ON THREE-PHASE MEASUREMENTS

When making three-phase measurements the natural tendency is to adjust the voltage on each phase just before taking a reading. This, however, means that when the three-phase supply is slightly unbalanced the three readings are taken for three different voltages on the cable as a whole. It has been found a better practise to hold the voltage on one-phase constant during all the measurements on all phases and let the others remain slightly high or low in voltage. In this way, readings on the three phases correspond to a given condition though they will not correspond to a balanced condition.

Again, any unbalance in voltages means that there may be slight phase shifts sufficient to make it impossible to analyze the individual wattmeter readings and get any idea of conditions in the insulation of each of the three conductors forming the load. In fact, it is not unusual to get negative readings on one phase, though the total is positive in value.

Appendix V

The Three-Phase Wattmeter Switch

To determine the power loss and reactive volt-amperes in a three-phase test circuit, using one wattmeter, requires a large amount of switching. Fig. 10 shows the operations to be made. Points *X*, *Y* and *Z* must all connect to *G*, either directly or through the current element of the wattmeter. With the current element of the wattmeter inserted between *X* and *G*, the potential element must connect to *A* and *X*, while the potential element for reactive volt-ampere measurements must connect to *B* and *C*. When the current element of the wattmeter is moved to connect between *Y* and *G*, the point *X* must again be connected to *G* and all the potential connections changed accordingly.

All of this switching with ordinary knife switches makes a very complicated and unwieldy arrangement. Accordingly, there has been developed at these Laboratories a simple-switching element which accomplishes in a single operation all of the necessary switching when changing from one phase to another. A diagrammatic view of this switch is shown in Fig. 11. The circles represent circular studs mounted in and insulated from the switch base. Connections are made to these studs as shown, the letters and numbers corresponding to those in Fig. 10. The upper or movable part of the switch rotates about the center and carries the spring contacts *H* and *K* which bear on the fixed studs. This switch in 60-deg. rotation provides correct connection of the wattmeter in each of the three phases.

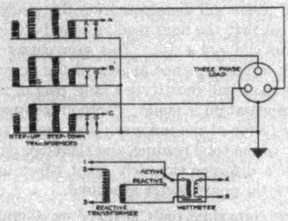

Fig. 10

Appendix VI

Shielding, Grounding, Etc.

In connection with all measurements made at high potential, shielding and guarding are important factors. In the three methods described in this paper, shielding is essential. All high-voltage parts, leads, etc., must be well shielded. All low potential wiring is best when done with metal-sheathed wire.

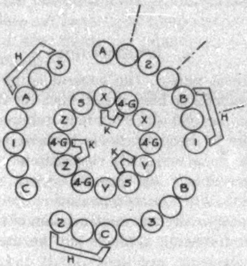

Fig. 11

Shielding is most readily done when one side of the testing transformer and the transformer tank can be grounded. All the single-phase test equipments described in this paper are operated with one side of the testing transformer grounded. The grounding of the transformer and shielding protects the operator against contact with high-voltage parts.

As an additional protection to the operator, these testing equipments have been designed as "one-man sets," that is, they are built for complete operation by a single operator. All are equipped with safety switches which make it necessary for the operator to be in his place at the instrument table in order to put voltage on any of the equipment.

Discussion

For discussion of this paper see p. 658.

VI—Compensation for Errors of the Quadrant Electrometer in the Measurement of Power Factor

BY D. M. SIMONS[1] and WM. S. BROWN[1]
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis;—*In a previous paper, the writers developed the equations for the main errors in the quadrant electrometer for the measurement of power factor, and checked their equations experimentally, the main sources of error being the unavoidable shunting capacity to ground of the test specimens and the charging current from the electrometer needle to the quadrants. In the present paper, methods of experimental compensations and elimination of these sources of error are described. Methods of compensation for capacity to ground have been presented before, and in this paper merely some additional refinements are given. The method of compensation for errors due to the charging current in the needle circuit is believed to be new; the method is described herein, the equation is derived in full, and an experimental check is given of the accuracy of the equation.*

* * * * *

I. Introduction

In a previous paper[2], the general equations of the electrometer have been given, both for the ordinary deflection method and for a new null method where the deflection is brought back to zero by inserting resistance in series with the electrometer needle. In order to save space, the previous work will not be described here, but the compensation for errors will be covered, practically as a continuation of the previous article. The following symbols will be used.

E = Voltage on the load taken as numerically equal to the transformer voltage
I = Load current or charging current of the specimen
I_h = Charging current flowing from the needle to the high quadrant
I_l = Charging current flowing from the needle to the low quadrant
r_1 = Resistance across the electrometer quadrants
r_2 = Resistance inserted in the needle circuit in the zero method, *i. e.*, the potential resistance
C_1 = Total capacity to ground of the high quadrant and all connected parts
C_2 = Capacity in the needle circuit
$\cos \phi$ = Power factor of the load
$\cos \psi$ = Power factor of the circuit in which I_h flows
n = Ratio of the transformer voltage to the needle voltage
f = Frequency
ω = $2 \pi f$

In the former paper it was shown that the main source of error was due to the capacity to ground of the "high quadrant" and all connected parts, including the capacity to ground of the high quadrant itself, all connected leads, the distributed capacity of the quadrant resistance, and the capacity to ground of the low-voltage electrode of the test specimen, the last usually being a large percentage of the total. Stated differently, a large part of the reading, either by the deflection or null method, is due to the capacity to ground. It was further shown that errors would be introduced by the charging current from the needle to the high quadrant, if the load current is so small that the needle current is no longer comparatively negligible.

In the previous paper, equations were given to include the effect of these two sources of error as well as others. In the present paper it is proposed to give methods of experimentally compensating for these two errors, so that these terms will disappear from the general equation, and so that the final reading may be taken as if these errors did not exist. This also means a great increase in accuracy, since in general the power factor of the unknown will constitute a large part of the reading, if compensation is made. Without compensation, the power factor of the unknown load may be a very small part of the total reading, and therefore any errors may have a disproportionately great effect upon the accuracy of the power factor measured.

II. Neutralization of Capacity to Ground, C_1

Considerable errors may develop in the measurement of power factor if a large part of the reading is due to capacity to ground, C_1. For that reason, there is often a real necessity for compensating for the greater portion, if not all, of the capacity to ground. This can be done in large part by raising the potential of the shield surrounding the low-voltage electrode of the test specimen to the same potential as that of the electrode itself. For example, in the measurement of small specimens of cable, the sheath must be surrounded by a grounded shield in order to avoid stray currents. If this shield is insulated and raised to the potential of the sheath, the capacity to ground of the sheath will be ineffective.

If desired, compensation may be even more perfect than that. If the quadrants consist of tinfoil pasted on glass, other sheets of tinfoil of the same size could be pasted on the other side of each piece of glass opposite the high quadrants and insulated from the instrument. All leads to the high quadrant are normally surrounded by grounded guards, which for this purpose should be insulated. The guards of the quadrants and of the leads may now be connected to the shield surrounding the low-voltage electrode of the test specimen, and all raised to the potential of this electrode. This will compensate for all capacity to ground except the distributed capacity to ground of the quadrant resistance which is normally very small. When n equals 2, under these conditions practically all of the reading will be due to the power factor of the load.

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[1] Both of the Standard Underground Cable Co., Pittsburgh, Pa.

[2] *The Quadrant Electrometer for the Measurement of Dielectric Loss*, D. M. Simons and W. S. Brown, Trans. A. I. E. E., 1924, p. 311.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Niagara Falls, N. Y., May 26-28, 1926.

Some experiments in compensation were performed where the load used was a short piece of cable. Fig. 1 gives the diagram of connections. The conductor of a similar piece of cable was connected to the high voltage and its sheath grounded through a resistance whose value was such that the potential drop across it was the same as that across the quadrant resistance. The sheaths of both specimens were therefore at the same potential. Compensation was then accomplished by connecting the sheath of the auxiliary specimen to the guard surrounding the load specimen.

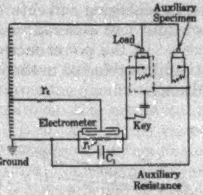

FIG. 1—DIAGRAM OF CONNECTIONS FOR COMPENSATION FOR CAPACITY TO GROUND IN QUADRANT ELECTROMETER

The accuracy of compensation can be checked by a method suggested by R. W. Atkinson. A key and condenser in series are inserted between the sheath of the auxiliary specimen and the guard surrounding the load specimen. A rough adjustment of the resistance in series with the auxiliary specimen is then made and a balance or deflection taken. If compensation is correct, no change in the balance or deflection will result on closing the key. If any change does occur, the resistance in series with the auxiliary specimen can be adjusted until the reading is independent of the position of the key. If a key were used without the series condenser, the balance would probably be too sensitive, due to possible differences in power factor of the load and auxiliary specimen.

Compensation for capacity to ground may be very advantageously applied to the method outlined in Section 5 of the former paper, where two balances are required, one on a zero-loss standard S and one on the unknown X, the total capacity to ground being kept constant. This can be accomplished in the following way. Insulate the guards of both S and X and connect them to the low-voltage electrode of the auxiliary specimen. Connect S to the high voltage, and the high-voltage lead of X to the low-voltage electrode of the auxiliary specimen, balance on S, compensation being made by means of the auxiliary resistance, series condenser and key. Then connect X to high voltage, connecting the high-voltage electrode of S to the low-voltage electrode of the auxiliary specimen, balance on X, compensating again. A different value of auxiliary resistance, of course, will be required if the charging currents of X and S are not the same. The difference between the two $r_2 C_2 \omega$'s will give the power factor of X. The great advantage of this method lies in the fact that the capacity to ground of the instrument and leads does not have to be compensated for, and the difficulty of the shunting capacities, C_a and C_b is entirely eliminated.

III. NEUTRALIZATION OF THE NEEDLE CHARGING CURRENT, I_h

The method used to neutralize the effect of I_h is shown in the diagram of connections, Fig. 2. The essential difference between this and the ordinary connection is that a graduated variable condenser C_3 and a variable resistance r_3, are inserted in parallel between the low quadrant and ground. The procedure follows:

1. Connect the low-voltage electrode of the test specimen to the high quadrant, leaving the high-voltage electrode floating, a convenient quadrant resistance r_1 being inserted as usual. An equal resistance, r_3, is then inserted between the low quadrant and ground and sufficient capacity C_3 is cut in until the instrument reads zero.

2. Apply high voltage to the specimen. Inasmuch as during the previous balance the total capacity to ground of the high quadrant was higher by C_{ab} (see Section 6 of first paper), this amount of capacity must be deducted from C_3. The deflection is now brought back to zero by the potential resistance r_2 as usual (or a deflection taken), and the power factor may be obtained by the following formula:

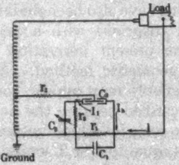

FIG. 2—DIAGRAM OF CONNECTIONS FOR NEUTRALIZATION OF I_h

$$\cos \phi + r_1 C_1 \omega - \frac{n-2}{2} \cdot \frac{I r_1}{E} + r_1 C_q \omega + \frac{I_h r_1}{E} = r_2 C_2 \omega \qquad (1)$$

in which C_q *is the capacity between the high and low quadrants themselves*, and C_1 is as usual the capacity to ground of the high quadrant and all connected parts, when the low quadrant is grounded. C_1, therefore, includes C_q. C_q is in most cases absolutely negligible, especially if the quadrants are made of tinfoil on glass, and the last term on the left is usually negligible. It will therefore be seen that equation (**1**) is practically identical with equation (**6**) of Part II of the previous paper, and that all effects of the charging current I_h have been neutralized. Furthermore, the equation is entirely independent of the load current if n equals 2, and therefore, if the compensation is made, the method of balancing first on a standard condenser and then on an unknown condenser could be made regardless of the relative sizes of the two condensers, except as influenced by the changes in capacity to ground as explained in the previous paper.

Another obvious method of avoiding the effects of I_h might be briefly mentioned. If the charging current to the high quadrant is approximately equal to that of the low quadrant, and the losses are the same on both sides, all effects due to them should be removed if the ground were placed at the midpoint of the quadrant resistance, instead of grounding one set of quadrants. We believe that there is one great difficulty with this method. The low-voltage terminal of the test transformer would, of course, have to be insulated. All stray currents therefore from the high-voltage end of the transformer to ground would return to the low-voltage terminal of the transformer through the ground connection and therefore through the lower half of the quadrant resistance, thereby introducing an indeterminate and, possibly, large correction.

IV. DERIVATION OF EQUATION FOR NEUTRALIZATION OF I_h

In the previous paper, an analytic method using the symbolic notation has been used. As stated in each case, the equations have also been derived geometrically from the vector diagrams with a perfect check. It appears that the present derivation is considerably simpler by the geometric method. At first thought, some of the geometry may appear too free, but it is believed to be justified because the actual vector diagram, if drawn to scale for a high-voltage measurement, especially with n equal to 1 or 2, would be so elongated that, for instance, all the vectors from the needle potential points, R, T, etc., to either quadrant are practically identical in absolute value. It is believed that practically all the substitutions about to be made are equivalent to the assumption that the angles are all so large or small that their sines or cosines, as the case may be, are equal to unity. An additional reason for confidence is that practically this same process and same assumptions completely checked the other equations derived analytically after the assumption that certain sines and cosines were equal to unity.

The vector diagram is shown in Fig. 3. $A\,B$ is the transformer voltage. $A\,N$ is the load voltage, the load current being the vector of reference. Angle $X\,A\,N$ = angle $A\,N\,M = \cos\psi$, the load power factor. B is the grounded point. The potential resistance, r_2, in Figs. 1 and 2 is connected to the transformer at any point, T.

In these first two figures r_1 and C_1 are as usual the quadrant resistance and the unavoidable shunting capacity and r_2 and C_2 are as usual in the potential circuit. It will be assumed that a resistance r_3, shunted by a capacity C_3, has been inserted between the "low quadrant" (the one usually grounded) and ground, no assumptions being made as to their present values. Let I_h and I_l be the charging currents from the needle to both the high and low quadrant respectively, and let $\cos\psi$ and $\cos\psi'$ be the power factors of these two currents respectively, as referred to the voltage between needle and quadrant.

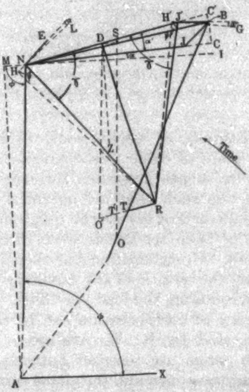

FIG. 3—VECTOR DIAGRAM FOR THE NEUTRALIZATION OF I_h

Returning to Fig. 3, $N\,J'$ is the resultant voltage across the quadrants to be mentioned later. O is the midpoint of the transformer high-voltage winding, and T is the point at E/n to which r_2 is connected. $T\,R$ is the voltage drop across r_2, and $R\,D$, perpendicular to it, is the effective needle voltage—assuming that C_2 is the capacity of the instrument itself, though actually an auxiliary potential condenser is used as explained in Section 3 of the earlier paper.

If there were no capacity to ground from the high quadrant and connected parts ($C_1 = 0$), the drop across the quadrant resistance r_1 due to the load current I would be in the direction $N\,I$, parallel to $A\,X$. Due to the capacity C_1, the drop lags by an angle α and may be represented by $N\,J$. $R\,N$ is the voltage between the needle and the high quadrant, angle ψ being angle $R\,N\,H$. The direction of I_h is therefore $N\,L$, $H\,N\,L$ being a straight line. The drop across the quadrants due to I_h lags behind this by the same angle α, and therefore the drop due to this current is $N\,E$. Supposing r_3 and C_3 to still be zero, the drop across the quadrants is $N\,B$, the vector sum of $N\,J$ and $N\,E$. If resistance and capacity are now added between the low quadrant and ground, namely r_3 and C_3, the diagram changes. $R\,J'$ is the voltage between the needle and low quadrant, angle $R\,J'\,H'$ is angle ψ', and $J'\,G$ is the direction of I_l the charging current to the low quadrant. The drop across the resistance r_3 due to I_l lags behind this by an angle α'' (not equal to α, unless $r_1 = r_3$, and $C_1 = C_3$). The total quadrant voltage due to I and I_h in r_1, and I_l in r_3 is the resultant of $N\,B$ and $B\,J'$, or is equal to $N\,J'$ as stated above, making the angle α' with $N\,J$.

The condition of zero deflection of the instrument is that the voltage across the quadrants shall be perpendicular to the voltage from the needle to the midpoint of the quadrants, *i. e.*, $R\,D$ is the perpendicular bisector of $N\,J'$, and the proof centers about the triangle $O'\,D\,R$, including the condition mentioned above and the fact that $O'\,R$ is perpendicular to $R\,D$.

Take S as the midpoint of $N\,B$, and draw $D\,S$, which will equal one-half of $J'\,B$. Draw $O\,S$, cutting $T\,R$ produced in T'. Draw $D\,O'$ parallel to $O\,S$.

$$O\,B = E/2$$
$$T\,B = E/n$$

Therefore,

$$O\,T = O\,B - T\,B = \frac{n-2}{2\,n}\,E$$

Therefore,

$$\frac{O\,T}{O\,B} = \frac{n-2}{n} = \frac{T'\,T}{S\,B}$$

Now,

$$S\,B = N\,B/2 = \frac{(I + I_h)\,r_1}{2}$$ (if cos $B\,J\,C$ and cos $B\,N\,C$ are taken as unity)

Therefore,

$$T'\,T = \frac{n-2}{n} \cdot \frac{(I + I_h)\,r_1}{2} \qquad (2)$$

Now,

$T\,R = r_2$ times current in potential circuit

$$= \frac{E\,r_2\,C_2\,\omega}{n} \qquad (3)$$

(Since the potential voltage T to D may be confused with $T\,B$ or E/n in scalar value)

Now,

$$O'\,T' = D\,S = J'\,B/2 = \frac{I_l\,r_3}{2} \qquad (4)$$

($O'\,D$ and $O\,S$ are parallel by construction, and in the actual diagram $D\,S$ and $O'\,T'$ are practically parallel)

$$\text{Angle } O'\,D\,R = 90 \text{ deg.} - \phi + a + a' \qquad (5)$$

(This may be seen by drawing a line $D\,Z$ from D perpendicular to $N\,I$. $O\,S$ is parallel to $A\,N$ because O and S are the midpoints of $A\,B$ and $N\,B$ by construction, and $D\,O'$, drawn parallel to $O\,S$, is also parallel to $A\,N$. Angle $O'\,D\,Z$, therefore, = 90 deg. $- \phi$. $D\,R$ is perpendicular to $N\,J'$, and $D\,Z$ to $N\,I$ by construction. Therefore, angle $Z\,D\,R = \alpha + \alpha'$).

$$\text{Sin } O'\,D\,R = \frac{O'\,R}{O'\,D} = \frac{O'\,T' + T'\,T + T\,R}{E/n} \qquad (6)$$

Substituting, (**2**), (**3**), (**4**), and (**5**) in (**6**), we obtain

$$\sin\,(90 - \phi + \alpha + \alpha') = \cos\phi + \sin\alpha + \sin\alpha'$$
$$= \frac{n\,I_l\,r_3}{2\,E} + \frac{n-2}{2} \cdot \frac{(I + I_h)\,r_1'}{E} + r_2\,C_2\,\omega \qquad (7)$$

$\sin\alpha = r_1\,C_1\,\omega$, and therefore only $\sin\alpha'$ remains to be determined.

An auxiliary angle γ will be introduced. γ = angle $R\,N\,J'$ = angle $N\,J'\,R$.

$$\sin\alpha' = \frac{CC'}{N\,C'} = \frac{C\,B + B\,C'}{N\,B}$$ ($N\,B = N\,C'$ if cos $B\,N\,C'$ is unity and $C'\,B\,C$ is practically a straight line)

$$= \frac{J\,B \sin C\,J\,B + J'\,B \sin C'\,J'\,B}{N\,B}$$

But $N\,B = (I + I_h)\,r_1$, $J\,B = I_h\,r_1$, and $J'\,B = I_l\,r_3$

Angle $C\;J\;B$ = angle $C\;N\;E$ = 180 deg. $- \psi - \gamma + \alpha + \alpha'$

Angle $C'\,J'\,B$ = angle $R\,J'\,C'$ − angle $R\,J'\,B$ = $(180 - \psi' + a'') = \psi' - \gamma - \alpha''$

Therefore,

$\sin\alpha' =$

$$\frac{I_h\,r_1 \sin(180 - \psi - \gamma + \alpha + \alpha') + I_l\,r_3 \sin(\psi' - \gamma - \alpha'')}{(I + I_h)\,r_1} \qquad (8)$$

Simplifying, dropping the sines of large angles and cosines of small ones, and rearranging terms, we obtain:

$$\sin\alpha' = \frac{I_h}{I}\cos\psi + \frac{I_h\,r_1 + I_l\,r_3}{I\,r_1}\cos\gamma$$
$$+ \frac{I_h}{I}\sin\alpha - \frac{I_l\,r_3}{I\,r_1}\cos\psi' - \frac{I_l\,r_3}{I\,r_1}\sin\alpha'' \qquad (9)$$

Since ψ and ψ' are circuit constants, and $\sin\alpha = r_1\,C_1\,\omega$, and $\sin\alpha'' = r_3\,C_3\,\omega$, the only unknown is the auxiliary angle γ, which must be evaluated.

From the figure,

$$\cos\gamma = \frac{N\,D}{R\,N} = \frac{(I\,r_1 + I_h\,r_1 - I_l\,r_3)/2}{E/n} \qquad (10)$$

Substituting (**10**) in (**9**), and (**9**) in (**7**) and algebraically simplifying:

$$\cos\phi + \left(1 + \frac{I_h}{I}\right) r_1\,C_1\,\omega + \frac{I_h}{I}\cos\psi$$
$$- \frac{n-2}{2} \cdot \frac{(I + I_h)r_1}{E} = r_2\,C_2\,\omega + \frac{I_l\,r_3}{I\,r_1}\cos\psi'$$

$$+ \frac{I_l r_3}{I r_1} r_3 C_3 \omega - \frac{n (I I_h r_1^2 + I_h^2 r_1^2 - I_l^2 r_3^2)}{2 E I r_1} \quad (11)$$

This is the general equation for cos ϕ in terms of the circuit constants including a resistance and capacity between the low quadrant and ground, no assumptions having been made as to their relative values.

In the actual process of neutralizing the charging current I_h as described in Section 3 of this paper, the first step was to insert r_3 between low quadrant and ground, making $r_3 = r_1$. Thus r_3 is then shunted by C_3 until the electrometer reads zero, the high-voltage electrode of the load not being connected. This means that the electrometer is measuring two equal loads, one through each set of quadrants at full voltage E/n on the needle. The losses measured on each side must, therefore, be equal. Therefore, from equation (**3**) of the previous article:

$$\frac{E}{n} I_h \cos \psi + \frac{E}{n} I_h r_1 C_1 \omega + \frac{I_h^2 r_1}{2}$$

$$= \frac{E}{n} I_l \cos \psi' + \frac{E}{n} I_l r_1 C_3 \omega + \frac{I_l^2 r_1}{2} \quad (12)$$

Dividing by $E\, I/n$:

$$\frac{I_h}{I} \cos \psi + \frac{I_h}{I} r_1 C_1 \omega$$

$$= \frac{I_l}{I} \cos \psi' + \frac{I_l}{I} r_1 C_3 \omega + \frac{n(I_l^2 - I_h^2) r_1}{2 E I} \quad (13)$$

And, since $r_3 = r_1$, (**11**) reduces to:

$$\cos \phi + r_1 C_1 \omega + \frac{I_h}{I} \cos \psi + \frac{I_h}{I} r_1 C_1 \omega$$

$$- \frac{n-2}{2} \cdot \frac{(I + I_h) r_1}{E} = r_2 C_2 \omega + \frac{I_l}{I} \cos \psi'$$

$$+ \frac{I_l}{I} r_1 C_3 \omega - \frac{n r_1 (I I_h + I_h^2 - I_l^2)}{2 E I} \quad (14)$$

If (**13**) be subtracted from (**14**), the equation reduces to equation (**6**) of the other article, except for a practically negligible term $I_h r_1/E$.

If this were so, our object would be accomplished, inasmuch as all terms containing the charging currents from the needle and the power factor of these currents would be removed. It was tried out experimentally on a known load and *did not check*. Finally, the error in reasoning was discovered. It has been assumed that C_1 and C_3 are the same in equations (**13**) and (**14**) respectively and also that C_1 is as usual the total capacity to ground of the needle and all connected parts. This is true *if the capacity between the quadrants themselves is negligible*, and only in that case. If this capacity, which is an essential part of the total capacity to ground, in the usual connection, is not negligible, then raising the potential of the low quadrant will diminish the capacity to ground of the high quadrant by a certain fraction of the capacity between quadrants, which will be called C_q. The fraction of C_q which will be taken away is inversely proportional to the potential of the quadrants, or since equal resistances r_1 and r_2 are used, inversely proportional to the currents flowing in these resistances.

We will, therefore, define C_1 as the total capacity between high quadrant and ground, when the low quadrant is grounded, and C_3 as the total capacity between low quadrant and ground, when the high quadrant is grounded. C_1 will therefore be the same as in all our other equations. The values of C_1 and C_3 given in the equations of this appendix are, therefore, different from this definition as explained above, inasmuch as they are capacities when equilibrium is reached.

Equations (**13**) and (**14**), as shown below, must, therefore, be rewritten and obtain equations in which these capacities agree with their definition in the beginning of this paragraph.

(**13**) becomes

$$\frac{I_h}{I} \cos \psi + \frac{I_h}{I} r_1 \left(C_1 - \frac{I_l}{I_h} C_q \right) \omega + \frac{I_h^2 r_1}{2}$$

$$= \frac{I_l}{I} \cos \psi' + \frac{I_l}{I} r_1 \left(C_3 - \frac{I_h}{I_l} C_q \right) \omega + \frac{n (I_l^2 - I_h^2)}{2 E I} \quad (15)$$

(**14**) becomes

$$\cos \phi + \frac{I_h}{I} \cos \psi + \left(1 + \frac{I_h}{I} \right) r_1 \left(C_1 - \frac{I_l}{I + I_h} C_q \right) \omega$$

$$- \frac{n-2}{2} \cdot \frac{(I + I_h) r_1}{E} = r_2 C_2 \omega + \frac{I_l}{I} \cos \psi'$$

$$+ \frac{I_l}{I} r_1 \left(C_3 - \frac{I + I_h}{I_l} C_q \right) \omega - \frac{n r_1 (I I_h + I_h^2 - I_l^2)}{2 E I} \quad (16)$$

If (**15**) be subtracted from (**16**) the following is obtained:

$$\cos \phi + r_1 C_1 \omega - \frac{n-2}{2} \cdot \frac{I r_1}{E} + r_1 C_q \omega + \frac{I_h r_1}{E} = r_2 C_2 \omega \quad (17)$$

This is the same as equation (**6**) of the former paper with the exception of the fourth and fifth terms, which in most instruments would be negligible, and therefore the effect of I_h has been practically neutralized by our method.

V. EXPERIMENTAL PROOF OF NEUTRALIZATION OF I_h

As mentioned in Section 4, it was thought at first that the method of neutralizing the effect of I_h outlined in Section 3 was practically perfect. It was tried out experimentally, a balance first being obtained on air condenser No. 1 in the usual manner, and then a balance was obtained after the procedure given in

Section 3 was performed. It was expected that the balance after I_h had been neutralized would be smaller than before, primarily due to the elimination of the term $\frac{I_h}{I} r_1 C_1 \omega$. Instead of that, however, the potential resistance required after the neutralization process had been performed was *larger*, which was puzzling. The effect of the capacity between the quadrants themselves, namely C_q, was then realized, and the derivation was changed as in Section 4. Finally equation (**1**) was obtained, including the effect of the capacity between quadrants. In our instrument, the quadrants instead of being tinfoil on glass, as is often the case, were cast aluminum plates, about 3/16 in. thick, with a beading along both the outside and inside and a rib at one point, which altogether made a by no means negligible area between adjacent quadrants, the separation between quadrants being about 1/16 in. C_q was calculated on the basis of 1/16-in. separation and the effective area between quadrants, and was also calculated from the balance described by means of equation (**1**), all other quantities being known. These two values of C_q checked to within about 10 per cent, which seems quite satisfactory, inasmuch as the actual capacity between quadrants could not be calculated accurately.

It was not convenient to measure this actual capacity and the easiest method of checking equation (**1**) was actually to shunt a known condenser between quadrants and measure its capacity by a series of readings, including our procedure for neutralizing I_h.

The four following readings were taken, at 14,000 volts with half voltage on the needle, 100,000-ohms quadrant resistance, r_1 and at 60 cycles.

1. Air condenser No. 2 alone with no compensation.
2. Air condenser No. 2 alone with I_h neutralized.
3. Air condenser No. 2 with an additional condenser of capacity C_A shunted across the quadrants, no compensation.
4. Air condenser No. 2 with the *additional* condenser C_A *across the quadrants*, I_h neutralized.

By "with I_h *neutralized*" in the readings 2 and 4 above, is meant that the process outlined in Section 3 was performed, namely that the low-voltage electrode of the load was connected to the high quadrant with the high-voltage electrode point floating. A resistance of 100,000 ohms was then inserted between the low quadrant and ground, and shunting this a variable capacity was connected, which was so adjusted that the reading of the electrometer was equal to zero. This variable condenser which was graduated and calibrated, was reduced in capacity by an amount equal to the C_{ab} capacity of the load, and the high voltage was then connected to the load, and a balance performed by means of the potential resistance as usual.

Following are the values of potential resistance used, and also $r_2 C_2 \omega$

Reading	r_2	$r_2 C_2 \omega$
1	18,570 ohms	0.02950
2	19,600 "	0.03113
3	25,100 "	0.03988
4	31,600 "	0.05020

Remembering that the power factor of air condenser No. 2 is zero, the two following equations, according to equation (**8**) of the other and equation (**1**) of this paper, are obtained from the readings 1 and 2 respectively.

$$0 + \frac{I_h}{I} \cos \psi + \left(1 + \frac{I_h}{I}\right) r_1 C_1 \omega + \left(1 + \frac{I_h}{I}\right) \frac{I_h r_1}{E} = 0.02950 \qquad \textbf{(18)}$$

$$0 + r_1 C_1 \omega + r_1 C_q \omega + \frac{I_h r_1}{E} = 0.03113 \qquad \textbf{(19)}$$

Remembering that during readings 3 and 4, the total capacity to ground as defined is equal to $C_1 + C_A$, and that the total capacity between quadrants is $C_q + C_A$, the two following equations are derived from both the equations (**8**) and (**1**) and the readings 3 and 4.

$$0 + \frac{I_h}{I} \cos \psi + \left(1 + \frac{I_h}{I}\right) r_1 (C_1 + C_A) \omega + \left(1 + \frac{I_h}{I}\right) \frac{I_h r_1}{E} = 0.03988 \qquad \textbf{(20)}$$

$$0 + r_1(C_1 + C_A)\omega + r_1(C_q + C_A)\omega + \frac{I_h r_1}{E} = 0.05020 \qquad \textbf{(21)}$$

Subtracting equation (**19**) from equation (**18**):

$$\frac{I_h}{I} \cos \psi + \frac{I_h}{I} r_1 C_1 \omega + \frac{I_h}{I} . \frac{I_h r_1}{I} - r_1 C_q \omega = -\ 0.000163 \qquad \textbf{(22)}$$

Subtracting (**21**) from (**20**)

$$\frac{I_h}{I} \cos \psi + \frac{I_h}{I} r_1 (C_1 + C_A) \omega - r_1 (C_q + C_A) \omega + \frac{I_h}{I} . \frac{I_h r_1}{E} = -\ 0.01032 \qquad \textbf{(23)}$$

Taking equation (**23**) from equation (**22**),

$$r_1 C_A \omega \left(1 - \frac{I_h}{I}\right) = 0.00969 \qquad \textbf{(24)}$$

Knowing that r_1 equals 100,000, I_h equals 30 microamperes, and I equals 0.568 milliamperes, and solving for C_A, the following is obtained.

$$C_A = 2.71 \times 10^{-10} \text{ farads}$$

It will be noted that C_A may also be obtained by subtracting equation (**18**) from equation (**20**), this being the ordinary method of measuring capacity as described in Section *B* of the previous paper. Making this subtraction

$$\left(1 + \frac{I_A}{I}\right) r_1 C_A \, \omega = 0.01038 \qquad \textbf{(25)}$$

From equation (**25**) we may calculate that $C_A = 2.61 \times 10^{-10}$ farads.

In order to check further their accuracy the authors sent the condenser C_A to an outside electrical laboratory whose measurements of the capacity were 2.53×10^{-10} farads. This check is not perfect, but it is believed to be quite satisfactory, especially in view of the difficulty with which the preliminary balance of the neutralization process with no voltage on the load was made, due to the lack of sensitivity of this instrument.

Discussion

For discussion of this paper see p. 658.

VII—Some Aspects of the Dielectric Loss Measurement Problem

BY B. W. ST. CLAIR[1]
Member, A. I. E. E.

***Synopsis.**—This paper points out some of the difficulties of making accurate power measurements at very low power factors. It deals with the lack of reference standards of known and constant power factor, especially for moderate sized samples and at high voltages. It also points out that a method of measuring an added loss as a check on a given test outfit's accuracy is not a check at all, although it has been used as such in several instances. Reference is made to calorimetrical methods of calibrating a testing outfit and to special forms of resistors that have very low time constant and are suitable for this class of test work.*

POWER measurements at very low power factors are admittedly quite difficult. The literature of the subject is quite voluminous, and many schemes, most of them modifications of two or three fundamental ones, have been proposed and used by various observers. It is not unusual to get widely varying results when a given sample is tested by different methods in different laboratories. Unfortunately, the real difficulties of this kind of measurement are not so widely appreciated by general testing engineers as perhaps they should be. It is the function of this short paper to point out some of these difficulties. No attempt is made, however, to have it cover the complete field of dielectric measurements.

A most difficult feature of this general problem centers around the lack of a reference standard of known loss. For work on very small samples, and at relatively high frequencies, this statement might be challenged. For power frequencies and for apparatus such as cables and condensers, there is no suitable reference standard, especially when consideration is given to the relatively high voltages under which tests must be made. Without an adequate reference standard, there can be no definite assurance that the results obtained by a given test set are correct, except as inference can be drawn from various auxiliary tests on the equipment or on the apparatus in test.

An air condenser, or an air condenser plus a known series resistance, has been repeatedly suggested as a suitable standard. There are two real difficulties in this suggestion. One is the difficulty of building an air condenser of adequate size and voltage rating, and the other is in knowing the losses of the condenser or proving the lack of losses at the high voltage necessary for tests on finished apparatus. Air condensers of small size and for low-voltage ratings have been built, but they do not lend themselves to either bridge methods or wattmeter methods on many of our present-day engineering samples. A number of air condensers designed for low loss at moderately high voltages has been built and are in service. Their capacitance, however, is quite low, especially when compared to reel lengths of cable or to condensers for power-factor correction. It is possible to build up a bridge with an air condenser as one of the arms, but the results of the bridge tests are still uncertain because of the very large ratio of the resistance arms necessary if the test work is on sizable samples and at the more usual voltages; and also there is no ready means of checking the results of distributed capacity and capacity between various high-voltage parts of the bridge and ground. A complete demonstration of the dependableness of a bridge scheme might lie in the interchangeability of the standard air condenser and the test sample. At present this is almost hopelessly impossible, at least with samples of moderate size.

Air condensers at present are also inadequate as

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reference standards for wattmeter methods. If a wattmeter is checked against a known air condenser at low voltages, it cannot be used at high voltages with any real assurance that the calibration can be translated from the low-voltage values. The currents that would circulate in the windings of wattmeters from air condensers at low voltage would be very different from those from actual cable samples or from static condensers. A calibration by air condenser is then generally carried over to another winding of the dynamometer with no really definite way of proving that this change can be made without change of instrument constants or characteristics, or the calibration is carried over to a much higher voltage without assurance that no change has occurred.

Another line of attack has been to eliminate all the known causes of error and then to attempt to prove the over-all accuracy of the equipment by checking a sample and then checking the sample with a known loss added to it as a series resistance. If the results corrected for the known losses were about the same as the results on the sample alone, the assumption was made that the measuring outfit gave correct results.

There are several reasons why this is not a dependable check of the test apparatus. If large changes are made in the losses, so that the dielectric loss plus the added losses are large in comparison with the dielectric losses, the difference is obtained by subtracting two large quantities. With only ordinary precision in the measurement of these large quantities the difference is subject to large uncertainties. Unless the change is made very great there is no check possible because of certain peculiarities of the trigonometric functions over the range of angles with which this paper deals.

From zero to five deg. the sine and tangent are for this purpose identical and have a linear relationship to the angle, thus

$$\sin X = 0.0174\,X$$
$$\tan X = 0.0174\,X \qquad (1)$$

where

X is expressed in degrees.

Now 5 deg. corresponds to a power factor of 8.7 per cent which is larger than is usually met in dielectric work on finished apparatus at power frequencies. The usual range is from 0.1 per cent to one or two per cent. The phase angle mentioned here is the departure of the current from an exact quadrature position with its voltage. Thus, the power factor of the sample is represented by the sine of this angle. Because of the linear characteristics and identity of the tangent and sine functions the addition of resistance to the test sample results in a change of angle proportional to this resistance and also in a change of power factor proportional to this resistance.

If the sample had no loss the phase angle would be zero. If to this ideal capacitance should be added a series of resistance R, the phase angle of the combination of capacitance and resistance would be

$$\alpha = \tan^{-1} \omega C R \qquad (2)$$

where

$\omega = 2\pi$ frequency
C = capacitance of sample
R = added resistance

The power factor of this group is also $\omega C R$, thus

$$\text{power factor} = \sin \alpha = \tan \alpha = \omega C R$$

and the losses in this resistance are equal to

$$I^2 R = E I \sin \alpha \qquad (3)$$

The normal losses of a condenser or cable or other dielectric sample can be considered as due to a series resistance added to a pure capacitance. Thus one speaks of the equivalent series resistance of a given sample. Its magnitude is such that $I^2 R$ losses would equal the dielectric losses. This resistance is denoted by R_1, so that the phase angle of the sample is

$$\beta = \tan^{-1} \omega C R_1 \qquad (4)$$

where

R_1 is this equivalent resistance

and its power factor is

$$\sin \beta = \tan \beta = \omega C R_1$$

The addition of resistance R to a sample of equivalent resistance R_1 results in a change of phase angle just equal to what the phase angle would be if the sample had no loss, that is, if R_1 were zero. This can be easily shown:

$$\text{phase angle} = \tan^{-1} \omega C (R + R_1) \qquad (5)$$

Because of the linear characteristic of the tangent function this can be written as

$$\text{phase angle} = \tan^{-1} (\omega C R) + \tan^{-1} (\omega C R_1) \qquad (6)$$

The difference between this and equation (2) is

$\tan^{-1} \omega C R_1$ which is equation (4)

The wattmeter deflection when the sample is tested alone is

$$W = E I \sin (\beta + \gamma) \qquad (7)$$

E is applied voltage
I is current in the series circuit of the wattmeter

γ is the phase displacement of the armature current of the wattmeter from its voltage. The difficulty of knowing the value of this angle is the chief difficulty in securing correct results in wattmeter methods at these low power factors. It is due to several things and may lag or lead its voltage. It is due to the inductance of the potential circuit resistance and of the armature of the wattmeters, to the capacitance currents flowing between various parts of the potential circuit and ground, and to eddy currents in the wattmeter structure or windings. The general tendency is to reduce this angle to as near zero as possible by careful attention to the set-up of the test equipment. It remains, however, a more or less unknown quantity as the accurate determination of the inductance of the resistances is made only with difficulty and the capacity currents cannot be computed.

Equation (7) for the wattmeter deflection becomes

$$W = E\,I\,(\sin\beta + \sin\gamma)$$

(At the small angles which are to be considered, the cosine can be taken as unity.)

When the series resistance R is added to the circuit, the wattmeter deflection becomes

$$W_1 = E\,I \sin(\alpha + \beta + \gamma)$$

which reduces to **(8)**

$$W_1 = E\,I\,(\sin\alpha + \sin\beta + \sin\gamma)$$

The difference in the two deflections is

$$W_1 - W = E\,I \sin\alpha$$

which is just equivalent to the I^2R loss in the resistance R, Equation (3).

Throughout this discussion, I is supposedly unchanged by the addition of R. In practise this is true.

At first, if viewed on a non-mathematical basis, this result seems paradoxical. A wattmeter incorrectly measuring a circuit of unknown loss has a correct increment to its indications when a known loss is added to the circuit. If there were no loss (*i. e.*, no phase angle) in the sample and if the phase angle of the wattmeter potential circuit were zero, the wattmeter would indicate zero. Its indications are directly proportional to these angles. Any angle in either circuit causes a deflection (positive or negative) that is proportional to the angle. The wattmeter can then be thought of as an instrument with an unknown zero point and a linear scale. The addition of a known quantity to the circuits of an instrument of this type results in a correct increment of indication without affording a clue to the total quantity being measured by the instrument. From the foregoing statements it is apparent that a wattmeter, although indicating incorrectly, will correctly show differences in two samples of the same capacitance. If the first sample is replaced by a second of identical capacitance the change in indication of the wattmeter is the difference in loss of the two samples, but no clue is given to the absolute magnitude of loss of either sample.

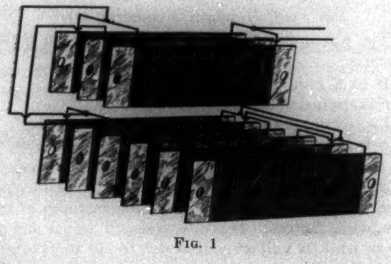

FIG. 1

This discussion has been applied to wattmeters where the armature is allowed to deflect. A similar treatment with the same result applies to those cases where phase changing methods are used to reduce the wattmeter deflection to zero.

To reduce the unknown phase angle of the potential circuit to as low value as possible the general tendency is to use guarded circuits of some sort. There are three main points to consider in laying out the potential circuit of an equipment of this sort—inductance of the resistances, distributed capacity and capacity to earth and other parts of the circuit. Some forms of "non-inductive" windings have rather large distributed capacitance. Some also have residual inductances that though low are difficult to compute. It is also difficult to measure these residual inductances as the time constant must be very small. If the phase angle is not to exceed 1 min. at 60 cycles, the time constant must be less than 7.7×10^{-7} sec. At any event the phase angle should be known to at least 1 min. if the over-all accuracy of the outfit is to be good when working on equipment like condensers. The error 1 min. causes 14 per cent error in losses when working with samples having a power factor of 0.2 per cent and 3 per cent when working on samples of 1 per cent power factor.

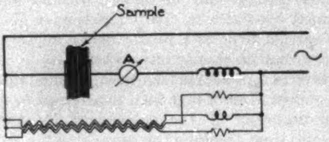

FIG. 2

A form of resistor that has low leakage, fairly low distributed capacitance, low inductance that is easily amenable to calculation, and is easily insulated for high voltage is the resistance card quite common in a-c. indicating instruments. A scheme of guard circuits that has proven very effective has been devised for this form of resistor. Every card is individually shielded as well as the case containing a group of cards. The general scheme is shown in Fig. 1 and the diagram of connections in Fig. 2.

In wattmeter methods either the potential losses or the $I^2 R$ loss in the series circuit of the wattmeter must be measured. In general, the series loss is by far the smaller of the two so that the scheme of connections shown in Fig. 3 is preferred to those of Fig. 4. If attempts are made to use very small samples, the inductance of these series windings may become excessive and must be allowed for in the computation of results.

The difficulties due to the absence of satisfactory no-loss standards of adequate size and voltage rating can be overcome in some cases where the test samples are of appreciable size so that the losses are large enough to measure by calorimetrical methods.

While fairly satisfactory results are possible, the testing work must be of very high order and unusual care must be taken to prevent the acceptance of erroneous results. Several different calorimeter methods have been used or proposed. In some of them the volume and temperature rise of water flowing over the sample in a suitable case have been used. In others the temperature rise of various parts of identical casings, one of them with an actual sample and the other with a resistor of known value, have been used as reference accuracy standards for dielectric-loss measuring outfits. In another method, identical temperature gradients with dielectric losses as one heating element and a resistor on direct current as the other have been used to demonstrate the accuracy of a testing equipment. This method is somewhat more elaborate than the others, and in the hands of a careful experimenter and with sufficiently good temperature conditions and thermometers may yield very good results.

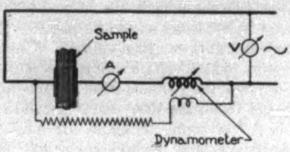

Fig. 3

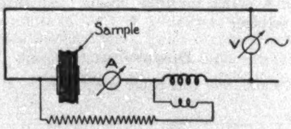

Fig. 4

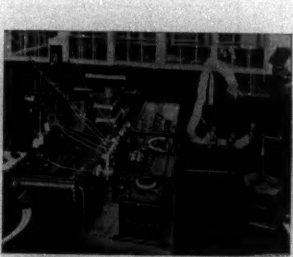

Fig. 5—Calorimeter Tests on Static Condenser Dielectric Losses

A photograph showing a test of this sort under way is shown in Fig. 5 and a diagram of the calorimeter is shown in Fig. 6.

The whole equipment was installed in a room the temperature variation of which throughout the calorimeter tests was not greater than 0.2 deg. cent. The temperature rise of the water jacket and of the oil above the arbitrary reference temperature was determined to 0.002 deg. cent. This generally required six or eight hours. Immediately thereafter, the condenser was removed from circuit and resistances that had been built into the same case were connected to a d-c. circuit. The losses in these resistances were varied until the same temperature distribution was reached.

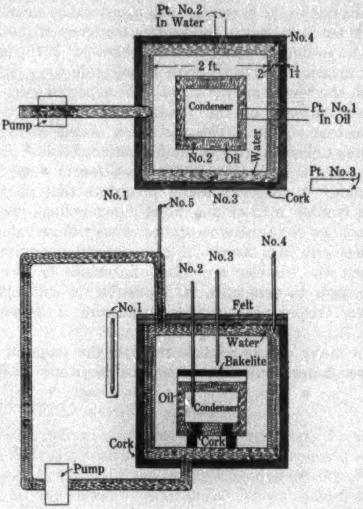

Fig. 6

At this point it was assumed that the a-c. losses and the d-c. losses were the same. All readings were taken at a distance by telescope to prevent body radiation influencing the thermometers. Accurate speed control of the circulating pump was also in effect although the pump did not add appreciable heat to the water stream and the connections were very well lagged to prevent undue radiation into the constant temperature room.

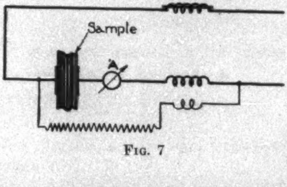

Fig. 7

Another source of disturbance is the change of wave shape of the current in the test sample. Because of the very low losses and the relatively large capacitance the tendency is to accentuate seriously any harmonics that may be present in the voltage wave.

The higher the harmonic the greater the amplification of that harmonic in the current wave. The phase position of the harmonic will also tend to advance beyond its normal position in the voltage wave and the losses due to the harmonics will in general be enhanced. This tends to reduce the leading tendency of the harmonic so that its phase shift is unknown. If the losses are measured correctly with appreciable wave distortion, the results are not those to be expected from sine wave currents of the fundamental frequency and measured voltage. This distortion of the wave also causes unknown error in the indications because of the unknown amplitude and phase position of these harmonics. A generally useful procedure that obviates this trouble is to obtain the required voltage rise by resonance or to use a reactance of very large value in series with the sample, if resonance is unadvisable. When the samples are small, resonance or a close approach to resonance is impossible or difficult at power frequencies. A circuit diagram is shown in Fig. 7.

In many checks of dielectric loss the shape of the power-factor curve with voltage is of more interest than is an exact knowledge of the actual loss. There is fortunately a fairly simple check that can be applied to a given wattmeter or bridge outfit to show its relative accuracy at varying voltages. This test consists in the selection of four samples of identical capacitance and apparent loss. These samples after check at a given voltage are connected in series multiple and tested at twice the original voltage. This results in applying the original potential to each sample. The combined capacitance is equal to each of the original samples. Because of the identity of the samples and the application of the original test voltage, the power factor of the group should be the same as the original four samples, and the measured losses should be just four times the loss of any unit at the original voltage. Thus, with an outfit to work to 2500 volts, a comparison of the relative voltage errors of the 400- and 800-, the 600- and 1200-, the 800- and 1600-, the 1000- and 2000-, and 1200- and 2400-volt points might be made. If nine identical samples were available, a check of the 400- and 1200-, and the 800- and 2400-voltage errors might be made, using three series multiple groups and three times normal voltage.

Discussion

For discussion of this paper see p. 658.

VIII—Zero Method of Measuring Power with the Quadrant Electrometer

BY W. B. KOUWENHOVEN* and PAUL L. BETZ†
Member, A. I. E. E. — Student Member, A. I. E. E.

Synopsis.—*This paper describes a zero method of measuring power by means of the quadrant electrometer developed by the authors, and which we believe to be new. Zero deflection of the electrometer is obtained by opposing the torque produced by the a-c. load by means of a counter torque set up by continuous potentials introduced into the electrometer circuit. The continuous potentials required are small in value and easily handled.*

We have derived the equations that apply to the method and checked their correctness by experimental observations on resistance, capacity, and inductive circuits.

THE importance of the quadrant electrometer as an instrument for measuring the power factor and loss in dielectrics and cables is well recognized. The quadrant electrometer wattmeter has also proved valuable in measuring iron losses in small specimens.

The usual connections for using the quadrant electrometer as a wattmeter are shown in Fig. 1. In Fig. 1, N is the needle; Nos. 1 and 2 are the respective quadrant pairs which are enclosed in the case of the electrometer. The deflection of the electrometer is proportional to the vector product of the load voltage V and the voltage drop across the quadrants, 1 and 2, caused by the load current I flowing through the non-inductive resistance R_1. If it is impossible to apply full voltage to the needle of the electrometer, a fractional part of the supply voltage may be applied to the needle as indicated by the dotted line in Fig. 1. Right and left deflections are taken at each load by means of the reversing switch or commutator, M, and the power consumed by the load is given by equation (1)[1].

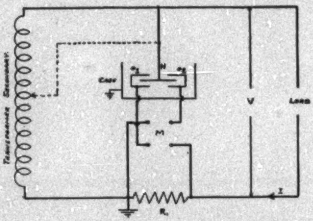

FIG. 1—QUADRANT ELECTROMETER WATTMETER

$$I\,V \cos\varphi = \frac{1 + R\left(\frac{V}{n}\right)^2}{2\,b_1\,R_1}\,n\,(\beta - \alpha) - \frac{2-n}{2}\,I^2\,R_1 \qquad (1)$$

Where

I = Load current in amperes
V = Load voltage in volts
$\cos\varphi$ = Power factor of the load
R_1 = Value of the non-inductive resistance in ohms
n = The factor by which the voltage applied to the needle must be multiplied to give the load voltage V
α = Deflection of the electrometer with the commutator vertical
β = Deflection of the electrometer with the commutator horizontal
R = An electrometer constant called the needle constant
b_1 = An electrometer constant

The quadrant electrometer wattmeter may be used as a deflection instrument as described above, or it may be used as a zero instrument by a method developed by Simons and Brown[2]. Simons and Brown showed that the deflection may be reduced to zero by the insertion of a resistance in the needle circuit of the instrument. The insertion of the proper value of resistance enables one to bring the voltage applied to the needle in quadrature with the voltage drop across the resistance R_1 and thereby reduce the deflection to zero. The power factor of the load may then be calculated from equation (2)[2].

$$r_2\,c_2\,\omega = \cos\varphi + \frac{2-n}{2}\;\frac{I\,R_1}{V} \qquad (2)$$

where

r_2 = The resistance inserted in the needle circuit
c_2 = The capacity of the needle circuit
f = Frequency
$\omega = 2\,\pi\,f$

and the other symbols have the same meanings as those used in equation (1).

The zero method of Simons and Brown possesses certain limitations, as pointed out by them,[3] on capacity loads of low power factor where n is large. This method is difficult to use on loads of high power factor because of the large value of resistance required for insertion in the needle circuit, and is not applicable to inductive

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1. For references see bibliography appended.

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circuits unless the needle voltage is reduced to a small fraction of the load voltage.

I. THEORETICAL

Zero or balance methods are in general more accurate than deflection methods, and their advantages are too well known to require discussion here. This paper describes a new zero method of measuring power and power factor with the quadrant electrometer. The method is applicable to inductive, resistance, and capacity loads and is independent of the value of the fractional part $(1/n)$ of the load voltage applied to the needle.

The zero method described here depends upon the use of continuous potentials to reduce the deflection of the electrometer to zero. A continuous potential is applied to the needle in addition to the a-c. load voltage and a second continuous potential is introduced into the quadrant circuit in addition to the drop across the resistance R_1, caused by the alternating-load current. The polarity of these two continuous potentials is arranged so that their effect opposes the deflection caused by the alternating potentials applied to the needle and quadrants respectively. The deflection is reduced to zero by adjusting the value of the continuous potentials to the proper value. The advantage of the method lies in its simplicity.

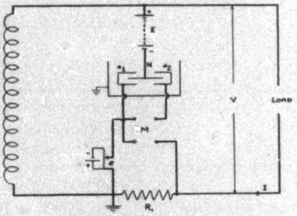

FIG. 2—CONNECTIONS FOR ZERO METHOD *A*

A number of arrangements of the connections for introducing the continuous potentials into the circuits of the electrometer have been investigated. Three of these arrangements, called, *A*, *B*, and *C*, respectively, are described in the paper.

Zero Method A. The diagram of connections for zero method *A* is given in Fig. 2, where *E* equals the continuous potential introduced into the needle circuit of the electrometer and *e* is the continuous potential introduced into the quadrant circuit; *V* and *I* represent the a-c. voltage and current, respectively, and R_1 is the non-inductive resistance.

Three methods of reducing the deflection caused by the a-c. load present themselves; (1) vary both *E* and *e*; (2) vary *E*, *e* constant; (3) vary *e*, *E* constant. Theoretically it makes no difference by which of these methods zero deflection is obtained, but from practical considerations it is simplest to vary *e*, which is on the grounded side of the instrument, keeping *E* in the high potential side constant. The variation in *e* is obtained by using a potentiometer rheostat as shown in Fig. 2.

In making a measurement, two readings are taken in order to eliminate certain electrometer constants. The operation is as follows: Assume that the commutator *M* is in a vertical position, and that the a-c. circuit is closed. As the instrument begins to deflect, the value of the continuous potential *e* is adjusted by means of the potentiometer until the deflection equals zero. The value of the potential *E* and of e_1 are noted. Then the commutator is placed in a horizontal position and the torque due to the a-c. power is again balanced by adjusting the continuous voltage applied to the quadrants. Let e_2 equal the value of the continuous quadrant voltage under these conditions. The experimental results show that the two values of the quadrant continuous potentials, e_1 and e_2, are very nearly equal and, in the equations, the average value *e* is used without introducing any appreciable error.

Before shifting the commutator from a vertical to a horizontal position, the a-c. circuit and the battery circuit of quadrant voltage *e* are opened. This prevents the electrometer needle from deflecting excessively and reduces the time between readings.

Equation (**3**) gives the general equation of the electrometer which was derived by Kouwenhoven in a previous paper.

$$(1 + R\,V_0^2)\,\theta = -\frac{b_1}{2}\,V_1^2 + \frac{b_1}{2}\,V_2^2 + b_1\,V_0\,V_1$$

$$-b_1\,V_0\,V_2 + c_0\,V_0 + c_1\,(V_1 - V_2) \qquad \textbf{(3)}$$

Here

V_0 = The instantaneous value of the voltage applied to the needle.

V_1 = The instantaneous value of the voltage applied to the quadrant pair No. 1

V_2 = The instantaneous value of the voltage applied to quadrant pair No. 2.

θ = Resulting deflection $(\alpha - \beta)$

α = Deflection with the commutator vertical

β = Deflection with the commutator horizontal

R = Needle constant of the electrometer

b_1, c_0 and c_1 = Electrometer constants

Let *v* equal the instantaneous value of the load voltage *V*, and *i* equal the instantaneous value of the load current *I*; then with the commutator vertical, we have

$$V_0 = v + i\,R_1 - E$$
$$V_1 = -\,e_1$$
$$V_2 = i\,R_1$$

With the commutator horizontal, the values of V_0, V_1 and V_2 are

$$V_0 = v + i\,R_1 - E$$
$$V_1 = i\,R_1$$
$$V_2 = -\,e_2$$

Substituting these values of V_0, V_1 and V_2 in the general electrometer equation (**3**) and solving, we obtain equation (**8**), the derivation of which is given in the mathematical part of the paper.

$$I\,V\cos\varphi + \frac{I^2 R_1}{2} = \frac{E\,e - \dfrac{e^2}{2}}{R_1} - k\,.\,\frac{e}{R_1} \qquad (8)$$

where k is an electrometer constant with a value of $\frac{c_1}{b_1}$ as shown in the mathematical part of the paper.

The term, $\frac{I^2 R_1}{2}$, of equation (**8**) is a correction term and equals half the loss in the resistance, R_1, which is measured by the electrometer.

If the polarities of the batteries E and e are reversed with respect to those shown in Fig. 2, we find that the power consumed by the load is given by equation (**9**).

$$I\,V\cos\varphi + \frac{I^2 R_1}{2} = \frac{E\,e - \dfrac{e^2}{2}}{R_1} + k\,\frac{e}{R_1} \qquad (9)$$

Equations (**8**) and (**9**) involve only the single electrometer constant, k. The other electrometer constants, that must be determined if the instrument is to be used as a deflection wattmeter, are eliminated.

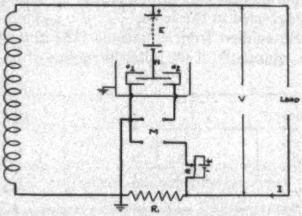

FIG. 3—CONNECTIONS FOR ZERO METHOD *B*

A study of equations (**8**) and (**9**) shows that this constant k may be eliminated if we take the mean of two sets of readings, the first set being taken with the connections as shown in Fig. 2, and the second set with the batteries E and e reversed. The constant k, however, can be evaluated very easily as shown in the experimental part of the paper, and the value of the term $k\,\frac{e}{R_1}$ determined in each case.

Equations (**8**) and (**9**) were checked experimentally and the results are given in the experimental part of the paper.

Zero Method B. The diagram of connections for zero method B is given in Fig. 3. Two readings are taken for this connection also; in the first the commutator is vertical, and in the second horizontal. When the commutator is vertical

$$V_0 = v + i\,R_1 - E$$
$$V_1 = 0$$

and

$$V_2 = i\,R_1 + e_1$$

With the commutator horizontal, V_0 has the same value as before, but now

$$V_1 = i\,R_1 + e_2$$

and

$$V_2 = 0$$

Proceeding as in connection A, we find that the power consumed by the load is given by equation (**12**) which is derived in the mathematical part of the paper.

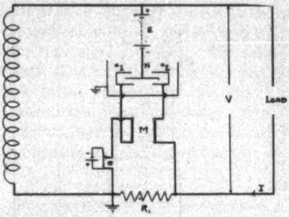

FIG. 4—CONNECTIONS FOR ZERO METHOD *C* COMMUTATOR VERTICAL

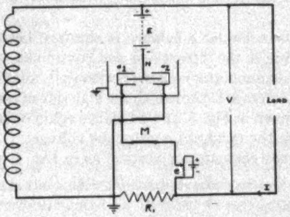

FIG. 5—CONNECTIONS FOR ZERO METHOD *C* COMMUTATOR HORIZONTAL

$$I\,V\cos\varphi + \frac{I^2 R_1}{2} = \frac{E\,e + \dfrac{e^2}{2}}{R_1} - k\,\frac{e}{R_1} \qquad (12)$$

Reversing the batteries E and e, we obtain

$$I\,V\cos\varphi + \frac{I^2 R_1}{2} = \frac{E\,e + \dfrac{e^2}{2}}{R_1} + k\,\frac{e}{R_1} \qquad (13)$$

Experimental proof of the correctness of equations (**12**) and (**13**) is given in the experimental part of the paper.

Zero Method C. The diagrams of connections for zero method C are given in Fig. 4 and Fig. 5, in which the potentiometers for varying e are not shown.

Zero method C, as may be seen from Fig. 4 and Fig. **5**, is a combination of the two previous zero methods, *A* and *B*. A study of equations (**8**) and (**12**) shows that the $\frac{e^2}{2}$ terms are of opposite signs; therefore, this term may be eliminated from the final result by combining the two methods.

In taking a measurement the operation is as follows; with the commutator vertical and the quadrant d-c. voltage in the grounded side of the quadrant circuit, as shown in Fig. 4, a balance is obtained between the torque due to the alternating and continuous voltages. Then the commutator is placed horizontally and the quadrant d-c. voltage is thrown to the high side of resistance, R_1, as shown in Fig. 5, and a balance again obtained by adjusting the quadrant continuous voltage.

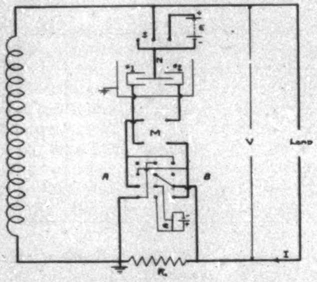

FIG. 6—SWITCHING ARRANGEMENT FOR ZERO METHODS

With the commutator vertical as in Fig. 4, we have

$$V_0 = v + i R_1 - E$$
$$V_1 = -e$$
$$V_2 = i R_1$$

and for Fig. 5

$$V_0 = v + i R_1 - E$$
$$V_1 = i R_1 + e$$
$$V_2 = 0$$

Substituting these values in the general electrometer equation (**3**) and solving, we obtain (**15**) the derivation of which is given in the mathematical part of the paper

$$I V \cos \varphi + \frac{I^2 R_1}{2} = \frac{E e}{R_1} - k \frac{e}{R_1} \tag{15}$$

If the polarities of the two batteries *E* and *e* are reversed, we get the relation (**16**):

$$I V \cos \varphi + \frac{I^2 R_1}{2} = \frac{E e}{R_1} + k \frac{e}{R_1} \tag{16}$$

Test data are given in the experimental part of the paper to prove the correctness of equations (**15**) and (**16**).

The battery *e* may be thrown from one side to the other side of the circuit by means of two double-pole, double-throw switches, or a single, four-pole, double-throw switch, and the connections shown in Fig. 4 and Fig. 5 are combined in a single switching arrangement shown in Fig. 6. It is possible to use zero methods *A*, *B*, or *C* by means of the connections shown in Fig. 6.

A single-pole, double-throw switch, *S*, is shown in Fig. 6 in the needle circuit. This switch when thrown to the left connects the needle directly to the line and permits bringing the mechanical and electrical zeros into coincidence. When the switch *S* is thrown to the right the battery *E* is connected in the needle circuit.

When the four-pole, double-throw switch shown in Fig. 6 is closed on the side marked *A*, the connections are such as to give zero method *A*. When closed on the opposite side, *B*, connections are made for zero method *B*. For zero method *C*, the four-pole, double-throw switch is closed on side *A* when the commutator *M* is vertical and on side *B* when the commutator is placed horizontal.

Method *C* eliminates the term $\frac{e^2}{2}$ from the equation and is therefore simpler to use from the standpoint of calculation. We can determine *k* by taking several electrometer readings in a circuit the loss of which is known. The constant *k*, is evaluated by this method in the experimental part of this paper, for the quadrant electrometer used in the tests.

It is also evident from equations (**15**) and (**16**) that *k* may be eliminated if we take the means of two sets of readings as described under method *A*, the first being taken with the polarities of the batteries as shown in Fig. 6, and the second with the batteries reversed.

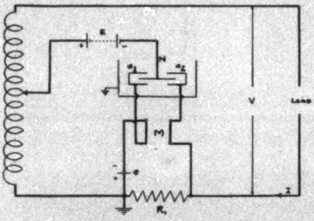

FIG. 7—FRACTIONAL VOLTAGE ON NEEDLE FOR ZERO METHOD *C* COMMUTATOR VERTICAL

We can simplify the equations for the purpose of calculation by writing them in a slightly different form. For example, we can write equation (**15**) in the form of equation (**17**).

$$I V \cos \varphi + \frac{I^2 R_1}{2} = \frac{(E - k) e}{R_1} \tag{17}$$

Zero Method—Fractional Voltage on the Needle. Any one of the three methods described may be used with fractional voltage applied to the needle. Method *C* requires the least amount of calculation and is probably the simplest method to use; therefore it will be the only method discussed with fractional needle voltage. The connections are shown in Figs. 7 and 8.

With the commutator vertical as in Fig. 7, we have

$$V_0 = \frac{v + i\,R_1}{n} - E$$

$$V_1 = -\,e$$

$$V_2 = i\,R_1$$

and for Fig. 8

$$V_0 = \frac{v + i\,R_1}{n} - E$$

$$V_1 = i\,R_1 + e$$

$$V_2 = 0$$

where n equals the factor by which the a-c. voltage applied to the needle must be multiplied to give the total alternating voltage of the circuit.

Substituting these values in the general equation (**3**) of the electrometer, and solving, we obtain (**18**) the derivation of which is similar to that of equations (**15**) and (**16**) which are derived in the mathematical part of the paper.

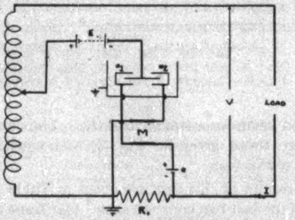

Fig. 8—Fractional Voltage on Needle—Zero Method *C* Commutator Horizontal

$$I\,V\cos\varphi + \frac{2-n}{2}\,I^2 R_1 = n\,\frac{(E-k)\,e}{R_1} \qquad (\mathbf{18})$$

It is evident from a study of equation (**18**) that if n equals 1, it reduces to equation (**17**).

The correctness of equation (**18**) was also verified experimentally.

II. Experimental

The electrometer used in these tests was a Dolezalek quadrant electrometer.

Determination of the Constant, k. The constant, $k = \dfrac{c_1}{b_1}$, was determined by using a resistance load in the circuit of Fig. 6. The load consisted of a non-inductive resistance, R_0, whose value was 260916 ohms. Method *C* was used in determining the constant. The applied a-c. voltage was 201 volts at 60 cycles. The continuous voltage, E, that is inserted in the needle lead, was made up of three 22.5-volt Radio *B* batteries. Its value was measured with a 150-volt d-c. voltmeter. The continuous voltage that is inserted in the quadrant circuit, was taken from a six-volt storage battery connected to a potentiometer rheostat. The value of the continuous quadrant voltage, e, that is actually used in the circuit was measured by a 3/15/150-volt Weston Laboratory Standard d-c. voltmeter. The voltmeters used in the experimental work were checked against a Weston Standard Cell.

The quadrant electrometer was set up and adjusted until its electrical and mechanical zeros coincided. Then the load was applied and readings were taken at different values of R_1, the non-inductive resistance across which the quadrants are connected. Two readings of e were made at each setting of R_1; e_1 with the commutator in a vertical position and e_2 with the commutator in the horizontal position. The value of e was adjusted each time until the deflection was reduced to zero, and e_1 is the value of the quadrant voltage for zero deflection and the commutator vertical, and e_2 for zero deflection and the commutator horizontal.

The readings are given in Table I below:

TABLE I
ZERO METHOD *C*—RESISTANCE LOAD R_0

A-c. Supply voltage	Load R_0-ohms	R_1 ohms	E volts	Commutator		Average e volts
				‖ e_1-volts	= e_2-volts	
201	260916	500	69.4	1.128	1.121	1.1245
		1000	69.45	2.284	2.208	2.246
		2000	69.45	4.63	4.31	4.47
		1000	69.4	2.29	2.20	2.245
		500	69.4	1.127	1.120	1.1235
		1000	69.35	2.284	2.205	2.245
		2000	69.4	4.62	4.32	4.47

The value of the alternating current in the circuit and the total loss measured by the electrometer, which in this case is the loss in the load R_0 plus one-half the loss in R_1, were calculated for each set of readings. From these results the constant, k, was determined from equation (**15**) as shown in Table II. Its value was found to equal 0.8154.

TABLE II
DETERMINATION OF CONSTANT k

Zero-Method *C* R_1 ohms	$I^2\left(R_0+\dfrac{R_1}{2}\right)$ Watts calculated	$\dfrac{E\,e}{R_1}$	Resistance Load R_0 $\dfrac{Ee}{R_1} - I^2\left(R_0+\dfrac{R_1}{2}\right) = k\,\dfrac{e}{R_1}$	k
500	0.1544	0.1561	0.0017	0.7559
1000	0.1540	0.1560	0.0020	0.8905
2000	0.1531	0.1552	0.0021	0.9395
1000	0.1540	0.1558	0.0018	0.8020
500	0.1544	0.1559	0.0015	0.6673
1000	0.1540	0.1556	0.0016	0.7130
2000	0.1531	0.1552	0.0021	0.9395
			Average,	0.8154

Experimental Proof of Zero Method A. Experimental proof of the correctness of the theory involved in zero method *A* and of equations (**8**) and (**9**) that apply to this method was obtained by using a resistance load as described in the determination of the electrometer constant k. The connections shown in Fig. 2 were used in this test, except that the polarity of the two batteries are opposite to that shown in the figure. Therefore, equation (**9**) applies to this test.

The results of the test are given in Table III. The watts consumed by the load R_0 and the loss in the resistance R_1 were calculated from the values used in the circuit as was the case in Table II.

A comparison of the results given in the last two columns of Table III shows good agreement between the calculated watts and the watts as measured by the electrometer.

As stated in the theoretical part of the paper, the constant k need not be determined if two sets of readings are taken, one with the battery polarities as shown in Fig. 2, and the other with the polarities reversed. In order to check this conclusion from the theory a number of sets of readings were taken at different values of R_1, using the resistance load, R_0, of 260916 ohms. The results for two sets of these readings are given in Table IV.

A comparison of the average watts as measured by the electrometer with the calculated watts shows an excellent agreement, and proves that this method may be used if desired to determine watts consumed by a load. A disadvantage of this method lies in the fact that four readings are needed.

In most cases, it is better to determine the electrometer constant k and calculate the power consumed by the load from a single set of readings.

The experimental results prove that Method A and the equations that apply are correct.

Experimental Proof of Zero Method B. The experimental proof of the correctness of the theory of zero Method B and of equations (**12**) and (**13**) was also obtained using a resistance load, R_0. The connections used are those given in Fig. 3, and equation (**12**) applies to this test.

The results of this test are given in Table V and a study of the last two columns shows that there is a good

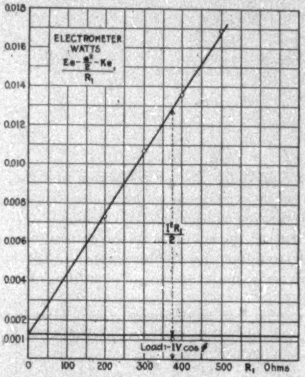

FIG. 9—CAPACITY LOAD (ZERO B METHOD)

TABLE III

ZERO-METHOD A—RESISTANCE LOAD R_0

A-c. Supply voltage	Load R_0 ohms	R_1 ohms	E volts	Commutator		Average e volts	$I^2\left(R_0+\frac{R_1}{2}\right)$ Watts calculated	Electrometer watts $\frac{Ee-\frac{e^2}{2}+ke}{R_1}$
				$\|\|$ e_1 volts	$=$ e_2 volts			
202	260916	100	69.1	0.23	0.216	0.223	0.1563	0.1556
202		500	69.0	1.148	1.112	1.13	0.1559	0.1565
202		1000	69.1	2.318	2.254	2.236	0.1552	0.1538
203		2000	69.0	4.72	4.6	4.66	0.1562	0.1572
203		4000	69.0	9.68	9.42	9.55	0.1544	0.1553

TABLE IV

A-C. SUPPLY VOLTAGE = 203 VOLTS

Remarks Batteries	R_1 ohms	E volts	Commutator		Average e volts	Electrometer Reading		Calculated watts $I^2\left(R_0+\frac{R_1}{2}\right)$
			$\|\|$ e_1 volts	$=$ e_2 volts		$\frac{Ee-\frac{e^2}{2}}{R_1}$	Average watts	
As in Fig. 2.....	100	69.1	0.298	0.16	0.229	0.1582	0.1575	0.1578
Reversed......	100	69.1	0.31	0.144	0.227	0.1568		
As in Fig. 2.....	2000	69.0	4.81	4.64	4.725	0.1574	0.1564	0.1562
Reversed......	2000	69.0	4.72	4.6	4.66	0.1554		

agreement between the calculated watts and the watts as measured by the electrometer.

The experimental results prove the correctness of the theory involved in Method B.

TABLE V

ZERO METHOD B, RESISTANCE LOAD R_0

A-c. Supply voltage	Load R_0 ohms	R_1 ohms	E volts	Commutator ∥ e_1 volts	Commutator = e_2 volts	Average e volts	Calculated watts $I^2\left(R_0+\frac{R_1}{2}\right)$	Electrometer watts $\frac{E\,e+\frac{e^2}{2}-k\,e}{R_1}$
201	260916	500	69.35	1.12	1.111	1.115	0.1544	0.1541
		1000		2.208	2.20	2.204	0.1540	0.1535
		2000		4.33	4.31	4.32	0.1531	0.1527
		1000		2.214	2.192	2.203	0.1540	0.1535

Experimental Proof of Zero Method C. Experimental proof of zero method C follows from the results of the tests of methods A and B and also from the measurements made to determine the value of the constant k which are given in Tables I and II. Therefore, no further readings will be given for the measurement by method C of the power consumed in a resistance load.

Capacity Load. Tests were made to determine the loss in a condenser using zero methods A, B and C. The condenser used in these tests was a 1/10 microfarad standard mica condenser. The results obtained by the three methods checked closely, and are given here for methods B and C only.

Data taken by method B, using the connections of Fig. 2, are given in Table VI.

TABLE VI

CAPACITY LOAD, ZERO-METHOD B

A-C. Applied Voltage = 199 Volts at 61 cycles

R_1 ohms	E volts	Commutator ∥ e_1 volts	Commutator = e_2 volts	Average e volts	Electrometer reading $\frac{E\,e+\frac{e^2}{2}-k\,e}{R_1}$ = watts
200	69.0	0.016	0.027	0.0215	0.00733
300		0.044	0.05	0.047	0.01069
400		0.079	0.08	0.0795	0.01356
500		0.13	0.118	0.124	0.01692

In order to eliminate the correction term $\frac{I^2\,R_1}{2}$ the electrometer[4] watts are plotted against the values of R_1 as abscissas. These points lie on a straight line as shown in Fig. 9. If this line is extended to the ordinate axis it will cut the axis at a value equal to the watts lost in the load.[5] From Fig. 9 we see that the loss in the condenser is 0.00125 watts.

A test was made with the same condenser using method C. In this test the loss in the condenser was calculated from the formula

$$I\,V\cos\varphi=\frac{(E-k)\,e}{R_1}-\frac{I^2\,R_1}{2}$$

The current I equaled 0.00754 ampere as calculated from the constants of the circuit. The readings are given in Table VII and the average loss was found to equal 0.00127 watt and the power factor was 0.085 per cent. The results of the tests by the two methods are in close agreement.

TABLE VII

CAPACITY LOAD, ZERO-METHOD C

A-C. Applied Voltage = 200 Volts at 60 Cycles

R_1 ohms	E volts	Commutator ∥ e_1 volts	Commutator = e_2 volts	Average e volts	Electrometer watts $\frac{(E-k)\,e}{R_1}$	$\frac{I^2\,R_1}{2}$ watts	Watts loss in condenser
200	69.0	0.023	0.018	0.0205	0.00698	0.00569	0.00129
300		0.05	0.036	0.043	0.00977	0.00853	0.00124

TABLE VIII

A-C. Applied Voltage = 62 Volts at 60 Cycles

Method	R_1 ohms	E volts	Average e volts	$E\,e$	$\frac{e^2}{2}$	$k\,e$	Electrometer reading. (Loss in Inductance plus $\frac{I^2\,R_1}{2}$)
A	200	69	1.707	117.8	−1.456	−1.392	0.575
B	200		1.6695	115.2	+1.393	−1.361	0.576
C	200		1.6955	117.0	0	−1.382	0.578

Inductive Load. Readings were taken with an inductive load using all three zero methods. The load consisted of an inductance of about 8. henrys and 1650 ohms resistance.

In this test the polarity of the batteries was such that equation (**8**) applies to the results of method A; equation (**12**) to method B; and equation (**15**) to method C.

The results are given in Table VIII and the loss as calculated includes half the loss in the shunt resistance R_1, which was the same for each method.

A comparison of the results of the three methods given in the last column of Table VIII shows that they give the same loss for the circuit within ± 0.3 per cent.

The experimental results prove conclusively the correctness of the equations as derived, and that the methods may be employed to measure the loss at any value of the power factor leading or lagging.

III. MATHEMATICAL

Derivation of Equation (**8**), *Method A.* Equation (**8**) is obtained by substituting the instantaneous values of the potentials applied to the needle and quadrants in the general equation (**3**) of the electrometer. The electrometer reads the integral of these voltages over a complete period T.

With the commutator vertical in Fig. 2 we have

$$V_0 = v + i\,R_1 - E$$
$$V_1 = -\,e_1$$
$$V_2 = i\,R_1$$

where $v = \sqrt{2}\,V \sin \omega t$

and $i = \sqrt{2}\,I \sin (\omega t \pm \varphi)$

Substituting these values and integrating over a complete period, we get the values of the terms in equation (**3**) as follows:

$$-\frac{b_1}{2} V_1^2 = -\frac{b_1}{2} e_1^2$$

$$+\frac{b_1}{2} V_2^2 = +\frac{b_1}{2}\frac{1}{T}\int_0^T R_1^2 . 2 I^2 \sin^2 (\omega t \pm \varphi)\, dt = +\frac{b_1}{2} I^2 R_1^2$$

$$+ b_1 V_0 V_1 = + b_1 \frac{1}{T}\int_0^T [\sqrt{2}\,V \sin \omega t + R_1 \sqrt{2}\,I \sin (\omega t \pm \varphi) - E][-\,e_1]\,dt = + b_1 E e_1$$

$$- b_1 V_0 V_2 = - b_1 \frac{1}{T}\int_0^T [\sqrt{2}\,V \sin \omega t + R_1 \sqrt{2} I \sin (\omega t \pm \varphi) - E]\,[R_1 \sqrt{2} I \sin (\omega t \pm \varphi)]\,dt = - b_1 (R_1 I V \cos \varphi + I^2 R_1^2)$$

$$+ c_0 V_0 = c_2 \frac{1}{T}\int_0^T [\sqrt{2}\,V \sin \omega t + R_1 \sqrt{2}\,I \sin (\omega t \pm \varphi) - E]\,dt = - c_0 E$$

$$+ c_1 V_1 = - c_1 e_1$$

$$- c_1 V_2 = - c_2 \frac{1}{T}\int_0^T \sqrt{2} V \sin \omega t\, dt = 0$$

With the commutator horizontal the values of V_1 and V_2 change as stated under zero method A, and by substituting these new values of V_1 and V_2 in the terms of equation (**3**) and integrating, we may determine the values of these terms for the commutator horizontal.

The signs and the values of the terms in equation (**3**) for the two positions of the commutator are given below in (**4**) and (**5**).

Pos of Comm.	Equa. No.	VALUE OF TERMS							
	(3)	$-\frac{b_1}{2} V_1^2$	$+\frac{b_1}{2} V_2^2$	$+ b_1 V_0 V_1$	$- b_1 V_0 V_2$	$+ c_0 V_0$	$+ c_1 V_1$	$- c_1 V_2$	$=(1+R\,V_0^2)\ \theta$
||	(4)	$-\frac{b_1}{2} e_1^2$	$+\frac{b_1}{2} I^2 R_1^2$	$+ b_1 E e_1$	$-b_1(R_1 I V \cos\varphi + I^2 R_1^2)$	$-c_0 E$	$-c_1 e_1$	-0	$=(1+R\,V_0^2)\ \alpha$
=	(5)	$-\frac{b_1}{2} I^2 R_1^2$	$+\frac{b_1}{2} e_2^2$	$+b_1(R_1 I V \cos\varphi + I^2 R_1^2)$	$-b_1 E e_2$	$-c_0 E$	-0	$+c_1 e_2$	$=(1+R\,V_0^2)\ \beta$

Taking the algebraic sum of equations (**4**) and (**5**) we get equation (**6**).

$$(1 + R\,V_0^2)\,(\beta - \alpha) = 2\,b_1 R_1 I\,V \cos \varphi + b_1 I^2 R_1^2 - b_1 E\,(e_1 + e_2) + \frac{b_1}{2}(e_1^2 + e_2^2) + c_1 (e_1 + e_2) \quad (6)$$

The experimental results show that e_1 and e_2 are nearly equal and that we may write $e = \frac{e_1 + e_2}{2}$ without introducing any appreciable error. Since the deflections α and β are each zero, we may write equation (**6**) equal to zero and solve for the power consumed by the load.

$$I\,V \cos \varphi + \frac{I^2 R_1}{2} = \frac{E\,e - \frac{e^2}{2}}{R_1} - \frac{c_1}{b_1}\,\frac{e}{R_1} \quad (7)$$

Equation (**7**) contains two of the electrometer constants b_1 and c_1, which may be combined as previously described, in a single constant $k = \frac{c_1}{b_1}$. Under these conditions we may write equation (**7**) in the form

$$I\,V\cos\varphi + \frac{I^2 R_1}{2} = \frac{E\,e - \dfrac{e^2}{2}}{R_1} - k\,\frac{e}{R_1} \qquad (\mathbf{8})$$

If we reverse the polarities of the two batteries, E and e, and proceed as in determining equation (**8**) we find that the power consumed by the load is given by equation (**9**).

$$I\,V\cos\varphi + \frac{I^2 R_1}{2} = \frac{E\,e - \dfrac{e^2}{2}}{R_1} + k\,\frac{e}{R_1} \qquad (\mathbf{9})$$

Derivation of Equation (**12**), *Method B*. The relations giving the power consumed by the load with the connections as in Fig. 3, method B, are derived from equation (3) in a manner similar to that used in finding (**8**).

Pos. of Comm.	Equa. No.	VALUE OF TERMS
	(*3*)	$-\frac{b_1}{2} V_1^2 + \frac{b_1}{2} V_2^2 + b_1 V_0 V_1 - b\ V_0 V_2 + c_0.V_0 + C_1 V_1 - c_2 V_2 \qquad = (1 + R\ V_0^2)\ \theta$
$\parallel$	(*10*)	$0 + \frac{b_1}{2}(I^2 R_1^2 + e_1^2) + 0 - b_1 (R_1 I\ V\cos\varphi + I^2 R_1^2 - E\,e_1) - c_0 E - 0 - c_1 e_1 \qquad = (1 + R\ V_0^2)\ \alpha$
=	(*11*)	$-\frac{b_1}{2}(I^2 R_1^2 + e_2^2) + 0 + b_1 (R_1 I\ V\cos\varphi + I^2 R_1^2 - E\,e_2) - 0 - c_0 E + c_1 e_2 - 0 \qquad = (1 + R\ V_0^2)\ \beta$

Taking the algebraic sum of equations (**10**) and (**11**) and solving for the power consumed by the load we obtain equation (**12**) for connection B.

$$I\,V\cos\varphi + \frac{I^2 R_1}{2} = \frac{E + \dfrac{e^2}{2}}{R_1} - k\,\frac{e}{R_1} \qquad (\mathbf{12})$$

Derivation of Equation (**15**), *Method C*. Method C is a combination of methods A and B, as stated above. With the commutator vertical equation (**4**) of method A holds and with the commutator horizontal we have equation (**10**) of method B. Taking the algebraic sum of equations (**10**) and (**4**) we obtain equation (**14**).

$$(1 + R\,V_0^2)\,(\beta - \alpha) = 2\,b_1 R_1 I\,V\cos\varphi$$

$$+ b_1 I^2 R_1^2 - b_1 E(e_1 + e_2) + \frac{b_1}{2}(e_1 - e_2) + c_1(e_1 + e_2) = 0 \qquad (\mathbf{14})$$

Assuming that e_1 is very nearly equal to e_2 the fourth term of the equation becomes negligible and we can write that the power consumed by the load is

$$I\,V\cos\varphi + \frac{I^2 R_1}{2} = \frac{E\,e}{R_1} - k\,\frac{e}{R_1} \qquad (\mathbf{15})$$

The Correction Term, $k\dfrac{e}{R_1}$. Several methods are possible for eliminating the correction term, $k\dfrac{e}{R_1}$ in addition to those mentioned in the paper. The elimination of this term may be accomplished in method C, for example, by reversing the polarity of the battery in the needle circuit, instead of reversing e as described under this method. This, however, introduces another correction term in the equation which involves E and is, therefore, not to be recommended.

The importance of the correction term, $k\dfrac{e}{R_1}$, can be reduced by use of larger values of the needle battery E. This reduces the value of e needed to balance the torque of the a-c. voltages.

IV. Errors

The sources of error present in any quadrant electrometer have already been ably discussed in an Institute paper[2]. The more important are:

1. A study of the diagram of connections of the quadrant electrometer shows that the electrostatic capacity formed by the two quadrant pairs and their leads is in parallel with the non-inductive resistance R_1. Therefore, the drop across R_1 and this condenser in parallel with it will not be in phase with the current. This error is naturally greater the higher the value of R_1. Taking this into account we find that the equation[6] for method C is as follows:

$$I\,V\cos\varphi + \frac{2-n}{2} I^2 R_1 + I\,(\omega\,c_1 R_1)\,V\sin\varphi$$

$$= \frac{n\,(E-k)\,e}{R_1} \qquad (\mathbf{19})$$

where c_1 is the electrostatic capacity of the quadrant pair circuit in farads.

In measuring the loss in the 1/10 microfarad condenser by methods B and C it was found possible to work with a value of R_1 as low as 200 ohms and still obtain good accuracy. Tests of this same condenser by the deflection method require a value of R_1 of the order of 1000 ohms to obtain satisfactory deflections. It is evident that the error introduced by the capacity of the quadrants depends upon the value of R_1, and that it is reduced in amount by the use of this zero method.

2. Further study of the quadrant electrometer circuit shows that the charging current I_1 from the

needle to the high quadrant flows through R_1 in addition to the load current, I.

This source of error is very small in low voltage electrometers and may usually be neglected.

3. The theory of the electrometer is based upon the assumption that the resistance of the needle circuit is zero, and the needle charging current flows through a pure capacity.

The use of the battery, E, in the needle circuit introduces a resistance, but its value in our tests was so small compared to the capacitance of the needle circuit that its effect was negligible.

4. If fractional voltage is applied to the electrometer needle, the e. m. f. of the portion of the transformer winding to which the needle circuit is attached, may differ in phase from the total voltage in the circuit. This source of error may become very important at low values of the power factor of the load, and in such cases must be corrected for[2].

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Discussion

SYMPOSIUM ON DIELECTRICS AND POWER-FACTOR MEASUREMENTS

(WHITEHEAD, CURTIS, HANSON, LEE, DOYLE AND SALTER, SIMONS AND BROWN, ST. CLAIR, KOUWENHOVEN AND BETZ)

NIAGARA FALLS, N. Y., MAY 26, 1926

P. L. Hoover: I should like to discuss briefly power-factor measurement in connection with bridge methods. There may be some doubt as to the accuracy of any bridge in measuring power factor, for, as Mr. Lee has pointed out, power factor is the ratio of the total power loss to the product of the voltage and the current. Now if a tuned vibration galvanometer is used for detecting the balance point of the bridge, only the losses due to a single frequency and not the total losses are measured. Fortunately in most cases the error that is introduced in this connection is probably very small and negligible. Nevertheless, most a-c. bridges cannot be regarded as precision power-factor bridges, since the power-factor balance is so critically dependent on an accurate capacity balance. For instance, with the Wien or the Schering bridge, if the capacity balance is off by one per cent the power factor may be off by as much as 50 or 100 per cent. Experimentally this means that the capacity balance must be made to a much greater precision than is required for the power factor. Such an experimental condition is to be avoided if possible.

A new type of bridge which is in use at the Harvard Engineering School and which was described recently before the Institute[9] avoids this difficulty. In this new bridge, a mutual inductance between the galvanometer circuit and one arm of the bridge measures the difference in phase of the two sides of the bridge so that the mutual inductance may be calibrated directly in power factor. Furthermore, with this new bridge the power-factor balance is not so critically dependent on an accurate capacity balance and it may thus be regarded as a precision power-factor bridge.

E. W. Davis: The problem of making dielectric loss measurements on reel lengths of cable as a part of routine testing in the process of the manufacture of a cable requires a method of great simplicity and of an accuracy equal to that of the knowledge of the variables involved. The use of direct-deflection wattmeters from the above point of view offers one of the simplest methods of making such measurements.

Two direct-deflection wattmeters recently developed for this service have been thoroughly tested and found satisfactory.

One of these meters is of the single-pivot type with no damping; the other, of special suspension construction wherein jewel and pivot friction losses have been reduced to a minimum. The constants of these instruments have been obtained to a great degree of accuracy and have been found to remain constant over the range of operation of the meters. Both meters have a very small phase angle between the current and potential circuits, an angle considerably smaller than is ordinarily found in commercial instruments. One of the meters is designed with an inverted scale so that while the maximum reading is 0.2 watt, a minimum of 0.01 watt can be read easily and accurately. Check measurements made between these instruments and an Irwin dynamometer, usually used in such tests, show the existing errors to be negligible. Results of tests on hundreds of reel lengths in our factory check very satisfactorily with tests made by more complicated methods in our laboratory.

The lack of suitable standards for the calibration or checking of various dielectric-loss sets has for some time been appreciated. Attempts are being made to construct or use suitable standards of glass or similar material. The use of such standards does not seem to promise high precision of determination of accuracy, but for the present the accuracy may be sufficient.

By use of the same cable sample and very careful determination and control of temperature, duration of application of voltage and mechanical handling of the sample, etc., we have checked three separate methods (two dynamometer and one bridge method) and found them to give results within five per cent of each other. This of course is not high precision but for the present seems to be sufficient.

The calorimetric method of checking dielectric losses discussed by Mr. St. Clair offers a wide field for investigation, but the chances of error are great and the complication and time required reduce it to a very special laboratory test.

The use of identical samples and various series-multiple connections of them does not work out very well in practise due to the impossibility of obtaining exactly identical samples, which of course means uneven distribution of voltages for the various series connections tried.

The paper by Mr. Hanson on the "Accuracy Required in the Measurement of Dielectric Power Factor" offers too much material for adequate discussion at this time. With the tremendous variation of the thermal and electrical constants of insulating materials and with the still greater variation of the thermal characteristics of the medium in which the cable is installed, it is rather difficult to conceive why such great accuracy in the power-factor measurement is required. Purely as a laboratory measurement, such accuracy might be desirable. As a factory routine test we do not believe that such accuracy is necessary.

A dynamometer-wattmeter method of measuring dielectric losses which is radically different from those discussed in this series of papers, was suggested by Prof. Dawes, of Harvard, some years ago and used for a short time in our laboratory.

In this method, the wattmeter and also the ammeter are in the high-tension leads, enclosed in suitable metallic shields which are placed on insulators. The shield for the wattmeter is made sufficiently large to hold the compensating resistances and capacitances.

The potential circuit is supplied by potential transformers, the case of which is also on insulators and at line potential. All leads are shielded as well as ends of the cable sample.

By this method, all leakage currents are eliminated or at least prevented from affecting the meter readings. No run is necessary to determine set losses. By grounding the high-tension lead and the low tension of the potential transformers supplying the wattmeter through a second potential transformer, the potential differences between coils in the meter is reduced to a minimum.

I. M. Stein: I should like to suggest that a complete bibliography be made a part of this symposium. Some authors have given references that they have used in their papers, but no one of the papers gives a complete bibliography.

Mr. St. Clair's talk of shielded resistances brought to mind a recent publication of the Bureau of Standards. This is Scientific Paper No. 516, "A Shielded Resistor for Voltage-Transformer Testing." The Bureau of Standards designed and has been using for potential-transformer testing for a number of years, a shielded resistance that should be of interest in some dielectric-loss measurements.

J. D. Stacy: We have used the phase-defect, compensation, dynamometer-wattmeter method in the measurement of power factor of capacitors very successfully, making several thousand measurements per year. The initial calibration of the instrument was laboratory work involving the method described by Mr. St. Clair. Our general results in the use of this instrument have been very satisfactory.

Delafield Du Bois: What we need most is a means of judging the merit of any given testing set, so that we may not only determine if an installed equipment is satisfactory for its continued use, but may also determine the most satisfactory new test set to install.

I can do no more than outline what such a measure of merit might be in the hope that some one will perfect it.

In judging the merit of a test set there are three primary considerations: accuracy, convenience and cost.

Obviously, accuracy must meet a certain minimum for commercial testing, but greater accuracy than this is usually desirable.

Under the heading of convenience, many things may be listed, but the most important are speed and the skill required to test.

Cost should include not only the first cost, but the cost of such spare parts as might be necessary to carry in stock in order to put the set back in service following a puncture of insulation during test.

Accuracy, convenience and cost may each be expressed numerically so that the sum of the three numbers will be the figure of merit of the test set considered.

The simplest means of reducing accuracy to a number is by establishing a standard test sample, such as the one Mr. Doyle has constructed. If a test set can measure the power factor of this test sample to within, say, B per cent accuracy, the figure of merit for accuracy will obviously be a function of B such, for instance, as $(C - B)\ D = A$ where C and D are well chosen constants. The actual values of C and D to be acceptable will, of course, have to be worked out by conference.

To arrive at a figure of merit for convenience let E be the time in man-minutes required to make two tests under widely different conditions. Then the figure of merit for convenience will be a function of E, such as $(F - E)\ G = H$.

If the skill required by the tester is greater than that of the usual laboratory assistant, E should be corrected in proportion to the increased cost of testing.

If the number of tests per day can be estimated, cost can be reduced to a fixed charge per test. This can be expressed in equivalent man-minutes, and the figure of merit for cost I would be a function of these man-minutes similar to that used in determining the figure of merit for convenience.

If the constants C, D, F and G are well chosen, then $A + H + I = M$ will express the merit of the set.

This is only a suggestion but I believe it expresses an actual need.

Brian O'Brien: In classifying dielectric losses is it desirable to distinguish between (1) normal conductivity and (3) anomalous conductivity? The weight of evidence would indicate that no measurable electronic conduction similar to metallic conduction occurs in dielectrics, and that all conduction here is of an electrolytic character, that is, ionic, even when the conduction obeys Ohm's law. If this be true, the distinction between class (1) and class (3) lies only in the magnitude of the disturbing factors which cause departures from Ohm's law. If such a distinction is useful for the practical treatment of the subject, it might be well to emphasize the fact that the two losses are probably identical as regards the fundamental phenomena.

Granting that no measurable hysteresis effect occurs in dielectrics in the sense of a lag in polarization independent of time as found in magnetic materials, and that thus no permanent polarization of the dielectric as a whole will, in general, exist, this does not necessarily exclude the possibility of permanent polarization of the molecules of the dielectric. Kelvin and others have shown that such a polarization may exist without manifesting itself as a polarization of the whole dielectric. Therefore in dismissing dielectric hysteresis this possible molecular polarization and its effects on other forms of loss, such as absorption, must not be overlooked.

N. L. Morgan: Mr. Lee referred to the method of measuring dielectric loss by the use of the wattmeter and the water-tube multiplier. We have been using this method since 1917. The water-tube multiplier which we have been using consists of a straight quarter-inch glass tube about five ft. long and fitted with short brass tubes for making the taps. The ratio of the taps is 1, 2 and 4. Sufficient common salt is dissolved in the water to obtain the most convenient conductivity.

The form of the water-tube resistance used by Mr. Lee, I understand, was of the coiled-hose type used by Ryan and others. I think that the form of the resistance may be the reason why the check tests shown in Figs. 6, 7, 8 and 9 do not agree. In this resistance, I assume that the hose is coiled. There is an appreciable capacity between turns, and the capacity current passes through the potential coils of the dynamometer. The resistance is shielded against capacity to ground, but not against capacity between turns. This would also account for the large discrepancy between the curves of Figs. 8 and 9 obtained by using the full resistance and a portion of it respectively.

In the circuit I am using, the variation of the resistance of the multiplier with temperature is of no consequence. An ammeter is placed in this circuit and this current and the line voltage are observed at the time of reading the deflection of the dynamometer. Then variation of the multiplier current can be placed in the equation of power, which is

$$W = K\ D\ E\ /\ a$$

where

W = power
K = a constant
D = deflection of dynamometer
E = time voltage
a = current in the potential circuit

We have not found that shielding of the water-tube against capacity to ground is necessary on cable of greater lengths than five ft., if grounded metal is not closer than three ft. to the resistance.

The accuracy has been very good. We have sent cables to various laboratories and they have checked within close limits. As for convenience, a man can take five readings in about ten minutes. The parts, outside of the transformer and dynamometer, cost about 20 cents.

I should suggest that when using this method the potential coil be compensated for eddy currents as described by Rosa in a bulletin of the Bureau of Standards, otherwise the constant K will vary with the line voltage E. I should suggest also that when determining the constant of the dynamometer, alternating current be used instead of direct. I think that with these precautions and proper shielding of the dynamometer and low-tension leads this method of measuring dielectric loss is sufficiently accurate for commercial testing.

R. Notvest: In connection with Dr. Whitehead's paper it seems to me that in describing the process of deterioration of composite dielectrics, the tendency prevails to investigate symptoms only and to pay little attention to the primary cause. In other words, all has been concentrated upon the heating effect and gaseous ionization which are secondary phenomena only when certain investigations led to the belief that a piezoelectric effect is the primary cause.

Very little work has been done in this field. Madame Curie has been the pioneer; also the 1919 A. I. E. E. TRANSACTIONS contain a very valuable paper on the subject by A. M. Nicholson of the Western Electric Co.

It is a fact that practically all insulation materials, with the exception of glass (an undercooled colloidal liquid), contain more or less crystalline substances. This is true of the many grades of porcelain, marble, lava, soapstone and slate. The multitude of synthetic insulation materials consist mainly of an electric inert mineral filler embedded in a plastic or semiplastic matrix of either a natural or artificial gum of the phenol-resin type. Any crystalline substance subjected to a mechanical stress will produce a potential parallel to an optical axis and when subjected to an electrostatic stress, will develop a certain amount of kinetic energy at right angles to the direction of the stress. Since there is a tremendous quantitative difference in the value of the piezoelectric effect of the various mineral substances utilized for insulation materials, in some instances the effect can be identified only through a superfine and tricky arrangement of the means of observation; yet a piezoelectric effect contributes to the deterioration of the paper insulation of power cables since cellulose always contains a certain amount of SiO_2, silicic acid.

Under the direction of Dr. Booth of Western Reserve University, I have spent considerable time within the past few years in investigating this problem and hope to be able to present a paper soon covering the subject.

The matter can be expressed as follows. Whenever an insulation material is of, or has in its structure, crystalline substances, it is subject to a piezoelectric effect. Where such crystalline substances are embedded in a plastic or semiplastic matrix and exposed to an electrostatic stress, the individual crystals have a tendency to reorient themselves so that some optical axis (the one having the highest piezo effect) is at right angles to the direction of the potential pressure. When the direction of the electrostatic stress changes rapidly, a gradual microscopical opening of the structure takes place, increasing the amount and volume of voids and through molecular friction and the emission of charged particles from the crystals themselves, ionizing the occluded or infiltrated gases which in time causes the mechanical rupture and breakdown of the specimen.

J. B. Whitehead: We have had in foregoing meetings a number of excellent papers on the quadrant electrometer as a high-tension wattmeter. We have two such papers in this program, but the greater number are devoted to the electrodynamometer wattmeter. I have worked a great deal with the former, and have found it of great value and accuracy above a certain value of phase difference; but in the very low range, it suffers the serious disadvantage that the fractional value of voltage on the needle introduces a phase error, troublesome to determine and to eliminate. While I have not worked extensively with the electrodynamometer, it would appear that it is subject to the same type of error, as well as others of electromagnetic character, in the voltage circuit.

I have been working recently with the Schering bridge, and I prefer it to all other methods for the measurement of very low loss and power factor at high voltage. It is very rapid and the possible errors are easy to detect and to eliminate; they arise principally in neutral capacity between the two sides of the bridge and to ground. Errors of this type are easily found and suitable screening will eliminate them. Particular attention must be given to the use of guard electrodes for both the sample under test and the air condenser, and to a screening system maintained at all times at the same potential as the test electrodes. This may readily be accomplished by resistance in the ground connection of the guards and their screening systems. Too little attention has been given to this source of error. I have found, for example, that 1200 ohms in a certain case was necessary in the guard-to-ground connection to ensure equality of potential with the test electrode. This connection in many cases is made directly to ground. In the case mentioned, a change of 100 ohms in 1200 was sufficient to unbalance the bridge. The principal disadvantage of the method is the requirement of a high-voltage air condenser, the capacity of which, for best accuracy, must increase with the capacity of the specimen to be measured. Thus the Schering bridge in its simple form is not suited to the measurement of the loss in long samples of cable. With a suitable air condenser and for relatively short cable lengths it possesses very great advantages.

Referring to Mr. Lee's historical review of the use of the electrodynamometer wattmeter, I wish to point out that in 1891, when I entered Rowland's laboratories, I found that in collaboration with Prof. Louis Duncan, he had developed a number of types of electrodynamometer, and was at that time investigating the value of the instrument for the measurement of dielectric loss. In a subsequent paper in the *American Journal of Science* he described a number of valuable methods for its use. Rowland's instruments were subsequently developed and sold by a well-known firm of manufacturers of electrical instruments.

H. L. Curtis: I should like to ask Prof. Kouwenhoven if the resistance of the battery in the electrometer method is negligible. It seems to me that in the final analysis it would have to be mentioned.

Mr. Hoover, in speaking of the bridge methods, stated that by a mutual-inductance method the sensitivity obtained in measuring the out-of-phase current was greatly increased. I regret that there are not a number of papers at this meeting on bridge methods. The bridge methods have much in common, the only difference being in the manner of making the adjustments. In all bridges it is necessary to vary the magnitude and phase of the current in one arm. What it amounts to is that the potential at the terminals of galvanometer is the same at each instant. There are a number of ways by which this can be accomplished. The Schering bridge has been used lately somewhat. This varies the phase by varying a condenser in parallel with the resistance in one arm. The adjustment, however, can be accomplished just as well and quite as conveniently by using the Rosa method where an inductance is in series with the resistance. The compensation can also be accomplished by putting a resistance in series with the condenser, or by a variable mutual inductance properly placed.

I cannot see that with any of these bridge methods or with the electrodynamometer method you are going to get away from the fact that you have to compensate for the magnitude of the current as well as its phase. If you try to avoid compensating one of these, you throw the burden on some other measurement. I also don't see that by using a mutual-inductance method you are necessarily going to gain in sensitivity over what you would obtain by any other method.

Mr. St. Clair mentioned the question of using two condensers in series as a check on the accuracy of measurement. I have always found that a very unsatisfactory method. It may work if you are using very large condensers where you can get them close together in value of capacitance and where the capacitance to ground is relatively unimportant; but with small condensers, the method of checking by using two in series is something which I have always found extremely difficult and feel that the check would be less satisfactory than some of the direct measurements which might be made.

There is one point in connection with Dr. Whitehead's suggestion about the potential of the guard plate of the condenser. It is essential that the potential of the guard plate and the guarded plate shall be the same at every instant. Generally, this condition is very closely approached by using a resistance between the guard plate and the earth, but if for any reason a phase angle is introduced in the measuring arm, an equal angle must be introduced also in the compensating arm.

C. F. Hanson: The particular phase of Mr. Lee's paper which I wish to bring to your attention is in regard to shunts generally supplied for use in connection with the dynamometer wattmeter.

The use of a shunt in measuring the power factor of a sample of cable 10 ft. long is not necessary, but it is necessary in measuring the power factor of a cable 500 ft. long, particularly if the same wattmeter is used for both jobs. The charging current of a 500-ft. cable may be of the order of one ampere corresponding to a voltage of 100 volts per mil, whereas the charging current of a 10-ft. sample of another cable may be only 0.002 ampere corresponding to a voltage of 40 volts per mil. A wattmeter which is sufficiently sensitive to measure power factor when 0.002 ampere is flowing, will not have sufficient current capacity to measure power factor when one ampere is flowing. A shunt is therefore necessary.

A shunt is usually constructed of two resistances and a capacitor of convenient dimensions. The capacitor is connected in series with the current coil of the wattmeter. The first resistance is connected in parallel with the capacitor. The magnitude of this resistance and the capacitor is so chosen that the current coil circuit behaves like a pure resistance at a given frequency. It is then a simple matter of connecting the second resistance, in the form of an Ayrton shunt, across the current-coil circuit containing the shunted capacitor.

The combination of the shunt and wattmeter coil may have a resistance of approximately 200 ohms when the shunt is set on 10, and 20 ohms when the shunt is set on 100. In other words, with a shunt setting of 10, a resistance of 200 ohms is connected in series with the cable and in some cases the error introduced may amount to 0.002 in the power-factor reading. With a shunt setting of 100 the error is only 0.0002 and is of no significance. The shunt settings chosen in this case typify those which might be used in an ionization test (the increase in power factor as the voltage stress is increased from 20 volts per mil to 100 volts per mil). Usually, by acquainting himself with his power-factor apparatus, the operator can avoid the foregoing error to a great extent if he exercises discrimination in the choice of his shunt setting.

One difficulty with most shunts is that the resistance coils in them are not provided with sufficient means for dissipating heat. This deficiency usually results in extraordinarily rapid deterioration.

The paper by Messrs. Doyle and Salter states that, in the case of three-phase measurements, they carefully control the voltage on a particular phase at the time when power factor is being measured on that phase. This voltage control does not eliminate a phase error, referred to in Appendix IV, which arises from inequality of voltage on the three phases. A similar error arises under two other conditions. The first is the condition of less insulation thickness on one conductor than on each of the other two, even though equality of voltage exists on all three phases. The second condition exists when the voltage vectors of the three phases are not equally spaced by 120 deg., even though the magnitude of the vectors may be equal and the insulation of the three conductors may be the same.

In Fig. 1 herewith are shown the vector relations between charging currents flowing in the cable and the voltage applied to each conductor. The three voltage vectors are equally spaced at 120 deg. and the cable is symmetrical including equality of insulation thickness on the three conductors. The cable is considered to have zero power factor. The vectors E_1 and E_3 are of equal magnitude but E_2 is greater in magnitude. In other words, transformers No. 1 and No. 3 are supplying voltages of equal magnitude but transformer No. 2 is supplying a voltage of greater magnitude.

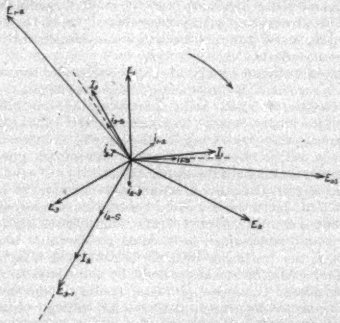

FIG. 1—THE VECTOR RELATIONS OF CURRENT AND VOLTAGES IN A THREE-CONDUCTOR CABLE WHEN IT IS CONNECTED TO A THREE-PHASE, Y-CONNECTED TRANSFORMER BANK

Fig. 1 is shown for a *Y*-connection of the supply transformers. The common point of the transformers is connected to the sheath of a three-conductor cable and to earth. The high-voltage terminals of the transformers are each connected to a conductor of the cable. The various vectors shown are as follows:

E_1	is the voltage to neutral on conductor No. 1.
E_2	is the voltage to neutral on conductor No. 2.
E_3	is the voltage to neutral on conductor No. 3.
i_{1-s}	is the charging current flowing from conductor No. 1 to sheath.
i_{2-s}	is the charging current flowing from conductor No. 2 to sheath.
i_{3-s}	is the charging current flowing from conductor No. 3 to sheath.
E_{1-2}	is the voltage between conductor No. 1 and conductor No. 2.
E_{2-3}	is the voltage between conductor No. 2 and conductor No. 3.
E_{3-1}	is the voltage between conductor No. 3 and conductor No. 1.
i_{1-2}	is the charging current flowing from conductor No. 1 to conductor No. 2.
i_{2-3}	is the charging current flowing from conductor No. 2 to conductor No. 3.

i_{3-1} is the charging current flowing from conductor No. 3 to conductor No. 1.

I_1 is the vector sum of i_{1-S}, i_{1-2} and i_{3-1} and is the total current flowing into conductor No. 1.

I_2 is the vector sum of i_{2-S}, i_{2-3} and i_{1-2} and is the total current flowing into conductor No. 2.

I_3 is the vector sum of i_{3-S}, i_{3-1} and i_{2-3} and is the total current flowing into conductor No. 3.

As the cable is considered to have zero power factor, the vector I_1 should be in quadrature with the vector E_1. The quadrature position is shown by the dotted line. Fig. 1 shows that I_1 is less than 90 deg. ahead of E_1. This condition exists because the vector E_2 is greater than E_1 and E_3, and indicates the error that will exist in the power factor as read on conductor 1. As the vector I_2 is in quadrature with the vector E_2 no error will exist in the power factor as read on conductor No. 2. The error of the power factor as read on conductor No. 3 will be of equal magnitude as the error existing in the conductor No. 1 power factor but of opposite sign.

The dielectric loss as read on conductor No. 2 will be abnormally high because the charging current I_2 is erroneously high. On conductor No. 1 dielectric loss will be abnormally high because of the erroneously high power factor. On the other hand, on conductor No. 3 dielectric loss will be abnormally low because of the erroneously low power factor.

In Fig. 1 of the paper by Messrs. Doyle and Salter the shielding problem is considerably simplified if the ground is removed from the lead sheath of the cable, and connected to the low-voltage terminals of the transformers. Any current then leaking over the transformer bushings and bus insulators will not flow through the current coil of the wattmeter but will return directly to the grounded terminals of the transformers.

In their Fig. 2, the ground cannot conveniently be removed from the lead sheath of the cable because it is not convenient to connect the current coil of the wattmeter into the high-tension lead from the transformer to a cable conductor. Therefore, in this case, the transformers must be insulated from ground. The case of each transformer should be connected to its low-voltage terminal. Likewise, all metal shields of the insulators of each bus should be connected to the low-voltage terminal of the transformer respective to the bus. Currents leaking over the transformer bushings and bus insulators will then return to the low-voltage terminals of the transformers without passing through the current coil of the wattmeter. These precautions are indicated in their Fig. 2 but hardly with sufficient clearness.

E. S. Lee: We have been told that there are no standards for these measurements. The question then is, how do we know we are accurate when we obtain a value?

In the General Electric Company, we used the dynamometer wattmeter first and obtained certain values. We had an opportunity to compare these with others observed more or less directly and they looked about right. Then we set up a Schering bridge and a year ago I reported our results. At that time I said that with the usual observers day in and day out we would check in general within about 0.002 on power factor, that is, 0.2 of one per cent. But we might expect at any time a difference of perhaps 0.004.

I always have wanted to see if we could not use some other method. In the paper he presented Mr. St. Clair told you of a calorimetric method which was used by himself and others in standardizing measurements made on capacitors. This was not an easy problem, either. It seems that there are difficulties at 2500 to 3000 volts, just as there are at the higher voltages.

From this work, it was found that the calorimeter method enabled these measurements as carried on in different laboratories to be standardized. Thus it occurred to us to apply similar methods to cables.

Two pieces of three-conductor cable, each 35 ft. long, were placed horizontally in a closed box as represented diagrammatically in Fig. 2 herewith. The box was 21 ft. long enclosing the central portion of these lengths.

Thermocouples were systematically placed along and around the sheaths of the cables over an 8-ft. length, the "hot" junctions being placed on one cable and the "cold" junctions on the other.

With copper-ideal thermocouples and a galvanometer of sensitivity 6.5 x 10^{-6} amperes per millimeter deflection at one meter scale distance, it was found that by using nine thermocouples in series for each unit, a temperature difference of 1/75 of 1 deg. cent. between the sheaths was indicated by a deflection of one cm. on the galvanometer scale. In watts, this means that one cm. deflection indicated a difference of power dissipation of 0.005 watts per ft.

Measurements were made by applying three-phase potential to one cable, and circulating direct current through the conductors of the other cable until there was thermal equilibrium with the same sheath temperature as indicated by zero deflection of the galvanometer. The d-c. watt input was then calculated and considered to be equal to the dielectric a-c. power loss.

At this time we have only completed one test, the results of which are shown in Table I.

There are three tests: Nos. 1, 2 and 3.

The values of three-phase power loss calculated from single-phase measurements in watts per foot are: 0.266, 0.276 and 0.275, or an average of 0.272.

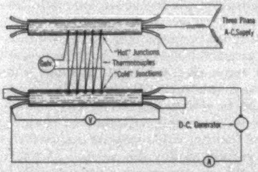

FIG. 2—SCHEMATIC DIAGRAM SHOWING LAYOUT FOR DETERMINATION OF DIELECTRIC POWER LOSS IN CABLES BY COMPARISON OF HEATING

By comparison of heating, the measured power loss in watts per foot for tests Nos. 1, 2 and 3 are: 0.297 which compares with 0.266; 0.289 which compares with 0.276; and 0.280 which compares with 0.275. The average is 0.282.

The differences are a little larger on the individual tests, but the average gives 0.272 three-phase power loss, calculated from single-phase measurements as against 0.282 by comparative heating, a difference of not quite four per cent.

Of course, if the watts check, the power-factor values should check. The value we obtain by taking an average of the results obtained in the three-conductor high measurement and the one-conductor high measurement, gives a value of 0.74 per cent. The power factor from the comparison of heating test was 0.72 per cent.

The voltage was a little bit higher in the comparison of heating tests: 36.0, 35.8 and 36.2 kv.

The results of these measurements are: first, our method of determining the power loss from three-phase supply from single-phase measurements seemed to give correct results; and second, that our standards in the outfit, our air capacitor and our method of compensation and our method of determining the watt constant evidently were quite correct.

The next thing we shall do will be to interchange the cables and then test at other voltages.

The thing that appeals to me is this; that here is a set-up that anybody can put up without very much trouble and be able to satisfy himself very adequately, I believe, of the accuracy of his own results by an independent method. The equipment is not considerable, though the time required is quite long, one or two days being required for each test.

Because of the most satisfying agreement obtained on these first measurements with the dynamometer-wattmeter equipment, it is our hope that other laboratories will provide similar set ups as a means both of checking their equipments and of learning more concerning this method.

The results are satisfying to us, and I believe that in general the method is simple enough so that it provides a satisfactory method which can be used by different observers in different laboratories to satisfy themselves of the reliabiity of their results.

TABLE I

COMPARISON OF RESULTS

DETERMINATION OF DIELECTRIC POWER LOSS IN CABLES

Tests on three-conductor, 350,000-cm. (Sector) Cable, 7/32in. by 3/32 in. Treated Paper Insulation 1/8 in., Lead 15 kv.

A. BY COMPENSATED DYNAMOMETER WATTMETER

Test	Three-Phase Power Loss (Calculated from single-phase measurements) Watts per Foot	Power Factor Per Cent		Voltage Kv.
		Three-Cond. High	One-Cond. High	
1	0.266	0.70	0.78	35.3
2	0.276	0.60	0.88	35.3
3	0.275	0.59	0.87	35.3
Average	0.272	0.74 per cent		

B. BY COMPARISON OF HEATING

Test	Measured Power Loss Watts per Foot	Power Factor	Kv.
1	0.297		36.0
2	0.289		35.8
3	0.260		36.2
Average	0.282	0.72 per cent	

E. H. Salter: Mr. Hanson has gone to great length to show what he terms the limits of accuracy in power-factor measurements of high-voltage cable. While a high degree of accuracy is desirable in such measurements, it seems that the stress is being laid on the wrong point when it is considered that two of the usual short samples of cable taken off the same end or reel of cable may show characteristics which differ by as much as 50 to 100 per cent, while the variation over a line, miles in length, may be much wider.

Mr. St. Clair suggested a method for checking dielectric-loss-measuring equipments by securing four identical condensers and after determining the characteristics of each, using series-parallel combinations. In precision measurements of the dielectric loss of condensers shielding and guarding play quite an important part. In making series-parallel combinations proper guarding would be difficult to obtain.

D. M. Simons: Mr. Lee's calorimetric method of comparing losses is of interest and should be of value. I think one warning should be included, and doubtless Mr. Lee had this in mind. Mr. Lee proved equality of losses in two cables by equality of temperature rise. This is true only if the two cables have equal thermal resistance from the sheath outward. The differences in surface thermal resistivity of different samples of leaded cable have been found to be great sometimes, though this effect should be minimized by the small temperature difference involved in Mr. Lee's method. This possible error can be eliminated by the method Mr. Lee has proposed of interchanging the alternating current and direct current between the two cables, or it could be checked by heating both cables with equal values of continuous current, and proving equal temperature rises.

I should like to confirm Mr. Hanson's conclusion of the necessity for accurate determination of current-carrying capacity, and especially the effect of having a large number of cables in a duct bank with very high voltage cables. 66,000-volt cable is now in operation in two places with two circuits per duct bank. I doubt if it would be economical to include another circuit in the same duct bank, though it may be necessary in certain special cases. For 132,000-volt cable, careful consideration must be given in each case as to whether or not it is economical to use more than one circuit in a duct bank; that is, each circuit can carry less power if two circuits are in one duct bank than if they were in two separate duct banks. The gain in carrying capacity obtained by using separate duct banks may be worth the cost of one additional conduit line.

Mr. Hanson's figures show a rather dangerous condition of instability of temperature in some of the higher-voltage cables, particularly with large numbers of cables per duct bank. I do not believe the actual conditions are as bad as shown; I am under the impression that Mr. Hanson figured the temperature rise based on the calculation of dielectric losses assuming a constant power factor throughout the entire body of insulation. In an actual single-conductor cable with thick insulation there is usually a considerable difference of temperature between conductor and lead, which means that the power factor of the entire cable is by no means that of the layers of insulation near the conductor, and therefore the actual watt losses may be considerably lower than those used by Mr. Hanson.

As pointed out elsewhere, I believe the temperature rise of conductor above lead due to dielectric loss should be figured in terms of a constant power factor of the insulation at the maximum conductor temperature. For the temperature rise of the lead sheath above the duct structure and of the duct structure above the original no-load temperature of the earth, the actual watt loss in the cable should be used, including the fact that there is a temperature gradient in the insulation. If this method of calculation is used, the conditions of stability shown by Mr. Hanson will be probably more favorable.

A few years ago[1] Mr. Brown and I showed that the quadrant electrometer could be used in a null method by bringing back the deflection to zero by the insertion of a resistance in the lead to the electrometer needle. If the initial deflection should be "negative," we had to adopt special means. Dr. Kouwenhoven's method is, I believe, a distinct improvement. He reduces the deflection to zero by the insertion of a continuous potential in the needle circuit (as well as across the quadrants), but he has the advantage of being able to reverse the polarity of the battery in the needle circuit, and can thus use his method to balance deflections which are initially negative. I have not had an opportunity to try out his method, and do not know which of the two is the simpler, but I am sure that all those who have used electrometers will appreciate this new and valuable method of attack suggested and worked out by Dr. Kouwenhoven.

B. W. St. Clair: I am in agreement with the comments of Dr. Curtis about series-parallel methods of checking the voltage accuracy of a test equipment, when the samples are of small capacitance when compared with the capacitance of leads and parts of the outfit to ground. With samples of appreciable size where currents of milliamperes or amperes magnitude circulate, I believe the method is an excellent one for checking relative voltage accuracy. As pointed out in the paper it does not give a clue to absolute accuracy but does give a definite check on the accuracy of the shape of the voltage-power factor curve of a given test equipment.

I have been much interested in the self-contained instrument for checking dielectric loss of reel lengths of cable that was mentioned by Mr. Davis. In the design of one of these we had to depart somewhat from the more usual portable-instrument practise. Ordinarily a wattmeter is built to have full scale deflection at about 60 per cent power factor at rated current and voltage. Under these conditions it is possible to have very good operating characteristics without excessive losses. It is possible by sacrifice of torque and with increased potential and current-circuit losses to build portable instruments for operation at 30 per cent or even 20 per cent power factor. To reduce this to the 1.5 per cent or 2 per cent necessary for cable tests means at least a ten-fold increase in instrument sensitivity. Reduction in operating torque below this 20-per cent point will result in unsatisfactory performance from bearing friction, and likelihood of trouble from stresses incidental to transportation. Any attempt to retain a reasonable operating torque by the use of greater ampere-turns on the armature results in increased armature weight or increased phase-angle corrections or increased potential losses. For a given weight of armature there is a minimum operating torque below which operation will be unsatisfactory. The net result is that there is no compromise in electrical characteristics possible with the standard bearing construction. Two alternatives are possible; the use of the monopivot construction or the use of a strip suspension that will serve the dual purpose of torque member and supporting member of the armature. I have preferred the latter method because it completely obviates the bearing trouble that is almost necessarily coincident with the rather heavy weights and low torque of a well damped sturdily built armature, even in a single-pivot construction.

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[1] TRANSACTIONS of A. I. E. E., 1924, p. 311.

The suspension arrangement is not a panacea for all instrument failings and it does impose double duty on the suspension members. Fortunately the development of special bronzes and processes incidental to good spring characteristics has brought forth sufficient knowledge of the limitations of such doubly stressed material as to permit easy design for satisfactory operation with exceedingly good electrical characteristics. The dependability of such devices will be inferior to the more robust double-pivot ones but will be satisfactory for checking losses on a factory or routine basis.

I am a firm believer in calorimetrical methods of checking the fundamental accuracy of a dielectric test outfit. It is the only direct method I know that measures the losses in terms of some physical quantity that is easily amenable to accurate measurement. Thermal work of this sort requires a high degree of skill and considerable patience. It is a method that has been carried to a high level of certainty and usefulness by biologists interested in general metabolism work. It is quite customary for them to make tests of heat output where the total heat flow is but a very few watts with an error not over five per cent. I have seen many tests where the claimed error was less than two per cent. It is possible that we could learn much from a survey of their present methods and equipment.

C. A. Adams: This thermal-balance method of measuring dielectric losses seems to me a very interesting one. Mr. St. Clair spoke of the difference between the location of the material in which the heat is generated and the conductor material, and Mr. Simons spoke of another possible error, the difference in thermal conductivity.

I feel sure that neither of these considerations affects the test as outlined by Mr. Lee. When I saw that he had a long conductor with various sections and that the flow of heat must be substantially radial at the central section, it was obvious that the relative thermal conductivities have no influence. The amount of heat dissipated from the surface is dependent wholly upon surface conditions. The heat is practically all dissipated by convection, and at those low temperature differences is dependent wholly upon the nature of the surface and possible obstructions.

The thermocouple leads may possibly offer some such obstructions, but apart from that, as between two lead sheaths, it is quite unlikely that there will be any significant difference.

The question as to whether the heat is generated in the conductor or insulation has nothing to do with the case as long as the heat flow is radial.

One of the authors spoke of the use of reactance in series with the supply transformer for smoothing out the e. m. f. wave. Ten years ago when I was making tests on some 25,000-volt cable in Boston, about five miles in length, where the charging current was some 1600 kv-a., we used that method for two purposes, to get double the transformer voltage on the cable and also to smooth out the wave.

We heard a great deal in the old days about the danger of resonance, but if you have resonance with the fundamental, you are a long way from resonance with the harmonics. The reactance in series, through the medium of almost infinitesimal harmonic current, will absorb practically all of the harmonic voltages, so that the actual voltage impressed upon the cable was practically sinusoidal, so nearly so in this case that it was impossible to detect the difference on the oscillogram.

The reactance proved useful in another direction, since any incipient failures in the joints which resulted in small transient currents gave distinct knocks in the reactance. You could put your ear to the core of this reactance and hear every little spit in the joints and there were a good many of them when the voltage was 50,000, which was double the normal operating voltage of the cables. That was some time ago when we didn't know as much about making joints as we do now. But in one case we stopped the test before failure and examined the junction boxes, 13 of which were found smoking. They had a semi-liquid filler compound through which these transient discharges had taken place.

Just one word in regard to this whole question of dielectric phenomena. There are few who realize how differently we employ words and what different meanings those words convey to different minds. I refer to some of the remarks concerning anomalous conduction. The fact is that we are dealing mostly with the superficial side of all of these phenomena. We don't know what actually is going on.

For example, take Maxwell's theory or hypothesis concerning the absorption current. That is what you might call an equivalent-circuit scheme and if it fits the facts it is very useful. And it may, as has been shown by Mr. Dunsheath, be transferable; that is to say, our knowledge of the absorption current may enable us to predict approximately what the dielectric loss is going to be; but that does not prove the hypothesis to be correct.

I think Dr. Whitehead is absolutely correct in saying that barring ionization (and in a good cable there should be very little ionization) the major factor of the dielectric loss is that which is represented by the absorption current, but these are all superficial considerations and tell us nothing of the ultimate nature of the phenomena.

There are a number of hypotheses that delve a little bit more deeply and have to do with the ionization of the dielectric material itself and with the tearing off of electrons from the molecules or atoms but we haven't yet come to the point where we can connect up the actual, practical phenomena that we observe with the fundamental laws of atomic structure.

There has been a great deal more written about this than most of us realize, but it has been done by scientists in their quiet way, and most of their articles are buried in the proceedings of the highly scientific societies. If we were to try to understand a little more thoroughly the ultimate nature of the phenomena and follow more closely the work that is being done by our scientific friends, we might perhaps arrive a little more quickly at some fundamental relationships which would enable us to predict from knowledge of the elements that go into a composite insulation what is going to happen under specified conditions.

As yet our work is too empirical, too superficial. Frankly, my real interest in this problem has to do almost wholly with that delving into the ultimate nature of things, trying to understand a little bit more the reason why, rather than merely collecting a lot of superficial information which in the long run requires much more time.

Delafield DuBois: At one time I was associated with the Russell Sage Institute of Pathology, where we had a calorimeter to determine the heat radiated by the human body. To give an idea of the extreme sensitivity of the apparatus, let me tell of an incident. The doctor in charge of the laboratory happened to be in the calorimeter flat on his back and not moving, but looking out through the window, he saw me enter the laboratory. Not long after that the assistant who was plotting the curve of heat generated vs. time called me over to the test table and showed me a slight kink in the curve that indicated the time when I came in. Apparently the mere sight of me made a slight difference in the heat generated by the doctor.

C. L. Kasson (communicated after adjournment): In regard to measurement of dielectric loss and power factor,—are not the results dependent on wave form, especially in short-length measurements? What agreement can be expected between the results on ten-foot samples and reel lengths?

Has the possibility of end losses in short samples been investigated as a source of error? These end losses must consist of leakage over the end and energy dissipated into the air and then picked up again on the lead sheath. The usual form of guarding does not take into consideration this latter factor which is important.

P. A. Borden (communicated after adjournment): Except, possibly, in those laboratories where the facilities of time and equipment are almost unlimited, the outlook upon the measurement of dielectric losses is at best a rather gloomy one; and we must thank Mr. Lee for having introduced a distinct ray of optimism at the psychological moment. The thermal-balance method, which he describes, while at once scientific and precise, is at the same time capable of application in almost any electrical laboratory, and with equipment which is neither expensive nor highly specialized in its construction or operation.

As an alternative arrangement, and as a possible improvement, I would suggest the replacement of the thermocouples by a resistance bridge, the varying arms of which would consist of a number of turns of enameled magnet wire wound upon the sheath of the cable. There is no doubt that the assembling of this circuit could be more quickly accomplished than the placing of the thermocouples which Mr. Lee has described; and there is the added advantage of the superior sensitivity of the circuit, as well as of increased precision due to the equivalent of an infinite number of points of thermal contact.

If, as has been anticipated by some, there be a source of error due to difference in the conditions of the lead surfaces, it would be possible to surround the cables with oil baths of small dimensions, or to immerse the two sections in a common bath, either of which measures should reduce such an error to negligible proportions.

J. B. Whitehead: The question was asked why, in my list of causes of phase difference in dielectrics, I had included three different types of conductivity, and why all types of conductivity are not the same. There is every evidence that electric conductivity in every instance consists of the motion of some kind of an electric charge, but there are many different kinds of charges,—for example, electrons as in the process of metallic induction, molecular ions and molecular aggregates of various sizes. The presence of any one of these types of ions will result in conductivity and to that extent they well might be included in one class, but the conductivities resulting from these several types of ions follow quite different laws as regards voltage, temperature, frequence, etc. Until these laws are brought together, therefore, and until the nature of dielectric absorption is better known, it appears to me advisable to separate the conductivities arising from different causes.

C. F. Hanson: Mr. E. W. Davis has expressed his doubt of the necessity of the high degree of accuracy which I have prescribed for power factor measurements of 132-kv. cable, in view of the variation existing in thermal constants and the inaccurate knowledge of them. Mr. E. H. Salter has expressed a similar doubt but for a different reason. His reason is that the power factor of a cable is too far from being uniform to warrant the accuracy prescribed. Both of them are correct in their views if the subject is considered from the point of view of present knowledge of thermal constants and the usual uniformity of power factor. This subject, as far as 132-kv. cable is concerned, should not be approached from that point of view. Rather, it should be considered from the point of view of the requirements of the accuracy of our knowledge of all the properties pertaining to the performance of a 132-kv. cable. If a 132-kv. cable is to perform in accordance with experience obtained with lower voltage cables, then I believe the accuracy I have prescribed is hardly too rigid, particularly for a six-cable duct bank. This statement, of course, implies that the power factor of the cable has to be uniform. The thermal properties of the cable and of the duct must also be uniform and known to a high degree of accuracy. If these conditions cannot be met, then it is possible that a 132-kv. cable will have to be replaced by a lower voltage cable.

I have used the term "uniform" loosely. In the case of power factor, it means a permissible variable excess above a certain value considered to be satisfactory. A deviation below this certain value is of no serious consequence because it does not impair the expected current capacity of the cable.

Power-factor uniformity varies with the operating voltage of the cable. In the case of 132-kv. cable the uniformity should be of the order of the accuracy required in the power-factor measurements. The case of lower voltage cables is quite a different story. For example, the permissible power-factor error and uniformity of a 13-kv. cable may be of the order of six per cent, depending upon the knowledge of the thermal constants. I am sure that this accuracy is not alarming and this uniformity is very likely well within the limits stated by Mr. Salter.

Mr. D. M. Simons has called attention to a rather important item in the matter of calculating the actual dielectric loss existing during the operation of a 132-kv. cable. The method which I did use is that prescribed by him in his paper which I have cited. I did not, however, follow the advice which he also gives in his paper, that in some cases a second calculation should be performed to obtain more nearly the actual dielectric loss. I have now done that for one cable.

Before proceeding to explain the method I employed in my second calculation, I shall present a few definitions in an attempt to clarify the subject.

1. The dielectric loss in a cable is usually considered to be that which exists when the entire insulation of the cable is at the same temperature. This is the loss measured in the laboratory according to American practise. I shall designate this loss as the "measured dielectric loss."

2. In actual use the entire insulation is not at the same temperature. The insulation near the conductor is at a higher temperature than the insulation near the sheath. The power factor of the insulation near the conductor, therefore, will be higher than the power factor near the sheath according to the power factor–temperature characteristic of the insulation. The dielectric loss of the cable under this condition, when the conductor is at the maximum permissible operating temperature, will be less than the "measured dielectric loss," and I shall designate this loss as the "actual dielectric loss."

3. The "actual dielectric loss" is distributed throughout the insulation of the cable. It is greater near the conductor than it is near the sheath, for two reasons:

   a. The electric stress is greater near the conductor than it is near the sheath, and

   b. The power factor is greater near the conductor than it is near the sheath.

In calculating current capacity it is difficult to deal with the "actual dielectric loss" as a distributed loss. It is much more convenient to deal with it as a concentrated loss on the surface of the conductor. This procedure may be followed by considering a fractional part of the "actual dielectric loss" as concentrated on the conductor surface. This fractional part of the "actual dielectric loss" I shall designate as the "conductor-equivalent dielectric loss."

The $R I^2$ loss in the conductor and the "conductor-equivalent dielectric loss" raise the temperaure of the conductor above that of the lead sheath. The $R I^2$ loss in the conductor, the "actual dielectric loss" and the sheath eddy-current losses, raise the temperature of the sheath above the base temperature of the earth.

In my calculations I used 50 per cent of the "measured dielectric loss" as the "conductor-equivalent dielectric loss."

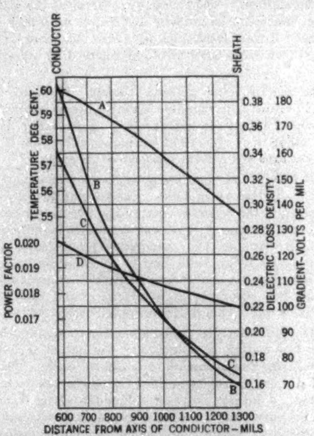

Fig. 3—Actual Condition Existing in Cable No. 15

A—Temperature deg. cent.
B—Dielectric loss density, watts per ft. per 0.06-in. thickness
C—Voltage gradient
D—Power factor

I shall demonstrate by example that this procedure is not always rigorous. For my example, I have chosen Cable No. 15, the power factor-temperature characteristic of which is shown in Fig. 5. This cable was calculated to have a current capacity of 187 amperes under the conditions specified.

I divided the insulation of the cable into a series of 12 concentric cylinders, each having a wall thickness of 0.06 in. (approximately 1/16 in.). I started with the inner cylinder against the conductor. I calculated the temperature and the voltage gradient of this cylinder 0.03 in. from the surface of the conductor. With these values I calculated the dielectric loss per foot for the first cylinder. The second cylinder from the conductor will have this dielectric loss flowing through it in addition to the copper loss. After having calculated the temperature and voltage gradient in the second cylinder, I calculated its dielectric loss. I proceeded in this manner until I had reached the lead sheath. The results of this series of calculations are shown in the accompanying Fig. 3.

A summary of the results follows:

The "measured dielectric loss" of the cable at 60 deg. cent. is 3.09 watts per ft.

The "actual dielectric loss" of the cable when the copper temperature is 60 deg. cent. is 2.91 watts per ft.

The "conductor-equivalent dielectric loss" was found to be 1.46 watts per ft., which is 50 per cent of the "actual dielectric loss" and 47.2 per cent of the "measured dielectric loss."

It appears, therefore, that I have gone wrong in my original calculations to the extent that I used 50 per cent of the "measured dielectric loss" instead of 47.2 per cent as the value for the "conductor-equivalent dielectric loss."

I believe that the summarized results show that my original calculations are approximately correct, although they are not absolutely correct. The difference is even not as much as it appears because there is a compensating error. In Fig. 5, I have shown the unstable power factor–temperature characteristic, *E F*, as a straight line. In reality this is not a straight line but is a curve which droops at the higher temperatures. This drooping probably offsets entirely the little advantage gained in the revised calculations. In conclusion, I believe so far as calculations are concerned, that my Figs. 5 and 6 show very nearly a correct picture. The thermal constants I have used may not fit all cases by any means, but they do most likely fit an average case.

Mr. Lee: As regards the nature of the surface of the cable in the heat comparator method, that certainly is very important and we must take it into account. There is, however, this fact; that there is only a one- or two-deg. rise which enters into the proposition and it is simplified in that regard.

There is one question I should like to ask Mr. Simons. It comes up in connection with the use of the electrostatic wattmeter, and is about the maximum voltage at which he knows it can be used. I have stated in my paper that we have to go up to 100,000 volts in these measurements. As I understand it, the electrostatic wattmeter has not been used much above 35,000 volts. Above that voltage several disturbing factors are introduced.

Also, Prof. Whitehead spoke regarding the size of a resistor that he had which wasn't very large. Again, I had in mind 100,000 volts.

W. B. Kouwenhoven: Dr. Curtis mentioned the effect of the resistance of the battery. The resistance of the battery which is inserted in the needle circuit of the electrometer in the method described by Betz and me has a negligible effect upon the compensation. Simons and Brown produced zero deflections of the electrometer by the insertion of a resistance in the needle circuit. The value of the resistance needed in their method is of the order of 20,000 ohms. The resistance of the battery that Betz and I used in the needle circuit was small and therefore had no effect upon the results obtained.

Messrs. Simons and Brown state on the third page of their paper that one way of eliminating the charging current to the needle is to put the ground at the mid-point of the quadrant resistance R_1. I have tried this method experimentally and found that it does not eliminate the trouble.

One advantage that the electrometer wattmeter possesses over the dynamometer wattmeter is its simplicity and range. If various sized samples are to be tested with a dynamometer wattmeter, either the instrument must be shunted or else several different current windings must be available. In the case of the quadrant electrometer wattmeter, different ranges may be obtained by changing the value of the resistance R_1, which shunts the quadrants. With the same electrometer I have measured losses in cables which were only a foot in length, whole reel lengths of cables and current-limiting reactors. In the measurements the values of the resistance R_1 varied from several thousand ohms to one one-hundredth of an ohm. The current ranged from a few milliamperes up to 200 amperes. This indicates clearly the wide range of the electrometer.

I used the zero method described by Betz and me for measuring the loss in the current-limiting reactor mentioned above. The measurements were made in the field and the results were quickly determined. The method worked very satisfactorily.

A number of different methods have been discussed here for measuring the power loss or power factor of cables and dielectrics. They are as follows:

Electrodynamometer wattmeter, bridge methods, quadrant electrometer wattmeter, and calorimeter methods. In using the bridge methods it is necessary to balance both for ratio and phase angle. The accuracy of the results depends to a large extent on the ratio of the bridge. The closer the ratio is to unity the greater the accuracy and ease of obtaining a balance. Consequently the bridges are limited to samples of approximately the same size. This is also true of the dynamometer wattmeter methods unless shunts are introduced with their attendant difficulties. In using calorimeter methods, care must be taken to see that the radiation coefficients of the lead sheaths of the cable under test and of the standard sample are the same. As pointed out, the quadrant electrometer has a wide range of application. Its use, however, also introduces those certain difficulties which have been discussed.

E. H. Salter: Mr. Hanson brought the question of the use of shunts in connection with the electrodynamometer wattmeter. We find that a dynamometer wattmeter which is sufficiently sensitive for use without a shunt on short samples of cable, can be used in connection with the shunt on cables up to the full reel. As an instance there we have made comparatively recently some tests using the compensated dynamometer or the phase-defect compensation method in which the apparent defect angle of an air condenser is compared with the apparent defect angle of the sample of cable.

In this particular case to which I refer, the air-condenser charging current was about 15 milliamperes, whereas the charging current of the cable sample which was a full reel ran in the neighborhood of one ampere.

In order to obtain sensitivity, the balance on the air condenser was made using the direct connection of the electrodynamometer in the circuit, whereas the balance on the full-reel sample was made with the shunt set for a multiplying factor of 1000.

We have used the shunted wattmeter over a complete range between those points, between the one multiplier and the 1000 multiplier, and have tried interchecks, shifting from one shunt position to another with the same voltage and the same cable sample and have found the result perfectly satisfactory.

Mr. Hanson also raised the question of the connection in Fig. 2 of our paper. I can state that the connections are made with the low-voltage end of these windings connected to the transformer case and that the transformer cases are insulated one from the other.

Mr. St. Clair raised a question regarding the statement: "With the present methods of determining compensation, it is believed that an accuracy of ± 0.05 per cent in power factor should be obtained." That refers to absolute power factor and not to percentage accuracy.

He raised a similar question with reference to the statement on the fourth page in the second column. There I think the percentage is clearly stated as measurements: "A careful operator should be able to make measurements to within ± 0.05 per cent power factor. The losses as computed from the current and power factor should be well within ± 5 per cent accuracy." There it is changing it over to the accuracy basis.

The Rectification of Alternating Currents with Steel-Enclosed Mercury Arc Power Rectifiers and their Auxiliary Devices

BY OTHMAR K. MARTI*

Synopsis.—*Recently many publications have been issued on mercury arc rectifier installations in this country and in Europe, as well as on the theory of their voltage and current performances. Nevertheless, it seemed that it might be of interest to give a condensed paper dealing with the most important theoretical treatments, as well as a description of a steel enclosed rectifier of modern design.*

The fundamental theory of these rectifiers is discussed, touching only slightly upon the physical phenomena, but treating particularly of the theory involved in the practical applications. The effects of various factors on the rectifier characteristics and operation are dealt with, and the methods used for calculating the relations of voltages, currents, transformer ratings, etc., are given and tabulated. Moreover, there are mentioned the latest developments of the rectifier proper, as well as the auxiliary devices. Included in this is the design of the transformers from a mechanical point of view, making them rigid enough to stand the mechanical stresses forced upon the windings under abnormal operating conditions; and from an electrical point of view, giving the characteristics of connections such as zig-zag windings, special polyphase windings, and the introduction of reactance absorption coils in the neutral connections of the transformer. Special attention is also paid to the anodes and their cooling equipment, and to the seals; several ingenious points being brought out in their construction.

The vacuum question is also considered one of the most important, not only from the standpoint of producing and maintaining the vacuum, but also from the standpoint of measuring it. The importance of properly measuring the degree of vacuum, as well as the amount of mercury and other vapor present in the cylinder, is brought out, since it was found that the well-known devices for measuring a vacuum, such as McLeod's vacuum gage, did not give the actual conditions existent in the cylinder, but only a relative indication, namely, the pressure of the perfect gases. A new vacuum-measuring gage of novel design, which only very recently has been developed to its present state of perfection, is described in detail, and its operation explained. This gage makes it possible to measure and record the absolute pressure of the gases and vapors contained in the rectifier cylinder.

The preparation of the rectifiers for service, and their operation, are described, and special attention is given to the numerous advantages secured by the use of rectifiers in substations. To simplify operation, and to assure continuity of service, a simple and reliable method of ignition and excitation had to be developed. Without touching upon the two methods formerly employed, i. e., d-c.—a-c. ignition and excitation, a new method, lately developed, which proved to be extremely serviceable in practise, is described and illustrated in detail.

The simplicity of starting and operation, the ease of control and the adaptability to full automatic operation, as well as the high efficiencies of these rectifiers, are dealt with, and all the advantages are recapitulated at the end. A somewhat exhaustive bibliography, which might be of value to future investigators, is also appended.

* * * * *

INTRODUCTION

MUCH information on mercury arc phenomena was published during the period 1892-1911, referring, however, only to the mercury arc in glass bulbs, while the theory of single-phase rectification was especially treated by Steinmetz and Cooper Hewitt. Over 20 years ago the latter was actually constructing the first rectifier of a practical design, which was received with much interest for a time, especially in this country. Steinmetz even gave a theoretical treatment of the two-phase rectifier, and discussed the internal phenomena with the help of oscillograms. After a comparatively long period of inactivity, this problem of rectification by means of the mercury-arc valve was again taken up, but this time in Europe. The large power rectifier was to a great extent made possible by the construction of an ingenious seal for use with steel tanks. Up to this time only glass vessels could be made sufficiently air-tight.

Inasmuch as in Europe alone several hundred rectifiers are already in successful operation, some of them for nearly 15 years, it can be seen that the steel-enclosed mercury arc rectifier is equal in its state of development to the rotary converter. Of course, there are still many developments possible, especially in increasing the d-c. voltage. The latest tests in this direction promise results which cannot even be foreseen as yet. The highest d-c. voltage rectifier commercially used,—*i. e.*, 4000 volts,—has been in successful operation for over a year and a half, and tests have been made with voltages as high as 8000 volts. Such high-voltage rectifiers, having an over-all efficiency of over 98 per cent, possess their greatest interest when considered with respect to railway electrification. It will be seen later on that among the chief advantages of these rectifiers are not only their high efficiency, especially at partial load, but also the decrease in the cost of maintenance and operation.

Although the physical phenomena involved in the operation of the mercury arc rectifier are not fully understood, the quantitative relations on which rectifier design is based are well established. These relations are given herein, particularly with a view to their practical applications, leaving out factors which would only complicate the treatment, and do not affect the final results of the computations to any great extent.

The presentation of the theory is based largely on a paper by Daellenbach and Gerecke (see Bibliography).

THEORY

The underlying principle of operation of the mercury arc rectifier is the valve action of the mercury arc in vacuum, which permits the flow of current in one direc-

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tion only, from the anode to the mercury cathode.

The fall of potential in the mercury arc is constituted of three parts; a drop of 5.7 volts at the cathode surface, a drop of 6.3 volts at the anode surface, and the drop in the arc proper which is 0.1 to 0.4 volts per centimeter of arc length. Provided the current density in the arc is not too great, the fall of potential in the arc is independent of the current and the voltage of the electrodes and depends only on the nature of the vacuum and on the nature of the electrodes. The voltage drop in a mercury arc is reduced slightly by the presence in close proximity of another arc in which the current is changing. For this reason, in polyphase rectifiers, where this condition exists during the period of overlapping of two consecutive phases, the anodes should be arranged in cyclic order so that the arc passes from anode to anode without skipping.

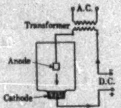


FIG. 1—SINGLE-ANODE RECTIFIER

When connected to the secondary of a transformer and an external circuit as shown in Fig. 1, current will flow in the direction indicated when the potential of the anode is positive with respect to the cathode. When a single anode is used, Figs. 1 and 2A, current will flow only during one-half of the cycle. By using two anodes with the midpoint of the secondary of the transformer as the negative d-c. pole, Fig. 2B, current will flow over one anode during the first half of the cycle and over the other anode during the second half of the cycle. The full voltage wave is thus utilized and with a sinusoidal pulsating alternating voltage on the a-c. side a continuously flowing pulsating direct current is obtained on the d-c. side. In polyphase rectifiers with more than two anodes, Fig. 2C-E, the current will flow over the anode with the momentarily highest potential and the arc will travel from anode to anode describing a cone with its apex at the cathode.

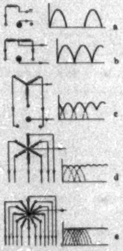


FIG. 2—SINGLE-PHASE AND POLYPHASE RECTIFIERS

It is seen from Fig. 2 that the larger the number of phases the better the form of the direct-current wave. However, as will be shown later, the number of phases used is limited by practical considerations. The frequency of pulsation of the direct-current wave is equal to the product of the a-c. frequency and the number of phases. These pulsations can be reduced to a negligible value by means of a choke coil in the cathode circuit.

In considering the current and voltage relations of a rectifier the following simplifying assumptions will be made: (1) The direct current wave is assumed to be a straight line. (2) The voltage drop in the arc is assumed to be constant at all loads. (3) The rectifier transformer ratio is assumed to be 1:1. (4) The magnetizing current of the transformer is neglected.

With these assumptions, the current and voltage relations of the rectifier will be derived: First, *neglecting the resistance and reactance of the transformer and line;* second, *the effect of the reactance of the transformer secondary will be considered.*

Symbols. Following is a list of the symbols used and their explanations:

A	Effective value of anode current
E	Effective value of phase voltage, primary and secondary
E_d	Average value of d-c. voltage
I	Constant direct current
I_p	Effective value of primary current
L	Inductance per phase of transformer secondary
P	Average d-c. power
P_1	Rating of transformer primary
P_2	Rating of transformer secondary
$X = 2\,\pi\,f\,L$	Reactance per phase of transformer secondary
$P.F.$	Power factor in line
a_1, a_2, etc.	Instantaneous values of anode currents
e_1, e_2, etc.	Instantaneous values of phase voltages
e_d	Instantaneous value of d-c. voltage
f	Frequency of a-c. supply
i_1, i_2, etc.	Instantaneous values of transformer primary currents
p	Number of secondary phases = number of anodes
t	Time
u	Angle of overlap
$x = \omega\,t = 2\,\pi\,f\,t$	

1. *Voltage and Current Relations with Zero Transformer Reactance.* We shall consider the general case of a p-phase rectifier, delivering a constant direct current I. The transformer is assumed to have zero reactance. The anodes then burn in sequence, one at a time, and each anode delivers the current I for an interval of $2\,\pi/p$. The anode current has the rectangular shape shown in Fig. 3. Its average value is I/p and its effective value

$$A = \sqrt{\frac{1}{2\pi} \cdot \frac{2\pi}{p} I^2} = I/\sqrt{p} \qquad (1)$$

The d-c. voltage, including the drop in the arc and the cathode choke coil, is equal to the voltage between the transformer neutral and the momentarily burning anode. Since the reactance drop is assumed to be zero, the d-c. voltage wave has the form shown in heavy outline in Fig. 3, and its average value

$$E_d = \frac{1}{2\pi/p} \int_{-\pi/p}^{+\pi/p} E\sqrt{2} \cos x \, d\, x = \frac{E\sqrt{2} \sin \pi/p}{\pi/p} \qquad (2)$$

The average d-c. power

$$P = E_d I = E I \sqrt{2} \frac{\sin \pi/p}{\pi/p} \qquad (3)$$

The rating of the transformer secondary windings

$$P_2 = p E A = E I \sqrt{p} = \frac{\pi/p \sqrt{p/2}}{\sin \pi/p} . P \qquad (4)$$

For a given transformer connection, the transformer primary and line current waves can be constructed from the anode currents. The effective values of the currents as well as the transformer primary ratings can then be computed.

For illustration we shall compute the voltages, currents, and transformer ratings of a six-phase rectifier with a 3 Y/6-phase transformer using the diametrical connection of secondaries.

Six-Phase Rectifier with Three-phase, Y-connected Transformer Primary. Fig. 4 shows a 3 Y/6-phase transformer connected to a six-phase rectifier. The numeral subscripts of the anode currents correspond to the order in which the anodes will burn. With the assumption of zero magnetizing m. m. f., the sum of the m. m. f's around a closed magnetic path in the transformer core is zero. With an assumed 1:1 transformation ratio, we can write:

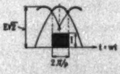


FIG. 3—ANODE CURRENT AND D-C. VOLTAGE WAVES OF A p-PHASE RECTIFIER, NEGLECTING TRANSFORMER REACTANCE

$$i_1 + a_1 - a_4 + a_6 - a_3 - i_2 = 0 \qquad (5)$$

$$i_1 + a_1 - a_4 + a_2 - a_5 - i_3 = 0 \qquad (6)$$

Also by Kirchoff's first law

$$i_1 + i_2 + i_3 = 0 \qquad (7)$$

Solving the above equations simultaneously for i_1, i_2, and i_3 we obtain

$$i_1 = -\frac{2}{3} a_1 - \frac{1}{3} a_2 + \frac{1}{3} a_3 + \frac{2}{3} a_4 + \frac{1}{3} a_5 - \frac{1}{3} a_6 \qquad (8)$$

$$i_2 = \frac{1}{3} a_1 - \frac{1}{3} a_2 - \frac{2}{3} a_3 - \frac{1}{3} a_4 + \frac{1}{3} a_5 + \frac{2}{3} a_6 \qquad (9)$$

$$i_3 = \frac{1}{3} a_1 + \frac{2}{3} a_2 + \frac{1}{3} a_3 - \frac{1}{3} a_4 - \frac{2}{3} a_5 - \frac{1}{3} a_6 \qquad (10)$$

From these expressions the primary-current curves have been constructed in Fig. 4. From equations (1) and (2) and from the diagram of Fig. 4,

$$A \quad = I/\sqrt{p} = I/\sqrt{6}$$

$$E_d \quad = E\sqrt{2} \frac{\sin \pi/p}{\pi/p} = E\sqrt{2} \frac{\sin \pi/6}{\pi/6} = \frac{3\sqrt{2}}{\pi} E$$

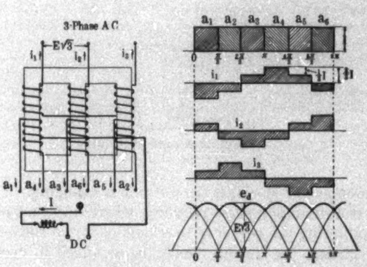


FIG. 4—ANODE AND TRANSFORMER PRIMARY CURRENTS OF 3-Y/6-PHASE RECTIFIER, USING THE DIAMETRICAL CONNECTION OF SECONDARIES

$$I_p \quad = \sqrt{\frac{1}{\pi} \cdot \frac{\pi}{3} \left[\left(\frac{1}{3} I\right)^2 + \left(\frac{2}{3} I\right)^2 + \left(\frac{1}{3} I\right)^2 \right]}$$

$$= \frac{\sqrt{2}}{3} I$$

$$P \quad = E_d I = \frac{3\sqrt{2}}{\pi} E I$$

$$P_2 \quad = p E A = E I \sqrt{6} = \frac{\pi}{\sqrt{3}} P$$

$$P_1 \quad = 3 E I_p = E I \sqrt{2} = \frac{\pi}{3} P$$

$$P.F. = \frac{P}{P_1} = \frac{3}{\pi} = 0.955$$

For other transformer connections the values of currents, voltages and transformer ratings can be computed similarly. These values for various transformer and rectifier connections are given in Fig. 8. Although computed on the assumption of zero transformer reactance and several other simplifying assumptions, *the values of this table are sufficiently accurate for most practical purposes.*

From Fig. 8, it is seen that the primary and secondary of the rectifier transformer have different ratings and that the relation between alternating and direct currents and voltages depends on the number of phases and the connections used. *The requirement of transformers of special design is therefore evident.*

Absorption Reactance Coil. The last two columns in Fig. 8 deal with rectifiers having an absorption reactance coil. The absorption reactance coil, Fig. 5, consists of two windings having a common core. One terminal of each winding is connected to one side of the split transformer neutral. The other terminals are connected together to the negative d-c. pole. The polarity of the windings is such that their m.m.f's. oppose each other.

This arrangement produces a strong choking effect causing both sections of the transformer secondary to deliver currents simultaneously. The effect is that the six-phase rectifier is converted into two parallel working three-phase sections, each section delivering one-half of the direct current.

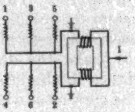


FIG. 5—ABSORPTION REACTANCE COIL

As a result, the effective values of the transformer currents and the transformer ratings are reduced as seen from Fig. 8, and the regulation of the rectifier is improved as will be shown later.

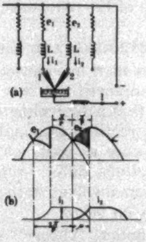


FIG. 6—ANODE CURRENT AND D-C. VOLTAGE WAVES OF A p-PHASE RECTIFIER, CONSIDERING REACTANCES OF TRANSFORMER SECONDARY

Effect of Reactance in Transformer Secondary. We shall return to the general case of a p-phase rectifier. However, each phase of the transformer secondary now has an inductance L. Due to this inductance the anode currents can no longer build up and die down instantly as previously assumed, but the currents of two consecutive phases overlap. Two adjoining anodes, therefore, have their arcs going simultaneously for a short interval, constituting an electrical connection between the open ends of the windings of the overlapping phases. This condition is shown in Fig. 6A.

Anode 1 carries the full current I until the point of intersection of the voltage waves e_1 and e_2, when anode 2 strikes its arc. The instant of this occurrence will be used as the time origin for the expression of the voltages and currents.

Applying Kirchoff's second law to the closed circuit formed by phases 1 and 2, Fig. 6A,

$$e_1 - L\frac{d\,i_1}{d\,t} + L\frac{d\,i_2}{d\,t} - e_2 = 0 \qquad (11)$$

(Since the voltage drops in the two arcs are equal they cancel each other and therefore do not enter into the above expression.)

Also

$$i_1 + i_2 = I \qquad (12)$$

$$e_1 = E\sqrt{2}\cos(\omega t + \pi/p)$$

$$e_2 = E\sqrt{2}\cos(\omega t - \pi/p)$$

Substituting for e_1 and e_2 in (**11**) and solving (**11**) and (**12**) simultaneously for i_1 and i_2 we obtain,

$$i_1 = I - \frac{E\sqrt{2}\sin\pi/p}{X}(1-\cos\omega t) \qquad (13)$$

$$i_2 = I - i_1 = \frac{E\sqrt{2}\sin\pi/p}{X}(1-\cos\omega t) \qquad (14)$$

where $X = \omega L$

The form of the anode current waves during the period of overlapping as determined by expressions (**13**) and (**14**) is shown in Fig. 6B. The overlapping of the currents lasts until i_1 has become 0, since the valve action of the arc prevents it from ever having a negative value.

The angle of overlap u can be determined by equating to zero the expression for i_1 with ωt replaced by u.

$$i_1 = I - \frac{E\sqrt{2}\sin\pi/p}{X}(1-\cos u) = 0$$

from which

$$\cos u = 1 - \frac{I X}{E\sqrt{2}\sin\pi/p} \qquad (15)$$

From (**15**),

$$\frac{E\sqrt{2}\sin\pi/p}{X} = \frac{I}{1-\cos u}$$

Substituting in (**13**) and (**14**) and replacing ωt by x we obtain

$$i_1 = I\left(1 - \frac{1-\cos x}{1-\cos u}\right) \qquad (16)$$

$$i_2 = I\,\frac{1-\cos x}{1-\cos u} \qquad (17)$$

The effective value of the anode current may be computed by means of the diagram in Fig. 6 and expressions (**16**) and (**17**).

$$2\,\pi\,A^2 = \int_0^u i_2^2\,d\,x + I^2\left(\frac{2\,\pi}{p} - u\right) + \int_0^u i_1^2\,d\,x$$

from which

$$A = \frac{I}{\sqrt{p}}\sqrt{1 - p\,\psi\,(u)} \qquad (18)$$

where

$$\psi\,(u) = \frac{(2 + \cos u)\sin u - (1 + 2\cos u)\,u}{2\,\pi\,(1 - \cos u)^2}$$

Compare (**18**) with (**1**)

$$P_2 = p\,E\,A = E\,I\,\sqrt{p}\,\sqrt{1 - p\,\psi\,(u)} \qquad (19)$$

The d-c. voltage, *i. e.*, the voltage between the transformer neutral and burning anode is now reduced by the drop in the inductance of the transformer secondary (See Fig. 6) which is equal to:

$$L\,\frac{d\,i_2}{d\,t} = \frac{\omega\,L\,I\sin x}{1 - \cos u} = \frac{I\,X\sin x}{1 - \cos u}$$

The average value of this drop

$$d = \frac{1}{2\,\pi\,/p}\int_0^u \frac{I\,X\sin x}{1 - \cos u}\,d\,x = \frac{I\,X}{2\,\pi/p} \qquad (20)$$

The average d-c. voltage considering the reactance drop is determined from equations (**2**) and (**20**)

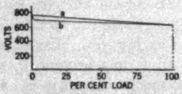


FIG. 7—VOLTAGE REGULATION CURVES OF RECTIFIER

a Without absorption reactance coil
b With absorption reactance coil

$$E_d = \frac{E\,\sqrt{2}\sin\pi/p}{\pi/p} - \frac{I\,X}{2\,\pi/p} \qquad (21)$$

$$P = E_d\,I = \frac{E\,I\,\sqrt{2}\sin\pi/p}{\pi/p} - \frac{I^2\,X}{2\,\pi/p} \qquad (22)$$

The transformer primary and line current waves can be constructed from the anode currents as was done previously. For a 3 Y/6-phase transformer the primary current wave can be constructed from the anode currents using equations (**8**), (**9**) and (**10**). The effective value of the primary current and the rating of the transformer primary can be calculated then.

Regulation. Equation (**21**) expresses the d-c. current-voltage characteristic of the rectifier. The first term in the expression represents the d-c. no-load voltage. The second term shows the variation of the voltage drop with load. It is seen that the larger the number of phases the greater is the voltage drop and, therefore, the poorer the regulation. It also explains how the absorption reactance coil improves the regulation. The reactance divides the six-phase transformer into two three-phase transformers having the characteristics of two three-phase transformers in parallel. By converting the six-phase rectifier into two parallel working three-phase sections, the voltage drop is reduced both on account of reduction in the number of phases and the current delivered by each section.

In Fig. 7 are shown regulation curves of a rectifier: *a*, without absorption reactance coil; and *b*, with an absorption reactance coil. In this particular case the reactance coil reduced the regulation from 12 per cent to 6 per cent. As seen from Fig. 7, when an absorption reactance coil is used the voltage curve drops steeply for very low values of current. This is due to the fact that for very low current the rectifier operates as a six-phase unit, the absorption reactance coil becoming ineffective, except to make the drop steeper by the addition of its reactance to the inherent reactance of the transformer. The current at which the bend in the curve occurs depends upon the design of the absorption reactance coil and is generally 15 to 20 amperes.

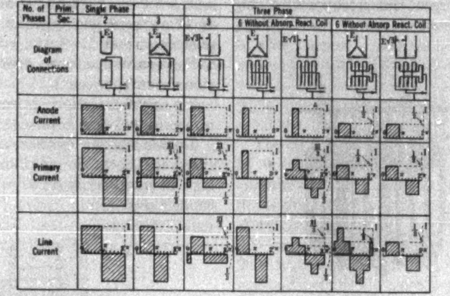
The transformer primary and line reactances have the same effects as the transformer secondary reactance, as far as the d-c. voltage, anode current, and transformer secondary rating are concerned, and could be replaced by an equivalent (not equal) reactance in the transformer secondary. The effect of the primary and line reactances on the transformer-primary current and rating depend on the type of primary connection used and the magnitude of the reactance.

RECTIFIER TRANSFORMERS AND THEIR CONNECTIONS

The transformers used in connection with mercury arc rectifiers are employed not only to step-down the the a-c. voltage so that a d-c. voltage of a desired value is obtained at the rectifier terminals, but they serve several other purposes as well, and, therefore, their design and connections differ from standard transformers. The factors involved in the design of rectifier transformers are given below.

The treatment under "Theory" showed that it is essential from certain points of view to have a large number of phases and anodes, respectively, (see Fig. 2) in order that the voltage on the d-c. side may be practically constant. Moreover, it could be seen that the rating for which a rectifier transformer must be designed is higher than the d-c. output which is desired from the rectifier. In designing the secondary winding, consideration must be given to the fact that the root-mean-square value of the anode current is considerably higher than the total current divided by the number of anodes. Furthermore, it is essential to keep in mind that in case of a short circuit on the d-c. side, the transformer connected to the rectifier is under more severe stresses than when connected to a rotary converter under similar conditions.

The rating of the transformer for a particular rectifier can be found under "Theory," and also in Fig. 8. Its mechanical construction has to be such that the high electric dynamic forces produced between the anode currents in the windings by a short circuit in the d-c. system are taken care of satisfactorily. Although it is not easy to meet these abnormal conditions, on account of the heavily unbalanced loads which occur, resulting in axial stresses on the coils, these conditions are admirably met in the transformer shown in Fig. 9 A, B. In this transformer the coils are always kept



No. of phases		Single-phase	Three-phase					
	Prim.		Delta	Star	Delta	Star	Delta	Star
	Sec.	2	3	3	6 without absorp. react. coil		6 with absorp. react. coil	
Anode current	Maximum	I	I	I	I	I	$\frac{I}{2}$	$\frac{I}{2}$
	Average	$\frac{I}{2}$	$\frac{I}{3}$	$\frac{I}{3}$	$\frac{I}{6}$	$\frac{I}{6}$	$\frac{I}{6}$	$\frac{I}{6}$
	Effective	$\frac{I}{\sqrt{2}} = \frac{I}{1.41} = 0.71\ I$	$\frac{I}{\sqrt{3}} = \frac{I}{1.73} = 0.58\ I$	$\frac{I}{\sqrt{3}} = \frac{I}{1.73} = 0.58\ I$	$\frac{I}{\sqrt{6}} = \frac{I}{2.45} = 0.41\ I$	$\frac{I}{\sqrt{6}} = \frac{I}{2.45} = 0.41\ I$	$\frac{I}{2\sqrt{3}} = \frac{I}{3.46} = 0.29\ I$	$\frac{I}{2\sqrt{3}} = \frac{I}{3.46} = 0.29\ I$
Primary current, effective value . .		I	$\frac{I\sqrt{2}}{3} = \frac{I}{2.12} = 0.47\ I$	$\frac{I\sqrt{2}}{3} = \frac{I}{2.12} = 0.47\ I$	$\frac{I}{\sqrt{3}} = \frac{I}{1.47} = 0.58\ I$	$\frac{I\sqrt{2}}{3} = \frac{I}{2.12} = 0.47\ I$	$\frac{I}{\sqrt{6}} = \frac{I}{2.45} = 0.41\ I$	$\frac{I}{\sqrt{6}} = \frac{I}{2.45} = 0.41\ I$
Line current, effective value.......		I	$I\sqrt{\frac{2}{3}} = \frac{I}{1.23} = 0.82\ I$	$\frac{I\sqrt{2}}{3} = \frac{I}{2.12} = 0.47\ I$	$I\sqrt{\frac{2}{3}} = \frac{I}{1.23} = 0.82\ I$	$\frac{I\sqrt{2}}{3} = \frac{I}{2.12} = 0.47\ I$	$\frac{I}{\sqrt{2}} = \frac{I}{1.41} = 0.71\ I$	$\frac{I}{\sqrt{6}} = \frac{I}{2.45} = 0.41\ I$
Power factor in line..............		$\frac{2\sqrt{2}}{\pi} = \frac{1}{1.11} = 0.90$	$\frac{3\sqrt{3}}{2\pi} = \frac{1}{1.21} = 0.83$	$\frac{3\sqrt{3}}{2\pi} = \frac{1}{1.21} = 0.83$	$\frac{3}{\pi} = \frac{1}{1.05} = 0.955$	$\frac{3}{\pi} = \frac{1}{1.05} = 0.955$	$\frac{3}{\pi} = \frac{1}{1.05} = 0.955$	$\frac{3}{\pi} = \frac{1}{1.05} = 0.955$
D-C. voltage.....................		$\frac{2\sqrt{2}}{\pi} E = \frac{E}{1.11} = 0.90\ E$	$\frac{3\sqrt{6}}{2\pi} E = \frac{E}{0.855} = 1.17\ E$	$\frac{3\sqrt{6}}{2\pi} E = \frac{E}{0.855} = 1.17\ E$	$\frac{3\sqrt{2}}{\pi} E = \frac{E}{0.74} = 1.35\ E$	$\frac{3\sqrt{2}}{\pi} E = \frac{E}{0.74} = 1.35\ E$	$\frac{3\sqrt{6}}{2\pi} E = \frac{E}{0.855} = 1.17\ E$	$\frac{3\sqrt{6}}{2\pi} E = \frac{E}{0.855} = 1.17\ E$
Transformer rating	Sec............	$\frac{\pi}{2} P = 1.57\ P$	$\frac{\pi\sqrt{2}}{3} P = 1.48\ P$	$\frac{\pi\sqrt{2}}{3} P = 1.48\ P$	$\frac{\pi}{\sqrt{3}} P = 1.81\ P$	$\frac{\pi}{\sqrt{3}} P = 1.81\ P$	$\frac{\pi\sqrt{2}}{3} P = 1.48\ P$	$\frac{\pi\sqrt{2}}{3} P = 1.48\ P$
	Prim.........	$\frac{\pi}{2\sqrt{2}} P = 1.11\ P$	$\frac{2\pi}{3\sqrt{2}} P = 1.21\ P$	$\frac{2\pi}{3\sqrt{2}} P = 1.21\ P$	$\frac{\pi}{\sqrt{6}} P = 1.28\ P$	$\frac{\pi}{3} P = 1.05\ P$	$\frac{\pi}{3} P = 1.05\ P$	$\frac{\pi}{3} P = 1.05\ P$
	Aver.........	$1.34\ P$	$1.35\ P$	$1.35\ P$	$1.55\ P$	$1.43\ P$	$1.26\ P$	$1.26\ P$

FIG. 8—VOLTAGE AND CURRENT RELATIONS FOR VARIOUS RECTIFIER CONNECTIONS, NEGLECTING TRANSFORMER REACTANCE AND RESISTANCE DROPS

under a certain pressure by means of a specially designed support. It is composed of pressure rings of cast steel placed above and below the winding and held together by long bolts and spiral springs, thus forming a solid unit. In order to distribute the axial pressure evenly between the low- and high-voltage windings, an auxiliary ring is placed between the steel-pressure ring and the end-distance pieces of one of the windings, making possible a proper adjustment by means of screws.

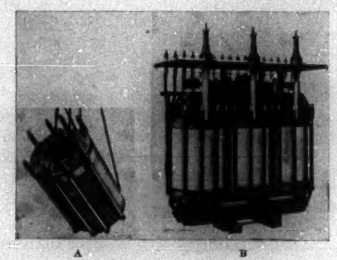


FIG. 9, A, B—RECTIFIER TRANSFORMER

In Fig. 9B is shown a rectifier transformer rated at 1750 kv-a., 60,000 to 1425 volts, 50 cycles, three-phase to 12-phase. The bushings are fastened to a common terminal frame so that the cover of the tank can be removed without taking off the terminals. Reference will also be made to the electrical design of the rectifier transformer in connection with regulation and parallel operation. The voltage regulation depends entirely upon the transformer design, or upon both the transformer and the auxiliary apparatus; since the rectifier itself has no regulating properties, the connections of the windings of rectifier transformers are therefore important. In Fig. 17 A, B, C, D are illustrated various transformer connections; the open terminals of the secondary windings lead to the rectifier anodes.

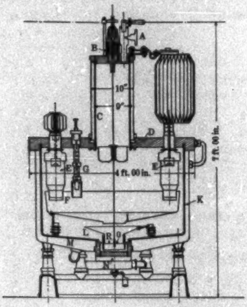


FIG. 10—CROSS-SECTION OF A RECTIFIER

The term "regulation of the rectifier" refers to the voltage drop between full load and no-load at the d-c. side and for several transformer connections usually employed the regulation performance is given below:

Double six-phase connection, primary three-phase, Y or delta, usually used with an absorption reactance coil, see Figs. 8 and 17A, has a regulation of about 10 to 12 per cent without and about 4 to 5 per cent with absorption coil;

Polygon connection, primary three-phase Y regulation about 6 per cent;

Zig-zag connection, primary three-phase Y, regulation as for polygon connection;

Double zig-zag connection, primary three-phase Y, (used for 12-anode rectifiers or two six-anode rectifiers in parallel) regulation as for the two preceding connections.

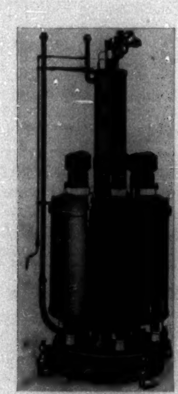


FIG. 11—GENERAL VIEW OF A RECTIFIER

CONSTRUCTION AND MECHANICAL DETAILS OF POWER RECTIFIERS

A cross-section of a Brown Boveri rectifier of recent design is shown in Fig. 10 and described below, although reference will be made to other makes when these differ in some essential point from the above mentioned make.

The large number of different types developed during the years of experimenting have now been condensed into three commercial types. The main differences between these are the difference in the sizes of the various parts, the spacing of the anodes and cathode, and the method of anode cooling. The foundation is very light compared to that required for rotary converters. A special earthquake-resisting foundation has also been developed, which successfully keeps the vacuum piping and other delicate parts intact, even in the countries most frequently subject to seismic disturbances. Six insulating feet, resting on the cast-iron foundation ring, support the rectifier proper. If for any reason it is impossible to lift the rectifier into position, the foundation ring is provided with casters, see Fig. 11, which enable the rectifier to be moved to the desired position. The casters are provided with means for raising the rectifier off the floor while moving it and for lowering it to the floor when it has been put into place.

Cylinder. In Fig. 10 the cylinder K, made of welded steel, is closed by the anode plate D, which supports the main anodes E, the excitation anodes G, and condensing cylinder C, at the top of which is mounted the ignition device. The cathode consists of a quantity of mercury in the receptacle R. From this the electrons travel through the openings in the funnels L and I, through the so-called arc guide F, to the anode E. The funnels L and I are insulated from each other and from the rest of the rectifier by means of special insulators, visible in the drawing. The arc guide F, shown in detail in Fig. 12, has baffle plates so arranged that no secondary high-vacuum phenomena at the cathode can influence the performance of the anode. The mercury is poured into the cathode receptacle through a small pipe, while a similar pipe supports a gage indicating the level of the mercury in the cathode receptacle.

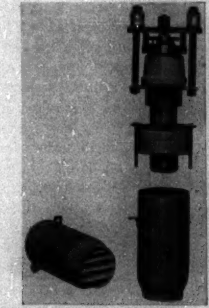


FIG. 12—ANODE WITH ARC GUIDE

The cooling water enters the rectifier at N, cooling the cathode and the main cylinder, which is specially treated to resist the action of the water for an indefinite period of time. The water then passes upward, through the anode plate D, into the jacket surrounding the condensing cylinder C, whence it is discharged.

The condensing cylinder C serves to cool the mercury vapor so that it becomes liquid and flows down along the walls of the cylinder into a trough, from which it is brought to a point near the wall of the main cylinder K, but outside the aprons L and I, and thence flows back into the cathode receptacle at the bottom of the main cylinder. To the top of the condensing cylinder is attached the ignition plunger, with its solenoid B, which in starting the rectifier acts upon the small iron piece just above it. The pipe to the vacuum pump also leaves the top of the condensing tower, being sealed in the same way as the current conductors.

Anodes. The main anodes are insulated with special porcelain bushings, the shape of which depends upon the voltage for which the rectifier is to be used. The shape of the anodes themselves is shown by the dotted lines. They are made of specially treated steel which has been found, after exhaustive tests, to be best suited to this purpose. The design of the anode cooling system for rectifiers to be used with voltages up to 1600 volts is shown on the right-hand side of the illustration, while the construction of the anodes for voltages as high as 3000 volts is shown on the left-hand side.

The difference in the cooling devices for the two types of rectifiers can easily be understood by taking into consideration the fact that the voltage drop of a rectifier is practically constant for all voltages. It follows, therefore, that while the heat losses in a 200-volt, 200-kw. rectifier are about 20 kw., in a 2000-volt, 200-kw. rectifier they are only about two kw. or about one-tenth of the low-voltage rectifier losses.

The excitation anode with its arc guide is shown on the left-hand side of the drawing—its function is explained later on.

Like the inner surface of the cylinder K, and all other metal parts inside the cylinder, the anode surfaces are treated in such a way that all occluded air and other gases can easily be removed. This shortens appreciably the time required for forming the rectifiers (about which more is said later on) before they can be put into service. The spacing of the anodes, their shape, and the special manner of treating them, make the rectifier insensible to momentary short circuits, as well as to heavy overloads, which would be disastrous in the case of rotary converters.

Sealing. The seals in the above mentioned make of rectifier are secured by means of mercury, and hence they are not only perfectly air-tight, but they also are not sensitive to the action of the mercury vapor and other gases which are developed in the cylinder. Furthermore, they can be equipped with a visible indicating device.

The Allgemeine Elektrizitats Gesellschaft of Berlin employs a composite packing of aluminum and lead, the former dealing with the high vacuum while the latter forms the seal against the atmosphere. An intermediate space is provided between the aluminum and lead seals which is itself evacuated, and serves to distribute the pressure in such a way that each seal has to deal only with a part of the total pressure difference. For inspection and testing purposes, however, it is necessary to fit each seal with a vacuum pipe, and in case of a leaky cylinder each vacuum pipe of the series must be connected to the gage in turn until the faulty seal has been discovered. Moreover, thorough investigation revealed the fact that mercury vapor and other gases will affect such metal seals to a certain extent, and unless iron is used, an amalgam is formed which may destroy the seals in a very short time. Furthermore, these investigations showed that due to aging of the metals all inner elasticity is lost after a year or two and that repeated tightening after successive intervals will not remedy the trouble.

The General Electric Company's glass seal which was recently mentioned in an article on rectifiers,* may prove to be very effective, and it will be interesting to learn how it stands up in practise. The description of it given in the article did not contain enough information to permit a thorough discussion of it. However, the disadvantage of these seals seems to be that there is no possibility of detecting any leakage, and that when the anodes are removed the seals are destroyed.

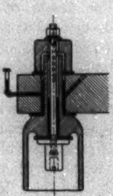


FIG. 13—MERCURY SEAL WITH INDICATING GAGE.

The construction of the Brown Boveri seal is shown in detail in Fig. 13. The joints are first fitted with special elastic washers, and thorough investigation, carried on for an extended period, showed that an asbestos washer (double cross-hatched shading) was most suitable. The mercury (solid shading) is then poured into the annular spaces between the anode plate and the insulating bushing, and the insulating bushing and the central conductor, respectively, thus sealing the vacuum, while the asbestos keeps the mercury from flowing into the cylinder.

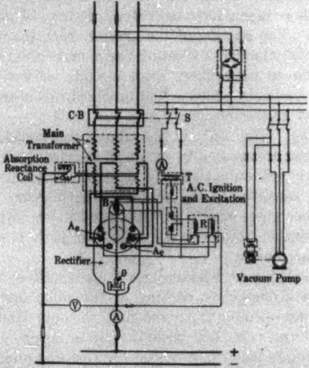


FIG. 14—TYPICAL DIAGRAM OF CONNECTIONS, SHOWING A NOVEL TYPE OF IGNITION AND EXCITATION EQUIPMENT

One of the most important features of the mercury seal is the fact that in case any mercury should filter past the asbestos packing it would only find its way to the mercury cathode at the bottom of the cylinder, doing absolutely no harm. At the same time the leak would be indicated at the mercury gage with which each seal is equipped. The gage, therefore, makes it possible to check instantly the condition of each seal, and if necessary the bolts can be tightened before the vacuum is broken.

Ignition and Excitation. The ignition of the arc is accomplished by means of a small plunger *O*, usually located, as in Figs. 10 and 14, within a few millimeters of the surface of the mercury cathode in the receptacle *R* and connected to a special metal rod extending from the top of the condensing cylinder. This plunger is controlled by the solenoid *B*, located at the top of the condensing cylinder, which pulls the plunger into the mercury cathode against the action of a spring. As soon as the plunger touches the mercury, the solenoid is deenergized and the plunger is withdrawn. In a fraction of a second an arc is drawn between the mercury cathode and either the excitation or the main anodes, provided the vacuum in the cylinder is sufficiently high and a load is connected to the d-c. circuit.

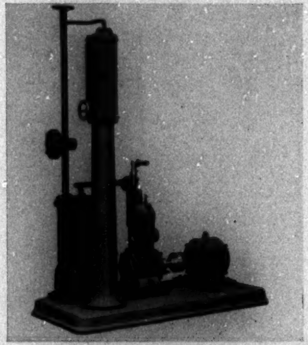


FIG. 15—VACUUM PUMP WITH McLEOD VACUUM MEASURING GAGE

When working at very light loads the main arc has a tendency to become unstable, and may be extinguished. Therefore, when the d-c. demand is highly fluctuating, as in traction systems, rolling mills, etc., where the current might drop below the value which is necessary to maintain the arc, the cylinders are equipped with excitation anodes. A cross-section through such an anode *G* is shown in Fig. 10 and it is indicated in Fig. 14, where it is denoted by A_e. The construction of it is similar to that of the main anode, and it is located at about three-quarters of the total distance between the cathode and the main anode. Much investigation had to be done in order to find the proper location and shape of the excitation anodes so as to prevent condensation of mercury at the main anodes in case the main arc should be extinguished for a considerable length of time. The latest development in this line is the recently developed pure *a-c. ignition device*, by which means the necessity for a special ignition converter is obviated. Trial tests in two Swiss installations demonstrated the superiority of this novel method which is of the utmost importance in connection with fully automatic substations. A full description of a plant equipped with such an ignition and excitation apparatus is given below under "Operation."

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*See Bibliography, reference 140.

AUXILIARY DEVICES

On a rectifier equipment of the simplest form there is only one important auxiliary provided, namely, the vacuum pump and measuring gages illustrated in Figs. 15 and 16A. A high vacuum is absolutely essential to the satisfactory operation of the plant.

The pumps used are of special design, and very efficient, so that there is no difficulty in producing a high vacuum in a properly sealed rectifier cylinder. There are several types of vacuum pumps in use, but, owing to the scope of the subject, a description of them will not be given here. Just as essential as the pump for producing the vacuum is the device for measuring it. An improvement of the utmost importance has recently been made in this line, which will be described below.

The familiar McLeod mercury column gage shown in Fig. 15, used in connection with rectifiers and in laboratory work, has been superseded by a direct-reading hot-wire vacuum gage, which in size and purpose is similar to the shunt of a precision millivoltmeter.

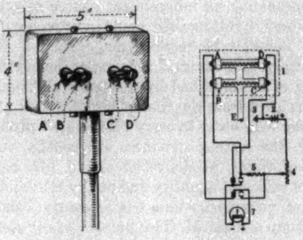
It may be recalled that McLeod's gage is based upon the fact that the pressure of a perfect gas times its volume is always constant. This law of Boyle's holds true, however, only for perfect gases, and therefore, the use of the above gage furnishes no information about the other gases and the vapors of water, mercury, etc., which the cylinder might contain. Another disadvantage of this gage is the fact that it is not direct reading, and cannot be used for automatic station control.

The development of the new direct-reading gage, therefore, made it possible for the first time to study scientifically several phases of the mercury arc rectifier and the influence of the degree of vacuum upon its operating conditions. The latest improvements in the operation of rectifiers are a direct result of the increased ability, made possible by this instrument, to measure high vacua.

Novel Hot-Wire Vacuum Gage. The arrangement of the direct-reading measuring device as shown in Fig. 16A and B, consists of the following important parts: a hot-wire sealed-vacuum gage 1, illustrated by Fig. 16A, a resistance 4, a shunt 5, and a precision moving coil voltmeter 6. The connection to the vacuum piping of the rectifier is shown at *E*.

The working principle of the hot-wire vacuum gage is based upon the fact that the thermal conductivity of gases depends upon the gas pressure. If two bodies at different temperatures are placed opposite to one another, a heat transfer takes place from the warmer to the colder body, partially by radiation and partially by conduction. It must be kept in mind that (1) the radiation is independent of the gas pressure existing between the two bodies, and that (2) the heat conductivity depends mainly upon the respective pressures. In an absolute vacuum the latter are zero. At first the conductivity rises in proportion to the increasing gas pressure, while the latter approaches asymptotically a constant limit of value. In the range of small pressures, the thermal conductivity is, therefore, a measure of the gas pressure.

The gage consists of four resistance wires connected together like a Wheatstone bridge. The wires stretched between the terminals *B C* and *A D* are located in the vacuum, while the wires *A B* and *C D* are exposed to the atmosphere, as shown in Fig. 16B. They are electrically heated with a constant current of about 100 milliamperes. When the branches *B C* and *A D* of the bridge are heated, their ohmic resistance and the pressure difference between the points *A* and *C* increase in proportion to the vacuum. Since this potential difference varies with the degree of vacuum, the deflection of the meter varies in proportion, giving a continuous indication of the vacuum in the rectifier cylinder. On account of the special bridge connection, fluctuations of the room temperature have practically no influence on the reading of the vacuum gage. For the safe operation of the voltmeter, the lead from terminal *D* is grounded at 3.



A B

Fig. 16A—Novel Electric Hot-Wire Vacuum Gage
B—Diagram of Connections for Gage and Recording Instrument

The millivoltmeter is designed as a contact voltmeter with two adjustable contacts which are set for predetermined low and maximum values of the vacuum, and, therefore, can be used for the automatic control of the air-pump set, making possible a full automatic control of a mercury arc rectifier substation.

The hot-wire vacuum gage is connected to the vacuum piping by means of a flange and a high-vacuum stuffing box. When connecting the apparatus, the vacuum gage is first fitted with the stuffing box, after which both parts are connected to the vacuum piping, and finally the stuffing box filled with the sealing mercury. This is essential in order to prevent the outer and inner parts of the vacuum gage from coming into contact with the mercury or the mercury vapor.

The arrangement with the battery 2 furnishing the direct current for the bridge has been superseded by an arrangement allowing the use of alternating current taken from the normal control circuit of the plant. The principle upon which the measuring device is based is the same, but this arrangement does away with the battery and makes the apparatus very simple and consequently fool-proof.

Next in order would be mentioned the ignition converter providing an independent source of current to start the arc. But recently made improvements on the Brown Boveri type rectifier made such a converter unnecessary, and, as can be seen under the next heading, starting is accomplished by means of alternating current furnished from the control circuit.

Cooling. In order to carry off the heat produced by the arc, the rectifier cylinders are usually cooled by means of water, as described above. In case fresh water of sufficient quantity and purity is available near the plant, no auxiliary device is necessary; otherwise, a small pump is required. This pump, usually electrically driven, continuously circulates the water through a recooling system, which consists of radiators mounted near the rectifier installation.

The method and arrangement described above for cooling the cylinders and the anodes are far superior to the methods employed by some companies which leave the water jacket open above the anode plate. Not only is a great deal of water vapor, which is harmful to the equipment in the substation, generated by that system, but the water has to be piped and sometimes pumped by a cumbersome and trouble-breeding method.

Another auxiliary of importance is the absorption or regulation reactance coil placed in the neutral point of the main transformer, its function being to limit the inherent regulation of the complete plant to a reasonable figure—usually of the order of five per cent from full to approximately no load. Without this coil the regulation may be as high as 10 to 12 per cent unless special transformer connections are used. Further details are given under "Theory" and "Transformers."

OPERATION OF RECTIFIERS: SINGLY, IN GROUPS, AND WITH ROTARY CONVERTERS

Before a mercury arc rectifier can be placed in service, a certain "forming" process has to be carried out. The forming itself consists of expelling the gases occluded in the metal parts of the rectifier, chiefly in the anodes. To be able to conduct this process intelligently, it is essential to have a measuring gage which indicates not only the pressure of the perfect gases, but also the pressure of other gases and vapors present. Some of the latest improved methods along this line have to be credited to the hot-wire vacuum gage previously described.

In order to shorten the forming process, it is important that all moisture and humidity be removed from inside the tank beforehand. This is done by the application of high heat, either by passing current through the rectifier, or by circulating hot water. If an electric current is used, it is gradually increased from a low value until the anodes are brought to maximum heat. At the same time the vacuum pump exhausts the gases liberated from the anodes. Various schemes were tried out to facilitate and hasten this preparation for the forming process. This preparation is of the utmost importance in that the gases occluded in the metals can otherwise not be expelled at all, or only very slowly, when the cylinder is being finally evacuated.

Forming is usually started by treating each anode individually, the d-c. energy being dissipated in a small grid resistance. This is followed by six-phase forming, during which time the rectifier may be connected to a commercial load. The time spent in forming at the factory varies between two and three days, and generally a similar procedure is carried out during erection at the power plant or substation. The forming process should be continued day and night with as few interruptions as possible, and therefore needs careful attention. It is understood that during this process the pumps have to be in operation continually, and also for a further length of time after the rectifier has been placed in service. Both processes have been very carefully worked out in every detail, and their successful application is a direct result of long and elaborate experimenting.

One of the difficulties that had to be overcome in connection with the satisfactory operation of the mercury arc rectifiers, both during forming and in actual service, was internal flashing-over or back-firing. Such internal disturbances, which paralyze the valve effect, were overcome after years of investigation. They have occurred sometimes when the forming process was not carefully carried out, or because of neglect in properly checking the material, especially that used for the anodes. However, if the proper precautions are not taken, such disturbances can occur during the forming process even now.

Ignition and Excitation Control of Novel Design. A typical diagram of connections is given in Fig. 14, which shows all the auxiliary gear of a rectifier controlled and operated in an ingenious way. The outstanding feature is that the controlling, starting, etc., are accomplished by means of alternating current, which is taken from the usual control circuit, fed from the high-voltage line through an auxiliary transformer. No direct current is needed to strike the arc nor to control the ignition anode. The fact that it was possible to dispense with the d-c. source, usually a specially designed converter set, and the development of a simple control arrangement for both ignition and excitation, account for the very simple layout of such a plant, which can be made fully automatic by merely adding two special relays.

The rectifier starts automatically as soon as the circuit-breaker *C B* on the high-voltage side connects the main transformer to the high-voltage line. This breaker is mechanically coupled to the switch *S*, which connects the excitation transformer *T* to the a-c. control circuit. As soon as the switch *S* is closed, the ignition solenoid *B*, see also Fig. 10, is energized, and the ignition anode consisting of plunger *O* is immersed in the mercury cathode, thereby closing another circuit through the right-hand secondary winding of the excitation transformer *T* and the two relay coils *R*. The right-hand relay, as soon as it is energized, in turn opens the ignition coil circuit, and the ignition anode is withdrawn from the mercury cathode by the action of the spring, thus striking an arc and enabling the rectifier to pick up load on the d-c. side at once. Should it happen that the polarity is not correct, the arc is immediately extinguished and the right-hand relay deenergized again, thus causing a repetition of the ignition process until the arc has the correct polarity. As soon as the excitation anodes are working the ignition circuit is broken by the action of the left-hand relay *R*.

The time which elapses from the moment the circuit-breaker relay receives an impulse to close the circuit breaker to the moment the rectifier is able to handle load is usually less than two seconds. The value of such a simple and quick operation cannot be appreciated enough in connection with automatic substations, as well as in other ways. For instance, in substations equipped with rotary converters, it is possible to have a rectifier as a spare set which can instantly be placed into service to take care of peak loads of such short duration that the rotary converters as reserve units would miss them because of the time involved in starting them.

The d-c. voltage is controlled either by step-by-step regulation from the primary side of the transformer or by an induction regulator, hand or automatically operated. The arrangement with the induction regulator is decidedly preferable for lighting supplies, as the voltage can be varied smoothly and to any extent. For the case of traction supplies, and where the question of regulation is not of primary importance, the rectifier can be given a shunt characteristic with a four-five per cent rise of pressure with falling load, by the use of specially designed transformers or an *absorption reactance coil* connected in the neutral of the main transformer, see Figs. 5, 7 and 14. Reference to the latter arrangement has already been made under "Theory" and further details of these two methods will be given below.

Parallel Operation. It is possible to feed two rectifier cylinders from one main transformer and in such a case it is essential that each cylinder take its proper share of the total load. Moreover, a number of rectifiers, all fed from the same primary supply, must be able to operate in parallel with each other. Furthermore, a rectifier might be added to an existing d-c. plant, and have to operate in parallel with existing rotary converter or mercury arc rectifier units. In all these cases certain conditions must be fulfilled in order that the units may supply power to the same bus-bars without interfering with each other, and that each may take a proportionate share of the total load. In considering the conditions necessary for satisfactory parallel operation of rectifiers, it has to be kept in mind that the rectifier cannot feed back to the a-c. line, and also that the inherent load characteristics of rectifiers are somewhat different from those of other converters. Some of the essential considerations relative to these facts are given below.

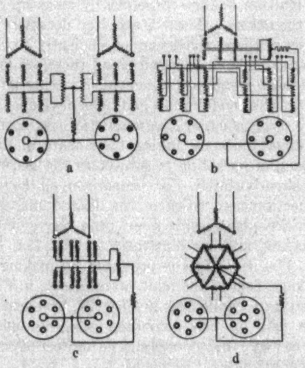


FIG. 17 A, B, C, D,—DIAGRAMS OF CONNECTIONS OF TWO RECTIFIERS IN PARALLEL

For all practical applications of rectifiers it can be assumed that the voltage drop in the arc in vacuum decreases with increasing current, and the load characteristic is therefore similar to that of an over-compounded d-c. generator. From the above, it can easily be seen that rectifiers feeding the same bus-bars and connected in parallel to the same source of alternating current may not operate satisfactorily. To meet this condition, a sufficiently large inductance has to be inserted in the anode circuit of each rectifier so as to obtain a suitable load characteristic which assures satisfactory parallel operation of each cylinder over a given range of load. For instance, a correctly dimensioned choke coil can be used to obtain such an inductance. Incorporating the arrangement of the connections shown in Fig. 17B, the inductance or the flux set up in the choke coil can be limited to that part produced by the difference in the anode currents. The same effect can be obtained by utilizing that part of the main transformer inductance which corresponds to the stray field, by using either a separate transformer for each rectifier, or a double six-phase arrangement, as given in Fig. 17A and C, respectively. However, the inductance set up in this way must be large enough to impress such a high voltage across the electrodes of a rectifier working in parallel with others but not taking a share of the load which will force it to pick up some of the load. The actual arc voltage, therefore, is less than the ignition voltage and when several rectifiers are operating in parallel the unit having the lowest ignition voltage consequently picks up the load before the others. The voltage drop necessary for the operation outlined above cannot efficiently be obtained by means of ohmic resistance, and therefore methods, such as those mentioned above, for inserting an inductance into the anode circuit had to be developed so as to produce a satisfactory characteristic. As soon as all the rectifier units are taking their share of the load, their parallel operation is subject to the general conditions already stated. For comparison, all the methods of connection used to effect the desired characteristic for rectifier plants with more than one cylinder are given below, and are illustrated in Fig. 17A, B, C, D. In Fig. 17A, the effect is produced by making use of the leakage field of the main transformer; in Fig. 17B, by inserting a choke coil; in Fig. 17C, by means of the leakage field of the secondary winding only; and finally, in Fig. 17D, by doubling the number of phases on the secondary side.

As already pointed out, it is also essential that each unit take a proportionate share of the load when working in parallel. Two units are assumed to have falling-load characteristics E_1 and E_2, illustrated in Fig. 18. It can be seen easily that the distance between the two points of intersection of any horizontal straight line with the characteristics of the two sets gives the total current "I" supplied. The distance from the middle vertical line to the points of intersection gives the currents supplied by sets No. 1 and No. 2, respectively. The voltage of the two parallel working units is indicated by E. From this curve it is evident that an increase of the total load will be distributed according to the slope of the load characteristic.

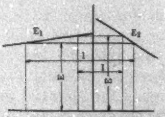


FIG. 18—PROPORTIONATE SHARE OF THE TOTAL LOAD BETWEEN A RECTIFIER AND A MOTOR GENERATOR

It is sometimes desirable to have a rectifier working in parallel with rotary converters or motor generators. The efficiencies of the two latter types of converters are lower at overloads and at partial loads than the efficiency of the rectifier. In order to obtain, therefore, a good annual average efficiency for the plant, it is desirable to design these two types of converting devices for such load characteristics that in parallel operation the rotary converter or motor generator is always working at practically full load, while the fluctuating peaks are taken care of by the mercury arc rectifiers. Assuming that the load characteristic of the motor generator is shown on the right-hand side of Fig. 18, and the load characteristic of the mercury arc rectifier on the left-hand side, it is evident from the figure that the rectifier will take by far the larger share of a given increase in load.

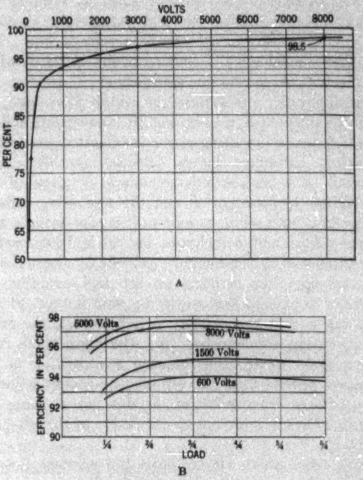


FIG. 19A,-B—OVER-ALL EFFICIENCIES OF RECTIFIERS FOR VARIOUS VOLTAGES

Efficiency and Power Factor. Apart from other considerations, the outstanding electrical characteristic of the rectifier is its efficiency, which remains nearly constant at all loads; but to obtain the true commercial efficiency we must include all losses between the high-tension terminals of the transformer and the d-c. bus-bars. These losses are the losses in the transformer, excitation equipment, vacuum pump, and reactance coils. The constant efficiency of the rectifier alone is due to the drop in the arc being approximately constant under all load and pressure conditions. This drop varies roughly between 16 and 23 volts according to the size of the rectifier, and is influenced by the vacuum to which it is directly proportional. The actual amount of watts dissipated in the rectifier is, therefore, simply the total drop multiplied by the current for any particular load, which gives a constant efficiency under all conditions. Thus when operating at 500-volt direct current, the efficiency of the rectifier is represented by

$$\frac{500}{500 + 20} = 95.5 \text{ per cent.}$$

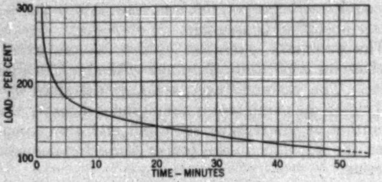


FIG. 20—OVERLOAD CAPACITY OF RECTIFIERS AS A FUNCTION OF THE TIME OF DURATION

These indications show that the higher the d-c. voltage, the better will be the efficiency. For voltages up to 8000 volts efficiency curves are plotted in Fig. 19A.

The over-all efficiency over the whole working range for 600-, 1500-, 3000-, and 5000-volt rectifiers of standard types is shown in Fig. 19B.

In general the power factor varies between 90 and 95 per cent. Reference was made to this under "Theory" and since it depends to a great extent on the external circuit further considerations are omitted.

Overloads. It is the very high momentary overloads that the rectifier can deal with that make it preeminently suited for traction conditions where the average load is below full load. The consequence of this is that rather smaller normally rated sets can be employed than if, for instance, rotary converters were used.

Traction rotaries must necessarily be amply rated, otherwise commutator trouble will be experienced under very heavy overload or short-circuit conditions. Sometimes this measure does not do, and some special schemes of load protection have to be employed.

Extensive load and short-circuit tests on rectifiers showed that practically any overload can be handled for a certain time with a complete absence of any bad effects on the rectifier proper. The permissible overload capacities of a Brown Boveri type rectifier set can be taken from the curve in Fig. 20.

As there are not any moving or wearing parts the life of the rectifier is still unknown, as none of the rectifiers installed more than 12 years ago have shown any signs of deterioration as yet. The arc takes place in a vacuum, and therefore there is not any appreciable chemical effect on the anode or on the cylinder. No change either in the size or the shape of the anodes, or of the inside surface of the cylinder, that would in any way affect the operation of the rectifier, could be observed. On the contrary, experience has shown that the longer the rectifier is in operation the better are its efficiency and its overload characteristic. The vacuum pump runs for such brief periods of time, and so infrequently, that the wear on its moving parts is negligible, while the development of the a-c. ignition has done away with any other rotating parts.

ADVANTAGES OVER ROTARY CONVERTERS

The floor space usually occupied by rectifiers is about the same as that required for rotary converters for d-c. voltages up to about 600 volts. Above that, the space required is considerably less. Special foundations are not required for any voltages except what is needed to bear the stationary weight. For this reason, and due to the fact that the operation of the rectifier is noiseless, new substations need only be of the lightest construction, and substations can, therefore, often be placed in locations that would not be at all suited for the installation of rotating machinery because of the vibration and noise.

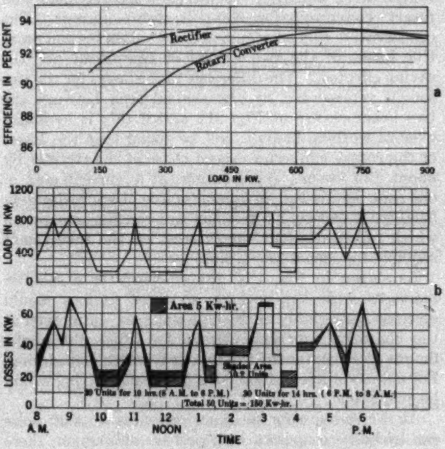


FIG. 21A—EFFICIENCY OF A 600-VOLT, 600-KW. ROTARY CONVERTER AND MERCURY ARC RECTIFIER

B—LOAD CHART AND REDUCTION IN LOSSES BY THE USE OF MERCURY ARC RECTIFIERS INSTEAD OF ROTARY CONVERTERS

Starting. Among the most striking advantages of the mercury arc rectifier over the rotary converter is the simplicity of the starting procedure. It takes only a few seconds to ignite the arc and put the rectifier into service. This makes it preeminently adapted to automatically controlled substations. Since a rectifier has no standby losses, like a rotary converter, it can be generally left connected to the high-voltage supply, and in case of a short circuit at the primary side of the transformer, the rectifier disconnects itself from the network at the moment the arc extinguishes, and it will pick up the load as soon as the a-c. voltage is restored. However, even in manually operated substations, the attendance is greatly reduced, as will be explained at length later on.

Efficiency. To illustrate what the superiority of the rectifier efficiency really means, especially the efficincey at partial loads, a typical case of an actual railroad in the United States has been taken and the annual saving effected worked out on the basis of the daily-load curve. The curve in Fig. 21A shows the comparative efficiency of mercury arc rectifiers and rotary converters for 600-volt direct current. The conservative efficiency curves do not alter the comparative losses of the two classes of equipment nor the saving effected. The difference in the losses for the two sets is represented by the shaded area between the two lower curves of Fig. 21B and if this is averaged over 24 working hours it will be found that the losses of the rotary converter exceed those of the rectifier by about 125 kw-hr., representing, at a current cost of one cent per kw-hr., an annual saving of approximately $450 for a 600-kw-hr. mercury arc rectifier substation which represents half the interest on the investment required for such a substation.

As shown in Fig. 19A, the efficiency of a rectifier increases with the voltage; so these savings would increase appreciably at the high voltages. But even at the comparatively low voltage for which the above figures were computed the saving is worth considering. For a d-c. voltage of 1500 volts the annual saving would be over $1000.

As d-c. motors for terminal pressures of 2000 volts can be built to operate with perfect satisfaction, there is no reason why d-c. line pressures of 4000 volts cannot become widely adopted for railway service. In fact, the Torino-Lanzo-Ceres Railway in Italy has already been using that voltage for some time, and has obtained the best kind of service from its rectifier.

Needless to say, such voltages would be impracticable with rotary converters. The increase in the voltages on the d-c. side successfully used with rectifiers during the past 15 years is shown by the heavy line in Fig. 22, while the broken line shows the voltages which have been tested successfully at long trial tests, and the light dotted line, the voltages obtained during short laboratory tests. Following the extension of the voltage curve, it may be concluded that the next few years will possibly give us rectifiers furnishing direct current at pressures suitable for high-power transmission.

Another advantage of note is the fact that for the parallel operation of rectifiers there is no elaborate synchronizing procedure required. Since polarity indicators are also not needed, another possible source of trouble is eliminated. These facts again decrease the amount of special equipment necessary for an automatically or remotely controlled substation, while in a manually operated substation the attendant's duties are further reduced.

Automatic Control. The equipment for the full automatic operation of mercury arc rectifier substations costs less than half as much as for a rotary converter substation of the same capacity. Not only is the equipment cheaper, but because it is also simpler a better operation is assured.

The automatic operation of the vacuum pump is likewise an important advantage of the rectifier equipment. The vacuum gage previously described opens and closes the pump-motor circuit independently of the operation of the rectifier set itself, and always maintains the vacuum at the proper stage for immediate starting.

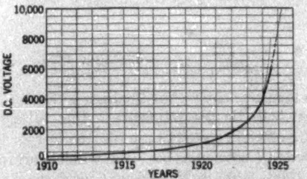


FIG. 22—MAXIMUM D-C. VOLTAGES OF RECTIFIER INSTALLATIONS IN SERVICE AND IN LABORATORY TESTS

Making allowance for interest, depreciation, and taxes on the additional investment required for full automatic control of rectifier substations, and taking into consideration the salaries for high grade inspectors to look after the automatic equipment periodically, there will be effected a saving of $500 per year even in a small substation of only 600-kw. capacity.

Another great advantage accruing from the use of mercury arc rectifiers is the fact that changes in voltage can be made without any change in the rectifier. For instance, a railroad equipped for using 600-volt direct current can be changed to 1200-volt direct current merely by changing the connections of the transformers, and making no changes whatever in the rectifiers. Needless to say, this could not be done with rotary converters. Furthermore, the additional traffic that can be handled by making this change would represent another great advantage, all without any extra cost, except the slight additional expense of having the transformers equipped with taps for making the change in the connections. As mentioned before, these rectifiers are able to withstand short circuits for a few moments without any harm, and can stand the severest overloads,

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taking instantaneously currents of even three times the normal value.

As an example of the short-circuit current one of the high-voltage rectifiers is capable of dealing with, it may be mentioned that a rectifier whose normal capacity is 1800 volts and 400 amperes successfully passed a momentary current of 8700 amperes and was able to resume operation immediately upon clearing the short circuit. Sixty similar short circuits were applied within two days, after which the rectifier was opened, and found to be in precisely the same condition as when sealed prior to these tests. In another instance, 10 short circuits were applied at one-minute intervals, and again there were no deleterious effects.

As mentioned under "Operation," the wear on the rectifiers is practically negligible. The advantages may therefore be summed up as follows:

1. Lightness of necessary foundations and substation structures, due to light weight and absence of moving parts
2. Speed and simplicity of starting
3. Simplicity of operation with a minimum of attention
4. Noiseless and vibrationless operation
5. High efficiency over the whole working range
6. Practicability of using high d-c. voltages
7. Facility of parallel operation without any necessity for synchronizing
8. Adapability to automatic control
9. Feasibility of raising the d-c. voltage, especially with electric traction equipment
10. Extremely high overload capacity and insensibility to short circuits
11. Negligible wear

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Discussion

R. H. Wheeler: Mr. Marti has been quite conservative in many statements in his paper. Progress in the development of the rectifier, together with its applications in commercial service, is going forward at a rapid rate. One thing that Mr. Marti did not bring out, and which from my angle is interesting, is that the rectifier is a piece of machinery which has the ability of being placed in practically any available space, rather than demanding a machine hall with ample provision for ventilation. The floor loadings required by the rectifier are less than 200 lb. per sq. ft., and permit of the rectifier being installed directly without special foundations.

I wish to confirm the statements of Mr. Marti as to the absence of noise. There are no reciprocating parts other than that of the vacuum pump which, when it is running, makes a chatter, but when one realizes that it is operated by a fractional h. p. motor, it is at once evident that the noise is not great and can easily be muffled by a cover if absolutely noiseless operation is desired.

Otto Naef: Mr. Marti mentioned that in order to obtain close d-c. regulation, special equipment must be added, and he quoted the interphase transformer, or absorption coil, as it is often called, as one of the means.

FIG. 1—EARLY TYPE OF STEEL-CLAD MERCURY ARC RECTIFIER (1908)

After considerable research, Brown Boveri has succeeded in bringing out a zig-zag-connected transformer winding which gives the same close regulation as is otherwise only obtained by the use of an interphase transformer. Because of the somewhat larger transformer required, this solution is not necessarily cheaper than the older method, but it has the advantage of greater simplicity.

D. C. Prince: Mr. Marti refers to an ingenious seal as having made possible the successful steel-tank rectifier. The accompanying Fig. 1 shows one of these earliest types of steel rectifiers, built about 1908. The operation of these early rectifiers compared favorably with that of any of the newer types.

In saying that the quantitative relations on which rectifier design is based are well established, Mr. Marti doubtless refers to the circuit characteristics. As far as I am aware, no data for metal rectifiers have been published including reproducible curves between volts and amperes at which arc-back occurs, such as shown in Fig. 2 herewith. This curve was made from a glass rectifier.

The extent to which engineering opinion differs regarding the fundamentals of steel rectifier design is brought out by comparing the accompanying Fig. 3 with the Brown Boveri rectifier shown in section in Mr. Marti's Fig. 10. It will be observed that the A. E. G. type has no arc guides and no condensing dome nor any elaborate anode hoods. The A. E. G. rectifiers are regarded as highly successful, and there are many installations. I understand that they have little if any seal trouble.

Even the figures accepted for some years for cathode drop have recently been overthrown[2]. The present figure is about 10 volts, part of which is consumed as latent heat of ionization of the mercury vapor. This part is responsible for most of the heating at the anodes where recombination occurs.

Too much cannot be said in praise of the paper by Messrs. Daellenbach and Gerecke referred to by Mr. Marti as the basis for his circuit theory. The part quoted by Mr. Marti, applying to regulation, is limited to the case of no resistance and reactance only in the individual anode leads. It is also limited to the case where not more than two anodes carry current simultaneously. The original paper by Messrs. Daellenbach and Gerecke is much more complete and will repay careful study, although the analytical methods pursued become rather involved for the more complicated cases met in practise. Their paper has been very useful in my work.

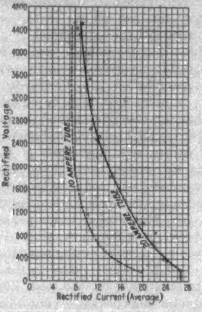

FIG. 2—VOLT-AMPERE CURVES MADE FROM GLASS RECTIFIER

E. B. Shand: A number of years ago the Westinghouse Company did a great deal of work on steel-tank rectifiers, and a resumé of this was given by the late B. G. Lamme in the discussion of "Power Rectifiers" by Milliken, *Proc.* Assoc. Iron & Steel E. E., 1921, p. 645. In this work, we started using mercury seals with the steel tanks in 1908 and developed them in 1909 to the general type used at present by the Brown Boveri Company. In 1915 considerable amount of work was done on plastic sealing cements. Our experience at that time indicates that the tightest possible joints can be made by this means, although the application to commercial use presents an additional problem. At this time we were able to operate rectifiers of commercial size with these seals for periods of somewhat over a month, without pumping. Welded tanks were introduced in 1910.

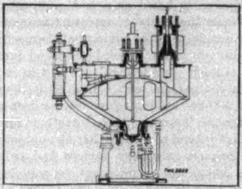

FIG. 3—A. E. G. TYPE OF RECTIFIER

I might add that the wire resistance gage for high vacuum, which is mentioned in Mr. Marti's paper (Figs. 16A and B), was developed during this work in 1916 or '17. I still have in the laboratory a gage of this sort which dates from the above period.

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[2] Günther-Schulze, *Engineering Progress*, Aug. 1925, (in English).

A single auxiliary anode was used for both starting and exciting of the rectifier. An external source of direct current was supplied to this anode. Small rectifiers are now available for the purpose of furnishing this power and which require no attention of any sort.

In connection with the bibliography, I believe that the interesting papers of M. Giroz[3] should be included.

In connection with the tabulated data on the sixth page of Mr. Marti's paper, I notice that the case of three single-phase transformers connected in star has not been included. In this case, the current ratios in the transformers secondaries and in the primary line are similar to the case of the delta-connected transformer with the balance coil.

It might be added that the balance coil or interphase transformer was developed in connection with the earlier work of the Westinghouse Company by Mr. C. L. Fortescue.

S. Q. Hayes: A point that may be of interest to a good many people is that the electrification of the railways in Japan now being carried outat 1500 volts direct current includes quite a large number of substations, and some of those substations have synchronous converters, some have motor converters, some have motor-generator sets, and some have mercury rectifiers; and they are operating the whole system together in parallel.

F. A. Faron: Mr. Marti infers in his paper that arc-backs occur during the bake-out or forming period. Are we to understand that arc-backs never occur during regular operation?

On the twelfth page Mr. Marti refers to the possibility of a rectifier starting to take peaks of such short duration that converter units would miss these peaks. What he says about starting may be true, but why should a machine be started to take care of such peaks? This is what the overload capacity of the machine is for. And if the load goes beyond this allowable limit, it is not usually a useful one and devices are provided to take care of limiting the output. With reference to regulation, it should be noted that it is possible, as brought out by Mr. Prince, to build rectifier equipment with practically flat d-c. voltage characteristics from low load to full load.

Mr. Marti indicates that traction loads are handled by converting equipment with drooping characteristics. This is not quite correct, for in 90 per cent of the traction substations in this country, compound-wound machines are used, having practically flat voltage characteristics from no load to full load.

Fig. 20 is apparently based on a maximum rated unit. Experience has shown that railway operators are not particularly interested in a maximum rating but in what the machine will do for a period of about two hours, which is the usual duration of peak service. This is fairly well brought out in Curve 21-*B*, on the same page. In comparison with Curve 20, it should be noted that a standard 500-kw. railway converter, for instance, would be rated 750 kw. for two hours and three times load for one minute.

Does Mr. Marti intend to convey that all equipment including rectifiers, transformers, auxiliary devices and switchboard for 600 volts take about the same space as complete converter equipments, or does he mean only the tanks? I am of the opinion that the complete installation of rectifiers, transformers, reactors, accessories, switchboard, etc., occupy more space than equivalent converter capacity for this voltage. I agree with Mr. Marti that, for higher voltages, the complete rectifier units require less space than converter units.

In traction work it is quite common to provide single-phase transformers which give the possibility of open-delta operation of converters in cases of emergency. This feature is not desirable in connection with a rectifier.

Under the subject of starting it should be noted that in automatic stations it is usually found more economical to remove the transformer from the line when the station is not in operation, to save the transformer excitation losses.

I have been unable to check the figures given under the heading *Efficiency*, as the converters offered by American manufacturers have higher efficiencies than those shown on the curves of Fig. 21A and actually cross the efficiency curve of the rectifier at about 600-kw. point, and show somewhat over one per cent higher efficiencies of the converter than Curve *A*, which reduces the hypothetical saving set forth by Mr. Marti.

I am fearful that "automatic control" is a rather abused and misunderstood term, but it should be understood as applying to a station wherein all functions of operation are performed without the aid of an attendant.

Certain functions necessary to the operation of a rectifier, which are absent with converter operations, must be properly incorporated in the control, and protection provided should the sequence of the device in operation be other than that intended.

There will be certain temporary shut-downs during operation which the control must take care of; such, for example, as low a-c. power supply, single-phase operation, overheating of the rectifier, failure of the exciting arc and overheating of the transformers.

Furthermore, such conditions as failure of cooling-water supply, continued arc-backs, heavy sustained overloads, etc., require that the equipment be shut down and locked out, and that the automatic control be designed to carry out these functions.

Incidentally, a very careful investigation has convinced me that there have been no rectifiers built which will not arc-back if the vapor-pressure conditions in the neighborhood of the anode are not correct.

Mr. Marti says: "For instance, a railroad equipped for using 600-volt direct current can be changed to 1200-volt direct current merely by changing the connections of the transformers and making no changes whatever in the rectifiers." It is one of the advantages of rectifiers to be able to use the same unit for higher voltages, provided it is suitably insulated;—but unfortunately the substation is only one thing that must be changed; in fact, changing the substation voltage, even where converters are used, is a relatively small expense compared to insulation of line, sectionalizing switches, motors, control, etc.

The substation of the average traction system in this country represents only an investment of about 20 per cent of the total investment in electrical apparatus and of this the converter and transformer represents about one-half, and it can readily be appreciated that changing the voltage at the substation is a small part of the total cost.

There have been cases in the past where it was as simple to arrange substations originally designed for 600 volts and which contained two or more machines to provide for 1200-volt operation, merely by insulating the base of one machine and connecting the units in series; in fact some of the first 1200-volt roads in this country contain substation equipments arranged in this way. I would therefore suggest qualification of this statement.

In another sentence of this paragraph attention is called to the rectifier being able to withstand "instantaneous currents of even three times the normal value." It should be noted that standard, 500-kw., synchronous converters for railway service are guaranteed to stand three times normal load for one minute.

From this discussion it might be assumed that I oppose rectifiers as against synchronous converters, but such is not the case. Rectifiers are a comparatively new type of apparatus in this country, and their operation will be watched with interest, for undoubtedly there are applications where rectifiers are desirable. There are some points in Mr. Marti's paper, however, with which I cannot agree.

Albrecht Naeter (communicated after adjournment): Having had the privilege of making an acceptance test of one of the early Brown Boveri rectifiers in the United States, the writer was very much interested in this article by Mr. Marti, for he found that in the early stages of operation, after the forming period of the electrodes of the particular set tested, considerable difficulty was encountered by the operating staff in maintaining a good vacuum. Frequently only a few minutes after a vacuum determination had been made by the MacLeod type of gage the vacuum would drop quickly to a value considered unsafe for continued full-load operation. Fortunately the development of the novel electric hot-wire vacuum gage, according to the paper under discussion, has helped to ameliorate this situation because of the further researches made possible through it. The writer feels that, from the standpoint of the operating personnel, the vacuum-gage indication should be a continuously recording one that is visible at all times. Naturally this would be taken care of through the automatic feature of the vacuum-pump set in complete automatic sets, provided the rectifier is kept out of service without an operator's attention when the vacuum is low.

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[3] Les Redresseurs a Vapeur de Mercure, *Bulletin de la Societe Francaise des Electriciens*, June, 1924, p. 463.

Les Redresseurs a Vapeur de Mercure Grand Debit, *Revue Generale de l'Electricite*, Nov. 20, 1920, p. 721.

La Chute de Tension Inductive des Redresseurs a Vapeur de Mercure, *Revue Generale de l'Electricite*, Feb. 14 and 21, 1925, p. 253 and 303.

The writer recognizes the advantages of rectifiers, and agrees heartily with their use for higher voltages such as are common in railway work. It seems that undue stress has been laid upon the relative merits of these rectifiers and synchronous converters. Mr. Marti points out in his paper that the best field of application of these rectifiers is that of higher voltages,—higher than those of the commercially practicable synchronous converters. If the efficiency curves in Fig. 21 had been plotted for 240-volt machines, instead of 600-volt, and for the same kilowatt rating, the rectifier would probably have been found at a disadvantage. The rectifier would have shown up still less favorably if the overall efficiency of a 3000- or 4000-kw. synchronous-converter set of 240 volts had been compared to that of the several rectifiers required for the same output. A number of years ago the technical press was full of articles on the relative merits of motor-generator sets and synchronous converters, on the assumption that was then accepted that these machines were equally applicable to the same field; but now it is recognized that each has its particular advantages that make its application desirable in certain cases.

Inasmuch as the author places emphasis on the merits of rectifiers, it would have been well, for the sake of completeness of the article, to summarize some of the disadvantages of rectifiers, particularly as compared to the converter, (since he carries out that comparison), such as low efficiency at 240 volts, limited maximum sizes now available, lack of neutral for three-wire systems, etc.

O. K. Marti: Referring to Mr. Prince's remarks regarding engineering opinion concerning the design of various types of rectifiers, I should like only to call attention to the fact that the Brown Boveri rectifier, a cross-section of which is shown in my paper in Fig. 10, outnumbers the A. E. G. type shown by Mr. Prince, in the proportion of ten to one, and it can be inferred from this fact that the eventual development will probably result in a type similar to the one shown in Fig. 10 in my paper.

With reference to Mr. Shand's discussion, it was interesting for me to note that two companies have followed, independent of each other, practically the same lines in developing the steel-enclosed rectifier. Regarding the two systems of ignition, it is certain that the a-c. system of ignition and excitation is far superior to the other system, since it does not require an auxiliary d-c. generating device.

I am obliged to Mr. Shand for the additional references for my bibliography and, at this opportunity, I should also like to mention a series of articles by Dr. Schaefer, published in the BBC Mitteilungen, 1919, Nos, 3, 5, 7, 8, and 9; incidentally, these articles discussed for the first time a method of compounding mercury arc rectifiers similar in principle to the one on which Mr. Prince's paper is based.

To Mr. Faron's remarks, I should like to add that, under normal operating conditions, a rectifier will not back-fire, but there is no question that back-fires may occur under abnormal conditions, such as those caused by inadmissibly high overloads, poor vacuum due to leakage or the use of improper material in the manufacture of the rectifier.

Regarding starting, I wish to bring out the fact that a rectifier can be started in less than two seconds, being then immediately able to pick up load. This surely is an advantage over a rotary converter which requires about thirty seconds for starting, and therefore cannot take care of any load before that time. To a railroad company, for instance, it means a great deal to have no delays of trains caused by lack of power.

Referring to the over-compound characteristics of traction conversion devices, it would have been interesting if Mr. Faron would have mentioned not only the fact that 90 per cent of the installed synchronous converters have over-compound characteristics, but would also have stated what was the percentage of converters with such a characteristic ordered during the last year. He would probably have found that it was about 50 per cent. This would have given a true picture of the status of the matter. Moreover, it would have been interesting if he would also have given the relative advantages of the two characteristics for railway service. I, personally, cannot see any, but will state here some of the many advantages of a drooping characteristic. It is realized by most railway men that the latter characteristic guarantees a far more flexible operation and a far better distribution of load between neighboring substations. This results in a better load factor, allows giving the same service with a lower station capacity, reduces the losses in the feeders, and does away, in most cases, with a load-resisting device. This last means nothing else than a complication of the system and a great waste of power as long as it is in operation.

As to the floor space needed by a rectifier, it is true that the actual floor space required by a 600-volt rectifier, with all its auxiliaries, is somewhat more than for a rotary converter of the same voltage and capacity. However, considering the smaller weight of the rectifier, it is possible to utilize the space available for the converting units more effectively,—for instance, by mounting the rectifier and its auxiliaries on different floors of a building. Moreover, for the same reason, a lighter foundation, and also a lighter building construction, can be used, so that both in the erection of new buildings and in the adaptation of old buildings to substation uses, a material gain is effected.

With reference to the efficiency curves shown in Fig. 21A of my paper, I should like to mention that they were derived from data published by American manufacturers, including the stray losses according to the A. I. E. E. Standards, given as one per cent of the output. The efficiency of the transformers was taken to be the same for both the rotary converters and the mercury arc rectifiers.

Regarding the automatic control, Mr. Faron states that auxiliaries will have to take care of abnormal conditions; but this holds true also for rotary converters. I wish to point out again that the automatic control of a rectifier is much simpler than that of a rotary converter, as most of the operations necessary in starting a rotary converter,—such as step-changing, raising and lowering the brushes, polarity check, etc.—are absent in the starting of rectifiers, for which the total starting operation consists of the natural sequence of closing the a-c. breaker, the automatic ignition, and closing the d-c. breaker. Due to the valve action of the arc, the protection which Mr. Faron recommends is not needed for the usual a-c. voltage supply.

I fully agree with Mr. Naeter's statements, and I think he is more than right when he compares with the present conditions the state in which electrical engineers were when the rotary converter came into prominence to take the place of the well established motor generator. There is no question but that a rotary converter will in many cases have an advantage over a mercury arc rectifier, just as a motor-generator set shows outstanding advantages over the rotary converter in certain applications.

Rectifier Voltage Control

BY D. C. PRINCE[1]

Fellow, A. I. E. E.

***Synopsis.**—Lack of voltage control is one of the objections which have been made to the use of rectifiers. The author describes a method of voltage control for rectifiers based on the use of a saturated core interphase transformer. Saturation of this device produces a gradual change from two three-phase rectifiers operating in parallel to one six-phase rectifier which has a higher inherent voltage ratio. The theory of operation is discussed and supporting oscillograms and observed regulation curves are shown.*

A RECTIFIER is a kind of electric valve which allows current to flow from an alternating supply into a d-c. load circuit. The instantaneous alternating voltage varies all the way from $\sqrt{2}$, the effective value in one direction, to the same amount in the other direction. There is, therefore, nothing startling in the idea that any voltage between these limits can be communicated to the load circuit. The problem is rather one of practicability than of possibility.

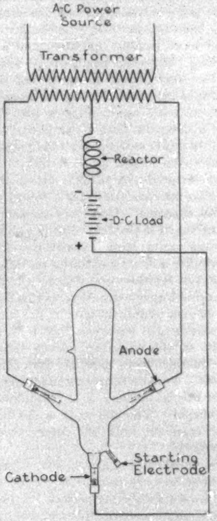

FIG. 1

The voltage communicated to the load circuit is not an instantaneous, alternating value but is an average over a considerable part of a cycle. Consider a simple, single-phase rectifier as shown diagrammatically in Fig. 1. The corresponding wave diagram is shown in Fig. 2. Current flows from the most positive anode to the cathode so that the cathode potential is below the most positive anode only by the amount of the arc drop which is practically constant and relatively negligible in rectifiers having any considerable output voltage.

This diagram assumes no inductance in the transformer or primary supply. The cathode traverses the series of positive lobes of two sine waves displaced 180 electrical deg. This potential is impressed upon the load and d-c. reactor. The counter e. m. f. load chosen will offer no impedance to the alternating components of the irregular cathode voltage. The inductance will offer no impedance to the d-c. component. The voltage wave is thus resolved into its components. The direct component is the average of voltage for a series of half cycles in one direction and is the same as the average of one-half cycle. Its value is $\frac{2\sqrt{2}E}{\pi}$ where E is the effective secondary voltage to neutral.

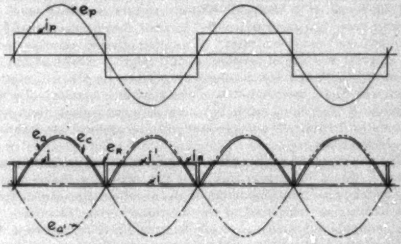

FIG. 2

If there were more phases, each anode would be the most positive for a shorter time and so the average voltage impressed on the load circuit would have to be taken for a shorter time. Fig. 3 shows some of the secondaries of a p-phase rectifier. The direction of each coil is the phase vector direction for that phase. For p phases, each anode will be most positive for the fraction of a cycle $\frac{2\pi}{p}$, that is, beginning at an angle $\frac{\pi}{p}$ before the maximum and ending $\frac{\pi}{p}$ after the maximum. The developed voltage wave is shown in Fig. 4.

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Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Niagara Falls, N. Y., May 26-28, 1926.

The average voltage, G, is found most easily by integrating the wave, that is, finding its area divided by the base.

$$G = \sqrt{2}\,E \times \frac{p}{2\,\pi} \int_{-\frac{\pi}{p}}^{+\frac{\pi}{p}} \cos\theta\, d\,\theta$$

$$= \frac{\sqrt{2}\,E\,p}{2\,\pi}\,[\sin\theta]_{-\frac{\pi}{p}}^{+\frac{\pi}{p}} = \sqrt{2}\,E\,\frac{p}{\pi}\,\sin\frac{\pi}{p} \quad (1)$$

For various numbers of phases, this equation gives the series of values in Table I. It is seen at once that by increasing the number of phases there results an increase in the average voltage. This indicates how voltage control may be approached. A method of changing smoothly the number of phases from two to six would give an increase of fifty per cent in the output voltage.

TABLE I

p	2	3	4	6	∞
G/E	0.90	1.17	1.27	1.35	1.41

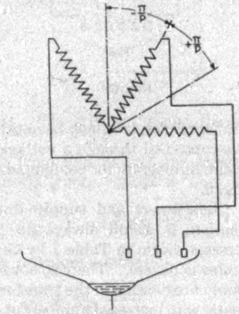

FIG 3

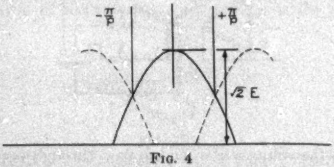

FIG. 4

Two three-phase rectifiers may be connected to a three-phase supply as shown in Fig. 5. These rectifiers have separate inductances in series with their cathodes but feed a common load. Since the two inductances absorb the pulsating components of the voltages of their respective rectifiers, there is nothing to prevent their feeding the load in parallel, each supplying one-half of the current all the time. If the switch S be closed, this condition is changed. There is no longer any impedance to prevent a current shift from one rectifier to the other. Current will naturally flow from the anode with the highest voltage, regardless of its location in a particular rectifier, so that six-phase operation replaces three-phase.

Fig. 6 is a wave diagram of the change. With the switch open, the two cathodes have potentials e_1, e_2 averaging e_a. With the switch S closed, the two cathodes follow the potential of the most positive anode, giving the wave e_6 shown by the heavy line.

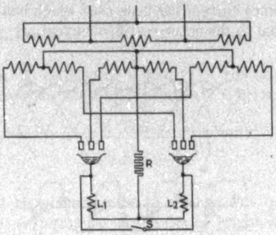

FIG. 5

Instead of the abrupt change from two three-phase rectifiers, the transition might have been made gradually. The two inductances L_1, L_2 may be so interwound that they present large impedance to current changes between rectifiers but low impedance to changes of the total current. The load voltage will then be the average e_a of the rectifier voltages, but only so long as both are carrying current. Should one stop carrying current, it could no longer influence the average voltage, for both windings of the choke system must carry a variable current in order to supply the inductive drops necessary for the voltage equalizing action.

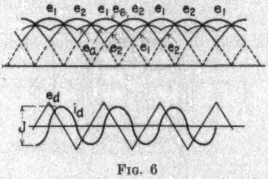

FIG. 6

The inductances, L_1 L_2, are not infinite, so that the tendency of the rectifier having higher voltage to take current from the other will cause a circulating current between the two rectifiers. In Fig. 6, e_d is the difference in voltage, i_d the circulating current. As long as this circulating current is less than the load current supplied by either rectifier, there can always be sufficient current changes to set up the difference voltage and preserve the average. If, however, the interchange current becomes equal to the load of one rectifier, one will momentarily be carrying all the current, and the voltage will no longer be the average, but will be the voltage of the higher rectifier.

With a further increase in interchange current, there will be a longer period during which there is no drop across the inductances $L_1 L_2$ and the output potential will follow the higher rectifier for a longer time. Fig. 7 shows the waves under this condition. A value of difference current has been used which has an amplitude equal to the entire rectified current, J. With this value, the load current is divided between the two rectifiers half the time and carried by one for half the time. To maintain the difference voltage, the difference current would have had to continue as shown by the dotted extension, but this it could not do because the current from one rectifier could not drop below zero.

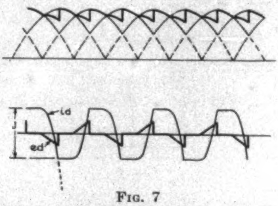

FIG. 7

Saturation forms a ready means of altering the interchange current as desired. Fig. 8 is a diagrammatic representation of a saturation regulator adapted for this use. It consists of a four-legged core on which two sets of windings are placed. One winding consists of two circuits surrounding the two central legs individually and connected so that the upper and lower halves of each will function as L_2 and L_1, respectively. The flux required for the difference voltage circulates in these two central legs and does not traverse the other two. The second winding used to saturate the core surrounds both central legs. The flux linkages are thus so balanced that current changes in either winding will induce no voltages in the other.

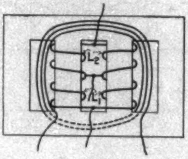

FIG. 8

To increase the interchange current which is the exciting current in the central windings a direct current is passed through the outer winding. The flux produced by this current passes in one direction through the central legs and returns through the outer legs. It therefore saturates the central legs which carry the alternating flux, and this saturation increases the interchange current and thereby the output voltage.

The direct current required to saturate the core may be obtained from the rectifier output in which case a compound rectifier results, or it may be obtained from a separate source controlled through a voltage regulator. The complete circuit diagram for compounding on load is shown in Fig. 9.

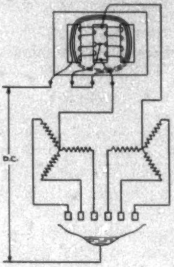

FIG. 9

If rectifier transformers and supply lines had no inherent regulation, it would always be possible to secure the increases shown in Table I by changing the equivalent number of phases. There is, however, a considerable amount of regulation in the transformer which tends to be greater with increased numbers of equivalent phases. Take the case of the three-phase rectifier shown diagrammatically in Fig. 10, having all the inductance X concentrated in the anode leads and an infinite choke L to keep the total current constant.

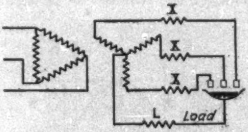

FIG. 10

When the voltages of a_1 and a_2 pass through equality, current will begin to change from a_1 to a_2. While the change is taking place the two anodes are at the same potential since otherwise they would not divide the current. The voltage difference between the two phases is absorbed in the inductances X. The current and voltage appear as shown in Fig. 11.

The rate at which current transfer will take place is proportional to the voltage causing the transfer, which is the vector difference between the phase voltages. At each transfer some of the average voltage is lost as shown by the crosshatched areas in Fig. 11. The curve of current transfer is a sine wave which lags 90 deg. behind the phase-voltage difference because the drop is inductive. At any instant the value of the rising current to one anode is independent of load because no current change takes place in the load circuit. When all the current has transferred in angle u the change ceases. The greater the load, the greater will be the angle u.

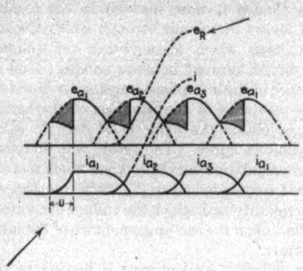

Fig. 11

It appears from Fig. 12 that the general expression for the phase-difference voltage is

$$\frac{e_d}{2} = \sqrt{2}\, E \sin \frac{\pi}{p} \qquad (2)$$

This voltage produces a current

$$i = \frac{\sqrt{2}\ E\ \sin \dfrac{\pi}{p}}{X}\ (1 - \cos \theta)$$

At the end of the lap angle u, the load of direct current J is reached so that

$$J = \frac{\sqrt{2}\, E}{X} \sin \frac{\pi}{p}\ (1 - \cos u) \qquad (3)$$

The curve of instantaneous lost voltage is the same as the half difference voltage used in making the current transfer. The average loss in output voltage represented by the hatched area is the wave e_R which has the same phase as the current i. Its value is

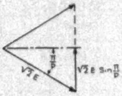

Fig. 12

$$e_R = \frac{p}{2\,\pi} \int_0^u \sqrt{2}\, E \sin \frac{\pi}{p} \sin \theta\, d\, \theta$$

$$= \frac{p}{2\,\pi}\ \sqrt{2}\, E \sin \frac{\pi}{p}\ (1 - \cos u) \qquad (4)$$

The open-circuit voltage has already been found to be

$$G_0 = \sqrt{2}\, E\, \frac{p}{\pi} \sin \frac{\pi}{p} \qquad (1')$$

so that the voltage under load is

$$G = G_0 - e_R$$

$$= \sqrt{2}\, E\, \frac{p}{\pi} \sin \frac{\pi}{p} \left(1 - \frac{1 - \cos u}{2} \right)$$

$$= G_0 \left(1 - \frac{1 - \cos u}{2} \right)$$

Equation (3) connecting J and $(1 - \cos u)$ can be used to replace the latter by the former. This results in

$$G = G_0 \left(1 - \frac{J\, X}{2\, \sqrt{2}\, E \sin \dfrac{\pi}{p}} \right) \qquad (5)$$

It would be advantageous to arrange this equation so that the performance of the device might be compared with that of more familiar apparatus. Let it be assumed that there will be a four per cent drop in the transformer when carrying an ordinary sine wave a-c. load equal to its rating for use with a rectifier. Then the short-circuit current per phase is $\dfrac{E}{X}$ or $25\, \dfrac{W}{p\, E}$ where W is the rated output of the rectifier. Equating these two values and solving for X gives

$$X = \frac{p\, E^2}{25\, W}$$

and the output voltage equation may now be written

$$G = G_0 \left(1 - \frac{J\, p\, E}{50\ \sqrt{2}\ W \sin\ \dfrac{\pi}{p}} \right) \qquad (6)$$

or making use of equation (1′)

$$G = G_0 \left(1 - \frac{\pi\, J\, G_0}{100\ \ W\ \ \sin^2\ \dfrac{\pi}{p}} \right) \qquad (7)$$

In the derivation of these equations it was assumed that all the reactance was in the secondary windings of the transformer with no mutual reactance. This is convenient and gives results of the right nature. The magnitude of the results may be wrong, however, for, unless all of the leakage reactance is actually located in the manner represented in Fig. 10, the inductance of the paths taken by the commutating currents may be appreciably less than the value obtained by considering short-circuit conditions in the transformer. This is due to the fact that in the latter case the currents have a mutual reaction which can greatly increase the apparent reactance.

The commutating currents on the other hand usually are not required to flow through circuits with much coupling between sections. No advantage of this kind occurs in the single-phase rectifier, but in the three-phase case, the reactance presented to the commutating currents is approximately half that determined by short-circuit measurements. The advantage in the six-phase case is hard to determine exactly. If the transformer has a primary and tertiary winding and all the reactance is in the primary, the advantage may be as high as six to one, but as a conservative estimate, let it be assumed that the reactance will be 40 per cent of the value determined from the simple measurements.

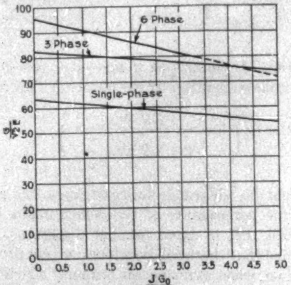

Fig. 13

Fig. 13 indicates the regulation curves given by equation (7) and takes into account the corrections just described. The regulation curve for the compound rectifier will fall between the curves for the six-phase and three-phase rectifiers. By increasing the saturation of the interphase transformer as the load is increased, it may be made to give high reactance at low loads and small reactance at heavy loads. As a result the output voltage will approach that of a six-phase rectifier under heavy loads and will be nearer to that of a three-phase rectifier under light loads. It will be seen that the range of voltage available for compounding is somewhat limited if only the three- and six-phase circuits are used. It is probable, however, that the limits are sufficiently wide for the great majority of applications. A much larger variation can be obtained by going from the single-phase to the six-phase connection. This transition is entirely practical but uses slightly more material than the three-phase to six-phase case and is therefore not desirable as long as the latter arrangement can be made to give the required characteristics.

Under extremely light loads, the direct current through the interphase transformer will be so small that it would be impractical to supply a transformer having sufficient exciting reactance so that its exciting current would always be smaller than the load current. Therefore, the effect of the interphase transformer will be lost as zero load is approached and the no-load voltage will be that corresponding to six-phase operation.

As long as the instantaneous current in the interphase transformer does not fall to zero, its reactance does not appear in the a-c. circuits. As soon as the current in one branch of the interphase transformer is actually interrupted, however, this transformer acts just as though it were inserted in the anode leads. Fig. 14 shows the circuit through which the transient current flows which accomplishes the commutation of the output current between anodes. The exciting inductance of the interphase transformer is added to the leakage reactance of the windings of the main transformer to obtain the value of X used in calculating the regulation curve. The result will therefore be a six-phase regulation curve dropping much more rapidly than that corresponding to the main transformer alone and the rapidity with which the voltage decreases will be dependent upon the exciting reactance of the interphase transformer.

After sufficient load current is flowing so that the interphase transformer is carrying current continuously, the output voltage will be that corresponding to the three-phase rectifier. As more load is applied the output current flowing through the saturating winding of the interphase transformer will decrease the reactance of the main circuit. The exciting current will then rise until it is as large as the load current and continuous choke action will be lost. The voltage will then be displaced from the three-phase value in the direction of the potential corresponding to six-phase operation. By this means the voltage may be held constant over the greater part of the load range or may even be made to increase slightly with load.

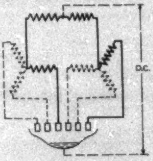

Fig. 14

In Fig. 13 part of the curve for six-phase operation is shown dotted. In this range the voltage is no longer given by equation (7) because the load is so great that three anodes carry current simultaneously during part of the time. Due to lack of complete data on the reactance of the circuits traversed by the commutating currents, it is not worth while to calculate the voltage under these circumstances. The three-phase rectifier curve will have a similar limit of applicability, but this limit is not reached in the range shown in Fig. 13.

Figs. 15, 16, 17 and 18 indicate the results of experimental work which serves as a check on the theory of operation of rectifiers with compounding. Fig. 15 shows the wave shapes obtained with an oscillograph when an experimental rectifier was operating as two separate three-phase units connected in parallel through an unsaturated interphase transformer. At time, t_1, the secondary voltage of the phase being measured became equal to that of the preceding phase in the same three-phase group, and it began to conduct current. Between t_1 and t_2 both phases were conducting current and the reactive drops caused by the changing currents averaged the induced voltages of the two phases so that their terminal voltages were nearly constant. At time, t_2, the current transfer was completed and the terminal voltage of the incoming phase again became equal to its induced voltage. Between t_4 and t_5, the current left the phase under observation, and this time the reactive drop prevented the terminal voltage of the phase from falling as rapidly as the induced voltage until after the

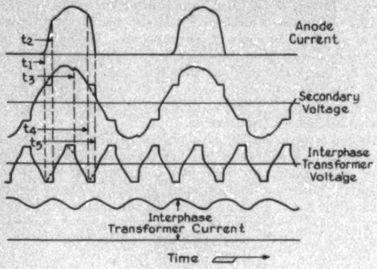
t2
t1
t3
t4
t5
Anode Current
Secondary Voltage
Interphase Transformer Voltage
Interphase Transformer Current
Time

FIG. 15

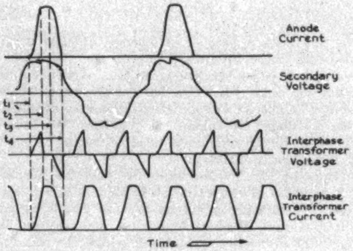
Anode Current
Secondary Voltage
t1
t2
t3
t4
Interphase Transformer Voltage
Interphase Transformer Current
Time

FIG. 16

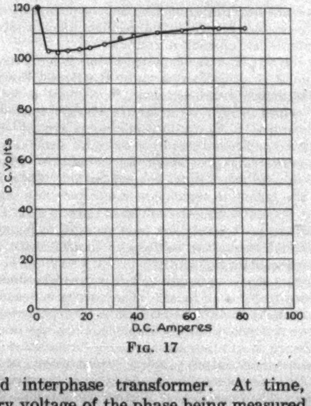

FIG. 17

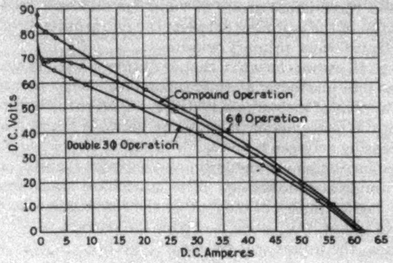

FIG. 18

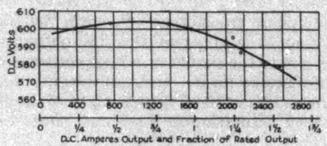

FIG. 19

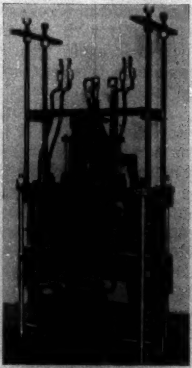

FIG. 20—INTERPHASE TRANSFORMERS

The 180-cycle units are provided with series d-c. saturating coils and variable gap in the d-c. core circuit.

transfer was completed. The dotted lines indicate the form of the secondary voltage and interphase transformer voltage waves, if they had not been distorted by the reactive voltages during current transfer. The slight irregularity in the secondary voltage wave at time t_3 was caused by commutation of currents in the second three-phase group of windings and was present because of the coupling between the two groups through the transformer primary windings.

Fig. 16 shows the wave shapes obtained with the core of the interphase transformer saturated. The operation is now that of a six-phase rectifier. Current is building up in the phase under observation between t_1 and t_2 and decreasing between t_3 and t_4. Not more than two anodes carry current simultaneously, and between t_2 and t_3 only one anode is conducting. The interphase transformer now carries pulses of current of the same shape as the anode pulses, but, since it carries the current of all the anodes, the pulses will be closer together. There is a voltage across the interphase transformer only during the time the current is transferring between anodes, which transfer must occur through it. As the reactance of the interphase transformer influences the period of commutation, it is apparent that the output voltage can never rise to the six-phase voltage corresponding to the transformer by itself. Instead, the output voltage will approach this value as the reactance of the choke is decreased by saturating it more and more, but can never reach it.

Fig. 17 shows an experimental compounding curve obtained from a small rectifier. After the first sudden drop to the normal three-phase voltage at six amperes, the voltage rises steadily with increase in load to 70 amperes, after which it again falls. To obtain this curve no changes were made other than changes in load. This rectifier had insufficient capacity to stand loading to short circuit. Accordingly, additional reactance was added enabling the curve of Fig. 18 to be taken. In Fig. 18 the load is increased to short circuit under three conditions, with no saturation of the interphase transformer, with the compounding apparatus in operation, and with the interphase transformer short-circuited. Although the compound curve never equals the six-phase curve except at short circuit, the six-phase voltage is approached sufficiently to give flat compounding from light load to 6½ amperes which is 10 per cent of the current under short circuit.

Tests which have been made on the performance of a 1000-kw., 600-volt, 12-phase rectifier further substantiate the correctness of the theories herein set forth. Fig. 19 is a regulation curve obtained with this rectifier adjusted to give substantially flat compounding out to full load, 1667 amperes. Fig. 20 is a photograph of the interphase transformer. Since the set is twelve-phase, there are three interphase units. The two lower units combine each two groups of three phases and have direct-current saturating windings. The small top unit combines the two six phase groups into one twelve-phase unit.

Discussion

R. H. Wheeler: The paper by Mr. Prince is a very interesting discussion of the potential possibilities of the use of the interphase transformer.

The interphase transformer has been utilized for a long time by Brown Boveri & Co., Ltd., and is incorporated in the circuit diagrams which the American Brown Boveri Electric Corporation has put forth in some of its proposals. We have not used the saturated-core type, however, for it long ago was demonstrated by many installations that the close regulation of the modern sets was sufficiently close for all traction purposes and very few industrial applications require the complications of the saturated interphase transformer.

Extended experience has also proved of late years that the compound rotary converter has not taken the position that it enjoyed some years ago when over-compounding was thought necessary. Wherever compounding is used of late it has been entirely of the flat compound type. To our knowledge, however, the shunt rotary converter has had a much greater application and appears to satisfy regulation and voltage conditions in all usual operations. To meet the flat compound or the over-compounded rotary converter, there has been introduced into the mercury arc rectifier installation circuits an induction regulator which quickly compensates for the drooping characteristic of the rectifier and permits it to parallel at all loads with compound rotary converters.

There has been a number of schemes utilizing a variable-reactance core in the interphase transformer. The d-c., saturated core described by Mr. Prince has been utilized in a number of installations some years ago, but has since been abandoned because of a number of factors which limited the degree of compounding demanded by commercial practise. We have found that the rising voltage characteristic is not sought by purchasers of such conversion machinery.

The paper by Mr. Prince points out that the regulation is largely a function of the variation in load as it increases and decreases. The scheme as offered does not permit an independent voltage control. We have found that there are possibilities of utilizing shunt connections which will permit the movement of the voltage curves vertically upward or downward, at the same time obtaining the benefit of the control Mr. Prince has described. The Brown Boveri Review of 1919, Nos. 7, 8 and 9, describes such an installation and discusses the possibility of shifting the load by voltage control of the rectifier where a rectifier and rotary converter are operating in parallel.

During the extended development of the mercury arc rectifier in large power sizes, as carried on by the Brown Boveri & Co. Ltd. of Switzerland, various schemes have been developed and patented, which permit of a flexible voltage control. These forms of control, however, include complications of circuits which demand additional reactance coils and machinery, and are not as commercially practicable as the circuits now offered, which provide for usual shunt characteristics. There is always available the induction regulator, transformer, tap changer, or similar device for changing the voltage of the sets.

The mercury arc rectifier has been installed in heavy electric traction service at varying voltages up to 4000 volts, d-c. The limit of the phenomena of conversion by the mercury arc rectifier does not seem to have been approached at the highest commercial d-c. voltages in operation today. The rectifier is causing a great deal of study to be made of its possibilities by engineers interested in the electrification of steam railroads, since the weights and dimensions are at a minimum not heretofore reached per kilowatt of substation output. No overhead cranes, heavy wall construction or heavy machine foundations are required in such substations. The operation of putting the machine in service or taking it out of service is no more complicated than throwing on the bank of transformers which serve the rectifiers, thus permitting the substation employee to be easily trained and of a type not essentially classed as "highly skilled." I can clearly visualize the use of the rectifier being established along the right-of-way of a steam railroad at much more frequent intervals than the rotating machinery type of substation providing much closer d-c. trolley regulation and obtaining alternating current from any commercial source of any commercial frequency, thus obviating the use of railroad-owned power stations and transmission systems.

It is interesting to note the control of the voltage and current curves as described in the paper by Mr. Prince. He sets forth the current characteristics in a very detailed fashion and we think that if it is possible to apply these schemes of voltage control to the twelve-phase rectifier, a very satisfactory current wave form will result.

Frequently we have been asked "What about radio interference?" because of the undulating character of the current wave form. Exhaustive tests were made by the largest utility company in the Middle West, using very sensitive radio receivers which were moved about the substation, and placed in all positions, to determine the extent of interference with radio reception. The results of this test proved definitely that there is no interference caused by the wave form emanating from the rectifier and through the circuits it feeds. We believe this test is of interest, as the wave form of the theoretical circuit might cause an apprehension upon the part of some engineers if the practical test had not been made. Furthermore, full appreciation had not been given to the fact that the superimposition of many phases of rectification causes practically a flat wave rather than a sharply undulating wave.

Otto Naef: I think Mr. Wheeler is right when he says that in Europe not much use has been found for the interphase transformer with d-c. magnetization. The method proved successful from the point of view of voltage regulation, but this asset was not considered important enough to set aside the disadvantage of higher cost, lower efficiency and power factor in a plant thus equipped. Besides, there has been a marked tendency in Europe in the last few years to eliminate, so far as possible, any complicated features.

I may mention here that instead of using a saturated interphase transformer, saturated transformers or choke coils in the anode circuits may be used with equal success. In addition to, or instead of, a series winding, a d-c. shunt winding may be put on the core, which, if excited from the d-c. mains, makes it possible to raise and lower the characteristic of the rectifier in much the same way as in an ordinary shunt-wound, d-c. machine.

Mr. Prince's curve shows a very steep rise in the voltage at low loads, which is a characteristic feature of the interphase transformer. It can be avoided by using a booster transformer instead of the interphase transformer. It may be excited from the low side of the rectifier transformer through a reactance coil, the inductance of which is varied by the application of d-c. magnetization. In this way it is possible to vary the boost and thereby obtain compounding of the d-c. voltage without getting that first kick.

F. A. Faron: Mr. Prince has removed one of the serious disadvantages of mercury arc rectifiers when applied to railway service. A considerable proportion of the rectifier applications will be additions to systems where revolving apparatus having very definite characteristics is already in use. It is hardly conceivable that the characteristics of an existing system would be changed to permit the application of rectifiers with naturally drooping characteristics. On many occasions, it is necessary to install an additional unit in an existing station containing converters, and a rectifier with drooping characteristics would divide loads properly with the existing units at *only one* point. Beyond that, the converter would have a tendency to take too great a share of the load and at low loads the rectifier would cause reverse current to flow through the converter, which would at least force the converter off the bus and possibly cause other difficulties. This condition is analogous to attempting to operate a shunt-wound and a compound machine on the same bus, which we all realize is a very unsatisfactory arrangement.

It has been proposed that the rectifier be operated with converters where the regulation of the converter is poorer than that of the rectifier. This is a very unusual condition, and I am not familiar with any place where this condition exists.

With the method of compounding suggested by Mr. Prince, the interphase transformer may be arranged so that the saturating winding may be equalized with the series field of the converter, insuring load division at all points.

Compounding is desirable from an economic standpoint, as none of the features considered desirable in a rectifier are impaired and holding practically flat voltage at the substation is a condition which requires a minimum amount of feeder copper. This is applicable to approximately 90 per cent of the railway substations in this country.

A rectifier without compounding is comparable to a shunt-wound converter having about the same regulation, whereas, with the compounding feature, the regulation is practically the same as that of a compound-wound converter. Other methods, such as tap changing or use of an induction regulator, while applicable possibly to an isolated station, are objectionable on account of external devices requiring maintenance, and the fact that no induction regulator is fast enough to follow load swings on the average railway substation. If possible to build a regulator of this sensitivity, the maintenance would necessarily be high, as the unit would be in practically continuous operation.

The method of regulation offered by Mr. Prince is as simple as the series field on a converter. It will find large application and may be considered one of the most valuable additions to information on rectifier circuits made in recent years.

E. B. Shand: I more or less agree with Mr. Wheeler that the shunt-wound converter is becoming more popular now for traction and the value of the over-compounding or flat compounding is not as great as it used to be.

There is one point to be made, however. It will be noticed that with the core unsaturated you get the effect of the ordinary balance coil and that the anode current flows for one-third of a cycle, but that with the balance coil saturated it flows for only one-sixth of a cycle. That means the transformer will have to be about 25 per cent larger; so it will be economically at a disadvantage.

D. C. Prince: In the discussion of this paper, three questions have been raised: 1. Can the method be employed to give voltage control independent of line-current variations? 2. Is excessive transformer secondary heating introduced by the operation of compounding? 3. Will its popularity be reduced by the trend toward substation layouts which limit the load on one substation by dropping its voltage?

In answer to the first question, I should say that the problem of saturating the interphase transformer from a separate source is simpler than that of obtaining the correct characteristics by self saturation. If the latter problem is solved, the former follows automatically.

As a rectifier is loaded, the current waves in the several secondary windings spread out so that their utility factors improve. In a flat compound rectifier, the tendency of the compounding to make the utility factor worse is offset by the normal tendency of this factor to improve with load. The light-load utility factors are three-phase 0.68, six-phase 0.55. The six-phase, short-circuit, secondary utility factor is 0.75 so that the transformer would not be expected to require materially greater secondary copper on account of compounding. Utility factor is defined as the ratio of no-load rectified voltage times rectified current to alternating sine-wave kv-a. for the same root-mean-square heating.

A very desirable thing about the compound characteristic is that by the use of high-reactance transformers it can be taken advantage of to hold substantially constant voltage to full load and yet with overloads the voltage breaks down more rapidly than that of a shunt machine so that the unit will shift its load to other substations. Since the rectifier cannot fall out of step, a generously designed set might be made to stand short circuit for a brief period without being tripped free of the lines.

Polarization of Radio Waves

BY E. F. W. ALEXANDERSON[1]
Fellow, A. I. E. E.

UNTIL rather recently, the practise of radio communication was confined to the use of long, earth-bound waves. These waves are preferred on account of the regularity of day and night operation and the absence of fading. The characteristics of the earth-bound wave were extensively explored and there was a tendency to generalize these results assuming that they apply to all radio wave transmission.

Usually the earth-bound wave is thought of as a moving electromagnetic field with horizontal magnetic lines sweeping parallel to the earth and vertical electrostatic lines terminating in the conducting earth. This theory has been supported by a great deal of practical evidence. Attempts to measure horizontally electrostatic lines have always given negative results and it almost seemed obvious that this should be so. It therefore seemed quite unnecessary to speak of polarization of radio waves, since the waves were supposedly always vertically polarized due to their nature of being earth-bound.

A great amount of data was collected and among these were some surprising observations. For instance, it seemed as if the wave did not always come in from the proper direction but wandered around, seeming to come at times from the side and sometimes even from behind. This led to a good deal of discussion, but no plausible explanation was found. Yet the evidence was accepted as incontrovertible that such direction changes did exist because the observations were confirmed by so many competent observers.

Recent investigations of the phenomena of radio wave propagation, however, have led to another explanation, the conclusion being that the observed irregularities were not actually changes in direction but changes in the plane of polarization. Whether such an explanation had been considered at some earlier date the author does not know, but if it was tentatively advanced, it was undoubtedly discarded on the ground of the experimental evidence, showing that the horizontal electrical lines did not exist. As a matter of fact the polarization theory did not originate with the study of long-wave phenomena and it requires a new conception of wave propagation, developed through recent work on short waves.

OBSERVATIONS ON SHORT WAVES

Our first experience in signaling with horizontally polarized waves was gained as a result of an incident in an investigation which had been organized to study directive radiation with short waves. One of the several directive antenna systems which were used was a combination of two large, square, vertical loops, tuned for a wavelength of 50 m. Each loop consisted of eight sections of conductor, separated by eight condensers so that each section was independently tuned. The composite antenna thus had four vertical conductors, two in each loop, and the object of the test was to regulate the direction and phase of the currents so that the composite antenna would give a unidirectional radiation in the plane of the loops. The only practical way to adjust the phase of the currents proved to be to make measurements of the composite radiation within a few hundred feet of the antenna. When the radiation diagram so obtained was in agreement with the theoretical expectations, it was assumed that the system was properly tuned. The experimental station at Schenectady was in radio communication with the stations of the Radio Corporation of America, at Riverhead, Long Island, and Belfast, Maine, where the signals were measured to ascertain whether the directive characteristics were the same at distant points as near the station. In one case it was found that no radiation whatever could be observed in the field around the station. But investigation showed that, by accident, one of the loops had been reversed so that while current was flowing in all the vertical conductors as indicated by the ammeters, the resultant effect in radiation was zero. No sooner had these facts been ascertained than a communication was received from Long Island that the signals were 50 per cent stronger than in previous tests. In view of the absence of any radiation near the station this surprising result called for further analysis. It was thus found that while the currents in the vertical conductors were in such a direction as to neutralize each other, the two top conductors carried current in the same direction. These conductors being horizontal, it followed that the radiation must have been horizontally polarized and that the failure of the local instrument to indicate any radiation was due to the fact that it was sensitive only to vertical radiation.

Since then, a number of antenna forms for horizontally polarized radiation have been tested and used in

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commercial service by the Radio Corporation of America. The practical conclusion reached through these tests, as well as the findings of the government laboratories and many amateurs, is that in most cases horizontal transmission and reception with short waves is superior to the old methods of using vertical polarization.

When one attempts to discuss the phenomena of wave polarization, he finds many theoretical and practical aspects and many questions which may be asked. For instance:

How does the wave acquire a horizontal polarization when it is radiated vertically and why does the wave become vertical when it is radiated horizontally?

Does the wave always twist in space or only under some circumstances, and what are these circumstances?

Why is horizontal polarization the rule on short waves and so difficult to discover on long waves?

How strong is static in the horizontal plane?

What relation has polarization to fading?

Does a wave fade in the horizontal and the vertical plane at the same time?

Can direction finders be made to compensate for errors due to polarization?

Many such questions may be asked and only a few can be answered definitely. If the idea of wave polarization is to be more than an empty phrase, we must try to form in our minds a picture of what the wave itself consists of, before we can intelligently discuss the peculiarities of its behavior.

THEORY OF WAVE MOTION

This subject is being approached from two angles: one, the classical theory of light radiation in the ether; the other, the electromagnetic theory as we know it in electrical engineering. Both of these theories have their difficulties. The only positive knowledge we have of electricity is the electronic theory. We know that the electron is the smallest element of matter. We know its mass, its electric charge and how fast it travels. But we do not know what magnetism is. As a matter of fact there is good reason to believe that there is no distinct force that can be called a magnetic field. Modern science also denies the existence of ether. We have tentatively substituted a conception of electromagnetic field, in which the energy appears sometimes in electric form and at other times in magnetic form. This is a mathematical rather than a physical substitution, but it is convenient because it permits us to use the equations for the electric and magnetic fields as used in electrical engineering. However, it is not an explanation. To an electrical engineer, a magnetic field is very real and in its nature is quite different from an electrostatic field. From this point of view, it requires a good deal of imagination to conceive that a magnetic field is nothing but a manifestation of a moving electrostatic field. Yet we may be forced to change our conception to that extent. This means the return to Faraday's idea of tubes of force which are material bodies with mass and elasticity.

The ordinary antenna consists of a vertical wire in which electrons periodically move up and down, and the space field, which accompanies the electrons, moves up and down with them. This moving space field is a physical reality. It has a mass and kinetic energy stored in its motion and it has elasticity. A physical model of such a field can be built. The field itself extends into space but the different portions of the field do not move simultaneously. This elastic, electric body moves in the only way in which one may conceive of a structure maintaining a periodic motion, that is, in waves proceeding from the center. The magnetic field is nothing but the kinetic energy of this moving structure. The electromotive forces which, according to our well-known equations, are induced by the change in the magnetic field, are nothing but the elastic forces which react against the inertia of this electrical body.

With this picture in mind, it is not difficult to see the physical meaning of wave polarization. The ordinary radiator has a vertical conductor in which the electrons move up and down and the accompanying fields move up and down with them within a radius of one-quarter wavelength. The inertia of this moving field, close to the conductor, is propagated through an elastic medium in accordance with ordinary laws of wave motion. In the wave which thus proceeds from the origin, every element oscillates in a vertical plane. Thus the vertically polarized radiation is closely analogous to a wave on the surface of water.

MECHANICAL MODEL SHOWING HOW A PLANE WAVE CHANGES INTO A SPIRAL

The radiator of horizontally polarized waves consists of a closed loop conductor in the horizontal plane. In this conductor the electrons circulate first in one direction and then in the other.

MECHANICAL MODEL FOR STUDYING WAVE POLARIZATION

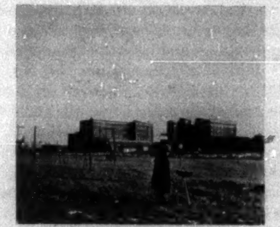
A mechanical model has been built for the study of wave polarization in the laboratory. The model consists of weights suspended in such a way as to make them free to move in all directions. Twenty-two of these weights are arranged in a row and connected by rubber bands. Each weight is suspended from a yoke and an equal weight hung on the other side of the yoke to serve as a counter weight. A screen is set up so as to hide the counterweights and avoid confusion in observing the wave motion. This model was set up especially to study the twisting of the plane of polarization and the experiment has strikingly confirmed the theory which it was intended to illustrate. This theory is briefly as follows:

Let us assume that the medium through which the radio waves pass has such characteristics that the velocity of propagation for a vertically polarized wave differs slightly from the velocity of the propagation for a horizontally polarized wave. For the present purpose, it is not necessary to try to explain the reason for this difference in velocity, but we may assume that the reason for it is the electrostatic or magnetic earth field or a retarding effect due to the closeness of the earth. Whatever the cause, we may assume that such a difference of velocity exists and the mechanical model has been constructed so as to reproduce such conditions. Thus wave motions in the horizontal or vertical planes can be studied independently, and these two wave motions may be adjusted for different velocities. A wave started in the vertical plane maintains itself vertically and a wave started horizontally maintains itself horizontally. If, however, a wave is started in a plane 45 deg. between the vertical and the horizontal, it is found that the wave motion proceeding therefrom assumes the shape of a spiral. The straight-line oscillation of the first weight is passed along as an elliptical motion which gradually widens into a circle. Then this circle again narrows down to an ellipse and finally to a straight line at right angles to the original line of oscillation. This is exactly in accordance with the theory. The point where the wave has shifted its plane of polarization 90 deg. is the point at which the faster of the two waves is half a wavelength ahead of the slower wave. From this point on, the wave proceeds, repeating this peculiar spiral motion.

Transmitting Loops Used in Tests which Revealed Horizontally Polarized Radiation

The fact that the twisting of the wave is due to different velocities in the two planes of polarization can also be demonstrated by this model. For such a purpose, the tension of the rubber bands between the counter weights is changed. The effect of this is to change the velocity of propagation in the vertical plane, whereas the velocity in the horizontal plane has not been affected because only the vertical motion is transmitted to the counter weights by the suspension yokes. The system can thus be adjusted so that the velocities in the horizontal and the vertical planes are exactly equal. After this has been done, it is found that the tendency to spiral motion disappears and the wave remains strictly in the plane in which it has been started.

While this mechanical experiment brings out no new facts unknown to the classical theory of wave motion, it helps us to visualize the main phenomena in the radio wave propagation which we are trying to explain. The phenomenon of a constantly shifting plane of polarization, discovered experimentally in the tests between Schenectady and Long Island, can thus easily be explained.


Transmitting Antenna for Horizontally Polarized Waves and Receiving Instrument Indicating Apparent Wave Direction at Right Angles to True Direction of Propagation

This conception of the wave motion also is a help in explaining the phenomenon of fading. There is much experimental evidence that fading is a phenomenon of wave interference. In other words, the fading is due to the fact that the radio waves arrive at a certain point through two paths. The waves will sometimes add to each other and sometimes neutralize one another. If one bears in mind the observations on the mechanical model and the fact that the waves in the two planes can be traced through, separately and distinctly, one may conclude that the two paths of the radio wave which produce fading are not necessarily two separate physical paths but may be the two paths in the horizontal and the vertical plane of polarization. For further illustration of this, a detector may be introduced into the mechanical model. If this detector is placed at a certain distance from the origin it is found that it gives no response when the system is adjusted for different velocities of propagation, whereas when the system is adjusted for equal velocity in the horizontal and vertical plane it gives a maximum response. Thus the phenomenon of fading has been reproduced mechanically through polarization in a single wave path.

By this, it is not suggested that in actual radio transmission the mechanical equivalent is sufficient to explain the fading. However, it is offered for what it may be worth as a help to interpret the many observations in actual radio transmission which are being accumulated.

IRREGULARITIES OF DIRECTION FINDERS

It has been known almost since the beginning of transatlantic communication with long waves that measurements of direction of wave propagation with rotating loops show peculiar irregularities from the time of sunset. It has also been known among aviators that direction finder bearings on an airplane are correct only if the plane flies in the line towards the observing station. If the plane flies at right angles, the direction finder gives false orientation as high as 45 deg. or more. This false orientation is greater if the antenna is trailing horizontally. It is therefore attempted to keep the antenna as nearly vertical as possible by a weight.

A third set of observations has been brought out through the research work in Schenectady on horizontally polarized waves radiated by horizontal loop. Measurements with a direction finder receiver usually give bearings approximately at right angles to the place where the station really is, but sometimes it gives no direction indication at all. Other measurements indicate that the direction of wave propagation is almost perpendicularly vertical. The observation that the wave appears to come straight down from above suggested an explanation that wave components radiated directly upwards had been reflected straight down by the Kennelly-Heaviside layer. However, in view of the other facts to be considered this explanation seems less likely.

Putting all these facts together it seems now that the old observations on the long wave, the airplane, and our recent work on the horizontal loop all may be explained as a characteristic behavior of the horizontally polarized wave. In all three cases, while the wave appears to come in from unexpected directions, it actually does not. When in the third case there is no direction indication whatever, and the wave appears to come in from above, this also is an illusion. The question is, what really does happen?

This is the problem on which the experiment with the mechanical model can throw some light. For this purpose let us return to the idea that the radio wave is a mechanical wave motion in the elastic electric medium. In the mechanical model, the weights represent the mass and the rubber bands the elasticity of this medium and the vertical as well as horizontally polarized wave can easily be reproduced. But in order to imitate a wave motion over the surface of the earth, one must also in some way imitate the presence of the earth. The earth is a conductor and therefore the elastic strains represented by the rubber bands cannot exist in the earth. On the other hand, displacement currents in the electric medium can induce conduction currents in the earth. These conduction currents are electrons in motion which can be represented by weights not tied together by rubber bands in the horizontal plane, whereas they are electrically associated with the electric medium above. To imitate this condition, weights may be hung by vertical rubber bands so that they are elastically associated with the wave medium but not connected to each other. If, now, a horizontally polarized wave is sent forth through this system, it is found that the wave motion is propagated to the vertically suspended weights producing elastic strains in the vertical rubber bands. It must be remembered that the elastic strains represent electromotive forces and these elastic strains so produced are of the same character as if they were a part of a vertically propagated wave motion. No such wave motion actually exists and these strains are only the electromotive forces which induce currents in the ground. If we now assume that a receiving antenna is set up in the form of a vertical loop with its plane at right angles to the wave motion, the primary wave motion does not induce any currents in the loop. However, the secondary electromotive forces, which induce currents in the ground, are in the plane of this loop and tend to induce such currents in the loop. In other words we may say that, inasmuch as the currents induced in the ground are at right angles to the wave motion, the loop is in the plane of those ground currents which, in their turn, induce currents in the loop.

EXPLORING ANTENNAS

If this theory is correct, it should be found that false indications of the direction finder and the apparently vertical wave propagation can be observed only in the proximity of the ground. Thus, if observations are made in airplanes high enough from the ground, the horizontally polarized wave should show a horizontal plane of polarization with a true direction of propagation.

Some measurements have been made which confirm these conclusions. By making frequent measurements to within ten miles of the station a set of tests was made exploring the characteristics of a wave radiated from a horizontal loop. The composite picture which was obtained from this test was a continuously twisting plane of polarization with alternate points of plane and circular polarization. At intermediate points, the polarization was elliptical. The plane polarization was indicated by sharp directional bearings and circular polarization was indicated by equal intensity from all directions. The observations indicating plane polarization gave bearings sometimes towards the transmitting station and sometimes at right angles.

Beside these measurements around the vertical axis, observations were made with the loop in the horizontal plane. On flat fields the horizontal position gave nearly zero response. At the top of a steep hill and a high bridge, the response in the horizontal plane was equal to the vertical.

These results indicate the presence of a horizontal and a vertical wave component with different velocity of propagation. Whenever the two waves are in phase, they give plane polarization. When they are 90 deg. out of phase, they give circular polarization. The observation with the loop in the horizontal position on the top of the hill and the bridge shows that even a moderate elevation is sufficient with short waves to reach the point at which the horizontal electromotive forces are not short-circuited by the ground.

All this leads the author to believe that horizontal polarization is not confined to short waves only. Direct observation of horizontal polarization at long waves could be made only at great heights but indirect observations through the effect of ground currents can be made by ordinary direction finders at any wave length. If this theory is correct it means that the irregularities of direction finder indications recorded on long waves can be explained by the presence of horizontally-polarized wave components.

Current Transformers with Nickel-Iron Cores

BY THOMAS SPOONER[1]

Member, A. I. E. E.

Synopsis.—*There has been developed recently a nickel-iron alloy called hypernik which has especially low hysteresis loss and high permeability at low inductions. Due to these properties the material is particularly suitable for the cores of current transformers. A ring-type series transformer with a core of hypernik has approximately one-third of the ratio and phase-angle errors exhibited by a similar transformer having a core of ordinary silicon steel.*

The relative performance of transformers with cores of the two kinds of material are shown by test and by calculation for various burdens and ratios. The minimum ratio for which satisfactory performance can be obtained with a through-type transformer having a hypernik core is about 200 to 5.

This new material is relatively expensive but its use will undoubtedly be warranted for certain applications.

THE accurate metering of power supplied at high voltages is difficult due to limitations in the performance of through-type current transformers. With a single turn for the primary the available m. m f. for low currents is very small. For a standard five-ampere secondary winding the number of secondary turns is fixed by the ratio; therefore, for a given burden inductions in the cores are high which means relatively large core losses and magnetizing currents. As the transformer ratio decreases the phase-angle errors rise very rapidly due to losses and the low permeability at low inductions for the ordinary silicon-steel core materials.

Some years ago L. W. Chubb recognized the possibility of improving the performance of current transformers by the use of certain nickel-iron alloys which have especially high permeability at low inductions.[2] Such a material has now become commercially available through the efforts of T. D. Yensen and P. H. Brace.

The purpose of this paper is to show some of the advantages which may be gained by the use of a particular nickel-iron alloy called hypernik for the cores of through-type transformers.

CURRENT TRANSFORMER CHARACTERISTICS

If the magnetic properties of the core of a current transformer and the leakage reactances of the windings are known, the ratio and the phase-angle errors for any secondary current or burden on the transformer may be calculated quite accurately. Since this particular discussion is limited to through-type transformers, the effect of leakage reactance will be neglected since this is practically negligible for this type of transformer.[3]

In order to calculate the performance of ring-type transformers, it is necessary then to know only the watt and wattless components of magnetizing current for the particular core at the desired frequency and induction as determined by the secondary current and burden. A method of measuring the core characteristics will be discussed later.

The relation between the magnetic properties of the core and transformer performance has been shown very clearly by Agnew.[4] Fig. 1 shows the vector diagram of a current transformer supplying an inductive load.

E_2 is the secondary voltage.

I_1 and I_2 are the primary and secondary currents, respectively.

n is the ratio of the number of secondary to primary turns.

φ is the phase angle of the secondary current.

M is the magnetizing component of the core exciting current necessary to produce the flux Φ.

F is the corresponding loss component of exciting current (due to hysteresis and eddy losses in the core).

Because of the magnetizing and loss components of the exciting current there will result (unless the exciting current is in phase opposition with the secondary current) a phase angle between the primary and secondary currents which will differ from 180 deg. by the angle θ. This is the phase-angle error of the transformer. The ratio error is proportional to the difference between I_1 and $n\,I_2$.

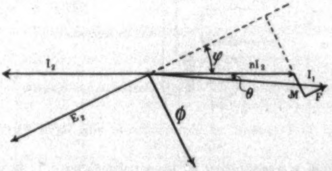


FIG. 1—VECTOR DIAGRAM OF CURRENT TRANSFORMER

Agnew gives the following simplified formulas for calculating the errors in current transformers.

$$\text{Ratio } (R) = n + \frac{M \sin \varphi + F \cos \varphi}{I_2} \qquad \textbf{(1)}$$

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[1] Research Engineer, Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

[2] U. S. Patent Number 1,277,384.

[3] Effect of Magnetic Leakage in Current Transformers. H. W. Price, C. K. Duff.

Papers on Current Transformers. *Bull.* No. 2, Section No. 4, University of Toronto, 1921, p. 169-171.

▶ *Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Niagara Falls, N. Y., May 26-28, 1926.*

[4] A Study of the Current Transformer with Particular Reference to Core Losses. P. G. Agnew, *Bull.* Bureau of Standards, Vol. 7, 1911, pp. 423-474.

$$\operatorname{Tan} \theta = \frac{M \cos \varphi - F \sin \varphi}{n\, I_2} \qquad (2)$$

Obviously if φ is 0, namely, if the burden on the secondary of the transformer is non-inductive,

$$R = n + \frac{F}{I_2} \qquad (3)$$

$$\operatorname{Tan} \theta = \frac{M}{n\, I_2} \qquad (4)$$

In other words, for this case and a given secondary current the ratio depends only on the ratio of the secondary to primary turns and the loss in the core. The phase angle depends only on the same turn ratio and the magnetizing current or permeability of the core material.

A material is desired, therefore, which has low losses and high permeability at low inductions. It is such a material which has now become available.

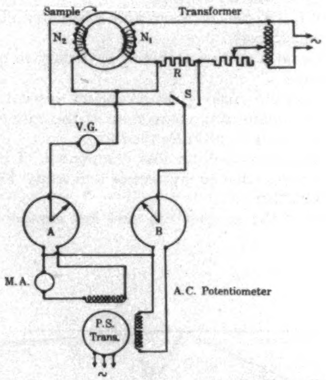

Fig. 2—Diagram of Connections for Ring Testing

Determination of Magnetic Properties of Core Materials

In order to predetermine the performance of a given design of a current transformer under definite conditions of load, it is desirable to have curves for the specific kind of core material which is to be used, giving the watt and wattless components of the exciting current plotted against induction. Such data can be obtained by supplying a sample of core material with a winding or windings, applying different voltages of the desired frequency and measuring the current and watts by means of sensitive indicating instruments. For very low inductions this is rather a difficult and tedious process since the wattmeter must be very sensitive and corrections must be made for the instrument losses.

A much quicker and simpler method is to use a Tinsley-Gall, a-c. coordinate potentiometer.[5] The use of this instrument for such a purpose has been described recently.[6] The method of test is shown in Fig. 2.

The a-c. potentiometer, itself, is shown schematically. It consists of two distinct potentiometers supplied with currents which are equal in magnitude but differ 90 deg. in phase. These currents are obtained from a phase-shifting transformer having a two-phase secondary winding. Voltage balance is indicated by means of a vibration galvanometer. The sample, which is shown in ring form, is provided with primary and secondary windings N_1 and N_2. A-c. current of the desired frequency is supplied from a convenient source through a regulating transformer and a series resistance. This source is the same as that which supplies the primary of the phase-shifting transformer. The primary current I_1 is first adjusted to some convenient value and switch S is thrown to the left, thus connecting the potentiometer to the secondary winding N_2. Potentiometer B is set to O and the phase-shifting transformer and potentiometer A are then adjusted for O reading of the vibration galvanometer $V\,G$. When this adjustment is completed, A gives the magnitude of the secondary voltage and from it the induction in the sample can be calculated as follows, assuming a sine wave of voltage, which is nearly true for low and moderate inductions assuming approximately a sine-wave supply of current. This assumption is far from true, however, for high inductions.

$$B = \frac{E_2 \times 10^8}{A \times N_2 \times f \times 4.44} \qquad (5)$$

where

B is the induction in gausses
E_2 is the voltage induced in the secondary
A is the cross-section of the specimen in sq. cm.
N_2 is the number of secondary turns
f is the frequency.

Next, S is thrown to the right connecting the potentiometer across the terminals of the shunt R. Then potentiometers A and B are adjusted for a vibration-galvanometer balance. The reading of A is now proportional to the loss component of the current in the primary and the reading of B is proportional to the wattless or magnetizing component of the current. If R equals 1 ohm, the readings of A and B are directly in amperes. This operation may be repeated for as many inductions as is desired. If more convenient, a single winding only may be used on the sample, provided it has a negligible resistance.

If now the magnetizing current is multiplied by the voltage and divided by the weight of the sample in pounds, there will result the wattless volt-amperes per pound for the material. Similarly the watt component of current gives the watts per pound. This is on the assumption that N_1 and N_2 are equal. By using a sufficiently large sample, data have been obtained by this method for a range of inductions from a few gausses to several thousand gausses, both at 25 and 60 cycles.

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[5] A New A-c. Potentiometer, by D. C. Gall, *Electrician*, Vol. 90, April 6, 1923, p. 360.

[6] Some Applications of the A-C. Potentiometer, T. Spooner, *Journ.* Optical Society of America and Review of Scientific Instruments, March 1926, p. 217.

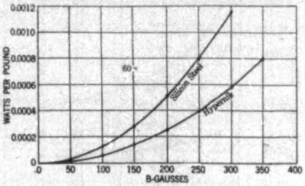

FIG. 3—CORE LOSS OF RING PUNCHINGS

Ring Dimensions = 5-7/8 in. by 3-5/8 in.

Figs. 3 to 6 show the watt and wattless volt-ampere for 60 and 25 cycles for a good grade of four per cent silicon steel as compared with a rather poor specimen of hypernik. The thickness of laminations was 0.014 in. (0.0356 cm.). These particular curves cover a rather limited range of inductions but indicate clearly the difference in the properties of the two kinds of material. The marked superiority of the hypernik is obvious. For actual design purposes it is more convenient to plot the results on double-log paper since a much larger range of inductions can be covered with less confusion of scales.

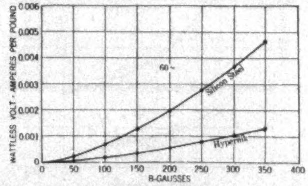

FIG. 4—MAGNETIZING CURRENT OF RING PUNCHINGS

Ring Dimensions = 5-7/8 in. by 3-5/8 in.

The Calculation of Performance. Using the magnetic characteristics of the material as given by such curves as shown in Figs. 3 to 6, and by using formulas (**1**) and (**2**), it is a very simple matter to predetermine the characteristics of a through-type transformer. The method will be illustrated by an example and the results compared with actual test values. The following constants apply to a transformer actually built and tested at 60 cycles.

Core Material.

Hypernik

Rings: $5\frac{7}{8}$ in. (14.9 cm.) outside diameter; $3\frac{5}{8}$ in. (9.2 cm.) inside diameter

Weight: 10 lb. (4.54 kilograms)

Cross-section of core (density 8.3) = 14.22 sq. cm.

Secondary Windings.

Turns: $(N_2) = 100$

Resistance = 0.109 ohm

Burden

15 volt-amperes at 80 per cent power factor (resistance 0.480 ohms)

Inductance = 956 microhenries.

The total resistance to be considered in calculating the performance must include the secondary resistance of the transformer winding; therefore,

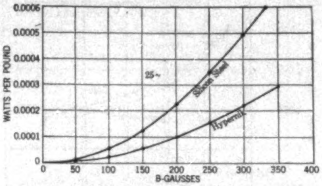

FIG. 5—CORE LOSS OF RING PUNCHINGS

Ring Dimensions = 5-7/8 in. by 3-5/8 in.

$R = 0.109 + 0.480 = 0.589$ ohm.

$X = 2\pi f \, 956 \times 10^{-6} = 0.360$ ohm.

$\sqrt{R^2 + X^2} = 0.690$ ohm

$\tan \varphi = 0.611$

$\sin \varphi = 0.521$

$\cos \varphi = 0.853$

One ampere in the secondary, therefore, corresponds to 0.690 volt and from this the inductions can be calculated by the ordinary transformer formula (**5**).

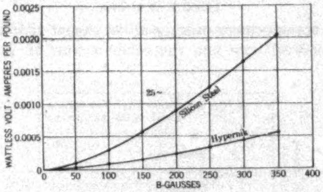

FIG. 6—MAGNETIZING CURRENT OF RING PUNCHINGS

Ring Dimensions = 5-7/8 in. by 3-5/8 in.

TABLE I

CALCULATION OF TRANSFORMER PERFORMANCE

			per lb.						
I_2	E_2	B	$V\,A$	W	M	F	R	$\tan \theta$	θ
0.5	0.345	91	0.00015	0.000049	0.435	0.142	1.0069	0.00590	20′
1	0.690	182	0.00047	0.00021	0.681	0.304	1.0060	0.00422	14′
2	1.380	364	0.00140	0.00086	1.014	0.624	1.0053	0.00270	9′
3	2.070	546	0.00259	0.00200	1.250	0.966	1.0049	0.00188	7′
4	2.760	728	0.00393	0.00348	1.424	1.261	1.0045	0.00141	5′
5	3.450	910	0.00540	0.00530	1.566	1.537	1.0042	0.00107	4′

The wattless volt-amperes per pound, $V A$, and the watts per pound, W, were obtained from curves similar to those given by Figs. 3 and 4, but covering a greater range of inductions. In order to obtain M and F, it is simply necessary to multiply $V A$ and W, respectively, by $\frac{N_2 W_t}{E_2}$ where W_t is the weight of the core material in pounds. R and $\tan \theta$ are then obtained directly from formulas (**1**) and (**2**).

This detailed analysis has been followed through since it has been observed that a person not familiar with this type of calculation is very likely to make a mistake in applying the various formulas.

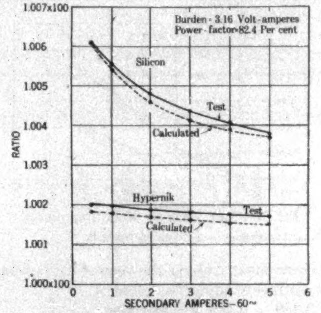

Fig. 7—500-5 Ring-Type Current Transformers

Burden = 3.16 volt-amperes
Power factor = 82.4 per cent

Check Values

Two transformers exactly alike, except that one had a silicon-steel core and the other a core of hypernik, were constructed according to the above specifications, each core weighing 10 lb. They were sent to the Bureau of Standards for check at 60 cycles and two burdens, namely 3.16 volt-amperes and 15 volt-amperes. It may be noted that the net inductions in the two cores were slightly different although the fluxes were identical since silicon steel has a density of about 7.5 and hypernik of about 8.3.

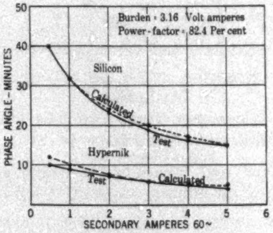

Fig. 8—500-5 Ring-Type Current Transformers

Burden = 3.16 volt-amperes
Power factor = 82.4 per cent

The test and calculated results are given by Figs. 7 to 10. The checks are probably about as close as could be expected and are in general very good. Before calculating the performance, the magnetic quality of the core material was checked on a completed transformer at two or three inductions by means of the a-c. potentiometer and where the results departed appreciably from the curves of Figs. 3 and 4 parallel curves were drawn from which the magnetic data were obtained.

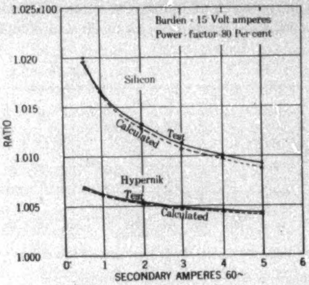

Fig. 9—500-5 Ring-Type Current Transformers

Burden = 15 volt-amperes
Power factor = 80 per cent

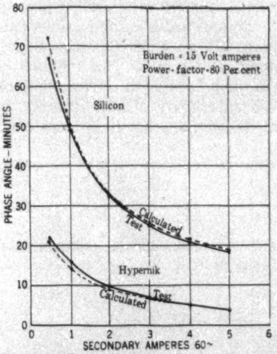

Fig. 10—500-5 Ring-Type Current Transformers

Burden = 15 volt-amperes
Power factor = 80 per cent

Relative Performance of Transformers for Various Ranges

Having shown that it is possible to calculate the performance of this type of transformer with approximately the same accuracy as it can be tested, we are now in a position to show by calculation what can be expected for various ranges. Figs. 11 to 15 give the comparative ratio and phase-angle curves for silicon and hypernik transformers for various ranges of secondary current from 200 to 1500 amperes, using various burdens and weights of core material. The very considerable superiority of hypernik over the silicon steel is seen for these conditions. In comparing the ratio curves it should be remembered that by means of turn compensation the actual ratio curves may be reduced much closer to unity by this means. It will be noted, however, that this compensation will not be nearly as effective for the silicon-steel transformers since the ratio curves are not nearly as flat as for the hypernik. The phase-angle

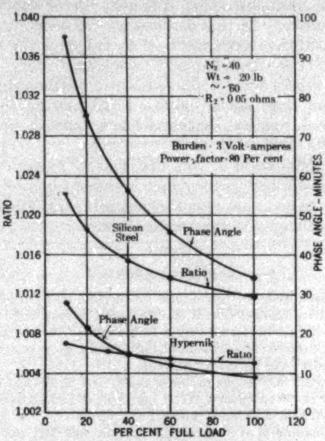

FIG. 11—200-5-CURRENT TRANSFORMERS

Burden = 3 volt-amperes
Power factor = 80 per cent
N_2 = 40
Wt. = 20 lb.
$\sim$ = 60
R_2 = 0.05 ohms

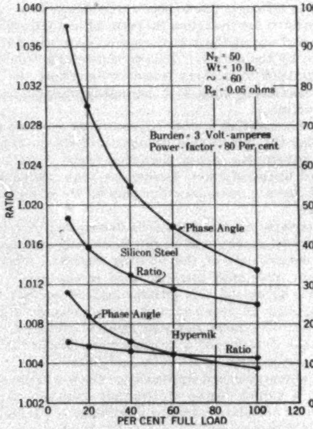

FIG. 12—250-5 CURRENT TRANSFORMERS

Burden = 3 volt-amperes
Power factor = 80 per cent
N_2 = 50
Wt. = 10 lb.
$\sim$ = 60
R_2 = 0.05 ohms

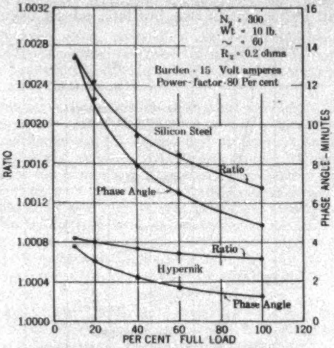

FIG. 13—1500-5 CURRENT TRANSFORMERS

Burden = 15 volt-amperes
Power factor = 80 per cent
N_2 = 300
Wt. = 10 lb.
$\sim$ = 60
R_2 = 0.2 ohms

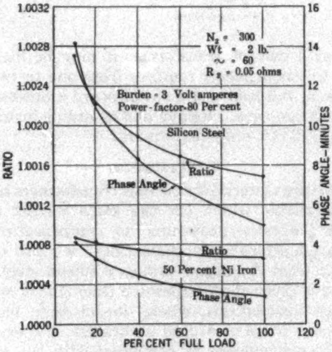

FIG. 14—1500-5 CURRENT TRANSFORMERS

Burden = 3 volt-amperes
Power factor = 80 per cent
N_2 = 300
Wt. = 2 lb.
$\sim$ = 60
R_2 = 0.05 ohms

errors will not be affected much by turn compensation. Of course, compensation for phase-angle and ratio errors can also be effected by means of parallel non-inductive or capacity shunts, but this is rather unsatisfactory since an adjustment has to be made for each burden.[2]

As an indication of the burdens which may be imposed on a current transformer it may be mentioned that a watthour meter requires from one to two volt-amperes at full load and a switchboard ammeter about three volt-amperes. Relays and recording instruments may take 15 volt-amperes or more.

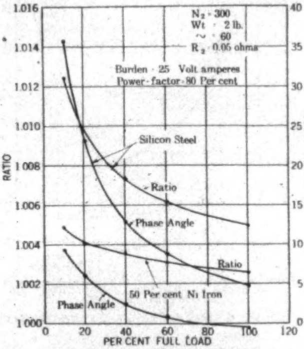

FIG. 15—1500-5 CURRENT TRANSFORMERS

Burden = 25 volt-amperes
Power factor = 80 per cent
N_2 = 300
$Wt.$ = 2 lb.
~ = 60
R_2 = 0.05 ohms

CONCLUSIONS

A new core material for current transformers has been made available which for the same weight of core reduces the ratio (assuming no compensation) and phase-angle errors to approximately one-third of those resulting from the use of ordinary silicon steel. This material is much more expensive than silicon steel but for large installations, where considerable blocks of power need to be measured accurately, the extra cost would be insignificant as compared with the value of the improved accuracy of measurement. Where there are space limitations, if desired, a smaller amount of core material could be used giving the same performance as for the silicon steel and at approximately the same cost, but resulting in a smaller transformer.

The gain to be expected by the use of hypernik for the cores of transformers, using L-shaped punchings, would not be as great as for ring-type punchings, due to the decreased permeability resulting from the gaps. Even here, however, for the same size of transformer a considerable improvement by the use of the new iron would result. The use of hypernik in through-type transformers will perhaps be the most important application since it is in this case that the transformer designer finds his chief difficulty in obtaining good performance. In this connection, however, it may be mentioned that hypernik is of no value for the cores of through-type transformers operating very heavy-burden relays since it saturates at a considerably lower induction than silicon steel. It is only at the lower inductions, namely, for meter loads, that its superiority manifests itself.

The application of this material for improving current-transformer performance is extremely simple since it involves no changes in the general transformer design and no changes in the existing meters.

Discussion

I. F. Kinnard: In the design of current transformers, it has long been recognized that a core material having low losses and high low-induction permeability is desirable. It is undesirable, however, to have the permeability of a transformer core changing very rapidly over its working range.

Wonderful strides have been made in perfecting magnetic materials in recent years. The material described recently by Messrs. Arnold & Elmen[1] known as "permalloy" has properties which recommend it very highly for the use Mr. Spooner has outlined. Mr. Spooner has not given us any specific magnetic or metallurgical data on hypernik, but it probably can be assumed that it is very similar in its properties to the series of iron-nickel alloys described by Yensen[2].

Permalloy in particular, and, to a lesser degree, other iron-nickel alloys, partially fulfill the requirements of an ideal core material. The reason they do not more completely fulfill these requirements is largely due to their rapid change in permeability. In fact, at a fairly low induction the permeability falls off so rapidly that the accuracy of the transformer is seriously impaired, as pointed out by the author. This is particularly troublesome in the through-type or bushing transformers where the operating density is necessarily high with secondary burdens usually met with in practise.

It is to be hoped that further improvement may make it possible to utilize the remarkable properties of these various alloys to greater advantage; that is, that their range of usefulness may be extended to transformers having less than 200 ampere-turns which will operate secondary burdens up to at least 15 or 20 volt-amperes.

I am interested in Mr. Spooner's description of utilizing the coordinate a-c. potentiometer for measuring the magnetizing and watt components of the exciting current. This is a big improvement over most methods used in the past. I believe it is possible to extend this general method so that a strictly null setting may be obtained and the possible accuracy made even greater. We must not lose sight of the fact, however, that the real criterion of a transformer's performance is the precision measurement of its ratio and phase angle.

Several laboratories are equipped to measure these quantities directly to a higher degree of accuracy than we can hope to reach by their calculation from a knowledge of exciting current and magnetic properties.

W. K. Dickenson: The ideal toward which every instrument-transformer engineer is working is to obtain a minimum error in the ratio of transformation of voltage or current, particularly that part of the error commonly called the phase angle, which is caused by the secondary voltage or current not being in exactly 180-deg. phase opposition to the primary voltage or current.

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[1] *Journal* Franklin Institute, May, 1923, pp. 621-632.

[2] A. I. E. E. TRANSACTIONS, 1924, p. 145.

In current-transformer design, this has usually been accomplished by the use of a relatively high number of ampere-turns (usually 1000 to 2000) and by a considerable cross-section of core iron. This materially limits the use of the through-type or single-turn-primary type of transformer, since, as there is only one turn available, the current has to be 1000 amperes or more to give a sufficient number of ampere-runs for a good transformer.

A very considerable improvement in the characteristics of transformers, particularly current transformers, has been obtained by the now very common use of silicon steels. It is very encouraging to note the further improvement in both the ratio and phase-angle errors by the use of high-permeability steels, such as hypernik and other nickel-alloy steels, such as nicaloi or permalloy. Mr. Spooner has shown that not only can the ampere-turns be reduced (he states a minimum of 200 ampere-turns) but the weight of core can also be reduced by the use of hypernik.

As pointed out by Mr. Spooner, however, it is to be regretted that only low secondary burdens may be operated by transformers having cores of these nickel-alloy steels, because of their characteristic of becoming saturated at quite low magnetic densities. It is unfortunate that in the particular application where the through-type or single-turn, primary type of transformer would be of greatest value, *viz.*, in large power stations, the secondary burden is likely to be fairly high, due to the necessary length of the secondary leads. These leads are seldom less than 100 ft. in length (200 ft. No. 10 A. w. g. wire equals 5 volt-amperes) and in one of the new stations in New York City the secondary leads are 1000 ft. in length, 2000 ft. of wire (50 volt-amperes for No. 10 A. w. g. wire).

It is to be hoped that means may be discovered that will enable us, through a wider range, to take advantage of the very encouraging results obtained by Mr. Spooner by the use of nickel-iron alloys.

Thomas Spooner: Mr. Kinnard is correct in assuming that hypernik is a nickel-iron alloy. The nickel content is approximately 50 per cent. Hypernik, however, is a very special alloy, in that it is made from very pure raw materials in a type of electric furnace which permits of no contamination. After being rolled to the proper thickness it is given a special, somewhat expensive heat treatment which was developed for this particular material.

Mr. Kinnard has perhaps misunderstood my purpose in using, to a large extent, calculated instead of test values for current-transformer performance. It was first shown that for through-type transformers, this is a reliable procedure. The calculated performance for a number of sizes of transformers was then determined, since this is much quicker than actually constructing the transformers and then testing them. If transformers are actually to be built, it is of course better to measure than to calculate their errors if accurate results are desired in order to take care of variations in the core material and, in any but through-type transformers, of the effect of joints in the magnetic circuit and of magnetic flux leakages.

Referring to Mr. Dickenson's remarks, while it is true that the nickel-iron alloys saturate at a considerably lower induction than do the silicon steels, the permeability of hypernik is nevertheless higher than that of the best laboratory-prepared silicon steel up to an induction of five or six kilogausses. This corresponds to a fairly large secondary burden, even for low-ratio through-type transformers, thus making hypernik superior to silicon steel even under these adverse conditions.

Circulation of Harmonics in Transformer Circuits

BY T. C. LENNOX[1]
Associate, A. I. E. E.

Synopsis.—*The paper describes the manner in which certain series of harmonic currents may be permitted to flow within a transformer network. In particular it is shown how the fifth and seventh harmonics of transformer exciting current may be eliminated from transmission lines. The extent to which the harmonic currents generated in a rectifier may be eliminated from the a-c. lines by means of phase multiplication is also indicated.*

* * * * *

THE action of a third harmonic of current or voltage in three-phase a-c. transformers has become familiar to engineers. Very exhaustive studies of the third harmonic of transformer magnetizing current, in particular, have been necessary, due to the excessive voltages that may result in Y-connected, three-phase circuits where the harmonic of current cannot flow, and to excessive interference with telephone circuits when it flows in a grounded neutral circuit.

It has not been so generally appreciated that this is but one of many similar phenomena that may be encountered if other harmonics than the third and other connections than the simple three-phase, Y, or delta are studied.

The case of the third harmonic of magnetizing current in three-phase transformer circuits is reviewed briefly. Here, it is usual to have three transformers or phases of similar characteristics so that the wave form of the magnetizing current is the same in each phase. Consequently, the third harmonic of current will be the same magnitude in each phase and have the same relation to the fundamental wave.

Bearing this in mind, we may take the case of a three-phase bank, connected in delta on one side and with excitation applied to this winding. Examining, then, the condition at one corner of the delta, it is found that the fundamental waves of exciting current of the two phases differ in phase by 60 deg. The third harmonics of current having three times the frequency of their fundamentals will differ in phase by three times 60 deg., or 180 deg. In other words, they will be equal and opposite and no third harmonic of current will flow from the supply circuit.

Next considering the Y connection, it is found that whereas at the neutral the fundamental waves are 120 deg. apart in phase, and consequently add up algebraically to zero, the third harmonics are three times 120 deg., or 360 deg. apart; or, in other words, are all in the same phase position and consequently add up to three times their average value and must flow into the neutral line. If no neutral is provided, no third harmonic of current can flow and consequently a third harmonic of voltage will appear from line to neutral in each phase.

The exciting current of a transformer contains higher odd harmonics,—the fifth, in particular, being quite prominent although smaller than the third. The seventh and some higher harmonics, although present, are of negligible amount. Considering these harmonics at a corner of the delta, the fifth harmonics will differ in phase five times 60 deg., or 300 deg., and will consequently have a resultant 1.73 times their average value, which must flow in the supply line. Similarly the seventh harmonics differ in phase 420 deg. or, subtracting 360 deg., 60 deg., and have a resultant in the line 1.73 times their average value. Similarly, the eleventh, thirteenth, and other odd harmonics not multiples of three must flow in the line, whereas the ninth, fifteenth, and others which are multiples of three, will equalize in the same manner as the third.

At the neutral of a Y connection, the fifth harmonics will differ in phase five times 120 = 600 deg. or 240 deg., and consequently will add up to zero the same as the fundamental. Similarly the seventh, eleventh, and other odd harmonics, not multiples of three, will equalize and may, therefore, flow in such a circuit without a neutral line. On the other hand, the ninth, fifteenth, and other harmonics which are multiples of three cannot flow.

Consider next the case of a Y-connected bank placed in multiple with a delta-connected bank, Fig. 1, in which all the phases are identical in characteristics, being different only in the turn ratio as necessary to obtain Y or delta connection. At each corner of the delta, three currents converge, so that the fundamental waves add to a resultant of twice the value of that in the Y-connected phase and in phase with it.

The third harmonic cannot flow in the Y connection and will equalize to zero between the two phases of the delta. A third harmonic of voltage exists from line to neutral in the Y-connected units, but if they have delta-connected, secondary windings, the voltage will result in a triple-frequency current in the delta which will reduce the voltage to a practically negligible value dependent on the impedance of the windings.

As the fifth harmonic can flow in the Y connection, there will be a fifth harmonic in each of the three phases converging at the corner of the delta. The delta phases being 30 deg. each way from the Y phase, their fifth harmonics will be 150 deg. out of phase with the fifth in the Y phase. This will result in the three adding up to zero with no fifth harmonic flowing in the line. Similarly with the seventh harmonic, the 30 deg. is multiplied to 210 deg. and the three equalized to zero. The ninth will act similarly to the third, flowing in the delta, and in the secondary delta of the Y-connected phases.

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[1] Transformer Engineering Department, General Electric Company, Pittsfield, Mass.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Niagara Falls, N. Y., May 26-28, 1926.

For the eleventh harmonic, the 30 deg. becomes 330 deg. and the three phases have a relation identical with that applying to the fundamental, so that the full value of the eleventh harmonic must flow in the line. Similarly for the thirteenth harmonic, the 30 deg. becomes 390 deg. and the full value of this harmonic must flow in the line.

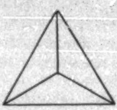

Fig. 1

Continuing the process, we find that all the odd harmonics except the eleventh, thirteenth, twenty-third, twenty-fifth, thirty-fifth, thirty-seventh, and others of that series will equalize within the network.

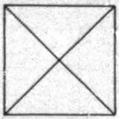

Fig. 2

This is of some practical interest in connection with power transformers or shunt reactors in cases where the fifth harmonic of exciting current may be thought to have some bearing on telephonic interferences.

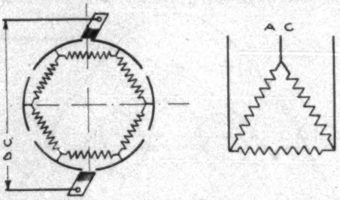

Fig. 3

By suitably arranging the banks so as to balance delta banks against Y banks, the fifth and seventh harmonics of exciting current may be largely eliminated from the lines.

If an analogous connection on a quarter-phase system consisting of a cross-square connection is taken, as in Fig. 2, it is found that the harmonics of the series 3, 5, 11, 13, 19, 21, 27, 29, etc., are equalized while those of the series 7, 9, 15, 17, 23, 25, etc., must flow in the line. A secondary square in the cross bank is not needed unless it is wished to eliminate even harmonics, in which case the second, sixth, tenth, and so on would flow in the squares.

An interesting case in this connection is a polyphase rectifier circuit such as is suitable for use for changing alternating current to direct current. Such a rectifier tends to draw current from the a-c. system having a wave form approximating the rectangular in shape. This wave consists of a series of odd harmonics the magnitude of which is in inverse ratio to their order, that is, in the familiar Fourier series.

Considering a six-phase ring, Fig. 3, with commutator and brushes or their equivalent arranged to operate in synchronism with the alternating voltage, the current in the ring will approximate the form shown in Fig. 4. The current in each phase has to reverse while the phase is short-circuited by the brush. This requires a definite time and explains the slanted sides in place of the vertical sides of a rectangular wave. A series of odd harmonics somewhat similar to that in a rectangular wave will, nevertheless, be present. If the primary of the transformer is connected in delta, the third and other harmonics which are multiples of three will equalize at the corners and the line current will consequently have only the fundamental and the remaining harmonics.

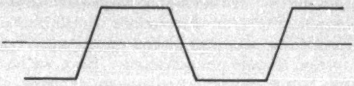

Fig. 4

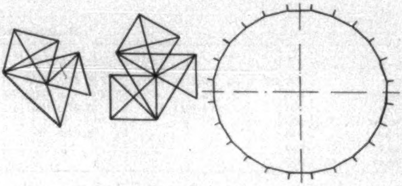

Fig. 5

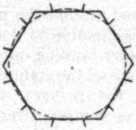

Fig. 6

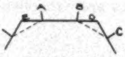

Fig. 7

If the number of sides in the polygon is increased to eight by the use of four separate transformers or phases with the necessary number of windings for this arrangement and the primaries are connected cross-square (Fig. 2), the harmonics of the series 3, 5, 11, 13, 19, 21, etc., are eliminated, leaving the remaining ones in the line.

If a twelve-sided polygon is used by means of six single-phase transformers or two three-phase transformers and Y-delta primaries are used with additional delta in the Y phases, all the harmonics except the 11th, 13th, 23d, 25th, and others of that series are eliminated.

For a still larger number of phases in a polygon obtained by the use of separate transformers or core legs, we cannot obtain a simple primary connection of the kind used above. However, for 24 phases using either 12 single-phase or four three-phase transformers, a set of compensating windings placed on the same cores as shown in Fig. 5 could be connected. Here, we have the necessary Y deltas and cross squares to carry all the harmonics except the 23d, 25th, 47th, 49th, 73d, 75th, and others of that series. The three-phase, a-c. connection to such an outfit would be made at any three points equally spaced around the polygon. In any of these polygons, it is possible to double the number of symmetrical phases by tapping out the leads as shown in Fig. 6, so as to have all the connection points fall on a circle intersecting the polygon at equal intervals. Here, the primary or compensating winding will eliminate only the same series of harmonics as for the smaller number of phases. However, some of the remaining harmonics are cancelled within the polygon itself, as for instance, in Fig. 7 which shows a part of a 12-phase connection from a six-phase ring. The current in *B C* is 150 deg. from that in *B A* but flows in coils *B D* on the same core. The fifth harmonic will be 750 deg. or 30 deg. from that in *B A* and, together with that in *A E*, will have a resultant equal and opposite to the fifth in *A B*. Similarly the seventh harmonics will be 330 deg. out of phase and will neutralize.

TABLE I

HARMONICS NOT ELIMINATED

	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	— — —	47	49	— — —
Square..........	100	33.3	20	14.3	11.1	9.1	7.7	6.67	5.88	5.26	4.76	4.35	4.0	3.7	3.45	3.23	3.03	2.86	2.7	— — —	2.12	2.08	— — —
Hexagon........	100		20	14.3		9.1	7.7		5.88	5.26		4.35	4.0		3.45	3.23		2.86	2.7	— — —	2.12	2.08	— — —
Octogon.........	100			14.3	11.1			6.67	5.88			4.35	4.0			3.23	3.03			— — —	2.12	2.08	— — —
12-Phase........	100					9.1	7.7					4.35	4.0					2.86	2.7	— — —	2.12	2.08	— — —
16-Phase........	100							6.67	5.88							3.23	3.03			— — —	2.12	2.08	— — —
24-Phase........	100											4.35	4.0							— — —	2.12	2.08	— — —
48-Phase........	100																			— — —	2.12	2.08	— — —

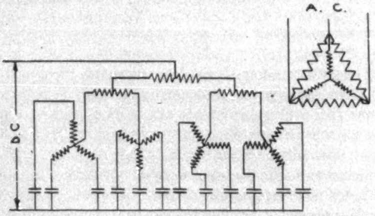

FIG. 8

As a result, no current of these frequencies will be required of the primary coils or line and the residue of harmonics to be taken from the line will be the same as for a polygon having the full number of separate phases.

Table I shows the magnitude of the minimum residue of odd harmonics for polygons of different numbers of phases, in per cent of the fundamental, for a rectangular wave in the polygon. From this it is evident that the minimum ripple resulting in the a-c. lines in such cases is composed of a series of odd harmonics comprising the numbers adjacent to the number of phases in the polygon and its multiples.

The extent to which this ideal condition may be approximated in the case of actual designs will depend principally upon the reactance of the circuits through which the harmonic currents are required to flow. In the arrangements outlined above, the primary windings or compensating windings, if used, must have small leakage reactance between them and the polygon if more than a few of the lower harmonics are to flow in appreciable amount.

Transformers for tube rectification are frequently connected in star rather than in a polygon. If there are an even number of phases with this arrangement all even harmonics will cancel. A series of odd harmonics will remain as in the polygon connection and will be eliminated in the same manner and to the same extent. The use of separate Y-connected banks, connected together by inter-phase transformers, as shown in Fig. 8, will not greatly alter the conditions, the minimum residue of harmonics from the 12-phase system shown being the 11th, 13th, 23d, 25th, etc., series. The wave forms to be expected in tube rectifiers are described in some detail by D. C. Prince, in the General Electric *Review* of September, 1924.

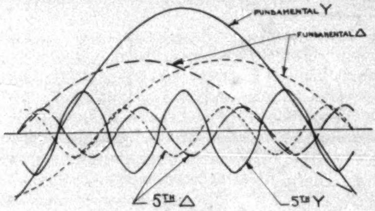

FIG. 9

To illustrate more clearly the action of the harmonic currents, Fig. 9 shows the fundamental and fifth harmonics at the corner of a Y-delta connection when the harmonic is in phase with the fundamental at its crest.

Discussion

D. C. Prince: There are, in general, two ways of regarding such waves as those set up by commutating devices whether of rotating or static character. Fig. 4, in Mr. Lennox's paper, is typical of the waves encountered. Such a wave may be regarded as made up of a series of constant-amplitude waves of different frequencies, as Mr. Lennox has done, or it may be regarded as a succession of simple states.

From this latter point of view, the wave shown consists of direct-current portions joined by portions of constant slope. Both these states must independently obey Kirchoff's laws. This enables the wave forms, at least in the simpler networks, to be made up, taking into account the various impedances.

In particular, the transition portions of rectifier waves have as their driving force the impressed sine-wave so that the transition currents are sine waves of fundamental frequency superimposed upon either steady or decaying direct currents. These lend themselves readily to quantitative analysis where the harmonic analysis may present some difficulty.

From the point of view of inductive interference, some error may be invited if the analysis method is used. For instance, a square wave under harmonic analysis may give waves of all frequencies which are multiples of the fundamental. Some of the frequencies with the indicated amplitudes might set up objectionable interference. Physically, however, the inductive effect is in the form of impulses sent out at half-cycle intervals. These impulses set up a series of damped wave trains in a resonant circuit or simply a series of pulses in a circuit damped beyond the critical point. Neither of these responses is necessarily objectionable. The difference is that, when a wave is analyzed, the effect of all the components must be included. Many of these will have cancelling effects which might be lost sight of in treating any given problem by harmonic analysis.

Measurement of Transients by the Lichtenberg Figures

BY K. B. McEACHRON*

Member, A. I. E. E.

Synopsis.—*The paper gives the results of a comprehensive study of the effect of transients on the size and appearance of Lichtenberg figures. Sixteen different rates of voltage rise were used, varying from about 20 minutes to 0.1 microsecond to reach a crest value of 25 kv. Results were obtained with transients the crest voltages of which ranged from 5 to 25 kv. The steeper wave fronts were checked with the Dufour cathode-ray oscillograph.*

Calibration curves are given, showing that the positive figures are not much affected by changes in wave front, while the negative figures vary considerably with changes in wave front, especially at the lower voltages.

The positive figures are divided into three types according to their appearance which is found to depend on the rate of voltage rise. It is concluded that for most conditions the size of the positive figures will give a determination of the crest voltage of the applied transient to within approximately 25 per cent.

UNTIL the cathode-ray oscillograph was applied to the measurement of transient phenomena, engineers were unable to determine accurately by experiment the form of a transient voltage or current, the wave front of which occupied only a few millionths of a second of time. Such transients may occur on transmission or distribution circuits during periods of disturbance,—as, for instance, during a lightning storm,—and may cause considerable damage to unprotected apparatus.

Although the cathode-ray oscillograph can be arranged to show the volt-time, ampere-time or volt-ampere relations, yet there is need of a device which may be used to record these disturbances with fair accuracy at the same time involving a small amount of equipment so that a large number of them can be spread over the systems of the country at a comparatively small cost.

LICHTENBERG FIGURES

If a sheet of insulating material, such as hard rubber or glass, is placed on a metal sheet and an electrode of any shape is allowed to rest on the upper surface of the insulating material, a Lichtenberg figure will be formed by the application of voltage between the electrode and the metal sheet. The figure may be made visible by the use of chalk dust which may be applied to the insulating surface before or soon after the application of voltage. A permanent photographic record of the figures can be made by placing a photographic film between the electrode and the insulating plate with the emulsion side in contact with the electrode.

A large amount of work has been done with these figures in the effort to discover their exact meaning and the variables which control the figures. Among those who have experimented with them may be mentioned P. O. Pederson[1], Toepler[2], Przibram[3], and others[4], all of whom have made notable contribution to the solution of the problem of the proper understanding of the Lichtenberg figures. Peters[5] applied the Lichtenberg figures to the study of transmission line transients, and suggested means for making connections to the transmission line to determine supplementary information about the transient. In order that the surges might be recorded with respect to time, Peters made use of a moving electrode, the complete instrument being termed the klydonograph.

Cox and Legg[6] later described some of the results obtained on transmission systems by using the revolving electrode type and described an improved klydonograph which made use of a roll film. As a result of tests with the klydonograph, Cox and Legg concluded that the relation between the size of figure and crest value of the transient was not appreciably influenced by the wave front between the limits represented by a 25-cycle wave and a wave whose crest is reached in 5 microseconds.

All investigators have found that the positive and negative figures were different in appearance and that when the electrode was positive a larger figure was formed than when the electrode was negative. At times the figures are complicated by tree-like growths which have been ascribed by some to over-loading the film, but this does not always seem to be the case as they are sometimes found at lower crest voltage than that which at other times does not produce them. In the practical interpretation of the figures, it seems to be necessary to neglect these effects until their meaning is known.

EFFECT OF WAVE FRONT

The author is not aware of any systematic work that has been done having for its object the calibration of the figures with respect to the form of the voltage transient. Up to the present time, the figure has been regarded as not being much affected by rate of voltage application and therefore the present investigation was undertaken to determine, if possible, the relation between the size of the figure and the rate of voltage application, as well as the crest value.

A transient, such as that shown in Fig. 1, consists of three major parts; the front, and the part we may call

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*Lightning Arrester Engineering Department, General Electric Company, Pittsfield, Mass.

[1] For all references see bibliography.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Niagara Falls, N. Y., May 26-28, 1926.

Also presented at the Pacific Coast Convention of the A. I. E. E., Salt Lake City, Utah, Sept. 6-9, 1926.

the body for want of a better name, and the tail. In the simple wave of this kind, presumably each of these parts may play a role in the production of the Lichtenberg figure. The investigation to be described in this paper is concerned altogether with the front of the wave, as the rate of voltage rise and the crest value of voltage appear to be the dominant factors in determining the characteristics and the size of the figures.

To make the work as complete as possible, tests have been carried out covering a wide range of wave fronts, the slowest being slower probably than any ever obtained in practise, while the fastest are probably as fast as any that ever occur in service, being but a few hundred feet in length. Sixteen different wave fronts at five different voltages were used and, except at the very slowest, oscillograms were taken of each voltage application. Every care was taken with the shorter waves to eliminate the effect of reflection and to keep the oscillation on the wave front to a minimum. This has been possible only through the use of the Dufour cathode-ray oscillograph[7] the voltage dividing system of which was connected directly across the electrodes producing the Lichtenberg figures, using leads only two or three feet in length.

It would have been desirable perhaps to have used a wave throughout having an initial value of de/dt kept constant up to the crest value of voltage; yet, with the steepest wave this limitation would have complicated the test seriously without adding much to the practical value thereof.

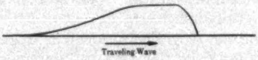

FIG. 1

VOLTAGE AND TIME CALIBRATION

The sphere-gap was used as the primary means of determining the crest value of voltage of the transient. With the slower waves the crest potential of which was reached in times longer than 10 seconds, it was possible to check the sphere-gap against a static voltmeter and also against meters located in some of the low tension circuits. The time for the slow waves was conveniently determined with a stop watch.

Within its range, the electromagnetic oscillograph record gave the time calibration of the transient, the time being based on a 60-cycle wave recorded on the same film. The transients above the range of the electromagnetic oscillograph were recorded with the Dufour cathode-ray oscillograph with time scale calibrated by the use of a wave meter. With the type of circuit used, the wave front could be calculated quite accurately and, except for the oscillations which in most cases were successfully removed, the oscillograms checked the calculations within the experimental error in practically every case except for the steepest waves where stray capacities become of importance. In all such cases, the oscillogram after being checked carefully was assumed to be correct.

CIRCUIT ARRANGEMENTS

The slowest waves were obtained with the circuit given in Fig. 2 in which the condenser C was charged slowly by the use of a motor driven regulator connected in the primary of the charging transformer. The reduction gears were such that, for the longest wave front, 22 minutes were required to reach a crest value of 25 kv. The limit of this method was about a 12-second front.

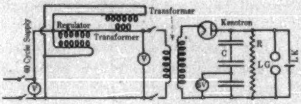

FIG. 2—CONNECTION DIAGRAM FOR SLOW WAVES USING MOTOR-DRIVEN REGULATOR
Time to reach crest 20 min. 55 sec.

Other circuits used were similar to that shown in Fig. 3, in that a capacity C_1 was allowed to discharge in another capacity C_2. By varying the amount of capacity employed, together with appropriate values for series resistance connected between the condensers, the desired changes in wave front were obtained. A sphere-gap was used in series between the two condensers for the steeper waves, while with the slower waves, using this type of circuit, a kenotron was connected between the two condensers and the transient initiated by heating the filament at the proper time.

Considerable difficulty was experienced with the steepest waves in decreasing the amount of oscillation on the wave front. The rate of voltage rise was extremely rapid, being 100 kv. per microsecond. The final circuit used was physically quite small with all connections made with conductors having a high uniform resistance to aid in damping out the oscillations. Such a wave front is shown in Fig. 13.

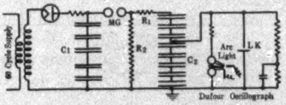

FIG. 3—CONNECTION DIAGRAM FOR VOLTAGE TRANSIENTS
Time to reach crest 1.0 sec.—0.1 microsecond

RESULTS—EFFECT OF CHANGES IN WAVE FRONT

The Lichtenberg figures were taken, using a piece of plate glass 20.3 cm. × 25.4 cm., 3.8 mm. thick, with a cylindrical brass electrode 1 cm. in diameter in contact with the photographic film. The films, which were East-man's super speed portrait films, were placed on the glass plate with the emulsion side in contact with the electrode. On the reverse side of the glass plate and opposite the electrode was a lead foil so connected that the electrode and foil were of opposite polarity. The parts were arranged in a light-tight box with the electrode projecting, to which connections were conveniently made, as shown in Fig. 4.

The results are plotted in terms of the radius of the figure plotted against slope of the wave front in volts per microsecond for five voltages, 5.5 kv., 9 kv., 12 kv., 17 kv., and 25 kv. (All voltages are crest values.) The curves for the positive figures are given in Fig. 5 and for the negative figures in Fig. 6.

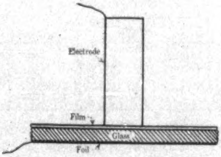

Fig. 4—Arrangement of Parts of Lichtenberg Camera

Notwithstanding the precautions taken to control all of the known variables, the plotted points show considerable variation which is most marked with the shorter waves and at 17 kv. and 25 kv. For all voltages, the positive figures increase in size as the wave becomes steeper, until a rate of rise of between one kv. per microsecond and 10 kv. per microsecond is reached. With wave fronts steeper than this, which corresponds to about a 60-cycle front, the lower voltage figures remain constant in size while the 17-kv. and 25-kv. figures decrease somewhat at the steepest points. Fig. 5 has been divided into three zones which are determined from the appearance of the positive figures and will be discussed in connection with the different types of figures.

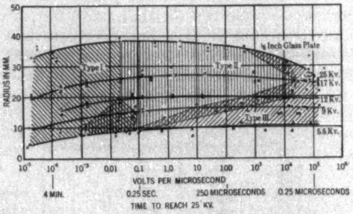

Fig. 5—Variation of Positive Lichtenberg Figures for Different Crest Voltages and Rates of Voltage Rise

The negative figures were less satisfactory at the slow rates of voltage rise than were the positive figures, it sometimes being difficult to separate what appeared to be corona from the Lichtenberg figure. The results do indicate a rather abrupt increase in the size of the negative figure between 0.001 volts per microsecond and 0.01 volts per microsecond. As the voltage rises faster and faster, the negative figure increases in size, the greatest percentage change occurring with the lower voltages. For instance, the figure at 5.5 kv. increases from about 0.2 mm. to 13. mm. when the front changes from 0.01 volts per microsecond to 100,000 volts per microsecond. At 9 kv. the radius changes from 0.5 mm. to 2.4 mm. when the wave front changes through a similar range. The radius at 25 kv. shows a sudden increase for fronts steeper than 10,000 kv. per microsecond.

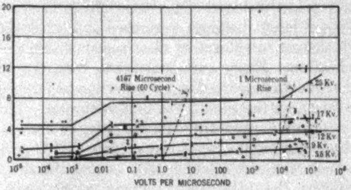

Fig. 6—Variation in Size of Negative Lichtenberg Figures for Different Crest Voltages and Rates of Voltage Rise

Using the curves in Figs. 5 and 6 as representing the average values from the data, calibration curves have been drawn as given in Fig. 7. These curves show that the average positive figures are not appreciably affected by wave front except for the highest voltage and steepest front. The variation between the slowest and fastest waves for the negative figures is much greater than for the positive and should be considered when determining the voltage from a negative figure. At 17 kv., an increase of nearly 50 per cent takes place when the wave front changes from that corresponding to a 60-cycle wave to one which rises to 17 kv. in 0.1 microsecond, while, for lower values of crest voltage,

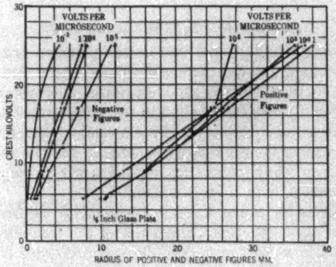

Fig. 7

the variation is much greater. Some idea of the wave front may be determined by the comparative sizes of the positive and negative, but there is sufficient variation among pictures at the same wave front to make such a procedure doubtful for everything except very approximate results.

APPEARANCE OF THE FIGURES

It is possible to divide the positive figures according to appearance into three definite type forms with respect to rate of voltage rise on the electrode. These three types are shown in Figs. 8, 9, and 10, type I being the slowest while type III represents the greatest rates of voltage rise.

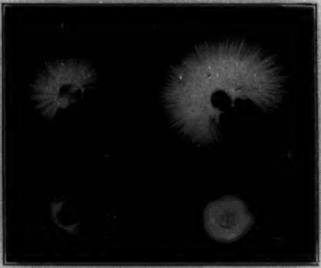

FIG. 8—LICHTENBERG FIGURES PRODUCED BY A VERY SLOW RISE IN VOLTAGE (TYPE I—POSITIVE FIGURES)

A. *P L*—20519 *R*
17 kv. in 840 sec.

B. *P L*—20512 *R*
25 kv. in 85 sec.

Type I consists of fine straight lines emanating from the center and perhaps extending only partially round the entire electrode. The appearance is much like that of fine hair being blown out by some force at the center. As the rate of voltage rise becomes smaller, both the number of hairs and their length decrease until, in one case with a 6-second wave at 5.5-kv. crest, only five hairs were found, their length varying from 7.5 mm. to 8 mm. However, with a 6-minute front at the same crest voltage, the length of the hairs had been reduced to 4 mm., while the number of hairs had increased to perhaps 30 or 40, bunched in two or three small tufts. Some representative type I, positive figures are shown in Fig. 8.

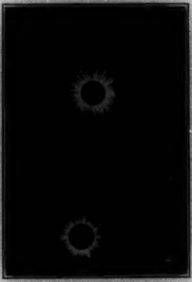

FIG. 9—LICHTENBERG FIGURES PRODUCED BY A FAST TRANSIENT (TYPE II—POSITIVE FIGURE)

P L—20573 *L*—21.6 kv. in 48 microseconds

The negative figures become smaller and smaller with decreasing rates of voltage rise until they disappear altogether or become so indefinite and indistinct that they cannot be measured.

In appearance, type II figures are quite different from type I, being characterized by crooked lines which are likely to have sharp turns or elbows near the ends and usually split or branching. Sometimes short projections like thorns appear, but these belong more to type III than to type II. Fig. 9 shows a typical figure of this type. The so-called slips are quite likely to have type II figures at the ends, even though the body of the figure is of another type. It should be noticed, also, that with type II figures, there is almost no crossing of lines even though the voltage was sustained for an appreciable length of time compared to the wave front.

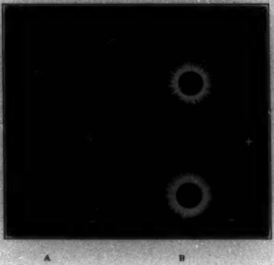

FIG. 10—LICHTENBERG FIGURES PRODUCED BY VERY FAST TRANSIENTS. (TYPE III—POSITIVE FIGURES)

A. *P L*—20607 *L*
7.3 kv. in 0.1 microseconds. Produced by transient shown in Fig. 12

B. *P L*—20605 *R*
17.8 kv. in 0.15 microsecond. Produced by transient shown in Fig. 13

The figures which have been called type III exhibit characteristics very different from either type I or type II. Type III figures as shown in Fig. 10 are recognized by their straight radial lines, having rather broad bases with splits and thorns. In type III figures, the lines do not cross and once seen it is not difficult to recognize a figure of this type indicating as a rule waves of quite steep fronts.

The voltage and frequency at which the three different types of figures are found are shown in Fig. 5 where the three types of figures are indicated. Some overlapping takes place where two types are found in the same figure. At the 5.5 kv., it is interesting to note that the range of wave fronts over which type II figures appear is very small, and in fact they are not found alone at this voltage, always being mixed with either type I or type III figures.

The negative figures undergo considerable change in appearance as the wave front becomes steeper, but the change is gradual and it is not possible, using the data now available to draw definite limits or to divide them into types. In Figs. 9 and 10 representative negative figures are given, corresponding to the wave fronts shown in Figs. 11, 12 and 13. The slow negative waves as in Fig. 8 show a rather hazy indefinite outline with a tendency to concentrate in tufts. These figures are difficult to measure and look much like photographs of corona.

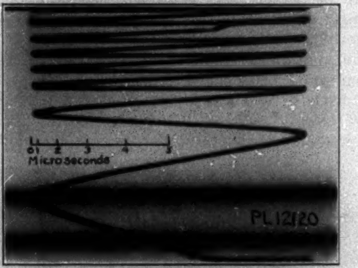

Fig. 11—Dufour Oscillogram of Transient Voltage

21.6 kv. in 48 microseconds. Corresponding to Lichtenberg figure shown in Fig. 9.

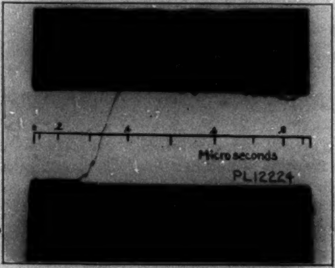

Fig. 12—Dufour Oscillogram of Transient Voltage Retouched

7.3 kv. in 0.1 microsecond. Corresponding to Lichtenberg figure shown in Fig. 10.

With steeper waves, as in Fig. 9, radial sectors of exposed film appear with more or less definite unexposed space between. Superimposed, many short crooked lines are frequently found not often extending beyond the exposed radial portion. While this form of negative is easier to measure than that obtained with the very slow waves, the limit of the figure is not sharply defined.

Very short waves produce a negative as in Fig. 10 in which the radial sectors are clearly defined and the limit of the figure is quite definite. Not only are the sectors more sharply defined but the width at the circumference of the figure is reduced, the space being occupied by a larger number of sectors. While crooked lines superimposed on the figure appear frequently, they are often less prominent than those seen with slower waves as in Fig. 9.

Oscillograms

Three oscillograms are shown in Figs. 11, 12, and 13 to indicate the form of waves actually applied to the photographic film when taking the Lichtenberg figures.

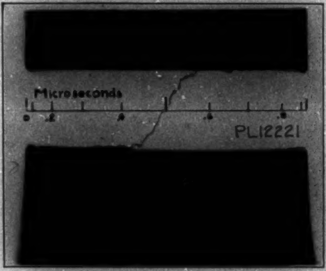

Fig. 13—Dufour Oscillogram of Transient Voltage

17.8 in 0.15 microseconds. Corresponding to Lichtenberg figure shown in Fig. 10.

These oscillograms were taken with the Dufour cathode-ray oscillograph and show some of the wave fronts used in making the Lichtenberg figures shown in Figs. 8, 9, and 10. In Fig. 11 is shown an oscillogram of the wave taken at the same time as film PL-20573L shown in Fig. 9. This film shows that the voltage rose to its maximum (21.6 kv.) in 48 microseconds the average rate of voltage rise being 450 volts per microsecond while the maximum rate is 1800 volts per microsecond. The film shows that the final voltage was attained without oscillation and that the voltage dropped very slowly compared to the front after reaching its crest. All of the waves used in this investigation, as a rule, had long tails as compared with the front, so as to be sure that the tail did not disturb the results.

As the time to rise to maximum voltage was reduced, the difficulty of getting a smooth wave increased, but that a fair degree of success was finally attained is seen in Fig. 12. This film shows that a voltage of 7.3 kv. was reached in 0.1 microsecond which is equivalent to an average rate of rise of 73,000 volts in one millionth of a second (Lichtenberg figure PL-20607L, Fig. 10). While some oscillations are present, their amplitude does not appear to be sufficient to make them a matter of importance.

Another film in which a still greater rate of voltage rise is recorded is shown in Fig. 13. Here the average rate of rise of potential is 120,000 volts per microsecond. The Lichtenberg figure corresponding to this wave front is shown in Fig. 10 film PL-20605R.

Films similar to these were taken for every voltage application when making Lichtenberg figures and it can be stated that for waves having a longer front than 10 microseconds any oscillations which were present were negligible and that for the shortest waves such oscillations did not exceed 20 per cent of the crest voltage.

It is interesting to note that when working with these figures the impression is gained that the presence of oscillations increases the distinctness and clarity of the figures and that this is particularly true of the negative. While this as yet is only an impression, the cleanest figures always were associated with the waves having oscillations on the wave front.

CONCLUSIONS

As a result of this investigation, it can be definitely stated that the size and appearance of both positive and negative Lichtenberg figures are dependent on the wave front as well as on the crest voltage.

Throughout the range of wave fronts probably found in service, the size of the positive figure is not much changed by a change in wave front only, except at voltages close to the upper limit of potential where a decrease in the size of figure is indicated with very abrupt fronts.

The positive figures may be divided into three type forms which are partly determined by wave front and partly by the value of the crest voltage. It is possible to gain some idea of the steepness of the front from the appearance of the positive figure.

The size and appearance of the negative figures are considerably affected by changes in wave front, the steepest waves always giving the largest figures. The percentage change with a constant crest voltage applied is greatest for the lower voltages. The change seems to be great enough so that it cannot be neglected. The negative figures change in appearance with increasing steepness of wave front, but the changes are so indefinite that it is only possible to state that a particular negative figure probably represents a fast wave or a slow wave.

No value is given for the possible variation in radius of figures with identical wave front from the average, although the usual variation is probably not greater than 25 per cent. Occasionally, one will be found where the figure is 50 per cent or more, larger or smaller than the average for that particular wave. There seems to be some tendency for the greatest variations to occur with the steepest wave fronts.

The curves which have been given apply to apparatus of certain characteristics and should not be applied to figures taken with a different dielectric than used here or for other different conditions. It seems likely, however, that similar results will be found with other apparatus constants although the values would be different.

Additional work is being done with the cathode-ray oscillograph in an effort to determine how the Lichtenberg figures grow and the reasons for the various characteristics which they exhibit.

The author wishes to express his appreciation of the work of E. M. Duvoisin and T. Brownlee, who have supplied all the figures and most of the oscillograms, and to E. J. Wade for his valuable assistance, particularlv in arranging the circuits so as to avoid oscillation.

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Discussion

J. H. Cox: I am glad to note that the Lichtenberg figures are receiving attention, from the engineering standpoint, from others. All of this work will hasten a more thorough understanding of the phenomenon.

I may state that the klydonograph, the surge recorder which utilizes this principle, was placed in use not with the contention that we knew all there was to be known about the phenomenon, but rather that we knew enough about it to be sure that valuable and sufficiently reliable information could be obtained by its use and that any information so widely desired as that on transmission-line surges should not be withheld until all of the less important points were determined. Of course, the sooner these points are learned the better.

Although from the standpoint of a surge recorder the data obtained by Mr. McEachron at the extremes of wave front are of no practical value, they are interesting in view of the figures in the practical range.

In general, Mr. McEachron's results agree quite well with our own. I am surprised that he obtained such large positive figures with slowly applied potentials. Our tests show that the more sloping wave fronts produce smaller figures. This effect was noticeable to a slight degree at 60 cycles, true to a greater extent with the negative figures but also evident with the positive figures whether half-cycles or the full wave was taken.

Dr. P. O. Pedersen, who has probably done more work than any other investigator on this subject, states the following: "The figures appear only if the potential difference across the Lichtenberg gap is altered in an impulsive or sudden manner, and not if the potential is raised gradually." This observation was made by Reis and Mikola: "At a certain stage of the charging of the electrode the intensity of the field between it and the plate reaches a value at which ionization by collision commences. The ionization current charges the surface of the insulating plate and thus keeps the intensity of the electric field between the electrode and plate below the high value necessary to form the Lichtenberg figure. With a rapidly varying potential there is, however, a possibility of obtaining sufficiently strong fields because it takes some time to establish the compensating charge on the insulating plate."

In our work at East Pittsburgh we were able to obtain some figures but they were smaller than the impulse figures. We found that there were variations due to conditions of the emulsion and it is possible that Dr. Pedersen's emulsion was so conductive that his failure to obtain figures with slowly applied potentials was due to conduction current rather than ionization currents.

The nature of the formation of the figure is still unknown, but we all agree that it is caused by the state of stress in the gas adjacent to the emulsion.

In the klydonograph, when a potential is impressed on the electrode there is an electrostatic field set up between it and the metal plate. The gradient is of course more intense near the electrode and, proceeding away from the electrode, a point is reached where the gradient is not sufficient to form the figure, and hence the boundary of the figure. If the emulsion surface were conducting there would then be a charge distributed over the surface of the plate and the stress would be entirely in the dielectric. Since there would be no stress in the gas there would be no figure.

It is evident that with emulsions such as we get, which are only slightly conducting, there is no time for this charge to creep out from the electrode under impulsive conditions, but the slower the application the more creepage is possible and hence a smaller figure.

Surface conductivity increases with moisture content and we have found that under humid conditions 60-cycle figures are smaller than under dry conditions. As far as we have been able to determine, there is not time enough for this effect under any application rapid enough to be important in surge conditions on transmission systems.

Mr. McEachron's positive figures agree with ours, and as he states, their appearance is only an approximate measure of the rate of application. I am unable to understand the difference between his negative figures and ours. Our abrupt-front negatives were very clean-cut with straight rays. The five-microsecond negatives were clean-cut, but clover-leafed. It was only in the longer wave fronts that they had the fuzzy appearance.

We have found in practise that the negative figures are much less satisfactory to deal with than the positive, and agree that they vary more with frequency.

Fortunately, the large majority of surges found in practise are positive. I see no reason for this in the case of switching surges, but it is nevertheless true.

In the case of an oscillation the negative is evident and can be measured, although smaller than the positive, except in the case of a sustained oscillation, when we are not interested in it because it would have no higher voltage than the positive.

It seems rather remarkable that the distinct breaks shown in Mr. McEachron's curves for negative figures come at the same point of wave front for all voltages. There seems to be no explanation why there should be such an abrupt break in a phenomenon of this sort. Is it not possible that this was caused by the change in the type of circuit used when going from the long to the short wave fronts?

As to accuracy, I cannot help but feel that a great part of the variation in the figures is due to applied voltage. My reason for believing this is that when a potential was impressed on six terminals in parallel usually no measurable difference could be detected in the figures. We found, as Mr. McEachron did, that an occasional freak figure would appear. This would be perhaps 50 per cent smaller than it should be, never larger. However, such a picture would occur in less than one per cent of the tests and therefore does not throw a great amount of doubt on conditions found from tests over any appreciable length of time.

I would like to ask what degree of accuracy is possible with the cathode-ray oscillograph. I noticed in some of Mr. Lee's work that the width of the zero line of an oscillogram was sometimes 20 per cent of the maximum deflection. I would also like to ask whether or not successive oscillograms taken with the same circuit with the same settings were identical in all respects.

It is desirable, of course, to know the accuracy of any instrument we use and therefore any work done to determine this is valuable. However, extreme accuracy in a surge recorder is not necessary. I believe it will be agreed that the operating engineer is interested only in the order of magnitude of the surge voltages on his line. Any instrument which will give this within 25 per cent is sufficiently accurate for practical purposes. I am confident that the accuracy of the klydonograph is well within that margin—within the frequencies present in actual lines. These frequencies range from one microsecond to a few hundred microseconds. I do not believe that we need worry about wave fronts steeper than one microsecond. Our tests have strengthened this belief.

(Discussion at Salt Lake City)

Joseph Slepian: The klydonograph is the only practical graphic surge recorder available at present, and its growing use is bringing us a wealth of information about abnormal voltages occurring on lines in practise. The proper interpretation of the figures obtained, therefore, is of highest importance, and Mr. McEachron is to be commended for his painstaking study of the relation between the figures and the surges which produce them.

As Mr. McEachron says, in spite of the interest which has been aroused by these figures, no adequate theory of them has yet been developed. However, there are a few facts which have been well established and which may be used in interpreting such data as Mr. McEachron presents.

First, we can be sure that the figures are produced by the high electrostatic stress in the air immediately adjacent to the film. Undoubtedly, the primary cause of the figures is some sort of electric discharge in the air. We know this because if a gas other than air is used or if the air pressure is changed, the figures are changed very radically both in appearance and size.

Second, we know that the figures are formed with extreme rapidity, in less than 10^{-8} seconds. In fact, the figures have been used to measure times as short as 10^{-11} seconds.

Now if the figures are produced by stress in the gas next to the photographic film, and in such very short times, it seems to follow inevitably that the figure size must accurately depend upon the peak stress in the gas, irrespective of the shape of the surge producing this stress up to the fastest surges with which we would be interested in practise. If, however, the figure size does vary under different circumstances with constant peak surge voltage in the line, it must be only because the peak stress in the gas does not bear a constant ratio to the peak voltage in the line. Such a variation in the ratio of stress in gas to line voltage is only to be expected if the electrostatic system in the neighborhood of the electrode on the plate is considered. It is evident that the electrostatic field at the electrode end will be determined not only by the voltage on the electrode but also by the dielectric properties of the glass, and by the surface electrical conductivity of the film. Evidently for extremely low frequencies, the surface conductivity of the film will play a significant part, and will operate to reduce the stress in the gas. Hence for these lower frequencies, from perhaps a thousand cycles downwards, the figures will come out too small. This agrees with Mr. McEachron's findings. For higher frequencies, however, the influence of this surface leakage should be negligible.

If there is any systematic variation in size of figure with frequency for higher frequency, then this is to be attributed to the dielectric properties of glass under single impulses.

I do not believe that Mr. McEachron has proved that there is a systematic change in figure size with frequency for the higher frequencies, even for negative figures, for which Mr. McEachron shows the greatest per cent variation.

Examining Mr. McEachron's Fig. 6, and considering only those points to the right of, say, his 10-volt per microsecond line, we see that it is extremely hazardous to draw any kind of straight line through the points. The upward tendency of the lines which Mr. McEachron has drawn is largely due to his including low-frequency points on the lines. However, I believe it is erroneous to assume that the undoubted upward tendency of the points at low frequencies exists also at the high frequencies. If, as is likely, this effect is due to surface leakage, it will disappear at the higher frequencies. I suspect that we have here an erroneous extrapolation of the effect of surface leakage.

All that we may conclude from Mr. McEachron's points at the right of his Fig. 6 is that he obtained considerable variation in the figure size. I have plotted these points of Mr. McEachron on a sort of shot-gun diagram, and find that for a given figure size, the extremes of voltage differ by 33 per cent from their average. This, then, would be maximum error for practically occurring surges if the arrangement of Mr. McEachron were used in a klydonograph.

It is interesting to compare this result with the results obtained by Westinghouse engineers. Mr. Peters, in his early work with photographic plates, found that the figures produced simultaneously by a single surge, on a single plate, never differed in size among themselves by more than five per cent. When the figures were produced on different plates, differences as much as 15 per cent were found occasionally. When figures were produced by successive discharges from the same condenser circuit, Mr. Peters also found variations which he was led to attribute to variation in the surge voltage.

The character of the discharge in a short-impulse circuit is affected greatly by the state of the initiating spark-gap, and Mr. Peters believed that variations in the surge voltage due to the varying state of the initiating spark-gap caused apparent variations in figure size.

On this account, therefore, I would like to ask Mr. McEachron whether an oscillogram was taken for each point shown in Fig. 6, and if not, whether some of the variability was not due to the discharges not repeating accurately.

I would like to ask Mr. McEachron about the nature of his voltage-dividing system for the Dufour oscillograph, which is indicated as being a resistance potentiometer in Fig. 3, and particularly whether, for the very steep wave fronts, the capacity to space of these resistors might not cause the oscillogram to fail to portray the voltage on the Lichtenberg-figure electrode. Also, I would like to know whether these resistors contributed to the damping out of oscillations at the oscillograph, and if so, whether these oscillations might not still be present at the Lichtenberg-figure electrode.

Herman Halperin: It appears to me that Mr. McEachron deserves great credit for his exhaustive calibration of the Lichtenberg figures. These figures, as recorded on the commercial device known as the klydonograph, have been used in investigating transient voltages on several large systems during the past few years. There has been considerable discussion as to the frequency of surges on transmission systems, and the nearest that we could come to the nature of the surges up to this time was to use the klydonograph figures and estimate whether the surge was "fast" or "slow." Now from the data given in Figs. 5 and 6, it appears that a much more definite idea can be obtained of the steepness of the surges that occur on commercial transmission systems.

The figures obtained in the experience of the Commonwealth Edison Company on its 12-kv., 60-cycle, underground cable system showed that the surges were of the upper range of Type II. Its experience also indicated that most of the surges recorded were positive in nature. The probable reason for this is that when a negative surge occurred, the length of the figure was so small that it was covered by the line made on the klydonograph plate by the positive alternations of the normal 60-cycle supply. As shown by Mr. McEachron's paper, the positive figures are several times as large as the negative figures for a given voltage. This meant that a negative surge would have to be several times normal voltage in order to be found. The experience of the members of the various companies represented on the A. E. I. C. Subcommittee on Transients on Underground Cable Systems showed that the highest three per cent of the transient voltages on underground systems were 3.0 to 4.7 times normal operating voltage. It appears from the paper that the negative surges of even three times normal operating voltage would be covered by the figures caused by normal operating voltage of the positive polarity.

P. B. Garrett: It was my privilege to take a small part in the practical tests which were made on the system of the Southern California Edison Co. with the klydonograph. My primary interest in the Lichtenberg figures, therefore, lies in the practical knowledge which has resulted from their use.

From the studies made by Westinghouse engineers, it was felt that the klydonograph was sufficiently accurate, within the range of surge frequencies encountered in practise, to make it an entirely feasible and practical instrument for system studies. We felt it to be accurate within 25 per cent and it is particularly gratifying to find that Mr. McEachron's very thorough investigation bears this out. I believe that Figs. 6 and 7 in Mr. McEachron's paper indicate very clearly that the figures produced within the practical range of frequencies are well within the 25 per cent error just mentioned. In fact, these figures show in some cases an accuracy within approximately 10 per cent in the negative figures and considerably less in the positive figures. In this connection, it seems a happy circumstance that by far the greater number of the figures encountered in our practical tests were positive in nature. We know of no very good reason why this should be true, but it nevertheless is the case. The negative figures being so much less satisfactory to deal with in every way than the positive figures makes us very grateful for the positive figures we find in such large majority in practise.

K. B. McEachron: In conducting this investigation every care was taken to insure reliable results, many of the figures at the different wave fronts being checked several times.

The films used were not subjected, for this particular study, to any conditioning process either to increase or decrease the moisture content, as this would introduce a condition different from that usually found in practise.

The reference made by Mr. Cox to the lack of clearness of the negative figures compared with those obtained in the tests he made, can probably be best explained by stating that the increased clearness of figures seems to be due to the presence of oscillations on the wave front. The statement in the paper about this point is to be found in the last paragraph before the conclusions.

The experience of myself and those associated with me is that it is extremely unlikely that any impulse circuit producing impulses having fronts of the order of one microsecond will not be free from oscillation unless some proper oscillographic device is used to guide one in improving the circuit.

It seems likely that the reason why the switching surge is usually positive is because in a damped oscillation the negative, although occurring first, is obscured in many cases by the larger positive figures even though the positive lobe may have a much smaller amplitude than the negative.

The abrupt change in the negative figures with slow waves mentioned by Mr. Cox has not been accounted for as yet. With such slow waves, it seems doubtful that any circuit changes could have any effect when the voltage and time relations are known in every case. The circuit can only have an effect when it modifies the wave front.

Concerning the accuracy of the cathode-ray oscillograph there is much to be said, but I am satisfied that for the results given in this paper the voltage is within five per cent and the time within one per cent of the correct values. The width of the trace on the photographic film is not troublesome as shown by the oscillograms given in the paper, the traces being as good as with the ordinary oscillograph, and frequently much finer and sharper lines are obtained.

It is true that successive impulses from the same circuit differ frequently, but since oscillograms were taken of every voltage application the changes in voltage and time relations could always be properly evaluated.

Dr. Slepian's discussion of the stress conditions in the air is interesting and agrees with our own analysis. I do not agree, however, that it is proper to consider only data beyond a certain arbitrary point such as the 10-volt per microsecond point in Fig. 6 without having experimental evidence to establish the existence of such a point. It is recognized that when dealing with Lichtenberg figures a curve or line drawn represents averages only, and that the true calibration is not a line but a band whose width is such that practically all the points fall within its boundary.

Dr. Slepian raises the same question as that asked by Mr. Cox as to the possibility of variation in succeeding impulses. The answer is the same, *i. e.*, an oscillogram was taken for each of the Lichtenberg figures.

It was early recognized in dealing with transients that stray capacities even of extremely small values could not be neglected. To make the dividing system function correctly without being dependent upon wave front it has been necessary to introduce proper compensation. The possibility of oscillations of disturbing magnitude existing between the dividing system and the surge recorder is rather remote because this possibility along with many others was foreseen and oscillograms were taken to determine whether or not such oscillations existed, and if found, the circuit was redesigned until the oscillations became small enough to be negligible.

A Flux Voltmeter for Magnetic Tests

BY G. CAMILLI[1]
Associate, A. I. E. E.

Synopsis.—*A voltmeter is described for a-c. circuits the voltage indications of which are directly proportional to the maximum flux density, regardless of the wave shape of the voltage. While the instrument is suitable for many varieties of magnetic tests, its most important application is to the reduction of transformer core-loss measurements to sine-wave basis. Test data indicate excellent accuracy for the meter in this application in comparison with other schemes or outfits used at the present time for that purpose. Losses determined by this new meter are, in general, appreciably larger than those determined by the older methods.*

The new meter makes it unnecessary to use, for reliable results, large generators to reduce the wave distortion caused by transformer excitation loads, and permits the use of any generator that will carry the load thermally.

INTRODUCTION

THIS paper describes a voltmeter for a-c. measurements designed so that its indications are directly proportional to the maximum value of a-c. magnetic flux regardless of wave shape. It is graduated in terms of equivalent effective sine-wave voltage, so that at any wave shape the maximum magnetic flux is the same as the maximum value of sine-wave flux generating a sine-wave voltage of the same effective value as the indication of this voltmeter. The instrument is particularly useful in core-loss measurement and for exploring alternating-flux magnetic networks. It is useful also for the determination of the form factor of a-c. voltages, and, with slight modifications, the form factor of alternating currents.

It is recognized that certain circuit characteristics are functions of the maximum value of the alternating wave. For instance, hysteresis loss and also magnetizing current to a large extent are functions of the maximum flux density, not functions of r. m. s. values of current, voltage or flux. The problem of measuring this maximum flux has not been satisfactorily solved by other methods though ballistic galvanometers and core-loss voltmeters or correction outfits have been developed and used for the purpose. The ballistic galvanometer is not well suited to a-c. measurements, and core-loss voltmeters have serious limitations as will be pointed out later.

The instrument developed by the writer for this purpose, called a flux voltmeter, consists essentially of a rectifier in series with a d-c. voltmeter. Across these two in series is impressed the voltage which is generated in a coil surrounding the core under test in which the alternating flux exists. Under this condition, the voltmeter indications, as will be proved later, are proportional to the maximum magnetic flux in the core.

THEORY OF THE INSTRUMENT

The principles underlying the functioning of the flux voltmeter are two:

First, in an a-c. circuit, the maximum alternating flux density is proportional to the arithmetic average value of the reactive voltage drop regardless of the wave shape of the voltage, provided the wave does not cross the zero line more than twice per cycle.

Second, if an alternating voltage is rectified without changing the wave shape, and is impressed on a d-c. voltmeter, the meter indication is proportional to the arithmetic average value of the rectified wave which is the same as the arithmetic average value of the unrectified a-c. wave.

The proof of the first theorem is as follows:

Let e be the instantaneous voltage consumed in the inductance of a circuit, and ϕ the flux associated with that circuit. By elementary theory,

$$e = K \frac{d\,\phi}{d\,t} \tag{1}$$

where K is constant of the circuit. It follows from (1) that

$$e\,d\,t = K\,d\,\phi$$
$$\int e\,d\,t = K \int d\,\phi$$

The integral of $d\,\phi$ is obviously the change in the flux; and its maximum value, that is, the maximum change in flux, is obtained by finding the maximum change in the integral of $e\,d\,t$. Referring to Fig. 6, the integral of $e\,d\,t$ is the area of the voltage wave (alternately positive and negative, plus a constant of integration), and the maximum change in $\int e\,d\,t$ is the area of half a cycle of the voltage wave. Thus,

$$\int_{t_1}^{t_2} e\,d\,t = K \int_{\phi \text{ lower limit}}^{\phi \text{ upper limit}} d\,\phi$$

$= K$ times the maximum change in flux.

The area of one-half cycle of the voltage wave may be also determined, however, if we know the average voltage, by the equation,

$$\int_{t_1}^{t_2} e\,d\,t = \text{area of half-cycle}$$

$$= e_{avg}\,(t_2 - t_1)$$

The maximum change in flux is therefore proportional to e_{avg} since $(t_2 - t_1)$ is a constant, being the duration of half a cycle dependent only on the frequency of the circuit. Since, also, the flux density is proportional to the total flux, the maximum a-c. flux density, B_{max}, must be proportional to the maximum total a-c. flux, ϕ_{max}, and thus we have,

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$$B_{max} \text{ is proportional to } e_{avg}.$$

The factor of proportionality is taken care of as follows: the meter scale being graduated in terms of effective sine-wave voltage, it follows that the alternating flux density in the circuit under test corresponds to an effective sine-wave voltage of the same equivalent value as the indication of the flux voltmeter. In other words, for any given wave shape of voltage, the flux voltmeter indicates its equivalent sine-wave value, which would cause the same maximum flux or flux density. On pure sine-wave voltage, the indications of the flux voltmeter are naturally identically the same as those of an ordinary r. m. s., a-c. voltmeter. In fact, the meter is calibrated in this way.

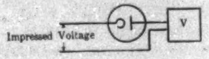

Fig. 1

The reason for the limit to wave distortion mentioned above, namely, that the wave must not cross the zero more than twice per cycle, may be understood as follows.

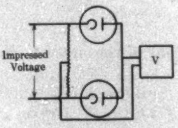

Fig. 2

Considering the positive half-cycle of the voltage wave, assume that this dips through the zero and has a negative portion. The positive portions of the half-cycle will add to the flux in accordance with the integral of *e d t*, and the negative portion will subtract by the same integral. Since the flux voltmeter rectifies the entire wave, this small negative portion will also be rectified and instead of reducing the voltmeter reading will increase it. The reading of the voltmeter, therefore, even though it still represents the arithmetic average of the voltage wave, will not truly represent the maximum of the flux wave. This limitation, however, is hardly a handicap because it is almost inconceivable that a commercial voltage wave could be so distorted as to cross the zero more than twice per cycle.

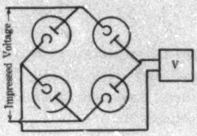

Fig. 3

Description of the Instrument. The practical construction of an instrument to meet these requirements is quite simple. Vacuum tubes are used for rectification, and a high-resistance, d-c. voltmeter to read the arithmetic average value of the rectified wave.

A diagrammatic sketch of the simplest possible construction is shown in Fig. 1 which shows a single rectifier tube in series with a d-c. voltmeter. The voltage to be measured is impressed across the voltmeter and rectifier in series; and the meter indication is then proportional to the integral of the impressed voltage.

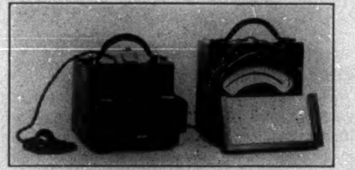

Fig. 4

Although the scheme of Fig. 1 is extremely simple, it suffers from two disadvantages: first, due to half-wave rectification the frequency of the pulsations of the rectified wave is one-half of that of the completely rectified wave, and it was observed that with 25-cycle impressed voltage, the needles of some voltmeters would vibrate like a frequency-meter reed. The second disadvantage is that with half-wave rectification the deflection of the meter is reduced to one-half. To increase the deflection would require a reduction in the resistance of the meter to one-half, double the current, or double the losses,—all of which would be undesirable.

The scheme shown in Fig. 2 using two tubes and an auto-transformer for rectifier yields full-wave rectification and better characteristics in general, except that the exciting current drawn by the shunt auto-transformer might be objectionable, especially when instrument transformers are used.

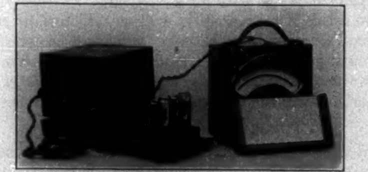

Fig. 5

The connection found most satisfactory is shown diagrammatically in Fig. 3, consisting of a four-tube, full-wave rectifier in combination with a d-c. voltmeter.

The instrument used for the investigations described below was constructed in accordance with the scheme of Fig. 3. Fig. 4 is a reproduction of a photograph of the outfit, showing the rectifier and voltmeter in separate cases, although some units now under construction incorporate the rectifier and voltmeter in one case. Fig. 5 shows the contents of the rectifier unit,—four vacuum tubes and one filament-current transformer. The purpose of the filament-current transformer is to obtain the necessary filament current from the lighting circuit instead of from the circuit under test. To draw this current from the circuit under test would be doubly objectionable: it would load the potential transformers in the circuit unnecessarily and reduce their accuracy, and it would change the calibration of the instrument with varying load voltage. Supplying the filament excitation from a lighting circuit eliminates both these difficulties, especially the latter, inasmuch as variations of the voltage of the lighting circuit, within the range of 110 to 125 volts, have no sensible effect on the calibration of the instrument.

The voltmeter is graduated in terms of equivalent effective r. m. s. sine-wave voltage, so that the maximum flux density in the core under test is the same as the maximum flux density for an r. m. s. sine-wave voltage of the same value as that indicated by the flux voltmeter.

The constants of the tubes, meter and transformer, are as follows:

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Tubes: Radio Corporation of America type, *U X*-120 tubes, well known to the radio public. Although this tube is of the three-element type, its grid and plate are connected together so as to act as a single electrode for rectification. The resistance of the tube in this connection is of the order of 5000 ohms. Rated filament voltage three volts, filament current 0.125 amperes.

Transformer: primary rated 110—125 volts, 25—60 cycles; four secondary windings, one for each tube, each rated 3—3.75 volts.

Voltmeter: d'Arsonval type, d-c. voltmeter, resistance 178,000 ohms. Scale 150 volts, equivalent sine-wave voltage.

EFFECT OF TUBE RESISTANCE

A voltmeter circuit always has a large series resistance incorporated within the voltmeter box. Tube resistance, therefore, not only is not a handicap but is a useful element, furnishing some of the desired resistance to the voltmeter circuit. In fact, were it not for the variability of tube resistance, it would have been desirable to use sufficiently high-resistance tubes to replace and eliminate all other series resistance. It is known that tube resistance varies with plate voltage, however, and therefore the tubes were chosen so as to make their resistance a small fraction of the total resistance of the voltmeter circuit. In the outfit described above, the tube resistance is only about five per cent of the total resistance, and therefore any little variations in the tube resistance are imperceptible in the total resistance.

ACCURACY OF THE FLUX VOLTMETER

The accuracy of the flux voltmeter in measuring the average value of any given voltage was tested both by the writer and by the Bureau of Standards and in both instances was found to be very satisfactory. Our tests consisted of (a) form-factor tests checked by oscillographic analyses, and (b) tests of application to core loss measurements.

FORM-FACTOR MEASUREMENTS

The form factor of a wave is defined as the ratio of the r. m. s. value of the wave to its arithmetical average value. Since ordinary a-c. voltmeters give the r. m. s. value, and the flux voltmeter the arithmetical average value, the form factor of a given voltage wave may be obtained by means of these instruments. The value so obtained may be checked oscillographically by computing from an oscillographic record of the wave both its effective and its average values in accordance with well-known methods. Since the flux voltmeter is graduated in terms of equivalent effective sine-wave volts, and since the form factor of a sine wave is 1.11, it follows that the form factor of a distorted wave is:

$$\text{form factor} = 1.11 \times \frac{\text{a-c. voltmeter reading}}{\text{flux voltmeter reading}}$$

The foregoing method of test and checking was applied to seven different wave shapes obtained on different generators under different conditions of loading, with results as follows:

		Form Factor		
Items	Exciting Current Load on the Generator	By Oscill.	By Flux Voltmeter	Figures
	Per Cent			
1	20	1.21	1.231	6
2	7	1.13	1.128	7
3	30	1.148	1.149	8
4	6	1.128	1.122	9
5	55	1.18	1.18	10
6	18	1.14	1.15	11
7	2.2	1.13	1.115	12

Those who are familiar with oscillographic analyses will appreciate the great difficulty of obtaining precise results. In view of this fact, and also in view of the possible small errors of the a-c. voltmeter used, the foregoing tests may be considered excellent commercial checks on the commercial accuracy of the flux voltmeter. The instrument is really much more precise than these tests seem to indicate, as the Bureau of Standards tests show.

The analyses of the oscillograms were made by a different person who had no knowledge of the form factor obtained by the flux voltmeter, in order that the observations might be unbiased.

MEASUREMENT OF TRANSFORMER CORE LOSS

The Institute rules provide that the efficiency rating of transformer shall be based on sine-wave operation. Although certain classes of transformer losses, such as the ohmic loss of the conductor, are not affected appreciably by ordinary variations in wave shape, it is known that core loss is seriously affected by the wave shape of the impressed voltage. Since it is difficult to obtain sine-wave voltage on a commercial scale for the testing of transformers, particularly for the condition of exciting current loads which tend to distort the generator voltage, some scheme that will reduce core-loss tests to a sine-wave basis is a necessity. The flux voltmeter was developed primarily for this purpose. The graduation of the scale in terms of equivalent effective sine-wave voltage is particularly useful for this purpose.

The principle of the application of the flux voltmeter to core-loss measurement is based on the fact that if the rated excitation voltage is set by the flux voltmeter at the rated frequency, the maximum flux density in the core corresponds to the rated sine-wave voltage excitation, even though the actual impressed voltage may be badly distorted.

The maintenance of the correct rated maximum flux density in the core by this means assures that the hysteresis loss of the core will correspond to the rated sine-wave voltage, because hysteresis depends on the maximum flux density.

Aside from the hysteresis loss, a core has also eddy-current losses.

Comparing the two types of losses we find that, in general,

(a) The eddy-current loss in laminated transformer cores is much smaller than the hysteresis loss, the eddy-current loss being of the order of one-quarter of the hysteresis loss. This difference in magnitude emphasizes the much greater importance of obtaining the hysteresis loss correctly than of obtaining the eddy-current loss correctly.

(b) While hysteresis loss varies as a complicated function of the density, the eddy-current loss varies substantially as the square of the effective r. m. s. voltage. So by means of the flux voltmeter we obtain the hysteresis loss correctly, and by means of an ordinary r. m. s. voltmeter we are able to make correction for the eddy-current loss.

(c) For a given line of sheet steel, the hysteresis loss per pound may vary a great deal depending on the annealing process and handling, but the eddy-current loss is relatively very slightly affected by these factors. This means that the eddy-current loss of a line of steel may be considered fairly constant without serious error, and hence, calculated corrections applied for it, when necessary, as described below.

When the excitation voltage is set by the flux voltmeter, the desired maximum flux density, and hence the correct hysteresis loss, is obtained. If the voltage wave is distorted, however, the a-c., r. m. s. voltage will be different from the flux voltmeter voltage, and therefore the actual eddy-current voltage under test will be different from that corresponding to the rated sine-wave voltage in the ratio

$$\frac{\text{True eddy loss}}{\text{Observed eddy loss}} = \left(\frac{\text{r. m. s. voltage}}{\text{flux voltmeter reading}}\right)^2$$

A correction, therefore, has to be applied for the changed eddy loss. The method used in the tests described below was as follows. If, in a normal sample at sine-wave voltage, the percentage values of the eddy and hysteresis losses are known over the desired range of densities, then, if the eddy-current loss is left unchanged and the hysteresis loss altered and if for the test purpose the voltage is set by the flux voltmeter, it follows that the

$$\text{True sine-wave core loss} = \frac{\text{Observed total loss} \times 100}{\text{Per cent hysteresis} + k \times \text{per cent eddy loss}}$$

where k is the square of the ratio of the a-c. voltmeter reading to the flux voltmeter reading. The relative values of the eddy and hysteresis losses in the iron sheets may be obtained from the Epstein test[2].

The fact that the percentage eddy loss varied at different densities is to be taken into consideration by using the characteristic curve of the material similar to the sample characteristic curve shown in Fig. 13. It is to be granted that in large built cores the ratio of eddy to hysteresis is not necessarily exactly the same as in small samples, due to increased intersheet eddies resulting from the closer contact of the sheets under pressure from highly tightened bolts. Large cores, however, have also increased hysteresis loss due to pressure of bolts and other reasons, so that in large cores the ratio of eddy to hysteresis probably does not increase as much as one might expect. Furthermore, it may be shown by an example that the error that may result from this variation is not very large.

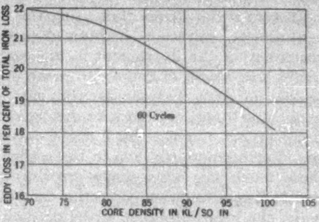

FIG. 13

EXAMPLE

Flux voltmeter setting	100 per cent rated voltage
Actual r. m. s. voltage	105 per cent rated voltage

$$k = \left(\frac{\text{r. m. s. voltage}}{\text{Flux voltmeter voltage}}\right)^2 = 1.10$$

$$\text{True core loss} = \text{Observed loss} \left(\frac{100}{\text{Per cent hyst.} + k \text{ per cent eddy}}\right)$$

If the eddy loss is 20 per cent,

$$\text{True core loss} = \text{Observed loss} \left(\frac{100}{80 + 1.10 \times 20}\right)$$

$$= \text{Observed loss} \quad \frac{100}{102}$$

The correction for error in the eddy-current loss thus amounts to two per cent. If the flux voltmeter were not used at all and no correction applied, the observed loss would be about ten per cent in error.

To show that the assumption of a considerably different percentage value of eddy loss would not affect the foregoing result appreciably, we may suppose that

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[2] Since the preparation of this paper, an excellent method used by the Bureau of Standards has come to the attention of the writer in which no assumption whatever is made about the percentage values of the eddy and hysteresis losses but two measurements are made at different wave shapes to obtain the separation of the actual eddy and hysteresis losses. The method, of course, involves the use of either the flux voltmeter or its equivalent in some form. The method is quite simple.

The core loss test is made at two different wave-shapes, setting the voltage by the flux voltmeter. Assume that W_1 and W_2 are the observed total losses, and E_1 and E_2 the corresponding observed effective a-c. voltages. Then the true total core loss W_s for the rated sine wave voltage E_s indicated on the flux voltmeter is

$$W_s = \frac{W_1(E_s^2 - E_2^2) + W_2\,(E_1^2 - E_s^2}{E_1^2 - E_2^2}$$

This is derived from the following basic equations:—

$$W_1 = H + W_0\,E_1^2$$
$$W_2 = H + W_0\,E_2^2$$

in which H is the hysterisis loss and is the same at both wave-shapes for the same flux voltmeter voltage; while W_0 is the eddy-current loss for unit effective voltage, and when multiplied by the square of the actual effective voltage gives the total eddy-current loss for that test.

the percentage is 30 instead of 20 and recalculate the true loss. We would then have

$$\text{True loss} = \text{Observed loss}\left(\frac{100}{70 + 1.10 \times 30}\right)$$

$$= \text{Observed loss}\,\frac{100}{103}$$

Comparing this with the previous result, it will be observed that a 50 per cent error in the percentage value of the assumed eddy loss has influenced the final result by only one per cent.

Test Data. In the data tabulated below, a given transformer was tested on two or more generators of widely different wave shape, thus using the degree of agreement among the results so obtained as a measure of the accuracy of the instrument in this application. An attempt was made, of course, to make one of the wave shapes in each instance as near sine wave as possible and the other, or others, as distorted as possible. Oscillograms of wave shape are shown for each instance. Furthermore, to give to the reader an idea as to how much the wave-shape error would have amounted to if no attempt at correction for wave shape were made, the losses observed when the voltage was set by an a-c., r. m. s. voltmeter are also given for each instance.

Since in these tests the comparative values of the losses rather than their absolute values are of interest, the loss data, to facilitate comparison, have all been reduced to percentage values calling one of the readings as 100 per cent.

TABLE I

Item	Trans. Kv-a. Rating	Generator Kv-a. Rating	Per cent Load on Generator	Core Loss with Flux-volt	Core Loss without Flux-volt	Oscillogram Fig.
				Per Cent	Per Cent	
No. 1	1,500	500	20	100	81.6	6
	"	1,500	7	100	94	7
No. 2	1,667	500	30	100	88.5	8
	"	3,000	5	100	94.2	9
No. 3	8,000	500	55	100	87.8	10
	"	1,500	18	100.5	92.2	11
	"	12,000	2.2	99.5	98.8	12
No. 4	10,000	500	50	100.3	83	—
	"	3,000	9	100	93	—
	"	12,000	2	100	101.5	—
No. 5	28,866	1,500	45	100	82	
	"	24,000	3	100	90.8	

These data would seem to indicate excellent commercial accuracy for the flux voltmeter, inasmuch as practically the same result is obtained no matter how distorted or how pure the wave shape is. The distortion of the wave in some instances was so great (see item No. 1) that more than 18 per cent error would have existed in the core-loss measurement if the voltage had been set by an r. m. s., a-c. voltmeter.

COMPARISON WITH OTHER METHODS

Two very similar methods have in the past been used for the purpose of reducing core-loss measurements to a sine-wave basis. It may be of interest, therefore, to present data as to how the accuracy of the flux-voltmeter method compares with theirs.

In the two older methods, use is made of a little "Standard" transformer the core loss of which has been calibrated on pure sine-wave voltage. In both methods, this "standard" transformer is excited, through potential transformers when necessary, in parallel with the power transformer under test. In one of the schemes, the voltage of the circuit is adjusted to a value such as to make the core loss of the "standard" the value corresponding to the desired sine-wave voltage. In the other scheme, the desired voltage is set by an r. m. s. voltmeter and the actual losses in both the power transformer and the little "standard" are observed. The factor which would convert the observed core loss of the little standard into its sine-wave value (known from its calibration curve) is used to convert the observed loss of the power transformer to sine-wave basis.

Tests indicated in Table I for the flux voltmeter were repeated, making corrections by these two schemes. The items in the following tabulation refer to the same apparatus and conditions as the corresponding items in Table I.

TABLE II

	Core Loss by		
	Flux-Voltmeter	Scheme No. 1	Scheme No. 2
	Per Cent	Per Cent	Per Cent
Item No. 1	100	90	91.7
	100	95	95.4
Item No. 2	100	93.3	94.2
	100	95.3	98.2
Item No. 3	100	95.2	98.8
	100.5	97.6	99.2
	99.5	99.2	102.
Item No. 4	100.3	95	96
	100	97	103.5
	100	109	111
Item No. 5	100	90.2	93.6
	100	97.4	97.4

It is interesting to note that (1) core-loss measurements made by the older schemes are influenced a great deal by differences in wave shape and that with badly distorted waves they would be very unreliable, and (2) that in eleven cases out of twelve, the losses obtained by the older methods were considerably less than those obtained by the flux voltmeter (and the one instance, in which they were more) might be somewhat doubted.

Outfits of this type have no doubt served an excellent purpose in the past, and even now, when the wave distortion is not large, they may give results which are entirely satisfactory. On the other hand, however, it appears that conditions have so changed in recent years that the average accuracy which was obtained by the aid of such outfits five or ten years ago is not readily obtained under present conditions. The reasons for this may be seen as follows:

1. These older schemes do not attempt to maintain a known flux density in the core, but aim at drawing inferences from the losses of a small unit to the losses of the main transformer. If the two transformers were operated at the same density, this might be possible, assuming that other conditions are favorable. It is not practicable, however, to duplicate the density of the main transformer in the auxiliary "standard" core, and therefore the greater the difference in density the less reliable is the result. Scheme No. 2 described above utilizes an extra theoretical correction based on the difference in the density of the two transformers and it will be observed that it gives, in general, more correct results than scheme No. 1. Still, it is not by any means as consistent and accurate as the flux voltmeter.

2. Transformer sheet steel is constantly being improved, so that hysteresis characteristics and the eddy-current ratio are modified as time goes on, and the assumed "standard" core, even when operated at the same density as the transformer under test, ceases to be representative in a short period of progress. Even at a given time, a great variety of sheet steels are used, and it would be impossible for a given "standard" core to duplicate the characteristics of them all.

3. Although the kv-a. capacity of transformers built in recent years has steadily increased, the kv-a. capacity of generating units used for testing the core-loss of the transformers has not proportionately increased, and as a result, the core-loss load on generators in testing departments is a much larger percentage of their capacity and as a consequence the wave distortion is on the average much larger than formerly. It may be noted that generator voltage wave distortion due to transformer exciting current load is far more than that due to other loads, on account of the fact that the former draws a badly distorted current. Even in a carefully designed generator, the harmonics of the transformer exciting current will produce a large distorted regulation through the usually high synchronous impedance of the generator.

APPLICATION OF FLUX VOLTMETER TO ALTERNATING FLUX NETWORKS

In a magnetic network, the flux density in different branches may be badly distorted, even with sine-wave line voltage and sine-wave total flux. In such cases, observing the r. m. s. voltages in exploring coils on different branches would not indicate the true flux densities in the respective branches, whereas voltage observations made by the flux voltmeter would definitely and correctly indicate the corresponding densities regardless of the shapes of the waves, provided, of course, that the voltage wave does not cross the zero more than twice per cycle. This application is too simple to require any further comments.

CONCLUSIONS

1. A voltmeter, suggestively called "flux voltmeter," is described for a-c. circuits, the indications of which are proportional to the arithmetic average value of the voltage wave and hence proportional to the maximum flux density, if the voltage is the reactive drop corresponding to the flux.

2. The primary application of the instrument is to the reduction of the core-loss measurements of transformers to sine-wave basis. The method is to set the desired value of the voltage by means of this instrument, in which case the maximum flux density in the core corresponds to the desired sine-wave voltage, even though the impressed voltage is badly distorted. Methods are indicated for taking into consideration the error in the eddy-current loss. Accuracy of the meter in this application is believed to be within one-half of one per cent.

3. In comparison with other and older schemes for the reduction of core loss to sine-wave basis, it is noted that the flux voltmeter has the following advantages:

   a. Its results are unaffected by the rated flux density of the transformer core.

   b. It is unaffected by the quality of the core material used.

   c. It is applicable to all commercial frequencies.

   d. In general, core losses obtained by other schemes are smaller than those obtained with the aid of the flux voltmeter.

   e. The flux voltmeter puts practically no load on the instrument transformers.

4. The flux voltmeter may also be used for exploring alternating flux networks.

5. The only limitation of the flux voltmeter seems to be in those instances in which the voltage wave crosses the zero line more than twice per cycle. Such distortions of voltage waves, however, are so rare commercially that it is difficult to conceive of encountering them in practical core-loss tests.

The author is indebted to Mr. C. A. Read for the taking and analysis of the oscillograms; to Messrs. M. G. Newman and A. Boyajian for advice in the progress of the investigation; and to Mr. L. de Nagy for assistance in making the tests.

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W. Becket Burnie, Iron Losses, *Electrician*, Vol. LXXVI.

H. M. Lacey and C. H. Stubbings, *Core Loss as Affected by Triple Harmonics*, A. I. E. E., Vol. LXVII.

R. C. Chicker, Transformer Core Losses as Affected by Triple Harmonics, *Electrician*, Vol. LXXVI.

Discussion

R. L. Sanford: For the determination of the average value of an alternating electromotive force a rectifier of some kind is necessary. While somewhat difficult to keep in good working condition, the rectifying commutator, either mounted directly on the generator shaft or driven by a synchronous motor, has been, up to the present time, the only satisfactory apparatus for the purpose. The commutator has a number of disadvantages which are obvious to any one that has had occasion to use it and the development of a really satisfactory method for measuring average volts represents a distinct advance in the art.

The device described by Mr. Camilli has the advantages of simplicity and portability. While the evidence of its reliability presented in the paper seems to be fairly conclusive, it was felt that a really crucial test would be a direct comparison with the rectifying commutator using a sine wave of voltage and also with various degrees of distortion. This was done at the Bureau of Standards. In making the test, the commutator brushes were adjusted for rectification at the zero point on the wave and its condition was checked by noting that the reading of an a-c. voltmeter was the same whether connected to the source directly or through the commutator. The average volts were read by means of a Brooks deflection potentiometer and a volt-box. Simultaneous readings were taken by different observers with the potentiometer and the flux voltmeter, and the values in all cases agreed well within the limits of the allowable error. It appears, therefore, that the flux voltmeter measures average volts with a satisfactory degree of accuracy and should prove to be a very useful instrument for a number of purposes.

As pointed out by Mr. Camilli, the total core loss with a distorted wave is somewhat high, as the eddy-current loss is proportional to the square of the effective voltage rather than to the average voltage. He has indicated an indirect method for making the correction. It would appear better, however, to determine the correction directly by making measurements with different degrees of distortion, noting the effective voltage in each case. The correction can then be made by simple calculation. This method has the advantage that no assumption as to the ratio of eddy currents to hysteresis is necessary.

W. H. Cooney: Any one who has endeavored to obtain a given core loss twice on the same transformer when using different generators on each test with consequent variations in wave form will appreciate the development of a meter which will give consistent results regardless of how distorted or how near sine wave the wave form is.

In the past, many core-loss correction outfits have been based on setting voltage by an a-c. voltmeter, thus getting the r. m. s. value. Since, as Mr. Camilli pointed out, only the eddy loss is a function of the r. m. s. value, the correction which had to be made was almost entirely in the hysteretic component. That this was not the proper end from which to tackle the problem can be realized by considering the general proportion of hysteresis and eddy losses.

Eddy losses are very rarely more than 20 per cent of the total loss and are usually much less. By the use of this scheme, which holds the average voltage corresponding to the effective sine-wave voltage desired, the correct hysteresis loss is maintained and the correction need be made only on the eddy component, which, as has been said before, is comparatively small.

Some of the previous core-loss corrections consisted essentially of a small "standard" transformer excited in parallel with the transformer under test, and one of two methods was used: (1) The available voltage was varied until the "standard" transformer held the desired core loss; or (2) the available voltage was adjusted to the desired root-mean-square value (regardless of the wave form) and the loss of the transformer under test was corrected in the same proportion in which it was necessary to correct the "standard" transformer in order to put the latter on a sine-wave basis.

The objections to the use of a small "standard" are quite apparent. The chief objection is that the "standard" is very seldom at the same density as the main transformer and thus is not operating on the same part of the density-loss curve which has a decided "knee" in it. It is necessary to maintain all sorts of calibration curves with the increased liability of introducing errors. There is finally the general objection that a small model in tests of this sort cannot be expected to duplicate the phenomena occurring in large apparatus.

The only correction which needs to be applied to this meter (for eddy loss) can be made very easily, as very few calibration curves will be necessary since the division of hysteresis and eddy losses can be determined closely enough for any given line of steel. The only possible case in which this meter will be inaccurate is where the wave goes through zero more than twice in a cycle, and this should never occur in commercial testing.

Aram Boyajian: The simplicity of the theory, construction and method of application of the flux voltmeter may tempt us to underestimate the excellent engineering which has been incorporated into this outfit. It is true that such parts of the outfit as d-c. voltmeters and vacuum-tube rectifiers have been available separately, and it is also true that the mathematical relation between maximum flux density and arithmetical average voltage has been known to the electrical art for a long time. It remained for Mr. Camilli, however, to bring together the theory and these pieces of apparatus into an outfit which accomplishes a new function in a new and very satisfactory manner. The accuracy and consistency of core-loss tests made with the aid of this outfit are surprisingly good, especially in comparison with the older schemes. The extent to which the older schemes underestimate the core loss under conditions of bad wave distortion will no doubt interest very strongly inspectors on acceptance tests as well as those engineers who draw up specifications.

T. C. Lennox: I should like to point out the advantage of this device for determining the core losses in interphase transformers for rectifiers. These interphase transformers have voltages which are not of sine-wave form and furthermore have different wave forms for different conditions of load so that it is difficult to obtain loss measurements which are representative of actual losses under load. The flux voltmeter could be very easily applied to obtain the actual flux densities under load conditions and thus aid greatly in obtaining a correct measurement of the efficiency of the equipment as a whole.

Variable Armature Leakage Reactance in Salient-Pole Synchronous Machines

BY VLADIMIR KARAPETOFF[1]

Fellow, A. I. E. E.

Synopsis.—*The electrical performance characteristics of a polyphase synchronous machine, that is, its voltage-current relations under load, depend essentially upon the nature and the extent of the magnetomotive forces of the armature currents. Broadly speaking, the effect of these magnetomotive forces is two-fold; i. e., (a) they oppose and distort the field magnetomotive force and (b) they create leakage fields linked with the armature conductors. The first influence is known as the armature reaction, and the second as the armature reactance. More specifically, in a machine with salient poles, the armature reaction may be resolved for purposes of computation into the direct reaction (along the center lines of the poles) and the transverse reaction, midway between the poles. In polyphase machines of usual proportions, the armature leakage reactance, x, usually plays a secondary role, and for most purposes is assumed to be constant and independent of the power factor of the load. The vector of the reactive drop, Ix, is simply drawn in a leading time quadrature with the current I.*

However, in machines with considerable armature reactance, or where higher accuracy is required, the assumption of a constant x leads to noticeable discrepancies between the computed and observed data. This is of particular importance in problems which involve hunting, instability, etc., and in which the torque (or displacement) angle must be predicted. This angle depends to a considerable degree upon the leakage reactance of the machine. It has been previously proposed by others to use two distinct values of leakage reactance, one when the leakage paths around a group or belt of armature slots are closed through the center of a pole face (maximum reactance), and the other when such slots are midway between the poles (minimum reactance). However, no account has been taken apparently of a gradual change in the reactance between the two extreme positions, nor have the results been properly correlated with the rest of the factors which enter in the performance of the machine.

In the present paper, the leakage inductance is assumed to consist of two parts, one of which is constant (the average inductance), and the other, varying harmonically at a double frequency, reaches a maximum opposite the centers of the poles. A magnetic linkage equation is written, and its derivative with respect to the time angle is taken to obtain the induced voltage. The result shows that the foregoing assumption leads to two reactive drops, one, the usual average Ix drop and another a supplementary drop, leading Ix by an angle 2ψ, where ψ is the internal phase angle at which the machine is operating. These quantities are introduced in the usual Blondel diagrams for the generator and the motor, and the relationships among the various quantities are established both graphically and analytically.

IN the performance characteristics of synchronous machines with salient poles, there are certain discrepancies between the computed and the measured electrical quantities, even when the usual Blondel diagram is used which is supposed to take into consideration the influence of salient poles on the armature reaction[2]. These discrepancies may be accounted for, at least in part, by taking into consideration the variations in the value of the permeance of the armature leakage flux during a cycle.

At zero power factor of the load, the armature leakage flux reaches a lower maximum value than with the same armature current at unity power factor, because in the latter case the armature slots, in which the current reaches its maximum, are almost opposite the centers of the poles, and the permeance of the leakage paths is greater. Hence, at high values of power factor, the machine should be expected to behave as if its leakage reactance were greater than that obtained from a short-circuit test. Often such is actually the case. Moreover, the torque (or displacement) angle[3] often comes out from test larger than from computation. This also indicates an actual increase in the equivalent reactive drop ix, as would be expected theoretically at higher values of power factor.

Several authors have pointed out the fact that the armature leakage reactance varies during the cycle, and some have used *two values of reactance*, one opposite the poles, the other midway between the poles[4]. However, to the writer's knowledge, *gradual changes* in the reactance have not as yet been considered quantitatively in their effect upon the Blondel diagram, and the latter is usually drawn with a constant ix drop. An attempt is made below to develop a method whereby such cyclic variations in the reactance can be taken into consideration in Blondel's theory of two armature reactions. Of course, a similar correction could be applied to the less accurate Potier diagram, but it would seem hardly logical to correct for the effect of salient poles upon the leakage reactance in a diagram in which the effect of salient poles on the armature reaction is disregarded altogether.

I. AVERAGE REACTANCE AND SUPPLEMENTARY REACTANCE

In Fig. 1, the curve $A\ B\ C\ D\ E$ represents values of the armature leakage inductance plotted with reference to the axis of abscissas $X\ X$. The time angle is denoted by α, where

$$\alpha = 2\pi f t = \omega t \qquad (1)$$

t being time and f the frequency. When $\alpha = 0$, let the

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[1] Professor of Electrical Engineering, Cornell University, and Engineering General Department, General Electric Co.

[2] For a theory of Blondel's diagram, see V. Karapetoff, *The Magnetic Circuit*, Arts. 48 and 49. The subject of direct and transverse armature reaction is also treated in detail in Chaps. 47 and 48 of the author's *Experimental Electrical Engineering*, Vol. II, 3rd ed., now going through the press. Extensive literature references will be found there.

[3] Mag. Cir., the angle $\beta + \phi_2$ in Fig. 40; this angle is denoted by θ in Fig. 2 of this paper.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Niagara Falls, N. Y., May 26-28, 1926.

[4] See, for example, C. P. Steinmetz, *A-C. Phenomena*, Chapter on Armature Reactions in Alternators.

group of armature conductors under consideration be opposite the center of a pole. The tooth-tip leakage is then at a maximum, and hence the leakage inductance reaches its maximum value. When $\alpha = 90$ deg., the same group of conductors is midway between the poles and the leakage inductance is a minimum.

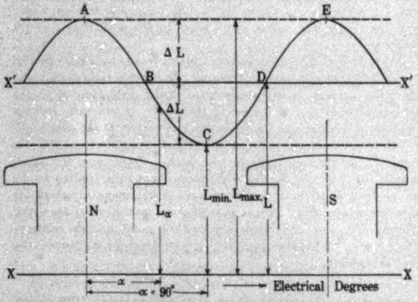

FIG. 1—VARIABLE ARMATURE LEAKAGE INDUCTANCE L_α AND ITS AVERAGE VALUE L

Thus, the leakage inductance varies at twice the frequency of the main induced voltage of the machine, and, as a first approximation, the inductance curve in this discussion will be assumed to be a sine wave of double frequency. Even when an actual test gives a curve departing from a sine wave, this experimental curve can be replaced by an equivalent sine wave, because higher harmonics in the inductance can produce only higher harmonics in the terminal voltage or in the current of the machine, and in this paper only the quantities of fundamental frequency are considered. The variable value of inductance, L_α, at an instant of time corresponding to the electrical angle α, shall therefore be expressed as

$$L_\alpha = L + \Delta L \cos 2\,\alpha \qquad (2)$$

Here L is the *average* value of the inductance over a cycle, and ΔL is the greatest departure from the average, that is, the amplitude of the sine wave with respect to the axis $X'\,X'$.

Assuming the armature current to be sinusoidal, it can be written in the form

$$i_\alpha = I \operatorname{Cos}(\alpha - \psi) \qquad (3)$$

where I is the amplitude of the current, and $\psi = Q\,O\,G$, Fig. 2, is the internal phase angle at which the machine is operating. The angle ψ characterizes the interval of time between the instant when a group of armature conductors is opposite the center of a pole and the instant when the current in the same group of conductors reaches a maximum[5]. For a generator, ψ is assumed to be positive when the current reaches a maximum *after* the corresponding group of conductors has passed by the center of a pole (although the current may be leading with respect to the terminal voltage). In a motor, ψ is greater than 90 deg., and may even be greater than 180 deg. when the motor is under-excited. Thus, in equation (3), the case of $\psi = 0$ corresponds to the generator working at such a power factor that the current I reaches its maximum when that group of conductors, through which I is flowing, is directly opposite the center of a pole. It can be shown that under these conditions the current is slightly leading with respect to the terminal voltage, but this fact is of no particular importance in this discussion.

Multiplying equations (2) and (3) term by term, in order to obtain the instantaneous magnetic linkages, gives

$$i_\alpha L_\alpha = L\,I \operatorname{Cos}(\alpha - \psi) + \Delta\,L\,I \operatorname{Cos} 2\,\alpha \operatorname{Cos}(\alpha - \psi) \qquad (4)$$

In the last term, the product of the cosines can be replaced by their sum and difference. Namely

$$\operatorname{Cos} 2\,\alpha \operatorname{Cos}(2\,\alpha - \psi) = 0.5 \operatorname{Cos}(\alpha + \psi) + 0.5 \operatorname{Cos}(3\,\alpha - \psi)$$

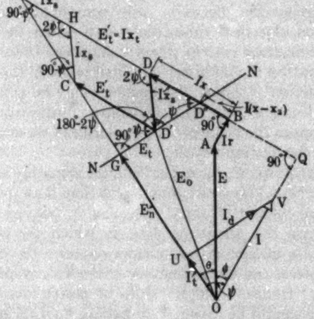

FIG. 2—A MODIFIED BLONDEL DIAGRAM OF A SYNCHRONOUS GENERATOR WITH THE AVERAGE REACTIVE DROP, Ix, AND SUPPLEMENTARY REACTIVE DROP, Ix_s

Leaving only the fundamental term containing α, and disregarding the term with $3\,\alpha$ (the third harmonic), *approximately*,

$$i_\alpha L_\alpha = L\,I \operatorname{Cos}(\alpha - \psi) + 0.5\,\Delta\,L\,I \operatorname{Cos}(\alpha + \psi) \qquad (5)$$

The voltage drop due to these linkages is

$$e_x = d\,(i_\alpha L_\alpha)/d\,t = \omega\,d\,(i_\alpha L_\alpha)/d\,\alpha \qquad (6)$$

Hence, taking a derivative of equation (5),

$$e_x = -\,\omega\,L\,I \operatorname{Sin}(\alpha - \psi) - 0.5\,\omega\,\Delta\,L\,I \operatorname{Sin}(\alpha + \psi) \qquad (7)$$

The term $-\,\omega\,L\,I \operatorname{Sin}(\alpha - \psi)$ represents a sine voltage which leads the current wave by 90 deg. and corresponds to the average constant reactance

$$x = \omega\,L \qquad (8)$$

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[5] *The Magnetic Circuit*, pp. 154 and 155; the angle ψ is marked in the diagrams there.

such as ordinarily used in the theory of synchronous machines. The last term in equation (**7**) corresponds to a *supplementary reactance*

$$x_s = 0.5\ \omega\ \Delta\ L \qquad (\mathbf{9})$$

The corresponding voltage drop, $x_s I \operatorname{Sin} (\alpha + \psi)$, *leads* the main reactance drop $I\,x$ by the angle $2\,\psi$, because

$$(\alpha + \psi) = (\alpha - \psi) + 2\,\psi \qquad (\mathbf{10})$$

Thus, the following expression is reached for the vector of total reactive voltage drop in the armature:

$$\text{Reactive drop} = I\,x\ \underline{/90^\circ} + I\,x_s\ \underline{/90^\circ + 2\,\psi} \qquad (\mathbf{11})$$

The quantities x and x_s are expressed by equations (**8**) and (**9**); the values of L and $\Delta\ L$ are shown in Fig. 1. The angle notation in equation (**11**) means that $I\,x$ leads the vector I by 90 deg. and $I\,x_s$ leads I by 90 deg. $+ 2\,\psi$. In other words, $I\,x_s$ leads $I\,x$ by $2\,\psi$.

II. A MODIFIED BLONDEL DIAGRAM

An application of these results to the Blondel diagram is shown in Figs. 2 and 3 which are generalized Figs. 40 and 41 in the "Magnetic Circuit." Fig. 2 represents a generator at a lagging current and Fig. 3 a motor at a leading current[6]. The current I is shown in both cases from the point of view of the machine itself (and not of the line) so that the phase angle ϕ, when the machine is operating as a motor, is greater than 90 deg. This method of representation is of advantage in that the same diagram and the same set of formulas cover operation both as a generator and as a motor.

Beginning with the line voltage E, Fig. 2, the ohmic drop $A\,B = I\,r$ is added in phase with the current, and the average reactive drop $B\,D = I\,x$ in leading quadrature with it. The supplementary reactive drop, $I\,x_s = D\,D'$, is added at an angle $2\,\psi$ to $B\,D$, in the positive (counter-clockwise) direction. In the case of the motor, ψ is greater than 90 deg., and $2\,\psi$ is greater than 180 deg. $E_0 = O\,D'$ is the total induced electromotive force. The direction $O\,C$ is that of the induced electromotive force at no load, or the center line of the pole. Hence, by constructing a right-angle triangle $D'\,G\,O$, with $O\,D'$ as hypotenuse, the net voltage $O\,G = E_n$ induced by the real poles is obtained, and the voltage $G\,D' = E_t$ induced by the fictitious (transverse) poles.

From equations (**83**) to (**85**) on p. 156 of the "*Magnetic Circuit,*" it will be seen that instead of drawing E_t normal to $O\,C$, a vector $D'\,C = E_t'$ can be drawn parallel to $I\,x$. This procedure is necessary when the angle ψ is not known, and in fact can be used to determine this angle, namely, by completing the parallelogram $D\,D'\,C\,H$ and extending $D\,H$ to its intersection with $O\,C$ at K[7]. In the triangle $K\,H\,C$ the angle at H is equal to $2\,\psi$ by construction; the angle at K is equal to 90 deg. $- \psi$, because $O\,Q\,K$ is a right triangle, and the angle at 0 is equal to ψ. Thus, in the triangle $K\,H\,C$ the remaining angle, at C, is also equal to 90 deg. $- \psi$. The triangle is isosceles and consequently $H\,K = i\,x_s$. This gives the following construction for the angles ψ and θ:

Lay off $O\,A$ and $A\,B$, and draw the direction $B\,K$, normal to the current. Lay off $B\,K = I\,x + E_t' + I\,x_s$, and connect point K to 0. This will give both the internal phase angle ψ and the displacement of torque angle θ. The latter is the electrical angle by which the pole structure of the loaded machine is advanced (or retarded) with respect to an identical unloaded machine connected to the same bus bars.

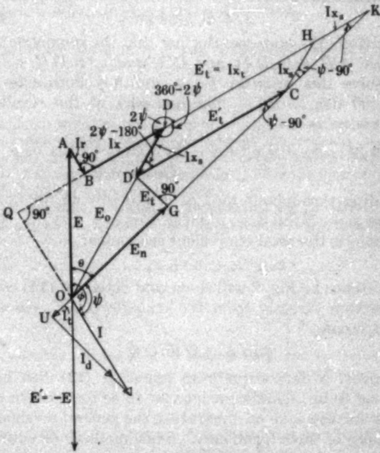

FIG. 3—A MODIFIED BLONDEL DIAGRAM, SIMILAR TO FIG. 2, FOR SYNCHRONOUS MOTOR

If it is required to compute the required excitation, complete the diagram by laying off $B\ D'' = I\ (x - x_s)$, and through D'' draw the straight line $N\,N$ normal to $O\,K$. This will determine the net induced electromotive force $O\,G = E_n$; from the no-load saturation curve, the corresponding net excitation, M_n, can be found. To this excitation, the direct armature reaction, M_d, is added [or subtracted, if $\sin \psi$ is negative, *ibid.*, equation (**79**)]. This will give the required field excitation, M_f, at that particular load.

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[6] The current and voltage notation has been changed from i and e to I and E; the induced voltage is denoted by E_0 in place of E. I has been defined above as the amplitude of the current. There is no objection, however, to considering I and E in the diagram as vectors of the effective values.

[7] The point K is of considerable importance in diagrams of synchronous machines, and Blondel uses the term Joubertian e. m. f. for the value of $O\,K$. It is convenient to call K the Joubertian point.

Most of the construction lines and angles indicated in Fig. 2 are also shown in Fig. 3, and the description given above, with slight modifications, may also be followed in Fig. 3.

III. ANALYTICAL SOLUTION

The heavily drawn polygon in Fig. 2 can be represented in the cissoidal complex form by the equation

$$E \,\text{Cis}\, \phi + I\, r \,\text{Cis}\, 0 + I\, x \,\text{Cis}\, 90 \text{ deg.} + I\, x_s \,\text{Cis}\, (90 \text{ deg.} + 2\,\psi) + I\, x_t \,\text{Cis}\, 90 \text{ deg.} = (E_n + I\, x_t \,\text{Sin}\, \psi) \,\text{Cis}\, \psi \qquad (12)$$

where

$$\text{Cis}\, \phi = \text{Cos}\, \phi + j \,\text{Sin}\, \phi = \epsilon^{j\phi} \qquad (12a)$$

and also

$$I\, x_t = E_t' = I\, k_t\, k_b\, m\, n\, v \qquad (13)$$

[*Magnetic Circuit*, p. 156, equation (**84**)], so that the *transverse reactance*, corresponding to the transverse armature reaction, is

$$x_t = k_t\, k_b\, m\, n\, v \qquad (13a)$$

Equation (**12**) expresses the fact that the geometric sum of $O\,A, A\,B, B\,D, D\,D'$ and $D'\,C$ is equal to $O\,G + G\,C$. To solve this equation for ψ, multiply both sides by $\text{Cis}\,(90 \text{ deg.} - \psi)$[8]. The real part of the resultant expression is

$$E \,\text{Sin}\, (\psi - \phi) + I\, r \,\text{Sin}\, \psi - I\, x \,\text{Cos}\, \psi - I\, x_s \,\text{Cos}\, \psi - I\, x_t \,\text{Cos}\, \psi = 0 \qquad (14)$$

Dividing throughout by Cos ψ and solving for tan ψ,

$$\tan \psi = (E \,\text{Sin}\, \phi + x_0\, I)/(E \,\text{Cos}\, \phi + I\, r) \qquad (15)$$

where x_0 is the total equivalent reactance:

$$x_0 = x + x_t + x_s \qquad (15a)$$

A reference to Fig. 2 will show that equation (**15**) could be written directly from the triangle $O\,Q\,K$, since in this triangle

$$\tan \psi = Q\,K / O\,Q \qquad (16)$$

However, a derivation from equation (**12**) has been deemed to be of sufficient interest to be included in this paper for the sake of illustrating the general method of solution of such problems. This method is entirely automatic, while the particular geometric relations in a given problem may or may not be evident. The displacement angle θ is determined from the relationship

$$\theta = \psi - \phi \qquad (16a)$$

for a motor θ is negative.

Equating separately the real and the imaginary parts on the two sides of equation (**12**), and solving each for the term with E_n, we obtain:

$$E_n \,\text{Cos}\, \psi = E \,\text{Cos}\, \phi + I\, r - I\, (x_s + 0.5\, x_t) \,\text{Sin}\, 2\,\psi \qquad (17)$$

$$E_n \,\text{Sin}\, \psi = E \,\text{Sin}\, \phi + I\, (x + 0.5\, x_t) + I\, (x_s + 0.5\, x_t) \,\text{Cos}\, 2\,\psi \qquad (18)$$

Knowing the angle ψ from equation (**15**), E_n can be computed from either equation (**17**) or (**18**). When ψ is near zero, it is preferable to use equation (**17**); when ψ is nearly 90 deg., equation (**18**) will give more accurate results.

A graphical interpretation of equations (**17**) and (**18**) is shown in Fig. 4. This diagram is identical with Fig. 2, except that a line, $G\,W$, is drawn perpendicular to $O\,Q$. The lengths $D'\,R$ and $R\,G$ are each equal to $0.5\, E_t'$. Equation (**17**) then simply means that

$$O\,W = O\,Q - W\,Q \qquad (19)$$

while equation (**18**) states that

$$W\,G = R\,G + W\,R \qquad (20)$$

Whether or not this graphical relationship will prove to be of further practical interest, remains to be seen. It has been deemed advisable to note it here, as a check on the formulas.

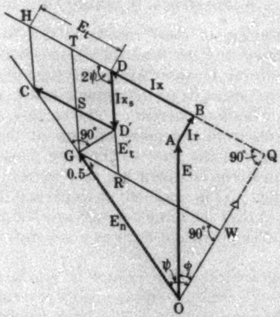

FIG. 4—A GRAPHICAL INTERPRETATION OF EQUATIONS (17) AND (18)

The theory deduced above has been checked on a machine with an exceptionally high leakage reactance. The assumption of a constant leakage reactance, deduced from the short-circuit test and from the design data of the machine, led to a wide discrepancy between the observed and computed performance characteristics. On the other hand, by assuming reasonable values of the average and supplementary leakage reactances, the current and voltage relations, as well as the torque angle θ, were found to agree quite closely with the test data[9].

In conclusion, the author wishes to express his appreciation to Mr. R. E. Doherty, Consulting Engineer of the General Electric Company, for suggesting this investigation and for encouragement while it was in progress.

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[8] Two Cis operators are multiplied by simply adding the angles; that is, Cis a. Cis b = Cis $(a + b)$. This follows directly from the exponential expression in equation (**12a**); see also V. Karapetoff, *The Electric Circuit*, p. 93, equation (**154**).

[9] For a different treatment of the subject see R. Bruderlin, *Drehfeldmaschinen mit veränderlicher Reaktanz, Arch. f. Elekt.*, 1924, Vol. 13, p. 12.

Discussion

P. M. Lincoln: It would have added to our information if Prof. Karapetoff had gone a little further into the details of the amount of departure of this particular machine which he cites in the last part of his paper.

The reactance of our large machines is becoming very much larger than it was ten or fifteen years ago. At that time, internal reactance of generators was something of the order of 10 per cent —from 8 per cent to 12 per cent; nowadays, on account of the very much greater capacity of the machines, it is necessary to limit the amount of short-circuit current that will occur in order to secure safety in switching devices. And the best method of limiting, of course, is to limit the reactance. The internal reactances of our large generators have gone from a matter of 10 or 12 per cent up to 15 to 25 per cent. It has more than doubled.

Now Prof. Karapetoff has indicated that this much larger internal reactance can be properly divided into two factors. One of them is a reactance which is independent of the position of the phase of the current-carrying coils with respect to the poles, and the other factor is a factor which depends upon that relationship.

I think it would add to the paper if Prof. Karapetoff would indicate the amount of departure between the tests and the assumption that the reactance is constant—how much is ΔL, what is the value of ΔL that he has discovered, what that relation is that it bears to the L.

E. B. Shand: Referring to Blondel's theory of two reactions, assume that the armature conductors of an alternator are carrying current of which the phase relation may be represented by the accompanying Fig. 1; the armature turns are concentric with the field winding so that they will either add to, or subtract from, the ampere-turns of the field. This component of current is that of direct magnetization. When the current is 90 deg. out of phase from the above condition, as is indicated in Fig. 2, the flux produced by the armature magnetizing force will pass into one side of the pole and out the other side. Thus flux has always presented some difficulty to me in the way of definition. If the ordinary transformer conception is adhered to, this flux, which does not interlink with the field winding, will be called leakage flux. On the other hand, the armature ampere-turns modify very materially the main flux of the machine and also may produce saturation effects so that it has always seemed to me that the definition "cross-magnetizing flux" in accordance with Blondel's ideas is the better term for it.

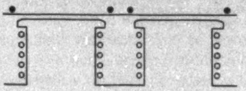

Fig. 1

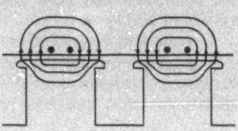

Fig. 2

Any analysis of the kind given in this paper is greatly dependent upon where the line is drawn between armature cross magnetization and armature leakage reactance. I should like Professor Karapetoff to give us his ideas on this matter and the assumptions he has made in connection with the paper.

J. F. H. Douglas (communicated after adjournment): Professor Karapetoff's article shows clearly how many factors must be considered in theory when accurately predicting the performance of a synchronous machine, factors which do not enter into the performance at zero power factor.

His treatment of linkages of flux has the merit of rigor which the arbitrary assumption of two "reactances" seems to lack. His equations (8) and (9) in conjunction with his Fig. 1 lead to an important relation

$$X_s/X = \Delta L/2\,L < 1/2 \tag{1}$$

on which it would be interesting to have an experimental check. He shows clearly that by considering theoretically derived constants X_t and M_d, and the more usually known reactance, $(X - X_s)$, values of the torque angle θ and of the internal phase angle ψ may be considerably in error.

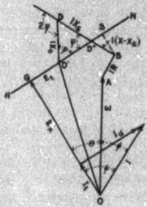

Fig. 3—Illustration to Prove that the Blondel Two-Reactance Diagram is Identical with the Diagram Including a Supplementary Reactance Drop

The graphical construction given in his Fig. 3 follows along lines advocated by me in the discussion of an article in the A. I. E. E. Journal for Jan. 1925,[1] with the added advantage, of course, of including the factor X_s.

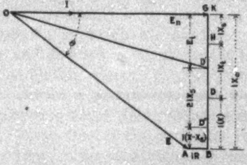

Fig. 4—Blondel Diagram for Condition of No Direct Reaction

Fig. 2 of his article is susceptible of interpretation in several ways. It is important to note that one interpretation is not inconsistent with the two-reactance theory in general and that of Dr. C. P. Steinmetz noted in Footnote 4 in particular. Consider the accompanying Fig. 3, which is Fig. 2 in the article with the point S added. The lines $D\,S$ and $B\,S$ are

$$D\,S = I\,(X + X_s)\,\mathrm{Cos}\,\psi = I_t\,(X + X_s) \tag{2}$$

$$B\,S = I\,(X - X_4)\,\mathrm{Sin}\,\psi = I_d\,(X - X_s) \tag{3}$$

We may locate the $I\,X$ drop as $B\,D + D\,D'$, or as $B\,S + S\,D'$. The latter interpretation of the figure is that the $I\,X$ drop consists of two separate components, (a) one caused by the direct component of the current I_d acting with a reactance, $X_a = X - X_s$, and (b) one caused by the transverse current I_t acting on the reactance $(X + X_s)$.

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[1] Douglas, Engeset and Jones, *Complete Synchronous Motor Excitation Characteristics.*

Another viewpoint might be to regard $S\,B$ as above and consider $S\,G$ as a sort of total transverse voltage drop, due to the transverse current I_t acting on the reactance $X_o = X + X_s + X_t$ using the notation of Karapetoff's equation (**15a**). Inasmuch as

$$X_o = X_a + 2\,X_s + X_t \qquad \textbf{(4)}$$

it will be seen that to consider X_a and X_t and neglect X_s may result in considerable error. The graphical construction is in no way influenced by these interpretations but the theory becomes very much simpler when we consider the constants M_d, X_o, and X_a, and disregard their components X, Xs and $X\,t$.

As is well known, the zero full-load saturation curve can be used to evaluate both direct reaction and armature reactance. The reactance evaluated is, I believe, that reactance I have called X_a, namely $(X - Xs)$. It would be most desirable to have an experimental method for the evaluating of the reactance $X_o = (X + X_s + X_t)$; the experiment which would be most useful would be to load a machine as an alternator, with a leading current such that the angle ψ was zero, that is, with the current wholly transverse. With this loading, the voltage terminal E, the torque angle θ, and the field current should be measured. Fig. 2 of the article then becomes the accompanying Fig. 4 below, and the reactance X_o and E_n can be computed by

$$X_o = (X + X_s + X_t) = (E \text{ Sin } \theta)/I \qquad \textbf{(5)}$$

$$E_n = E \text{ Cos } \theta + I\,R \qquad \textbf{(6)}$$

The value of E_n should check with field current I_f if the theory is correct, and the value of X_o could be obtained.

I wish to record the conviction that the attempt to separate Xo into components X, X_s and X_t, will be very difficult. First, physically, where shall we picture the $(X + X_s)$ linkages as ending and the X_t linkages as beginning? Experimentally X_o alone can be measured. If we calculate X_t theoretically from design data, and the residual $X + X_s$ is small, then there will arise the question whether our transverse coefficient is accurately known. For example, the whole effect could be attributed to an error in the theoretical derivation of the constant X_t. Stated in a somewhat different way, I could, with a single value of armature reactance X_a, and a somewhat larger value of X_t than Prof. Karapetoff uses, predict the same performance characteristics that he does, with the use of the three constants X, X_s and X_t. (I am referring to the phase angles θ and ψ, for the field current another constant, that of direct armature reaction would have to be used.)

Vladimir Karapetoff: In reply to Mr. Shand's question, I should say that from a physical point of view we have in an alternator a doubly excited magnetic circuit, which is excited by the field winding and by the combination of the polyphase windings on the armature. There is a complicated flux which varies in space and in time, and only part of which moves synchronously with the poles, the rest corresponding to harmonics moving at different velocities, some in the direction of rotation of the machine, some against it. Any division into various armature reactions and leakage reactances is only a practical makeshift which perhaps is not rational from the physical point of view. The best we can do here is to play the game straight and not figure out the same fluxes twice.[2]

The particular makeshift which Blondel proposed many years ago consists in resolving the armature currents into two components in time, one component which reaches its maximum when that particular group of conductors is opposite the center of a pole and the other group which reaches its maximum in the position midway between the poles. The magnetomotive forces due to these two components are considered separately and also in combination with the field m. m. f. For details see the references given above.

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[2] V. Karapetoff, "The Magnetic Circuit," p. 150; Doherty and Nickle, *Synchronous Machines*, presented at A. I. E. E. Annual Convention, White Sulphur Springs, June, 1926.

Fire Protection of Water-Wheel Type Generators

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and

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Associate, A. I. E. E.

Synopsis.—*In recent years, much attention has been given to fire protection of turbo generators, resulting in a trend towards closed ventilating systems and the use of inert gas to smother combustion. Water-wheel type generators are not so well adapted to this method of protection. In water-power plants, water is the most readily available means for fire extinguishment, but its indiscriminate use through perforated pipes or nozzles may cause damage equal to that of the fire.*

The present paper describes a system of fire protection for water-wheel type generators which is designed to limit both fire and water damage to the section of the generator immediately adjacent to the point of origin of the fire. This is accomplished, first, by air baffles which control the flow of ventilating air around the armature end projections, and second, by the use of fusible sprinkler heads which permit the application of the water solely to the region of the fire. The means for preventing and detecting fires in such generators are also outlined. Tests made during design to establish the effectiveness of the scheme and devices used are also described.

THE problem of protecting generators against fires of internal origin is closely associated with that of their ventilation. This is obvious from the fact that the cooling medium ordinarily employed, namely, air, contains the oxygen required to support combustion. This fact has been recognized in that one method employed for extinguishing fires in generators is to control the composition of the atmosphere within the ventilating system. In closed ventilating systems, such as are now becoming standard for steam turbo generators, this may be accomplished in various ways. For instance, the fire may be permitted to burn until enough of the oxygen contained in the closed ventilating system is used up so that the remaining gases will no longer support combustion, or an inert gas such as carbon dioxide may be admitted to the ventilating system in sufficient concentration to lower the free oxygen content to a point where combustion will stop. A third method which has been proposed is to maintain at all times in the closed ventilating system an inert gas, such as hydrogen, as a cooling medium instead of air. Methods of protection, however, which have been worked out for steam-turbine-driven generators are not in general applicable without modification to water-wheel type generators.

The purpose of this paper, therefore, is to point out certain factors involved in the problem of fire protection of water-wheel type generators, and to record the principles and devices employed in the fire protection of the 65,000-kv-a. Niagara generators.

Let us therefore first catalog some of the essential points in which water-wheel type generators differ from steam turbo generators from the standpoint of fire protection.

1. Water-wheel type generators are of relatively large diameter. This means that the armature windings occupy a considerable circumferential length. For instance, in the 65,000-kv-a. generators to which reference was made, the circumferential length of the armature windings is approximately 75 ft., whereas in turbo generators the diameters are small and the windings concentrated into a much smaller space. The significance of this fact is that it ought not to be necessary to burn up a whole winding, say 75 ft. long, on account of a fire starting at one point.

2. Water-wheel type generators are usually of open construction and employ an open system of ventilation as contrasted with the closed systems now usually employed with turbo generators. It is, indeed, becoming more common to employ semi-enclosed systems of ventilation for water-wheel generators in which either the air inlet or air outlet is enclosed, but it is seldom that both inlet and outlet are enclosed, and no case is known to the writer where a completely closed system of ventilation with air coolers is employed with a water-wheel generator, although such a case may exist.

3. In turbo generators it is now usual to employ forced ventilation in which the air is moved by an external fan and directed in definite paths through the generator ventilating ducts. In water-wheel type generators, however, a natural system of ventilation is usually employed in which the air is moved by means of the natural blower effect of the generator rotor, sometimes assisted by fan blades attached to the rotor, and sometimes, also, by an external blower the only function of which, however, is to bring the air to, or remove it from, the generator. This method usually results in a considerable circumferential motion of the air around the armature windings, particularly about those portions of them which project beyond the iron core. These end projections are frequently insulated with inflammable materials, and on account of this and their extent and exposure to the whirling air currents, they constitute the principal element of fire hazard in the generator. In many cases of fires which have occurred in generators of this type, it has been the experience that a fire originating at one point of the circumference has been communicated very quickly throughout the entire circumference of the generator through the agency of these whirling currents of air. When this occurs, it usually results in the loss of the entire winding, if, indeed, damage does not also result to other parts of the

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structure. In large machines this involves a very considerable direct loss which, however, is usually exceeded by the cost of the loss of use of the machine while repairs are being made. The latter is of especial significance in hydroelectric plants where it is unusual to find spare capacity available.

Figs. 1 and 2 are cuts made from photographs of the results of fires in the armature end projections of two horizontal-shaft water-wheel type generators. Both of these started from faults in the armature coils, and within a few seconds after starting, the flames had been swept completely around the circumference of the machine. Eye-witnesses state that within one minute after these fires started, the iron shields covering the end projections of the armature windings were at white heat. One of these fires occurred in a machine with completely closed ends and the other in one with partially closed ends, but the results were the same in both cases.

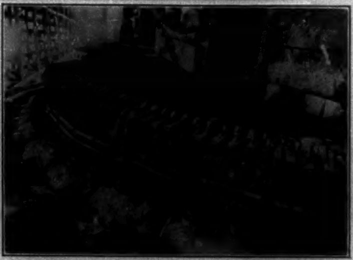

Fig. 1

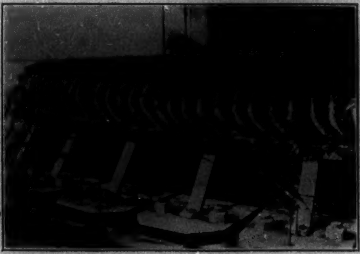

Fig. 2

Figs. 1-2—Effect of Whirling Air Currents on Fire in End Windings of Generators

In view of these considerations, it would appear that, whereas in steam turbo generators the most promising element with which to work for fire protection is the *composition* of the ventilating atmosphere, in water-wheel type generators the *directing* of the ventilating atmosphere in its passage through the machine appears to offer the most promising opportunity for the control of fire.

In any complete system of fire protection there are four necessary elements:

1. Prevention; that is, means should be provided to prevent, whenever possible, the starting of a fire.
2. Control; that is, assuming that a fire has started in spite of preventive methods, means should be provided to confine the fire to as small an area as possible.
3. Detection; means should be provided for promptly detecting the presence of a fire, and it is desirable that such means should also give an indication of its location.
4. Extinguishment; a fire having occurred, means for its prompt extinguishment should be provided.

PREVENTION

The only inflammable material built into a generator is the insulation. The use of non-combustible insulation, if that were possible, would therefore be the best fire preventive. Even Class "B" insulation as now employed, however, contains a considerable amount of inflammable material used as a binder, and the requirement for flexibility often dictates the use of fibrous or Class "A" materials on the armature end projections. Generator windings often also accumulate, between cleanings, considerable quantities of oily dirt of an inflammable nature, so that even the use of fireproof insulation cannot always be depended upon to make a fire impossible.

Generator fires are usually caused by insulation breakdown. One obvious means of fire prevention, therefore, is to use a high factor of safety in the insulation of the windings.

Should a failure occur, however, the next obvious thing to do is to disconnect the generator from the system and remove its excitation as quickly as possible. This is best accomplished by means of differential relay protection in a manner now well understood and almost universally used. Such a differential relay system should be sensitive, and, from the fire prevention standpoint, should be arranged to de-energize the machine as promptly as possible. Reference is made to the recently published Relay Handbook for details of such applications.

CONTROL

In spite of all that may be done in the use of fire-resisting insulation, in preventing insulation failures and in de-energizing the machine in the event of a failure, a fire may nevertheless occur. In one case known by the authors, a stubborn blaze was initiated in a mica-insulated generator winding by an insulation breakdown on over-potential test with a 50-kw. testing transformer equipped with an instantaneous trip. Owing to the energy stored in the magnetic circuits of the generator, it is impossible to de-energize a machine instantaneously, and the writer therefore sees no reason to believe that even the differential relay system will necessarily prevent the starting of a fire.

Assuming, therefore, that a fire may be started, the next problem is to prevent such a fire from spreading, or in other words to confine it to the smallest possible zone. Here appears the virtue of controlling the direction of flow of the ventilating air as it passes the generator windings. The minimum fire-spreading effect will result if the air currents are directed radially through the generator winding with complete suppression of the circumferential component of the movement. The requirements for cooling, of course, necessitate that some air shall be allowed to pass through and around the armature windings. Through the core, ventilating ducts are provided for this purpose. It has been customary, however, to permit the air to flow about the projecting ends of the armature coils in any manner which might result from the chance arrangement of the parts without any specific provision for guiding it in any particular direction, radial or otherwise. The result has been the production of a large circumferential component of air velocity in the neighborhood of the armature end projections.

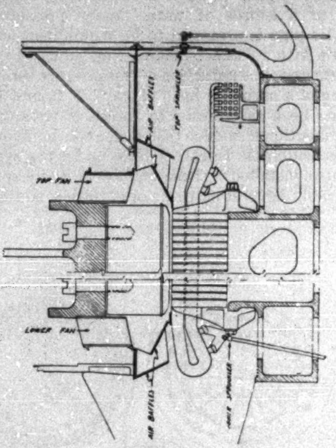

FIG. 3

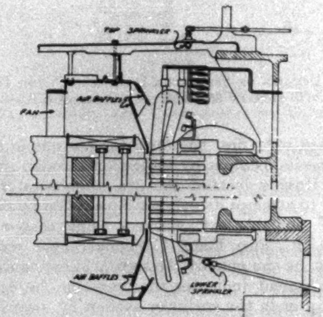

FIG. 4

FIGS. 3-4—CROSS-SECTIONS SHOWING APPLICATIONS OF FIRE BAFFLES TO TWO DESIGNS OF 65,000-KV-A., VERTICAL SHAFT GENERATORS

To rectify this condition, the idea was conceived of placing between the rotating field and the armature end projections a stationary structure of some sort for the purpose of guiding the air currents in a radial direction through and between the armature end projections. Such a structure might be of a number of different forms, such as, for instance, a perforated wall, or a series of vanes which might be of any required shape, or an arrangement of baffled passages. After considerable study of these three alternatives, an arrangement of passages between two inclined walls was adopted for the 65,000-kv-a. generators. Figs. 3 and 4 show the application of this scheme of air control to the two makes of 65,000-kv-a. generators installed by The Niagara Falls Power Company, and Fig. 5 is a reproduction of a photograph showing the baffles installed at the lower ends of the coils in one of the generators. In order to prevent the generation of eddy currents in these baffles due to their presence in a strong varying magnetic field, they are made of insulating material.

The effectiveness of these baffles in directing the air through the end windings is very marked, the hurricane of air usually found about the armature end projections in conventional machines of this type being entirely absent from the machines equipped with these baffles.

The effectiveness of the radially directed air currents in preventing the spread of a fire was demonstrated by means of tests on a full scale model of a small section of one of these generators. These tests will be hereinafter described, and seem to indicate that by this means a fire may be confined to a circumferential length of not over two ft.

DETECTION

A fire having been started, it is, of course, desirable to detect its presence immediately, and since we have now found means of confining a fire to a small section of the winding, means of locating it in the machine are also desirable. In the case of the 65,000-kv-a. machines, the heated ventilating air is collected in a steel housing which surrounds the armature frame and is thence blown out of the building by means of separate motor-driven blowers. This arrangement would lend itself very readily to a system of smoke detection involving the use of a photo-electric cell. Such a system has not been installed, however. Various detectors based on abnormal temperatures might also be employed; one device suggested consists of a fuse wire carried around the generator and arranged to give an indication upon being fused at any point. Another possible device would be a vapor tension thermometer with its bulb in the form of a long slender tube extending completely around the machine. This would indicate the maximum temperatures at any point throughout the length of the tube. A third system of giving the indication of the presence of a fire is the use of sprinkler heads in which case the means of detection and of extinguishment are combined. With any system the provision of hand holes at frequent intervals about the circumference of the machine is desirable both for the purpose of locating the fire and for allowing access to it by means of hand extinguishers or a fire hose. The combination of sprinkler heads and hand holes was used in the 65,000-kv-a. generators (Fig. 6).

Fig. 5—Fire Baffles Installed at Lower end of Armature of 65,000-Kv-a. Generator

Extinguishment

A number of mediums are now available for extinguishing fires in electrical apparatus, the principal ones being water, steam, carbon dioxide, carbon tetrachloride, "Fire Foam," and the well-known soda acid extinguisher. These may be applied in various ways; for instance, soda acid, "Fire Foam," and carbon tetrachloride are usually applied by hand or portable tank extinguishers. Water or steam may be applied through a hose; or water, steam, "Fire Foam," and carbon dioxide may be applied through permanent piping.

Fig. 6—Hand Holes and Sprinkler Heads at Top of 65,000-Kv-a. Generator

In the case of the 65,000-kv-a. generator at Niagara, the means provided for extinguishment consist, first, in a series of hand holes giving access to the windings for the use of hand or portable extinguishers or water hose. There is also installed at suitable points near the armature end projections, a series of sprinkler heads connected by piping through a manually-operated lever valve to a source of water under pressure. This sprinkler pipe is arranged to be maintained under air pressure with a contact-making gage adjusted to sound an alarm upon the reduction of this pressure through the blowing of a sprinkler head (Figs. 7, 8, 9, and 10).

It has been ascertained by means of tests upon the above mentioned model that a fire can be maintained for a considerable length of time under the system of air control installed in these machines without spreading appreciably beyond its point of initiation, so that upon occurrence of a fire, the operator has ample time to determine its location by inspection through the hand holes. Having located the fire, the operator is instructed to attempt its extinguishment first by means of hand or portable extinguishers using carbon tetrachloride or "Fire Foam." Should these prove insufficient, he can employ the soda acid extinguisher, or if his judgment indicates the necessity, he can turn on the water in the sprinkler system by means of the above mentioned lever valve. In this event the application of the water will be limited to one or two sprinklers which will have opened immediately adjacent to the fire, so that the water damage will be confined to as small a portion of the machine as possible.

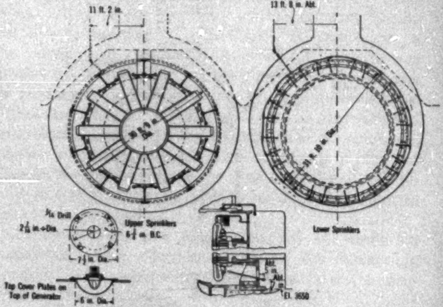

Fig. 7—Details of Sprinkler Head and Piping Installation in 65,000-Kv-a. Generator

If the use of carbon dioxide for fire extinguishment is perfected to a point where it is applicable to machines of this type, there would appear to be no reason why the general system of protection installed in these machines would not be readily adaptable to its use. It would appear that the baffles between the rotor and armature coils might be of material assistance toward the effective use of carbon dioxide.

DETAILS OF INSTALLATION ON 65,000-KV-A. GENERATORS

These are shown in Figs. 7, 8, 9, and 10. The air baffles are constructed of bakelized canvas or asbestos supported by suitable castings with radial vanes forming passages for the cooling air.

The sprinklers are of modified standard design and are enclosed in baskets of sufficiently fine wire mesh to retain the loose parts resulting from a blow-off.

Separate piping systems are provided for the top and bottom ends of the armature windings with separate alarm signals so that a fire may be immediately located with respect to the two ends of the generator.

The manually-operated control valves shown in Figs. 9 and 10 are normally sealed in the closed position and are so arranged that any leakage of water into the sprinkler piping which may take place is caught in a sump and may be periodically drained off. The opening of the valve for draining the sump will automatically test the alarm system by lowering the air pressure in the sprinkler system.

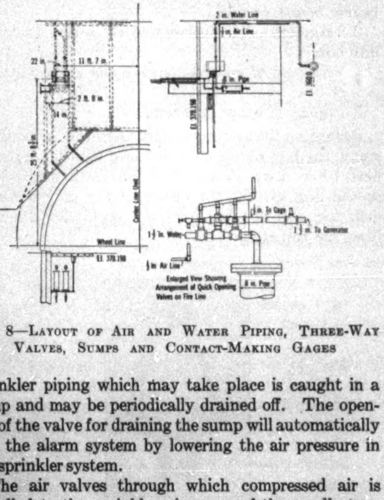

FIG. 8—LAYOUT OF AIR AND WATER PIPING, THREE-WAY VALVES, SUMPS AND CONTACT-MAKING GAGES

The air valves through which compressed air is supplied to the sprinkler pipes are of the needle type and may be kept slightly open so as to automatically maintain the air pressure so long as all sprinkler heads are intact. Upon the blowing of a head, the pressure will drop due to the throttling effect of the needle valve and the alarm will be given.

The contact-making pressure gage is of standard design arranged to close a contact upon a drop in pressure to a predetermined value. Any desired form of alarm can be used.

TESTS

Before adopting the above described system of protection for the 65,000-kv-a. generators, a series of tests was made to determine the best type and arrangement of sprinklers, to determine whether the temperatures produced by a fire would be sufficient to operate standard sprinkler heads, and whether the air control devices proposed would be effective in confining the fire to a small area.

The first series of tests was taken to determine the following points:

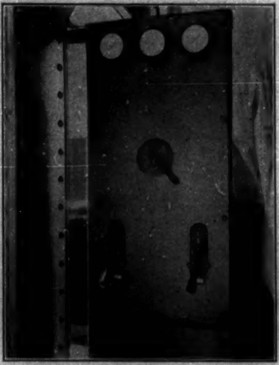

FIG. 9—ARRANGEMENT OF HANDLES OF THREE-WAY VALVES AND CONTACT-MAKING PRESSURE GAGES

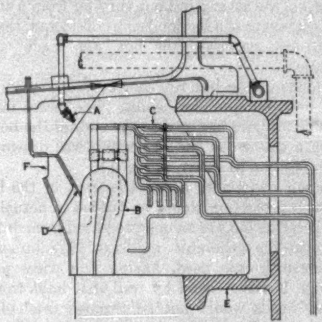

FIG. 10—SECTIONAL VIEW OF GENERATOR

Showing relative location of end windings, connections and sprinkler. A—sprinkler, B—stator coil, C—stator connections, D—fire bafflers, E—stator frame, F—air inlet

1. Location of sprinkler heads with respect to generator windings.
2. Design of sprinkler-head deflector.
3. Design of screen to catch links or parts which fly out from sprinkler heads when they operate.

It was impossible to make the proposed tests on one of the 65,000-kv-a. generators; therefore it was desirable to make the conditions under which the tests were to be taken as near like actual conditions as possible.

Fig. 10 shows a section of the generator with location of sprinkler head as originally estimated to be approximately correct. Fig. 10 also shows the relative locations of the stator-coil end windings, stator connections, and fire baffles, and also shows the relatively small amount of space above the electrical parts in which to locate the sprinkler heads.

In order to see exactly how water would spray from the sprinkler heads under different conditions, a dummy section of the generator was made up in the form of a wooden box as shown in Fig. 11. This box represented at full scale one of the generator sections in which a sprinkler head was to be located. The length of the box represented the length of one of these sections measured on the circumference of the generator at the air-gap. On account of the size of the 65,000-kv-a. generators, the air-gap did not vary far from a straight line in the length of a section being considered; therefore for our test purposes, the box was made without curvature.

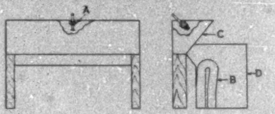

FIG. 11—DUMMY SECTION OF GENERATOR USED IN THE WATER-SPRAY TESTS

A—Sprinkler, B—stator coil, C—screen, D—beaver board

The ends of the box represented the barriers to be placed between sprinkler heads. Side pieces made of beaver board were fastened to the ends of the box, and on each a stator-coil end projection was drawn in its relative position as shown in Fig. 11.

A screen was placed across the front of the box as some such protection would be needed in actual practise to keep the links of an operated head from dropping down into the generator windings. Medium-weight wire screening was used, having five wires per in. each way, it being found by trial that links from the sprinkler heads would not go through mesh of this description, while they would go through mesh having wires of the next greater standard spacing.

In this first set of tests no fire was used, as the principal object was to determine how well water would be sprayed into the generator windings and connections under different conditions. As fires in generators usually occur in the stator-coil connections or stator-coil end projections, it was necessary in the tests to find an arrangement in which water would reach these parts effectively.

Before taking the tests it was quite obvious that water could be sprayed in large quantities on the stator windings and connections directly in front of the sprinkler, but it was not certain just what position of sprinkler head and type of deflector would give the best water distribution over the entire area to be considered. From Fig. 11 it can be seen that the area to be covered was much longer than it was wide, but it was just as important that the water reach the coils at the ends of a section, or in other words, that it reach coils midway between two sprinklers, as it was to reach the coils directly in front of the sprinkler.

In the sprinkler tests, over two dozen different special deflectors were used, and several locations of sprinkler and angle of spray were tried. The water pressure, also, was varied to determine the effect of such a change on the water distribution.

Under each test the water distribution was observed and rated in five ways as follows:

1. Distribution and distance water sprayed directly in front of sprinkler head.
2. Amount and distribution of water sprayed onto beaver boards at side.
3. Amount and distribution of water backward into box.
4. Amount of water to reach upper corners at each side of box.
5. Spray of water in general.

Regarding the location of sprinkler head and angle of spray, the best results were obtained with the sprinkler head tilted down 45 deg. from horizontal and located in the box so that the center of the sprinkler-head deflector was five in. from the back and three in. from the top of the box.

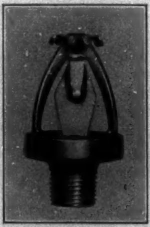

FIG. 12—STANDARD SPRINKLER HEAD USED IN FIRE PROTECTION OF BUILDINGS

If the sprinkler head was tilted more than 45 deg. from horizontal, an undue amount of water was sprayed into the box; and if the angle was made less than 45 deg., a larger portion of water than necessary was sprayed out directly in front.

Fig. 12 shows a common type of sprinkler head as generally used in fire protection of buildings. It will be noted that the deflector used at the top of this sprinkler has projections or ears around its outer edge which are bent downward. It was quite obvious that this type of deflector would not meet our needs, as the bent ears have a tendency to spray a larger portion of the water backward.

After many tests were made in which a large number of different deflectors was used, it was found that a deflector as shown on sprinkler head in Fig. 13 gave the best results. This deflector, which is approximately 13/16 in. in diameter and is cupped upward slightly, happens to be a deflector which is used on many sprinkler heads in mills where the projecting ears of the standard sprinkler are objectionable. When used in mills, however, the deflector is cupped downward instead of upward.

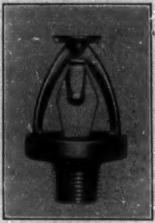

Fig. 13—Sprinkler Head Used in 65,000-Kv-a. Generators

In order to show how the design of a deflector affects the water distribution, a few of the deflector tests will be described.

1. Deflector in accordance with Fig. 13. The water was sprayed a distance of six ft. out in front of the box, and the beaver boards at the sides were well covered. Some water was sprayed backward into the box. With this deflector, the water was discharged in a fine spray which almost resembled a fog. The distribution of water over the desired area was very good.

2. Same as (1) except that deflector of larger diameter was used. Water was sprayed farther out in front than in case (1), but the spray was not as fine and the general distribution was not as good as in case (1).

3. Deflector in accordance with Fig. 12 except that all but two ears on opposite side were cut off. (It was thought that the two remaining ears might help throw more water out to the sides.) Water was sprayed farther out in front than in case (1). Water spouted up at ears, but distribution at sides was not as good as in case (1).

4. Same as (3) except that all ears were cut off. Results were about the same as those of (3).

5. Same as (1) except that deflector was inverted and therefore cupped down instead of up. Less water was sprayed out in front than in case (1). Practically no water reached the beaver boards at the sides. Large amount of water discharged back into the box.

The deflector tests such as described above were taken with a medium water pressure of 43 lb. per sq. in. With deflector used in case (1), the water pressure was increased to 100 lb. per sq. in. With this pressure the spray became extremely dense, and the distribution of water remained good.

By decreasing the water pressure it was found that the quantity of water and the distribution of water were satisfactory down to a pressure of 30 lb. per sq. in. With a pressure of 25 lb. per sq. in. the results were only fair; therefore it was felt that in practise the water should be held up to at least 30 lb. per sq. in. This matter of water pressure can be easily taken care of in case of the 65,000-kv-a. generators, as the normal water pressure in the station is approximately 80 lb. per sq. in.

The principle of operation of the sprinkler heads used may be seen from the construction shown in Figs. 14 and 15. Fig. 14 shows the cross-section of a sprinkler head and position of links before the head has operated, and Fig. 15 shows the same head with position of links shortly after the head has operated.

It may be noted that the links of a sprinkler head are small levers so assembled that they can be held together in the normal position before operation by a very small amount of solder. By using solders of different melting points, the sprinkler heads can be made to operate over a wide range of temperatures.

Ratings of sprinkler heads have been standardized so that a head may be obtained having a rating of 155,212,286 or 360 deg. fahr. For each different rating the composition of the solder is such that the heads will operate at a temperature corresponding to the rating.

In order to make sure that sprinkler heads located in the 65,000-kv-a. generators as previously determined would operate satisfactorily in case of a generator fire, further tests were considered.

Actual fire tests seemed desirable, as it was not certain that heat sufficient to operate the sprinkler heads would have a chance to reach them in case of a generator fire. The ventilation of these generators is such that a current of air passes in a horizontal direction through the stator connections and end windings and then out through openings in the stator frame. It seemed possible that in case of a fire in the end windings or con-

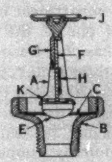

Fig. 14—Cross-Section of Sprinkler Head

Showing position of links before operation

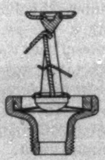

Fig. 15—Cross-Section of Sprinkler Head

Showing position of links after operation

A—Yoke, B—body, C—diaphragm, E—glass valve, F—main strut piece, G—hook link, H—key link, J—deflector nections of one of the generators, sufficient heat to operate a sprinkler head might not pass up through this current of air.

Another point in question regarded the wire screen which was to be used to keep the links of an operated head from dropping down into the generator windings. It is known that a wire mesh offers some resistance to the passage of heat, and it was thought desirable to check this point in a test to find out if heat sufficient to operate a head in case of a generator fire would pass through the proposed screen.

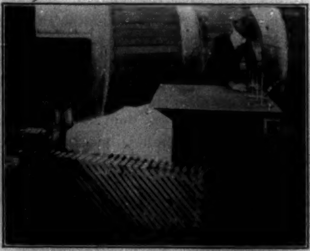

FIG. 16—DUMMY SECTION OF GENERATOR USED IN FIRE TESTS

In order that the second series of tests could be made conveniently, a dummy generator section was made of sheet metal. The construction of this compartment was such that, turned one way, it represented a section of the generator at the top of the machine, and inverted, it represented a bottom section of the generator. Dummy stator connections and stator-coil end projections were also constructed, these being made of wood. All parts were made to full scale.

Fig. 16 shows the dummy compartment in a position representing a top section of one of the generators. It may be noted that the door has been removed and the dummy coils taken out of the sheet metal compartment and placed in the foreground so that the construction may be seen. In the same figure, an end view of the dummy stator-coil connections may be seen in position in the compartment. The dummy connections and coils were made of boards one in. thick and spaced ½ in. apart. With this construction the wooden parts could be easily ignited for the fire tests.

A motor and exhaust fan were connected to the sheet metal compartment as shown in Fig. 16, in order that air might be circulated through the stator coils and connections in the normal way.

Provision was made for the mounting of a sprinkler head in the test compartment as previously determined. Four thermometers having 150-deg. cent. scales were suspended through small holes in the top of the compartment as shown. With the use of the thermometers, temperatures could be read at time intervals between the starting of the fire and the operation of the sprinkler head. By means of a hose, air pressure was placed on the sprinkler heads so that the operation of a sprinkler head would be known immediately by the rush of air.

Fig. 17 shows a cross-section of the test compartment described above with door removed and the dummy connections and stator end projections in their normal position. The location of the wire screen and fire baffles is also given. The small glass window in the end of the dummy section was provided so that the intensity of the fire could be watched with the metal door in place and the compartment entirely closed except for the air inlet in the front and the air outlet in the rear.

Shutters were placed in the air inlet and a damper in the air outlet so that the flow of air through the compartment could be regulated if desired.

With the test equipment arranged as shown in Fig. 17, the first set of fire tests was made. Fire was started at different places in the dummy connections and coils by means of a small rag soaked in kerosene. After a sprinkler head operated, the wooden parts were taken from the compartment and the fire was put out by means of steam.

In Fig. 18 some representative temperature curves are given showing the results of a few of the tests made with the fire started in the dummy coils. In these curves, temperature in degrees centigrade is plotted against time in minutes. In the first four tests shown, the shutters in the air inlet and the damper in the air outlet were open and the fan was running. In the last test shown, the fan was shut down. The four different curves for each test represent the readings of the four different thermometers which were numbered one to four from right to left facing the front of the dummy compartment. Temperature readings were taken every quarter of a minute.

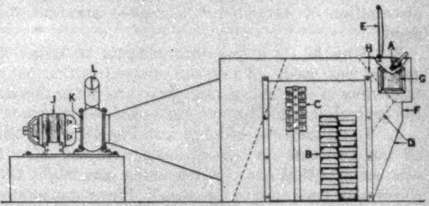

FIG. 17—CROSS-SECTION OF SHEET-IRON DUMMY COMPARTMENT USED IN FIRE TESTS

In tests 2, 5, and 7, 100-deg. cent. sprinkler heads were used and in tests 3 and 6, 68-deg. cent. heads were used. Facing the front of the compartment, the fire was started in the left end of the dummy coils in tests 2 and 3, and in the middle portion in tests 5, 6, and 7.

It should be noticed that in these tests the temperature curves become very steep close to the time of head operations. In fact, the temperature increased so fast that the thermometers were raised out of the compartment in order to keep them from going off scale and breaking. This accounts for the fact that the curves do not in all cases extend to the vertical line which indicates the time of head operation.

As would be expected, the curves show that under similar conditions the 68-deg. heads operate sooner than the heads rated 100 deg., although the difference in time is small. By comparing test 2 with test 3, it may be seen that with the fan running and with the fire started in the left end of the dummy coils, the 100-deg. head operated in three min. and the 68-deg. head in 1¼ min.

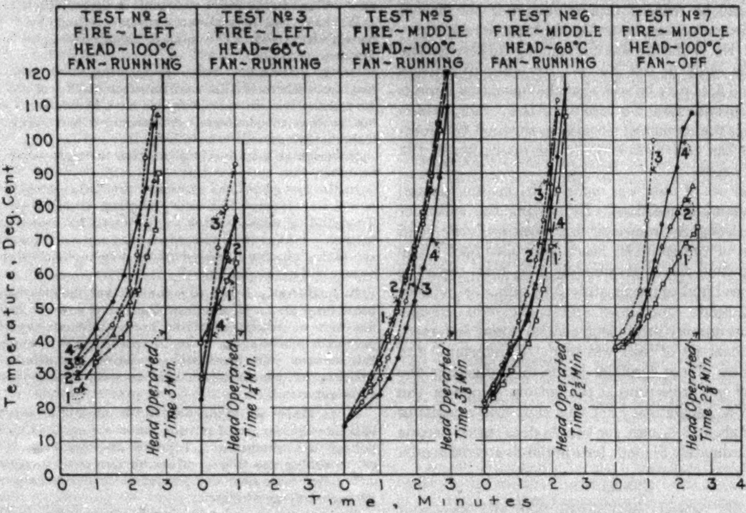

FIG. 18—REPRESENTATIVE TEMPERATURE CURVES

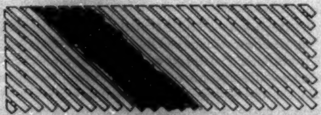

FIG. 19—DUMMY COILS SHOWING BURNED PORTION

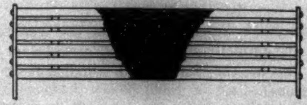

FIG. 20—DUMMY CONNECTIONS SHOWING BURNED PORTION

Tests 5 and 6 show that under conditions similar to those described above except that the fire is started in middle of the dummy coils, the 68-deg. head operated in 2½ min. and the 100-deg. head in 3⅛ min.

It may be noted that in some of the tests the fire got started a little sooner than it did in other tests. This was due to the fact that steam was used to extinguish the fire and in some cases the dummy coils were left more moist than in others. By comparing tests 5 and 6, it may be seen from the curves that soon after the fire was started it required ½ min. longer in test 5 than it did in test 6 to reach a certain temperature. By making correction for this difference, the 100-deg. cent. head of test 5 would have operated about ½ min. sooner, or in approximately 2 ⅝ min., had the fire developed to raise the temperature as in test 6 where a 68-deg. cent. head was used. In other words, the difference between 2 ⅝ min. and 2½ min., or ⅛ min., gives approximately the difference in time between operation by a 100-deg. cent. head and a 68-deg. cent. head, under such conditions as existed in test 6.

In all cases it may be noted that the temperatures as measured with the thermometers exceeded the sprinkler head ratings before the heads operated. This does not mean that the heads were rated incorrectly but rather that with the temperature increasing rapidly, the time lag is greater in the sprinkler heads than in the thermometers.

Fig. 19 shows the way the dummy coils were burned and charred in Test No. 5, this being a typical case.

In this test the fire was started in the middle of the dummy coils, and it should be noted that the fire followed up the diagonal wooden members, but due to the strong current of air that was being blown through the dummy coils, the fire did not spread to the right nor to the left appreciably. Fig. 20 shows the way the dummy connections were burned in the same test. Later a similar test was made in which the fire was allowed to burn for 15 min. in the dummy coils. The results were about the same in that the fire spread very little. This is a very important point since the fire damage can be reduced to a minimum if the fire in coils of generators can be prevented from spreading.

In test No. 7 the fan was not running; otherwise the test is the same as test No. 5. By comparing tests Nos. 7 and 5, it may be seen that the operating time is 2 5/8 min. in one case and 3 1/8 min. in the other, which shows that the current of air passing through the compartment has very little effect in the operation of the sprinkler heads.

Another set of tests was made with the fire located in the dummy connections. By moving the sprinkler heads back slightly from their first position, very good operation of the sprinkler heads was obtained. The operating time was approximately 1 3/4 min. for the 68-deg. head and approximately 2 1/2 min. for a 100-deg. cent. head.

With the dummy sheet metal compartment inverted, an additional set of tests was made to determine the effectiveness of the sprinkler heads in protecting the stator-coil end projections at the bottom of one of the 65,000-kv-a. generators. For this case, the operating time was about 2 1/4 min. for the 68-deg. cent. heads and approximately 3 1/2 min. for the 100-deg. cent. heads.

CONCLUSION

The tests proved conclusively that the proposed scheme of fire protection as described would be very satisfactory for the 65,000-kv-a. generators for the following reasons:

1. Sprinkler heads arranged in the generator as shown and properly spaced would operate for a very small fire in the stator end windings or stator connections.
2. Sprinkler heads would operate in spite of the air currents, which, in case of the coil projections and connections at top of generator, would tend to carry the heat from a fire away from the sprinkler heads.
3. With current of air flowing radially through the end windings and connections, a fire started in these parts is confined to a small section even after the fire has burned for as long a time as 15 min.
4. If a fire started in the generator windings cannot be extinguished easily with hand fire extinguishers, the operator can then turn a valve which will allow water to be sprayed effectively in the region of the fire only, thereby doing minimum damage to the remainder of the machine.

The authors wish to acknowledge the valuable assistance given by Mr. Ira Knight of the General Fire Extinguisher Company, who furnished equipment and helped to carry out the water spray tests.

Discussion

(DISCUSSION AT NIAGARA FALLS)

H. C. Don Carlos: We have found in some cases where machines were designed as semi-enclosed or totally enclosed machines that on account of the lack of fire protection it was necessary to change these to open-type machines, so that we could get to the fire to put it out in case of failure.

We have had several cases where windings have been completely destroyed. The damage is expensive and the outages are long.

On the other hand, with open-type machines we have had very few cases where the fire resulting from failure of coils has been very extensive. In most cases we have been able to control the fire, so that the damage is confined to a few coils in the immediate neighborhood of the failure.

In designing a system of protection there are several things to be taken into consideration.

In the first place, the system of protection must be such that it will not damage the entire winding, as has been suggested. The winding might as well be destroyed by fire as by the protection. While water protection may not permanently damage the entire winding it may damage it to such an extent that the outage will be intolerable.

B. L. Barns: I wish to point out that the illustrations which have been given of windings that were entirely destroyed by fire were of generators of the horizontal-shaft type having no provision for bringing them to rest quickly in emergencies of this nature. Vertical-shaft generators are usually provided with brakes by means of which it is possible to stop them in about three minutes.

H. U. Hart: I do not believe the method proposed counterbalances the danger of putting water connections in large high-voltage a-c. generators. I prefer the introduction of a fire-extinguishing gas to generators by means of storage tanks instead of a water sprinkler system which would cause damage to the high-voltage windings.

The damage to ends of coils due to fire in a-c. generators can be greatly minimized by using mica tape on the ends of the coils instead of cotton tape. The cost is greater, but this would lessen the amount of inflammable material.

R. B. Williamson: The method proposed in this paper is designed to do two things: One is to localize the flame by means of guards or baffles that force the air to flow in radial jets against the coils and prevent it from being swept around in a circle; the other is to localize the part wet by the water. In a great many cases where large machines have been wet all around it has been a very long, tedious job to get them dried out and any method that will limit the damage to a restricted area is highly desirable. In the early days of vertical machines no protection was provided in the way of brakes for shutting down promptly. I recall one instance of a slow-speed vertical machine of about 5000 kw., which had just been installed; in fact, it never had been on the load. When it was brought up to voltage it developed that there had been a connection damaged or broken in shipment which had not been detected, and this started a small arc which under the influence of the fans developed into a flame that swept around the winding and completely burned the insulation off the ends of all the coils. If the machine had been equipped with brakes they would have helped materially in limiting the damage. Of course brakes are now the universal custom on machines of this type. The limitation of the area in which the water is applied is very important, and the method described by Messrs. Johnson and Burnham is an excellent one for large water-wheel type generators.

L. W. Riggs: I should like to inquire if any one has had experience in applying carbon dioxide to such machines, that is, not including turbo alternators and, if so, what amount of leakage of air can be tolerated; how tight does the ventilation system have to be?

S. Q. Hayes: Bearing on this same subject it might be interesting to note that the Duquesne Light Company in Pittsburgh is using carbon dioxide in connection with the putting out of fires on some synchronous converters. If I remember correctly it has been figured that a mixture of about 20 per cent of carbon dioxide is sufficient to prevent the combustion being carried on. Up to the present time, the Duquesne Light Company has had no fire. They were just rearranging some of their synchronous converters, completely enclosing them, and they provided for this with the Lux Company in New York. They have a rather interesting scheme to automatically open a number of cylinders that will squirt carbon dioxide into this enclosed machine. They have it so arranged that the air circulation will be shut off automatically and the carbon dioxide turned on in case of trouble. I think there was a description in one of the recent technical papers prepared by one of the Duquesne Light Company's men covering this proposition.

These machines, so far as I remember, are 7500-kv-a., 60-cycle, synchronous condensers, fairly high-speed.

M. W. Smith: When carbon dioxide is used for extinguishing fires in rotating machinery it is common practise to close both the air inlet and air outlet in order to retain this gas within the machine. With the air outlet closed, the gas pressure, and consequently the leakage, increases. The amount of leakage will be influenced by the construction of the end-bell enclosures. One precaution that is usually taken in the application of carbon dioxide is to provide means for short circuiting the ventilating system after the normal inlets and outlets have been closed. This allows the gas to circulate internally from the outlet to the inlet, and the pressure is largely consumed inside the machine with the reult that the leakage is materially reduced.

H. C. Don Carlos: I should say that fires result from different causes. One common cause is poor connections at the end of the winding. These may escape detection on account of the insulation covering and the difficulty of inspecting all of the joints, which emphasizes the importance of extreme care in making up joints at the end of the windings.

Another cause, of course, is the failure of insulation, and extensive fires will usually result from the failure of insulation at the end of a winding. In my experience, failure of insulation in the slot is not likely to develop a serious fire.

James A. Johnson: That would indicate that the oily dust, or an accumulation of that nature, is not really the main item to be guarded against. The prevention of these fires really goes farther back,—before the machine even leave the plants. That is the proper time to put the preventive measures into effect.

L. W. Riggs: It seems to me that the operators have a great deal to do with the prevention of fires. I not do think they are likely to find loose connections in modern machines.

Any oily dirt accumulated on the end windings should be cleaned off. In the first place it adds to the inflammability of insulation; and in the second place it plugs up the air ducts thereby allowing the machine to become much hotter than contemplated thus promoting the starting of fires. The operating force should not allow a machine to get dirty even if they feel that it will not catch fire.

There is another point which one of the speakers brought up in regard to the volume of air that is contained in the machine in a given instant. On high-speed synchronous machinery, it is surprising how little air there is in the machine at any one time. I think that most people would be surprised if they calculated the volume of air.

(Discussion at Salt Lake City)

E. H. Freiburghouse: Fires which are not caused by the electrical failure of the insulation sometimes occur; hence the system and equipment which Messrs. Johnson and Burnham describe would function in a desired manner whereas the common differential-relay system for the protection of a generator would not set in motion alarms or mechanisms for extinguishing fires until the insulation had arced through. Their system also has a very valuable feature whereby the application of water would be selective and localized to the actual fire area. Sprinkler heads could be made to act as relays in the application of carbon dioxide within totally inclosed generators of the steam turbine type and would protect the generator from fires originating in the surface of the insulation before electrical breakdown occurred.

I believe, however, that in most cases of fire within totally inclosed generators the carbon dioxide could and would be applied much more quickly by means of the electrically operated differential-relay system. Evidently protective equipment of the differential-relay type has prevented many fires even from starting. By opening the armature and field circuits most of the flux in the magnetic circuit is eliminated within one second and this extinguishes the dynamic arc within a second or two, whereas the data which the authors present indicate that it might be two or three minutes after the fire started before the sprinkler heads would operate.

Generators having open or semi-enclosed systems of ventilation which the authors have described do not readily permit of instantaneous flooding of the entire generator with carbon dioxide. Therefore, the detective feature of the sprinkler head and the localized application of water obtained by it are valuable.

We somewhat question the application of radial vanes in the discharge air from fans used in the ventilation of a generator since the vortex angle of the air leaving the fan blades is far from radial and would therefore make the ventilation system less efficient. This would be especially so in the case of high-speed turbine generators in which the ventilation is obtained by means of fans.

J. M. Buswell (communicated after adjournment): I shall be interested indeed in knowing whether the Johnson-Burnham scheme of baffling the stator frame so as to stop the hurricane produces a very noisy machine.

I am also interested in knowing if there have been any modifications to the method of applying those fire baffles and whether or not there has been any modification of suggestion as to the material of which these fire baffles may be made.

I have a further thought that the two- or three-minute period dealt with by Johnson and Burnham, in their paper, is sufficient to burn out the fire baffles themselves and produce a blast-furnace effect and that some more rapid fight against a fire will have to be provided.

It seems that there is some possibility that a modification of the Johnson-Burnham suggestion might work very well. Specifically, my thought is that by tilting the fire baffles slightly in the direction of the air currents thrown off by the rotor, there would not be so much air flutter and eddy; and then by arranging for the quick release of carbon dioxide or carbon tetrachloride in the section in which the fire occurs, we could get a combination faction which might be very effective.

I believe that the damaging fires which have occurred have swept around the armature and even before sweeping around the armature attained at the source a tremendously high heat in just a few seconds, and it is my belief that whatever fire fighting is done must be started immediately after the original arc starts, the first move, of course, being to kill the field and disconnect the armature, and by killing the field I mean not only disconnecting it but actually killing it.

G. A. Fleming (communicated after adjournment): The research work in controling the spread of fire in generators through the proper design of air ducts as described by Messrs. Johnson and Burnham, is one more example of constructive thought in machine design, reducing operating troubles and the additions of undesirable accessories.

It is true that most water-wheel type generators employ an open or semi-enclosed system of ventilation, but two 45,000-kv-a. waterwheel-driven generators are now being installed by the Southern California Edison Company that will have a completely closed ventilating system similar to those used on turbo generators. Better fire protection was one reason for the choice of this system so a few remarks on the company's practise in arranging fire equipment should be of interest as discussion of this paper.

Carbon dioxide is used as the extinguisher medium and is injected as directly as possible into the enclosed air system. Since no damage is done to the generator by the CO_2 it is possible to arrange for its automatic release even with a relatively small increase in air temperature. Also, a scheme by which the differential-relay system for internal trouble will release the CO_2 when the machine is de-energized is being seriously considered for cases where too much outside equipment is not included in the differential loop. One present installation includes this feature but, due to unreliable relays, it has not been put in service.

The authors refer to the fact that the energy stored in the magnetic circuit makes it impossible to de-energize instantaneously a machine even with differential relays and therefore it is impossible to prevent fire with them alone. But since the relays act ahead of the fire, it will surely be of tremendous advantage to have them open the fire extinguishers as well as all electrical circuits and thereby anticipate it as much as possible. It is our practise to arrange for an initial release of CO_2 under atmospheric pressure of approximately 50 per cent the volume of the ventilating system and to follow this with delayed charges at intervals timed to maintain a minimum 25 per cent saturation until there is no danger of the fire spreading or again starting from the rotation of the machine, or otherwise which we believe to be approximately 30 min. for a large generator. In the use of such a system it is of importance to have the ventilating system as tight as possible to prevent the escape of the CO_2 and suction of outside air at points where the pressure is less than atmospheric pressures due to the fan action.

J. A. Johnson: The use of carbon dioxide in this type of machine where large volumes are involved will mean in many cases a change in the usual system of ventilation. A closed system of ventilation or a system of ventilation which can be converted quickly into a closed system appears at present to be necessary for the use of carbon-dioxide extinguishment. Completely open ventilating systems are impracticable in large vertical machines.

As to the question of brakes, with the system of air control described it probably is better to allow the machine to continue to rotate in order to maintain the radial movement of the air, as that in itself has the effect of limiting the spread of the fire. However, these machines are equipped with brakes and can be stopped if desired.

One of the fires shown in the pictures was caused by a fault between strands of one of the end connections which started a small arc inside of the insulation, developed sufficient heat to burn a hole through the insulation and started a fire. I am not sure about the other case.

It is quite true that oily dirt should be removed, as Mr. Riggs suggests. Under Niagara Falls conditions, however, with daily load factors above 90 per cent and even monthly load factors 85 per cent or higher, opportunities for shutting down for cleaning, involving dismantling to the extent necessary in machines of this character, without serious loss of revenue, do not occur as frequently as might be desired. Manufacturers can help by designing oiling systems which will not leak oil, and the baffles described in this paper will undoubtedly assist in depositing suspended oil and dirt before they reach the windings.

Mr. Freiburghouse and Mr. Buswell both question the use of radial vanes, one suspecting reduced efficiency and the other increased noise. Both of these suspicions are apparently based on a misconception as to the location of these vanes. They are not placed immediately in front of the fan blades, but either at some distance away or at the side, and pass only a small part of the air supplied by the fans. There is no particular virtue in *radial* vanes except that they are easy to make and serve well enough. Under certain conditions curved vanes might be better, the criterion being that the resulting air currents through the end windings shall be substantially radial or a combination of radial and axial but in no degree whatever circumferential. The installations described produce no noise and we believe reduce losses by elimination of some unnecessary air movement.

In reply to Mr. Buswell's thought that the fire baffles themselves might be burned out it may be pointed out that the current of air through the baffles will blow the flame directly away from them, and that the air serves to keep the baffles themselves cool. Since the baffles are made of materials which will hardly support combustion, the only danger to them is the general rise of temperature in the compartment which might ultimately reach a value which could damage them. However, long before this condition could become serious the sprinkler heads will blow, the alarm be given, and extinguishing means be brought into play.

E. J. Burnham: The accompanying drawing shows a method of using fusible sprinkler heads in generators having a closed system of ventilation, such sprinkler heads being used for detecting fire and causing, at the same time, automatic release of CO_2 into the ventilating system. Refering to the drawing, notice that air pressure is normally kept on the sprinkler system, and that the operation of a sprinkler head causes a reduction in pressure in the piping system, thereby causing the contacts on the pressure gage to close, which, in turn, operates an electrical release, this causing a weight to drop, that lets the CO_2 into the generator ventilating system. In using this scheme for releasing CO_2, suggestion was made that the generator differential relays be connected so that either the operation of a sprinkler head, or operation of the differential relays would automatically release the CO_2.

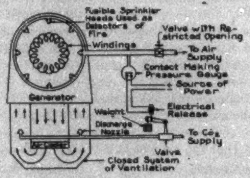
Fusible Sprinkler Heads Used as Detectors of Fire
Valve with Restricted Opening
Windings
To Air Supply
Contact Making Pressure Gauge
+ - Source of Power
Generator
Electrical Release
Weight
Discharge Nozzle
To CO_2 Supply
Valve
Closed System of Ventilation

FIG 1.—DIAGRAM OF FIRE PROTECTION OF GENERATORS

Referring to Mr. Hart's discussion, he states that he prefers the use of a fire-extinguishing gas, to water. Also, Mr. Buswell suggests the possible modification in baffles, with arrangement so that CO_2 may be quickly released in sections where fire is located. I wish to point out that we are not proposing to use the sprinkler method of extinguishing fires where the CO_2 method can be easily applied, but rather, for use with large, vertical, slow-speed generators using open-type ventilation, where the use of CO_2 would be very ineffective.

Mr. Freiburghouse, in his discussion, states that in most cases of fires in enclosed generators, CO_2 would be applied quicker by relays than by sprinkler heads. The sprinkler-head method of releasing CO_2 in generators having a closed system of ventilation is not expected to supersede the method by which the CO_2 is automatically released by the differential relays. It is suggested that the two methods be combined so that CO_2 will be released for surface fires, or other cases in which the relays fail to function before damage becomes excessive.

The Retardation Method of Loss Determination
as Applied to the Large Niagara Falls Generators

BY J. ALLEN JOHNSON[1]
Associate, A. I. E. E.

Synopsis.*—Institute literature is apparently lacking in information regarding retardation tests. The purpose of this paper is to point out certain advantages of the method and to describe the procedure followed in testing the Niagara generators.*

The basis of the method is first given with the fundamental formula upon which it depends. Since it requires the use of the moment of inertia, or "fly-wheel effect," W R², methods of determining this value are given, and it is pointed out that it can be determined by test.

The methods of determining the speed of rotation and its rate of change are given, with a description of the devices used for automatically recording the necessary data and the method of interpreting such data. It is shown how the determination of losses over a range of speeds increases the accuracy of the determinations.

General methods of testing are described, with a connection diagram and an outline of the necessary precautions. The detail procedure for tests for different losses is given.

It is shown how this method of test furnishes data for the separation of the friction from the windage. The division of mechanical losses between generator and turbine in a specific case is also described.

The procedure in determining the electrical losses from the test data is given and some observations made regarding the apparent desirability of the further refinement of the method.

WHILE the retardation method of determining generator losses has long been known and is recognized by the Institute Standards, the Institute records are peculiarly lacking in literature on the subject, a search of the Index having failed to disclose a single paper on, or specific reference to, retardation tests. It seems rather strange that a method of testing which has found its way into the Institute Standards should have received so little attention in its literature. The explanation probably lies in the inherent simplicity of the method and the comparative rarity of its use in past times. It would appear that the method has been, and is yet, considered as a "last resort" method, to be used only when no other method is possible. It is the writer's purpose to show that the method has advantages and virtues of its own, not possessed by the usual and better known methods. It has, in the writer's opinion, never received the attention or publicity it deserves, nor been subjected to the intensive study for the perfection of its technique that is warranted by its possible accuracy and its ease of application.

The writer's attention has been focussed upon this method by the necessity of determining the conventional efficiencies of a considerable number of large water-wheel driven generators. The usual or standard method of determining generator losses is by driving the generator with a motor of known efficiency, and measuring the input to the motor under the various desired conditions. This method is usually impracticable in the case of large water-wheel generators. The most practicable and available method of making loss tests on such installations is by means of the retardation method.

The lack of repute which has apparently been attached to this method is probably due to two factors: first, that it depends upon the fly-wheel effect, $W R^2$, of the rotating parts, the determination of which has been considered to lack accuracy since it has been assumed that this value had to be calculated from the design: second, that the accurate measurement of speed and time are attended with certain difficulties which require the use of more refined methods and instruments than the usual stop-watch and indicating tachometer.

As a matter of fact, the fly-wheel effect can be determined by test; and the rate of retardation can be determined by automatic recording devices far more accurately than by the tachometer and stop-watch. In fact, the method lends itself naturally to automatic graphic recording methods, which may be made of almost any degree of accuracy desired.

This paper, therefore, will describe the procedure followed in the determination of the losses in the large generators of the Niagara Falls Power Company in an attempt to demonstrate thereby the practicability of the retardation method for the determination of the losses in such units.

The Retardation Method of Loss Test. The retardation method is based upon the fundamental law for the kinetic energy of a rotating mass, *i. e.*:

$$E = 1/2 \frac{W R^2 \omega^2}{g} \qquad (1)$$

where E is the energy

$\frac{W}{g}$ is the mass of the rotating body

R is the radius of gyration, and

ω is the angular velocity.

If energy is added, the angular velocity or speed of rotation must increase, and if energy is abstracted, the speed must decrease. Obviously the rate at which the speed decreases must be a function of the rate of abstraction of energy, that is, of the power loss, and will be given mathematically by the first differential of

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Presented at the Regional Meeting of District No. 1, of the A. I. E. E., Niagara Falls, N. Y., May 26-28, 1926.

equation (**1**) with respect to time. The equation for power loss then takes the form:

$$\frac{dE}{dt} = \frac{WR^2 . 2\,\omega\, d\,\omega}{2g \;\; dt} \qquad (\mathbf{2})$$

Putting $\omega = 2\pi s$ (where s is speed rev. per sec.)

$$\frac{dE}{dt} = \frac{WR^2}{2g} . 8\,\pi^2 s . \frac{ds}{dt} \qquad (\mathbf{3})$$

Expressing this equation in convenient units for the purpose of generator-loss measurements, we have:

$$\text{Loss in kw.} = \frac{4\,W R^2 \pi^2 S}{32.2\,(3600 \times 33000 \times 1.34)} . \frac{ds}{dt} \qquad (\mathbf{4})$$

where

W is weight in pounds
R is radius of gyration in ft.
S is rev. per min.
$\frac{ds}{dt}$ is rate of retardation in rev. per min. per min.

32.2 is the acceleration of gravity in ft. per sec. per sec. and the expression in parenthesis in the denominator contains the necessary conversion factors to convert from sec. to min. and from ft-lb. per sec. to kw.

Assuming WR^2, the "fly-wheel effect," to be known, it is then only necessary to determine the speed (S) and the corresponding simultaneous value of the rate of retardation $\left(\frac{ds}{dt}\right)$ to determine the corresponding instantaneous value of power loss.

For convenience it is desirable, for each generator tested, to combine all of the constants into a single constant which may be called the "retardation loss coefficient" and which may be designated by the letter D. The loss in kw. for that particular generator will then be:

$$\text{Loss in kw.} = D\,S\,\frac{ds}{dt} \qquad (\mathbf{5})$$

Determination of WR^2. The usual method of determining WR^2 is by calculation from design, or from design corrected by actual measurement of weights. This is a rather laborious process, but one which can, if necessary, be carried out to almost any desired degree of accuracy. The usual method is to divide the mass up into simple forms, the weights and radii of gyration of which can be easily calculated, the summation of all the parts giving the desired value for the whole. The generator manufacturers are usually required to state this value for use in the governor design, and it is therefore usually available.

The value of WR^2 can, however, be determined by means of a retardation test in connection with the direct measurement of the friction, windage and iron loss by input. In this test the generator is driven as a synchronous motor by a second generator connected to its terminals. Readings of input to the driven machine are made, also of the corresponding values of current, voltage and speed. From these readings, the values of combined friction windage and iron losses are determined. Upon the completion of these tests, the driving generator is disconnected, after raising the speed slightly, and a retardation test carried out while values of voltage, speed and rate of retardation are carefully obtained. By substituting in formula (**4**) above, the corresponding values of loss, speed and rate of retardation obtained from these two tests, the value of WR^2 can be readily calculated. It will usually be found, if the work is carefully performed, that the value of WR^2 so found, will check very closely with that obtained by calculation from design.

Determination of speed (S) *and rate of retardation* $\left(\frac{ds}{dt}\right)$. In the earlier tests, these quantities were measured by means of the tachometer and stop-watch, but it was found by experience that such methods are apt to give inaccurate results, particularly at the higher speeds where the speed is most rapidly changing, due to the inaccuracies of tachometers and the difficulty of accurate timing by such means.

In the later tests, automatic means were employed for recording graphically the quantities from which the speed and rate of retardation are deduced. These means comprised three devices, namely:

1. A contact-making revolution counter
2. A contact-making timer
3. A two-element chronograph

The revolution counter consisted merely in a train of gears arranged to be driven directly by the rotor of the generator under test, and with a contact-making device arranged to make contact every 10 revolutions of the generator. Any other number of revolutions per contact could, of course, be chosen if desired.

The timer used in these tests was one driven by a clock and arranged to make contact once every five sec. omitting a contact once every minute. Any other time interval within reasonable limits could likewise be provided by suitable means.

The chronograph used in these tests was a rather crude device in which the paper used for record was wrapped about a cylinder and glued to make a continuous cylinder of paper. The two pens were mounted upon the armatures of two small solenoids, so arranged that upon making circuit through the coils the pen is moved a millimeter or two, parallel to the axis of the cylinder, returning to its original position upon breaking the circuit. The cylinder is mounted upon a coarse screw thread and driven by a small synchronous motor, and advances along its axis one in. (25.4 mm.) with each revolution which it makes. In the tests to be described, the paper was driven at a speed of about three in. (75 mm.) per min. Experience has shown, however, that a greater speed would be preferable in order to give greater accuracy in determining the generator speed.

Fig. 1 shows, *a* Timer, *b* Chronograph, and *c* Revolution Counter. The chronograph shown is not the one used in the tests herein described, but one built for future use. Fig. 2 is a reproduction of a typical chronograph record of a retardation test.

Interpretation of Chronograph Records. By drawing straight lines across the record at suitable equal time intervals, the number of revolutions occurring in each of such time intervals can be accurately measured, thereby determining the average speed of rotation during each such interval. Tabulating these and taking the differences between successive readings, gives the average rate of retardation occurring between such successive speeds. If the time intervals are taken sufficiently short the average speeds for each interval can, with sufficient accuracy, be taken as occurring at the middle of the intervals, and the corresponding values of $\frac{d\,s}{d\,t}$ as simultaneous with the times dividing the intervals. Also the speed at the division points can with sufficient accuracy be taken as the average of the preceding and succeeding intervals. In illustration, the following tabulation, Table I, is from the chronograph record of a test for friction, windage and iron loss, on a 65,000-kv-a. generator.

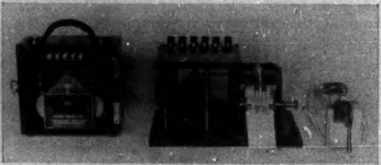

FIG. 1—THE CHRONOGRAPH BY WHICH TESTS WERE MADE

a. Timer
b. Chronograph
c. Revolution counter

The curve shown in Fig. 3 is a plot of these losses against speed, from which the desired value at normal speed can be read. These results are quite consistent, and no great error can be made by this method, since errors in one direction in one or more readings must be compensated by errors in the other direction in succeeding ones. Hence, the individual errors can confidently be eliminated by plotting the results graphically.

TABLE I

FRICTION, WINDAGE AND IRON LOSS BY RETARDATION

Time Minutes	Av. rev. per min.	(S) Rev. per min.	$d\,s/d\,t$ Rev. per min. per min.	Kw. ($0.525\,S\,d\,s/d\,t$)	Remarks
0					
	17.6				
1		111.00	13.2	769	These values of kw. are obtained by using the value of $W\,R^2$ as calculated from design, *i. e.*, 68,255,000 lb.-ft. (Retardation loss coefficient = 0.525)
	104.4				
2		98.40	12.0	620	
	92.4				
3		87.35	10.1	463	
	82.3				
4		77.85	8.9	363	
	73.4				
5		69.35	8.1	295	
	65.3				
6		61.60	7.4	239	
	57.9				
7		54.70	6.4	184	
	51.5				

In the usual application of the retardation method, as set forth in the handbooks, it has been customary to determine $d\,s/d\,t$, the slope of the retardation curve, at normal speed only. This has resulted in a lack of accuracy and is undoubtedly in part responsible for the lack of confidence in the method. By the method of determination here set forth, this inaccuracy is eliminated by the device of using the entire curve to eliminate the errors in individual points, which is particularly applicable in this case since the curve as a whole cannot depart far from the truth.

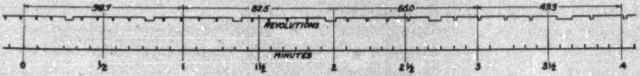

FIG. 2—CHRONOGRAPHIC RECORD OF RETARDATION TEST FOR FRICTION, WINDAGE AND LOAD LOSSES ON 65,000-KV-A. GENERATOR

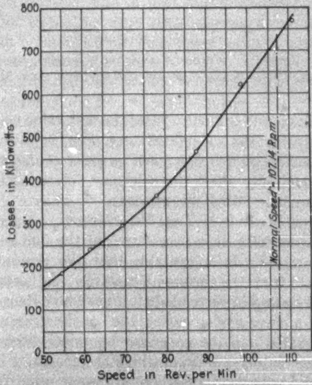

FIG. 3—FRICTION, WINDAGE AND IRON LOSSES OF 65,000-KV-A. UNIT WITH EXCITATION CORRESPONDING TO 11,100 VOLTS AT 107.14 REV. PER MIN.

This same test will illustrate the method, mentioned above, of determining by test the value of $W R^2$. The same machine showed friction, windage and iron losses as measured by input, at normal speed of 107.14 rev. per min., and at the same voltage as obtained at normal speed during the retardation test of 695 kw., see Fig. 4. The value of $d\,s/d\,t$ at the same speed, from the retardation test is 12.99 rev. per min. per min. Therefore, from equation 4:

$$W R^2 = \frac{695 \times 36000 \times 33000 \times 1.34 \times 32.2}{4 \times 9.87 \times 107.14 \times 12.99}$$

$$= 64{,}700{,}000$$

$W R^2$ as calculated from design 68,255,000

Difference 3,555,000

or about 5 per cent. Much closer agreement than this is usually found, in many cases the test value checking exactly with the calculated, see Fig. 6. However, in a machine of this size, even an error of this magnitude could only affect the efficiency determination by less than one-tenth of one per cent.

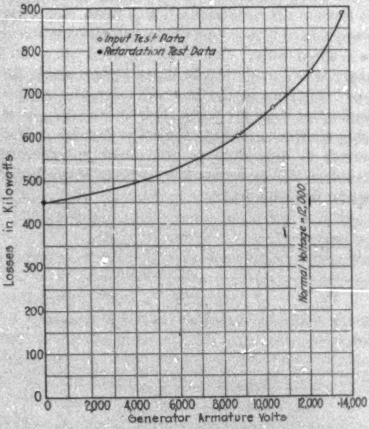

FIG. 4—FRICTION, WINDAGE AND IRON LOSSES OF 65,000-KV-A. UNIT—NORMAL SPEED 107.14 REV. PER MIN.

General Methods of Test. Before starting a series of retardation tests, it is necessary that the unit be properly prepared. It is desirable that the generator be separated from the turbine, if possible. In horizontal-shaft units and in vertical-shaft units where the generator has a lower guide bearing, the coupling can usually be separated and the generator operated without the turbine. In vertical units where the generator has no lower guide bearing, or where, for some other reason, the coupling cannot be separated, it becomes necessary to permit the turbine to rotate with the generator. This introduces losses which cannot be charged to the generator and considerable judgment and ingenuity is then required to apportion the observed losses between generator and turbine.

When it is necessary to include the turbine rotor with the generator, precaution should be taken to guard against injury to the turbine by overheating bearings or glands or runner rim by possible rubbing at the latter point, due to too close clearance. A small amount of water can usually be supplied to such points to keep them cool. In such a case, however, it is essential that conditions of water flow, etc., be kept constant during the tests in order to eliminate any variable factors from this source.

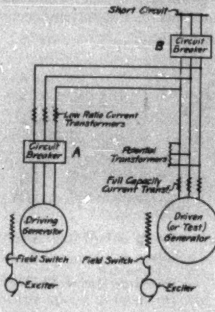

FIG. 5—DIAGRAM OF CONNECTIONS FOR RETARDATION TESTS

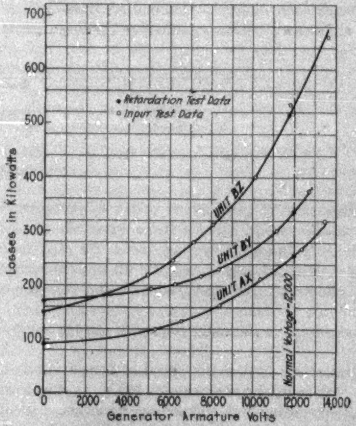

FIG. 6—FRICTION, WINDAGE AND IRON LOSSES OF THREE 32,500-KV-A. UNITS—NORMAL SPEED 150 REV. PER MIN.

The simplest and most practical method of bringing the generator up to speed is to drive it as a motor from another generator of the same voltage. The driving generator can be of considerably smaller capacity than the one to be driven, no difficulty whatever having been experienced with a ratio of ratings of one to four. The driving and driven machines are synchronized from rest in a manner now well understood. Fig. 5 shows the connections as usually employed for all tests.

In the leads from the driving machine there should be an oil circuit breaker, and current transformers. For the latter, a primary ampere capacity of about 6 per cent of the rated current of the driven machine, with 5-ampere secondary, has been found satisfactory. Such a size is large enough to carry the current required to accelerate the driven machine (if not accelerated too rapidly), and is small enough to give good readings on standard 2.5- to 5-ampere ammeters and wattmeters when measuring the input to the driven machine. In any specific case the required size should be calculated from the estimated friction, windage and iron losses. Suitable means should also be provided for measuring voltage and power input. It is assumed that proper calibrations of all instruments and instrument transformers will be taken care of in the standard manner.

It is preferable that the machine under test be separately excited in order to eliminate complications. However, if a direct-connected exciter is used, allowance must, of course, be made for the output and losses thereof.

Procedure. It has been found most convenient to make tests in the following order:

1. Friction, windage and iron loss by input
2. Friction, windage and iron loss (or $W R^2$) by retardation
3. Friction and windage by retardation
4. Friction and windage and load loss by retardation

1. Friction, Windage and Iron Loss by Input.

Close fields of both machines and circuit breaker A (leaving B open,) Fig. 5, and synchronize from rest. Bring machines up to normal speed gradually so as not to dangerously overload current transformers. Raise voltage to 10 or 15 per cent above normal for first reading. Take readings of voltage, current, power input, field current and speed at successive voltages down as low as field controls will permit. Usually readings can be carried down to about half voltage. Take at least 10 readings at each point at 15-sec. intervals. Before each series of readings see that field current of driving machine is adjusted so that armature current is a minimum, with driven machine field adjusted for desired voltage.

2. Friction, Windage and Iron Loss by Retardation.

At completion of input test, as above, raise speed to about 15 to 20 per cent above normal, adjust field current for normal open circuit voltage at normal speed, as per saturation curve, and open circuit breaker A, Fig. 5. As speed decreases record revolutions and time by means of chronograph as described above. Read volts, field current and speed by tachometer, (for check only, if a reliable one is available) at time intervals synchronized with chronograph. Field current should remain constant throughout test. Records should continue until speed has decreased at least to half normal value.

This test can be repeated for as many values of field current or generator voltage as desired. However, a single test at rated voltage will usually be sufficient to check the input test and the value of $W R^2$. Should a good check not be obtained, however, it is desirable to repeat and correct the value of $W R^2$ if the value calculated from design appears to be persistently in error.

3. Friction and Windage by Retardation.

Synchronize from rest and bring to 15 or 20 per cent above normal speed. Open oil circuit breaker between machines and immediately open field switch of test machine. As speed decreases, record revolutions and time on chronograph and read speed on tachometer (if available) at times synchronizing with chronograph record. Record should be continued to well below half speed in order to obtain as much of this curve as possible for a purpose to be explained later. A single test should be sufficient to determine friction and windage.

4. Friction, Windage and Load Losses by Retardation.

With circuit-breaker B open, close circuit breaker A and synchronize from rest as usual. Raise speed to above normal as before. Quickly open circuit-breaker A, open field switch, close circuit-breaker B, close field switch and quickly adjust field current to give desired armature current as per closed circuit saturation curve. These operations can and should be completed before speed has come down to 10 per cent above normal, in order that steady conditions may exist while speed is passing through normal value. Record revolutions and time by chronograph, and make synchronized readings of speed and of field and armature currents. This test should be repeated at various armature currents in order to obtain data for a curve of stray-load loss versus armature current.

Separation and Determination of Friction and Windage. The nature of the friction and windage losses and the consequent laws governing the form of the friction and windage retardation curve furnish a means of separating the "apparent friction" from the "apparent windage" losses.

With a constant coefficient of friction, the friction loss should vary directly as the first power of the speed.

The windage loss, however, in accordance with the well-known laws governing the movement of air, should vary directly as the third power of the speed. It would seem, therefore, that the curve of windage and friction losses against speed, should have the form:

$$\text{Total friction and windage loss} = F S + W S^3 \qquad (6)$$

where F may be called the coefficient of friction loss and W the coefficient of windage loss, S being the speed in rev. per min.

Upon plotting such a curve from the record of a friction and windage retardation test, it is found to conform exactly to this law.

TABLE II
FRICTION AND WINDAGE BY RETARDATION

Time Minutes	Av. rev. per min.	(*S*) Rev. per min.	(ds/dt) Rev. per min. per min.	Loss kw.	Remarks
0					
	115.4				
1		111.25	8.3	485	
	107.1				
2		103.20	7.8	423	
	99.3				
3		96.25	6.1	309	WR^2 (from design) 68,255,000 lb.-ft.^{2} (Retardation loss coefficient = 0.525)
	93.2				
4		90.20	6.0	285	
	87.2				
5		85.50	5.4	240	
	81.8				
6		79.30	5.0	209	
	76.8				
7		74.65	4.3	169	
	72.5				
8		70.55	3.9	145	
	68.6				
9		66.75	3.7	130	
	64.9				
10		63.05	3.7	123	
	61.2				
11		59.95	2.5	78.7	
	58.7				
12		57.00	2.8	84.0	
	55.6				
13		54.45	2.3	66.0	
	53.3				
14		52.15	2.3	63	
	51.0				
15		49.85	2.3	60.2	
	48.7				
16		47.60	2.2	55	
	46.5				
17		45.60	1.8	42.2	
	44.7				
18		43.75	1.9	43 6	
	42.8				
19		42 05	1.5	33.1	
	41.3				
20		40.35	1.9	40.1	
	39.4				

Table II is the data from the graphic record of such a test, and Fig. 7 shows the values plotted from Table II.

The coefficients are readily found by substituting in equation (**6**) the values of total loss and speed for two points of the curve and solving simultaneously. For example, taking the values at 100 and 60 rev. per min. from a trial curve drawn for the test points of Fig. 7 and substituting in equation (**6**) we have the simultaneous equations:

$$372 = 100\,F + 1{,}000{,}000\,W$$

$$94 = 60\,F + 216{,}000\,W$$

Solving for *F* and *W* we have:

$$F = 0.355$$

$$W = 0.0003365$$

Recalculating for successive values of *S*, we have:

TABLE III
FRICTION AND WINDAGE LOSSES BY CALCULATION EQUATION (6)

(*S*) Rev. Per Min.	Friction Loss in Kw.	Windage Loss in Kw.	Total Loss in Kw.
40	14.2	21.6	35.8
50	17.7	42.0	59.7
60	21.3	72.7	94.0
70	24.9	115.5	140.4
80	28.4	172.0	200.4
90	31.9	245.0	276.9
100	35.5	336.5	372.0
110	39.1	347.0	486.1
120	42.6	581.0	623.6

Plotting these values we find the resulting curve, Fig. 7, to correspond with the test values with practical exactness at all speeds. This curve is quite sensitive to small changes in the values of the coefficients of friction and windage losses, and several trials are sometimes necessary before the correct coefficients are found to make the curve fit the test points at all parts of the curve. When this has been done, however, one can be quite confident that the resulting values of total losses are correct, whatever the separate values of "apparent friction" and "apparent windage" may represent, see Fig. 8.

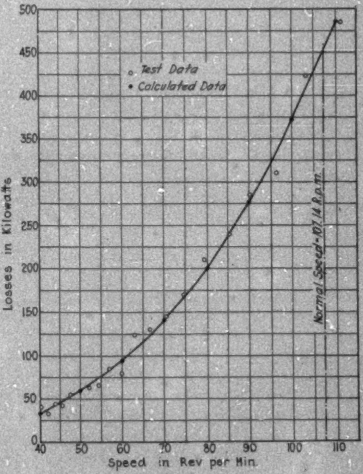

FIG. 7—FRICTION AND WINDAGE LOSSES OF 65,000-KV-A. GENERATOR

It is indeed probable that even in the case of a test upon a generator alone, separated from its turbine, the "apparent friction" loss may include some of the windage and the "apparent windage" some of the bearing friction, since it seems probable that a small part of the bearing losses are of such a nature as to vary with the third power of the speed. However, while there may be some doubt as to the exact composition of the two kinds of losses differentiated in this manner, their separation is nevertheless well worth while for the purpose (if for no other) of detecting the presence of any possible abnormal friction loss, such as a tight bearing or gland, or a rubbing turbine seal, which would surely show up in an abnormally large first power loss.

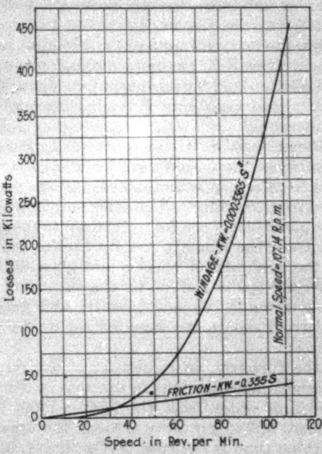

FIG. 8—FRICTION AND WINDAGE LOSSES OF 65,000-KV-A. GENERATOR

However, the author believes, from experience in testing several different types and makes of generators by the above methods, that the separate "apparent friction" and "apparent windage" found by this method are very close to the actual friction and windage losses, when not complicated by other factors such as "disk friction" or "pumpage" losses in the turbine, due to the presence of water for cooling purposes.

Division of Friction and Windage Between Generator and Turbine. In those cases where tests are made with the turbine connected, the friction losses as above determined, will include turbine friction in bearings, glands and seals, and the "apparent windage" loss will include windage in the turbine and probably another loss due to the presence of a small amount of water in the turbine, which may be classed as "disk friction" or "pumpage." These losses can, at present, only be approximated, but by separating the first and third power losses by the method described above, these approximations can be handled with much more confidence than when the mechanical losses are all lumped together.

In an isolated case of a single unit, the division of the mechanical losses between generator and turbine is quite difficult, if not impossible. In a case, however, where several units of similar but different characteristics are being tested, it is often possible, by careful observations and comparisons, to make approximations which give results approaching the truth closely enough for practical purposes.

As an illustration of what may be done along this line, the steps taken in allocating the mechanical losses in three units of 32,500-kv-a. capacity, comprising three different combinations of turbines and generators will be described. If we designate the two turbine manufacturers by the letters *A* and *B*, and the three generator manufacturers by *X*, *Y* and *Z*, then we can designate the three units as *A X*, *B Y* and *B Z*.

The friction and windage losses of these three machines, all of the same capacity rating, and the steps by which they were allocated, are shown in the following tabulation:

TABLE IV
ANALYSIS OF FRICTION AND WINDAGE OF THREE 32,500-KV-A. UNITS

	Unit *A X*	Unit *B Y*	Unit *B Z*
Total Friction, Windage & Pumpage	91.65	171.5	151.05
Friction *H*	18.75	28.0	28.25
Windage and Pumpage	72.90	143.5	122.80
Analysis of Friction			
Weight of Generator Rotor and Shaft *A*	271,000	311,000	318,000
Weight of Turbine Runner and Shaft *B*	73,500	73,500	73,500
Weight on Thrust Bearing during Test *C*	344,500	384,500	391,500
Hydraulic Thrust	100,000	100,000	100,000
Weight on Thrust Bearing under Load *D*	444,500	484,500	491,500
Thrust Friction under Load $\left(\frac{D}{444500} \times 15\right)$ *E*	15.0 kw.	16.35kw.	16.6 kw.
Thrust Friction during Test $\left(\frac{C}{D} \times E\right)$ *F*	11.62	12.95	13.25
Generator Share of *F* $\left(\frac{A}{C} \times F\right)$ *G*	9.15	10.46	10.76
Turbine Share of *F* $\left(\frac{B}{C} \times F\right)$	2.47	2.49	2.49
Total Friction from retardation test *H*	18.75	28.0	28.25
Total Thrust Bearing Friction during test *F*	11.62	12.95	13.25
Other Friction, guide bearings, etc., (*H* − *F*) 1	7.13	15.05	15.00
Generator Share of *I* *J*	2.0	2.0	2.0
Turbine Share of *I*	5.13	13.05	13.0
Total Generator Friction (*G* + *J*)	11.15	12.46	12.76
Analysis of Windage & Pumpage			
Total Windage and Pumpage (Ret. Test)	72.9	143.5	122.8
Generator Windage (Est.)	50.0	93.5	72.8
Turbine Windage (Est.)	10.0	10.0	10.0
Turbine Pumpage (Est.)	12.9	40.0	40.0
Total Generator Friction & Windage	61.15	105.96	85.56

That portion of the loss which is proportional to the third power of the speed was at first thought to be entirely windage, but upon further investigation it was found that there might be a loss in the turbine of the nature generally called disk friction which also varies as the third power of the speed, and, owing to the different construction of the two types of turbine, it is reasonable to expect that under test conditions these losses might differ very markedly between the two types. This presumption is further corroborated by the very great discrepancy between the measured so-called windage losses of *A X* and *B Y* and *B Z*, these losses in the case of *B Y* being just about double the corresponding losses of *A X*, see Fig. 6. It hardly seems reasonable to charge this discrepancy entirely to true windage.

The same sort of reasoning applies also to the friction losses. The thrust and guide bearings of the three units do not differ materially, but there is a considerable difference in the nature and area of the turbine seals between the *A* and *B* designs, and it would, therefore, seem reasonable to expect that that portion of the friction loss which takes place in the seals would be somewhat in proportion to the areas of the contiguous surfaces.

Analysis of Friction Loss. At the time when Unit *A X* was placed in service, the bearing manufacturer's engineers made a determination of the loss in the thrust bearing of this unit and stated that this loss was apparently about 20 h. p., or 15 kw. This value has been used as a basis for the analysis of the friction loss, it being assumed that the loss in these thrust bearings will vary in direct proportion to the weight that they carry. This loss has also been further divided between the generator and turbine in proportion to their weights.

Upon subtracting the value thus determined for the thrust bearing on Unit *A X* from the total friction losses as determined by test, there remains but slightly over seven kw. to be divided between the generator and turbine-guide bearings and turbine seals. In consideration of the smallness of this loss, the nature of the parts which create it, and the relation which might be expected to exist between the generator guide and thrust-bearing losses, it was concluded that two kw. would be a reasonable portion of this loss to assign to the generator-guide bearings, the remainder being charged to the turbine.

In the case of the other two units, the total measured friction losses were considerably greater and the thrust-bearing friction was also somewhat greater, due to the increased weights of these machines over that of Unit *A X*. There appeared, however, no reason why the guide-bearing loss should be any greater than on Unit *A X*, and the assumption checks with the observed fact that the guide-bearing oil temperatures of the three units do not materially differ from each other.

There is then left chargeable to turbine friction approximately 13 kw. in Units *B Y* and *B Z*, or about two and one-half times the corresponding loss in Unit *A X*, which ratio would appear reasonable from a consideration of the two turbine designs. This results in the friction losses of the three generators differing by only such amounts as are accounted for by their different weights.

Analysis of "Windage" Loss. In the case of the so-called windage losses—all those losses varying as the third power of the speed—the total of such losses in the case of Unit *A X* was but 72.9 kw. As stated above, there is too great a discrepancy between this value and those of similar losses determined for the other two units to be all charged to generator windage, and it is accordingly necessary to find some other difference between the two turbine designs to account for a part of this discrepancy. The value of this loss is about 20 kw. greater for Unit *B Y* than for Unit *B Z*. This difference is believed to be reasonably accounted for by the fact that in Unit *B Y* a large volume of air was allowed to discharge around the end windings of the armature, no attempt having been made to limit the discharge at these points to merely that necessary to maintain the temperature within safe limits, whereas, in the case of Unit *B Z* the corresponding openings have been partially blocked for this purpose. The additional air moved in the case of Unit *B Y* might account for the 20-kw. difference.

There then remains a difference of approximately 50 kw. between these losses on Units *A X* and *B Z*. Part of this difference is undoubtedly in the generator and a part in the turbine. The designer of the Generator *X* was particularly careful to decrease to the greatest possible extent the windage loss by careful fan design and by the addition of sheet-metal shrouds on the generator-rotor spider and fan for the purpose of directing the air currents, while apparently such provisions are not carried out to the same extent in the case of Generator *Z*.

As to the turbines, an inspection of the respective runner designs indicates that in the case of Turbine *B* the lubricating water from the lignum-vitae bearing is much more apt to get away through the upper seals of the turbine than is the case of Turbine *A*, and this might readily account for a considerable part of the observed difference in losses.

A formula was given in the water-wheel test code for determining the turbine windage, but in this case this formula gave a value of approximately 50 kw., which, in the case of Unit *A X*, is obviously far too high, as it would leave but about 23 kw. for generator windage. There is, however, undoubtedly some windage loss in the turbines under test conditions, and this was approximated by reference to fan data. It was found that the shut-off power of a fan was in the neighborhood of 20 per cent of the full-load power, and it was assumed that the 50 kw. resulting from the above mentioned formula would represent the full-load power of the turbine runner as a fan, which appeared reasonable from a consideration of its dimensions and in comparison with fan data. It was then assumed that the shut-off power or windage with turbine gates closed would be 20 per cent of this, or 10 kw. This was assumed to be the same for all three turbines. There was then left, in the case of Unit *A X*, 62.9 kw. to be divided between generator windage and turbine disk friction. Of this, 50 kw. was arbitrarily charged to generator windage, as this seemed a reasonable value, and left an amount of 12.9 kw. chargeable to turbine disk friction. In the case of Units *B Y* and *B Z*, it was thought that the disk friction might be approximately three times that of Unit *A X*, and was accordingly put at 40 kw., leaving for generator windage 93.5 kw. in the case of Unit *B Y*, and 72.8 kw. for Unit *B Z*, as compared with the 50 kw. assumed on Unit *A X*. The value of 50 kw. (40 + 10) for turbine windage and disk friction happens to be the same as would have been assigned to turbine windage had the test code formula been used, hence, should not be open to serious criticism.

The final values of generator friction and windage then become for Unit *A X*, 61.15 kw.; Unit *B Y*, 105.96 kw.; and Unit *B Z*, 85.56 kw. We have two checks on these figures; first, the designer of Generator *Z* stated that he estimated the friction and windage of his generator at 80 kw., and second, the designer of Generator *Y*, that of his at about 109 kw. We have a further check on the accuracy of the resulting values of generator efficiency in that the efficiencies of the turbines of Units *B Y* and *B Z*, based on the values of generator efficiency thus arrived at, checked each other exactly.

SEPARATION AND DETERMINATION OF ELECTRICAL LOSSES

Iron Loss. Obviously the iron loss is determined by subtracting the friction and windage from the iron-loss friction and windage as determined either by the input or by the retardation test. Fig. 4 shows iron loss, friction and windage data for the 65,000-kv-a. unit, and Fig. 6 for the three 32,500-kv-a. units. The values of the friction and windage losses, of course, determine the termini of these curves on the zero voltage axis, thereby giving the necessary data for the completion of the lower part of the curves. Fig. 9 shows iron losses alone for the three 32,500-kv-a. generators and Fig. 10 for the 65,000-kv-a. generator.

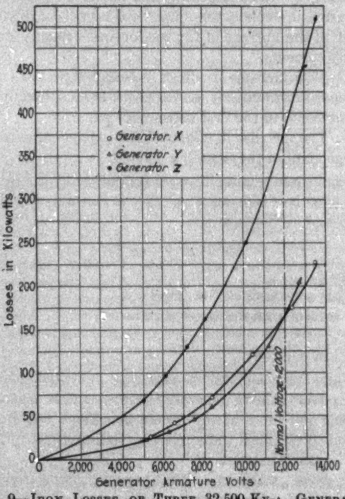

FIG. 9—IRON LOSSES OF THREE 32,500-KV-A. GENERATORS

Stray-Load Losses. Similarly the stray-load loss is determined by subtracting the friction and windage and armature $I^2 R$ loss from the appropriate test values. This has been done for the 65,000-kv-a. generator; the various steps by which the stray-load loss has been determined are indicated in Fig. 11.

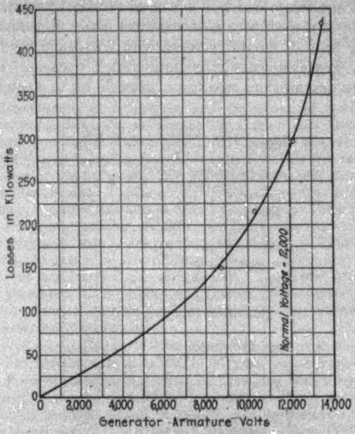

FIG. 10—IRON LOSSES OF 65,000-KV-A. GENERATOR

Following the same general principles used in determining the friction and windage, and the iron loss, the test points are first determined and plotted for the delineation of a curve of loss against speed as obtained by the retardation test. In Fig. 11 these test points for armature currents of 1554 amperes, 2145 amperes, and 3156 amperes are shown respectively at *a*, *b* and *c*.

These points do not look very encouraging as almost any kind of a curve can, seemingly, be drawn through them. However, in delineating these curves, we are entitled to use all the information which we have, so, before drawing in the curves, it may be indicated on the sheet just what is known about the losses which are to be deducted. We, therefore, draw in the friction and windage loss curve, as per Fig. 7.

As to the $I^2 R$ loss, it is known both from theory and observation, that the short-circuit current remains practically constant at all speeds, hence, the $I^2 R$ loss is constant throughout the test. It is possible, therefore, to add a constant $I^2 R$ loss to the friction and windage-loss curve, obtaining the corresponding curves shown in Fig. 11.

The difference between this curve and the total-loss curve is the "stray-load" loss, and there should be some consistent relationship between the stray-load loss and speed. Three assumptions are possible:

1. Stray-load loss decreases as speed increases,
2. Stray-load loss constant as speed increases,
3. Stray-load loss increases as speed increases.

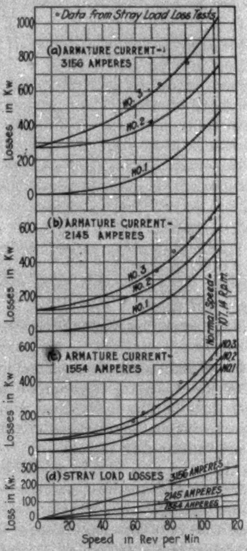

FIG. 11—RETARDATION TEST FOR STRAY LOAD LOSSES OF 65,000-KV-A. GENERATOR

Curve No. 1—Friction and windage losses
No. 2—Curve No. 1 plus $I^2 R$ losses
No. 3—Curve No. 2 plus stray load losses

If the stray-load loss is caused mostly by eddy currents in the copper, as seems most likely, it would appear that it should increase with frequency, and the test points indicate in general that it does so. If so, three assumptions are again possible as to a possible exponent of the power of the speed by which it varies, that is, if it is assumed that:

$$\text{Stray-load loss} = C\, S^n \qquad (7)$$

where C is a constant,

n may be

1. less than unity
2. equal to unity
3. greater than unity

In order to determine which of these assumptions most nearly agrees with the test data, trial curves were drawn and the speed-time curve reconstructed from points taken from the trial curves. The reconstructed curve based upon the assumption of $n = 1$ shows the better agreement, and this assumption was consequently used in drawing the final curve through the test points. The differences, however, between the results of the three assumptions are very slight, and none of them are probably scientifically true.

In drawing the curves, a point was selected on each of the trial curves through which it appeared the curve would have to pass whatever its form. From these values, the corresponding friction, windage and iron losses were deducted, the residual values being plotted on a rectangular scale and straight lines drawn through these points and the origin. These three lines, shown at *d* Fig. 11, are then the most probable curves of stray-load losses for different speeds at the three values of armature current. These three curves were then added to the friction, windage and $I^2 R$ loss curves to produce the total-loss curves as shown. It was found that the stray-load loss at any given speed varied almost exactly as the square of the current, the chance agreement of the trial points in this respect being so close that the final curves were intentionally so drawn.

These results would *seem* to indicate the following conclusions:

1. The short-circuit stray-load losses are due (at least mostly) to an increase in the effective resistance of the armature conductors, due, no doubt, to a displacement of the current in the conductors—usually called "eddy currents."
2. This increase in effective resistance varies with the speed or frequency, and appears to be substantially proportional to the first power thereof.
3. The increase in effective resistance is apparently independent of the armature current.

From these results, it would appear that, practically, the effective resistance of the armature would be expressed by the formula:

$$R_f = R\,(1 + K f) \qquad (8)$$

where

R_f is resistance at frequency f

R is resistance for direct current

and

K is a constant.

While these conclusions *seem to be* indicated, it is, of course, not safe to generalize from a single test, and further and more accurate determinations will be required to establish these relations definitely, if indeed, they actually exist at all.

However, in this case, it is apparently true that, for all practical purposes, the determination of the short-circuit stray-load loss for any current and any speed may be made from a single accurate test at one current and one speed. If formula (8), or one similar to it, should prove to be generally true, it would appear that some independent method of predetermining the coefficient (K) would be extremely useful.

CONCLUSIONS

1. The retardation method for determining certain of the losses in large generators is a useful, accurate, consistent and easily applied method, and deserves more extensive use and fuller official recognition.

2. The retardation method of measuring friction and windage losses contains within itself the means of separating these losses from each other. This is a very useful property when it is necessary to allocate such losses between the generator and its turbine. It is also useful for detecting abnormal friction or windage.

3. The determination of a loss at different speeds, which is a natural application of the retardation method, furnishes a means not only of more accurately determining such loss at normal speed, but also furnishes a means of throwing new light upon the nature of certain of the losses.

4. Development of apparatus and refinement of methods of carrying out retardation tests are desirable. An instrument to measure the rate of retardation directly, should be a possibility not difficult to realize.

Discussion

R. B. Williamson: The testing of large generating units after they have been installed in the power station is becoming more common than was formerly the case because the great increase in size has rendered satisfactory factory testing very costly; in many cases it is, in fact, impracticable because the machines are not completely wound and assembled at the factory. There is no doubt that a great deal of money has been expended in the past on unsatisfactory factory tests that could have been made to much greater advantage after installation. Any methods of testing, therefore, that are of advantage when machines have been set up ready for use are worthy of very careful study and we are much indebted to Mr. Johnson for the present paper which shows the application of the retardation method to some of the largest hydroelectric units so far built.

In factory testing the usual method of determining losses is by means of a calibrated driving motor, but this is often difficult to apply to large vertical units. When using a driving motor it is frequently difficult to hold the source of power steady enough to avoid periodic swings in the power input to the motor and this affects the accuracy of the readings. When large machines are tested in the power house by measuring the input with the generator operating as a motor supplied from a second water-wheel unit, the source of power is much steadier and correspondingly better results can be obtained. Even here, however, difficulties are sometimes met due to periodic variations caused by slight hunting of the water-wheel governor. One great advantage of the retardation method is that, after the machine has been run up to or slightly over speed, the driving power is cut off and all fluctuations due to variations in driving power are also cut off.

In applying the method to water-wheel units where it is necessary to leave the wheel connected and thus rotate the runner in the water-wheel casing, it is important that the casing be drained and also opened by removing the manhole covers. I recall one instance where a retardation test was being made and unaccountable variations were found in the speed. The casing had presumably been drained but the vents had not been opened and it was found that a considerable quantity of water was being thrown around in the casing. The manhole covers were then removed, the casing completely drained well below the level of the runner, and there was no further trouble.

As pointed out by Mr. Johnson, it is not essential that the fly-wheel effect of the rotating parts be known in advance provided means are at hand for measuring the kilowatts required to drive the machine as a motor at known speed. If the loss is thus measured, the fly-wheel effect for use in the other tests can then be calculated with a sufficient degree of accuracy. Usually, however, there is no difficulty in obtaining a close estimate of the fly-wheel effect from the manufacturer and the retardation tests are sufficient to obtain the losses by the methods described.

The analysis of the friction and windage of the three 32,500-kv-a. units in Table IV is of special interest. Unit *A X* as mentioned is equipped with shrouded steel-plate fans with curved blades, and baffles are provided to prevent eddies and short-circuiting of air around the fan. On account of the limitations imposed by the generator design, these fans have an efficiency considerably lower than that of regular blowers provided with a spiral casing, but at the same time it is true that a generator of this type with such fans and baffles will show in general less windage loss than one where the air is free to eddy around. Where the peripheral speed of the rotor is fairly high, as in the present instance, the difference in windage loss may be considerably in favor of the machine with fans as in Table IV, whereas the general feeling of many users is just the opposite, *i. e.*, that fans increase the windage loss. The spider arms of large generators stir up vigorous air currents which involve loss in power; hence shrouding the arms to cut off these currents is frequently well worth while.

It is of special interest to the designer to note that these tests indicate that the stray-load loss at a given speed varied as the square of the current. This, I believe, has been generally assumed to be the case and tests on many machines made by driving by means of a calibrated motor show this relation to be quite closely true. It is important, however, to have this confirmed by tests made on such a large generator and by a different method. The greater part of the stray loss is undoubtedly due to so-called eddy currents in the windings, but a considerable part is also due to stray induced currents in various parts of the machine structure caused by stray fluxes from the windings. It is fortunate, therefore, that notwithstanding the complicated make-up of the stray loss, it is found to be practically proportional to the square of the current.

We fully agree with Mr. Johnson that with the use of suitable instruments, such as he describes, the retardation method can be made of much more use than has been the case in the past. We also feel sure that with the more extended use of methods such as these, tests of machines after installation will supersede many of the factory tests now made on large or even moderate-sized units, because such tests will be less expensive and very much more satisfactory.

E. M. Wood and G. D. Floyd: At the Queenston plant of the Hydro-Electric Commission, we have had the same problem of determining the conventional efficiency of large water-wheel driven alternators, and have in general solved it in a manner similar to that described by Mr. Johnson. Some of our conditions have been different from those described and have given rise to variants in the procedure, some of which may be interesting.

All our tests were made with the turbine uncoupled from the unit under test, as the units have two guide bearings. The units were started as described in the paper. Field current for approximately normal voltage at no load was required on each of two similar units for sure starting. Starting currents at times swung to ½ rated current of one generator of 55,000-kv-a. rating.

Manufacturers' calculations of $W R^2$ were taken but were checked by wattmeter measurement of input to the unit as synchronous motor, whenever the governor of the driving unit would hold speed sufficiently steady to give reliable readings. The calculation checked closely with the test values.

When conditions permitted this test, however, a complete curve of losses by wattmeter was taken down to approximately half voltage. Under proper conditions, these results are possibly more dependable than those from the retardation test but in most cases (but not all) the governor allowed the speed to hunt so that the results were of doubtful value.

The current transformers used were rated at about four per cent of the current rating of the generator and were connected across the blades of gang-operated disconnecting switches to protect them during starting and during changes of test conditions. A man was stationed at the switch with an ammeter and given instructions to close the disconnects quickly in case of a sudden rise of current.

On the first units tested, retardation tests were made using a stop watch and hand tachometer, varied by use of a tachograph and in some cases by use of a high-speed graphic voltmeter for the core-loss curves. We found these not very satisfactory for the reasons stated in the paper.

On the last three units tested, a Cambridge chronograph adapted by Messrs. Borden and Floyd was used. Two elements record on a tape the following:

1. One jog for each half-revolution of the generator.
2. One jog for each revolution of a machine running at synchronous speed (187½ jogs per minute).

From the tapes, speed-time curves were plotted, the rate of retardation at synchronous speed was found and the losses calculated. The average loss between one per cent above and one per cent below synchronous speed was calculated directly in most cases, as the average speed-time curve had been obtained very accurately between 110 per cent and 90 per cent of synchronous speed. In those cases where a curve of loss against speed was obtained, the loss at synchronous speed checked the calculated loss very closely.

The method of reading the record, tabulating the results and calculating the losses given in the paper is very neat and simple. The long intervals were apparently allowable due to the high fly-wheel effect and slow retardation. In our case, the intervals had to be much shorter. This could be varied by using longer overlapping intervals.

Curves of friction, windage and short-circuit losses were taken on one of our units by the retardation method with good results. The switching incident to this test is somewhat involved and must be carried out in a very short time, requiring careful preparation. In this test, it is of importance to observe the temperature of the armature windings.

Mr. Johnson's practise of running the retardation test to below 50 per cent of rated speed is new, and he has used it to obtain much interesting information.

V. Karapetoff: I am glad to hear a practical operating man recommend the retardation method because I have been urging its use for over 20 years. I learned about it in Europe where it has been used much more than here. The principal reason why, perhaps, it has not been used so much is that with small machines the rotating part stops too rapidly to allow an accurate measurement of retardation, and the second reason is the necessity for knowing the moment of inertia.

Of course these difficulties are not insuperable and those interested in this method will find both in the Continental and in the British literature, quite a number of ingenious and indirect methods of measuring retardation and the moment of inertia. The general theory will be found in my *Experimental Electrical Engineering*, Third Edition, Vol. I, pp. 408-416.

Mr. Johnson suggests that a convenient and simple method be developed for directly measuring the rate of retardation. I wish to mention a condenser device which was proposed quite a number of years ago for measuring acceleration and retardation of railway trains, and which, with modern measuring instruments, should be satisfactory for the purpose desired. Consider a capacitor connected to a source of direct voltage. Then we have the relation: $q = C\,e$, where q is the charge on the condenser, C is its capacitance, and e is the applied voltage. Taking a derivative of both sides of this equation with respect to time, we get $d\,q/d\,t = C\,d\,e/d\,t$. Call this current i; then $i = C\,d\,e/d\,t$.

Suppose that you have some device, such as a d-c. magneto generator, belted or otherwise connected to the generator under test, and let the induced voltage of this device be proportional to its instantaneous speed. Then $e = k\,v$, where v is the instantaneous velocity, and k a coefficient of proportionality. Substituting this value of e in the equation for the condenser current, we find that $i = C\,k\,(d\,v/d\,t)$.

The value of $d\,v/d\,t$ is the retardation of the machine, and the current flowing into or out of the condenser is proportional to it. In other words, if you have a magneto in series with a galvanometer and a condenser, the instantaneous indication of the galvanometer is a direct measure for the instantaneous retardation of the machine.

W. J. Foster: Mr. Johnson has described the retardation test and he has developed it, I think, to a higher degree of perfection than any other person.

There are three methods of testing large generators: the motor method, the retardation and the calorimetric. The first that I had experience with was the motor method and fairly good results were obtained, but it failed completely in the matter of the load losses or short-circuit losses. Since those losses have become very important the motoring method by itself is now obsolete.

The retardation method was developed and practised several years ago quite extensively in the shops of the General Electric Company. It was found to be a very convenient method when the speed could be determined with accuracy and the $W\,R^2$ from calculation, but when the rotor contained castings there were always some uncertainties about the $W\,R^2$.

In our work we sometimes found it well to obtain the speed electrically, which is especially easy where there are direct-connected exciters and a battery is used to excite the field so as to hold the excitation perfectly constant.

I regard the calorimetric method, however, as the coming method in most machines. Our hydraulic generators are going to become more and more totally enclosed and it is not in the far distant future when we will have the closed system, the same air being returned over and over again. So I think that the most accurate method and the method by which we will obtain the real efficiency will be the calorimetric method.

I say the real efficiency because the matter of taking the short-circuited losses as the load losses is an approximation to accuracy. It has been standardized and when it is used by the designers in calculations and in giving guarantees it, of course, should be measured in the test, and the total losses should include for load losses the short-circuited losses. But by the calorimetric method the actual load losses are obtained and that is extremely important.

I think that in such generators as Mr. Johnson has worked with, where he has housings around them, he can very well check up the results obtained by the retardation method. I wish he would do so as a supplement before this paper is published in the A. I. E. E. TRANSACTIONS. It wouldn't take over two days on one of those machines to do all of it. He should vary the power factor of the machine under test from unity down to 80 per cent, and thus be able to furnish extremely valuable data for designers as well as for users.

I can't speak in too high terms of the work done by Mr. Johnson. I have been in contact with it somewhat and he certainly has used his brains.

It was just a trifle amusing to me to notice his remarks with reference to the unusual opportunities that he had in having three generators of different design and also some combinations of hydraulic turbines to work with.

No doubt the engineers of all of the manufacturing companies will agree that each has some compensation for the disappointment he experienced in failing to receive the order for all three units,—in the excellent test data and the improved retardation method furnished by Mr. Johnson.

Mr. Johnson discusses the effective resistance and gives formula (8). I wish to remind you that there are other losses, eddy-current losses in the magnetic material, and what is needed now is for data to be collected that will look towards the determination of the effects of end magnetization.

P. A. Borden: It is particularly interesting to learn that in his tests of generators, Mr. Johnson has been making use of the chronograph for the precise determination of speeds through a cycle of velocity values. A method almost identical with Mr. Johnson's was developed independently by the Hydro-Electric power Commission of Ontario, and since then has been employed in all its tests of large units where there was required a permanent and accurate record of all speed values during the time of observation.

L. A. Doggett: (communicated after adjournment): In connection with the study of the losses of induction motors supplied with non-sinusoidal electromotive forces, some retardation runs have just recently been made at the Pennsylvania State College. These runs were on machines of 5 to 20 h. p. Although these machines are pigmies compared to Mr. Johnson's 65,000-kv-a. unit, it would seem worth while to have on record some of the problems occurring at the other end of the scale of sizes.

First, as to speed measurements, it soon developed that the attachment of a tachometer, whether of the mechanical or electrical type, so altered the friction that the retardation curve was materially affected. For example, where it took 52 sec. to reach zero speed without the tachometer, it took 37 sec. to reach zero speed when a tachometer of the electric type was applied to the end of the shaft throughout the run, all other conditions remaining the same. To avoid this error a stroboscopic method was used, in which an arc was supplied by a current of known frequency. The light from this arc was directed upon a disk attached to the shaft. On this disk were arranged in the usual way circular rows of spots at uniform intervals, such as 45 deg. and 30 deg. The time was noted when these spots reached apparent rest. With this arrangement very satisfactory speed-time curves were obtained.

Second, as to $W R^2$ measurement, this value was determined in two ways. The first is as described by Mr. Johnson and involved a separate measurement of the friction and windage at a definite speed. This in conjunction with the retardation run gives a value for $W R^2$, which for one five-h. p. induction motor was 2.36 lb-ft^2. A second value was determined by another method. In this method the rotor was removed from the stator and suspended vertically as a torsion pendulum and in this condition its time of swing was determined. The time of swing was again determined when an additional known $W R^2$ was attached to the rotor. From these data the $W R^2$ came out 2.26 lb-ft^2. The torsion pendulum method is considered the more reliable.

J. Allen Johnson: Several speakers have made suggestions as to the technique of the tests such as the necessity for draining the wheel case and opening vents, the use of disconnecting switches to short circuit the small current transformers during adjustments, etc. These are desirable precautions. Messrs. Wood & Floyd have used a generator running at synchronous speed to measure time. It would seem to the author that much more accurate means of measuring time are available in the form of clock mechanisms.

The electrical accelerometer outlined by Professor Karapetoff is extremely interesting and if it could be worked out into an instrument suitable for field use might be of value. At present it appears to the author as a scheme which might readily be used in the laboratory but hardly a tool for powerhouse use. It has, however, interesting possibilities, and the author looks forward to the time when the inherent merits of the retardation method will have created sufficient demand for such an instrument to justify some manufacturer in undertaking its development.

Anyone who has carried out retardation tests and has observed the almost majestic steadiness with which these huge machines slow down under the influence of their own losses, completely unaffected by any outside influence, will share with him, the author believes, not only his conviction that in this method we have something elemental in its simplicity and trustworthiness, but also his desire for a means of making the necessary measurements of speed and rate of retardation as simple and direct and elemental as the method itself. Such a means seems to be provided in the use of a chronograph, revolution counter and timer. The author doubts if he could have equal confidence in the results obtained by any method less simple and direct, such, for instance, as electrical tachometers or accelerometers involving the accurate knowledge of electrical constants and the necessity of accurate calibration.

Mr. Foster calls attention quite rightly to the fact that in measuring the short-circuit losses we are only making an approximation to the actual stray-load losses. The author frankly recognized this fact in stating in this paper that these tests were to determine the "conventional" efficiencies of the machines in question.

Mr. Foster proposes the calorimetric test to determine the *actual* load losses. The author sincerely hopes that successful results may be obtained in this manner, but confesses to some skepticism in the matter, when recalling a test of this character, in which he assisted, made on a 7500-kw. generator in 1905 or 1906. The losses by radiation, the difficulties in determining accurately the average temperature of large volumes of air and of determining the mass and velocity of the air are a few of the practical obstacles which the author sees in applying the calorimetric method. He agrees with Mr. Foster, however, that no other method of determining the actual stray losses seems possible, and the desirability of finding out what these losses actually are probably justifies considerable trouble in perfecting the calorimetric method. He is of the opinion, however, that nearer two months than two days would be required to get any worthwhile results on the large generators by this method.

Mr. Doggett's method of determining $W R^2$ by using the rotor as a torsional pendulum is very interesting. To apply this method to a rotor weighing 400 tons, however, would seem to invoke some practical difficulties.

Some Notes on Electricity Transmission and Distribution Practise in Europe with Comments on High-Tension Substations and Switchgear

BY G. F. CHELLIS[1]
Member, A. I. E. E.

***Synopsis.**—This paper describes some of the more recent overhead and underground electricity transmission developments in Europe and is intended to convey a general idea of the status of that art in some of the European countries.*

No attempt has been made to include all the recent high-tension lines in the countries touched upon, but those included are typical of the work being done and the voltages in use.

A most interesting part of the paper is a summary and analysis of the recent report of the Weir Commission of England recommending a policy with regard to superpower development in that country. This report covers a program extending to 1940 and proposes a plan of interconnection and base-load plant construction intended to tie together practically all parts of England.

* * * * *

GENERAL

A CONSIDERABLE amount of electrical plant construction has been going on in Europe during the last five years to meet increases in power demand, resulting principally from industrial growth.

Water-power has been developed wherever possible, but regardless of high fuel cost and uncertainty as to fuel supply, steam plants have been necessary in certain areas.

Recent statistics indicate that the electricity consumption per capita in some of the European countries is higher than in the United States, but such statistics are confusing and do not present a true picture of electrical progress, except where industrial and domestic consumption are separately shown. This point is clearly brought out by considering the total unit consumption in certain mining and industrial communities where it amounts to 50,000 watts or more per capita, but with relatively low domestic consumption.

A good deal of work must be done to educate the public to a greater use of electrical energy, and growth of domestic load in Europe should, therefore, be slower than in American communities which are being energetically exploited.

Although there are wide variations in climatic and topographical conditions, government regulations, and engineering points of view in Europe, there seems to have been produced in the different countries, types of line construction well suited to local conditions.

There is considerable diversity in the design of transmission line supports which, in general, are somewhat more flexible than the usual American designs, while spans are shorter, with relatively higher safety factors in conductors.

SWEDEN

As the distances over which power has to be transmitted in the European countries are relatively shorter, such extensive transmission line networks as those which have been developed in the United States, and Canada, are not found.

The Swedish State Lines, planned by the Royal Board of Water Falls, are perhaps an exception to this general statement, as these lines form the nucleus of an extensive future superpower system designed to work at

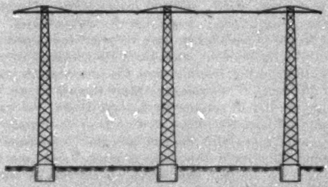

FIG. 1—TYPE OF STANDARD STEEL SUPPORT FOR TWO 200-KV., THREE-PHASE CIRCUITS

(TROLLHATTAN-VASTERAS LINE OF SWEDISH STATE POWER SYSTEM)

Height of support above ground 52.5 ft. (16 m.) approx.
Distance between mast centers 39.4 ft. (12 m.)
Conductor spacing 19.7 ft. (6 m.)

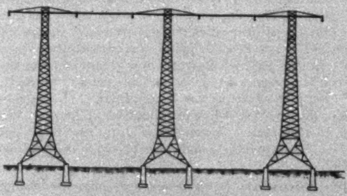

FIG. 2—TYPE OF STEEL ANCHOR STRUCTURE FOR TWO 220-KV., THREE-PHASE CIRCUITS

(TROLLHATTAN-VASTERAS LINE OF SWEDISH STATE POWER SYSTEM)

Height of support above ground 52.5 ft. (16 m.) approx.
Distance between mast centers 39.4 ft. (12 m.)
Conductor spacing 19.7 ft. (6 m.)

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[1] Whitehall Securities Corporation, Ltd., London, England.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Niagara Falls, N. Y., May 26-28, 1926.

220 kv.; but those built to date will be operated at a reduced voltage until the higher voltage is required.

Two types of towers used in the construction of the Western Trunk Line of the Swedish State System between Trollhattan and Vasteras are shown in Figs. 1 and 2.

ITALY

Italy shares with Switzerland the run-off from the Swiss-Italian Alps, which provides an abundant source of water-power, and here, as well as in Switzerland, rapid progress has been made in hydroelectric development and transmission lines.

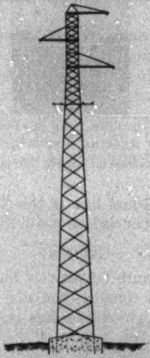

FIG. 3—TYPE OF STANDARD STEEL TOWER FOR ONE 125-KV., THREE-PHASE CIRCUIT

(TEMU-CEDEGOLO LINE OF THE SOC. GEN. ELLETTA DELL 'ADAMELLO)

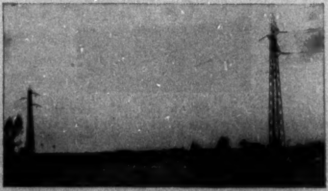

FIG. 4—TYPE OF STANDARD STEEL TOWER FOR ONE 130-KV., THREE-PHASE CIRCUIT

(REGGIO-EMILIA LINE OF THE SOC. ELETTA. INTERREGUIALE CISALPINA)

Height to lowest cross arm	69 ft. (21 m.)
Average span	820 ft. (250 m.)
Conductor spacing	17.5 ft. (5.3 m.)
Weight of tower	4800 lb. (2200 kilos)

Because of the rugged topography of Northern Italy, transmission lines must be largely confined to the narrow valleys, in some of which there are as many as ten parallel multicircuit lines of voltages ranging from 2 kv. to 130 kv.

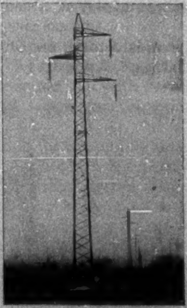

FIG. 5—TYPE OF STANDARD STEEL TOWER FOR ONE 140-KV., THREE-PHASE CIRCUIT

(OVESCA-ARQUATA LINE OF THE SOC. GEN. ITALIANA "EDISON DI ELETTA" MILAN)

Average span 656 ft. (200 m.)

The types of tower used on three of the more interesting lines recently built in Italy are shown in Figs. 3, 4, and 5.

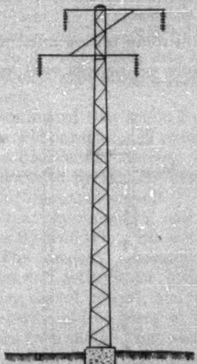

FIG. 6—TYPE OF STANDARD STEEL TOWER FOR TWO 132-KV., 16-2/3-CYCLE, SINGLE-PHASE CIRCUITS

Length of line	134 mi. (216 km.)
Height to lowest conductor	28 ft. (8.5 m.)
Average span	737 ft. (216 m.)

Conductors aluminum steel reinforced.
Insulators, suspension type, seven units.
Ground wire, copper clad steel.

SWITZERLAND

The most important recently constructed transmission lines in Switzerland have been built by the Swiss Federal Railways, in conjunction with the electrification of that system. Certain of these lines are jointly owned by the Federal Railways and the Swiss Power Transmission Co., Ltd., of Berne.

These lines are of interest on account of the high single-phase voltages employed, which are 66 kv. and 132 kv.

The types of standard tower used on three of these lines are shown in Figs. 6, 7, and 8.

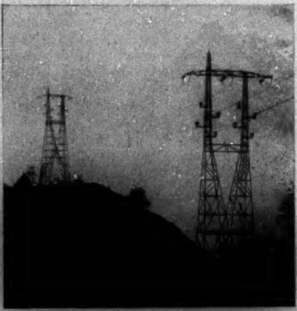

Fig. 7—Type of Standard Steel Tower for One 135-Kv., Three-Phase, 50-Cycle Circuit and Four 66-Kv., Single-Phase 16-2/3-Cycle Circuits

(Amsteg-Immensee Line of the Swiss Federal Railways)

Length of line 29 mi. (47 km.)
Average span 787 ft. (240 m.)
Maximum span 1610 ft. (490 m.) with 620 ft. (190 m.) difference in elevation of supports
Conductors aluminum steel reinforced except bronze on long spans
Ground wire galvanized steel

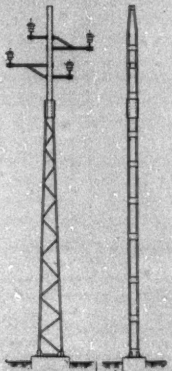

Fig. 8—Type of Steel Tower Used on Standard Transmission Lines for Two 66-Kv., Single-Phase, 16-2/3-Cycle Circuits

(Swiss Federal Railways)

Length constructed, 155 mi. (250 km.)
Average span, 410 ft. (125 m.)
Conductors, stranded copper
Ground wire, galvanized steel

SPAIN

Concrete poles, Fig. 9, with concrete cross-arms cast integral are used on the 70-kv. lines of the Cia Anonima Mengemor, Spain.

A photograph of one of the anchor towers of the 130-kv. Albaseta - Dos Aquas line of the Sociedad Anonima Hidroelectrica Espanola is shown in Fig. 10.

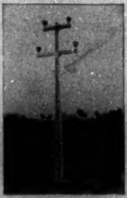

Fig. 9—Type of Concrete Line Support on 70-Kv. Lines of Cia Anonima Mengemor, Spain

Average span 328 ft. (100 m.)

GERMANY

The 100-kv. Lauta-Trattendorf line in Germany is worthy of serious consideration as a precedent in the construction of high-tension lines on public highways.

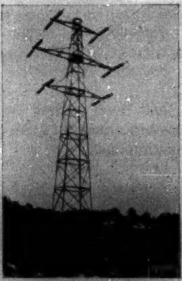

Fig. 10—Type of Steel Anchor Tower on 130-Kv. Albaseta-Dos Aquas Line of the Sociedad Anonima Hydroelectrica Esponola

Average span 656 ft. (200 m.)

While few municipal authorities would sanction the construction of lines of such voltages in the public street, the factor of safety is no doubt higher than that common to overhead contact wires for tramway systems.

A typical view of this line is shown in Fig. 11.

ENGLAND

In England, with the exception of one group of companies in the North, which operates about 1000 mi. of high-tension lines, extensive overhead transmission systems have not been developed. This is perhaps due to the wide distribution of its coal deposits which renders transmission less effective than in places where power markets are widely separated from sources of fuel.

A further explanation of the relatively few overhead lines constructed in England is to be found in the well-established practise of placing power lines underground, which has to a large extent been forced by strict Government and Municipal regulations, and by the difficulty and delay experienced in securing suitable rights of way.

Numerous attempts have been made in England to establish a policy with regard to national superpower developments, beginning as early as 1914. The principal schemes have been included in reports made in 1918 by the Electric Power Supply Committee (known as "The Williamson Report") and the report of the Commission appointed by the Minister of Transport to review the national problem of the supply of electric energy (known as The Weir Report) which was published early in the year 1926.

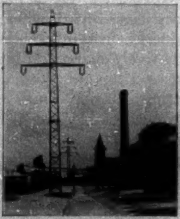

Fig. 11—Type of Steel Support on 100-Kv. Lauta-Trattendorf-Moabit Line in Germany

Height of tower, 92 ft. (28 m.)
Height to lowest cross-arm 65 ft. (19.8 m.)
Conductor spacing (3.1 m.)
Conductors—copper
Insulators, two strings of six Kapp units
Minimum clearance between line and buildings (26 m.)

The scope of the report covers a period to the year 1940 when the consumption in Great Britain, now growing at the rate of 19 per cent per annum, would be raised from the present figure of 110 units per capita to 500 units, the rate now obtaining in the United States; but it is expected that the 500-unit figure may be reached before that date.

The basic recommendations of the Committee are as follows:

a. To reduce to 58 in number 438 of the power stations in Great Britain, many of which are small and unsuited to generate power economically. The 58 stations, 28 of which would be of capital order and 30 of secondary order, would be designed to produce electrical energy at a minimum cost. (These 438 stations do not include 48 stations owned by railway companies, and operated particularly for railway electrification.)

b. To establish a gridiron (high-tension transmission network) for the purposes of interconnecting the 58 power stations.

c. Provision for the output of the 58 power stations to be turned into the gridiron at a predetermined price per unit with privilege of purchasing for local supply, at the price billed to the gridiron.

d. Provision for the closing down of nearly 400 generating plants which is expected to be brought about automatically as the companies operating them would be able to purchase power from the gridiron at a cost less than that of private generation.

e. Provision for the establishment in communities, not at present supplied with electricity, for local authorities, who would distribute power purchased from the gridiron in such areas.

f. Recommendations in favor of the standardization of frequency affecting a few of the larger suppliers. Subject to further investigations this may be undertaken at public expense at a total gross cost approximately £10,500,000, and a net cost of £8,000,000 after allowing for expenditure represented by useful additions to plant capacity.

It is estimated that the total expenditure to 1940 would extend upward to £250,000,000 and that with proper coordination and assistance from the Government, this figure is substantially less than would result if developments were allowed to continue under existing conditions.

It is further estimated that one-half of this amount of new money would be required in connection with the development of distribution systems, this portion of capital to be provided by local authorities, power or distribution companies, as at present.

It is stated that the remaining half of the capital can be conveniently divided into two sections:

First. The amount necessary to construct the high-tension network for bulk transmission, plus working capital, reserve, and interest capitalized for a period of five years, at which date the net earnings from the gridiron would be expected to be sufficient to cover all obligations.

It is estimated that the sum of £25,000,000 would be required for gridiron expenditure at the end of five years from the date the scheme might be put into operation.

Second. The amount necessary to provide local authorities, or other authorized persons with capital required for the erection of new capital generating stations, or additions to present stations. Within certain limits capital and interest would be guaranteed by the Government.

The report points out the following conditions obtaining in Great Britain at the time of its issue.

Average price charged to consumers for all purposes	2.047d. (pence)
Gross revenue	£34,256,000
Revenue per £100 invested	£21.2
Plant installed	3,096,535 kw.
Maximum load	1,844,000 kw.
Spare plant	68 per cent of maximum load
Units sold per annum	4,016,000,000
Units sold per head of population	110
Annual load factor	24.9 per cent
Capital invested per kw. for generation	£23.8
Capital invested per kw. for distribution	£28.5
Total capital investment per kw. for generation and distribution	£52.3

It is estimated that as a result of economies to be effected by the recommendations of the committee, the position in the year 1940, or whenever the national consumption has been raised to 500 units per head, would approximate the following:

Units sold per capita	500
Maximum load	8,135,000 kw.
Kw. installed	10,000,000 kw.
Spare plant	25 per cent
Units sold	21,385,000,000 (kw-hr.)
Annual load factor	30 per cent
Total capital invested for generation	£127,000,000
Capital invested for "Grid Iron" transmission	£29,000,000
Capital invested for distribution	£243,500,000
Total revenue	£88,100,000
Average price per unit	1 d. (pence) or under
Number of capital stations	28
Number of secondary stations	30
Total number of stations	58

In the annex to Appendix I of the report there appears an interesting estimate of the cost of power stations of 133,000-kw. installed capacity, capable of meeting maximum load conditions of 100,000 kw., which is said to be based on contracts for the Barking Station of the County of London Electric Supply Co., Ltd., one of the principal superpower stations constructed in England to date.

	£ per kw.
308 acres of ground at Barking	0.733
Civil engineering works, piling, wharfs, railway sidings, water culverts, screens and foundations	1.880
Buildings and cranes	1.722
Turbo alternators and accessories	3.950
Boilers and accessories, and ash-removal plant	3.464
Switchgear and cables	0.683
Step-up transformers and station auxiliaries	0.593
Piping	0.465
Coal-storage equipment, consisting of cranes, locomotives, and cars	0.148
Sundries, including batteries and charging sets, oil coolers and pumps, oil filter plant, auxiliary sets and wiring and fittings	0.301
	£13.939
Engineering supervision, legal and incidental expenditure (5 per cent)	0.697
	£14.636

It is estimated that the average total cost of generation per kw. on the basis of coal at 16s. (shilling) per ton would be 0.3456d. (pence) with an additional capital and operating charge on main transmission lines of 0.0291d. (pence) or a total of 0.3747d. (pence) per unit sold over the gridiron.

Local distribution expenditure, including capital charges, local management and taxes, is further taken at 0.5000d. or 0.8747d. per unit at the distribution system, which is raised to 1.029d. on the basis of 85 per cent distribution efficiency.

It is understood that the assumed efficiency of 85 per cent has caused some amount of criticism.

According to the report the average cost of 1d. per unit would not be a uniform price maintained throughout Great Britain, but would vary according to local generating conditions, the amount of transmission network involved, and the distribution expense.

The estimated figure for Scotland is given as 1.038d. per unit, and that for Central England as 1.029d. per unit.

A striking feature of the report is a recommendation that the transmission network be owned and controlled by a new body created by Act of Parliament, termed the Central Electricity Board, which would purchase and dispose of energy from all of the interconnected stations, or alternatively charge for the use of the transmission network, or a further alternative according to which all generating stations would be purchased and the less efficient ones closed down, leaving the best stations to be operated by the Board, thus bringing all generation and transmission under the control of a single body.

Objection has been raised in some quarters on the ground that acceptance of the latter recommendations might be a step toward government ownership. On this account an opposition movement is on foot to prevent the bill being passed by Parliament.

The report is noteworthy as constituting the largest scheme ever proposed for the unification and extension of electric service for the definite purpose of reducing the cost of electric energy to a community.

In general, the measure seems to be popular, and there is every indication that some such Bill will be authorized by Parliament in the near future as it appears to be agreed that only through the intervention of Government interest can such a system be brought into being on account of the great diversity of interests involved.

The report shows typical curves of load and load factor, but no maps or other data are included to show details of the interconnecting network.

A transmission voltage of 33 kv. seems to be considered sufficiently high for average English conditions, at present, and has been more or less standardized for both overhead and underground lines. Some 66-kv. lines have been built and still higher voltages will no doubt be employed if extensive electrification plans are adopted.

Typical views of some of the high-tension transmission lines recently constructed in England are shown in Figs. 12, 13 and 14.

Fig. 15 shows the dead-end structure of a 66-kv. line and its junction with 66-kv., single-conductor cables leading to the transformers.

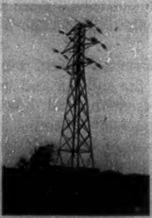

Fig. 12—Semi-Anchor Tower, 66-Kv. Dunstan-Bedlington Line Northeast Coast Power Co., Newcastle-on-Tyne, England

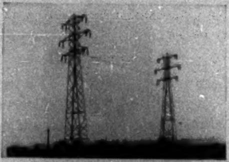

Fig. 13—Straight Line Railway Crossing 66-Kv. Dunstan, Bedlington Line, Northeast Coast Power Co., Newcastle-on-Tyne, England

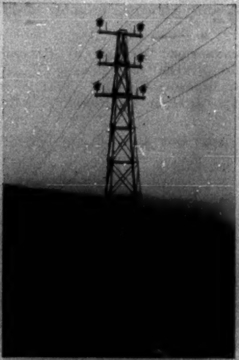

Fig. 14—Type of Standard Steel Tower on 33-Kv. Glen Valley Line of North Wales Power Company

A type of steel line support which works out somewhat cheaper than the usual type of steel tower is shown in Fig. 16.

This structure consists of steel-tubular members mounted in steel-socket pieces, and held rigidly in place by means of high grade steel guys of large diameter.

High-Tension Caples

Among the European power companies operating

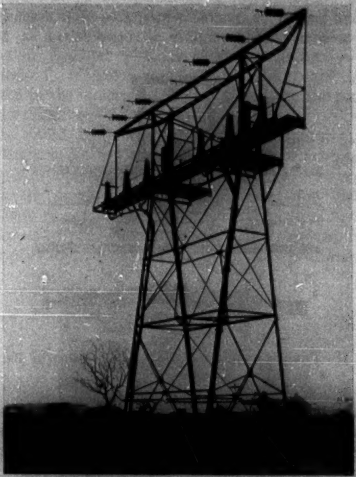

Fig. 15—Dead-End Structure of Double Circuit 66-Kv. Line Showing Junction with 66-Kv. Single Conductor Cables Leading to Transformers. Northeast Coast Power Co., Newcastle-on-Tyne, England

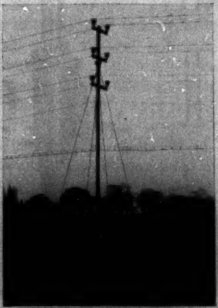

Fig. 16—Straight Line Kay Pole for Two 11-Kv., Three-Phase Circuits

Note telephone cable carried on messenger wire.
Average span 330 ft. (100 m.)

cable transmissions of relatively high voltage are the North East Coast Power Co., Newcastle-on-Tyne, England, 66 kv.; Union D'Electricité Paris, France, 60 kv.; and Provincial Electricity Works, Bloemendaal, Holland, 55 kv.

There is also a considerable amount of 33-kv. cable in operation in England.

In Italy a 130-kv. experimental cable of the Pirelli type has been in operation for something over a year with promising results.

One source of trouble which has been responsible for a considerable number of breakdowns in high-tension cable installations is the result of pressure differences within the sealed cable, where considerable difference in elevation exists throughout a part or whole of its length.

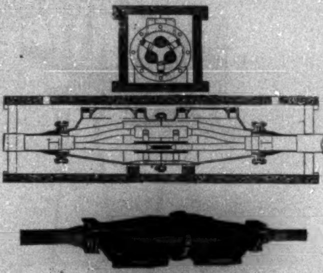

FIG. 17—THREE-CONDUCTOR, 50-KV. CABLE JOINT OF BRITISH MANUFACTURE

Pressure differences are created if there is any movement of compound used in the cable or its joints and any voids caused by such movement of the insulating compound must necessarily weaken the insulation, resulting in failure.

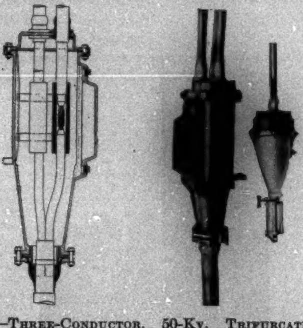

FIG. 18—THREE-CONDUCTOR, 50-KV. TRIFURCATING BOX WITH SINGLE-CONDUCTOR TAILS AND SEALING END, OF BRITISH MANUFACTURE

This has led to the development of a type of joint design for filling under pressure, which is expected to ensure complete filling and to prevent voids, but the writer is not able to say whether or not this method has been successful.

Investigations are in progress to determine the feasibility of constructing supertension cables in which a uniform potential gradient would be maintained by means of metallic sheathes, placed between layers of insulation and connected to taps in the transformer winding.[2]

Cables so constructed would be expected to run high in cost, and the splicing and sealing of the ends would seem to present unusual difficulties. However, it is not improbable that continued investigation may develop a cable with solid insulation for supertension voltages at a cost no greater than that of a steel tower line, particularly if there is a saving in cost of right of way, as might be expected in congested districts.

A three-conductor, 50-kv. cable joint of British manufacture is shown in Fig. 17 and in Fig. 18 and a 50-kv. trifurcating box with its single-conductor tails and sealing end.

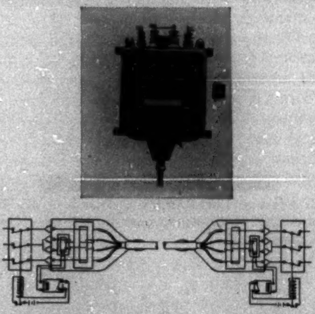

FIG. 19—CONNECTION DIAGRAM OF CALENDER HUNTER FOUR-CONDUCTOR SYSTEM OF CURRENT BALANCED CABLE PROTECTION AND INTERIOR VIEW OF A SEALING END SHOWING PHYSICAL ARRANGEMENT OF BALANCING TRANSFORMER

Practically all distribution in England is accomplished by means of underground cables.

The principal distribution problem is to be found in the London area which is served by about 80 power plants.

A frequency of 50 cycles predominates, but there is considerable diversity as regards phase and voltage.

Low-tension distribution is direct current, two- and three-wire, and alternating current with various systems of connections. Lighting voltages vary from 100 to 240 volts, while power service voltages extend to something over 500 volts. Bulk supply is sold at high-tension voltage.

The main feeder cables of power systems are frequently protected by means of current balance, this being accomplished in various ways. In the case of two parallel feeders, the current in one is balanced against the current in the other by means of differential relays which open the circuit-breakers upon a very slight difference in current value.

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[2] See paper by A. M. Taylor in *Journ. I. E. E.*, London, England, No. 315, February, 1923.

Cables are in use with six conductors, three of which in a three-phase system are similarly balanced against the remaining three.

A new form of cable specially designed for balanced protection, and which costs somewhat less than six-core cable, has recently been brought out. This system utilizes a special form of four-conductor cable in which two conductors have one-half of the cross-section of the remaining two, and current transformers of special design contained in the sealing ends at each end of the cable. These transformers function to cause a current to flow in a separate winding when the current in the different phases becomes unbalanced. The resultant current is employed to operate a relay arranged to open the circuit breakers.

FIG. 20—SHILBRUGG SUBSTATION OF THE SWISS FEDERAL RAILWAYS, SHOWING 66-KV., SINGLE-PHASE CONSTRUCTION

FIG. 21—12,000-KV-A. OUTDOOR TRANSFORMER INSTALLATION OF THE NORTHEAST COAST POWER CO. AT DUNSTAN, ENGLAND

A diagram showing the connections of the special balancing transformers and relay and the interior of one of the sealing ends is shown in Fig. 19.

HIGH-TENSION EQUIPMENT

Until quite recently European engineers have conformed to a universal practise of placing high-tension equipment indoors, but with the use of larger units and higher voltages, the outdoor substation is beginning to make its appearance.

The recently constructed substations of the Swiss Federal Railways are outdoor type and are relatively simple because of the use of the single-phase system of power transmission and distribution to track circuits.

A photograph of the Shilbrugg substation of the Swiss Federal Railways near Zurich is shown in Fig. 20.

Fig. 21 shows a recent outdoor transformer installation in England with its 66-kv., single-core cable connection. The sealing end for the 66-kv. cable may be seen in the foreground, bolted directly to the transformer tank.

The more prominent electrical manufacturers in England and on the Continent have developed complete lines of equipment for voltages up to 130 kv., while some are prepared to offer equipment for higher voltages.

Lightning protection seems to have received considerable attention on the Continent. The predominating type of arrester has horn gaps in series with water columns contained in vertical porcelain receptacles, which are mounted on high-tension insulators. In some cases arresters of the electrolytic and oxide film types of American manufacture are used.

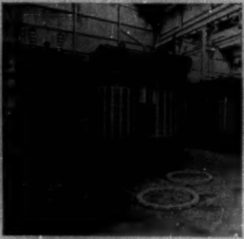

FIG. 22—12,500-KV-A., 110-12-KV., SINGLE-PHASE, WATER-COOLED TRANSFORMER UNITS OF BRITISH MANUFACTURE

Lightning protection seems to have caused very little concern in England. A length of cable at the end of a high-voltage line is considered by many British engineers to be an excellent method of lightning protection.

On the Continent there seems to have been some fear regarding the solid grounding of high-tension neutrals, although the advantages of grounding are recognized. In some instances a compromise is resorted to, by placing a horn gap in series with the neutral-ground connection. The horn gap is set sufficiently close to break down if a ground occurs.

It is almost universal practise on the Continent to design transformers for forced oil cooling, as this type of transformer costs somewhat less than the water-cooled type. Water-cooled transformers are, however, built in considerable quantities, both on the

Continent and in England, as they are sometimes to be preferred.

Fig. 22 shows part of a group of twelve 110-kv., 12,500-kv-a. single-phase water-cooled transformers recently built in England for shipment to India.

Fig. 23—Portion of 33-Kv., Iron-Clad Switchgear Installed at Barking Station of the County of London Electricity Supply Co.

(On back of Photo)

Showing one-quarter of the ultimate switchgear. In the foreground is seen the sealing off of the end of the busbars and the tail end of the vent pipe leading down through the floor to the outside of the building, but in this case the drawout portion of the main oil circuit breaker is shown removed and suspended above its panel. The operation is interlocked with the crane to ensure that all conductors are dead before it commences.

Fig. 24—Hand-Operated, Armor-Clad, Flame-Proof, Draw-Out Type Feeder Pillar for Industrial and Mining Service

Circuit breaker tank opened for inspection

There have been developed in England very complete designs of fully enclosed, or co-called "Armour clad" switchgear, for voltages as high as 33 kv.

Fig. 25—Electrically-Operated, Armour-Clad Switch-Gear of Draw-Out Type for Power Station Service, 500,000 Arc Kv-a. Rupturing Capacity

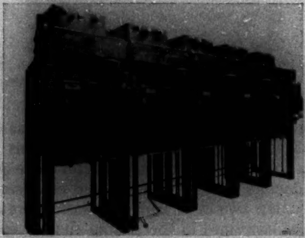

Fig. 26—Electrically-Operated, Armour-Clad Switchgear of the Drop Tank Type of 250,000 Arc Kv-a. Rupturing Capacity Installed at Talbot Road Power Station, Nottingham, England

When circuit breaker is open tank may be lowered for inspection by means of the crank shown in the foreground. During the lowering process all leads are disconnected and live parts protected automatically.

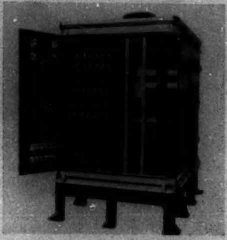

Fig. 27—Self-Contained, Armour-Clad Distribution Substation of British Manufacture

The recent switchgear installation at the Barking Station of the County of London Electricity Supply Co. introduces one of the most modern developments of this type of gear, which is designed for an operating voltage of 33 kv. and rupturing capacity of 1,500,000 arc kv-a.

Fig. 23 shows one quarter of this gear in place, with one of the circuit breakers withdrawn. In the foreground may be seen the sealing ends of the compound-filled busbar components and the gas-vent pipes leading through the floor to the outside of the building.

One of the features of the Barking Station gear is the very complete system of interlocks which has been developed to such a point that it is practically impossible to perform any switching operation in the wrong sequence. To make the system fully complete each section is interlocked with the crane so that no circuit-breaker mechanism can be removed from its position until all connections have been made dead.

This type of gear has been extensively developed to cover all classes of power, mining and industrial requirements.

Three examples of anchor-clad switchgear are shown in Figs. 24, 25 and 26.

A complete armour-clad substation containing high-tension switches, distribution transformer and low-tension fuses is shown in Fig. 27.

CONCLUSION

The writer wishes to express his appreciation to all who furnished or assisted in securing information or illustrations contained in this paper.

Lightning and Other Experience
with 132-Kv. Steel Tower Transmission Lines, and its Bearing on Tower-Line Design from the Continuity of Service Standpoint

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Synopsis.—*A large number of transmission lines have been erected without adequate consideration having been given to the ability of the line to stay in circuit electrically during the occurrence of lightning or other disturbance conditions. These come, however, within the range of the transmission engineer's field of action.*

The authors' experience with a 55-mi., 132-kv. transmission line since 1917 to date is cited, the particular line being one which has been very successful from a standpoint of continuity of service. Further experience with other lines built with variations on the original design and placed in service in 1924 and 1925, is given. A detailed analysis is given of eighty-eight cases of lightning trouble on one of these lines during 1925, the various steps taken to reduce the frequency of this trouble, the effect of increased clearances and separations, of additional insulators, of flux control units, and of arcing horns and shields on the amount of trouble and on the damage to insulator and conductor. These are discussed both from the standpoint of actual experience and from the laboratory analyses and investigations made.

A general discussion of the ground wire and its effect on lightning voltages is given and the arguments for and against it are discussed. The importance of paying attention to the mechanical side of the ground wire and the effect of such attention on continuity of service is pointed out.

A summation is given of the analysis and experience of the authors in operation of 132,000-volt lines as applied to transmission line design from the standpoint of continuity of service.

I. TRANSMISSION LINES AND THEIR OBJECT

THERE is nothing new in pointing out that transmission lines have as their object the transmission of electrical energy; it has perhaps not been emphasized often enough that a necessary corollary of the transmission facilities is that they be able to transmit this electrical energy continuously. The degree of continuity depends a good deal, of course, on the territory and the type of service supplied through the transmission line but there is no question that regardless of the territory, the standards of service required and expected by the power consumers in general have been growing more rigid. Further, it is quite obvious that even though a certain load or a group of loads may be of such a nature that continuity of service is not absolutely essential and the load can well afford to go along on, say, 99 per cent continuous service, that if the nature of the line trouble is such as to bring about an interruption that will use up the allowable 1 per cent shut-down in service at any one time, then even for that type of load, the line can not be considered as rendering adequate service.

The disadvantages of interruptions to service have been well understood generally by utilities operating in large urban communities where facilities for providing the maximum continuity of service have been in common use for years, and where they have been maintained in spite of all other radical changes that have been made in the systems. These disadvantages, however, have not always been realized by the utilities operating in medium sized or small industrial communities and it has very often happened that the transmission engineer or the power engineer has overlooked them. That these disadvantages can be serious, again goes without saying. The seriousness can extend all the way from a small money loss due to a halt in production extending the length of the interruption, to an extremely heavy money loss, which would result, say, in the case of freezing of an electrochemical process or of a process dependent upon electric power for maintaining solutions or melts in a liquid state, and can finally reach the peak in the case of steel mills or mines, where serious loss of life can result due to a loss of power, if proper precautions are not taken to take care of such an emergency.

All these matters as a general rule have not been given adequate enough consideration by the transmission engineer and where they have been, the tendency has always been to stress the structure part and its ability to sustain itself under extreme conditions of ice, wind and storm at the expense, very often, of its ability to stay in circuit electrically under lightning or other disturbance conditions.

Now lightning disturbances, if they are electrostatic disturbances, are, of course, beyond the control of the engineer but the effects of these disturbances on the transmission system are partly within his control and are within his province of action, and any transmission line that is not designed with a view to properly standing up under lightning conditions, has not, of course, been adequately designed. It is an undoubted fact that a good many lines have been designed in the past, and even in the last five years, where the factor of the ability of the line to stay in circuit under lightning disturbances has not been adequately taken into consideration and it has taken actual operating experience to disclose that fact.

II. 132-KV. LINES CONSIDERED

The authors' first extensive experience with a 132-kv. line was with the 55-mi. transmission line built on double-circuit steel towers and running from the Windsor Power Plant of the Ohio Power Company and of the West Penn Power Company, located approximately 12 mi. north of Wheeling, W. Va., to Canton, Ohio, the line running in an approximately straight northwestern direction. A sketch of this tower as well as pertinent design data is shown in Fig. 1. This line is equipped with two ground wires, ⅜ in. diameter Siemens-Martin, and was placed in operation late in 1917. The line is insulated with 10 disk units at suspension points and 12 units at dead-end points. When this line was put into operation there were operating four 30,000-kv-a. steam turbine units at the Windsor Plant and these constituted practically 80 per cent of the capacity connected to the 132-kv. transmission system of which these two lines formed a part. The line has had altogether 41 interruptions due to lightning since it was put into service, and divided by years as follows:

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*Both of American Gas & Electric Co., 30 Church St., New York, N. Y.

Presented at the Regional Meeting of District No. 1, of the A. I. E. E., Niagara Falls, N. Y., May 26-28, 1926.

1917	2	1921	4
1918	4	1922	6
1919	3	1923	4
1920	5	1924	6
1925	7		

In 10 per cent of the cases both lines were affected. All but four of these interruptions were such that they did not affect in any way the insulators or the conductor nor did it require any work to repair these. On the four other occasions mentioned the insulator string was flashed and damaged to a point necessitating a change and the conductor dropped to the ground. The ground wire was removed from over all the railroad crossings on the Windsor-Canton line during 1924, the work being completed by the end of August. In 1925, 12 mi. of ground wire were removed at each end of the line, this being done as a precautionary measure. The wire was actually not in very bad condition but atmospheric conditions at both ends of the line were rather bad and there was danger of deterioration developing to a point that would be dangerous. Up till now this wire has not been replaced. It will be noticed from the operating record that the number of interruptions apparently did not in any way increase due to the removal of this ground wire although perhaps a sufficient time has not elapsed for this point to be known definitely. This much is known, however; all the trouble encountered in 1925 was on sections where ground wire was installed. It will be noted that in general the history of operation of this line has been a highly successful one, and that the line has given such uniform continuity of service as to make possible the supplying of a load to the city of Canton amounting to well over 60,000 kv-a. dependent entirely upon the transmission line for its power.

During 1923 and 1924 the construction of three additional lines in Ohio was undertaken as follows:

1. A 73-mi., double-circuit line from the Philo Plant of the Ohio Power Company, located at Philo, Ohio, on the Muskingum River, south of Zanesville, and running to Canton, Ohio.
2. A 15-mi., single-circuit line, but strung on the same type of towers as the Philo-Canton line, running from Philo to Crooksville, Ohio.
3. A 45-mi., single-circuit line from Fostoria, Ohio, running approximately southwest to Lima, Ohio. This line, too, was to be built on a double-circuit transmission tower with only a single circuit strung.

During 1924 and 1925 there was again constructed a 129-mi. line of the same type of structure, with only a single circuit strung but utilizing 397,500-cir. mil, A. C. S. R. instead of 336,400 A. C. S. R. used on the three lines previously mentioned. This line runs from Lima, Ohio, to the Twin Branch, Indiana, plant of the Indiana & Michigan Electric Company which is located on the St. Joseph River, approximately 8 mi. east of the city of South Bend.

The tower utilized on all these lines is shown in Fig. 2. Comparing Figs. 1 and 2, it will be noticed that a number of changes were made between 1915 when the Windsor-Canton tower was designed and 1922 when the Philo-Canton tower was designed. It will be seen that the height of the bottom crossarm was raised from 50 ft to 64 ft, the vertical crossarm spacing was raised from 12 ft. to 13 ft. and the height of the upper crossarm was raised from 74 ft. to 90 ft. Further, while the Windsor-Canton tower was designed and erected with two ground wires, the Philo-Canton and other towers were all designed for but one possible ground wire and none was actually installed. A large number of factors brought about this change in the design:

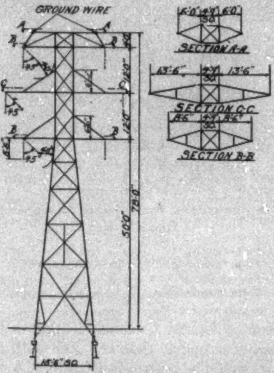

Fig. 1—Straight Suspension Tower—Windsor-Canton Line (1916)

1. In the first place, transmission line construction

costs increased greatly between 1917 and 1922. The increase was not only brought about by the increased cost of materials and the higher cost of labor, but also by the increasing cost of right of way. A definite conviction had come about that lines were costing too much and that an attempt should be made to lower the cost, and the most logical and natural suggestion was the lengthening of the average span, making possible, of course, fewer points of anchorage along the right of way, fewer points of suspension and less steel in towers, etc.

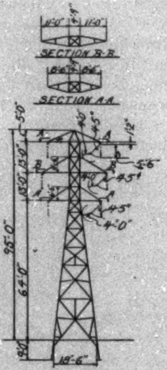

FIG. 2—STRAIGHT SUSPENSION AND SMALL ANGLE (FIVE-DEG.) TOWER

Philo-Canton
Philo-Crooksville
Fostoria-Lima
Twin Branch-Lima
} Lines (1923-1925)

2. It seemed logical to conclude that a line with fewer points of support, that is a line with fewer insulators in it, would be better electrically.

3. By lengthening the spans, it was possible to take full advantage of the additional mechanical strength obtainable in a steel-reinforced aluminum cable which, from a conductivity standpoint, it was possible to purchase on a cheaper basis at the time these lines were built than straight copper.

4. The experience that had been obtained in the operation both of the Windsor-Canton line and in the operation of a large number of 66,000-volt lines, some of which had been equipped with ground wires and others of which were not, as well as the correlation of similar operating experience of other engineers over that same section of the country, seemed definitely to indicate that the ground wire was doing more harm than good on the lines about which anything was known, on operating voltages up to and including 132,000 volts. Definite examples extending over years were cited by the operating people to show that removal of the ground wire over sections of line where such ground wire had been giving trouble due to the ground conductor getting entangled in the main conductors under storm conditions or to breakage in other ways, had materially improved system operation and caused in no case any increased trouble. In fact, there did not seem to be any cases of trouble that could definitely be ascribed to the omission of such ground wires. In short, operating experience, although perhaps not properly evaluated, seemed to show that the ground wire was doing more harm than good and that an improvement in operation could be obtained by its omission. It was known, of course, that where the ground wire had given trouble it was brought about by mechanical weakness and that it was possible to install a ground wire free from such trouble, but when the expense of this was taken into consideration and balanced against the then existing experience, it did not seem worth while to make this additional expenditure.

The Philo-Canton and Philo-Crooksville lines were put into service in September, 1924, the Fostoria-Lima line was put into service August 1924, and the Twin Branch-Lima line was put into service September 1925, all of them being put into operation initially at the rated voltage, that is 132,000 volts, with the exception of the Fostoria-Lima line, which was operated at 66,000 volts until September 1925 when it was cut over to 132,000 volts. Very little trouble was experienced with the Philo-Canton line during 1924 although definite records are not available. It is believed, however, there were actually no flashovers. The history of this line during 1925 is shown in the tabulation under Table I which gives in as complete detail as was possible to obtain, 88 cases of trouble on the line. The experience falls naturally into four groups:

1. The group covered by cases 1 to 15 inclusive
2. The group covered by cases 16 to 35 inclusive
3. The group covered by cases 36 to 68 inclusive
4. The group covered by cases 69 to 88 inclusive

III. ANALYSIS OF THE FOUR GROUPS

An examination of the first 15 cases of trouble shows that all but one case of trouble were exclusively in the top conductor. In 11 of the 15 cases one or more insulators were shattered and in 13 of the 15 cases considerable damage was done to the conductor. The question of the arcing and the points between which the arc took place has never been definitely cleared up although the table attempts to show how this actually happened. A check-up on this information, however, disclosed that the field inspectors in many cases were able to get no definite evidence and in a good many cases guessed as to what happened, very often not guessing correctly. However, the information was the best obtainable and has been allowed to stand for whatever light it may throw on the situation. It is perhaps pertinent to point out that the question of

TABLE I
REPORT ON INSULATOR FLASHOVERS, PHILO-CANTON LINE, 1925

Case No.	Pole or Tower No.	Type	Circuit No.	Angle & degree	Type insul. assem.	Units in string	Cross-arm ext.	Ground wire	Top, middle or bottom conductors	Arcing protection	Damage to insulator
1	134	C	1	R-29	Strain	12	No.	No.	Top	2 horns	East string flashed—4 units shattered
2	133	A	1	0	Susp.	10	"	"	"	None	Three top units shattered
3	133	A	2	0	"	10	"	"	"	"	Entire string flashed
4	211	B	1	0	"	10	"	"	"	"	None
5	176	B	1	0	"	10	"	"	"	"	Entire string flashed top & bot. units shattered
6	130	B	2	0	"	10	"	"	"	"	Entire string flashed two top units shattered
7	230	B	1	0	"	10	"	"	"	"	Entire string flashed top unit shattered
8	230	B	2	0	"	10	"	"	Top & mid.	"	
9	227	A	1	0	"	10	"	"	Top	"	Entire string flashed top unit shattered
10	230	B	1	0	"	10	"	"	Mid.	"	Entire string flashed top unit shattered
11	202	A	1	0	"	10	"	"	Top	"	Entire string flashed two top units shattered
12	203	A	2	0	"	10	"	"	"	"	Entire string shattered
13	234	B	2	R-7	"	10	"	"	"	"	Entire string flashed none shattered
14	152	C	1	0	Strain	12	"	"	"	2 horns	None
15	152	C	2	0	"	12	"	"	"	"	
16	126	B	2	0	Susp.	9	"	"	"	Flux con.	Two insulators shattered
17	125	A	1	0	"	9	"	"	"	"	Top insulator flashed
18	125	A	2	0	"	9	"	"	"		Top insulator flashed
19	85	C	2	L-15	Strain	12	"	"	"	None	Two units shattered on Philo side
20	100	A	2	0	Susp.	9	"	"	"	Flux con.	Entire string flashed—two top units shattered
21	167	B	2	0	"	9	"	"	"	"	Two top units shattered
22	167	B	1	0	"	9	"	"	"	"	None
23	176	B	2	0	"	9	"	"	"	"	Entire string flashed—two top units shattered
24	194	B	2	0	"	9	"	"	"	"	Top unit shattered
25	201	C	2	L-30	Strain	12	"	"	"	None	Two units shattered on Philo side
26	202	A	1	0	Susp.	9	"	"	"	Flux con.	Entire string flashed—2 top units shattered
27	203	A	1	0	"	9	"	"	"	"	Entire string flashed—1 top unit shattered
28	207	C	2	R-14	Strain	12	"	"	"	None	5 units shattered on inside string
29	209	B	2	0	Susp.	9	"	"	"	Flux con.	Entire string flashed—2 top units shattered
30	211	B	1	0	"	9	"		"	"	Two top units flashed
31	206	C	1	0	Strain	12	"		"	None	One string of insulators burned into
32	206	C	2	0	"	12	"		"	"	Several insulators flashed
33	50	A	2	0	Susp.	9	"		"	Flux con.	Entire string flashed—2 top units shattered
34	49	A	2	0	"	9	"		"	"	2 top & 2 bot. units shattered
35	66	A	2	0	"	9	"		"	None	Entire string flashed—3 units broken
36	249	B	2	0	"	9	"		"	Flux con.	Entire string flashed—top unit shattered
37	271	B	1	0	"	9	"		"	"	Entire string flashed—2 top units shattered
38	150	A	2	0	"	9	"		"	"	Top unit flashed
39	223	B	2	0	D-susp.	..	"		"	"	Bot. unit broken—top unit flashed
40	271	B	1	0	Susp.	9	"		"	"	Entire string flashed—2 top units broken
41	249	B	1	0	"	9	"		"	"	Entire string flashed—2 top units broken
42	197	B	2	0	"	9	"		"	"	Top unit shattered
43	88	A	2	0	"	9	"		"	"	Top unit shattered
44	69	B	2	0	"	9	"		"	"	Top unit burned
45	40	A	2	0	"	9	"		"	"	Entire string flashed—3 top units broken
46	NSS	SS	1	..	Switch	9	"		..		..
47	NSS	SS	1	..	Bus		"		..		..
48	110	B	1	0	Susp.	9	"		Top	Flux con.	Entire string flashed—3 top units broken
49	237	C	2	0	Strain	12	"		Mid.	Arcing horns	1 string burned off—all but 3 units burned
50	272	B	2	L-1-38	Susp.		"		Top	Flux con.	2 top units broken—3 units flashed
51	167	B	2	0	"	9	"		"	"	Top unit broken
52	19	B	1	0	"	9	"		"	"	Entire string flashed—top unit broken
52	19	B	1	0	"	9	"		Mid.	None	Entire string flashed—top unit broken
52	19	B	1	0	"	9	"		Bot.	"	Entire string flashed—top unit broken
53	19	B	2	0	"	9	"		Top	Flux con.	Entire string flashed—2 top units broken
54	19	B	2	0	"	9	"		Mid.	None	Entire string flashed—top unit broken
55	153	A	1	0	"	9	"		Top	Flux con.	Entire string flashed—4 bot. units broken
56	210	B	2	0	"	9	"	No	"	"	2 top units flashed
57	166	C	1	0	D-susp.	9		"	"	"	2 top units broken
58	172	C	2	0	Strain	12	No	"	"	None	None
59	212	C	2	0	Strain	12	No	No	Top	None	2 units broken—1 flashed—both strings
60	NSS	Sub	1		Pillar	3		"	Top No. 1	"	
61	57	A	2	0	Susp.	9	No	"	Top	Flux con.	1 unit broken
62	270	C	1	0	Strain	12	"	"	"	None	None
63	200	C	1	0	"	12	"	"	"	"	1 unit broken
64	260	A	1	0	Susp.	10	"	"	Mid.	"	Top unit broken—3 units flashed
65	260	A	2	0	"	"	"	"	"	"	Top unit flashed
66	262	B	1	0	"	"	"	"	Bot.	"	2 units broken
67	262	B	1	0	"	"	"	"	Mid.	"	4 units broken
68	262	B	2	0	"	"	"	"	"	"	2 units broken
69	46	B	2	0	"	"	"	"	"	"	Entire string flashed—top & bot. units broken
70	132	A	2	0	"	"	"	"	"	"	Entire string flashed
71	132	A	2	0	"	"	"	"	Bot.	"	Entire string flashed top & bot. units broken
72	132	A	1	0	"	12	Yes	"	Top	Flux con.	Entire string flashed
73	132	A	1	0	"	10	No	"	Mid.	None	Entire string flashed—top unit broken
74	93	B	1	0	"	12	Yes	"	Top	Flux con.	4 top units broken
75	93	B	1	0	"	10	No	"	Mid.	None	Entire string flashed—top & bot. units broken
76	93	B	2	0	Susp.	10	No	No	Mid.	None	Entire string flashed—top unit broken
77	93	B	2	0	"	12	Yes	"	Top	Rings & horns	None
78	199	C	1	0	Strain	"	No	"	"	"	Entire string flashed—5 units broken
79	229	B	1	0	Susp.	"	Yes	"	"	Flux con.	6 units flashed
80	229	B	2	0	"	"	"	"	"	Rings & horns	Entire string flashed
81	230	B	2	0	"	"	"	"	"	Flux con.	Top unit broken
82	262	B	2	0	Susp.	12	Yes	No	Top	Flux con.	2 top units broken
83	276	A	2	0		12	Yes	No	Top	Flux con.	Entire string flashed—2 units broken
84	207	C	2	R-15	Strain	"	No	"	"	Ring & horn	1 unit broken—4 units flashed
85	151	C	2	0	"	"	"	"	"	"	None
86	232	B	2	0	Susp.	"	Yes	"	"	Flux con.	None
87	234	B	1	R-7	"	"	No	"	Bot.	None	Bot. unit broken
88	275	B	2	0	"	"	Yes	"	Top	Ring & horn	Entire string flashed—1 unit shattered

field information in all these cases is generally the most difficult question to take care of and is one of the reasons, perhaps, why so much loose thinking has been done on the subject. Typical strings of damaged insulators are shown in Fig. 3.

The second group covering cases 16 to 35 embodies experience on the same line with the so-called flux control units. The theory underlying it has been set forth by Mr. A. O. Austin in a group of papers which is given in the bibliography. The underlying idea is that an insulator placed at the end of the horn and mounted below the insulator string will give a screen effect and will prevent the formation of so-called streamers on the conductor and subsequent ionization and

TABLE I Continued
REPORT ON INSULATOR FLASHOVERS, PHILO-CANTON LINE, 1925

Case No.	Damage to arcing protection	Damage to conductor	Power arcover string	Arc			
				From	To	From	To
	¾" burned off ends both arcing horns	None	Yes	Horn	Horn	..	
1	....	Pitted aluminum over wide stretch	"	Cond.	Brace	..	Cage
2	....	Cond. badly burned—4 str. steel holding	"	"	"	..	"
3	....	Badly burned 25' north of clamp 3 str. stl. Holding	No	"	"	..	"
4	....	None	Yes	Clamp	Top cap	..	
5	....	Badly pitted 45' toward Canton	"	Cond.	"	..	
6	....	Outer layer A1 burned toward Philo 12'	"	"	Cage	..	Top cap
7	....	Outer layer A1 burned toward Philo 12'	No	"	"	..	Brace
8	....	3 strands A1 burned 3" from clamp	Yes	"	Top cap	..	
9	....	Outer layer A1 burned 12' toward Philo	"	Cond.	"	..	
10	....	Alum. burned thru stl. core not dam.	"	"	"	..	
11	....	Top layer alum. burned 5' toward Philo	"	"	Cage	..	Top arm
12	....	Top layer alum. burned 5' toward Philo	"	"	Top arm	..	
13	None	Cond. badly burned in loop	No	Loop	Cage	..	Top arm
14		Cond. badly burned in loop	"	"	"	..	"
15	Flux control insul. broken—arm burned	Aluminum burned thru	Yes	Fluxcont.horn	Top	..	
16	....	Conductor pitted	"	Cond.	Cage	..	
17	....	Conductor pitted	"	"	"	..	
18	....	None	"	"	Top arm	..	
19	1 flux control unit shattered hole burned in arm	Cond. damaged 25' toward Philo	"	"	Cage	..	Top arm
20	1 flux control unit shattered	None	"	F C A	Top cap	..	
21	....	Cond. damaged 7' toward Philo	No	Cond.	Cage	..	
22	....	Cond. damaged 6' toward Canton	Yes	"	Top	..	
23	1 flux control unit burned	Cond. damaged 30' toward Canton	"	Flux cont. arc	Top cap	..	
24	....	Cond. badly burned in loop	"	Loop	"	..	
25	Flux control arm pitted	Cond. badly burned 15' toward Canton	"	Cond.	Cage	..	Top arm
26	1 flux control unit damaged	None	"	flux cont. arm	"	..	
27		Cond. badly burned in loop	"	Cond.	Top arm	..	
28	1 flux control unit & arm burned	Cond. burned both sides of clamp	"	F C A	Cage	..	Top arm
29	1 flux control unit broken	Cond. damaged 25' toward Canton	"	"	Top	..	
30	....	Cond. badly burned in loop	"	Loop	Top cap	..	Top arm
31	....	Cond. badly burned in loop	"	"	Top arm	..	
32	1 flux control unit shattered	Cond. damaged 20' toward Philo	"	Cond.	"	..	
33	1 flux control unit shattered arm burned	Cond. damaged 10' on Canton & 4' on Philo side	"	F C A	Cage	..	Top arm
34	....	Cond. pitted 10' toward Canton	"	Cond.	Top arm	..	
35	1 flux control unit shattered—arm burned	Cond. burned 2' from clamp	"	F C A	Cage	Cond.	Top cap
36	1 flux control unit shattered	Cond. burned 10' toward Philo	"	Clamp		..	"
37	1 flux control unit shattered—arm burned	None	"	C C A	Cage	Cond.	"
38	1 flux control unit shattered end of arm	None	"	"	"	..	"
39	1 flux control unit shattered hole burned in arm	Cond. damaged both sides	"	"	"	Cond.	"
40	1 flux control unit shattered	Cond. damaged 2' toward Philo	"	"	"	"	"
41	1 flux control unit shattered	None	"	"	..	..	"
42	1 flux control unit shattered hole burned in arm	None	"	"	..	..	"
43	1 flux control unit flashed	None	"	"	Cage	..	"
44	1 flux control unit shattered arm burned off	Cond. damaged 10' toward Philo	"			..	"
45	....	....		?	?	?	?
46	....	....					
47	1 flux control unit shattered arm burned	Cond. burned 15' toward Canton	Yes	F C A	brace&cage	Cond.	Top & cap
48	Arcing horn tower end 1" burned off	Cond. slightly pitted in loop	Yes	Horn	Horn	..	Tower cap
49	None	Cond. slightly pitted	"	Clamp		Cond.	Top cap
50		Cond. badly burned on Philo side	"	Cond.	Cage	..	"
51	Flux control unit flashed—arm burned	Cond. pitted 328' toward Philo	"	"	..	..	"
52	....	Cond. pitted 320' toward Philo	"	"	..	..	"
52	....	Cond. pitted 175' toward Philo	"	"	..	..	"
52	None	Cond. burned 2' toward Philo	"	"	..	..	"
53	....	Cond. burned 2' toward Philo	"	"	..	..	"
54	Flux control arm burned off	Cond. damaged 10' toward Canton	"	F C A	Cage	Cond.	"
55	Flux control unit burned	Cond. damaged 5' toward Philo	"	Cond.	Top cap		Hanger
56	Flux control unit shattered	None	"	F C A	Top arm		Top cap
57	....	Loop badly burned	No	Loop	Cage		Top arm
58	....	Loop burned	Yes	Clamp	Tower yoke		Tower cap
59			Yes	Blade contact	Ground Steel		
60	None	Cond. burned toward Canton	No				
61	....	Loop and 10' of cond. toward Philo dam	Yes	Cond.	Midarm Brace		
62	....	None	"				
63	....	Cond. burned both sides of clamp	Yes	Cond.	Top cap		
64	....	None		Not determin	ed		
65	....	Cond. burned 20' toward Canton	"	Cond.	Top cap		
66	....	Cond. burned 20' toward Canton	"	"	"		
67	....	Cond. burned 20' toward Canton	Yes	"	"		
68	....	None	"				
69	....	Cond. burned 15' toward Philo	"	Cond.	Top cap		Bot. top Phase
70	....	Cond. burned 15' toward Philo	"	"	"		Mid phase
71	None	None	"	"	"		
72	....	None	"	"	"		
73	None	None	"				
74	....	None	"	Clamp	Top cap		
75							
76	....	Cond. damaged 15' toward Philo	"	Cond.	Top cap		
77	....	Cond. damaged 15' toward Philo	"	Mid. cond.	Top cond.		
78	Ring marked—3"	None	"	Ring	Top caps		
79	None	Cond. damaged 40' toward Philo	"	Cond.	Top arm		
80	None	Cond. damaged 15' toward Philo	"	Ring	Horn		
81	None	Cond. damaged 30' toward Philo	"	Cond.	Top cap		
82	....	Cond. damaged 3' toward Canton—6' toward Philo	Yes	Cond.	Top cap		
83	2 flux control units broken	Cond. dam 2' toward Philo	Yes	F C arm	Top arm		
84	None	Loop & cond. toward Canton pitted	No	Loop	Top cap		
85	None	Cond. pitted in loop	"	"	Top arm		
86	1 flux control unit broken	None	"				
87	....	Cond. slightly damaged	Yes				
88	None	Cond. burned 5' toward Philo	"	Ring	Horn		

eventual breakdown of the air surrounding the insulator. Regardless of the apparent weakness of the theory, it was nevertheless decided to give this system a trial in view of the fact that information (and this information again had to be obtained in a hurry with no chance for proper investigation or check-up) given by other operating companies having systems of the same voltage was very emphatic in the statement that the ap-

TABLE I Continued
REPORT ON INSULATOR FLASHOVERS, PHILO-CANTON LINE, 1925

Case No.	Date of failure	How trouble was located	Line opened automatic or manuel	No. times line opened	Line out at (time)	Line in at (time)	Did line open at both ends?	Means taken to correct	Weather Conditions
1	3-26	Call by employee	Auto.	1	5:48 P M	5:51 P M	Yes	Remove arc horns	Rain & Lightning
2	3-26	Call by employee	"	1	5:52 P M	5:53 P M	"	Flux control	" "
3	3-26	Call by employee	"	1	"	"	"	Flux control	" "
4	4-2	Call by farmer	"	1	6:00 P M	6:33 P M	"	Flux control	" "
5	4-16	Normal patrol	. .	0	. .	. .	. .	Flux control	" "
6	4-19	Normal patrol	Auto.	1	11:28 A M	11:29 A M	Yes	Flux control	Rain, Light; Wind
7	4-25	Normal patrol	"	1	8-22 P M	8:31 P M	Canton	Flux control	. .
8	4-25	Normal patrol	"	1	"	8:30 P M	"	Flux control	"
9	4-25	Normal patrol	"					Flux control	"
10	4-25	Normal patrol	"					Flux control	"
11	4-25	Special patrol	"					Flux control	"
12	4-25	Normal patrol	"	1	8:24 P M	8:30 P M	Philo	Flux control	"
13	?	Normal patrol	"					Flux control	"
14	5-16	Call by employee	"	1	9:28 P M	9:23 P M	Canton	Remove arc horns	"
15	5-16	Call by employee	"	1	"	"	Yes	Remove arc horns	"
16	5-16	Normal patrol	"	1	7:45 P M	7:47 P M	"	Flux control	"
17	?	Call by employee	"	1	9:26 P M	9:28 P M	"	Flux control	"
18	?	Call by employee	"					Flux control	"
19	5-21	Special patrol	"	2	3:52 P M	3:55 P M	Yes	Remove arc horns	"
20	5-23	Call by employee	"	1	9:26 P M	9:27 P M	"	Flux control	"
21	5-21	Normal patrol	"	1	3:52 P M	3:53 P M	"	Flux control	"
22	5-21	Normal patrol	"					Flux control	"
23	5-21	Normal patrol	"	1	3:54 P M	3:55 P M	Yes	Flux control	"
24	5-23	Normal patrol	"	1	9:26 P M	9:27 P M	"	Flux control	"
25	5-23	Normal patrol	"	1	10:25 P M	10:26 P M	"	Remove arc horns	"
26	5-21	Normal patrol	"	1	10:43 P M	10:44 P M	"	Flux control	"
27	5-21	Call by employee	Auto.(#2)	1	10:51 P M	10:52 P M	"	Flux control	"
28	6-6	Normal patrol	Auto.(#2)	1	12:02 P	12:04 P	"	Remove arc horns	"
29	6-6	Normal patrol	Auto.(#1)	1	12:04 P	12:05 P	"	Flux control	"
30	?	Normal patrol	Auto.(#1)	1	12:04 P	12.06 P	"	Flux control	"
31	6-6	Normal patrol	Auto(#1,2)	1	12:06 P	12:08 P#2	"	Remove arc horns	"
32	?	Normal patrol	Auto.(#2)	1	132 P	1:33 P	"	Remove arc horns	"
33	6-6	Normal patrol	Auto.(#2)	1	158 P	1:59 P	"	Flux control	"
34	?	Normal patrol	Auto.(#2)	1	2:04 P	2:05 P	"	Flux control	"
35	3-26	Normal patrol	Auto(#1,2)	4			"	Flux control	"
36	6-24	Normal patrol	Auto.	1	11:30 P M	11:32 P M	"	Add units X-arm ext. R. & H.	"
37	7-2	Normal patrol	"	2	10:29 A M	10:31 A M	"	Add units X-arm ext. R. & H.	"
38	6-27	Call by employee	"	1	3:12 P M	3:13 P M	"	Add units X-arm ext. R. & H.	"
39	"	Normal patrol	"	1	"	"	"	Add units X-arm ext. R. & H.	"
40	7-2	Normal patrol	"	2	10:29 A M	10:31 A M	"	Add units X-arm ext. R. & H.	"
41	"	Normal patrol	"	2	"	"	"	Add units X-arm Flux control	"
42	7-7	Normal patrol	"	1	2:47 P M	2:48 P M	"	Add units X-arm R. & H.	"
43	"	Normal patrol	"	1	2:52 P M	2:53 P M	"	Add units X-arm flux control	"
44	"	Normal patrol	"	1	2:54 P M	2:55 P M	"	Add units X-arm R. & H.	"
45	7-4	Normal patrol	"	4	4:10 P M	4:12 P M	"	Add units X-arm flux control	"
46	7-6 to 11	Redman	. .	. .	. .	. .	. .	. .	"
47	7-6 to 11		. .	. .	. .	. .	. .	. .	"
48	7-2	Normal patrol	Auto.	2	10:29 A M	10:31 A M	Yes	Add units X-arm ext. R.&H.	"
49	7-10	Report by Hare	"	1	3:36 A M	3:37 A M	"	None	"
50	7-10	Report by Hare	"	1	3:52 A M	3:53 A M	"	Add units X-arm ext. R. & H.	"
51	7-10	Normal patrol	"	2	4:02 A M	4:03 A M	"	Add units X-arm flux control	"
52	7-10	Normal patrol	Manuel	1	8:00 A M	8:04 A M	Philo	Add units X-arm flux control	"
52	7-10	Normal patrol	"	1			"	None	"
52	7-10	Normal patrol	"	1			"		"
53	7-10	Normal patrol	Auto.	1	8:00 A M	8:03 A M	Yes	Add units X-arm ext. R. & H.	"
54	7-10	Normal patrol	"	1	"	"	"	None	"
55	7-25	Call employee	"	1	9:19 P M	7:20 P M	Philo	Add units X-arm ext. R. & H.	"
56	7-16	Normal patrol	"	1	1:04 P M	1:05 P M	Yes	Add units extend arm	"
57	7-25	Call E*	"	1	7:19 P M	7:20 P M	Philo	Add units extend arm	"
58	7-7	Normal patrol	"	1	2:54 P M	2:55 P M	Yes	Extend arm	"
59	7-16-25	Normal patrol	Auto	1	12:34 P M	12:35 P M	Yes		"
60	7-25	Call E*	"	1	7:10 P M	7:20 P M	Philo		"
61	7-16-25	Call E*	"	1	1:04 P M	1:05 P M	Yes	R. & H.* Add units extend arm	"
62	7-25-25	Call E* Hare	"	1	7:19 P M	7:20 P M	Philo	R. & H*	"
63	8-5	Call E* Hare	"	1	10:53 P M	10:55 P M	Yes	R. & H.*	"
64	"	Call E*	"	1	"	"	"	None to date	"
65	"	Call E*	"	1	"	11:01 P M	"	None to date	"
66	?	Call E*	Probably ca	used from				None to date	"
67	?	Call E*	Trouble of	8-5-25				None to date	"
68	8-10	Call E*	Auto.	2	3:31 P M	7:43 P M	Canton	None to date	"
69	8-18	Special patrol	"	1	10:10 P M	10:11 P M	"	9-1-25 add 2-units	"
70	"	Special patrol	"	1	"	"	"	9-1-25 add 2-units	"
71	"	Special patrol	"	1	"	"	"	None to date	"
72	8-12	Special patrol	"	1	11:05 P M	11:06 P M	Yes	9-1-25 add 2 units to mid. cond.	"
73	"	Special patrol	"	1	"	"	"	9-1-25 add 2 units to mid. cond.	"
74	9-3	Call E* Sowell	"	1	4:03 A M	4:05 A M	Canton only	9-1-25 add 2 units to mid. cond.	"
75	"	Call E* Sowell	"	2	4:07 A M	4:46 A M	Yes	9-1-25 add 2 units to mid. cond.	"
76	9-3	Special patrol	Auto.	7	4:03 A M		Yes	9-1-25 add 2 units to mid. cond.	"
77	"	Special patrol	"	7		5:11 A M	"		"
78	"	Special patrol	"						"
79	"	Special patrol	"	Caused by	trouble				"
80	"	Special patrol	"	of	9-3				"
81	"	Special patrol	"						"
82	9-3	Special patrol	Auto.						"
83		Normal patrol							Unknown
84		Call E*							"
85		Normal patrol							"
86		Normal patrol							"
87		Normal patrol							"
88		Normal patrol							"

plication of this system had resulted in an almost complete elimination of the flashovers on their systems. It therefore seemed advisable, while the lightning season was on and in view of the fact that this equipment could be installed in very quick time, to give it a trial and find out exactly what it was worth. Consequently the two top conductors of the Philo-Canton line were equipped with these controls, one of the 10 units being removed to allow its installation without lessening the ground clearance. A typical view of such an installation is shown in Fig. 4. It shows, however, 12 units in the string, this being a change which will be discussed later. The original installation, however, went in with nine disk units.

An examination of cases 16 to 35 shows that the installation of these controls in no way minimized the trouble nor was there any change in shifting of the trouble from the top conductor to the other two conductors; in other words, all cases of trouble in this group were exclusively in the top conductor. The one marked change was that the shattering of the insulators seemed to be confined more closely to the two top units, the control assembly acting apparently as a not overwell designed horn and as such being subject to considerable burning of its own. This is shown in Figs. 5 and 6 which are quite typical as to what happened to some of the horns after flashover.

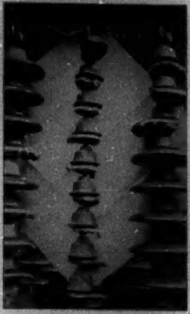

FIG. 3—TYPICAL STRINGS OF DAMAGED INSULATORS PHILO-CANTON LINE (1925)

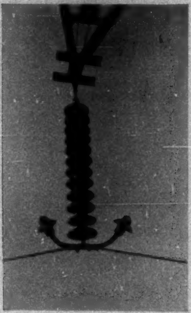

FIG. 4—INSTALLATION OF "FLUX-CONTROL" ON PHILO-CANTON LINE

The change to 12 units was a later change, the original installation consisting of 9 units

The next group, which covers cases 36 to 68, covers a period during which a new series of changes was made on the line in an attempt to see whether, without resorting to a ground wire, flashover could not be prevented or reduced to a point where its effect would be harmless or practically so. In view of the fact that the so-called flux-control unit very definitely did not seem to do what the sponsors claimed it would do, it was decided to test out three other changes and these consisted of the following:

1. The top-arm was extended two and one-half feet in each direction, making it the same length as the middle arm and the hanger of the middle arm was lowered approximately 2 ft., 10 in. on the cage. This is shown in Fig. 7. This made it possible to increase the insulation on the top arm, where the trouble again seemed to be centered, from the original 10 units to 12 units, and this change was made on the entire line. This, of course, was based on the theory that the lightning voltages were not very much in excess of the flashover voltage of the top string and it was thought possible that the installation of two additional units would raise the flashover of the string to such a point that the induced voltage would not exceed the flashover voltage of the string in at least, say, 90 per cent of the cases.

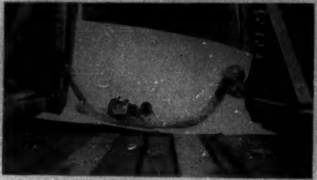

FIG. 5—TYPICAL BURNING OF "FLUX-CONTROL" HORNS AND INSULATORS PHILO-CANTON LINE (1925)

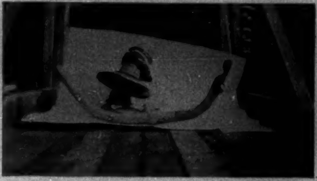

FIG. 6—TYPICAL BURNING OF "FLUX-CONTROL" HORNS AND INSULATORS PHILO-CANTON LINE (1925)

2. The moving out of the point of attachment of the top string of insulators resulted in a greater clearance between it and the second arm hanger. The object of this, of course, was to increase the flashover distance to the surrounding structure.

3. In addition to these changes which were carried through not only on the Philo-Canton line, but also on the Philo-Crooksville, Fostoria-Lima and Lima-Twin Branch lines, there was installed on the Philo-Canton line the following equipment:

An 18-in. times 22-in. cast-iron arcing ring was employed on the lower end of the upper string and a 24-in. arcing horn mounted parallel to the conductor. This assembly was used on only one side of each tower and the flux control assembly was maintained on the opposite side, alternating the position of the two on successive towers so that for each circuit there were installed an equal number of arcing rings and "flux controls." There was no change made in the flux control assembly from what was originally employed with the exception of the fact that three insulator units were added (making 12 in all); also where any damage had been done to the assembly the damage was repaired.

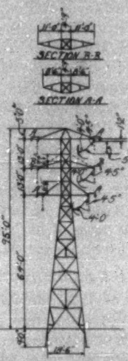

Fig. 7—Changes Made in Upper and Middle Arms Philo-Canton Line (1925) to Obtain Larger Clearance

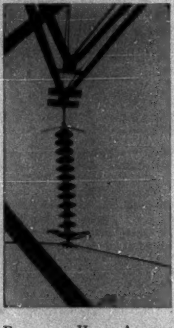

Fig. 8—Arcing Ring and Horn Assembly, Philo-Canton Line (1925)

Fig. 8 shows a straight-arcing ring and horn-suspension assembly, and Fig. 9 is a drawing of this same assembly.

In the case of dead-end assembly a 33-in. diameter-pipe ring with a double horn was employed and an assembly of this is shown in Fig. 10; Fig. 11 is a photographic view of such an assembly after the flashover described under case No. 78.

While the troubles listed under cases 36 to 68 were taking place, the changes enumerated above were being made all at the same time, the changes being made on only one circuit at a time during the daytime and the circuit being put back into service at night. It will be noticed that in 24 of the 33 cases, the trouble was again on the top conductor and that in all but five of the cases the flux control was employed. In general the type of damage was the same as that in group two, consisting of considerable damage to the string, to the conductor and to the flux-control unit. Where no arcing protection of any kind was employed, that is on the middle and bottom conductors, the damage in some cases was more severe and in other cases less than in the two upper conductors. It is to be noted that among this group, while there were some units in service with the arcing ring protection, there were

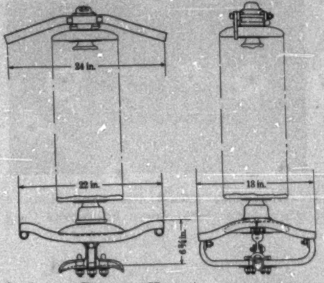

Fig. 9—Arcing Ring and Horn Assembly for Suspension Tower Philo-Canton Line (1925)

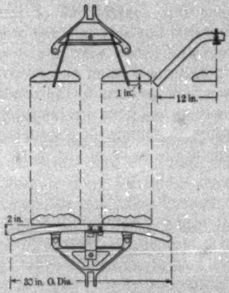

Fig. 10—Dead End Arcing Ring and Horn Assembly Philo-Canton Line (1925)

no cases where any damage was sustained to units with such protection.

The last group, namely cases 69 to 88 inclusive, extended over a period of time of well over two months and during the course of this period no changes of any kind except those described previously, were made. It will be seen that of the 20 cases, 13 occured on the top conductor, five on the middle conductor and two on the bottom conductor. Of the 13 top-conductor failures, seven were cases in which flux control had been employed and six in which rings and horns had been employed but with this difference: in five of the seven cases employing flux control one or more insulator units were shattered but only in three of the six cases, employing rings and horns, were any insulators damaged. Again, while in four of the seven flux-control cases there was damage done to the conductor and in the case of the rings and horn combination three of the six cases resulted in damage of conductor, the extent of the damage in the case of the horn combination seems to have been less than in the case where the flux control has been employed.

FIG. 11—DEAD END ARCING RING AND HORN ASSEMBLY PHILO-CANTON LINE (1925) AFTER FLASHOVER CASE NO. 78

(See Table I.)

The troubles encountered on the Twin Branch-Lima and the Fostoria-Lima lines were almost negligible in comparison with the trouble that was encountered on the Philo-Canton line. In the case of the Fostoria-Lima line only one flashover was experienced and that was on a top conductor during a period when the line was operated at 66,000 volts. The conductor was slightly damaged but no damage was sustained to the insulators. In the case of the Twin Branch-Lima line only three cases of flashovers due to lightning were experienced in 1925, two being on the top conductor and one on the middle conductor. In each of these cases at least two insulators were shattered but no damage was done to the conductor. On the Philo-Crooksville line in a period of considerably over a year, there were two cases of flashover but in neither case caused trouble of serious consequence.

IV. RELAY PROTECTION

At the beginning of 1925 the following scheme of protection was employed on the Philo-Canton line. At the Philo end current-balance relays were employed, operating only in case both lines were in circuit, and separate overload relays were employed on each of the two lines. The balance relays were operating on a so-called cross-connection and the lines were interlocked with a locking-out relay to render the second line non-automatic for a period of five sec. in case of the functioning of the switch on the first line. At the Canton end, cross-connection with a duo-directional reverse power relay was employed with fast and slow overload relays, the fast relays operating only with both lines in. The two feeders here were also interlocked with a five-sec. locking-out relay. As the lines continued to be damaged some changes were made in

TABLE II

RECORD OF REVISION OF RELAY SETTINGS ON PHILO-CANTON LINES

	Philo end							Canton end						
	Balanced protection				Separate overload			Balanced protection				Separate overload		
Date of Revision in Relay Settings	Amp. tap instant.	Normal Kv-a. setting	Time in cycles for 5 times normal	Time in secs. for interlock relays	Amp. tap & lever setting	Normal Kv-a. setting	Time in cycles for 5 times normal	Amp. tap & lever setting	Normal Kv-a. setting	Time in cycles for 5 times normal	Time in secs. for interlock relays	Amp. tap & lever setting	Normal Kv-a. setting	Time in cycles for 5 times normal
1/ 1/25	5	45700	Inst.	5	10 *a* #4	91500	73	5 *a* #2	40000	39	5	8 *a* #4	64000	73
5/27/25	"	"	"	0	"	"	"	5 *a* #1	"	24	0	"	"	"
6/ 4/25	"	"	"	⅓	"	"	"	8 *a* # ½	64000	12	⅓	"	"	"
8/20/25	"	"	"	3½	8 *a* #6	73200	113	"	"	"	3⅓	"	"	"
10/13/25	"	"	"	"	10 *a* #6	91500	"	"	"	"	"	"	"	"
10/26/25	8	73200	"	"	"	"	"	8 *a* Inst.	"	Inst.	"	"	"	"
11/ 9/25	"	"	"	"	"	"	"	"	"	"	"	10 *a* #6	80000	113
At present	"	"	"	"	"	"	"	"	"	"	"	"	"	"

REMARKS

Philo end { For balanced protection-current balanced relays with locking out relays used.
For separate overload-induction overload relays on each line used.

Canton end { For balanced protection-duo-directional reverse power relays in conjunction with induction overload relays with locking out relays used up to 10/26/25. Since 10/26/25 current balanced relays with locking out relays used.
For separate overload-duo-directional reverse power relays in conjunction with induction overload relays used.

the direction of speeding up the interlocking time and also in the direction of speeding up the relays at the Canton end. In October, there were installed at the Canton end of the line a set of current-balance relays similar to those at the Philo end but the duo-directional reverse power relay was maintained to select between the Windsor-Canton and Philo-Canton lines in case of one-line operation throughout. In general, the relays functioned entirely satisfactorily and it is not believed that the severe burning which took place was caused in any way by the failure of the relays to clear the circuits quickly enough, the relay settings being as fast as could possibly be obtained without endangering normal operation. Table II gives the complete relay set-up from the beginning up to the present time.

V. Action in Regard to Design of New Towers Taken During Period of Trouble

While the changes associated with the trouble discussed under Group 3 were being made, another 132,000-volt transmission line was in course of construction, a line over extremely rough country in West Virginia, running from Logan to Glenlyn, W. Va. A typical profile of the country is given in Fig. 12 which shows a group of spans for a line running from Logan to St. Albans, the territory from Logan to Glenlyn being, if anything, a little rougher. Here, of course, it was feared that lightning trouble would be particularly severe due to the distance of the line from the earth. Reasoning along the lines described in connection with these changes, it appeared logical at that time to design the new line with more clearance to allow the use of more insulators and to give more ground clearance. Accordingly, the tower shown in Fig. 13 was designed. It will be noticed that in this case provision was made for the use of a 5-ft., 6-in. string of insulators, the equivalent of a 14-unit string, and an arrangement made to give a 6-ft. clearance to ground under practically extreme conditions of deflection of conductor. To make this possible, the bottom of the top arm had to be raised two ft. over that shown in the tower, Fig. 2. Such a line, it was thought, insulated if necessary with 14 units, would be able to withstand practically any lightning voltage that might be induced in the line. The reasoning behind this may be summed up as follows:

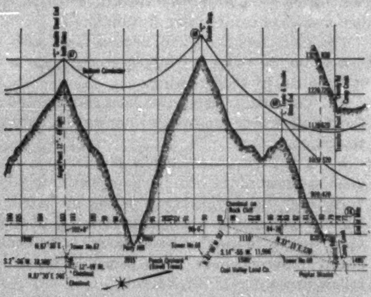

Fig. 12—Typical Profile along Logan-Glenlyn (W. Va.) Line

Let the lightning voltage come to whatever value it will, the line will be insulated sufficiently heavy to withstand almost anything that can come on it short of a direct stroke.

It may be pointed out that the design of this line has subsequently been modified considerably but this will be discussed in its place later.

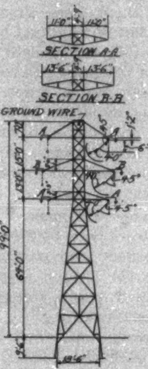

Fig. 13—Logan-Glenlyn 132-Kv. Suspension Tower

VI. A More Detailed Analysis of Troubles

While these troubles were occurring it was impossible to get any accurate data as to what was happening. Lines were flashing-over; insulators were being shattered; the conductor was being burned; and the field forces were too busy trying to place the line in an operating condition with a minimum delay to be able to give very much attention to the details of what was happening. Things were happening so thick and fast that there was no opportunity to stop and consider the matter calmly. With the approach of the end of the lightning season a little more time could be taken to determine what had actually happened and to make an intelligent analysis.

First toward that end was the gathering together of data with regard to the trouble shown in the tabulation under Table I. This tabulation was very carefully gone over and a survey was made in the field to supplement, if possible, the information given therein. In the course of this investigation it was found how very little knowledge as to what actually happened was obtainable and the smallness of the amount was really surprising. It was found, as a careful review of the troubles tabulated in Table I will show, that there existed a good deal of conflicting data and that a good many of the reports which were supposed to have been based on observation at the time the repairs were made, were entirely unreliable. However, this much stood out with bold clearness. The Philo-Canton line had been subjected to an unusual amount of punishment from lightning. That it was due to the fact that the area which it traversed was a particularly stormy area there was very little doubt. There also did not seem to be any doubt that the measures that had been taken up till then were none of them completely effective and some of them were almost negligibly so. Further, a calm review of what had been done and the lines that had been followed brought home clearly the fact that designing a line merely to withstand lightning voltages was certainly not a proper procedure since it resulted in placing insulation on the lines of such a value that it could not possibly be matched economically by that of the transformers, oil switches and of other apparatus connected to the system.

Simultaneous with the gathering of the field data a very thorough review was made of the work in connection with this subject done by Steinmetz, Creighton, Austin, Peek and others, and particularly their work in connection with the ground wire. It is believed that some of the salient points brought out in some of these papers might bear restating.

Just when the ground wire first came into use is very much of an unsettled question, but Creighton believes that for power work it was first employed by C. C. Chesney in 1891. While the date of its first use is in doubt, there is no doubt that it has ever since been a subject of great controversy among engineers. It is quite surprising, for example, how as late as 1921 engineers have held that "unless the ground wire strengthens the towers materially it should be left off, as it is a potential source of trouble and does not improve conditions as far as the insulator is concerned." On the other hand, Steinmetz, Creighton and Peek have always been firm advocates of the ground wire. Perhaps the most comprehensive work done on the subject was contained in Creighton's paper of 1916 in which he points out what were then considered the three recognized uses of the ground wire; namely:

1. Lightning protection
2. A mechanical support for towers and poles
3. A test circuit

Of these three, two can, of course, be entirely neglected by the transmission engineer but the third is probably as valid today as it was then. In the same paper Creighton pointed out that induced potential was proportional to the height of the conductor above the earth and worked out mathematical relationships between the configuration of the circuit, the cloud inducing the stroke, and the induced potential in the wire within the field of the discharge.

Creighton also laid down two laws for obtaining the best protection possible with a single ground wire, and these are:

1. String the power wires as near the earth as practicable,
2. String the ground wire above but as near the power wire as safe mechanical spacing permits.

This work has subsequently been checked by Peek experimentally on a laboratory scale and in general the agreement between the two is quite close.

Viewing the troubles encountered on the Philo-Canton line, after an analysis had been made of all the available field data, and in the light of the work of the other investigators mentioned, the explanation that seemed to stand up best came down to the following:

1. The Philo-Canton line traverses an unusually stormy country and the year 1925 seems to have been an unusually severe year from the standpoint of lightning.
2. The height of the particular tower was considerably higher than any that had been previously constructed on the system and this resulted in potentials being induced in the upper two conductors during lightning higher than those that were encountered on the Windsor-Canton line by at least 300,000 to 400,000 volts.
3. The omission of the ground wire subjected the insulator strings to this full voltage and this value was sufficient often enough during 1925 to give the large number of cases of trouble that are tabulated.
4. The fact that the Twin Branch-Lima and the Fostoria-Lima lines were not subject to these flashovers can be explained by the following:
    a. The first of these two lines did not operate for a sufficiently long period during the lightning season.
    b. The Fostoria-Lima line was in for a whole year but most of the time it was operating at 66,000 volts and its performance when insulated for 132,000 volts with a follow-up voltage of 66,000 volts would, of course, be expected to be considerably different from what it would be with a follow-up voltage of 132,000 volts.
    c. From general observations taken, it appeared indisputable that the lightning in the territory traversed by these two lines was not anywhere near the same severity as that encountered along the Philo-Canton line.

Viewed thus, a satisfactory solution seemed attainable by adopting the following methods:

1. Preventive. Installation of ground wire to reduce the lightning voltages.
2. Remedial. Where either because of the severity of the lightning or because of the great importance of the load no *chances* could be taken, the installation of arcing rings and horn devices to clear the insulator string and the conductor in case of spill-over.

Before definitely embarking on this course, it seemed advisable to check this proposed solution and consequently a series of tests was carried out with the lightning generator of Peek to determine the effectiveness of the ground wire on the tower shown in Fig. 2. These tests showed definitely that a reduction amounting on the average to about 50 per cent in the value of the induced voltage could be obtained by the installation of one ground wire at the peak of the towers. This is shown in Fig. 14 which shows the induced potential in A, suspension, B, moderate angle, and C, dead-end towers—the height of the crossarms is the same in each case and is shown in Fig. 2—with and without ground wires, under conditions of a potential gradient of 50 kv. per foot of height, and of 25 kv. per foot of height.

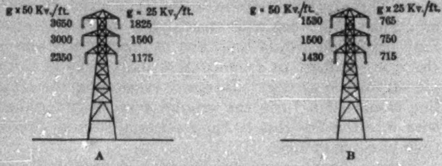

Fig. 14—Ground Wire Tests on Philo-Canton 132-Kw. Tower

A. No ground wire B. With ground wire

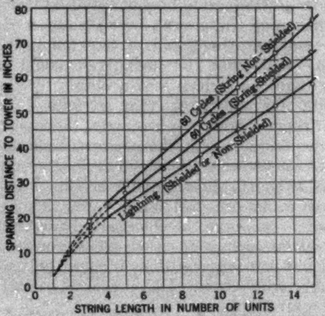

Fig. 15—Relation between Insulator Length and Tower Clearance for Balance between the Two for 60 Cycle and Lightning Flashover

Distance in inches through which spark over to tower takes place for various length strings of insulators

Another phase of the investigation was the determination of the relationship between the length of insulator string and clearance of string to ground to obtain a balance between the two, some of this work being summarized in Fig. 15. On the basis of the data shown in Fig. 15 it would appear that the original design shown in Fig. 2 was quite ample from the standpoint of clearance; that the design proposed in Fig. 13 would have a clearance that would be altogether out of proportion, and beyond what would be necessary for a string, say of 10 units; and that the clearance distances in Fig. 13 can be materially decreased.

The third phase of the investigation was a series of tests in connection with the arcing rings and horns which it was thought advisable to employ on the towers traversing territory particularly susceptible to lightning. Before working out the designs, a series of tests was carried out in the high-voltage engineering laboratory at Pittsfield, to determine the specific shapes and sizes of horns and rings and their placement on the string to clear a power arc from the insulator string and from the conductor if flashover should take place and should be followed by a power arc. The rings and horns that were worked out as well as their relations to the string and to the conductors are shown in Figs. 16 and 17 which show the designs for a straight suspension string and for a straight dead-end string. A similar design was worked out for the double suspension string.

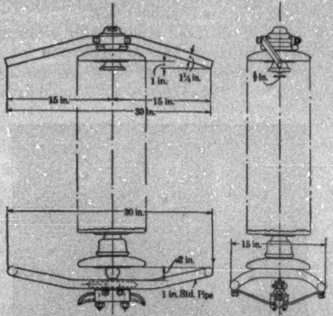

Fig. 16—Arcing Ring and Horn Assembly (1926 Installation) for Single Suspension String—Philo-Canton[Line]]

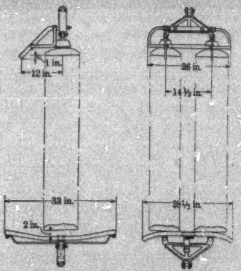

Fig. 17—Arcing Ring and Horn Assembly (1926 Installation) for Dead End String Philo-Canton Line

Fig. 18 shows a 60-cycle power arc on a string of 10 units protected with a hand-made model of the combination shown in Fig. 16. This combination gave an arcing characteristic that upon repeated laboratory tests cleared consistently and uniformly and in no case gave a cascaded arc. Of course, it is not certain that this performance would be repeated under actual line conditions with the variables of weather and wind entering in but the probability of getting an approximation of this performance is at least much greater in this case than it would be if the ring and horn had been designed on a purely paper basis.

As a result of all these studies and investigations, ground-wire installation was decided upon for all 132-kv. steel tower lines that had been erected since the beginning of the Philo-Canton line, or that were in process of erection by the end of 1925, all of which were erected without ground wire. In addition, arcing rings and horns were laid out for the Philo-Canton line and for a number of other lines where either, as in the case of Philo-Canton, it was known that lightning conditions were very bad, or where, because of the existence of but a single circuit between stations, it was imperative to keep the outage time in that circuit as near zero as possible.

Fig. 18—60-Cycle Power Arc on 10 Unit String Protected with Assembly Shown in Fig. 16

VII. Design of Ground Wire and Connections

It is conceded among engineers that the disrepute into which the ground wire had fallen was due primarily to the fact that it had been put up more or less on the basis that it was merely an appendix to the line, and since it was not carrying any energy, and therefore not earning its way, that it ought therefore to be put in with as little expense as possible; this resulted in almost every case, in the installation being as poor a one as possible. The conductors employed were very seldom anything but galvanized steel wire, and that was generally of a very much smaller diameter than the main conductor and of a lesser number of strands. The attachment was generally of some clamp type; the sagging of the ground wire was given very little consideration; and all in all the thing was put in in a most non-engineering manner. As a consequence troubles of all sorts were encountered with the ground wire. Three or four years after the installation the wire as a rule would begin to rust and a short time after that breakages would crop out in various sections of the line resulting in grounds and short circuits being placed on the system. In many cases, too, the main line would swing into the ground wire or the ground wire into the line due to their unequal sags or due to their unequal swings.

It has always been known that if a ground wire similar to the main conductor were employed and the same attention given to its installation as given to the main conductor, that these troubles could be avoided but with the exception of a very few cases the authors do not know of any installations that were put in on that basis. In view of the fact that the decision to install the ground wire was reached after considerable trouble without it and after a conviction that the ground wire would remedy a considerable portion of this trouble, it was deemed advisable not to repeat the mistakes of handicapping the ground wire through poor installation. On the other hand, it was felt that if there were any ways that the installation could be made cheaper than that which would obtain by the use of a straight line conductor, that advantage should be taken of these means. Accordingly, there was worked out in conjunction with the conductor manufacturer, a special 159,000-cir. mil A. C. S. R. conductor consisting of twelve 0.1151-in.-diameter aluminum strands and seven 0.1151-in. steel strands and this conductor was found to have mechanical characteristics so closely similar to that of either 336,400 or 397,500-cir. mil that under most conditions it could be used interchangeably with them. Other characteristics of this conductor are as follows:

Weight per ft.	0.398 lb.
Weight of ice per ft.	0.6585 lb.
Weight of ft. ice and cable	1.057 lb.
Wind pressure per ft.	1.051 lb.
Weight per ft. iced-cable in plane of resultant	1.491 lb.
Diameter	0.576 in.
Area of cable	0.1977 sq. in.
Modulus of elasticity	16,750,000 lb. per sq in.
Coefficient of expansion	0.0000086

It was found that in the case of an eight-pound wind this special conductor would deflect to an angle of 44 deg. where 397,500 cir. mil would swing 41 deg. With a 12-lb. wind the angle of swing was 56 deg. for the special conductor and 53 deg. for the 397,500-cir. mil conductor. With this conductor, it was possible to work out a stringing condition under which maximum stress developed in it in no case exceeded 7500 lb. developed, and gave ample clearance under all conditions between it and the two upper conductors. In the case of extra long spans where the possibility of vibration was feared, no attempt was made to utilize the smaller conductor but a conductor similar to the main conductor was employed for ground wire purposes.

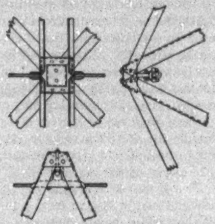

FIG. 19—METHOD OF ATTACHING GROUND WIRE IN SUSPENSION TOWERS

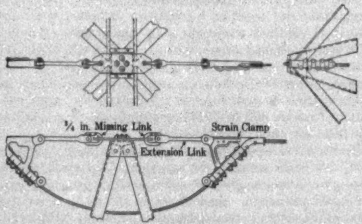

FIG. 20—METHOD OF ATTACHING GROUND WIRE AT DEAD END TOWERS

More attention than is usual was given to the hardware and to the method of attachment of the ground wire and details of this for a suspension and dead-end tower are given in Figs. 19 and 20, respectively.

VII. APPLICATION OF EXPERIENCE TO TOWER LINE DESIGN

The authors believe that theoretical analysis and their experience in operation applied to tower-line design with a view particularly of designing the lines for continuous service can be summed up as follows:

1. It is essential that a balance be maintained between the height of tower, the length of span and the lightning voltage that may be expected in the conductor span. As a general rule the cost of the line can be decreased by increasing the length of span, but carried beyond a certain point the continuity of service to be expected from the line is bound to be decreased materially in the raising of the height of the conductors. The standard suspension insulator has been developed to such a point at the present time that the increased number of points of attachment on the line will have materially less effect on lowering the continuity of service than will the raising of the height of the conductor.

2. A proper balance should be maintained between the number of insulators employed per string, the insulating values employed on other apparatus on the system and the clearance to ground. To insulate a line sufficiently high to prevent flashover in a great majority of cases, will result as a general rule (except perhaps in certain types of lines operating at 220 kv. or higher voltages) in overinsulating the lines at the expense of the rest of the equipment and will result in failures at station and substation points.

3. Unless the line is very low, it is believed that steel-tower structures carrying power conductors of 44,000 volts and over should in general be designed and installed with a ground wire. It is fully realized that there may be locations and conditions under which this is not practicable; for example, in territory where the soil conditions are such that the ground resistance is very high, and where consequently the effectiveness of the ground conductor is minimized to an almost negligible value; but these are special cases. There may also be cases in territory that is particularly lightning free where no ground wire would be justified. In no case, the authors believe, should the use of a ground wire be dismissed without giving full consideration to the known lightning conditions of the territory over which the line has to traverse, to the height of the tower, to the insulation proposed for the tower and to their relations to the lightning voltages that might be expected on the power conductors.

4. If a ground wire is installed it should be accepted as an essential part of the line and its installation treated the same as that of the main conductors, full attention being given to the material of the conductor itself and to its method of attachment and stringing. There is no reason why the main power conductor should be designed for a possible 50 years of life and the ground conductor designed for a life equal to 10 per cent of that.

5. It is possible that a line will have to be built of so great a height because of the contour of the country or for some other reason, that even with the use of a ground wire, sufficient protection will not be obtained to keep the number of flashovers down to a low enough point. In these cases, the authors believe it would be well to install remedial devices in the form of arcing rings and horns or their equivalents to prevent conductor burning and insulator shattering in case of a flashover.

6. The line must be designed so that its switching arrangement is correct; that is, the line must be so designed that it can be relayed properly and disconnected from the rest of the system quickly in case of trouble.

VIII. FUTURE DATA

The authors are carrying on an investigation at the present time to determine the most economical type of tower for 132 kv. for various types of country, keeping fully in mind the principles that they have laid down above and it is hoped to have some of these towers erected within the next year. At the present time there are operating on systems of subsidiaries of the American Gas and Electric Co. approximately 695 mi. of 132-kv. steel-tower transmission line (933-circuit mi.) and within a year this amount will be 902 mi. (1169-circuit mi.). All of these lines will be equipped with ground wires and some of them in addition with the protective features in the form of rings and horns. An attempt will be made to gather full and complete information on the behavior of these lines within the next two years during lightning periods and it is hoped at that time to be able to summarize further experiences before the Institute.

Acknowledgment is hereby made to Mr. F. W. Peek, Jr., and Mr. W. L. Lloyd, Jr., of the General Electric Company, and to Mr. A. O. Austin of the Ohio Brass Company for their assistance in carrying out some of the investigations, and to Mr. Frank Howard, of the Ohio Power Company, for his assistance in carrying through some of the developments, and in obtaining the field data.

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Discussion

F. W. Peek: Messrs. Sindeband and Sporn's paper gives some very good data on practical lines in confirmation of experimental and theoretical work which I have been doing on this subject.

I have been very much interested as to the voltages that may occur on transmission lines due to lightning, the nature of these voltages and the ability of the insulation to withstand them. The spark-over voltage of an insulator is quite different for lightning voltages and 60-cycle voltages. Fortunately the lightning spark-over voltage is much higher than the 60-cycle, spark-over voltage and is not affected by dirt, rain, water or other foreign material on the insulator. About what voltages should be expected on a transmission line due to lightning and what is the insulator strength? If these factors are known it is easy to predict whether or not trouble is likely on any line. My investigation made partly in the field and partly in the laboratory on models with the lightning generator shows that the maximum voltage that can occur on a transmission line depends upon the height of the conductor above ground. In fact, the lightning voltage above ground on a line is equal to a constant times the height of the line in feet. Thus:

$$V = g \alpha H = G H$$

where H is the height of the line in feet, g is the gradient in volts per foot at the instant before discharge while α depends upon the rate of discharge of the cloud and the size of the cloud. It approaches unity for a large rapidly discharging cloud. The highest value of g is 100,000 volts per foot. The maximum voltage that can occur on a conductor insulated above ground is thus 100,000 times the height of the line in feet. For theoretical reasons it is easy to see that that value will apply only in case of a direct stroke. Since direct strokes are not a very common occurrence, data obtained on actual lines during flashover are of greater interest. Values of the apparent gradient of from 20,000 to 30,000 are quite common, while values of 50,000 have been observed.

The practical lesson that can be drawn from this formula, I think, is best illustrated by an example. I shall take height above ground and conductor separation approximately equal to Mr. Sindeband's case. Assume 40, 50 and 60 ft. as the average height of the conductors above ground. Assume further that the storm is about a quarter of a mile away and G is 25,000. The voltage on the top conductor is then $G H$ = 1,500,000 volts. On the next conductor down it is 1,250,000 volts and on the bottom conductor just 1,000,000 volts. If a 10-unit string of insulators is used the lightning spark-over voltage is 1,400,000.

For this particular storm and this particular insulator and tower the top insulator would spark over, while the middle and bottom ones would not. With a more severe storm the top and middle would spark over. At G = 35,000, all three insulators would spark over.

In confirmation, Mr. Sindeband's data show that most trouble occurs on the top conductor, next on the top and middle and finally on all three.

Incidentally, such data offer a method of measuring these voltages. If the top insulator sparks over and not the other two the voltage is known within the difference between 60 and 50 or about 20 per cent. From the voltage and height of conductor the gradient is known. If the top and middle insulators spark over, a similar measurement is made.

From the standpoint of lightning alone it is undesirable to have a high tower.

If the above conductors were placed in a horizontal plane 40 ft. high, the voltage above ground for this particular storm would be 1,000,000. There would be no potential difference between conductors as in the case of the vertical arrangement. For the storm in question there would be no spark-overs.

How about the ground wire? Theoretically and also from studies made on models in the laboratory a single ground wire approximately cuts the lightning voltages in half if it is installed under favorable conditions. If it is assumed that the above factor holds in practise it is equivalent, as far as lightning is concerned, to doubling the insulation of the line, but with one very decided advantage. Doubling the line insulation doubles the voltage on the apparatus while the ground wire cuts it in half. If it functions properly, there is thus a double gain in using the ground wire.

Whether or not there is trouble on a line from lightning is a matter of the height of the tower and the insulation and is entirely independent of the line voltage except as it governs the number of insulators and the height of the tower. Since high-voltage lines are better insulated, less trouble should be expected.

I wish to say a few words on Mr. Sindeband's work and experience. Arc-overs have been experienced first principally on the top insulator, then on the top and middle, and next on all three. Instead of putting a great many more insulator units on the line, Mr. Sindeband has installed ground wires hoping that the ground wires will materially reduce the voltages and thus reduce the trouble. In addition, he has added the device at the end of the insulator string, shown in Fig. 18. This device is known as a grading ring or shield and was first developed for 220,000-volt lines in California. Its object is, primarily, to make the voltage distribute evenly over the string and make each insulator take its share of the voltage. In this part of the country, where there are heavy lightning voltages, it causes them to divide evenly over the string and makes each insulator take its share of the voltage. However, another very important function of a device of this kind is in case of an arc-over. It seems impracticable to make most low and moderate voltage lines absolutely lightning-proof, though it can be done if the cost is justified by making the tower low, using a number of ground wires and extra insulators. It is very important, therefore, to have some device that will cause the arc to clear the string and prevent burning and cracking of the insulators until the arc is suppressed by sectionalizing or otherwise. A horn gap might be used for this purpose. Studies we have made with the horn, however, show that it is very difficult to make it effective. The reason is quite obvious. It is this: With the horn there is no appreciable grading; about 25 to 30 per cent of the voltage is on the line unit. The sudden application of a lightning voltage causes the bottom unit to arc over, then the next and so on up the string. A complete cascade results. Even though a horn is adjusted at 60 cycles to clear very well it does not clear when the arc is started by lightning unless the separation is greatly reduced. We have not found this to be the case with the ring; it causes an even distribution at the start and there is no tendency for the initial arc to cascade. It clears the string and the 60-cycle arc follows the path of the lightning arc. If the relays operate quickly there is no damage to the insulators.

There have been very few actual operating data on the value of the ground wire. However, recent work indicates that the ground wire functions as the laboratory experience and the theoretical work show. You will observe in Messrs. Sindeband and Sporn's study, as would be expected, that most trouble occurred on the top conductor. In a similar tower equipped with a ground wire there would be practically the same voltage on all conductors. Data on a line with a ground wire should, therefore, show a more even distribution of trouble. That is exactly what has been found.

L. E. Imlay: We have one important tower line supporting six circuits on which we have five grounded guard wires. We have had no lightning disturbances on this line except in two instances when the lightning came in from connecting lines having two ground wires. From our experience we believe that five grounded wires on the top of these towers give practically complete protection.

L. C. Nicholson: The Niagara, Lockport & Ontario Power Company operates a system of some 2000 circuit miles, mostly at 60,000 volts with some 110,000-volt circuits. The practise of the company is to use ground wires. We have had experience both with and without ground wires. Our experience without them was very undesirable.

The construction that has gone up in the last ten years is of the A-frame suspension-structure type with square dead-end towers using two ground wires, the ground wires being used not only for electrical purposes but also, for mechanical reasons, to support the A-frame structures. The ground wires used are substantial in size and well grounded and have given little if any trouble.

While we can't say that flashover from lightning on lines equipped with ground wires is nil, yet it doesn't amount to a great deal. I think it would average, in rough figures, one flashover per summer per hundred miles on a double-circuit line. From five to seven insulator units are used on 66,000-volt lines. The 110,000-volt system is not very extensive and has not been in service very long so I am unable to give any conclusive figures on that.

The burning of conductors sufficiently to cause them to drop is almost unknown. I attribute this to the fact that the arc does not remain in one position long enough to burn the conductor in two or seriously damage it. I believe there have been occasions when small conductors dropped within a time period after the occurance, but the matter of burning of conductors is not a serious one.

Insulators do not puncture. This is another great measure of progress which has been made in the last 10 or 15 years. When we first started operating that was our main trouble. They now occasionally flashover, but are seldom destroyed or disabled.

E. S. Healy: There is one important phase of flashover trouble that should not be lost sight of. Under all reasonably possible wind conditions, adequate clearance must be maintained on every tower in a transmission line.

In rolling country, it is very easy to encounter special conditions that give short clearances. Any special construction requires a careful study.

Of course, any difficulties caused by insufficient clearance must be eliminated before a study of lightning effect can be made. In the two cases of flashover troubles that I have been entirely familiar with, inadequate clearance was at least a contributory cause. One of these was a tree which had probably caused occasional trouble for nearly two years before it was discovered. Aluminum and small copper conductors will blow out and carry the insulator into seemingly impossible positions.

S. S. Hertz: An entirely new engineering contribution in Institute papers has been made by the authors, and that is the analysis and publication of their *mechanical* experiences with overhead ground wires. Their contribution of field experience in its *electrical* aspect is also especially valuable, adding to the fund of data in several previous Institute papers on the electrical aspects of the overhead ground wire, such as those contributed by Dr. Steinmetz, Mr. Peek, Mr. Creighton and others. At the Midwinter Convention of 1922, Dr. Steinmetz strongly emphasized the electrical protective value of the overhead ground wire for certain types of lines, giving his preference to the overhead ground wire over the lightning arrester, principally because the advantage of the ground wire is that its function is preventive while that of the lightning arrester is merely curative.

Mechanically, the overhead ground wire has gradually undergone an evolution which is of value to note here. Starting years ago with barbed-wire and almost no standards of construction, the overhead ground wires have been gradually improved until today it is generally agreed that the overhead ground wire circuit should enjoy the benefits of the same standards of construction as are used in designing the power conductors. That is exactly what the authors have done in the lines described. Operating difficulties and troubles from mechanical failures of overhead ground wires which, on few properties, have prejudiced the use of any overhead ground wire, can be fully solved by improving construction standards. The overhead ground wire, if used at all, should rigidly adhere to at least the same standards of material and construction as used in designing the power conductors with which they are to be placed.

The following improvements in construction standards are the ones which appear most needed to bring the trend of overhead ground wire design to par with the general design of power conductors. These standards have been employed on several of the recently constructed transmission lines:

a. Use of suspension clamps and strain clamps for the ground wire of the same general design as used for the power conductors.

b. Placing slightly less sag in the ground wire than in the power conductors (in sleet districts). This is advisable to provide ample separation when the ground wire is carrying sleet while the power conductors have shed their sleet load.

c. Use of non-rusting wires, so that the natural life of the ground wire will be at least that of the power conductors.

d. Assigning a slightly greater factor of safety to the ground wire than is given to the power conductors. For example, where the power conductors are designed with a safety factor of 2, it may be advisable to have a factor of safety of about 2,25 in the ground wire. In emergencies affecting the supporting structures, the ground wire would have to bear the greater part of the strain and in any occurrence of trouble it is good practise to have the ground wire safer than the power conductors.

H. B. Vincent: I'd like to ask Mr. Sindeband two questions: Has he determined the most efficient setting of the arcing ring with respect to the wet flashover of the string? In other words, what percentage of the wet flashover of the insulators is the wet flashover of the arcing ring? If so, at what ohmage water is the wet string figured?

Has he any record of troubles which actually occurred during rain storms?

Is the flashover shown in Fig. 18 a wet or a dry flashover?

N. J. Neall: Something that has been said in the discussion prompts me to mention a feature of lightning-arrester performance that has been observed during the last year in connection with some comparative tests of lightning arresters on an 11,000-volt system where a number of circuits are carried in a parallel horizontal plane arrangement to a given substation over an H-frame construction. It was found that the arresters attached to the upper transmission wires, particularly those just under the overhead ground wires, were the principal ones to discharge and the inference, pending further information, is that we have a sort of succession of protective zones beginning with the overhead ground wire and then downward from conductor plane to conductor plane.

A. O. Austin: With the rapid changes in transmission work, there is a tendency to make an installation which is copied in many instances before operating records are available which will show up difficulties or advisable improvements or modifications. Hundreds and even thousands of miles are put in before any real operating experience can be obtained. This time lag is unfortunate and leads to a serious situation because line trouble of any description affects not only the operation and subsequent expenditures but financing as well.

In 1916 I presented a paper before the Toronto Branch calling attention to the dangers which were likely to occur with an increase in the size of the system, unless the factors of safety were materially improved. The discussion today brings this out clearly, hence the extended interconnection of large networks should be given careful attention as the increased trouble frequently more than offsets any economic advantage. The splitting up of systems, during storm periods or where the margin of safety is rather small or the circuit breakers likely to blow up, should also have more attention. A short circuit on the line today is far more serious than it was a few years ago owing to the fact that conductors are generally larger and the amount of current fed into a short circuit will create far more damage, not only to the line but to the possible connected apparatus. The time of clearing a short circuit also tends to increase with the size of the system in order to obtain selective relay operation.

Today plants are penalized for poor power factor, which results in loading the motors more heavily so that a drop in voltage frequently allows much apparatus to drop out of step. This causes heavy losses in some plants and inconvenience in all, warranting considerable expense where even momentary drops in voltage or bumps on the system can be eliminated.

In addition to the increase in number and seriousness of transmission interruptions, the cost of transmission lines is becoming more of a factor in the total cost of power. Additional parallel circuits are required to offset interruptions where lines are subject to flashover. The additional cost must be incurred whether or not the line can be cleared with a relay and circuit breaker. On some of the large systems considerable time is required following what would normally be considered a momentary interruption on a small system, before the line can be put back into service. There can be little question but that the success of future transmission systems demands that flashovers be eliminated as any other method is too costly and uncertain of results.

The whole subject needs the most careful consideration. If we apply common sense and carry the analysis far enough it will be found that there is general agreement as to the fundamentals necessary for the construction and operation of a transmission system. Above all we should not expect results which are not justified either partially or wholly by consideration of fundamentals.

The ground wire is a good example in point. A poorly installed ground wire may cause much mechanical trouble and while it may be of some benefit in reducing induced voltage or in absorbing a surge, the trouble which it causes may more than offset any advantage. Where the ground wire is installed on a wood pole it may so reduce the insulation that flashovers will result even though the maximum voltage may be reduced. While the shattering or burning of poles may be prevented, the increase in bird troubles and the elimination of the insulation furnished by the wood may more than offset any advantage in the use of a ground wire in this case.

Where a ground wire is installed on a steel structure the situation is somewhat different since there is no insulation in the structure to cut out. If the ground wire has good mechanical reliability it will, of course, reduce the maximum voltage tending to flash the line and, in general, conditions will be improved. There are some cases where the ground resistance is exceedingly high and if this resistance is materially reduced by the use of a ground wire, short circuits from lightning and birds will be increased.

An analysis of the field conditions, as well as tests in the laboratory, shows that a very great improvement may be expected from the complete impregnation of poles which will eliminate a low-resistance core. This low-resistance core tends to cause the pole to be shattered under lightning. This shattering can be eliminated only by increasing its resistance or by providing a shunt path. The former may prove most economical for new lines whereas the latter may be necessary for lines already installed. A down lead or lightning rod on the side, while saving the pole, tends to eliminate the insulation unless the ground resistance is high. In some cases the use of a gap in this circuit may be used to advantage in increasing the effective insulation for a surge or to prevent bird or squirrel trouble.

In the construction of a new system better operating conditions will be obtained if the cost of a good ground wire is used to provide more insulation and tower clearance or a more favorable electrostatic field around the conductor or live parts in the vicinity of the tower.

The old method of using a compass in tower design to determine clearance should be abandoned, as two towers having the same clearance may set up widely different electrostatic-field conditions so that one tower may have many flashovers while another tower having the same clearance may have few, if any, under the same conditions.

The ground wire is an old device and outside of improving its mechanical reliability, its application has developed little, if any, over a number of years. Practically the only improvement has been the limited use of the insulated ground wire which has given very good results.

It would seem that an improved application of the ground wire is very necessary and desirable if we wish to improve its effectiveness, and it is predicted that a great increase in its efficiency will be possible in a short time.

Bird trouble is a serious problem on some systems but it may be readily eliminated if wood poles can be kept from shattering or burning. This subject is receiving careful attention and reports from the field as well as experimental results would indicate that material improvements are possible in this direction. I am certainly not against the use of a ground wire. If, however, we find an improved method of using it or increasing its efficiency we may do much to reduce trouble and the cost of transmission lines.

The discussion today apparently leans toward the low structure. While I do not believe that this is essential where the necessary factor of safety is provided to offset the greater voltage induced on the line, it has certain advantages. The low structure means more supports and more insulators. More supports will permit the use of lighter insulators with much longer economic life which more than offsets the increased quantity. Where the low structure with its lighter mechanical loads is used it is possible to provide towers or structures which will set up a more favorable electrostatic field and thereby provide a higher flashover voltage between conductor and ground for a given length of insulator.

Owing to the fact that the electrical problems in preventing flashover on the transmission line are not so generally understood as the mechanical problems, there is a tendency to develop the mechanical end of the structures at the expense of the electrical performance of the line.

Unless the ground wire is so situated that an effective shielding exists, the upper conductor receives the greatest stress. While this is generally recognized there are few lines which provide more clearance and insulation for this conductor. On the contrary, the upper conductor usually has the lowest flashover stress for operating conditions. This results in much more trouble than would be the case were the insulation and clearance graded in proportion to the stress imposed, whether this be the top conductor or the lower conductor. If the location of the ground wire reduces the stress on the top conductors so that the lower conductors have a higher maximum induced voltage we should provide greater insulation and clearance for the lower conductors. An arc from conductor or live surface to a tower member or ground is bound to be destructive if there is any appreciable power connected to the system and the successful line of the future will be designed to prevent arcs to ground rather than minimize their damage after they form.

In some cases it is necessary to minimize the destructive effect of flashovers, as it may not be possible to eliminate flashovers without incurring a prohibitive cost. The success, however, of some rather low-cost lines leads us to believe that a great improvement may be possible through a proper consideration of tower design and clearance and a better realization and use of the ground wire itself.

While the ground wire has been improved mechanically, its electrical efficiency has not been improved over the earlier installations. In some cases I have used a ground wire effectively to eliminate troubles which were very serious. In others trouble has been materially reduced by the removal of the ground wire. Hence, its economic importance depends upon the particular application.

More attention must be given to the tower as many of the newer and more expensive lines have made a poorer electrical showing than some of the earlier and cheaper lines. This is natural in a way as building costs have gone up tremendously and the tendency has been to offset the increased cost by building towers of smaller clearance or greater mechanical strength to handle the larger conductors. Since fixed charges form an appreciable part of the cost of power on many systems we are likely to use designs which cut the factor of safety, causing increased troubles, particularly where there is no precedent upon which to draw. There is a tendency to use a relatively shorter arm with the increasing size of conductor. This, together with the high braces and wide face of the tower in the direction of the line, while economical from the tower standpoint, makes the problem of preventing flashovers exceedingly difficult. Tower clearances in the past have been laid out by means of a compass rather than by studying the electrical field set up.

Much time and thought has been given to this subject over a period of years and I feel that within the next year a material improvement will result in the construction of transmission lines as the relative performance of different constructions can be determined experimentally to a very large extent and it is gratifying to know that results in the field apparently conform to tests in the laboratory.

It is exceedingly difficult to improve the standard of an existing system although it is evident that if a system having a low standard of operation is connected to another portion having a high standard, both of them will operate on the lower standard. Usually anything within reason which will raise the standard of the old system is justifiable and much time has been spent upon this problem.

It would seem that a new system should go in with ample clearance. I should rather see a new line go in with ample clearance and no ground wire rather than have the system installed with a smaller clearance and an expensive ground wire, as the latter can always be installed later whereas increasing the clearance after installation is a difficult and costly proceeding. Owing to the more severe conditions which may arise through interconnection, a ground wire which was unnecessary on the smaller system may prove advisable and can be installed at little additional expense over that necessary were it incorporated in the first installation.

As systems grow the demand and value of greater reliability increases and if this is hampered by too small a clearance or poor field conditions at the tower, the problem of providing greater reliability will be exceedingly difficult. While certain fundamentals are recognized clearly in the laboratory, it seems to me more difficult to recognize the application of the same fundamentals in preventing flashover on the transmission line.

On the transmission line the problem is to prevent an electrical discharge between the conductor or live part and the tower and support. The problem is to prevent the break-down of the air; hence, anything which will affect the maximum stress on surfaces which will normally discharge will have an effect upon the flashover voltage. It is obvious that the density of the field surrounding a conductor or a live part may necessarily vary greatly for a given clearance with the wide range in tower designs so that general statements based upon a specific instance may be very misleading. In most lines the discharge starts probably from the conductor or live part while in others there is reason to believe that the discharge actually starts from the tower or ground side owing to the fact that the greater density in the air is produced on this side of the gap. Where the latter condition exists, increasing the size of the charged surface on the conductor will necessarily tend to lower the flashover voltage and increase troubles, although in other cases this might be a very material advantage. While this is too large a subject to go into detail, a consideration of the conditions between electrodes will illustrate the problem fairly well and not only show the inconsistencies but, what is even more important, a means for increasing the flashover voltage for a given clearance. While this is fairly simple between ordinary electrodes, the problem is more difficult in service owing to the effect of rain and mechanical considerations.

Referring to Fig. 1, if we have a grounded electrode, *A*, and a live part, *B*, with a given distance between them, it is comparatively easy to determine their flashover voltage for conditions which are likely to exist on the line. If we maintain the same clearance between electrodes but use electrode *C* in place of *B*, the flashover voltage under the same conditions will be increased, particularly if we pay no attention to polarity. If, on the other hand, *A* remains the same and we replace the electrode *C* by electrode *D*, the flashover voltage will be reduced and may be even lower than that for *A* and *B*, owing to the greater concentration of stress in the air around electrode *A*. If we replace electrode *A* by electrode *E* on the ground side in combination *A-B* as in *E-B*, the flashover voltage may still be the same as in the previous case *A-D*, but will be lower than for *A-B*. If *E* represents the electrode on the ground side, and *D* on the line side, the flashover voltage will be materially increased over any of the previous cases. There can be no question about this and it only remains to make the application to the transmission line in order to improve the flashover voltage materially with the resulting decrease in the number of flashovers. Up to the present time, there has been no line application which corresponds to the gap *E-D*, the applications being more on the basis of *A-C* which must necessarily have limitations compared to the possibilities of the gap *E-D*. There is one other case which it is well to consider, and that is the condition where we have a gap similar to *F-G*. In this case, while the electrodes apparently give a better field, their projection into the field or reduction in clearance reduces the flashover voltage until it is as low if not in some cases lower than the normal gap *A-B*. When we can make use of a dielectric which has a greater strength than air, together with some of the favorable conditions above outlined, it is possible to raise the flashover even for a given clearance far above anything now in use. Air gradient and not string gradient is the real problem.

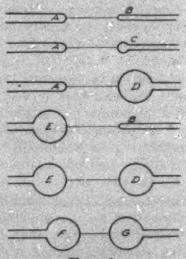

FIG. 1

There are three general schemes which may be used to advantage in connection with steel towers. One is to increase the clearance and improve the field conditions as discussed above. The second is to lower the maximum induced voltage by the use of a ground wire or lower the height of the structure. The third is to control the field in the vicinity of the conductor or at the upper end of the string so that the maximum flashover may be developed for any stress which may exist in operation. This is readily solved by the use of a sphere-gap in the laboratory and it may be approximated for transmission work. In the case of wood supporting structures the problem may be somewhat different as most wood structures furnish insulation which will provide a higher flashover voltage than exists on the largest steel tower line now in operation. The problem in this instance then is to make use of this insulation and increase the life of the structure. Complete impregnation apparently reduces the tendency to shatter and at the same time increases the life. In some cases it is further necessary to prevent burning due to leakage or due to discharge and experimental results would indicate that great improvement in this direction will be available shortly.

In the case of the steel tower, the economic problem is to develop the break-down strength of the air path and in the case of the wood structure to prevent burning or shattering. Much time has been spent on this over a period of years and possibilities in these directions are far greater and more nearly at hand than generally supposed. In many cases it may be advisable to effect a compromise of the several means to get the best economic results. It goes without saying, however, that improved flashover voltage over existing lines must be provided for the large system if high-powered transmission networks are to be extended much further. It seems that this problem together with the use of a lower transmission frequency are the most important ones facing the long high-powered line of the future.

J. H. Cox: Among engineers there seems to be a tendency to question the theory of a phenomenon and to question substantiating laboratory data until the case is proved by actual experience in the field. The theory of the ground wire has long been known. With typical arrangements, this theory indicates that by the use of a ground wire, surge voltages induced by lightning are reduced from about 40 per cent on the top wire to about 25 per cent on the bottom conductor, with vertical configuration. Mr. Peek's work with his lightning generator indicates a still greater protection, or about 50 per cent. Finally, these field experiences of Messrs. Sindeband and Sporn should establish beyond question the usefulness of the ground wire.

It seems strange, with theory indicating as it did a large measure of protection, that when trouble was encountered due to mechanical reasons the obvious solution—that of better mechanical installations—was not practised.

The proper proportion between line insulation and apparatus insulation has received too little consideration in the past. The authors bring up this point. Obviously it is less serious to have a line flashover than a bushing flashover or insulation puncture. During the past two years the Westinghouse Company together with the operating companies, has conducted surge investigations with the klydonograph on a great many power systems. During these investigations, we have conducted tests on 66-kv. lines having line insulation ranging from pin type, or three 10-in. disks to ten 10-in. disks. In spite of the ground wire, we shall always encounter a certain number of flashovers due to direct strokes and the highest induced strokes. More attention should be given this matter so that apparatus is not subjected to these higher potentials by over-insulating the line. On the other hand too low a value of insulation permits an excessive number of flashovers at a voltage lower than necessary and too low to permit the operation of lightning arresters.

There is a point in this question which though evident might bear mentioning. The proportional protection from flashovers afforded by a ground wire is far greater than that indicated by the reduction in surge voltages. If the number of surges between various voltages over a long period are plotted against voltage and a curve drawn through the points a curve is obtained resembling a probability curve as shown in Fig. 2.

If the full line represents the number of surges of various voltages on a particular line without a ground wire, a curve of the surges encountered on the same line with a ground wire would be displaced by the amount of the reduction accomplished by the ground wire, giving the dotted line. A vertical line may be drawn at the impluse-flashover voltage of the line in question. The flashovers of the line are caused only by the surges lying to the right of this vertical line. It is clearly seen that the flashover region under the dotted line is less than the flashover region under the full line by more than the percentage reduction.

Our tests with the klydonograph checks the authors' experience in that the highest potentials from lightning were always on the highest conductor and the lowest on the bottom conductor.

There were two points brought out in the discussion towards which I might be able to contribute. The question was asked as to whether flashovers were encountered at some distance from the lightning stroke, indicating a higher potential at some distance from the source. In our experience with the klydonograph, we have never encountered this situation except in the case of a flashover on an isolated-neutral system. We have encountered surges of the order of one million volts and at a distance of 50 mi. no potential greater than normal line potential was recorded. Where the lightning caused a flashover on an isolated-neutral line an over-voltage would be recorded at all stations no matter how long the line. This, however, was due to the acting ground set up by the flashover.

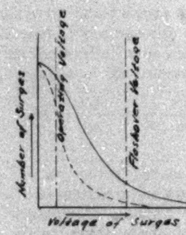

FIG. 2

Another question brought up was whether or not the upper conductors afforded protection to the conductors beneath them. From the theory of the ground wire this is not possible except where the upper conductors flashover. The only protection afforded by the ground wire is due to the fact that its charge dissipates to ground with the discharge of the cloud and the presence of a conductor remaining at ground potential reduces the potential induced on the neighboring conductors. The only way in which the upper conductors could shield the lower conductors is by discharging their charge to ground either by means of the flashover or by lightning arresters.

V. Karapetoff: In considering the influence of the ground wire, it is necessary to distinguish between its effect in reducing the over-voltage and the over-charge. These two effects are expressed numerically by different ratios. Consider the simplest case of a single conductor without any ground wire, with a cloud above charged positively. Then you have a negative bound charge on the wire, with the positive charge conducted to the ground through leakage. When a lightning flash takes place, the bound charge becomes free, so that the protective equipment is subjected, first of all, to an over-voltage, and secondly, there is a definite bound charge which has to be conducted to the ground, say, through a lightning arrestor.

Now consider a ground wire near the conductor. Certain equations may be established for the bound charge, etc. A solution of these equations gives the over-potential to which the line wire is subjected immediately after the stroke of lightning, and also the value of the charge released on this wire as a result of this stroke. The new values of over-voltage and of released charge may be compared with those obtained before, without the ground wire.

As a result of some computations which I have had an opportunity of performing lately, I find the ratio of the two over-voltages to be somewhat different from that of the two charges. Depending upon what you are interested in, that is, whether it is the maximum instantaneous over-voltage or the discharge capacity of the protective equipment (which is bound with the magnitude of the charge and not of the voltage), your judgment about the protective value of the ground wire will be different. My computations, unfortunately, have been made only for this simple combination. It would be desirable to continue these computations so as to include, say, three or six line conductors and a ground wire.

Herman Halperin: In my work as chairman of the A. I. E. E. Subcommittee on Lightning Arresters during the past year, it appeared that the members of the subcommittee and other engineers had many opinions as to the means of coping with lightning disturbances on high-voltage lines, especially those operating between 66 kv. and 154 kv.; but there were little definite data of operating experience, which is the final criterion, to show just what was accomplished by certain types of protection or methods of diminishing the magnitude of lightning voltages. The subcommittee members felt that more operating companies should gather complete data regarding the lightning on their lines, analyze the data, and submit papers along the lines of the one submitted by the authors today. Their paper is especially timely, as it concerns a voltage on which there is considerable discussion. I am sure that the industry will look forward to another paper in two years to compare the coming two years' experience of the authors with ground wires with the last two years, when ground wires were not used on certain lines.

In the future investigations of the authors, it might be well to obtain some data as to the magnitude of transient voltages on their lines, by means of a recording device such as the klydonograph. These data, when correlated with the operating experiences, will give a more definite idea of the effectiveness of the various designs in coping with lightning on their lines.

M. L. Sindeband: Mr. Cox mentioned the problem of insulation. In this paper, we mentioned the matter of over-insulating the transmission line and the bad effect this would have on the equipment in the substations resulting, as it would, in pushing the high voltage from the line into the stations. Of course, the ground wire would leave the transmission-line insulation unchanged but by cutting down the induced voltage it would have a particularly beneficial effect on the substation equipment since, as a general rule, the oil switches and transformers are not insulated as highly as the transmission line.

When we first started this problem and considered the idea of adding more insulators, the manufacturers of equipment objected since they argued that if the insulation of the line is going to be boosted, the insulation of the equipment should be boosted at the same time and they were afraid that this would lead to large expense. If that were done,—that is, if the line insulation were raised,—it does not necessarily mean that the equipment insulation has to be raised likewise since the station and substation equipment has, as a general rule, the benefit of a lightning arrester but the moment you go into that you enter into another controversy—namely, the lightning arrester—and there is almost as much disagreement about that as there is about the ground wire.

Mr. Vincent asked what particular feature of the rings governed their design. We experimented with the design of the rings only in so far as the clearing of lightning flashovers and 60-cycle flashovers were concerned. The wet proposition was not considered at all.

Now as to the table listing the troubles: On our original table sent in to the Institute we had some marks indicating which were under rain conditions and which under purely lightning conditions. Unfortunately, they were left out in printing. However, as we recall it, practically all of these troubles occured during rain periods, that is, they were a combination of rain and lightning.

Fig. 18 is a 60-cycle dry flashover test.

In conclusion, I wish to state that I believe that the operating companies should contribute their bit in helping the manufacturers of equipment with this problem, because certainly the manufacturing companies do spend a great deal of time and money for this purpose.

Notes on the Vibration of Transmission-Line Conductors

BY THEODORE VARNEY[1]

Associate, A. I. E. E.

Synopsis.—This paper describes tests made with a graphic recorder to show the vibration of transmission-line conductors under various conditions of wind velocity, conductor tension and span length. The method of taking the records is discussed. Formulas are given for determining the velocity of propagation of transverse waves along a conductor, the wavelength of a vibration, and the frequency of vibrations caused by "eddies" formed at a conductor subjected to air currents.

If a wire is suspended freely between supports and is struck near one of the supports, a wave will run along the wire to the other support, be reflected and return to the starting point. If the supports are rigid, that is, possessed of infinite mass, the wave will be entirely reflected and it will pass back and forth until the viscosity of the wire damps out the wave. With a decrease in the mass of the support, the amount of the wave energy reflected is lessened. Part of the energy passes into the support, either storing energy therein to be given back to the latter if the support is elastic, or becoming dissipated by the viscosity of the support.

Assuming that the tension in the wire is constant, the velocity of propagation of the transverse wave is:

$$v = \sqrt{\frac{P}{m}} = \sqrt{\frac{P\,g}{w}} \qquad (1)$$

when

v = Velocity in feet per second
P = Total tension in the wire
m = Mass per ft. of wire
w = Weight per ft. of wire in pounds
g = Acceleration due to gravity

The time required for the wave to travel twice the length of the span is the fundamental time period. If at the instant the first impulse reaches the starting point the wire is struck again, the vibration will be sustained. If at the instant the impulse reaches the second support the cable is struck again, the two crests meet at the center, producing a node. If the frequency of the exciting force is increased, the span breaks up into a series of nodes and loops. If resonance occurs, there must be a whole number of loops between supports, provided the mass of the wire is uniform.

The velocity remaining the same, the product of wavelength and frequency is a constant and is equal to the velocity; that is,

$$l\,f = v \qquad (2)$$

where

l = Twice the distance between nodes
f = Frequency in cycles per second

In cases where resonant vibration of conductors has been observed, it was traceable usually to the wind blowing across the line and always at low velocities. A strong wind broke up the resonant conditions and merely swayed the span as a whole. The vibrations were in a vertical plane. Resonance appeared more frequently in the early morning or near sundown. Resonance also occurred over a wide range of cable tensions.

Resonance in a transmission cable is a very elusive thing; it begins without warning and ceases abruptly, and, while the conditions at the moment as regards temperature, wind velocity and direction may be noted, it is impossible to maintain or reproduce them at will.

It was at first thought that a span vibrating with fixed nodes and loops would not be affected by moving the point of support to the first node. If this were true, then the behavior of a single loop could be investigated on a small scale, experimentally, and would afford means of observing the behavior of a full sized span. Accordingly, an attempt was made with several sizes of stranded cables to produce resonant vibration by mounting the cable transversely in an aviation wind tunnel. The tunnel used was 8 ft. square and the wind could be varied from zero to 75 mi. per hour. The cables were supported at the sides of the tunnel and the tension in the cable was varied over a wide range.

No resonant vibrations could be produced. The cables vibrated rapidly with a very small amplitude, not over one one-hundredth or two one-hundredths of an inch. These small vibrations were noted by means of a reflecting mirror and a stroboscope and were found to agree fairly well with the eddy frequency.

The meaning of the expression "eddy frequency" will be explained later.

The probable reason why resonance and considerable amplitudes could not be obtained in the wind tunnel is that it is difficult to maintain constant wind velocities over the whole length of span. Moreover, upon further thought, it is more difficult to maintain approximately resonant conditions over a short span than it is over a long one. If, to assume an example, the wind velocity were adjusted so as to produce one loop in an 8-ft. span, it would require a change in wind velocity of 100 per cent to create the next larger whole number of loops, that is two, in the span. Any intermediate value of wind velocity would not produce resonance. If, on the other hand, the span were 80 ft., a 10 per cent change

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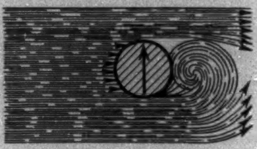
[1] Aluminum Company of America, Pittsburgh, Pa.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Niagara Falls, N. Y., May 26-28, 1926.

in velocity would be necessary to produce either nine or eleven loops in the span, while in a span 800 ft. long, only 1 per cent change would be necessary to produce 99 or 101 loops in the span.

In actual long spans of transmission lines, the nodes have been observed to shift back and forth in position with the slight changes in wind velocity, the vibrations, however, persisting with considerable amplitude for long periods of time.

In 1921 an investigation made by E. F. Relf and E. Ower and covered by a report to the British Aeronautical Research Committee, showed that the singing note produced by wires moving rapidly through the air corresponded with the periodic eddies produced behind the wire.



Eddy Formation Produced by Wind Blowing on a Wire

When a fluid medium, such as air or water, flows past an obstruction in its path, eddies are produced behind the obstruction. If the obstruction is of symmetrical cross-section such as a circle, the behavior of the eddies formed will be similar upon each side of the obstruction. As the fluid flows past the obstruction, the friction on the two sides is not exactly the same at any particular instant and this tends to slightly lower the velocity of the fluid passing on one side. The fluid on the other side, continuing in its normal velocity, creates a slight difference of pressure back of the obstruction, the lower pressure area being on the side where the higher velocity exists. This causes a flow of the fluid from the opposite side to fill this rarified area, and the action of the fluid takes the form of swirls or eddies. As this rarified area is restored to normal density by the inflow of the eddy, the velocity on that side of the wire is reduced and the inrush of the eddy accelerates the motion of the fluid on the other side and presently the eddy ceases on the first side and begins on the other, thereby repeating the cycle of events.

The result of this alternating-eddy formation is to produce an alternating force on the obstruction in a plane at right angles to the flow of the fluid. In aircraft work, the obvious means to prevent the formation of these eddies is to "stream-line" the section of the obstruction, thereby allowing the air to flow down both sides and join at the rear edge of the section without the formation of these eddies. The figure shown herewith illustrates this eddy phenomenon.

Experiments in both air and water were carried out from which the truth of the following general expression was established:

$$f = \frac{V}{D} \text{ function } \left(\frac{V\,D}{e} \right) \qquad (3)$$

where

f = Frequency in cycles per sec.

V = Velocity of the medium with respect to the cylindrical wire

D = Diameter of the wire

e = Coefficient depending upon the medium, being 0.000159 for air and 0.0000122 for water

Further investigations gave the following data: When

$$\frac{V\,D}{e} = 100, \text{ function } \left(\frac{V\,D}{e} \right) \text{ equals } 0.125$$

$$\frac{V\,D}{e} = 150, \text{ function } \left(\frac{V\,D}{e} \right) \text{ equals } 0.150$$

$$\frac{V\,D}{e} = 300, \text{ function } \left(\frac{V\,D}{e} \right) \text{ equals } 0.172$$

$$\frac{V\,D}{e} = 600, \text{ function } \left(\frac{V\,D}{e} \right) \text{ equals } 0.185$$

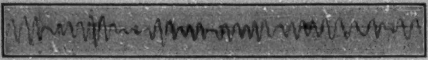
When

$\frac{V\,D}{e}$ is greater than 600, function $\left(\frac{V\,D}{e} \right)$ equals 0.185

It will be noticed from the foregoing that function $\left(\frac{V\,D}{e} \right)$ is a constant for values of $\frac{V\,D}{e}$ of 600 or greater. For a wind velocity of one mile per hour, the minimum value of D would be 0.78 in. For two miles per hour the minimum value of D would be 0.39 in. Therefore for usual transmission line conditions, a constant value for the function $\left(\frac{V\,D}{e} \right)$ can be used. Curves of frequency plotted against wind velocity become, therefore, a series of straight lines—one for each diameter of conductor.

As a check upon the application of this theory, the following results tabulated from a long series of painstaking observations upon a certain transmission line are of interest. The cable in this case was 1 in. diameter, weighing 0.858 lb. per ft. and was supported upon steel towers.

The accompanying charts were obtained by attaching one end of a string to a transmission wire and the other end to a light wooden block arranged to slide in a slot in a vertical board which was fastened to a board resting on the ground. The lower end of this block had attached to it a light spring which served to keep the string taut and yet permitted the block, with the pencil attached, to move up and down in response to the



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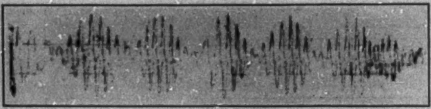

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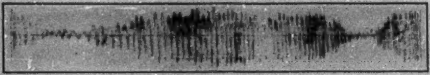

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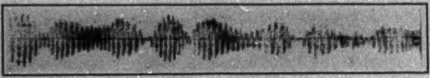

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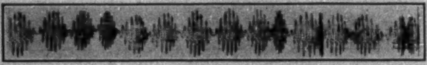

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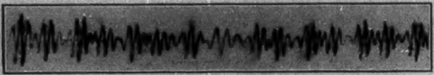

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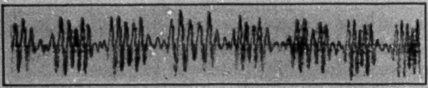

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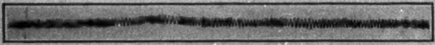

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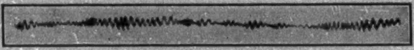

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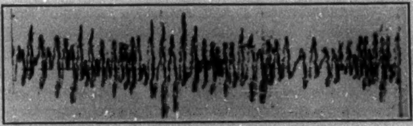

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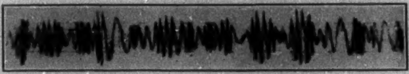

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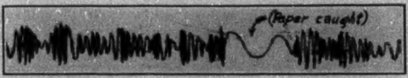

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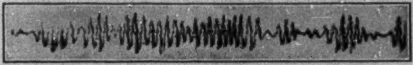

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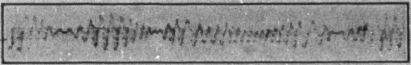

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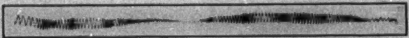

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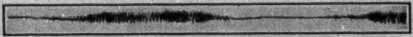

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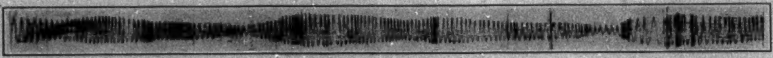

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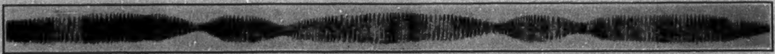

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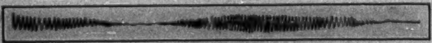

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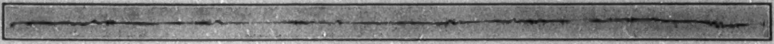
RECORDS OF TRANSMISSION-LINE VIBRATIONS
(See table for detailed information)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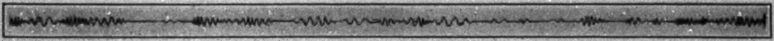
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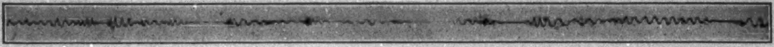
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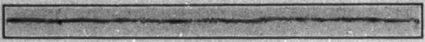
No. 23—Length: 5 seconds

Records of Vibration of Transmission-Line Conductors
(See table for detailed information)

DATA RECORDED IN VIBRATION TESTS ON TRANSMISSION LINES

The conductor upon which the tests were made was 1 in. in diameter, weighed 0.858 lb. per ft. and was supported upon steel towers.

1	2	3	4	5	6	7	8	9
Chart No.	Span (ft.)	Tension (lb.)	Wave Velocity (ft. per sec. calculated)	Wind Velocity (M. P. H., Measured)	Eddy Frequency (cy. per sec. calculated)	Vibration Frequency Recorded on chart (cycle per sec.)	Amplitude of Vibrations Recorded on chart (1/32 inches)	Loop Length (ft.)
1	1200	8500	565	5.2	17.0	9.5	12	29.7
2	"	"	"	7.7	25.0	12.5	28	22.6
3	"	"	"	8.0	26.0	12.5	26	22.6
4	"	"	"	8.5	27.5	21.5	22	12.8
5	"	"	"	8.9	29.0	18.3	24	15.4
6	"	"	"	10.0	32.8	18.3	12	15.4
7	"	"	"	11.0	36.0	20.0	16	13.2
8	"	7930	545	10.0	32.8	33.0	4	8.3
9	"	7700	538	3.5	11.5	13.0	12	20.5
10	"	7530	532	3.9	12.8	14.0	24	19.2
11	"	"	"	4.8	16.0	11.3	18	23.4
12	"	"	"	5.2	17.2	10.5	14	25.7
13	"	"	"	5.6	18.7	12.0	20	22.5
14	"	"	"	7.8	25.5	22.0	12	12.2
15	"	"	"	8.0	26.0	22.0	6	12.1
16	"	"	"	19.1	62.5	23.0	6	11.6
17	"	"	"	21.3	69.5	22.0	18	12.3
18	"	"	"	21.3	69.5	22.0	20	12.3
19	"	"	"	22.5	73.0	22.5	8	11.8
20	984	5600	460	3.40	11.0	14.0	2	16.4
21	"	"	"	5.00	15.0	15.0	6	15.3
22	"	"	"	5.70	19.0	15.0	6	15.3
23	"	"	"	5.90	20.0	20.0	1	11.5

vibrations of the transmission-line wire. The string was attached as nearly as possible at the middle point of the first node from the insulator clamp.

A wooden slide with a strip of paper attached to it was then moved in a direction at right angles to the movement of the pencil and was timed with a stopwatch. The device was crude but the charts afford a means of determining quite accurately the total number of vibrations produced in a certain time.

In the cases described herein the cable was quite large and the vibration had sufficient force so that the string could be pulled quite taut without damping the vibrations. This prevented, to a considerable extent, interference due to the fluttering of the string by the wind.

In other cases where the wire was smaller and the fluttering of the string interfered with proper results, a temporary platform was erected on the tower and by cutting off the current on the line, some charts have been obtained by attaching a pencil directly to the wire. The first described method has the advantage that it can be used on a live line and charts can be made on short notice, whenever the wind conditions are observed to be favorable, without taking time to cut the power off the line and arranging for a series of observations. However, as noted, it is very often difficult to obtain satisfactory results by this means in the case of small line wire. The amplitudes of the vibrations are obtained by measuring the charts directly.

In the tabulations given herewith, the figures in column 1 refer to the number of the charts as appearing on the reproduction of the charts themselves. The second column gives the length of span where these particular charts and observations were made. In this case, the supports were practically level. It will be noted that the observations are made upon two-span lengths. The third column gives the total tension in pounds in the span at the time of the observations. These tensions were calculated from the observed sags. The fourth column gives the theoretical velocity of propagation of transverse waves in the cable as described in equation (**1**).

Column 5 gives the wind velocity in miles per hour as noted by a portable anemometer used during the experiments. Column 6 gives the theoretical eddy frequency calculated from the size of wire and observed wind velocity, using equation (**3**). In equation (**3**) the values of V and D must be in the same units; that is, V is feet per second and D is diameter in feet of the conductor. Also, these units might be used in terms of inches per second and inches of diameter.

Column 7 gives the frequency in cycles per second as taken directly from the charts. Column 8 gives the amplitude also measured directly from the chart. Column 9 is the calculated loop length using equation (**2**). The values of V in this equation are taken from column four and the values of F are taken from column 7. The values in column 9 obtained by this means are, therefore, theoretical but they check with a fair degree of accuracy with the actual observed distances between nodes at the time of the observations.

On account of the variable character of the wind under most conditions, however, these node points were observed to shift constantly back and forth. Usually, however, they corresponded to values sometimes smaller and sometimes greater than the loop length given in column 9. It is assumed, therefore, that the theoretical values given in column 9 are fair assumptions.

Chart 8 is particularly interesting because of the absence of "beats" and the exact agreement between the eddy and observed frequency. This is interpreted as indicating a practically uniform wind velocity throughout the entire span. It is likely also that the time period of oscillation of the tower supports acts to amplify or damp the line vibration.

A device at the support which has a period corresponding to a certain proportion of the natural period of the conductor will act as a damper. Such a damper having a period of about seven cycles per second would probably be effective in greatly reducing the amplitude of vibrations in the conductor throughout the range of wind velocities noted in the case described herein.

It is reported that the device recently described in the technical press[2] by Mr. G. H. Stockbridge, of Los Angeles, California, has been found effective. Further investigations on transmission lines in service are at present going on.

The writer is indebted for various courtesies and assistance in obtaining the data described in these notes from Capt. Wm. McEntee and Dr. A. F. Zahm of the United States Navy Yard, Washington, D. C.; Mr. H. A. Barre and Mr. H. Michener of the Southern California Edison Co., Los Angeles, Calif; Mr. A. E. Silver, Electrical Engineer, Electric Bond & Share Company, New York; and to Messrs. M. E. Noyes, J. P. King, Walter Hays and C. B. Owen, Aluminum Company of America.

Discussion

A. E. Knowlton: The principle of aerodynamics which Mr. Varney cites to account for the vibration of the transmission-line conductors is undoubtedly the same one which is found applied in the propulsion of Dr. Floettner's rotorship. The one difference is that he rotates the rotor and keeps the pressure always in one direction. That is not, of course, the case with the line conductors. The instability there results in alternation of the pressure and consequent vibration of the conductor.

Another and more important difference is that Dr. Floettner has made a useful application of it and one can't say as much in the case of the line conductors. The fact is that we get two distinct results, one useful and the other deleterious.

A committee of operating engineers is attempting to assemble the facts that have been observed and perhaps have been recorded in connection with this phenomenon, so that we can draw conclusions not merely as to the rigidity of the theory as Mr. Varney has set it up, but also upon the efficacy of certain preventive measures.

Theodore Varney: In closing, I think Prof. Knowlton's comment regarding Dr. Floettner's rotorship is appropriate. While Dr. Floettner turns the eddy effect of the wind to good account, the same influence accomplishes no good result in a transmission line conductor. It is not correct to say, however, that this effect is always deleterious in the latter case because cases are on record where vibration has been going for over ten years without interference in any respect to the continuous and successful operation of the line.

As the speaker views the matter, destructive effects will result from vibration only when the direct stress in the conductor increased by the stress due to vibration exceeds the endurance limit of the material of the conductor.

This is a complex problem, but it appears that the stress due to vibration is directly proportional to the stored energy in one half of each vibrating loop and inversely proportional to the distance over which that energy is dissipated. This energy is expressed by the mass of the half loop, the square of the amplitude and the square of the frequency of vibration. This energy passes back and forth freely between loops in the span away from the supports. At the supports there is an inevitable bump and increase of stress, which can be greatly reduced by care in clamp design.

The most beneficial remedy is to reduce the amplitude of vibration by dampers.

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[2] *Electrical World*, December 26, 1925, p. 1304.

Sag Calculations for Transmission Lines

BY H. B. DWIGHT[1]

Fellow, A. I. E. E.

***Synopsis.**—A set of sag formulas is presented, in the form of convergent series which give accurately the results of the hyperbolic catenary formulas. The series are useful for calculating almost any practical transmission line span, long or short.*

Trial and error methods are not used. First, the sag or deflection is calculated for a given maximum tension in the cable. Then, changes in temperature corresponding to changes in deflection and loading are computed. This procedure is followed whether the supports are at equal heights or at unequal heights.

A formula is presented for the deflection from the line joining the supports, when the elevations are unequal. This is useful in sighting from tower to tower, as shown in Fig. 2.

A few examples are worked out and some derivations of formulas are given. In the last appendix instructions are given with regard to allowing for the change in weight per ft. with change in length of cable.

SAG calculations for transmission lines are frequently made by means of formulas based on the assumption that the curve of a cable in a span is a parabola. Formulas for the calculations are also published, which use the equation for the catenary, a curve so named because it is the shape of a suspended chain. This equation involves hyperbolic cosines.

In this article is presented a group of convergent series which will be found convenient and accurate for making sag calculations. Provided enough terms are computed, these give the results of the hyperbolic catenary formula as accurately as desired. Usually, two or three terms are all that are required. Since the series are convergent series derived from the expansions of hyperbolic functions, they can be said to be themselves hyperbolic formulas. The first term of the series is in many cases the same as the well-known parabolic formulas[2], the majority of which consist of only one term. The series herewith presented should therefore be easily understood and applied by those accustomed to the parabolic formulas.

The series moreover give directly and automatically the percentage error involved by using the parabolic formulas. Even where the latter have good accuracy, it is always worth while estimating the amount of their error in a given case. Often the worst feature of an approximate formula consists in the fact that the amount of its error is not indicated and remains unknown, so that in some more or less unusual case the error may be unexpectedly large. An approximate formula in the form of a convergent series is much safer to employ, since when the terms do not become smaller and smaller, the last one being negligibly small, it is obvious that the formula is not applicable to the case considered.

If, therefore, a formula is really the first term of a convergent series, it is practically always advisable to publish two or three terms of the series, so that the appropriateness of the formula for a given case may be quickly estimated.

The series are given in the form in which they can be most conveniently used for practical sag calculations. A few examples of their use are described, and in an appendix there is given the derivation of some of the series. The derivation of the others can be obtained by following the method of the examples given.

The description of the engineering problem is usually as follows: A certain maximum tension T is specified for a cable and it is desired to know what will be the sag corresponding to this tension under given conditions of temperature and wind and of ice loading. Further, it is desired to know what changes in temperature and tension correspond to given changes in sag or in loading. These results are required when the two ends of the span are supported at the same height or at unequal heights.

In preparing the formulas, the use of trial and error methods has been avoided. These involve more work than direct calculations, and have a greater liability of error. If values of temperature are assumed, such as 40 deg. or 60 deg., a trial and error process is required at some stage of the calculations. Accordingly, values of sag or tension are assumed, and temperatures are obtained by direct calculation.

This provides data for drawing a series of curves of sag plotted on temperature, separate curves being drawn for different lengths of span. These lengths of span may be the actual lengths for the transmission line being designed, or they may be even hundreds of ft., such as 200- 400- 600 ft., etc. In the latter case, new curves may be plotted from the first set, so as to show sag plotted against span, separate curves being plotted for specified temperatures such as 20 deg., 40 deg., 60 deg., etc., and for specified loadings.

These procedures do not involve trial and error processes. The drawing of the curves described above deals only with the final results, after temperatures have been calculated. This is not likely to produce errors in the same way as the use of curves for intermediate stages of the calculations.

Attention should be called to a paper by J. S. Martin,[3] containing a very complete table for calculating sags

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[1] Professor of Electrical Machinery, Massachusetts Institute of Technology, Cambridge, Mass.

[2] See articles on sag calculations by L. E. Imlay in the *Electric Journal* of February, 1925, and succeeding issues.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Niagara Falls, N. Y., May 26-28, 1926. Complete copies available to members on request.

[3] "Structural Engineering Problems in Transmission Line Construction," by J. S. Martin, *Proceedings*, Engineers' Society of Western Pennsylvania, November 1922, page 309.

when the supports are at equal heights. Formulas are given for using the data in the table when the supports are at unequal heights but it is stated that the precision is not as great as for the case of equal heights, on which the calculations for the tabulated data were based. A method of successive approximations is described for obtaining sags at specified temperatures. While this involves some work, it is an orderly procedure leading to the desired result.

SAG FORMULAS

It is to be particularly remembered that in the formulas in this paper, l represents a half span only.

SUPPORTS AT EQUAL HEIGHTS

$$\text{Sag or deflection} = d = l\left[\frac{1}{2}\frac{w\,l}{T} + \frac{7}{24}\left(\frac{w\,l}{T}\right)^3 + \frac{241}{720}\left(\frac{w\,l}{T}\right)^5 \ldots\right]\text{ft.} \qquad (1)$$

where $l = \frac{1}{2}$ span $= \frac{1}{2}$ horizontal distance between supports in ft.

T = tension in pounds, in cable at supports, where the tension is greatest.

w = resultant loading in lb. per ft. of cable. Note that $w^2 = v^2 + h^2$ where v is the vertical force per ft. acting on the cable due to gravity on the cable itself and on the ice covering, if any, and where h is the horizontal pressure in lb. per ft. due to wind. The deflection d is not vertically downward. The distance vertically downward by which the lowest point of the cable is lower than the supports is $\frac{v\,d}{w}$ ft.

When the deflection, d, is given,

$$T = w\,l\left[\frac{1}{2}\frac{l}{d} + \frac{7}{6}\frac{d}{l} - \frac{4}{45}\frac{d^3}{l^3}\ldots\right]\text{lb.} \qquad (2)$$

The unstressed length of cable in the span is

$$L_u = 2\,l\left(1 + \frac{2}{3}\frac{d^2}{l^2} - \frac{14}{45}\frac{d^4}{l^4} + \frac{278}{945}\frac{d^6}{l^6}\ldots\right) - \frac{wl^2}{A\ E}\left(\frac{l}{d} + \frac{5}{3}\frac{d}{l} + \frac{4}{9}\frac{d^3}{l^3}\ldots\right)\text{ft.} \qquad (3)$$

where A = area of cross-section of cable, in square inches and E = modulus of elasticity in pounds per square inch.

The first line of formula (**3**) gives the actual perimeter of the catenary, and the second line gives the stretch in the conductor.

To find the changes in temperature, t, corresponding to changes in deflection, d, and loading, w, find values of L_u corresponding to certain values of d and w. The changes in L_u are due to temperature, if the cable is assumed fastened to rigid supports, and

$$t = \frac{L_u - L_{u0}}{a\,L_{u0}}\text{ deg.} \qquad (4)$$

where t is the change in temperature,
L_{u0} = value of L_u at the lower temperature,
L_u = value of L_u at the higher temperature, and
a = temperature coefficient of expansion per degree.

If a is specified per degree Fahrenheit, then t is in degrees Fahrenheit. Note that

$$L_u = L_{u0}\,(1 + a\,t)\text{ ft.} \qquad (5)$$

When a is defined as the increase per degree above a certain temperature, then L_{u0} should be that temperature, in using equation (**4**) or (**5**).

Curves can be plotted of d against t. Different curves for different values of w can be plotted.

To find changes in load, w, corresponding to changes in d at a given temperature, insert values of d in equation (**3**). The value of w which will give the value of L_u for the given temperature, is obtained directly. L_u is given by equation (**5**).

Changes in temperature corresponding to changes in tension T are given by the following equation, and by equation (**4**).

$$L_u = 2\,l\left[1 + \frac{1}{6}\left(\frac{w\,l}{T}\right)^2 + \frac{7}{40}\left(\frac{w\,l}{T}\right)^4 + \frac{241}{1008}\left(\frac{w\,l}{T}\right)^6 \ldots\right] - \frac{2\,w\,l^2}{A\,E}\left[\frac{T}{w\,l} - \frac{1}{6}\frac{w\,l}{T} - \frac{7}{120}\left(\frac{w\,l}{T}\right)^3 \ldots\right]\text{ft.} \qquad (6)$$

Curves of tension, T, against temperature, t, can be plotted.

SUPPORTS AT UNEQUAL HEIGHTS

It is usually desirable to find first a solution of the catenary for a given maximum tension T at the higher support where the tension is the greatest, as was done in the case when the supports were at equal heights. Then, changes in temperature, loading and tension can be calculated, corresponding to changes in deflection.

Let v and h be the vertical and horizontal forces per foot acting on the cable, as previously defined. Then, $w^2 = v^2 + h^2$. Let p be the vertical height by which one support is higher than the other and let $2\,l$ be the horizontal distance between the supports.

$$q = \frac{p\,w}{v}\text{ ft.} \qquad (7)$$

$$2\,k = \sqrt{4\,l^2 - q^2 + p^2} = 2\,l - \frac{(q^2 - p^2)}{4\,l} - \frac{(q^2 - p^2)^2}{64\,l^3}\ldots\text{ft.} \qquad (8)$$

These dimensions are shown in Fig. 1, which is drawn in the plane of the cable and not in a vertical plane.

If h, the wind pressure, is zero, then $q = p$ and $k = l$.

Equation (**8**) is based on the fact that the square of the distance between the supports is equal to $4\,l^2 + p^2$ and to $4\,k^2 + q^2$.

Let

$$b = \frac{q}{2\,k} \tag{9}$$

$$\begin{aligned} m = {} & k\left(1 - b^2 + \frac{2}{3}\,b^4 - \frac{8}{15}\,b^6 \;.\;.\;.\right) \\ & + \frac{T}{w}\left(b - \frac{2}{3}\,b^3 + \frac{8}{15}\,b^5 - \frac{16}{35}\,b^7 \;.\;.\;.\right) \\ & - k\left(\frac{w\,k}{T}\right)\left(\frac{2}{3}\,b + 0 + \frac{2}{45}\,b^5 \;.\;.\;.\right) \\ & - k\left(\frac{w\,k}{T}\right)^2\left(\frac{2}{3}\,b^2 + 0 \;.\;.\;.\right) \\ & - k\left(\frac{w\,k}{T}\right)^3\left(\frac{16}{45}\,b + \frac{8}{9}\,b^3 \;.\;.\;.\right) \text{ ft.} \end{aligned} \tag{10}$$

The deflection, $d + q$, is found from equation (**1**) putting $l = m$.

The distance vertically downward from the upper support to the lowest point of the cable is $\dfrac{v\,(d + q)}{w}$ ft.

For finding the effect of changes of temperature and loading, it would be possible to assume different values of T and find values of m and d as shown above. However, it will be a little shorter to assume values of H the horizontal tension and find m from the following:

$$\begin{aligned} m = {} & k + \frac{H}{w}\left(b - \frac{1}{6}\,b^3 + \frac{3}{40}\,b^5 - \frac{5}{112}\,b^7 \;.\;.\;.\right) \\ & - \frac{k}{6}\left(\frac{w\,k}{H}\right)\left(b - \frac{1}{2}\,b^3 + \frac{3}{8}\,b^5 \;.\;.\;.\right) \\ & + \frac{k}{30}\left(\frac{w\,k}{H}\right)^3\left(\frac{7}{12}\,b - \frac{17}{24}\,b^3 \;.\;.\;.\right) \text{ ft.} \end{aligned} \tag{11}$$

The deflection in this case is found from

$$d = n\left[\frac{1}{2}\,\frac{w\,n}{H} + \frac{1}{24}\left(\frac{w\,n}{H}\right)^3 + \frac{1}{720}\left(\frac{w\,n}{H}\right)^5 \;.\;.\;.\right] \text{ft.} \tag{12}$$

$$\text{where } n = 2\,k - m \text{ ft.} \tag{13}$$

After m, n and d are found, using either (**10**) or (**11**) for m, the unstressed length of cable is found by applying equation (**3**) to each part of the span separately, as follows:

$$\begin{aligned} L_u = {} & m\left[1 + \frac{2}{3}\left(\frac{d + q}{m}\right)^2 - \frac{14}{45}\left(\frac{d + q}{m}\right)^4 \right. \\ & \left. + \frac{278}{945}\left(\frac{d + q}{m}\right)^6 \;.\;.\;.\right] \\ & - \frac{w\,m^2}{A\,E}\left[\frac{1}{2}\left(\frac{m}{d + q}\right) + \frac{5}{6}\left(\frac{d + q}{m}\right) \right. \\ & \left. + \frac{2}{9}\left(\frac{d + q}{m}\right)^3 \;.\;.\;.\right] \\ & + n\left[1 + \frac{2}{3}\left(\frac{d}{n}\right)^2 - \frac{14}{45}\left(\frac{d}{n}\right)^4 \right. \\ & \left. + \frac{278}{945}\left(\frac{d}{n}\right)^6 \;.\;.\;.\right] \\ & - \frac{w\,n^2}{A\;E}\left[\frac{1}{2}\,\frac{n}{d} + \frac{5}{6}\,\frac{d}{n} + \frac{2}{9}\left(\frac{d}{n}\right)^3 \;.\;.\;.\right] \text{ft.} \end{aligned} \tag{14}$$

Curves for sag, temperature, loading and tension can now be drawn, as described for the case of supports at equal heights. If a value of H has been assumed, the tension at each support can be found by applying equation (**2**) or (**21**) to each part of the span. The tension at a support is somewhat greater than H, and is greater at the higher support.

If there is no horizontal part of the curve between the two supports, then m is greater than $2\,k$, and n is a negative quantity. See equation (**13**).

Where a value of H is assumed, a table, not too condensed, of hyperbolic sines can be used with equation (**18**) to find m. However, this would not give a direct calculation for m in terms of T, such as is given by equation (**10**). If the difference in elevation of the supports is unusually great compared with the length of the span, the series may not converge rapidly enough, and a table of hyperbolic sines and cosines may be required. In such a case, trial and error methods may be necessary in order to obtain the desired results.

An illustration of this type of calculation is given in Example V. Since it is a trial and error method, different values of H must be assumed until a satisfactory value of T is obtained.

When the supports are at equal heights, values of H may be assumed and a table of hyperbolic sines and cosines used in a somewhat similar manner to Example V. This usually requires more work than to use the series. This is a trial and error method if one is working to a specified value of T. It may be necessary to do this if the series do not converge rapidly enough, as with an unusually large ratio of sag to span, but this is not likely to occur with practical transmission-line spans having supports at equal heights. One should also refer to the table by J. S. Martin[4].

Deflection from Line of Supports

After the location of the lowest point O, Figs. 1 and 2, and, therefore, the complete equation of the catenary for one set of conditions, have been found, it is possible to find by a direct calculation the vertical distance, PQ, from the line of the supports A and C to a tangent BD which is parallel to AC.

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[4] *Loc. Cit.*

In Fig. 2, $PQ = AB = CD$, and the two latter distances may be measured on the towers and the line BD be used for sighting to determine the correct amount of sag to give the cable when stringing it. For this purpose, assume that there is no wind or ice load, and so the cable hangs vertically. The calculation would be practically the same if wind load were included.

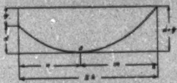

FIG. 1—SPAN WITH SUPPORTS AT UNEQUAL HEIGHTS

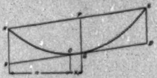

FIG. 2—DEFLECTION OF CABLE FROM LINE OF SUPPORTS

Let the equation of a line parallel to the line of the supports be $y = g\,x + f$ where g is known but where f is unknown. For this line, $\dfrac{d\,y}{d\,x} = g = \dfrac{p}{2\,l}$. The height of any point on the catenary above O is given by

$y = \dfrac{H}{w}\left(\cosh\dfrac{w\,x}{H} - 1\right)$. See Appendix I.

$$\frac{d\,y}{d\,x} = \sinh\frac{w\,x}{H}$$

If the line $B\,D$ is tangent to the catenary at Q, which is the point (x_1, y_1),

$$\sinh\frac{w\,x_1}{H} = g$$

Then

$$\cosh\frac{w\,x_1}{H} = \sqrt{1+g^2}$$

$$y_1 = \frac{H}{w}\left(\sqrt{1+g^2} - 1\right) \qquad \textbf{(15)}$$

$$x_1 = \frac{H}{w}\sinh^{-1} g$$

$$= \frac{H}{w}\left(g - \frac{1}{2\times3}g^3 + \frac{1\times3}{2\times4\times5}g^5 - \frac{1\times3\times5}{2\times4\times6\times7}g^7 + \ldots\right) \qquad \textbf{(16)}$$

Thus, numerical values of x_1 and y_1 can be obtained.

The height of P above O is $d + \dfrac{p\,(n + x_1)}{2\,l}$ and therefore,

$$P\,Q = A\,B = C\,D = d + \frac{p\,(n + x_1)}{2\,l} - y_1 \qquad \textbf{(17)}$$

all parts of which are known.

Example I. Find the difference in temperature for the following two sets of data for the same span:

$$2\,l = 800 \text{ ft.}$$

$$\frac{w\,l}{T} = 0.13128$$

$$\frac{w}{A\;E} = 0.0000007572$$

For the higher temperature, without wind or ice load,

$$\text{sag} = 23.1760 \text{ ft.}$$

$$\frac{w}{A\;E} = 0.0000002976$$

Temperature coefficient 0.0000096

By formula (**6**),

$$\begin{aligned} L_{u0} &= 800\,(1 + 0.0028724 + 0.0000519 \\ &\qquad + 0.0000012\ \ldots) \\ &\quad - 800\,(0.0023070 - 0.0000066 \\ &\qquad + 0.00000004\ \ldots) \\ &= 800 \times 1.0006252. \end{aligned}$$

By formula (**3**),

$$\begin{aligned} L_u &= 800\,(1 + 0.0022381 - 0.0000035 \\ &\qquad + 0.00000001\ \ldots \\ &\quad - 0.0010273 - 0.0000057 - 0.00000001\ \ldots) \\ &= 800 \times 1.0012016 \end{aligned}$$

By formula (**4**),

$$t = \frac{0.0005764}{0.0000096 \times 1.0006252} = 60.1 \text{ deg. fahr.}$$

This checks the result given in Mr. Martin's[5] paper.

Example II. Find the deflection for the following span[6]:

$2\,l = 2000$ ft.

Supports at equal heights.

$T = 70{,}000$ lb.

$w = 4.700$ lb. per ft.

$$\frac{w\,l}{T} = \frac{4.700 \times 1000}{70000} = 0.067143$$

$$\frac{l}{2}\left(\frac{w\,l}{T}\right) = 33.571$$

By equation (**1**), $d = 33.57 + 0.088 + 0.0005 = 33.66$ ft.

This agrees with the value of 33.6 ft. given in reference 6. It is seen that the series gives quickly and directly a precise solution of this problem, and the degree of precision of the calculation is indicated by the convergence of the series. The first term is the well-known parabolic formula.

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[5] Problem IV of the article by J. S. Martin, *Loc. Cit.*

[6] Problem 2, p. 11, Transmission Line Design by F. K. Kirsten, 1923, Bulletin No. 17, University of Washington, Seattle, Wash.

Example III. Find the horizontal point for the following catenary[7]:

$2\,l = 2700$ ft.

Supports at unequal heights, $p = 179$ ft.

$T = 60587$ lb. at the higher support.

$h = 1.321$ lb. per ft.

$v = 2.870$ lb. per ft.

$w = 3.158$ lb. per ft.

$$q = \frac{p\,w}{v} = 197.0$$

$$2\,k = 2698.75$$

$$b = \frac{q}{2\,k} = 0.0730$$

By equation (**10**),

$m = 1342.2 + 1395.6 - 4.6 - 0.02 - 0.01 = 2733.2$ ft.

$n = 2698.7 - 2733.2 = -34.5$ ft.

The above paper gives $m = 2608.8$ ft. and $n = +91.2$ ft., but it neglects the fact that the cable lies in an oblique plane, and the dimension $p = 179$ ft. does not lie in that plane.

Example IV. Find the deflection for the following span[8]:

$2\,l = 4279$ ft.

Supports at unequal heights, $p = 185.5$ ft.

$T = 33000$ lb. at the higher support.

$h = 2.036$ lb. per ft.

$v = 2.623$ lb. per ft.

$w = 3.322$ lb. per ft.

$$q = \frac{p\,w}{v} = 235.0 \text{ ft.}$$

$$2\,k = 4276.57 \text{ ft.}$$

$$b = \frac{q}{2\,k} = 0.05495$$

By equation (**10**),

$$m = 2132 + 545 - 17.0 - 0.2 - 0.4$$
$$= 2659 \text{ ft.}$$

By equation (**1**),

$$d + q = 356 + 15 + 1$$
$$= 372 \text{ ft.}$$

The paper referred to above states that the deflection from the upper support is

$$9933.77 - 9615.44 = 318.33 \text{ ft.}$$

and that the deflection from the lower support is

$$9786.91 - 9615.44 = 171.47 \text{ ft.}$$

The difference between these deflections is 146.86 ft. which is less than the inequality in height of the supports, namely 185.5 ft. The difference of the deflections in the oblique plane should, however, be greater than 185.5 ft. In the appendix of the above paper, the equation $k_0 = k \cos\theta$ should be $k_0 = \dfrac{k}{\cos\theta}$, and formula (**3**) should be $k\left(\dfrac{1}{\cos\theta} - 1\right)$ instead of $k(\cos\theta - 1)$

$$\left[k = 185.5 \text{ ft. and } \cos\theta = \frac{2.623}{3.322}\right]$$

Example V. To illustrate the use of a table of hyperbolic sines and cosines,

Let $H = 31940$ in Example IV.

$$\frac{w\,k}{H} = \frac{3.322 \times 2138.28}{31940} = 0.22240$$

$$\sinh 0.22240 = 0.22424$$

$$\sinh\frac{w\,(k-n)}{H} = \frac{q\,w}{2\,H \times 0.22424}$$

$$= \frac{235.0 \times 3.322}{2 \times 31940 \times 0.22424}$$

$$= 0.05450$$

$$\frac{w\,(k-n)}{H} = 0.05447$$

$$n = 2138.28 - \frac{0.05447 \times 31940}{3.322}$$

$$= 1614.6 \text{ ft.}$$

$$m = 4276.57 - 1614.6 = 2662.0 \text{ ft.}$$

The value of T can now be calculated, and it will be slightly different from 33,000 since a value of H was assumed as part of the trial and error method.

The complete paper contains an appendix, here omitted, giving derivations of formulas.

Appendix

DERIVATION OF FORMULAS

I. Sag, or deflection, when supports are of equal height.

Let

$l = \dfrac{1}{2}$ length of span in ft.

w = force in pounds per ft. acting on the cable, due to gravity and wind pressure,

H = horizontal component of tension in cable.

T = tension in cable at support, in lb.

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[7] Problem with cable loaded, Table VII, Transmission Line Design, by G. S. Smith, JOURN. A. I. E. E., Dec., 1925, p. 1352. See complete paper.

[8] "Mississippi River Crossing," by H. W. Eales and E. Ettlinger, JOURN. A. I. E. E., Oct., 1925, first problem in the appendix. See complete paper.

By the well-known catenary formulas[9],

d = sag or deflection

$$= \frac{H}{w}\left(\cosh\frac{w\,l}{H} - 1\right)$$

and

$$T = H\cosh\frac{w\,l}{H}$$

$$\frac{T}{w\,l} = \frac{H}{w\,l}\left[1 + \frac{1}{2}\left(\frac{w\,l}{H}\right)^2 + \frac{1}{24}\left(\frac{w\,l}{H}\right)^4 \cdots\right]$$

$$\frac{w\,l}{T} = \frac{w\,l}{H}\left[1 - \frac{1}{2}\left(\frac{w\,l}{H}\right)^2 + \frac{5}{24}\left(\frac{w\,l}{H}\right)^4 \cdots\right]$$

(by long division or by multinomial expansion)

$$\left(\frac{w\,l}{T}\right)^5 = \left(\frac{w\,l}{H}\right)^5 \cdots$$

$$\left(\frac{w\,l}{T}\right)^3 = \left(\frac{w\,l}{H}\right)^3\left[1 - \frac{3}{2}\left(\frac{w\,l}{H}\right)^2\right]$$

$$= \left(\frac{w\,l}{H}\right)^3 - \frac{3}{2}\left(\frac{w\,l}{T}\right)^5 \cdots$$

$$\frac{w\,l}{H} = \frac{w\,l}{T} + \frac{1}{2}\left(\frac{w\,l}{H}\right)^3 - \frac{5}{24}\left(\frac{w\,l}{H}\right)^5 \cdots$$

$$= \frac{w\,l}{T} + \frac{1}{2}\left(\frac{w\,l}{T}\right)^3 + \frac{3}{4}\left(\frac{w\,l}{T}\right)^5 - \frac{5}{24}\left(\frac{w\,l}{T}\right)^5 \cdots$$

$$= \frac{w\,l}{T} + \frac{1}{2}\left(\frac{w\,l}{T}\right)^3 + \frac{13}{24}\left(\frac{w\,l}{T}\right)^5 \cdots$$

$$d = l\frac{H}{w\,l}\left[\frac{1}{2}\left(\frac{w\,l}{H}\right)^2 + \frac{1}{24}\left(\frac{w\,l}{H}\right)^4 + \frac{1}{720}\left(\frac{w\,l}{H}\right)^6 \cdots\right]$$

$$= l\left[\frac{1}{2}\left(\frac{w\,l}{H}\right) + \frac{1}{24}\left(\frac{w\,l}{H}\right)^3 + \frac{1}{720}\left(\frac{w\,l}{H}\right)^5 \cdots\right]$$

$$= l\left[\frac{1}{2}\frac{w\,l}{T} + \frac{1}{4}\left(\frac{w\,l}{T}\right)^3 + \frac{13}{48}\left(\frac{w\,l}{T}\right)^5 + \frac{1}{24}\left(\frac{w\,l}{T}\right)^3\right.$$

$$\left. + \frac{3}{48}\left(\frac{w\,l}{T}\right)^5 + \frac{1}{720}\left(\frac{w\,l}{T}\right)^5 \cdots\right]$$

$$d = l\left[\frac{1}{2}\frac{w\,l}{T} + \frac{7}{24}\left(\frac{w\,l}{T}\right)^3 + \frac{241}{720}\left(\frac{w\,l}{T}\right)^5 \cdots\right] \quad (1)$$

II. Equation (1) can be transformed by a similar process to give a series for T in terms of d.

$$\frac{d^4}{l^4} = \frac{1}{16}\left(\frac{w\,l}{T}\right)^4 \cdots$$

$$\frac{d^2}{l^2} = \frac{1}{4}\left(\frac{w\,l}{T}\right)^2 + \frac{7}{24}\left(\frac{w\,l}{T}\right)^4 \cdots$$

$$= \frac{1}{4}\left(\frac{w\,l}{T}\right)^2 + \frac{14}{3}\frac{d^4}{l^4}$$

$$\left(\frac{w\,l}{T}\right)^2 = 4\frac{d^2}{l^2} - \frac{56}{3}\frac{d^4}{l^4}$$

$$\frac{d}{l} = \frac{1}{2}\frac{w\,l}{T}\left[1 + \frac{7}{3}\frac{d^2}{l^2} - \frac{98}{9}\frac{d^4}{l^4}\right.$$

$$\left. + \frac{241}{360} \times 16\frac{d^4}{l^4} \cdots\right]$$

$$\frac{T}{w\,l} = \frac{1}{2}\frac{l}{d} + \frac{7}{6}\frac{d}{l} - \frac{4}{45}\frac{d^3}{l^3} \cdots \quad (2)$$

III. Perimeter of catenary in terms of sag.

$$\text{Perimeter} = 2\,l\frac{H}{w\,l}\sinh\frac{w\,l}{H}$$

$$= 2\,l\left[1 + \frac{1}{6}\left(\frac{w\,l}{H}\right)^2 + \frac{1}{120}\left(\frac{w\,l}{H}\right)^4\right.$$

$$\left. + \frac{1}{5040}\left(\frac{w\,l}{H}\right)^6 \cdots\right]$$

As in I,

$$\frac{d}{l} = \frac{1}{2}\frac{w\,l}{H} + \frac{1}{24}\left(\frac{w\,l}{H}\right)^3 + \frac{1}{720}\left(\frac{w\,l}{H}\right)^5 \cdots$$

$$\frac{d^6}{l^6} = \frac{1}{64}\left(\frac{w\,l}{H}\right)^6 \cdots$$

$$\frac{d^4}{l^4} = \frac{1}{16}\left(\frac{w\,l}{H}\right)^4 + \frac{1}{48}\left(\frac{w\,l}{H}\right)^6 \cdots$$

$$= \frac{1}{16}\left(\frac{w\,l}{H}\right)^4 + \frac{64}{48}\frac{d^6}{l^6} \cdots$$

$$\left(\frac{w\,l}{H}\right)^4 = 16\frac{d^4}{l^4} - \frac{64}{3}\frac{d^6}{l^6} \cdots$$

$$\frac{d^2}{l^2} = \frac{1}{4}\left(\frac{w\,l}{H}\right)^2 + \frac{1}{24}\left(\frac{w\,l}{H}\right)^4 + \frac{1}{720}\left(\frac{w\,l}{H}\right)^6$$

$$+ \frac{1}{576}\left(\frac{w\,l}{H}\right)^6 \cdots$$

$$= \frac{1}{4}\left(\frac{w\,l}{H}\right)^2 + \frac{2}{3}\frac{d^4}{l^4} - \frac{8}{9}\frac{d^6}{l^6} + \frac{4}{45}\frac{d^6}{l^6}$$

$$+ \frac{1}{9}\frac{d^6}{l^6} \cdots$$

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[9] For the derivation of these, see "Principles of Electric Power Transmission" by L. F. Woodruff, Chap. VI, or other texts.

$$\left(\frac{w\,l}{H}\right)^2 = 4\,\frac{d^2}{l^2} - \frac{8}{3}\,\frac{d^4}{l^4} + \frac{124}{45}\,\frac{d^6}{l^6}\ \ldots$$

$$\text{Perimeter} = 2l\left(1 + \frac{2}{3}\,\frac{d^2}{l^2} - \frac{4}{9}\,\frac{d^4}{l^4} + \frac{62}{135}\,\frac{d^6}{l^6}\right.$$

$$\left. + \frac{2}{15}\,\frac{d^4}{l^4} - \frac{8}{45}\,\frac{d^6}{l^6} + \frac{4}{315}\,\frac{d^6}{l^6}\ \ldots\right)$$

$$= 2\,l\left(1 + \frac{2}{3}\,\frac{d^2}{l^2} - \frac{14}{45}\,\frac{d^4}{l^4} + \frac{278}{945}\,\frac{d^6}{l^6}\ \ldots\right)$$

as in equation (**3**).

IV. Stretch of Cable.

At any horizontal distance x from the lowest point of the span, the stress is

$$\frac{T}{A} = \frac{H}{A}\cosh\frac{w\,x}{H} \text{ pounds per square inch}$$

where A = area of cross-section.

Let u = perimeter of the catenary as far as the horizontal distance x.

Increase in length of $d\,u$ is

$$\frac{T\,d\,u}{E\,A} = \frac{H\ \cosh\dfrac{w\,x}{H}}{E\,A}\,d\,u$$

$$\text{Now } d\,u^2 = \left[1 + \left(\frac{d\,y}{d\,x}\right)^2\right] d\,x^2$$

$$= \left[1 + \sinh^2\frac{w\,x}{H}\right] d\,x^2$$

$$d\,u = \cosh\frac{w\,x}{H}\,d\,x$$

Increase in length of $d\,u$ is

$$\frac{H\ \cosh^2\dfrac{w\,x}{H}}{E\,A}\,d\,x$$

Integrate from $x = -\,l$ to $x = l$ to obtain the increase in length of the cable. See "Short Table of Integrals" by B. O. Peirce, Nos. 460 and 663.

Increase in length of cable

$$= \frac{H^2}{2\,E\,A\,w}\left(\sinh\frac{2\,w\,l}{H} + \frac{2\,w\,l}{H}\right)$$

$$= \frac{H^2}{2\,E\,A\,w}\left[\frac{4\,w\,l}{H} + \frac{4}{3}\left(\frac{w\,l}{H}\right)^3 + \frac{4}{15}\left(\frac{w\,l}{H}\right)^5\ \ldots\right]$$

$$= \frac{2\,w\,l^2}{E\,A}\left[\frac{H}{w\,l} + \frac{1}{3}\,\frac{w\,l}{H} + \frac{1}{15}\left(\frac{w\,l}{H}\right)^3\ \ldots\right]$$

Now from I,

$$\frac{d}{l} = \frac{1}{2}\,\frac{w\,l}{H}\left[1 + \frac{1}{12}\left(\frac{w\,l}{H}\right)^2 + \frac{1}{360}\left(\frac{w\,l}{H}\right)^4\ \ldots\right]$$

$$\frac{H}{w\,l} = \frac{1}{2}\,\frac{l}{d}\left[1 + \frac{1}{3}\,\frac{d^2}{l^2} - \frac{2}{9}\,\frac{d^4}{l^4} + \frac{2}{45}\,\frac{d^4}{l^4}\ \ldots\right]$$

$$= \frac{1}{2}\,\frac{l}{d}\left[1 + \frac{1}{3}\,\frac{d^2}{l^2} - \frac{8}{45}\,\frac{d^4}{l^4}\ \ldots\right]$$

$$\frac{w\,l}{H} = \frac{2\,d}{l}\left(1 - \frac{1}{3}\,\frac{d^2}{l^2}\right)\ \ldots$$

$$\left(\frac{w\,l}{H}\right)^3 = \frac{8\,d^3}{l^3}\ \ldots$$

Increase in length of cable due to tension

$$= \frac{2\,w\,l^2}{A\,E}\left(\frac{1}{2}\,\frac{l}{d} + \frac{1}{6}\,\frac{d}{l} - \frac{4}{45}\,\frac{d^3}{l^3} + \frac{2}{3}\,\frac{d}{l} - \frac{2}{9}\,\frac{d^3}{l^3}\right.$$

$$\left. + \frac{8}{15}\,\frac{d^3}{l^3}\ \ldots\right)$$

$$= \frac{w\,l^2}{A\,E}\left(\frac{l}{d} + \frac{5}{3}\,\frac{d}{l} + \frac{4}{9}\,\frac{d^3}{l^3}\right) \text{ as in equation (3)}$$

V. Supports at Unequal Heights[10].

Referring to Fig. 1,

$$d = \frac{H}{w}\left[\cosh\frac{w\,n}{H} - 1\right]$$

$$d + q = \frac{H}{w}\left[\cosh\frac{w(2\,k-n)}{H} - 1\right]$$

$$q = \frac{H}{w}\left[\cosh\frac{w\,n}{H} - \cosh\frac{w\,(2\,k-n)}{H}\right]$$

$$\cosh a - \cosh b = 2\sinh\frac{a+b}{2}\,\sinh\frac{a-b}{2}$$

$$q = 2\,k\,\frac{H}{w\,k}\,\sinh\frac{w\,k}{H}\,\sinh\frac{w\,(k-n)}{H}\ \text{ft.} \qquad \textbf{(18)}$$

Let

$$b = \frac{q}{2\,k}$$

$$\sinh\frac{w(k-n)}{H} = b\left[1 + \frac{1}{6}\left(\frac{w\,k}{H}\right)^2\right.$$

$$\left. + \frac{1}{120}\left(\frac{w\,k}{H}\right)^4\ \ldots\right]^{-1}$$

$$= b\left[1 - \frac{1}{6}\left(\frac{w\,k}{H}\right)^2 + \frac{7}{360}\left(\frac{w\,k}{H}\right)^4\ \ldots\right]$$

$$\text{also} = \varphi + \frac{1}{6}\,\varphi^3 + \frac{1}{120}\,\varphi^5 + \frac{1}{5040}\,\varphi^7\ \ldots$$

---

[10] See Reference 9.

where

$$\varphi = \frac{w\,(k-n)}{H}$$

$$\varphi^7 = b^7 \ldots$$

$$\sinh^5 \varphi = \varphi^5 + \frac{5}{6}\varphi^7 \ldots = b^5 \left\{ 1 - \frac{5}{6}\left(\frac{w\,k}{H}\right)^2 \ldots \right\}$$

$$\varphi^5 = b^5 \left\{ 1 - \frac{5}{6}\left(\frac{w\,k}{H}\right)^2 \ldots \right\} - \frac{5}{6} b^7$$

Similarly, it is found that

$$\varphi^3 = b^3 \left\{ 1 - \frac{1}{2}\left(\frac{w\,k}{H}\right)^2 + \frac{17}{120}\left(\frac{w\,k}{H}\right)^4 \ldots \right\}$$

$$- \frac{1}{2} b^5 \left\{ 1 - \frac{5}{6}\left(\frac{w\,k}{H}\right)^2 \ldots \right\} + \frac{37}{120} b^7$$

$$\varphi = \frac{w(k-n)}{H} = \sinh \varphi - \frac{1}{6}\varphi^3 - \frac{1}{120}\varphi^5 - \frac{1}{5040}\varphi^7$$

Substituting the values given above and putting $n = 2\,k - m$, equation (**11**) is obtained.

VI. The following series may prove useful in certain cases:

Supports at Equal Heights

$$H = w\,l \left[\frac{1}{2}\frac{l}{d} + \frac{1}{6}\frac{d}{l} - \frac{4}{45}\frac{d^3}{l^3} \ldots \right] \text{lb.} \qquad (\mathbf{19})$$

$$H = T\left[1 - \frac{1}{2}\left(\frac{w\,l}{T}\right)^2 - \frac{7}{24}\left(\frac{w\,l}{T}\right)^4 - \frac{241}{720}\left(\frac{w\,l}{T}\right)^6 \ldots \right] \text{lb.} \qquad (\mathbf{20})$$

$$T = H\left[1 + \frac{1}{2}\left(\frac{w\,l}{H}\right)^2 + \frac{1}{24}\left(\frac{w\,l}{H}\right)^4 + \frac{1}{720}\left(\frac{w\,l}{H}\right)^6 \ldots \right] \text{lb.} \qquad (\mathbf{21})$$

Supports at Unequal Heights

$$\frac{d}{n} = \frac{n}{2\,k}\left[\left(\frac{q}{2\,z}\right) \left\{ 1 - \frac{1}{6} b^2 + \frac{3}{40} b^4 \ldots \right\} - \frac{1}{6}\left(\frac{q}{2\,z}\right)^3 \left\{ 1 - \frac{5}{6} b^2 \ldots \right\} + \frac{3}{40}\left(\frac{q}{2\,z}\right)^5 \ldots \right]$$

$$+ \frac{n^3}{24\,k^3}\left[\left(\frac{q}{2\,z}\right)^3 \left\{ 1 - \frac{1}{2} b^2 \ldots \right\} - \frac{1}{2}\left(\frac{q}{2\,z}\right)^5 \ldots \right]$$

$$+ \frac{n^5}{720\,k^5}\left(\frac{q}{2\,z}\right)^5 \ldots \qquad (\mathbf{22})$$

where $z = k - n$. Thus, a value of n can be assumed and d found directly. However, it seems somewhat preferable to assume a value of H and then find m, n and d by (**11**) and (**12**).

Series (**22**) should be used only when the terms of higher orders which it contains, are all quite small.

VII. If a bare cable is suspended between two fixed supports, an increase in temperature will increase the length of the cable. However, the total weight of the cable remains the same, and so w, the weight per foot of cable, decreases by the same percentage that the actual perimeter of the catenary increases. This can be easily allowed for in connection with some of the formulas in this paper for finding differences of temperature.

The first line of formula (**3**) gives the actual perimeter of the catenary. In finding L_{u0} and L_u, corresponding to two values of d, the value of w used to obtain L_u may be decreased in inverse proportion to the actual perimeters in the two cases. A similar process can be used with formulas (**22**) and (**14**) for supports at unequal heights.

The effect of change in w with change in perimeter of catenary has been included in the calculations in the paper by G. S. Smith[11] to which reference has been made.

An investigation has been made to find the proprotionate effect of this feature in a number of practical cases, including data very similar to that in the paper by G. S. Smith. The effect on the calculated value of tension was extremely small. The value of temperature, which is rather sensitive to the precision of the calculations, was changed by only a small fraction of one degree. Accordingly, w is shown as a constant in the formulas in the main part of this paper. Allowance for the change in w can be made, if desired, as indicated in this appendix.

Discussion

M. G. Lloyd: I want to say something about the problem in general, and how it has been handled elsewhere. There are two ways in which it may come up.

In a line that is already constructed, if you know what your sag is, you can get the tension very simply by this formula (**2**) which the author has given and which expresses the tension in terms of the sag, the loading, and length of span.

I think the more usual case, however, comes up in the design of a line where you want to know how to string the wire so that under the worst condition of loading which it will experience or which may be assumed, it will not be stressed beyond a definite fraction of the strength. In that case you start out with your loading and with your definite tension and you want to find the sag. Formula (**1**) does that very simply for that loaded condition.

The construction man puts it up, however, under some other condition, and he wants to know what the corresponding sag is under stringing conditions, and that is the thing which gives a lot of trouble.

As the author here points out, by using these formulas, one can work back by a method of trial and error or successive approximations, or you can, using formula (3) and others, work out a number of cases and get a set of curves from which then you may be able to read off directly the particular value which you want.

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[11] *Loc. Cit.* See Reference 7.

I want to speak, however, of another method of doing that which we have found more simple and time-saving in the long run. And I might say that at the Bureau of Standards I think perhaps we have done as much sag computing as anywhere in the country, in connection with the sag tables and curves for the National Electrical Safety Code.

In an Institute paper[1] which I think we may call one of the classics on this subject, Percy H. Thomas showed how, in plotting a curve between sag and stress, one could represent what we might call a generalized condition, by expressing sag as a percentage of span length and applying the loading in unit terms. That corresponds to the first formula of the paper and it is all right for the simple case which does not involve conditions of changing temperature, because it is changing temperatures that bring in the greatest difficulty.

A little later, two students, Messrs. Melvin and Wynne at the Massachusetts Institute of Technology, with whose thesis work I presume Prof. Dwight is not familiar, pointed out how a series of curves could be plotted to assist greatly in solving this problem. They plotted a whole family of curves which apply to different loadings of a given material, and then plotted on the same sheet of paper, another set of curves which are the stretch curves at different temperatures. The two families of curves intersect. (They used the parabolic relation in getting these but in the later work we have used the catenary relation in order to have them more accurate.)

What we call the stretch curve will represent conditions corresponding to some single temperature when the load on the wire is varied.

Now suppose the load is entirely removed from the wire; (and in that load I include the weight of the wire itself); we come all the way down on the stretch curve to the axis (the limiting member of the sag-stress curve being comprised of two axes) just as though the wire had no weight or load, and could be strung without sag or tension.

In using the old Thomas method, one would compute what we are now doing graphically; that is, take the load off the wire, get the condition of the wire with no load, then assume a change in temperature and find the change of the unstressed wire in length due to change of temperature. Upon arriving at that new length at the new temperature, apply the load again and then find the condition of the wire in sag and tension

The method of using the new charts is this: Pick out the point representing the stress,—that is, the tension reduced to pounds per square inch,—and the corresponding sag. You find what that is for your loaded condition.

Now then, if you want to find what the sag would be at some different temperature under the stringing condition of load,—that is, no load except the weight of the wire itself,—you follow the stretch curve to the axis.

The stretch curves are plotted for definite intervals of temperature, say, ten degrees. If the coefficient of expansion is a constant, it doesn't matter what the temperature is; one can shift the temperature scale to suit. Say you have taken the loaded condition at zero degrees and the stringing temperature at 60 deg.; you follow the axis for 60 deg. and return on another constant-temperature line to the load represented by, say, the weight of the conductor only. This point gives the stress and the sag for your stringing condition.

That gives a very rapid method of computing sags without any successive approximations and will fit any temperature and any loading. The only requirement necessary is that you have your chart to begin with.

If one has a great deal of this to do, it is worth while to compute the chart and have it on hand. Of course if one has only one problem to work out, it is easier to use the formulas. The author mentions a paper by Martin presented before the Engineers Society of Western Pennsylvania in which he has worked out some tables giving the catenary functions. They are very useful in a single case; in fact, we have sometimes found them preferable even in doing a lot of such work, since working over curves like these and attempting to interpolate between the drawn curves to obtain accurate results requires very close attention and careful work and it becomes very tiresome and exhausting if kept up for any length of time. On that score, it is easier to do computing, if you can do it, in a mechanical way with computing machines, as it takes a great deal less mental effort and less eye work.

I might say also that we have worked these charts out with the principal materials used, such as copper, aluminum and steel. In working them out, we used formulas somewhat similar to these in the paper, expressing the catenary functions in series.

H. B. Dwight: Has the work with curves for sag calculations, as described, been done for the case of supports at unequal heights or for cases of supports at equal heights only?

Mr. Lloyd: What I said applied to equal heights only.

H. B. Dwight: Is there any work on the other as yet?

Mr. Lloyd: No, not to work it out in the same way, except that with unequal heights, in considering the curve of the wire extended, you have always equivalent cases of equal heights of a span of greater length, of which your actual span is merely a portion; and you can always work it out in that way.

E. V. Pannell: The high degree of accuracy being striven for in transmission line calculations is most noteworthy, but the question is whether the mathematicians have not gone a long way ahead of those responsible for testing the fundamental properties of material. While sag and tension calculations are being made to four, five, or even six significant figures, it must be admitted that knowledge of the physical properties of the wire does not come anywhere near this degree of exactitude.

The fundamental property in all calculations of wire extension is the modulus of elasticity. For copper wire, this is variously reported as from 16,000,000 to 18,000,000 lb. per sq. in. Here is evidently a difficulty and a possible error of 12½ per cent. In the case of aluminum, various investigators have reported a modulus of 9,000,000 and 9,500,000 and 10,000,000 lb. per sq. in., involving a possibility of error almost as great as for copper.

The properties of the wire when stranded into cables of various lengths of lay and characteristics become still more involved and except in the case of certain German laboratories, I know of no thorough tests having been made on a scientific basis by which attempt could be made to establish these properties.

It would seem as if the great degree of accuracy possible in a mathematical manipulation described by Dr. Dwight and other investigators is very nearly useless until the physical constants of wire and cable are more soundly established.

H. B. Dwight: The discussion by Mr. Pannell brings out a matter of considerable engineering importance. The physical characteristics of stranded cables, such as modulus of elasticity and temperature coefficient of expansion, are not known with exactness for standard conductors. Accordingly, further tests should be made to determine the average values of these constants and their usual amount of variation.

It is not necessarily obvious at first sight what is the effect of a certain percentage change in one of the constants, and it would appear to be a good engineering procedure to repeat a sag calculation using maximum and minimum values of a constant, so as to find what effect the variation has on the engineering decisions which depend on the calculation. If this were done for the two constants mentioned above, important changes in the results would be obtained.

In general, one should not write the values of engineering quantities with a precision greater by more than one significant figure than the precision of the measurements on which the quantities depend. Exceptions to this rule, however, are sometimes justified. For instance, one is justified in assuming the length of a span to be exact, as 800.00 feet. The sag calculation really deals with small changes in length. If the calculation should be repeated for a span of 801.00 feet the difference would be minute. If the length of span is considered to change after the application of a load, as by deflection of the towers, this should be taken up in a separate calculation.

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[1] Trans. A. I. E. E. 30, p. 229 (1911).

When one wishes to compare two methods of calculation, one is justified in assuming for that purpose that all the constants are known with precision. The results are useful for the purpose intended.

Even where a constant is not definitely known, it is good engineering practise to adopt a standard value so as to design all the spans of a transmission line to have the same factor of safety rather than by irregular designing to have one span weaker than the others. Thus, a standard value of ice load or wind load is adopted even though the probable maximum value is uncertain. So also for the sake of uniform design of all the spans, it is proper to take a standard or average value of modulus of elasticity. The effect of possible variations in this constant can be made the subject of a separate investigation, as previously mentioned.

Where average values have been taken for several quantities, their deviations will probably cancel out to some extent in their effect on the final result, and so it is justifiable to carry out the computations based on the average values, with a moderate degree of precision.

The above mentioned reasons for making precise computations do not justify adding more than about one or two significant figures except in the case of the length of the span, or in the case of comparing two mathematical methods. In Table I of the paper by Dr. G. S. Smith[2], the modulus of elasticity is given as 29,000,000; that is, to two significant figures. This is multiplied by some quantities depending upon the section of the cable, and the result is given in section (13) of Table I as 3,153,306,102; that is, to 10 significant figures. This is due apparently to the plan of writing the result of all multiplications with as many significant figures as a calculating machine will give, but when this is done, it does not seem possible to tell what figures have use and meaning and what have no meaning. Such a procedure masks the precision of the different parts of the calculation, and gives the impression that no attention is being paid to the relative precision of the calculation. It would seem better to give only such figures as are intended to be used.

The use of charts has a place in the calculation of sags, as described by Dr. M. G. Lloyd or as described in a number of A. I. E. E. papers. A chart method may give a sufficient degree of precision for a certain class of work, but this should be carefully determined. It should be remembered that a reading taken from a curve is correct to a certain number of significant figures, and the final result cannot be accurate to a greater degree than that determined by the curve reading. In sag calculations, especially where temperatures are involved, the discrepancies resulting from taking readings on curves may be greater than are desirable in designing.

The discrepancy between the values of deflections obtained in my paper and in that of reference (8) is due to the fact that two different catenaries have been assumed in the two papers, for the case of unequal supports with wind load. While the deflections are different, since they are measured to different lines, the difference in the stresses obtained by the two calculations is not appreciable for practical spans.

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[2] TRANSACTIONS A. I. E. E., 1925, p. 938.

The Cosmic Harness of Moving Electricity

President's Address

BY M. I. PUPIN
Fellow, A. I. E. E.

MOVING ELECTRICITY THE MOTHER OF ELECTRICAL ENGINEERING

THE Annual Convention brings us together for the purpose of advancing our knowledge of each other and of the ideals of our Society. We believe that our ideals are in harmony with the ideals of American science and American engineering, and, moreover, we believe that we have a place of honor among those whose mission is the cultivation and amplification of these ideals. They furnished the motive power for the rapid advancement of the science and the art of electrical engineering during the last hundred years. The advancement has been very rapid, but, nevertheless, no other art has a scientific foundation which is so deep, so broad, and so firm as the foundation of electrical engineering. In no other department of human knowledge are science and art so closely welded together. These statements, I know, many will consider as somewhat too bold. I shall try to justify them by referring briefly to the outstanding events in the history of the art of electrical engineering and of the science to which it is welded.

The very meaning of the word engineering implies an art which guides the activities of physical forces into channels of useful service. When the Galileo-Newton philosophy had disclosed the laws of motion of terrestrial as well as of celestial bodies, a new universe was revealed to man, a universe of orderly motion of matter in obedience to forces acting in accordance with laws of child-like simplicity. This philosophy suggested to the engineer new sources of power and service, and to the natural philosopher a new and apparently most comprehensive view of physical phenomena. Some philosophers, thrilled by the beauty of the new knowledge, believed that the whole future history of the universe could be foretold by the Galileo-Newton philosophy if we only knew at any given moment the configuration, the state of motion of every one of its parts and of the forces acting between these parts. That was the mechanistic view of the universe which flourished soon after the triumph of Newton's great achievements. These achievements, however, misled some enthusiasts into the belief that all physical phenomena are reducible to orderly motions of matter under the action of gravitational forces. But as soon as man had discovered that other processes, not expressible in terms of motion of matter, formed an essential part of physical phenomena, that belief was abandoned.

Among these processes, the motion of electricity stands foremost. The new universe revealed by our knowledge of the motions of electricity appeals to our imagination so strongly today that many would not hesitate to rewrite the first sentence of the book of Genesis as follows: "In the beginning God said, 'Let electricity move, and the embryo of the Universe began to form.'" Perhaps in a hundred years from now such a glorification of the motion of electricity will appear just as extravagant as the old mechanistic view of the universe. There is no doubt, however, that the nineteenth, and the first quarter of the twentieth century, will long be remembered as the epoch which revealed to us the hidden powers of electrical motions and their exalted position in our present knowledge of the universe. Who could have foretold all this when Stephen Gray, less than two hundred years ago, modestly announced that electricity can move any distance over conductors and that it does move with enormous rapidity? The world paid small attention to Gray's great discovery, and it might have continued its indifference if Franklin, instructed by his Leyden jar discharges, had not inferred that lightning is a motion of electricity. Gray's modest terrestrial experiment received from Franklin a celestial illustration which commanded attention, although it was ridiculed by some learned members of the Royal Society. The motion of electricity which, in Gray's experiment, was detected by a tiny electroscope, assumed a sublime aspect when its flash in the heavens blinded the eye, deafened the ear, and shattered many stable structures of man.

Franklin's discovery of the electrical character of lightning was a great stimulus to the study of the motion of electricity. One may compare it to the stimulus which the Copernicus-Kepler revelation concerning the motion of the planets gave to the study of the motions of terrestrial bodies which Galileo inaugurated. Just as Copernicus and Kepler gave us a Galileo and a Newton, so Gray and Franklin were destined to be succeeded by an Oersted and a Faraday. But it required a Volta to introduce Oersted; it required large electrical motions to reveal the magnetic forces of moving electricity which Oersted discovered.

Prior to Oersted, the engineer moved material bodies and guided their motions into channels of useful service by providing a material connection between the driving and the driven body. Oersted showed that a material body which is the seat of electrical motions can make other bodies move without a material connection between them. The magnetic flux is the invisible coupling. Oersted's discovery of the magnetic field which accompanies electrical motions promised, therefore, to give birth to a new type of engineering, employing a new type of coupling. This promise was

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one of the great incentives to the advancement of the new knowledge. The mechanical action of an electrically charged body upon other bodies gave a similar promise, but it failed, and it was destined to fail to make that promise good. The promise of the Oersted discovery blossomed out into a reality more beautiful than the fairest dream which prompted that promise. Electricity in motion offered to the engineer a moving force which proved much more powerful than that offered by electricity at rest. The lifting power of Henry's electromagnets was immeasurably greater than the lifting power of the gravitational action of the material out of which his electromagnets were made. One can imagine today what an impression that new fact must have made upon the mind of the engineer of a hundred years ago. Today one can say that electrical engineering is the science and the art which tell us how to make material bodies move by employing an invisible harness hitching up these bodies to moving electricity. It was born when Oersted made his discovery, but its growth was destined to be slow as long as the Voltaic battery was the only powerful means of generating and sustaining electrical motions. To Faraday belongs the glory of discovering a new and much more powerful instrument than the Voltaic battery. It was his clear vision which prophesied a reciprocal relation between moving electricity and moving magnetism. The prophesy was probably the offspring of the intuition which suggested that since moving electricity moves magnets, it is reasonable to expect that moving magnets will move electricity. This expectation proved correct, and it offered to the engineer an ideally simple and powerful method of setting electricity in motion. The promise of Oersted's discovery to the engineer assumed a new meaning after Faraday's discovery, and electrical engineering began its career which placed it in the exalted position it has today among the engineering sciences. The efforts of the electrical engineer to render useful service by hitching up material bodies to moving electricity resulted in the creation of the dynamo, the motor, the transformer, the telegraph, the telephone, and other epoch-making devices which have revolutionized the material conditions of human life. Grateful mankind responded with a generous support of the science which gave birth to and nursed the young art of electrical engineering and, like a wise mother, gave it its exalted ideals. These ideals are the bond of union between electrical engineering of today and its trusty guide, the electrical science. The progress of one brings quickly an equal progress of the other, because hand in hand they always walk together with equal step. One cannot contemplate their stately walk without recalling to mind the well-known line from one of the odes of Horace:

"O matre pulchra, filia pulchrior!".

THE HARNESS OF MOVING ELECTRICITY

The enormous lifting power of electromagnets was the great scientific sensation of a hundred years ago. It excited the lively imagination of Joseph Henry, at that time a young engineer, and he was the first to give it a novel service when he designed the first electromagnetic telegraph which gave the first real job to the electrical engineer of a hundred years ago. The enormous lifting power of the electromagnet furnished also a new job to the natural philosopher when it forced upon him the question: What is the invisible coupling through which this force is transmitted from the stationary to the movable part of the electromagnet? Faraday was the first to suggest an answer to this question. His discoveries and visions detected what one may call an invisible electromagnetic harness to which all material bodies in the universe are attached and which is always available to be employed by moving electricity in useful service. Faraday and Maxwell taught us that this harness is woven out of the electrical and magnetic tubes of force. Change the electrical elements of this cosmic harness in any part of space and its magnetic elements will also be changed in accordance with Maxwell's extension of Ampère's law. Change its magnetic elements and its electrical elements will be changed in accordance with Maxwell's extension of Faraday's law of electromagnetic induction. It is by these changes that an action is transmitted from one part of free space to another. A more complete and, at the same time, ideally simple description of the operation of the invisible harness than that given by Maxwell was unthinkable sixty-one years ago. It became the foundation of the electrical science as well as of the electrical art, that is, of electrical engineering; it welded the two to each other. Faraday and Maxwell performed the welding process. Their mode of thought appealed to the engineer because it expressed the motion of energy from one part of space to another in terms of the action of the invisible coupling between them, furnished by the tubes of force. No elaborate mathematical process was required to aid our understanding of this action and yet the Faraday-Maxwell electromagnetic theory was often accused of being too mathematical, because its fundamental laws, mentioned above, when expressed mathematically were called Maxwell's equations. This conveyed the idea that the theory is a mathematical apparatus which cannot be operated by the mathematical skill of an ordinary electrical engineer and therefore of no use to him. Nothing can be more erroneous than this notion. Nothing is more concrete and simple than the Faraday-Maxwell electric and magnetic flux and nothing is more extensively used by the electrical engineer than these fluxes and the simple laws which govern their activity. No elaborate mathematical apparatus is necessary in order to understand that the Faraday-Maxwell science revealed to us the most accurate understanding of not only the mode of operation of the invisible coupling in ordinary electrical power generation and transmission, but also in the transmission of radiant energy from the distant stars to our terrestrial globe.

This understanding gave us the first glimpse of that unity of the universe in which the invisible harness, joining every one of its parts to every other part, is always ready to transmit service which moving electricity makes available.

Faraday and Maxwell, however, had not spoken the last word concerning the invisible cosmic coupling. New explorers of the boundless region, revealed by the visions of Faraday and Maxwell, have delivered and are still delivering new messages from this region, unfolding many of its secrets. What are these secrets and how does their unfolding affect the views of the electrical science and its art, electrical engineering?

Franklin was the first to profess the belief that all electricity has its origin in material bodies. Faraday's discovery, that to each atomic valency there is attached a definite electrical charge, gave Franklin's belief a more intelligible form, which appealed to our imagination more and more as the conviction grew stronger, that all chemical reactions are due to the activities of the atomic charges. It was this conviction which suggested the name "electron" to the smallest unit of atomic charges long before its independent individual existence had been demonstrated by actual experiment. Roentgen's discovery of the X-ray suggested the hypothesis that these rays are excited by the impact upon the anode of tiny projectiles, shot forth with enormous velocities from the cathode of a high vacuum tube. Experiment proved that these projectiles are the individual electrons, the existence of which in the atomic structure had been suggested by Faraday's electrochemical discoveries; experiment also determined their electrical charge and inertia. This is the foundation of modern *electron-physics* and it is so broad that it furnishes new support to the foundation of the Faraday-Maxwell electromagnetic theory, to chemistry, astrophysics, meterology, biology, and, above all, to electrical engineering. It has created a new electrical industry and a new type of electrical engineering. It is the busy electron in the amplifying vacuum tube which gives life to the radio broadcasting industry and supplies new problems to the electrical engineer, the so called radio engineer; it carried conviction to those who were inclined to think that the tiny electron was only a fiction of a supersensitive scientific imagination.

THE ELECTRON, THE PRIMORDIAL UNIT OF POWER GENERATION

The marvelous success of this new electrical industry and of the electrical engineering which guides it, directed our attention to the function of the electron in all electrical power operations. The result is that today the electron and its positive partner, the proton, have become the fundamental concepts in the science of modern physics and in the art of electrical engineering. The tubes of electrical force between them are the primordial electrical flux, the fundamental and the only substance in the web of the cosmic harness. The relative motion of that primordial flux manifests itself as the magnetic flux which measures the momentum of this relative motion. Relative to what? Relative to the observer who is measuring that momentum. A charge moving with the observer has no momentum relative to the observer and is not accompanied by a magnetic field which the observer can detect.

Electron-physics made a fundamental contribution to the achievements of Oersted, Faraday and Maxwell when it demonstrated the individual existance of the electron and the proton and pointed out that the electrical flux, which unites the two, is the primordial flux, the cosmic bond of union between all electronic granules in the universe. This is the invisible harness to which all parts of the cosmic space are hitched up. To the electrical engineer who is a disciple of the Faraday school of thought the electronic granules are unintelligible except as local convergencies of the primordial flux. It is the activity of the flux which tells him the story of energy movement from one part of space to another and without this energy transference, the motion of the isolated electronic granules would have but a very small interest for him. He is, it is true, interested in the cosmic processes by which heavier atoms are evolved out of lighter atoms by a suitable grouping of the electronic granules, but that which interests him incomparably more is the energy liberation in these processes and the invisible harness along which the liberated energy is transmitted, destined to perform some useful service in some distant part of space. He is also interested in the energy which is stored up in the formation of the atomic nucleus, and how much of it can be made available when the structure of the nucleus is changed as in radioactivity.

Electron-Physics interpreted in terms of Faraday's visions and Maxwell's quantitative formulation of them suggests to the electrical engineer a universe which reminds him of a power distribution system in which there is an endless number of power stations all interconnected by the primordial flux. Material bodies, from the smallest atoms to the biggest stars, are, according to this picture of the universe, local aggregations of electronic centers in the all-embracing, primordial flux. This cosmic structure, however, is not a static but a dynamic one. Every one of its electronic centers is in a state of activity, receiving energy from its busy neighbors and giving it out without cessation or rest. It is pulled by or is pulling at the cosmic harness to which it is inseparably attached. It is doing its share of service in the evolving universe, and how much of this service is to benefit man depends upon the man himself; upon his science and art of electrical engineering. It is the problem of the electrical engineer to transform the activity of the infinitely numerous, but infinitely small, electronic toilers in the cosmic power stations into orderly service for the uplift of the life of man. He is the coordinator of the restless activity of these toilers; they follow his bidding as if guided by the magic wand which Faraday and Maxwell and their disciples gave him; they are his obedient servants. Here, they heat an electrical furnace and there, they guide chemical reactions; here, they drive the propellers of a battleship, and there, they turn the busy wheels of an industrial plant; here, they speedily carry the weary industrial toiler to his home and there, they make it cozy and comfortable by their light-giving service; here, they record the cheerless figures of the stock exchange ticker and there, they carry sweet melodies and soul stirring language to the millions of eager listeners on this hopeful continent.

It is a master mind, indeed, that can thus control the activity of an infinitely numerous army of toilers. No vulgar rule of the thumb can find a lasting place in the logic of such a mind; its art is an exact science and its science is supported by an art the experience of which, through many generations, has been tested by methods of measurement of astronomical precision. No vague and hazy notions obscure the lucidity of the electrical engineer's operations. The enormous electrical efforts of his million-kilowatt power station are just as lucid to him as the feeble efforts of the tiny electrical power which brings us the wireless message from distant Australia. Both of these efforts are huge in comparison with the efforts of a single electronic toiler in terms of which the electrical engineer can express every electrical effort. He knows the numerical value of the labor of these tiny workers and he also knows that it is their toil by which the lily, without toiling or spinning, arrays itself in beauty which far surpasses Solomon in all his glory. It is their toil which promises to the civilization of man a beauty and glory which will far surpass the beauty of the lily. The mission of the electrical engineer is to make this promise good. In the performance of this mission, he will keep always in mind the words of St. Luke who glorified the blessedness of the indolent lily. The apostle said:

> "And seek not ye what ye shall eat, or what ye shall drink, neither be ye of doubtful mind."
>
> "But rather seek ye the kingdom of God; and all these things shall be added unto you."

The men who made the science and the art of electrical engineering did not seek what they should eat or drink. But in their thirst and hunger for the eternal truth they did seek and find, in part at least, the kingdom of God, which resides in the beauty of their science and its art, and in the beauty of the universe which they reveal. That science and its art are the creation of a new philosophy which we call the philosophy of idealism in science. It is the simplest philosophy ever constructed by the mind of man and represents the essence of scientific experience of centuries; an experience which was always guided by a definite motive, a definite mental attitude, and a definite method of work. The motive was the unselfish longing for God's eternal truth; the mental attitude always demanded an open minded and unprejudiced interpretation of nature's language; the method of work is that by which our patron saints, Gray, Franklin, Volta, Oersted, Ampère, Faraday, Maxwell, Roentgen and their disciples, created the science and the art of electrical engineering. This motive, mental attitude, and method of work is the firmest foundation of the scientific idealism which is the idealism of our profession and we have always been the leaders in the propagation of its gospel. We were the earliest apostles who converted the American industries, so that today they worship at the altar of the Idealism of Science. We must impress that idealism upon all phases of our national life, in order to assist our nation in the solution of the many complex problems of modern democracy.

Research

Report of Technical Committee on Research[*]

J. B. WHITEHEAD, Chairman

To the Board of Directors:

The year just closing has revealed the usual activity in the field of electrical engineering research. This activity extends from laboratory investigations of purely scientific character, outward to the development of all equipment, and to the study and improvement of the performance of the largest types of machinery and transmission systems. The importance of scientific research to industry in all its branches is now clearly recognized. The idea of research is "sold". Moreover, it appears to have been a bargain based on good value, for the article sold is in constant use and there is increasing demand for it. Industrial research laboratories are numbered by hundreds. Problems demanding solution are continually appearing, and there is increasing need for skilled research workers.

THE NATIONAL RESEARCH ENDOWMENT

There has, however, been one new and striking note during the year which has immediate bearing on the future of engineering research. This is the shifting of the emphasis in public discussion from the importance of applied and industrial research to the importance of protecting and stimulating purely scientific research. Advocates of the value of industrial research have always pointed to its intimate dependence on a fundamental scientific basis. Industrial research laboratories have realized from their beginnings their dependence upon trained research workers. Recently, however, Secretary Hoover has pointed out that the tremendous development of industrial research has resulted in two tendencies which threaten to dry up the sources of inspiration for research, and to diminish the number of those who are fitted to carry it on. The first of these is the stripping of university laboratories of their men who are trained in research because of attractive offers from industrial laboratories. The second is the great increase in university student attendance which has compelled universities to limit research activities in order to meet the mere volume demand for elementary education. Recognizing this tendency, Secretary Hoover has placed the weight of his great influence back of the suggestion of the National Academy of Sciences that a national research endowment be accumulated, whose chief purpose should be the promotion of purely scientific investigation in American universities, the natural nurseries for the training of competent scientific research workers. As a result, Herbert Hoover, Elihu Root, Chas. E. Hughes, Owen D. Young, John J. Carty, Wm. H. Welch, and other distinguished associates are serving as trustees of the National Research Endowment, whose purpose is to raise twenty million dollars to aid American universities in carrying on fundamental research during the next ten years.

This movement is one of the healthiest and most promising events in recent years for the further development and progress of research in all fields of industry. Not only will it tend to conserve the sources of supply of research workers, but it is certain to result in an elevation in the general standards of ability, and in the increase of researches of scientific value, emanating not only from universities but also from industrial research laboratories. Industry itself has been quick to recognize the importance of the move, for the heads of many great corporations have entered actively into the campaign to raise the endowment, and it is not unlikely that industry itself will make substantial contributions.

ELECTRICAL ENGINEERING RESEARCH

Naturally, electrical engineering with its rapid development and expansion, its dependence on physical laws only uncovered in recent years, is always a heavy contributor in the field of engineering research. While there is no single result of outstanding novelty and importance, the past year has nevertheless seen many advances of interest. The range of problems studied has been much the same, but some shifting of emphasis in the various fields may be noted. Thus, for example, in overhead high-voltage transmission, attention has centered more particularly on analytical studies of the regulation, stability, and power limitations of high lines and large systems. At the same time there has been somewhat less attention than in foregoing years to experimental studies, in laboratory and field, of transients, protective apparatus, and high-voltage problems, although these have continued in some measure. The laws of corona, its physical character, and its bearing on transmission-line performance are still unanswered questions, and are therefore receiving some attention.

In the field of electrical machinery, the intensive study devoted to fuel economy, with the resulting improvements in steam-power producing machinery, is so striking as to warrant its inclusion here as an important research activity. More directly in the analytical and laboratory class are continued studies of the problem of the ventilation of large rotating units, and of the general relations between temperature and machine rating. Other studies of note are in connection with machinery and the application of delicate vibrating instruments to problems of balance, and the like, the mathematical similarity between mechanical and electrical systems, and the proposal to investigate the former electrically through the equivalent investigation of the mechanical forces arising in short circuits in synchronous machinery. There have been fewer papers than usual on the properties of iron as related to the question of core losses.

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[*] Committee on Research:

John B. Whitehead, Chairman, Johns Hopkins University, Baltimore, Md.
Edward Bennett,
V. Bush,
E. H. Colpitts,
E. E. F. Creighton,
W. F. Davidson,
W. A. Del Mar,
B. Gherardi,
V. Karapetoff,
A. E. Kennelly,
M. G. Lloyd,
F. W. Peek, Jr.,
Harold Pender,
E. W. Rice, Jr.
D. W. Roper,
C. H. Sharp,
C. E. Skinner,
Harold B. Smith,
R. W. Sorensen.

Presented at the Annual Convention of the A. I. E. E. at White Sulphur Springs, W. Va., June 21-25, 1926.

There has been a continuation of the marked activity in experimental studies in the field of electric communication. Papers have been presented to the Institute reporting studies of the quality, the recording, and the reproducing of sound, on the importance of loading telephone circuits, and on new methods of carrier-current transmission. The publications of the Bell Telephone Laboratories show an extremely wide range of subject of investigation. Some of them are scientific research of the purest character, and the range extends to the experimental development of equipment and methods for meeting the increasing modern demands for facility in communication.

Results in the radio field are numerous and important. New tubes are being developed, new circuits devised, and new methods adopted for improvement in all directions of this highly specialized branch of electrical engineering. Perhaps the most interesting activity has been that of studying the behavior of short-wave transmission and the resulting new knowledge that has come as to the conducting properties of the upper atmosphere.

In the field of electrical measurements, among other noteworthy advances, may be mentioned the adaptation of the cathode ray oscillograph to the measurement of very short-time intervals, simpler and more convenient forms of oscillograph, and a greatly increased attention to the methods of measurement of dielectric loss at low-power factor and high voltage.

Studies which lie close to the field of pure physical research, and which may prove to have a practical bearing, are those in which the cathode stream of electrons has been brought through the walls of the tube into the surrounding air, the study of atomic hydrogen in its adaptation to arc welding, the obtaining of copper in large crystals showing 13 per cent increase in normal conductivity, and the continued study of the structure of crystals by the means of X-rays.

DIELECTRICS AND INSULATION

The Committee on Research has continued to devote its principal attention to the subject of dielectrics and insulation. It serves as a consulting committee in Electrical Engineering to the National Research Council. Several of its members are also members of the Committee on Electrical Insulation of the Division of Engineering and Industrial Research of the National Research Council, the two committees having the same chairman. The program of the Committee on Electrical Insulation has been outlined in foregoing reports. During the past year considerable progress has been made by the subcommittees in the respective divisions. The Subcommittee on Dielectric Absorption and Theories of Dielectric Behavior has made a report which was also presented at the Midwinter Convention of the Institute. It is a comprehensive survey of existing knowledge of the anomalous properties of dielectrics and of theories that have been offered in explanation, and it includes suggestions of directions in which further experiment will probably result in new knowledge important to the control of the properties of insulation. As a result of this report the Committee on Electrical Insulation is prepared to suggest problems for investigation and research. It also hopes that some plan may be worked out whereby joint and coordinated work of a number of investigators may be undertaken. The report of the subcommittee on Dielectric Strength may be expected in the near future. The literature on these subjects is very extensive, and requires careful reading and discrimination. The members of the Committee are giving their services voluntarily and this work of necessity takes a secondary place in their busy programs.

The Committee is glad to record continued activity in the experimental investigation of the properties of insulating materials of all characters, and believes that its own interest in this field of study has stimulated the interest which lies back of the present activity. During the year there have been presented to the Institute important papers on gaseous ionization in paper-insulated cables, on the theory of dielectric absorption, the measurement of dielectric losses, and a method for the convenient and quick measurement for the absorption in commercial insulation. Attention should also be called to important contributions from abroad. A noteworthy paper on high-voltage, impregnated-paper cables has appeared in England, and there has been a number of papers from Germany bearing on the dielectric strength of different materials with special reference to the mechanism of the failure of high-voltage insulation.

ORGANIZED RESEARCH

It is a conspicuous feature of research activity in general that it is highly organized in industrial laboratories, but proceeds practically without organization in university laboratories. This situation is natural. The work in an industrial laboratory is usually directed towards the solution of particular questions, and if they are sufficiently important the whole resources of the laboratory may be diverted to their solution. In university laboratories, individual workers find such time as they can for research outside of crowded programs, and usually do research without material compensation and only for the love of it.

Considering both extremes, it will be seen that the situation is not particularly conducive to the production and publication of important results of research. In a few notable instances industrial research laboratories are making substantial efforts in the field of pure physical research, and it is possible to point to results of great value emanating from these efforts. These examples are few, however, and on the whole it can not be said that results of fundamental scientific value are to be expected in large quantities from laboratories in this class. In only a few university laboratories is any considerable proportion of the resources of men and materials devoted to original investigation. Such as is accomplished usually comes from men who are willing to sacrifice time outside the educational program, and who are prompted to it solely by the love of it.

An exceptional and promising example of organized research is that being carried out at Harvard, Johns Hopkins, Massachusetts Institute of Technology, and the University of Wisconsin, on the impregnated-paper insulation of high-voltage cables. The work is being done under the auspices of the Committee on Cable Insulation Research of the National Electric Light Association, whose member companies have subscribed sufficient funds to ensure the active prosecution of the work. Results of importance and value are already beginning to appear.

It would appear, then, that the several national organizations which have among their principal purposes the encouragement and support of engineering research, might do well to undertake actively the organization of research in university laboratories and the provision of material support of the necessary experienced research workers. The national engineering societies might consider also with propriety, these things as lying among their normal functions. The National Research Endowment is pledged specifically to support pure scientific research in university laboratories. There are some problems of pure research which must be attacked by research engineers, but it appears doubtful whether appeal for work in the engineering field will for sometime receive consideration by the National Research Endowment. It is highly desirable, therefore, that engineering foundations interested in research and the national engineering societies should consider how the sources of the training of research engineers can best be conserved, and how activity in engineering research may be encouraged.

Standards

Report on the Work of the Committee on Standards*

H. S. OSBORNE, Chairman

GENERAL REVISION OF THE STANDARDS

The past year has marked the practical completion of the revision of the Institute standards which has been actively under way during the past three years. The revised standards are published in the form of a number of sections, each section dealing with standards for a particular type of apparatus or a particular branch of the electrical industry. This arrangement has very great advantages which were discussed in the report of the work last year. At the present time, 21 of these sections have been completed, covering the great bulk of material previously incorporated in the Institute Standards and also some valuable new material, discussed in last year's report. A number of the other sections are in preparation, the more important of these being Standard Definitions and Symbols, Standards for the Measurement of Test Voltages in Dielectric Tests, Standards for Lightning Arresters, Standards for Storage Batteries, Standards for Electrical Measuring Instruments and Standards for Air Circuit Breakers. Working committees of the Standards Committee are also busy on new material, the most important of which is a proposed section on Standards for Graphical Symbols.

During the past year, on the initiative of the Russian engineers, the Institute Standards were translated into Russian. A translation of the revised standards into Spanish is now being carried out under the direction of the Standards Committee. It is thought that this translation will be of great value in leading to closer relations with Latin America.

COOPERATION WITH INTERNATIONAL ELECTROTECHNICAL COMMISSION

A very considerable part of the activities of the Standards Committee during the year has been directed toward the preparation of recommendations to the United States National Committee of the I. E. C. for material presented at the April meeting of the I. E. C. in New York. A large amount of material was prepared covering recommendations on traction motors, high-pressure tests, standard pressures, rating, graphical symbols, nomenclature. About 50 representatives of the Institute were appointed as delegates or technical

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H. E. Farrer, Secretary, 33 W. 39th Street, New York,
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C. R. Harte,
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G. L. Knight,
A. M. MacCutcheon,
J. Franklin Meyer,
F. D. Newbury,
Harold Pender,
F. L. Rhodes,
L. T. Robinson,
C. H. Sharp,
C. E. Skinner,
R. H. Tapscott,
H. R. Woodrow.
Ex-Officio
Chairmen of Working Committees
Chairmen of A. I. E. E. delegations on other standardizing bodies.

Presented at the Annual Convention of the A. I. E. E., at White Sulphur Springs, W. Va., June 21-25, 1926.

advisors to the Advisory Committees of the I. E. C.

COOPERATION WITH AMERICAN ENGINEERING STANDARDS COMMITTEE

The work of the Standards Committee under the rules of the American Engineering Standards Committee has continued to expand, the Institute being appointed sole or joint sponsor for eight projects during the year, including six sections of the revised Institute standards. The Institute has also appointed representatives on additional Sectional Committees for which it is not a sponsor.

During the year, the American Engineering Standards Committee approved as American Standard the Standards on Industrial Control Apparatus and the Standards on Instrument Transformers.

A very important step in the development of standardizing work in the electrical field has been taken in the decision to appoint an Electrical Advisory Committee for the American Engineering Standards Committee. This has arisen from the joint study given throughout the year by a Joint Committee of the A. I. E. E. and of the Electrical Manufacturers' Council to the mutual problems involved in electrical standardizing work. Based on the recommendations of this Committee, the boards of the two organizations recommended to the A. E. S. C. that an Electrical Advisory Committee be appointed to make recommendations to the A. E. S. C. on all matters pertaining to electrical standards coming before them under the constitution and procedure of the A. E. S. C. This recommendation has been accepted by the A. E. S. C., who have invited the Institute and the other interested organizations to nominate representatives on the committee.

OTHER MATTERS

An interesting step which was taken by the Institute during the year consisted of the appointment of a subcommittee of the Standards Committee on Nomenclature. The function of this committee is not to define established terms, but to consider the need of additional nomenclature to represent new concepts or to more clearly represent concepts not at present adequately expressed. It is proposed that the recommendations of this committee should be made in general through the medium of technical papers and that the proposed terms might not require, in many cases, exact definition or standardization until their significance has become established by usage.

The cooperation of the Standards Committee with Technical Committees of the Institute and with other technical organizations has continued to be very close throughout the year. An illustration is the acceptance by the Standards Committee of an invitation to become responsible for the Electrical Measurements Section of the Power Test Codes of the American Society of Mechanical Engineers.

CONCLUSION

Briefly summarized, the present situation marks an important stage of progress in the practical completion of two matters which have occupied the major attention of the Standards Committee for the last few years; namely, the extensive revision and rearrangement of the A. I. E. E. Standards and the establishment of closer and better defined arrangements for cooperation with other bodies interested in electrical standards. With these matters well in hand, it is believed that the Standards Committee is in a position to make great future progress in furthering the cause of electrical standards in fields which have not as yet been actively covered.

Discussion

H. M. Hobart: It seems to me that we are only just beginning a much larger activity in standardization. Now that the Standards are being brought out in a large number of different pamphlets, it seems to me that we are just at the beginning of an era when hundreds of people can be doing the standardizing work.

Those people should be doing the work who are specialists in particular subjects. It seems to me that the most natural way to bring this about is for each Technical Committee to take on automatically those Standards relating to the subjects coming under the purview of that technical committee. Each committee would probably have two or three Standards that would be its special property, and it would be its special responsibility to keep those up to date.

Some progress has been made in that direction. In the Electrical Machinery Committee we have a subcommittee on Standards, of which Mr. J. C. Parker is Chairman, and during the past year this committee has done a good deal of very vigorous work in connection with the further revision of standards.

There is one Standard entitled Synchronous Alternators which has been adopted for less than a year but since that time very many suggestions have come from very many people. Ambiguities have been pointed out; the attention of the committee has been called to incorrect statements; the need for revision in various limits has been urged. Take, for instance, in the company where I work, there used to be just a few people interested in standardization. Now that we have Standards in separate pamphlets, the designing and commercial departments are interested and they have made at least a dozen suggestions since the revision suggesting that the Standards Committee, when opportunity presents itself, should have certain changes made.

That ought to be going on all over the country in all of the manufacturing companies. They could either come straight to the Standards Committee with those suggestions or send them to the particular Technical Committee that would deal with the subjects in which they are interested.

I want to suggest that now it isn't necessary for a few of us to be working hard on standards, but it is arranged so nicely and naturally that the subject falls into the hands of hundreds of members of the Institute in respective committees, if the committees see fit to turn their attention in that direction.

Instruments and Measurements

Report of Committee on Instruments and Measurements*

A. E. KNOWLTON, Chairman

To the Board of Directors:

The Committee on Instruments and Measurements undertook to continue this year two of the studies begun in 1924 and to start a third major study.

Power and Energy Measurement

One of the first two was that of the measurement of energy and power under the following conditions:

1. A study of methods of measurement of variable power, with particular regard to cases of commercial importance, such as efficiency tests on large machines.
2. A study of methods of measurement of energy in large blocks where the high value of the product makes every practicable improvement of the method desirable.

The subcommittee originally consisted of H. B. Brooks, Chairman, F. V. Magalhaes, and J. R. Craighead; because of weight of other business, Mr. Brooks resigned as chairman and Mr. Craighead was appointed chairman, and G. A. Sawin added to the committee. It was decided that, in both problems, great advantage could be secured by improvement in watthour meters and the subcommittee took the first steps toward accumulating data on the present performance of watthour meters as a basis upon which to proceed. The presentation in February, 1925, of the paper by Messrs. Kinnard and Faus on *Temperature Errors in Induction Watthour Meters* indicated such an advance in the field of watthour meters that it was considered unnecessary to accumulate the data referred to above; this branch of the work was dropped, therefore, because that and other work now in progress makes it seem probable that meters of substantially improved accuracy, particularly with regard to errors with varying temperature, would be available.

In connection with the further study of the first problem, the paper by Mr. E. S. Lee, presented in May 1925, entitled *Measurement of Electrical Output of Large A-C. Generators,* covered to a satisfactory degree the necessary details of application of indicating instruments to the testing of variable power.

In the measurement of energy in large blocks, present commercial methods require the use of instrument transformers and watthour meters. Therefore, the subcommittee has prepared the following short discussion of methods and devices in use:

This discussion refers to the measurement of energy on three-phase circuits at large supply or interchange points, where the value of the energy is large enough to justify any reasonable complication or expense to improve the accuracy and certainty of measurement.

When power is measured in large blocks, the current and voltage are practically always of such values as to require instrument transformers, both for convenience in application to meters and instruments and for protection of operators. The systems are usually three-phase. Grounding conditions of the systems vary, but for metering purposes may be classified in three groups:

1. Ungrounded,
2. Having one or more grounds on only one side of the metering point,
3. Having grounds on both sides of the metering point or a fourth wire so that there is the possibility of currents passing the metering point outside the three-line conductors.

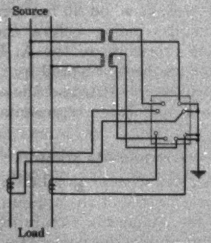

Fig. 1—Connection for Metering on Three-Wire, Three-Phase System with not more than One Point Grounded.

1. Ungrounded Systems. The usual form of metering is by the use of two potential and two current transformers with a polyphase meter shown in Fig. 1. The advantages lie in the fact that there are only four transformers and one meter to care for, and that the meter will rotate forward under all conditions where the energy flow is in a given direction. The disadvantages lie in the necessity of having elements with closely similar characteristics and with negligible interference, and in the fact that the power factor under which the elements actually operate differs from the line power factor due to the use of voltages which in a balanced circuit are dephased 30 deg., one in the lagging and one in the leading direction from the position representing the true power factor of the three-phase circuit. For this use, therefore, the meter selected should have:

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*Committee on Instruments and Measurements:

A. E. Knowlton, Chairman
F. V. Magalhaes, Vice-Chairman
H. B. Brooks, Secretary
O. J. Bliss,
Perry A. Borden,
W. M. Bradshaw,
J. R. Craighead,
W. A. Del Mar,
E. D. Doyle,
W. M. Goodwin, Jr.,
C. M. Jansky, Jr.,
W. B. Kouwenhoven,
P. M. Lincoln,
W. M. McConnhey,
W. J. Mowbray,
H. A. Perkins,
L. T. Robinson,
Bryon W. St. Clair,
G. A. Sawin,
I. B. Smith,
Roy Wilkins.

Presented at the Annual Convention of the A. I. E. E. at White Sulphur Springs, June 21-25, 1926.

a. Close balance of elements and negligible interference between elements,

b. Excellent power-factor characteristics.

The substitution of two single-phase meters for the polyphase meter is sometimes advocated. This does not change the situation in regard to power factor, but substitutes the adjustment of the two meters for the balance of elements. Where the effect of low power factor or of unbalance of load at somewhat higher power factors causes one meter to run backward, the system will be in error due to the light load adjustment on the reversed meter, which will produce torque in the wrong direction. This may be met by the use of separate meters to record the backward reading, with a ratchet arrangement which allows each meter to record energy delivered in one direction only, as has been done in cases where the direction of the total flow of energy is expected to reverse at intervals. The polyphase meter is usually considered preferable.

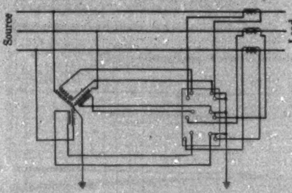

Fig. 2—Connections For Metering on Four-Wire, Three-Phase System or Three-Wire, Three-Phase System with Ground on Both Sides of the Metering Point

2. Systems having grounds on one side of the metering point. These may be treated as ungrounded systems, using the methods described above, or the system shown in Fig. 2 may be used. Here three potential transformers are connected with primaries in *Y*, the common point grounded, and three current transformers are used with a three-element, polyphase meter, or with three single-phase meters. The three-element, polyphase meter has the advantage of simplicity as compared with the three single-phase meters, but it should be assured that interference between elements is negligible and that a satisfactory balance between elements is assured; once obtained, the balance among elements is quite permanent, while the balance among the three single-phase meters is subject to the usual small variations requiring occasional adjustment.

This method has the advantage of having the power factor of each element of the three-phase meter, or of each single-phase meter, the same as the line power factor on a balanced circuit, approaching the line power factor under unbalanced conditions more closely than the methods previously described.

To secure good results, it is necessary that the grounding of the primary circuit be sufficient to maintain the neutral at a satisfactory balance at all times among the three phases. If this is not accomplished, the voltages on the individual potential transformers vary with circuit conditions and with the characteristics of the potential transformers themselves, involving certain errors in the potential transformer ratio and phase angle and in the meters.

In some cases this method has been proposed for application to ungrounded circuits, but the uncertainty of the position of the neutral renders it unsatisfactory for the best accuracy. For other reasons it is undesirable also to ground the neutral of the primaries of the potential transformers when the circuit is not otherwise grounded. It is possible to omit the ground on the potential transformer primary and obtain an artificial neutral by balancing the secondary burdens of the potential transformers, but the method requires care in adjustment and therefore, is not to be recommended.

3. Systems having grounds on both sides of the metering point, or a fourth wire. These systems should be treated as four-wire, three-phase systems, using the method shown in Fig. 2. In the unlikely case of an ungrounded, four-wire, three-phase system, the common point of the potential transformer primaries should be connected to the fourth wire.

In some cases, switching operations under different conditions of load and supply cause important variations in the ground connections. Here the following two fundamental principles govern the method selected:

a. The minimum number of current transformers and corresponding meter elements must be such that under any condition, where a correct record is desired, no more than one possible path for return current past the metering point shall be without a transformer and meter element; in other words, in the circuit of *n* wires or paths, the minimum number of current transformers and corresponding meter elements required is $(n-1)$.

b. The connection of potential transformers must be such that the unbalance of voltages on their primaries is not in excess of the unbalance of the delta voltages on the primary lines.

In illustration of the first principle, consider a three-phase, three-wire circuit, having a permanent neutral ground on one side of the metering point and an occasional neutral ground on the other; the connection of Fig. 1 cannot be properly used because of possible ground current past the metering point during the presence of the second ground but the connection of Fig. 2 is correct.

An illustration of the second principle: Suppose that, at times, the entire circuit is ungrounded; if the connection of Fig. 2 is used, the voltages on the potential transformers will be subject to unbalance because the relation of the various lines to ground is no longer definitely controlled. Assuming the practise once used in emergency of disconnecting the system ground to continue operation when one line is grounded be followed, two transformers will receive delta voltage and the third transformer no voltage. In this case, the ground must be removed from the potential transformer primary neutral and a voltage balance be obtained by adjusting secondary burdens. These latter practises have been recognized as harmful for other reasons as well, and they are practically obsolete for the type of circuits under consideration. They are mentioned as an illustration of an extreme case only.

SELECTION OF INSTRUMENT TRANSFORMERS

The errors of an instrument transformer appear as an error of ratio and as a phase angle by which the secondary voltage or current departs from its theoretically correct phase for metering. Since the rate of the watthour meter and its lag are both adjustable, it is possible to offset known errors, to some extent, by special adjustment of the watthour meter. Since, however, the ratio and phase angle of ordinary instrument transformers vary appreciably with voltage, current, burden, etc., any adjustment of the watthour meter to meet these errors is based upon an estimated average condition and therefore cannot be very accurate. It is, therefore, desirable to select transformers and adjust conditions to give the best possible accuracy without corrective adjustment of the watthour meter. This applies particularly to phase-angle errors of which the variation with power factor makes satisfactory correction impracticable.

POTENTIAL TRANSFORMERS

Standard commercial potential transformers are designed to cover a range of burdens. In the higher voltage sizes, used on most circuits of large power, the accuracy is usually somewhat better than on low-potential transformers. The no-load phase angle is ordinarily negative (secondary voltage reversed leading primary voltage), and several types of transformers are available where this does not exceed 8 minutes at 60 cycles. Non-inductive current drawn from the secondary windings tends to change this angle in the positive direction and lagging current in the negative direction. In the best transformers, it is possible, by increasing the non-inductive current, to reduce the phase angle practically to zero by proper adjustment of the secondary burden. Even without this adjustment the phase angle with two watthour meters and necessary leads as sole burden may generally be kept within 10 to 12 minutes at 60 cycles.

The no-load ratio is usually low, but this error in many high-voltage transformers is not more than 0.2 per cent at 60 cycles. Either non-inductive or lagging current drawn from the secondary tends to increase ratio, so that frequently the reduction of phase angle by burden adjustment referred to above is accompanied by an increase in accuracy of ratio. The variation in errors of a potential transformer for changes of voltage within the limits caused by ordinary regulation is very small. Whatever ratio error remains may, therefore, be fairly considered a constant error subject to correction by compensation in the watthour meter or by the application of a correction to the final result.

Plate 1, a and b, show average ratio errors and phase angles of a number of transformers from 22,000 to 66,000 volts at 60 cycles, and c and d show test results on a single 33,000-volt transformer at 25 cycles.

CURRENT TRANSFORMERS

Standard current transformers are designed to cover a range of burdens. The ratio errors and the phase angle between primary and (reversed) secondary currents vary appreciably with current, secondary burden and large changes of frequency. The phase angles are usually positive, and are increased by increase in non-inductive burden. It is desirable, therefore, to keep both burden and its power factor as low as possible. This means low resistance leads and only the necessary one or two watthour meters for the best results. The variation of ratio and phase angle with current must also be considered. It is desirable to obtain transformers having as little variation of these errors with current as possible.

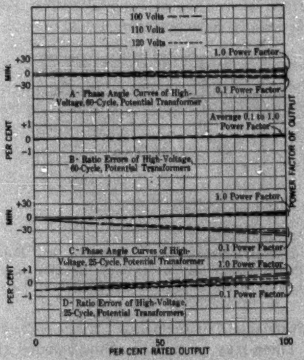

PLATE 1—RATIO ERROR AND PHASE-ANGLE CURVES OF HIGH-VOLTAGE 25- AND 60-CYCLE POTENTIAL

On account of the range for which the transformer is designed, the ratio is usually low at low burdens to prevent it from being too high for reasonable accuracy at high burdens. The amount by which the ratio is low on the lowest practicable burdens varies with the practise of the various manufacturers, but does not usually exceed 0.8 per cent maximum. No attempt should be made to bring a low ratio up by additional burdens, as it requires very careful adjustment of the amount and power factor of the burden to produce satisfactory results on ratio and phase angle at a single value of current, while the resulting phase angle (and sometimes ratio) at other values of current becomes rapidly worse. This difference between treatment of potential and current transformers should be noted as resulting from the fact that the potential transformer must be accurate at a roughly constant voltage and current, while the current transformer must be accurate at widely varying currents and voltages. The same results may be obtained by adjustment of the watthour meter as by a change in the ratio of the current transformer.

Improvement of current-transformer accuracy by special means has been proposed in several forms. A method recently brought out, for which apparatus is beginning to be available, is the two-stage transformer with special watthour meter. ("The Two-stage Current Transformer," Brooks and Holtz, A. I. E. E. JOURNAL, June 1922.) The two-stage transformer is, in principle, a combination of two transformers in one, in which the second transformer corrects the errors of the first by the use of a simple corrective winding in the watthour meter. By this arrangement the errors of ratio and phase angle due to the current transformer are greatly decreased as compared with standard current transformers. Typical curves show ratio error held within 0.2 per cent and phase angle within about 6 minutes for a test from 10 per cent to 100 per cent current at 60 cycles with a burden of two watthour meters in the corrected circuit.

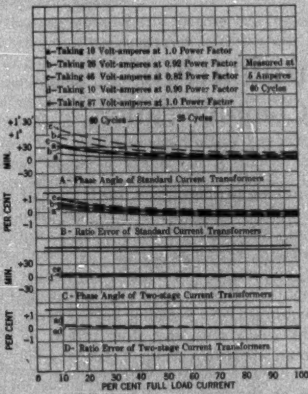

PLATE 2—RATIO ERROR AND PHASE-ANGLE CURVES OF STANDARD AND TWO-STAGE CURRENT TRANSFORMERS AT 35 AND 60 CYCLES, WITH STATED BURDENS

In Plate 2, A and B show average ratio errors and phase angle at 25 and 60 cycles of standard current transformers, and C and D show test results on a two-stage transformer designed for a working pressure of 32,000 volts.

SECONDARY CONNECTIONS OF INSTRUMENT TRANSFORMERS

To assure the best results, it is preferable that separate leads be used for all secondary circuits, and that the length of leads be kept as short as possible. All secondary circuits of current and potential transformers should be connected to a common ground.

WATTHOUR METER

The watthours per disk revolution of a watthour meter will depart somewhat from the nominal value for load current values other than those for which it has been adjusted. On this variation are superposed whatever errors result from changes of temperature, power factor, frequency, voltage and wave form. Certain work now in progress (such as that of Kinnard and Faus, already cited) makes it seem probable that meters of substantially improved accuracy, particularly with regard to errors with varying temperature, are to be available. Temperature compensation is more readily obtained for meter elements operating at unity power factor; inasmuch as one of the two elements of the ordinary polyphase meter usually operates at low power factor, it is greatly to be desired that such studies be prosecuted with a view to obtaining temperature compensation for all conditions of phase departure within the meter elements.

The power-factor error is partly compensated by lag adjustment. The frequency and voltage errors are negligible for the variations occurring in the usual circuits under operating conditions. If special wave-form conditions, serious enough to cause error, exist, the greater part of the error may be eliminated by calibrating the watthour meter in place by the use of indicating instruments the wave-form error of which is negligible.

The watthour meter in general service is one of the most reliable devices in measurement work. In view, however, of the large value of the energy measured at the points here under consideration there are cases where it may be advantageous to use two watthour meters similarly connected to assure a proper record in case of the accidental failure of one meter. In such cases it is proper to designate one meter as the regular standard, and to substitute the reading of the second meter only when reasonable proof of the failure of the first meter to record correctly has been obtained.

Owing to the recent rapid improvement in the watthour meter, detailed data on performance are omitted.

HIGH FREQUENCY MEASUREMENTS

The second of the studies undertaken in 1924 was in the field of measurement of high-frequency quantities; progress is being made in this investigation but the subcommittee (C. M. Jansky, Chairman, E. D. Doyle, B. W. St. Clair) makes no report at this time.

DIELECTRIC LOSSES AND POWER FACTOR

The matter of measurements in connection with dielectrics, especially solid dielectrics, was surveyed by the committee and a subcommittee appointed as follows: W. A. Del Mar, Chairman, O. J. Bliss, and W. N. Goodwin, Jr. This committee arranged a symposium of papers presented at the First District Regional Convention at Niagara Falls May 26, 27, 28, 1926. The papers submitted are as follows:

The Power Factor of Dielectrics and Insulation, by J. B. Whitehead, Johns Hopkins University

The Mechanism of Breakdown of Dielectrics, by P. L. Hoover, Harvard University

Standards for Measuring Power Factor of Dielectrics, by H. L. Curtis, Electrical Testing Laboratories

The Significance of Errors in Dielectric-Loss Measurements, by C. F. Hanson, Habirshaw Electric Cable Co.

Use of Dynamometer Wattmeter for Measuring Dielectric Power Loss, by E. S. Lee, General Electric Co.

Commercial Dielectric-Loss Measurements, by R. E. Marbury, Westinghouse Elec. & Mfg. Co.

Three Methods of Measuring Dielectric Power Loss and Power Factor, by E. D. Doyle and E. H. Salter, Electrical Testing Laboratories

Compensation for Errors of the Quadrant Electrometer, by D. M. Simons, Standard Underground Cable Co.

The Dielectric-Loss-Measurement Problem, by B. W. St. Clair, General Electric Co.

Zero Method of Measuring Power with a Quadrant Electrometer, by W. B. Kouwenhoven and P. L. Betz, Johns Hopkins University

It is believed that these papers in the total give a reliable index of the present state of the art of measurements involving the higher voltages and small phase angles.

Electrical Communication*

Report of Committee on Communication

H. P. CHARLESWORTH, Chairman

To the Board of Directors:

In presenting this report it is the aim of the Committee on Communication not so much to present a complete report of all the developments which have been made during the last Institute year in the art of electrical communication, but rather to single out those advances which it is thought will be of greatest interest to the members of the Institute.

PRINTING TELEGRAPHY

The year has been marked by a rapid growth in the use of printing telegraph apparatus by the telegraph companies, railroads, news associations for distributing news items, and by general business concerns in connection with private line business. There is a general tendency toward the operation of the apparatus at higher speeds, thus increasing the speed requirements of the line circuits and requiring the use, especially with long and complicated circuits, of repeaters which reform or regenerate the line signals.

Printing telegraphs are now being used in connection with switching arrangements, both manual and automatic. These arrangements have been found valuable for sending information from a central location to one or a number of receiving stations. A recent installation of this type consists of a number of printers controlled by automatic switching apparatus so arranged that any printer equipped with a sending keyboard can communicate individually with any other printer so equipped, or communicate with a group of receiving-only printers. Dials, similar to those used with telephone instruments, are used at the transmitting stations for setting up any desired operating condition.

The ever widening field of usefulness of printing telegraphs was illustrated by a paper on ciphering and deciphering arrangements for secret wire and radio communications read by Mr. G. S. Vernam before the Midwinter Convention of the Institute in New York last February.

In the message business of the telegraph companies there has been a marked tendency to use tape printers instead of page printers. To facilitate the subdivision of the tape into single lengths and the attachment of it to message forms for delivery to customers, effective cutting and gumming devices have been developed.

In addition to the application of tape printers to the Multiplex circuits used in trunk line service, a simplified combination of such a printer with a keyboard transmitter is coming into extensive use for branch office circuits and other short-line service. The application of these printers will probably be quite extensive due to their accuracy, traffic handling capacity, and economy. This type of printer is primarily intended for single-line operation, affording intermittent single-channel service in both directions. It also lends itself readily to use on duplexed circuits when this method is desirable. Printers of this type have a field in private offices of patrons where the amount of business handled warrants their use. Maintenance requirements are few, and the attention required is sufficiently small to make this service desirable.

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*Committee on Communication:

H. P. Charlesworth, Chairman

F. L. Baer,
O. B. Blackwell,
L. W. Chubb,
Charles E. Davies,
H. W. Drake,
Major P. W. Evans,
R. D. Evans,
E. H. Everit,
L. F. Fuller,
D. H. Gage,
S. P. Grace,
P. J. Howe,
F. H. Kroger,
N. M. Lash,
Ray H. Manson,
R. D. Parker,
H. S. Phelps,
Lieut. Com. B.B. Ralston,
F. A. Raymond,
Chester W. Rice,
J. K. Roosevelt,
H. A. Shepard,
John F. Skirrow,
E. B. Tuttle,
F. A. Wolff,
C. A. Wright.

Presented at the Annual Convention of the A. I. E. E. at White Sulphur Springs, June 21-25, 1926.

"TICKER" TRANSMISSION

During the past year the automatic tape transmission system for telegraphic tickers was installed and put into operation at Cleveland, Chicago, Los Angeles, and San Francisco. Long-distance distribution of ticker service was greatly extended from New York, Cleveland, and Chicago.

A particularly note-worthy feature was the establishment of telegraph ticker service in important business centers on the Pacific Coast which heretofore obtained market quotations only by Morse on brokers' private leased wires. Full market quotations are now supplied to Pacific Coast brokers from the New York Stock, New York and New Orleans Cotton Exchanges, and the Chicago Board of Trade. Provision is also made for "dropping" the quotations at various cities along the route of the main circuit which extends from Chicago to San Francisco via Los Angeles, a wire distance of 2895 miles. New York stock market quotations reach the brokers on the Pacific Coast in 30 to 45 seconds from the time that they appear on the tickers in Wall Street.

The system developed for this and similar services elsewhere includes modifications of the two-channel, Multiplex printing telegraph apparatus and repeaters suitable for use on the long lines involved.

The transmitters are controlled by perforated tapes similar to those used in other automatic telegraph systems. Instead of the more familiar five-unit code, a six-unit code is employed, the sixth pulse of each character serving to distinguish between letters and figures. The receiving instruments at customers' offices are self-winding tickers of a type already widely used in Eastern cities.

The output of the channel printer system in characters per minute per channel is the same as that obtainable with self-winding ticker systems used on shorter circuits having but one channel.

SUBMARINE TELEGRAPHY

The great success of the new type of submarine telegraph cable loaded with permalloy that was mentioned in last year's report has led to the rapid extension of this type of cable, and during the year five important telegraph cables of this type were under construction. The technical features of this type of cable were described in a paper by Mr. Oliver E. Buckley which was presented at the Convention last June and which appeared in the August JOURNAL.

MACHINE SWITCHING TELEPHONY

With the continued steady growth in the application throughout the country of machine-switching telephone apparatus have come further developments in this form of apparatus. In the panel system, which is the type of system used for large cities, a simplified form of tandem switch has been developed by which, after one selection at the calling office, the final selection of the called office can be made by apparatus located at a distance from the originating office, and used to collect traffic from a number of offices routed over common groups of trunks. This results in a material saving in the trunk plant.

A more efficient method of associating a sender with the calling subscriber has been developed. This results in an appreciable saving in the number of senders required, uses more economical apparatus, and reduces the number of types of apparatus required in an office.

In the step-by-step system, which is the type usually used for the medium and smaller sized cities, a line-finder system has been developed which is similar in principle to that being used with success in panel offices. This employs the same selector that is used in the rest of the switching train, effects an improvement in service to the subscriber, and lends itself more readily to efficient equipment layout.

A machine-switching tandem system, employing step-by-step equipment, has been developed for completing toll calls within a 50-mile radius of any given central office area. All calls completed through this system are handled directly by the originating operator over dialing trunks. A tandem system of this kind probably will find its principal application in areas employing step-by-step machine-switching equipment. An installation of this type recently has been put into service in Los Angeles and serves some 75 central offices having a total of approximately 400,000 subscribers. The principal new engineering feature of this system is the means which was developed for dialing and signaling over phantom toll lines.

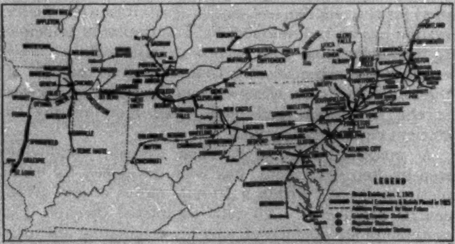
New types of frames for mounting step-by-step equipment have been developed which take advantage of the ceiling heights ordinarily found in central office buildings, thus effecting a material reduction in floor space requirements.

A cordless "B" switchboard has been developed for completing calls from manual offices to machine offices where the number of manual offices involved makes the use of dialing devices at the manual offices prohibitive in cost. This switchboard employs new trunking principles which result in very efficient and simplified operating equipment.

TELEPHONE TOLL CABLES

An event of prime importance during the year was the completion on August 11 of the telephone toll-cable system connecting New York and other Atlantic Seaboard cities with Chicago. This system of cables runs from Chicago through South Bend, Toledo, Cleveland, Detroit, Pittsburg, and Harrisburg to Philadelphia and New York, connecting with the cable system extending from New York to New Haven, Hartford, Springfield, Providence, Worcester, Boston, and Albany, and from Philadelphia to Baltimore and Washington. Extensions of this cable westward from Chicago carry the toll-cable system to Milwaukee, and by the end of this year the system will extend to Peoria, Springfield, and St. Louis. Numerous shorter branches carry this network to many other points near the large centers mentioned.

In addition to this very extensive network in the northeastern part of the country, smaller networks are starting in other parts of the country, notably in various parts of the Pacific Coast and in Florida. The rapidity of growth of this type of telephone plant is illustrated by the fact that during the year 1925 over 1000 miles of toll cable were placed in service, and during the year 1926 more than 1500 miles will be installed. The map reproduced below shows the routes of the present and proposed toll cables.



Toll Cable Routes in Northeastern Section of the Country

The possibility of providing satisfactory telephone transmission over very long circuits in cable has come about through a series of remarkable developments in telephony which have been discussed from time to time before the Institute. During the year the network mentioned above was the subject of a paper presented before the New York Section in December by Mr. J. J. Pilliod, and was discussed at other section meetings.

The development of toll-cable networks means a great deal in improving and increasing the service given to the public. Practically the entire country is subject to severe sleet storms from time to time and toll cables furnish a means for providing great security against interruption from such storms or other weather conditions. Furthermore, they furnish a practical solution for meeting the very great demands for service between large urban centers of the country. A single cable contains between 200 and 250 telephone circuits and would require five or six open-wire toll lines with former methods of construction, the largest number of circuits which can be provided by this means; furthermore, furnishing means for supplying circuits so liberally as to give a faster service. As indicative of the increased facilities provided, it may be mentioned that the New York-Chicago toll-cable system contains about 510,000 miles of wire, while at the time this project was started, the facilities for serving the corresponding points consisted of less than 100,000 miles of open wire.

In last year's report reference was made to certain papers describing the development of telegraph systems suitable for use in long telephone cables. The application of these systems has been considerably extended during the past year. The voice-frequency carrier system is of special interest, since, through the development of apparatus for separating to a very high degree currents of different frequencies, it has become possible to operate 12 independent telegraph circuits over a single telephone cable circuit, each of the 12 circuits using a different frequency ranging from 425 to 2300 cycles per second. There are now in operation about 25,000 channel miles of telegraph circuits operated by this system in the telephone cable plant.

Radio Telegraphy

A new radio telegraphy service between the Dutch East Indies and San Francisco has been established, and a new service from New York to Holland has been introduced for handling through traffic from Java.

The work of decreasing the use of spark transmitters with their corresponding interferences, commenced in 1924, has progressed rapidly to such an extent that over 150 American merchant ships are now equipped with tube transmitters.

The use of the radio direction finder for navigation is rapidly increasing and has passed the experimental stage. The Great Lakes are leading in this respect, due to the demonstration of the particular value of this apparatus in the difficult navigation problems inherent in the Great Lakes.

Transatlantic Radio Telephony

The development work which has been in progress during the past few years on transatlantic radio telephony reached a point of considerable interest last winter in the attainment of two-way telephonic conversations. In the earlier tests, telephonic transmission had been but one-way, from the United States to England, with the study of transmission of radio waves in the reverse direction being made by the utilization of radio telegraph stations in England. Two-way talking was made possible the winter of 1925-26 by the completion of a transmitting station by the British General Post Office. The British radio telephone transmitter is, in general, similar to that employed on the American side. Both transmitters are of the carrier-suppression, single-side, band type, which makes the maximum use of the transmission power and which minimizes the frequency band which is required. Receiving on both sides is carried out by means of directional receiving antennas of the wave-antenna type. Reception on the American side has been further improved by establishing a receiving station at a higher latitude where the atmospheric interference is less, located at Houlton, Maine. The Houlton reception is extended into New York by means of wire telephone circuits. Likewise, the voice currents originating in New York are transmitted to the Rocky Point transmitting station over wire circuits. Similarly, in England the radio transmitting and receiving stations are connected into London by telephone circuits. In this way, an integral two-way circuit is established with New York and London as the terminals.

The carrying on of two-way talking tests between England and the United States created much general interest and arrangements were made to enable a group of American Press representatives in New York, and a similar group of English newspaper men in London, to participate in the tests by conversing with each other. These tests represent the first time that groups of people have been able to converse with each other across the Atlantic.

Papers by Messrs. A. A. Oswald, J. C. Schelleng, and R. A. Heising, describing important technical features of the apparatus used for transatlantic radio telephony were published in the June 1925 *Proceedings* of the Institute of Radio Engineers.

RADIO PROPAGATION TESTS

A great deal of activity has been shown in studying the properties of the medium, intervening between transmitter and receiver, through which the radio waves must pass. This information is essential before we can utilize, to the best advantage, this remarkable but eccentric medium which nature gives us.

Extensive propagation tests have been reported on long, medium, and short wave lengths, together with a large amount of theoretical work towards explaining and correlating the apparently contradictory behavior found in different parts of the radio spectrum. This work has confirmed the existence of a positive ionization gradient in the upper atmosphere, postulated independently, many years ago, by Kennelly in this country and Heaviside in England.

The Naval Research Laboratory published the results of experiments which indicate that short waves may, in effect, hurdle the first few hundred miles of their journey and be received at a distant point with a much greater strength than would be expected from the past experience with long waves. In the range of frequencies under exploration (down to about 30 megacycles, or 10 meters) the "skipped" distance appears, in general, to increase with increase of transmission frequency and to be greater at night-time than at day-time. These results have been confirmed by other experimenters. Theories have been proposed (among others, by Baker and Rice in a paper presented before Mid-winter Convention of the A. I. E. E., 1926) to explain these phenomena which assume that waves received at a distance are deflected downward from some upper ionized region of relatively low index of refraction and of low attenuation.

The observed indications of rotation in the plane of polarization and directional errors find a qualitative explanation when due account is taken of the fact that the earth's magnetic field, acting upon the free electrons, will change the velocity of the wave and produce rotation. This effect is especially marked in the vicinity of 214 meters, which corresponds to the resonant frequency of an electron in the earth's magnetic field.

Careful investigations of fading and signal distortion, so exasperating to the broadcast listener, have shown that they are closely related to the high degree of frequency selectivity exhibited by the radio medium.

During the total eclipse of January 24, 1925, many observations were recorded which further tended to confirm the recent electron dispersion theory of the radio medium.

An observed variation of signal strength during magnetic storms is also significant. Here the effect is a reduction of the normal signal strength by night and an increase above normal by day.

Increasing use is being made of the short wave range of frequencies as is evidenced by the continued increase in the number of short wave licenses issued to commercial concerns. Experimental work indicates the possibility of twenty-four-hour telegraph service with short waves over distances of several thousands of miles. The transmission varies largely with the time of day, however, one frequency transmitting much better for one period of the day and another frequency for another period, so that the use of several wave frequencies appears to be required to give uniform operation.

RADIO BROADCASTING

Continued improvement has been made in radio broadcasting on both the receiving and the transmitting sides. The receiving set improvements are, in general, characterized by better quality of reproduction, which means higher grade amplifiers and better loud speakers; but there is still much to be desired in these respects in receiving sets as a whole. The fact that the reliable, high-quality service range of the average radio broadcast transmitting station is relatively short is being more generally recognized and is leading to a more extensive use of long distance wire lines for enabling good programs to be made available at a number of widely separated broadcast stations, and is leading also to the use of greater power in transmitting. As many as 17 stations, spread over the northeastern quarter of the United States, are now regularly supplied for a portion of their time with programs transmitted over special long-distance wire circuits. A number of stations are now transmitting with five-kw. power, and one station which is capable of transmitting with as much as 50 kw. has been established during the year.

Another important improvement in radio transmitting stations introduced during the year is the use of master oscillators with piezoelectric frequency control for the purpose of reducing quality distortion in the received signals. In general, there was a considerable improvement in the maintenance of the frequency assignments of broadcast stations, due to a more general appreciation of the need for frequency stability and to the efforts of the Radio Inspection Service in checking up conditions and of the Bureau of Standards in the work of frequency standardization.

The problem which exists in the regulation of radio, in the general public interest particularly in the allocation of wave bands, has been reflected in submitting to Congress several bills designed to strengthen the Government's control of radio, and by the holding of the Fourth National Radio Conference at Washington during the fall of 1925. This Conference recognized the fact that the number of transmissions which can be accommodated in the range of frequencies available for broadcasting is definitely limited, and recommended that the Department of Commerce undertake to limit the number of broadcast stations to be licensed.

ELECTRICAL TRANSMISSION OF PICTURES

The commercial service for the transmission of pictures by wire between New York, Chicago, and San Francisco, has continued throughout the year. The results have been promising and it has been decided to add five more complete stations to the picture-transmission network. This extension should enable a thorough test to be made of the commercial demand for this type of service.

An interesting experiment was conducted for the U. S. Signal Corps, in October 1925, when pictures were taken from an aeroplane over Fort Leavenworth, Kansas, developed on the plane, dropped by means of a parachute, and transmitted from Fort Leavenworth to New York, Chicago, and San Francisco, simultaneously. The entire experiment was so successful that army officers in New York City viewed the picture 29½ minutes after the snapping of the camera on the plane at Fort Leavenworth, approximately 1500 miles away by wire. This is an indication of the usefulness of telephotographs for war purposes.

During the year from March 1925 to 1926, a series of tests of radio transmission pictures has been conducted between Carnarvon (Wales), San Francisco, Honolulu, and New York. Specific mention should be made of the pictures sent by radio from Honolulu to New York during the Naval maneuvers of our fleet there in May 1925. As a result of the successful completion of this experimental work, commercial service has been established between England and New York and also between Hawaii and San Francisco.

INDUCTIVE COORDINATION

Last year, the report of the Committee announced the formation of the American Committee on Inductive Coordination, embracing representatives of all wire-using utilities interested in this problem. During the year, the committee has issued a very complete and extensive bibliography of publications bearing on inductive coordination. This bibliography includes not only American, but also foreign references and contains a total of about 900 items. It is expected to be of great value to all concerned with inductive coordination work. Another item of interest in this field is the issuance of a report by the joint, general committee of the National Electric Light Association and the Bell Telephone System, covering the principles and practises adopted by those organizations for the joint use of wood poles by supply and communication companies.

During the year this joint general committee continued its progress in research work on a large number of specific problems in inductive coordination. The study of these problems has been facilitated by the development of improved measuring apparatus and methods particularly adapted to the wave analysis of power-circuit voltages and currents and of induced voltages and currents in telephone circuits.

Two papers dealing with the subject of inductive relations between power-distribution circuits and telephone circuits were presented at the Seattle Convention in September, in connection with a symposium on power-distribution.

TELEPHONE SERVICE FOR THE DEAF

Another interesting telephone development relates to improved service for deaf people. Many partially deaf people, who have difficulty in hearing ordinary conversations, use the telephone with ease. Others, who are somewhat more deaf, however, have not been able heretofore to use the telephone. In order to make the telephone service available in these special cases, apparatus which greatly increases the volume of sound received has been developed for the use of deaf people. Although in cases of extreme deafness, no amplification of sound, however great, makes hearing possible, this new device makes it possible for many people of impaired hearing who would otherwise be unable to use it to obtain telephone service. The apparatus is designed, of course, only for those having impaired hearing.

CHARACTERISTICS OF SPEECH

The work carried on by telephone research engineers, in the study of the characteristics of speech, has continued to produce important and interesting results which have been described in a number of publications during the year. Of particular interest is the paper published in the A. I. E. E. JOURNAL for March 1926, by Messrs. Maxfield and Harrison entitled "Methods of High Quality Recording in Reproducing of Music and Speech Based on Telephone Research." This paper tells of the application to the improvement of phonographic reproduction of speech of some of the methods developed in the study of the quality of telephone transmission.

LOADING OF TELEPHONE CIRCUITS

The year 1926 marks the completion of the first 25 years of the application of loading to telephone circuits, in the form of inductance coils inserts in the line at periodic intervals. A paper presented by Shaw and Fondiller at the Midwinter Convention reviewed the progress which has been made in this country, in both the development and application of loading, covering particularly the last 15 years. The outstanding developments in that period are the compressed-powder iron-dust core material, the loading systems for the long toll cables and the more economical systems for local exchange area trunk circuits. The extent of the application of loading is indicated by the estimate given in that paper, of 1,250,000 loading coils in this country, as of January 1, 1926, and by the expectation that this number will be doubled by 1930.

FIRE-ALARM, POLICE-SIGNAL, AND TRAIN-CONTROL SYSTEMS

The National Fire Protection Association has continued its work of standardizing and improving emergency signalling systems. At its last annual convention, completely revised "Regulations for Municipal Fire Alarm Systems" and partially revised "Regulations for Protective Signalling Systems" were adopted, and work for the further revision of the latter regulations has been carried on throughout the year.

The International Association of Municipal Electricians has continued the compilation and promulgation of standard specifications for wires and cables for municipal signalling. Important additions were made to these specifications during the past year.

Another important development has been the completion of arrangements by some leading railroad systems for automatic train-control systems. This promises to become a very important field of operation.

The field of police emergency signals and traffic signals is constantly growing in importance. Many cities are now installing an elaborate system for one or both of these. In some cases, the systems include a combination of police-alarm, fire-alarm, and traffic signals, whereby officers on traffic duty may receive due warning of fires and escape of bandits in automobiles, and thus facilitate the movement of fire apparatus in the one case and block all traffic in the other. Further advances have been made also in the installation of extensive modern manual fire-alarm systems in a number of important cities.

Status of Electric Lighting in 1926

Report of Committee on Production and Application of Light[*]

PRESTON S. MILLAR, Chairman

To the Board of Directors:

As required by provision of the By-laws, this report constitutes a brief resumé of of the progress of the lighting art, including recent developments in electric lighting which appear to have significance as indicating an actual trend in the art.

ILLUMINANTS

In electric illuminants there have been no new developments of a radical character during the past year. Among tungsten filament incandescent lamps there has been developed a new standard line in which the gas-filled type has been introduced in the 50- and 60-watt sizes heretofore generally of the vacuum type. These lamps, standardized thus far from the 15- to the 100-watt sizes of the 115-volt range, are characterized by bulbs of a pleasing contour (Fig. 1), lightly frosted upon the inner surface, providing a measure of diffusion of light, while retaining the cleanly smooth surface heretofore to be had only in the clear bulb and "natural glass" lamps. The adoption of this new standard line of lamps in place of a variety of types heretofore employed is a conspicuous example of the principle of simplification which the Department of Commerce has urged successfully and constructively upon industry in this country.

Arc Lamps. The employment of arc lamps in this country for general illumination is confined largely to the magnetite lamp which continues in successful operation in some localities, and which in its higher powered form has been utilized in several recent installations where high-intensity street lighting is desired.

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Charles F. Scott,
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C. J. Stahl,
G. H. Stickney.

Presented at the Annual Convention of the A. I. E. E. at White Sulphur Springs, June 21-25, 1926.

For the projection of motion pictures in large houses, the high-intensity arc is quite generally used. In medium-sized theatres carbon arcs, both alternating and d-c., operating at currents from 30 amperes to 80 amperes, are quite generally in use. A 10- to 30-ampere, horizontal carbon arc used with a reflector instead of a lens as a condenser has entered into use recently.

Practically all interior motion picture photography is now done with the aid of electric arcs for illumination. Four types of arc are used: the high-intensity arc (150 amperes), the white-flame arc, the ordinary carbon arc, and the mercury vapor arc. There is steady improvement in all forms of studio arc-lighting apparatus.

The high-intensity arc has been brought to its highest degree of perfection in military searchlight work and is now used universally by both the Army and Navy.

White-flame arcs are still quite generally used in photo-lithography and color reproduction work. Improved units are occasionally appearing in this field.

The use of white-flame arcs in portrait photography is increasing and several small, efficient arc mechanisms are now being used in this field.

FIG. 1—NEW STANDARD LINE OF TUNGSTEN FILAMENT LAMPS

Small Gaseous Conductor Lamps. A distinctly new development is the small gaseous conductor lamp[1] devised by Moore for indicator or marker purposes rather than for purposes of illumination. These lamps operate at 115 volts alternating current or direct current. They consume about one-half milliampere for the T-4 bulb size, and one milliampere for the G-10 bulb size, and have an efficiency of about one-third lumen per watt. The life is said to be 3000 to 5000 hours. One form of the lamp, fitted with a G-10 bulb, has an over-all length of two in. and a diameter of one to one-fourth in. This is provided with a wire resistance of about 35,000 ohms in the base, which is of the medium screw type. Another form has a T-4 bulb, an over-all length of about one to one-fourth in., and a diameter of one-half in. This is fitted with a candelabra screw base in which there is a composition paste resistance of about 70,000 ohms. The lamps are said to contain neon, helium, and argon in certain proportions. When excited by electric pressure at the electrodes, the gas becomes luminous at the cathode. On direct current, the gas around only one electrode glows; on alternating current the gas glows near both electrodes at a frequency depending upon the supply current.

The lighting and extinction of these lamps is practically instantaneous. They thus have properties which are peculiarly desirable for such instruments as stroboscopes, synchroscopes, etc. If 230 volts be applied, they merely glow a little more brightly than on 115 volts.

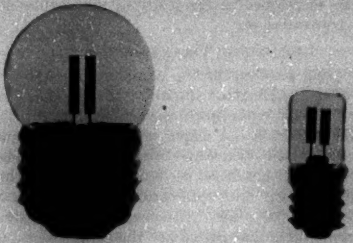

FIG. 2—GASEOUS CONDUCTOR LAMPS FOR 115-VOLT CIRCUITS

The two forms of this lamp are illustrated in Fig. 2. Application of the lamp to an electric flat iron is shown in Fig. 3. The small power consumption makes it practicable to use the lamp in a wide variety of service to indicate that the circuit is alive, or to indicate locations of switches, polarity of d-c. circuits, etc. For other applications see paper to which reference has been made.

FIG. 3—GASEOUS CONDUCTOR LAMP TO INDICATE CLOSED CIRCUIT ON FLATIRON

Double Filament Automobile Lamp. The past year has witnessed the rather extensive introduction of the double filament automobile headlight lamps. In ordinary practise the lower filament provides the principal driving beam. When the upper filament is switched into circuit the beam is depressed by two to three degrees. A further discussion of this development in relation to headlight practise appears under a later section of this report.

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[1] "Recent Developments of Moore Gaseous Conductor Lamps" by Moore and Porter, *Transactions*, Illuminating Engineering Society, Feb. 1926, p. 176.

Prefocusing Lamp Socket. To meet the requirements for precise location of filaments of incandescent lamps used in motion picture machines, stereopticons, signals, etc., there has been developed a special type of base and socket which insures correct operating position for the filament in such apparatus. Each lamp is based in an optical jig to secure exact filament location with respect to both axial alignment and light center length once the projection apparatus is properly fitted. This device eliminates the necessity of refocusing on renewing a lamp, and procures for users of projectors maximum screen illumination. While this equipment is not regularly listed by manufacturers it is understood that it can be procured when desired.

LIGHTING EQUIPMENT

For Residence Lighting. The trend in luminaries for home lighting in recent years has been largely in the direction of the candelabra type of equipment employing frosted or coated incandescent lamps which in large part have been without shades and wholly exposed to view. In illuminating engineering circles this trend has been regarded as contrary to the public interest in that it has brought within the field of ordinary vision sources of excessive brightness which have produced glare. With a view to correcting this condition much educational work is being done by central stations and other branches of the industry to impress the public with the desirability of properly shading light sources, and new types and finishes of shades are being made available by some manufacturers to encourage this practise.

For Street Lighting. In street lighting equipment there has been a pronounced trend in the direction of employment of directive equipment, usually of prismatic glass, surrounded by lightly diffusing outer glass envelopes. The purpose has been to secure a considerable measure of favorable redirection of light while keeping brightness and glare within bounds. A variety of equipment of this kind has been placed in service. In some cases effort is confined to redirection into the lower hemisphere of some of the light flux which otherwise would be directed upward. In other cases this has been combined with a latitudinal redirection along the street of some of the light flux which otherwise would fall upon building fronts.

ORGANIZED STUDY OF ILLUMINATION

Illumination and Production. Under the aegis of the National Research Council an effort is being made to ascertain some of the relations between changes in illumination and changes in industrial production. Investigations to date[2] have indicated that improved illumination has secured industrial advantage in the way of increased production, diminished shrinkage, and more favorable working conditions. The purpose of the present investigation is, if possible, to ascertain under independent auspices the facts in certain representative industries. Thus far, the committee having this enterprise in charge, and operating under the chairmanship of Professor D. C. Jackson, has engaged largely in developing methods with a view to the formulation of a procedure and the establishment of systems of control which when applied to an industry which is largely dependent upon vision, may be expected to determine the facts of relationship between illumination and industrial production.

Street Lighting. A committee of the Illuminating Engineering Society is endeavoring to develop a method of appraising the effectiveness of street lighting which will make it practicable to determine, at least approximately, the relative merits of two different street lighting systems. It is an interesting and notable fact that despite the extensive attention which has been devoted in recent years to the subject of street lighting, there is today nothing like consensus among the leaders of the art as to the best manner of locating and equipping street lamps and of distributing the light for purposes of street illumination. If the present attempt fails in its ultimate object, it may at least succeed in diminishing the divergence of views among street lighting engineers. There is included in the program a project for study of the elusive subject of glare in street lighting.

ORGANIZED MOVEMENTS FOR THE IMPROVEMENT OF LIGHTING PRACTISE

Electric lighting is receiving proportionately more attention at the hands of electric service companies throughout the country than has been the case in recent years. The potentialities of revenue from increased lighting load and a growing sense of responsibility for making available to the public the benefits of improved illumination have combined to command a greater degree of specialized attention to illumination.

Training Course for Lighting Men. At the 1924 Convention of the National Electric Light Association, a Committee to Promote Central Station Illuminating Engineering reported upon the importance of the opportunity in the lighting field afforded central stations, and described a training course for prospective central station illuminating engineers then in course of preparation. This course, under the joint auspices of the National Electric Light Association and the Illuminating Engineering Society, was arranged for the autumn of 1924 and was carried through with success. It made available to a training group the kind of instruction which in recent years had been given lighting students in training courses by the larger manufacturers of incandescent lamps, and supplemented this by the experience of central station companies who have pioneered in the organization and operation of lighting service departments and by a carefully chosen inspection trip.

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[2] Report of Committee to Promote Central Station Illuminating Engineering, Lighting Bureau, Commercial National Section, Presented at National Electric Light Association Convention, May, 1924. (Table III)

Selection of Lighting Equipment. A further indication of central station concern for the improvement of electric lighting is furnished by action of the Lamp Committee of the Association of Edison Illuminating Companies, which heretofore has functioned successfully in connection with incandescent lamps themselves, but has not concerned itself with lighting equipment. It is understood that this Committee has taken cognizance of unsatisfactory lighting conditions in residences, and has requested the Illuminating Engineering Society to formulate a statement of principles and perhaps some simple specifications which may be looked to for the guidance of central stations which desire to select satisfactory lighting equipment for sale to the public. It is further understood that the Illuminating Engineering Society is endeavoring to comply with this request and has a committee at work formulating such a statement of principles which may govern the selection of residence lighting equipment.

Central Station Lighting Departments. Whereas in the spring of 1924 only nine central station companies were known to have lighting service departments, today there are 42 central stations which have such departments and the number is fast growing. The principal limitation is the unavailability of trained men competent to organize and operate such activities.

This awakening of central stations to their opportunities and responsibilities in lighting is of large significance to the country. Most classes of illumination, and particularly residence, industrial, and street lighting, suffer needlessly from ineffective and inappropriate lighting which fails to take advantage of recent advances in the art. The light and power industry can contribute materially to the welfare of the country by lending its great influence to betterment of these conditions. It is a matter of gratification to this Committee that definite progress in this direction is indicated by the developments of the past year.

I. E. S. and Central Station Lighting Men. During 1925 the National Electric Light Association, in reorganizing its Commercial Section for greater effectiveness, dispensed with its Lighting Sales Bureau. In September, 1925, on the day preceding the opening of the Illuminating Engineering Society's annual convention, there was a gathering of central station lighting men out of which there was evolved a plan for greater activities in connection with central station lighting under the auspices of the Illuminating Engineering Society. In this connection it is understood that there is in course of preparation a Lighting Service Department Manual which will make available the experience of central stations which have assumed the lead in such work. This will include a brief survey of the lighting field and chapters on the organization of a lighting service department, its scope, needed equipment, and engineering and commercial features of its work.

Lighting Demonstrations. Interest in electric lighting has been enhanced and in all probability practise has been much improved as a result of demonstrations made in the elaborate Lighting Educational Centers of the incandescent lamp manufacturers at Nela Park and Harrison. So successful have these been that less elaborate demonstrations have been set up and operated in a number of cities. Notable among these are demonstrations in Boston, Chattanooga, Cincinnati, Detroit, Kansas City, Knoxville, Louisville, Nashville, Philadelphia, Pittsburgh, and Providence. Others are understood to be in preparation.

MOTOR VEHICLE LIGHTING

Specifications for automobile headlamp and rear-lamp performance have been prepared by the Illuminating Engineering Society and revised as of 1922. These headlamp specifications cover laboratory tests for headlamps and headlighting devices and prescribe candle power limits for different parts of the beam. The headlamp specifications were endorsed in 1922 by the Society of Automotive Engineers, which organization, however, supplemented its endorsement by promulgating, within such limits, recommended practise calling for a higher maximum beam candle power and a higher candle power in the beam spread to the right and to the left than is stipulated in the specifications. The I. E. S. headlamp specifications were approved as a "Tentative American Standard" in 1922. They are in use by the Eastern Conference of Motor Vehicle Administrators in which are represented all the New England States, (except Massachusetts) New York, New Jersey, Pennsylvania, Maryland, Delaware, Virginia, District of Columbia, Ontario, and Quebec. The specifications are also in force, either wholly or in part, in Ohio, Wisconsin, Iowa, Utah, Nebraska, Texas, California, and Oregon. Also they are recommended in the "Uniform Act Regulating the Operation of Vehicles on the Highways" prepared under the direction of Secretary Hoover in the National Conference of Street and Highway Safety.

The rear lamp specifications developed by the Illuminating Engineering Society cover the general relation of the lamp and license plate holder and the quantity and uniformity of illumination upon the license plate. At the present time the rear lamp specifications used by the State of Massachusetts are substantially in accord with those prepared by the Illuminating Engineering Society.

During the past year the Automotive Lighting Association has prepared specifications covering stop and direction signals and the depressed beam from controllable headlights or auxiliary driving lights. Subsequently somewhat different specifications for depressed beam lighting have been formulated tentatively by the Illuminating Engineering Society's

Committee on Motor Vehicle Lighting. The matter also is understood to be under consideration by the Eastern Conference of Motor Vehicle Administrators.

A further advance made in the practise of the Eastern Conference of Motor Vehicle Administrators is a requirement for excellence of headlamp construction not formerly imposed. Another forward step was taken by the National Conference on Street and Highway Safety in formulating a Uniform Vehicle Code intended to promote uniformity of regulatory action in the several states.

The depressible beam, fast coming into use in motor car lighting, is a step in the direction of securing the advantages of the desirable headlighting characteristic required by the specifications now in general use with the element of advantage otherwise secured through dimming. It is a step further in the direction of doing away with the disadvantage inherent in each practise, measurably avoiding the glare to which the approaching driver is subjected under the first practise and the hazard of passing vehicles with lights dimmed involved in the second.

The depressed beam has become a practicable device through the development of a lamp having twin filaments, one placed 9/64 in. above the other and suitably coordinated with lens or reflector elements. By switching from one filament to the other, the beam is depressed by from two to three deg. in a simple, positive, and easily applied manner.

Although but few of these devices as thus far developed are understood to have been approved by the Eastern Conference, more than 20 car manufacturers have adopted such equipments for their new cars.

Research. As a result of discussion following the 1925 Conference on Street and Highway Safety, a joint Steering Committee on Headlight Research has been formed by the Society of Automotive Engineers and the Illuminating Engineering Society.

The first objective of the Committee is to stimulate and guide experimental research directed toward the determination of the most satisfactory methods of automobile headlighting and, in accordance with the results of research, to formulate a code of recommended practise with respect to headlight equipment, adjustment, and use.

A program has been projected by the Committee including collection of data on visibility of objects under various conditions, experimental studies of visibility under conditions of actual driving, collection of data on quality and condition of lamps as used and the possibilities of bettering them, and a demonstration of the lighting which various automobile manufacturers consider most satisfactory for all-around use.

Relation to Street Lighting. In this connection it may be interesting to observe that the Committee on Street Lighting of the Illuminating Engineering Society is of the opinion that urban streets ought to be sufficiently well lighted to make it practicable to do away entirely with powerful headlights on automobiles, as is the practise in New York City, and to a limited extent in some other cities. It is held that streets which support considerable traffic ought to be sufficiently lighted to make this practicable, and that where such condition exists, the safety and convenience of all concerned will be promoted. The Committee is engaged in establishing street lighting minima above which it can feel confident in recommending the abolition of headlights.

RESIDENCE LIGHTING

One of the serious obstacles to the provision of better lighting in the home is the general lack of adequate outlets and the apparent difficulty and expense of making additions.

In the 1925 report of this Committee, reference was made to the Red Seal Campaign which undertakes to establish a minimum limit of adequacy of wiring according to the needs of each community. This plan has been adopted by nineteen different local electric leagues operating in six hundred and seventy-six communities and reaching nearly ten million people. One manufacturer is undertaking to establish a nationwide standard of quality and adequacy, and it is probable that others will follow along similar lines. It is therefore becoming the fashion to provide suitable wiring.

Although home-lighting equipment is still being selected with the main emphasis on the artistic features of metal working, with inadequate attention to the artistic and utilitarian values of the illumination itself, there yet seems to be a slow but general progress toward a better understanding of the merits of good lighting. The more common use of such terms as "shaded lights" indicates the trend.

SCHOOL LIGHTING

Preliminary tests have been made for the city of Newark, New Jersey, to secure an indication of the quantity and character of artificial light desirable for special public school classes of pupils having defective vision.

Observations of reading and writing suggested 15-ft. candles, or about 50 per cent more than ordinarily recommended. Comparison seemed to show semi-indirect and direct lighting, with large diameter diffusing glassware, to be equally acceptable, when supplementing daylight.

RAILWAY LIGHTING

The first edition of a Manual of Lighting Practise for Railroads has been practically completed by the Association of Railway Electrical Engineers.

The following topics have been covered: Fundamentals of Illumination and General Design, Design Data, Railway Shop and Roundhouse Lighting, Office and Drafting Room Lighting, Freight and Passenger Station Lighting, Warehouse and Pier Lighting, Yard Lighting, and Car Lighting.

Bus Lighting

The 12- to 16-volt systems are rapidly displacing the six- to eight-volt systems formerly used in bus lighting. A 21-c. p. incandescent lamp in an S-11 bulb designed for 300 hours of life has been standardized particularly for this purpose. It is interesting to note that some of the modern double-deck motor coaches have as many as 42 lamps for interior lighting. Special types of enclosing units and opal glass reflectors have been developed for motor coach lighting. Enclosing units are being used in the majority of new buses.

Illumination of Railroad Classification Yards

The employment of flood lights for illumination of railroad classification yards is becoming general. The Committee on Illumination of the Association of Railway Electrical Engineers reported that in 1924, 35 railroads had 90 yards equipped with an aggregate of about 2100 flood lighting units. Since that time the practise has been extended, notable recent installations being in the Selkirk Yard of the New York Central Railroad and the Markham Yard of the Illinois Central Railroad.

The necessity for adequate and proper illumination is enhanced by the introduction of the so called mechanical car-retarder system for controlling the speed of cars in the gravity type of classification yard since the operators of such equipment must have a good view of the entire yard from their control towers.

The above mentioned Committee has received a report from one railroad covering operating records of yards for several months before and after equipment with flood lighting, there previously having been no artificial illumination in the yard. This showed an increase of 15.5 per cent in number of cars handled at night with a decrease of 21 per cent in the average cost of damage suffered by cars and with entire elimination of personal injuries attributable to inadequate illumination during the months in which the records were studied.

Lighting for Advertisement

Electric signs and illuminated displays are a constantly growing factor in the advertising field. There has been within the last year a noted increase in the diversity of form of such displays together with a more rational design of the illuminated pattern based upon new engineering information.

Signal Lighting

Traffic Signal Lighting. With the large increase in the use of electric traffic signals, there has come fortunately an approach toward uniformity of practise in their use. Red quite generally now means "Stop" and green means "Go". Amber signifying "Caution" is employed in many cities for the assistance of the pedestrian.

A lens, a glass reflector, and an incandescent lamp are the usual equipment for each signal. Styles of signals are becoming more nearly uniform. Flexibility of control for isolated units and for large interlocked installations is a feature of the latest developments. A centrally controlled progressive system of signal operation has been developed and placed in operation in Chicago, which is greatly expediting the flow of traffic through the "Loop" district. Many of the signals installed in the past have not been sufficiently bright to have the necessary attention value when seen against the sky near the sun. Experience indicates that this is a subject to which considerable attention must be given to insure the efficacy of the signal under all conditions.

The proper solution of the traffic problem in any given case requires real engineering analysis.

With the multiplication of traffic signal lights, there comes a possibility of confusion which merits further study. Red traffic signal lights are sometimes placed at elevations of not more than 10 ft. above the roadway; there are red tail lights; and sometimes red stop lights on street cars and port running lights on motor buses. In some districts there is also a red signal at fire alarm boxes. Police patrol stations and building exit markers further complicate the situation. This is a matter which clearly requires care and attention if we are to avoid complexity of signal lights which may result in confusion and increase the accident hazard.

The 1926 Conference of Street and Highway Safety, taking cognizance of the above described situation in regard to traffic signals, adopted by majority vote a recommendation that yellow be employed as a rear lamp signal for motor cars.

Railway Signal Lighting. Progress is reported in the development of electric signals both to replace the kerosene lamp for night use and to serve as a full color signal both day and night. The development has involved specialized design of lamp filament and bulb, of optical system, of a universal accurate focusing mechanism, and of means for directing the light as desired.

Aviation Lighting. After much experimentation, particularly on the part of the United States Air Mail Service, it appears that the following is likely to become general practise in aviation lighting: a 500,000,000-c. p., high-intensity arc searchlight, visible in clear weather for 150 mi., is located every 250 mi. along the route. Every 25 mi. between these beacons are located 60-ft. steel towers supporting a 24-in. rotating beacon, employing a 30-volt, 900-watt, tungsten filament lamp. This beacon has a beam c. p. of approximately 7,500,000, and in clear weather is visible for 75 mi. The beam is elevated at an angle of one to one-half deg. above the horizon, and rotates at a speed that provides a flash every 10 seconds. In very hilly or mountainous sections, routes are marked with smaller beacons consisting of four automobile headlamps equipped with 12- to 16-volt, 21-c. p. tungsten filament lamps. These beacons are rotated at a speed of 10 rev. per min.

The boundaries of the principal landing fields are marked by 60-c. p. series lamps spaced 150 ft. apart around the edge of the field. Emergency landing field boundaries are marked by two-c. p., three- to four-volt lamps, 28 of these being connected in series around the boundary of the field. They are operated from gasoline-electric equipments. Each fitting has a relay which introduces an equivalent resistance into the circuit in case of a lamp failure. It is usual to place a number of flood-lighting equipments along two sides of the field to illuminate it for landing purposes. These have 120-deg. Fresnel lenses and are equipped with 900-watt, 30-volt, tungsten filament lamps. To some extent 10-kw. ribbon filament lamps are also used in very large Fresnel type lenses for flood lighting landing fields.

With the development of commercial flying it is probable that there will arise requirement for well defined routes and landing fields. The total lighting load at a typical air mail field is 38 kw.

A New Application of Light

The transmission by wire or wireless of actual scenes or so called "television" is understood to have been accomplished experimentally. With the aid of the photoelectric cell and the Moore gaseous conductor lamp it has been found possible to use relatively simple apparatus for the transmission and reception of pictures within the interval imposed by the human eye as a requirement for sustained vision. It is expected that this new art will be brought to commercial development at a relatively early date.

Illumination Nomenclature and Symbols

Nomenclature, abbreviations, and symbols in the illumination field are now fairly well fixed in this country. The report of the Committee on Nomenclature and Standards of the Illuminating Engineering Society, having been submitted some time ago to the procedure of the American Engineering Standards Committee, was adopted as an "American Standard."

Illumination Items in the Proceedings

Under the auspices of this Committee there have appeared in the proceedings from time to time brief articles designed to keep the membership posted as to significant developments in the lighting field. Evidences of appreciation having been received, this practise is being continued and may be recorded as a supplementary activity of this technical committee.

Conclusion

The Committee on Production and Application of Light, having thus reviewed the field of lighting within its purview, is in a position to report to the Institute that electric lighting is undergoing a sound and wholesome development under the influence of forces largely commercial in character but greatly beneficial from a public point of view.

The Illuminating Engineering Society is intent upon developing the science and the art of illumination and is making measurable progress in its activities. Associations of manufacturing and operating interests are contributing notably through progressive engineering and business development which derives adequate sanction from the fundamentally favorable consideration that every improvement in equipment and practise in the lighting field is mutually advantageous to the commercial interests and to the public.

Discussion

W. E. Beaty (Cincinnati): The application of individual motor drive to varied machines, and particularly to the machines requiring 5 h. p. and below, has introduced the need for indicating lamps in connection particularly with automatic control of such machines. The gaseous lamp shown may meet that problem, and I should like to ask Mr. Millar what the voltage limitations are on a lamp of that character? I am wondering if the lamp can be used on all circuits up to, say, including 550 volts?

W. N. Goodwin: We are quite familiar with these gaseous lamps for advertising purposes in tubes several feet long employing a few thousand volts. I understand that in those, the the voltage is frequently a function of the distance between the electrodes, so that it should be quite easily possible to adopt them for any voltage from 115 volts up to several thousand.

P. S. Millar: The Committee Report records the achievement of operating these small lamps on a voltage as low as 115. It should be easy to make them operate on 220 or 550 volts.

Electrical Machinery

Annual Report of Committee on Electrical Machinery*

BY B. L. BARNS, Chairman of Subcommittee on Annual Report

To the Board of Directors:

The activities of this committee during the past year have been carried out on a plan of organization comprising a number of subcommittees as outlined in Mr. H. M. Hobart's report to the President and Board of Directors on the Development of the Activities of the A. I. E. E. Technical Committees under date of January 19, 1925. This report is under consideration by the Committee on Technical Activities to determine the advisability of recommending it in whole or in part for all of the technical committees. The scope of the Electrical Machinery Committee's activities has been based on the recommendation contained in a report made by Mr. A. W. Berresford as chairman of a Special Committee to Review Technical Activities of the A. I. E. E. under date of January 23, 1924.

During the past year the Electrical Machinery Committee has been more active in standardization work. A subcommittee on standardization has been organized under the chairmanship of Mr. J. C. Parker which is functioning in an advisory capacity to the Institute's Committee on Standardization in matters concerning electrical machinery. The committee on Electrical Machinery, through a subcommittee under the chairmanship of Prof. V. Karapetoff, is also sponsoring research work affecting standardization and advancements in the art and is collaborating with the Committee on Research.

That part of the Electrical Machinery Committee's work which has resulted in the most tangible accomplishments has been the procuring and reviewing of suitable papers on various subjects which have been presented under the auspices of this committee at the regular and regional meetings. The following table shows the number of papers under different classifications that have been presented during the year.

Factors which affect design of electrical machinery...	5 papers
Generator design and construction..................	8 papers
Motor design and construction......................	4 papers
Transformer design and construction...............	2 papers
Total........................	19 papers

In preparing a review of the development or advancement of the electrical machinery art one is inclined to turn first to those new types of machines that have developed and those machines which represent new records for high rating or size or voltage. The research work, whether it be mathematical, chemical, or physical, by which new types and high ratings are made possible, is less spectacular but of fundamental importance. The enumeration of new high water marks of ratings may not of itself appear to be an index of an advancement of the art but there are often many details and problems involved in the successful design and manufacture of such machines involving real pioneer work. Nor is all the engineering pioneer work confined to setting new records of large size. If we would but examine them carefully we would find research and ingenuity written all over the smaller apparatus that we see about us every day and take for granted.

It is, of course, beyond the scope of this report to record even briefly the solution of the many problems involved in the perfecting of new developments, for time and space would not permit, nor is such detailed information available for that purpose. In considering the remarkable things that have been accomplished in building huge machines we should not overlook the importance of the development and improvement of the small apparatus upon which the expansion and growth of the electrical industry is so largely dependent.

Again we take a considerable measure of satisfaction in recording new attainments reached in higher ratings of machines which have been put into successful operation. New records have been made for large capacity all along the line. Larger ratings than ever before obtained have been built in transformers, synchronous motors, induction motors, d-c. motors, horizontal-shaft alternators and turbo alternators.

The following review is classified under headings for convenience in preparation and reference and an attempt has been made to include under each a bibliography of the more important articles that have been published during the year. An attempt has also been made to cover the more important developments that have taken place in Europe as well as in America (the United States and Canada). An apology is offered if important articles or developments have been omitted. Such omissions are entirely accidental.

RESEARCH

This committee now has the following subjects under consideration and it is probable that as investigations advance sufficiently to mention definite results they will be described in the form of papers for presentation at

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*Committee on Electrical Machinery:

H. M. Hobart, Chairman, General Electric Co., Schenectady, N. Y.
J. C. Parker, Vice-Chairman, Brooklyn, N. Y.
C. A. Adams,
H. C. Albrecht,
B. F. Bailey,
B. L. Barns,
B. A. Behrend,
A. C. Bunker,
James Burke,
Walter M. Dann,
L. L. Elden,
G. Faccioli,
W. J. Foster,
Harold Goodwin, Jr.
J. I. Hull,
V. Karapetoff,
A. H. Kehoe,
A. E. Kennelly,
P. M. Lincoln,
A. M. MacCutcheon
F. D. Newbury,
N. L. Pollard,
R. F. Schuchardt,
C. E. Skinner,
A. Still,
R. B. Williamson.

Presented at the Annual Convention of the A. I. E. E., at White Sulphur Springs, June 21-25, 1926.

Institute meetings. These topics are mentioned not so much to record the actual progress made by the Committee as to indicate problems of importance brought to the Committee's attention.

Effect of Altitude on the Dielectric Strength of Insulations. An investigation has been proposed to determine whether it is desirable to draw a distinction between apparatus that is to be used at sea level and high altitudes as regards the insulation tests.

Influence of Expansion and Contraction of Conductors on Insulation Deterioration in Long Machines. This subject has already received a great deal of attention and some valuable experimental work has been done. This subject is becoming of greater importance with the increasing capacities of turbo alternators. Further investigation has been suggested as desirable.

Surge Tests of Insulation. An investigation of this subject is being undertaken to determine whether it is desirable or practical to standardize a test of this sort on the insulation of machines that may be subjected to transient high-voltage conditions.

Hot Spots in Cores of Alternators. Observations made by engineers in Europe have indicated that higher temperatures may be reached in the slots at the ends of a turbo generator core than in the slots at the center of the core. Manufacturers in the United States have reported that they have been unable, from tests made on machines of American make, to substantiate this statement. This may be due to the difference in the design of end windings in Europe and in America.

Evaluation of Conventional Losses. The present A. I. E. E. Standards assign conventional values to certain losses that are known to exist. It is felt that methods should be devised by which these losses may be determined by tests so that the design and tests may be reduced to a more exact science and due credit may be given to designs that are especially meritorious in this respect.

Calorimetric Method of Determining Losses in Alternators. The feasibility of determining the losses and efficiency of alternators by calorimetric methods which are based on the temperature rise of the cooling medium is being investigated.

Stability of Alternators. As distribution systems increase in size and the use of long, high-voltage transmission lines is extended, the subject of stability of generators becomes of greater importance. A study of the possibility of the evaluation or definition of generator stability is being made so that this characteristic may be specified.

Relation between Dielectric Tests on New and Used Machines. This subject has arisen from a consideration of recommended practise in checking the condition of the insulation of a machine that has been in operation for some time.

Non-Destructive Physical Tests on New and Used Insulation. Physical tests of insulation have been suggested as being desirable; accordingly, the subject is being investigated.

During the past year research work has been done on the following subjects that come within the scope of this committee's activities:

Effect of Altitude on Ratings of Electrical Machinery. This investigation was undertaken for the purpose of providing a satisfactory revision of the present A. I. E. E. rules which have been recognized as not being entirely satisfactory.

Copper Eddy-Current Losses. A theoretical foundation for the calculation of no-load copper eddy-current losses in machines has been presented.

Motor Band Losses. Methods of calculating losses in the binding bands of armatures of direct-current machines have been made practicable.

Heating Curves of Electrical Machinery. A further study of the prediction of the heating curves of machines has been presented.

Alternator Short Circuits. The calculation of the transient phenomena resulting from short circuits of the armature windings of alternators has been a subject of study for many years and an accurate method of predetermining the value of the short-circuit current is becoming more and more important as the generating capacity of power systems increases from year to year. Two papers have been presented to the A. I. E. E. during the past year dealing with this subject. One of these papers treats the subject on the basis of Kirchoff's law that the vector sum of all the currents at a junction is equal to zero. The other paper is based on the constant linkage theorem, viz: "In a circuit having zero resistance, the algebraic sum of the flux linkages of the circuit must remain constant."

Measurement of Stray Load Losses in Alternators. A paper has been presented which has suggested a different method of determining stray load losses than that specified in the present standards, and results of tests have been given.

Ventilation of Turbo Alternators. During the year this work has been carried forward to a conclusion by the use of small models. Previous calculations and empirical formulas have been checked so that now a complete solution of the complex problem of measuring static pressures in a fluid moving at considerable velocity can be successfully obtained.

Theory of the Synchronous Induction Motor. This subject has received considerable attention in Europe and a number of excellent treatises have been written.

Hydrogen as a Cooling Medium. A further investigation of the problems incident to the use of hydrogen as a cooling medium has been undertaken.

Characteristics of Synchronous Machines. Blondel's theory of two reactions has been extended to include the effect of harmonics and of field excitation in the quadrature axis.

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VENTILATION OF ELECTRICAL MACHINERY

Ventilation, one of the fundamental considerations in the design of electrical machinery, has been a subject of very active study and experimentation. Because the design of turbo alternators involves a relatively greater need of a knowledge of the phenomena incident to the flow of gases through irregular passages, experimental work has been carried out to determine the forms of fans, bafflers and internal ducts which would give the desired cooling effect with a minimum of windage losses by introducing into new machines alternative ventilation schemes which were tried out when they came to test. In this way much valuable and accurate information has been obtained which has been free from some of the uncertainties of theoretical considerations of a very complicated problem. Nevertheless, considerable help has been obtained by building small models and testing them under laboratory conditions to check and verify principles that have been deduced by a theoretical treatment of the problem. Models have been constructed to represent the vent ducts and air passages in the stator core of a turbo alternator and tested to determine the drop in pressure in the several parts of the passage ways. In this way data were obtained by which close approximation can be made of the amount of pressure required to force a given quantity of air through a given machine. Other models have also been tested to determine the most efficient shapes of slot retaining wedges in stators and the best arrangement of spacers in the internal ventilation ducts. All of this work has contributed toward better designs not only of turbo alternators but also of other types of machines.

This committee's report of last year made mention of an investigation into the use of hydrogen as a cooling medium for rotating machinery. The results of this have been published and this method seems to have many advantages that would make it desirable for commercial application and some operating engineers are looking forward to the time when it will be found feasible to use hydrogen for this purpose. Some of the new machines that are now being purchased are being arranged so that they may be readily adapted to the use of hydrogen when the problems peculiar to its use are finally solved.

A few years ago the recognized need of providing clean air for the ventilation of machines, especially large capacity turbo alternators, was met by the use of air cleaning and air washing apparatus which took air from outside the building, cleaned it, and discharged it outside again after it has passed through the machine, probably cleaner than when it came from the washer. This veritable throwing away of perfectly clean air appeared to be an uneconomical procedure and during the past year or more a more rational arrangement of a closed system has been used particularly for large turbo alternators. This system recirculates the same air, cooling it by passing it over water cooling pipes. The advantage of this arrangement has been recognized during the past year for application to other types of machines, it having been adopted for hydroelectric plants and substation equipment. The closed ventilating system necessarily presupposes an abundance of cooling water at low cost.

During the past year an ingenious arrangement has been devised for cooling heavy duty bearings of substation machinery where the water supply for such purposes is expensive. This scheme consists of a closed water circulating system so arranged that the water is cooled by the air used to ventilate the machine.

Although for a number of years sales campaigns have been carried on advocating the use of small fans to increase the heating capacity of radiators in very cold weather, this idea has not been applied to the cooling of transformers until this year. By the use of jets of air directed over the surfaces of self-cooled transformers the convection of heat has been greatly accelerated. This development gives promise of many possibilities.

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TESTING ELECTRICAL MACHINES

A renewed interest has been shown in the accurate determination of load or stray losses of rotating machines. The assumption that is made in the present A. I. E. E. Standards that the measured short-circuit core loss is equal to the load loss is generally considered to be far from correct in certain types of alternators and there is a general desire for a more accurate method of determining these losses that would be suitable for application to commercial testing. A paper which was presented at the Midwinter Meeting advocated a method of test which was based on measuring the synchronous motor power input when running light under conditions of leading and lagging currents. The discussion of this paper indicated that a calorimetric method of measuring losses has been in use in one of the large factories for several years and that this method is being studied by others. The calorimetric method makes use in different ways of the observed temperature rise of the air passing through the machine. Since closed ventilating systems are being more generally used, the determination of the losses by the measurement of the heat absorption by the air cooling equipment seems to offer a satisfactory method of test. These calorimetric methods do not, however, offer means of direct measurement of the so called load losses but are more suitable for the determination of the total losses under actual load condition. This committee is desirous that suitable commercial test methods be developed for a more accurate determination of the efficiency of large synchronous machines than is now described in the A. I. E. E. Standards.

The testing of large machines on power house settings such as water wheel driven alternators often involves the measurement of losses by the retardation method. During the past year a chronographic method has been devised for obtaining a simultaneous record of time and revolutions on a paper ribbon.

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TURBO ALTERNATORS

In this Committee's report of last year, mention was made of a 62,500-kv-a. 1200-rev. per min. turbo generator which had recently been put into operation as the largest single-shaft unit that had been built. During this year two machines of this capacity but of 1800 rev. per min. have been put into successful operation. The step up from 1200 rev. per min. to 1800 rev. per min. has been made possible by the more skillful design of the electrical parts, a better proportioning of the mechanical parts, and more extended multiple ventilation. This increase in speed makes it possible to further extend the upper limit of rating for single-shaft machines to such an extent that two 60,000-kw. units are now under construction and 75,000-kw., 80 per cent power factor generators are being considered. A machine of this rating at 1200 rev. per min. would present difficulties in manufacture because of the great weight of the rotor. Another large turbo alternator that was put into service the past year is rated 66,660 kv-a., 1500 rev. per min., 25 cycles. In France one of 60,000 kv-a., 1500 rev. per min., 50 cycles, is about ready to be put into operation. The description of this generator in a paper presented before the Institute brought out in the discussion some interesting information on the relative design practises in Europe and the United States. The French machine employed deep slots in the stator core for the dual purpose of increasing the reactance and providing axial paths for ventilation. Another feature is the much smaller ratio between saturation and impedance ampere turns than has been considered good practise in America. American practise has considered it necessary to maintain this ratio at near unity while European practise allows it to be as low as 0.5. This difference may possibly be accounted for by the difference in operating practise on the two continents. In America the apparent desire of the operators is to get the maximum output from the generator throughout its period of life, while in Europe, it is said, a generator is usually operated at slightly less than its rated capacity. Another factor has been the general practise in Europe to employ voltage regulators to care for the swings in load and supply the necessary excitation to prevent the generators from falling out of step, while in the United States operating people have insisted upon generators that may be hand regulated and which are not dependent upon automatic regulation for their stability.

It is interesting to note that designing engineers now predict the possibility of building machines having ratings of 75,000 kw., 80 per cent power factor at 1800 rev. per min., 100,000 kv-a. at 1500 rev. per min., for 50 cycles and 40,000 kv-a. at 3000 rev. per min. for 50 cycles. A 25,000-kw., 3000-rev. per min. machine has been built in Europe. It is the opinion of a leading engineer that if greater capacities than 75,000 kw. are demanded by the operating companies they will be obtained by advances in the design and materials applied to the four-pole generator rather than a return to a six-pole design. For very large units there are advantages in the cross-compound arrangement or multiple-shaft design involving two or more generators. A unit of this type having a house generator, in addition to the two main generators and with a total rating of 77,000 kw. has been completed during the past year and another of 80,000 kw. is under construction. An order has recently been placed for a two-generator unit rated at 90,000 kw. Triple shaft units with three generators of combined capacities of 160,000 kw. and 200,000 kw. are being considered.

These very high generator ratings have involved problems concerning mechanical stresses in the rotor due to centrifugal forces and deflection, insulation of the rotor windings and ventilation of the stator. Centrifugal stresses have been provided for by the development of special alloy steels. The limitations of rotor deflection in its relation to critical speed have been largely eliminated by a better knowledge of the mechanics of the problem. Whereas before this was so well understood it was believed that the rotor speed should be well below the critical speed, now the designers are concerned only that the normal speed must not approach any multiple of the first critical speed. Experience has shown that the critical speed is not dependent only on the weight and length of the rotor but that it is influenced by the type and proportions of the stator frame and bearing supports and the design of these parts offers a means of control. The very great pressures on the rotor winding insulation coupled with the expansion and contraction resulting from changes of temperature has necessitated the development of types of insulation for the end coils that will withstand these destructive conditions. In the matter of ventilation, schemes of providing a number of multiple paths have been devised to obtain the equivalent of a short core machine. In some cases as many as 28 parallel paths are provided. For the ventilation of the smaller generators, fans mounted on the shafts inside the end bell housing have been used although separate motor driven fans would be more efficient. This arrangement has the advantage, however, of eliminating a separate auxiliary which if it failed to operate would cause the removal of the generator from service. The fan capacity required for the very large generators has exceeded the limit of the type that can be mounted on the rotor, and separate fans or blowers are now a necessity. With well designed external blowers, having an efficiency of 65 per cent, a saving in the total losses of the machine would amount to approximately 0.1 to 0.2 per cent at rated load. On the larger machines it is often feasible by the use of external blowers to build them with smaller diameters than would be possible with fans on the rotor. Under such conditions the external blower ventilated machine may show an efficiency of from 0.3 to 0.4 of a per cent higher than the machine with fans attached to the rotor. Besides the better efficiency this arrangement affords another important advantage in that the air passes through the air coolers of a closed system after leaving the fans and before it enters the generator, thus extracting the heat put into it by the fans. A material reduction in the temperature of the air entering the generator is obtained as compared with its temperature when the fans are assembled on the shaft. The closed ventilating system with fin-type radiator coolers is now being used almost universally for turbo alternators.

During the year there have been improvements in the Emmet mercury turbine with resulting higher efficiencies.

The 3000-kw. 3600-rev. per min., 20-stage turbine for operation at a steam pressure of 1200 pounds which was mentioned in our report of last year has been put into successful operation.

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WATER-WHEEL DRIVEN ALTERNATORS

American hydroelectric practise still favors the vertical shaft arrangement of alternators to such an extent that about 80 per cent of the water wheel generators are of this type and the arrangement is more or less standardized with a spider or bridge spanning the top of the armature frame for supporting the thrust bearing which carries the weight of the generator rotor and the water wheel runner. Two years ago several units were built with the thrust bearing beneath the revolving field and no bearing or superstructure above the rotor except a cover or housing. Although there seemed to be a number of advantages to be gained by this so called umbrella type, no further machines have been built 'til this year. A 6250-kv-a., 138½ rev. per. min. generator of this type is now under construction. One manufacturer is using a fabricated construction of welded steel plate to a large extent for the armature frames. The closed ventilation system has been adopted in one large water power station.

During the year a rather formidable number of large capacity alternators both horizontal and vertical shaft have been under construction or installed and the following incomplete list will serve to indicate to what extent machines of large ratings are being used.

3-37,500 kv-a.	12,000 volts,	120 rev. per min.	vertical shaft
3-18,750 kv-a.	6,600 volts,	100 rev. per min.	vertical shaft
2-29,000 kv-a.	.... volts,	300 rev. per min.	vertical shaft
2-25,000 kv-a.	13,200 volts,	450 rev. per min.	vertical shaft
2-20,000 kv-a.	11,000 volts,	200 rev. per min.	vertical shaft
1-17,000 kv-a.	6,600 volts,	375 rev. per min.	vertical shaft
1-17,500 kv-a.	6,600 volts,	100 rev. per min.	vertical shaft
2-25,000 kv-a.	6,600 volts,	90 rev. per min.	vertical shaft
2-54,000 kv-a.	12,000 volts,	187½ rev. per min.	vertical shaft
2-13,500 kv-a.	6,600 volts,	277 rev. per min.	vertical shaft
4-16,000 kv-a.	11,000 volts,	133 rev. per min.	vertical shaft
3-20,000 kv-a.	11,000 volts,	330/360 rev. per min.	vertical shaft
2-15,625 kv-a.	11,450 volts,	257 rev. per min.	vertical shaft
2-25,000 kv-a.	12,000 volts,	300 rev. per min.	vertical shaft
2-18,750 kv-a.	12,000 volts,	150 rev. per min.	vertical shaft
2-15,000 kv-a.	6,600 volts,	171 rev. per min.	vertical shaft
1-21,000 kv-a.	11,000 volts,	138½ rev. per min.	vertical shaft
1-12,000 kv-a.	12,500 volts,	128½ rev. per min.	vertical shaft
1-12,000 kv-a.	2,400 volts,	250 rev. per min.	vertical shaft
8-12,500 kv-a.	14,000 volts,	100 rev. per min.	vertical shaft
2-10,600 kv-a.	12,000 volts,	120 rev. per min.	vertical shaft
2-36,000 kv-a.	6,600 volts,	100 rev. per min.	vertical shaft
2-30,000 kv-a.	6,600 volts,	112½ rev. per min.	vertical shaft
2-10,000 kv-a.	6,600 volts,	180 rev. per min.	vertical shaft
2-33,000 kv-a.	11,000 volts,	360 rev. per min.	horizontal shaft
1-33,000 kv-a.	13,200 volts,	360 rev. per min.	horizontal shaft
2-25,000 kv-a.	11,000 volts,	450 rev. per min.	horizontal shaft
1-20,000 kv-a.	11,000 volts,	375 rev. per min.	horizontal shaft
1-14,444 kv-a.	6,600 volts,	600 rev. per min.	horizontal shaft
1-13,000 kv-a.	12,000 volts,	500 rev. per min.	horizontal shaft
1-13,125 kv-a.	4,400 volts,	225 rev. per min.	horizontal shaft

Although the 65,000-kv-a. units at Niagara Falls are still the largest machines that have been built, larger alternators have been under consideration and will probably be built in the near future. The manufacturers are prepared to build the following maximum rated water-wheel driven alternators of the vertical shaft type and designed to meet the usual run-away speed requirements:

10,000 kv-a. at 720 rev. per min.
20,000 kv-a. at 600 rev. per min.
30,000 kv-a. at 514 rev. per min.
55,000 kv-a. at 400 rev. per min.
80,000 kv-a. at 300 rev. per min.
110,000 kv-a. at 200 rev. per min.
130,000 kv-a. at 100 rev. per min.

The use of welded steel plate stator frames and the substitution of a few separate sole plates for a continuous base ring has appreciably reduced vertical shaft generator weights. Comparisons of construction and actual weights indicate that American designs are at least twenty-five per cent lower than similar European machines.

The past year has set a new record in the maximum capacity of horizontal shaft water-wheel driven generators. Two alternators rated 33,000 kv-a. at 360 rev. per min. were built in the United States for a Brazilian plant and a third was built for a California plant. In Europe a 30,000-kv-a., 500-rev. per min. horizontal shaft alternator has recently been built which has an unusual rotor construction consisting of twin rotors each with its pole pieces and windings mounted side by side on the shaft in a common armature core. This construction was adopted to conform with certain shop and transportation facilities.

The use of automatic or remote control of hydroelectric units is extending. During the past year some 9000 kv-a. units have been put into successful operation, being the largest yet constructed for this type of control.

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Diesel Engine Driven Alternators

There has been considerable activity during the past year in exploiting the use of Diesel engines as prime movers for stand-by plants. An installation of this type which is worthy of note consists of three 3125-kv-a. 80 per cent power-factor, 25-cycle, 125-rev. per min. units for the Panama Canal which are probably the largest generators driven by this type of engine.

Transformers

An important consideration in transformer work is the provision of means of dissipating the losses with sufficient rapidity to keep the temperature of the transformer within safe limits. In the past this has been accomplished by one of four methods, *viz:*

1. Air-blast, in which air is blown around and through the core and windings.

2. Self-cooled, in which the heat is transferred from the core and coils to the casing by an oil bath and then dissipated principally by radiation.

3. Water-cooled, in which the heat is removed from the oil bath by water cooling coils.

4. Oil-cooled, in which the oil bath is circulated through a cooling apparatus independent of the transformer.

A new method involving an old principle has gained much favor this year. This consists of directing jets of air upward and over the radiating surfaces thereby removing a larger part of the heat by convection than can be done by the natural circulation of air. Some very large transformers arranged for cooling by air jets have recently been completed. The rating of these oil-insulated, air-cooled distributed core type transformers is 30,000 kv-a., 220,000 volts Y, 125,000 volts Y, 10,640 volts delta at 60 cycles and 55-deg. cent. temperature rise by resistance.

The method in which the oil is cooled outside of the transformer tank is more generally used in Europe than in America. This year, however, some very large transformers arranged for this method have been built and installed in America. They are rated 20,000 kv-a., 50 cycles, 72,000-Y, 11,000-volt single-phase, and sea water is used as the cooling medium. Although air-blast transformers are not used to as great an extent as they were many years ago there is still a demand for them for installation in certain localities where the presence of large quantities of oil is considered to be a fire hazard. The largest air-blast transformer yet built was installed this year. It was rated 18,500 kv-a., 25 cycles, 11,800-3,300 volts three-phase and weighed about 43 tons.

The largest self-cooled single-phase transformers ever built were completed this year. They are rated at 20,000 kv-a. 72,450-Y, 13,800 volts, 60 cycles. Likewise the largest water-cooled transformers both in point of rating and physical size have been built this year. These are rated 28,866 kv-a., 220,000-Y, 66,000-Y, 10,750 volts, 60 cycles, single-phase. As will be noticed in the ratings of the above self-cooled and water-cooled transformers, tertiary windings are being used to a large extent in large high-voltage power transformers. The tertiary windings, being connected delta, permit using the star connection for both primary and secondary windings. The tertiary winding also provides a means of connecting a condenser into the system for transmission line voltage regulation without using an independent transformer where the secondary voltage is too high for a condenser winding.

During the past year the largest and highest voltage transformers ever built in the British Empire were completed in Canada. These are designed for a rating of 25,000 kv-a., single-phase, 60 cycles, 154,000 volts and weigh 80 tons each. Twelve of these transformers were built on one order.

A German manufacturer has built the largest capacity transformer ever constructed. By removing the aluminum windings of a 60,000-kv-a. transformer that had been in operation for several years and substituting copper windings, a three-phase transformer having a rating of 75,000 kv-a., 110,000 volts was produced. This transformer is now in successful operation.

Probably the most important development pertaining to transformer construction has been the further improvement of and extended use of transformers equipped with load ratio control or devices for changing taps without interrupting the load. Transformers of this type are being extensively used to regulate circulating currents between systems where the flow of energy is in either direction and where each system must maintain its voltage independently of the direction of the flow of energy.

One of the largest units of this latter type is a three-phase self-cooled auto-transformer rated 36,000 kv-a., 66,000 volts, 60 cycles. It is used as a tie between two 60,000-volt systems. The taps are arranged to give 10 per cent buck and 10 per cent boost in nine steps. The total weight of this transformer with operating mechanism and circuit breakers is about 73 tons.

The electrical circuit of this unit is novel in that double transformation is employed to secure voltage control. Electrically, the unit consists of two distinct transformers each provided with a high-voltage and low-voltage winding. The high-voltage windings of each transformer are connected to the 66,000-volt line, one being connected in shunt to the line and the other in series to the line. The shunt transformer contains the necessary taps and ratio adjusters by means of which fractions of the low voltage winding on the shunt transformer can be connected to the low-voltage winding on the series transformer. By varying the taps, the desired amounts of voltage either bucking or boosting can be inserted in the high-voltage line through the series transformer. By thus placing ratio adjusters in a separate low-voltage circuit, the switching apparatus is protected from abnormal voltages which may arise in the high-voltage lines.

It sometimes becomes desirable for the sake of greater insurance against interruption of service to separate the regulating function into a separate transformer. An interesting example of this consists of banks of three single-phase, self-cooled, power transformers rated 20,000 kv-a., 12,600-13,800 volts, 60 cycles, a three-phase, self-cooled, regulating transformer of 60,000 kv-a. capacity.

Transformers having arrangements for tap changing under load are also being extensively used for electric furnace work.

The extension of the idea of rural electrification has opened up a new field of design in small distribution transformers for very high voltages. Some 100-kv-a., 110,000-volt transformers have recently been built for this purpose.

During the past year the largest potential transformers were built for use in America. These were designed for use between line and neutral of a 144,000-volt system. They have only one high-voltage bushing, the neutral being grounded to the cover.

The development of successful high-voltage transformers is assisted greatly by accurate knowledge of very high-voltage phenomena which can be obtained only by experimentation at voltages much higher than the actual transformer potentials. During the year a study has been made of transformer and other high-voltage insulation with a 2,000,000-volt lightning generator for the purpose of obtaining practical information in designing apparatus to resist the effects of voltages induced in transmission lines by lightning. High-voltage testing sets to provide voltages as high as 2,100,000 volts have been constructed. These consist of 350,000-volt transformers connected in series or in chains, each succeeding unit being insulated from ground by supporting it on insulating cylinders.

Improvements have been made in the temperature-indicating devices that are being used more generally to check the actual operating conditions of service transformers. A model has been produced which is suitable for use with subway transformers.

Of special interest to radio telephone engineers and enthusiasts is the development of a transformer for use with the ignition systems of oil heaters which successfully eliminates radio interference trouble, a serious objection to electric ignition in these devices heretofore. This transformer is provided with an internal magnetic blocking filter as well as low-voltage and high-voltage filters. During the past year the largest high-voltage oil-immersed reactors were constructed that have ever been built. They were rated 1300 kv-a., 3000 volts for operation on a 73,000-volt, 60-cycle circuit. A distinctive feature of these reactors is that they were provided with shielding coils to prevent the stray magnetic field entering the steel tank and setting up additional losses. Air-cooled reactors have presented a serious difficulty due to the fact that they would flash over when a failure occurred on the circuit in which it is connected if any conducting material were lodged between the turns. An investigation has been carried out to determine a suitable insulation for the conductor of this type of reactor which indicates that a heavy asbestos covering will prevent flashovers from this cause.

A new transformer connection having 100 per cent apparatus economy for transformation from two-phase to three-phase, or vice versa, has been devised which has been described in a paper presented to the Institute.

Some very large single-phase transformers that are quite novel in certain respects have been built recently in Europe for 16⅔-cycle railway work. They have three windings so arranged that power may be supplied from any one winding to either one or both of the other two. One of the two cores carries a 132,000-volt winding, the other carries a 66,000-volt winding and a 15,000-volt winding is divided into four parallel sections on both cores. It is stated that with this arrangement the bus bars and oil switches are not necessary on the 15,000-volt side. These transformers are rated 11,000 kv-a. and the total weight including oil is 117 tons.

A new protective device for large transformers has been developed in Germany which is called the Buchholz Relay. The operation of this relay is based on the fact that practically any trouble in an oil-insulated transformer will create gases which rise to the surface of the oil. A small amount of gas will cause the relay to operate. This relay will detect incipient troubles that will not actuate a current operated device and will disconnect the transformer before great damage is done. This relay can be used where the differential relay scheme cannot be applied.

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D-C. Machines

A new record has been set in direct-current motor capacity by the construction of an 8000-h. p., 40-rev. per min., single-armature blooming mill motor. Several 7000-h. p. reversing mill motors have been built and the power for the first edging rolls to be equipped with electric drive is supplied by two motors of 2000 h. p. each. The simplicity and flexibility of speed control of direct-current motors make them very desirable for variable speed drives and there has been considerable activity in the construction of large motors for steel mill work.

A new type of armature winding has been developed and applied to many commercial machines. This winding, because of the shape of the coils, is called "Frogleg" and is a combination of a wave winding and a multiple winding arranged so that the armature is cross-connected by the wave winding and all cross-connections at the commutator and at the back of the armature are eliminated. This type of winding provides complete cross-connection which is very desirable for good commutation. The winding is arranged so that each slot contains four bars, two of which are in the wave winding circuit and two in the multiple winding circuit, and each commutator bar is connected to both circuits.

At a recent exhibition in Europe a direct-current generator rated at 12,000 amperes and 5 volts was shown.

The use of multiplex windings has, in general, been avoided due to the failure of these types to properly commutate in certain instances and to the uncertainty of satisfactory results. A paper has been presented calling attention to certain rules that must be observed when multiplex windings are used.

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FREQUENCY CHANGER SETS

During the year a 40,000-kw. frequency changer set of the cascade induction motor type has been built and put into service and two sets of this capacity having both machines of the salient pole type are under construction, one of which is arranged with a spring support for the 25-cycle generator so that it may be used for single-phase power. A 7500-kv-a. set for converting from 60-cycles, three-phase to 25-cycles, single-phase is being built with the stator of the single-phase generator spring supported. The purpose of the spring support is to minimize the effect of the vibration incident to single-phase operation at low frequencies.

An automatic control equipment has been arranged to remove the field excitation of the motor and reduce that of the generator of a frequency changer set when a disturbance occurs on the motor circuit and again to restore the excitation when the line voltage is normal. This type of control called for a special design of the pole face winding to enable the motor to resynchronize while the generator is carrying part load at the reduced voltage. The benefit of this arrangement has been a greatly improved service due to a material reduction in the length of the interruptions.

SYNCHRONOUS CONDENSERS

Considerable advancement has been made recently in the design of synchronous condensers toward the reduction of losses. In sizes above 5000 kv-a., the losses are now as low as from 1.75 per cent to 2 per cent. The extension of high-tension distribution systems and higher transmission voltages has expanded the field of line voltage regulation by the use of synchronous condensers very rapidly. The largest condenser ever built, rated at 40,000-kv-a., 600 rev. per min., was recently put into operation in connection with a 220-kv. transmission line. The important consideration of keeping the losses down to a minimum has resulted in a strong tendency toward higher speeds. A considerable number of machines of 25,000 kv-a. to 40,000 kv-a. at 600 rev. per min. and of 15,000 kv-a. at 720 rev. per min. has been built. The disadvantage of the noise at these high speeds has been overcome largely by a totally enclosed construction.

An interesting development in the automatic control of synchronous condensers for line voltage regulation has been an installation of a 1000-kv-a. condenser provided with a voltage regulator arranged to start and stop on voltage indication. The condenser is at the end of a long transmission line and remains connected to the line through the starting tap in case of complete failure of power, thus providing a reactance for line protection when power is restored.

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SYNCHRONOUS MOTORS

The past year has been marked by a further extended use of synchronous motors for many applications where induction motors have formerly been considered necessary. This extended use has been reflected in the development of complete lines of general purpose synchronous motors. The construction of a 9000-h. p., 6600-volt motor is of particular interest because of its application to the main roll of a steel mill and because it is the largest continuous rated motor of any type to be applied to industrial purposes. About 1½ per cent higher efficiency and the absence of about 4000 reactive kv-a. are the principal advantages obtained over a corresponding induction motor.

Another interesting development is a single-phase synchronous motor as part of a motor-generator set for use on heavy duty electric locomotives to supply direct current power to the main motors of the locomotives. These motors are rated 1200 kv-a., 2300-volt, 25-cycle, single-phase for the freight locomotives, and 500 kv-a. for the smaller switching engines. These sets are brought up to about 80 per cent of synchronous speed by a starting motor of the repulsion type and then the motor is connected to the line at full voltage after which it quickly comes up to full speed and the excitation is automatically applied by a relay actuated by the current induced in the field during the starting period.

INDUCTION MOTORS

No outstanding developments in the electrical design of general purpose induction motors have been brought out during the past year but the attention of designing engineers has been directed more particularly to the improvement of constructional features as relating to mechanical design and economical manufacture. A number of manufacturers have developed complete new lines with improved mechanical features.

The relative merits of sleeve and ball or roller bearings is a subject over which there is considerable discussion and difference of opinion. While a few manufacturers feature ball or roller bearings exclusively, the larger firms are prepared to supply either type according to the preference of the purchaser. New lines having tapered roller bearings have been developed which have overcome the assembly difficulties that previously made this type of bearing appear to be impractical for this application. General improvements have been made in the construction of the sleeve type of bearings to overcome the oil throwing and to exclude dirt and dust from entering the bearing which were objectionable features of the older design.

One manufacturer has adopted a fabricated construction of the mechanical parts of induction motors to the exclusion of cast parts except for the end bells. Another is using pressed steel end frames and riveted feet for moderate and small-sized motors. The motor manufacturers have in general retained the conventional casting construction for standard lines. General purpose motors having cast steel parts have been featured by one manufacturer while another has used the welded construction for large special motors.

Improvements in the power factor characteristics of induction motors have been accomplished by the adoption of more precise manufacturing methods. For instance, a grinding process has been adopted for finishing the stator and rotor cores of small machines to obtain uniformity of air-gap. The use of larger bearings and shafts has permitted reductions of the air-gaps resulting in better power factor. However, there is a strong conviction among designing engineers that better practise in application engineering presents the greatest possibilities for the improvement of power factor conditions in general power distribution systems. It is particularly important to avoid over-motoring, *i. e.*, installing motors of needlessly large capacity and operating them at low loads and consequently low power factor.

The ever extending use of electric power is presenting new and more severe conditions affecting the insulation of motor windings. Improvements in insulating materials and processes are being continually sought for the purpose of minimizing the destructive effects of dust, vibration, acid and alkaline vapors, and moisture. Form wound coils which are completely insulated before they are assembled in the slots present the least difficulties in that the interior of the coils can be completely filled with varnishes or compounds, but the so called random wound coils in which the wires are not bonded together have presented a more difficult problem. A recent improvement in this type of winding has been obtained by encasing the complete end windings outside of the slots in a plastic insulating material.

During the year considerable attention has been given to theoretical studies of the performance characteristics of the synchronous induction motor and the compensated induction motor which combine the power factor characteristics of the synchronous motor with the starting torque characteristics of the wound rotor induction motor. For a number of years a type of synchronous induction motor arranged to receive its excitation from an external source has been in general use in Europe.

As mentioned in this committee's report of last year two new types of synchronous self-excited, induction motors have been developed in America. Another type of compensated motor has been developed this year in Europe which uses the Leblanc principle with one rotor having separate windings in separate slots. Although the use of the self-excited synchronous induction motor is being extended in America, it has not reached anywhere near the proportions of the use of the separately-excited motor in Europe. American practise accepts the ordinary induction motor for general use supplemented by a relatively few synchronous machines for the purpose of maintaining good power factor conditions.

A European manufacturer is building a complete line of induction motors up to 225 h.p. with centrifugal-operated starters which have the same efficiency and power factor characteristics as the usual wound rotor type. This arrangement offers greater simplicity of starting than the ordinary manually operated compensator or controller. Compared with this, the general practise in America has been to avoid the use of centrifugal devices and built-in resistances in polyphase motors. A recent European development has been a line of water cooled, totally enclosed motors primarily for pump drives. These motors are capable of being rated up to 70 per cent of the rating of the open type frame of the same size.

An interesting new development has been the design of a high-resistance squirrel-cage motor used largely for driving sugar centrifugal hydro extractors, etc.

The principal part of the load on this machine being the starting and stopping of a load of high inertia, it follows that the principal part of the losses in the motor are in the squirrel cage winding. The rotor of this motor has been designed with a high resistance section to the squirrel-cage winding which is built in the form of a fan to effectively dissipate the heat developed in the squirrel cage without communicating the heat to the rest of the motor.

A notable step in large-sized induction motors has been the building of a blooming mill motor having a continuous rating of 8000 h. p. at 13,200 volts. This is the largest continuous rating at the highest voltage yet obtained in induction motors.

Full voltage starting of induction motors is being urged by some engineers who claim for it greater efficiency, greater simplicity and lower cost, but there is a marked reluctance to abandon the time honored use of auto-transformers for reduced voltage starting. The so called double squirrel-cage motors or those having high reactance rotor windings for full voltage starting or applications requiring high starting torque seem to be growing in popularity especially for high-speed ratings where the inherent reactance of the standard construction is low.

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HIGH-FREQUENCY MACHINES

During the past year a new development in high-frequency alternators has resulted from relatively new developments in the use of high-frequency power. The successful operation of air-core induction furnaces for melting copper, brass and nickel, and silver alloys has called for the development of generators suitable for supplying power at about 480 cycles for one type of furnace and at about 2000 cycles for another type. Current of 2000 cycles is also used for heating long objects of small sections such as boiler tubes. Generators for these purposes have been built in capacities up to 600 kw., single-phase, 480 cycles, and 150 kw., single-phase, 2000 cycles.. A further application of high-frequency power has been to obtan very high motor speeds for wood cutting and grinding operations and generators for three-phase power up to 420 cycles have been built for this purpose. These generators have been of the standard salient pole type, which is preferred to the inductor type because of the better characteristics. The high speeds and large number of poles has necessitated some special types of windings and mechanical details. A consideration of the fact that with a limiting peripheral velocity of 15,000 feet per minute the pole pitch of a 2000-cycle alternator is only ¾ inch will give some idea of the limitations of the field structure of such an alternator. The use of two-pole induction motors on frequencies of from 300 cycles to 420 cycles producing speeds of 18,000 rev. per min. to 25,000 rev. per min. has presented problems in motor construction more of the mechanical nature than of an electrical nature. The present limitation of speed is due to bearing troubles but the problem of the lubrication of high speed bearings is being studied with the hope that higher speeds can be used successfully if they are found desirable.

A recent European development for obtaining high motor speeds for wood working machinery consists of a combination motor having two stators and two rotors which are concentric. This motor develops a speed of 6000 rev. per min. on 50-cycle current. American practise has used the simpler high frequency motor supplied with power from frequency changer sets.

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SYNCHRONOUS CONVERTERS

Conditions in England have led to a careful study and development of motor-converters and apparent success has been obtained with a larger output per pole than has been obtained with d-c. machines in America. American practise has pinned its faith to the conventional six-phase synchronous converter. The largest capacity converter set that has been installed for heavy duty, 1500-volt, d-c. traction service was built during the past year and consists of two 1500-kw. rotary converters connected in series. Limitation of space available for the installation of a 4200-kw. synchronous converter led to a rather unique arrangement. It was desired that the transformer should be set on the base of the converter at the collector end. In order to make use of the space on both sides of the shaft, the transformer was split into two units each of half the total capacity and each connected six-phase and the converter was built with twelve collector rings for twelve-phase operation, taking advantage of higher efficiency, smaller space and lower cost. This is the largest twelve-phase converter of which we have knowledge.

An ingenious arrangement for obtaining a stepless voltage control of a direct-current power supply for crane motor operation has been devised by a European manufacturer which consists of a small three-phase synchronous converter which is direct connected to a self starting synchronous motor or booster. The stator of the booster is arranged so that it can be shifted angularly thereby varying the direct-current voltage of the converter from zero to plus or minus full voltage. The complete outfit is of small enough dimensions to be mounted in the crane cab.

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INDUCTION DISK PHONOGRAPH MOTOR

Further applications of induction disk motors have recently been made to low-power, low-speed equipment. These motors operate under the same principles as used in the customary induction-type alternating-current watthour meter. The rotor consists of a conductive disk, through which a shifting flux is produced by one or more split phase electromagnetic elements. Eddy currents are generated in the disk and the reaction between the eddy currents and the flux produces the driving torque.

The chief applications of induction disk motors heretofore have been as timing devices in demand meters, time switches, etc. However, a larger motor has now been perfected for phonograph drive and has been adopted by two of the most prominent phonograph producers. The chief advantages of this motor are silence, constant speed, lack of vibration and reliability. The rotor runs on a vertical shaft, the upper end of which also carries the phonograph turn table. No speed reducing gears are therefore necessary, the rotor turning at slow speed and driving the turn table directly. The motor has high starting torque, giving it rapid acceleration. The speed is controlled by a very sensitive fly-ball governor, operating through friction. The normal speed is 78 to 80 revolutions per minute, at which a torque of 6 inch ounces is produced, this being sufficient to drive any standard phonograph. The input to the motor is approximately 35 watts.

An important advantage of this electric phonograph motor over the spring motor lies in the fact that the former supplies a constant driving torque regardless of the length of time it is operated, whereas the spring motor delivers a varying torque. The torque of the spring motor starts at a rather high value just after it has been fully wound and tapers down to a low value as the spring unwinds.

The induction disk motor is desirable, of course, only for low power applications having a low efficiency. It is very reliable and silent, however, owing to its relatively slow speed and simple construction.

INDUCTION VOLTAGE REGULATORS

A new development in connection with induction voltage regulators has been the combination of a series transformer and a transfer switch with the regulator and so arranged that the polarity of the series transformer may be reversed thereby doubling the range of the regulator. The cycle of operation of the combination produces the same results as would be obtained by a regulator of twice the capacity. The switch is geared to the regulator which may be automatically operated. An outdoor type of regulator has been developed with all the auxiliaries including current and potential transformers mounted within the auxiliary casing. Induction regulators have been developed for use on 12,000-volt single-phase circuits. The noise of single-phase regulators has been materially reduced by the use of a cushion support for the regulator in its tank and a similar cushion between stator and rotor. An indicator has been developed to show the boosting or lowering position of the regulator rotor.

MERCURY-ARC RECTIFIER

During the past year the mercury-arc rectifier has been further developed and notable improvements have been made particularly with regard to the automatic upkeep of the vacuum and to simplifying the control and auxiliary apparatus. This piece of apparatus would appear to approach the ideal for the rectification of alternating current in that the function is performed without transforming the energy into magnetic and mechanical energy with the attendant losses. The loss affecting the efficiency of the rectifier is the arc drop within the tank and since the arc drop is approximately constant the rectifier does not show as high efficiency at rated load as does the rotary converter for large low voltage capacities. For heavy duty traction purposes involving voltages above 600 volts the rectifier has better efficiency characteristics. American manufacturers are now offering rectifier equipments for railway substations and industrial power uses that may operate in parallel with rotary converter or motor generator equipments and arranged for complete automatic operation. Rectifiers are usually supplied with a slightly dropping characteristic, which may be adjusted to match the characteristics of shunt wound rotary converters operating in parallel. Rectifiers may also be compounded in order to obtain voltage control; however, it must be appreciated that this control will slightly lower the over-all efficiency and power factor of the rectifier unit. Equipment has been developed to include the application of automatic voltage regulators for maintaining constant d-c. voltages. A peculiar advantage of the rectifier over other types of converters from a commercial point of view is that the rating of a unit is independent of the frequency of the alternating current and within certain limits a given rectifier is suitable for operation at various voltages, the kilowatt capacity being greater the greater the d-c. voltage. While rectifiers have been extensively used in Europe for more than one decade, there are only two commercial installations in America which have been in commercial service for more than one year. A few more were added during the past year, and a large number of equipments are on the books of American manufacturers and will go into service this year.

The voltage limitations of the rectifier are much higher than those of synchronous converters and motor-generator sets, for they are built in capacities as large as 3000 kw. at 4000 volts d-c. One 4000-volt plant has been in successful operation for more than two years on a standard gage railroad in Europe. These capacities are characterized by very high efficiencies. Since the loss is due only to the resistance drop of the arc the efficiency is inherently higher for the higher voltage ratings. Other advantages which have been claimed are that there is no synchronizing operation and there are no moving parts. The absence of noise and vibration makes the rectifier peculiarly desirable for substations in congested and residential districts where quiet operation is necessary. The successful operation of a rectifier depends upon maintaining almost a perfect vacuum and equipment is available which will accomplish this automatically. The operation of this equipment is based upon the principal that the transfer of heat from one body to another is accomplished by radiation and conduction but the conduction is dependent upon the gas pressure and is zero in an absolute vacuum. This device is called a hot wire vacuum gage.

The operation of rectifiers of this type is liable to be interrupted by an occasional arc-back which is a short-circuit between anodes and occurs when one or more anodes function temporarily as a cathode. Since this occurrence causes no damage to the equipment except in very rare cases, the rectifiers may immediately be restarted by reclosing the breakers either manually or automatically. The cause of arcing back is being studied and it is hoped that this difficulty will be overcome.

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I wish in conclusion to express my appreciation of the assistance of Messrs. W. J. Foster, R. B. Williamson, V. Karapetoff, V. M. Mountsinger and others, in the preparation of this report.

Discussion

H. B. Dwight: The report states that the committee desires that suitable commercial test methods be developed for a more accurate determination of the efficiency of synchronous machines than is given by the present Standardization Rules. Possibly one of these items is the amount of core loss corresponding to full-load conditions.

At present the Standardization Rules specify the core loss for full load and rated voltage to be practically the same as that for no load and rated voltage, the small correction for resistance drop being almost negligible. If a satisfactory, precise test for determining the internal voltage of a synchronous machine were to be devised, full-load core loss could be taken equal to the reading on the core-loss curve corresponding to the internal voltage. This under full-load conditions would mean an increase in core loss of from 20 to 50 per cent, which is of considerable importance.

The internal voltage is equal to the vectorial sum of the terminal voltage and the reactive drop in the winding. What is desired, therefore, is a dependable test for the leakage reactance of a synchronous machine. There is a possibility that for certain types of machines and for certain purposes, the leakage reactance may be taken equal to the measured reactance of the armature with the rotor removed. While this may not be precisely equal to the true leakage reactance, it is nearly equal to it for many usual machines, and it has the advantage that it is a definite, easily measured quantity.

Two students of the Massachusetts Institute of Technology, C. F. Kirsch and M. L. Libman, have arranged a sample machine so that they could make a direct measurement of the leakage reactance and compare it with the reactance when the rotor was removed. To do this, they isolated part of the winding which was large enough to be representative of the total winding. They put a load on the main part of the winding and measured the difference in the terminal voltages of the main part and the isolated part. This difference, corrected for the number of coils in each part, was equal to the reactance drop in the winding, and the voltage on the unloaded part was proportional to the internal voltage of the machine. The difference between the reactance measured in this way and the reactance of the main part of the winding when the rotor was removed, was about 20 per cent.

This "search winding" method is not intended to be a commercial measurement of the reactance of a complete machine, since it requires part of the winding to be set aside as a search winding. It is a means for judging other methods of measuring or calculating the reactance of synchronous machines.

Power Generation

Annual Report of Committee on Power Generation[*]

VERN E. ALDEN, Chairman

To the Board of Directors:

Last year's report of this Committee reviewed the many important developments in the art of power generation and dealt particularly with advances in steam station design and operation. As a committee we were just a little prone to believe that there could not be the advance during 1925 that had taken place during 1924. Nevertheless, substantial progress has been made.

IMPORTANT TECHNICAL ACHIEVEMENTS OF THE LAST YEAR

1. The 3000-kw. turbine designed for operation with a steam pressure of 1200 lb. per sq. in., which was referred to in last year's report, was placed in operation in the Edgar Station of The Edison Electric Illuminating Company of Boston in December 1925. This turbine, receiving approximately 125,000 lb. of steam per hr. from a single high pressure boiler, exhausts its steam first through a reheating superheater built into the same setting with the high pressure boiler. The steam is then delivered to the main steam header of the station at a pressure of approximately 350 lb. per sq. in. The most annoying trouble encountered in connection with this unit has been vibration incident to double winding on rotor and possibly undue flexibility in the rotor shaft of the generator which runs at 3600 r. p. m.

The rotor was originally of the built-up type with squirrel-cage winding in addition to the usual externally excited field. This arrangement was adopted as a precaution in case there should be governor trouble arising from the use of extremely high pressure steam. As the governing and synchronizing have proved to be perfect, however, the generator has been equipped with a solid field and all vibration has been entirely eliminated.

The action of the unit is perfectly normal and indicates that equipment of this character is suitable for regular commercial service, where economically justifiable.

This unit has had over 1800 hours of commercial operation including one run of over three weeks' time without a shutdown.

The performance of the Edgar Station has been highly satisfactory. Operated with a favorable load factor and good coal, the 350-lb. pressure plant has a record of one lb. of coal per kw-hr. which means practically 14,000 B. t. u. per kw-hr.

With the single high-pressure boiler and the 3600-kw., 1200-lb. pressure turbine in service, the average coal consumption per kw-hr. for the whole station is reduced three per cent. This indicates that if all four boilers were of the high pressure type, the fuel consumption per kw-hr. would be reduced 10 per cent, to a value of 0.9 lb. of coal per kw-hr.

2. With the exception of moderately high outages of the generating units, the operation of the Philo Station of the Ohio Power Company, the Twin Branch Station of the Indiana and Michigan Electric Company, and the Crawford Ave. Station of the Commonwealth Edison Company, all at a pressure of 550 lb. per sq. in. and a steam temperature of 725 deg. fahr. with the reheating cycle, has been quite satisfactory. The troubles encountered have been of a nature not entirely chargeable to the use of the higher steam pressure and temperature and the use of the reheating cycle. The indications are that these stations may be counted upon for thoroughly satisfactory and reliable service.

3. The Columbia Power Company's new 90,000-kw. station designed for operation with steam at a pressure of 550 lb. per sq. in. and a temperature of 725 deg. fahr. and for operation on the reheating cycle was started in December of last year.

4. Two more turbines have been purchased for the Crawford Ave. Station in Chicago, and a turbine has been purchased for Waukegan Station of the Public Service Company of Northern Illinois, all for operation at a steam pressure of approximately 550 lb. per sq. in. Two new stations have been designed for operation with this same pressure. There is accordingly either in operation at a pressure of 550 lb. per sq. in. or now projected a total of 925,000 kw. of power station capacity.

5. The Milwaukee Electric Railway and Light Company is installing in its Lakeside Station for operation at 1200 lb. pressure a boiler of modified Stirling type which will deliver steam to a 7000-kw. turbine, the steam temperature at the throttle being 720 deg. fahr. The steam will be exhausted through a reheating superheater into the main steam header of the station, maintained at a pressure of approximately 315 lb. per sq. in.

6. Of particular interest has been the increase in the size of turbo generator units as exemplified by the

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Presented at the Annual Convention of the A. I. E. E., White Sulphur Springs, W. Va., June 21-25, 1926.

purchase during the last year of the following machines:

a. A 77,000-kw. and a 90,000-kw. unit purchased from the General Electric Company for the Crawford Ave. Station in Chicago.

b. An 80,000-kw. Westinghouse turbine for the Hudson Ave. Station in Brooklyn.

c. A 160,000-kw. Brown Boveri turbine for the Hell Gate Station of the United Electric Light and Power Company of New York.

This trend towards the use of still larger units is further indicated by the projected use of turbo generator units of 200,000-kw. capacity in the New State Line Station near Chicago. It is to be remembered that the largest machine which we had to report upon twelve months ago was of 60,000-kw. capacity.

7. The first large furnace of the Fuller Well type designed for pulverized fuel firing with turbulent mixing of coal and air has been placed in operation under a 11,400-sq. ft. boiler in the River Station of the Buffalo General Electric Company. Three more boiler and furnace units of the same design are now being installed in the River Station.

8. The Combustion Engineering Corporation is advocating the use of an entirely new type of boiler which will completely surround a furnace built for pulverized fuel firing. It is proposed to transfer to an air preheater an extraordinarily large percentage of the work usually performed by the last passes of the boiler in scrubbing heat from the flue gases. This will result in the boiler itself being of relatively small area. Brickwork with its troublesome problem of maintenance is to be almost entirely eliminated.

It is reported in the technical press that the Combustion Engineering Corporation has contracted to install a total of 15 of these new boiler and furnace units in the stations of eight different companies. Each unit will be capable of evaporating approximately 100,000 lb. of steam per hour.

9. As indicative of the trend towards the use of very large boiler units we have the operation of one of the 26,470-sq. ft. Ladd boilers in the River Rouge Plant of the Ford Motor Company. The furnace for this boiler was rebuilt during the last year, water-cooled walls were installed, the pulverized coal feeders and burners were revamped, and the boiler was equipped with air heaters and induced draft fans of ample proportions. Since being rebuilt, this boiler unit has evaporated a total of 480,000 lb. of water per hr. which would correspond in a well designed steam generating station to a gross generation of 45,000 kw. in the turbine room.

10. The Milwaukee Electric Railway and Light Company has built a plant for the processing of coal, prior to its use in the boiler plant of the Lakeside Station. Coal in pulverized form while falling through two vertical retorts will mingle with rising currents of hot air and gas which will drive off a considerable amount of gas including the valuable by-products of the coal. The coke residue still in pulverized form, after being cooled in the hopper at the bottom of the lower retort, will be pumped to the pulverized fuel bunkers in the boiler house and burned in the furnaces. Equipment has been provided for removal of tar and benzol from the gas.

Experimental work extending over almost a year's time has indicated that no major difficulties will be encountered in operation. This installation in Milwaukee is well beyond the laboratory stage and provision is made for the expansion of this processing plant so as to take care of all the coal burned in the station.

This installation is of interest, as the successfulness of its operation will indicate the advantages which will accrue from the joint operation of our steam stations and low-temperature carbonization plants by means of which coal may be processed before being burned.

TRENDS IN STEAM GENERATING STATION DESIGN

Aside from the individual achievements referred to above there are certain definite trends in power station design and operation which should be noted:

1. The heating of the feed water to a temperature of from 350 to 400 deg. fahr. by means of steam bled from three or more stages of the turbine.
2. The removal of the heat in the flue gases after they leave the boiler by means of an air heater which returns this heat to the furnace.
3. The elimination of the economizer.
4. The wide-spread use of steaming surface for furnace walls and the elimination of a considerable part of the furnace brickwork.
5. The use of automatic control in the boiler house with consequent improvement in day-in and day-out operating efficiencies.
6. The use of separate ventilating fans in connection with very large generators of 62,500 kv-a. and higher capacities.
7. The use of relatively smaller surface condensers made possible by more careful design of the condensers themselves and by a better understanding of the relation of turbine performance to condenser performance.

DEVELOPMENTS IMMEDIATELY AHEAD

The present combination of boilers and water-cooled furnaces which are, as one engineer expressed it, "fearful and wonderful jobs of plumbing," probably forecast certain changes in boiler and furnace design which will result in simplification and decreased investment, while at the same time maintaining what appears to be the most logical development of the boiler surrounding the furnace.

We are probably on the verge of being forced into the use of higher voltages in connection with the very large generators now contemplated.

AUXILIARY POWER SUPPLY

Two phases of the auxiliary power supply problem are worthy of study by the members of the Institute:

1. Is the use of 2300 volts for distribution to the station auxiliaries the best choice, involving as it does the use of expensive oil-immersed switching equipment occupying expensive space? The alternative is a voltage of the order of 440 and the substitution of carbon circuit breakers and contactors for oil switches. This alternative offers distinct advantages from the standpoint of reduced investment in equipment and in building, and in greater ease of inspection and maintenance.

2. What is the best solution for variable speed drive of the forced and induced draft fans in our new stations? The difficult nature of the problem is exemplified by an installation in one of the new stations now being built: Two 200-h. p. motors drive the forced draft fans of each boiler and two 550-h. p. motors drive the induced draft fans for each boiler. The speed of the fans must be varied by 50 steps through a range from seven to one and the total h. p. input to the four fans varies from 1500 to 15. In this case the motors for the fans in connection with one boiler cost approximately $12,000 and the electrical control equipment for these motors cost approximately $20,000 and occupied a considerable amount of rather expensive space. If electrical engineers can not work out a simpler and less costly solution for this admittedly difficult problem, power station designers may have to return to the use of steam turbine drive for these fans, much as they may wish to avoid this solution, with its attendant use of reduction gears.

USE OF STEAM AT HIGHER TEMPERATURES

The use of higher steam temperatures of the order of 800 deg. fahr. and perhaps as high as 900 deg. fahr. is being forecasted by scattered operating experience, most of it unpremeditated. Due in the main to errors in superheater design the steam temperatures have been higher than anticipated in connection with some of the new stations. Some of these stations have operated for appreciable periods of time with steam temperatures in excess of 775 deg. fahr. One turbine operated for a time with steam at a temperature of approximately 1000 deg. fahr. There have been no serious indications of distress as a result of operation at these high temperatures. It has recently been reported in the technical press that a 40,000-kw. turbine in the Gennevilliers Station in France operated for 135 hr. during the months of October, November and December of last year with steam at temperatures ranging from 775 to 914 deg. fahr. An inspection made December 25th showed no indications of deterioration of the parts in contact with high-temperature steam. The turbine was returned to service and has continued to operate satisfactorily with steam at temperatures varying all the way from 700 deg. to 900 deg. fahr.

These experiences together with the experience gained in the operation of oil stills tend to show that we can avail ourselves of the attractive possibilities incident to the use of steam temperatures considerably in excess of the present accepted limit of 750 deg. fahr.

JOINT USE OF STEAM STATIONS AND WATER-POWER PLANTS

Worthy of attention are three examples of the broad general solution of power supply by the combined use of steam generating stations and water-power plants in:

1. The construction of the 350,000-h. p. hydroelectric plant at Conowingo for joint use with 520,000 kw. of capacity in steam stations of the Philadelphia Electric Company.

2. The construction of a new 70,000-kw. steam generating station by the Southern Power Company.

3. The completion of a 128,000-kw. addition to the Long Beach Steam Station of the Southern California Edison Company and the starting of work by this same company on a new station of at least 600,000-kw. ultimate capacity with an initial development of 94,000 kw. The Southern California Edison Company has approximately 350,000 kw. installed in water-power plants.

OPERATING RELIABILITY AND MARGIN OF SPARE CAPACITY

The results of studies made jointly by a number of the operating companies of this country having for their purpose the determination of the operating reliability of our large generating units, are of considerable interest.

An analysis of the operating records covering the calendar year of 1925 for 191 steam turbines aggregating 5,627,000-kw. capacity showed that on an average these machines were in service 65.1 per cent of total hours in the year and that they generated 44 per cent of the maximum possible number of kw-hr.

These machines were idle because not needed 21.25 per cent of the hours in the year. For 13.63 per cent of the hours in the year, however, they were out of service for overhauling, inspection, maintenance work and cleaning, these outages being allocated as shown below:

Turbine outage............	— 7.29 per cent
Generator outage..........	— 1.85 per cent
Surface condenser outage...	— 3.51 per cent
Other causes..............	— 0.98 per cent
Total outages.........	—13.63 per cent

Comparing the foregoing results with the results of similar analyses made during previous years covering operation for the period from 1914 to 1923 inclusive, three things stand out in striking fashion:

1. There is little if any evidence to indicate the decrease which we would like to see from year to year in turbine outages.

2. Prior to 1923 the generator outage averaged 2.8 per cent of the hours in the year. For 1923 and 1925 the generator outage has averaged 1.8 per cent. These statistics are concrete evidence as to what the generally adopted closed system of ventilation has accomplished.

3. The surface condenser outage has increased from

1.2 per cent to 3.51 per cent of the total hours in the year. This increase serves perhaps as an index of pollution of the water in our rivers and harbors.

A study of the fact referred to above, that the outages due to causes other than the turbine, the generator and the surface condenser, aggregated only 0.98 per cent of the total hours in the year and that this item is of only seven per cent of the relative importance of the outages due to other causes, is cause for reflection. It is not recorded that Oliver Wendell Holmes was an outstanding engineer but any one of us would have been proud to have been the designer of as perfectly proportioned a piece of equipment as his "One Horse Shay." On an average the investment in the turbine room with its equipment is only 40 per cent of the total investment in the station, yet 93 per cent of the outages of station capacity are chargeable to the turbine room. Perhaps we have been failing to strike the proper balance between installed capacities, with their corresponding investments in the turbine room and in the boiler house with its related coal handling equipment.

Turning to a consideration of water-wheel driven generators, an analysis made by the Hydraulic Power Committee of the National Electric Light Association shows that the total outage time on these units is appreciably less than on steam turbines. The analyses covered the operating records for 1924 on 56 water-wheels aggregating 950,000 h. p.

These machines were in operation 77 per cent of the hours in the year and generated 48.1 per cent of the maximum possible number of kw-hrs. They were idle because not needed for service 17.62 per cent of the total hours in the year and were out of service for 5.4 per cent of the total hours in the year for reasons indicated below:

1.	General hydraulic causes	−0.64 per cent
2.	Water-wheels and auxiliaries	−3.01 per cent
3.	Generators and appurtenances	−1.55 per cent
4.	Electrical causes beyond the generator, related to switching equipment and outside transmission	−0.2 per cent
	Total outages	−5.40 per cent

Outage time is related directly to the need for spare capacity and the fixed charges on spare capacity constitute perhaps as much as 10 per cent of the total annual cost of power generation in steam generating stations. It behooves us to find ways of reducing this item of cost. A major responsibility in this connection lies with the equipment manufacturer. He must build equipment that will be capable of operation for all except a very few hours per year. Particularly must the manufacturer guard against any inherent weakness which will cause trouble involving unexpected shut-downs even though these outages be of short duration. Much can be accomplished by the engineer who designs the power station in foreseeing and eliminating certain features which may constitute the cause of an outage. The operating engineer can do much to control the necessary margin of spare capacity by the choice of the best operating methods, the careful training of personnel and by the careful scheduling of inspection, preventive maintenance and cleaning. With careful planning, a large percentage of outage time can be made to occur when the capacity is not needed for service.

PROBABLE USEFUL LIFE OF STEAM STATIONS NOW BEING BUILT

One of the most troublesome problems facing the executives of our large electric light and power companies today is "what to do with the old steam generating stations." Many of our large companies have sizeable blocks of capacity in steam generating stations in connection with which the coal consumption per kw-hr. generated is two pounds or higher. The costs of labor and maintenance are high. Viewed from the standpoint of operating costs (not including fixed charges), it seems nothing short of a crime to generate power in these stations, bearing in mind that the modern stations being operated by these same companies will generate power at a fuel consumption slightly in excess of one pound of coal and with much lower costs for operating labor and maintenance.

There is, of course, the possibility in some cases of rebuilding in part, at least, in order to improve the station performance. Notable examples of such programs of rebuilding are exemplified by the substitutions of unit coal pulverizers for the obsolete stoker equipment in the Brunots Island Station in Pittsburgh and in the Ashley Street Station in St. Louis. Some executives raise the objection, however, that such a procedure would, in connection with their stations, be sending good money after bad.

It is not our purpose to discuss this question of "What shall we do today with our old stations?" but rather to ask "What of the stations being built today in relation to our operating problems of 20 and 30 years from now?"

Assume, for example, that a company which had a system load of 400,000 kw. in December 1925 places in operation this year the initial 50,000-kw. unit of a new 300,000-kw. steam generating station. We will assume that the average annual load growth is nine per cent, that the yearly load factor is 56 per cent, and that the shape of the load duration curve is typical of load conditions in almost any one of our large cities on the Atlantic seaboard. An additional 50,000-kw. turbine must be installed each year until 1931 when the station will be completed. Since the station will be half completed in 1929, we may take the beginning of its useful life from that date. If we follow conventional lines of thought, we will consider the useful life of this station to extend from 1929 to 1949. What shall we do with this station in 1949? Conceivably the new stations being built in 1949 may generate power for 9000 B. t. u. per kw-hr., whereas the very best our hypothetical station, started in 1926, can do, even on the basis of a good load factor, is 14,000 B. t. u. per kw-hr.

The system load has grown, however, from 400,000 kw. in 1925 to 3,185,000 kw. in 1949. As the result of the inevitable law which pushes the old station a little higher each year into the peak of the load duration curve, we find that our 300,000-kw. station will generate in 1949 only 300,000,000 kw-hr. corresponding to an annual use factor of 11.4 per cent. As the result of the low load factor, the B. t. u. per kw-hr. has been pushed up to 17,000. The annual coal consumption is 190,000 tons and the annual fuel cost is, let us say, $1,300,000. By scrapping this station and substituting in its place a modern station of the 1949 vintage, we can reduce the annual fuel cost in connection with the 300,000,000 kw-hrs. of generation carried by this station from $1,300,000 to $800,000, with an annual fuel saving of $500,000. Few of us would dare to predict that this 300,000 kw. of new capacity built in 1949 will cost less than $18,000,000, or that the investment bankers of that day will be willing to finance new construction work at rates which would permit the increase in fixed charges incident to the construction of the new station to be less than $2,200,000.

The major increase on load on most of our large systems comes about as a result of increase in load density. Consider therefore the strategic position which the station built in 1926 will probably occupy with respect to the load as of 1949 or 1959.

It would appear that the engineers and executives of 1949 are going to have a very difficult time justifying the scrapping of our 300,000-kw. station of 1926 vintage. Such a step will be even more difficult to justify in 1959, for by that time the annual fuel cost in connection with this station will have dropped from $1,300,000 as of 1949 to approximately $125,000 per year corresponding to an annual station use factor of one per cent. The function of this station will have changed from that of generating kilowatt-hours to one of supplying kilowatts of capacity. It is to be understood that the foregoing analysis is based not on this station being held merely in reserve but on its carrying its proportionate share of the system peak load.

This analysis rather places the burden of proof on us to show good reasons why we should not design our stations on the basic assumption that they will be used and useful not merely for a period of 20 years but for 100 years, 200 years, or until such a time as the use of electrical energy as related to the life of our large cities and our nation has become no longer necessary.

This statement is not to be construed as an argument for the doing away with renewal reserves. Sound business principles indicate the advisability of accumulating ample renewal reserves.

It is intended to make the engineer who lays down the design of a new generating station ask himself certain questions:

1. Is the equipment well adapted to the load conditions which will obtain in connection with this station after it has become 20 or 30 years old? These load conditions will involve starting and stopping all turbines in the station twice each day, and the picking up of large blocks of load at a rapid rate. The boilers must be banked frequently and they must be able to pick up load quickly.

2. Is the design of the station such that the costs of operating labor and maintenance will be reasonable?

In the example outlined above, after 1955, the costs of operating labor and maintenance will exceed the cost of coal and in 1959 will be at least five times the cost of the fuel.

During its first 100 years of life, our hypothetical station will probably burn $90,000,000 worth of coal and require the expenditure of $80,000,000 for operating labor and maintenance.

Consider the tremendous advantage in connection with this type of operation which a station of consistent design with duplicate units of interchangeable parts, would have.

3. Will it be necessary, solely from a standpoint of being able to install more kw. of station capacity on the available ground area, to do a costly job of rebuilding in the future? Perhaps the very policy now being pursued by some companies in building stations in which extra space is provided in the initial section of the station, so that later units installed may be increased in size, is laying the ground work for an expensive rebuilding job 20 or 30 years from now.

4. With changed conditions which will exist fifty or one hundred years from now, and with the city pressing in on our station from all sides, will we still be able to live peaceably with our next-door neighbors?

5. Is the station design such as to form the proper economic balance between fixed changes and operating costs on the basis of one hundred or even two hundred years' life?

These and a host of other questions press for answers the moment you admit the possibility that the foregoing analysis may be correct. Perhaps those of us who are power station engineers and the executives making decisions bearing on power station design, had better try to find the answers to some of these questions. If we don't, we will probably be damned cordially by the men of the coming generation who will carry on our work.

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Transmission and Distribution

Annual Report Committee on Power Transmission and Distribution[*]

PERCY H. THOMAS, Chairman

To the Board of Directors:

INTRODUCTORY

The present report of the Committee on Power Transmission and Distribution has been prepared through the cooperative efforts of the members of the Committee and does not represent the views of any particular member or group of members. Each of the several topics found below has been considered by those particular committee members who are the best qualified to discuss that subject and the subject matter herein is substantially that prepared by such members.

The purpose of the report is to cover the present state of the art of power transmission and distribution, or the progress during the year, in such a way as to present a clear view of what has been accomplished, this being intended for the consideration not only of transmission and distribution engineers but of other members of the Institute interested in different branches of the electric power field.

Where appropriate, at the end of each section there is added for purposes of discussion a series of topics covering subject matter now being investigated or subject to controversy.

TRANSMISSION LINE STRUCTURES

The trend of the year in transmission structures is a continuation of the study to move large blocks of power over considerable distances. Attention has been given to the design of lighter, cheaper, and simpler structures by careful utilization of materials and by great attention to prevention of corrosion or other forms of rapid depreciation.

A number of new construction methods and ideas have arisen during the year for 220-kv. lines. In eastern Pennsylvania a single-circuit, 220-kv. tower line has just been completed, designed for a general condition of one-in. radial ice, with exposed sections at higher elevations designed for a loading of one and one-half in. radial ice. To limit the stress that can be thrown upon the tower by unbalanced conductor pull due to unequal loading or a broken conductor, a clamp has been designed to slip at a moderate value. By this means it is possible to secure economies in the weight of tower steel and a decrease in the strain on and consequently the cost of foundations and still assure the safety of the structure.

The same line has brought forward a new structure for heavy angles and dead-ends. To avoid the heavy expense of the tower and footings of the conventional self-supporting structure, use has been made of a guyed mast. This structure is pivoted at the bottom and guyed at the top so that all load upon the tower is vertical and all side pull from the conductors is carried to suitable anchorage through tension members.

In connection with this job a new splice for the steel reinforced aluminum conductor has been developed. This splice combines the advantages of small diameter, ease of make-up in the field, and high efficiency. The usual twisted sleeve for the joining of the steel cores has been replaced by a soft iron compression sleeve. A single-piece aluminum sleeve is then compressed over the iron sleeve and the aluminum strands.

In California an extension to the Pit River—Vaca Dixon, 220-kv. lines has brought out novel construction for the crossing of the Sacramento and San Joaquin Rivers and the interlying lowlands. The crossing has an over-all length of approximately 24,000 ft. and is accomplished by the use of twelve suspension structures and four anchor towers. The special suspension towers (the conductor resting in a saddle supported by suspension insulator strings) vary in height from 200 ft. to 460 ft., while the anchor towers and three suspension towers on the island are standard structures. The two river crossing spans are respectively 4135 and 3175 ft., but the adjacent structures bring the total distance between anchor towers for these sections to 8000 and 8500 ft. Insulation and clearance to towers for the crossing sections have been materially increased above the general transmission design values to reduce hazard of electrical trouble at these points. The entire project is particularly novel because of the difficulties of obtaining suitable footings and because of the extreme height of the crossing towers.

In the Philadelphia region there has been a novel development in double-circuit tower design for operation at 132 kv. Throughout this line the horizontal offset of the middle conductor has been toward the center line of the tower rather than outward, as is customary. In other words, the top and bottom crossarms are longer than the middle one. This was done to provide greater clearance, the minimum for the top conductor being six in. greater than for the other two conductors, and the effect of the batter of the tower in reducing clearance for the bottom conductor being avoided. This line has been designed with special care to maintain clearance to tower equal to the length of the insulator string even under a condition of 30-deg. side swing.

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[*] Committee on Power Transmission and Distribution.

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This matter of conductor spacing and clearance to tower members deserves special consideration at this time. Clearance and spacings for the vertical configurations now in common use for double-circuit towers have become somewhat standardized, but for the new types of towers using horizontal configuration a new problem arises. If the double-circuit tower with horizontal arrangement of conductors, such as is now in operation in New England, comes into general use, it would be handicapped should unnecessarily large spacing be adopted at the beginning. We have experience on steel-tower, long-span construction with horizontal configuration at 220 kv., 165 kv., and 150 kv., and at lower voltages with wood pole construction. It seems desirable to give attention to this matter of horizontal spacing for long-span, moderate voltage, double-circuit lines and to determine the relationship of length of span and operating voltage to width of spacing necessary for reliable transmission.

DISTRIBUTION

A-C. Low Voltage Networks. The subject of a-c., low-voltage networks, particularly as applied to underground distribution in heavily loaded areas, has received fully as much attention during the current year as any other distribution problem. A number of installations has already been made and a number is being contemplated. The main features of the problem relate particularly to (a) the method of protection against failure of the transformers and primary feeders supplying the network, in order to insure proper continuity of service, and (b) the most selective isolation of any portion of the system which has developed fault.

During the year there have been developed new types of network protectors consisting of low voltage circuit-breakers with relay and automatic reclosing features, and also submersion-proof, automatic oil circuit breakers for primary voltages up to 15,000 volts. These two types of equipment are used respectively with two general methods of supply for a-c. networks:

1. Interlaced radial primary feeders with secondary network breakers and reverse power protection.

2. Loop primary feeders sectionalized by circuit breakers and balanced pilot wire protection.

Secondary Distribution Systems and Voltages. Secondary a-c. systems and voltages have received extended attention. This has arisen primarily from network situations and the desire to establish combined lighting and power secondaries.

Owing to the square-root-of-three relation which exists between phase, voltage and normal neutral of a three-phase system, it is necessary either to sacrifice an entirely balanced loaded system, or else to deviate from either or both of the lighting or polyphase motor voltage standards, as the ratio between these voltages is two to one. This has resulted in some engineers advocating the modification of the present standard motor voltages, the development of special distribution equipment such as translators, etc.

Often the problems of secondary distribution are greatly influenced by local conditions, principally by the type of system already in existence, when the inherent economies of the different systems do not differ widely. Also the costs of changeover may outweigh the potential savings.

System Grounding Practise. There has been considerable discussion with reference to grounding practise on distribution systems, particularly on the 4000-volt, three-phase, grounded neutral primary systems with a common neutral for both the primary and secondary, and having distributed grounds. This type of system, which it is claimed offers very considerable economies and decreased operating hazards, is receiving increased attention on the part of central station engineers and has been the subject of papers before the Institute during the past year.

High-Voltage Distribution. Using the term "high voltage" with reference to primary distribution voltages above the 4000- to 4800-volt class, the past year has seen more extensive use of the higher voltages and the improvement of methods of construction and protective devices for minimizing costs and operating hazards. There is an increased tendency toward the utilization of voltages of the magnitude of 13,200 volts, which often are the generated voltages, and even higher, for the supply of power for general distribution instead of only for large power consumers. The simplicity and economy of this method in territory having sufficiently great density of load are apparent from the saving in substation investment, the avoidance of one transformation, less complication, and decreased primary distribution copper, as compared with the lower voltages. The problems of voltage regulation, insurance to service, and decreased operating hazards have been given such study that very favorable results have been reported.

Street Lighting Distribution. The advantages of supplying the street lighting system from the general distribution system instead of from specialized equipment located in substations have become increasingly apparent. The past year has witnessed a greater application of decentralized supply for street lighting. The choice between multiple lighting with individual or group supply from secondary mains, and series circuits supplied from pole or subway type constant-current transformers, depends very largely upon local conditions such as the availability of lighting mains over the entire territory. The development of reliable outdoor control equipment for both multiple and series systems has made substantial progress.

Power Factor. Power factor conditions on distribution systems have received increased attention. This has led to the development of motors for industrial service which operate at higher power factors. Increased attention has also been given to the possibility of power factor correction by means of synchronous or static condensers. The use of these latter devices in smaller sizes than heretofore considered feasible is now being advocated in many instances. There is a general tendency to raise the power factor on the individual distribution feeder as resulting benefits will be broader than with correction at substations.

Single- Vs. Multiple-Conductor Cable. Particular attention has been directed toward the possibility of using single rather than multiple-conductor cable for distribution purposes. Where local conditions warrant, the single-conductor cable has many advantages, among which may be noted its superiority as regards connection to apparatus, localization of faults, and possibility of replacement of only a single phase of the circuit rather than the entire circuit. These points are particularly true in regard to distributing mains, secondary mains, and services.

For primary feeders the tendency is still to favor the multiple-conductor cable, although in several instances, depending upon local conditions, it is probable that some saving in yearly operating costs could be made by using single-conductor.

Continuity of Service. Continuity of service to customers has been improved by improved methods in fusing of distribution transformers and by the greater utilization of automatic reclosing feeder circuit breakers.

As distinguished from the earlier practise of fusing transformers mainly to protect them against overloads, the practise now tends toward higher fusing, to clear the transformers only in case of short circuits and for the protection of the primary circuit. With proper coordination with the relaying of the feeder breakers in the substation, there is increased assurance that failure of a transformer will not trip the entire circuit and, with radial secondary mains, only the customers supplied from the faulty transformer are interrupted.

Continued experience with automatic reclosing equipment on substation feeder circuit breakers has shown that the continuity of service to customers is greatly improved owing to the interruptions being usually momentary in the event of trouble on the circuit such as tree contacts, momentary heavy overloads, short circuits, etc.

Group Feeder Voltage Regulation. The past year has shown a tendency to make use of group, rather than individual, regulation for distribution feeders. This results in a simplification of the apparatus necessary in the substation and accordingly affords some economies. Group regulation is made possible for those feeders having approximately the same per cent drop from the substation to the load center and which carry approximately the same magnitude and type of load, such as in an a-c. network.

INTERCONNECTION OF POWER UTILITIES

The practise of interconnecting adjacent utilities and of the transfer or exchange of power continues. This practise differs fundamentally from the interchange of power between power plants in a given system in that, in place of the unitary operation and one-man control of the single system, interconnection usually involves independent operation and independent control of the separate utilities. The result of actual operation indicates that unless attention is paid to the synchronizing characteristics of the interconnections and sufficiently close cooperation is secured in operation, each company must be prepared, at least on occasion, to care for its own load, since at times of disturbance it sometimes happens that synchronism between the utilities is lost and the systems separated. This difficulty is especially to be feared in territory subject to disturbance from lightning.

While the benefits of continuity of operation may of course be obtained with interconnected utilities by the same technical methods used in the operation of single large systems, namely, by one-man control and the proper design of governor and other apparatus and connecting lines, such coordination has not so far been undertaken on any large scale.

One of the most notable and extensive interconnections exists in the southeastern states and recent experience in this district is important. There has existed for several years a connection at 110,000 volts between the Alabama Power Company system and the Georgia Railway & Power Company system, the connection being between Gadsden, Alabama and Lindale, Ga. In the late fall of 1924 interconnection was completed at 110,000 volts between the North Auburn Substation of the Alabama Power Company and the Columbus Electric and Power Company's substation at West Point, Ga. An interconnection at 110,000 volts was also completed in 1925 between Columbus Electric & Power Company's Bartlett's Ferry plant and the Central Georgia Power Company at Macon, Ga. During the dry season of 1925, these interconnections were used to the limit of their capacity in order to transmit power from the steam plants in Alabama to supply the deficiency of the three above mentioned companies in Georgia and also to Augusta, Ga., to which point the Georgia Railway & Power Company extended one of their 110,000-volt circuits.

In 1925 no particularly new problems in operation came up and it was found feasible to interchange blocks of power as large as 40,000 kw. over tie lines which were originally constructed for capacity of 25,000 kw.

The principal features to be worked out in connection with the interchange of large blocks of power as far as we have seen in the south are the maintenance of proper frequency and the absolute necessity of the receiving system taking care of not only the wattless current of their own system, but also part of that for the sending system. The necessity of having synchronous condensers or other regulating equipment in order to maintain satisfactory voltage conditions at the receiving load centers if entirely satisfactory service was to be given was clearly demonstrated. Practically none of the interconnected systems in the south have sufficient condenser capacity for conditions such as existed during the drought of 1925. It was frequently necessary to operate large hydro generators as condensers in order to take care of wattless current and voltage regulation. Not being primarily designed for such work, these generators, of course, are not as satisfactory as synchronous condensers.

It was demonstrated that many of the apparently difficult problems of coordination and of technical details as to handling of an individual company's load in a group of interconnected but independently operated systems largely disappeared when it became a question of subordinating these things to the necessity of obtaining power to take care of one's customers. The dry season of 1925 further showed the possibility of steam plants taking care of a serious shortage of hydro power. Power was actually relayed a distance of approximately 500 mi. at 110,000 volts. It was found necessary in some cases to revise relay set-ups of the interconnected companies from their normal set-up in order to take advantage of the maximum interchange. Necessity made many things possible which would not have been done under normal operating conditions. An Operating Committee has been functioning among the power companies of the southeast for some two years, exchanging data on load and generating conditions. This committee worked exceedingly well coordinating to make the maximum use of the power available. Through this committee, load dispatchers gained a better perspective of the interconnected systems, and while each system was operated independently, they were so coordinated that there was no difficulty in making the maximum use of the transmission lines and generating plants.

Speaking generally, it is felt to be proved with reasonable satisfaction that the interconnection of independent systems has been of material benefit in reducing the period of outage due to serious trouble on any one system. Without absolutely unified control of the operation, it is somewhat questionable just how far the interconnection of systems has operated to prevent momentary outages. Unquestionably, it has reduced the duration of outages, however. During the progress of interconnection, from experience in the southeast, the following seem to be the most important items to be considered:

1. A more careful investigation of relay protective equipment of each individual system and their coordination with other systems so that the maximum benefit of interconnection may be made use of.
2. The installation of more synchronous condenser capacity at most centers to take care of wattless current and for voltage regulation.
3. A more thorough understanding on the part of load dispatchers and operating superintendents of each other's system requirements so that shortages may be foreseen as far in advance as possible.
4. A careful study of the value of booster transformers at the point of interconnection between systems, these transformers to be equipped with tap circuits so that the voltage may be changed as required to supply current in either direction without disturbing the balance of either of the two interconnected systems.

The following additional topics for discussion are of general importance:

5. What is the most feasible method of interconnecting large systems, whether under one-man control or not, so that continuity of supply may be assured to all from a base load plant?
6. How should the interchange of power between particular systems and the power factor at various points be controlled where three or more independently operated systems are mutually interconnected by several links?

UNDERGROUND CABLES

The outstanding advance in the field of underground transmission during the past year has been the development of a radically new type of single-conductor cable for operation at 132 kv., three-phase. The New York Edison-United Companies and the Commonwealth Edison Company have placed orders for such cable for lines about 10 and 6 mi. long, respectively, for installation this year. The carrying capacity will be about 90,000 kv-a. per circuit.

The cable will have a 600,000-cir. mil. hollow conductor insulated with 23/32 in. of impregnated-paper insulation. This thickness is less than has been used on single-conductor cables recently installed on 66- and 75- kv., three-phase circuits in this country. The over-all diameter will be about 3.1 in. The cable will be impregnated with a thin oil which will be under pressure during operation, in order to minimize the possibility of the formation of voids in the insulation.

The lengths will be connected so that the hollow space will be continuous through the ordinary joints. The line will be divided into sections about one mi. long, each section ending in barrier joints, wherein the copper conductors will be connected, but the central hollow spaces of adjacent sections will be blocked from each other. Elevated reservoirs will be connected by pipe lines which connect through the barrier joints to the hollow space in the conductor and thereby maintain a hydrostatic pressure on the oil. When the cable becomes warm during operation, oil will flow from it into the reservoir and vice versa.

An unusual feature of this cable is the large charging current which amounts to about 2400 kv-a. three-phase per mi. of line at a leading power factor of about one per cent.

Several manufacturers are developing other types of 132-kv. cable and making length for short experimental installations.

There has been an increasing interest in three-conductor cable which has a metallic covering over the insulation of each conductor and no belt insulation. A number of commercial installations is at present being made in this country at 27 and 33 kv.; and it is reported that several installations of similar type cable have been made in Europe at various voltages including higher voltages.

The data accumulated by the Electrical Testing Laboratories on their inspection and tests of several million feet of high-tension cable as reported by F. M. Farmer at the Madison meeting, A. I. E. E., May 6th and 7th, show a marked improvement in the quality of the cable made during the last few years. This is shown by the increase in the dielectric strength and reduction of damage to insulation due to bending test. The improvements in quality shown by such test data, combined with operating experience, have led to considerable reductions in the insulation thickness used by some operating companies, and the use of higher voltages for given thicknesses of insulation.

Several American manufacturers are now using wood pulp paper for a portion or all of the insulation in the cable. In some cases the paper is entirely wood pulp, while in other cases the paper is a combination of wood pulp and manila hemp stock. Some manufacturers report that this paper will lead to improved dielectric strength and decreased cost.

It has been learned that one of the principal causes of the failures in high-voltage cable was the use of impregnating compounds that were unstable under the dielectric stresses and temperatures of normal operation. Some of the manufacturers and the Electrical Testing Laboratories have developed tests which will aid in the selection of compounds free from this difficulty.

Some questions now under discussion are as follows:

1. The use of wood-pulp paper insulation instead of paper made from manila hemp rope fiber.
2. The use of three-conductor cable which has a metallic covering over the insulation of each conductor and no belt insulation for voltages above 33 kv.
3. The limiting voltage for three-conductor cables of standard construction.
4. Changes in the test requirements now included in specifications for high-tension cables so as to secure more satisfactory operation.
5. Possibilities of increasing the maximum operating voltage of underground cables by changing the type of construction.

STABILITY AND LOAD LIMIT IN LONG TRANSMISSION LINES

During the past three years, very considerable progress has been made in the subject of transmission stability and load limit, and it seems appropriate at this time to make a resumé of the present status of this problem.

It is now generally recognized that stability constitutes an important problem in the transmission of power over long distances, or for large amounts per circuit. Furthermore, many of the interruptions to service on existing systems have been due to instability at times of transient disturbances, which situation has been recognized as a result of increased knowledge of system operations during and after an abnormal disturbance condition. The extensive investigations which have been made during the last few years have increased our technical knowledge of the stability problem, so that at the present time, given the necessary basic data and machine performance circuit constants, etc., the performance of any system in regard to load limit or static or transient limits can be predicted with a sufficient degree of accuracy for the present purposes. It is believed that the various groups which have been working on the problem would give substantially the same results for identical assumptions as to the layout of the power system, and as to the rate of variation of prime mover governors, kind of voltage regulators, duration of disturbance, etc. The way in which the stability problem affects the design of layout is understood from the technical side, and a number of schemes has been proposed for increasing the stability of systems.

In the actual operation of power systems, the way in which stability studies will manifest themselves is by reducing the number or duration of interruptions to service or by permitting the increased amount of power to be handled without reducing the standard of service. At the present time, the principal problem is not one of determining how the system will act for a definite set of conditions, but to what extent it is desirable to modify the design of systems in order to improve them from this standpoint. In order to do this, it will be necessary to get the cooperation of the operating companies so as to interpret on an economic basis the costs of interruptions to service. When this has been done, it will be a relatively simple matter to determine the additional expense in remedial measures for improving stability which should be employed in any particular case.

The practical value of means to increase the stability or load limit of any system may be evaluated on the basis of the amount of additional power that may be delivered over the system based on the conditions that the transient stability or load limit of the system will be the same for a given fault condition as it would be for the system without the means with the lower quantity of power delivered. The determination of the actual load at which the systems will be run is a matter for the operating engineers themselves to decide, as this involves the income value of the good will obtained by increased reliability of service which is difficult to define and which will vary with different localities and at different times. In connection with the stability problem, there are certain speculative factors which can be evaluated only by the statistical method. Such factors include the value of fault resistance, the average duration and character of various types of disturbance, etc. In this connection, it should be pointed out that the ordinary records obtained on systems are entirely unsuitable for interpreting the stability conditions, because such instruments are not adapted for recording the high-frequency electromechanical transients which take place at times of a disturbance. Of particular significance is the move taken by one or two power companies to install specially designed recording apparatus for obtaining adequate records on operating experience from the standpoint of stability. This will logically lead to increased information as to the operation of existing systems, and will utimately result in the obtaining of sufficient statistical data from which one can predict the operation of future systems.

During the past year, much progress has been made in increasing our technical knowledge of the subject of transmission stability and load limit. Probably the most important investigations are those of tests on an actual system. These tests included both switching operations and short-circuit tests, and are described in a paper presented at the Midwinter Convention by Mr. Roy Wilkins. At the Seattle Convention, two papers dealing with the general aspects of the stability problem were presented, one by Messrs. Doherty and Dewey and another by Mr. Fortescue. At the Midwinter Convention, Messrs. Nickle and Lawton presented the results of recent laboratory and shop tests on stability. Miss Clarke presented a method for the determination of static stability limits. Mr. Wilkins presented the paper describing the stability tests on the system of the Pacific Gas & Electric Company. Messrs. Evans and Wagner presented a method for the determination of static stability limits and gave the results of calculations by this method with comparisons of the test results obtained on the P. G. & E. Company system.

Certain power limits have been defined in these papers and it is recognized now that with absolutely fixed excitation for both sending and receiving systems there is a definite power limit under steady load conditions. This power limit depends in this case upon the synchronous impedance of the machines at the sending and receiving ends. If we take the value of excitation which gives the desired terminal voltage at the power limit, we have the static stability or load limit with "fixed excitation" or what is termed hand regulation. It is also recognized that the load limit under transient conditions may be lower. It is not determined by the synchronous reactance solely, but is affected also by what is known as the transient reactance, and the "leakage reactance" of a machine in varying degrees depending upon conditions. It may be stated perhaps more consistently, however, that this action depends upon the actual field being maintained essentially constant for a short period due to internal currents induced in the damper and field windings. In the case of machines operated with automatic voltage regulators there is a theoretical possibility of further extending the limits of stable operation over those obtainable with hand operation. Full agreement on this point has not been reached. One group reports that considerable increase in the limits has been obtained experimentally and the other group reports inability to secure any increase in the limit over that obtained with "fixed excitation."

In view of the fact that the operation of power systems at times of transient disturbances is a relatively complicated phenomenon, it is quite natural that the extensive discussions which have taken place on this subject should have tended to emphasize the relatively minor points on which full agreement has not been secured, and has tended to obscure the major points which have been established. It has, therefore, seemed desirable to present at this time a statement of those underlying principles requisite for good stability conditions. They are as follows:

1. Low series reactance.
2. The supply of reactive kv-a. as close as practicable to the point at which the demand originates.
3. The maintenance of internal voltages in machines by special excitation schemes, or by the use of special machine characteristics.
4. Such a layout of the system as to reduce to a minimum the effect on stability of any abnormal condition. For example, the avoidance of a concentration of power in any particular piece of equipment such that a single fault could render a large part of the system ineffective.
5. The rapid and selective isolation of faults.

In line with the policy which has been laid down for the Annual Convention of presenting committee reports and inviting discussions, the following topics are suggested as worthy of further study and suitable for discussion at the Annual Convention:

1. Further investigations of the stability problem require a more exact knowledge of the characteristics of machines under transient conditions, including among other factors the effect of damper windings.
2. Further work will be required in connection with schemes of excitation designed for the purpose of maintaining internal voltages in machines constant during transient conditions.

3. Special designs of machines such as compensated machines will be subjects for further discussion and study.

4. A number of measures for increasing stability has been investigated, but are of such great importance that actual operation experience will be required before their true value in improving stability can definitely be ascertained. In this category is included the intermediate condenser station, the special governor control, and the compensated machine.

5. A somewhat different line of discussion would be advantageous to bring out the relation between the probability of outages and system loads.

6. Also the determination of the safe operating angle between internal voltages of synchronous machines, or the relation of this angle to the probability of outage.

INSULATORS

During the year there have been no outstanding changes in the design of line insulators either of the pin or suspension types, though the art appears to be developing slowly but surely toward increased reliability in service. This improvement is largely in the mechanical sense and relates principally to suspension insulators. On the electrical side, there is a gradual crystallization of ideas on the means of safeguarding high-voltage insulator strings from flashover.

Mechanical. Owing to the relatively severe requirements of heavy high-voltage lines, it has been necessary for both manufacturers and prospective users to make extensive tests on the behavior of insulator units, particularly under long sustained mechanical loads. As a result, there are already in use insulator units of so-called high strength type which are capable of supporting for short periods loads of the order of 25,000 lb. Units of this type, when subjected to a continuously applied load, will sustain 16,000 to 17,000 lb. for periods of a year or more without undue failures. Types of still higher duty have also been developed, and it is possible to purchase units having a combined electrical and mechanical strength of approximately 45,000 lb. Results of these studies are also reflected in a marked increase in strength of some of the standard 10-in. units.

Although such tests have been able to furnish much information upon the ultimate strength of insulators under various loading conditions, little is known of the factor of safety that should be employed. It has been customary to assume a maximum safe working load of 3000 lb. on the ordinary 10-in. suspension unit of 11,000 lb. ultimate strength. A disposition to use somewhat higher loadings has now become evident. Likewise, in the case of the above described high-strength units capable of indefinitely withstanding test loads as high as 16,000 lb., the assumed maximum safe loads in service have been fixed by some companies at 6000 to 8000 lb. The question naturally arises as to whether it might not be safe to use much higher assumed maximum loadings.

The design of pin-type insulators is tending toward the use of fewer insulator shells for a given voltage and this practise leads to an important saving in cost without lowering the dry flashover value and without appreciable sacrifice in reliability. The latter is made possible because of perfected methods in manufacture leading to a uniformity of product not heretofore attainable.

The subject of porosity has practically ceased to be an important factor in insulators manufactured by any of the older and well established companies.

One manufacturer in this country is engaged upon the problem of commercially producing Pyrex insulators. Apparently much developmental work yet remains to be done, but the material has considerable promise, particularly because of its high immunity against shattering when exposed to heavy arcs.

Foreign development in insulators is different from American practise, in that much effort has been devoted to avoiding the use of cement in assembling. The German manufacturers have been particularly active in this direction and have produced various cementless types, of which the "Kugelkopf" and "Motor" are typical. The latter insulator consists of a heavy cylindrical section of porcelain carrying a porcelain skirt at one end and either a porcelain skirt or a metal shield at the other end. Mechanical loads are supported by means of a metal cap attached at each end of the cylindrical section by means of a lead alloy. In pin-type insulators, foreign practise has adhered largely to the use of hemp in binding the shells together and for attaching pins.

Electrical. For extra high voltage lines, much work has been done during the past year in connection with the grading and shielding of insulator strings. The results indicate that it is entirely feasible to safeguard insulator strings for operating voltages up to 220 kv., but entire agreement has not yet been reached as to the most satisfactory method to apply. Discussion turns upon whether such protection should take the form of arcing horns, rings or shields which also provide a reasonable degree of potential grading, or whether the protection should consist of insulated horns, which arrangement has been designated as "flux control." Generally speaking, these two methods are predicated upon opposing theories as to the origin of certain types of flashover. This is a subject upon which it is expected operating data will be rapidly collected.

General. Pillar insulator designs have been changed but little during the past year, but a pressing demand exists for insulators of this type having greater mechanical strength both in the low-voltage and high-voltage ranges. It is possible to obtain certain types of pillar insulators of considerable strength which are suitable for heavy duty at the low voltages, but these are quite expensive.

During the year some of the highest voltage catenary insulators yet designed have been installed.

Closely related to the subject of insulators is the development during the past two or three years of large coupling condensers for carrier current work on high-tension transmission lines. These condensers, while experimental at first, have later demonstrated a reliability in keeping with insulator bushings and other similar high-voltage equipment.

INDUCTIVE COORDINATION OF POWER AND COMMUNICATION CIRCUITS

During the past year the work of the Joint General Committee of the N. E. L. A. and the Bell Telephone System has been continued and progress has been made by the Joint Subcommittee on Development and Research in its studies of a number of specific problems, among which may be mentioned coordination of transpositions in power and telephone circuits, induction under conditions of jointly used poles including the effects of unbalances in local telephone circuits and power distribution circuits, the origin and regulation of harmonies in power circuits, survey, composition, and effects of noise in telephone circuits, residual voltages and currents of power systems, induction in carrier communication channels, and energy level of telephone circuits. The studies of many of these subjects have been facilitated by the development of improved measuring apparatus and methods, particularly in connection with the wave analysis of power circuit voltages and currents and induced voltages and currents in telephone circuits, and also in connection with the measurement of carrier frequency currents on working power lines.

The organization of the American Committee on Inductive Coordination has been completed and the committee has issued a preliminary report.

Two papers dealing with the subject of inductive relations between power distribution circuits and telephone circuits were presented at the Seattle Convention in September as part of a symposium on power distribution.

RURAL DISTRIBUTION

Interest in the satisfactory and economic supply of energy to the farmer has increased considerably in the past year. Rural supply has undoubtedly been most highly developed on the Pacific Coast, yet the year has seen a greater study given the subject both in the middle west and in the east. Twenty states have set up organized analyses of the situation and experimental projects are now under way.

The small number of customers per mile (averaging perhaps two to three) makes necessary a cheap type of construction which must, however, be sufficiently sturdy to demand little maintenance and assure fair service. Three-phase, four-wire main feeders with single-phase and neutral legs to outlying farms seem to be a reasonable answer to the problem, and have come into use in the middle west. On these single branches galvanized iron wire is used in some sections of the country for obvious economies, the neutral is stapled directly to the pole top, and the phase wire is carried on a bracket with suitable insulator. These extensions are connected to the main feeders through expulsion fuses. While this represents the cheapest type of construction, there are many cases where it is adequate.

Transformers suitable for farm work have been on the market for some time but even this new apparatus has been expensive. Some transformer makers have now come forward with good designs for this service and there are now available transformers that are in line with the general economies needed to make a rural line a paying proposition.

There has been given considerable thought to the development of portable equipment for transient needs and many new ideas have been evolved to render this service. Portable transformer, metering, and motor apparatus is generally available to furnish power to the farmer for wood cutting, threshing, and the like. This equipment is now generally furnished by the power company and a service man usually supervises its use, the general trend being away from the ownership of such apparatus by cooperative groups.

FOREIGN PRACTISE

The Committee has secured this year one paper on foreign practise on power transmission and distribution, this paper being presented at the Niagara Falls Regional Meeting. More variety of design and much less uniformity of construction is noticeable in this country than abroad.

One of the most notable events of the year abroad is perhaps the Wier Report, recommending the adoption of a universal electric power distribution system serving all England and southern Scotland, with some exceptions as regards the city of London. It is proposed in this report that all the power for use by the system shall be generated in some 50 to 60 power houses, this constituting about 10 per cent of the present number of power houses. It is proposed that the National Government own the transmission lines and buy and sell the power at a price based essentially on cost allowing a fixed return on capital,—6½ per cent plus depreciation on money now invested. The distribution systems and the power generating plants, although subject to close government control, would be privately owned. The most important of the generating plants are proposed as mine mouth plants.

It is reported that a bill has passed the second reading in the House of Commons which is intended to carry out some such scheme although departing somewhat from the recommendations of the Wier Report.

Conclusion. In general it may be said that steady progress is being made in many directions in the field of power transmission and distribution and that a continuous expansion in the number and capacity of transmission systems may be expected.

Discussion

W. S. Lee: Referring to interconnection in the Southern States, this interconnection has been in effect something like 15 to 18 years. I am afraid that this committee is going to get on dangerous grounds when they try to have the entire area controlled by devices from one point.

You should consider that the Southern Systems are interconnected for over 1000 mi. and there are six or seven different companies operating in a somewhat local territory. From the Georgia line to the Virginia line the Southern Power Company system covers something over 400 mi. It is necessary to have it cut up into divisions. The head dispatcher rules certain base or main lines and each local dispatcher controls a certain territory operating around him. Any impression that this paper might give of trying to get one operator to manipulate all the interconnecting companies, I think would be wrong. That is a nice theory, but I think it is hardly practical.

In North Carolina we have an interconnection with the Georgia Railway on the West, the Carolina Power on the East, and in case of any trouble or interruption, the dispatchers pick up a load without referring the matter to an executive.

It can readily be seen that the point on cooperation referred to is pretty well carried out by the group of Southern companies.

The power plants, or undeveloped sites today, are very large as a usual thing; the smaller ones, the ones that are easily developed, have been developed. Oftentimes on account of lack of load or financial arrangement, there is a tendency to develop a plant for a small amount of money. Sometimes it is not a complete development of the project.

I suggest that the committee consider the possibility of steam generation by electricity. I am quite mindful of the fact that we cannot produce steam by electricity at a profitable price, but if we have these large projects that should be completely developed economically, then why not use steam generation to help carry the system along during the earlier stages?

The usual method of developing a water power is to find, as you know, three things; site, money and load.

Now it is a very common practise to go out and make long-term contracts at low rates with the idea of inducing capital. If these big projects can be developed by taking part load as a steam load at a very low price it is just putting an implement of earning into that system that may be useful. As other customers develop, take off steam production and sell for regular commercial service.

Another company with which I am connected has just completed a plant of 540,000-h. p. installation. The plant was financed in a very unusual way. One customer made a 40,000-h. p. contract for primary power and a 160,000-h. p. contract for making steam. That steam contract was cancelable in twelve months.

That contract was canceled before it went into operation and notice was given of the cancelation, but it did furnish a certain implement of revenue that made possible the carrying out of the project in a big economical and fully developed way.

The manufacturers are not much interested in making steam apparatus. It is very simple but they should encourage it with the idea of what it will make possible in other developments.

W. A. Del Mar: I should like to discuss the following statement in this report: "It has been learned that one of the principal causes of the failures in high-voltage cable was the use of impregnating compounds that were unstable under the dielectric stresses and temperatures of normal operation."

I think the committee was going ahead of their exact knowledge in making that statement. I think that it is more than doubtful whether failures in high-voltage cables are caused by the instability of impregnating compounds.

As I pointed out in a recent paper[1] it is far more likely that the decomposition of the compound and the failure of the cable are both results of conditions, such as low hydrostatic pressure in the cable, which promote ionization.

It is also well known that 25,000-volt cables in which the oil has been considerably decomposed and so far as the eye could judge, in some cases entirely converted into *X*, have continued in operation for years.

Another somewhat optimistic statement is that some of the manufacturers and the Electrical Testing Laboratories have developed tests which will aid the selection of compounds, from the point of view of stability. I believe there is some hope that such tests may be developed, but at the present time, the ones which have been tried have been very unsatisfactory and uncertain results have been obtained. It is impossible for one laboratory to check another one and the discrepancy in this regard is very marked.

Another strange fact illustrating our general ignorance of the subject is that some 25,000-volt cables made with highly unstable oil, as determined by the above tests, have not developed *X* after three or four years' operation, whereas others made with comparatively stable oil, very much more stable, indeed, have developed *X* after a few months of operation.

I share the opinion of our European friends that American cable operators study the specification and manufacture of cables too much and their operation too little, and that American cable manufacturers have been equally delinquent in their interest in the installation and jointing of cables.

R. D. Evans: The subject of special characteristics of machines and their effect upon stability is pretty well understood. The special characteristics may be utilized in two ways, either to increase the margin of stability or to increase the amount of power that may be transmitted with the same margin of stability.

Recent studies have indicated that under some conditions it is possible to obtain both advantages with a lower total investment cost. I wish to make the point clear that these studies indicated it is possible to use machines of more expensive construction and yet obtain a system including machinery at a lower total cost.

As the result of such studies, some of the recent hydroelectric undertakings have involved decisions to use machines with special characteristics. I mention this point because it seems to me that it may be indicative of a trend, or new practise in regard to application of such machines.

Many years ago machines of normal reactance were employed, and in order to improve regulation, machines with better inherent characteristics were used. Then the advent of the automatic voltage regulator made it possible to obtain good regulation with machines of more nearly normal design proportions.

Now it appears from the two contracts that there may be a trend returning to the use of low-reactance machines for the special applications of high-power and long-distance transmission. Such a trend may also include quick-response excitation systems.

One of the subjects suggested in the Committee Report as suitable for discussion is concerned with collection and analysis of data relating to stability. In this connection it may be pointed out that the angle-outage curve is a very useful index for comparing in a general way the relative stability characteristics of power systems.

H. R. Summerhayes: I think this subject of stability is perhaps wrongly named. It should be named "power limitations." With that name it would be more readily understood.

I have a little hobby on this subject to improve the limit of power that can be transmitted over a line, and that is what I call a "compensated line."

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[1] *The Effect of Internal Vacua upon the Operation of High-Voltage Cables*, A. I. E. E. JOURNAL, July 1926, p. 627.

To balance the reactance of each section of line we employ a unit of series capacity, and the voltage drop over the reactance is neutralized by the voltage drop over the capacity for each section and then you have the compensated line in which all that remains is the resistance of the line. As compared with a line having reactance only, and the same generating and receiving apparatus, the amount of power transmitted will be greatly increased.

Of course, there are certain practical difficulties in the use of these series condensers which I hope will be solved. I think they will because the advantage of using the series condenser is so great that there is bound to be a good deal of study devoted to eliminating the practical difficulties in their use.

Another objection is the expense. In time that may come down, so that I am just bringing this in as a prediction that some day, with these very large lines of very large power, shall see the use of series condensers with the power going through the condensers.

It is the sort of thing that doesn't present any great advantage in small powers and therefore it is difficult to make a commercial experiment because we like to start with the small powers in making experiments, and as there is no great advantage in the smaller units we have to make our experiments on large power scales. Personally I think we shall have to develop it in the laboratory first and then supply it on a large scale. That, of course, requires a good deal of courage.

Another point which I want to mention is the standardization of voltages for transmission. The Public Policy Committee of the N. E. L. A. this year adopted a resolution in which they urged as a matter of policy the further interconnection of systems and that the managers or executives of neighboring systems study the advantages and the methods of interconnection. In that resolution they also advocated the standardization of voltages above 60,000.

That may come as a surprise to a good many people, because I think many of us have thought that we had standard voltages above 60,000, but apparently that is not the case. It is a very large issue with the N. E. L. A. just now, and I understand that it is to be taken up by the Institute as well—just what voltages should be standardized and what should be the zones of standard voltages.

This is a very important subject and I believe it should be given careful consideration as it is of great economic importance.

In the report of the Committee on Transmission and Distribution, there was recommended a careful study of the value of booster transformers at the point of interconnection between systems. That means voltage-ratio control under load. The ratio may have to be controlled and reversed, perhaps, and you have to be able to make a considerable change in the voltage of the transformer with power going in either direction. There have been important developments in that line and a great many kw. of transformers have been put out and installed with ratio control.

Several systems have been in use and it appears that no one method is adaptable for all voltages and all powers. No doubt experience will show just what method will survive. Probably there will be several methods in use that can be made adaptable to different conditions. At any rate, the subject is of very great importance since it involves the control of wattless current, power factors, etc., and has a most important bearing on interconnection.

The paper mentions the fact that there is a tendency to extend the time and thermal capacity of current-limiting reactors. That used to be 2 sec. but now it is up to 5 sec. and the tendency is to extend the time still further. Putting copper into the reactor and thus extending the thermal capacity is something that you do without really increasing the total cost.

You may pay more for your reactor but you gain that over the losses so that if the load factor is at all good your annual cost may not be increased at all. You get something for nothing there. That tendency is very noticeable.

Another thing that was brought out at the meeting of the N. E. L. A. Apparatus Committee is the new development in starting compensators. Perhaps that should have been noted in the report of the Protective Devices Committee. There is now under way the development of a time-delay under-voltage release attachment to put on hand-starting compensators. All of you are familiar with the shutting down of motors due to momentary line disturbances and there is a lot of trouble that really need not occur. In many cases the under-voltage devices on the starting compensators have been removed altogether with greatly improved continuity of service.

This time-delay attachment itself, I believe, will eliminate a number of shutdowns. The time delay is as short as 1.5 sec., but it will be enough to take care of many of the transient disturbances.

M. I. Pupin: I should like to ask the last speaker whether he has another hobby which is more popular in Schenectady than this hobby. They use it in wireless telegraphy. It is compensation, not by capacity in series, but by inductances in parallel. Inductances are cheaper as a rule than capacities and require much less attention.

That scheme is 29 years old. A friend of mine, Charles S. Bradley, took out a patent on the scheme. I worked out the mathematical theory for him and then he sold the patent to the General Electric Company. The General Electric, so far as I know, has never used that scheme of balancing the line for large power transmissions, but it is using it today at the wireless station on Long Island for balancing a wireless transmission line, one that runs parallel to the ground, and credit is being given to Alexanderson and not to Bradley.

H. R. Summerhayes: I don't know that I can answer that very well, but I do know that the use of the shunt inductance has been suggested a number of times and preliminary propositions have been made out on it, but in almost every case synchronous condensers were used instead because of the ease of adjustment and control of the value.

A field control was considered so important as compared with having the number of inductances with switches, (which were expensive), that we used synchronous condensers. I think Mr. Nickle can tell something about that.

C. A. Nickle: If we have a long line with distributed capacity, it may be represented by a reactance with shunt capacity at either end. It is true that this shunt capacity effect can be cancelled by the use of shunt reactors, and this has been tried on small set-ups.

Now if we have a machine of zero reactance at the sending end of this line, distributed capacity is an advantage. However, when we use a machine of finite reactance, distributed capacity becomes a disadvantage. In fact, if this capacity is sufficiently large, the charging current of the line will be sufficient to excite the generator and the generator will require no other excitation.

It is well known that a generator with no excitation furnished by an exciter can put out no power, so that with the finite generator, there is always a certain sized generator on a certain sized line which can put no power over the line.

Furthermore, a standard generator which we would ordinarily furnish would have so much reactance, and this charging current of this condenser might furnish so much excitation, that the machine, when transmitting full rated power over the line, would run under its proper excitation. In other words, if we could put more current on the field we should be able to get more power if the machine is running below its rating.

When we put this reactor across the line, the excitation can be increased because we have removed the charging current; thereby we can get more power. This reactor, however, costs money and there is another means of making this generator take more excitation which doesn't cost anything.

If the machine is running below its maximum rating so far as field heating is concerned, why put on a reactor to raise the field current when we can merely make the air-gap bigger and accomplish the same result? There is no difference whatsoever except that this takes out material and costs nothing and the reactor does cost something.

When the synchronous reactance of a generator is equal to the charging reactance of the line, the parallel reactance is infinite; therefore, the generator will put out no power.

For exactly the same reason a given size of generator, connected to transmission lines, has always a most favorable number of lines for that generator. If I can get so much power over one line and I put another line in parallel, the power may be increased. But on the other hand, we may find that the power is decreased because we are increasing our charging current which is making our generator weaker instead of stronger.

M. I. Pupin: That old scheme which somebody introduced of replacing the parallel line by condensers at either end has done more mischief than many things of which I know. That is wrong and you cannot do it.

C. A. Nickle: This result is true for one point only; I will admit that; but that is the point that we are studying. At the middle of the line the scheme does not give the same conditions.

M. I. Pupin: I think it is well to bring this out in this discussion, although it is somewhat remote from the main object.

This scheme is particularly useful in the case of transmission of power over cables. When it comes to air lines, it is not quite so important, excepting in wireless telegraphy. When you have a long line, you introduce inductance at periodically recurring points. There are two things which must be observed; the first is the wavelength—the electrical wavelength which is to be transmitted. These inductances must be separated by only a small part of a wavelength.

Then in the second place, these coils must be very highly efficient; that is, the reactance must be very large in comparison with the resistance. In fact, so far as my experimental investigations go, and the theory itself, it is practically impossible to use a coil which has an iron core; and that is where Bradley's invention, which was sold to the General Electric Company, fell to the ground. He didn't know that and the Genral Electric didn't know it.

But the General Electric Company found it out in the wireless work on Long Island. This is exactly what they do on Long Island. They use coils connecting the wireless antenna to the ground. There is no other scheme that will do the same thing that this does. It is an admirable scheme.

The laboratory experiment represented a cable between Buffalo and Syracuse and the frequency was 110 cycles. It reduced the charging current down to one-seventh; there was a very large inductance, and it does not affect your terminal generation in any respect whatever.

It will make no difference in this case how many lines you have connecting with your generator; the charging current will be reduced in each compensated line.

Mr. Nickle: So far as building the generator and testing is concerned, that is done exactly as we predicted. We do get an increased maximum power. Furthermore, when the reactance is equal to the charging current we find we get no power. We have tried it on the quarter-wave line.

On a long-distance line there is no doubt but that, where you can put in the proper number of lines, the charging current will put the excitation up.

What I have been talking about was current-compensated lines. When you put in more lines there you defeat your ends. You should forget about compensation here. If we put in more lines we do increase the charging current and when we increase the charging current in some cases it gets so high that we can't get any power out of it.

R. N. Conwell: I am glad that Mr. Lee raised the question about the one-man control, because, after going over the report again, I find it is subject to misinterpretation.

Interconnections fall generally into about three classes:

The first one which we have considered is the so-called relay interconnection where relatively small amounts of power are interchanged between systems. Usually the lines are small, the capacity small and the interchange for emergency use only.

There is a second class that has become common and that is where the lines are of high voltage and the distances are great. Usually these are used primarily for firm power interchange.

There is a third class that is being considered at the present time. The lines are relatively short, connecting systems closely situated, but generally for high power, something like 200,000 or 300,000 kv-a. The third class will permit the interchange of the full diversities of adjacent systems attaining the advantages of generation reserve and the most economical operation of the plants in the individual systems. You can readily see that in this third class of interconnection, one-man control is almost an essential if all the advantages are to be obtained.

D. W. Roper: At a time when the art and science of the manufacture of high-tension cable with impregnated paper insulation is advancing as rapidly as it has been for the past few years, it seems quite impossible to prepare a statement that says anything with which the engineers of all the manufacturers and operating companies will agree. The statement to which Mr. Del Mar takes some exception appeared to represent the consensus of opinion of the majority of the members of the committee at the time it was prepared. While Mr. Del Mar and others have since developed several different theories, the statement to which he takes exception is still the opinion of several members of the committee.

One operating company has high-tension cable made to practically the same specifications and furnished by seven different manufacturers. The two makes of cables which have not failed in service have a compound which will pass the test developed by the Electrical Testing Laboratories, but the compounds taken from those five cables that have failed in service do not pass the test. In the face of this agreement of service records with the test indications, it does not appear expedient to discard the test or to condemn it in the light of present information.

It will probably take considerably more work in the laboratories, together with a careful examination of several years' service records, to determine definitely the value of any particular theory and the relative importance of the several steps in the process of drying, evacuating and impregnating the paper insulation.

If the selection of the proper compound is such a simple matter and of so little importance, it seems difficult to understand why some manufacturers are changing their impregnating compound so frequently.

Present Practises in Protection

Annual Report of Committee on Protective Devices*

E. C. STONE, Chairman

To the Board of Directors:

FOREWORD

Because of the many developments during the past few years within the scope of the Committee on Protective Devices, it was held desirable to present, this year, a survey of the present state of the art in the field of protective devices for power systems, and the following annual report has been prepared with this purpose in view. The Committee has also, during the past year, given careful attention to the possibilities of further standardization in the field of protective devices, and has certain definite recommendations to make.

As heretofore, the work has been carried on by subcommittees, each under the direction of its own chairman. The subjects covered and the chairmen in charge are as follows:

Automatic Substations. W. H. Millan, Union Elec. Lt. & Pwr. Co., St. Louis, Missouri.

Current Limiting Reactors. E. A. Hester, Duquesne Light Company, Pittsburgh, Pa.

Lightning Arresters. H. Halperin, Commonwealth Edison Co., Chicago, Illinois.

Oil Circuit Breakers. J. M. Oliver, Alabama Power Company, Birmingham, Alabama.

Protective Relays. J. A. Johnson, Niagara Falls Power Co., Niagara Falls, New York.

The Subcommittee on Grounding of Systems, which reported last year, was not continued this year, as it was felt that it had completed its work, for the time at least.

Following are some of the more important subjects dealt with in the report:

Full automatic stations of the following types are in successful operation:

Hydroelectric generating stations
Railway substations for city and heavy traction, as well as interurban service
Edison d-c. substations
Mining and steel mill substations
Alternating-current substations (reclosing breakers)
Synchronous condenser stations.
Mercury arc rectifiers for arc lighting and power service are now being provided with automatic switching equipment.

Thorough periodic inspection and maintenance are essential to the successful operation of automatic stations.

There is a marked trend toward the use of reactors with insulated conductors, in order to obtain protection from external interference.

Lightning protection on low-voltage circuits (230-115) in general appears to be unnecessary. For distribution circuits up to 6.6 kv., having numerous transformers connected, satisfactory protection is being obtained from available types of arresters in many cases.

On circuits rated at 73 kv. and below, the general tendency is to install arresters, usually of the high discharge rate type. The installation of arresters on circuits from 73 kv. to 154 kv. is debatable, but there is a growing tendency to provide some means to reduce over-voltages due to lightning. On circuits at 154 kv. and higher, the tendency is to omit lightning arresters.

The Dufour oscillograph and klydonograph for measuring transient phenomena are expected greatly to increase our knowledge of lightning arrester performance and requirements for lightning protection.

Relative interrupting capacity ratings for oil circuit breakers under various operating duties have been agreed upon by the N. E. L. A. and Electric Power Club, and are presented herein.

Progress in the problem of short-circuit interruption is recorded in further short-circuit tests on operating systems, additional testing facilities in manufacturer's plants, and improvement in details of design. The problem still remains one of outstanding importance, however, and seems far from satisfactory solution.

In relay practise, definite trends are reported as follows:

a. Integration of entire system into one unit, so interconnected and relayed that its integrity shall not be broken by a fault.

b. Fault isolation by use of balanced relay systems or systems responsive to location or specific nature of the fault.

c. Use of "back-up" protection, in the form of a second system of relays to function in case of failure of the first system.

d. Use of devices for detection of approaching faults before they actually occur.

The committee has given active attention to the obtaining of papers for Institute meetings. Reference is made herein to a number of important subjects which should be covered by papers next year.

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*Committee on Protective Devices:

E. C. Stone, Chairman
F. L. Hunt, Vice-Chairman
W. S. Edsall,
H. Halperin,
F. C. Hanker,
S. E. M. Henderson,
R. A. Hentz,
E. A. Hester,
George S. Humphrey,
J. Allen Johnson,
M. G. Lloyd,
H. C. Louis,
A. A. Meyer,
W. H. Millan,
J. M. Oliver,
N. L. Pollard,
E. J. Rutan,
E. R. Stauffacher,
H. R. Summerhayes.

Presented at the Annual Convention of the A. I. E. E., at White Sulphur Springs, W. Va., June 21-25, 1926.

AUTOMATIC STATIONS

Survey. A brief survey of the present status of the automatic station art seems desirable at this time. For convenience, this survey will be considered under several pertinent topical headings.

Automatic hydroelectric generating stations have been the subject of extensive development during the past three years. Stations containing two 9000-kv-a. units are now in successful operation, while many ranging from 100- to 5000-kv-a. are regularly being installed and operated. Automatic synchronizing has been perfected and is now used where large units are connected to small systems. Governor design has been improved to meet the rigid requirements of automatic station operation. In fact, practically all small hydroelectric developments and many moderate sized ones are now designed for automatic operation and uniformly successful results are reported.

Railway substations now being purchased are invariably automatic with rare exceptions due to local conditions. They are not only considered standard for interurban service but also standard for heavy city service and electrified portions of steam railroads. All kinds of service find them economical and reliable and their use is now being rapidly extended not only by the installation of new stations but also by the conversion of existing stations from manual to automatic operation.

Automatic substations for Edison three-wire networks have now been in successful operation on small and moderate sized systems for over four years. Their first application to two of the largest systems has just been completed. Their success for this class of service seems assured although more operating experience is desirable before extensive installations are made on important projects.

Mining substations now being purchased are all either completely or partially automatic as individual local conditions warrant. The various designs offered have been subjected to operating service for over five years. As a result, the automatic substation is today considered standard by the electrical profession for the motor generators and synchronous converters used in the coal and metal mining industries for electricity supply.

Steel mills have been the latest industry to adopt automatic stations. One small installation of partially automatic-controlled synchronous motor-generators has been in successful operation over five years. Another extensive installation of completely automatically-controlled synchronous motor-generators has been in service almost two years. About eight additional extensive installations are now in progress so that it would seem as though steel mill electrical engineers have started toward adopting automatic stations as standard for their service.

Automatic feeders for alternating-current supply constitute by far the largest field for completely automatic operation. In practical application, the art is still in an initial state of development. The equipments so far offered have simulated manual practise with the only improvement of more prompt operation. Successful a-c. motor-operated mechanisms are now being applied to the largest oil circuit breakers, thus eliminating solenoids and batteries for reclosing service. Automatic reclosing relay and oil circuit breaker combinations are now in operation which permit clearing transient feeder faults without appreciable service interruption. These so far, however, have been applied only to moderate sized, relatively low pressure service, although in one particular case 44,000-volt feeders are being automatically reclosed on a relatively large capacity power system.

Synchronous condensers for power factor correction were first made automatic about ten years ago. Only a few installations have been made but each has been uniformly successful and the use of this type of automatic switching equipment is spreading with a relative degree of rapidity.

Mercury arc rectifiers for arc lighting and power service are now being provided with automatic switching equipment. The smaller sizes with glass tube rectifiers have been in service a sufficient length of time to demonstrate their success. Those for larger units using metal tanks are just being placed in service so that next year's report may contain further data concerning their performance.

Railway signal and automatic train control substations are the latest addition to the rapidly growing list of automatic applications. Some of these have been in successful operation for over three years. Many are now being installed to furnish power for the operation of track signals and train control circuits.

Symbols. Symbols for automatic station wiring diagrams have been attracting increased attention during the past year. Data which will give the existing practise are being collected by the subcommittee. It is then planned to correlate the various symbols, adjust differences where they exist, and finally prepare a simple, agreeable set which may be submitted to the Standards Committee for approval and promulgation.

Nomenclature. Nomenclature for various types of a-c. reclosing feeders was suggested in the 1925 report. These suggestions have been before the electrical engineering public for about a year and no unfavorable comments have been heard or received by the committee. Instead, it has been found that many of the designers and purchasers of automatic d-c. feeders have adopted the proposed nomenclature. It has, therefore, been presented to the Standards Committee for approval and inclusion in the A. I. E. E. Standards.

Protection. Recommended protection for railway and Edison automatic stations with synchronous converters and synchronous motor generators, as well as for automatic switching equipment in a-c. feeder service, was given in the 1925 report. These recommendations have been adopted as standard by the principal manufacturers and users of these classes of automatic stations. This year there have been developed recommendations for the protection of mining and steel mill automatic stations using synchronous converters and motor generators as well as synchronous condensers in voltage regulating substations and hydroelectric generating stations using synchronous generators.

Steel mill substations are quite generally provided with synchronous motor generators. Some are partially automatic as when an attendant is constantly on duty in the vicinity. Others are completely automatic as when the stations are non-attended and locked. The partially automatic stations are similar in design to the completely automatic ones except that certain protective and re-starting features are omitted. The items to be omitted vary from installation to installation so that it is difficult at this time to make any complete recommendation for the partially automatic. For this year's report, therefore, consideration will be given only to the completely automatic synchronous motor-generators in steel mill service. It is recommended that these be provided with protective features as follows:

A-c. undervoltage
Severe a-c. overload
Single-phase starting
Excess temperature due to sustained moderate overload
Imperfect start
Loss of field of a-c. machine
Loss of field of d-c. machine
Reversed phase rotation
D-c. reverse power
Excess bearing temperature
Machine overspeeding

Mine substations are usually equipped with synchronous motor-generators although some stations have synchronous converters and a few have induction motor-generators, with mercury rectifiers beginning to be used. The recommended protection for synchronous motor-generators in mine substations is the same as that for synchronous motor-generators in steel mill service. Few induction motor-generators are now being applied and only two or three mercury rectifiers have been installed in mine service so that recommendations for these are omitted for the time being. For synchronous converters in mine service it is recommended that protection be given as follows:

A-c. undervoltage
Severe a-c. overload
Single-phase starting
Excess temperature due to sustained moderate overload
Imperfect start
Loss of field
D-c. overload
Incorrect polarity
D-c. reverse power
Excess bearing temperature
Overspeeding

The protection for automatic hydroelectric stations with synchronous generators and for synchronous condenser stations is quite similar. For synchronous generators in automatic stations, the recommended protection is as follows:

A-c. undervoltage
Severe a-c. overload
Excess temperature due to sustained moderate overload
Single-phase operation
Imperfect start
Loss of a-c. machine field
Exciter overvoltage
Excess bearing temperature
Machine overspeeding

Synchronous condensers usually have, in addition, a-c. undervoltage and single-phase starting protection. They are usually provided with automatic voltage regulators so that the exciter overvoltage protection included for synchronous generators is in that case omitted.

Storage Batteries. The majority of automatic stations are not provided with storage batteries. If they are automatic hydroelectric stations, a small turbine or waterwheel directly connected to a generator seems to be the preferred source of control power. If they are substations, then the source of power suitably transformed is used to supply the operating current.

Some designs of automatic stations, particularly those used for certain classes of Edison service and for all classes of remote supervision, require relatively small operating batteries. Practically all types of standard batteries have been used with an equal measure of success although some are better adapted than others for this service. In general, the batteries are trickle charged through rectifiers or motor-generators. In a few unusual designs, a motor-generator is provided to carry the operating load with a battery floated on it for stand-by purposes.

Oil Circuit Breakers. Oil circuit breakers are used in automatic stations between incoming lines and the station equipment and between the station equipment and outgoing lines and feeders. The two applications require quite different treatment in the selection of the oil circuit breaker equipment if a maximum of reliability is obtained.

When used between the incoming line and the station equipment, the oil circuit breaker is closed each time the machine in the station is started and opened each time it is stopped. Usually such an oil circuit breaker is operated most frequently to make or break currents less than its normal continuous current carrying capacity. However, it may be required to operate from two to one hundred times each day depending upon the service conditions.

Oil circuit breakers used between the station equipment and outgoing feeders are normally closed. They are opened usually only in case of trouble or to permit of circuit maintenance. These oil circuit breakers will consequently operate infrequently. Usually, however, when they do operate, they will be called upon to interrupt currents approximating their circuit interrupting capacity.

In the first case, a so called "one-shot" breaker suffices if its mechanical parts permit the required frequent operation. In the second case, the circuit interrupting capacity on successive interruptions is the more important function with the mechanical life of secondary consideration.

To meet the frequent operating requirements of the breaker located between the incoming line and the station equipment, there have been developed new designs of universal motor operating mechanisms. Some of these have been in service in excess of five years. They eliminate the difficulties experienced with the older type of operating mechanisms and give a very much improved automatic station equipment.

For improving the requirement of successive circuit interruption to clear feeder faults, various schemes have been approved and are being tried. Double tanks, slow release of gases, ventilating ducts, and explosion chambers are all in use with no marked preference conceded at this time.

Reclosing Cycle. The reclosing cycle used for oil circuit breakers which protect feeder service in automatic stations varies from application to application. In one extreme case, each time a feeder breaker opens, it is held open until closed from a central operating point; *i. e.*, it is closed under the supervision of an operator so that in case the trouble persists the operator can immediately report the difficulty to a central load dispatcher. At the other extreme is the application where an oil circuit breaker may reclose on a fault five or six times in rapid succession.

So far as is known by the committee, there are only two installations where the operation of the breakers is supervised after each automatic opening and only one installation where the breakers reclose five or six times in rapid succession on a circuit fault. By far the majority of automatic reclosing feeders in alternating-current service are reclosed two, three, or four times on a fault before being locked out.

The reclosing cycle having three reclosures before lock out is almost universally standard. The time intervals, between reclosures, however, vary between relatively wide limits. One group of operators, which is rapidly growing, prefers to have the breakers reclose immediately after the first or initial circuit interruption. They want this time made as short as practicably consistent with positive relay and oil circuit breaker operation. The majority of operating companies, however, appear to be satisfied with a time interval ranging from five to twenty seconds between the time the breaker opens initially and the time of the first reclosure. Practically all classes of operators are content with automatic reclosing the second time from fifteen to thirty or forty seconds after the second circuit breaker opening. Those who use third and fourth reclosures appear to be satisfied with a time interval between the second and third and the third and fourth reclosures of about one minute each. There are some unusual conditions, however, where the time intervals between the first and second reclosure and the second and third reclosure are even longer, being as much as three minutes and five minutes respectively, but these are again unusual.

As a result, it is seen that there is a wide range of operating adjustments in the automatic reclosing of alternating-current feeder circuits indicating that the art has not yet become stabilized.

Relays. Relays of many varieties form the primary basis on which successful operation of automatic stations depends. The devices have been the subject of much study, development and observation in automatic station practise. Relays are used not only for performing protective functions, but are also used for controlling operating sequence and as preventatives against avoidable damage to service and apparatus.

The early designs of automatic stations included relays which had been developed for use in manual stations. Gradually, however, most of these have been replaced by relays or their equivalents specially designed to meet the arduous service and strict regulation required for automatic station operation. However, not all of the relays or other similar devices employed in automatic stations today can be considered perfect. Service records indicate though, that as a result of relay development, automatic stations are today equally, if not more, reliable than the equivalent manual stations.

Inspection. Periodical inspection of automatic stations is essential to their successful operation. The frequency and extent of such inspection depends upon service requirements. Experience indicates that automatic stations provided with rotating machines and supplying important service should be inspected daily. Such inspection rarely requires over an hour in the station after the inspector has become proficient. For less important service, automatic stations with rotating machines may be inspected weekly while transformer stations with reclosing feeders only are inspected at even greater intervals. This class of inspection serves to detect any tendency for the devices in the station to change their adjustment or operating characteristics and prevents unnecessary service interruptions.

Inspection as thus outlined must not be confused with periodical maintenance. Quite different personnels are required for the two sets of functions although where the automatic stations are small the same staff may perform both duties.

Most of the systems having several or more automatic stations make it a rule for each inspector to file a written report after each inspection of an automatic station. This report summarizes the work done in the station together with the observations which have been made. The reports differ from organization to organization depending upon local conditions. In general, however, these reports contain the time and date of the inspection, together with a list of devices inspected and comments on any device which requires attention. In some cases, the reports list the readings of the counting devices in the station and indicate the number of operations of the principal functions. Of course, reports are properly signed and dated and otherwise identified.

Maintenance. Periodical maintenance is equally as important as periodical inspection. The frequency of maintenance will depend upon the number of operations the automatic station is called upon to make. If only six or eight operations a day constitutes the cycle, thorough examination of the equipment with such cleaning of contacts and replacing of worn parts as is necessary ought to be made bi-monthly. If several hundred operations per day is the schedule, then weekly examination with such maintenance as is needed is generally recommended.

Maintenance reports are usually prepared on the moderate sized and large systems which use automatic stations. Their form follows, in general, the inspection reports mentioned above, except that they may elaborate on the details of devices which require more than usual maintenance as well as list the renewal parts which are installed.

A clear distinction should be made between inspection and maintenance. One reviews, casually, the condition of each device or piece of apparatus in the automatic station. The other should be a minute inspection accompanied by such cleaning and adjusting and replacing of worn parts as the service may require. Every effort should be made not to confuse these two, and in many systems the distinction has been drawn so clearly that inspection is charged to the operating account while maintenance is charged to the usual maintenance account.

Records. Records of automatic station performance are quite necessary for their efficient adjustment and operation. These records may be divided into two classes as follows:

1. Records of device and functional performance.
2. Records of station performance.

Device and functional performance records are usually made chronological and recorded in a log book kept in the station. Here are listed the date and time of the visits of inspectors, maintainers, and others, with their observations and the adjustments and changes they may make from time to time in the station equipment. Brief summaries of the station log may be prepared and forwarded to a central supervisor from time to time and from these may be prepared condensed operating reports.

Station performance records are usually obtained automatically by curve drawing or recording instruments and meters. Sometimes the instruments and meters form a permanent part of the automatic station equipment. Ofttimes portable curve drawing instruments or meters are used at regular intervals to check service requirements and station operation. This portion of the automatic station art is just being developed and not many definite data are available as a result of service experience.

Fire Extinguishment. Automatic fire extinguishment in automatic stations has been considered by the committee for the past two years. Practically nothing has been done in automatic stations along this line. The art of automatic or even partially automatic fire extinguishment in manual stations is even now just in the experimental stage. The committee is watching all installations quite carefully and expects to report further progress next year.

Remote Supervision. Remote supervision of automatic stations is becoming more and more important. A number of systems is in the open market and some of the operating companies have manufactured their own systems. A general review of the situation was presented by Mr. Chester Lichtenberg under the auspices of the Protective Devices Committee at the Midwinter Convention in New York, February 1926, and reference is made to his paper for an up-to-date survey of the situation.

Telemetering. Telemetering is just now being developed particularly for automatic station service. There are about one-half dozen installations in partial or complete service, but this, too, is a new development in the art which will probably serve as the subject of further comments next year.

Ventilation. Ventilation of automatic stations forms a very important part of the design of these stations. In general, automatic stations are smaller than the equivalent manual stations. Therefore, a less volume of air is available for carrying off the heat generated in the station. This means that there must be larger openings provided for the air to enter and leave the station if the apparatus is to be maintained at a reasonable temperature. Very thorough studies of this subject are now being made by several members of the Institute and at least one member has promised to prepare a paper on this important topic for presentation to the Institute in the near future.

Other Topics. Other topics not yet studied by the automatic station subcommittee are as follows:

Nomenclature for types of automatic reclosing a-c. feeders.

Terminology for operations in automatic stations.

Protection against failure of cooling mediums.

Automatic warning of approach of apparatus failure.

Dielectric failure of oil where used as an insulating or cooling medium.

Loss of oil in transformers.

CURRENT LIMITING REACTORS

A review of the status of current limiting reactors in the light and power industry during the past year reveals nothing revolutionary either in application or in manufacture and design. Preceding reports have indicated a general stabilization of both application and design practises and these tendencies have become even stronger. The practise of sectionalizing generating stations with reactors is on the increase and their use at other points on the system to localize faults and reduce short-circuit currents is becoming quite general.

The practise of insulating the coil conductors is increasing in popularity, and tests have been made which show the efficacy of this method of preventing trouble from external sources.

There is some activity in the development of oil immersed, steel enclosed reactors, and it is expected that the next year will show considerable progress. There has been one installation of this type of reactor operating successfully at 66,000 volts for some time.

The mechanical and electrical reliability of reactors seems to be quite definitely established in the minds of operating engineers. Cases of failure due to inherent weakness continue to occur but with such infrequency that they may be attributed to accidents of manufacture. It is felt that with reasonable care in installation and inspection for freedom from foreign objects, the reactor may be counted among the most reliable of electrical equipment. Of the serious failures which have occurred recently, the greater number may be attributed to the presence of something foreign to the reactor.

Out of a total of almost 5000 reactors installed during the past five years, only thirty-seven failures have been reported and only a few of these could not be attributed to something other than defective design or manufacture. These results were reported by 43 operating companies. Of these 43, 11 reported that no reactors had been installed during the last five years.

In presenting last year's report, the subcommittee made several recommendations of subjects requiring investigation.

The matter of thermal capacity and conductor cross-section continues to furnish material for discussion. Most operating companies appear to be specifying a thermal capacity of about five seconds and require a cross-section having the same carrying capacity as the cables to which the reactor is connected. This is a somewhat more liberal allowance than was usual some years ago where two seconds were allowed and a smaller cross-section used. It hardly seems logical to make the reactor the weakest link in the system and the longer time is the result of the more general use of the so called "back-up" protection.

The two-second idea seems to have grown out of the fact that this was the maximum allowable time for relay settings. Should the circuit breaker fail, then the reactor was as good a place as any for the short circuit to burn clear. Now, there is usually at least one back-up circuit breaker and in the case of extreme contingency the time may run well above two seconds.

Attention has been given to the matter of providing reactors with taps and it is the general opinion of the subcommittee that this is undesirable except in cases of extreme necessity. It is recognized that occasions will arise when there is no alternative, as for instance, in the balancing of feeders, but it is suggested that each case be made the subject of special treatment. The manufacturers are willing to supply such reactors on special order but are unwilling to attempt to make them a standard product on account of manufacturing difficulties. Finally, there is the objection to having many dissimilar reactors on the system with resulting chances of confusion.

Under the subject of shielded reactors two general classifications may be made, first, those having each individual conductor insulated, and second, those which are oil immersed and totally enclosed.

The insulated conductor reactor has proven quite popular and effective and it is predicted that they will largely replace the bare type on new installations, especially in generating stations. The number of failures due to the presence of foreign material is very likely to show a marked reduction.

There is not a great amount of data available on oil-immersed, totally enclosed reactors although there are some in operation. In order that the status of this type of apparatus may be put before the industry, it is hoped to present a paper during the coming year, covering operating experience and possibilities.

It has been suggested that your subcommittee consider the use of current-limiting reactors with static condensers to aid in smoothing out surges, to limit the fuse current, and to increase the capacity of the condenser. Your subcommittee feels that, since a reactor considered in this light becomes a part of a specific device, it should be studied by the committee under whose jurisdiction static condensers are placed. The function of this subcommittee is to study the reactor as an individual piece of apparatus and its effect on the system as a whole.

There has been much discussion in the past few years as to whether the use of reactors actually increases the duty on oil circuit breakers. In the light of recent developments and tendencies in practise it would appear that the duty is actually decreased rather than otherwise. The fact that the installation of reactors in a great number of cases makes it possible for the oil circuit breaker to interrupt the short-circuit current is clear indication that the gain in smaller current over-balances the disadvantage of a possible higher recovery voltage.

There are still two schools on the question of the efficacy of resistance-shunted reactors. The one side contends that the resistance is quite effective in absorbing the surge voltage while the other maintains that it has no appreciable effect. There are some of both types in service and a few tests have been made, but thus far the results have not been very conclusive. It is hoped that with the assistance of the Dufour oscillograph and the klydonograph, described in another section of this report, some definite settlement of the question may be obtained. A paper covering operating experiences with the two types is also recommended.

Some trouble has been experienced because of circulating currents in two-winding reactors due to unbalance in windings. Consideration of this brought out the fact that it is purely a manufacturing problem and therefore the concern of the particular manufacturer and the user experiencing the difficulty.

The effect of reactors on nearby magnetic materials has been mentioned as a subject for investigation. Your subcommittee is of the opinion that this is a fairly definite thing and that it has been well taken care of by the manufacturers in the printed specifications covering each type of reactor. Special cases must be subject to special attention by the manufacturer.

The arrangement of single-phase reactors on three-phase circuits and the position of the reactor with relation to the oil circuit breaker is sometimes brought up for discussion. This, however, is so often subject to local conditions that no set rules can be laid down with reference to physical arrangement. Obviously, the reactor must be between the source of power and the circuit breaker whose duty is to be limited but this again is subject to a great many variable factors and no standard can be applied.

The problem of standardization in reactors is a rather difficult one as is obvious from the foregoing considerations. Something may possibly be done on thermal capacity and conductor cross-section and your subcommittee is now considering this with the manufacturers.

The wide variety of reactance values and current ratings now in use is rather astonishing and it is suggested that the succeeding subcommittee look into the possibility of reducing these to some standard basis. It would seem feasible to standardize certain sizes with respect to current, voltage, and reactance values, as has been done with other types of equipment.

In reviewing papers which have been presented during the past few years a few points were found to be rather incompletely covered. This suggests the desirability of a few additional papers and the following are recommended:

1. Status and Operating Experience on Oil Immersed Reactors.

2. Effect of Shunting Resistance in Reducing Surge Voltages.

3. Further Operating Experience with Insulated Conductor Reactors.

LIGHTNING ARRESTERS

General. In general, during the past year there have been no radical changes in the fundamental principles of lightning arrester design, and the efforts of manufacturers and operating companies have been directed mainly to increase in reliability of the arrester and economy in its use. One modification is the development by a manufacturer of a liquid type of arrester with an electrolyte, which is claimed to have a freezing temperature of about minus 48 deg. cent.; this development has been also accompanied by some changes in the construction of the arrester. Other recent developments have been more in the nature of increased sturdiness, additional refinements, and increased attention to special features imposed by operating conditions. The general tendency is to install arresters outdoors wherever possible.

For high-voltage networks, it has been found that the transient overvoltages which appear on the high-tension side of a transformer may also be induced on the low-tension side (and lines connected thereto) by means of electrostatic induction. This effect varies with the physical dimensions of the transformers, and there have been cases for transformers of large dimensions, that the induced voltage on the low-tension side has been high as compared with normal overvoltages on that side; therefore, it appears that some consideration must be given to reducing such voltages.

Development and Practise. The A. I. E. E. Standards state that "a lightning arrester is a device for protecting circuits and apparatus against lightning or other abnormal potential rises of short duration." In operation the device must shunt or divert the transient current in a sufficient amount so that the resulting rise in the voltage will not be above what the apparatus will withstand. Since maximum shunting occurs when the impedance of the circuit is zero, it is evident that the most effective protection during the transient over-voltage will be secured when the impedance of the arrester and its connections to the ground is minimum.

The earliest efforts in over-voltage protection consisted in providing air-gaps connected between line and ground and so arranged that when the transient voltage appeared, the gap would break down and practically connect the circuit directly to earth. Usually the arc through the arrester would not break until the generator voltage and resulting system voltage became quite low. Neglecting the effect of the voltage fluctuation, this might have been satisfactory on some of the small systems, especially some years ago when generators of high impedance were used; but with the larger systems which have generators with lower impedance, the arc would be maintained and furthermore there was the possibility of accompanying high voltages due to what was in effect an arcing ground. The operating difficulties were so great that later some impedance, usually resistance, was introduced in the connection to ground in order to facilitate the breaking of the arc; but this reduced the efficiency of the arrester as a protective device.

The operation of systems with inadequate protection was found to be impractical on account of many interruptions of the service, and, as the art developed and the customer grew to place more and more dependence upon continuity of electric power supply, greater and greater efforts were exerted by the operating companies to eliminate interruptions and large voltage fluctuations. The persistance of this demand led to the development of arresters with characteristics which provided a comparatively free path for flow of surge current at voltages above the operating voltage, but which did not permit current in detrimental amounts to flow, except during overvoltage. This important characteristic has been described as an "electric valve action" and arresters of these characteristics are known as "valve type" arresters.

The earliest of these was the electrolytic arrester, which, after some modification made through service experience, had very good protection characteristics. It had certain mechanical disadvantages, such as the use of oil and the use of an electrolyte which froze, and made the arrester practically inoperative at very low temperatures. There was some expense to operating these arresters on account of the periodic maintenance and daily charging during the warm weather, and, if the arrester was remotely located, then the expense became considerable.

There is now almost a universal agreement that for a high degree of protection, a lightning arrester should have a high discharge rate, a short dielectric spark lag, and the ability to interrupt the dynamic current. This is borne out by theoretical considerations which have been presented on several occasions and tests and operating data reported by manufacturing and operating companies.

According to Peek,[1] "an induced stroke of lightning may produce a voltage on an overhead line equal to about 100 kv. per foot of elevation of the wires above ground. In the average case, this factor has been found to be 30 kv. per foot." On the low-voltage lines, the line insulation is a smaller portion of the probable usual voltage induced by lightning than it is for the high voltage lines; for instance, on a 220-kv. line without a ground wire, the ratio of the usual highest lightning voltage to the insulator spark over voltage is about 1.1 while the corresponding ratio for a similarly designed 70-kv. line would be over two and for still lower voltages, this ratio would become larger. Direct strokes of still greater severity may be expected occasionally.

The present practise is approximately as follows:

1. Low-Voltage Circuits. Due to the fact that on such systems lightning arresters are spaced rather closely, a high individual discharge rate is not as essential for low-voltage circuits as for high-voltage lines. A variety of arresters are being used with comparative success, and among other factors the simplicity of design is of considerable importance.

Experience on one system has shown that where the wires of low-voltage secondaries are placed directly under the wires of higher-voltage primaries, say, rated at four kv., and the length of the secondaries are not more than about 600 feet, it has been found unnecessary to place arresters on secondaries when the primary wires are well protected from lightning by means of arresters connected to them. In the experience on another system it has been found that secondaries under 1000 feet in length, whether under protected primaries or not, do not require arresters. Theoretically, there is some question as to the degree of protection offered to secondaries by protected overstrung primaries, but there is no doubt that the general practise of grounding one secondary wire affords effective protection to the secondary circuit. Where the length of the secondaries is longer and there are no wires above them to protect them, it has appeared necessary to put arresters on the secondaries in order to obtain protection from lightning.

2. High-Voltage Circuits (0.6 to 154 Kv.). On high-voltage circuits, the lightning arresters are usually spaced a great distance apart so that the individual arrester, to be of any value, should have a large discharge rate which limits to a few types the number of arresters suitable for this purpose. The general tendency is to install arresters on circuits rated at 73 kv. and less. The installation of arresters on circuits from 73 kv. to 154 kv. seems to be debatable, but there is a growing tendency among engineers to provide some means to diminish the over-voltages due to lightning. This is done by either the rearrangement of the phase wires into a horizontal configuration in order to diminish the value of the induced voltage from lightning, or by the use of the ground wires to further reduce the induced voltage, or both, and also by the use of lightning arresters in order to protect the apparatus and circuits.

3. Extra High-Voltage Lines. There is a tendency to omit lightning arresters from lines operating at 154 kv. and higher, the main argument being that the line insulation is able to withstand the induced over-voltages and the direct strokes of lightning cannot be handled effectively by existing arresters.

As no arrester will insure an absolute protection against a direct stroke of lightning, a certain amount of damage to existing equipment is to be expected, its amount being dependent upon the degree of protection provided by a given installation of arresters. The installation of arresters by operating companies has been shown by Roper, Atherton, and others to be warranted only on a basis of the improvement and reliability of service to the customer, as the cost of installing and operating an arrester is many times the revenue that would be lost by the operating companies during the time of interruptions.

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[1] Address by F. W. Peek, Jr., at meeting of N. E. L. A., Overhead Systems Committee, February 11, 1926, Kansas City, Missouri.

The degree of protection to be selected for each case depends primarily on the economic considerations, and the problem can be treated in a manner similar to that described in an article by A. L. Atherton.[2] As an example, the Commonwealth Edison Company has found for business and residential sections with a high density of load, that on its 4000-volt distribution system, there is slight advantage in having more than 135 arresters per square mile. The less expensive line types of arresters are used on the lower voltages, except on important installations where the larger station type is installed. Above 50 kv., only the station type arresters are used.

NEW DEVICES

The outstanding recent event in the field of lightning arresters has been the application in this country of two excellent tools for investigations, that is, the Dufour cathode ray oscillograph and the klydonograph.

The Dufour oscillograph, which is essentially a laboratory instrument, was constructed in Europe several years ago, but it was not used in this country until recently. It will make records of voltage or current transients of a very much shorter duration than is possible by the ordinary vibrating type of oscillograph. (For further details see "Dufour Cathode Ray Oscillograph".) In the work of the subcommittee in the past few years, the measurement of the voltage-time curve (see JOURNAL of A. I. E. E., June 1924, p. 575) was recommended as a basis of comparing various lightning arresters. Until recent time, there have been no facilities available to determine this curve experimentally, but now this can be done with the Dufour oscillograph. This apparatus also affords a means of determining the relative time lag of insulations which is also important in the study of lightning protection.

The klydonograph makes it possible to obtain records of transient voltages as they appear on the systems. It makes a record which gives a direct indication of the polarity and the approximate magnitude of the over-voltage and an approximate indication of the wave front of the surge. (Further details are given in "Klydonograph".)

Both of these devices will give more definite information as to the nature of the surges due to lightning and other sources, and also determine the effect of lightning arresters on reducing the surges.

FURTHER WORK

Some further work to be done is as follows:

1. Standardization of technique for using lightning generators for testing lightning arresters.

2. Determination of voltage-time characteristics of lightning arresters, including rate of discharge and the dielectric spark lag.

3. Statistical data of operating experience on high voltage lines, especially those ranging above about 73 kv., should be gathered and correlated for systems which have one or more of the following conditions that affect the amount of the over-voltages due to lightning and the methods of coping with these voltages:

a. Wires of a given line in a horizontal arrangement.

b. Wires in other configurations.

c. Lines with and without arresters.

d. Line protected by ground wires (See Peek's[3] article.)

e. Lines constructed so that corona acts as lightning arrester.[4]

DUFOUR CATHODE RAY OSCILLOGRAPH

For the first time in the history of the art of lightning arrester protection, it is possible to secure oscillograms showing the voltage, current, and time relations of an electrical transient which may occupy a time of 1/100,000,000 of a second or less. This achievement has been made possible through the application of the Dufour cathode ray oscillograph[5] to the measurement of transient phenomena encountered in the art of lightning protection.

This oscillograph makes use of a beam of electrons moving at high velocities which produce a photographic impression by impinging directly on the photographic film located inside of the vacuum chamber. Since such a beam of electrons can be deflected by both electrical and magnetic fields, it is possible to take volt-ampere curves of any device operating on a transient by super-imposing on the electron stream two fields at right angles, one proportional to the current and the other proportional to the voltage. Such curves may be taken in a millionth of a second and give information on the operating characteristics of apparatus subjected to transients which has not been obtainable in the past.

If the volt-time or current-time curves are desired, it is necessary to supply time axis usually perpendicular to the direction of deflection of the transient. For very slow speeds this may be conveneintly done by the use of a moving film. For high speeds the moving film is impracticable because of the tremendous peripheral velocities involved; therefore, means are provided to move the electron stream at a uniform rate across the stationery photographic film. This is done by the use of a magnetic field whose change of intensity is suitably controlled. The upper limit of velocity, which is about ¾ mile per second, is reached because of the increasing difficulty in timing the unknown transient so as to get the record on the films while the spot moving at the ¾-mile per second rate is traversing a film four in. by five in. in size. The timing accuracy obtained in usual operation is 50 microseconds.

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[2] *Lightning Arrester Application from the Economics Standpoint*, A. L. Atherton, TRANS. A. I. E. E., 1924, page 581.

[3] *Lightning and Other Transients on Transmission Lines*, F. W. Peek, Jr., JOURNAL A. I. E. E., August 1924, page 697.

[4] *The Corona as Lightning Arrester*, J. B. Whitehead. JOURNAL A. I. E. E., October 1924, page 914.

[5] *Studies of Time Lag of Needle Gaps*, K. B. McEachron and E. J. Wade, JOURNAL A. I. E. E., January 1926, page 46.

For phenomena which take place in 50 microseconds or less, the time scale must be greatly magnified, which is accomplished by the substitution of a high-frequency field at right angles to the direction of deflection of the unknown transient. Thus the time taken to cross the film is determined by the frequency of the source and films have been taken using a frequency of 1,000,000 cycles, the time to cross the film being 1/2,000,000 of a second.

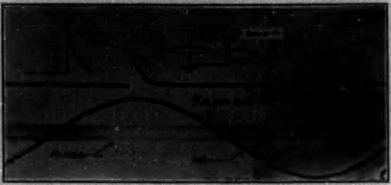

Fig. 1

Frequently it is desirable to draw out the high-frequency wave so as to render the results more intelligible, thus giving a zero line which may be many feet in length on the four-in. by five-in. film. This is accomplished by applying a uniform time motion perpendicular to the motion of the high-frequency timing wave. The transient being studied also produces its motion in the same direction as that of the uniform time motion but is usually very fast in comparison.

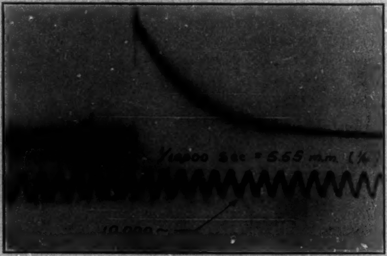

Fig. 2

To show the use of this oscillograph of measuring transient phenomena, a series of films has been prepared of which four of the most typical films are shown here. A surge generator, similar to that used by Peek[6], was used to send current impulse of known characteristics through non-inductive resistance across a part of which were connected the leads of the oscillograph. With such an arrangement, the voltage impulse across the oscillograph leads will be exactly similar in shape to the current surge in the main circuit. The first two films were taken on the same wave front. The first film (Fig. 1) shows the limit of speed of the ordinary Duddell or Blondel oscillograph. The second film (Fig. 2) shows the cathode ray oscillogram of the same transient and the same 10,000-cycle wave, the time being such that 1/10,000 of a second equals 5.5 mm. It should be noticed that no distortion appears in the transient voltage wave such as found in the first film due to inertia effect of the ordinary oscillograph.

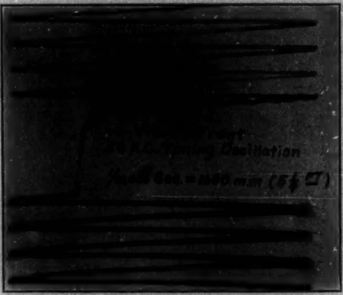

Fig. 3

Fig. 4

The second group of films shows the use of the high-frequency timing wave and the uniform sweeping motion combined; they were taken on the same transient. The first of the group (Fig. 3) used the 50,000-cycle timing wave, which represents a multiplying factor of 1600 compared with that of Fig. 2. When taking the oscillograms shown in Fig. 4, the timing frequency was increased to 300,000 cycles and now it is possible to get an accurate picture of the wave front itself. The transients shown on Figs. 3 and 4 can be replotted on rectangular co-ordinates to eliminate the distortion effect of the timing oscillation on the shape of the wave front; however, such error is usually so small that it is not necessary. It will be noticed that there are high-frequency oscillations on the wave front of the main impulse which are due to reflections in the connecting leads between the oscillograph and the impulse circuit. However, in order to measure these high-frequency oscillations on the wave front, a timing frequency of 1,000,000 cycles would be necessary and for such conditions, the time required to traverse the film once is but 1/2,000,000 of a second. The oscillations on the wave front are of the order of 100,000,000 cycles and with such timing frequency, it is possible to get some idea of the wave shape of some of these oscillations. With Dufour oscillograph a magnification of 30,000 times that obtained with the ordinary oscillograph is possible without disturbing inertia effects.

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[6] *The Effect of Transient Voltages on Dielectrics*, F. W. Peek, Jr., Trans. A. I. E. E., Vol. XXXIV, 1925, page 1857.

As may be readily appreciated, from a study of these films, this oscillograph is destined to be of tremendous value in the study of transient phenomena since it is now possible to depict accurately the operating characteristics of any device under the action of an impulse. Studies which are being made will lead to a determination of the characteristics of all kinds of protection equipment, will give information concerning the breakdown of insulation with very short times of application, and will determine the nature of the overvoltages which appear on transmission and distribution systems.

KLYDONOGRAPH

The klydonograph[7] developed by J. F. Peters is a surge recorder which utilizes the principle of the Lichtenburg figures. When a voltage impulse is impressed on a terminal in contact with the emulsion of a photographic plate between it and a grounded metallic plate, upon development of the photographic plate there will be a figure which by its size indicates the magnitude of the voltage impressed and by its appearance indicates the nature of the voltage, that is, polarity and the approximate steepness of wave front. This instrument lends itself readily to application where a continuous graphic record of transient voltages is desired.

Uses. 1. The use to which the klydonograph is best adapted is the recording of surges on transmission lines. Since the range of voltages which may be applied directly to the klydonograph is from 2 kv. to 20 kv., in order to apply it to high-voltage lines, a potentiometer, or multiplier, is necessary. This may be conveniently accomplished by taking the klydonograph potential from the middle plate of two suitably adjusted air condensers connected between the high-tension conductor and ground.

2. A klydonograph connected as above to give the nature of the surge, and another connected to measure the voltage across a non-inductive resistance inserted in the ground lead of a lightning arrester will give the discharge current of the arrester and the nature of the surge which caused it.

3. The klydonograph may be used to measure indirectly the abruptness of a disturbance. To do this on a transmission line the instrument is connected to measure the voltage induced in an antenna loop. In the laboratory, the klydonograph is connected to measure the potentials across a non-inductive resistance and a concentrated inductance. From these the frequency of the disturbance may be calculated.

4. By the speed of propagation of the figures on the plate it is possible to measure time intervals of the order 10^{-8} of second. Thus it can be used to estimate the time lags of spark-gaps, insulators, etc.

5. Finally, the klydonograph lends convenience to all measurements where heretofore spark-gaps have been used. This is for the reason that the klydonograph will indicate the correct voltage with one trial, while a spark-gap indicates only minimum values.

Results. 1. Extended field tests have added to information on the magnitudes of surges due to lightning switching, arcing ground, etc. On three 120- and 140-kv. systems, three direct strokes of more than 1,000-kv. crest value were recorded in one season. Induced lightning up to 650 kv. was recorded on a 140-kv. system and exceeding 400 kv. on a 66-kv. line. In general they were single impulses, and the few oscillatory records were highly damped. The latter were probably line oscillations. On a 120-kv. system, switching surges with a maximum voltage to ground of 390 kv. were recorded, and were rarely oscillatory.

Fig. 5 shows a section of film taken during an actual test on a transmission line.

OIL CIRCUIT BREAKERS, SWITCHES, AND FUSES

Papers. In 1918, at the Midwinter Convention of the A. I. E. E., Messrs. Hewlett, Mahoney, and Burnham presented a paper on *The Rating and Selection of Oil Circuit Breakers.* Since then much additional experience has been gained in the design and operation of oil circuit breakers. Factory and field tests have been conducted, methods of determining short-circuit current have received further attention, and new decrement curves have been prepared. Preparation of another paper bringing this information up-to-date is very desirable. The three larger manufacturers of oil circuit breakers have under consideration the preparation of such a paper and it is hoped that this paper will be available within the next year.

Standardization. Revised sections of the A. I. E. E.

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[7] *Klydonograph and Its Application to Surge Investigations,* J. H. Cox and J. W. Legg, JOURNAL A. I. E. E., October, 1925, page 1094.

Standards dealing with Oil Circuit Breakers (number 19) and Disconnecting and Horn Gap Switches (number 22) were adopted in June 1925 and are now available. The revised Standards cover these subjects more completely than ever.

As a result of its work during the present committee year, the Protective Devices Committee recommends the following additions to the above mentioned section of the Standards for Oil Circuit Breakers:

a. Rule No. 19-102. Add to present rule—"By rated voltage is meant the voltage from line to line as distinguished from line to neutral."

b. Rule No. 19-104. Add to present rule—"By normal voltage is meant line to line voltage as distinguished from to neutral."

c. In referring to interrupting capacity ratings, the term "arc amperes" is often used, and should be defined as—"The r. m. s. value of the current taken during the first half-cycle of arc between contacts during the opening stroke."

FIG. 5

Definitions of Normal (or Working) Voltage, Normal (or Working) Current, and Interrupting (or Recovery) Voltage should be adopted by the A. I. E. E. A tentative definition of Recovery Voltage, not approved by the Protective Devices Committee, is given here as a matter of interest:

> Recovery Voltage is defined as the maximum peak value, measured from normal zero, of the voltage divided by $\sqrt{2}$ which may occur on the live side of the breaker after the interruption of the arc and before normal voltage conditions are restored.

Attention is called to the following typographical errors in the revised Standards, Sections 19 and 22:

19-63—use "Isolating" instead of "Isolated."

22-55—use "Disconnecting" instead of "Disconnection."

22-59—second line—incorrect reference to "oil circuit breaker."

22-155—second line—incorrect reference to "oil circuit breaker."

Future revisions of the A. I. E. E. Standards should take into account the factors which determine the interrupting duty which may be imposed upon oil circuit breakers. No reference to these factors, or so called "Prescribed Conditions," is made in the revised Standards; however, the available fund of information upon this subject is limited and much further study will be necessary before definite recommendations can be agreed upon. Factory and field tests should materially assist in clearing up this matter. Particular attention is called to a paper by Mr. E. C. Stone on *The Oil Circuit Breaker Situation from an Operator's View Point*, presented at the Annual Convention of the A. I. E. E. at Saratoga Springs in June, 1925 (see A. I. E. E. JOURNAL, July, 1925, page 756). In this paper some of the factors affecting current to be interrupted and recovery voltage are set forth and discussed.

Standardization of interrupting ratings has progressed to the point where definite recommendations have been made for uniform standard interrupting capacity ratings. The recommended steps of rating, based on the standard operating duty (2-OCO), for power house indoor oil circuit breakers at 15,000 volts and below are given in the following tabulation:

Arc Amperes	Rated Volts	Arc Kv-a.
2,500	4,500	20,000
2,000	7,500	25,000
1,500	15,000	40,000
2,500	15,000	65,000
3,500	15,000	90,000
5,000	15,000	125,000
7,000	15,000	175,000
10,000	15,000	250,000
14,000	15,000	350,000
20,000	15,000	500,000
30,000	15,000	750,000
40,000	15,000	1,000,000
60,000	15,000	1,500,000
100,000	15,000	2,500,000

Definite progress is being made toward recommendations of standard steps of interrupting rating on oil circuit breakers rated from 15,000 to 220,000 volts. Standardization of this kind, it is believed, will materially reduce the present extensive lines of breakers and great variety of interrupting capacity and voltage ratings which manufacturers are forced to carry, ultimately leading to reduction in oil circuit breaker costs.

Progress is being made toward recommendations of definition of high-voltage fuse ratings. Steps have also been taken for preparation of a uniform test procedure for testing the interrupting rating of high-voltage fuses, similar to the procedure recently prepared for testing oil circuit breakers.

Research is under way for the determination of relative accuracy of current transformers and shunts to be used in connection with oscillographic records of short-circuit tests; results and recommendations will probably be available within another year.

Interrupting Capacity Ratings. The following relative interrupting capacity ratings for oil circuit breakers have been approved by the N. E. L. A. and the Electric Power Club:

a.	One-unit operating duty...... 100 per cent to Rating varies between limits given with design of breaker.	125 per cent
b.	Two-unit operating duty, two-minute interval (Standard operating duty)................	100 per cent
c.	Four-unit operating duty, two-minute intervals................	70 per cent
d.	Four-unit operating duty, one-half minute intervals................	60 per cent
e.	Four-unit operating duty, no time intervals....	25 per cent
f.	300-unit operating duty, 15-minute intervals...	30 per cent
g.	Four-unit operating duty, successive intervals of 0, 30, 75 seconds................	30 per cent
h.	Four-unit operating duty, successive intervals of 15, 30, 75 seconds................	40 per cent
i.	Three-unit operating duty, one-minute intervals................	70 per cent

1. With reference to operating duties (d), (e), (g) and (h), while there are no known limitations which prevent the general application of these operating cycles to all oil circuit breakers, still in view of lack of operating experience and possible hazard, it is recommended that these operating cycles be confined to breakers having interrupting rating on operating cycle (b) of not over 250,000 kv-a. and having voltage ratings of 37,000 and lower.

2. Zero means no time delay between full open position and start of closing.

3. The present factor which the operators are using for operating duty (e) is 30 per cent, but it is the opinion of the Power Club that 25 per cent is more suitable.

In the above tabulation a unit operating cycle is understood to consist of a closing of the circuit breaker followed immediately by its opening without purposely delayed action.

Oil Circuit Breaker Tests. High capacity testing facilities are now available at the plants of two large manufacturing companies. Plans are also under way for several operating companies to undertake a series of cooperative oil circuit breaker tests, interchanging results among the companies participating, all tests to be made according to recommendations of the uniform test procedure. It is believed that much valuable data as to oil circuit breaker performance, and conditions affecting interrupting duty, will be secured from such tests.

General. Improvements in the details of design have been made by the various manufacturers who are continuing the development of their oil circuit breakers on the basis of different design features as follows:

High-speed contacts,

Explosion chambers,

Multiple contacts in series,

Resistance introduced into breaker circuit to reduce energy released by arc in the breaker,

Various methods of relieving gas pressure.

Armored, or metal clad, switch gear which has been used extensively in Great Britain and Continental Europe has been introduced in this country. The unit of metal clad switch gear is the three-phase circuit breaker with mountings and all auxiliaries. Each unit contains, within itself, the main oil breaker, bus bars, disconnecting devices, instrument transformers, cable potheads, and necessary minor features. All of the live conductors involved in this assembly are either immersed in oil or solidly imbedded in insulating compound and the whole is enclosed within grounded metal casings which form the exterior covering. Disconnection is provided by withdrawal, in a horizontal direction, of the carriage type oil circuit breaker.

The field of truck mounted breakers has been extended and high interrupting capacity outdoor breakers, up to circuit voltages of 154,000, are now being built with this type of mounting.

Working toward a better interchangeability of breakers of the smaller indoor type, single-pole elements are now being mounted on trucks and arranged for installation in a uniformly constructed steel housing; this will serve in place of the usual cell construction and will allow a more liberal interchange of elements and, because of the single- rather than triple-pole element, will permit a smaller stock of spare units to be carried. Each element may be controlled by a separate protective relay, allowing one pole to trip while the other two remain closed, continuing a single-phase supply over the circuit which the switch controls.

Another step toward universality is being made by one manufacturer who is arranging to build certain high-voltage breakers with provision for interchanging manual, solenoid, and motor operating mechanisms.

The application of motor mechanisms has been considerably widened to cover a greater number of breaker capacities and to meet a larger diversity in control circuits as to a-c. voltage, and frequency, and varying d-c. voltage.

Another manufacturer employs high-speed arcing contacts in high-voltage oil breakers. In the 110,000-, 132,000-, and 187,000-volt breakers the arcing contacts are of the high-speed bayonet type. A cam in the guide mechanism of the arcing contacts provides for a straight line motion of the arcing contact latch until the main contacts have parted a pre-determined distance, when the cam releases the arcing tips from the latch of the moving contacts. The release of the arcing tip is followed by a quick return to the normal position. The 25,000-, 37,000-, 50,000- and 73,000-volt breakers employ finger type high speed arcing contacts. The main contacts open at a moderate speed and when they reach a pre-determined separation, the arcing contacts are released at a high rate of speed. The contact parts are so arranged as to take advantage of the magnetic blow-out action of the current-carrying loop of the breaker. Another class of breakers, used for central station service, employs the double tank feature, the purpose being to prevent oil throwing and to provide additional assurance that a tank will not be burned through. Breakers with interrupting ratings up to 1,500,000 kv-a., of the indoor type, are being provided with the resilient double tank and also each pole unit is designed in two sections so as to reduce the area which might be exposed to high gas pressures.

The multiple series break principle for 4, 6, 8, and 10 breaks is being employed on breakers from 15,000 to 220,000 volts.

Considerable interest has been manifested by a number of companies in the interrupting capacity of both high- and low-voltage fuses. Several companies have tested fuses in the 250- to 600-volt class.

In one series of tests on open-link fuses in sizes from 100 to 1000 amperes, with the fuses both initially hot and cold prior to test, it was found that in every case the fuse operation was selective between two fuses of adjacent current rating, the two fuses being in series for each individual test.

A series of tests conducted on renewable and one-time enclosed cartridge fuses indicated that the rupturing capacity of the fuse is largely a function of the length of the fuse element and the volume of the cartridge fuse case. The time for interrupting a circuit was found to be very selective between fuses of different sizes. The melting time of the fuse was a direct function of the thermal capacity of the element and inversely to the heating effect of the short-circuit current.

Tests have been made by a number of companies on high-voltage fuses for protection of transformer banks, sectionalizing lines, and similar service. Standards have been fairly well established on the liquid types of fuses. On the open link expulsion type fuses, a wide variety of results has been secured.

PROTECTIVE RELAYS

In view of the recent publication of the Relay Handbook, this year's report of the relay subcommittee consists principally of a resumé and analysis of the present status and trend of protective relay practise.

As the publication of the Relay Handbook has given to the industry a comprehensive picture of the relay art in detail, this report will refer in detail only to developments which have occurred since the Handbook went to press, and only generally to developments occuring prior to that date.

Present State of Protective Relay Practise. A "relay" in the electrical art is an automatic device whose function is to receive information and initiate action in response thereto. It is estimated that the total number of types of such devices used in the electric power and telephone industries alone is in excess of 60,000. However, this subcommittee, being a branch of the Committee on Protective Devices, deals primarily with relays for the protection of power circuits and apparatus. The function of the relay as a protective device is to receive information of some abnormal condition and initiate action to correct the abnormal condition, or to disconnect the apparatus or circuit involved, from the source of energy.

Probably the oldest form of protection in use on electrical circuits is the fuse. From this developed the so called "overload" relays designed to act in response to excessive flow of current. At first these acted instantaneously, but as power systems, protected by "overload" relays, grew in magnitude and complication, it became necessary to differentiate between relays in different parts of the circuit by introducing the element of time. We therefore have today, as our fundamental system of protection against faults producing abnormal currents, a system depending on overcurrent relays with differing time adjustments, such time adjustment, in general, becoming longer the closer the generating station is approached. This has resulted, with the enormous and complicated power networks now existing, in the appearance of various limitations to the use of this system, due to the fact that the time differentials between relay settings cannot be reduced below a certain minimum on account of the inherent time required for relays and circuit breaking devices to function.

To meet this objection, a second general system of protection against abnormal current producing faults has developed, based upon the principle of balancing out of relay circuits all current except the fault current, thereby causing the protective relays to function in response to the fault current alone.

This general idea has crystallized in the system of protecting individual pieces of apparatus by balancing out of the relay circuits all current except that resulting from a fault within the particular piece of apparatus. There are two methods of applying this principle: first, by means of a series differential connection in which the two ends of a circuit are balanced against each other, and second, a parallel differential system which may be used where a circuit consists of parallel paths which may be balanced against each other.

In addition to systems of protection using current alone, we have a number of systems which depend upon the use of both current and voltage. Some of these systems, such as directional ground relaying systems, use both the fault current and the fault voltage; others, such as directional current and directional power systems, utilize the circuit voltage and total current, and certain other devices, such as the impedance relay, utilize the total current and fault voltage.

It should be observed that we have in the above, in general, two systems of current relaying, one depending upon total current and time for selective action, the other depending upon fault current and location for selective action.

In addition to protection against faults involving current, many other abnormal conditions are now protected against specifically by means of relays, such as abnormal voltage, temperature, speed, etc., single-phase operation of motors and generators, cessation of flow of oil, water and air for lubrication or cooling purposes, and other conditions too numerous to mention. Development of many of these additional protective devices has been largely accelerated by the advent of the automatic station, in which, there being no operator to act in case of trouble, it is necessary to provide automatic devices.

Trend of Protective Relay Practise. The above outline of the types of protective relaying systems now in use, together with the chronological order of their development, indicates certain quite definite trends in the protective relay art. These may be briefly stated as follows:

First: With the development of large interconnected power systems supplied from numerous generating sources of varying efficiencies, there appears to be a growing trend towards the integration of all such sources into one comprehensive system so interconnected and relayed that its integrity shall not be broken by the occurrence of faults.

Second: Since the system of relaying which depends upon differential timing for selective fault isolation has not proved sufficiently flexible to promote the maintenance of the maximum integrity of such systems, there appears to be a strong trend towards obtaining such fault isolation through the use of balanced relay systems, or systems responsive to the location or specific nature of the fault.

Third: There also seems to be a rather strong trend towards the use of what may be called "back-up" protection, or a second line of defense, in the form of a second system of relays to function in case of failure of the first system, applied either to the same circuit breakers as the primary defense, or to others nearer the sources of energy.

Fourth: There also seems to be a strongly developing tendency toward the development and use of devices for the detection of abnormal conditions and approaching faults before such faults actually occur.

Summarizing the apparent trend of relaying practise in general, therefore, it would appear that the tendency is towards the maintenance of system integrity by the setting up of three lines of defense:

1. Relays which will detect and give warning of approaching faults or conditions which may cause faults,
2. Relays which respond to the abnormal conditions resulting from the occurrence of faults to isolate the particular piece of apparatus or circuit at fault,
3. A back-up system (consisting in its commonest form of overcurrent relays) adjusted with such time delay as to function only upon the failure of the first and second lines of defense.

The relay art has played an important part in the evolution of the modern superpower system and has itself evolved in sympathy therewith. This evolution is still going on and must continue to do so. We are confident that it will go on until every requirement for continuous electric service has been met.

Developments During the Past Year. A number of new developments has occurred during the year, most of which are in the nature of detail improvements devised to keep abreast of the evolving art. The following specific developments are noted for the purpose of recording the advances in the art and as information as to new devices available.

1. Speed indicating relay for application to automatic stations. Makes separate contacts at definite per cent under-speed, at synchronous speed and at definite per cent over-speed.

2. Automatic network relay. For the control of alternating-current network breakers. Connects transformers to network when capable of supplying load and disconnects them on reversal of energy flow. Will operate on magnetizing current of a transformer.

3. Ratio differential relay. For the differential protection of a-c. generators and transformers. Tripping current varies with load, allowing the relay to be set for close protection at normal loads. Does not require balancing auto-transformers for transformer protection.

4. Direct-current polarized relay. Has inverse definite minimum characteristics. A complete line is available including over-current, under-current, over-voltage, under-voltage, reverse power, polarized potential, and resistance measuring relays.

5. Duplex impedance relay. A combination of a directional impedance relay and a ground relay in one case.

6. Automatic reclosing relay. Has been modified to permit any desired duty cycle. Time intervals may be varied between five seconds and two minutes.

7. Over- and under-voltage relays. Have been modified by the addition of a voltage adjusting resistor controlled by a pointer and scale. Permits relay to be adjusted for any voltage within a definite range.

8. An over-voltage relay. The same as existing over current relay except that a voltage winding has been substituted for the current winding and certain other minor alterations.

9. A power directional over-current relay, offering power directional protection against ground faults, and against phase-to-phase faults where for any reason single-phase directional elements are preferred to a polyphase relay.

10. A network relay for protecting low-voltage a-c. networks against a faulty distributing transformer or feeder.

11. Power directional relays for sensitive protection against ground faults in grounded neutral circuits.

12. Differential frequency relay consisting of two induction frequency elements arranged in opposition on a single shaft. For application in connection with

two circuits that are interlinked by means of a rotating machine.

13. A phase unbalance relay for use in a polyphase circuit for protection against faults producing an unbalancing of currents in the several phases.

14. Synchronism indicating relay comprising an induction type differential element for preventing interconnection of two systems at one point unless they are already connected at some other point.

15. Motor-operated timing-relay for controlling the time elapsing between certain operations. Intended primarily for use with automatic reclosing of circuit breakers.

16. Auxiliary relay. A plunger type relay particularly adapted to automatic switching for introducing time in starting up and in shutting down machines. Can also be used in any a-c. or d-c. circuit where instantaneous pick-up and long time drop-out are required.

17. Electrically-reset instantaneous over-current and auxiliary relays. Equipped with a solenoid for resetting.

18. Flashing protective relay. Has a heavy winding for connection from d-c. machine frame to ground to protect against flashing or grounding of winding to frame. Instantaneous.

19. Plunger type trip free relay for use with electrically operated circuit breakers to prevent the breaker being held closed with over-current in the circuit.

20. Locking relay plunger type. Primarily for preventing a circuit breaker from opening on an excessively heavy over-current.

21. Locking relay. Primarily for preventing the opening of one circuit breaker due to surge conditions resulting from the opening of another circuit breaker.

Recommendations. Since the efforts of Institute Technical Committees are directed towards research, standardization, and publicity, it is believed that this subcommittee can be of most service this year by suggesting a few subjects on which research, standardization, or publicity appear desirable.

As to research, the following studies are recommended:

A. Study of pilot wire transmission line relay systems employed in Europe, to determine their proper field of usefulness. It is understood that many successful installations are in use and it is thought that American practise may benefit by the investigation thereof.

B. Study of characteristics of current transformers at very high overloads. The characteristics of current transformers at very high overloads are of importance in many relay applications, and the information at present available is somewhat meagre. It is recommended that the manufacturers be urged to investigate this matter and present the information to the industry in the form of papers, or otherwise.

It is recommended that standards be formulated and adopted for the following:

1. Relay operation nomenclature. The selection of terms to define the characteristics of relay operation are very chaotic at the present time, probably no two companies having the same conception of what is required, or using exactly the same terms. In order that material for publication regarding operating experience may have a common language of expression, it is recommended that standard nomenclature for relay operation be adopted.

2. Relay acceptance tests. At the present time there are no Institute standards of overpotential tests, temperatures, etc., for relays. It is recommended that such standards be established.

3. Relay designs and ratings. At the present time there is no purposeful coordination of standard relay designs among the different manufacturers. This fact imposes considerable hardship upon the relay users, in view of the difficulty introduced thereby, of making relay applications involving relays of different manufacturers. It is therefore recommended that, in so far as appears feasible, standard ratings and operating characteristic curves be established for the more common types of relays.

4. Current transformer characteristics. A. Coordination between transformers of different types and voltages: Considerable difficulty arises in the application of balanced protection where transformers of more than one type or voltage rating are involved. It is recommended that in so far as possible the designs of transformers of different types and ratings be so coordinated that differing types can, without serious difficulty, be used for differential protection.

B. Coordination of characteristics as between different manufacturers: It is also recommended that in so far as may be practicable, current and potential transformer designs of different manufacturers of the same general type and voltage ratings, be so coordinated that like types will have like characteristics.

As to papers for publication, due perhaps to the above noted confusion as to nomenclature for relay operation, there has been little published information regarding the operation, successful or otherwise, of existing relay systems. It is therefore recommended that at a future general Institute meeting, an appropriate amount of time be alloted for the presentation of relay operating experience. It is suggested that this presentation might well be made under three general divisions:

Time differential systems, or systems in which selectivity is obtained principally by timing.

Current differential systems, or systems employing various current balancing plans for selective operation.

Voltage differential systems in which the abnormal voltage conditions resulting from a fault are utilized to assist selective operation, such as impedance relay applications.

In the advancement of the above suggested program the following sub-subcommittees have been appointed:

1. On Relay Operation Nomenclature and Experience.

2. On Relay Acceptance Test Specifications and Standards.

3. On Current and Potential Transformer Characteristics.

These sub-subcommittees have not had time to complete their work this year, but it is hoped that they will be continued next year and continue their work to a useful conclusion.

Appendix

REPORT OF SUBCOMMITTEE ON AUTOMATIC STATIONS

Since the presentation of the Automatic Stations Subcommittee Report this year, three points have developed on which is submitted supplementary information, including points as follows:

1. Minimum safe protection of power apparatus in automatic stations.

2. Device function numbers for use in automatic stations.

3. Device symbols for use on automatic station wiring diagrams.

Minimum Safe Protection. In its reports of 1925 and 1926, the subcommittee presented its recommendations for the protection of synchronous converters of railway, Edison, and industrial types; synchronous motor generators of Edison and industrial types; synchronous generators (in automatic hydroelectric applications) and synchronous condensers. These recommendations have been tabulated and are presented herewith. It is urged that they be transmitted to the Committee on Standards with the recommendation that they be accepted and incorporated in the Standards of the Institute.

Device Function Numbers. During the year 1924 the subcommittee submitted a set of device function numbers for use in automatic stations. Through the efforts of the subcommittee, this was accepted by the principal manufacturers and users and is now almost universal. This idea has practically standardized itself, but it is felt that these device function numbers should be transmitted to the Committee on Standards, who should be urged to accept and incorporate them in the Standards of the Institute.

Device Symbols. The subcommittee also submits a set of device symbols for use on automatic station wiring diagrams. These have just been accepted by the two principal manufacturers and will be used by them in the future. It is urged that these wiring diagram symbols be transmitted to the Committee on Standards for acceptance and incorporation in the Standards of the Institute.

WALTER H. MILLAN,
Chairman.

MINIMUM PROTECTION
FOR POWER APPARATUS IN AUTOMATIC STATIONS

Protection	Synchronous Converters			Syn. Motor Gen.		Gen.	Cond.
	Rwy. 600-volt	Edison 250-volt	Mining & indust.	Edison 250-volt	Mining & indust.	Synch. hydro.	Synch.
A-C. Under-voltage	x	x	x	x	x	x	x
A-C. Over-voltage						x	x
Incorrect Polarity	x	x	x				
Single-Phase Starting	x	x	x	x	x		x
Single-Phase Operation						x	x
Loss of Field, A-C. Machine	x	x	x		x	x	x
Loss of Field, D-C. Machine				x	x		
D-C. Reverse Power	x	x	x	x	x		
D-C. Overload	x	x	x	x			
Excess Temperature (Sustained Overload)	x	x	x	x	x	x	x
Imperfect Start (Lock-out)	x	x	x	x	x	x	x
Machine Overspeeding (Lock-out)	x	x	x	x	x	x	
Severe A-C. Overload (Lock-out)	x	x	x	x	x	x	x
Grounding Protection (Lock-out)	x	x	x	x		x	x
Excess Bearing Temp. (Lock-out)	x	x	x	x	x	x	x

General Power Application

Annual Report of Committee on General Power Application[*]

A. M. MacCUTCHEON, Chairman

To the Board of Directors:

Continuing the policy of last year the committee organized its work along the following lines:

1. To secure and present papers covering general power applications not previously on record before the Institute.
2. To study the field of general power application with a view to indicating such parts of the field as are not now adequately served and to point the way toward future developments.
3. To summarize in the Annual Report the progress in General Power Applications during the past year.
4. To present a bibliography which may be referred to by those who desire more complete and detailed information.

The program of the Regional Convention at Cleveland was largely prepared under the direction of this committee. The subject of sectionalized electric drive as applied to paper machines was presented in a series of three papers. Electric refrigeration from the standpoint of the refrigerator manufacturer and the Central Station was discussed in two papers.

The committee has requested the assignment of a session at the 1927 Winter Convention and has already arranged for a paper on a-c. elevator motors and a paper on sectionalized group drive applied to the manufacture of steel wool. It is suggested that continued attention in future years be given to this subject of sectionalized group drive where the different motors on the group or train must be synchronized in speed. In the discussion of the committee report it is hoped that other subjects will be suggested, indicating those fields of application in which the members of the Institute will be most interested.

This year the committee can do no more than initiate a consideration of such of the industrial fields as are not now adequately served. Information of this nature must largely come from the user. It is considered that a very real service will be rendered to industry by recording such needs in the proceedings of the Institute. As examples, the manufacturers of refrigerating machines feel that there is a real need for developing a motor for household refrigerators which shall have adequate starting torque and yet be considerably less expensive. This is not a simple problem but deserves the most careful consideration. There is opportunity for rendering a real service to industry in developing a control system for synchronizing the various motors of a group or train drive, which shall have many of the advantages of, and be less expensive than, the extremely accurate control systems developed for paper mill applications. Several suggestions have been received that the central stations increase the possibilities of small polyphase motor applications, where such motors might be less expensive, more reliable, and of better operating characteristics than single-phase motors.

It is requested that all members of the Institute record with the General Power Committee the unsatisfied needs of the electrical industry as such needs come to their attention.

The following have been suggested as suitable subjects for papers. Discussion is invited.

"Power Factor Correction" by the Use of Condensers.

"Texrope" Drive for Short Belt Centers.

Induction Motor Control under Starting Conditions by the Use of Resistance Starters.

Double Squirrel-Cage Rotor Induction Motors.

A-C. Elevator Drive.

Experience with Antifriction Bearings Applied to Electric Motors.

Electric Welding of Long Pipe Lines in the Field.

Isolated Small Power Plants for Standby Purposes to Insure Continuity of Service.

Co-operation between Power and Telephone Companies in Eliminating Problems of Inductive Interference.

A summary of the progress in general power applications can never be complete. This report records only those applications which have come to the attention of the committee. The thanks of the committee are extended to Allis-Chalmers Mfg. Co., American Brown Boveri Elec. Corpn., D. H. Braymer Equipment Co., Can. Gen. Elec. Co., Century Elec. Co., Condit Elec. Mfg. Co., Elec. World Pub. Co., Fairbanks, Morse & Co., General Electric Co., Goodyear Tire & Rubber Co., Haughton Elevator & Machine Co., Howell Elec. Motors Co., Lincoln Elec. Co., National Elec. Condenser Co., Ohio Bell Telephone Co., Pittsburgh Elec. Furnace Corpn., Reliance Electric & Engineering Co., Ridgway Dynamo & Engine Co., Westinghouse Elec. & Mfg. Co., who have rendered most valuable aid in the preparation of the report.

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Presented at the Annual Convention of the A. I. E. E., at White Sulphur Springs, W. Va., June 21-25, 1926.

LUMBER AND WOOD WORKING MACHINERY

Electric Dogs. The application of electric motor drive to the dogging devices and set works of reversing saw carriages is not only a distinctive forward step from a safety standpoint, but also it makes possible numerous operating economies not obtainable with any form of manual or mechanical operation. For electric dogging and tapering devices, high torque motors can be furnished for either a-c. or d-c. operation, although the former is most generally used. The motors have been thoroughly tried out in this service and proved themselves admirably adapted to the work. Due to the reversing motion of the carriage and operation in other than the horizontal position, the motor bearings are of the waste-packed sleeve type. Each dogging and tapering device is equipped with three motors of the same size and all are interchangeable, which greatly reduces the stock of wearing parts needed.

The control for electric dogs is constructed to withstand the heavy shocks and jars of logs slammed on the carriage by the nigger. The contactors for controlling all motions on the carriage may be housed in a steel cabinet, to protect them against damage and prevent the entrance of dust, and the push button stations mounted on the most convenient locations.

Electric Set Works. Although electrically-driven set works have been available for several years, the motors used on this equipment were formerly belted to the mechanism and ran continuously. For this scheme, a clutch was necessary to apply the motive power for setting or receding the carriage knees.

Present practise favors the use of a direct-connected motor which requires less space, eliminates the friction clutch, and operates only during the setting or receding operation. An electrically-operated brake is used in connection with the direct-connected squirrel-cage motor, which acts as a recoil mechanism and makes it unnecessary to use a recoil pawl. The action of the brake is entirely automatic. When the motor is not running, the brake is set but as soon as power is applied to the motor the brake releases.

The control for the direct-connected motor may be either push-button or drum-controller operated, and consists of magnetic reversing contactors designed particularly to function reliably in the rapid operation of the log carriage.

Band Mill Drive. A 9-ft. band mill has been built with the lower wheel shaft direct-connected to a 300-h. p. motor through a flexible coupling. This is the first large band mill ever built with a direct-connected motor drive.

RUBBER MILLS

The synchronous motor has made greater strides in general application in 1925 than probably during any other one year, and in no industry is this more true than that of the rubber industry.

Equipment is now being built for several automatic across-the-line starters with dynamic brake for synchronous motors driving rubber mill lines. These starters are unique in that three-pole, double-throw, oil-immersed, 2200-volt contactors will be used to throw the motors across the line when the contactor is in the upper position. The lower or dynamic braking contacts of the contactor are closed by gravity assisted by spring action. This is the first installation in which a double-throw contactor will be used for both dynamic braking and across-the-line starting.

A synchronous motor has been used to drive a rubber-hog, a machine used to cut up old tires, scrap rubber, etc. We believe this is the first time a synchronous motor has been used on this machine.

BAKERIES

For dough mixer drive up to and including 50 h. p., two-speed, squirrel-cage motors have certain desirable characteristics. When starting, the motors are thrown across the line using the slow-speed connection and automatically transferred to the high-speed connection. This reduces the starting current below that of a single-speed motor, and also makes available two running speeds. Magnetic across-the-line starters are used with the proper automatic sequence obtained so that the motors will always start on the low connection.

PETROLEUM INDUSTRY

Two-speed, wound-rotor induction motors of 20/50-h. p. and 25/65-h. p. capacity with suitable controllers have been developed to meet increased requirements of pumping service. Similarly, to meet the demand of shallow light pumping wells, particularly in foreign fields, a 10/25-h. p. two-speed motor and complete controller has been developed. There are gaseous oil fields where protected-type electrical equipment must be employed in drilling service, and in some instances protected motors and controllers are desired on pumping wells. This condition, together with the fact that any two pumping motors may be used to form a twin-motor, cable-tool drilling equipment, has resulted in developing enclosing parts for the slip rings of the entire line of pumping motors.

On the protected equipment, the motor slip rings are enclosed in a steel housing on the end of the shaft. The housing is designed to meet the specifications of the Bureau of Mines for explosion-proof apparatus. No parts of the motor subject to sparking are left exposed.

The controllers for the protected pumping motors have been made with all arcing contacts oil-immersed. This includes the drum cam contactor-type controller, pole-changing switch, and circuit breaker, which are the only parts presenting a fire hazard due to sparking.

IRRIGATION

Hollow-shaft, squirrel-cage induction motors are at present being extensively used for driving turbine or centrifugal type pumps used in the irrigation fields along the Pacific Coast. The pumps are of small diameter so that they can be lowered in the well casing and submerged in the water. As the water level lowers, the pump is lowered and the shaft is extended to meet the new conditions.

The motor consists essentially of a standard vertical squirrel-cage motor with a hollow shaft, the inside bore being sufficiently large to accomodate, with clearance, the extension of the pump shaft. The motor drives this shaft at the upper end by means of a coupling. The pump half of the coupling is threaded to the pump shaft so that the proper adjustment and alinement of the pump impeller at the other end of the shaft can be made conveniently. The upper ball bearing is sufficiently large to handle the thrust weight due to the motor rotor and the pump impeller and shaft, while the lower ball bearing of the motor acts as a guide bearing for the motor rotor. A cover or hood, which is easily removable for adjusting purposes, encloses the coupling and shaft end and also protects the motor from dripping water without interfering with the ventilation.

The principal demand for these motors is in sizes from five h. p. to 50 h. p. of four and six poles with a tendency to go to higher speeds, that is, two poles and larger horse powers. These motors are so designed that they may be started directly on the line.

Chemical and Electrochemical

In this field one large installation consists of a large, geared, turbine-driven, d-c. electrolytic unit, rated at 2800 kw., 136½ volts. This unit, the largest of its kind, is representative of the most efficient type of electrolytic power unit obtainable, where the power plant and electrolytic-cell room are adjacent.

One copper mining company has purchased 11,500 kw. in synchronous motor-generator sets to supply d-c. power for obtaining copper electrolytically from the leached ore.

A new type of drive for centrifugal extractors in chemical and other industries involved the use of a direct-coupled squirrel-cage motor to secure high starting torque with low power consumption and good performance at full speed. This motor has a high resistance section in the rotor winding built in the form of a fan and located at the top of the motor. The heated air developed is thus expelled without passing through the motor proper. Adjustments in acceleration and speed are obtained by use of double busses, one high-voltage and one low-voltage, the accelerating being done on the high-voltage bus, the motor running on the low-voltage bus.

Electric Furnaces

A 15-ton, electric arc furnace requiring 5000 kv-a. electrical equipment has been applied for melting cold scrap, and another 25-ton electric arc furnace requiring 5000 kv-a. electrical equipment, for refining steel.

A 25-ton furnace for duplexing special steels from open hearths for ingot production is of interest. A large industrial plant has installed two 15-ton furnaces for duplexing blast furnace iron direct. In another plant, open hearth to electric furnace duplexing is being carried on with a 1½-ton electric furnace with most satisfactory results, although this size is somewhat smaller than is usually employed.

Continuous melting and the introduction of permanent molding machines has created a necessity for a continuous supply of hot metal. Batch operation is usually unsuitable for these improved molding methods.

A large percentage of recent electric furnaces put into operation have been installed for high-quality, close-grained, strong, electric-furnace, gray irons.

Of particular interest is a recent double electric furnace installation on the duplexing of malleable iron from cupolas on a continuous process. While this installation is unusual on account of the large tonnage handled, the results already obtained indicate that they will have a marked effect on malleable practise generally.

Since its invention, about a decade ago, the high-frequency induction furnace has been exploited on a basis of operation at approximately 12,000 cycles, this frequency being obtained by means of mercury-arc oscillators. In the last year the use of 500-cycle generators in melting copper, brass, nickel-silver, and similar metals has been quite successful. Sets of 2000 cycles also have been supplied.

Arc furnaces, originally supplied with single-voltage equipments, were superseded by two-voltage ones and during the past year three-voltage furnaces were under construction. These latter use the high voltage for breaking down the scrap, an intermediate voltage for completing the melting, and the customary refining potential of approximately 90 volts.

Elevators

The first full-automatic floor-landing elevator equipment for high-speed passenger service was produced and is now in operation. This equipment automatically brings the car to the floor after the operator has initiated the stop by moving the car switch to the "off" position. This constitutes an important improvement over previous elevator operation in that the car stops at the floor, but does not stop before reaching the floor and then creep to it, nor does it overrun the floor and then creep back.

Tunnel Ventilation

While motor application to ventilation problems is not new, there is one outstanding instance of such to be found in the furnishing of 90 motors, totaling 6000 h. p., to drive the fans which will ventilate the Holland tunnel, a vehicular passageway under the Hudson River connecting New York and Jersey City. The tunnel has a capacity of 2000 vehicles per hour in each direction, is approximately 8500 feet long and is the largest tunnel of this type in the world. The motors are standard wound-rotor induction motors and will be installed late in 1926.

CEMENT MILLS, CRUSHERS, ETC.

There is a tendency toward the application of slow-speed, direct-connected motors to grinding mills. Two 900-h. p., 180- rev. per min. clutch-type synchronous motors have been sold to operate 7-ft. by 40-ft. compartment tube mills. These will be among the largest motors for tube-mill drive ever manufactured.

The application of clutch-type synchronous motors has made rapid progress, particularly in the case of grinding and crushing machinery in the cement and copper industries, but installations in flour mills, on pumps, and on other applications have also been made. This motor has been developed for use in applications where the well-known advantages of the synchronous motor are desired and where high starting torque with low starting current is essential. The motor is a combination of a synchronous motor of standard type and a magnetic clutch arrangement to form a compact self-contained unit. The motor rotor can be revolved independently of the load and after synchronous speed is reached the load can be started by exciting the clutch.

Controllers have been developed for this motor ranging from manual control of both motor and clutch to automatic motor starting and automatic clutch engagement. All forms of controllers have both the clutch and motor control built together as a unit.

While the motors above are of the clutch type, there have also been applied motors of the same rating (900 h. p. at 180 rev. per min.) of the induction synchronous type for direct connection to compeb mills without the use of a clutch.

High-speed, gyratory crushers driven by vertical direct-connected induction motors have been placed on the market by a large manufacturer.

AGRICULTURE

Applications of electricity to agriculture greatly increased during the year. Much attention has been given to this branch both on the continent and in America. Some of these applications include electrical plows, electrical treatment of ensilage, fodder, etc., and general farm and dairy machinery.

ARC WELDING

The past year has seen an interesting development in the field of arc welding. While for some years past there have been occasional applications of arc welding in the building industry, the past year has shown several striking applications which no doubt will lead to its adoption, eliminating riveting in many instances. Tests that have been conducted show conclusively that an arc-welded joint made in the fabrication of structural steel is both stronger and cheaper than a riveted joint of the same class of construction. Besides being cheaper and stronger, the elimination of the nerve-racking din of the pneumatic hammer alone will make this process welcome to the inhabitants of the larger cities where building operations are most active.

The introduction of a new and improved automatic arc-welding head for production work is another of the notable advances in the field of arc welding. This head is much simpler than anything yet produced and promises to be a decided advance in this class of work.

The construction and application of automatic welding machines for the commercial production of tanks, range boilers, etc., was continued and a variety of mechanisms adapted to particular classes of work were constructed.

INDUSTRIAL HEATING PROGRESS

Industrial electric heating made marked advancement in 1925, both in increased connected load and in new developments. It is estimated that throughout the country more than 200,000 kw. was added to central station lines not including arc welding.

The use of electric heat is increasing rapidly in such processes as japanning, core baking, drying, heat treating and annealing metals, glass annealing, melting steel, brass, and the soft metals. The application of small units to process machines is increasing the rate of production in many lines and the use of small devices such as glue pots, soldering irons, and immersion units is becoming standard practise.

New developments were carried on in cloth singeing, sheet-metal, tinning, heating mine drills, and in baking bread and other food products.

PAPER MILLS

A new rotary-contactor regulator for sectional paper-machine drive has been applied to both fourdrinier and cylinder-paper machines and is operating successfully. This regulator is fully described in a paper presented by Mr. S. A. Staege before the Cleveland Section of the A. I. E. E. during the March 1926 meeting. During the coming year a number of paper machines under control of this rotary-contactor regulator will be put into operation. These machines make a variety of papers at maximum speeds varying from 450 to 1400 ft. per min.

A specially-developed d-c. drive for paper cutters has been applied with a resulting appreciable increase in production and uniform quality of product. The motor has a speed range of three or four to one by field control, and the control is designed so as to permit uniform acceleration to a predetermined speed and uniform retardation to the full field speed of the motor. The operation is obtained entirely from the push-button station, and permits the operator to slow down immediately when necessary to remove poor sheets. Also, the cutter can be driven at the maximum speed suitable to the product.

An automatic control equipment designed to govern the synchronous motor driving two magazine pulp grinders has been developed. This control, in case of failure of the power supply to either of the feed motors, continues to function and holds approximately half load on the other stone. It also maintains automatic control on one stone when the other is not in use, and permits of the use of the feed mechanism as a hoist, raising and lowering the magazine when it is necessary to change stones.

In Canada, synchronous motors have been applied with success for direct connection to screens and vacuum pumps of the Nash type. In the past, these have been driven by chain or belt, but, on account of the low-speed, direct-connected induction motors, have not been satisfactory. Higher efficiencies and smaller floor space are the principal advantages.

PRINTING

The standard drive for printing presses is a two-motor arrangement consisting of a large driving motor and a small motor for threading the paper through the rolls at a very low speed. In a certain installation, a special drive was provided which involved the use of one wound-rotor motor for each press instead of the customary two. The necessary speed variation was secured by means of two synchronous motor-generator sets for supplying current to the driving motors, the generators being of low and high frequency. Full automatic control was provided, and by pressing the proper push button any of the driving motors could be made to run at a threading speed of 1/15 to 1/20 of the normal speed.

LAUNDRY MACHINERY

An improved control for reversing washing machines in laundries has been developed. The control consists of a drum driven by the machine which is to be reversed. The machine makes a given number of revolutions in one direction, bringing the drum to the point where the connections are changed and the motor reversed. A special motor is used, and the combined design is such that the motor is brought to rest gently and started again in the opposite direction without shock to the machinery, thus combining the advantages of electric control with the cushioned effect of the belt-driven machine.

STEEL MILLS

A 2500-h. p., 257-rev. per min., synchronous motor has been applied for tube-mill drive. This is apparently the first installation of synchronous motors for this work.

A marked tendency has been evidenced to get back to the use of d-c. motors where variable-speed drive is required.

As regards main-roll drives, the year was one of unprecedented activity in the application of electric motors. One large manufacturer supplied a continuous capacity of 133,000 h. p. in motors for this service.

An unusual equipment was required by the Youngstown Sheet & Tube Company. Instead of driving the several stands of a Morgan Mill from one common line shaft, the stands are combined in several groups, each group driven by a separate adjustable-speed motor. Both a-c. and d-c. adjustable drives are used. The roughing train will be driven by a 3600-h. p., adjustable-speed Kraemer drive, and the intermediate train by a 7500-h. p. similar drive, while the last three stands will each be driven by a 2000-h.p., d-c. motor. The equipment is unique in that the machines can be regrouped in many combinations, providing high efficiencies even on the lighter loads.

The first application of a decidedly large synchronous motor to main-roll drive is being made by the McKinney Steel Company, where a 9000-h.p., unity-power-factor, 107-rev. per min., 6600-volt, 25-cycle unit is used. The motor will drive a 10-stand Morgan continuous sheet bar mill. In addition to being the first large synchronous unit so applied, this motor has a higher continuous rating than any other motor so far applied to industrial purposes in this country and possibly abroad. The complete operation of starting and throwing on the line is entirely automatic and under control of a master switch.

A new and interesting application in the steel industry is the use of individual roll drive on run-out tables, etc. The motors themselves are of a small capacity and of the squirrel-cage induction type, but the installations are interesting in that speed variations of as high as three to one are required in some cases while two to one is very common. The speed variation is accomplished by supplying variable frequency to the motors, this being obtained from a motor-generator set, the motor of which is usually of the adjustable-speed, d-c. type. The alternator voltage follows the frequency changes and the roll motors have the characteristic of constant torque.

RAILWAYS

The Ward Leonard system of control was applied for the first time to the operation of a lift and turnover dumper for railroad coal cars. This dumper can handle a 120-ton car and the cradle hoist requires two 450-h. p., d-c. motors supplied with power by two synchronous motor-generator sets. The Barney haul for bringing the car up to the cradle also uses two 450-h. p. motors with similar control. The dumping is entirely automatic.

A number of five-h. p. motors was applied in the operation of car retarders, a service heretofore secured pneumatically. The function of these retarders is to stop railroad cars at the proper place in a classification yard and the retarding action is secured by wedging the car wheels between two sections of rail which are actuated by the five-h. p. motors. Through remote control the car dispatcher has the entire government of the cars.

The application of oil engines in combination with generators and motors for the propulsion of electrically-driven motor cars has advanced considerably in the past year. New installations have been made by the Canadian National Railway and by the New York Central.

Gas-electric busses, while not new, made large gains in the number of applications.

Mining. A new application of the so called super-synchronous motor is to be found in installations for driving mine ventilating fans. The characteristics of these fans are such that the driving motor must be able to start and bring the fan up to speed under full load.

Tap-Changing Transformers. Considerable publicity has been given to the use of tap-changing transformers arranged for changing ratio under load. This type of transformer has found a special field for electric furnace work and is being manufactured by several companies.

Synchronized Group Drive. Reference is made under Paper Mills to the continued improvement in the highly specialized and accurate systems of control used in paper mill drives.

In the use of the much simpler and less expensive but likewise less accurate "dancer roll" control, experience has shown that it is much easier to keep the various motors in step during the period of acceleration by the use of the Ward Leonard system. As in the paper mill drive, the speed change of the group as a whole is accomplished by changing the line voltage, leaving only the correction in speed of the individual motors to be accomplished by field change.

Several applications of train drive have been made to continuous strip mills where the speed-torque characteristics of the various motors have been so related as to permit the elimination of all corrective rheostatic action. The various motors are held in step by the strip of steel being rolled under tension.

Probably the most novel application of train drive during 1925-26 was in connection with the manufacture of steel wool. Ten motors are kept in synchronism by a novel type of dancer roll which permits the maintenance of an adjustable but uniform tension on the wire which is being cut. This application will be described in detail in a paper to be presented next winter by Crosby Field.

Anti-Friction Bearings. The use of antifriction bearings on motors has increased rapidly. While ball-bearing motors have been supplied for years there is a continued improvement in the method of mounting and provision for disassembly. Standard lines of general purpose motors are now available with roller bearings. While possibly this subject does not come within the scope of this committee, it is referred to as being of unusual interest to the user. The steel mill electrical men have been particularly active in investigating the advantages and disadvantages of this type of bearing.

CONTROL

A time-element magnetic control has been used for d-c. motors. Definite time intervals between the closing of successive accelerating contactors are secured by relays which have a lag in their operation due to the self inductance of a short-circuited coil.

Mine-hoist control in the past has been provided with current-limit acceleration relays. Definite time-accelerating relays have lately been employed with success.

Considerable progress has been made in the development and application of magnetic switches for throwing induction motors directly on the line. Special switches have been designed for dust-tight service in cement mills, etc.

A reversing master switch intended for use with magnetic switch starters and providing undervoltage protection in both forward and reverse directions has been applied for shipper-rod control of machine tools.

Temperature type overload relays have superseded older forms and are now developed for circuits of 2000 amperes.

Manually operated contactors have been applied replacing knife switches on motor circuits. These will safely interrupt ten times their continuous rated current and may be used on either a-c. or d-c. systems.

An electrically-operated pressure governor used in connection with a pressure-regulator control has been applied where water is either not available or not suitable for operating the hydraulic type of regulator.

One manufacturer reports the development of a new type of speed-regulating controller. With this equipment it is possible to maintain a substantially constant low speed on a slip-ring motor independent of load variations. The speed is maintained constant by means of an oil pump geared to the motor. The pressure developed by the pump varies with the speed and this change in pressure causes secondary contactors to open and close, thus regulating the speed. The same device provides automatic acceleration of the motor. It also disconnects the motor from the line at zero speed after it has been reversed to obtain a quick stop.

Inductive time-limit controllers have been applied not only to all types of mill auxiliaries, but also to bucket cranes.

Development was completed on a new type of space heater. In the new design, the resistor is imbedded in an insulating material of the refractory type, which produces a sturdy unit and one which can be overloaded with less danger of burn-out.

There has been placed in operation a new type of liquid slip regulator. In this design, the electrodes are stationary and the liquid level is varied by forcing low-pressure air into and out of a displacement chamber. The regulator is remote-operated, a master switch controlling the primary contactors, the air compressor, and solenoid-operated valve which admits air to the displacement chamber. A second valve provides slip regulation by bleeding air from the displacement chamber. In this way the liquid level is lowered on heavy loads and rises again as the load decreases.

A new line of a-c. controllers has been put on the market utilizing a new type of thermal overload relay. This relay consists of a pair of heaters enclosed in molded insulating material. An overload causes the melting of a special alloy and allows a contact mechanism to open the control circuit. The alloy hardens after the overload is removed and the contact mechanism can then be reset manually. The relay is provided with adjustment for varying the tripping point.

The application of Dean motor-operated valve units has been extended. A smaller unit has been brought out which has found its principal application in oil refineries. An explosion-proof station for this service has also been designed.

Some novel applications of magnetic clutches have been reported. A number of these clutches has been used to drive elevators, feeding gravity conveyors. If material backs up in the gravity conveyor, the circuit to the clutch is opened, thus stopping the elevator and preventing the feeding of additional material into the jammed conveyor.

Marine. Electrical propulsion of naval craft received a decided impetus in the launching of the airplane carriers *U. S. S. Saratoga* and *Lexington.* These are the largest naval craft afloat and each will carry propelling equipment of approximately 180,000 s. h. p.

The application of electrical drive to operate unloading machinery as cited in the case of the *"T. W. Robinson,"* a 13,000-ton Great Lakes steamship, which itself is equipped with turbine electric drive, was a new departure.

In the past, numerous ferry boats have been provided with electric propulsion through fore and aft propellors. In 1925, for the first time, Diesel-electric drive was applied to the operation of a side-wheel ferry boat.

The city of Houston placed the first order for a Diesel electrically-driven and equipped fire boat.

The greatest advance in the marine field is probably the large increase in electrically-driven auxiliaries.

The first successfully operated electric platform hoists were installed on the S. S. *George Washington* and *Robert E. Lee.*

FOREIGN DEVELOPMENTS

The economical viewpoint involved in motor application when considered in connection with low speed driven machines has resulted in the development of a specially designed built-in, oil-immersed, high precision reduction gear, located in the end shield of the motor so as to form a unit. This application has been made to three-phase induction motors ranging from 1/8 to five h. p. and to three-phase and single-phase commutator motors from 5 to 15 h. p. A notable application has been to ring spinning frames in textile mills.

A-c. motors with built-in centrifugal starting device have found an increasingly wide application, being used to cover horse powers from 2½ to 250. The unusually difficult problems encountered in starting and accelerating sugar centrifugals have been successfully met by the use of this type of starting device on vertical motors.

The further development of three-phase and single-phase commutator motors with series characteristics has received a great deal of attention. Likewise, extensive research work has been carried on in the development of 25-cycle, high-powered commutator motors with suitable controls for traction service.

MISCELLANEOUS

The a-c. brush shifting motor was applied for the first time to the operation of punch presses, draw presses, and shears. These machines are ordinarily driven by a high-resistance type of squirrel-cage motor but the ability to adjust speed to the work in hand is very desirable.

The sale of "Texrope" drives increased tremendously. This is not particularly an electrical development but is of interest in the application of motor drive where the motor is placed close up to the work.

An interesting application is that of small induction motors operating on frequencies of 300 to 400 cycles and at speeds of 18,000 to 25,000 rev. per min. to grinding and polishing machinery. Even higher speeds are anticipated.

Synchronous motors of the vertical-shaft type have been applied for driving plate-glass grinders.

Refrigerating motors have been further developed by several companies. There is a real need of a satisfactory low-priced motor for this service, comparable if possible to automobile-starting motors.

Application of power apparatus for household purposes goes on apace. A new phonograph motor has recently been developed and is in use by some of the larger companies.

An improved type of mooring tower erected at the Ford Airport, Dearborn, Mich., afforded a new application for electrically-driven elevator and mooring mechanism.

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SHIPS

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STEEL MILLS

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TEXTILE

Elec. Wld., Aug. 22, 1925, p. 365. Power Factor Correction in Hosiery Mill.

Iron and Steel Production

Annual Report of Committee on Applications to Iron and Steel Production*

F. B. CROSBY, Chairman

To the Board of Directors:

To anyone who has followed closely from its insignificant status at the beginning, on through the first and into the second quarter of this 20th century, the application of electricity in the iron and steel industry, the steady growth to its present gigantic proportions has always seemed inevitable, though none the less amazing.

In some respects, the year 1925-26 has seen all previous records broken in the rate of electrification. The applications of electric power have become so diversified that it renders entirely out of the question more than the briefest outline of outstanding features in the developments of the past year.

I. GENERATION OF POWER

The steam turbine continues to prove the most reliable of prime movers in the steel plant power station. With the increased economies in boiler room practise, with blast furnace gas as fuel, and the further use on the mills of cooling water from the condensers, the production cost per kw-hr. from the turbine driven generator closely approaches that of the gas engine. Coal or oil can be used as auxiliary fuel supplementing the supply of blast furnace gas and thus avoiding delays which may arise due to shortage of gas during periods of castings or furnace repairs.

The thermal efficiency of the gas engine is very attractive and where an ample supply of blast furnace gas is available this prime mover finds warm support. Five new installations of gas-engine-driven blowers, and one gas-engine-driven generator are reported for the current year.

The ideal arrangement appears to be a combination of steam turbines, gas engines and purchased power, with dependence upon the gas engine and central station for the base load while the turbines float on the line carrying the peaks. Under these conditions a large block of power can be purchased at favorable rates owing to the comparatively low maximum demand charge, especially if hydropower happens to be available.

One steel company which is putting through a complete electrification program of old and new mills is building a modern power plant with two 12,500-kv-a., 6600-volt, 60-cycle, turbo generators to supply power for the plant. Steam will be obtained from blast furnace gas and powdered-coal-fired boilers designed for 300-lb. pressure at 180 deg. superheat.

There is no question but that blast furnace gas should be used as efficiently as possible and wherever possible, before any other fuel is considered. Considerable economies can yet be made in the use of gas for heating blast furnace stoves. This will leave more gas available for the generation of power, either through gas engines or boilers and turbines, for the production of steam for plant uses, or when mixed with coke-oven gas, for use in heating furnaces throughout the plant.

Coke-oven gas finds a ready application in practically every heating, reheating and annealing furnace operation in the plant, but where a large city offers a ready market, it is often necessary to restrict its use in the steel plant to effect an economic balance between city and plant consumption.

The steel industry is still looking to the central stations for an answer to the question of pulverized versus solid coal. In this field the opinion is prevalent that powdered fuel research and application are in the incubation stage and that developments in the near future may revolutionize present standards of boiler practise.

II. DISTRIBUTION

Many steel plants have reached and passed the economic limit of 2200 volts for distribution. The large powers often required result in prohibitive current values at this voltage. This is reflected in excessive costs of protective oil switches and line losses even in the relatively short transmission distance from the load center to the numerous mill motors.

A distribution voltage of 6600 has become quite usual, so that only in the smaller plants, and in some cases where power is purchased, is 2200 volts standard. The increasing loads in the larger plants are taxing even the 6600-volt systems, but, so far, only one plant is reported as having main roll motors fed directly from a 13,200-volt system.

III. MAIN ROLL DRIVE

Many of the mills laid out previous to the advent of the steel mill motor, now, each year, are reaching a point where, because of competition with recent equipment, it is necessary to throw them out or re-build them so far as possible along modern lines. The first consideration is usually the replacement of the original engine by a suitable motor and control.

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A. G. Pierce,
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Presented at the Annual Convention of the A. I. E. E., at White Sulphur Springs, June 21-25, 1926.

One electrical manufacturer reports the sale of an aggregate of 33,270 h. p. of motors replacing engine drives during the past year. During this period the company has installed and placed in operation 31,000 h. p. (continuous rating) of reversing-mill motors and has taken orders for six additional reversing equipments aggregating 24,600 h. p. Three of these six units are replacing engine drives of existing mills, while two are for new mills which are to replace engine driven mills.

Notable among the reversing equipments installed during the year are two 7000-h.p. motors, the largest single armature machines thus far built for this service. An 8000-h. p. unit of slower speed but nearly 50 per cent larger in physical dimensions is, however, nearing completion in the factory.

Developments in the frequency converter type of alternating-current, adjustable speed drives provide for operation at constant torque above and below synchronism. This method of speed adjustment requires only two rotating machines,—the main motor and the frequency converter,—both on the same shaft. The slip energy of the induction motor is changed to line frequency in the frequency converter and returned to the line through speed adjusting transformers. Three units of this type, one of 770 h. p. and two of 1600 h. p. each, are now being built.

A 5000-h. p., 99-rev. per min., 60-cycle, induction motor built for a continuous billet mill drive is of interest particularly in that it is designed for 13,200-volt operation. Heretofore, except in one instance (see Report for 1925) main-roll motors have not been built for voltages above 6600 as it was thought that insulation troubles on higher potentials would be aggravated by the dirty atmosphere and conducting dust usually found in steel plants.

The necessity of maintaining constant speed relation between the several drives of modern tandem mills has brought about the development of a new method of control for compound wound, adjustable speed, direct-current motors. This involves a series exciter and a variable potential field rheostat mechanically connected to the main shunt field rheostat, with resistance so proportioned that the series excitation is of the correct value to give very nearly zero speed regulation at any speed setting. This method of control is being applied to a 10-in., semi-continuous, merchant mill in three sections which are driven by 1000-, 1200- and 2000-h. p., d-c. motors respectively.

Another electrical manufacturer reports unprecedented activity in its steel mill business for the past year with an aggregate h. p. rating of mill motors sold from 1st June, 1925 to 1st May, 1926, of 102,910 h.p. This company also reports placing in operation a 4000-h. p. reversing drive in which the single motor is supplied from a fly-wheel motor-generator consisting of two 750-volt generators permanently paralleled, a scheme first suggested by them in 1922 and now generally adopted.

This same electrical manufacturer also reports a new tonnage record for electrically-driven blooming mills made by a 40-in. mill, driven by one of its reverse equipments; namely 90,175 tons of ingots rolled in one month. A repeat order duplicating the electrical equipment was obtained for a 46-in. mill, using a motor rated 7000 h. p., 50 to 100 rev. per min.

Another of its installations of especial interest is a 9000-h. p. unity power factor, 107-rev. per min., 6600-volt, 25-cycle synchronous motor which, from the standpoint of continuous h. p. capacity, is the largest motor of any type used for industrial purposes in the United States or, so far as can be learned, in any other country. It is also notable in that it is the first really large synchronous motor to be used for driving a rolling mill. The motor may be started, stopped, or reversed from a master switch exactly the same as any other type of rolling mill motor. The Korndorfer system of control is used and the complete operation of starting on the low voltage tap of the auto-transformer, switching to a higher tap, applying field at the correct time, and throwing on the line, is taken care of automatically. In the design of the motor, particular attention has been given to obtaining very good starting torque characteristics. The excitation for the motor field is derived from a separate motor-driven exciter.

One of the best examples of motor drive for so-called tandem mills was put in operation about the first of the year. Practically every stand is driven by an individual direct-current motor, furnished with power from several synchronous motor-generator sets; a wide range of speeds is obtained by a combination of generator voltage and motor field control.

The aggregate capacity of the several motors driving this mill amounts to 15,850 h. p. (40 deg. cent.) The electrical apparatus is located in the best possible surroundings, emphasizing the growing importance which the mill operators are assigning to the electrical part of their equipment.

While the cost of such an elaborate drive is relatively high, the equipment is, on the whole, very economical; the flexibility and the wide speed range permit of rolling on this mill a great variety of products which would otherwise require at least two separate mills of a less flexible type.

The following tabulation includes only main mill motors on a continuous rated basis, in units above 300 h. p. as reported by the three principal electrical manufacturers in this country, up to 1st May, 1926.

	1923	1924	1925	1926
60-cycle	452840	478390	543440	586440
25-cycle	475825	490225	538450	582430
Direct-Current	299670	324860	430610	530060
	1228335	1293475	1512500	1698930

IV. AUTOMATIC CONTROL

The year 1925 showed great activity in the application of automatic control to cranes and auxiliary mill machinery. Its dual functions of *control* and *protection* are now universally accepted so that there is now little opposition to its general use. Although there were no new developments of remarkable importance, a most interesting feature was the rapidly increasing use of the inductive time element control system.

This system of control was commercially introduced in 1924 by one manufacturer and shortly afterwards a similar system was brought out by another. The essential features of these systems of control are: (1) the short circuiting of accelerating resistance within a definite time, regardless of the load, and (2) the securing of a time interval purely by electro magnetic means. In the first system, the time interval is secured by making use of the transient voltage induced in a transformer or "inductor" due to changes in the motor current. In the second system, use is made of the slowly decreasing magnetism of shunt-wound relays when their coils are short-circuited. Both systems fill a long-felt need. Theoretically, current-limiting control provides a valuable protection to the motor. Practically, the protection actually afforded in many instances is almost nil since the current-limiting relays must be adjusted to start the motor under the heaviest load which may be encountered. With current-limiting control, therefore, the motor starts slowly under heavy load and quickly under light load, in either case subjecting the motor to excessive current. On the other hand, time-element control imposes excessive current on the motor only when the motor has an excessive load to accelerate and thus gives the motor much better average protection.

Although the larger portion of these installations has been for the control of mill tables and other auxiliary drives, a very considerable number has been installed for the control of open-hearth charging machines, soaking-pit cranes and standard cranes, particularly for the bridge motions. Installations of particular interest include a dynamic lowering controller for a bucket hoist and an ore-bridge trolley controller. Recent installations placed in control rooms entirely separated from the mill proper with control panels mounted in continuous switchboards, resistors mounted overhead in tiers on structural supports, with foot walks between, and with ample provision for ventilation, mark the growing appreciation of the fact that proper and substantial installation contributes probably as much as first class equipment to continuity and economy of operation.

A very interesting development of the past year which will surely have an important effect on crane control is the increasing use of roller bearings. One large installation has been made of cranes equipped throughout with roller bearings even including the sheave bearings in the hook block. An obvious effect of these anti-friction bearings is to increase the free running speed of bridges and trolleys and the light hook hoisting speeds. This imposes a greater responsibility than ever before on the control equipment to protect the motors and gearing in plugging and emphasizes the necessity for reliable brakes and limit switches for the hoists. Brakes on the trolley motors, hitherto rather unusual, would appear to be necessary on a trolley equipped with roller bearings. In some cases the hoist control will need to provide some means of limiting the light hook hoisting speed.

Automatic control of the electro-pneumatic type is being applied to the control of two 275-h. p., compound-wound motors in parallel for the drive of a large bloom shear. The cycle for this machine requires a complete operation in about six seconds and the pneumatically operated contactors of the large capacities involved are much faster and more positive in action than magnetic contactors of similar capacity. The electro-pneumatic contactors are assembled in steel frame work, and make a more compact and sturdy controller than the usual contactors mounted on slate or other insulating panels.

V. AUTOMATIC SUBSTATIONS

The use of automatically controlled substations in steel plants has increased very greatly since the first installation in 1924. An installation recently put into operation provides for the automatic control of two 1500-kw. synchronous motor-generator sets and of twelve 4000-ampere, d-c. feeders. This automatic equipment is in a substation with several large reversing mill drives with an attendant always present, but it was felt that the more reliable operation was of sufficient value to warrant the installation of automatic equipment just the same. Several other installations are being made for the automatic control of motor-generator sets and a-c. and d-c. feeders.

VI. YARD ELECTRIFICATION

The Diesel electric locomotive of which special mention was made in this report last year has grown rapidly in favor as experience has demonstrated its possibilities. Larger sizes up to 60 tons or more are now available and with their proved economy and reliability, appear to have successfully solved the dangerous problems of third rail or overhead trolley in the steel plant.

VII. ILLUMINATION OF YARDS AND BUILDINGS

This past year has seen a marked impetus in the increasing recognition of the value of good lighting in mill buildings and yard. More attention has been given to the specific requirements of different kinds of mills and to the different operations within a given mill.

The kw-hrs. required for lighting range from 10 to 15 per cent of the total power load and are an appreciable item in the total cost. More efficient lighting units have been developed of larger wattage yielding more light per unit current consumption and, incidentally, requiring less time for cleaning.

VIII. Electric Heating

The use of electricity for heating is increasing rapidly in a great variety of applications. One instance of especial interest is the heating of ingot hot tops.

Early in 1925 an equipment was installed for heating the tops for ingots,—primarily of monel metal and nickel. Before using this equipment, 3800 lb. of metal was poured to obtain a 3000-lb. ingot, the waste averaging 20 per cent to 30 per cent. Now 3250 lb. of metal are poured and 85 per cent of the ingots are used with very little cropping while a large percentage of the remaining 15 per cent is worked into commercial products.

The electrical equipment consists of one 1800-kv-a., three-phase transformer designed to supply power to the ingot heating equipment and also to serve as a spare for a six-ton arc melting furnace. Other equipment consists of one 10-per cent, three-phase, air core reactor for use in the 2200-volt circuit, instrument and control panel; also six automatic, single-phase, electrode regulating equipments for controlling the input to the arcs used to heat the top of the ingot.

IX. Electric Furnace

Last December, several units of the first commercial installation of Northrup induction furnaces operating at 480 cycles were put in operation. This particular installation was made for melting nickel silver and similar alloys, but these equipments have many possibilities in the heating and melting of ferrous metals and alloys, several applications being under consideration at present.

The initial installation consists of two motor-generator sets, each delivering 600 kw., single-phase, 1875 volts, 480 cycles, to six Northrup high-frequency furnaces, each generator being driven by a 3-phase, 60-cycle, 2300-volt synchronous motor, with direct-connected exciter. To compensate for the low power factor and obtain best circuit conditions, a bank of capacitors is included with each furnace.

Last fall, the equipment for the first three-voltage-control arc melting furnaces was sold. These equipments are used with six-ton, three-electrode furnaces, one originally installed with a 1200-kv-a. bank of transformers and the two others with a 1500-kv-a. bank of transformers.

Each new equipment consists of a 2550-kv-a. bank of transformers which will deliver full output at 165, 156 or 147 volts and a lower capacity at 138, 120, 95, 85, 80 or 70 volts. Air core reactors of 25 per cent capacity for connection in the 11,500-volt circuit, together with suitable switching equipment, were also included.

The melter has a choice of several voltages to start the heating, probably using 2550 kv-a. at 165 or 156 volts, 2400 kv-a. at 138 for the intermediate voltage and a still lower kv-a. at the refining voltage,—probably 85 or 90 volts, depending upon the furnace equipments.

Marked economies are effected in furnace operation, especially from the standpoint of the life of the side walls and roof, together with possible reduction in electrode consumption.

In conclusion, I wish to give full credit to the members of this committee for their effort as individuals in gathering the data which forms this report.

Electricity in Mine Work

Annual Report of Committee on Applications to Mining Work*

F. L. STONE, Chairman

The Committee on Applications to Mining Work has done very little constructive work during the past year. From the nature of things, the committee can only watch the development of the use of electric power in mines. This use is becoming more general every year and electric motors are supplanting every other kind of motive power. This is due, of course, to their higher efficiency and flexibility of control, combined with the fact that they are just as reliable, if properly designed for mine use, as any of the other forms of drive. I think that no mine operator who is contemplating the opening of a new mine would consider any other method of drive.

In coal mines there always exists the hazard of explosion from gas, and in the bituminous mines from both gas and coal dust. These explosions have destroyed hundreds of lives and ruined millions of dollars' worth of property. The operators consequently are making strenuous efforts to eliminate the initial cause of such explosions.

A small percentage of these explosions has been traceable to the electric arc. After such an explosion has occurred, all traces of its initial cause are usually obliterated. Consequently many explosions have been attributed, for the want of a better explanation, to electric arcs, without any real justification.

The hazard, however, does exist and it is entirely wrong for electrical engineers to attempt in any way to dodge the issue. It is unquestionably unsafe to operate open motors at the face of any mine that may become gaseous with little or no warning.

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G. M. Kennedy,
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A. B. Kiser,
Carl Lee,
John A. Malady,
Charles H. Matthews,
D. C. McKeehan,
W. F. Schwedes,
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Presented at the Annual Convention of the A. I. E. E., White Sulphur Springs, W. Va., June 21-25, 1926.

This condition, which is fully recognized by mining engineers, is putting a new problem to the electrical engineers. They must design apparatus which, to say the least, will be safer than that in use at the present time. The apparatus must be such that when its free space is filled with explosive gas or coal dust in suspension and this mixture ignites, sufficient heat will not be transmitted to the outside of the motor to cause the ignition of explosive mixtures surrounding the motor. The connections to the apparatus must be such that they cannot be tampered with or opened while power is on the motor, thereby drawing an arc. It is not the purpose of this report to outline in detail the many problems that are now up to the electrical engineer to produce apparatus which will be safe, even in gaseous atmospheres.

The United States Bureau of Mines has rendered very valuable assistance to the manufacturers in describing in detail what constitutes safe apparatus, and will issue to manufacturers what is known as approval plates, which means that this particular piece of apparatus for which the plate is issued has been tested by them and has been found to be safe for use in explosive atmospheres in so far as they can ascertain. Quite a long list of such apparatus has been approved by the Bureau of Mines alreadу and this list is increasing continually. The Chairman of this Committee believes that it will not be many years before state legislation decrees that all electrical apparatus used at the face of any coal mine must be similar to that which is now known as "approved apparatus."

Due to the great number of producing bituminous mines, the bituminous industry finds itself in a very precarious position. As a whole, the potential capacity of the industry to produce is very greatly in excess of the demand. Consequently, the price of the product is extremely low, and it would seem as though only the most efficiently operated mines can exist. This condition has led mining engineers to reconsider and intensively study their mining conditions with a view to reducing costs. The result of this study is that several new methods of mining have been evolved all of which require electric motors in some form or other. We have several successful mechanical loading devices where the coal is loaded mechanically into the cars after being shot. We have other schemes where the coal is drawn to the entry in large scraper buckets and there loaded directly in cars. There are also innumerable conveyor schemes which carry the coal from the face to the mine car, the conveyors being more or less portable and so arranged that they can follow the face as it recedes.

The electrical equipment for these entirely new devices does not present any very serious problem beyond the proper selection of motors to meet the conditions indicated.

In the effort to reduce further the costs at the mines, we find more and more fully automatic substations being installed for supplying direct current and also fully automatic pumping stations. We find the electrical engineers of the coal mines watching their peak demands and installing meters much more freely than in the past.

The electrification of power shovels continues at an ever increasing rate, the general scheme being the use of Ward Leonard controlled generators, using a motor and generator on each of the motions.

The metal mines have been fairly active this year, and electrification of these mines has proceeded along standard and well-defined lines.

Marine Work

Annual Report of Committee on Application to Marine Work[*]

L. C. BROOKS, Chairman

To the Board of Directors:

The work of the Committee on Application to Marine Work for this year has been very largely a continuation of the work of the last year, in connection with the compiling of the Revised Rules, which we hope will be ready for publication during the present summer.

As has been the case in the past several years, the work of this committee has been divided into sections, each in charge of a subcommittee, including propulsion, power apparatus, interior communication, fixtures and fittings, wire and cable, radio, historical, and editing. Each subcommittee prepared revisions of the Marine Rules applicable to their work, which revisions were discussed and approved by the main committee, and afterwards turned over to the editing committee for final compiling.

The work of these subcommittees is to be commended, especially from the fact that there are many avenues of thought which needed to be correlated into a unified final decision by the main committee. The entire revision is now in the hands of the editing committee, and it is anticipated that it will be ready for publication very shortly.

At the present time the marine industry is not active, of course, but due to the fact of more stringent requirements of insurance, it is very important that these Revised Rules may be issued as soon as practicable, in order that the United States may have some definite standard as a basis.

A. I. E. E. Marine Rules are now accepted as standard, as far as applicable, by the American Bureau of Shipping, and nearly all of the naval architects and shipbuilding companies.

In addition to the work on the Marine Rules as above mentioned, there were several committees consisting of other lines of thought that were allied with the marine work, as follows:

Personnel Committee, working in conjunction with the Department of Commerce and new commanding officer in charge of the steamboat inspection service; also working in conjunction with the law officer of the Shipping Board. There is considerable optimism that some definite results will be obtained in connection with the proper recognition of electrical engineers on board ships.

This committee has also given assistance in connection with the Department of Commerce, which is endeavoring to establish a uniform practise in fire alarm laws, which is a very important subject in connection with marine navigation. This phase of the question will include also electrical systems or devices in the form of automatic steering arrangements, navigating instruments, sounding machines, etc.

In connection with the committee as a whole, it is with regret that we chronicle the death of W. F. Meschenmoser of the Russell & Stoll Company. Mr. Meschenmoser has been for many years an active member of this committee and chairman of the subcommittee on fixtures and fittings; his work will be greatly missed in future activities and meetings of this committee.

Also, it is reported with keen regret that Mr. Maxwell W. Day, who has been a member of this committee since its inception, has found it necessary to retire from all business connections, and resigned from the committee early in the year. Mr. Day's personality and valuable technical experience have been a great asset to this committee through its entire life, and he has been greatly missed.

During the past year one paper has been presented at the Pacific Coast meeting, on the subject of electric propulsion. No doubt the coming year will see other papers prepared for presentation at some of the Regional meetings.

As to the work of the future, it is believed that the work of this committee will be very important in keeping these rules up to date, to conform to changes that develop with the advance of time and the development of the art. This will apply especially to interior communication apparatus, wire and cable, and personnel. It is also believed that considerable work is possible and desirable in connection with standard approval of fittings.

A very important factor in connection with the work of this committee is the cooperation between the work of this committee and other committees covering the manufacture of apparatus and appliances. The application to marine work requires certain refinements and rigid construction not necessary in ordinary apparatus, and for that reason the work of the Marine Committee must of necessity cover construction details more than is ordinarily considered the function of other industrial committees.

In conclusion, the chairman wishes to thank the committee as a whole, and the chairman of the subcommittees in particular, for their good work during the past year, and the results which they have obtained.

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[*] Committee on Applications to Marine Work:

L. C. Brooks, Chairman
H. F. Harvey, Jr., Vice-Chairman
J. S. Jones, Secretary

R. A. Beekman,
J. F. Clinton,
C. S. Gillette,
William Hetherington, Jr.
H. L. Hibbard,
Edward C. Jones,
A. Kennedy, Jr.
M. A. Libbey,
E. B. Merriam,
W. F. Meschenmoser,
I. H. Osborne,
Arthur Parker,
G. A. Pierce,
H. M. Southgate,
W. E. Thau,
C. P. Turner,
A. E. Waller,
J. L. Wilson,
R. L. Witham.

Presented at the Annual Convention of the A. I. E. E. at White Sulphur Springs, W. Va., June 21-25, 1926.

Electrochemistry and Electrometallurgy

Annual Report of the Committee on Electrochemistry and Electrometallurgy[*]

G. W. VINAL, Chairman

To the Board of Directors:

The Committee on Electrochemistry and Electrometallurgy of the Institute makes its annual report dealing primarily with a review of some of the outstanding developments within the field of the Committee's work. At the beginning of the administrative year, the Committee was enlarged to include members representing as far as possible the many diverse fields that are included within the scope of electrochemistry and electrometallurgy.

The Committee has endeavored to bring about a closer cooperation between its own work and that of the American Electrochemical Society with which it should be closely identified. The work of our Committee does not overlap that of the Electrochemical Society, and it is apparent that a still greater degree of cooperation between the two organizations would be valuable[1].

In 1925 Mr. F. E. Smith, President of the London Physical Society, advocated the replacement of the present international electrical units by the absolute cm. g. sec. units. This is not a new idea, but it is an indication that we are approaching a time when such a step may be considered possible. The accuracy of the electrical standards must keep ahead of the demands of engineering. Improved facilities and technique for performing experiments within the physical laboratory are making possible a step which is of particular interest within the field of electrochemistry where the transformation of energy in its various forms plays an important part.

The international electrical units upon which the fundamental measurements of electrical engineering are based are defined by standards which are essentially electrochemical. The mercury ohm which serves as the standard for the measurement of resistance involves chemical and electrochemical processes for purifying the materials. The silver voltameter, whose electrolytic deposits serve to define the ampere, is obviously an electrochemical device. The standard cell, whose voltage has been determined by experiment from the international ohm and the international ampere, is essentially a small voltaic cell. Engineers become so accustomed to dealing with volts and amperes and ohms that the fundamental standards for their values are taken more or less as a matter of course. These fundamental standards were re-defined by the International Conference on Electrical Units and Standards which met in London in 1908. They were devised to represent as nearly as possible the absolute units based upon the centimeter, the gram, and the second. It is highly desirable for reasons that are too obvious to require repetition that they should represent these absolute units as closely as possible. With the increasing facilities for experimental work and improved technique, it is now possible to realize by experiment the absolute values of the fundamental units with a higher degree of accuracy than had been obtained prior to 1908. Experimental work is now in progress in the United States to verify the conclusions of experimenters at the national laboratories of England and Germany within the past few years, which have indicated that the mercury ohm does not represent the absolute ohm with as high a degree of accuracy as was previously supposed to be the case. Fifteen years have elapsed since the international experiments using the silver voltameter were made. During that time our electrical units have been carried forward by means of the working standards of the laboratory, that is to say, the wire-resistance standards and the standard cells. Developments in engineering practise are making it more than ever important that the fundamental electrical standards, maintained with the highest possible degree of accuracy, should be in accord with our cm. g. sec. system of measurements. Therefore, a difference between the value of the international ohm and the absolute ohm amounting to five parts in 10,000, together with its effect upon the value for the standard cell, must inevitably give rise to serious consideration.

An outstanding development in electrodeposition during the past year has been made in chromium plating. Although it has long been known that chromium could be electrodeposited, it is only within the past few years that the conditions for its commercial deposition have been defined. There are still many problems requiring study before its general application will be entirely feasible. There are at least twenty laboratories in the country now engaged in the study of chromium plating, and while it is not yet in extensive use, the indications are that it will soon find many important industrial applications. Chromium is a material of extreme hardness and is also notable for its resistance to tarnish and to chemical action. It has been successfully applied to plates used for printing paper currency at the Bureau of Engraving and Printing, and it has materially increased the useful life of these plates. It will probably find application also in other forms of printing where very great numbers of copies are required. It has been suggested also for use on dies and gages. Although the reflecting power of chromium is appreciably less than that of silver, its resistance to tarnish may make it of decided value on reflectors.

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[*] Committee on Electrochemistry and Electrometallurgy:

G. W. Vinal, Chairman
Lawrence Addicks,
Arthur N. Anderson
T. C. Atchison,
Farley G. Clark,
Safford K. Colby,
F. A. J. Fitzgerald,
W. F. Hendry,
Walter E. Holland.
F. A. Lidbury,
Wm. A. Moore,
Charles H. Moritz,
Carl G. Schluederberg,
Magnus Unger,
J. B. Whitehead,
J. L. Woodbridge,
J. L. McK. Yardley.

[1] The committee is indebted to Dr. Fink, Secretary of the American Electrochemical Society, for furnishing certain information used in this report.

Presented at the Annual Convention of the A. I. E. E., White Sulphur Springs. W. Va., June 21-25, 1926.

In an effort to prevent corrosion, research has been in progress to improve the quality of nickel plating and eliminate the pits and pores which have been the chief cause of its failure to prevent the corrosion of underlying iron or steel.

A process recently developed and known as "galvannealing," is said to be an improvement over the familiar processes of galvanizing. It is claimed that the zinc covering is more uniform and flexible.

The protection of iron by cadmium would not be anticipated from the potential relations of these metals as usually given. Experiments, as well as actual service, indicate that cadmium can be used successfully as a protective material against corrosion of iron and steel.

Chromium plating, while not affording an absolute protection against corrosion unless the deposits are unusually thick, may be used in combination with underlying coats of copper and nickle, to afford a high degree of protection. Efforts have also been made to develop electroplating of simultaneous deposits of copper and nickle similar to monel metal. The importance of corrosion research to industry in general is very great and will doubtless assume even greater importance.

The progress of electrothermics during the past few years has been so rapid that publications of even recent date often must be used with caution lest they be misleading as to present practise in melting and heat-treating furnaces. During the past year, the development in the use of electricity for heating and industrial purposes has been rapid, but the idea sometimes expressed that all industrial heating processes are potentially valuable outlets for electrical energy has been considerably modified. In general, engineers are beginning to realize that, while there are many processes which can be done electrically, there is, nevertheless, a choice to be made which depends upon practical and economical aspects of fuel costs and the present state of manufacturing processes.

Soaking pits, electrically heated by resistors, have been found to yield ingots that can be more easily and economically rolled in certain plants than similar ingots from gas-fired pits.

Results obtained by the use of electric furnaces in the production of steel, ferro-alloys, and other operations of a similar nature depend very largely on the control of the electrical energy applied to the furnace. The electric furnace is not an inherently efficient piece of apparatus unless the electric energy is properly applied. This means that furnaces should operate in such a manner that the power is fully utilized with respect to time factor, load factor, and, when alternating current is employed, power factor. Aside from the more obvious advantages of high power factor, some electrochemical operations may be adversely affected by variations in power factor for reasons that are not at once obvious. Current density often plays an important part in metallurgical operations and when the control of the furnace is accomplished by means of current regulators, material variations of power input may occur. Thus, in operating a three-phase furnace, the variations in power factor during the melting-down period may keep the load in a state of unbalance with respect to the phases. It is sometimes observed that one electrode heats up unduly without, however, doing its share of the work. The electrical energy supplied is the important factor from a thermodynamic standpoint in metallurgical operations where a transformation of one kind of energy to another occurs. For both economic and metallurgical reasons, the control of the energy input is of important consideration. Modifications to meet practical conditions have been made in certain plants. Current control has been replaced by watt control, and individual motors for shifting the electrodes have been replaced by a hydraulic system which offers advantages in the accuracy with which an electric balance can be maintained during the period of melting. Some furnaces are now operating on 98 per cent of the energy supplied to the primary of the transformer, the two per cent loss being divided between the transformer, bus-structure, and electrode holders. The unequal power requirements during the various periods of charging, melting, refining, and tapping of steel furnaces have in some cases been nearly equalized by staggering the operation of a group of three furnaces, having one transformer unit and one control mechanism. A more continuous load is thus maintained and the labor somewhat diminished, as measured by the weight of steel produced per man.

Because of the various states of oxidation of the metallic constituents of the ferro-alloys and the losses encountered in the reduction of them, some modifications in the usual method of charging the furnaces have been proposed. By very rapidly raising the oxides and reducing agents to the reducing temperature, the formation of intermediate oxide states is avoided. This is applicable to many types of reduction and is accomplished by a continuous charging of the material in a small stream into the hottest part of the furnace.

Up to a few years ago, the common practise in brass-rolling mills was to melt brass in crucibles, using coal or oil for heating. These crucibles generally held 600 lb., and some held as high as 900 lb of metal.

In the period from 1917 to 1922, the introduction and development of the induction type furnace, particularly in the Naugatuck Valley, met with considerable success, as it gave a better control of temperature for pouring, the metal was better (less contamination from gases, etc.), the metal losses were lower, and the cost of furnace linings was less than the equivalent cost of crucibles. These induction furnaces are rated 60 to 75 kw., single-phase, 25 or 60 cycles, and 220, 440, or 550 volts, and can melt 600 to 900 lb. of heat. Pouring temperatures needed for the general run of production requirements are within the working limits of the lining material used in these furnaces. For certain metals, such as nickel, silver, high copper alloys, etc., higher pouring temperatures are necessary, and difficulty has been experienced with the present lining materials.

Because of this situation, experiments have been carried on with a high-frequency induction furnace, and as a result, twelve of the furnaces have just been put into commercial operation in Waterbury. It is interesting to note that the first experiments with high-frequency furnaces were based on the use of 12,000 cycles; standard type generators were not available for this frequency, and mercury-arc oscillators were used, but these were limited in regard to power output. The experiments referred to above showed that much lower frequencies could be used. The high-frequency furnaces are rated to 100-kw. capacity, single-phase, 480 cycles, 1850 volts. For the twelve furnaces, two special frequency-changer sets, each rated 600 kw., are used. Because of the low power factor at which the furnace operates (10 to 12 per cent), a bank of capacitors, rated approximately 950 kv-a., is used with each furnace. The resultant power factor as measured at the frequency-changer set, is close to unity.

The construction of the furnace is relatively simple; it consists of a crucible which holds 600 lb. of metal, surrounded by an edge-wound copper coil, together with a blower, this assembly in turn arranged in a frame in such a way as to permit of tilting and pouring the metal. These furnaces have been in successful operation since the first of the year and additional furnaces are now being installed elsewhere.

An electric furnace for tempering steel parts and employing a forced convection, for obtaining uniform temperature conditions throughout the furnace, has been described recently. Experimental work is again in progress on a zinc furnace which embodies an electrothermic distillation process.

Other uses for electricity in electrometallurgy that have been noted recently include the electric heating of ingots after casting to keep the metal fluid and prevent piping. Heating for this purpose takes the form of an arc playing on the top of the ingot.

Also there has been an increasing use of electricity for annealing purposes, it is significant that the range of objects which are subjected to an electric annealing process is steadily increasing. Annealing of the non-ferrous metals without oxidation has now become possible. Development of the mechanical operation of furnaces for enameling, to provide a continuous cycle of operation, is desirable.

Efforts are being made to improve the life of resistor products, and developments in the design of some furnaces to permit making repairs without the necessity of cooling the furnace are significant of possible development of continuous operation.

In discussing electrical power for chemical plants, it has recently been pointed out that these are not all dependent upon cheap power such as might be obtained from a hydroelectric development. Many of those which are not thus dependent produce, or could produce, cheap power for their own motor drives as a by-product of process steam. Chemical plants may be differentiated into those which require large quantities of cheap power and those which require large quantities of steam. The former include electrochemical and electrometallurgical plants for electrolytic processes and furnace work, while the latter class include those which require heating at lower temperatures, such as may be obtained from steam. The tendency at the present time in relation to the use of power by chemical plants has been summarized in the following paragraphs.

Where considerable steam is required for heating and process work, the chemical plant should generate its own steam at high boiler pressure and then obtain electrical power for its mechanical drives and electrolytic circuits from that steam before turning it over as exhaust or bled steam to the processes.

Very highly efficient turbo-generator units of medium size are now available.

Where the boiler and prime mover rooms are in close proximity to the electrolytic cell or tank room, the electrolytic power should be generated as d-c. power.

Very highly efficient geared d-c. turbo-generating units are now available.

Where appreciable distance intervenes between the boiler and prime mover rooms and the tank house or cell room, or where appreciable steam is not required for heating and process work, so that the electrical power is purchased, then, in the great majority of cases, the best economy will be obtained by converting the electrolytic power from alternating current by means of synchronous converters.

The inefficiency of the motor-generator set as a means of conversion from alternating current to direct current as compared to the transformer synchronous converter unit, is all the more apparent when the operating conditions of the plant drop below full-load conditions.

The power factor of an electrochemical or electrometallurgical plant is relatively high, owing to the large proportion the electrolytic load at unity power factor bears to the total motor load. It is, therefore, seldom necessary to provide power factor corrective means within such plants to bring the over-all power factor within the usual commercial range.

One of the arguments advanced by those who urge use of cheap hydroelectric power for electrochemical and electrofurnace processes is that these industrial operations afford very high annual capacity load factor. They are, therefore, a type of user who justifies the relatively high investment per unit of electrical capacity that is necessary with hydro plants.

Several papers of interest to electrical engineers were presented at the Chattanooga meeting of the American Electrochemical Society. The feature of the meeting was a symposium on the relation of the electrochemical industry to the production of fertilizers. Recent developments in nitrogen fixation are significant to the engineer, because the power requirements are changing as a result of the work of the chemist. Synthetic ammonia, which is one of the forms of fixed nitrogen, is not supplying a product for fertilizer purposes as may be generally supposed, but rather is displacing ammonia from other sources for refrigeration and chemical use. The arc process for the fixation of nitrogen has found very little use in this country, but research on the atomic states and modes of energy transfer suggest possible new developments in this line. Concentrated fertilizer materials including ammonia, urea, nitric acid, phosphoric acid, and their compounds, were discussed. The power supply and economics of the situation are only a part of the nitrogen problem. Other factors that vitally affect the electrochemical side of this industry are the physical characteristics of the materials produced, their suitability for a variety of crops, transportation charges, and, not least, the reaction of the farmer himself.

The electrical properties of the copper-nickel-manganese series of alloys have recently been investigated with reference to resistivity, the temperature coefficient, and the thermoelectric power against copper and mechanical properties. A paper describing this work was presented before a recent meeting of the Electrochemical Society. It seems possible that resistance alloys superior to those now in common use may become available. This is important to the electrical engineer, particularly in connection with instruments which may be subjected to wide variations in temperature.

The development of radio has stimulated the production of batteries, including both dry batteries and storage batteries. A new form of caustic-soda battery which requires only the addition of water has also appeared. The battery industry has watched with some concern the increasing efforts to provide battery eliminators for radio sets, but as yet it seems probable that little of inroads on the battery industry has been made. Coincident with this, there has been a marked improvement in the performance of certain types of small rectifiers. The life of the aluminum rectifier has been greatly increased and it is now used as one form of battery eliminator, and, together with several storage cells, makes possible the operation of radio sets from a-c. power. Also, other types of small rectifiers for trickle charging of storage batteries have been developed.

Chlorine, which is a product of the electrochemical industry, was discussed at length in a symposium on the subject held by the Electrochemical Society at its recent meeting in Chicago. Papers included the economics of chlorine production, discussion of transportation problems, the chemistry of bleaching powder, and other uses for it.

Synchronous Machines

I—An Extension of Blondel's Two-Reaction Theory

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Associate, A. I. E. E. — Associate, A. I. E. E.

***Synopsis.**—Blondel treated salient pole machines by resolving the fundamental space component of m. m. f. along the two axes of symmetry—the direct axis of the pole, and the quadrature axis between poles. Using this idea, and applying harmonic analysis, Blondel's theory has been extended in the present paper to a comprehensive system of treatment, in which the effect of harmonic m. m. fs., as well as the fundamental, and also of field m. m. f. in the quadrature axis, as well as in the direct, have been taken into account.*

It is shown that the "armature leakage flux" which causes reactance voltage drop in synchronous operation, comprises all fluxes, due to armature currents, which generate fundamental voltage, except the space fundamental component, the latter constituting the total flux of "armature reaction."

Impressing upon the variable air-gap permeance those space harmonics of m. m. f. which are due to the fundamental time component of current and which therefore rotate at various fractional speeds, produces odd space harmonics of flux rotating at many different speeds and in opposite directions. Some of these, listed in Table I, produce fundamental voltage, but most of them generate time harmonics. The former, which are reactive voltages, are only those of the nth space order rotating at one nth speed—that is, those which correspond in space order and speed to the harmonic m. m. fs. The corresponding reactances are definitely defined in Appendix C in terms of permeance coefficients, and means are outlined for quantitative determination of such coefficients from graphically constructed field plots. Although strictly there are as many field plots required as there are significant m. m. f. harmonics, an approximation, developed in Appendix B, is given in which only one plot is necessary, other permeance waves being derived therefrom.

It is shown that only the average term and the second space harmonic of the permeance series affect the fundamental voltage. Hence, unless it is required to calculate the harmonic voltages, only those two terms of the permeance series need to be determined.

In the application of the results, the fundamental voltages thus produced by the armature currents are superposed upon that due to current in the field winding, which latter has been previously treated. This gives the vector diagram, Fig. 19, from which the steady state relations are set down in equations.

In Part II, the steady state angle-power relations are developed, including an interpretation of the "reluctance term" in the power or torque equation.

In Appendix D, the vector diagram for salient pole machines is interpreted in terms of the well-known Potier diagram for cylindrical rotor machines. Also the effect of saturation both on angle-power relation and on the value of excitation required under load is discussed.

Subsequent papers in the near future will present results which have been obtained from the application of the method and point of view here outlined, to the solution of problems relating to abnormal operating conditions of synchronous machines.

INTRODUCTORY

THE ever widening application of synchronous machines and their great importance as a factor in the operation of power systems have created the occasion for an extended study of their behavior under the various normal and abnormal conditions. The results of the investigation comprise not only the determination of such operating characteristics, but also extensions in fundamental theory and in mathematical facilities for applying it. In other words, the existing theory, which, like most theories, has been gradually built up by a refinement here and another there, has been in a sense coordinated in this paper, and extended to a system of analysis which has proven to be competent for most of the present day practical problems relating to synchronous machines. It is the authors' purpose to present the results of the investigation in a series of papers, the present one dealing with the theory relating to steady state, polyphase operation.

The present work is essentially an extension of Blondel's† treatment, and an application of the results to definite problems. His fundamental conception of resolving the armature m. m. fs. and fluxes in salient pole machines into two space components—one in line with the pole axis, the other in quadrature thereto—is the basis of the present study. The fundamental premises are extended, and new constants which are necessary to characterize the performance are defined and incorporated in the equations. As in any engineering analysis, this has involved certain assumptions, carefully defined, which would simplify the equations to a practical degree, but which would not throw the calculated results beyond the limit of reasonable accuracy. That is, the predominating factors which practically determine the characteristics have been included, and those have been omitted which complicate the equations without significantly affecting the calculated result. The final equations are thus relatively simple, considering the nature of the phenomena related.

Lyon‡ has treated the *flux* distribution, voltage and power of a synchronous machine by an interesting application of harmonic analysis to the flux waves which must exist. He starts with a general equation for the complicated resultant flux wave, hence the resulting series contain coefficients which it would be very difficult to determine. However, it is true, as Prof. Lyon says, that it is "useful to know what harmonics may be present even when their magnitudes can not be calculated." The present paper, while

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†Bibliography 11.

‡Bibliography 7.

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applying the same general method of attack, *i. e.*, harmonic analysis, nevertheless starts with the resolution of *m. m. f.* waves, instead of *flux* waves, and takes advantage of the symmetry embodied in salient pole machines, by making the resolutions along the two axes of symmetry,—*i. e.*, applies Blondel's idea. This treatment gives m. m. f. waves spread over definite permeance distributions, thus making it possible to determine the coefficients of the significant harmonics of the flux waves.

Thus several aspects of the problem have been treated during the investigation: an extension in fundamental theory, comprising an harmonic analysis of the phenomena in the air-gap, and the inclusion of exciting field ampere turns in the quadrature axis, the definition of characteristic constants, and thus, to this extent, a basis for their determination; the formulation of characteristic equations; the excitation-voltage load characteristic; angle-power relations under steady and transient states; short-circuit phenomena under steady and transient states, single-phase and polyphase; and mechanical forces and torques due to electromagnetic reactions under various conditions.

The present paper is in two Parts. The first interprets and extends the theory relating to steady state conditions, including definitions of the essential constants; constructs the excitation-voltage-load diagram, and shows the relation of this to the corresponding familiar diagram for cylindrical rotor construction. Part II treats the steady state power-angle relations.

I. General Theory

Percentage Representation of Quantities. It is convenient to express the quantities involved in percentage (as a fraction) of their normal values. While the base of the fraction for each quantity has definite dimensions, the fraction is, of course, a numeric, and its use avoids cumbersome conversion factors in the equations. Nevertheless, the results thus expressed by fewer symbols, are none the less definite. It is believed that many mathematical treatments of engineering problems could be immensely simplified in this way.

On this basis:

Unit voltage is normal rated voltage.
Unit armature current is normal rated current.
Unit power is normal three-phase power.
Unit impedance is that impedance through which normal voltage at normal frequency causes normal current to flow.
Unit field current is that field current which induces normal fundamental, open circuit voltage at normal speed.
Unit space fundamental flux is the flux which rotating at normal speed induces unit voltage.
Unit flux density is the maximum density of the unit fundamental flux wave.
Unit space fundamental magnetomotive force is the resultant fundamental magnetomotive force produced by normal three-phase armature current.
Unit permeance coefficient is that coefficient which, multiplied by unit m. m. f., gives unit flux density.
Unit time is the time required for one electrical radian at fundamental frequency.
Unit angle is one radian.
Unit speed is normal synchronous speed.
All armature voltages and currents are per phase.

Armature Current Phenomena. It has been shown by Arnold and others* that in a synchronous machine, the *space* distribution of *m. m. f.* due to each armature coil can, under certain reasonable assumptions†, be resolved into rotating space harmonics. The superposition of these m. m. f. distributions due to all coils, produces an interesting and highly convenient result—namely, a number of space sinusoids of m. m. f. of different pole spans, rotating at different speeds, some forward, some backward. Thus, for instance, in a balanced three-phase system the fundamental sinusoid alone rotates at synchronous speed with respect to the armature, *i. e.*, with the field poles; the third harmonic vanishes; the fifth, with a pole span of one-fifth, rotates backward at one-fifth of synchronous speed; the seventh, with a one-seventh span, forward at one-seventh speed; the ninth vanishes; the 11th, backward and so on. Stated in other words, for balanced current (sine wave in time) in a three-phase armature, there are a number of different, independent, *space* sinusoids of *m. m. f.*, each rotating at a different speed; and they exist as sinusoids, of course, regardless of the salient pole character of the magnetic circuit of the rotor, so long as the poles are symmetrical. Fig. 1 shows the fundamental and fifth of such a system at one instant of time.

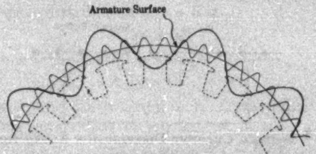

FIG. 1—TRAVELING SPACE SINUSOIDS OF M. M. F.

While the method is here discussed in terms of a three-phase machine, the general theory obviously applies to any polyphase system. In two-phase, the only difference is in the number of harmonics dealt with. For this case, the space harmonics of m. m. f. are as follows:

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*Bibliography 2; and 4, chapter VII.

†The particular assumptions involved in order that m. m. f. with respect to the rotor may be definite at each point of the armature surface are: infinite permeability of the armature iron; no slots; armature current concentrated at points along the armature surface; and symmetrical field structure.

Subject to these assumptions the armature m. m. f. becomes equivalent to a distribution of magnetic potential along the armature surface.

the fundamental, forward at synchronous speed; the 3rd, backward at $\frac{1}{3}$ speed; 5th, forward at $^1/_5$ speed, etc.—all of which may be treated in the foregoing manner. In single-phase, the method of analysis is the same, but the results involve some additional constants which enter the problems of transient state, hence this case will be treated in a later paper.

If there are time harmonics in the current, each one of them will produce its own system of space sinusoids of m. m. f., which will be like that produced by the fundamental term in all respects except that the speed of rotation of each sinusoid will be increased by the order of the time harmonic of current. For instance, if the current in each phase comprise a fundamental and a fifth harmonic, there will be two systems of space sinusoids: one due to the fundamental, as described in the foregoing paragraph; and another in which the sinusoids of full pole span (*i. e.*, fundamental in space) rotate at five times synchronous speed, the third vanishes, the fifth (one-fifth pole span) at five-fifths speed, and so on.

It is possible to superpose the armature m. m. fs.* as in the foregoing, because they are all assumed† to be distributed along the armature surface. However, the field pole m. m. f. is not. It is distributed radially and at some distance from the armature currents. Hence the superposition of the armature and field m. m. fs. is not permissible. But all of the *fluxes* at any point on the armature surface, produced by all of these m. m. fs., can be superposed; likewise, the corresponding voltages.

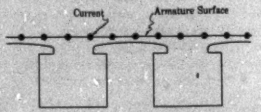

FIG. 2—ASSUMED CURRENT DISTRIBUTION ALONG THE ARMATURE SURFACE

So the problem is to determine all of the space harmonics of flux which are produced by all currents of both the field and armature, and which generate voltage in the armature conductors; then to superpose such of them as are significant, in order to determine the resultant voltage. In addition, there are voltages due to those fluxes which do not appear in the air-gap—namely, slot leakage and end-winding leakage. These fluxes are practically proportional to, and in phase with, the armature current, and thus produce corresponding reactive voltages. The resultant of all of the foregoing voltages is, of course, the terminal voltage.

Turn now to the flux waves produced by the rotating sinusoids of m. m. f. Under steady state operation the fundamental space wave of the armature m. m. f. rotates at synchronous speed with the field poles, at some definite angle of displacement. Hence whatever may be the shape of the flux wave thus produced, it is nevertheless steady in value and fixed with respect to the poles. All of the space harmonics of m. m. f., on the contrary, rotate at speeds different from synchronous speed, and therefore travel over magnetic paths of varying permeance, each wave of m. m. f. producing a pulsating, tufting, rotating flux—rotating, as it were, at a non-uniform velocity with respect to the poles, yet, at the same time, pulsating in magnitude. This is, of course, a complicated phenomenon, but it can be handled analytically as follows:

1. Resolve each rotating space harmonic of m. m. f. into two stationary pulsating waves with respect to the poles.
2. Determine the two corresponding stationary (with respect to the poles), pulsating *flux* waves produced by (1).
3. Combine the flux waves of (2) into *rotating* space harmonics of flux.
4. Determine the component voltages which the space harmonics of flux in (3) generate.

The same method of analysis, of course, applies to the space fundamental of m. m. f. as well as to the space harmonics of m. m. f., the essential difference in result being that the speed of the space harmonics of *flux* due to the space fundamental of m. m. f. is zero with respect to the poles.

Before proceeding to such analysis, it is necessary to resolve the system of three-phase, balanced, sine wave (in time) currents into two component three-phase systems: One in which the current in each individual phase reaches maximum at the instant the axis of the field pole coincides with the axis of magnetization of the phase under consideration (this is called the *direct* component because it produces the direct component of armature reaction) and another in which the current in each individual phase reaches maximum at the instant its axis of magnetization is in line with the axis midway between poles, that is, one-quarter cycle later. This is the *quadrature* component. Each of these complementary three-phase systems has its own set of rotating space sinusoids of m. m. f. as described in the foregoing.

While the general case is treated in Appendix A, the physical significance is necessarily somewhat obscured in such a treatment. Hence it appears desirable to illustrate by carrying through the analysis of some particular harmonic, say the fifth. The procedure is the same for any harmonic, including the fundamental.

The first step is to define the references clearly. Since the resolution of m. m. fs. is to be made with respect to the poles, the space reference will be taken as the axis of a field pole, and time will be reckoned from the instant the axis of magnetization of some particular phase, say phase 1, lines up with the pole axis. Thus, referring to Fig. 3A, the electrical angle α is measured from the axis of pole a in the direction of rotation. The position of the axis of phase 1 with reference to pole a is shown for the instant $t = 0$. In Fig. 3B, the position of phase 1 with respect to pole a is shown for

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*Neglecting the magnetic reluctance of the armature.

†The assumption is that the m. m. f. wave of each coil is approximately a square wave, and hence that the total current in each slot is concentrated in a conductor in the center line of the slot, at the surface of the armature, as illustrated for a particular case in Fig. 2.

$$t = \frac{\pi}{2} \text{ for which } \alpha = -\frac{\pi}{2}$$

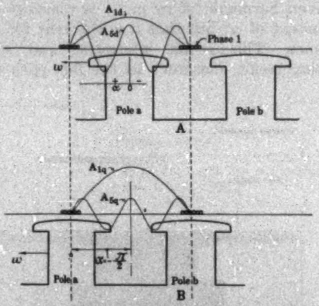

FIG. 3—SPACE AND TIME RELATIONS BETWEEN THE FUNDAMENTAL AND 5TH SPACE HARMONIC OF M. M. F.

A—For direct component of armature current
B—For quadrature component of armature current

Now consider the time-space relations of the m. m. f. waves. By hypothesis the *direct* component of current is maximum at the instant for which the space relations of the m. m. fs. are shown in Fig. 3A, that is, at $t = 0$. That is, the instantaneous value of this current is

$$i_d \cos t \qquad (1)$$

Thus at that moment, the fundamental[*] A_{1d} and the fifth A_{5d} space harmonics of m. m. f. have the space relations with respect to the poles as shown. Be it remembered that the fundamental is fixed in magnitude and position with respect to the pole; whereas the 5th, like the other harmonics, is rotating with respect to the pole.

The *quadrature* component of current, on the other hand, is a maximum at a *later* time, that is at $t = \frac{\pi}{2}$

Thus the instantaneous value is

$$i_q \sin t \qquad (2)$$

and the space relations of its corresponding m. m. f. waves *at that instant* are shown in Fig. 3B. But obviously this position is *not the space position which the 5th had at* $t = 0$, since the wave is rotating with respect to the poles. The *quadrature* fundamental, like the *direct* fundamental, is of course stationary with respect to the pole, and of constant magnitude.

To analyze the 5th it is necessary to obtain an expression for each of the two components of the rotating 5th space sinusoid of m. m. f., namely a_{5d} and a_{5q}. The known characteristics are (*a*) the amplitude, (*b*) the space distribution, or position of the rotating wave at one instant, and (*c*) the speed and direction of rotation. Take the *direct* component first:

(*a*) amplitude, A_{5d}

(*b*) maximum value of wave coincides with pole axis at $t = 0$

(*c*) rotates backward with respect to the armature at $^1/_5$ speed, and with respect to the poles at $^6/_5$ speed, thus at an electrical angular velocity (referred to its own wavelength) of 6 times normal.

The equation for such a rotating wave is

$$a_{5d} = A_{5d} \cos \{ 5 (\alpha + \psi) + 6 t \} \qquad (3)$$

at $t = 0$, it is

$$A_{5d} \cos 5 (\alpha + \psi)$$

But from the assumed boundary condition in Fig. 3A, the space distribution at $t = 0$ is

$$A_{5d} \cos 5 \alpha$$

Hence $\psi = 0$. Thus the wave is

$$a_{5d} = A_{5d} \cos (5 \alpha + 6 t) \qquad (4)$$

Likewise, the 5th rotating m. m. f. wave due to the *quadrature* component of current is

$$a_{5q} = A_{5q} \cos \{ 5 (\alpha + \psi') + 6 t \} \qquad (5)$$

At $t = \frac{\pi}{2}$, it is

$$A_{5q} \cos \{ 5 (\alpha + \psi') + 3 \pi \}$$

which must represent the wave in the particular position shown in Fig. 3B, that is

$$- A_{5q} \sin 5 \alpha$$

Thus

$$\cos \{ 5 (\alpha + \psi') + 3 \pi \} = - \sin 5 \alpha$$

Hence

$$\psi' = - 0.1 \pi$$

and the equation of the wave is, by (5),

$$a_{5q} = A_{5q} \cos \{ 5 (\alpha - 0.1 \pi) + 6 t \}$$

or

$$a_{5q} = A_{5q} \sin (5 \alpha + 6 t) \qquad (6)$$

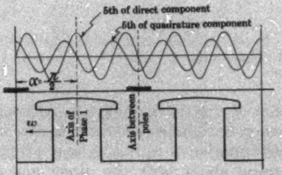

FIG. 4—THE DIRECT AND QUADRATURE COMPONENTS OF THE 5TH SPACE HARMONIC OF M. M. F. AT $t = O$. EQS. (4) AND (6) RESPECTIVELY

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[*] The waves as shown are due not only to current in phase 1, but also to currents in the other two phases, the resultant waves of all the currents having, *at that instant*, the same space distribution as the corresponding harmonics produced by phase 1 alone. The magnitudes are different, of course.

The waves represented by (**4**) and (**6**) are shown in Fig. 4 for the instant $t = 0$.

The stated plan is to resolve each of the two waves (**4**) and (**6**) into two pulsating components, stationary with respect to the poles, and then determine the corresponding stationary, pulsating flux waves. Thus, by direct trigonometric transformation, (**4**) becomes

$$a_{5d} = A_{5d}\,(\cos 5\,\alpha \cos 6\,t - \sin 5\,\alpha \sin 6\,t)$$

which comprises the two stationary pulsating waves

$$a'_{5d} = A_{5d} \cos 5\,\alpha \cos 6\,t \qquad (\mathbf{7})$$

and

$$a''_{5d} = -\,A_{5d} \sin 5\,\alpha \sin 6\,t \qquad (\mathbf{8})$$

Similarly, (**4**) becomes

$$a'_{5q} = A_{5q} \sin 5\,\alpha \cos 6\,t \qquad (\mathbf{9})$$

and

$$a''_{5q} = A_{5q} \cos 5\,\alpha \sin 6\,t \qquad (\mathbf{10})$$

The next step is to obtain the flux waves produced by these stationary, pulsating waves of m. m. f. The product of the m. m. f. at any point along the armature, by the permeance coefficient applying to the magnetic path at that point, gives the flux density at that point. But the permeance* coefficient, as well as the m. m. f., is a function of the space angle α. Although it is different for each harmonic and is in any case a somewhat complicated function of α, it is at least a function which is symmetrical about both the direct and quadrature axes, in all cases of symmetrical salient poles, and can therefore be represented by a Fourier Series of even cosines.† Fig. 5 shows the distribution of m. m. f., permeance coefficient and resultant flux for the zero harmonic; Fig. 6, the same for the fundamental, cos α distribution of m. m. f.; Fig. 7, for fundamental, sin α distribution of m. m. f.; Fig. 8, for 5th harmonic, cos 5 α distribution of m. m. f.; Fig. 9, for the sin 5 α m. m. f.

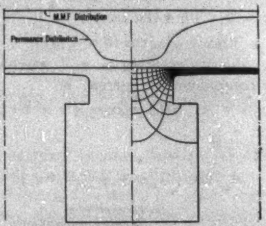


FIG. 5—A—FLUX AND PERMEANCE DISTRIBUTION FOR THE ZERO HARMONIC (CONSTANT M. M. F.), GRAPHICALLY DETERMINED

These flux distributions were determined in one case by graphic method, and in the other by test. Also the permeance distributions were determined in one case from the field plot, and in the other, from the curve in Fig. 10, derived in Appendix B. It will be noted that there is a very reasonable agreement. It is shown in Appendix B that for a uniform air-gap, the permeance coefficient determining the radial component of flux for any given harmonic of m. m. f., is constant (that is, independent of α) but that it is different for different harmonics. This is an interesting and somewhat surprising result, inasmuch as the flux path near the point of zero m. m. f., as illustrated in Fig. 8, would at first appear to be of greater permeance on account of its progressively shorter length as the point is approached. But its width, on account of the change in direction from radial to tangential, also becomes progressively smaller, thus giving constant permeance. By appropriate modification of the result for uniform air-gap, the permeance distribution for salient poles may be plotted, as in Figs. 11 and 12, and analyzed.

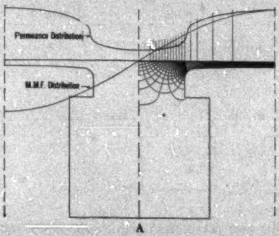
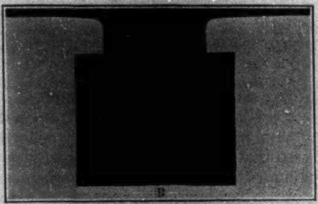


FIG. 6—A—FLUX AND PERMEANCE DISTRIBUTION FOR FUNDAMENTAL, COS α DISTRIBUTION OF M. M. F., GRAPHICALLY DETERMINED



B—FLUX DISTRIBUTION BY TEST

The two permeance coefficient series, one for the cosine, the other for the sine, distribution of the 5th harmonic m. m. f., are respectively,

$$p^{V}_{c} = p^{V}_{0c} + p^{V}_{2c} \cos 2\,\alpha + p^{V}_{4c} \cos 4\,\alpha + \ldots \qquad (\mathbf{11})$$

---

*Question may be raised whether it is proper to consider the m. m. f. and permeance thus distributed, since the air-gap, in the case of salient poles, is extremely variable, and hence the total m. m. f. indicated at any point may not be entirely consumed in the corresponding permeance, that is, the return path from pole to armature may consume more or less than the path from armature to pole. It is permissible because, over a pole pitch, the distributions are symmetrical, thus giving a corresponding return path in a similar position somewhere within the pole pitch.

†Even cosines, because a cycle of the resultant permeance wave is one pole pitch, thus one-half cycle of fundamental space angle.

and

$$p^{\mathrm{v}}_{s} = p^{\mathrm{v}}_{0s} + p^{\mathrm{v}}_{2s} \cos 2\,\alpha + p^{\mathrm{v}}_{4s} \cos 4\,\alpha + \ldots \quad \textbf{(12)}$$

the subscript *c* and *s* indicating cosine and sine respectively, and the numeral v (thus avoiding the appearance of an exponent) indicating that the series is for the 5th space harmonic of m. m. f.

The flux waves are, therefore, obtained by multiplying (**7**) an by (**11**), and (**8**) and (**9**) by (**12**). Thus,

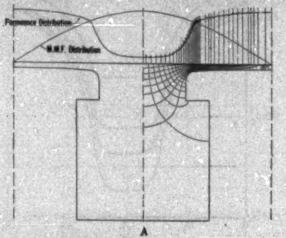
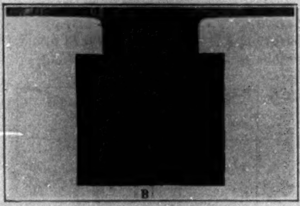
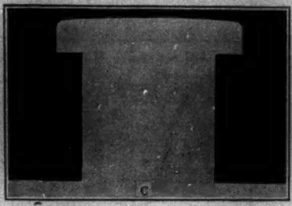


FIG. 7—A—SAME AS FIG. 6, EXCEPT FOR SIN α DISTRIBUTION OF M. M. F.



B—FLUX DISTRIBUTION BY TEST



C—FLUX DISTRIBUTION FOR FUNDAMENTAL M. M. F. OF SIN α DISTRIBUTION, *i. e.*, ZERO M. M. F. AT CENTER OF POLE. THIS IS THE SAME AS *B*, EXCEPT THAT IT SHOWS THE WHOLE POLE.

$$\begin{aligned}\beta'_{5d} &= a'_{5d}\, p^{\mathrm{v}}_{c}\\ &= A_{5d}\,(p^{\mathrm{v}}_{0c} + p^{\mathrm{v}}_{2c} \cos 2\,\alpha\\ &\qquad + p^{\mathrm{v}}_{4c} \cos 4\,\alpha + \ldots)\cos 5\,\alpha \cos 6\,t \qquad \textbf{(13)}\end{aligned}$$

$$\begin{aligned}\beta''_{5d} &= a''_{5d}\, p^{\mathrm{v}}_{s}\\ &= -A_{5d}\,(p^{\mathrm{v}}_{0s} + p^{\mathrm{v}}_{2s} \cos 2\,\alpha\\ &\qquad + p^{\mathrm{v}}_{4c} \cos 4\,\alpha + \ldots)\sin 5\,\alpha \sin 6\,t \qquad \textbf{(14)}\end{aligned}$$

$$\begin{aligned}\beta'_{5q} &= a'_{5q}\, p^{\mathrm{v}}_{s}\\ &= A_{5q}\,(p^{\mathrm{v}}_{0s} + p^{\mathrm{v}}_{2s} \cos 2\,\alpha\\ &\qquad + p^{\mathrm{v}}_{4s} \cos 4\,\alpha + \ldots)\sin 5\,\alpha \cos 6\,t \qquad \textbf{(15)}\end{aligned}$$

$$\begin{aligned}\beta''_{5q} &= a'_{5q}\, p^{\mathrm{v}}_{c}\\ &= A_{5q}\,(p^{\mathrm{v}}_{0c} + p^{\mathrm{v}}_{2c} \cos 2\,\alpha\\ &\qquad + p^{\mathrm{v}}_{4c} \cos 4\,\alpha + \ldots)\cos 5\,\alpha \sin 6\,t \qquad \textbf{(16)}\end{aligned}$$

Expanding these equations, they become

$$\begin{aligned}\beta'_{5d} = \frac{1}{2} A_{5d}\,[&(p^{\mathrm{v}}_{4c} + p^{\mathrm{v}}_{6c}) \cos \alpha + (p^{\mathrm{v}}_{2c} + p^{\mathrm{v}}_{8c}) \cos 3\,\alpha\\ &+ (2\,p^{\mathrm{v}}_{0c} + p^{\mathrm{v}}_{10c}) \cos 5\,\alpha\\ &+ (p^{\mathrm{v}}_{2c} + p^{\mathrm{v}}_{12c}) \cos 7\,\alpha + \ldots]\cos 6\,t \qquad \textbf{(17)}\end{aligned}$$

$$\begin{aligned}\beta''_{5d} = -\frac{1}{2} A_{5d}\,[&(p^{\mathrm{v}}_{4s} - p^{\mathrm{v}}_{6s}) \sin \alpha + (p^{\mathrm{v}}_{2s} - p^{\mathrm{v}}_{8s}) \sin 3\,\alpha\\ &+ (2\,p^{\mathrm{v}}_{0s} - p^{\mathrm{v}}_{10s}) \sin 5\,\alpha\\ &+ (p^{\mathrm{v}}_{2s} - p^{\mathrm{v}}_{12s}) \sin 7\,\alpha + \ldots]\sin 6\,t \qquad \textbf{(18)}\end{aligned}$$

$$\begin{aligned}\beta'_{5q} = \frac{1}{2} A_{5q}\,[&(p^{\mathrm{v}}_{4s} - p^{\mathrm{v}}_{6s}) \sin \alpha + (p^{\mathrm{v}}_{2s} - p^{\mathrm{v}}_{8s}) \sin 3\,\alpha\\ &+ (2\,p^{\mathrm{v}}_{0s} - p^{\mathrm{v}}_{10s}) \sin 5\,\alpha\\ &+ (p^{\mathrm{v}}_{2s} - p^{\mathrm{v}}_{12s}) \sin 7\,\alpha + \ldots]\cos 6\,t \qquad \textbf{(19)}\end{aligned}$$

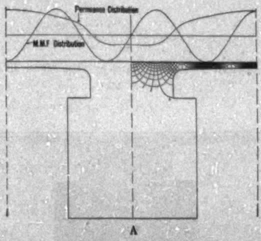
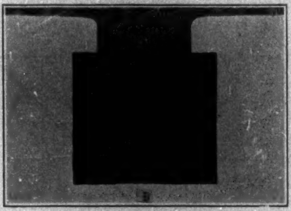


FIG. 8—A—FLUX AND PERMEANCE DISTRIBUTION FOR THE 5TH HARMONIC OF M. M. F. COS α DISTRIBUTION, GRAPHICALLY DETERMINED



B—FLUX DISTRIBUTION BY TEST

$$\beta''_{5q} = \frac{1}{2} A_{5q} \left[(p^{V}_{4c} + p^{V}_{6c}) \cos \alpha + (p^{V}_{2c} + p^{V}_{8c}) \cos 3\,\alpha \right.$$
$$+ (2\,p^{V}_{0c} + p^{V}_{10c}) \cos 5\,\alpha$$
$$\left. + (p^{V}_{2c} + p^{V}_{12c}) \cos 7\,\alpha + \ldots \right] \sin 6\,t \qquad (20)$$

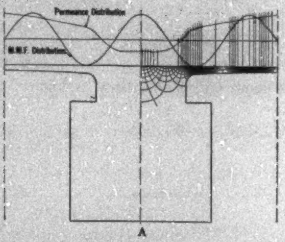
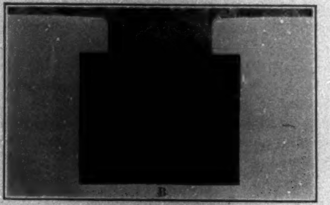


Fig. 9—A—Same as Fig. 8, except for sin α Distribution of M. M. F.



B—Flux Distribution by Test

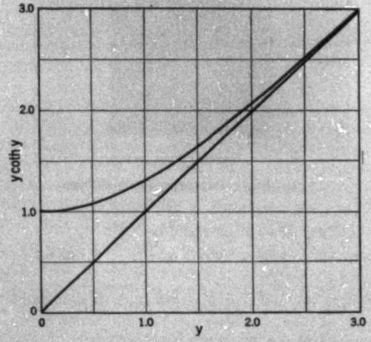


Fig. 10

Thus the stationary pulsating 5th space harmonic of m. m. f. a'_{5d}, impressed on the variable (in space) permeance, produces space waves of *flux* of *all* odd harmonics, including the fundamental, there being a definite progression in the subscripts of the permeance constants for the increasing order of the odd space harmonic.

It will be noted that if the corresponding harmonics of (**17**) and (**18**), and of (**19**) and (**20**) be paired, the result will be that for each space harmonic of the series,

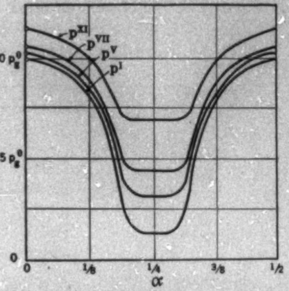


Fig. 11—Permeance Distributions Determined from Figs. 10 and 21

Analysis of these Waves Gives:

$p_0^{I} = 0.66\,p_g^0$
$p_0^{V} = 0.73\,p_g^0$
$p_0^{VII} = 0.79\,p_g^0$
$p_0^{XI} = 0.94\,p_g^0$

$p_2^{I} = 0.44\,p_g^0$
$p_2^{V} = 0.37\,p_g^0$
$p_2^{VII} = 0.33\,p_g^0$
$p_2^{XI} = 0.25\,p_g^0$

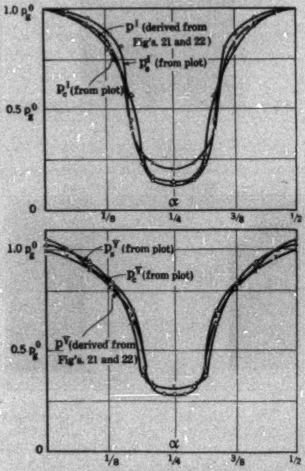


Fig. 12—Comparison of Derived and Plotted Permeances for 1st and 5th Harmonics

there will be two flux waves in space and time quadrature, *but of different amplitude.* Now just as the rotating 5th harmonic m. m. f. wave was resolved into two equal stationary pulsating waves in space and time quadrature, so in the present case the equal components (*i. e.*, two amplitudes equal to the smaller) can be composed into one main rotating wave; and the difference between the amplitudes constitutes a residual, single, stationary, pulsating wave, which can be resolved, according to the well-known scheme, into two equal waves of one-half amplitude, rotating in opposite directions. Thus one of these must rotate with the foregoing main wave, and in space phase with it, and the other alone in the opposite direction. In Fig. 13 the two waves

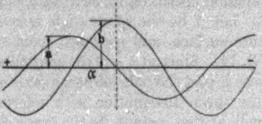


FIG. 13

$b \cos \alpha$ pulsating at $\cos t$

and

$a \sin \alpha$ pulsating at $\sin t$.

constitute two rotating waves as follows:

$$a + \frac{(b-a)}{2} = \frac{b+a}{2}$$

rotating in the direction* of positive α,

and

$$\frac{b-a}{2}$$

rotating in the opposite direction.

Thus equations (**17**) and (**18**) together contain two rotating *flux* waves for each odd harmonic of space distribution, the speeds of rotation being different for all harmonics; hence a corresponding difference also in the magnitude and order of the harmonic voltages thus generated; likewise, (**19**) and (**20**). These flux waves are obtained by pairing the corresponding space harmonics of (**17**) and (**18**), and of (**19**) and (**20**), and expanding them.†

Eqs. (**17**) and (**18**) therefore give:

$$\beta_{5d1} = \frac{1}{4} A_{5d} [(p^{V}_{4s} - p^{V}_{6s}) + (p^{V}_{4c} + p^{V}_{6c})] \cos (\alpha + 6\,t)$$

$$+ \frac{1}{4} A_{5d}[-(p^{V}_{4s} - p^{V}_{6s}) + (p^{V}_{4c} + p^{V}_{6c})] \cos (\alpha - 6\,t) \qquad \textbf{(21)}$$

$$\beta_{5d3} = \frac{1}{4} A_{5d}[(p^{V}_{2s} - p^{V}_{8s}) + (p^{V}_{2c} + p^{V}_{8c})] \cos (3\,\alpha + 6\,t)$$

$$+ \frac{1}{4} A_{5d}[-(p^{V}_{2s} - p^{V}_{8s}) + (p^{V}_{2c} + p^{V}_{8c})] \cos (3\,\alpha - 6\,t) \qquad \textbf{(22)}$$

$$\beta_{5d5} = \frac{1}{4} A_{5d} [\,(2\,p^{V}_{0s} - p^{V}_{10s})$$

$$+ (2\,p^{V}_{0c} + p^{V}_{10c})\,] \cos (5\,\alpha + 6\,t)$$

$$+ \frac{1}{4} A_{5d} [-\,(2\,p^{V}_{0s} - p^{V}_{10s})$$

$$+ (2\,p^{V}_{0c} + p^{V}_{10c})\,] \cos (5\,\alpha - 6\,t) \qquad \textbf{(23)}$$

$$\beta_{5d7} = \frac{1}{4} A_{5d} [\,(\,p^{V}_{2s} - p^{V}_{12s})$$

$$+ (p^{V}_{2c} + p^{V}_{12s})\,] \cos (7\,\alpha + 6\,t)$$

$$+ \frac{1}{4} A_{5d} [-\,(p^{V}_{2s} - p^{V}_{12s})$$

$$+ (p^{V}_{2c} + p^{V}_{12c})\,] \cos (7\,\alpha - 6\,t) \qquad \textbf{(24)}$$

And (**19**) and (**20**) give:

$$\beta_{5q1} = \frac{1}{4} A_{5q} [(p^{V}_{4s} - p^{V}_{6s}) + (p^{V}_{4c} + p^{V}_{6c})] \sin (\alpha + 6\,t)$$

$$+ \frac{1}{4} A_{5q} [(p_{4s}{}^{V} - p^{V}_{6s}) - (p^{V}_{4c} + p^{V}_{6c})] \sin (\alpha - 6\,t) \qquad \textbf{(25)}$$

$$\beta_{5q3} = \frac{1}{4} A_{5q}[(p^{V}_{2s} - p^{V}_{8s}) + (p^{V}_{2c} + p^{V}_{8c})] \sin (3\,\alpha + 6\,t)$$

$$+ \frac{1}{4} A_{5} [(p^{V}_{2s} - p^{V}_{8s}) - (p^{V}_{2c} + p^{V}_{8c})] \sin (3\,\alpha - 6\,t) \qquad \textbf{(26)}$$

$$\beta_{5q5} = \frac{1}{4} A_{5q} [\,(\,2\,p^{V}_{0s} - p^{V}_{10s})$$

$$+ (2\,p^{V}_{0c} + p^{V}_{10c})\,] \sin (5\,\alpha + 6\,t)$$

$$+ \frac{1}{4} A_{5q} [\,(\,2\,p^{V}_{0s} - p^{V}_{10s})$$

$$- (2\,p^{V}_{0c} + p^{V}_{10c})\,] \sin (5\,\alpha - 6\,t) \qquad \textbf{(27)}$$

$$\beta_{5q7} = \frac{1}{4} A_{5q} [\,(p^{V}_{2s} - p^{V}_{12s})$$

$$+ (p^{V}_{2c} + p^{V}_{12c})\,] \sin (7\,\alpha + 6\,t)$$

$$+ \frac{1}{4} A_{5q} [\,(p^{V}_{2s} - p^{V}_{12s})$$

$$- (p^{V}_{2c} + p^{V}_{12c})\,] \sin (7\,\alpha - 6\,t) \qquad \textbf{(28)}$$

The third subscript is the order of the space harmonic *flux* wave. Thus, β_{5q7} is the 7th harmonic space flux produced by the 5th space m. m. f. due to i_q.

Consider the meaning of these waves. Each one has an electrical angular velocity of 6 times normal (referred to its own wavelength) with respect to the pole. The wave moves forward if the sign of t is minus, and backward if it is plus.* Thus, (**25**) comprises two waves, one (the first term) rotating backward at 6 times synchronous speed with respect to the poles, 5 times speed with respect to the armature; the other, rotating forward at 6 times speed with respect to the poles, thus 7 times with respect to the armature. These waves are of full pole span, hence the former generates a 5th harmonic, the latter a 7th harmonic voltage in the armature conductors. Eq. (**26**) likewise comprises two 3rd harmonic space waves, one forward, one backward, with respect to the pole at 6/3 or 2 times speed, thus generating in the conductors a 9th and a 3rd harmonic voltage. Similarly, (**27**) comprises two 5th harmonic space waves which generate a fundamental and 11th. And the waves of (**28**) generate a fundamental and 13th harmonic voltage. Similarly, the waves due to the *direct* component 5th, in eqs. (**21**), (**22**), (**23**) and (**24**) produce corresponding harmonic voltages. It will be noted that the only fundamental voltages resulting from the 5th space harmonic of m. m. f. are those due to the backward rotating 5th and the forward rotating 7th space flux waves it produces. All other space fluxes which it produces give higher harmonic voltages.

---

*If $\cos t$ is associated with $\sin \alpha$, and $\sin t$ with $\cos \alpha$, this would reverse the direction of rotation and also change the sign of one of the waves. This resolution is given merely for illustration of the physical relations. The method employed in the following is purely mathematical.

†They could be obtained also by the scheme used to illustrate the physical meaning of the waves, as in Fig. 13.

It is shown in Appendix B that for the 5th and higher space harmonics of m. m. f., the permeance series for the cosine distribution is approximately—almost exactly—the same as that for the sine distribution. Thus, making the approximation that

$$p^{V}_{ms} = p^{V}_{mc}$$

then, all of the waves due to the 5th m m. f., which produce fundamental voltage are, from (**23**), (**24**), (**27**) and (**28**)

$$\beta_{5d5} = A_{5d}\, p^{V}_{0} \cos (5\,\alpha + 6\,t) \tag{29}$$

$$\beta_{5q5} = A_{5q}\, p^{V}_{0} \sin (5\,\alpha + 6\,t) \tag{30}$$

$$\beta_{5d7} = \frac{1}{2} A_{5d}\, p^{V}_{2} \cos (7\,\alpha + 6\,t) \tag{31}$$

$$\beta_{5q7} = \frac{1}{2} A_{5q}\, p^{V}_{2} \sin (7\,\alpha + 6\,t) \tag{32}$$

What will be the magnitude and phase of the voltages generated by these waves of flux? In general, voltage is, by definition, *minus* the rate of change of magnetic linkages.† Thus

$$e = -\frac{d\,\Omega}{d\,t} \tag{33}$$

The plan is to change the reference of the rotating flux waves from the poles to the armature; then to resolve each wave into two stationary, pulsating components, involving cosine and sine space distributions. The former will produce linkages with phase 1; the latter will not. The linkages thus obtained may then be applied in (**33**).

For illustration, take (**29**). This 5th harmonic wave moves backward at $^6/_5$ speed with respect to the poles, hence at $^1/_5$ speed with respect to the armature, thus giving an angular speed, on its own wavelength, equal to normal. Let γ be the space angle measured along the armature surface, and reckoned from the axis of phase 1, Fig. 14. Since at $t = 0$ the armature reference (axis of phase 1) and the pole reference (axis of the pole) coincide, the space distribution with reference to the armature will, at that instant, be the space distribution with respect to the pole. Thus the rotating wave expressed with respect to the armature is,

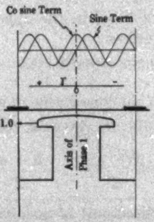


FIG. 14—SPACE REFERENCE ON THE ARMATURE, SHOWING THE POSITION OF THE POLE AND FLUX WAVES AT $t = 0$

$$\beta_{5d5} = A_{5d}\, p^{V}_{0} \cos (5\,\gamma + t) \tag{34}$$

Resolve this by expansion into

$$\beta'_{5d5} = A_{5d}\, p^{V}_{0} \cos 5\,\gamma \cos t \tag{35}$$

and

$$\beta''_{5d5} = -\,A_{5d}\, p^{V}_{0} \sin 5\,\gamma \sin t \tag{36}$$

These are shown in Fig. 14. The flux expressed in (**35**) produces linkages with phase 1, but (**36**) does not.

The per cent flux given by (**35**) at $t = 0$ is

$$\Phi = \frac{A_{5d}\, p^{V}_{0} \int_{-\pi/2}^{+\pi/2} \cos 5\,\gamma\, d\,\gamma}{\int_{-\pi/2}^{+\pi/2} 1.0 \cos \gamma\, d\,\gamma} = \frac{A_{5d}\, p^{V}_{0}}{5}$$

This is the maximum value of flux, which pulsates at cos t, thus

$$\phi_{5d5} = \frac{A_{5d}\, p^{V}_{0}}{5} \cos t \tag{37}$$

The linkages are

$$\Omega_{5d5} = C_5\, \phi_{5d5} = \frac{A_{5d}\, C_5\, p^{V}_{0}}{5} \cos t \tag{38}$$

---

*For instance, in a rotating wave, say (**25**), $\sin(\alpha + 6\,t)$ must equal a constant. That is, $\alpha + 6\,t = C$, and $\frac{d\,\alpha}{d\,t} = -\,6$. Thus velocity is backward at 6 times normal speed.

†Linkages of a circuit are understood to be positive when they agree in sign with those produced by a positive current in the circuit.

where C_5 is the reduction factor* due to pitch, distribution and connection of the coils. The instantaneous voltage, expressed as a fraction of normal voltage, is by (**33**) and (**38**)

$$e_{5d5} = \frac{A_{5d}\, C_5\, p^{\nu}{}_{0}}{5} \sin t \tag{39}$$

But

$$A_{5d} = \frac{C_5}{5} A_{1d}$$

Hence

$$e_{5d5} = \frac{A_{1d}\, C^2{}_5\, p^{\nu}{}_{0}}{25} \sin t \tag{40}$$

Similarly†

$$e_{5q5} = - \frac{A_{1q}\, C^2{}_5\, p^{\nu}{}_{0}}{25} \cos t \tag{41}$$

$$e_{5d7} = - \frac{1}{2} A_{1d}\, p^{\nu}{}_{2} \frac{C_5}{5} \frac{C_7}{7} \sin t \tag{42}$$

$$e_{5q7} = - \frac{1}{2} A_{1q}\, p^{\nu}{}_{2} \frac{C_5}{5} \frac{C_7}{7} \cos t \tag{43}$$

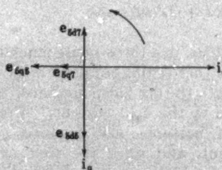


FIG. 15—VECTOR DIAGRAM OF VOLTAGES PRODUCED BY THE 5TH HARMONIC M. M. F.

Consider the phase of these voltages. From eqs. (**1**) and (**2**), i_d is a cosine function of time, and i_q a sine function. But e_{5d5}, produced by i_d, is a *plus* sine function. Hence it *lags* a quarter period behind i_d. Similarly, e_{5q5} lags a quarter period behind i_q, which produces it; e_{5d7} leads i_d by a quarter period; and e_{5q7} lags behind i_q by a quarter period. These relations are shown in Fig. 15. It will be noted that e_{5d7} (fundamental) voltage produced by 7th space harmonics of flux, (itself produced by the 5th space harmonic of m. m. f.) is a reactive voltage of the same phase relation to the current producing it, as the reactive voltage across a condenser, whereas e_{5d5}, e_{5q5} and e_{5q7} (produced by 5th space harmonics of flux) are of the same nature as the reactive voltage across an inductance. By taking the vector sum of the fundamental voltages due to *all* harmonics‡ (including the fundamental), produced by i_d, and, in addition, the reactive voltages due to the slot and end-winding leakages, the result will be the total reactive voltage produced by i_d, and will thus determine the synchronous reactance x_d for the *direct* component of current. Similarly, x_q is determined for the quadrature component.

The general equations for all harmonics due to the total sine wave (in time) current are given in Appendix A as eqs. (**45a**) and (**52a**). Making the approximation, justified in Appendix B, that the permeance series is the same for the cosine and sine distribution of m. m. f. for each harmonic, but remembering that it is a different series for the different harmonics, these equations become

$$e_{nd} = \sum_{m=0}^{m=\infty} - (-1)^{\frac{1+n}{2}} \frac{A_{1d}}{2n} C_n\, p^{n}{}_{m} \left\{ \begin{array}{l} C_{(n+m)} \dfrac{(k_n + m)}{(n + m)} \sin (n + m) \dfrac{\pi}{2} \sin (k_n + m)\, t \\ + C_{(n-m)} \dfrac{(k_n - m)}{n - m} \sin (n - m) \dfrac{\pi}{2} \sin (k_n - m) t \end{array} \right\} \tag{58a}$$

$$e_{nq} = \sum_{m=0}^{m=\infty} + (-1)^{\frac{1+n}{2}} \frac{A_{1q}}{2n} k_n\, C_n\, p^{n}{}_{m} \left\{ \begin{array}{l} C_{(n-m)} \dfrac{(k_n + m)}{(n - m)} \sin (n - m) \dfrac{\pi}{2} \cos (k_n - m)\, t \\ + C_{(n+m)} \dfrac{(k_n + m)}{(n + m)} \sin (n + m) \dfrac{\pi}{2} \cos (k_n + m) t \end{array} \right\} \tag{59a}$$

These two equations contain much information. Aside from giving the phase and magnitude of *all* harmonic voltages, they show such rather unexpected facts as that a space harmonic m. m. f. of one order, say the 5th, produces a space harmonic of flux of another order, say the 7th, which generates a fundamental voltage. Likewise the 7th m. m. f. harmonic produces a 5th flux harmonic which generates fundamental voltage. Moreover, the 11th and 13th, and the 17th and 19th, are similar pairs in this respect. The equations show also that the only values of m (the order of the space harmonic of permeance) which produce fundamental voltages are zero and 2.0. This is clear from an inspection of the time angle $(k_n \pm m)\, t$. In this, k_n is always $\pm$ 1.0. *This is of great practical importance.* It means that in the practical application of these results, only the average permeance term, and the 2nd harmonic term need to be determined, thus immensely simplifying the problem.

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*C. A. Adams, Bibliography (10).

†Or these equations may be obtained from the general equations (**58a**) and (**59a**).

‡The voltages in Fig. 15 are produced by the 5th space harmonic of m. m. f. alone. There is a similar set of voltages for the 7th, 11th, 13th, 17th, 19th, etc., harmonics of m. m. f.

From these equations the various fundamental voltages* have been computed, and are tabulated in Table I, and are shown in Fig. 16 as vectors in relation to the other voltages due to armature current. From these results, the expressions for the reactances are derived in Appendix C.

It will be helpful to sum up the points thus far established. The general plan, it will be remembered, is to determine, first, all of the component voltages produced by armature currents, such voltages comprising (a) all of those generated by flux emanating from the armature surface, *i. e.*, the air-gap flux, (b) that due to flux crossing the slots, (*i. e.*, slot leakage) and (c) that due to end winding leakage; and, second, all voltages due to current in the field winding; then, by superposition of all significant components, to determine the resultant, which is the terminal voltage. In the foregoing, the voltages due to the air-gap fluxes which are produced by armature current have been determined. The voltages due to slot and end winding leakage have been previously determined.† Hence all voltages due to armature current are accounted for. In Appendix C, the important ones of these are assembled in expressions for reactance.

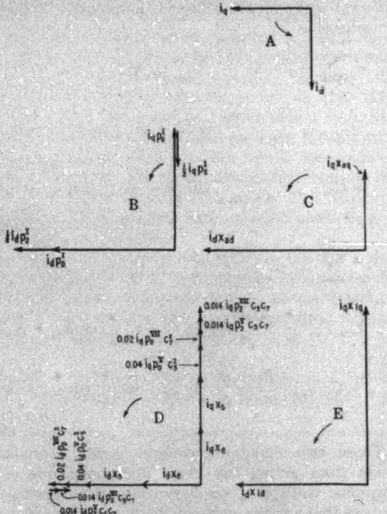
i_q
A
i_d
$i_q p_0^I$
$\frac{1}{2} i_q p_2^I$
B
$\frac{1}{2} i_d p_2^I$
$i_d p_0^I$
$i_q x_{aq}$
C
$i_d x_{ad}$
$0.014\, i_q p_2^{VII} c_5 c_7$
$0.014\, i_q p_2^{V} c_5 c_7$
$0.02\, i_q p_0^{VII} c_7^2$
$0.04\, i_q p_0^{V} c_5^2$
$i_q x_s$
$i_q x_e$
D
$0.02\, i_d p_0^{VII} c_7^2$
$0.04\, i_d p_0^{V} c_5^2$
$i_d x_s$
$i_d x_e$
$0.014\, i_d p_2^{VII} c_5 c_7$
$0.014\, i_d p_2^{V} c_5 c_7$
$i_q x_{lq}$
E
$i_d x_{ld}$

FIG. 16—FUNDAMENTAL VOLTAGES DUE TO ARMATURE CURRENTS

A—Components of armature current
B—Component armature reaction voltages
C—Resultant armature reaction voltages
D—Component leakage reactance voltages
E—Resultant leakage reactance voltages (D and E magnified in scale, compared with B and C)

Field Current Phenomena. The other source of flux is the field current. This current is due to the exciter voltage, assumed to be constant, and to such harmonic voltages as are short-circuited by the field winding. The rotating space harmonic m. m. fs. produced by the armature current are of odd order, and rotate at odd fractions or multiples of synchronous speed with respect to the armature. Hence their electrical angular speed (on the harmonic base) with respect to the field pole and field winding must be of even order; because one times speed is always added to or subtracted from the odd term. For instance, the 5th rotates backward at 1/5th speed with respect to the armature, thus 6/5th with respect to the pole. Hence it generates a 6th harmonic. The 7th rotates forward at 1/7th speed with respect to the armature, hence 6/7th with respect to the poles. Hence it also generates a 6th harmonic in the field winding.

Therefore, the general form of the field current will be a Fourier Series of even harmonic terms, thus

$$I = I_0 + I_2 \cos 2\,t + I_4 \cos 4\,t + \dots$$
$$+ I_2{}' \sin 2\,t + I_4{}' \sin 4\,t + \dots \qquad \textbf{(44)}$$

But inasmuch as, in a three-phase machine, the 5th is the lowest order‡ of rotating harmonic m. m. f. (excepting the fundamental), the 6th harmonic must be the lowest order harmonic in the field current. Moreover, since the 7th m. m. f. harmonic also produces only a 6th in the field current, and since the next existing space harmonics are the 11th and 13th, both of which generate a 12th harmonic in the field winding, and next the 17th and 19th, which generate an 18th, it follows that the only harmonics in the field winding are multiples of the 6th.

But these are all of practically negligible magnitudes. While the 6th harmonic ripple may, in rare cases, be detected in the field current under the condition of balanced three-phase currents, and although it does very slightly affect* the fundamental voltage, its influence is practically insignificant. Hence only the effects of the average term I_0, which is sustained by the constant exciter voltage, are considered.

The voltage due to the flux produced by I_0 has been

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*While only the fundamental voltages are derived in detail, equations (**58a**) and (**59a**) contain the harmonics.

†Bibliography 8.

‡There is an exception to this in the case of a delta-connected machine in which the circulating currents (third harmonic and its multiples) produce space third harmonics of m. m. f. and odd multiples thereof. However, these m. m. fs. rotate at such a speed as to induce in the field winding 6th, 12th, 18th, etc., time harmonics of current.

adequately treated before.[†] The flux wave thus produced is of the general shape indicated in Fig. 17, and hence contains a fundamental and odd harmonics. The component harmonic waves are, of course, stationary with respect to the poles and therefore generate replica voltage waves in the individual armature conductors. The flux wave is determined, as in Figs. 5, 6, 7 and 8, by graphic field plot from a scale drawing of the armature and field pole.

If there should be a field m. m. f. in the *quadrature* axis, there would be another flux wave produced, symmetrical about that axis, hence containing only odd terms—fundamental and odd harmonics.

Significance of Various Voltages. Consider now some aspects of the various fluxes and voltages. Which of them are important? Which of the fluxes determine the leakage reactance; which, the armature reaction?

Take the harmonic voltages first. These are due, as already explained, to (a) a set of rotating flux waves produced by a sine wave (in time) current; (b) an additional set of rotating waves for each time harmonic in the armature current. In the various sets, the speed of rotation of the respective *space* harmonics (*i. e.*, of the same pole span) is proportional to the order of the time harmonic producing it, and the amplitude is proportional to the amplitude of the time harmonic current; (c) the space harmonics of the field flux; and (d) the effects of permeance waves due to slots.[‡]

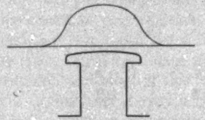

FIG. 17

How important are these harmonic voltages? Some are of no importance because they are of negligible magnitude. This may be due either to small amplitude of the flux harmonic, or to reduction by the connection and space phase of armature conductors. It is well known, for instance, that either a Y connection or $^2/_3$ coil pitch, eliminates the third harmonic; $^4/_5$ pitch, the 5th; $^6/_7$ pitch, the 7th, etc.; and the distribution of coils greatly reduces any which may remain[§]. So in many cases the harmonics may be neglected because they are not appreciable.

Moreover, if a machine is connected to a bus of sine wave voltage, any harmonics which are generated in the machine are thus short-circuited—that is, any corresponding harmonic current is a short-circuit current. Hence no useful power could be developed by such harmonics. However, if there are corresponding harmonics in the terminal voltage and in the current, power can be developed by such harmonics. For instance, a 7th time harmonic in the current will produce a 7th harmonic space flux wave traveling at synchronous speed forward, that is, stationary with respect to the poles. It could therefore synchronize with the 7th space harmonic of field flux. In the main, however, time harmonics are not of significant importance. They may be, of course, in certain rare cases of resonance, and in problems of telephone interference.

Therefore, since the harmonics do not significantly influence the regulation and power relations, principally to be considered here, they will be neglected. If their evaluation, for other purposes, is ever required, the foregoing treatment, particularly eqs. (**58a**) and (**59a**), constitutes a basis for their determination.

Consider the fundamental voltages. They are due to

(a) fundamental space component of flux due to the *direct* component of armature m. m. f., *i. e., direct* armature reaction.

(b) fundamental space component of flux due to the *quadrature* component of armature reaction.

(c) 5th space harmonic flux,[*] rotating backward at $^1/_5$ speed, and due to the *direct* component of armature current.

(d) 5th space harmonic,[*] rotating backward, and due to the *quadrature* component of current.

(e) two corresponding components (*i. e., direct* and *quadrature*) of the 7th space harmonics, rotating forward at $^1/_7$ speed.

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[*] Its action may be approximately illustrated by the 5th harmonic m. m. f. as follows: The rotating 5th m. m. f. of constant magnitude tends to change the flux linkages of the field winding at $6\,\omega$ by a small amount depending on the magnitude of the 5th m. m. f. wave. But the closed field winding maintains constant linkages, by adjusting the field current at all instants to maintain that condition. The resulting flux at the air gap is due to the superposition of the pulsating, field-flux wave, of a shape similar to that in Fig. 17, upon the fluxes due to the armature current. But the wave in Fig. 17, contains among others a 5th and 7th space harmonic. Being a stationary pulsating wave, the pulsating component gives forward and backward rotating waves of equal amplitudes, equal to ½ of the amplitude of the pulsation. Thus each space harmonic in the wave, including the 5th and 7th, rotates. These two are in phase with the 5th and 7th which cause the 6th harmonic current in the field. Hence they may change the magnitude of resultant 5th and 7th flux waves, and by that much, affect the fundamental voltage generated by these. But the reactance to the 6th harmonic current is extremely high, and almost completely damps out this current. It is rarely large enough to detect in the oscillogram of the field current.

[†] Bibliography 7, and Appendix of 8.

[‡] This is neglected in the present treatment. It is a different factor from the ripples due to current concentration, the latter being taken into account. The former have been treated by Lyon. See Bibliography 7.

[§] In most modern alternators the harmonics are practically all thus screened out, leaving the terminal generated voltage almost a true sine wave. Bibliography 10.

[*] Each of these, *i. e.*, (c), (d) and (e), comprises two flux components: one due to the 5th space harmonic of m. m. f.; the other, to the 7th. See Table I.

(f) two components of the 11th rotating backward at 1/11 speed;† and so on.

(g) slot leakage flux produced by both components (*direct* and *quadrature*).

(h) end-winding leakage flux produced by both components.

(i) fundamental space component of flux due to field m. m. f. in the *direct* axis.

(j) fundamental space component of flux due to field m. m. f. in the *quadrature* axis.

Consider these in detail. The terms (a) and (b) are the usual vectors of voltage produced by the flux of armature reaction; these are $i_q x_{aq}$ and $i_d x_{ad}$ in diagram, Fig. 18. (i) corresponds to the *nominal* voltage e_d. (j) is an added similar term e_q, Fig. 19, for the quadrature axis. (g) is the usual slot leakage $i_q x_s$, and $i_d x_s$, Fig. 16D, which is independent of currents in any other slot, and is therefore the same for single or polyphase currents for full pitch coils. They are, however, different for fractional pitch. It may therefore be treated as external reactance‡, provided the slot contains current of one phase only. Slot reactance is obviously the same for the *direct* and *quadrature* components of current. (h) corresponds to the usual end winding leakage reactance $i_q x_e$ and $i_d x_e$, Fig. 16D. There is some mutual reactance between phases, and hence it is not the same for single-phase and polyphase currents, and hence can not be considered as external reactance. It is here assumed to be the same for the *direct* as for the *quadrature* component of current.

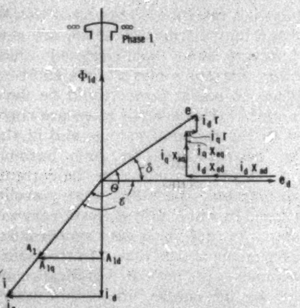

FIG. 18—VECTOR DIAGRAM FROM EQUATIONS. NEGLECTS SATURATION

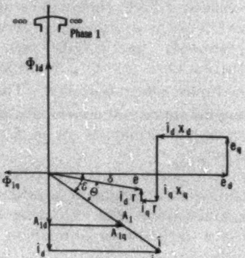

FIG. 19

In Appendix C the following expressions for the various reactances are derived:

$$x_{ad} = p_0^{\mathrm{I}} + \frac{p_2^{\mathrm{I}}}{2} \tag{1c}$$

$$x_{aq} = p_0^{\mathrm{I}} - \frac{p_2^{\mathrm{I}}}{2} \tag{2c}$$

$$x_{ld} = x_s + x_e + \frac{C^2_5\, p_0^{\mathrm{V}}}{25} - \frac{C_5\, C_7}{70}\,(p_2^{\mathrm{V}} + p_2^{\mathrm{VII}}) + \frac{C^2_7\, p_0^{\mathrm{VII}}}{49} + \frac{C^2_{11}\, p_0^{\mathrm{XI}}}{121} - \frac{C_{11}\, C_{13}}{286}(p_2^{\mathrm{XI}} + p_2^{\mathrm{XIII}}) + \frac{C^2_{13}\, p_0^{\mathrm{XIII}}}{169} + \ldots \tag{3c}$$

$$x_{lq} = x_s + x_c + \frac{C^2_5\, p_0^{\mathrm{V}}}{25} + \frac{C_5\, C_7}{70}\,(p_2^{\mathrm{V}} + p_2^{\mathrm{VII}}) + \frac{C^2_7\, p_0^{\mathrm{VII}}}{49} + \frac{C^2_{11}\, p_0^{\mathrm{XI}}}{121} + \frac{C_{11}\, C_{13}}{286}(p_2^{\mathrm{XI}} + p_2^{\mathrm{XIII}}) + \frac{C^2_{13}\, p_0^{\mathrm{XIII}}}{169} + \ldots \tag{4c}$$

$$x_d = x_{ad} + x_{ld} \tag{5c}$$

$$x_q = x_{aq} + x_{lq} \tag{6c}$$

REACTANCE AND VECTOR DIAGRAMS

The most recent contribution relating to the general subject treated here is a paper by Karapetoff§ on "Variable Leakage Reactance." In his historical references, attention is called to the fact that in Blondel's diagram, the leakage reactance drop is represented by a single vector; and that although other authors[*] had used a different value of reactance for currents opposite the pole than for currents between poles, none had accounted vectorially for the variation of reactance between those positions. His treatment *assumes* this variation to be harmonic, and therefrom he establishes an interesting relation, namely that the leakage reactance voltage comprises two terms, one corresponding in phase relation to the usual reactive drop, and another whose phase relation changes with the phase of the total current.

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†When irregular armature windings are used, for instance fractional slots per pole, then additional harmonics occur. These are important since they may create serious vibration in the stator frame; and, in the case of induction motors, may significantly increase the leakage reactance. Their computation is beyond the scope of this paper.

‡The effect of currents of different phases in the same slot, as in fractional coil pitch, is accounted for by appropriate factors in the calculation of slot leakage.

§Bibliography 12.

The present general treatment justifies his assumption. Indeed, it is clear from eqs. (**58a**) and (**59a**), as already mentioned, that the *only* permeance terms which produce fundamental voltages due to harmonic flux waves (leakage reactance) are those of zero and of second space order, *i. e.*, $m = 0$ and $m = 2.0$. In other words, his assumption is one of the necessary conclusions of the present treatment.

However, the impression should not be left that this variation in *leakage reactance* is of a large order of magnitude. Indeed it is insignificant in a large percentage of salient pole synchronous machines. In the first place, a large portion of the leakage reactance consists of the slot leakage and end winding leakage which are practically independent of pole position. The rest of the leakage consists of the tooth tip and belt leakage (comprising the space harmonic fluxes which generate fundamental voltage), and most of this is independent of pole position, leaving only a very small fraction of the total which does depend on pole position.

From the foregoing, and the results in Appendix C, the vector diagram, neglecting saturation, can be constructed. The diagram, of course, will contain the same quantities as the well-known Blondel diagram, with the rather trifling exception discussed above, regarding the leakage reactance voltages.[*]

From Table I, the voltages due to armature reaction, *i. e.*, to the space fundamentals of flux, are

$$- j \underset{\cdot}{A}_{1d} \left(p_0^{\mathrm{I}} + \frac{1}{2} p_2^{\mathrm{I}} \right) = - j \underset{\cdot}{i}_d \, x_{ad}$$

and

$$- j \underset{\cdot}{A}_{1q} \left(p_0^{\mathrm{I}} - \frac{1}{2} p_2^{\mathrm{I}} \right) = - j \underset{\cdot}{i}_q \, x_{aq}$$

The voltages due to leakage reactance, derived in Appendix C, are

$$- j \underset{\cdot}{i}_d \, x_{ld}$$

and

$$- j \underset{\cdot}{i}_q \, x_{lq}$$

Taking the fundamental flux ϕ_{1d}, *direct* axis, as the time reference vector, the vector diagram in Fig. 18 is constructed. By the convention adopted in the equations, $\underset{\cdot}{i}_q$ was taken as lagging with respect to $\underset{\cdot}{i}_d$. This corresponds to a *motor* load, hence in Fig. 18, according to the definition, the power factor angle θ, is greater than 90 deg., *i. e.*, $> \frac{\pi}{2}$. Redrawing Fig. 18 in Fig. 20, the conventional relations are shown, in which i_q is reversed, thus a generator load. Also the reactances are combined, since saturation is in this particular case neglected.

$$x_d = x_{ad} + x_{ld}$$
$$x_q = x_{aq} + x_{lq}$$

The vectors in Fig. 20 are the *induced*, not the consumed, voltages.

Introducing an excitation current in a field winding around the quadrature axis, brings in an additional voltage e_q as shown in Fig. 19. The general power-angle relations are derived in Part II from this diagram. An approximate method due to Blondel, for taking saturation into account, will be referred to in Part II and also in Appendix D, in which Fig. 20 will be interpreted in terms of the familiar Potier diagram for cylindrical rotor machines.

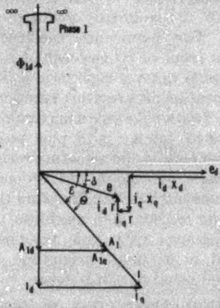

FIG. 20—VECTOR DIAGRAM—USUAL CONVENTIONS

An idea of relative phase and magnitude of voltages may be obtained from an inspection of Table I. Referring to Fig. 16D, in which the phase relation of the various vector voltages of Table I are shown[†], it will be noted that the only ones which do not have the same phase relation with respect to the current which causes them, are those which involve products $C_5 C_7$, $C_{11} C_{13}$, etc.—that is, those terms in which a flux of one space order is produced by m. m. f. of another space order. The diagrams show nothing above the 7th, since the next term involves the 11th and 13th, and is entirely negligible. They are, in other words, only those terms which involve second harmonic permeance coefficients for m. m. f. space orders > 1.0, such as p_2^{V}, p_2^{VII}, etc.

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[*] Bibliography 5.

[*] In the present treatment, these comprise all fundamental voltages except those produced by the two space fundamental flux waves.

[†] In the diagram, currents are substituted for corresponding armature reactions.

They are given in eq. (**7c**) Appendix C. It will be observed in Fig. 16D that the only such terms are

$$0.014\, i_d\, p_2^{V}\, C_5\, C_7$$
$$0.014\, i_d\, p_2^{VII}\, C_5\, C_7$$

These are *reversed* with respect to the other voltages produced by i_d. The corresponding terms produced by i_q are

$$0.014\, i_q\, p_2^{V}\, C_5\, C_7$$
$$0.014\, i_q\, p_2^{VII}\, C_5\, C_7$$

which *add on* to corresponding voltages produced by i_q. The vector sum of these four terms* constitutes the variable phase $I\,X_s$ vector in Karapetoff's treatment. Its phase with respect to the current causing it, changes through 180 degrees as the total current changes from all i_d to all i_q, *i. e.*, through 90 deg. It thus changes as twice the angle between the resultant current and the zero reference, as his equation specifies.

What about the magnitude? The product $C_5\, C_7$ may be of any value between zero and a maximum value = 1.0 for a three-phase machine. It is zero if the coil pitch is either $^4/_5$ or $^6/_7$, and greatly reduced for other pitches.† It is greatly reduced also by coil distribution‡. The permeance coefficients p_2^{V} and p_2^{VII} are of the order of 50 per cent of p_0^{I}. Thus the maximum possible value of each term is of the order of $^1/_2$ of one per cent of the armature reaction voltage due to i_q or to i_d. This would give a maximum variation in leakage reactance voltage with pole position, of the order of ± 1 per cent of the armature reaction voltage; and in many cases, on account of fractional pitch, etc., it would be practically zero. Therefore the conclusion is that in many salient pole machines the variation in leakage reactance with pole position is entirely negligible, and is of small magnitude in any case approaching the usual design practise.

PRACTICAL APPLICATION OF RESULTS

The application of the method given here requires numerical data for the zero and second space harmonic of permeance for each significant harmonic m. m. f. The harmonic m. m. fs. are easily computed, and are all expressed in Table I in terms of the fundamental m. m. f. component. But the permeance coefficients must be determined from field plots§ such as shown in Figs. 5, 6, 7, 8 and 9. From such plots, the permeance curves are determined and analyzed to obtain the required value of the average and the second harmonic coefficients. Partial analysis of these particular plots is shown in Fig. 21.

In order to avoid having to plot, as above, the flux distribution for each significant harmonic, including the fundamental, an approximation is made as follows: A plot is made for the zero harmonic, as in Fig. 5, which results are shown also on Fig. 21. In addition, the latter shows a curve of "equivalent air-gap," taken as the reciprocal of the permeance for the zero harmonic. From this and the curve in Fig. 10, derived in Appendix B, the permeance curves for any harmonic can be obtained, as in Fig. 11. These are then analyzed for the zero and second harmonic. The results thus obtained for the particular case here illustrated are compared in Table II with those obtained directly from the plots, and are also compared in complete curves in Fig. 12. It will be noted that the agreement is very satisfactory. The only discrepancy worth noting is between p_{2c}^{I} and p_{2s}^{I}, which are, from plots, respectively 0.43 and 0.47. The value derived from Figs. 10 and 21 is 0.44.

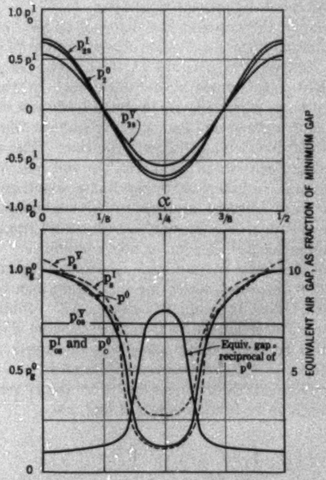

FIG. 21—PERMEANCE DISTRIBUTION DETERMINED FROM FIELD PLOTS. ALSO EQUIVALENT AIR-GAP

The above simplifying assumptions are fully discussed in Appendix B.

The general plan, therefore, is to obtain a permeance curve for the zero harmonic for the machine under consideration, and from that, the "equivalent air-gap" curve. Then with the aid of Fig. 10, plot a group of curves as in Fig. 11. An analysis of these will give the required p_0'' and p''_2.

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*And, if they were appreciable, others involving 11th and 13th, and 17th and 19th. But the large denominators in these terms, Table I, indicate the very small magnitude.

†Of course it would be zero also for a pitch of either $^2/_5$ or $^4/_7$, etc.

‡C. A. Adams, Bibliography 10.

§Comprehensive treatments of flux plotting are being prepared by engineers of the General Electric Company, for presentation at an early date.

II. Steady State Power-Angle Characteristics

The angle-power relations developed in the following are important in connection with the parallel operation of frequency converter sets; the natural oscillating frequency of machines where the circumstances are such as to approach steady state conditions—such as high resistance in the field circuit, or very low oscillating frequency; and in the problem of stability. A future paper will treat the case of transient conditions.

1. Single Machine on Infinite Bus. Referring to Fig. 19, the time diagram for a three-phase generator, the following relations may be obtained.

$$e_d = e \cos \delta + i_q r + i_d x_d \qquad (45)$$

and

$$e_q = i_q x_q - e \sin \delta - i_d r \qquad (46)$$

Since the component of terminal voltage in phase with i_d is $e \sin \delta$, and the component in phase with i_q is $e \cos \delta$, the electrical power output of the generator is

$$P = e\, i_d \sin \delta + e\, i_q \cos \delta \qquad (47)$$

Solving (**45**) and (**46**) simultaneously for i_d and i_q

$$i_d = \frac{x_q\,(e_d - e \cos \delta) - r\,(e_q + e \sin \delta)}{r^2 + x_d\, x_q} \qquad (48)$$

and

$$i_q = \frac{r\,(e_d - e \cos \delta) + x_d\,(e_q + e \sin \delta)}{r^2 + x_d\, x_q} \qquad (49)$$

Substituting (**48**) and (**49**) in (**47**)

$$P = \frac{\begin{array}{c}(e\, e_d\, x_q - e\, e_q\, r) \sin \delta + (e\, e_d\, r + e\, e_q\, x_d) \cos \delta \\ + \dfrac{e^2}{2}\,(x_d - x_q) \sin 2\,\delta - e^2\, r\end{array}}{r^2 + x_d\, x_q} \qquad (50)$$

To obtain the mechanical power input, the copper loss of the armature must be added. This loss is

$$P_L = i^2 r = (i_q^2 + i_d^2)\, r \qquad (51)$$

Substituting (**48**) and (**49**) in (**51**)

$$P_L = \frac{\begin{array}{c} r\,(x_q^2 + r^2)\,(e_d - e \cos \delta)^2 \\ + r\,(x_d^2 + r^2)\,(e_q + e \sin \delta)^2 \\ + 2\, r^2\,(x_d - x_q)\,(e_d - e \cos \delta)\,(e_q + e \sin \delta)\end{array}}{(r^2 + x_d\, x_q)^2} \qquad (52)$$

The total mechanical power input is then eq. (**50**) plus eq. (**52**). This equation is rather complex to apply in practise. However, the armature resistance, which is generally small, has only a small effect on the power-angle characteristic, and for practical purposes, the armature resistance may be assumed to be zero.

Equation (**52**) then becomes zero and eq. (**50**) becomes

$$P = \frac{e\, e_d}{x_d} \sin \delta + \frac{e\, e_q}{x_q} \cos \delta + \frac{e^2\,(x_d - x_q)}{2\, x_d\, x_q} \sin 2\,\delta \qquad (53)$$

For a given terminal voltage and displacement angle, the power thus comprises three independent components. The first is proportional to the excitation in the direct axis, the second is proportional to the excitation in the quadrature axis, while the third is independent of any excitation and may be termed the "reluctance power," due to the salient pole construction. Generally there will be no rotor m. m. f. in the quadrature axis under *steady state*, *i. e.*, e_q is zero. Eq. (**53**) then becomes

$$P = \frac{e\, e_d}{x_d} \sin \delta + \frac{e^2\,(x_d - x_q)}{2\, x_d\, x_q} \sin 2\,\delta \qquad (54)$$

It may be noted that the first term of this equation is the same as would be obtained using the cylindrical rotor theory. However, the second, or reluctance term, may be fully as important as the first for salient pole machines, and it is due to this fact that the power-angle characteristics obtained by the two-reaction theory differ so markedly from those obtained by the cylindrical rotor method. In Fig. 22, power-angle curves are shown for a typical salient pole machine.

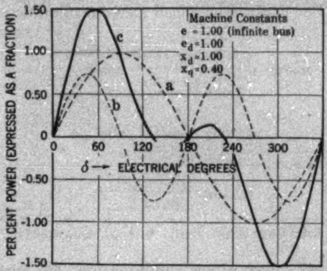

FIG. 22—POWER-ANGLE CHARACTERISTICS OF A SALIENT POLE SYNCHRONOUS MACHINE

The equations given above have been derived for a generator.

In the case of a motor the angle, δ, is negative. This makes the electrical power output negative, *i. e.*, power *input* to the motor. Thus, with δ negative, P is also negative, but P_L remains positive. That is, the copper loss reduces the mechanical power output. When there is no resistance, the mechanical power output as a motor is the same as the electrical power output as a generator for numerically equal displacement angles. Hence, for no resistance, equations (**53**) and (**54**) apply to operation either as a motor or as a generator.

The first term in equation (**54**) is that obtained by cylindrical rotor theory and has a maximum when

$\delta = 90$ deg. However, when the second term is added, having its maximum at 45 deg., the maximum value of the resultant curve occurs at some other value of δ between 45 deg. and 90 deg.

Differentiating* equation (**54**) with respect to δ, and considering e constant, *i. e.*, that the machine is connected to an infinite bus,

$$\frac{d\,P}{d\,\delta} = \frac{e\,e_d}{x_d} \cos \delta + e^2 \frac{(x_d - x_q)}{x_d\,x_q} \cos 2\,\delta \qquad (\mathbf{55})$$

Since

$$\cos 2\,\delta = 2 \cos^2 \delta - 1 \qquad (\mathbf{56})$$

$$\frac{d\,P}{d\,\delta} = \frac{e\,e_d}{x_d} \cos \delta + 2\,e^2 \frac{(x_d - x_q)}{x_d\,x_q} \cos^2 \delta$$

$$- e^2 \frac{(x_d - x_q)}{x_d\,x_q} \qquad (\mathbf{57})$$

The power has a maximum when $\frac{d\,P}{d\,\delta} = 0$ or

$$\cos^2 \delta + \frac{e_d\,x_q}{2\,e\,(x_d - x_q)} \cos \delta - \frac{1}{2} = 0 \qquad (\mathbf{58})$$

or

$$\cos \delta = \frac{-\,e_d\,x_q}{4\,e\,(x_d - x_q)}$$

$$\pm \frac{1}{4\,e\,(x_d - x_q)} \sqrt{e_d^2\,x_q^2 + 8\,e^2\,(x_d - x_q)^2} \qquad (\mathbf{59})$$

The two values of $\cos \delta$ in equation (**59**) correspond in general to two maximum values of power, the positive sign giving the first and largest maximum as δ increases from zero. Since we are not interested in the second maximum, occurring in the vicinity of $\delta = 180$ deg., we may take

$$\cos \delta = \frac{-\,e_d\,x_q + \sqrt{e_d^2\,x_q^2 + 8\,e^2\,(x_d - x_q)^2}}{4\,e\,(x_d - x_q)} \qquad (\mathbf{60})$$

In the above the core loss and friction and windage losses have been neglected. These terms introduce practically a constant term and may be taken into account by making a final correction in the results obtained.

Reactance external to the machine proper, but included between the source of constant voltage and the machine, may be taken into account by adding it to both the leakage reactance in the direct axis and the quadrature axis.

Saturation has also been neglected in the foregoing. It is approximately taken into account in Appendix Saturation in the pole body does not change the angle-power relation at the same condition of voltage, load and power factor.

2. General Case of a Generator Supplying Power to a Synchronous Motor. From Fig. 23

$$i_{d2} = i_{q1} \sin \delta' - i_{d1} \cos \delta' \qquad (\mathbf{61})$$

$$i_{q2} = i_{q1} \cos \delta' + i_{d1} \sin \delta' \qquad (\mathbf{62})$$

$$e_{d1} - i_{d1}\,x_{d1} + i_{d2}\,x_{d2} \cos \delta' - i_{q2}\,x_{q2} \sin \delta' = e_{d2} \cos \delta' \qquad (\mathbf{63})$$

$$i_{q1}\,x_{q1} + i_{d2}\,x_{d2} \sin \delta' + i_{q2}\,x_{q2} \cos \delta' = e_{d2} \sin \delta' \qquad (\mathbf{64})$$

Substituting (**61**) and (**62**) in (**63**) and (**64**)

$$e_{d1} - i_{d1}\,x_{d1} + i_{q1}\,x_{d2} \sin \delta' \cos \delta' - i_{d1} \cos^2 \delta'$$

$$- i_{q1}\,x_{q2} \sin \delta' \cos \delta' - i_{d1}\,x_{q2} \sin^2 \delta' = e_{d2} \cos \delta' \qquad (\mathbf{65})$$

$$i_{q1} x_{q1} + i_{q1} x_{d2} \sin^2 \delta' - i_{d1} x_{d2} \sin \delta' \cos \delta' + i_{q1}\,x_{q2} \cos^2 \delta'$$

$$+ i_{d1}\,x_{q2} \sin \delta' \cos \delta' = e_{d2} \sin \delta' \qquad (\mathbf{66})$$

$$- i_{d1} \left[x_{d1} + \frac{x_{d2} + x_{q2}}{2} + \frac{x_{d2} - x_{q2}}{2} \cos 2\,\delta' \right]$$

$$+ i_{q1} \frac{x_{d2} - x_{q2}}{2} \sin 2\,\delta' = e_{d2} \cos \delta' - e_{d1} \qquad (\mathbf{67})$$

$$- i_{d1} \frac{x_{d2} - x_{q2}}{2} \sin 2\,\delta' + i_{q1} \left[x_{q1} + \frac{x_{q2} + x_{d2}}{2} \right.$$

$$\left. - \frac{x_{d2} - x_{q2}}{2} \cos 2\,\delta' \right] = e_{d2} \sin \delta' \qquad (\mathbf{68})$$

Solving (**67**) and (**68**) for i_{d1} and i_{q1}

$$i_{d1} = \frac{-\,e_{d2}(x_{q1} + x_{q2}) \cos \delta' - e_{d1} \frac{(x_{d2} - x_{q2})}{2} \cos 2\,\delta' + e_{d1} \left(x_{q1} + \frac{x_{q2} + x_{d2}}{2} \right)}{x_{d1} x_{q1} + x_{d2} x_{q2} + \frac{(x_{d1} + x_{q1})\,(x_{d2} + x_{q2})}{2} + \frac{(x_{q1} - x_{d1})\,(x_{d2} - x_{q2})}{2} \cos 2\,\delta'} \qquad (\mathbf{69})$$

$$i_{q1} = \frac{e_{d2}\,(x_{d1} + x_{q2}) \sin \delta' + e_{d1} \frac{(x_{d2} - x_{q2})}{2} \sin 2\,\delta'}{x_{d1}\,x_{q1} + x_{d2}\,x_{q2} + \frac{(x_{d1} + x_{q1})\,(x_{d2} + x_{q2})}{2} + \frac{(x_{q1} - x_{d1})\,(x_{d2} - x_{q2})}{2} \cos 2\,\delta'} \qquad (\mathbf{70})$$

From Fig. 23, the component of the terminal voltage in phase with i_{d1} is $i_{q1}\,x_{q1}$ and the component in phase with i_{q1} is $e_{d1} - i_{d1}\,x_{d1}$.

---

*The rate of change of power with angle, as given by (**55**), is usually referred to as "synchronizing power."

Hence,

$$P = (e_{d1} - i_{d1}\,x_{d1})\,i_{q1} + i_{q1}\,x_{q1}\,i_{d1} \tag{71}$$

or

$$P = e_{d1}\,i_{q1} + i_{d1}\,i_{q1}\,(x_{q1} - x_{d1}) \tag{72}$$

Substituting (**69**) and (**70**) in (**71**) and (**72**)

$$P = \frac{e_{d1}\,e_{d2}\,(x_{d1} + x_{q2})\sin\delta' + \dfrac{e_{d1}^2}{2}\,(x_{d2} - x_{q2})\sin 2\,\delta'}{A}$$

$$+ \frac{(x_{q1} - x_{d1})}{A^2}\left[e_{d2}\,(x_{d1} + x_{q2})\sin\delta' + e_{d1}\frac{(x_{d2} - x_{q2})}{2}\sin 2\,\delta'\right]\left[e_{d1}\left(x_{q1} + \frac{x_{q2} + x_{d2}}{2}\right) - e_{d2}\,(x_{q1} + x_{q2})\cos\delta'\right.$$

$$\left. - e_{d1}\frac{(x_{d2} - x_{q2})}{2}\cos 2\,\delta'\right] \tag{73}$$

where

$$A = x_{d1}\,x_{q1} + x_{d2}\,x_{q2} + \frac{(x_{d1} + x_{q1})\,(x_{d2} + x_{q2})}{2} + \frac{(x_{q1} - x_{d1})\,(x_{d2} - x_{q2})}{2}\cos 2\,\delta' \tag{74}$$

Expanding (**73**),

$$P = \left(e_{d1}\,e_{d2}\left[(x_{d1} + x_{q2})\,(x_{q1} + x_{q2})\,(x_{q1} + x_{d2})\right.\right.$$

$$\left. - \frac{1}{4}\,(x_{q1} + x_{q2})\,(x_{d2} - x_{q2})\,(x_{q1} - x_{d1})\right]\sin\delta'$$

$$+ \left[\frac{e_{d1}^2}{2}\,(x_{d2} - x_{q2})\,(x_{q1} + x_{q2})\,(x_{q1} + x_{d2})\right.$$

$$\left. - \frac{e_{d2}^2}{2}\,(x_{q1} - x_{d1})\,(x_{d1} + x_{q2})\,(x_{q1} + x_{q2})\right]\sin 2\,\delta'$$

$$\left. - \frac{e_{d1}\,e_{d2}}{4}\,(x_{q1} - x_{d1})(x_{d2} - x_{q2})(x_{q1} + x_{q2})\sin 3\,\delta'\right)$$

$$\div\left[x_{d1}\,x_{q1} + x_{d2}\,x_{q2} + \frac{(x_{d1} + x_{q1})\,(x_{d2} + x_{q2})}{2}\right.$$

$$\left. + \frac{(x_{q1} - x_{d1})\,(x_{d2} - x_{q2})}{2}\cos 2\,\delta'\right]^2 \tag{75}$$

If

$$x_{d1} = x_{d2} = x_d$$
$$x_{q1} = x_{q2} = x_q$$
$$e_{d1} = e_{d2} = e_d$$

eq. (**75**) reduces to

$$P = \frac{4\,e_d^2\,x_q\sin\delta'}{(x_d - x_q)^2\cos 2\,\delta' - 4(x_d^2 - x_q^2)\cos\delta' + 3\,x_d^2 + 3\,x_q^2 + 2\,x_d\,x_q} \tag{76}$$

3. *Special Case of Zero Field Current.* The vector diagram for this case is shown in Fig. 24. The voltages and currents shown are, of course, all for phase 1, and under the condition of constant terminal voltage e impressed. Although these relations are, of course, all included in the general case, they are somewhat obscure in combination with other quantities.

The power is

$$P = e_{ad}\,i_q - e_{aq}\,i_d \tag{77}$$

but

$$e_{aq} = i_q\,x_q = -\,e\cos\delta \tag{78}$$

and

$$e_{ad} = i_d\,x_d = -\,e\sin\delta \tag{79}$$

Hence

$$P = e^2\,\frac{(x_d - x_q)}{2\,x_d\,x_q}\sin 2\,\delta \tag{80}$$

This corresponds to the third term in (**53**), that is, the reluctance term due to salient poles. It is, of course, zero if $x_d = x_q$, which is the condition applying to a cylindrical rotor of uniform air-gap.

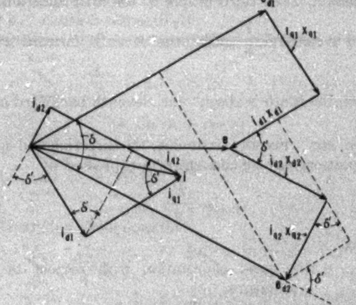

FIG. 23—VECTOR DIAGRAM FOR TWO MACHINES

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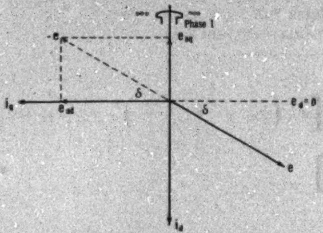

Fig. 24—Vector Diagram for a Single Machine on an Infinite Bus, and Without Field Excitation

Appendix A

General Equations for Harmonic Fluxes and Voltages Due to Armature Currents

A three-phase current which produces a space fundamental m. m. f., A_{1d}, rotating in synchronism with the rotor and in space phase with the direct axis, also produces in general, space harmonics of m. m. f., A_{nd}, rotating in space at $\frac{1}{n}$ times normal speed, the direction of rotation depending on the order of the harmonic. The third is absent, the fifth rotates backward in space at $\frac{1}{5}$ speed, the seventh forward at $\frac{1}{7}$ speed, the ninth is absent, the eleventh backward at $\frac{1}{11}$ speed, etc. For any harmonic, the direction and speed of rotation may be conveniently expressed by

$$\omega_{an} = \frac{k_n}{n} \tag{1a}$$

where

ω_{an} = the speed of rotation with respect to the armature

n = the order of the harmonic

and

$$k_n = -\frac{1}{\sqrt{3}} \tan \frac{n}{3} = \pm 1.0 \tag{2a}$$

The speed of the m. m. f. wave with respect to the rotor is the difference between its speed with respect to the armature and the speed of the rotor with respect to the armature, or

$$\omega_{rn} = \frac{k_n}{n} - 1.0 = -\frac{(n - k_n)}{n} \tag{3a}$$

The equation for a rotating wave of amplitude, A_{nd}, distributed as an nth harmonic in space and rotating at a speed $-\frac{(n - k_n)}{n}$ is

$$a_{nd} = A_{nd} \cos [n (\alpha + \psi) + (n - k_n) t] \tag{4a}$$

where

$n \psi$ = the angular displacement of a peak of the wave from the direct axis when $t = 0$ and is measured on the harmonic base.

$n\alpha$ = electrical angle, reckoned from the direct axis in the direction of rotation and measured on the harmonic base.

When $t = 0$, *i. e.*, when the axis of phase 1 coincides with the direct axis, each rotating m. m. f., due to the direct component of armature current, has one of its maxima in the direct axis. Hence $\psi = 0$ and the equation for the wave is

$$a_{nd} = A_{nd} \cos [n \alpha + (n - k_n) t] \tag{5a}$$

Expansion of equation (**5a**) gives two components,

$$a'_{nd} = A_{nd} \cos n \alpha \cos (n - k_n) t \tag{6a}$$

and

$$a''_{nd} = - A_{nd} \sin n \alpha \sin (n - k_n) t \tag{7a}$$

Equation (**6a**) is an m. m. f. wave of distribution $\cos n \alpha$ and stationary with respect to the rotor. Likewise equation (**7a**) is stationary with respect to the rotor, but of $\sin n \alpha$ distribution. Each of these waves pulsates in time at a frequency $(n - k_n)$ times normal. The m. m. fs., a_{nd}' and a_{nd}'', will each produce a flux wave,

$$\beta'_{nd} = A_{nd}\, p_c{}^n \cos n \alpha \cos (n - k_n) t \tag{8a}$$

and

$$\beta''_{nd} = - A_{nd}\, p_s{}^n \sin n \alpha \sin (n - k_n) t \tag{9a}$$

where $p_c{}^n$ is the permeance series for an nth space harmonic of m. m. f. of distribution $\cos n \alpha$, and $p_s{}^n$ is a similar permeance series for an nth space harmonic of m. m. f. of distribution $\sin n \alpha$. These series have been shown in the body of the paper to consist of an average term and cosine terms of even multiples of α, or

$$p_c{}^n = p_{0c}{}^n + p_{2c}{}^n \cos 2 \alpha + p_{4c}{}^n \cos 4 \alpha + \ldots \tag{10a}$$

$$p_s{}^n = p_{0s}{}^n + p_{2s}{}^n \cos 2 \alpha + p_{4s}{}^n \cos 4 \alpha + \ldots \tag{11a}$$

The general terms of these series are $p_{mc}{}^n \cos m \alpha$ and $p_{ms}{}^n \cos m \alpha$, respectively, which substituted in equations (**8a**) and (**9a**), give

$$\beta'_{nd} = \sum_{m=0}^{m=\infty} \frac{A_{nd}}{2} p_{mc}{}^n \{ \cos (n + m) \alpha \cos (n - k_n) t + \cos (n - m) \alpha \cos (n - k_n) t \} \tag{12a}$$

$$\beta''_{nd} = \sum_{m=0}^{m=\infty} - \frac{A_{nd}}{2} p_{ms}{}^n \{ \sin (n + m) \alpha \sin (n - k_n) t + \sin (n - m) \alpha \sin (n - k_n) t \} \tag{13a}$$

where m is any positive even integer, including zero.

Expansion of (**12a**) and (**13a**) gives

$$\beta'_{nd} = \sum_{m=0}^{m=\infty} \frac{A_{nd}}{4} p_{mc}{}^{n} \cdot \left\{ \begin{matrix} \cos[(n+m)\alpha - (n-k_n)t] + \cos[(n+m)\alpha + (n-k_n)t] \\ + \cos[(n-m)\alpha - (n-k_n)t] + \cos[(n-m)\alpha + (n-k_n)t] \end{matrix} \right\} \quad \textbf{(14a)}$$

$$\beta''_{nd} = \sum_{m=0}^{m=\infty} \frac{A_{nd}}{4} p_{ms}{}^{n} \cdot \left\{ \begin{matrix} \cos[(n+m)\alpha + (n-k_n)t] - \cos[(n+m)\alpha - (n-k_n)t] \\ + \cos[(n-m)\alpha + (n-k_n)t] - \cos[(n-m)\alpha - (n-k_n)t] \end{matrix} \right\} \quad \textbf{(15a)}$$

Collecting similar terms,

$$\beta_{nd} = \sum_{m=0}^{m=\infty} \frac{A_{nd}}{4} (p_{mc}{}^{n} + p_{ms}{}^{n}) \left\{ \begin{matrix} \cos[(n+m)\alpha + (n-k_n)t] \\ + \cos[(n-m)\alpha + (n-k_n)t] \end{matrix} \right\} \quad \textbf{(16a)}$$

$$+ \sum_{m=0}^{m=\infty} \frac{A_{nd}}{4} (p_{mc}{}^{n} - p_{ms}{}^{n}) \left\{ \begin{matrix} \cos[(n+m)\alpha - (n-k_n)t] \\ + \cos[(n-m)\alpha - (n-k_n)t] \end{matrix} \right\}$$

For any given values of m and n, equation (**16a**) comprises four rotating flux waves. The order of the harmonic space distribution is given by the coefficient of α and the speed and direction of rotation with respect to the rotor are given by the *negative* ratio of the coefficient of t to the coefficient of α.

A similar set of flux waves will be produced by the *quadrature* component of armature current. The m. m. f. waves produced by this component of current will have the same speed and direction of rotation with respect to the rotor as the corresponding m. m. f. waves produced by the *direct* component, *i. e.*,

$$\omega_{rn} = - \frac{(n - k_n)}{n} \quad \textbf{(17a)}$$

The equation for a rotating wave of amplitude, A_{nq}, distributed as an nth harmonic in space and rotating at a speed $- \frac{(n - k_n)}{n}$ is

$$a_{nq} = A_{nq} \cos [n (\alpha + \psi) + (n - k_n) t] \quad \textbf{(18a)}$$

where α and ψ have the same significance as explained for equation (**4a**). When $t = \pi/2$, the quadrature component of fundamental m. m. f. and phase 1 are both in the quadrature axis and a_{nq} has one of its maxima in this axis, *i. e.*,

$$a_{nq} = A_{nq} \cos n (\alpha + \pi/2) \quad \textbf{(19a)}$$

$$t = \pi/2$$

Substituting $t = \pi/2$ in (**18a**) and equating to (**19a**)

$$n\,\alpha + n\,\psi - \pi/2\,k_n + \pi/2\,n = n\,\alpha + \pi/2\,n \quad \textbf{(20a)}$$

or

$$\psi = \pi/2 \frac{k_n}{n} \quad \textbf{(21a)}$$

Equation (**18a**) is then

$$a_{nq} = A_{nq} \cos \left[n \left(\alpha + \pi/2 \frac{k_n}{n} \right) + (n - k_n)\,t \right] \quad \textbf{(22a)}$$

Expansion of equation (**22a**) gives two components,

$$a'_{nq} = A_{nq} \cos n \left(\alpha + \pi/2 \frac{k_n}{n} \right) \cos (n - k_n)\,t \quad \textbf{(23a)}$$

and

$$a''_{nq} = - A_{nq} \sin n \left(\alpha + \pi/2 \frac{k_n}{n} \right) \sin (n - k_n)\,t \quad \textbf{(24a)}$$

Since

$$\cos (n\,\alpha + \pi/2\,k_n) = - k_n \sin n\,\alpha \quad \textbf{(25a)}$$

and

$$\sin (n\,\alpha + \pi/2\,k_n) = k_n \cos n\,\alpha \quad \textbf{(26a)}$$

(k_n is always unity, equation (**2a**), its sign depending on the order of the harmonic considered.)

$$a'_{nq} = - A_{nq}\,k_n \sin n\,\alpha \cos (n - k_n)\,t \quad \textbf{(27a)}$$

and

$$a''_{nq} = - A_{nq}\,k_n \cos n\,\alpha \sin (n - k_n)\,t \quad \textbf{(28a)}$$

These two components of m. m. f. are stationary with respect to the rotor and pulsate in time at $(n - k_n)$ times normal frequency. a'_{nq} is of sine space distribution and meets the permeance $p_s{}^n$. a''_{nq} is of cosine space distribution and meets the permeance $p_c{}^n$.

The flux waves produced by a'_{nq} and a''_{nq} are then

$$\beta'_{nq} = - A_{nq}\,k_n\,p_s{}^n \sin n\,\alpha \cos (n - k_n)\,t \quad \textbf{(29a)}$$

$$\beta''_{nq} = - A_{nq}\,k_n\,p_c{}^n \cos n\,\alpha \sin (n - k_n)\,t \quad \textbf{(30a)}$$

The general terms of $p_s{}^n$ and $p_c{}^n$ are $p_{ms}{}^n \cos m\,\alpha$ and $p_{mc}{}^n \cos m\,\alpha$ respectively.

Substituting these terms in (**29a**) and (**30a**)

$$\beta'_{nq} = \sum_{m=0}^{m=\infty} - \frac{A_{nq}}{2} k_n\,p_{ms}{}^n \cos (n - k_n)\,t\,[\sin (n - m)\,\alpha + \sin (n + m)\,\alpha] \quad \textbf{(31a)}$$

$$\beta''_{nq} = \sum_{m=0}^{m=\infty} - \frac{A_{nq}}{2} k_n\,p_{mc}{}^n \sin (n - k_n)\,t\,[\cos (n - m)\,\alpha + \cos (n + m)\,\alpha] \quad \textbf{(32a)}$$

Expanding these equations,

$$\beta'_{nq} = \sum_{m=0}^{m+\infty} - \frac{A_{nq}}{4} k_n\,p_{ms}{}^n \left\{ \begin{matrix} \sin[(n-m)\alpha - (n-k_n)t] \\ + \sin[(n-m)\alpha + (n-k_n)t] \\ + \sin[(n+m)\alpha - (n-k_n)t] \\ + \sin[(n+m)\alpha + (n-k_n)t] \end{matrix} \right\} \quad \textbf{(33a)}$$

$$\beta''_{nq} = \sum_{m=0}^{m=\infty} - \frac{A_{nq}}{4} k_n\,p_{mc}{}^n \left\{ \begin{matrix} \sin[(n-m)\alpha + (n-k_n)t] \\ - \sin[(n-m)\alpha - (n-k_n)t] \\ + \sin[(n+m)\alpha + (n-k_n)t] \\ - \sin[(n+m)\alpha - (n-k_n)t] \end{matrix} \right\} \quad \textbf{(34a)}$$

Collecting similar terms of equation (**33a**) and (**34a**),

$$\beta_{nq} = \sum_{m=0}^{m=\infty} - \frac{A_{nq}}{4} k_n \left(p_{mc}{}^n + p_{ms}{}^n\right). \left\{ \begin{array}{l} \sin\left[(n-m)\,\alpha + (n-k_n)\,t\right] \\ +\sin\left[(n+m)\,\alpha + (n-k_n)\,t\right] \end{array} \right\}$$

$$+ \sum_{m=0}^{m=\infty} \frac{A_{nq}}{4} \left(p_{mc}{}^n - p_{ms}{}^n\right) \left\{ \begin{array}{l} \sin\left[(n-m)\,\alpha - (n-k_n)\,t\right] \\ +\sin\left[(n+m)\,\alpha - (n-k_n)\,t\right] \end{array} \right\} \qquad \textbf{(35a)}$$

For any given values of m and n, equation (**35a**) comprises four rotating waves. The order of harmonic space distribution is given by the coefficient of α and the speed and direction of rotation is the negative ratio of the coefficient of t to the coefficient of α. These waves are in space quadrature (on the harmonic wavelength) with the corresponding waves in equation (**16a**).

The velocity of a wave with respect to the armature is the sum of the speed of the rotor and the speed of the wave with respect to the rotor, or

$$\omega_{an} = 1.0 + \omega_{rn} \qquad \textbf{(36a)}$$

By means of this relation, waves rotating with respect to the rotor may be readily converted into waves rotating with respect to the armature. Consider the first rotating wave in equation (**16a**). Its speed with respect to the rotor is $-\dfrac{(n-k_n)}{n+m}$. From (**36a**) its speed with respect to the armature is

$$-\frac{(n-k_n)}{n+m} + 1.0 \text{ or } \frac{m+k_n}{(n+m)}$$

Also at $t = 0$ it is of cosine distribution with respect to the direct axis of the rotor and is also of cosine space distribution with respect to the axis of phase 1 since at this instant these two axes coincide. The other rotating waves of (**16a**) may be treated in a similar way giving the following rotating waves with respect to the armature.

$$\beta_{nda} = \sum_{m=0}^{m=\infty} \frac{A_{nd}}{4} \left(p_{mc}{}^n + p_{ms}{}^n\right). \left\{ \begin{array}{l} \cos\left[(n+m)\,\gamma - (k_n+m)\,t\right] \\ +\cos\left[(n-m)\,\gamma - (k_n-m)\,t\right] \end{array} \right\}$$

$$+ \sum_{m=0}^{m=\infty} \frac{A_{nd}}{4} \left(p_{mc}{}^n - p_{ms}{}^n\right). \left\{ \begin{array}{l} \cos\left[(n+m)\,\gamma - (2\,n+m-k_n)\,t\right] \\ +\cos\left[(n-m)\,\gamma - (2\,n-m-k_n)\,t\right] \end{array} \right\} \qquad \textbf{(37a)}$$

where

γ = electrical angle reckoned from the axis of phase 1 in the direction of rotation of the rotor and is measured on the fundamental base.

Similarly the waves of equation (**35a**) may be converted into rotating waves with respect to the armature. When $t = 0$, these waves are of sine distribution with respect to the direct axis of the rotor and are therefore of sine distribution with respect to the axis of phase 1 since at this instant the two axes coincide. The rotating waves thus obtained are

$$\beta_{nqa} = \sum_{m=0}^{m=\infty} - \frac{A_{nq}}{4} k_n \left(p_{mc}{}^n + p_{ms}{}^n\right). \left\{ \begin{array}{l} \sin\left[(n-m)\,\gamma - (k_n-m)\,t\right] \\ +\sin\left[(n+m)\,\gamma - (k_n+m)\,t\right] \end{array} \right\}$$

$$+ \sum_{m=0}^{m=\infty} \frac{A_{nq}}{4} k_n \left(p_{mc}{}^n - p_{ms}{}^n\right). \left\{ \begin{array}{l} \sin\left[(n-m)\,\gamma - (2\,n-m-k_n)\,t\right] \\ +\sin\left[(n+m)\,\gamma - (2\,n+m-k_n)\,t\right] \end{array} \right\} \qquad \textbf{(38a)}$$

When equation (**37a**) is expanded, each rotating wave gives two stationary components with respect to the armature, but pulsating in time. One of these components is of cosine space distribution and produces flux linkages with phase 1, while the other is of sine distribution and produces no flux linkages with this phase. Hence the only terms which induce voltage in phase 1 are the cosine terms which are

$$\beta'_{nda} = \sum_{m=0}^{m=\infty} \frac{A_{nd}}{4} \left(p_{mc}{}^n + p_{ms}{}^n\right). \left\{ \begin{array}{l} \cos\,(n+m)\,\gamma\,\cos\,(k_n+m)\,t \\ +\cos\,(n-m)\,\gamma\,\cos\,(k_n-m)\,t \end{array} \right\}$$

$$+ \sum_{m=0}^{m=\infty} \frac{A_{nd}}{4} \left(p_{mc}{}^n - p_{ms}{}^n\right). \left\{ \begin{array}{l} \cos\,(n+m)\,\gamma\,\cos\,(2\,n+m-k_n)\,t \\ +\cos\,(n-m)\,\gamma\,\cos\,(2\,n-m-k_n)\,t \end{array} \right\} \qquad \textbf{(39a)}$$

The net flux which enters the armature in a pole span of phase 1 is the area of equation (**39a**). The flux contained in a fundamental wave of unit amplitude is the area of a cosine wave. Hence the per cent flux contained in equation (**39a**) is

$$\phi_{nda} = \frac{\displaystyle\int_{-\pi/2}^{+\pi/2} \beta'_{nda}\, d\,\gamma}{\displaystyle\int_{-\pi/2}^{\pi/2} 1.0 \cos\gamma\, d\,\gamma} = \frac{1}{2} \int_{-\pi/2}^{\pi/2} \beta'_{nda}\, d\,\gamma \qquad \textbf{(40a)}$$

Therefore,

$$\phi_{nda} = \sum_{m=0}^{m=\infty} \frac{A_{nd}}{4} \left(p_{mc}{}^n + p_{ms}{}^n\right).$$

$$\left\{ \frac{\sin (n+m)\, \pi/2 \cos (k_n + m)\, t}{n+m} + \frac{\sin (n-m)\, \pi/2 \cos (k_n - m)\, t}{n-m} \right\}$$

$$+ \sum_{m=0}^{m=\infty} \frac{A_{nd}}{4} (p_{mc}{}^n - p_{ms}{}^n) \cdot \left\{ \frac{\sin (n+m)\, \pi/2 \cos (2\,n + m - k_n)\, t}{n+m} + \frac{\sin (n-m)\, \pi/2 \cos (2\,n - m - k_n)\, t}{n-m} \right\} \qquad \textbf{(41a)}$$

The flux linkages which this flux produces are

$$\Omega_{nda} = C_{(n \pm m)}\, \phi_{nda} \qquad \textbf{(42a)}$$

where

$C_{n \pm m}$ = reduction factor due to pitch, distribution, and connection of the armature coils for the *flux* distribution considered. It is the product of the individual factors and is expressed as a fraction of that reduction factor which is applied in calculating voltage generated by a fundamental flux wave. This differs in general from C_n, the corresponding factor for the m. m. f., as indicated by the subscripts.

Then,

$$\Omega_{nda} = \sum_{m=0}^{m=\infty} \frac{A_{nd}}{4} (p_{mc}{}^n + p_{ms}{}^n) \cdot \left\{ \frac{C_{n+m} \sin (n+m)\, \pi/2 \cos (k_n + m)\, t}{n+m} + \frac{C_{n-m} \sin (n-m)\, \pi/2 \cos (k_n - m)\, t}{n-m} \right\}$$

$$+ \sum_{m=0}^{m=\infty} \frac{A_{nd}}{4} (p_{mc}{}^n - p_{ms}{}^n) \cdot \left\{ \frac{C_{n+m} \sin (n+m)\, \pi/2 \cos (2\,n + m - k_n)\, t}{n+m} + \frac{C_{n-m} \sin (n-m)\, \pi/2 \cos (2\,n - m - k_n)\, t}{n-m} \right\} \qquad \textbf{(43a)}$$

The per cent voltage induced is

$$e_{nd} = - \frac{d\, \Omega_{nda}}{d\, t} \qquad \textbf{(44a)}$$

or

$$e_{nd} = \sum_{m=0}^{m=\infty} \frac{A_{nd}}{4} (p_{mc}{}^n + p_{ms}{}^n) \cdot \left\{ C_{(n+m)} \frac{(k_n + m)}{(n+m)} \sin (n+m)\, \pi/2 \sin (k_n + m)\, t + C_{(n-m)} \frac{(k_n - m)}{(n-m)} \sin (n-m)\, \pi/2 \sin (k_n - m)\, t \right\}$$

$$+ \sum_{m=0}^{m=\infty} \frac{A_{nd}}{4} (p_{mc}{}^n - p_{ms}{}^n) \cdot \left\{ C_{(n+m)} \frac{(2\,n + m - k_n)}{(n+m)} \sin (n+m)\, \pi/2 \sin (2\,n + m - k_n)\, t + C_{(n-m)} \frac{(2\,n - m - k_n)}{(n-m)} \sin (n-m)\, \pi/2 \sin (2\,n - m - k_n)\, t \right\} \qquad \textbf{(45a)}$$

In a similar manner, the voltages produced by the flux waves in equation (**38a**) are obtained. When this equation is expanded, each rotating wave gives two stationary components with respect to the armature, but pulsating in time. One of these components is of cosine space distribution and produces flux linkages with phase 1, while the other is of sine distribution and produces no flux linkages with this phase. Hence, the only terms which induce voltage in phase 1 are

$$\beta'_{nqa} = \sum_{m=0}^{m=\infty} \frac{A_{nq}}{4} k_n (p_{mc}{}^n + p_{ms}{}^n) \cdot \left\{ \cos (n-m)\, \gamma \sin (k_n - m)\, t + \cos (n+m)\, \gamma \sin (k_n + m)\, t \right\}$$

$$+ \sum_{m=0}^{m=\infty} - \frac{A_{nq}}{4} k_n (p_{mc}{}^n - p_{ms}{}^n) \cdot \left\{ \cos (n-m)\, \gamma \sin (2\,n - m - k_n)\, t + \cos (n+m)\, \gamma \sin (2\,n + m - k_n)\, t \right\} \qquad \textbf{(46a)}$$

The net flux which enters the armature in a pole span of phase 1 is the area of equation (**46a**). The flux contained in a fundamental wave of unit amplitude is the area of a cosine wave. Hence, the per cent flux contained in equation (**39a**) is

$$\phi_{nqa} = \frac{\int_{-\pi/2}^{+\pi/2} \beta'_{nqa}\, d\, \gamma}{\int_{-\pi/2}^{\pi/2} 1.0 \cos \gamma\, d\, \gamma} = \frac{1}{2} \int_{-\pi/2}^{\pi/2} \beta'_{nqa}\, d\, \gamma \qquad \textbf{(47a)}$$

Therefore,

$$\phi_{nqa} = \sum_{m=0}^{m=\infty} \frac{A_{nq}}{4} k_n (p_{mc}{}^n + p_{ms}{}^n) \cdot$$

$$\left\{\begin{array}{l} \dfrac{\sin (n-m)\,\pi/2 \sin (k_n - m)\,t}{n-m} \\[2ex] + \dfrac{\sin (n+m)\,\pi/2 \sin (k_n + m)\,t}{(n+m)} \end{array}\right\}$$

$$+ \sum_{m=0}^{m=\infty} - \frac{A_{nq}}{4} k_n (p_{mc}{}^n - p_{ms}{}^n)\,.$$

$$\left\{\begin{array}{l} \dfrac{\sin (n-m)\,\pi/2 \sin (2\,n - m - k_n)\,t}{n-m} \\[2ex] + \dfrac{\sin (n+m)\,\pi/2 \sin (2\,n + m - k_n)\,t}{n+m} \end{array}\right\} \qquad \textbf{(48a)}$$

The flux linkages which this flux produces are

$$\Omega_{nqa} = C_{(n \pm m)}\, \phi_{nqa} \qquad \textbf{(49a)}$$

Then,

$$\Omega_{nqa} = \sum_{m=0}^{m=\infty} \frac{A_{nq}}{4} k_n (p_{mc}{}^n + p_{ms}{}^n)\,.$$

$$\left\{\begin{array}{l} \dfrac{C_{(n-m)} \sin (n-m)\,\pi/2 \sin (k_n - m)\,t}{n-m} \\[2ex] + \dfrac{C_{(n+m)} \sin (n+m)\,\pi/2 \sin (k_n + m)\,t}{n+m} \end{array}\right\}$$

$$+ \sum_{m=0}^{m=\infty} - \frac{A_{nq}}{4} k_n (p_{mc}{}^n - p_{ms}{}^n)\,.$$

$$\left\{\begin{array}{l} \dfrac{C_{(n-m)} \sin (n-m)\,\pi/2 \sin (2\,n - m - k_n)\,t}{n-m} \\[2ex] + \dfrac{C_{(n+m)} \sin (n+m)\,\pi/2 \sin (2\,n + m - k_n)\,t}{n+m} \end{array}\right\} \qquad \textbf{(50a)}$$

The per cent voltage induced is

$$e_{nq} = - \frac{d\,\Omega_{nqa}}{d\,t} \qquad \textbf{(51a)}$$

$$e_{nq} = \sum_{m=0}^{m=\infty} - \frac{A_{nq}}{4} k_n (p_{mc}{}^n + p_{ms}{}^n)\,.$$

$$\left\{\begin{array}{l} C_{(n-m)} \dfrac{(k_n - m)}{(n-m)} \sin (n-m)\,\pi/2 \cos (k_n - m)\,t \\[2ex] + C_{(n+m)} \dfrac{(k_n + m)}{(n+m)} \sin (n+m)\,\pi/2 \cos (k_n + m)\,t \end{array}\right\}$$

$$+ \sum_{m=0}^{m=\infty} + \frac{A_{nq}}{4} k_n (p_{mc}{}^n - p_{ms}{}^n)\,.$$

$$\left\{\begin{array}{l} C_{(n-m)} \dfrac{(2\,n - m - k_n)}{(n-m)} \sin (n-m) \\ \qquad\qquad \pi/2 \cos (2\,n - m - k_n)t \\[2ex] + C_{(n+m)} \dfrac{(2\,n + m - k_n)}{(n+m)} \sin (n+m) \\ \qquad\qquad \pi/2 \cos (2\,n + m - k_n)t \end{array}\right\} \qquad \textbf{(52a)}$$

Making $p_{mc}{}^n = p_{ms}{}^n = p_m{}^n$, as concluded in Appendix B, equations (**45a**) and (**52a**) become

$$e_{nd} = \sum_{m=0}^{m=\infty} \frac{A_{nd}}{2} p_m{}^n\,.$$

$$\left\{\begin{array}{l} C_{(n+m)} \dfrac{(k_n + m)}{(n+m)} \sin (n+m)\,\pi/2 \sin (k_n + m)\,t \\[2ex] + C_{(n-m)} \dfrac{(k_n - m)}{(n-m)} \sin (n-m)\,\pi/2 \sin (k_n - m)\,t \end{array}\right\} \qquad \textbf{(53a)}$$

$$e_{nq} = \sum_{m=0}^{m=\infty} - \frac{A_{nq}}{2} k_n\, p_m{}^n$$

$$\left\{\begin{array}{l} C_{(n-m)} \dfrac{(k_n - m)}{(n-m)} \sin (n-m)\,\pi/2 \cos (k_n - m)\,t \\[2ex] + C_{(n+m)} \dfrac{(k_n + m)}{(n+m)} \sin (n+m)\,\pi/2 \cos (k_n + m)\,t \end{array}\right\} \qquad \textbf{(54a)}$$

In a square wave the amplitude of any harmonic m. m. f. expressed as a fraction of the fundamental m. m. f. is

$$A_n = \pm \frac{A_1}{n} \qquad \textbf{(55a)}$$

When each phase comprises more than one coil, this must be multiplied by C_n or

$$A_n = \pm \frac{A_1}{n} C_n \qquad \textbf{(56a)}$$

The sign of A_n depends on the value of n and may be expressed

$$A_n = - (-1)^{\frac{1+n}{2}} \frac{A_1}{n} C_n \qquad \textbf{(57a)}$$

Substituting (**57a**) in (**53a**) and (**54a**),

$$e_{nd} = \sum_{m=0}^{m=\infty} - (-1)^{\frac{1+n}{2}} \frac{A_{1d}}{2\,n} C_n\, p_m{}^n.$$

$$\left\{\begin{array}{l} C_{(n+m)} \dfrac{(k_n + m)}{(n+m)} \sin (n+m)\,\pi/2 \sin (k_n + m)\,t \\[2ex] + C_{(n-m)} \dfrac{(k_n - m)}{(n-m)} \sin (n-m)\,\pi/2 \sin (k_n - m)\,t \end{array}\right\} \qquad \textbf{(58a)}$$

$$e_{nq} = \sum_{m=0}^{m=\infty} (-1)^{\frac{1+n}{2}} \frac{A_{1q}}{2\,n} k_n\, C_n\, p_m{}^n .$$

$$\left\{ \begin{array}{l} C_{(n-m)} \dfrac{(k_n - m)}{(n - m)} \sin (n - m)\, \pi/2 \cos (k_n - m)\, t \\ \\ +C_{(n+m)} \dfrac{(k_n + m)}{(n + m)} \sin (n + m)\, \pi/2 \cos (k_n + m)\, t \end{array} \right\} \qquad \textbf{(59a)}$$

where m = any positive even integer, including zero. For certain values of m, $(n - m)$ and $(k_n - m)$ may be negative. As a result, the subscript of C may be negative, which has no significance. However, any other negative sign introduced in the equation by such values of m is significant and must be retained.

These are the final voltage equations in terms of the space fundamental m. m. f. Although it is possible to obtain from these equations all harmonic voltages produced, under the assumptions, by the fundamental (in time) current, nevertheless the voltages of immediate interest are only those of fundamental frequency, which thus become a factor in the reactances.

Table I has therefore been prepared giving all fundamental voltages produced by the armature current, their magnitude and phase, and the harmonic m. m. fs. and fluxes which produce them. When the phase of a voltage is given, it is with reference to the component of armature current producing it.

TABLE I

FUNDAMENTAL VOLTAGES

A. Due to Direct Component of Armature Current.

n	m	k_n	Order of Flux Wave	Amplitude and Phase of Fundamental Voltages
1	0	1	1	$-j\, A_{1d}\, p_0{}^{\mathrm{I}}$
1	2	1	1	$-j\, \dfrac{A_{1d}\, p_2{}^{\mathrm{I}}}{2}$
5	0	−1	5	$-j\, \dfrac{A_{1d}\, C^2{}_5\, p_0{}^{\mathrm{V}}}{25}$
5	2	−1	7	$+j\, \dfrac{A_{1d}\, C_5\, C_7\, p_2{}^{\mathrm{V}}}{70}$
7	0	1	7	$-j\, \dfrac{A_{1d}\, C^2{}_7\, p_0{}^{\mathrm{VII}}}{49}$
7	2	1	5	$+j\, \dfrac{A_{1d}\, C_7\, C_5\, p_2{}^{\mathrm{VII}}}{70}$
11	0	−1	11	$-j\, \dfrac{A_{1d}\, C^2{}_{11}\, p_0{}^{\mathrm{XI}}}{121}$
11	2	−1	13	$+j\, \dfrac{A_{1d}\, C_{11}\, C_{13}\, p_2{}^{\mathrm{XI}}}{286}$
13	0	1	13	$-j\, \dfrac{A_{1d}\, C^2{}_{13}\, p_0{}^{\mathrm{XIII}}}{169}$
13	2	1	11	$+j\, \dfrac{A_{1d}\, C_{13}\, C_{11}\, p_2{}^{\mathrm{XIII}}}{286}$

B. Due to Quadrature Component of Armature Current

n	m	k_n	Order of Flux Wave	Amplitude and Phase of Fundamental Voltages
1	0	1	1	$-j\, A_{1q}\, p_0{}^{\mathrm{I}}$
1	2	1	1	$+j\, \dfrac{A_{1q}\, p_2{}^{\mathrm{I}}}{2}$
5	0	−1	5	$-j\, \dfrac{A_{1q}\, C^2{}_5\, p_0{}^{\mathrm{V}}}{25}$
5	2	−1	7	$-j\, \dfrac{A_{1q}\, C_5\, C_7\, p_2{}^{\mathrm{V}}}{70}$
7	0	1	7	$-j\, \dfrac{A_{1q}\, C^2{}_7\, p_0{}^{\mathrm{VII}}}{49}$
7	2	1	5	$-j\, \dfrac{A_{1q}\, C_5\, C_7\, p_2{}^{\mathrm{VII}}}{70}$
11	0	−1	11	$-j\, \dfrac{A_{1q}\, C^2{}_{11}\, p_0{}^{\mathrm{XI}}}{121}$
11	2	−1	13	$-j\, \dfrac{A_{1q}\, C_{11}\, C_{13}\, p_2{}^{\mathrm{XI}}}{286}$
13	0	1	13	$-j\, \dfrac{A_{1q}\, C^2{}_{13}\, p_0{}^{\mathrm{XIII}}}{169}$
13	2	1	11	$-j\, \dfrac{A_{1q}\, C_{13}\, C_{11}\, p_2{}^{\mathrm{XIII}}}{286}$

In the above, $A_{1d} = i_d$ and $A_{1q} = i_q$. See "notation."

Appendix B

DETERMINATION OF THE PERMEANCE OF AN AIR-GAP OF UNIFORM LENGTH

When the magnetomotive force of the armature winding is divided into sinusoidally distributed components of different numbers of poles, the question arises as to whether the permeance presented to all the harmonic components is the same. For, if the air-gap is of appreciable length in comparison with the pole pitch of the magnetomotive force, the flux is not truly radial in crossing the gap, but a certain part of it flows in a peripheral direction and returns to the armature without entering the rotor.

In order to determine the permeance of the air-gap for all harmonics of m. m. f., it is necessary to derive the equations for the flux distribution in the air-gap, in accordance with Laplace's equations. It is unnecessary, besides being rather laborious, to determine this permeance variation for a salient pole machine, as it is an independent phenomenon from the variation due to the poles themselves.

In all that follows, therefore, the harmonic m. m. f. will be assumed to act over a uniform gap. With these conditions in mind, let

R = radius to any point in the air-gap space in cm.

R_a = radius to armature surface nearest the air-gap in cm.
R_f = radius to rotor surface in cm.
λ = mechanical angular displacement on armature periphery in radians.
γ = electrical angular displacement on the armature periphery.
q = number of poles on the rotor.
n = order of the space harmonic m. m. f. distributed on the armature surface.
τ = fundamental pole pitch in cm.
g = radial (constant) length of air-gap in cm.
A = magnetomotive force in gilberts.
p = permeance $= \dfrac{\beta}{A}$.
β_R = radial component of flux density at any point in the air-gap in gausses.
β_λ = tangential component of flux density at any point in the air-gap in gausses.
ϕ = total flux per pole per unit length of core in maxwells.

Knowing that the lines of induction are continuous, or that the total flux entering any volume is equal to that leaving it, we may derive the fundamental differential equation

$$\frac{\partial \beta_\lambda}{\partial \lambda} + \beta_R + R\frac{\partial \beta_R}{\partial R} = 0 \qquad \textbf{(1b)}$$

Also, since the line integral of the m. m. f. around any area in a cross-section of the air-gap is zero (since there are no currents flowing through the area in a direction perpendicular to its plane), we can derive the second fundamental differential equation

$$\frac{\partial \beta_R}{\partial \lambda} - R\frac{\partial \beta_\lambda}{\partial R} - \beta_\lambda = 0 \qquad \textbf{(2b)}$$

This last equation is only true when the permeability is constant throughout the area, as in the case of the air-gap we are considering.

Differentiating (**1b**) and (**2b**) with respect to R and λ,

$$\frac{\partial^2 \beta_\lambda}{\partial R\,\partial \lambda} + 2\frac{\partial \beta_R}{\partial R} + R\frac{\partial^2 \beta_R}{\partial R^2} = 0 \qquad \textbf{(3b)}$$

$$\frac{\partial^2 \beta_\lambda}{\partial \lambda^2} + \frac{\partial \beta_R}{\partial \lambda} + R\frac{\partial^2 \beta_R}{\partial \lambda\,\partial R} = 0 \qquad \textbf{(4b)}$$

$$\frac{\partial^2 \beta_R}{\partial R\,\partial \lambda} - R\frac{\partial^2 \beta_R}{\partial R^2} - 2\frac{\partial \beta_\lambda}{\partial R} = 0 \qquad \textbf{(5b)}$$

$$\frac{\partial^2 \beta_R}{\partial \lambda^2} - R\frac{\partial^2 \beta_\lambda}{\partial \lambda\,\partial R} - \frac{\partial \beta_\lambda}{\partial \lambda} = 0 \qquad \textbf{(6b)}$$

Eliminating $\dfrac{\partial^2 \beta_\lambda}{\partial R\,\partial \lambda}$ from (**3b**) and (**6b**) and $\dfrac{\partial^2 \beta_R}{\partial \lambda\,\partial R}$ from (**4b**) and (**5b**), we get

$$\frac{1}{R^2}\frac{\partial^2 \beta_R}{\partial \lambda^2} + \frac{\partial^2 \beta_R}{\partial R^2} + \frac{2}{R}\frac{\partial \beta_R}{\partial R} - \frac{1}{R^2}\frac{\partial \beta_\lambda}{\partial \lambda} = 0 \qquad \textbf{(7b)}$$

$$\frac{1}{R^2}\frac{\partial^2 \beta_\lambda}{\partial \lambda^2} + \frac{\partial^2 \beta_\lambda}{\partial R^2} + \frac{2}{R}\frac{\partial \beta_\lambda}{\partial R} + \frac{1}{R^2}\frac{\partial \beta_R}{\partial \lambda} = 0 \qquad \textbf{(8b)}$$

Eliminating $\dfrac{\partial \beta_\lambda}{\partial \lambda}$ from (**1b**) and (**7b**) and $\dfrac{\partial \beta_R}{\partial \lambda}$ from (**2b**) and (**6b**), we get

$$\frac{1}{R^2}\frac{\partial^2 \beta_R}{\partial \lambda^2} + \frac{\partial^2 \beta_R}{\partial R^2} + \frac{3}{R}\frac{\partial \beta_R}{\partial R} + \frac{\beta_R}{R^2} = 0 \qquad \textbf{(9b)}$$

$$\frac{1}{R^2}\frac{\partial^2 \beta_\lambda}{\partial \lambda^2} + \frac{\partial^2 \beta_\lambda}{\partial R^2} + \frac{3}{R}\frac{\partial \beta_\lambda}{\partial R} + \frac{\beta_\lambda}{R^2} = 0 \qquad \textbf{(10b)}$$

These equations are of the same form and have the same solutions. These solutions are

$$\beta_R = [B_1 R^{\rho-1} + C_1{}^{-\rho-1}] \sin \rho\,(\lambda - \lambda_1) \qquad \textbf{(11b)}$$

and

$$\beta_\lambda = [B_2 R^{\rho-1} + C_2{}^{-\rho-1}] \sin \rho\,(\lambda - \lambda_2) \qquad \textbf{(12b)}$$

At the surface of the armature, *i. e.*, $R = R_a$, the peripheral component of flux density must equal the peripheral rate of change of m. m. f., assuming unit permeability. If the m. m. f. at the armature surface is

$$a_n = A_n \cos n\,\gamma = A_n \cos \frac{n\,q}{2}\,\lambda \qquad \textbf{(13b)}$$

then

$$[B_2 R_a{}^{\rho-1} + C_2 R_a{}^{-\rho-1}] \sin \rho\,(\lambda - \lambda_2)$$

$$= \frac{d\left(A_n \cos \frac{n\,q}{2}\,\lambda\right)}{d\,(R_a\,\lambda)} = -\frac{n\,q}{2\,R_a} A_n \sin \frac{n\,q}{2}\,\lambda \qquad \textbf{(14b)}$$

From (**14b**)

$$B_2 R_a{}^{\rho-1} + C_2 R_a{}^{-\rho-1} = -\frac{n\,q}{R_a} A_n \qquad \textbf{(15b)}$$

and

$$\rho\,(\lambda - \lambda_2) = \frac{n\,q}{2}\,\lambda \qquad \textbf{(16b)}$$

When $\lambda = 0$, $\lambda - \lambda_2 = 0$, hence

$$\lambda_2 = 0 \qquad \textbf{(17b)}$$

and

$$\rho = \frac{n\,q}{2} \qquad \textbf{(18b)}$$

Then

$$\beta_\lambda = \left[B_2 R^{\frac{nq}{2}-1} + C_2 R^{\frac{-nq}{2}-1} \right] \sin \frac{nq}{2} \lambda \qquad \textbf{(19b)}$$

The potential of the rotor surface is zero at all points and hence the peripheral flux density at the surface of the rotor, *i. e.*, $R = R_f$, is always zero. Then from (**19b**),

$$B_2 R_f^{\frac{nq}{2}-1} + C_2 R_f^{\frac{-nq}{2}-1} = 0 \qquad \textbf{(20b)}$$

or

$$C_2 = - B_2 R_f^{nq} \qquad \textbf{(21b)}$$

Substituting (**21b**) in (**15b**) and solving for B_2,

$$B_2 = - \frac{n q A_n R_a^{\frac{nq}{2}}}{2 (R_a^{nq} - R_f^{nq})} \qquad \textbf{(22b)}$$

From (**21b**),

$$C_2 = \frac{n q A_n R_a^{\frac{nq}{2}} R_f^{nq}}{2 (R_a^{nq} - R_f^{nq})} \qquad \textbf{(23b)}$$

Substituting (**22b**) and (**23b**) in (**19b**),

$$\beta_\lambda = - \frac{n q A_n R_a^{\frac{nq}{2}}}{2 (R_a^{nq} - R_f^{nq})} \cdot \left[R^{\frac{nq}{2}-1} - R_f^{nq} R^{\frac{-nq}{2}-1} \right] \sin \frac{nq}{2} \lambda \qquad \textbf{(24b)}$$

Substituting (**24b**) in (**2b**),

$$\frac{\partial \beta_R}{\partial \lambda} = - \frac{n^2 q^2 A_n R_a^{\frac{nq}{2}}}{4(R_a^{nq} - R_f^{nq})} \cdot \left[R^{\frac{nq}{2}-1} + R_f^{nq} R^{\frac{-nq}{2}-1} \right] \sin \frac{nq}{2} \lambda \qquad \textbf{(25b)}$$

or

$$\beta_R = \frac{n q A_n R_a^{\frac{nq}{2}}}{2 (R_a^{nq} - R_f^{nq})} \cdot \left[R^{\frac{nq}{2}-1} + R_f^{nq} R^{\frac{-nq}{2}-1} \right] \cos \frac{nq}{2} \lambda + D \qquad \textbf{(26b)}$$

D must be zero in order that eq. (**1b**) be satisfied. Hence,

$$\beta_R = \frac{n q A_n R_a^{\frac{nq}{2}}}{2 (R_c^{nq} - R_f^{nq})} \cdot \left[R^{\frac{nq}{2}-1} + R_f^{nq} R^{\frac{-nq}{2}-1} \right] \cos \frac{nq}{2} \lambda \qquad \textbf{(27b)}$$

Since we are interested primarily in β_R at the armature surface, *i. e.*, $R = R_a$, we get

$$\beta_{R = R_a} = \frac{n q A_n}{2 R_a} \frac{R_a^{nq} + R_f^{nq}}{R_a^{nq} - R_f^{nq}} \cos \frac{nq}{2} \lambda \qquad \textbf{(28b)}$$

Eq. (**24b**) and (**27b**) are true whether R_a is greater or less than R_f. Eq. (**27b**) shows that the radial flux density at any point in the gap has a truly sinusoidal distribution exactly proportional to the m. m. f., just as would be expected. But, the ratio of the flux density to the m. m. f. depends on the order of the harmonic, and so it is clear that the permeance of a given machine is different for every harmonic m. m. f.

Since the ratio of R_a to the gap is ordinarily very large, it is interesting to find the permeance for the condition $g/R_a = 0$. The permeance is the coefficient of the m. m. f. in eq. (**28b**) or

$$p = \frac{nq}{2 R_a} \frac{Ra^{nq} + R_f^{nq}}{R_a^{nq} - R_f^{nq}} \qquad \textbf{(29b)}$$

Since

$$R_f = R_a - g \qquad \textbf{(30b)}$$

$$p = \frac{nq}{2 R_a} \frac{R_a^{nq} + (R_a - g)^{nq}}{R_a^{nq} - (R_a - g)^{nq}} \qquad \textbf{(31b)}$$

$$= \frac{nq}{2 R_a} \frac{1 + \left(1 - \frac{g}{R_a}\right)^{nq}}{1 - \left(1 - \frac{g}{R_a}\right)^{nq}} \qquad \textbf{(32b)}$$

Since the fundamental pole pitch is

$$\tau = \frac{2 \pi R_a}{q} \qquad \textbf{(33b)}$$

$$\frac{nq}{2 R_a} = \frac{n \pi}{\tau} \qquad \textbf{(34b)}$$

and

$$n q = \frac{2 \pi g n}{\tau} \frac{1}{g/R_a} \qquad \textbf{(35b)}$$

eq. (**32b**) becomes

$$p = \frac{n \pi}{\tau} \frac{1 + \left(1 - \frac{g}{R_a}\right)^{\frac{2 \pi g n}{\tau} \frac{1}{g/R_a}}}{1 - \left(1 - \frac{g}{R_a}\right)^{\frac{2 \pi g n}{\tau} \frac{1}{g/R_a}}} \qquad \textbf{(36b)}$$

As g/R_a approaches zero as a limit,

$$p = \frac{n \pi}{\tau} \frac{1 + e^{\frac{-2 \tau m g}{\tau}}}{1 - e^{\frac{-2 \tau m g}{\tau}}} \qquad \textbf{(37b)}$$

or

$$p = \frac{n \pi}{\tau} \frac{e^{\frac{n \pi g}{\tau}} + e^{\frac{-n \pi g}{\tau}}}{e^{\frac{n \pi g}{\tau}} - e^{\frac{-n \pi g}{\tau}}} = \frac{n \pi}{\tau} \coth \frac{n \pi g}{\tau} \qquad \textbf{(38b)}$$

$$p = \frac{1}{g} \frac{n \pi g}{\tau} \coth \frac{n \pi g}{\tau} \qquad \textbf{(39b)}$$

This is of the form

$$p = \frac{1}{g}\, y \coth y \qquad \textbf{(40b)}$$

The function, $y \coth y$, is plotted in Fig. 10.

When R_f is zero, corresponding to the condition of rotor removed, eq. (**29b**) becomes

$$p = \frac{n\,q}{2\,R_a} \qquad \textbf{(41b)}$$

$$R_f = 0$$

or from (**34b**)

$$p = \frac{n\,\pi}{\tau} \qquad \textbf{(42b)}$$

$$R_f = 0$$

These equations show that, when the air-gap is small compared to the pole pitch of the harmonic wave, the permeance is simply equal to the reciprocal of the air-gap length, as ordinarily assumed. When the air-gap is very large, *i. e.*, $R_f = 0$, the permeance is a function of the order of the space harmonic of m. m. f. as given in eq. (**41b**).

These are the results which were sought in this appendix. It may be interesting, however, to connect up the results with some other things that are closely related to them. Rudenberg[*] utilized the same differential equations to determine the distribution of flux density in an armature core of known radial depth, and constant permeability. His initial conditions were the same as those used above, except that, instead of making β_λ equal to zero, he made β_R equal to zero, when R is equal to R_f. In other words, he considered the case of the flux entering a core of high uniform permeability within which it was strictly confined, while, in this appendix, the case of an air-gap bounded at its surface by a circular field structure of infinite permeability has been considered. Rudenberg's equations were used by Alger and Eksergian[†] to determine the losses in an armature core and the total flux per pole produced by a sinusoidal m. m. f. in such a core. The results they derived for the case of a zero inner radius of core are exactly the same as those here derived for the case of a zero radius of field structure. Both investigations show that the flux per pole per unit length of core produced by a sinusoidally distributed m. m. f. applied to an air core is exactly equal to the maximum of the m. m. f. wave, or

$$\phi = A \qquad \textbf{(43b)}$$

This is a very interesting result and one that is extremely useful in the calculation of the reactance of a stator with rotor removed, or of a rotor with stator removed. For example, if it is desired to determine the leakage reactance of an induction motor which cannot be assembled, it is possible for relatively long core lengths to measure the reactance of the rotor and stator separately and to subtract from each the reactance indicated by equation (**43b**), and thus to obtain the total leakage reactance of the machine.

The final form, eq. (**39b**), for permeance is based on the assumption that the ratio of the air-gap to the radius of curvature of the armature surface is very small.

In the application of the results to salient pole machines, a curve of "equivalent air-gap" is obtained by plotting the field distribution for constant magnetic potential difference between the armature surface and the field pole surface, as shown in Figs. 5 and 21. This approximation gives reasonably accurate results when applied to the case of zero armature curvature, as shown in the results in Table II, derived from the plots in Figs. 5, 6, 7, 8 and 9, and shown in Figs. 21 and 11.

TABLE II

COMPARISON OF PERMEANCES OBTAINED DIRECTLY FROM FIELD PLOTS, AND DERIVED FROM FIGS. 10 AND 21.

	Values from plots	Values Derived from Figs. 10 and 21
p_{0c}^{I}	$0.67\ p^0_g$	$0.66\ p^0_g$
p_{0s}^{I}	$0.67\ p^0_g$	
p_{2c}^{I}	$0.43\ p^0_g$	$0.44\ p^0_g$
p_{2s}^{I}	$0.47\ p^0_g$	
p_{0c}^{V}	$0.74\ p^0_g$	$0.73\ p^0_g$
p_{0s}^{V}	$0.73\ p^0_g$	
p_{2c}^{V}	$0.38\ p^0_g$	$0.37\ p^0_g$
p_{2s}^{V}	$0.37\ p^0_g$	

$$\frac{\text{min. gap}}{\text{pole pitch}} = 0.02 \qquad \frac{\text{pole arc}}{\text{pole pitch}} = \frac{2}{3}$$

$$\frac{\text{max.}}{\text{min.}}\ \text{gap} = 1.5 \qquad p_g^0 = 1.5\ p_0^{\mathrm{I}}$$

If the ratio of the maximum equivalent air-gap to the radius is less than, say, 0.15, the error in the calculated permeance will probably be less than 10%, the calculated value being too low. If the ratio is greater than 0.15, eq. (**29b**) should be used. However, only a few machines—such as 4- or possibly 6-pole machines—require such special treatment.

It might seem that the permeance series for a space harmonic m. m. f. would be considerably different for $\cos n\,\alpha$ and $\sin n\,\alpha$ distribution. However, as already shown, the permeance for a uniform gap is constant for a given order of harmonic, *i. e.*, it is the same at a given point on the armature surface regardless of the manner in which the harmonic m. m. f. is distributed with respect to that point. Hence, if any variation of permeance along the armature surface is introduced by salient pole construction, it would seem reasonable to assume that such variation is a function of only the physical dimensions and more or less independent of the manner in which the harmonic m. m. f. is distributed with respect to the rotor. This assumption should, of course, be checked by a study of actual cases. Figs. 6, 7, 8 and 9 show field plots which have been made for sine and cosine distribution of fundamental and fifth harmonic m. m. fs. For the fifth, analysis of the permeance distribution shows that the series is practically identical for either sine or cosine distribution. For the fundamental m. m. f., the agreement is not quite so good, but, for the case assumed, is quite within practical limits of accuracy. These results for the first two terms of the permeance series are shown in Table II. For higher harmonics, the agreement would be practically perfect since, as the gap becomes larger in comparison with the wavelength of the m. m. f. considered, the effect of salient pole construction on the permeance becomes less pronounced as shown in Fig. 11.

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[*] *E. T. Z.* 906, p. 109.

[†] Bulletin 31, Vol. 56, Mass. Inst. Tech., Feb., 1926.

In view of results which have been obtained, it seems justifiable to use the same permeance series for either a cosine or sine distribution of m. m. f. or

$$p_c^n = p_s^n$$

Appendix C

REACTANCE EQUATIONS

General equations have been derived in Appendix *A* for voltages induced in the armature due to the fact that the armature winding is not sinusoidal. Since only fundamental voltages enter in the calculation of reactance, these have been tabulated in Table I and expressed in terms of the space fundamental m. m. fs. On the percentage basis, A_{1d} and A_{1q} are numerically equal to i_d and i_q respectively. Hence the coefficients of A_{1d} and A_{1q} are per cent reactances (expressed as a fraction).

In this treatment, voltages induced by space fundamental m. m. fs. existing within the limits of the armature core are taken as the reactance voltages of armature reaction and voltages produced by space harmonic m. m. fs. within these boundaries are classed as leakage reactance voltages, ordinarily referred to as due to tooth tip and belt leakage.

The first two terms of parts A and B, Table I are voltages induced by space fundamental m. m. fs. Hence the equivalent reactances of armature reaction in the direct and quadrature axes are

$$x_{ad} = p_0^{\mathrm{I}} + \frac{p_2^{\mathrm{I}}}{2} \qquad \textbf{(1c)}$$

$$x_{aq} = p_0^{\mathrm{I}} - \frac{p_2^{\mathrm{I}}}{2} \qquad \textbf{(2c)}$$

All other terms in Table I are produced by space harmonic m. m. fs and are thus leakage reactance voltages. In addition there are also leakage reactance voltages due to slot and end turn fluxes which are independent of rotor position. Designating slot and end turn leakage reactances by x_s and x_e, the total leakage reactances in the two axes are respectively

$$x_{ld} = x_s + x_e + \frac{C^2{}_5\, p_0^{\mathrm{V}}}{25} - \frac{C_5\, C_7}{70}\,(p_2^{\mathrm{V}} + p_2^{\mathrm{VII}})$$

$$+ \frac{C^2{}_7\, p_0^{\mathrm{VII}}}{49} + \frac{C^2{}_{11}\, p_0^{\mathrm{XI}}}{121} - \frac{C_{11}\, C_{13}}{286}(p_2^{\mathrm{XI}} + p_2^{\mathrm{XIII}})$$

$$+ \frac{C^2{}_{13}\, p_0^{\mathrm{XIII}}}{169} + \dots \qquad \textbf{(3c)}$$

$$x_{lq} = x_s + x_e + \frac{C^2{}_5\, p_0^{\mathrm{V}}}{25} + \frac{C_5\, C_7}{70}\,(p_2^{\mathrm{V}} + p_2^{\mathrm{VII}})$$

$$+ \frac{C^2{}_7\, p_0^{\mathrm{VII}}}{49} + \frac{C^2{}_{11}\, p_0^{\mathrm{XI}}}{121} + \frac{C_{11}\, C_{13}}{286}(p_2^{\mathrm{XI}} + p_2^{\mathrm{XIII}})$$

$$+ \frac{C^2{}_{13}\, p_0^{\mathrm{XIII}})}{169} + \dots \qquad \textbf{(4c)}$$

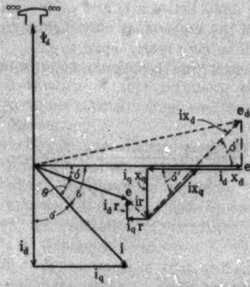

FIG. 25—SHOWING THE SALIENT POLE DIAGRAM IN TERMS OF THE CYLINDRICAL ROTOR DIAGRAM. FULL LINE VECTORS ARE FOR SALIENT POLES. DOTTED VECTORS ARE FOR A CYLINDRICAL ROTOR MACHINE OF EQUAL x_d

The total reactances in the two axes are

$$x_d = x_{ad} + x_{ld} \qquad \textbf{(5c)}$$

and

$$x_q = x_{aq} + x_{lq} \qquad \textbf{(6c)}$$

Subtracting equation (**3c**) from equation (**4c**)

$$x_{lq} - x_{ld} = \frac{C_5\, C_7}{35}\,(p_2^{\mathrm{V}} + p_2^{\mathrm{VII}})$$

$$+ \frac{C_{11}\, C_{13}}{143}\,(p_2^{\mathrm{XI}} + p_2^{\mathrm{XIII}}) + \dots \qquad \textbf{(7c)}$$

The order of this difference is in general quite small and is discussed in the body of the paper.

Appendix D

COMPARISON OF EXCITATION CALCULATIONS BY THE CYLINDRICAL ROTOR AND TWO-REACTION THEORIES

Fig. 25 illustrates both methods of calculating excitation. The solid lines refer to the two-reaction method and the dotted lines refer to the cylindrical rotor method. Since the axis of e_d is perpendicular to the axis of i_d and since $i\,x_d$ is perpendicular to i, the angle σ' is equal to the angle σ. The projection of $i\,x_d$ on the axis of e_d is thus $i\,x_d \cos\sigma' = i\,x_d \cos\sigma$. But $i \cos\sigma = i_d$. Hence the projection of $i\,x_d$ on the axis of e_d is $i_d\,x_d$, and e_d is the projection of e_{d0} (nominal voltage for a cylindrical rotor machine) on this axis.

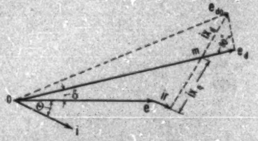

FIG. 26—EXCITATION DIAGRAM CONSTRUCTED FROM THE POTIER DIAGRAM, NEGLECTING SATURATION

Since the angle between e_d and e_{d0} is usually small, the cosine of the angle differs only slightly from unity; hence e_d will be very nearly equal to e_{d0}.

It may be seen from the diagram that the intersection of the line e_d with the line $i\,x_d$ occurs at a point distant in amount $i\,x_q$ from the base of the $i\,x_d$ vector. This construction affords a simple and rapid method for determining the true diagram by modification of the cylindrical rotor diagram.

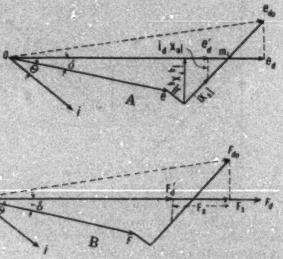

FIG. 27—EXCITATION DIAGRAM TAKING INTO ACCOUNT, APPROXIMATELY, THE EFFECT OF SATURATION

A—Voltage diagram
B—Corresponding m. m. f. diagram

Although the effect of salient pole construction on the magnitude of required excitation is *very* small, nevertheless, its effect on the displacement angle is important. This effect is discussed in Part II of the paper.

The construction of the salient pole diagram from the diagram for the cylindrical rotor may be made clearer by the following: construct the usual Potier diagram, for the case of negligible saturation, shown as the dash lines in Fig. 26. Locate on the $i\,x_d$ vector the point m, distant from the end of the $i\,r$ vector by the length $i\,x_q$. The line $o\,m$ gives the direction of the actual nominal voltage e_d. Its length is determined by dropping a perpendicular to it, from the end of the vector e_{d0}. Hence the magnitude of e_d is practically the same as e_{d0}.

Effect of Saturation. The plan is to find the total *direct* component of flux—*i. e.*, the component which passes through the pole body, linking the field winding. Then the effects of saturation in the pole body may be approximately taken into account, as in the following, but it is assumed that the saturation in the armature iron and in the cross field path is negligible. This is a reasonable assumption in the usual machine, since most of the saturation is in the pole body. Where there is saturation in the other paths, a more complicated method, still an approximation, and due also to Blondel* may be utilized.

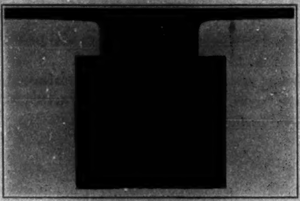

FIG. 28—FLUX DISTRIBUTION SHOWING DISTORTION FOR $\alpha = \pi/4$

Fig. 27A shows the same voltages as Fig. 26, and, in addition, the *direct* and *quadrature* components. Fig. 27B shows the corresponding diagram in terms of m. m. f. The total actual flux in the main axis is proportional to the virtual voltage e_d'. This would require, without saturation, a field excitation F_d', Fig. 27B which is proportional to e_d'; and the armature reaction, represented by the difference between e_d and e_d', would require the field excitation F_a, as usual. If saturation exists in the pole body, a field excitation larger than F_d' would be required to sustain the flux necessary for e_d' in the saturated path. Hence, in addition to F_d' and F_a, there must be an excitation F_s supplied to compensate for the m. m. f. drop in the pole body, thus maintaining at the pole face the same magnetic potential with respect to the armature as existed in the case of no saturation. Therefore the total field excitation would be F_d.

It is interesting and important to note that the saturation in the pole body does not, under the assumptions, change the displacement angle δ under the same conditions of terminal voltage, current and power factor.

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*Bibliography (11) and (13).

NOMENCLATURE

A_{nd} = Amplitude of the nth space harmonic of m. m. f. produced by the *direct* component of the armature current, and is expressed as a fraction of the amplitude due to normal current.

A_{nq} = Corresponding amplitude for the *quadrature* component of armature current.

a = Instantaneous values of m. m. f. at a given time and point.

C_n = Reduction factor due to pitch, distribution and connection of the armature coils. It is the product of the individual reduction factors for the pitch, distribution and connection; and is expressed as a fraction of that reduction factor which applies in the calculation of voltage generated in the armature winding by fundamental flux wave.

e = Voltage expressed as a fraction of normal voltage. Subscripts have the same meaning as those for flux densities.

e_d = Fundamental, "nominal" voltage due to current in the field winding, *direct* axis.

e_q = Corresponding voltage for current in a winding, *quadrature* axis.

e_{ad} = Total reactive voltage due to *direct* component of armature reaction, *i. e.*, the voltage $i_d\, x_d$.

e_{aq} = Corresponding voltage, *quadrature* component.

e_{d0} = Nominal voltage of a cylindrical rotor machine, corresponding to e_d on a salient pole machine.

e_{d1} = Nominal voltage of machine 1.

e_{d2} = Nominal voltage of machine 2.

e_d' = Virtual voltage.

f = Electrical frequency, in cycles per second.

I = Current in the field winding, expressed as a fraction of the current which will produce normal fundamental voltage on open circuit.

i_d = maximum instantaneous value of the *direct* component of armature current, expressed as a fraction of the maximum instantaneous value of normal current.

i_q = Corresponding value for the *quadrature* component.

i_{d1} = Component of current in the direct axis of machine 1.

i_{q1} = Component of current in the quadrature axis of machine 1.

i_{d2} = Component of current in the direct axis of machine 2.

i_{q2} = Component of current in the quadrature axis of machine 2.

k_n = Coefficient = ± 1.0 depending upon value of n.

m = Order of space harmonic of permeance coefficient.

n = Order of the space harmonic m. m. f. When used with p, as p^n, it is expressed as a Roman numeral to avoid possible confusion with use as an exponent. For the fifth harmonic, p^n is p^{V}.

P = Power expressed as a fraction of some convenient value of k-va. chosen as a base. When dealing with a single machine, the rated kv-a. is usually taken. In any case it must correspond to the voltage and current on which the reactance percentages are based. For instance, for two unlike units, the base may be taken as the sum of the two ratings, and the per cent reactances determined accordingly.

p = Permeance, expressed as a fraction of that permeance, which, multiplied by unit m. m. f. gives unit flux density.

p_c^n = p for the cos $n\,\alpha$ term of the nth space harmonic m. m. f.

p_s^n = p for the sin $n\,\alpha$ term of the nth space harmonic m. m. f.

p_θ^0 = Permeance at minimum air-gap, for constant m. m. f. (zero harmonic) along the armature surface, expressed as a fraction of unit permeance.

t = Time expressed as a fraction of the time to pass one electrical radian at fundamental frequency. It is therefore also the time angle. Unit time is thus $\frac{1}{2\,\pi\, f}$

x_d = Synchronous reactance, *direct* axis, expressed as a fraction of that reactance which would consume normal voltage with normal current flowing.

x_q = Synchronous reactance, *quadrature* axis, expressed as a fraction on the same basis as x_d.

x_{ld} = Leakage reactance of armature, *direct* axis, expressed as a fraction as in the case of x_d.

x_{lq} = Leakage reactance of armature, quadrature axis, expressed as above.

x_{ad} = Effective reactance of that armature reaction which is due to i_d. Armature reaction flux is understood to be that portion of the fundamental flux wave which exists within the length of the armature core. This excludes any portion due to current in the end winding, the latter being classed as end winding leakage. That is, $x_{ad} + x_{ld} = x_d$.

x_{aq} = Corresponding value for i_q. That is, $x_{aq} + x_{lq} = x_q$.

x_{d1} = Reactance of machine 1 in the direct axis.

x_{q1} = Reactance of machine 1 in the quadrature axis.

x_{d2} = Reactance of machine 2 in the direct axis.

x_{q2} = Reactance of machine 2 in the quadrature axis.

x_l = Total leakage reactance of the armature, neglecting the leakage due to second harmonic permeance terms. (See discussion under Reactance).

α = Electrical angle reckoned from the axis of the pole in the direction of rotation, and is expressed as a fraction of one radian, fundamental base.

β = Flux density, expressed as a fraction of the maximum density of a unit fundamental flux wave.

β_{nd} = Flux density produced by the *n*th space harmonic m. m. f. due to the *direct* component of armature current.

β'_{nd} = Stationary component, $\cos n\,\alpha$ distribution, of β_{nd}.

β''_{nd} = Stationary component, $\sin n\,\alpha$ distribution, of β_{nd}.

β_{nq} = Corresponding flux density for the *quadrature* component of armature current. This has stationary components corresponding to those of β_{nd}.

γ = Space angle in terms of the fundamental, reckoned from the axis of phase 1, Fig. 14, and expressed as a fraction of one radian.

δ = Electrical angle, expressed as a fraction of one radian (fundamental base), by which the terminal voltage e lags behind the nominal voltage e_d. It is the "displacement angle."

δ' = Displacement angle between e_{d1} and e_{d2}.

ϵ = Angle by which i lags behind e_d.

θ = Angle by which the terminal voltage leads the current. $\cos\theta$ = power factor.

τ_n = Pole pitch of *n*th harmonic. τ_1 = unity.

Φ = Maximum value of the instantaneous flux ϕ.

ϕ = Fraction of the flux contained in that fundamental flux wave which corresponds to normal voltage at normal frequency. Instantaneous value.

ψ_n = Electrical angular displacement (on fundamental base) along the armature surface, of the maximum of the *n*th harmonic m. m. f. wave from the axis of phase 1 at $t = 0$. On the harmonic base it would be $n\,\psi$.

Ω = Flux linkages, expressed as a fraction of the linkages, which, at normal frequency, would generate normal voltage.

ω = Fraction of normal electrical angular velocity. Normal angular velocity = $2\,\pi f$ where f = normal frequency.

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13. A. Blondel, *Revue Gen'l. d'Elec.*, 1919, Vol. V, pp. 811 and 843.

Discussion

C. A. Adams: In nearly every piece of electrical machinery we have two or more magnetomotive forces. There are two general methods of dealing with these m. m. fs. They may either be combined into a resultant m. m. f. and the corresponding flux computed therefrom, or the several fluxes produced by the several m. m. fs. acting separately may be computed and combined into a resultant. These two methods yield the same result, provided the reluctances of the magnetic circuits in which they act are constant.

In the part of the machine under consideration—namely, the air-gap region—reluctances may be assumed approximately constant, although varying from point to point along the air-gap.

I have never quite liked the method of dealing with the total apparent reactance of the armature of an alternator, either for the salient-pole or the non-salient-pole machine, since it deals with a hypothetical flux in a very complicated magnetic circuit. I much prefer the method in which the field and armature m. m. fs. are combined or compounded. In the case of the non-salient-pole machine, this method may be applied as follows:

Leaving the local variation of gap reluctance, due to the presence of the teeth, to be considered separately in terms of the resultant tooth harmonics, the air-gap permeance may be considered constant from point to point around the periphery. First, compute the peripheral distribution of field m. m. f. and resolve this into its space fundamental and space harmonics. Combine this space fundamental with the space fundamental of the armature m. m. f. in their proper space-phase relation. The resultant sinusoidal space distribution will yield a sinusoidal flux which determines the fundamental of the armature e. m. f.

The space harmonics of the field m. m. f. will yield corresponding harmonic fluxes, which will generate harmonics e. m. fs. in the armature, the magnitude of which will depend otherwise upon the nature of the winding. In most non-salient-pole machines these harmonic e. m. fs. should be small, particularly in three-phase machines with an armature coil pitch of about 86 per cent.

The harmonics of the armature m. m. f. distribution considered separately will yield space harmonic fluxes revolving at speeds inversely as the orders of the harmonics; that is, they will induce in the armature winding e. m. fs. of fundamental frequency. Moreover, as these fluxes are proportional to the armature current, the resulting e. m. fs. will be in quadrature with the armature current and therefore reactive e. m. fs., the sum of which is nothing more nor less than what has been called the belt-leakage e. m. f., as Mr. Doherty has already pointed out. The question as to whether this includes the tooth-tip leakage e. m. f. depends upon the definition of tooth-tip leakage. Personally, I prefer to consider the tooth-tip leakage as involving only that flux which passes between the tops of two adjacent teeth by way of such iron surface as may partially close the circuit on the other side of the air-gap, which in large alternators is usually small and may be included as an extra term in the individual slot leakage.

This method would leave only three items for the reactance voltage of the machine: the slot leakage, including a small amount of tooth-tip leakage above described; the coil-end leakage; and the belt leakage, which is nothing more than the e. m. f. induced in the armature conductors by the space harmonics of the armature m. m. f. This latter in a three-phase machine with balanced load and 5/6 coil pitch is a fraction of 1 per cent.

In the case of the salient-pole machine, there is no method available superior to Blondel's two-reactance method, although the method outlined above can be applied with certain rather crude approximations to take account of the lack of uniformity in air-gap reluctance in different parts of the periphery.

Mr. Doherty's amplification or extension of the Blondel method is certainly a very interesting and valuable contribution to this important subject.

P. L. Alger: There are three points about this paper on which I think it is worth while to enlarge somewhat. In the first place, the paper is very long and it might appear that it would be possible to combine all these long series into a few terms and get shorter equations which would be simpler to use. The answer to that is that we have a great many problems which require very complete analyses and others which require only incomplete analyses, so that we need a theory which will satisfy either demand.

The old approximate theories are sufficient for ordinary problems, but if we have to abandon them and go to another extensive theory for special problems, it is too inconvenient to tolerate. Therefore, by developing the whole problem in terms of series in which the first few terms are important, later terms less important and the last terms very unimportant, we can solve all problems by the same method, no matter how complicated they may be, by carrying the calculations only as far as each case demands. Thus we have a complete theory for all purposes and yet the work done in each case is suited to the particular need at hand. So I think this method of carrying the whole analysis out in infinite series is by all means the best, and the one which should be followed in most problems of a similar degree of complication.

The second point I should like to enlarge upon is that of showing the need for such extended solutions in every-day life. Nowadays we try to make a study of the conditions and make a machine to fit those conditions. Thus we have a great many freak machines or machines that are not at all good electrically, but yet are so designed as to serve a particular purpose most economically. For example, in manufacturing we desire to have only a few dies, and to keep in stock only a few sets of punchings, and yet to be able to apply these to all cases. This requires that we have fractional slots per pole on almost all machines. We put up with the inconvenience of having irregularities in the winding to get the greater convenience of manufacturing simplicity.

This leads to the result that our armature windings are sometimes unbalanced by a small amount, and also they are nearly always irregular.

Messrs. Doherty and Nickle assume throughout that each pole is like every other pole; that is, that the number of slots per pole is integral. We actually have a variation of armature reaction around the periphery which makes each pole different from its neighbors, but which balances around the whole machine. These variations lead to additional harmonics and other things not touched upon by this paper.

But the method is such that these features can be added at any convenient time by simply calculating the *variation* of armature reaction around the periphery and treating that as an additional source of harmonics, additional fluxes and voltages without disturbing the structure of the whole theory.

One case where a practical problem to which this theory lends great aid arises is in making a synchronous machine without external excitation. We ordinarily think of synchronous machines as requiring some supply of d-c. excitation from outside, although by making a salient-pole machine such as Mr. Nickle has shown, a synchronous speed can be obtained with a certain amount of power without external excitation.

Suppose we make a machine with one large harmonic in addition to the fundamental in its m. m. f. distribution, and with a variable permeance as well. By so selecting the harmonics of the m. m. f. and permeance distribution that the flux made by the fundamental m. m. f. acting on a permeance harmonic will have the same number of poles as the harmonic m. m. f. acting on the average permeance, we can produce a new type of machine.

For this machine will have two separate revolving fields produced by different sources whose relative speeds depend on the speed of the rotor, and will synchronize at a certain rotor speed. That will give a synchronous machine operating with quite a large torque without any external excitation. It can be done, to be sure, only if the machine operates below the synchronous speed for the fundamental itself, which leads to high losses; but it is quite feasible for small motors such as those used for driving control instruments, so that there is quite a field for this kind of machine.

This paper gives a theory which explains and enables us to solve quite completely many of these special problems.

The final point which I should like to bring out is that touched upon by Professor Adams and Mr. Doherty; this is the problem of calculating the reactance due to the slots. Both Professor Adams and Mr. Doherty have given the impression that the reactance due to the slot harmonics, themselves, is very small; but if we build a machine with very few slots per pole, this reactance may become a large part of the whole, and a very important feature of the machine.

In the case of an induction motor, the theory in the articles such men as Arnold and Professor Adams have worked out and published, involves the calculation of the effect of the slot openings, as that of the average overlapping. These formulas are rather hard to apply, and they are indefinite in the case of a large ratio of slots. By taking another point of view, a much simpler and more accurate solution can be obtained. This point of view is to consider that each coil in a slot makes a rectangular m. m. f. wave and a corresponding flux distribution, and to show that the total flux is simply made up of the sum of all the various rectangles made by all the coils. It is evidently possible to calculate the total voltage made by each rectangle of flux, and hence the total voltage produced in the winding by the total air-gap flux. Subtracting from the total the voltage due to the useful or fundamental flux, which is easily calculated, the difference gives the reactance voltage due to all the harmonics of the air-gap flux.

The formula so derived for the total zigzag leakage reactance is very simple. It is this:

$$X_T = \frac{\pi^2}{10}\left(\frac{P^2}{S_1^2} + \frac{P}{S_2^2}\right) X_M$$

where P is the number of poles, S_1 and S_2 the number of primary and secondary slots, respectively, and X_M is the magnetizing reactance corresponding to the fundamental of the air-gap flux.

By zigzag leakage I mean the harmonics of air-gap flux produced by the localization of the m. m. f. in slots, which are the harmonics that have distribution and pitch factors equal to the corresponding factors for the fundamental.

This formula shows the reactance due to the tooth harmonics in the flux, obtained by subtracting the useful from the total e. m. f. produced by the air-gap flux. The analysis assumes, of course, that the slot m. m. fs. are concentrated at points rather than distributed across the slot openings.

In the infinite series of air-gap flux harmonics, those which have unity values of distribution and pitch factors with respect to the fundamental are included in my formula. They are much more important usually than the other harmonics which correspond to the true belt leakage.

The belt leakage is due to the winding arrangement as regards pitch, and number of phases; and is nearly independent of the zigzag reactance with which I have been dealing. It is best to separate the two and keep them distinct.

By taking the same idea of a rectangular m. m. f. applied over the permeance distribution worked out by Mr. Doherty, we hope to obtain a similar formula for the tooth-tip leakage reactance of synchronous machines. We know this reactance may be large and in some cases we find it is a vital element in the design of a machine. So, by utilizing these ideas of permeance distribution calculated from flux plots, and m. m. f. waves made up of certain totals and certain useful fundamentals, we hope to derive simpler formulas for the reactance than have heretofore been available. If our plans go well, a paper along these lines will be presented to the Institute in the near future.

C. A. Adams: Mr. Alger must have misunderstood what I was saying. I was referring exactly to that same point of view. I spoke of the armature combination of magnetomotive forces and the separation of the total magnetomotive forces and the fundamentals. Those harmonic fluxes are proportional to the currents and generate reactance voltages, but it was a cross-gap flux to which I referred when I spoke of the particular case of less than 1 per cent.

M. I. Pupin: An alternator or an a-c. motor may sometimes look like a queer machine, depending upon the purpose for which you wish to use it.

Now I shall give a brief description of an alternator that I have been working with for a number of years. Consider an alternator with suitably laminated field and armature cores. Send a small direct current through the field coils and rotate the armature, the armature being short-circuited. What do we get? We get alternating electromotive forces in the armature as well as in the field coils, not one of them but an infinite series in each; the components of the series are harmonically related to each other.

I have treated this scheme mathematically and find that, mathematically, the problem can be solved. You get two beautiful series, each of which is convergent. The coefficients of the various harmonics are also infinite convergent series.

Suppose I want to use that machine for the purpose of modifying the feeble field current by a cable signal. As long as you have a constant current in the field the machine will generate an alternating high-frequency current of constant amplitude. Now superpose the cable signal and you will have an alternating current with amplitudes varying in accordance with the variation of your cable signal. The machine generates a high-frequency e. m. f. modulated by the cable signal and it is obvious that it offers a means of amplifying low-frequency cable signals. It works beautifully except for some of the difficulties the gentlemen brought out this morning.

If the air-gaps vary, or if the field core and the armature core vary from point to point, then you get fluctuations, even without the cable signals. Therefore, when rectified, you will not get a straight line but a wavy line, and that is absolutely useless for cable signaling purposes. It may be useful for some other purpose, but not for cabling. Experimentation with a machine of that kind will show how difficult it is to make an alternator which will give a constant electromotive force of definite frequency and amplitude. It is practically impossible as long as you use iron in the field and in the armature. I have tried it and failed.

Then I did this: I used an alternator without any iron in the field or in the armature. There you don't get any variations due to the material of which the magnetic field is made; the variation due to the variation of the air-gap is negligibly small because the whole machine is an air-gap.

You will say that this is a very inefficient machine. It is, but it does the trick, and you don't care how inefficient it is because you can use the amplifier and amplify to any desirable limit.

R. W. Wieseman: I should like to say a few words about the graphical solution of plotting magnetic flux distribution as shown on the fifth and sixth pages of this paper.

It is sometimes thought that the predetermination of flux-density curves is purely a theoretical operation. On the contrary, it is a very practical problem and it is used extensively in the design of dynamo machinery. A number of flux-distribution coefficients can be obtained by the graphical solution and their values check test results very closely. A good example of this is the predetermination of the characteristic curves of a two-speed, salient-pole, synchronous motor.[1]

In the near future, I shall present a paper which will show the practical application of the graphical solution of plotting flux-density curves to the design of synchronous machines.

R. D. Evans: I should like to suggest to those interested in the matter of space and time harmonics, comparison of the method presented in this paper with that given by C. L. Fortescue in his paper on *Symmetrical Coordinates*.[2]

With reference to the variation in leakage reactance caused by the non-cylindrical rotor construction salient-pole machines, I note that the authors pointed out that the variation is quite small. In such investigations that I have carried out for other purposes, my experience would indicate that such variations are small.

In regard to the power-angle diagrams, which are quite important for such studies as stability, the use of a two-reaction method of some sort is, of course, very necessary. Otherwise, as pointed out, large discrepancies in the relation of rotor position to terminal voltage would arise. The use of a two-reaction method seems to be particularly desirable for transient investigations because the paths of the direct-component flux and the cross-component flux are physically different and the resolution into two parts would permit taking them into account separately. In connection with the determination of the relation between rotor position and terminal voltage and excitation, Mr. Wagner and I carried out some investigations measuring the rotor position by another machine on the same shaft. We obtained very close checks of rotor position relative to terminal voltage from test results with the classical method of Blondel.[3]

John F. H. Douglas (communicated after adjournment): This paper is an important step forward in the theory of synchronous machines, since it considers for the first time the effects of the harmonics in the magnetomotive force wave, upon the characteristics of the machines. One striking conclusion of this paper is the fact that the reactance and reaction coefficients can be obtained from the analysis of the wave of air-gap permeance. Another, shown by Figs. 5-11, is that the permeance in the interpolar regions of these machines, is considerable. It is not generally appreciated that the permeance of the air-gap is different for field and armature magnetomotive forces, and that it is different for different orders of the armature harmonics of m. m. f.

The accompanying Fig. 1 shows a method of testing air-gap permeance used by the writer and E. W. Kane of Marquette University. While our tests gave waves of the zero order, our values check closely with those given by the authors of this paper. It is noteworthy that Table II in conjunction with equations (1c) and (2c) yield a ratio of transverse to direct reaction of approximately 50 per cent. This ratio is substantially higher than that obtained by those authors who neglect the permeance of the interpolar regions.

It is a fact that in predicting reactance, the flux paths assumed lead to values less than the experimental value. Considerable discrepancy exists between rational and experimental values of reactance. It is to be noted that the third and fifth terms in eq. (3c) may in some cases increase values of reactance calculated by some 30 per cent. Thus the refinement in theory is well justified, if more accurate estimates of reactance can be obtained.

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[1] A. I. E. E. Trans., 1925, p. 436.

[2] A. I. E. E. Trans., 1918, p. 1027.

[3] Studies of Transmission Stability, by R. D. Evans and C. F. Wagner, A. I. E. E. Journal, April, 1926, p. 374.

Some of the results of this paper are negative. For instance eq. (7) indicates that direct and transverse reactance are substantially equal. This conclusion, however, is based on the definition of reactance as due to local fluxes, and fluxes in relative motion to the field structure. A more useful definition is that "reactance" and "reaction" are those vectors proportional to the current in the e. m. f. and the m. m. f. diagram respectively. Since only tests can determine this question, the existence of a "variable leakage reactance" must still be regarded as an open question.

The authors say, "The effect of salient poles on the magnitude of required excitation is very small." While this is generally the case, there are exceptions under conditions of abnormal conditions of operation, as for example in the "nose" region of the synchronous-motor characteristic and near the unstable point of generator operation. Consider Fig. 2 herewith for a generator carrying heavy charging current of a transmission line and no other load. The drop $I X_q$ is less than the terminal voltage, and this in turn is less than the drop $I X_d$. The Potier diagram gives a positive, the Blondel, a negative excitation. Fig. 2 also shows the complete zero-leading power-factor characteristics predicted by both methods.

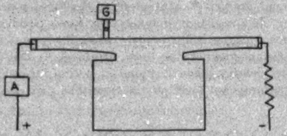

FIG. 1—METHOD OF TESTING AIR-GAP PERMEANCE

Fig. 27 is based on three assumptions; saturation in the armature and cross field negligible, and a transverse reactance identical with direct reactance. In deciding whether a given component of the effect shall appear in the m. m. f. diagram the question is whether experiment shows this factor as more constant as an e. m. f. or an m. m. f. vector. The writer knows of no way to settle this point theoretically. It is conceivable, for example, that all the cross fluxes, not only those represented by X_{aq} but those of X_{1q}, might be affected by saturation.

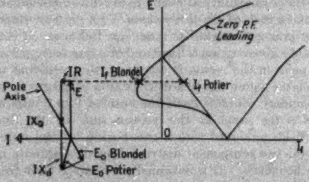

FIG. 2—COMPARISON OF BLONDEL AND POTIER DIAGRAMS NEAR UNSTABLE REGION

The data given by the writer in a paper on "Synchronous Motor Characteristics"[4] indicated at least that X_{aq} is greatly affected by saturation.

Quentin Graham (communicated after adjournment): Messrs. Doherty and Nickle have dealt in a comprehensive way with a problem that is treated usually in a much less rigorous manner. The method of harmonic analysis is certainly the only one that is adequate for the treatment of many of the more complex problems of synchronous machines. It seems to me that the chief usefulness of the paper is not in a refinement of reactance and reaction calculations, with which Part I is concerned, but rather in the solution of innumerable special problems in which a similar method of attack is required.

Since the assumption of uniform permeance is a common one in considering synchronous-machine performance, it may be well to emphasize some of the differences that a variable permeance produces. It is shown in the paper that fundamental voltages are set up in the armature conductors by the following fluxes:

a. The flux which results from the fundamental of the armature m. m. f. and the zero-order component of the permeance series.

b. The flux resulting from the fundamental of the armature m. m. f. and the second-order component of the permeance series.

c. The flux resulting from the combination of any higher harmonic of the armature m. m. f. and the zero-order of the permeance wave.

d. The flux resulting from the combination of any higher harmonic in the armature m. m. f. and the second-order term in the permeance wave.

e. The fundamental component of the no-load field form.

In a machine having a cylindrical field member with uniform permeance the fluxes *b* and *d* are absent. There is left then the flux *a*, which is the usual armature reaction flux, and *c* which is a wave of short span traveling at slow speed whose effect is more conveniently grouped with the reactance fluxes. With the assumption of uniform permeance, that is, with only the zero-order term present in the permeance expression, we may say that all harmonics in the armature m. m. f. wave, including the fundamental, set up fluxes which generate fundamental voltage in the armature. Further, no voltages of other frequency are generated or, expressed differently, no time harmonics appear.

The introduction of salient poles and variable permeance is responsible for the flux *b*, which adds to the armature reaction flux, and *d* whose effect is grouped with the reactance. The fluxes under *d* are of various wavelengths but travel at such speeds as to give fundamental voltages. In addition, there are the various fluxes having combinations of wavelength and speed of rotation such as to produce frequencies which are multiples of the fundamental. It is the variable permeance that is responsible for the presence of time harmonics except, of course, those that are due to the shape of the field flux form and which are present even though there is no current flowing in the armature.

These facts are all contained in the original paper but are repeated here in different form and with a slightly different point of view.

There are two points of possible importance which the authors appear to have omitted in their paper. The first is the effect of a damper winding on the magnitude of the various harmonic fluxes. Any m. m. f. wave which has a velocity other than synchronous may set up secondary currents in the rotor which will have the effect of reducing the magnitude of the flux waves set up by that m. m. f. The paper contains a discussion of the induced currents in the field winding and concludes that they may be neglected. It is probable that the same conclusion was reached concerning the induced currents in damper bars although I believe these currents should be considered in cases where particular time harmonics are of importance. Their effect on reactance in the steady state is probably imperceptible.

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[4] Douglas, Engeset and Jones, A. I. E. E. TRANS., 1925, p. 164.

The other point to which I wish to call attention is the phase sequence of the fundamental voltages generated by the various fluxes. Since the fifth space harmonic, for example, travels against rotation and produces a flux wave with the same direction of travel, the voltage induced in the three-phase armature winding has opposite sequence to the main induced voltage. This negative sequence voltage which is grouped among the reactances appears at the terminals as an unbalance in the three voltages. In Fig. 15 of the paper, e_{5d7} and e_{5q7} are voltages produced by positively rotating flux while e_{5d5} and e_{5q5} are the result of fluxes rotating negatively. The same is true of part of the fundamental voltages arising from the 11th and other space harmonics of the armature m. m. f. It is surely incorrect to add these various voltages in the final reactance equation just as though they were all induced by positively rotating flux waves. I realize that the voltages calculated are for one phase and that equations (**58a**) and (**59a**) could give correct values for the voltage of, say, phase 1. But since the time relation of between voltages induced by positively rotating and negatively rotating fluxes is different for phase 2 and phase 3, the same equations would not apply.

I have been particularly interested in the footnote on the thirteenth page which refers to the harmonics of irregular windings since I have in the course of preparation a paper which covers in some detail the calculation and the effects of these harmonics.

R. E. Doherty: Professor Adams has said that it is immaterial, so far as final results are concerned, whether fluxes or m. m. fs. are superposed. When saturation does not exist, such quantities which exist at the same point may be superposed. All armature m. m. fs. are distributed along the armature surface and hence may be superposed. The field m. m. f., however, is located at some distance from the armature surface and hence cannot in general be superposed directly with armature m. m. fs.

I am just raising the point that every now and then we are likely by habit to superpose quantities that properly cannot be superposed. If we assume a fictitious m. m. f. which, if located at the armature surface, would produce the same effect as the actual field winding, then of course we may superpose this fictitious m. m. f. with those due to the armature since they are now at the same point. In cylindrical-rotor machines, this fictitious m. m. f. would be practically the same as the actual m. m. f. and it would be immaterial which we use. In salient-pole machines, however, the fictitious m. m. f. would be greatly different from the actual m. m. fs. In such cases it becomes much simpler to superpose fluxes than to attempt to obtain the correct m. m. f. to use.

As to the question of fractional slots which Mr. Alger has brought up, we especially stated that the treatment of this is beyond the scope of this paper. I hope that some one in the future—perhaps Mr. Alger—will treat this case.

Mr. Graham has stated that the fifth-harmonic flux, rotating backward with respect to the armature, will induce a negative-sequence fundamental voltage in the armature winding. For a fundamental wave, phases 1, 2, and 3 are located at 0 deg., 120 deg. and 240 deg. respectively. For a fifth-harmonic wave these angles become 0 deg., 600 deg. and 1200 deg. which of course are the same as 0 deg., 240 deg., and 120 deg., respectively. This phase rotation for a fifth harmonic is thus opposite to the phase rotation for the fundamental. Hence it is necessary that a fifth-harmonic flux wave travel in a direction opposite to that of the fundamental wave in order to generate fundamental voltages having the same sequence as the fundamental.

Mr. Graham also raises a question regarding the effect of a squirrel-cage winding. The present paper does not cover this case. However, it is important to call attention to the fact, as Mr. Graham has done, that squirrel-cage currents may have an important effect on the value of the harmonic flux waves, and therefore upon the leakage reactance of the machine, even under steady-state conditions. It is the authors' opinion that the effect of such currents is not negligible, and an appropriate approximation would have to be made in case a squirrel-cage winding is present.

Waldo V. Lyon (communicated after adjournment): This paper by Doherty and Nickle is interesting to the writer since it applies, as they courteously say, "the same general method of attack" that he used in his 1918 paper on harmonic analysis. It is particularly gratifying to have this method which he had come to believe of academic interest only, taken down from its dusty shelf and developed in such a comprehensive manner.

Although the general methods of analysis that have been employed in these papers are the same, the fundamental physical principles upon which they rest are somewhat different. Thus it is of interest to compare the methods critically so that we may the better determine the advantages and disadvantages of each. The writer depended upon the principle of the superposition of magnetic fields, whereas the present authors depend upon the principle of the superposition of distributions of magnetic potential—two principles which are of course closely allied. In his 1918 paper, the writer's statement of the former principle was as follows: "If a constant magnetizing force produces at any point a certain magnetic flux density when it alone is acting, and another constant magnetizing force produces at the same point another flux density when it alone is acting, then when both magnetizing forces act together the resultant flux density at the point in question is the vector sum of these two flux densities, provided the permeability at every point in the region concerned is the same in each of the three cases." A similar statement of the latter principle might be made as follows: If the entire boundary of a given region is fixed and the magnetic permeability at every point in the region is constant, the resulting magnetic field within the region due to a distribution of magnetic potential on the boundary which is itself the sum of two component distributions is the same as the vector sum of the individual magnetic fields produced within this region by the independent action of the two component distributions of magnetic potential along the boundary. It is assumed, of course, that there is no current within the given region.

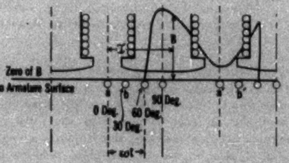

Fig. 1

Fig. 1 is drawn to show how these two principles may be applied to the analysis of the flux distribution in a synchronous machine. By an application of the first principle we know that the resultant flux density at any point in the air-gap, or in the field or armature iron if that is desired, due to the joint action of any field or armature coils, is the vector sum of the flux densities at the point due to the individual actions of the same coils. By an application of the second principle we know that if there is produced on the surface of the field and armature a given distribution of magnetic potential which is itself a sum of component distributions of potential, the resulting flux density at any point in the air-gap is the vector sum of the flux densities produced at that point by the individual action of these component distributions of magnetic potential. If it be permissible to assume that the field current is concentrated in conductors that lie in the surface of the field poles as indicated in Fig. 1 in a manner similar to that in which the authors assume that the armature currents are arranged, then the writer cannot agree with the authors when they say, "Hence the superposition of the armature and field m. m. fs.[5] is not permissible." The writer believes, however, that it would not be advisable to apply the principle of superposition to the field and armature distributions of magnetic potential. It is much better first to determine and then to combine, as the authors have done, the component flux distributions due to the field and armatue distributions of potential.

Let us now examine in detail the two methods of analysis. Consider Fig. 1. Imagine that the armature coil $a-a'$, when opposite a pole, has a sufficient m. m. f. to establish a distribution of flux in the air-gap, the maximum density of which is unity; (see author's definition of unit flux density). Determine the normal flux density at the surface of the armature either by experiment or by the methods that the authors describe. Analyze this distribution of flux for its harmonic components. In this first position of the coil there will be only sine components due to the symmetry of the magnetic circuit. Move the coil to some new position such as $b-b'$ and again determine the sine and cosine harmonic components of the flux density. For each new position of the coil there will be new sine and cosine harmonic components. Plot the amplitude of each of these components against coil position and analyze the *variation* of each of these amplitudes for *its* harmonic components. It is in this way that the writer determined the a and c coefficients of his 1918 paper. Refer again to Fig. 1. The flux density, B, at any point x in the air-gap due to this one armature coil is:

$$B = \Sigma A_q \sin_q (x - \omega t) + \Sigma C_q \cos_q (x - \omega t)$$

The amplitudes of the sine and cosine components are:

$$A_q = a_{q0} + a_{q1} \cos 2\,\omega t + a_{q2} \cos 4\,\omega t + \ldots a_{qp} \cos 2_p\,\omega t \qquad (1)$$

$$C_q = c_{q1} \sin 2\,\omega t + c_{q2} \sin 4\,\omega t + \ldots c_{qp} \sin 2_p\,\omega t \qquad (2)$$

Under the same conditions as laid down by the authors, only the coefficients a_{q0}, a_{q1} and c_{q1} need be determined. If it is deemed permissible to neglect all of the terms in (1) and (2) beyond $\cos 8\,\omega t$ and $\sin 8\,\omega t$ then these necessary coefficients can be calculated from plots of the flux density due to the single coil when it is successively placed in the positions marked 0 deg., 30 deg., 60 deg. and 90 deg. Let A_{q0}, A_{q30}, A_{q60}, and A_{q90} be the amplitudes of the sine components of the qth harmonic flux density at the given coil positions. Also let C_{q30} and C_{q60} be the amplitudes of the cosine components of the qth harmonic flux density at the corresponding coil positions. Then it may be readily shown that

$$A_{q0} = \frac{1}{6} \left\{ A_{q0} + 2A_{q30} + 2A_{q60} + A_{q90} \right\}$$

$$A_{q1} = \frac{1}{3} \left\{ A_{q0} + A_{q30} - A_{q60} - A_{q90} \right\}$$

$$C_{q1} = \frac{1}{\sqrt{3}} (C_{q30} + C_{q60})$$

Thus the required coefficients are determined readily by plotting the flux density in the air-gap due to a single armature coil placed at four different positions with respect to the field poles. For this reason the writer cannot agree with the authors' comment on the computation of these coefficients when they say, "Hence the resulting series contain coefficients which would be very difficult to determine." The difficulty is at least no greater than would be experienced in determining the corresponding coefficients that the authors use.

An approximate correction for the effect of saturation of the field poles can be made by assuming that it reduces the flux distributions due to the armature coil in the 0-deg. and 30-deg. positions but does not affect the distributions at the 60-deg. and 90-deg. positions. Saturation would thus increase the relative magnitudes of the A_{q60}, A_{q90} and C_{q60} coefficients. In his 1918 paper the writer gave a numerical estimate of the approximate effect of saturation on the values of the required coefficients. From a theoretical standpoint the authors should take into account the effect of saturation by recognizing that while the distribution of m. m. f., or magnetic potential, due to an armature coil carrying a fixed current is the same when the coil is opposite a pole as when opposite the interpolar space, the polar surface is at zero potential when the coil is in the latter position but not when it is in the former position. It would probably be preferable, however, to make some simpler approximate correction, such, for example, as the authors have suggested.

There is a problem of some interest that the authors did not mention. It is that of a synchronous generator which, when feeding an open circuited transmission line, loses its field excitation. If R and X represent the equivalent resistance and reactance of such a line on open circuit then it is readily shown for this case that

$$(R + r)^2 = -(X + x_q)(X + x_d)$$

This shows that X is capacitive and lies between x_q and x_d. Since the values of x_q and x_d depend upon the saturation, a graphical solution of the forgoing relation can be made which will determine the saturation, and from that the terminal voltage can be found.

After all, the particular method of arriving at a given solution is not of paramount importance so long as it follows generally accepted physical principles. Both the authors and the writer have shown different methods of obtaining the same result. The necessary numerical constants can be determined equally well for one method or the other. Whether or not one of the methods seems to arrive at the solution in a simpler way depends a good deal upon the scientific bringing up of the user.

No doubt this idea of a multitude of harmonic distributions of flux of different wavelengths—the writer has called them phantom poles—rotating in different directions at different speeds is a difficult phenomenon to visualize. It is a powerful method of analysis, however, and the authors are to be congratulated on the effort they have made to present it in such a comprehensible form.

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5. The writer understands that the authors use the term "m. m. f." in the same sense that he uses the term "magnetic potential."

Some Graphical Solutions of A-C. Circuits
Founded Upon Non-Euclidian Geometry

BY F. W. LEE[1]

Associate, A. I. E. E.

***Synopsis.**—1. A graphical method for the solution of alternating-current circuits is developed and discussed.*
2. The application of these transformations is shown graphically by numerous examples.
3. Certain characteristics of circuits are identified which greatly simplify the graphical solutions of circuits.

INTRODUCTION

THE purpose of this paper is to show how, with the aid of simple functional transformations, it is possible to obtain a mental picture of the limitations, as well as the influence, of each factor in an a-c. circuit upon the resultant voltages and currents. Steinmetz first explained how the influence of the individual factors of a circuit may be computed with the aid of his complex operator. Arnold also indicates how they may be used for constructing loci diagrams. The usual representation of an electric circuit is by the impedance or the admittance diagram; these operate upon the current or voltage in question and also determine the relative phase relations and magnitudes of the voltages and currents in the circuit. The method of analysis now presented has the limitation of the admittance or impedance diagrams for its objective.

Fundamentally, this idea embraces the operation carried out upon a function instead of a particular value. The function has all particular values as special cases. Also, if the functional transformations are simple, the individual operations may be traced after each functional change has been made. In this particular discussion the functions are circles and each particular solution may be followed with great rapidity. The effect of each functional change may be seen and any alteration of each particular factor visualized under various circuit conditions.

The nature of these solutions all have circles for their functions, but the parts of the circle which are physically operative may be greatly attenuated by circuit limitations. Nevertheless, the circle has a very well defined physical, and also a mathematical, relation to the circuit in question. Although the following transformations transform into similar figures, the circle, as will be seen, is a natural consequence in alternating circuit modifications. This operation upon a circle may take various forms; these are best understood when the equation of a circle is considered.

OPERATIONAL CHANGES

The function for simplicity is taken as an impedance and is represented by Z. It may also, however, take the value of a voltage, a current, or an admittance, as conditions require. In Fig. 1, a circle of radius $|r|$ is shown.

The equation of this circle is $Z = |r| \epsilon^{j\theta}$. Every point on this circle is uniquely defined; for example, $Z_1 = |r| \epsilon^{j\theta_1}$. The center of the circle coincides with the $X\,X$ and $Y\,Y$ axis. Certain operational changes may be made upon this function Z which will give rise to another function W; for example, a constant complex value $a = |a| \epsilon^{j\theta_a}$ may be added to every point upon the circle shown in Fig. 2.

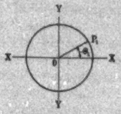

FIG. 1

$$W = Z + a$$

or

$$W = a + |r| \epsilon^{j\theta}. \tag{1}$$

From Fig. 2, it is seen that the diameter of W is the same but the center has been shifted for O to O'. Here it may also be seen that W is a more general expression for a circle.

Again consider the circle $Z = |r| \epsilon^{j\theta}$ but let it be multiplied by $\epsilon^{j\theta_1}$. The new function W now is

$$W = Z\,\epsilon^{j\theta_1} = |r|\,\epsilon^{j\theta} \times \epsilon^{j\theta_1} = |r|\,\epsilon^{j(\theta + \theta_1)}. \tag{2}$$

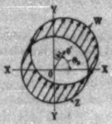

FIG. 2

This is shown in Fig. 3; the circle has the same radius as in Fig. 1, but the points have all been displaced by θ_1 deg.; or the circle has been rotated in a positive sense.

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Had Z been multiplied by $| b | \epsilon^{j\theta_b}$,
as demonstrated by Fig. 4,
then

$$W = | r | \times | b | \epsilon^{j(\theta_b + \theta)}. \tag{3}$$

The original circle Z, having a radius $| r |$, will change

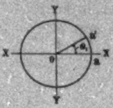

Fig. 3

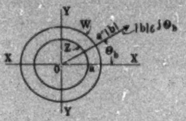

Fig. 4

into circle W, having a radius $| r | \times | b |$, or the product of the radii. The circle W would also be rotated with reference to Z by an angle of θ_b degrees. Point a will go into a', etc.

If a more general equation for a circle had been

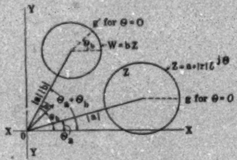

Fig. 5

considered, as $Z = a + | r | \epsilon^{j\theta}$, where a is a complex number, as shown in Fig. 5, and had Z been multiplied by $b = | b | \epsilon^{j\theta_b}$, the circle Z would have gone into

$$\begin{aligned} W &= Z | b | \epsilon^{j\theta_b} = [| a | \epsilon^{j\theta_a} + | r | \epsilon^{j\theta}] [| b | \epsilon^{j\theta_b}] \\ &= | a | \; | b | \epsilon^{j(\theta_a + \theta_b)} + | r | | b | \epsilon^{j(\theta + \theta_b)} \\ &= c + | r' | \epsilon^{j(\theta + \theta_b)}. \end{aligned} \tag{4}$$

Notice that the circle Z has been moved through an

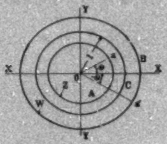

Fig. 6

angle θ_b and has also been revolved about its center by θ_b. Its new radius $r' = | r | \times | b |$ and its new distance from the origin $c = | a | \times | b |$.

To define a circle, three factors are necessary after a transformation, viz., (a) the radius or diameter, (b) the distance and direction from the origin, and (c) the value of θ_b or its zero reference point.

Another equally important transformation is the reciprocal of a circle presented by Fig. 6. $Z = | r | \epsilon^{j\theta}$ is a circle about the origin.

Then

$$= \frac{1}{Z} = \frac{1}{| r | \epsilon^{j\theta}} = \left| \frac{1}{r} \right| \epsilon^{-j\theta}. \tag{5}$$

From this equation a new circle arises, having a radius $\left| \dfrac{1}{r} \right|$ with angles taken in the negative sense. Circle Z will go into circle W and vice versa; point a would correspond to a', a circle inside of the unit circle will fall outside, and a circle outside will fall inside, upon inversion. The unit circle will invert into itself but the points on it will not coincide. Should, however, the center of the circle not coincide with the

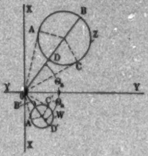

Fig. 7

origin, Fig. 7, then the inversion will also transform the circle into another circle.

Here

$$Z = | a | \epsilon^{j\theta_a} + | r | \epsilon^{j\theta}$$

$$W = \frac{1}{Z} = \frac{1}{a + | r | \epsilon^{j\theta}} = | a' | \epsilon^{-j\theta_a} + | r' | \epsilon^{j\theta'} \tag{6}$$

θ and θ' bear no linear functional relationship. This transformation may be proved from similar triangles upon elemental portions of the two circles. The angles, however, do not transform uniformly as a

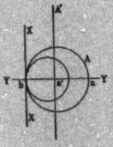

Fig. 8

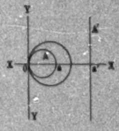

Fig. 9

linear relation since the interval $A\,B\,C$ is compressed in the much smaller arc $A'\,B'\,C'$. Thus, if the circle were considered as a rubber band, this would mean that the band had shrunk and that the tension in the band was no longer uniform; *i. e.*, some points would be stretched and others compressed. The location of this circle is best determined from a line drawn through the center of circle $A\,B\,C\,D$ from the origin. After the transformation, this ray will also embrace the diameter of the new circle. The angle θ_a will give the direction, since, after the transformation, the angle changes into $-\,\theta_a$. The radius of

$$W \text{ is } \frac{1}{2}\left[\frac{1}{|a|-|r|}-\frac{1}{|a|+|r|}\right]=\frac{r}{a^2-r^2}$$
$$= |r'| \qquad (7)$$

The distance of the center from the origin of W is

$$a' = \frac{1}{2}\left[\frac{1}{|a|-|r|}+\frac{1}{|a|+|r|}\right]=\frac{a}{a^2-r^2} \quad (8)$$

It should be remembered that for every point on the circle Z there always will be a corresponding point upon the circle W, and that the points are continuous on the circle.

If the circle touches the origin, Fig. 8, and lies on the $X\,X$ axis, the reciprocal will be the straight line A'. Since point a will go into a' and b will go to infinity, the radius will be one-half of infinity; hence A' will be a circle of infinite radius, or a straight line.

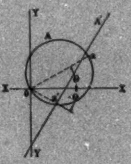

FIG. 10

Circles inside of the unit circle, Fig. 9, will go into straight lines outside of the unit circle, since the reciprocal of a is greater than unity. Also a circle outside of the unit circle will go into a straight line within the unit circle. All of these circles (Fig. 8 and Fig. 9) touch at the origin O, or have one point in common; hence after the transformation they again will have one point in common or the parallel straight lines will touch at one point, at infinity.

Other circles A, Fig. 10, located at an angle θ with respect to the origin, will transform as a reciprocal, into the straight line A'. The distance $o\,a' = \dfrac{1}{o\,a}$ is determined by the diameter $o\,a$. An interesting case of these transformations is shown in Fig. 11.

As before, circle A will go by inversion into the straight line A' and circle B into the line B'. Also the point of intersection between the two circles a will, after the transformations, have the point a' in common, or the intersection of the lines A' and B'. Since the circles intersect again at the origin the two straight lines will intersect again at infinity, from which it follows that two straight lines intersect at two points and that the direction of infinity from the origin is not definite.

The converse of this is also true; a straight line by inversion will go into a circle represented in Fig. 12.

The reciprocal of the perpendicular distance $o\,a$ will determine the diameter $o\,a'$. The direction of the diameter is determined by the angle θ; therefore, the length $o\,a$ and its direction are immediately determined by the circle locus.

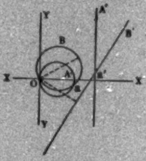

FIG. 11

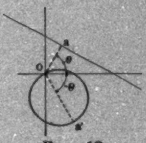

FIG. 12

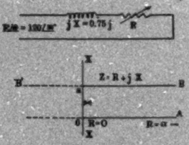

FIG. 13

APPLICATIONS

Consider the very familiar circuit of Fig. 13 which has a reactance X in series with a variable resistance R. The equation of the current in the circuit

$$I = \frac{E}{R + j\,X} = E\,.\,\frac{1}{R + j\,X} = E\,.\,\frac{1}{Z} \quad (9)$$

where R ranges from O to infinity.

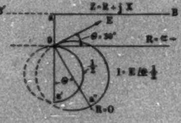

FIG. 14

The locus of R is shown by the line $O\,A$, beginning at the origin and extending in a positive direction to the right; $Z = R + j\,X$ is shown by the line $a\,B$. This line $a\,B$ may be continued to the left; this would be a negative resistance and therefore not a physical part of the problem. The equation of I shows that Z occurs as a transformation in that it must be inverted (see equation 6). It is very obvious that the inversion of the straight line $a\,B$ is a circle, Fig. 14, with a diameter $o\,a' = \dfrac{1}{o\,a}$.

The region shown by the full line corresponds to the portion of the straight line $a\,B$ and the dotted portion to $a\,B'$ which latter portion has only an interest in carrying through the transformations.

This value of $1/Z$ must be multiplied by $E\underline{/\theta}$ which will give the locus of I. Thus the diameter of $1/Z$ is changed and the circle is turned by θ degrees, as shown. This gives the resultant value of current and shows that a circuit in general which has a straight line as a function or a circle as a variable will lead to other circles, after transformations have been made.

The circuit as explained may be modified so that it is composed of an impedance in series with a resistance (Fig. 15), as, for example, a non-inductive load on a feeder circuit.

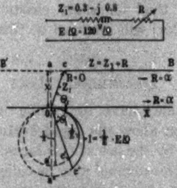

FIG. 15

In this case, as before, draw $O\,X$, the locus of R, and to this locus add at every point Z_1; this is the impedance $Z_1 + R$.

Now $I = \dfrac{1}{Z_1 + R}\,E\underline{/0}$ from which it is seen that $Z_1 + R$ must be inverted. Extend the line $c\,B$ to B' and at the point a let it cut the $Y\,Y$ axis. The reciprocal of this inversion is the circle whose diameter is $O\,a^1$. The line only has physical significance from C on toward B. This corresponds to $R = 0$ at C, and $R = \infty$ at B. The point C upon inversion will determine the active part of the circle. Notice that C' on the circle has the same angle as Z_1 here in the negative sense and the range is from C^1 to O. This circle must be multiplied by $E\underline{/0}$ to obtain the current whose diameter is $O\,A''$ and whose limitations are again between the points C'' and O.

The voltage on the load is deduced from the relation E_L = load voltage $= E - I\,Z$. Instead of a particular current the loci circle I is multiplied by $-\,Z_1$ and then added to $E\underline{/0}$, as shown in Fig. 16.

From the foregoing it is seen that this method of analysis is applicable to all types of circuits which may be reduced to a simple equivalent circuit.

A further illustration is shown in Fig. 16A of a circuit similar to that of Fig. 15 in which the inductive reactance is equal to twice the resistance of the circuit. It is desired to reduce the short-circuit current by one-half with added reactance from a choke coil and to compare the maximum current possibilities at various power factors.

The impedance vector Z_1 corresponds to the circuit before the reactance $a\,b$ has been added and Z_2 represents a distance numerically twice as long as Z_1. Draw a circle about Z_1 as a diameter and also one about Z_2 as a diameter. These circles are the loci of all minimum impedances possible with any desired power factor of load Z_L. Consider the general case, here represented as 50 per cent p. f. leading; the lines $o\,k$ and $o\,m$ are perpendicular to $a\,k$ and $b\,m$ respectively and represent the shortest resultant impedance of line and load. Before the reactance is added, the current in the circuit is proportional to $\dfrac{1}{Z_1 + Z_L}$ and afterwards to $\dfrac{1}{Z_2 + Z_L}$ where, for example, Z_L is taken as a 50 per cent power factor locus.

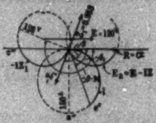

FIG. 16

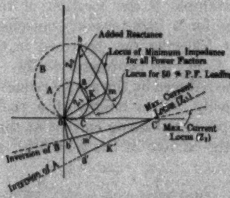

FIG. 16A

It is evident that there is a short-circuit current for $Z_L = o$, but from the diagram it is seen that this may not be the maximum current possible in the circuit. For a definite range of power factors touching the circles between $a\,k\,c$ and $b\,m\,c$, maximum currents greater than the short-circuit current are possible. The inversion of the circles A and B indicates the locus of these maximum currents which are the straight lines $a'\,c'$ and $b'\,c'$. These two lines intersect at c', corresponding to the intersection at c of the two circles which is the resonant point in both instances. For this point the ratio of the short-circuit currents corresponding to line impedance Z_1 and Z_2 is $\frac{o\,c'}{o\,c'} = 1$; for $Z_L = 0$ the ratio is $\frac{o\,b'}{o\,a'} = 0.5$; at 50 per cent power factor leading it is $\frac{o\,m'}{o\,k'} = 0.633$. Here again the dotted portions of the loci represent the mathematical possibilities and the full lines the physical limitations. The points c and c' are the same for any reactances which may have been added to Z_1; hence they may be called invariant points in the comparison.

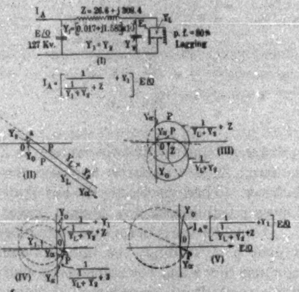

FIG. 17

Another example is the location of the generator current locus for a high-tension transmission line, in this case the 500-mile line described in the JOURNAL of the A. I. E. E. for September, 1924. Fig. 17 (I) shows the line reduced by the simple reduction formulas of Dr. Kennelly. It is loaded at a constant power factor of 80 per cent lagging current at the load. The equation of the current at the generator is

$$I = \left[\frac{1}{\frac{1}{Y_L + Y_2} + Z} + Y_1 \right] E\underline{/0}\,. \qquad \textbf{(10)}$$

The successive operations which are necessary are indicated in this equation. The variable Y_L is the load admittance; the other factors are constants which are indicated as per Fig. 17, (I). The graphical operations are accomplished in the order in which equation (**1**) is derived and are indicated as follows:

Y_L and $Y_L + Y_2$ [Fig. 17, (II)]

$\frac{1}{Y_L + Y_2}$ and $\frac{1}{Y_L + Y_2} + Z$ [Fig. 17, (III)]

$\frac{1}{\frac{1}{Y_L + Y_2} + Z}$ and $\frac{1}{\frac{1}{Y_L + Y_2} + Z} + Y_1$ [Fig. 17, (IV)]

$$I = E\underline{/0} \left[\frac{1}{\frac{1}{Y_L + Y_2} + Z} + Y_1 \right] \text{[Fig. 17, (V)]}$$

It will be seen from Fig. 17, (V) that the current is a minimum at P. In order to determine the exact value of load Y_L for this minimum current, this value of P may be traced through each transformation and is indicated by P on every locus.

From purely physical considerations of this circuit it is evident that the voltage at open circuit is the same, irrespective of the value of admittance Y_L as gradually decreased to zero. Also, the voltage at a short circuit is zero irrespective of how Y_L was increased to infinity. Hence there are two invariant points in this system common to any kind of loading, that for open circuit $Y_L = Y_0$, and that for short circuit $Y_L = Y_\infty$. All of the current loci will intersect at these two invariant points.

DISCRIMINANT LOCI

Certain loci may have something in common with other loci which allow the rapid evaluation of a large number of loci by the use of this locus. A locus which can be used in this manner may be called a discriminant locus. For example, suppose it were desired to compute all the generator current loci for all power factors of loading Fig. (18) and of circuit Fig. 17 (I).

Here are shown various power factors of loading Y_a = 50 per cent leading, Y_b = 86 per cent leading, Y_c = 86 per cent lagging, and Y_d = 50 per cent lagging as illustrated. It would be, with the above simple method of current evaluation, rather tedious to carry through each evaluation as per Fig. 17. From Fig. 18 (II) it is seen that $Y_L + Y_2$ produces, for every power factor, a circle locus 123056; this circle has one point in common for every power factor in addition to the invariant points Y_0 and Y_∞; it is a discriminant locus. Now by treating this circle as a special type of load and carrying it through as per Fig. (17), a locus of I is found, Fig. 18, III. All power factors, Y_a, Y_b, Y_c, and Y_d, have the points 5, 6, 2 and 3 respectively, on this circle. Since the current loci must also pass through Y_0 and Y_∞ in addition to a point on this locus, it is possible to draw a new circle through these three points, for example, point 2, Fig. 18 (IV), to determine the current locus at 86 per cent power factor. The small shaded area shows the change from 80 per cent power factor of Fig. 17 to 86 per cent power factor, or the range of generator current due to change of power factor at the load. The phase angle of the current I at the generator and the voltage $E\underline{/0}$ can be obtained directly from the diagram. Because all current loci have their centers in line $M\,N$, this line is also a discriminant locus. It can be seen from Fig. 18, (V), that a change of load power factor from 50 per cent to 80 per cent, leading, corresponding to loci Y_a and Y_b, produces a much greater change of current at the generator than a similar change from 50 per cent to 86 per cent lagging, or circles Y_c and Y_d.

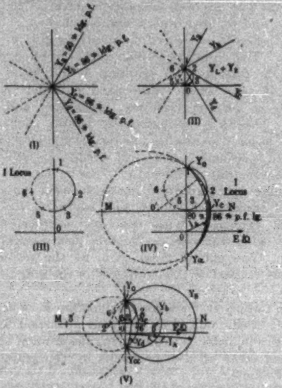
(I)
(II)
$Y_L + Y_2$
I Locus
(III)
M
I Locus
N
86 % p.f. lg.
$E\underline{/0}$
(IV)
Y_a
Y_c
M
N
Y_b
(V)

FIG. 18

In order to determine the voltage E_L at the load, the relation

$$E_L = E\underline{/0} - [\,I_A - E\underline{/0}\;Y_1\,]\,Z \qquad (\mathbf{10})$$

is used, and the transformations are followed as indicated in the equation, (see Fig. 19). All of the generator current loci of Fig. 18 (V) were operated upon as indicated in equation (**10**). The discriminant line $M\,N$ of Fig. 18 (V) transforms into the line $M'\,N'$ and

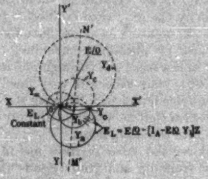
Y'
N'
$E\underline{/0}$
Y_d
Y_c
Y_∞
X
X'
0
E_L Constant
Y_b
Y_0
$E_L = E\underline{/0} - [I_A - E\underline{/0}\,Y_1]Z$
Y_a
Y
M'

FIG. 19

allows a very rapid determination of the voltage at the load E_L. This diagram shows what values of Y_L must be chosen if the system is to have a constant voltage at the load end in conjunction with a constant voltage at the receiving end of a transmission line.

The well-known asynchronous machine can be graphically visualized with these transformations. The standard circuit of the induction machine is shown in Fig. 20 (I).

Remembering that the slip

$$s = \frac{\omega_1 - \omega_2}{\omega_1} \qquad (\mathbf{11})$$

in which ω_1 is the synchronous speed and ω_2 the speed of the rotor, in the equivalent circuit,

$$s = \frac{r_2}{R + r_2} \qquad (\mathbf{12})$$

or

$$R = \left(\frac{r_2}{s}\right) - r_2 \qquad (\mathbf{13})$$

where r_2 is the rotor circuit resistance and R is the resistance equivalent to the mechanical load on the rotor.

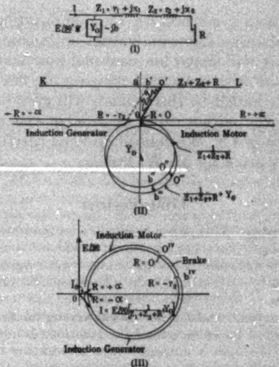
I
$Z_1 = r_1 + jx_1$
$Z_2 = r_2 + jx_2$
$E\underline{/0}$
Y_0
$-jb$
R
(I)
K
a'
b'
O'
$Z_1 + Z_2 + R$
L
$-R = -\infty$
$R = -r_2$
0
$R = 0$
$R = +\infty$
Induction Generator
Induction Motor
Y_0
$\frac{1}{Z_1+Z_2+R}$
O''
b''
O'''
b'''
$\frac{1}{Z_1+Z_2+R} + Y_0$
(II)
Induction Motor
$E\underline{/0}$
O^{IV}
$R = 0$
Brake
b^{IV}
I_0
$R = +\infty$
$R = -r_2$
0
$R = -\infty$
$I = E\underline{/0}\left[\frac{1}{Z_1+Z_2+R} + Y_0\right]$
Induction Generator
(III)

FIG. 20

The range of speed for the induction motor is

$$0 < \omega_2 < \omega_1$$

for which R varies from 0 to ∞ resistance.

The range of speed for the induction generator is

$$+\infty > +\omega_2 > \omega_1$$

for which R varies from $-r_2$ to $-\infty$.

The range of the frequency converter or brake is

$$-\infty < \omega_2 < 0$$

for which R varies from $-r_2$ to 0.

Here the critical locus is R, which has all values from $+\infty$ to $-\infty$ with singular points at O and $-r_2$ as shown in Fig. 20 (II). The effect of adding the primary and secondary impedance is shown in the line $K\,L$. Upon inversion the circle through the origin is realized and the points $R = +\infty$ and $R = -\infty$ are the same point at O.

The current delivered to or from this circuit is shown in Fig. 20, (III)

$$I = \left[\frac{1}{Z_1 + Z_2 + R} + Y_0\right] E\underline{/90}\,.$$

The voltage is chosen at 90 deg. to make the diagram similar to the conventional representation of the Heyland diagram.

Further application of these methods to transformers, generators, filter circuits, vacuum tubes, etc., is obvious, once the view point is ascertained.

CONCLUSIONS

This analysis indicates how, with the aid of a few elementary transformation theorems, the operation of electric circuits, machinery, and vibrations of all kinds may be visualized and their limitations and possibilities discerned without the aid of extensive mathematical formulas.

It indicates a method for the computation of charts showing changes upon individual units comprising the system. It will show, for example, how a system may be operated with greatest efficiency or flexibility and what units added to this system will give best results.

The writer wishes to acknowledge the many kind suggestions of Dr. Whitehead and the careful comparisons of the proof by M. W. Pullen.

Discussion

M. I. Pupin: This paper recalled to my mind the first paper which I read relating to an a-c. circuit in which a simple harmonic force is impressed and the resulting current is complex harmonic. It was a paper written by the late Professor Rowland of Johns Hopkins, about thirty-eight years ago.

He pointed out—and I think he was among the first to do so—that in a circuit in which one of the reactions is not simple harmonic, you may get a current which is complex harmonic although the impressed e. m. f. is simple harmonic. He considered the case of an induction coil with an iron core going through a cycle of magnetization and de-magnetization. Since the magnetic property of that core goes through a complete cycle in which the core does not act the same at any two consecutive stages on account of the variable permeability, you have a reaction due to an inductance which varies periodically. All you can say about that reaction is that it is a harmonic function having a fundamental period—equal to that of the impressed motive force. By Fourier's theory, you can expand that function in an infinite series of harmonics which are odd multiples of the fundamental.

We have a great many cases like that in electrical engineering—cases of circuits in which one of the electromagnetic constants is a harmonic function of the time. For instance, in an induction motor the mutual inductance between armature and field is a periodic function of the time which is determined by the speed of the motor. You cannot treat this case in the same way in which you treat ordinary a-c. circuits, although many engineers do when they employ the graphical method.

I do not use the graphical method in that way and I explain to my students—graduate students, to be sure—why I don't use it,—because it is misleading and does not give the real physics of the reactions of the operation of the induction motor. That is true of the single-phase induction motor, and to a great extent of the polyphase induction motor. You can't treat these circuits the same as you treat circuits in which all of the electromagnetic constants remain invariable. You can, by the use of a graphical method, get approximately good results, but you don't get exact results. Exact results demand going back to Maxwell and a treatment of the circuits in the Maxwellian fashion. To be sure, that is not the way to avoid complicated mathematics. But it may be complicated for some people and not complicated for a student who has been tolerably well trained in mathematical operations, and we have quite a number of them.

In our discussion of electrical circuits we deal with differential equations. The equation of reactions is a differential equation and it is the fundamental equation in the theory of electrical circuits. It is a linear differential equation with constant coefficients as long as resistance, inductance and capacity are constant. I think a sophomore can handle such an equation.

Some of us have an idea that all equations expressing the law of actions and reactions in electrical circuits are differential equations with constant coefficients. That is not the case. Very often in some very important problems, as in induction motors, the law of equality of actions and reactions does not lead to differential equations with constant coefficients. They are linear but with variable coefficients and demand a different treatment. You cannot treat them by the graphical method, with all due respect to the excellency of the graphical method in some cases.

You can't apply it in all cases, as you know, of course, and the only object of my talk is to call your attention to that as forcibly as I can in order to avoid giving the student the impression that he can treat all a-c. problems by the graphical method. It cannot be done.

There is one very interesting case I shall mention here, which probably will not be familiar to all of you, and it is this: It is a vacuum-tube oscillator in wireless telegraphy. There is a new piece of apparatus which is just as important as any generator. Why? Because it is the foundation of radio broadcasting, and that is a great thing. That is one of the great achievements of the electrical engineer's profession. The ability to transmit the spoken word from any place in the United States to any other place in the United States, and perhaps to any other place in the world, is a great achievement—a tremendous achievement, and it is made possible by the vacuum-tube oscillator invented by a student of mine, Armstrong, in my laboratory at Columbia.

Another student of mine, of the General Electric Company, came almost as near to it as Armstrong did. I think that is due, to a certain extent, to my talking to them about circuits that cannot be treated by the graphical method. In fact, I lectured about a method of creating a-c. oscillations of different frequencies by means of induction machines. Armstrong was listening intently and when I got through one day he came to me and said, "Professor, your theory is very fine, but I can do the same thing in a much simpler way by using the vacuum tube.—" And he did. I decided that he was right and my mathematics and machine could go into the scrap heap.

I think that particular case justifies me in wanting you to impress upon the students and your friends that there are a-c. circuits which cannot be treated by the graphical method.

W. P. Dobson: Mr. Lee has described methods admitting of a wide variety of application, a feature which most graphical methods do not possess.

In La Cour and Bragstad's "Theory and Calculation of Electric Currents," Chapter III, methods similar to those of Mr. Lee, are described. Impedance and admittance are represented by vectors and the operations $I.\,Z$ and $Y.\,E$ are carried out graphically in essentially the same manner as indicated in Figs. 1 to 5 of Mr. Lee's paper. Thus the voltage curve at the terminals of a constant impedance is similar to the current curve but corresponding vectors are displaced by the angle $\varphi = \tan^{-1} \frac{n}{r}$ and the lengths are in the ratio $Z : 1$. If new axes be chosen rotated in the negative direction through an angle φ and if suitable scales for current and voltage be chosen then the current curve will be the voltage curve referred to the new axes. In the case of a circuit having variable impedance La Cour and Bragstad make use of the geometrical principle of inversion. Thus the admittance curve is the inverse of the image in the axis of reals of the impedance curve. In Mr. Lee's first example, Fig. 14, the circle $o\ a'$ is the inverse of the image of $a\ B$. This is the admittance curve. It will also be the current curve (to a suitable scale) referred to a new set of axes displaced 30 deg. in the negative direction. The same curve may also represent the voltage on the load $E - I\ Z$. Referring to Fig. 15 it may be shown that if c^1 is taken as the origin and $c^1\ o$ the axis of reals, the curve $c^1\ o$ will represent, to a certain scale, the voltage on the load. This renders the construction in Fig. 16 unnecessary. The utility of La Cour's method lies in the fact that only one diagram is necessary. There are two sets of axes, one for voltage and the other for current.

The application of graphical methods to the calculation of electric circuits is often unsatisfactory unless power can be represented on the diagram. La Cour and Bragstad obtain constructions for both power and efficiency.

The chief advantage of graphical methods in most cases lies in the power of visualization which they supply. For numerical work they seldom offer any advantage over arithmetical methods. They frequently do not yield sufficiently accurate results and their application is limited by the quantities which we desire to investigate. An excellent example of the utility of graphical methods is Mr. C. H. Holladay's chart for transmission lines[1] in which voltage, currents and power are represented on the same diagram.

C. A. Adams: I have been tremendously interested in what Dr. Pupin has said and agree that in our engineering teaching we are apt to slur over the more exact methods of analysis in favor of approximate methods. I think it is safe to say that nearly all methods of solving engineering problems as presented to students of engineering are approximate methods and in most cases very crudely approximate. Moreover, the student, in ninety-five cases out of a hundred, doesn't know that the solution is approximate or what the approximations are. The solution is good for a narrow range within the desirable practical degree of accuracy. When he gets outside of that range, he is lost. That this is so has been proved time and time again by the failure of the better engineers to solve satisfactorily fairly simple problems presented to them. So I am in hearty sympathy with the spirit of Dr. Pupin's remarks.

I wonder, however, if in the case of the induction motor and other pieces of electrical machinery, it is humanly possible to be at all exact in our analysis? You may try to take account of the variability of some of the factors we ordinarily assume constant. They vary in some cases in a manner which can be expressed only by a long series of terms, and even then only approximately. The properties of the materials are never known accurately. No mathematical analysis, however complicated, can predetermine with accuracy the core losses or the magnetic-circuit relations in a piece of rotating electrical machinery. It is merely a question of degree, and a question of judgment as to whether a more extended mathematical analysis is warranted by the resulting gain in accuracy.

The mistake we make in engineering education is in failing to emphasize the approximations, in failing to provide the student with a scientific foundation on which he can build his own solutions and make his own approximations with his eyes open and with full knowledge of the errors involved and the range of practise within which these errors are of negligible importance. Then when he gets into some other range of experience he will be able to carry his refinements farther, or make different approximations.

Of the approximate methods of analysis and visualization, some are tremendously valuable; as, for example, the circle loci of Mr. Lee's paper. In this connection I refer Mr. Lee to a series of articles I wrote for the *Harvard Engineering Journal* in 1905-1906, in which I discussed the circular loci of induction motors and incidentally used almost identically the same method as that employed by Mr. Lee. These loci are very useful in providing a simple method of visualizing the approximate relations between the constants of the motor and its operating characteristics.

As long as the student or engineer, in making his applications, knows where the approximations have been made, the magnitude of the resulting errors and how accurate it is necessary to be in his solution, he can use mathematics or graphical methods intelligently.

That is really the important point at issue, and the mistake which we have made in the past is exactly that pointed out so clearly by Dr. Pupin. We carry our students through a lot of superficial approximation methods of solving problems without sufficient emphasis on the approximations and without giving them the ability to go more deeply into the analysis. In fact, in many cases they have a notion that they are getting a deep, theoretical treatment of the subject. Most of them think it is too theoretical, when as a matter of fact the old complaint of the employers in industry that our men were too theoretical was all nonsense. The only difficulty was that really they were not theoretical enough, because had they been more so they would have obtained good practical results.

D. C. Prince: We have had two general points of view presented as to methods of solving problems. One is that we take our problem and divide it up into the form of an infinite series. The other is that we take the outstanding phases of it and produce a graphical solution for them.

The principal advantage of the graphical method is that it gives a general picture which you are very likely not to get by infinite series. The advantage of the infinite series is accuracy, but at the same time you set up your equations and get out certain results which are all right provided your series is truly convergent and has been properly taken.

For instance, Doctor Pupin has raised the question of applying a series to the vacuum tube. I have seen a great many of those analyses, and as long as the current does not stop, it is possible to make a very nice convergent series which will cover the case. If you press the output and go in the direction of a highly efficient oscillating circuit where the current is in the form of jabs occupying only a few degrees of arc, you find you have repeated points of discontinuity appearing, which present points of difficulty. If the current is zero for a considerable time, we then have in our series a large number of different waves, all of which add to zero and therefore all of which are not there for a considerable part of the time.

Therefore there are problems which seem to demand coming back to a more or less laborious first principle in which you work the thing out. In a paper in the *Journal* of the Radio Institute[2] we presented such an idea of going after in a more or less graphical way the method of working out the oscillator because the series methods, if sufficiently developed to be exact, seemed to lead off to a point where you could not tell which terms were valuable and which were not.

M. I. Pupin: Somebody couldn't tell, but others could. Armstrong could.

D. C. Prince: Yes, but he is working almost altogether with cases where he does not press efficiency to a point that strikes discontinuity.

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[1] TRANS. A. I. E. E., 1922, p. 785.

[2] June, August and October, 1923.

M. I. Pupin: Discontinuities such as a man can make with his voice are discontinuities which happen after the stationary state has been reached and you don't care about them. You are dealing then with the stationary state. The hand of man is slow and his voice is slow and these oscillations go perhaps a million times per second. You are dealing with a stationary state there anyway.

D. C. Prince: I referred particularly to discontinuities that occur at the frequency of the oscillation.

M. I. Pupin: But there are no discontinuities of that kind, so far as I know.

D. C. Prince: In the ordinary amplifier you always have to provide for proportional current so you may get some sort of current variation which will be a continuous procedure with harmonics of various sorts. As soon as you begin to chop the waves down to zero so the current actually flows for a small part of a cycle, then you have discontinuity repeating as fast as the cyclic variation. These intervals of zero current last as long as three-quarters of a cycle or more. In oscillating circuits, if you make an analysis of the possible circuit efficiency, it approaches 50 per cent on the basis that the waves shall come down and be tangent to the axis. We get efficiencies of 80 per cent and more.

M. I. Pupin: Those are periods that are equal periods and follow others of equal intervals of time. That gives you simply another series which is a continuous series.

D. C. Prince: You can produce a continuous series, but what it means is that you are making out of a blank a continuous series of waves which is not there at all.

M. I. Pupin: That doesn't make any difference.

Harold Pender: Not long ago I was talking to the representative of one of the leading publishers of technical books in this country. I asked him why he didn't publish some really good textbooks on electrical engineering. His reply was that his best books on electrical engineering subjects did not sell. This is typical of the present tendency in engineering teaching. Apparently what is desired by a great many teachers is a type of book which will make the teaching of electrical engineering easy; a book that will require a minimum amount of thought on the part of the student and a minimum amount of effort on the part of the instructor. That is one of the reasons why graphical methods are so much in vogue. The teacher seems to wish to find a means of getting over the difficult fundamental part and to give the student something with which he can play; some nice little vectors with which he can juggle as he would with toys.

Over and over again in teaching the elements of alternating currents I have seen that the students find great difficulty in understanding the basic theory based on the instantaneous values. But just so soon as they think they understand the vector representation of alternating currents, everything (to their mind at any rate) becomes extremely simple and easy. I think if we can get the student to think of things as actually happening and not to think of alternating currents, for example, as so many vectors which he must draw on a sheet of paper and by some juggling get an answer, the more thoroughly will the student understand what it is all about.

There is a book which appeared about twenty years ago, by Professor Bedell, on alternating currents. I think it is used still at one university, although I am not sure of that. I know it was in use a few years ago. Most people think that that book is very old-fashioned. I would suggest that some of the men who are teachers get a copy of the book and read it. The author of the book is a man who grew up on instantaneous values. He knows his subject and he is not inclined to avoid difficulties by using vectors and graphical solutions.

R. E. Doherty: I should dislike to have the idea firmly entrenched in the minds of people here that graphical methods are in general of no very great use. I agree completely with Dr. Pupin, Mr. Adams and Dr. Pender in what they have said with respect to graphical solutions, and with their idea of what should be given out in general to students; nevertheless, I wish to say, that I have found graphical solutions of problems to be of great help. I understand that Dr. Pupin refers specifically to quantities which vary as sine waves; that is, sinusoids.

M. I. Pupin: To circuits which contain variable elements—variable reactions.

R. E. Doherty: Differential equations, of course, which cannot be solved by any functions familiar to us can sometimes be found very conveniently by carrying out graphical construction.

F. W. Lee: Dr. Pupin's points are well taken because ordinary circuit analysis does not consider the variation of resistance with the frequency and the current density in a conductor; also it neglects the variation of energy dissipation and change of induction with variable magnetic intensities, nor will it account for changes of capacity and power factor as well as previous cyclic conditions of condenser systems. It has been the custom to combine these phenomena into effective resistances and reactances, taken sufficiently large to insure safety. At best the linear differential equation with constant coefficients is an approximation.

I am sure that the electrical profession would be delighted to have a new system of mathematics which takes cognizance of all these facts and which would be able to give quick accurate solutions. It should not be forgotten, however, that although the solution of the linear differential equation with constant coefficients as presented by Rowland, was available before Dr. A. E. Kennelly first introduced the complex vector in 1893 in a paper on "Impedance,"[3] this, followed by the application of the complex vector to Kirchhoff's law and electrical machinery by Dr. Steinmetz, did more to clarify and simplify the understanding of a-c. phenomena than any other single factor. Most engineers are very loath to abandon the electrical analysis founded upon these conceptions unless something simple and better is offered as a substitute.

Also I do not wish to detract from the classical work of Dr. Arnold upon graphical presentations of a-c. circuits. Sufficient amount of fundamental material was introduced in order to give ample background to a clear understanding of the complete contents of the paper.

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[3] A. I. E. E. Trans., 1893, p. 116.

Remote-Controlled Substations
of the New York and Queens Electric Light and Power Company

BY W. C. BLACKWOOD[1]
Member, A. I. E. E.

***Synopsis.**—This paper describes a unit type of distribution substation which has been designed for a metropolitan district. Although of much lower capacity than other stations on the same system, its cost per kv-a. is approximately the same as the larger stations. The station is unattended and all operations are controlled from a distant attended station. The cost of operation per unit of capacity is no greater than for a much larger attended station. Remote control was adopted rather than automatic operation because of the nature of the load and territory served where service with minimum interruption is demanded.*

THE New York and Queens Electric Light and Power Company serves four wards in the Borough of Queens, New York City, covering approximately 100 sq. mi. This territory is developing very rapidly both with industrial establishments of all types and sizes and residences varying from small one-family houses to very large apartment buildings.

Energy is purchased from the United Electric Light and Power Company at 13,200 volts and 26,400 volts and is stepped down for 2300/4000-volt, four-wire, distribution in substations located in Long Island City, Flushing, Maspeth and Jamaica, the ultimate capacity of these stations being 75,000 kv-a., 40,000 kv-a., 60,000 kv-a. and 30,000 kv-a. respectively.

In 1925 it was necessary to provide additional substation capacity in the district supplied by the Jamaica Substation, and three means of accomplishing this were considered—first, to increase the size of the present substation at Jamaica; secondly, to build another similar attended station; and thirdly, to build a smaller unattended station for automatic operation or remote control, and to provide for future growth by the erection of other similar unit unattended stations.

A study of these plans indicated that the cost per kv-a. of a large attended station and a small unattended station with automatic features or remote control was approximately the same due to the greater simplicity of the smaller station and the possibility of using switches of lower interrupting capacity than would be required in a large substation where the concentration of power would be greater. It was also found possible to resort to bus regulation in the smaller station, because of the shorter distribution feeders which would be supplied by it, while in the large station it would be necessary to install regulators on the individual feeders with the attendant complication of spare feeder equipment and transfer bus.

If operators were employed the operating cost per kv-a. of the smaller station would, of course, be greater than that of a large station but, if unattended, the operating cost per kv-a., considering periodic inspections which must be made, is estimated to be less than that in the large attended station.

In considering whether an automatic or a remote-controlled substation would be built, the governing factor was the quality of service to be supplied. As demanded in a metropolitan section, it has always been the company's policy to furnish service as nearly free from interruption as possible. Also, the increasing demands upon electric service for such devices as oil-burning equipments, electric clocks, operation of radio and other uses are beginning to make the furnishing of service with 100 per cent continuity imperative. An automatic substation without supervision and control from an attended station would not give the quality of service demanded, and, in order to supply such service, it was decided that all of the operations in the substation should be under the direct and immediate control of an attendant at some point on the system. With such supervision, it was not considered necessary to install automatic features in the station, and the final decision was to build a station with all switches remote-controlled from the nearest attended station and with all operations indicated at that station.

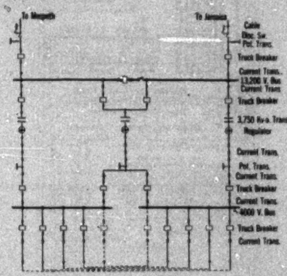

FIG. 1—ONE-LINE WIRING DIAGRAM WOODHAVEN SUBSTATION

The first substation of this type was erected in Woodhaven at a location about 2½ mi. from the Jamaica substation. Three 3750-kv-a., three-phase, oil-insulated, self-cooled transformers were installed and the rated capacity of the station is 7500 kv-a., one transformer being a spare. The station is fed by a 13,200-volt, three-phase, 500,000-cm. underground cable from the Jamaica substation and a similar cable from

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[1] Of the New York and Queens Electric Lt. & Power Co., Long Island City, New York.

Presented at the Annual Convention of the A. I. E. E., White Sulphur Springs, W. Va., June 21-25, 1926.

the Maspeth substation approximately 5½ mi. away.

Referring to the wiring diagram, Fig. 1 will show that each incoming cable is connected through oil circuit breakers to a 13,200-volt bus, which is sectionalized at its midpoint by an oil circuit breaker. One transformer is connected to each section of this bus through oil circuit breakers, and the third transformer is arranged with double breakers for connection to either section of the bus. The switching connection on the 4000-volt side of the transformers is similar to that on the primary side, with the exception that the 4000-volt bus is operated in two sections without a tie breaker. There are four 4000-volt loop distribution feeders, the two ends of each feeder being connected to opposite sections of the 4000-volt bus. Three 135-kv-a., single-phase, automatic induction voltage regulators are connected in the secondary leads of each transformer. The distribution feeders are rated at 400 amperes for a loop feeder, although the switches on each end of the loop have a capacity of 600 amperes.

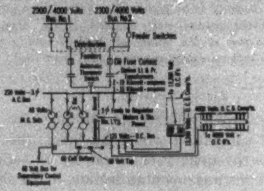
2300/4000 Volts Bus No. 1
2300/4000 Volts Bus No. 2
Feeder Switches
Distribution Feeders
Oil Fuse Cutout
Station Lt. & Pr. Transformers
220 Volts—3 ϕ A. C. Bus
48 Volts M. G. Sets
3 ϕ Feeds to Regulator Motors & Sta. Power
125 Volts—D. C. Bus
4000 Volts O. C. B. Comp'ts.
To 4000 Volt O. C. B's.
60 Cell Battery
48 Volt Tap
48 Volt Bus for Supervisory Control Equipment

Fig. 2

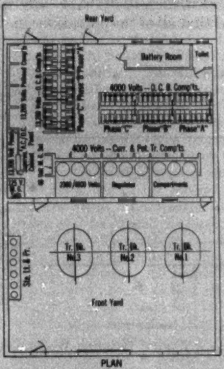
Rear Yard
Battery Room
Toilet
13,200 Volts Pothead Comp'ts.
13,200 Volts—O. C. B. Comp'ts.
Phase "C" Phase "B" Phase "A"
4000 Volts—O. C. B. Comp'ts.
Phase "C"
Phase "B"
Phase "A"
4000 Volts—Curr. & Pot. Tr. Comp'ts.
2300/4000 Volts
Regulator
Compartments
Tr. Bk. No. 3
Tr. Bk. No. 2
Tr. Bk. No. 1
Sta. Lt. & Pr.
Front Yard
PLAN

Fig. 3—Plan of Woodhaven Substation

Each transformer bank is protected by differential relays and the bus sections are similarly protected so that any fault on the bus or its connections will open all breakers connected to the section in trouble. With the loop arrangement of feeders, this will cause no interruption to service.

The bus and switch structures were greatly simplified by the installation of single-phase, truck-type, oil circuit breakers, grouped according to phases both on the 13,200-volt bus and the 4000-volt bus. The transformer switches and 13,200-volt switches are controlled as three-phase units, by electrically connecting the solenoids.

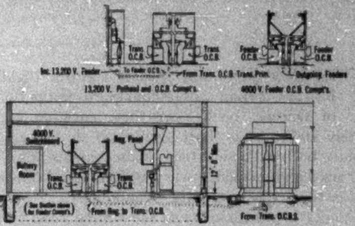
Trans. O.C.B.
Trans. O.C.B.
Feeder O.C.B.
Feeder O.C.B.
Inc. 13,200 V. Feeder
To Feeder O.C.B.
From Trans. O.C.B. Trans. Prim.
Outgoing Feeders
13,200 V. Pothead and O.C.B. Compt's.
4000 V. Feeder O.C.B. Compt's.
4000 V. Switchboard
Reg. Panel
Battery Room
Trans. O.C.B.
Trans. O.C.B.
12'-0" Min.
From Reg. to Trans. O.C.B.
From Trans. O.C.B.S.

Fig. 4—Sections of Woodhaven Substation

The 4000-volt feeder breakers are operated single-phase. As will be noted from the sectional drawings of the switch compartments, the grouping of all the switches on one phase makes it unnecessary to install three buses and avoids the usual connections from the bus to the switches which must, in the case of two of the phases, cross other buses if they are arranged in vertical formation.

Fig. 5—Panel of Jamaica Substation for Woodhaven Control

Fig. 2 is a diagram of the control connections, in the design of which consideration was given to the fact that the station would be unattended and that therefore continuity of the d-c. supply was essential. Two banks of station light and power transformers are fed from two 4000-volt feeders, tied to opposite bus sections. The secondaries are connected through an automatic throw-over switch to a 220-volt, three-phase bus from which two 5-kw. motor-generator sets are fed. Normally, one set is in operation, trickle-charging the battery, and relays are provided which, in case of failure of this set, will immediately place the other set in operation.

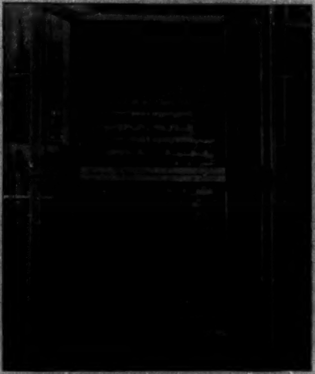

FIG. 6—SUPERVISORY CONTROL RELAY CABINET AT JAMAICA SUBSTATION FOR WOODHAVEN CONTROL

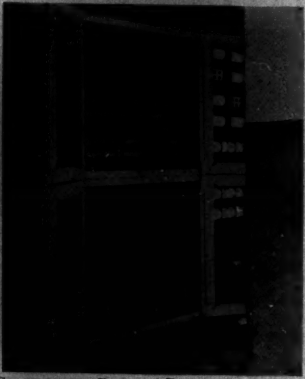

FIG. 7—SUPERVISORY CONTROL RELAY CABINET AT JAMAICA SUBSTATION FOR WOODHAVEN CONTROL

A one-kw. motor-generator is provided to supply the 48 volts, direct current, required to operate the supervisory control system. In case of failure of this set, connection is automatically made to a 48-volt tap on the station battery. The motor-generator set is used for normal operation instead of the battery tap to avoid unequal discharge of the battery.

Sectionalizing switches, but no fuses, are used in the d-c. connections to the oil circuit breaker compartments and all wiring is insulated with 5/32-in. special rubber compound, so installed that grounds or short circuits are practically impossible. This plan was adopted because the blowing of a fuse in an unattended station would probably not be discovered until an attempt was made to operate a breaker and a serious interruption might result if the d-c. supply to a trip coil was open at the time of a feeder short circuit.

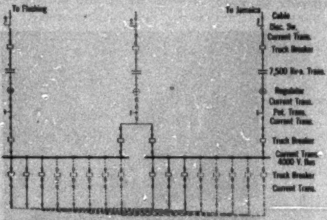
To Flushing
To Jamaica
Cable
Disc. Sw.
Current Trans.
Truck Breaker
7,500 Kv-a. Trans.
Regulator
Current Trans.
Pot. Trans.
Current Trans.
Truck Breaker
Current Trans.
4000 V. Bus
Truck Breaker
Current Trans.

FIG. 8—ONE-LINE WIRING DIAGRAM—HOLLIS SUBSTATION

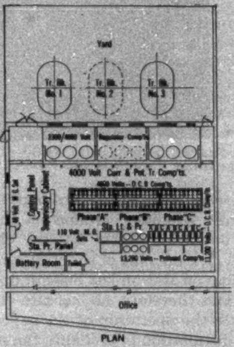
Yard
Tr. Bk. No. 1
Tr. Bk. No. 2
Tr. Bk. No. 3
2300/4000 Volt
Regulator Comp'ts
4000 Volt Curr. & Pot. Tr. Comp'ts.
4000 Volts—O.C.B. Comp'ts.
Phase "A"
Phase "B"
Phase "C"
48 Volt M.G. Set
Control Panel
Supervisory Cabinet
110 Volt M.G. Sets
Sta. Lt. & Pr.
Sta. Pr. Panel
Battery Room
Toilet
13,200 Volts—Pothead Comp'ts
13,200 Volts—O.C.B. Comp'ts
Office
PLAN

FIG. 9—PLAN OF HOLLIS SUBSTATION

All operations in the station are controlled from a panel in the Jamaica substation shown in Fig. 5. This panel is made of furniture steel and the control keys and indicating lamps are arranged in accordance with the wiring diagram of the Woodhaven station. The control is the Westinghouse Synchronous Relay Visual System, which was described in a paper by Mr. Chester Lichtenberg, presented at the Midwinter Convention in New York, February 8-11, 1926. The relay cabinets at the Jamaica and Woodhaven stations are shown in Figs. 6 and 7. The control cable, which is installed in underground ducts throughout, is ten pair No. 19 B&S, paper-insulated and lead-encased.

The supervisory equipment controls 21 single-pole, 13,200-volt breakers and 12 single-pole, 4000-volt breakers, in groups of three, but with single-pole indication at the Jamaica station and 24 single-pole, 4000-volt feeder switches with single-pole indication at the Jamaica station.

It is also possible to read at Jamaica the current and voltage of each transformer in the Woodhaven station and to operate the regulators from Jamaica to equalize the voltage on the transformers when one is being cut into service.

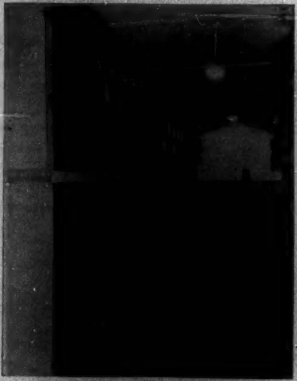

Fig. 10—Woodhaven Substation 4000-Volt Oil Circuit Breakers and Control Panels

Indication is given at Jamaica of high temperature in the transformer windings, ground on the d-c. control system, the operation of the battery charging motor generators and the opening of the station door.

The Woodhaven station has been in operation only a few months and therefore no definite conclusions can be drawn as to the operation of the supervisory equipment. The installation is probably the most extensive and complicated that has yet been placed in service and there was naturally a number of difficulties encountered in starting it. These were mostly matters of relay adjustment and changes in connections which were found to give back feeds or sneak paths which caused incorrect relay operations. These have been corrected and it is believed that the reliability of operation will be at least equal to that of an attended station.

Construction of a similar station at Hollis approximately eight miles from Jamaica has been started and while the connections and arrangement are substantially the same as at Woodhaven, certain changes have been made which are considered improvements in the design.

The Hollis station will have twice the capacity, or 15,000 kv-a. supplied by two 7500-kv-a. transformers with one 7500-kv-a. spare unit. The diagram of connections is shown in Fig. 8. Each transformer will be supplied by a 13,200-volt incoming feeder with no high-tension bus. With this exception the connections are the same as at Woodhaven. There are, of course, eight-instead of four-loop, 4000-volt feeders.

The arrangement of the bus and switch structures shown in the plan Fig. 9, is slightly different and a control board has been provided at the end of the 4000-volt bus structure on which are located the control switches for the feeder breakers which at Woodhaven are located on the panels directly above the breakers. The relays for each feeder are located on panels above the breakers as at Woodhaven but these panels have been faced in the opposite direction so that access is had from the top of the bus structure instead of from ladders in the switch aisle.

The supervisory control equipment is identical with that used at Woodhaven and the control panel will be placed alongside the Woodhaven panel in the Jamaica substation.

The growth of load in the Borough of Queens is taking place in a number of communities or centers some of which are widely separated and it is probable that the growth will be served by the erection of stations similar to those described above, at or near the load centers. Supervisory control equipment, although it has progressed rapidly in the past few years, has been developed principally along the lines of supervision and indication of the operations of automatic stations and has not been depended upon entirely for the complete operation of stations. Much more reliable equipment is needed when the station is not arranged to operate itself automatically in case of failure of the supervisory equipment and while the equipment now available may prove to be satisfactory, it will probably be found necessary to change its design in some respects to make it more sturdy before the reliability of operation which is demanded in metropolitan districts, is fully realized.

Fig. 11—Woodhaven Substation, 13,200-Volt Oil Circuit Breakers

Discussion

C. M. Gilt: The load and service requirements in the metropolitan districts are such that there has been some hesitancy in the introduction of purely automatic stations. In these territories it is common practise to maintain crews on 24-hr. duty for the promptest possible restoration of service in case of feeder outages. It is therefore important that the repair crews learn of the outage of any particular feeder as quickly as possible after it has occurred.

The purely automatic station does not provide this information without special indicating equipment approaching remote control in cost and complications. In some cases indicators have been installed to show that a feeder in a particular substation is out and in other cases the indication has shown in addition which feeder is open.

The remote-control equipment not only indicates which feeder has gone out for any cause, but makes it possible for the operator to reclose the breaker under directions from the repair crews as soon as they find the trouble or sectionalize the feeder. It also provides some indication of load characteristics and distribution as well as voltage conditions which frequently are of great value in maintaining proper service.

As indicated by this paper, equipment is available for accomplishing the results over comparatively few wires, and in spite of the fact that it appears light and delicate to the man accustomed to powerhouse equipment, it seems to have a remarkably satisfactory record of performance. It is to be hoped that the manufacturers will continue their efforts in developing simpler and more sturdy devices for this very important type of control.

F. B. Johnson: Mr. Blackwood spoke of bus regulation and if I understand him correctly, that refers to bus regulation on the 4000-volt side of the substation. I would like to ask him how large blocks of power are regulated through bus regulation?

Chester Lichtenberg: The remotely-controlled substations which have been described seem to be only an intermediate step in the development of the art. They seem to be merely an extension of the manual operation of substations and, therefore, are only a step towards the ultimate which probably is a completely automatic system.

In many parts of the country, central station companies have adopted the use of automatic substations for the class of service described by Mr. Blackwood. Some of the stations are remotely supervised where this has been deemed necessary.

A consideration of remotely-controlled substations brings with it a very important question. It is the hazard introduced if equipment designed for supervisory purposes is used for remote-control purposes.

Remote-control equipment in common use today has a distinctive feature. It is provided with at least one wire between the controlling point and the outlying station for each device to be controlled. Supervisory equipment, on the contrary, uses only a few wires for controlling a number of devices in an outlying station. It uses these wires in a predetermined fashion and as a result there may be quite an appreciable time lag between the operation called for by a supervisory equipment and the functioning of the devices at the remote end. Besides, this time interval may vary from one to two seconds up to eight or ten seconds. It is therefore difficult to operate a supervisory system as a remote-control system unless the limitations of the supervisory equipment are realized.

G. O. Brown: We have advanced beyond the remote-controlled substations by the use of entirely automatic substations.

In about 1920 the Kansas City Power & Light Company put in two substations entirely automatic and during the next year added only the indicating part of the supervisory control so the load dispatcher would know what was going on in those stations.

Within the last two years we have added two automatic substations of the same type and this fall we shall convert the last two of our manually operated stations into automatic. This will make all substations within Kansas City entirely automatic.

Our experience with this method of operation, using the supervisory-control devices for indication only, has been highly satisfactory. There have been less interruptions, and the time required to restore service has been much less than in the manually controlled stations.

E. K. Huntington: Our experience has been somewhat similar to that mentioned by Mr. Brown, in having two fully automatic, a-c. substations built about three years ago. Later we installed a simple form of remote indication which advises the system operator of trouble in the stations. This indication divides the trouble into three classes: transformers and station equipment; feeder circuits; and street-lighting circuits, so that the proper gangs may be sent out. Inasmuch as we have to rely upon circuits available from the local telephone company, I fear that at times entire control of the station by this means would be rather unsatisfactory. Much better service could be obtained by the use of a direct line, preferably underground, between the substation and system operator's office.

I fully agree with what Mr. Lichtenberg has said in connection with the automatic station having a number of advantages over the station which is entirely remote-controlled with no automatic equipment whatsoever in the station.

R. J. Wensley: The two stations described by Mr. Blackwood presented many new problems in design of the supervisory control equipment. The requirements of the induction-regulator control and of the single-pole breakers on the three-phase, four-wire circuits presented the most difficult problems. In the solution of these and other new problems incident to the remote control of the stations described, it is believed that a reasonably satisfactory solution of a complex problem has been obtained.

The author gives as his conclusion that supervisory control equipment should be made more sturdy. He is misled by the belief popular among engineers used to dealing with heavy power equipment, that size constitutes sturdiness. There are many cases where increase in size or weight of parts actually makes the device less suitable for its duty.

In his discussion of the Lichtenberg paper on the subject of supervisory systems, (presented at the Midwinter Convention, February 1926), F. R. McBerty, President of the North Electric Manufacturing Co. and a telephone engineer of many years' standing, gives an excellent brief for the telephone relay as a reliable device. He shows that the design tests on this class of equipment run into many millions of continuously successful operations. In the equipment described by Mr. Blackwood, the telephone-type relays have been improved by the addition of twin contacts. These operate on the halves of a split spring, each half of which is free to move independently. The chance of failure due to simultaneous contact trouble is small. This twin-spring construction has been standard with one manufacturer for about three years.

The upward swing of the sales curve of supervisory control equipment is so pronounced that it is felt that the idea is now well established and that there is a large future for this class of equipment. So many possibilities in the flexible and economical control of power distribution systems are revealed when these problems are examined as potential applications of supervisory control that many large power systems are being studied to determine where this new art may be used to advantage. Another year will undoubtedly show a great increase in number of equipments in service as well as some improvement in general design. This improvement will probably show mostly in simplification rather than in change in design of essential parts.

W. C. Blackwood: I shall close by answering the question as to the blocks of power which are bus-regulated. In the

Hollis Station we have three 7500-kv-a. transformers and the total station load of 15,000 kv-a. will be bus-regulated. The area served by the station is approximately a circle with a radius of from about 2 to 2 ½ miles.

The High-Speed Circuit Breaker in Railway Feeder Networks

BY J. W. McNAIRY[1]

Non-member

Synopsis.—*A method of isolating grounded sections of extensive feeder networks supplying power to railways without disturbing the power supply to other sections has long been desired by railway engineers. This has been successfully accomplished by utilizing the inherent characteristics of the magnetic type of high-speed circuit breakers in the manner described in this article. These characteristics are (a) high-speed operation, (b) discriminating characteristic, (c) reduction in trip point with reduction in line voltage, (d) polarized characteristic.*

A description of an experimental investigation on an equivalent network for a representative section of feeder system, made up by using reactors and resistors, has been given. Complete selective operation, isolating defective feeder without disturbing the power supply to interconnected feeders, was obtained for all locations of the short circuit.

Breakers of the type used successfully on d-c. networks can be applied to a-c. networks and the advantages of high-speed protection realized on this type of system. High-speed operation is very effective in reducing telephone interference during short circuits on the a-c. railway system.

INTRODUCTION

THE development of the high-speed circuit breaker and its application to the d-c. railway system has contributed greatly toward reduced maintenance and increased reliability of the power supply of this system. The effectiveness of the high-speed operation of this device in reducing the duration of short circuits and limiting the current, thereby eliminating flashovers of commutating apparatus and reducing the damage from arcs at points of fault, has been well recognized for a number of years. The elimination of flashovers and the reduction in the damage to windings of motors and other apparatus, connecting cables and mechanical equipment coming into contact with fallen contact lines or faulty cables, has reduced to a minimum the duration of power interruptions from this source.

With a few minor exceptions, high-speed circuit breakers applied to d-c. railway networks have been of the magnetic type, one form of which is shown by Fig. 1. This type involves the principle of a contact arm held in the closed position against a strong spring by a holding armature across the poles of an electromagnet. A tripping coil is interposed between the holding coil and the armature and is so connected that current through this coil exerts a m. m. f. to reduce the flux in the retaining armature, at the same time increasing the flux through a suitable magnetic by-pass circuit without an appreciable change in the flux through the core of the main holding coil. As the tripping current is increased, the flux is shifted in this manner from the holding armature to the magnetic by-pass circuit without the necessity of changing the flux interlinking the main holding coil. The effectiveness of this arrangement is such that the armature is released practically instantaneously when current in the trip coil reaches a predetermined value, regardless of the rate at which this current is increasing.

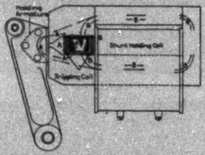

FIG. 1A

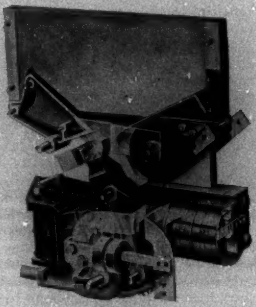

FIG. 1 B

1A—MAGNETIC CIRCUIT OF THE HOLDING AND TRIPPING COILS OF THE HIGH-SPEED CIRCUIT BREAKER

1B—PHANTOM VIEW OF A 600-VOLT, D-C., HIGH-SPEED CIRCUIT BREAKER

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[1] Railway Equipment Engineering Department, General Electric Co., Schenectady, N. Y.

Presented at the Annual Convention of the A. I. E. E., White Sulphur Springs, W. Va., June 21-25, 1926.

There are certain inherent characteristics in this type of circuit breaker which introduce possibilities for selective operation between breakers applied to a d-c. railway network, in case of short circuits, in such a way as to isolate defective feeder sections without the interruption even momentarily, of the power supply, to interconnected sections. The results obtained by proper utilization of these characteristics are comparable to those obtained on the usual a-c. networks by selective relay systems of the various well-known types.

The remainder of this article furnishes a detailed explanation of these characteristics and the methods of utilizing them, together with a description of the results obtained during an experimental investigation on a representative network.

CHARACTERISTICS OF THE MAGNETIC TYPE HIGH-SPEED CIRCUIT BREAKER

The following inherent characteristics of the magnetic type of high-speed circuit breaker contribute most to the solution of problems of selective operation on a d-c. railway network:

a. High-speed operation,

b. Discriminating characteristic providing a reduction in the trip point during a rapid current rise,

c. Reduction in trip point with a reduction in line voltage,

d. Polarized characteristics.

I. HIGH-SPEED OPERATION

In order to fully protect commutating apparatus in the d-c. railway substation, machine breakers must operate sufficiently fast to prevent flashover in case of short circuits inside of the station. While short circuits inside of station feeder breakers occur infrequently, without high-speed operation of protecting breakers, the current is relatively great and the resultant damage to substation apparatus is likely to be serious. This consideration automatically precludes the use of relays or any other selective system which delays the operation of such circuit breakers even slightly under short-circuit conditions.

The selective operation of the magnetic type of high-speed breaker is based upon an arrangement whereby the current is limited by the breaker or breakers supplying the faulty feeder directly, before the trip point of similar breakers supplying interconnected feeders, or the machine breakers in the substation is reached. During d-c. transients with an initial current rise of hundreds of thousands of amperes a second, the speed of operation of the breakers feeding such a short circuit must be relatively high. This is essential in order that advantage may be taken of conditions or arrangements whereby the trip current of one breaker is reached in advance of the others. These breakers must operate fast enough to limit the current before the trip point of the remaining breakers is reached. The speed of operation of the magnetic type of high-speed breaker designed to operate sufficiently fast to prevent flashovers of commutating substation apparatus is sufficient for such an application.

II. DISCRIMINATING CHARACTERISTIC

The discriminating characteristic of the magnetic type of high-speed circuit breaker is obtained by connecting an inductive shunt in multiple with the tripping coil, the total current through the breaker being distributed between these two circuits. The distribution of a slowly rising current between the trip coil and shunt is determined by the resistance, (and of a rapidly rising current by the inductance,) of the two parallel circuits.

The trip point of a circuit breaker so equipped can therefore be made a function of the rate at which the current is increasing.

CALCULATIONS OF CURRENT DISTRIBUTION BETWEEN TRIPPING COIL AND ITS SHUNT

Where the constants of the external circuits are known, the current through the trip coil at any instant can be calculated as follows:

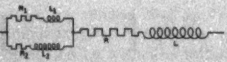

FIG. 2—GRAPHICAL REPRESENTATION OF THE RESISTANCE AND INDUCTANCE OF THE TRIP COIL OF A HIGH-SPEED CIRCUIT BREAKER IN PARALLEL WITH ITS SHUNT AND THE COMBINATION IN SERIES WITH A FEEDER

Referring to Fig. 2,

L_1 = Inductance of tripping coil
L_2 = Inductance of tripping coil shunt
L = Inductance of the external circuit
R_1 = Resistance of tripping coil
R_2 = Resistance of tripping coil shunt
R = Resistance of external circuit
I_m = Final current external circuit
= E/R where E applied voltage
t = time

$$I_1 R_1 + L_1 \frac{d I_1}{d t} = I_2 R_2 + L_2 \frac{d I_2}{d t} \qquad (1)$$

$$I_2 = I - I_1 \qquad (2)$$

$$\frac{d I_2}{d t} = \frac{d I}{d t} - \frac{d I_1}{d t} \qquad (3)$$

L_1, L_2, R_1 and R_2 are negligibly small in comparison with L and R;

$$\therefore I = I_m \left(1 - e^{-\frac{R}{L} t}\right) \qquad (4)$$

Substituting (4) in (2):

$$I_2 = I_m \left(1 - e^{-\frac{R}{L} t}\right) - I_1 \qquad (5)$$

Also from (4):

$$\frac{d I}{d t} = I_m \frac{R}{L} e^{-\frac{R}{L} t} \qquad (6)$$

Substituting (**6**) in (**3**):

$$\frac{d\,I_2}{d\,t} = I_m \frac{R}{L} e^{-\frac{R}{L}t} - \frac{d\,I_1}{d\,t} \qquad (\mathbf{7})$$

Substituting (**5**) and (**7**) in (**1**):

$$L\,(L_1 + L_2)\frac{d\,I_1}{d\,t} + L\,(R_1 + R_2)\,I_1$$

$$= I_m\,(L_2 R - R_2 L)\,e^{-\frac{R}{L}} + I_m R_2 L \qquad (\mathbf{8})$$

Solving for I_1:

$$I_1 = \left[\frac{I_m\,(L_2 R - R_2 L)}{L\,(R_1 + R_2) - R\,(L_1 + L_2)}\right] e^{-\frac{R}{L}t}$$

$$+ \frac{I_m R_2}{R_1 + R_2} + c\,e^{-\left(\frac{R_1 + R_2}{L_1 + L_2}t\right)}$$

Substituting the value of c, when at $t = 0$, $I_1 = 0$:

$$I_1 = \left[\frac{I_m\,(L_2 R - R_2 L)}{L\,(R_1 + R_2) - R\,(L_1 + L_2)}\right]\left[e^{-\frac{R}{L}t} - e^{\frac{R_1 + R_2}{L_1 + L_2}t}\right]$$

$$+ \frac{I_m R_2}{R_1 + R_2}\left[1 - e^{-\frac{R_1 + R_2}{L_1 + L_2}t}\right]$$

The time of maximum I_1 can be found from the relation:

$$\frac{d\,I_1}{d\,t} = I_m\left(\frac{R_1 + R_2}{L_1 + L_2}\right)\left(\frac{L_2 R - R_2 L}{L(R_1 + R_2) - R(L_1 + L_2)}\right)e^{-\frac{R_1 + R_2}{L_1 + L_2}t}$$

$$- I_m\left(\frac{R}{L}\right)\left(\frac{L_2 R - R_2 L}{L\,(R_1 + R_2) - R\,(L_1 + L_2)}\right)e^{-\frac{R}{L}t}$$

$$+ \frac{I_m R_2}{L_1 + L_2} e^{-\frac{R_1 + R_2}{L_1 + L_2}t} = 0$$

Or:

$$t = 2.3026\left(\frac{L\,(L_1 + L_2)}{R\,(L_1 + L_2) - L\,(R_1 + R_2)}\right)$$

$$\log_{10} \frac{(L_1 + L_2)\,(L_2 R - R_2 L)}{L\,(R_1 L_2 - R_2 L_1)}$$

The curves of Fig. 6 were calculated in this manner.

TRANSIENT CURRENTS DURING SHORT CIRCUITS AND UNDER NORMAL LOADS

The ratio of inductance to resistance of the usual railway circuit fed by a given circuit breaker involving the motors of cars or locomotives under maximum normal load conditions is usually considerably higher than that of the feeder circuit alone when a short circuit occurs at the maximum distance from the circuit breaker. A typical comparison is shown by the curves of Fig. 3. By taking advantage of this difference the magnetic type of high-speed circuit breaker can be equipped with an inductive shunt proportioned to trip the breaker at current values considerably lower than the steady load setting with a short circuit through the maximum length of feeder without danger of too frequent operation by transient currents encountered under normal load conditions.

In addition, the mechanical speed at which the magnetic type of circuit breaker opens after the trip point is reached is affected, to some extent, by the amount the trip coil current is increased in excess of that required for tripping the breaker. The force for accelerating the contact arm of the circuit breaker is the difference between the tension of the main springs and the holding effect of the armature flux. As the current through the tripping coil is increased, a greater resultant force is available for accelerating the contact arm mechanically. The curve of Fig. 4 shows this relation between the force available for accelerating the contact arm and the current through the trip coil.

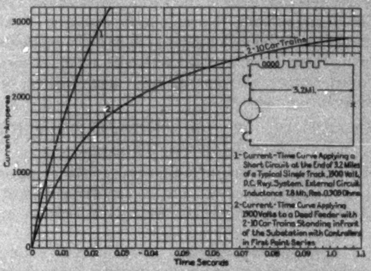

FIG. 3—CURVES SHOWING THE DIFFERENCE IN RATE OF CURRENT RISE CAUSED BY APPLYING A SHORT CIRCUIT AT THE DISTANT END OF A FEEDER AND ENERGIZING A DEAD FEEDER WITH LOAD IN THE OTHER SUBSTATION

The inductive shunt in multiple with the trip coil is, therefore, not only effective in reducing the trip point of the circuit breaker under short-circuit conditions, but is also effective in forcing a greater percentage of the line current through the trip coil after the trip point is reached with a consequent greater mechanical speed of operation of the circuit breaker under conditions where selective operation is desired.

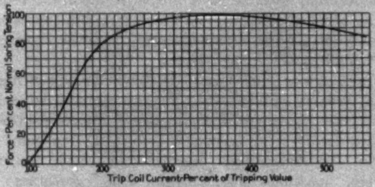

FIG. 4—RESULTANT FORCE FOR ACCELERATING CONTACT ARM OF THE MAGNETIC TYPE OF HIGH-SPEED CIRCUIT BREAKER

A breaker of this type can be made to operate fast enough, in many instances, to limit the maximum current during a short circuit to a value below the normal overload setting. Such a breaker will "discriminate" between normal overloads and short circuits, and this characteristic is frequently referred to as the discriminating characteristic of the circuit breaker. The amount of discrimination is limited only by the transient currents, encountered under normal load conditions.

This discriminating characteristic contributes greatly to the selective operation between breakers, both in substations and in the complete feeder network.

Selective Operation Between Feeder and Machine Breakers

For selective operation in the substation, it is the practise to equip the feeder breakers with inductive shunts which impart a decided discriminating characteristic. The high-speed breakers for machine protection have no discriminating characteristic; that is, their trip point is independent of the rate at which the current is rising. A short circuit on the feeder of a substation so equipped, in most cases, will be cleared by the feeder breaker without tripping the machine breakers and interrupting power to the station bus and other feeders. This statement applies for a majority of short circuits where only one machine is operating in the substation and both machine and feeder breakers carry the same short-circuit current because of the reduction in trip point and faster operation of the feeder breaker.

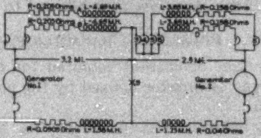

Fig. 5—Equivalent Network for a Typical Double-Track, 1500-Volt, D-C. Railway System, Showing Location of Circuit Breakers

Where the total machine capacity connected to the station bus is considerably in excess of the capacity of the feeder as is usually the case during important traffic periods of the large systems, short circuits within several hundred feet of the station on the ordinary feeder will open the feeder breakers without operating the machine breakers.

Selective Operation Between Breakers in the External Network

By examining Fig. 5, the function of the discriminating breaker in the external network may be better understood.

Nos. 1, 2, 7, and 8 are substation feeder breakers and Nos. 3, 4, 5, and 6 are sectionalizing tie breakers.

Assuming a ground occurring at *S* on feeder *B* near the tie station, the rate of increase of current and the current at any instant through breaker No. 3, due to the short circuit, will be the sum of currents through the three breakers, Nos. 1, 7, and 8. Breaker No. 3 will, therefore, trip at a very much lower value than that required to trip either of the three breakers Nos. 1, 7, or 8, and will operate fast enough to limit the total current to a value considerably below the total required for tripping any of these three breakers feeding the short circuit through the tie station.

The short circuit, under such a condition, will be cleared by breaker No. 3 at the tie station and breaker No. 2 at the substation without interruption of power to feeders *A*, *C*, or *D*. The discriminating characteristic is effective under these conditions in reducing the trip point and in speeding up the operation of breaker No. 3 so that the short-circuit current is limited to a value considerably below that necessary for tripping the other feeders.

It also reduces the trip point of breaker No. 2 sufficiently to operate it even though the resistance of the feeder limits the current to a value below the steady overload setting.

It can be stated in general that as the number of feeders interconnected through a tie station is increased, the selective operation between the breakers feeding a short circuit directly and the breakers feeding interconnected feeders is more easily obtained. This may be explained as follows: With a certain number of interconnected feeders, when a short circuit occurs on one feeder a certain proportion of the current on this feeder will flow from each other feeder. With a larger number of interconnected feeders, the current in this feeder will be larger in proportion to the current in each of the other feeders. Also the current in this feeder will more quickly rise to the tripping point of this feeder's breaker. And this in effect means that the selective action of the system increases when the number of feeders is increased.

III. Reduction in Trip Point with Reduced Line Voltage

The holding coil of the magnetic type of high-speed breaker can be excited from the main feeder circuit when desired. When so connected, the trip point is a function of the line voltage. When breakers having their holding coils so excited are connected in the feeder network at some distance from the substation during a short circuit, the trip point of any breaker will depend on its distance from the fault, the nearest breaker having the lowest trip point. Breakers which are identical in every respect, connected in the same feeder at some distance from each other and subjected to the same short-circuit current, can be operated selectively in this manner in such a way that only the breakers adjacent to the fault open.

Under such conditions when a short circuit occurs at a sufficient distance from the substation to include a sectionalizing breaker, such as on a feeder near the sectionalizing tie station of Fig. 5, the current rise is sufficiently slow to allow time for the desired change in the holding-coil current and results in the operation of the tie breaker before the short-circuit current reaches the trip point of a similar breaker near the station, the holding-coil current of which is not appreciably affected by the short circuit.

The curves of Fig. 6 may be taken as typical and apply to the system shown by Fig. 5 with a short circuit on feeder *A*, with power fed from station No. 1 only, breakers No. 1, 2, 3, and 4 only closed.

The rate at which the holding current is reduced under such conditions is easily controlled by regulating the time constant of the holding-coil circuit, and by choosing the portion of the saturation curve of the holding-coil magnetic circuit over which the breaker operates under normal conditions. The breaker can be designed with a highly saturated holding magnetic circuit, so that the trip point is not greatly affected by voltage reductions occurring in normal service, but will have its trip point lowered very rapidly by voltage reductions in excess of a predetermined maximum.

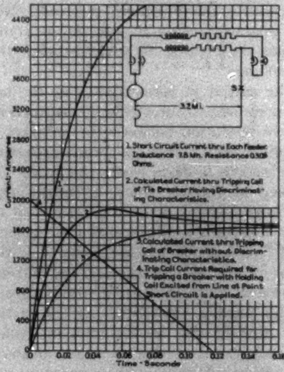

FIG. 6—CURRENT-TIME CURVES APPLYING A SHORT CIRCUIT OPPOSITE A SECTIONALIZING TIE-STATION 3.2 MI. FROM SUBSTATION ON A DOUBLE-TRACK SYSTEM WITH STUB-END FEED. BREAKERS NO. 2 AND NO. 3 TRIPPED

In general, exciting the holding coils from the line does not place objectionable limitations on the transfer of power through a tie station. Referring again to Fig. 5, this point may be illustrated by the following limiting cases:

(1) With a tie station at the extreme end of a double-track system, stub-end feed, the tie breakers will carry only one-half the load under the worst conditions when the load is all on one feeder.

(2) On a large system with many feeders interconnected through a tie station, an excessive load on any one feeder will not greatly reduce the voltage at this point because of the large number of feeders in multiple.

(3) Where heavy loads occur on all feeders near a tie station simultaneously—a condition which results in the greatest voltage reduction—the exchange in current through the tie breakers will be small and the reduction in trip point will not be objectionable. The effects of reduction of voltage at the tie station under normal conditions on the trip points of breakers the holding coils of which are excited from the line is therefore of no great importance for the usual applications.

PROTECTION OF EXTREMELY LONG FEEDERS

In addition to the selective possibilities of such an arrangement when applied to large network, reduction in trip point with reduction in line voltage is useful for applications involving unusually long feeders where short circuits at the extreme distance from the station may result in a current rise and a final current no greater or even less than that encountered under normal load conditions. If such a feeder is sectionalized at some distance from the station by a high-speed breaker with holding coil excited from the line, the trip point will be lowered sufficiently to operate on short circuits at the extreme end of the line, thereby preventing annealing of feeder copper or other damage which might otherwise occur if the short circuit is allowed to persist.

IV. POLARIZED CHARACTERISTICS

The direction of current required for tripping the magnetic type of breaker is fixed by the polarity of its holding coil.

Referring again to Fig. 5, all of the holding coils of the sectionalizing tie breakers 3, 4, 5, and 6 are excited from the station bus. The polarity of these breakers is fixed so that they trip on current flow from the tie bus to the feeder only. A short circuit occurring in the vicinity of the sectionalizing tie station, such as at *S*, reduces the trip point of all breakers located at this point. The direction of the short-circuit current is correct to trip No. 3 breaker, but assists in holding in breakers 4, 5, and 6. The short circuit under such conditions will, therefore, be cleared by breaker No. 3 without having interrupted the tie between feeders *A*, *C*, and *D*. The polarized characteristic is, therefore, of great assistance in preventing the operation of the breakers feeding power to the tie bus.

TESTS ON EQUIVALENT NETWORK

A series of tests was made on the equivalent network of Fig. 5 to demonstrate the selective possibilities of the magnetic type of high speed circuit breaker on a representative system.

For approximately uniform spacing between substations and sectionalizing tie stations, selective operation is more easily obtained on networks of the general form of Fig. 5 as the number of tracks and feeders increase. The transient current through that section of the individual feeders between the substation and the fault is not greatly affected by the number of parallel feeders, and the performance of station feeder breakers clearing short circuits on the feeder to which they are connected is therefore not affected. At the sectionalizing tie station the ratio of current rise in the breaker feeding the short circuit direct to the current rise in the several feeders feeding the short through the tie station is increased as the number of feeders is increased, this greater difference resulting in more positive selective operation.

The section of double-track system, the equivalent circuit for which is shown on Fig. 5, was, therefore, taken as representative of the more difficult type of system for selective operation and was used for the experimental investigation.

Equivalent circuits were set up for each feeder using cast-iron resistors and air-core inductance coils.

While both mutual and self-inductance of the parallel copper conductors comprising each feeder of network are easily calculated, calculations of inductance of the rails of the track return are difficult because of skin effect and variations in magnetic permeability.

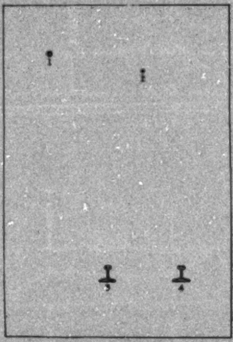

Fig. 7—Cross-Section of a Feeder, Trolley Wire and Rails Identified for Calculating the Rate of Current Rise

The values on which the calculations for the equivalent network were based were taken from the data obtained during a series of short-circuit tests on an actual system involving a rate of increase of current of the same order of magnitude as that of the equivalent system. The method consisted in calculating the inductance of the copper conductors of the feeder on which the tests were made and determining the inductance of conductors which at the rail locations gave a total inductance equal to that shown by initial current rise.

Air-core inductance coils were necessarily used to represent the total inductance of the positive and negative sections of the feeders.

The self-inductance of these coils was adjusted to give a counter e. m. f. of inductance on the initial current rise equivalent to the total calculated induced voltage of the complete feeder with the same current rise. This voltage is generated both by the self-inductance of the conductors and by the mutual inductance from the other conductors.

The rate at which the current increases in each conductor was determined by solving the following simultaneous equations.

Referring to Fig. 7, in which the conductors are given numbers that are used as subscripts in the following equations, let E = voltage, L = self-inductance indefinite radius, M = mutual inductance:

$$L_1 \frac{dI_1}{dt} \pm M_{1-2} \frac{dI_2}{dt} \pm M_{1-3} \frac{dI_3}{dt} \pm M_{1-4} \frac{dI_4}{dt}$$

$$= L_2 \frac{dI_2}{dt} \pm M_{2-1} \frac{dI}{dt} \pm M_{2-3} \frac{dI_3}{dt} \pm M_{2-4} \frac{dI_4}{dt} \quad (1)$$

$$L_3 \frac{dI_3}{dt} \pm M_{3-1} \frac{dI_1}{dt} \pm M_{3-2} \frac{dI_2}{dt} \pm M_{3-4} \frac{dI_4}{dt}$$

$$= L_4 \frac{dI_4}{dt} \pm M_{4-1} \frac{dI_1}{dt} \pm M_{4-2} \frac{dI_2}{dt} \pm M_{4-3} \frac{dI_3}{dt} \quad (2)$$

$$E = L_2 \frac{dI_2}{dt} \pm M_{2-1} \frac{dI_1}{dt} \pm M_{2-3} \frac{dI_3}{dt} \pm M_{2-4} \frac{dI_4}{dt}$$

$$+ L_4 \frac{dI_4}{dt} \pm M_{4-1} \frac{dI_1}{dt} \pm M_{4-2} \frac{dI_2}{dt} \pm M_{4-3} \frac{dI_3}{dt} \quad (3)$$

$$\frac{dI_1}{dt} + \frac{dI_2}{dt} = \frac{dI_3}{dt} + \frac{dI_4}{dt} \quad (4)$$

The inductive voltage in the positive and negative conductors is then calculated and the air-core coils adjusted for an equal voltage during the initial current rise.

The equivalent circuit involving air-core inductance and cast-iron resistors approximates the actual circuit only because, in the actual system, the time constants of the individual parallel conductors are not exactly the same and as a result the voltage induced by the flux of mutual inductances varies as the current increases. The rate of change of current $\frac{di}{dt}$ decreases more rapidly in those conductors having a relatively high resistance than in those of lower resistance. This is apparent since the voltage for increasing the current is

$$E_L = E - R\,I \pm M_1 \frac{dI_1}{dt} \pm M_2 \frac{dI_2}{dt}$$

The mutual inductance (M) is relatively small. E = voltage applied to any conductor and I = current.

The error introduced by the approximate method of using the self-inductance of air-core coils to represent the total inductance of the feeder is well within the limits of accuracy of calibration of commercial breakers for the circuit in question.

Separate calculations for each location of the short circuits were made because of differences in magnitude and direction of the mutual inductance effects with short circuits at different locations. The air-core inductances were adjusted accordingly for each test. The values given on Fig. 5 apply for short circuit at the tie station and were modified slightly for short circuits at other locations.

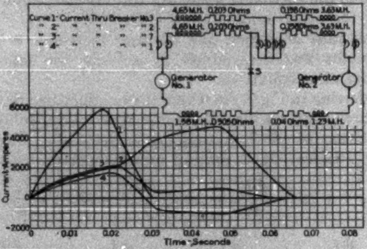

Fig. 8—Current Through Various Breakers During Short Circuit on Feeder Near the Tie Station of Typical Double-Track, 1500-Volt, D-C. Railway System. Breakers Nos. 2 and 3 Tripped

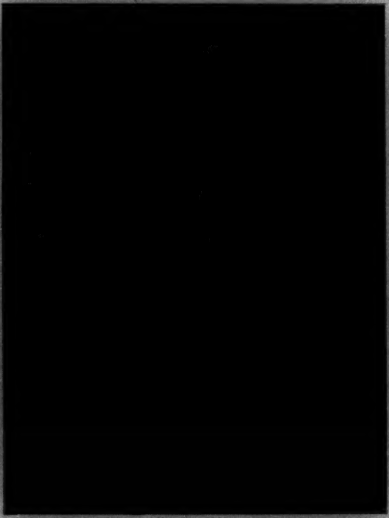

Fig. 9—Oscillogram Obtained when Applying a Short Circuit to the Equivalent Circuit of a 1500-Volt, Double-Track System, Stub-End Feed at the Maximum Distance from the Substation. Short Applied at *S*, Fig. 5, with Substation No. 1 Supplying Power; Breaker No. 4 Closed. Breakers Nos. 2 and 3 Opened by the Short Circuit

During the tests the holding coils of substation feeder breakers were excited from an independent constant potential source while sectionalizing tie breakers were excited from the line.

The resistance of the individual feeder of this network is such that the maximum current through each feeder with a short circuit at the sectionalizing tie station is less than the normal load setting required (6000 amperes). The maximum load condition in the individual feeders on a single breaker determined from the train schedules is encountered when restoring power to a dead feeder on which two ten-car trains are standing immediately in front of the substation with their controls in the first point series. The transient currents for these two limiting conditions are shown by the curve of Fig. 3. The current rise when closing the circuit of the two ten-car trains with the control in the first point was interpolated from oscillograms obtained during a series of tests made on a 600-volt system.

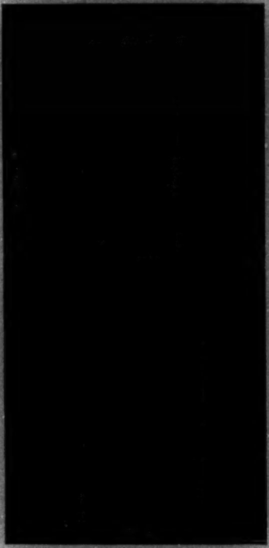

Fig. 10—Oscillograms Obtained when Applying a Short Circuit to the Equivalent Network Shown in Fig. 5. Directly in Front of Substation No. 1 on Feeder *B*, with All Breakers Closed. Breakers Nos. 2 and 3 Opened by the Short Circuit

Breakers having identical constants were selected for all locations on the experimental network. The sectionalizing tie breakers were duplicates of the substation feeder breakers with the exception that their holding coils were excited from the feeders, the holding coils of substation feeder breakers being excited from an exciter bus.

A 1500-kw., 1500-volt generator was used as a power supply for each substation. All breakers were set for a steady overload of 6000 amperes. Selective operation completely isolating the faulty feeders without interrupting power supply to interconnected feeders was obtained for all locations of short circuits applied to the experimental network during the series of tests.

Referring again to Fig. 5, the most important locations tested were as follows:

1. Short circuits on all feeders both directly in front of substations and the sectionalizing tie station,
2. Short circuit on feeder *B* at the tie station while carrying a 2000-ampere load in the center of feeder *D*,
3. Short circuits one-half way between substations and tie stations,
4. Short circuit at the substation and at the tie station with a stub-end feed from a substation to the tie station.

The curves of Fig. 8 were taken from oscillograms of a short circuit at *S* of Fig. 5. Several of the actual oscillograms are shown on Figs. 9 and 10. It is interesting to note that as the short is disconnected from the the tie bus the current increases rapidly through breaker No. 2 speeding up its operation.

While it was not possible to simulate all of the conditions of load and short circuits encountered during the operation of such a system, the results of the tests indicate that it is possible to obtain a high degree of selective operation on d-c. railway feeders by the proper utilization of the inherent characteristics of the magnetic type of high-speed circuit breakers.

Several important installations involving a complete equipment of high-speed breakers designed for selective operation based on the principles discussed in this article have been made in the last few years.

THE HIGH-SPEED CIRCUIT BREAKER IN A-C. RAILWAY NETWORK

The discussion in the preceding paragraphs of this article has been confined to the d-c. railway network. A method of operation has recently been developed whereby the same type of high-speed breakers applied successfully to the d-c. railway system for a number of years may be applied to the a-c. network.

By the proper application of a-c. breakers of this type many of the advantages of high-speed protection can be realized on a-c. railway feeder networks. The duration of short circuits and resultant damage to apparatus, particularly in case of an arc, can be limited to a small percentage of that resulting under similar conditions with the usual slow-speed protective apparatus. High-speed protection is also of particular value in eliminating telephone interference during a-c. short circuits.

Furthermore, by taking advantage of the characteristics of the magnetic type of high-speed breaker, a high degree of selective operation can be obtained on a-c. circuits, isolating short circuits in a fraction of the time required with the usual type of protective system involving slow-speed breaker, inverse time relays, pilot wires, and the like.

Discussion

J. J. Linebaugh: Very few engineers other than those actively engaged in railway substation and distribution work appreciate the great advance made during the last few years in the protection of substation apparatus and railway networks, due to the development of the high-speed breaker.

It should be appreciated that the high-speed breaker operates and opens the circuit so quickly that practically all of the phenomena regarding amperes, volts, etc., are actually transients. This opens up possibility of obtaining selectivity and protection which has not been possible in the past.

Another important feature of the high-speed breaker described by Mr. McNairy is the characteristic that it only operates with current in one direction. This feature makes it possible to obtain maximum selectivity in railway networks under short circuits.

It is now possible to tie as many tracks together as desired, utilizing all the copper in regular operation to the best advantage, and isolate any section if it should become grounded or short-circuited without disturbing other tracks or sections. This is particularly applicable and advantageous on systems of two or more tracks.

The high-speed breaker has practically eliminated short circuits as we originally knew them on substation apparatus and distribution systems; that is, the breaker operates so quickly after short circuit occurs that the current is prevented from reaching dangerous values which would cause serious damage. This is due to the speed with which it attacks the circuit rather than the speed or time of rupturing the circuit which is about the same as with an ordinary breaker.

The advantages of high-speed breaker protection are becoming more evident as their use is extended. Figures now available indicate that they very materially decrease locomotive and substation maintenance.

Chester Lichtenberg: The circuit breakers described have been of the so-called magnetic type. In addition, there is the latched-in type of high-speed breaker. Emphasis is laid upon the difference between these two types because in applying them different parameters must be recognized. Those of the latched-in type were the first high-speed circuit breakers developed. The magnetic type followed. Both have been very successful where their inherent characteristics were recognized and applications correctly made.

A prime characteristic of the latched-in type is that it will always trip at a definite current depending upon the setting of its overload feature. It always trips at the same current whether the rate of rise of current is slow or rapid.

The magnetic type described in the paper has a quite different characteristic. It is provided with a so-called bucking bar, which gives it a discriminating characteristic; that is, if set for tripping at a definite, steady current, it will trip at a lower current. The difference between the point of tripping and the point of setting will depend upon the rate of rise of the current.

This characteristic can be and sometimes has been incorporated in the latched-in type of high-speed breaker and wherever this discriminating feature is incorporated in the breaker, allowance must be made in its application.

This point is quite important in the application of high-speed breakers because if they are not correctly adjusted or set, they will open when they should not. The discriminating characteristic is a point of very great importance because it extends their application.

Regenerative Braking for Direct-Current Locomotives

BY A. BREDENBERG, JR.

Non-member

Synopsis—*This paper gives a comparative study of several d-c., regenerative braking systems now in successful operation. The general characteristics of regenerative braking, including advantages, limitations and functions, are briefly discussed. The principal points covered in the descriptions of the various systems are: method of field excitation during regeneration, compensation for sudden changes in line voltage or grade, method of control, operating characteristics and relative complication of equipment.*

Several systems are described in which the regenerating motor fields are excited by a separate generator provided for that purpose, and then other systems are described in which the fields are excited by one or more of the traction motors. Under the former heading are included C. M. & St. P. freight locomotive with a line driven motor-generator set for excitation, the Mexican locomotive with a dyna-motor-driven, motor-generator set for excitation and the axle generator system. The applications described under the latter heading are the C. M. & St. P. gearless passenger, the Paulista and the Spanish Northern locomotives.

A comparison is made of these two methods of field excitation, based on a six-motor, two-speed, 3000-volt locomotive. Curves are included illustrating the speed-braking characteristics and also the motoring characteristics. The conclusions drawn from this comparison are that, in general, the separate excitation system is to be preferred on account of the greater braking effort and greater speed range provided. Since the motor excitation system can be provided with less additional expense, it is desirable to consider this method when the expense of the separate excitation system does not seem justifiable.

IT is not the purpose of this paper to give an exhaustive discussion of the question of regenerative braking, but to make a comparative study of some of the systems now in successful operation on direct-current locomotives.

GENERAL CHARACTERISTICS

Whenever it is proposed to electrify a railroad, where grades exceeding 0.6 per cent form an appreciable part of the line, usually the possibility of applying regenerative braking is considered on account of its marked advantages. These are, briefly, as follows: (a) Reduced wheel- and brake-shoe wear; (b) increased safety due to reduced tire heating and brake-shoe wear and to the fact that duplicate braking systems are provided; (c) higher average speed descending grades, since a very uniform speed can be maintained; (d) elimination of delays due to inspections, worn-out brake shoes, etc.; (e) saving of power returned to the line, and (f) increased comfort to passengers as a result of the uniform speed and elimination of noises and shocks caused by the air-brake system. Against these advantages must be balanced the increased weight, cost, and maintenance of the equipment.

Although regenerative braking may be used to reduce train speeds to a certain extent, and systems have been designed to slow down a train or car to very low speeds, yet the primary purpose of this method of braking as applied to heavy traction service is to hold the train at a uniform speed on a given down grade. Regenerative braking should not be overemphasized as an emergency braking system. This method of braking can only be applied to the locomotive, whereas, in the air-brake system, the brakes may be applied to every car in the train. Therefore, regeneration does not replace the air-brakes or lessen their required reliability but it should be considered as a means of braking, supplementing the air-brakes with some very decided advantages as outlined.

Of course, regenerative braking is particularly applicable to long heavy mountain grades and is generally used in such service because of its obvious advantages. It is applicable to comparatively light or short grades provided the results obtained are commensurate with the extra cost incurred.

In order to raise the voltage of a d-c. series motor to make it return power to the line, it is necessary to excite separately the series field and to control this excitation so that smooth operation is obtained regardless of changes in line voltage or grade.

METHODS OF FIELD EXCITATION

There are two general methods of exciting the motor fields and both of these have been successfully applied. One method is by the use of a generator which may be driven by a motor operating from line voltage or from a dynamotor. Also, it might be mechanically driven from an idle axle of the locomotive.

The other method is to use one or more of the traction motors to excite the fields of the remaining motors. This method may be subdivided further according to the method of controlling the exciter field. Thus, in one case, the exciter field is controlled by contactors, resistors, etc., especially provided for that purpose, while in the other case, control of the exciter field is obtained by means of the same contactors and resistors that are used in accelerating the locomotive.

C. M. & ST. P. FREIGHT LOCOMOTIVES

The method of exciting the regenerating motor fields by means of a motor-generator set operated from the line voltage was first put into commercial use on the

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1. Railway Equipment Engineering Dept., General Electric Co., Schenectady, N. Y.

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Chicago, Milwaukee and St. Paul 3000-volt freight locomotives. By reference to Fig. 1, which shows the regenerative connections of one-half of one of these locomotives, it may be seen that the exciter is so connected that the exciter armature current is the sum of the motor field current and the regenerated current. Control of the exciter field is obtained by means of a motor-operated rheostat, automatically controlled by a current-limit relay which is connected in the exciter armature circuit. The current-limit relay thus is responsive to the sum of the motor armature and field currents. Control of the braking speeds is obtained by changing the setting of the current-limit relay by means of the braking handle on the master controller.

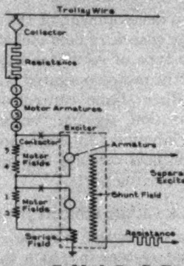

FIG. 1—DIAGRAM OF C. M. & ST. P. FREIGHT LOCOMOTIVE

Compensation for sudden surges in current, due to changes in line voltage or grade, is obtained by a combination of several factors, viz., the differential series field of the exciter, the exciter armature reaction, and the resistance drop in the exciter armature circuit. For example, assume a sudden decrease in line voltage. This will tend to rapidly increase the regenerated current and consequently the current through the exciter armature. The compensating factors will then operate as follows: The differential series field and the exciter armature reaction will tend to weaken the exciter field and reduce the exciter voltage, thus reducing the motor field current, while the increased resistance drop will cause the exciter armature circuit to absorb more of the exciter voltage and thus further reduce the field current and generated voltage of the traction motors. Likewise, a sudden decrease in regenerated current will be compensated for in just the opposite manner. Thus, the inherent characteristics of the exciter circuits prevent excessive surges of current and torque and allow time for the circuits to readjust themselves to the changed operating conditions.

MEXICAN LOCOMOTIVES

Another application of the method of exciting the motor fields by means of a separate motor-generator set is that of the Mexican Railway, 3000-volt locomotives. In this case, the regenerative exciter is driven by a 1500-volt motor, which is connected to the midpoint of a 3000/1500-volt dynamotor. Connections for this locomotive are shown in Fig. 2. Here the exciter carries the motor field current only. Control of locomotive speeds is obtained by adjusting the exciter voltage by means of a variable resistance in the exciter field circuit. This adjustment is obtained directly by means of the braking handle of the master controller.

Compensation against sudden surges in line current is provided for by balancing resistances which carry the sum of the field and armature currents. Thus, if the line voltage drops suddenly, it will tend to produce a rapid increase in regenerated current. The increase in current causes an increased voltage drop across the balancing resistance which causes a decreased voltage and current in the motor field circuit. This reduces the regenerated voltage of the motors, thus preventing an excessive change in regenerated current. Similarly, when passing from one control step to another, the balancing resistance aids in producing smooth operation by reducing current and torque increments between steps.

COMPARISON OF C. M. & ST. P. AND MEXICAN LOCOMOTIVES

By comparing these applications, it will be seen that in one case the exciter carries the sum of the motor armature and field currents while in the other case the exciter carries the field current only. This means that, in the latter case, the exciter armature current will be halved. At the same time, the exciter voltage will be increased as the exciter must overcome the voltage drop in the motor fields and also in the balancing resistance. Experience has demonstrated the fact, however, that the drop through the balancing resistance can be held to such a value that the net result is a reduction in kilowatt capacity required of the exciter.

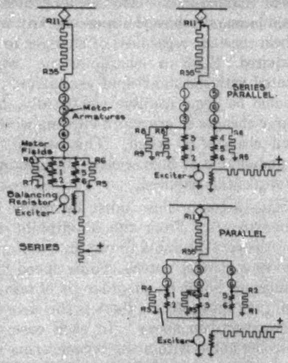

FIG. 2—DIAGRAM OF MEXICAN LOCOMOTIVE

In the first system described, compensation is obtained by means of exciter differential field, armature reaction and resistance drop. In the second system, a resistance alone is used since tests and operating data have demonstrated that a very effective compensation can thus be provided. The second system will tend to be less efficient on account of the losses in the balancing resistances. A much simpler control is thus provided, however, and furthermore it is very easy to make adjustments by providing taps in the balancing resistances.

Compared with the other type, the system used on the Mexican locomotives has the characteristic of relatively small changes in speed with regard to changes in the braking effort required. The result is that changes in grade or curvature will cause smaller speed changes in this case than in the C. M. & St. P. System. In brief, the Mexican system tends toward a constant speed characteristic while the C. M. & St. P. System tends toward a constant torque characteristic.

The ideal characteristic for regenerating motors would be a characteristic similar to that obtained in motoring operation with a series motor where the armature and field currents are equal throughout the operating range. This would give a very stable electrical characteristic but an extremely unstable speed characteristic, as the braking effort would decrease very rapidly with increases in speed.

The ideal characteristic from a mechanical standpoint would be one in which the speed on any controller notch is held constant regardless of changes in braking effort required. Such a characteristic would be approximated with constant field excitation. Such a system would be very unstable electrically, however, as very large changes in armature current would occur with small changes in speed. This would be particularly undesirable at high speeds on account of the relatively weak fields required.

In practise, a compromise must be made between these two extremes. From the standpoint of train operation, it is very desirable to obtain a uniform speed for any given controller notch. Some speed variation must be allowed, however, in order to obtain the desired stability of the motors. Such is the case with the two systems described above. In each case, for any given controller notch within the regenerating range of the motors, with increases in speed a sufficient increase in braking effort is obtained to give mechanical stability, but, at the same time, the speed regulation is not made so close as to sacrifice the desired electrical characteristics.

With the more constant speed characteristic of the system used on the Mexican locomotives, very little manipulation of the master controller is required to hold the train at a practically constant speed. This has been demonstrated in actual operation on the Mexican Railway. The electrified section of this railway is 45 kilometers long and regeneration is obtained for about two-thirds of this distance, the grade being about 4.7 per cent the greater part of the way.

The principal advantage of the Mexican system is its simplicity. Compensation for line-voltage changes is obtained by means of a simple resistance, and control of the regenerating speeds is obtained by hand control of a resistance in series with the exciter field. No relay or motor-driven rheostat is used. This will reduce the first cost of the system and keep the maintenance required down to a minimum.

DEVELOPMENT OF METHOD OF CONTROL OF EXCITER FIELD

The steps in the development of the method of controlling the exciting generator field included experiments with the sensitive type of regulator used with stationary units. Later, the motor-operated field rheostat was used and finally direct hand control of the field was adopted, compensation against voltage and speed fluctuations being obtained by the inherent characteristics of the motor field circuits without the addition of any regulator or moving parts.

AXLE GENERATOR SYSTEM

The axle generator system may be considered as a motor-generator set system in which the motor of the motor-generator set is replaced by the axle driving mechanism. The same method of control may be used in either case.

The principal differences between an axle generator system and the corresponding motor-generator set system are that the axle generator is necessarily a slow-speed machine, it varies in speed with the speed of the locomotive, and the driving torque of the axle generator is available to aid in braking the train.

Assuming that the same connections are used for an axle generator system as are used on the Mexican locomotive (Fig. 2), it may be readily seen that, with an increase in speed due to an increase in the grade, the regenerated voltage and current will tend to increase due to the increase in speed of the motors themselves and also to the increase in speed of the axle generator which thus tends to increase the motor field current. This helps to give a very close speed characteristic for any given excitation of the axle generator field, such being desirable provided the electrical stability of the motors is not lessened.

Regarding the added braking effort of the exciter, this might prove an advantage in freight service where it is often desired to handle a considerably heavier train down grade than could be hauled up the same grade by one locomotive. In this case, however, it will probably be necessary to use the air brakes on the train in conjunction with the regenerative braking so that little would actually be gained from the additional braking effort obtained from the axle generator. Furthermore, there must be an idle axle available to drive the exciting generator. This is usually the case with passenger locomotives. On freight locomotives, however, there are very often no idle axles, as illustrated by the Mexican and Spanish Northern locomotives and the Paulista freight locomotives, where every axle drives a traction motor.

Another limitation of the axle generator is that, due to its low speed, it must be of relatively large size. In addition, it must deliver practically the same output over a wide range of speeds, which will tend to compromise the design and complicate the control. Furthermore, the axle generator is mounted close to the track and thus is subject to more abuse, dust particles, etc., than a generator mounted in the locomotive cab.

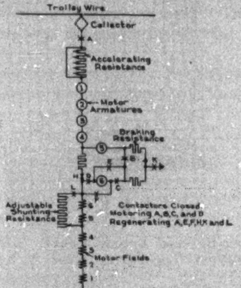

FIG. 3—DIAGRAM REGENERATING CONNECTIONS OF C. M. & ST. P. GEARLESS PASSENGER LOCOMOTIVE

C. M. & ST. P. GEARLESS PASSENGER LOCOMOTIVES

In order to eliminate the use of a separate generator for field excitation, the method of using one or more traction motors as a regenerative exciter was developed. The first application of this system to heavy-traction service was on the C. M. & St. P., 3000-volt, gearless, passenger locomotives. Fig. 3 shows the regenerating connections for one-half of the locomotive. The two halves may be operated in series or in parallel when regenerating so that two combinations of regenerating motors are obtained, one with eight motors in series and the other with two parallel groups, of four motors in series, in each. In each six-motor group, two of the traction motors are used to excite the fields of all six motors. Control of the excitation is obtained by means of a variable resistance, which shunts the fields of the exciting motors. This resistance is adjusted by means of contactors controlled by the braking handle of the master controller.

From the connections, it will be seen that the sum of the field and armature currents of the regenerating motors passes through the two exciter armatures which are connected in multiple and that an external resistance is connected in series with each of the exciter armatures. This system thus provides an inherent regulation against line-current surges which is in effect a combination of the C. M. & St. P. freight and the Mexican systems. That is, the sum of the armature and field currents of the regenerating motors passes through balancing resistances and also through the exciter armatures. An effective means of compensation against surges is thus provided.

A further simplification of the apparatus required for regeneration may be obtained with the traction-motor exciter system by using the same resistances and contactors for controlling the exciter during regeneration that are used during acceleration of the locomotive. This system is now in successful operation on the Paulista locomotives (four-motor, 3000-volt) and on the Spanish Northern locomotives (six-motor, 3000-volt). On the Paulista locomotives, one traction motor is used to excite its own field and the fields of the other three motors which are connected in series to the line. On the Spanish Northern locomotives one traction motor is used to excite its own field and the fields of the other five motors which are connected in series to the line.

PAULISTA LOCOMOTIVES

Fig. 4 shows the schematic connections of the Paulista locomotive both motoring and regenerating. It will be seen that there are two motor combinations motoring, one with all four motors in series with 750 volts applied per motor and the other combination with two groups of two motors in series in each with 1500 volts applied per motor. There is one regenerating combination with the three regenerating motors connected in series with a normal voltage of 1000 volts per motor. By referring to the resistance symbols it will be seen that part of the accelerating resistance is connected in series with the regenerating motors. This resistance is used in connecting the motors to the line when establishing the regenerative connection. It is short-circuited during regeneration. The remainder of the accelerating resistance is connected in the motor field circuit and is

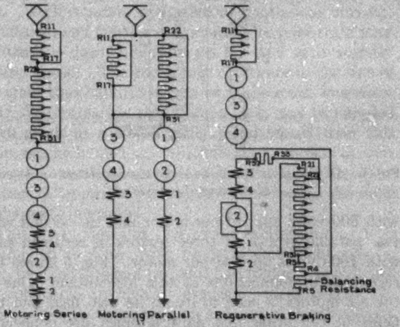

FIG. 4—DIAGRAM OF SCHEMATIC CONNECTIONS OF PAULISTA FREIGHT AND PASSENGER LOCOMOTIVE

thus used to control the regenerative operation. Some additional resistance and additional contactors are required but it is to be noted that an appreciable saving in equipment is made by using the accelerating resistance and contactors for control of the exciter field.

Practically the same method of compensation for line current surges is used as on the Mexican locomotives. A balancing resistance is connected in the circuit so as to carry the sum of the line current and the exciter field current. This exciter field current is also part of the regenerating motor field current so that the compensation is obtained in two ways; directly by the effect of the balancing resistance on the regenerating motor fields, and indirectly by the effect of the balancing resistance on the exciting motor field.

SPANISH NORTHERN LOCOMOTIVES

On the Spanish Northern locomotives there are two motoring combinations, one with all six motors in series with 500 volts applied per motor and the second with two parallel groups of three motors in series in each with 1000 volts applied per motor. Fig. 5 shows the regenerating connections of this locomotive. One of the six motors is used to excite its own field and the fields of the other five motors which are connected in series to the line with a normal voltage of 600 volts per motor. The regenerating system is otherwise identical with that of the Paulista locomotives.

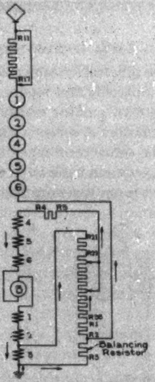

FIG. 5—DIAGRAM OF REGENERATING CONNECTIONS OF SPANISH NORTHERN LOCOMOTIVE

COMPARISON OF METHODS OF FIELD EXCITATION

In order to obtain a comparison of the system using a separate generator for field excitation and that using traction motor excitation, a study was made of the operating characteristics of a locomotive similar to the Spanish Northern, six-motor, two-speed, 3000-volt locomotive as compared with the same locomotive with a motor-generator set for field excitation. For motor excitation, then, there will be one regenerating combination with one traction motor exciting the other five, all connected in series. For separate excitation there will be two regenerating combinations, one with all six motors in series and the second with two parallel groups of three motors in series in each, these combinations being the same as those obtained while motoring.

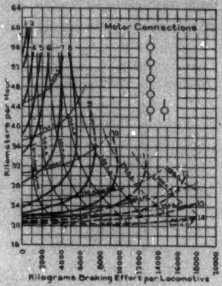

FIG. 6—SPEED-BRAKING EFFORT CURVES FOR 86-TON, 6-MOTOR, 3000-VOLT LOCOMOTIVE WITH TRACTION MOTOR EXCITATION

In comparing the operating characteristics of these two systems of regeneration, the main points to be considered are as follows: range of operating speeds, train weights which may be handled, slope of grades, mileage on which regeneration can be used, and reliability.

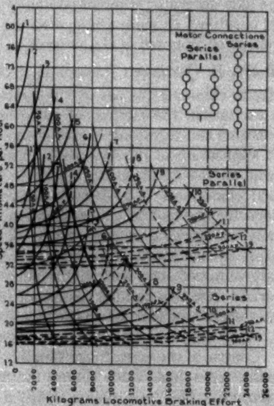

FIG. 7—SPEED-BRAKING EFFORT CURVES FOR 86-TON, 6-MOTOR, 3000-VOLT LOCOMOTIVE WITH SEPARATE EXCITATION

Figs. 6 to 8 inclusive illustrate the speed-braking effort characteristics obtained with the two systems. Figs. 6 and 7 give the relation between speed and braking effort for each step of the braking handle on the master controller, and Fig. 8 is a composite diagram which shows the range of speeds and braking efforts which may be obtained in each case. The shaded areas in Fig. 8 give the continuous operating ranges while the unshaded areas give the ranges within which the locomotive may regenerate for short periods. This diagram also shows the speed-traction effort curves for motoring operation in each of the two motoring connections. In each case the full field and two reduced field characteristics are given. The tractive effort and speeds at the continuous current rating are also indicated.

Two regenerating speed ranges are obtained with the separate excitation system whereas one speed range is obtained with motor excitation, this latter speed range being intermediate to the two speed ranges with separate excitation.

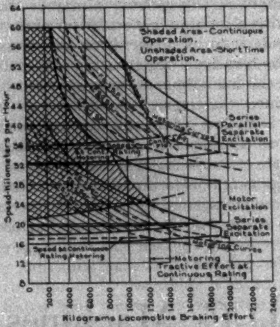

FIG. 8—COMPARISON BETWEEN SEPARATE AND TRACTION MOTOR EXCITATION FOR 86-TON, 6-MOTOR, 3000-VOLT LOCOMOTIVE

In analyzing these curves, it is to be noted that with the same line voltage, the same motor combination, and the same armature and field current, a slightly higher speed will be obtained when regenerating than when motoring. The reason for this is the reversal of the effect of resistance drop in the motor circuits. Under the same conditions the braking effort regenerating will be appreciably greater than the corresponding tractive effort motoring on account of the reversal of the effect of the motor losses on the torque at the driving wheels. Furthermore, with a given torque at the driving wheels, it will be possible to handle a considerably heavier train down grade than up grade since the train and curve friction oppose the locomotive ascending the grade while they assist the locomotive when descending.

With the separate excitation system, therefore, the regenerating speeds will be a little higher than the motoring speeds with the same motor combination, voltage, and currents. With motor excitation of fields, in this particular case, the regenerating speed at the continuous-current rating of armatures and fields will be about 70 per cent of the corresponding motoring speed. As a rule, the double-speed range is to be preferred unless it is desired to descend the grade at a lower speed than used in ascending. In the latter case the speed range of the motor excitation system will probably be more suitable.

From an analysis of the curves, it may be seen that for a given armature and field current of the regenerating motors a greater braking effort is obtained with the separate excitation system than with the motor excitation system. The reason for this is that, in the former system, the full braking effort of all six motors is available while in the latter the full braking effort of five motors is available and a fraction of the full braking effort of the exciting motor. The result is that greater train weights can be handled down a given grade with the separate-excitation system than with the motor-excitation system.

The following conclusions in regard to operating characteristics may be stated as a result of the comparison of these two systems:

1. With motor excitation in general, on all grades, the same train can be handled down hill as up hill, but at reduced speeds. This reduction in speed will be comparatively small for light grades but will increase as the grade increases.

2. With separate excitation at continuous current, armatures, and fields, a considerably heavier train may be handled down hill than up hill and at a slightly greater speed. The percentage difference in the size of the train thus handled decreases as the grade increases.

3. With separate excitation, a train may be handled down grade at a considerably higher speed than this train can be hauled up the same grade, assuming that the safe operating speed is not exceeded. The percentage difference in these speeds decreases as the slope of the grade increases.

This study indicates that motor excitation may best be applied to a line with comparatively light grades; *e. g.*, one and one-half per cent and under. It will still be applicable to heavier grades if these grades are short and little time is lost on account of slower speeds, or if the slower speed obtained will not handicap the schedule speed desired.

Regeneration with separate excitation is particularly applicable to heavy grades; *e. g.*, two per cent and over, when the length of these grades is an appreciable part of the total length of the line. It is also preferable to use this system for lighter grades when it is necessary to take advantage of the highest speeds possible down grade.

Concerning reliability of operation, there will be little difference if any, between the two systems, assuming that the same rugged type of equipment is used in each case and that the same method of compensation is used as has already been described.

It is evident that the additional expense of providing for regeneration is less with motor excitation than with separate excitation since in the former case less additional equipment is required.

It is to be noted that the foregoing comparison covers the specific case of a two-speed, six-motor locomotive. The same general considerations would apply to a similar comparison for any two-speed locomotive. For locomotives with more than two motoring speeds or combinations, there will be cases where motor excitation cannot be so readily applied on account of the limited speed range which can be obtained. This would be the case, for example, with a three-speed locomotive such as the Mexican locomotive. The logical regenerating combination in this case would be that of one motor acting as an exciter for the other five motors in series. The speed range thus obtained would be quite low in comparison with the highest motoring speed which is obtained with three parallel groups of two motors in series in each. Other combinations could be worked out for this type of locomotive, but with a reduction in braking torque which would, of course, be undesirable. To other types, such as the C. M. & St. P. gearless passenger locomotive, motor excitation can be more readily applied. This locomotive has four motoring combinations; viz., with twelve, six, four and three motors in series across the line. Two regenerating combinations are obtained with eight regenerating motors in series and two parallel groups of four motors in series. Thus, a better speed-range regenerating is obtained than with the six-motor, three-speed locomotive cited above.

It is not within the scope of this paper to discuss all possible combinations to which motor excitation may be applied. From the foregoing discussion, however, the conclusion may be drawn that this type of regenerative braking may be readily applied to two-speed locomotives in general and in some cases to locomotives with a greater number of motoring combinations.

In the foregoing discussion, it has been attempted to present the characteristics of certain types of regenerating systems which have been successfully applied to d-c. locomotives. In general, it may be stated that a system with a separate generator for excitation of the motor fields is to be preferred as this gives the same motor combinations regenerating as motoring and furnishes the full torque of each motor for braking the train. Since the motor excitation system can be provided with less additional expense, however, it is desirable to consider this method when the expense of the separate excitation system does not seem justifiable. In any application, before a decision is reached as to the type of braking system to be used, a thorough study should be made of all the contributing factors.

Multiplex Windings for D-C. Machines

BY CARL C. NELSON[1]
Enrolled Student

Synopsis.—With the increase in capacity of d-c. machines, the question of their successful operation has become more and more important. It is thought that multiplex windings offer creditable advantages if they are designed so that cross-connections can be suitably applied and the circuits thereby kept balanced, a feature which was not given thorough consideration when multiplex windings were first tried.

This paper describes machines in which the proper design may be secured when standard construction is used. Both the lap and wave types of windings are considered. The principles are applied to duplex windings in this paper, but if so desired they may be applied also to triplex and other multiplex windings.

A brief description of "frog-leg windings," a recently developed type of multiplex winding, is also given.

THE use of multiplex windings in large d-c. machines offers marked improvements provided the windings can have armature cross-connections properly applied to them, so that the armature circuits can be kept balanced; that is, the currents in the circuits may be kept equal. This result has been successfully accomplished with multiplex windings on practical machines with the ordinary type of cross-connections in a few cases (see references 6 and 7), and it seems probable that the same good results can be obtained in other cases.

Multiplex windings have, in general, more parallel paths and, consequently, less current per coil than the corresponding simplex windings which are usually used. Accordingly, the current to be commutated is much smaller and therefore the amount of sparking is reduced. The fact that an increase in the number of parallel paths in an armature winding improves commutation, (other things being equal,) is exemplified by the

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well-known case of an armature which is of such rating that either a wave or lap winding could be used. The latter is distinguished by more paths in parallel, more and narrower commutator bars and better commutation, as is well known from practical experience. If, then, multiplex windings, having even more paths in parallel than the ordinary lap windings, can be safely used and kept properly balanced, another step in the improvement of commutation may be expected. The advantages to be obtained justify further study and trial of multiplex windings.

The windings used in practise at the present time are usually simplex wave, simplex lap, and, occasionally, an eight-pole, duplex wave winding. This is a very limited assortment.

A number of machines tried in the past with multiplex windings was not designed properly and armature cross-connections could not be applied suitably. The desirable condition would be the interconnection of equipotential points in one individual part of a multiplex winding to equipotential points in another individual part of the multiplex winding, and so on. In this way all the individual parts would be joined together. In addition, the winding should be so arranged that the resulting potential distribution on the commutator would be uniform and the volts per bar would be of a low value. Arnold has advocated the use of cross-connections and has shown triplex wave windings[2] with the cross-connections joining equipotential points of the three individual parts of the winding, which is the desirable condition. In describing Fig. 12 (reference 7), it is stated that triplex lap windings have been made with efficient cross-connections on one end of the armature and have operated successfully. Arnold, however, has not shown duplex lap and triplex lap windings with simple cross-connections joining the individual parts of the winding together. In his early work he has shown duplex lap windings[3] so arranged that equipotential points in the same part of the duplex windings were joined, thereby giving a potential distribution on the commutator such that bars in pairs were at the same potential. This has the same effect as using half the number of bars and therefore is not desirable. In his later work, Arnold has shown duplex lap windings with equalizing rings[4] both on the front and back of the windings and with connections from the front to back joining these rings, thus interconnecting the individual parts of the duplex winding. This method introduces the complication of cross-connections at both front and rear ends of the armature, and of connectors along the shaft. It would be more desirable to use the arrangement proposed in this paper, in which all the cross-connections are at the rear end of the armature.

It is very desirable that multiplex windings be used where they are successful, since by means of them d-c. machines can be designed to have as many and as narrow commutator bars as possible. This results in a commutator bar in large machines from 3/16 to 3/8 in. in width, which is considerably narrower than many in use. The benefit of the smallest current per circuit as well as a corresponding improvement in commutation will thus be obtained.

A preliminary investigation as to the number of slots and bars and their relations in certain machines using multiplex windings has been made by the author in the Electrical Engineering Department of the Massachusetts Institute of Technology. It is his wish to duly acknowledge a number of valuable suggestions made by Professor H. B. Dwight of this department.

The windings used in the figures for purposes of illustration have been shown as applied to cores with extremely few slots and should therefore not be considered as actually usable windings. This has been done so as to present clearly the principles which are to be applied to practical windings. The naming of the types of windings herein is similar to that used in the *Standard Handbook for Electrical Engineers.*

Multiplex windings were proposed in the early writings on d-c. machines, but little or nothing was said about cross-connections. Where there are four or more parallel paths through the armature, these are usually necessary. The cross-connections must be made between points having mathematically the same voltage, the flux being the same in all poles. This is provided for in the windings suggested in this paper.

The type of winding proposed is an ordinary multiplex winding so arranged that a system of cross-connections, all at one end of the armature, can be conveniently applied to equipotential points, thereby joining to one another the individual parts of the winding[5] and producing a uniform distribution of potential on the commutator. These connections are shown on the diagrams of this article.

The shape of the coils of a duplex lap winding is the same as that of the coils for a simplex lap winding and no departure is made from the usual construction. All the coils for any one machine are of the same size and shape. The ends of the coils in a duplex lap winding are connected to bars 1, 3, 5, ... and so on to all the odd numbered bars. Another part of the winding placed in the slots connects to bars 2, 4, 6, ... and so on to all the even numbered bars. There are two cases to be considered. In Case I, when the total number of commutator bars is even, the two parts of the winding are separate and independent except as they are joined by equipotential or cross-connections. A two-part winding of this type is known as doubly reentrant and will be referred to herein as such. It is represented by the four-pole diagram, Fig. 1, and by the symbol ○○ .

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[2] See p. 87, *Die Gliechstrommaschine Arnold—la Cour,* Third Edition, 1919, Julius Springer, Berlin.

[3] See p. 177, *Die Gleichstrommaschine, Arnold,* Second Edition, 1906, Julius Springer, Berlin.

[4] See p. 82, Same reference as 1, and Fig. 14, p. 604, *Standard Handbook for Electrical Engineers,* Edition of 1922.

[5] See p. 244, *The Dynamo,* C. C. Hawkins, Volume I, Sixth Edition, 1922, Sir Isaac Pitman & Sons, Ltd., London.

In Case II when the number of commutator bars is odd and the coil ends are connected to every other bar, the first part of the winding goes around the armature and leads into the second part of the winding, which, after going around the armature, connects to the first part, resulting in a continuous winding. This winding is referred to as singly re-entrant and is represented by the six-pole diagram, Fig. 2, and by the symbol ◎ .

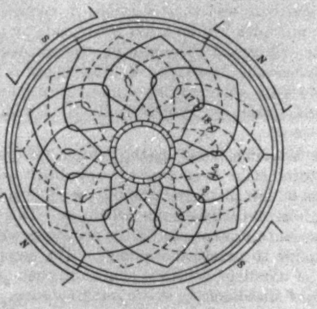

FIG. 1 (CASE I)—DUPLEX LAP WINDING, DOUBLE RE-ENTRANT

Four poles, 18 slots, 18 bars, 8 paths. Cross-connections as shown.

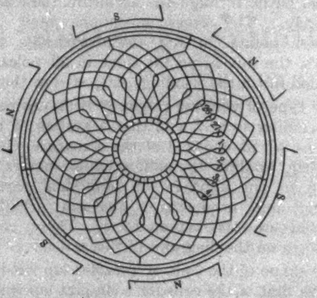

FIG. 2 (CASE II)—DUPLEX LAP WINDING, SINGLY RE-ENTRANT

Six poles, 27 slots, 27 bars, 12 paths. Cross-connections as shown.

The diagrams show the cross-connections joining the two parts of the duplex lap winding, which is a desirable feature. The commutator potential is shown to progress fairly uniformly around the commutator, which is also desirable. To arrange the cross-connections properly and to secure these results certain numbers of slots and bars must be used. The method of cross-connecting, as shown, can be used with duplex lap windings on the following machines when the choice of slots and bars is made as indicated in Table A.

TABLE A

DUPLEX LAP WINDINGS

Number of Poles	Slots per Pair of Poles	Total No. Slots	Bars per Slot	Total No. Bars	Re-entrancy	
a. 4	Odd no.	Even no.	Odd no.	Even no.	Double	○○
b. 6	Odd no.	Odd no.	Odd no.	Odd no.	Single	◎
c. 8	Odd no.	Even no.	Odd no.	Even no.	Double	○○
d. 10	Odd no.	Odd no.	Odd no.	Odd no.	Single	◎
e. 12	Odd no.	Even no.	Odd no.	Even no.	Double	○○

There may be a cyclical change in the number of slots per path but this is quite possibly not harmful, for it occurs with two-circuit windings which operate successfully. The great improvement in commutating characteristics obtained by having only half as much current per path to be reversed at one time would seem to be the important consideration in connection with this type of winding.

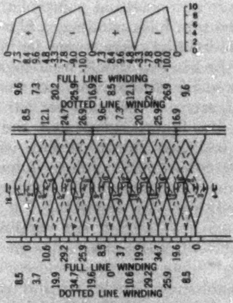

FIG. 3 (CASE I)—VOLTAGE DISTRIBUTION DIAGRAM FOR DUPLEX LAP WINDING, DOUBLY REENTRANT

Four poles, 18 slots, 18 bars, 8 paths, (same winding as Fig. 1).

In determining the voltage distribution (see Fig. 3) the developed winding is laid out and a field form as might naturally be expected is plotted above the winding. As the voltage in a conductor is proportional to the flux it cuts, an excellent idea of the voltage distribution may be secured by summing up the voltages induced throughout the winding. In doing this, it is assumed that any conductor in a slot generates a voltage proportional to the flux directly above the center line of that slot.

Another type of winding is the duplex wave winding having four parallel paths through the armature. This type would give an intermediate step between the simplex wave and the simplex lap winding and although the benefits to be derived from the use of this type are not expected to be as great as those obtained with the duplex lap winding, certain cases may justify the use of this type. An eight-pole, duplex wave winding is used rather extensively at the present time and has proven satisfactory. Such a winding with cross-connections joining equipotential points of the individual parts of the duplex winding is advocated and shown by Arnold.[6]

Cross-connections joining the two parts of a duplex wave winding are possible in certain cases. These connections should join points having mathematically the same voltage when all poles have an equal amount of flux. They can be used on the following machines when the choice of slots and bars is made as indicated in Table B.

TABLE B
DUPLEX WAVE WINDINGS

Number of Poles	Total No. Slots	Bars per Slot	Total No. Bars	Re-entrancy
f. 8	Even no.	Odd no.	Even no.	Single or double according to whether commutator pitch is odd or even.
g. 12	Even no.	Odd no.	Even no.	
h. 16	Even no.	Odd no.	Even no.	

Of course these windings must be such that the commutator pitch, Y, is a whole number where

$$Y = \frac{\text{number of bars} \pm 2}{\text{pairs of poles}}$$

The cross-connections in the windings of the above table will span half the number of poles, or in other words, they are diametral. When the number of pairs of poles is odd, such as in the six- or ten-pole machine, cross-connections to equipotential points diametrically opposite are not possible. This can be explained as follows: Suppose that the choice of slots is made so that a point of one part of a duplex winding has a certain position relative to a pole with one polarity; then a corresponding point in the other part of the duplex winding has a position under the influence of a pole of opposite polarity, a condition which does not lend to cross-connections. It may be possible that six- and ten-pole, single re-entrant windings of the duplex wave type would be successful even though they do not have cross-connections.

Triplex windings could be designed on the same basis as the duplex windings herein described, but only duplex windings have been treated in this paper, since they involve the least departure from simplex windings, and so should be the first to be tried out.

Since the preparation of this article was commenced, a very interesting paper[7] on frog leg windings by W. H. Powell and G. M. Albrecht has appeared. This illustrates the general principles of using a multiplex wave winding in the same slots with a lap winding either simplex or multiplex to a less degree than the wave winding, so that both windings produce the same voltage at the brushes and are connected to the same set of commutator bars. The wave winding connects the circuits of the lap winding and the lap-wound coils connect the circuits of the wave winding thereby producing a thoroughly cross-connected winding. The name *frog leg* is used because of the peculiar shape of the coils which consist of a lap-wound coil and a wave-wound coil taped together. The advantages obtained by the authors are first, better cross-connections, and second, the use of multiplex windings.

In Part III of the above described paper, a triplex winding with external cross-connections as previously mentioned, and not of the frog leg type, is mentioned as having been made and operated successfully. If triplex windings, not of the frog leg type as referred to above, have been successful, then a duplex winding should also operate well when the choice of slots has been made properly and the cross-connections applied efficiently as outlined herein.

It is possible that duplex lap windings with ordinary cross-connections as shown will give some of the advantages of the frog leg winding without the complication of having two complete windings in the slots.

Discussion

H. B. Dwight: Part of the work done by Mr. Nelson in preparing his paper was a detailed study of all editions of Professor Arnold's books as well as a study of the few other books which have referred to multiplex windings on cross-connected d-c. armatures.

While Arnold's recommendations were valuable, they were not, by any means, complete. They consisted mainly of isolated examples. A clear description of the types of windings recommended and the types which should be avoided was not given. Some of the earlier editions of Arnold's books contained arrangements of multiplex windings with armature cross-connections which were unworkable or inadvisable, and which were not included in the later editions. Probably they were recognized by Professor Arnold as being inadvisable. Other types of windings, which Mr. Nelson's paper states appear favorable and should be tried out by the industry on practical machines, were not described in Arnold's books.

J. L. Burnham: The impression I get from reading this paper is that the entire emphasis is given to the necessity of equalizing at points 360 electrical degrees apart and devising windings that will give interconnecting points exactly 360 degrees apart in the different windings employed. The necessary condition for obtaining windings that can be so equalized involves the use of an odd number of armature slots per pair of poles, thereby giving armature circuits having unequal numbers of conductors in parallel between adjacent sets of brushes of opposite polarity. This point is mentioned in the first paragraph of the third page and passed over casually as possibly of no importance. I believe that this is a very vital objection to these arrangements of multiplex windings. The intimate connection through the brushes of these adjacent circuits having unequal voltages varying in a cyclical manner to produce a continuous exchange of current through the brushes, is harmful to commutation, causing excessive local currents in the brush contact surfaces. I have observed performance of such machines in which the brushes were severely burned in sharply defined zones, causing very rapid wear of brushes and roughening and pitting of the commutator bars.

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[6] See p. 86, *Die Gleichstrommaschine Arnold—la Cour*, Third Edition, 1919, Julius Springer, Berlin.

[7] P. 345, *Iron and Steel Engineer*, September 1925.

The arrangements of windings described in this paper are not new. It seems strange that such arrangements of windings have not been used commercially if they have the merits claimed. It is probable that they have been tried and found inferior to the more usual types of simple windings.

The frog-leg winding, to which reference is made at the end of the paper, has been discussed considerably of late. With this arrangement, which requires four layers of windings in the armature slots, the objections mentioned above have been diminished, but it is my feeling that well equalized, simple windings, giving equal circuits between adjacent sets of brushes, are to be preferred. The conductors of such windings may be split to reduce eddy-current losses to any degree equivalent to that obtained in the frog-leg winding. When commutation limits are reached with the simple coil, it is possible to split the individual coils to reduce the reactance voltage, giving the equivalent for which investigators have been striving in devising the multiplex windings without the objectionable features that preclude good performance in the type of windings described in this paper and the objectionable structural features of the four-layer frog-leg winding.

H. B. Dwight: The fact that duplex wave windings are successfully used at the present time by designers of machines in the United States and Canada, is one of the strongest reasons for giving a thorough practical try-out to other multiplex windings, which are in some ways very similar to the approved duplex wave windings.

Duplex wave windings consist of two wave windings side by side but cross-connected and built according to the rules described in the paper by Mr. Nelson. Duplex wave windings, as well as simplex wave windings, have a cyclic variation in the number of conductors per path. This does not seem to increase the sparking at the brushes. On the contrary, duplex wave windings have been proved by test to have less sparking than the corresponding simplex windings of the same voltage and amperage rating but with fewer and wider commutator bars. The improvement in commutation that goes with more, and narrower, commutator bars, is due, probably, as stated by Mr. Nelson, to the substantial decrease in amperes per bar and the resulting decrease in the number of amperes that are reversed when a brush leaves a commutator bar. An example of this is commonly observed by designers; namely, when they change a two-circuit, wave-wound, four-pole armature so as to give the same voltage and amperage with a multiple winding. The latter requires twice as many commutator bars and the improvement in commutation is most marked.

One criterion of the commutation characteristics of a winding is the width of the commutator bar, other things being equal. If the narrow bars are obtained by using a duplex wave winding, which is as irregular as any winding described in Mr. Nelson's paper, it has been shown by test that the improvement in commutation is obtained. Nothing has been shown to prove that the improvement will not be obtained by using other multiplex windings, such as the duplex lap winding, in order to have narrow commutator bars. Consequently, such windings should be given a trial.

Some failures with multiplex windings 25 or 30 years ago were due clearly to the lack of cross-connections. The inequality in length of paths was probably not a contributing factor. It is not peculiar that these failures put all kinds of multiplex windings into disrepute, for the action of armature cross-connections was not well known at that time, when they were just beginning to be used with simplex lap windings.

The improvements due to the frog-leg winding are probably that it gives thorough cross-connection and reduces the amperes per commutator bar. It cannot be said that it does away with inequality in length of paths, for it is made up of usual simplex and multiplex windings. Reference 7 specifies that the number of bars divided by the number of pairs of poles should be an integer, but not necessarily an even number. Thus, in Part II of Reference 7 there are 52½ slots per pole in the 1000-kw. generator, and so the number of coil sides between two adjacent brushes continually varies between 52 and 53.

A. A. Nims: The cross-connections in multiple or multiplex armature windings should connect points which are at equal potential for all positions of the armature. To make sure of this, a general method of examination is preferable to one based upon a definite position of the armature.

Such a general method is indicated by the fact that the several coil e. m. f's. are essentially alternating and therefore may be represented by vectors. Though these coil e. m. f's. are likely not to be sinusoidal, they are identical in wave-shape and amplitude, disregarding any minor effect of different positions of the various conductors in a slot. Therefore, some convention may readily be devised[1] whereby the coil e. m. f's. may be represented by vectors of equal length, but in proper phase.

To apply this scheme to some of the windings illustrated in the paper, it will be found more convenient to use coil-side voltage vectors rather than coil voltage, as thereby the potentials of the rear noses of the coils, to which cross-connections are attached, become apparent.

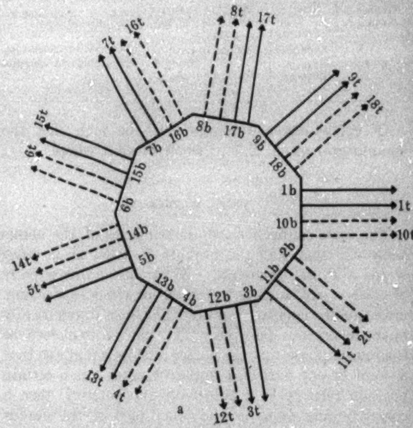

FIG. 1A—COIL-SIDE VOLTAGE VECTORS SEPARATE. DOTTED LINES REFER TO ONE WINDING; SOLID LINES TO OTHER

Fig. 1A shown herewith then represents vectorially the conductor voltages of the winding of Case I, Fig 1 of the paper. For clearness, the several vectors are radiated from a nine-sided polygon instead of from a point, since there are nine slots per pair of poles. The vectors labeled with the slot number followed by the letter *b* represent the voltages of the bottom coil-sides or conductors, assumed to be shown at the left side of each slot in the original paper, while *t* suffixed to the slot number designates the voltages of the top coil sides assumed to be shown at the right of the slots.

In Fig. 2B herewith these coil-side voltage vectors are combined in the order shown in Fig. 1 of the original paper; the outer points of the star correspond to commutator bars, and the inner points to the rear noses of the armature coils. It is then evident that for every commutator bar or rear nose in one of the windings there is a commutator bar or rear nose respectively at the same potential in the other winding. This is in agreement with the statement in the paper that equipotential connections may all be located at the rear end of the armature.

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[1] *Electrical World*, September 28, 1912, p. 660.

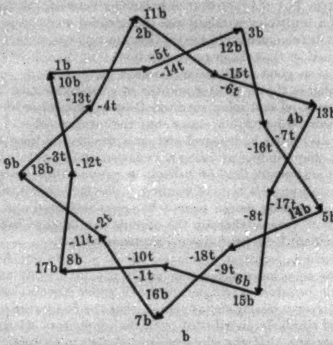

11b
2b
3b
12b
-5t
-14t
-15t
-6t
1b
10b
-13t
-4t
13b
4b
-7t
-16t
9b
18b
-3t
-12t
-17t
14b
5b
-8t
-2t
-11t
17b
8b
-10t
-18t
-9t
6b
-1t
16b
15b
7b
b

FIG. 1B—COIL-SIDE VOLTAGE VECTORS COMBINED IN THE ORDER SHOWN IN WINDING OF FIG. 1, ORIGINAL PAPER

The vector diagrams for both windings coincide; thus all lines appear solid.

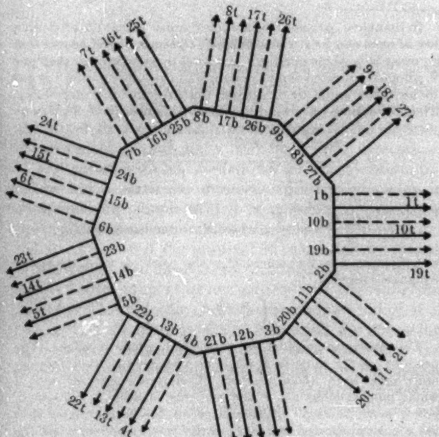

8t
17t
26t
7t
16t
25t
9t
18t
27t
24t
15t
6t
8b
17b
26b
9b
18b
27b
7b
16b
25b
24b
15b
6b
1b
10b
19b
1t
10t
19t
23t
14t
5t
23b
14b
5b
2b
11b
20b
22b
13b
4b
21b
12b
3b
2t
11t
20t
22t
13t
4t
21t
12t
3t
a

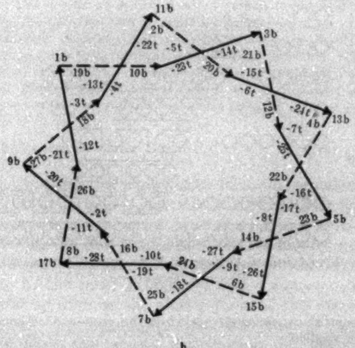
FIG. 2A—COIL-SIDE VOLTAGE VECTORS SEPARATE. DOTTED LINES REFER TO COIL-SIDES AT BOTTOM OF SLOTS

Cross-connections are often used between the equipotential points of a simplex lap winding. These points are under alternate poles as in the case of the duplex winding. But it is to be noted that in the winding proposed as typical, duplexing the winding has not increased the number of points having any given potential. There is only one point in each winding having any given potential, and the cross-connections still unite only two points.

Fig. 2, A and B, are corresponding diagrams for Case II of the original paper, showing that there are three commutator bars or rear noses having a given potential. The cross-connections now tie three points together, as in simplex lap winding for six poles. Since there are only two windings, these three points must be in one winding, or two in one winding and one in the other. With single re-entrancy as in this case, the distinction between the two windings practically disappears, and since the equipotential points divide the winding into three equal parts, the symmetry appears to be complete.

In the earlier treatises on armature windings, practically the only consideration imposed on multiplex windings, either lap or wave, seemed to be the selection of a proper number of commutator bars, armature coils or coil sides to permit re-entrancy. Later study brought out the importance of symmetry between circuits, and in 1916 there was added to the usual winding formulas the condition that the ratio of poles to armature circuits must be integral[2].

This condition limits windings to the usual simplex lap and wave windings, the duplex wave winding in four-pole machines, the triplex in six-pole machines, the duplex and quadruplex in eight-pole machines, the duplex, triplex and sextuplex in twelve-pole machines, and so on, always keeping to a degree of multiplicity which is an exact factor of the number of bipolar elements of the machine.


11b
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-6t
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27b
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-12t
-25t
-20t
22b
26b
-16t
-2t
-8t
-17t
23b
5b
-11t
14b
8b
16b
-10t
-27t
17b
-28t
-19t
24b
-9t
-26t
6b
25b
-18t
7b
15b
b

FIG. 2B—COIL-SIDE VOLTAGE VECTORS COMBINED IN THE ORDER SHOWN IN WINDING OF FIG. 2, ORIGINAL PAPER

Multipolar machines may be regarded as an aggregation of bipolar elements. If each of these elements be regarded as a unit in arranging the series-parallel grouping, the numerical reason for the condition that the ratio of poles to armature circuits must be integral is apparent from Fig. 3, which shows the possible combinations of different numbers of units up to 16 poles, except for 14 poles, which has the same combinations as 10 poles.

It is now suggested that some of these parallel combinations may be duplexed or even triplexed, thereby making the number of circuits greater than the number of poles. This, of course, violates the condition of an integral ratio of poles to paths. Such a procedure appears to give symmetrical windings that can be cross-connected conveniently, and records of satisfactory performance are cited. Evidently some modification of the condition just mentioned is indicated.

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[2] S. P. Smith, *London Electrician*. June, 1926.

Experience with four-circuit, six-pole windings has been such as to warrant staying pretty close to the rule that the ratio of poles to paths must be integral when the paths are fewer than the poles. But if more paths than poles are desired, the indications are that the ratio of paths to poles should be integral.

C. C. Nelson: At the present time we are not in possession of any complete and conclusive evidence to show that multiplex windings will not work. The knowledge we have on such windings is of a fragmentary nature and consists mostly of what we have gained from the observation of a few isolated cases. In regard to Mr. Burnham's discussion, I grant that there may be a cyclical change in the number of conductors per path but there has not been any direct work done in connection with this to show that it is a vital objection. We do have the cross-connected duplex wave winding, however, which now operates well but which was not satisfactory before cross-connections were applied, indicating that the cross-connections have made it a useful type of winding, commercially, whereas formerly it was objectionable. As pointed out by Professor Dwight, this winding is as irregular as any of those described, and yet it has given a satisfactory improvement in commutation when the cross-connections were suitably applied.

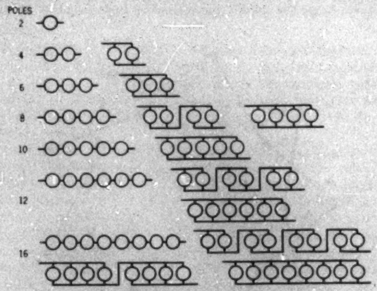

Fig. 3—Multipolar Armature Windings; Combinations of Bipolar Elements

To my knowledge, the duplex wave winding with cross-connections is the only multiplex winding which has gained a great deal of commercial importance before the introduction of the recent frog-leg winding. For this reason the other types described in the paper should be tried, with a view to possibly increasing the number of types of windings which are to be considered useful. If designers are to be satisfied with only simple windings we cannot hope to progress very far in the armature-winding art.

Professor Nims' general method of pointing out the equipotential points may be preferable to one based upon a definite position of the armature. The conclusions reached by the two schemes are in agreement, indicating that the general method may be of some value.

On the first doubly re-entrant wave windings used without cross-connections there was considerable unbalancing between the two parts of the winding and hence cross-connections joining the two parts of the winding were put on to equalize and balance the circuits. Before cross-connections were used, the results secured with the singly re-entrant winding may have caused it to gain some favor due to the fact that the singly re-entrant winding is continuous and not separated like the doubly re-entrant type and hence would probably not unbalance so much as the doubly re-entrant type.

Professor Nims states that the number of equipotential points with the duplex lap winding is not greater than the number of equipotential points secured with the simplex lap winding. This is true but the fact that unbalancing between the separate parts of a multiplex winding was experienced when these parts were not interconnected indicates that multiplex windings should be designed so that the cross-connections join the separate parts and cause as good equilization between the circuits as possible. It is probable that a greater number of cross-connections would be necessary on multiplex windings than on simplex windings.

Professor Nims further states that the ratio of poles to armature circuits must be integral and cites the case of the four-circuit, six-pole winding as being a violation of this rule. I infer from his statement that he believes experience has shown this to be an unfavorable type of winding. The four-circuit, six-pole winding of regular design cannot be cross-connected and hence there are no means whereby the circuits can be kept balanced. This is evident from the following explanation.

To produce points of equipotential it is necessary for one conductor to occupy a position under one pole while another conductor occupies a similar position under another pole of similar polarity, assuming, of course, that the flux in the poles is equal and similarly distributed. As the conductors are in slots, it is necessary that the slots occupy similar positions. If n_x denotes the number of slots per x poles, where x is necessarily an even number if similarly situated slots are to be obtained, then the total number of slots is given by

$$N = n_x \frac{p}{x}, \text{ where } p = \text{the total number of poles.}$$

In the above, p/x must be a whole number so that the total number of slots can be a whole number, remembering of course that n_x must be a whole number. Also p/x must be greater than one if similarly situated slots are desired, or in other words x is not to be taken equal to or greater than the number of poles, p. Then for duplex wave windings the commutator pitch is given by

$$Y = \frac{n_x \frac{p}{x} B_s \pm 2}{\frac{p}{2}} = \frac{\text{number of bars} \pm 2}{\text{pairs of poles}}$$

$$= \frac{2\, n_x B_s}{x} \pm \frac{4}{p}, \text{ where } B_s = \text{the bars per slot,}$$

and because $p = 2\,P$ where P = the pairs of poles, the above equation becomes

$$Y = 2\left(\frac{n_x B_s}{x} \pm \frac{1}{P}\right).$$

The commutator pitch, Y, must be a whole number. The product, $n_x B_s$, must be a whole number because each individually must be a whole number. Therefore Y can be only a whole number when $x = P$ and as x must be even, P, the pairs of poles, must be even. Furthermore, the similarly situated slots are x poles apart and as $x = P$, where P = numerically half the number of poles, the similarly situated slots are necessarily diametrically opposite. The above explanation is general and holds for all duplex wave windings.

If the nature of the four-circuit, six-pole winding was such that cross-connections could be conveniently applied so as to balance the circuits it then probably would be a favorable type of multiplex winding.

The Mechanism of Breakdown of Dielectrics

BY P. L. HOOVER[1]
Associate, A. I. E. E.

***Synopsis.**—In attempting to analyze various experimental data that have been obtained in researches on dielectric phenomena in high-voltage cable insulation and other dielectrics, the various existing theories of dielectric behavior have seemed inadequate. A critical study has therefore been made of these theories in an attempt to obtain a working hypothesis that more nearly meets the stringent requirements of experimental facts.*

The logarithmic formula is shown to give erroneous results if applied to high-voltage cables when they are operating under high stress. The gradient in a cable must be calculated from the volt-ampere characteristic of the dielectric when stresses above the elastic limit are used. For stresses below the elastic limit it makes no difference which method is used, but at high stresses an entirely different gradient distribution is obtained when calculated from the volt-ampere characteristic.

Likewise, when an insulation is operated above the elastic limit the stress ceases to be a critical factor, but the strain is of utmost importance. In comparing cables that are operating under high voltages, therefore, the strain at the core should be considered rather than the stress at the core. Stress is given by the voltage gradient and strain by the polarization or the current density in the dielectric.

Since there is always a conduction current flowing, there must be mobile or free ions present. It is assumed that these free ions or electrons come from the molecules of the dielectric and that the number that are present depends on a condition of equilibrium existing between the molecules and the free ions. There exists then a state of kinetic equilibrium between the molecules and the free or mobile ions. Any change in external or internal conditions will disturb the equilibrium and thus change the electrical behavior of the dielectric. Thermal effects and corona effects are accounted for on this basis.

Breakdown occurs when the equilibrium conditions are so disturbed that the insulation as a whole becomes unstable, electrically. High stress or strain and high temperature affect the conditions of equilibrium decidedly.

Corona in gases, oils, and solids consists of minute disruptive discharges that are initiated by rapid changes in equilibrium conditions when the dielectric is overstressed. In solids, however, where the ion mobility is very low and the ion friction high, there will not in general be a corona effect observed because there can be no rapid readjustment of equilibrium conditions. If the insulation is not homogeneous, or if it is composite, there will likely be internal discharges, a corona effect, when the weaker dielectric is overstressed. Moreover, since the molecules of a solid can not readjust themselves quickly to the new conditions of equilibrium imposed by a high voltage suddenly applied, there will be high local stresses and strains set up which may result in mechanical deterioration of the dielectric, that is, chipping or cracking of the dielectric.

Breakdown, therefore, will take place when the insulation is rendered unstable by disturbing the equilibrium conditions, regardless of whether it is due to mechanical strains, electrical strains, or to thermal effects. The tri-fold nature of the phenomenon must be considered in the complete analysis of the problem.

* * * * *

INTRODUCTION

THE mechanism of breakdown of dielectrics is of the highest importance and yet at the present time it is very little understood. It is the purpose of this paper to discuss and to extend some of the theories of breakdown which have been presented heretofore.

The mechanism of breakdown of single-conductor cables has a particular interest, since the geometry is comparatively simple and it is possible to overstress parts of the insulation without complete rupture taking place. That is, if the gradient is calculated by means of the well known logarithmic formula for a cable with a ratio of outer to inner radius greater than the Naperian base *e* and for a voltage near the breakdown value, it is found that the inner layers of insulation are operating at gradients considerably higher than any that the insulation would stand if it were made up in flat sheets. This same effect is brought about whenever a dielectric is in a non-uniform field, although in general it is not possible to derive a simple mathematical formula for the gradient distribution of a complex field.

There is, however, considerable justification for skepticism regarding the possibility of overstressed insulation, that is, regarding the validity of the logarithmic formula when used for calculating gradients when the insulation is stressed beyond the elastic limit. In using the logarithmic formula for calculating gradients it is tacitly assumed that the dielectric has the same electrical constants at and beyond breakdown that it has at low gradients. Reasoning by analogy from other physical phenomena, we would not expect this to be the case; in fact, we would expect new laws to enter as soon as the elastic limit of the material is exceeded. For example, Hooke's law is used in determining the stress distribution in beams and supports of all kinds as long as the elastic limit is not exceeded but beyond the elastic limit Hooke's law does not hold and cannot be used.

HISTORICAL REVIEW

To account for these phenomena of overstressed insulation, various theories have been proposed, such as the maximum stress theory, the average stress theory, Fernie's[2] minimum stress theory, Russell's[3] theory, and Osborne's[4] theory. These theories, and in particular the minimum stress theory, have been discussed at length by Simons[5] and will not be considered in detail in this paper.

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[1] Research Fellow, The Harvard Engineering School.

Presented at the Annual Convention of the A. I. E. E. at White Sulphur Springs, W. Va., June 21-25, 1926.

[2] F. Fernie, Insulating Materials, *Beama*, 1920, p. 244.

[3] A. Russell, *Dielectric Strength of Insulating Materials and the Gradient of Cables*, JOURNAL of A. I. E. E., Vol. 40, p. 6, 1907.

[4] H. S. Osborne, *Potential Stresses in Dielectrics*, JOURNAL of the A. I. E. E., Vol. 29, p. 1553, 1910.

[5] D. M. Simons, *On the Minimum Stress Theory of Cable Breakdowns*, JOURNAL of the A. I. E. E., Vol. 41, p. 557, 1922.

Differing substantially from these theories just mentioned is the pyroelectric or thermal theory of breakdown as developed by Wagner[6] and by Hayden and Steinmetz[7]. Quoting Wagner,

"It is a well known fact that the resistance of the insulation decreases rapidly with increasing temperature. The hot spots therefore take up a larger current than the adjoining cooler spots, which are subjected to the same tension. It is evident that the hot places receive more energy than the cooler ones, with the result that the temperature difference further increases. It is easy from this to picture that at a certain tension an unbalanced condition takes place, and the hottest spot finally burns."

Quoting Hayden and Steinmetz,

"The mechanism of breakdown of solid insulation is essentially different from that of air. Solid insulators do not have a definite dielectric strength or breakdown gradient at which they change to conductors, as in the case of air. They are really high-resistance conductors with a very high negative temperature coefficient of resistance. One characteristic of a substance having a high negative temperature coefficient of resistance is its instability at constant potential above a certain critical voltage or voltage gradient. At and above this voltage the decrease of resistance by the increase of temperature increases the current density and thus the temperature indefinitely, thereby short-circuiting the constant potential supply and destroying the insulator. Below this critical voltage the decrease of resistivity and increase of temperature limit themselves and the conditions are stable."

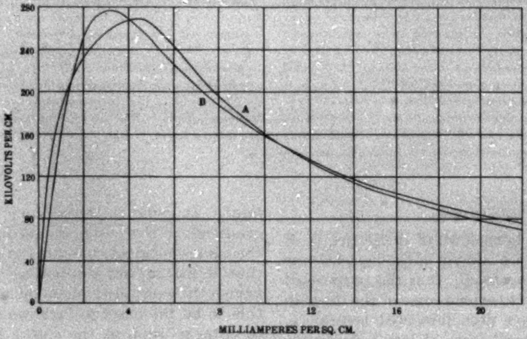

Fig. 1—Volt-Ampere Characteristics

A. From data of Wagner (Oiled Paper)

B. $e = \dfrac{175\, i}{1 + 0.1\, i^2}$

The pyroelectric theory thus assumes that solid insulators have volt-ampere characteristics similar to those shown in Fig. 1, and that this type of characteristic is due solely to the high negative temperature coefficient of resistance of the material. With an insulating material of this kind operating at low-current densities there would be, on the basis of the pyroelectric theory, a certain increase in temperature, the increase of temperature being limited by the dissipation of heat into the surrounding medium. At higher current densities, however, there comes a point where the rate of generation of heat exceeds the rate at which heat is dissipated and the temperature increases indefinitely. At this point, dynamic rupture of the insulation takes place, for the current would increase indefinitely with no further increase of voltage. This point is given by the maximum of the volt-ampere characteristic, Fig. 1, and is determined mathematically by $dE/dI = 0$.

The question arises, however, as to whether or not this type of volt-ampere characteristic is entirely the result of a high negative temperature coefficient of resistance. Might it not be due, at least in part, to change in resistivity with current density due to some molecular phenomenon? In any case, breakdown will occur when $dE/dI = 0$, for at this point the current will increase to the short-circuit value of the supply.

Günther-Schulze[8] has made a study of the phenomena of dielectric breakdown and states that different definitions of dielectric strength may be given according to the way in which the problem is approached. To quote from his paper:

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[6] K. W. Wagner, *The Physical Nature of the Electrical Breakdown of Solid Dielectrics*, Journal of the A. I. E. E., Vol. 41, p. 288, 1922.

[7] J. L. R. Hayden and C. P. Steinmetz, Insulation Failure—A Pyroelectric Effect, *Electrical World*, October 1922, p. 865.

[8] A. Günther-Schulze, The Dielectric Strength of Liquids and Solids, *Jahrbuch der Radioaktivität und Elektronik*, 1922, Vol. 19, p. 92.

"Experience has taught us that perfectly non-conducting substances do not exist. Even at low voltages there is always a current flowing in the dielectric, be it ever so small. This proves that mobile charges, ions or electrons, are always present in the dielectrics. As long as the current is not capable of producing ions itself but has to depend on other sources, its strength is limited by the number of ions present in the dielectric. However, as soon as the voltage gradient is sufficiently high for the original ions to produce new ions by collision with the molecule of the dielectric, the current begins to increase very rapidly and soon becomes excessive: the breakdown or flash-over is accomplished.

"Accordingly, the dielectric strength is the lowest potential gradient at which the current through the dielectric is replaced by an independent discharge which is self-increasing (a spark)." (*Translated*) This gives what Günther-Schulze calls the dielectric impact strength (Stossfestigkeit).

As a second point of view he gives the following: "It should be immediately obvious that the dielectric displacement, *i. e.* the displacement relative to each other of the charges in a molecule or an atom cannot be driven beyond a certain limit; on the other hand, there must be, for each kind of molecule and for each state of aggregation of the same molecule, a certain characteristic field at which the molecule or the atom is torn to pieces so that oppositely charged ions are formed.

"This critical field strength is a material constant just as important as the specific inductive capacity and like this gives information on the fields inside the atoms.

"These considerations result in the following definition of the dielectric strength: the dielectric strength of a dielectric is the lowest potential gradient at which the bonds between the charges in a dielectric are severed so that a discharge passes through the dielectric." (*Translated*) This he calls the dielectric tensile strength (Reissfestigkeit).

Günther-Schulze then considers various data and concludes or assumes that breakdown in liquids is really a gas discharge in disguise. He states:

"Some further reflections will show that this assumption is decidedly logical. It is just as impossible to make liquid dielectrics entirely free from ions as gases. But the ions in liquids are not capable of producing new ions through collision, because the mean free path is not sufficient.

"Thus, if an increasing field is applied to the liquid dielectric, the velocity of the ions in the field will increase. But this again means an increased ion friction and an increased heating of those parts of the dielectric that surround the path of the ion. If the increasing field is continued, the heating finally becomes so great that the ions leave an extremely minute, sub-microscopic vapor track in the dielectric. If an ion comes into this vapor track, it is capable of producing new ions through collision, thus initiating the spark discharge, provided the potential drop along the path of the ion is sufficiently high." (*Translated*)

For solid dielectrics he states that the phenomena are somewhat similar. The extremely high ionic friction causes heating effects that result eventually in breakdown. It is, then, the dielectric impact strength that determines the stability of the insulation, the dielectric tensile strength playing a relatively insignificant part in breakdown phenomena.

Thus, dielectric breakdown, according to Günther-Schulze, is a pyroelectric effect. The concept of dielectric impact strength, as he describes it, is only a physical picture, in terms of ions and atoms, of the pyroelectric theory of breakdown.

The concept of dielectric tensile strength, however, should not be thrown aside so hastily. A further consideration of essentially this same idea, but modified considerably and from a different point of view, leads to some very interesting conclusions and results.

THEORETICAL CONSIDERATIONS

As was first pointed out by Maxwell, the total current in a dielectric consists of the polarization current and the conduction current, the two currents being super-imposed. Is it not possible that there is some relation between them since they are contemporaneous phenomena?

At low gradients there is always a current flowing and this current is due to moving ions or electrons. There is also a certain polarization of the dielectric. At higher gradients the polarization increases and so does the conduction current. That is, the number of mobile charges increases with the degree of polarization of the dielectric. This suggests that the mobile ions must come from the molecules of the dielectric. If this is the case, there must exist a state of equilibrium, a kinetic equilibrium, between the mobile or free ions and the molecules of the dielectric. Increasing voltage gradient would increase the polarization and establish new conditions of equilibrium, the tendency being to increase the number of free ions and thus increase the conductivity of the dielectric. Ultimately a gradient will be reached where the number of ions required to establish the equilibrium will be so great that the molecular bonds will be destroyed and dynamic rupture of the insulation will take place.

This concept of dielectric behavior may be regarded as a dissociation theory of conduction and breakdown because of its similarity to the ideas of dissociation of poly-atomic gases with temperature. The dissociation of molecules and atoms in electrolytes also presents a somewhat analogous case.

The idea of assuming molecular dissociation at comparatively low-voltage gradients may seem rather bold at first sight, but a little thought will show that it does not demand any radical departure from the already well established theories of molecular behavior. In the first place, with liquids, and especially with solids, the inter-molecular fields are quite as important as the intra-molecular fields. The great tensile strength of solids can only be explained on the basis of strong inter-molecular fields. Any change in the molecular structure of a solid produces corresponding changes in all of its physical properties due to the changes in the inter-molecular fields. Consequently, it must be assumed that, in solids at least, the electrons in the atom are influenced not only by their own nuclei but also by the fields of the neighboring atoms or molecules. Under such conditions it is probable that the atoms can be ionized; that is, an electron can be removed with much weaker external fields than would be necessary if the atom were isolated in space.

Furthermore, molecular theory demands that the molecules be in motion except at the absolute zero, so any theory dealing with molecular behavior must necessarily be a kinetic theory. With these ideas in mind it is easy to picture an electron in a dielectric moving about with its nucleus, but also under the influence of the strong inter-molecular fields.

The effect of a superimposed external field is now obvious. The molecular fields will be changed and there will be a force on the electrons tending to move them in the direction of the external field. Some of the electrons will drift from molecule to molecule and thus a conduction current will flow. The electrons that do not drift along will be acted on by the same forces, however, so the atoms or molecules will be distorted; that is, the dielectric will be polarized. The external field thus merely changes the equilibrium conditions and causes a general drift of electrons.

For low gradients the number of ions drifting, that is, the magnitude of the conduction current, will be proportional to the gradient. At higher gradients, as the polarization of the molecules increases, the molecular bonds will become weaker and weaker and the number of electrons which are drifting will increase more rapidly than the gradient increases, and there will be an increase in the conductivity of the dielectric. At some critical gradient the molecular bonds will be entirely broken and dynamic rupture will take place. The phenomenon throughout is essentially that of a kinetic equilibrium between the free or drifting electrons and ions and the polarized molecules of the dielectric.

On the basis of these ideas, the volt-ampere characteristic would have the form of the curves of Fig. 1. Curve *A* is from some experimental data of Wagner[9] on oiled paper, while Curve *B* is an empirical curve made to fit Curve *A* as nearly as possible in order that some numerical computations could be made. The equation of Curve *B* is

$$e = \frac{175\, i}{1 + 0.1\, i^2} \qquad (1)$$

where e is kv. per cm.
and i is milliamperes per sq. cm.

Referring to Curve *B* of Fig. 1, it is seen that at low-current densities, *i. e.*, when $0.1\, i^2$ is negligible in comparison to unity, the current is proportional to the voltage. At higher current densities the current increases faster than the voltage and the curve takes the form shown. The equilibrium conditions are perfectly stable up to the maximum point of the characteristic, at which point rupture occurs. If some means be taken to prevent an excessive current, however, it is possible to maintain stable equilibrium in the dielectric even though it be operated beyond the maximum point of the volt-ampere characteristic. By using a high-resistance, mosaic electrode, Wagner was able to determine experimentally the Curve *A* of Fig. 1.

The concept of overstressed insulation follows as a simple extension of this idea of equilibrium between the ions and the molecules. For if a dielectric is in a non-uniform field, the gradient or current density at any point will determine the equilibrium conditions for that point. It might well be that a part of the insulation is operating beyond the breakdown conditions, but if the remaining dielectric prevents a large flow or drift of ions the insulation as a whole will be stable. Upon decreasing the voltage, the dielectric will revert to its initial state of equilibrium unless some change in chemical constitution has taken place due to the extreme polarization. In this case we would have the phenomenon of deterioration due to high stress which is observed repeatedly in certain types of insulation. This form of deterioration would be expected in those materials that are more or less unstable chemically.

CABLE BREAKDOWN

If a cable is constructed of an insulating material having a volt-ampere characteristic similar to those of Fig. 1, it follows that at low current densities the current will be proportional to the voltage. At higher current densities and voltages, when the inner layers begin to be overstressed, the current through these layers will begin to increase more rapidly than the voltage across these layers. Ultimately, the inner layers will be operating at a point beyond the maximum of the volt-ampere characteristic, but the outer layers, not being overstressed, will prevent complete rupture of the cable. Nevertheless, when the rate of increase of voltage with current for the cable as a whole becomes zero, the insulation will be unstable and the cable will puncture.

The possibility of such an analysis by means of the volt-ampere characteristic was recognized and suggested by Peaslee.[10] It is the purpose of the remainder of this paper to carry the analysis still further and to compare the results with certain experimental data.

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[9] K. W. Wagner, Loc. Cit. See also Electric Cables, by W. A. Del Mar, p. 79.

In the appendix it is shown that the voltage across a cable made from a material with a volt-ampere characteristic given by equation (1), Curve *B* of Fig. 1, will be

$$E = \frac{175\,I}{4\,\pi}\,\text{Log}\left(\frac{0.1\,I^2 + 4\,\pi^2\,R^2}{0.1\,I^2 + 4\,\pi^2\,r^2}\right) \qquad (2)$$

where

E = Total voltage in kv.
I = Milliamperes per cm. length of cable
R = Outer or sheath radius (cm.)
r = Inner or conductor radius (cm.)

From equation (**2**), the volt-ampere characteristics of cables of this material can be plotted. Fig. 2 gives the volt-ampere characteristics for cables of inner radius of one cm. and with various outer radii. These characteristics have the same general shape as those of Fig. 1. This, of course, is to be expected, and it is seen that the cable insulation as a whole increases in conductivity as the current increases. The insulation, however, remains stable, regardless of the stress relations in the inner layers, until the maximum point of the particular cable characteristic is reached. This point can be found by setting the derivative of equation (**2**) equal to zero, or,

$$\text{Log}\left(\frac{0.1\,I^2 + 4\,\pi^2\,R^2}{0.1\,I^2 + 4\,\pi^2\,r^2}\right)$$

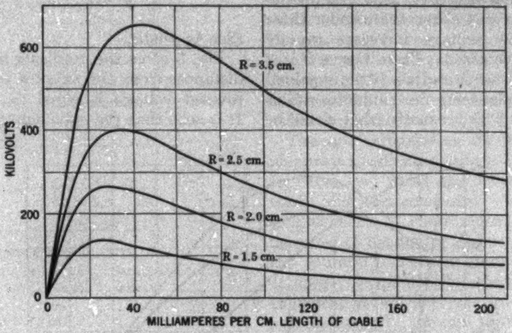

FIG. 2—VOLT-AMPERE CHARACTERISTICS FOR CABLES
Inner radius 1 cm.

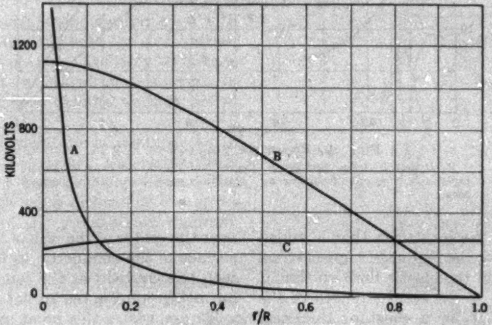

FIG. 3—PUNCTURE VOLTAGE OF SINGLE-CONDUCTOR CABLE vs. r/R

A. r = Constant
B. R = Constant
C. $R - r$ = Constant

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[10] W. D. A. Peaslee, Discussion, JOURNAL of the A. I. E. E., Vol. 41, p. 620, 1922.

$$= \frac{0.8\,\pi^2 I^2 (R^2 - r^2)}{(0.1\, I^2 + 4\,\pi^2 R^2)\;(0.1\, I^2 + 4\,\pi^2 r^2)} \qquad (3)$$

From equations (**2**) and (**3**), the puncture voltage of the various cables can be calculated. Fig. 3 shows curves of puncture voltage, plotted as a function of the ratio r/R and obtained in this way. Curve *A* is for cables with a constant-conductor radius of 0.158 cm.; Curve *B* is for cables with a constant sheath radius of 5.03 cm.; and Curve *C* is for cables with a constant insulation thickness of one cm.

An examination of Curve *A* shows that, under those conditions, the puncture voltage increases rapidly as the ratio of r to R is decreased. From Curve *B* it is seen that for any given sheath radius, little is gained in puncture voltage by decreasing the conductor radius beyond a certain value. This is exactly what would be expected physically, since the insulation that is added by this procedure is added to the regions of highest stress and will be strained beyond the elastic limit so that it cannot possibly add much to the total strength of the cable. Curve *C* shows that for a constant thickness of insulation, the breakdown voltage decreases as the core becomes smaller and smaller in radius. This again is precisely what we know to be a fact, physically.

In Fig. 4 are curves plotted from data obtained experimentally by various observers. Curves *A* and *B* resemble the *A* and *B* curves of Fig. 3. Curves *B′* and *B″* resemble *B* of Fig. 3, except for the tendency to droop at small values of r/R. The five points in the center of the plot and numbered 1, 2, 3, 4, and 5 are from data by Fernie and correspond to a constant insulation thickness. The points are so erratic in their positions that no representative curve can be drawn. They indicate, however, a rough agreement with Curve *C* of Fig. 3.

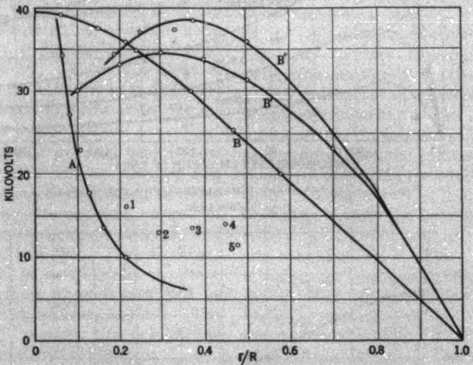

FIG. 4—PUNCTURE VOLTAGE VS. r/R

A, *B* and *B′* from Middleton, Dawes and Davis; *B″* from Wiseman. Points 1, 2, 3, 4, and 5 from Fernie. (Multiply ordinates by 10 for Fernie's data.)

These experimental checks are hardly sufficient to establish positively the theory as developed, but the agreement is certainly sufficiently good to warrant further study.

It is of interest to investigate the question of voltage gradient in various parts of the cable insulation on the basis of this theory. The gradient is given by

$$c = \frac{2\,\pi\, 175\, r\, I}{4\,\pi^2 r^2 + 0.1\, I^2} \qquad (4)$$

(See Appendix.)

Fig. 5 gives the gradients in kv. per cm. at various distances from the axis for two cables, when the impressed voltage is equal to the breakdown voltage. It is seen that the maximum gradient is at some point near the center of the wall of insulation and is not at the surface of the conductor, as is ordinarily assumed. In fact, the gradient at the conductor is less than that at any other point in the insulation.

To emphasize this point and to show the change in gradient distribution with increasing voltage, the curves of Fig. 6 are given. It is seen that for low voltages the maximum gradient is located at the core and that the gradient follows practically the logarithmic law. At higher voltages, and hence higher current densities, the gradient departs from the logarithmic law and has the form shown by the curves. It is seen that the gradient never exceeds a certain definite maximum. This maximum value is determined by the volt-ampere characteristic, Fig. 1, and for the case illustrated is 178 kv. per cm. The position of this point of maximum gradient moves gradually from the surface of the conductor towards the sheath, but puncture occurs before the sheath is reached.

It is important at this time to investigate the stress-strain relations in the dielectric. It seems evident that stress is given by the voltage gradient and strain by the polarization or probably by the current density. We have then the stress proportional to the strain for low stresses, but for higher stresses the strain increases faster than the stress. Finally the ultimate strength is exceeded and the strain increases indefinitely even with a decrease in stress. The analogy with the stress-strain relations of mechanics is obvious, the only difference being that the elastic limit and the ultimate strength in dielectrics are probably not as definite as they are in mechanics.

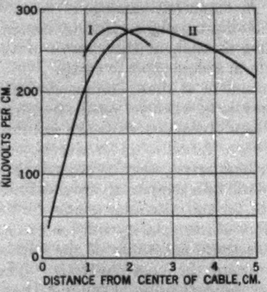

FIG. 5—GRADIENT DISTRIBUTION IN SINGLE-CONDUCTOR CABLES AT THE BREAKDOWN VOLTAGE

I $r = 1.0$ cm.
$R = 2.5$ cm.
II $r = 0.16$ cm.
$R = 5.03$ cm.

The elastic limit may therefore be defined, as in mechanics, as that stress at which the stress-strain diagram begins to deviate from a straight line. The ultimate strength likewise being defined as the stress at the maximum point of the stress-strain diagram. The stress-strain diagram of a dielectric is its volt-ampere characteristic.

From the foregoing it appears that in cables operating at high stresses, the logarithmic formula fails completely to give the correct gradient distribution. Furthermore, it seems that stress or voltage gradient is not the best criterion to use in judging the quality of various insulations when they are operating beyond the elastic limit. Strain is obviously the factor that is of utmost importance when once the elastic limit is passed. Consequently, in comparing various cable insulations, the strain at the core should be used rather than the stress at the core, the strain being given by the current density. This concept is thought to be very important.

There is, however, some question, when alternating currents are used, as to whether the total current or simply the in-phase current should be used to determine the strain. The polarization of the dielectric determines the quadrature current and the conductivity and energy loss determine the in-phase current. As long as both of these two components increase uniformly with the stress, the total current will increase uniformly with stress. However as soon as one of the components begins to deviate from a linear relation, the total current will correspondingly deviate from a linear relation with stress. Furthermore, if a stress is reached where either component of the current increases indefinitely with no further increase of stress, then the total current must likewise increase indefinitely. Consequently it seems that it is immaterial whether the total current or simply the in-phase current is used in determining the strain in the dielectric and it is not evident at this time which of the two will give the best indication of the true strain in the dielectric.

The one outstanding effect that has not been explained is the drooping of the curves B' and B'' in Fig. 4. In these two cases the sheath radius was kept constant and the conductor was made of smaller and smaller radius. There seems to be a point where further decrease of conductor radius not only fails to increase the puncture voltage but actually diminishes it. It hardly seems possible that an increase in insulation thickness should decrease the breakdown voltage, but such seems to be the case so far as these particular data are concerned.

Osborne's[11] theory explains this effect by assuming "... that a solid dielectric, when overstressed, is not disrupted uniformly, but that the material is affected as though it had been pricked by a number of needlepoints."

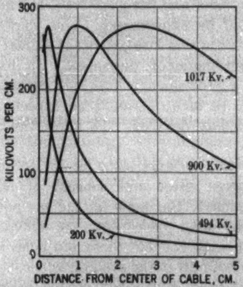

FIG. 6—GRADIENT DISTRIBUTION IN SINGLE-CONDUCTOR CABLE

Breakdown voltage = 1017 kv.
$r = 0.16$ cm. $R = 5.03$ cm.

The ends of these needlepoint ruptures then become points of very high stress and therefore cause rupture of the entire dielectric. Considerable evidence exists supporting the needlepoint theory but little is known as to the origin, nature, and effects of these needlepoint ruptures. Their general character would suggest a high frequency discharge and this suggestion seems to be borne out by further analysis.

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[11] H. S. Osborne, Loc. Cit., p. 1577.

Corona, in fact, is a high-frequency phenomenon. If a thick-walled glass tube is filled with mercury and connected to a high voltage source, there will be a corona discharge between the glass tube and any grounded objects near the tube. If the hand is brought near the tube, a strong corona discharge will take place between the hand and the glass tube, but no sensation of shock will be experienced. This shows that the corona current in this case consists of a great number of minute, high-frequency discharges. In making this statement, high-frequency does not necessarily mean oscillatory. In fact, it would be expected that the pulses are highly damped. To quote Steinmetz[12] on this point:

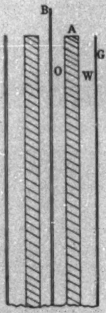

Fig. 7—A, Glass Tube; B, Brass Rod; O, Transformer Oil; W, Water Electrode; O, Glass Tube

> "The corona, my experience led me to believe, also is a disruptive effect and consists of innumerable minute streamers, the same as the pin points which perforate the solid dielectrics, the difference being merely that with the air those pin holes heal again, but they do not heal in the solid dielectric."

Much has been written on corona in oils and in solids, but there is still cause for considerable speculation as to the fundamental nature of the phenomenon. It probably is a high-frequency rupture also. An experimental arrangement designed to show the nature and effects of corona in oils is illustrated in Fig. 7. Voltage was applied between the brass rod and the water sheath. At about 20 kv., numerous discharges took place between the brass rod and the inside of the glass tube A. The apparatus, being transparent, afforded opportunity for visual observations. It was noticed that the discharges resembled ordinary spark discharges, except that near the interface between the oil and the glass the spark discharge, or corona, spread out over considerable area. That is, the spark is not strictly radial but seems to consist of streamers that spread throughout an appreciable volume of the oil. Although corona in oil thus consists of disruptive discharges, there is some question as to their needle-like character. In fact, it seems as if these discharges are propagated outward and get thinner and thinner as they go. A somewhat analogous case would be a lightning bolt discharging a cloud by numerous streamers extending throughout the volume of the cloud but uniting to form a single bolt to earth.

The mechanism of these corona discharges in oil is readily understood when we recall the physical picture of a kinetic equilibrium between the mobile ions and the molecules when the oil is under electric stress. If the oil is under high stress, near or beyond the breakdown point, there will be a comparatively great number of ions free to move, for the molecular bonds will be practically broken. Ions of one polarity will tend to move toward the outer electrode and those of the opposite polarity will move toward the inner electrode. The glass tube prevents any concentration of the ions that are moving toward the outer electrode but from the inner electrode sparks or corona discharges will radiate outward and discharge a finite volume of the ionized dielectric. Heterogeneity of the oil accounts for the concentration of the ion flow and the resulting spark discharge, rather than a more uniform corona glow.

Nevertheless, regardless of the nature of these discharges in the oil, considerable damage is done to the dielectrics. The oil is decomposed, gases being evolved quite rapidly, and the inner surface of the glass tube is badly cracked and chipped. Fig. 8 is an enlarged photograph of a glass tube after a few minutes' application of voltage. The various markings are chips and cracks along the surface of the inner bore of the tube. This tube did not puncture, although some of the cracks extend fully halfway through the wall of the tube.

Fig. 8

This chipping and cracking of the glass tube may be due to a rather high temperature locally, or it may be due to the high-frequency character of the discharges. Peaslee[13] has obtained somewhat similar effects in porcelain. He, however, superimposed high-frequency pulses on the 60-cycle voltage and therefore knew definitely the cause of the phenomena which he observed.

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[12] C. P. Steinmetz, Discussion, Journal of the A. I. E. E., Vol. 29, p. 1606, 1910.

Another very interesting set of experiments on cracking of dielectrics due to electric polarization has been performed by Schumacher.[14] In these experiments use was made of the ordinary two-electrode vacuum tube. In case a crack developed in the glass, the change of pressure inside the tube caused a great change in the ionization current through the tube. This gives an extremely sensitive method of determining when a crack develops. The lead-in wires to the filament and the plate were sealed in the base as nearly as possible one-eighth in. apart. A potential of 400 volts was impressed between the filament and plate leads and the polarity reversed every 15 seconds. The number of reversals that could be made before a crack developed was recorded for various kinds of glass.

Even with these low gradients, 3200 volts per in., cracks developed after a sufficient number of reversals of the voltage and Schumacher correlates the susceptibility to crack with the chemical composition of the glass. To quote his summary:

"A study has been made of the susceptibility to crack development shown by five different kinds of glass when these were subjected to the action of an electric current. The results indicate that the tendency to crack increases with increasing alkali content of the glass and with increasing electrical conductivity. These conclusions are shown to be in accord with previous observations made on the conductivity of glass."

This effect must be due entirely to the mechanical stresses and strains set up in the glass due to the reversing of the polarization of the dielectric. The currents through the glass at these low gradients are extremely small and the heating effects are certainly negligible. These experiments also indicate that in solid dielectrics the mobility of the molecules is very low and so we would not expect to find the phenomenon of corona.

Fig. 9

In order to check up on the corona effect in solids, glass thermometer tubes were used. Mercury formed the inner electrode, which was 0.2 mm. in diameter, and water was used for the outer electrode. The ratio of R/r was 33. This tube was known to puncture between 35 and 40 kv., so 34 kv. was applied and held for half an hour. During this interval nothing happened. There were no visual signs of stress or other effects to indicate that the tube was operating near the breakdown voltage. Due to the large ratio of R to r the inner layers must have been strained far beyond their breakdown point.

Upon increasing the voltage gradually during 30 seconds to 37 kv., the tube punctured. Examination showed that there were three distinct places where the tube punctured. Furthermore, throughout the length of the tube the inner wall was chipped and cracked. Figs. 9 and 10 are magnified photographs of parts of the tube. Fig. 9 shows two of the three breakdown points, one at division 26 and one at division 32. The third was at division 50. In Fig. 10 is shown a portion of the tube that was not punctured, but the chips and cracks along the inner bore of the tube are plainly seen. This effect is undoubtedly a high-frequency phenomenon. There is some question, however, as to whether the high-frequency surges are a consequence of breakdown or are the actual cause of breakdown.

Fig. 10

The high-frequency character of the phenomenon is suggested by the fact that several distinct ruptures were observed and also by the numerous chips and cracks along the inner bore of the tube. It is difficult to conceive of this chipping and cracking being due to a simple release of the stress but, on the other hand, a rapid alternation of stress would very likely result in cracking due to mechanical strains set up as a result of the rapid change in the polarization of the dielectric.

These experiments indicate that corona does not occur in solid dielectrics even when they are operated beyond the elastic limit. Thus the low mobility of the molecules and ions in the solid dielectric tends to prevent the formation of corona and likewise increases the stability of the insulation even when it is operated beyond the elastic limit. This extreme stability probably will not be found, even in solids, if there is any appreciable nonhomogeneity of the dielectric. For this reason, composite or laminated insulations are particularly subject to corona phenomena, *i. e.*, internal discharges, when the weaker dielectric is over-strained. The complexities introduced by composite insulation

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[13] W. D. A. Peaslee, *Insulation Failures Under Transient Voltages*, Journal of the A. I. E. E., Vol. 35, p. 1187, 1916.

[14] E. E. Schumacher, Crack Development in Glass Under Electrical Stress, *Journal* of the American Chemical Society, Vol. 46, No. 8, 1924.

are so numerous that they will not be discussed at this time.

The drooping of the Curves B′ and B″ in Fig. 4 is explained, therefore, by assuming that the inner layer of insulation could not have been homogeneous. In both of these cases very small conductors were used, less than one mm. in diameter. With these very small conductors, it was, no doubt, impossible to get uniform vulcanization of the rubber insulation or uniform solidification of any insulating material. The resulting heterogeneity of the dielectric would cause internal discharge within the inner layers and thus the stability of the dielectric as a whole would be decreased; that is, the puncture voltage would be lower than would be the case if there were no internal discharges.

In this connection, however, it is well to mention some work by Kennelly and Wiseman[15] where they found that for varnished cambric there was a diminution of 14.8 per cent in puncture voltage when an 18.1-sq. cm. electrode was used in place of a 1.13-sq. cm. electrode. However, if sixteen of the smaller electrodes were connected together so as to have the same total area as the larger electrode and if some resistance was inserted between the different small electrodes, the puncture voltage for the combination was the same as for a single small electrode. On the other hand, if the small electrodes were connected by a low resistance as by fastening them into a brass plate, the puncture voltage was practically the same as with a single large electrode. It was suggested at the time that this effect might be due to high-frequency surges, the resistance serving to damp them out and thus give a higher puncture voltage. It would seem that in this case the high-frequency surges took place just before breakdown and were the cause of breakdown taking place at a lower voltage than otherwise would be the case.

CONCLUSIONS

The mechanism of breakdown is not a simple phenomenon. The one fundamental concept is that in the dielectric there is a kinetic equilibrium between the mobile charges and the molecules. However, if there is any appreciable heating effect due to the conduction current or to dielectric losses, the equilibrium conditions will be changed. Therefore, in the complete analysis of the phenomena, the thermal effect must be considered. Then again, if the field is not uniform or if the dielectric is composite or heterogeneous, there is the possibility that part of the insulation will be overstrained and internal discharges are then likely to initiate high-frequency effects that disturb the stability of the dielectric as a whole. All of these three effects are undoubtedly present in every breakdown, but in many cases one or even two of them may be negligible. Experiments can be designed to show any one effect by reducing to a minimum the effects of the other two. It seems that in the past this tri-fold nature of the phenomenon has not been fully appreciated and has been the cause of much confusion. The term tri-fold may be misleading, for they are not three separate effects, but three manifestations of essentially the one phenomenon of kinetic equilibrium between the ions and the molecules of the dielectric.

The author is indebted to the members of the Electrical Engineering staff of the Harvard Engineering School, and especially to Professors H. E. Clifford and A. E. Kennelly for suggestions in the preparation of this paper. The work has been done at the Harvard Engineering School under the auspices of the Cable Research Committee, which is a sub-committee of the appropriate committees of the National Electric Light Association, the American Institute of Electrical Engineers, and the Association of Edison Illuminating Companies.

Appendix

In order to obtain the total voltage on a cable it is necessary to integrate the voltages across the successive layers of insulation. That is, the total voltage is the integral of the gradient, or

$$E = \int_r^R d\,E = \int_r^R e\,d\,r$$

where

E = total voltage on the cable

e = voltage gradient

r = distance from the center of the cable

By hypothesis, Curve B of Fig. 1 is taken as the volt-ampere characteristic of the insulation. That is, the gradient is given by

$$e = \frac{175\,i}{1 + 0.1\,i^2} \tag{1}$$

where i is the current density.

The total voltage on the cable is, therefore,

$$E = \int_r^R \frac{175\,i}{1 + 0.1\,i^2}\,d\,r$$

But $i = \dfrac{I}{2\,\pi\,r}$

where I is the current per unit length of cable. Therefore:

$$E = \int_r^R \frac{2\,\pi\,r\,(175\,I)}{4\,\pi^2\,r^2 + 0.1\,I^2}\,d\,r$$

or

$$E = \frac{175\,I}{4\,\pi}\log\left(\frac{0.1\,I^2 + 4\,\pi^2\,R^2}{0.1\,I^2 + 4\,\pi^2\,r^2}\right) \tag{2}$$

Equation (2) thus gives the voltage across the cable in terms of the current per unit length of cable and the inner and outer radii of the wall of insulation.

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[15] A. E. Kennelly and R. J. Wiseman, The Apparent Dielectric Strength of Varnished Cambric, *Electrical World*, Vol. 70, p. 1138, 1917.

Now, breakdown occurs when $d\,E/d\,I = O$, and

$$\frac{d\,E}{d\,I} = \frac{175}{4\,\pi}\log\left(\frac{0.1\,I^2 + 4\,\pi^2\,R^2}{0.1\,I^2 + 4\,\pi^2\,r^2}\right) - \frac{17.5\,I^2}{2\,\pi}$$

$$\left(\frac{(4\,\pi^2\,R^2 - 4\,\pi^2\,r^2)}{(0.1\,I^2 + 4\,\pi^2\,R^2)\;(0.1\,I^2 + 4\,\pi^2\,r^2)}\right)$$

Or for breakdown:

$$\log\left(\frac{0.1\,I^2 + 4\,\pi^2\,R^2}{0.1\,I^2 + 4\,\pi^2\,r^2}\right) =$$

$$\frac{0.8\,\pi^2\,I^2\,(R - r^2)}{(0.1\,I^2 + 4\,\pi^2\,R^2)\;(0.1\,I^2 + 4\,\pi^2\,r^2)} \qquad (3)$$

The gradient in the insulation is given by equation (**1**); in terms of I, the current per unit length of cable, this becomes

$$e = \frac{2\,\pi\,r\,(175\,I)}{4\,\pi^2\,r^2 + 0.1\,I^2} \qquad (4)$$

The above equations therefore characterize the cable completely and enable us to calculate voltages, gradients, and current densities in any part of the insulation.

Discussion

W. A. Del Mar: Mr. Hoover's main thesis is based on certain experimental curves obtained by K. W. Wagner, showing the relation between volts and amperes in various dielectrics. In a discussion of another paper which took place the same year, Mr. Peaslee[1] indicated that these curves could be interpreted in terms of volts and current density. Mr. Hoover makes a third step; namely, that these same curves may be interpreted in terms of potential gradient and current density, and he proceeds to build on that. It is disturbing to me that it took three men to take those three steps. I cannot help wondering how many other things we almost know, yet do not quite know, that will require two or three men to elucidate.

There are, however, two points about this conversion of the Wagner volt-ampere curve into the stress-strain curve which require further study. Wagner found in his experiments that the breakdown voltage is proportional to the thickness of insulation, and this either consciously or unconsciously has been assumed by Mr. Hoover when he converted voltage into voltage gradients. On the other hand, a good many experimenters have found that that relation, instead of being always linear, is generally a curve with a tendency to flatten toward the axis of thickness. Hence, there must be some limitation to the extent to which we can convert these volt-ampere characteristics into stress-strain characteristics.

A similar argument applies to the area. As Mr. Hoover points out, some experiments by Kennelly and Wiseman indicate that unless certain precautions are taken, the breakdown voltage per unit thickness is not the same for insulation of different areas. Yet in converting Wagner's amperes to amperes per square centimeter, it has necessarily been assumed that changing the square centimeters does not affect the stress or the breakdown voltage. That is another point that has been jumped over.

I do not wish to intimate that these things invalidate Mr. Hoover's results. I think, on the contrary, that it simply means that something has been rather jumped over in his presentation.

As Mr. Hoover said, a great many inventions are shown up by this presentation. The various types of graded cables which depend on the old logarithmic formula are all left "up in the air," so to speak. Those of us who have tried to make cables of these various types know that they do not show the increase in dielectric strength that is expected by their inventors. In fact, in my experience neither the inter-sheath type of cable nor the type that is graded with three or four different dielectrics shows any of the expected increase in dielectric strength but in the past we never quite understood the reason. Mr. Hoover has furnished the answer.

There is a weak point that Mr. Hoover has skipped over and that is that all of the experiments by Middleton, Dawes and Davis were made with alternating currents, whereas his own theory is worked out for direct currents, as the Wagner volt-ampere characteristics were obtained by experiments with direct voltages. That simply shows there is a gap which remains to be filled. It may indicate that the characteristic a-c. and d-c. curves are identical. Probably that is the explanation, but it is a point that needs to be bridged properly and not jumped over.

The question of high frequencies being the ultimate cause of failures under many conditions, is one which has been in the mind of a great many people for years, and Mr. Hoover cites the very interesting and seemingly conclusive experiments by Kennelly and Wiseman which favor this theory. Some experiments by Gewecke and Krukowski[2] have, however, thrown a little doubt on these conclusions and the situation needs further elucidation.

There is an interesting statement made by Steinmetz[3] that I think is of some interest in connection with Osborne's theory of needle points, and that may bear on Mr. Hoover's experiments with glass. He said, "A particle of higher specific capacity than the surrounding material will concentrate the line of electric force toward itself, creating points or edges of excessive flux density at the poles of the particle. The insulation would char at these points or edges and the shape of the product of chemical decomposition would tend toward the form of a conducting needle with excessive voltage gradients at its ends, gradually piercing the dielectric until final puncture occurs between the terminals. Thus, in laminated insulation consisting of very many layers, a foreign particle in one of the layers, though originally forming only an insignificant part of the total thickness of the dielectric, may gradually but cumulatively in course of time, pierce and destroy the insulation by its electrostatic cutting edges, the average voltage gradients within the dielectric being still very low compared with the testing dielectric strength of the material."

W. F. Davidson: The relation of the several factors determining the voltage distribution or stress within complex insulation is a point which we have neglected in many of our studies on insulation. The present paper indicates a method for determining the stress-strain distribution in a cable on the basis of information called to our attention by K. W. Wagner.

As Mr. Del Mar has brought out, the normal distributions are based on the assumption of continuous potentials, while we are faced in many of our problems with alternating potentials. In the case of alternating potentials, we have, in addition to the cases mentioned by Mr. Hoover, the influence of electrostatic capacity which is of predominant influence in many cases.

I wish that Dr. Whitehead would tell of some of the work that he has been doing at Johns Hopkins University in connection with the same general research activities that Mr. Hoover has been following. However, it may be worth while to indicate some of the results. Working with small cables made under carefully controlled laboratory conditions, he has been investigating specifically the relation between voltage and the power factor of the charging current in samples which have been impregnated with different quantities of air and more recently with quantities of moisture left in the paper. The results are still qualitative but seem to indicate that the way in which the air is distributed through the dielectric has a great influence on the power factor. If the air is in layers, then the power factor increases rapidly after the voltage reaches a certain critical value; if the air is diffused through the entire body, the influence is small. Moisture seems to have an effect something like that of air in layers, but to date he has been unable to isolate the two effects.

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[1] TRANS. A. I. E. E., 1922, p. 620.

[2] *Archiv. Elektrotechnik* 1914, Vol. 3, p. 63.

[3] TRANS. A. I. E. E., Vol. 42, p. 1035.

Because of such influencing factors, I feel that we can do little more with any theory in connection with cable and similar types of complex insulation than take it as a general guide.

Another point of special interest is the development of the importance of the inter-molecular in closely compacted materials. It has seemed to me that we might go a considerable way toward verifying or disproving the assumption if we take some material which could be studied in several different states. Possible carbon dioxide, which is easily obtained as gas, liquid, or solid, or sulphur, which might be obtained in several crystal forms as well as a liquid, might afford a suitable material for investigation.

R. J. Wiseman: Some think dielectric breakdown is distinctly a heat phenomenon resulting from the energy consumed in the dielectric. I have never had that feeling. I have always felt that the heating is an after effect and not a cause. I think Mr. Hoover is also beginning to bring that idea out.

If we assume, as Mr. Hoover has done, that a dielectric has a volt-ampere characteristic as shown in Fig. 1, we are taking a possible shape of volt-ampere curve and may be justified in doing it. That, however, is not a general curve and I believe applies only to the material with which Wagner has experimented. Each insulating material or dielectric will have its own kind of curve. This volt-ampere curve will differ for each dielectric as well as for every possible combination of insulating materials such as occur in rubber compounds and various combinations of oil and paper.

Mr. Hoover's Figs. 2, 3 and 4 are very interesting and afford us a fine chance to study the mechanism of breakdown as he views it as well as from our own points of view. It will be necessary to study Figs. 3 and 4 very carefully in order to appreciate their significance because they tend to tie in the close relationship of curves of breakdown obtained by formulas based entirely on voltage stress and those using voltage and current characteristics. They also bring out that breakdown is fundamentally a density action of ions or current or whatever you wish to call it. Instead of calling it strain, I think we ought to call it current density or ionic density. Strain is more of a mechanical consideration and not so much an electrical one.

Several years ago I did some work on the dielectric strength of cables. At that time I tried to explain why the points did not fall along the theoretical curve. I did not work along the lines that Mr. Hoover followed,—namely, current density,—but studied the variation in the maximum voltage stress at the conductor itself. I developed an empirical formula which is really only a correction of the logarithmic formula that we can develop theoretically. In this empirical formula I assumed that the dielectric stress at the conductor, instead of being a constant, was a variable with the size of the conductor itself. I believe it is quite probable that the same condition will be found if the volt-ampere characteristic of the material is obtained. It would be necessary to make tests with single-conductor cables. You cannot take the dielectric in plain sheet form and bring out the characteristic of material in cables unless you know the relationship between the two methods of testing.

The empirical formula, which I believe helps us to explain the departure from theory, is based on ionization. If we take K as the voltage stress at the conductor and r the radius of the conductor and assuming $K = K_0\left(1 + \frac{a}{\sqrt{r}}\right)$ by computing back to K_0 for various diameters of conductor, we shall obtain very closely a constant which I believe is the true dielectric strength of the material. Since obtaining the above formula I have improved it by dividing K as obtained by the theoretical formula by $\sqrt{R}$ where R is the radius of insulation. The voltage formula now becomes:

$$V = K_0\left(1 + \frac{a}{\sqrt{r}}\right) r \sqrt{R} \ln R/r \text{ volts,}$$

Whether the above is the same thing Mr. Hoover is describing, I do not know. I think a study of both is well worth while and intend to make it later myself as I am particularly interested in the subject.

On Fig. 6 we notice the drooping of curves B' and B''. B'' happens to be a curve I obtained. I should like Mr. Hoover to explain not the drooping so much as the departure from the theoretical law, because B'' was obtained with constant insulation radius R and varying conductor radius r. According to theory, we get a maximum voltage breakdown when $r/R = 0.434$. Curve B'' does reach a maximum and decreases, but all values are above the theoretical, based on a constant value of K. This is also true of curve B' obtained by Messrs. Middleton and Davis. I believe it is all a question of ionization. Had we the volt-ampere characteristic of the material I used, we should very likely get the same curve if we computed breakdown in the manner proposed by Mr. Hoover. I believe it adds emphasis to the fact that we must study each kind of dielectric before we are able to arrive at any conclusions.

Mr. Hoover believes that the shape of the curve for B' and B'' may be due to using extremely small conductors. In my own case the smallest conductor was about 1/16 in. in diameter and gave a ratio r/R about 0.2. I doubt if this is small enough to influence the results by possible mechanical considerations. As I recall it, I was able to get very good samples with small conductors.

Referring to the formula:

$$V = K_0\left(1 + \frac{a}{\sqrt{r}}\right) r \sqrt{R} \ln R/r,$$

it is true that a is a factor depending on the kind of dielectric. We can go even further and say that it may change from plus to minus. If a is zero, we get a straight line horizontal to the abscissa axis if we plot as ordinates $K/\sqrt{R}$ and $1/\sqrt{r}$ as abscissa. This is the condition of a perfectly pure homogeneous dielectric. As the dielectric departs from purity, the line begins to slope, crossing the ordinates positive at $1/\sqrt{r} = 0$ and giving a plus sign to a. As the material departs further from purity or homogeneity, a increases and finally takes on a negative sign. In rubber compounds we get large values of a and they may even go negative due to their distinct non-homogeneity. For oil and paper a may or may not go negative according to the nature of each, as well as the amount of air and moisture left in them.

R. W. Atkinson: Many observers have carefully dissected, layer by layer, cable insulation which previously has been subjected to voltage stresses, and obtained thereby very important first-hand knowledge of the phenomenon of breakdown of cable dielectric. Such dissections of insulation in various stages of incipient breakdown have furnished much information about the "mechanism" which results in final breakdown.

A characteristic type of failure now becoming widely known is that resulting from stresses in a three-conductor cable in the filler spaces; that is, outside of the bounds of the tape insulation of the individual conductors. The stresses at this point, while lower than at the surface of the conductor, are not perpendicular to the layers of insulation and result in relatively low strength and incipient failures at this point before there is serious stress adjacent to the conductor surface. It is much easier to obtain visual evidence of this action than in the case of failure of the laminated or taped-on insulation because the time from the inception of failure to the completion is so much greater that many more samples can be found where the process of failure has started but has not been carried to the point of destruction of the evidence. With this type of failure, the steps which take place are now revealed very clearly. Initially, minute discharges take place circumferentially around the insulation. The energy in these intial discharges is so minute that the destruction caused by them is exceedingly slow. It seems probable that the destruction is not directly a thermal one but rather a chemical one caused by the bombardment of the electrons, or perhaps by ultra-violet light. By a little stretching of the imagination, it can be considered that the bombardment results in local high temperatures, the spots of high temperature being practically molecular in dimensions. (On the other hand, although I believe that initially the temperature is not greatly raised in areas larger than those of substantially molecular dimensions, the evidence is insufficient to prove that the destruction is not caused by high temperature in regions of exceedingly small extent, yet of vastly greater dimensions than the molecular.) Undoubtedly the discharges in these spaces are of a high-frequency character and therefore the destruction occurs much more rapidly than would be the case at commercial frequency. When the local destruction has been carried to the point of making the insulation in that region partly conducting, the local area is extended circumferentially and the discharge gradually takes place between portions of the insulation spaces where there would be normally a considerable difference in potential. The magnitude of these potential differences has been investigated and reported by myself and others, and it has been shown to be a considerable fraction of the voltage applied to the cable. This process is carried on until it results in the development of an amount of energy which is sufficient and so localized that at the ends of these circumferential conducting paths, there are regions of high temperature that will gradually destroy the insulation. Indeed, we have then developed the "needle-point," which is discussed by Osborne and which produces ultimate failure in the way that he has described.

The evidence shows that in many cases failure in portions of the laminated insulation having only radial stresses normally may be caused in a somewhat similar manner; that is, the initial failure is small in extent and results in a gradual local injury, the magnitude of this injury increasing first in severity and then in the area affected.

I hope that it will be possible for Mr. Hoover to continue and extend his experiments with glass. In some ways this material lends itself exceedingly well to an investigation of this kind, inasmuch as it is possible to obtain a very wide range of ratio of the inner to the outer diameter of the cylindrical insulator and to obtain exceedingly high stresses without necessitating either large actual physical dimensions or high voltages. I hope Mr. Hoover will be able to make measurements with the same type of glass and with different wall thicknesses, at the same time taking measurements of the volt-ampere characteristics of the materials so as to make a direct determination as to whether or not there is a departure of the stress from that obtained by the simple mathematical expression and what is the extent of that departure. Mr Hoover says that in this experiment the chipping and cracking near the bore of the thermometer tube is undoubtedly a high-frequency phenomenon. Why may this not be something simpler? For instance, a thermal effect as a result of local heating, or perhaps mechanical failure due to mechanical stresses set up by the high electrical stress? Of course, if the glass has zero thermal coefficient of expansion, such as pyrex glass has, the probability of the failure resulting from thermal effect would be small. It will be of interest, however, to know Mr. Hoover's reasons for attributing this chipping and cracking to high-frequency phenomena, as such reasons may be entirely adequate for the conclusion given.

Mr. Hoover brings out in impressive manner the important affect on the stress distribution, of variation with voltage of the effective specific inductive capacity or the resistivity of the insulating material. Undoubtedly, in a number of cases, this is a factor of some importance. On the other hand, we have a large number of measurements of these constants of cable insulation up to stresses that will produce failure during application for a period of a few minutes to a few hours, and in these samples find no change in properties of importance in this connection. It is perfectly practical to make a full investigation of these properties for any given material and to verify or disprove the application of this theory for any particular kind of material.

P. L. Alger: I should like to call attention to the analogy that seems to exist between the theory of cable insulation stress and the theory of the stress in a gun. The old idea of a gun was that when the pressure inside increased, failure occurred simply by stretching the periphery. You could figure the whole problem out by plane or two-dimensional theory. But later it was found that when you apply stress in any direction to a solid, a stress occurs also at right angles, so the problem becomes one of three dimensions.

It appears to me, therefore, that a study of the three-dimensional theory of failure of cylinders might illuminate some of the problems of cable insulation.

W. A. Hillebrand: In considering the mechanism of the breakdown of a dielectric there are several possibilities to be taken into account. Free electrons may be pulled out of the material itself, atoms may be disrupted by the electrostatic field imposed or additional ions may be formed by collision with free electrons present in impurities in the dielectric up to the point where a conducting stream is established.

The first of these requires electric intensities considerably in excess of the rupture value of dielectrics, and in the second case, the electric stresses rise to values several hundred times greater, so that both of these possibilities would seem to be ruled out of consideration. If this is true, the problem of producing a satisfactory dielectric resolves itself into one of eliminating impurities, chiefly air and water.

E. W. Davis (communicated after adjournment): The dielectric strength of solid insulating material cannot be stated as a definite value but can only be given within comparatively wide limits. The reason for this can be summed up in one word, homogeneity, or rather lack of homogeneity. The non-homogeneity may be mechanical,—*viz.*, impurities or treatment; it may be thermal, chemical, or electrical, or combinations of any or all of these.

In cables, this inherent erraticism of dielectrics is further complicated by the nature of the electrostatic field. The uneven distribution of stress across the insulation tends to emphasize non-homogeneity still further.

It was recognized at an early date that, in a single-conductor cable, up to a definite point the addition of more insulation increased the dielectric strength of a cable; beyond this point, no apparent advantage was obtained by putting on more insulation; that is, the maximum stress as determined on thin insulation could not be applied to the calculation of voltage strength for thicker insulation.

In this present paper the author has attacked the single-conductor cable problem from the pyroelectric theory point of view, and has extended this theory to apply to the non-homogeneous field present.

There is no doubt that practically all breakdowns in dielectrics, at some point in the phenomena, are due to pyroelectric or local heating effects. In uniform fields the pyroelectric effect may be the only one present.

In irregular fields, however, the pyroelectric effect may be of a secondary nature and form the final step in the chain of events leading up to and ending at failure. The concentration of electrostatic flux beyond the ability of the dielectric to withstand would start, or cause to be started, abnormal heating effects of sufficient magnitude to begin the pyroelectric cycle and ultimately cause complete breakdown.

This brings us to the point considered by the author; that is, the "electrical elastic limit" or the critical point between stress and strain. It has been shown in the past that insulation can be "over-stressed" and permanently injured without causing actual breakdown; that is, the "elastic limit" has been exceeded but the cycle of events has been interrupted before actual breakdown has occurred. This would tend to point to the conclusion that the pyroelectric effect, in some cases, is a secondary phenomenon.

In a dielectric, the ratio of stress to strain is inversely proportional to the specific inductive capacity of the material and so long as this remains constant, the ratio of stress to strain also is constant. The elastic limit of a dielectric then is the point at which the direct proportionality of stress to strain ceases to hold. Above this point, if the stress is held constant and the strain increases, the dielectric constant must increase. This of course is based on a perfect dielectric in which no leakage currents or dielectric losses exist. All dielectrics, however, have conduction currents and losses under voltage which further complicate the problem by adding a heating effect. The increase of conduction current with stress is proportional to the stress up to the point where the heating and the negative coefficient of the dielectric cause an increase in conductivity and also where the increase of strain with stress becomes greater than the increase of stress. At such a point, failure will result. This means that the electric "elastic limit," the pyroelectric effect and homogeneity are so tied into the mechanism of breakdown that all must be considered. The dielectric strength of insulation then may not be a definite value but rather a broad band wherein the various controlling phenomena exert accumulative effects upon the dielectric.

P. L. Hoover: It has been pointed out that certain assumptions have been made in the conversion of Wagner's data from potentials to stresses. I have assumed that a slab of dielectric of thickness t may be divided into elementary slabs of thickness $d\,t$, and that the voltage across any two of these elementary slabs is twice the voltage across one, if the current density is uniform. That seems to be logical but, of course, what seems logical may not be true.

W. A. Del Mar: The point I tried to raise there is that there may be limitations to that assumption. They seem to be perfectly correct as far as you have gone, but you have made us question how far you can go without going to a limiting point.

P. L. Hoover: There may be limitations. When breakdown voltage is plotted against thickness, we do not get, ordinarily, a linear relation. On the other hand, Wagner did find breakdown voltage proportional to thickness. Therefore, from Wagner's data, the assumption is justified and I have simply gone ahead and made it. Deviations from the linear relation between breakdown voltage and thickness must be accounted for, then by non-uniform fields, non-homogeneity of the insulation, etc.

It was also brought out that the volt-ampere characteristic will be different for every material and that in making calculations for the a-c. breakdown the a-c. characteristic should be used. Such is indeed the case. Wagner measured the d-c. characteristics of various materials and obtained a different curve for each material. All of the volt-ampere characteristics however, have the same general shape and there is no essential qualitative difference between them. Equation (1) will be different therefore for each material and the other equations will be changed correspondingly. Assuming that the theories advanced are correct, the contribution of this paper is not from data given on cables with insulation of a characteristic set forth by equation (1), but a method for calculating the breakdown voltage and the gradient in any cable with any insulation from a fundamental and characteristic equation of that particular insulation. The data given are to be regarded as illustrative of the method and relatively unimportant as compared to the significance of the method itself.

Mr. Wiseman has stated that according to theory the maximum breakdown voltage is expected when r/R equals 0.434. It must be remembered that the theory to which he refers is that which accepts the logarithmic formula for the gradient in a cable. I have attempted to show that the logarithmic formula does not hold at high stresses and therefore the ratio $r/R = 0.434$ has no significance.

Mr. Hillebrand stated that the value of electric intensity required to pull an electron out of an atom or molecule is considerably in excess of the rupture value of dielectrics, and therefore rules out this phenomenon from consideration when discussing dielectrics. I cannot agree with that. If we are considering a single atom in space, or any number of atoms, providing they are far apart as compared to their actual size, then it is true that the field required to pull out an electron is considerably in excess of the rupture value of dielectrics. In solids or liquids, however, we do not have isolated molecules. The intermolecular fields are quite large and of the same order of magnitude as the intramolecular fields. For this reason an electron may be removed from a molecule, passed on to the next molecule, and so on through the dielectric with an external field considerably weaker than that which would be required to extract an electron from the same molecule if it were isolated in space.

In regard to the effects of transients on breakdown, much can be said. The experiments of Prof. Kennelly and Mr. Wiseman, as cited in the paper, seem to show that small oscillations occur just before breakdown and cause breakdown to take place at a somewhat lower voltage than otherwise would be the case. However, as Mr. Del Mar stated, these data should be examined critically. In comparing breakdown values for large and small areas the question of the probability of finding weak spots should be considered. It seems to me that in the experiments of Kennelly and Wiseman the question of probability has been ruled out. They first measured the puncture voltage of some material using an electrode area of about 1 sq. cm. Then an electrode of 16 times this area gave a diminution of about 15 per cent in the puncture voltage. This same diminution was found regardless of whether the electrode was a single large plate or a group of 16 small electrodes connected together by a low resistance lead. This agrees with the predictions of the probability theory and this general effect of decreasing puncture voltage with increasing electrode area is often considered a strictly probability phenomenon. However, the probability theory received a serious jolt when Kennelly and Wiseman connected the 16 small electrodes together through a small resistance,—an ohm or so. When this small resistance was inserted between each electrode, there was no diminution in puncture voltage with increasing area. This is not in accord with the probability theory and an explanation must be found on some other basis; at least for these particular data.

To say that the probability theory is of no use in explaining phenomena of this kind would be going too far. If the material being tested was not very homogeneous, the probability factor undoubtedly would be large. The magnitude of the probability factor is therefore a variable and the importance of this factor in any particular set of data will have to depend on personal opinion unless conclusive experiments are performed to decide the case.

The peculiar effects that were found in puncturing the thermometer tubes have been ascribed to high-frequency phenomena. That such must be the case seems to follow from the fact that the effects observed are similar to those known to result from high-frequency pulses. The effects of heating and mechanical strains also seem to be ruled out by the fact that the tube was held very near the puncture voltage for half an hour. This gave ample time for heating and mechanical effects to show up but none was observed. As stated in the paper there is no evidence in this case to show whether the oscillations or pulses were set up just before breakdown or whether they were the result of breakdown.

It is easy to see how a small oscillation might cause dynamic rupture of the whole insulation. If we suppose that the particular cable or insulation is operating at a point on the positive slope but near the maximum of the volt-ampere characteristic, a small impulse superimposed on the existing stress might very easily carry the insulation over to a point on the negative slope of the curve. We would then have a state of unstable equilibrium, and dynamic rupture would follow.

It is for this reason that oils are particularly bad as a constituent of composite insulation. If the insulation as a whole is operating near the rupture point, the oil will probably be overstrained and small discharges will take place within the oil. Aside from any deterioration of the insulation due to these internal discharges, the stability of the insulation as a whole will be so disturbed that the operating voltage will be reduced materially.

Mr. Atkinson has discussed at length the effects of internal discharges in cables and impregnated paper insulation. Impregnated paper is not a homogeneous insulation, and as a result, the field is not strictly radial but has longitudinal and circumferential components. Furthermore, since the dielectric strength is less in a longitudinal or circumferential direction, due to the fact that the paper offers little or no barrier action to the motion of ions, it seems very likely, as is the case, that internal discharges will take place in a longitudinal or circumferential direction. These discharges are the case of "tree designs." They undoubtedly deteriorate the insulation and may lead to breakdown as described by Mr. Atkinson. I do not agree with Mr. Atkinson however, when he states that these discharges are "needle-points."

In order to have a discharge from one point to another, it is necessary to have a considerable quantity of charge at those two points. It seems unlikely that such an accumulation of charge exists within a cable insulation. The condition that more likely exists is that a certain region discharges into another region of lower potential. The flow from one region to the other may be quite concentrated and result in a charred path, but these discharges cannot end at a point. They must spread out and discharge a finite volume of dielectric. We cannot picture therefore, the needle-point as piercing it way through the insulation due to the high flux concentration at a point, for there is no point. Visual observations on discharges in oil support this view and so it seems that the needle-point concept must be modified. On the other hand, these internal discharges do lead to progressive deterioration due to the fact that the charred path becomes a conducting filament that connects the two regions. In this manner the low-potential regions will progress inward toward the conductor and the high-potential regions will progress outward toward the sheath. Dynamic rupture of the cable is the final result.

As regards the communication of Mr. Davis, I wish to state that I have not taken the pyroelectric point of view. I do believe that there are many cases wherein the heating effects are directly responsible for breakdown. On the other hand, I agree with Mr. Davis that there are many cases also in which the heating effects are of a secondary nature, if not entirely negligible.

Since the publication of this paper, it has been called to my attention that Fig. 1 does not check the original data of Wagner. This is due to a misprint in the source of my material which resulted in my taking an erroneous value for the area of the electrode. This means that equation (1) does not give the volt-ampere characteristic of the oiled paper that Wagner used. Equation (1), however, gives a curve that resembles the curves which Wagner actually did get and the conclusions arrived at in the paper are by no means invalidated. As stated before, it is the method and the qualitative nature of the conclusions that are most important.

Mercury Arc Rectifiers

BY D. C. PRINCE[1]
Fellow, A. I. E. E.

Synopsis.—*Mercury rectifiers have been known for about twenty-four years, but until the last few years their principles of operation have not been understood with any certainty, and all are not yet conclusively proved. In the first part of this paper, the probable mechanism of the electron source or cathode spot is outlined, the source of the various losses is indicated, and the probable mechanism of arc back, that is, failure to rectify, is described.*

The second part of the paper is devoted to the principles of simple rectifier circuits, while the third shows a variety of rectifiers of different kinds and sizes.

THE mercury arc rectifier has been known for about twenty-four years, but, in spite of the length of time which has elapsed, there are a great many things about it which are not understood with any degree of certainty. A good deal has, however, been found out, especially in the last few years, and since interest in rectifiers is growing, it may be worth while to go over some of the principal features so as to get a general idea of the rectifier and its problems.

Fig. 1 represents a standard, 50-ampere, 100-volt mercury arc rectifier. This form is shown for purposes of explanation on account of its simplicity. It consists of an evacuated glass bulb containing, in this case, two main anodes composed of graphite located in the anode arms projecting at either side of the bulb; two auxiliary anodes also made of graphite lower down and on the front of the bulb; a starting anode made of mercury at the lower right; and a mercury cathode pool at the bottom.

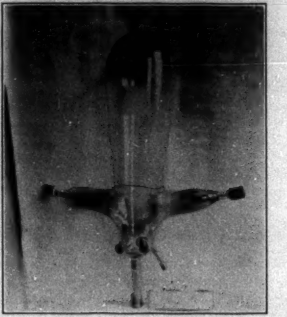

FIG. 1—50-AMPERE LOW-VOLTAGE RECTIFIER

The principle of operation in its barest essentials is similar to that of the vacuum tubes used in radio reception, with which most people are now familiar. In a vacuum tube a filament is heated until the electrons in it have so much energy that they are able to break through the metallic boundary into the surrounding space. If the anode (plate) is positive, these electrons are attracted toward it and current flows. If the anode is negative with respect to the filament, the electrons will be drawn back upon the filament and no current can flow. Rectifier or check-valve action is thus established, since current can be carried in one direction only. In the high-vacuum rectifier the electrons filling the space between anode and filament produce a charge known as the "space charge" which tends to drive the electrons back. This causes a loss which becomes very high if it is attempted to draw considerable current.

In the mercury rectifier in place of the filament we have a small, bright, dancing spot called the "cathode spot" which is the source of the electrons. Electrons are drawn from this spot to the anodes when they are positive just as in the high vacuum tube, but when the anodes are negative they do not constitute a source of electrons, so that no current can flow and rectification is obtained. There is some difference of opinion as to what occurs in this spot. The electrons proceeding from it strike neutral molecules of mercury vapor and ionize them; that is, one electron is removed from the molecule and the remainder of the molecule then has a positive charge. These positively charged molecules, called positive ions, are attracted toward the cathode, and, since they are quite heavy, their striking represents considerable energy and the spot where they strike is heated. There is thus some action analogous to the heating of a filament. In addition to this heating, the positive ions, being heavy, move slowly, and a large accumulation of them near the mercury surface produces a high potential gradient which tends to draw electrons from the mercury surface at a temperature lower than that at which they would be able to leave it if there were no such strongly attractive force. The electrons drawn out ionize new molecules which heat the surface of the mercury and draw electrons from it, so that the process once started is self-continuing provided the potential necessary to make the current flow is present. To start the spot in the first place, an arc is drawn by tilting the tube so that the mercury in the cathode makes contact with the mercury in the starting anode. The potential is impressed on the starting anode through a small resistance. When the tube is returned to the vertical position, this circuit is broken and the resulting arc initiates the cathode spot.

Measurements have been made of the current density in the cathode spot, and it is believed to be of the order

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of 26,000 amperes per sq. in. The heat due to bombardment of the positive ions evaporates a great deal of mercury. From this and other causes a pressure is built up at the cathode spot of the order of two atmospheres, although the vessel as a whole has been evacuated as perfectly as possible. The large glass bulb operates as a condenser to condense this jet of evaporated mercury. The high-pressure vapor is thus projected into a condenser where the pressure is of the order of 50 microns, that is, 5/100 of a millimeter.

The temperature of the cathode spot has not been definitely established. Some authorities give it as 2000 deg. cent., which would be the temperature required to produce the electron stream by heat alone. The actual temperature is probably much lower, although the amount has not been established. The potential drop required to maintain the cathode spot is approximately 10 volts. Approximately half of this 10 volts is consumed in latent heat of ionization of the mercury (work function 4.4 v.) and in the energy of the individual electrons as they escape from the pool. The remaining five volts appear as heat at the surface of the mercury pool, about half being used in evaporating mercury while the other half is conducted away through the liquid mercury in the pool. When the electrons combine with molecules, either on the walls of the vessel or at the anodes, the five volts represented by their energy are returned also as heat. Such electrons as combine with positive ions to form neutral molecules in the space give up some of their energy in the form of the greenish light so characteristic of the mercury arc rectifier.

About four or five amperes are required to maintain a stable cathode spot. For higher values of current the area of this spot increases, and above 40 amperes more than one spot may exist simultaneously. The spot, or spots, always moves very rapidly from place to place due to the strong blast of mercury vapor and impinging positive ions. In order to make a rectifier operate down to zero load current, the auxiliary anodes are provided which draw a current of approximately five amperes so that the cathode spot is maintained in readiness even though the load is disconnected. These auxiliary anodes were not provided in some of the earlier rectifiers, but their provision facilitates many things and may be considered standard in future rectifiers, both large and small.

The 10-volt cathode drop does not make up the entire loss. In the high-vacuum tube a drop is produced due to the presence of the electrons in the space. In the mercury arc this drop is eliminated by the presence of the positive ions. Whenever a drop tends to exist at any point, the electrons are accelerated until they are able to ionize neutral molecules. The resulting positive ions are of the opposite sign to the electrons so that the net space charge is reduced to approximately zero but does not disappear completely for some drop is necessary to replace ions lost by recombination and collection by the walls.

Fig. 2 shows the form of arc drop curve for a small rectifier. The two lower curves apply to a tube of the same general form as that shown. That is, it has short, straight arms. The lowest curve gives the arc drop characteristic of the bulb cooled only by natural air circulation. To determine whether the anode area is important, the current has been collected by one anode, or by two, and it is observed that the total drop is almost exactly the same in both cases. The characteristic drop reduces to a minimum at 10 amperes and then rises. Below 10 amperes, therefore, it has the nature of a negative resistance. In order to operate a mercury rectifier, it is, therefore, necessary to include an impedance sufficient to make the average impedance positive; otherwise the tendency would be for the current to rise indefinitely or fall until the arc was extinguished. The rise in the drop beyond 10 amperes is due to excessive vapor pressure produced by heat. The next higher curve which has its minimum at about 28 amperes was obtained from the same tube merely by cooling the condensing bulb with a 12-in. desk fan. A still greater increase can be maintained by more positive cooling methods such as oil or water. Beyond the minimum points, although the arc drop rises with increase in current, the response to instantaneous variations in current is still that of a negative resistance, so that even though rectifiers are operated beyond the point of minimum drop, two anodes still cannot be paralleled without some impedance to force a division of current.

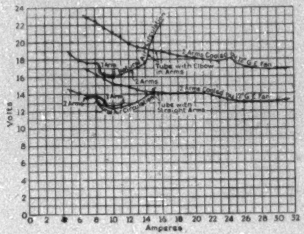

FIG. 2—ARC DROP CURVES OF 10-AMPERE TUBES TAKEN WITH DIRECT CURRENT

The two higher curves on this figure are corresponding curves for a tube of the same size but having an elbow in the anode arm. Such a tube will be shown in a later figure. It appears that increasing the length of the path adds a nearly constant voltage drop. Since the drop in the rectifier is a function of current only, the capacity and efficiency are greater the higher the voltage used.

There is naturally a limit to the voltage that can be applied, and Fig. 3 shows the nature of this limit.

This figure shows what is known as the "arc back characteristic" of a 20-ampere glass rectifier with natural air circulation. It appears that for any current, there is a maximum voltage which the tube will rectify. Above this voltage one of the anodes becomes a cathode, so that the unidirectional conductivity of the device is lost.

Like the point of minimum arc drop, the current at which such a failure or arc back occurs can be increased by cooling. The probable mechanism is substantially as follows. When the anode is negative, some of the positive ions will be attracted toward it and will strike it with considerable velocity. Under normal conditions very few of these collisions will produce electrons from the anode. Some few will be drawn out, however, and these will proceed toward the cathode, ionizing further molecules as they go. The additional positive ions produced in the neighborhood of the anode will return toward it, and they also will produce a certain small number of electrons. The higher the negative voltage on the anode, the more of these electrons will be produced. The higher the vapor pressure, the more collisions each electron will make and the more positive ions they will produce. Thus a point is reached where, by increasing vapor pressure with temperature or by increasing voltage, so many electrons will be drawn from the anode that it will become a cathode and rectification ceases.

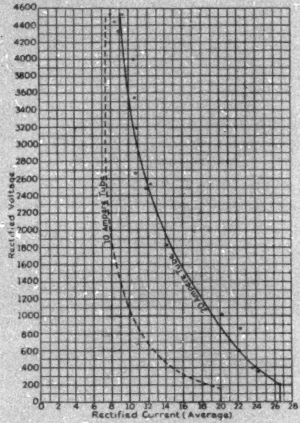

FIG. 3—ARC BACK CURVE FOR 20-AMPERE TUBES WITH BENT ARMS

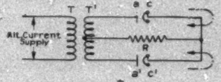

FIG. 4—IDEAL SINGLE-PHASE, FULL-WAVE RECTIFIER CIRCUIT

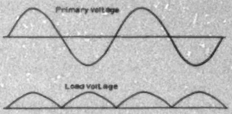

FIG. 5—WAVE SHAPES OBTAINED WITH CIRCUIT OF FIG. 4

This covers in a general way the principal phenomena of the rectifier. Under normal conditions it gives an almost perfect unidirectional action with a relatively small drop, so that in circuit determinations we may neglect all but the unidirectional effects.

Fig. 4 shows the most elementary rectifier circuit. Power is supplied from any alternating-current source to transformer primary *T*. The secondary *T'* is connected at its terminals to anodes *a* and *a'*. In this case high-vacuum rectifiers are indicated. The cathodes *c* and *c'* are connected together and to the midpoint of transformer secondary *T'* through the load *R*.

Fig. 5 shows the primary voltage and voltage and current across load resistance *R*. During the half-cycle that anode *a* is positive, electrons flow from *c* to *a*; that is, current in the usual sense flows from *a* to *c* and returns through *R* in the direction shown by the arrow.

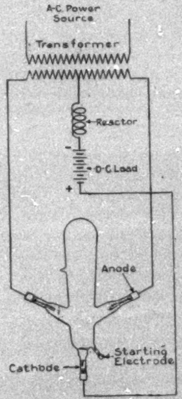

FIG. 6—MERCURY ARC RECTIFIER CIRCUIT WITH SMOOTHING INDUCTANCE IN OUTPUT CIRCUIT

During the other half-cycle, *a'* is positive and current flows from *a'* to *c'* and returns through *R* in the direction shown by the arrow, that is, during both of these half-cycles, current has flowed in the same direction through R and rectification has been obtained. Since there is no other form of impedance present, the current through R and the voltage drop across R are both sinusoidal in form, but, due to the rectifier action, both half-waves are in the same direction. There would be no object in supplying rectified current to a resistance, so these figures are primarily interesting to show the rectifier action in its simplest terms.

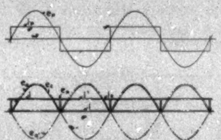

FIG. 7—PRIMARY AND SECONDARY VOLTAGES AND CURRENTS OF THE RECTIFIER SHOWN IN FIG. 6

To remove the current pulsations, an inductance may be connected in series with the load. Fig. 6 shows a circuit similar to that of Fig. 4, but including such an inductance and also including a battery as load in place of the resistance. A mercury rectifier is shown instead of the high-vacuum, hot-cathode rectifier. The inductance prevents the current from falling to zero, so that the cathode spot is maintained. A mercury rectifier could not be used in the circuit shown in Fig. 4 because it allows the cathode current to fall to zero.

If this smoothing reactor has a large value of inductance so that the rectified current is held constant, the various waves have the form shown in Fig. 7. The sinusoidal primary voltage is impressed through the transformer on the two anodes a and a'. The cathode will assume a potential just enough less than the most positive anode to cause current to flow. The cathode potential is shown as trace e_c. This cathode potential is impressed upon the load circuit consisting of the storage battery and the reactor. Since the reactor maintains the current constant, there can be no variation in voltage across the battery. The entire voltage variation appears, therefore, across the reactor. The steady rectified current is shown at i_R. This current flows from whichever anode is positive for the moment. i and i' are, therefore, the respective anode currents. Neglecting transformer exciting currents, the transformer primary current must be of the same shape as the secondary. i_p is, therefore, the form of primary current obtained.

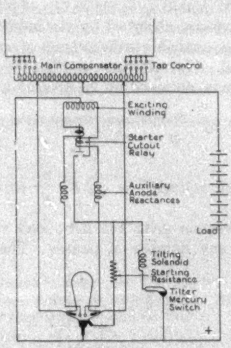

FIG. 8—WIRING DIAGRAM OF BATTERY-CHARGING RECTIFIER

For storage battery charging, it is not necessary to employ a steady direct current. If the mercury rectifier is equipped with auxiliary holding anodes supplied from a separate source, a battery charging equipment can be arranged as shown in Fig. 8. With this arrangement a small current of approximately five amperes is maintained by an auxiliary transformer winding. The battery to be charged is then connected between the midpoint of the main compensator winding and the rectifier cathode. Current will flow to charge the battery only while the anode voltages are higher than the battery counter-electromotive force. Between times, the cathode spot is maintained by the auxiliary winding through auxiliary anode reactors which serve both to maintain the current at a steady value and to limit that value. This diagram also shows the arrangements for automatic starting. A relay which is normally closed is connected with its holding coils in the two auxiliary anode leads. When voltage is thrown on the rectifier, current flows through the contacts of this relay to a tilting solenoid and mercury switch which set up a rocking motion in the tube. A circuit is also made to the starting anode through a resistance. As soon as the cathode spot has been formed, the relay opens the circuit to the starting mechanism, and the rectifier is then ready for business. It is kept in an active condition by the auxiliary anodes whether the load is connected or not a very desirable state where an unknown battery which may be badly sulphated or even open-circuited is to be charged.

FIG. 9—MERCURY ARC RECTIFIER SET FOR CHARGING BATTERIES OF INDUSTRIAL TRUCKS

Fig. 9 is a photograph of one of these rectifiers.

Fig. 10 is a diagram of a three-phase rectifier. The transformer windings T_1, T_2, and T_3 are connected in delta and supplied from a three-phase, alternating-current source. The transformer secondary windings t_1, t_2, and t_3 are connected so as to form a Y. The outer ends of the Y are connected to anodes a_1, a_2, and a_3. The cathodes are connected through an inductance L, and the load R to the Y point of the transformer. Hot cathode rectifiers are indicated although no change would be involved in inserting a mercury rectifier.

Fig. 11 shows the wave forms obtained by this arrangement. The three-phase or anode voltages are shown as e_1, e_2, and e_3. The cathode follows these in succession, always assuming a potential sufficiently below the more positive anode to cause the current to flow. The total current is maintained constant by the inductance L so that the drop across the load is the average cathode potential and is shown at E. Since the total current is constant as shown at I and current flows always to the more positive anode, the three anode currents are as shown in i_1'', i_2'', and i_3'', each one having full value of the rectified current for ⅓ of a cycle. Since the transformer will not transform direct current, the transformer primary will carry only the alternating component of these waves. These components are indicated as i_1', i_2', and i_3'. Combining these currents in pairs at the corners of the delta gives the three line currents i_1, i_2, and i_3. It is observed that both line and delta currents are not symmetrical. This indicates that with this connection there are even harmonics of current present. Even harmonics have an opposite phase rotation from the normal and so are still more objectionable than odd harmonics. Two three-phase rectifiers may be connected as shown in Fig. 12, however, to form a six-phase rectifier. With this arrangement, instead of providing two inductances L, a single compensator or "interphase transformer" can be used provided the two three-phase rectifiers are connected 180 deg. out of phase as shown. The resulting six-phase rectifier is the one which is usually used for power installations.

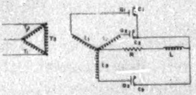

Fig. 10—Circuit Diagram of Three-Phase Rectifier

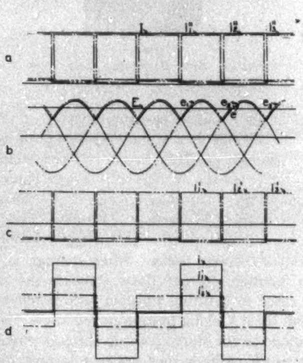

Fig. 11—Primary and Secondary Voltages and Currents of the Rectifier Shown in Fig. 10

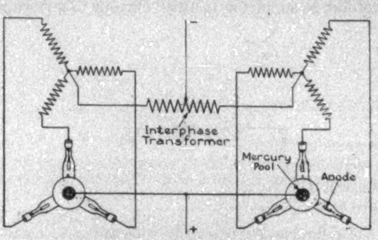

Fig. 12—Double Three-Phase Rectifier

The foregoing wave diagrams are made on the assumption that there is no inductance in the alternating circuits. On this same assumption a mercury rectifier has a perfectly definite voltage ratio. If we have a rectifier of p phases, the cathode will follow the anode of one phase for the fraction of a cycle $\frac{2\pi}{p}$. Since the pulsations are absorbed by the inductance L, the direct voltage available is the average of a sine wave for this interval. By referring to Fig. 13, the average voltage may be obtained by integrating a cosine wave from $\frac{-\pi}{p}$ to $\frac{+\pi}{p}$. This gives a value for the direct-current voltage $G = \sqrt{2}\, E \frac{p}{\pi} \sin \frac{\pi}{p}$. This general expression gives the light load voltage for a rectifier of any number of phases. The rectifier in Fig. 12 was made up of two three-phase units and therefore has an output voltage corresponding to three phases.

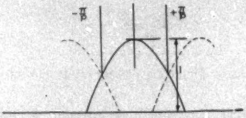

Fig. 13

The perfectly square current waves shown in the current diagrams could only exist in an inductanceless circuit, since otherwise it would require infinite voltage to produce an instantaneous change of current. Actually, therefore, the current does not change instantaneously. Instead, when the point of the cycle is reached at which two anodes have the same potential, the current begins to shift from one to the other. Since this shift is brought about by a sinusoidal voltage difference between the two anodes, the changing component of current will be sinusoidal, and, if the primary impedance is nearly all inductive, as it usually is, the current will start to change just as though an alternating voltage were suddenly impressed on a circuit carrying a direct current. The condition is shown in Fig. 14. A steady current is maintained at a value I by the inductance L up to the time t_0, at which time there is no difference of potential between the anodes a and a'. At this point the current will begin to change as though it were going to follow the dotted curve i. This curve is exactly the curve that would be obtained in impressing a voltage on a circuit of inductance and small resistance; that is, it has an axis u which has the decrement factor $\epsilon^{\frac{-rt}{L}}$. The alternating component has a constant amplitude. Of course, this current cannot continue to flow, as by the time t', the current to the anode a has been reduced to zero, and it is prevented from reversing by the unidirectional conductivity of the rectifier tube. We are therefore interested in only a part of this transient curve which takes place between times t_0 and t'.

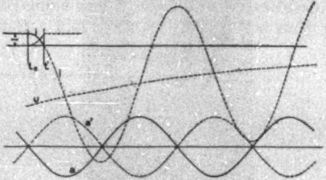

FIG. 14—GRAPHICAL REPRESENTATION OF CONDITION ARISING FROM THE SUPPOSITION THAT THE TRANSFORMER SHOWN IN FIG. 6 HAS LEAKAGE REACTANCE

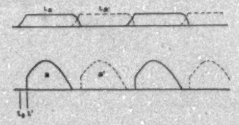

FIG. 15—OUTPUT CURRENT AND VOLTAGE WAVES, SHOWING EFFECT OF 26-DEG. PERIOD OF SHORT CIRCUIT DURING WHICH CURRENTS ARE TRANSFERRED AND NO VOLTAGE IS PRODUCED BY THE RECTIFIER

During the interval t_0 and t' during which current flows from two anodes, those two anodes must be at the same potential. The transformer secondary is thus in effect short-circuited for this interval, and there is no output voltage. This relation is shown in Fig. 15. i_a and i_a' represent the anode currents. a and a' represent the corresponding anode voltages. The cathode now follows these lines, and it, therefore, has an

FIG. 16—10-AMPERE, 300-VOLT GLASS RECTIFIER

FIG. 17—THREE-PHASE, 20-AMPERE GLASS RECTIFIER

FIG. 18—50/100-AMPERE, THREE-PHASE RECTIFIER

average value somewhat less than in the case where no inductance is present.

This is the mechanism which acts for rectifier voltage regulation. Elaborate equations have been worked out to give the amount of this regulation, but they will not be dealt with at this time.

Leaving now the rectifier circuits, we shall review some of the forms of rectifiers which have been made.

FIG. 19—250-AMPERE GLASS RECTIFIER

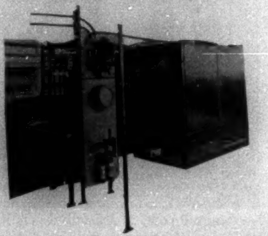
FIG. 20—100-KW. RECTIFIER USING TWO 250-AMPERE TUBES

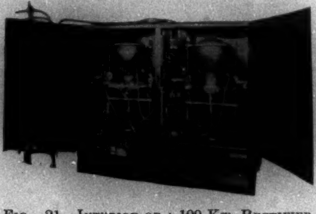
FIG. 21—INTERIOR OF A 100-KW. RECTIFIER

Fig. 16 shows the smallest mercury rectifier now made by the General Electric Company. This rectifier has a rating of 10 amperes and 300 volts. It has an overall height of approximately 8 in.

Fig. 17 shows a three-phase, 20-ampere glass rectifier.

Fig. 18 shows a 50/100-ampere, three-phase rectifier. This rectifier is approximately 30 in. high and is the largest glass rectifier made by the company before 1925. The two ratings given are one for natural air cooling and the other for fan cooling. The current ratings apply to approximately 220 volts and must be reduced somewhat if higher voltages are used. It is quite feasible to use voltages as high as 3000 for these tubes.

FIG. 22—EARLY IRON RECTIFIER

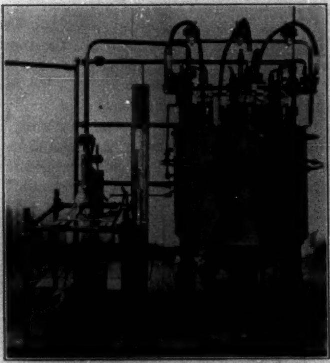
FIG. 23—MODERN IRON RECTIFIER

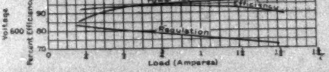

FIG. 24—CHARACTERISTIC CURVES OF AN IRON RECTIFIER

Fig. 19 shows the new 250-ampere glass rectifier developed during the last year. The normal rating is 250 amperes, 250 volts fan cooling. One of this type was operated in oil at 200 amperes, 3000 volts and withstood repeated short circuits opened by a high-speed breaker.

Fig. 20 shows a 100-kw. rectifier now supplying a shop circuit in the Schenectady works. This rectifier employs two of the 250-ampere glass tubes. The interior of it is shown in Fig. 21. From the earliest days, attempts were made to make rectifiers of iron in large sizes. Fig. 22 shows one of these early rectifiers which was modeled quite closely in form after the glass rectifier. As iron is not subject to the same limitations as to form, later rectifiers have been made similar to Fig. 23, which is a photograph of the rectifier shown at the Atlantic City convention during the fall of 1925.

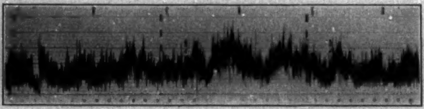

Fig. 25—Typical Street Railway Load

Fig. 24 shows the characterisitc performance of such a rectifier operating a street railway load. The high efficiency under light loads is particularly significant as the load factor of street railways is notoriously poor.

Fig. 25 shows the kind of load factor met in this service.

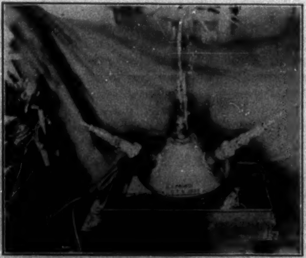

Fig. 26—Small Iron Rectifier with All Joints Welded

One objection to the iron type of rectifier as usually constructed is that some leakage always occurs with bolted joints, although the amount may be very small. For this reason, it is necessary to have a vacuum pump attached to the rectifier, although it need not be operated at all times. Attempts have been made to avoid the need for a vacuum pump by the use of seals fused directly to the metal parts. The rectifier shown in Fig. 23 has no clamped joints between metal and insulation, and, by the use of well fitted metal-to-metal joints, the danger of leakage is greatly reduced. It is possible to go still further and make rectifiers in which the sealing is complete. Fig. 26 is a small one of this kind which showed no indications of impaired vacuum three months after being evacuated and sealed.

Discussion

A. Odermatt: While it is true that there are still some phenomena in connection with the mercury arc rectifier which we don't quite understand, it is also true that we know enough about the process taking place inside the rectifier to design perfectly reliable converters of both the glass-bulb and the iron-clad types.

Statistics covering about 800 rectifiers in service, with a total output of over 400,000 kw. supplied by the European Brown Boveri Company, show that the percentage of failures with rectifiers is smaller than with most of the other classes of electrical machinery such as synchronous converters, generators and motors. This refers to big rectifiers in iron casings but probably also holds good for the glass-bulb type. With the latter, of course, the risk of breakage should be taken into account, but on the other hand, it is easy to have available spare tubes for replacement.

I mention this in order to emphasize that the mercury arc rectifier, after having gone through a period of development of over twenty years, is today a thoroughly reliable apparatus, and its many advantages at higher d-c. voltages especially give the static mercury vapor converter a marked superiority over the rotating type.

The Brown Boveri Company is still using the condensing chamber, which has given excellent results in preventing the mercury from being extracted by the air pump. This rather long chamber of comparatively small diameter acts indeed as a kind of safety device in that direction, the piping through which the air and gases are exhausted being connected to the top of the condensing chamber.

It has been mentioned previously that the operating engineers in Chicago do not feel very enthusiastic about the results obtained with the rectifiers there. So far as the Brown Boveri rectifiers installed by the Commonwealth Edison Co. are concerned, I should like to make the following statement:

Two rectifier substations were installed there in the summer of 1925, both of which have given excellent results until the spring of 1926, when there was some trouble with one of them. The other one has given entire satisfaction all the time. The trouble which occurred was due to an external cause and had nothing to do with the proper working of the rectifier. It was water entering the working chamber, as a result of corrosion at the welding seam, that caused the breakdown; and not only rectifiers, but most electrical apparatus would stop working satisfactorily under such conditions.

Although it is easy to prevent the repetition of a breakdown of that kind, it is most unfortunate that this should have happened in one of the few plants installed in this country, because it is likely to put the reliability of the rectifier in a wrong light. I can give assurance, however, that rectifiers installed in great numbers—I mentioned the number of 800 before—have given such excellent results that the percentage of failures is actually smaller than with other electrical equipment. The 800 rectifiers of which I am speaking are spread over the five continents of the world and the fact that over 50 of them are operating very satisfactorily, as far away as Australia and Japan, is the best evidence of the great reliability of this apparatus.

C. P. Osborne: About twenty-three years ago, the company with which I am associated was the first to try out the mercury arc rectifiers for street lighting. These sets were installed by the General Electric Company and some of them are still in operation.

The operating engineer today is greatly interested in mercury arc rectifiers not only for street lighting but for low-voltage distribution. Operating companies will no doubt install more d-c. distribution in the future than in the past, if it can be done with the mercury arc rectifier.

Our company is confronted at this time with a program which necessitates moving motor-generators which feed our d-c., three-wire Edison system from one location to another. At present the system is fed by motor-generator sets and it would be much more convenient if mercury arc rectifiers could be used and not have rotating apparatus in the new building.

I feel that the 1400-volt mercury arc rectifier will be a success and that the manufacturers' further developments and research will succeed in building lower voltage sets at a reasonable cost so they can be considered instead of converters and motor-generator sets for d-c. systems.

We visited in Chicago a substation where the mercury arc rectifier was furnishing energy for railway operation, and we did not find a great deal of enthusiasm among the men from the results they were getting from the mercury arc rectifier. The sets had just been started, however, and no doubt there will be changes which will be found necessary in order to get the desired results.

We operating men are responsible to the public for good service and in trying out new equipment we feel the manufacturers should be sure of its good operation before it is placed in service for the public. We are anxious to give the public good service in the cheapest way, but we must consider that the service comes first.

J. A. Cook: A small power application of mercury arc rectifier tubes is important to electric utility companies in street-lighting work. Glass tubes for this service have an average life of several thousand hours. Individual tubes may last double normal life or may fail after a few hours' operation. Such failure is attended by darkness on those city streets where lamps are located which depend upon this tube for their supply of current. It follows that failures of tubes should be anticipated so far as possible. C. M. Green, at West Lynn, advances the theory that failure can in some instances be anticipated. He uses a 1-to-1 ratio current transformer designed to operate at open circuit without excessive heating. The secondary of this transformer is connected through suitable resistance to the vibrator of an oscillograph. This gives a view of the ripples superimposed on the direct current. When certain tubes are about to fail there will be noted a pronounced peak which occurs on an otherwise flat portion of the wave. I should like to ask Mr. Prince if he can tell us of a test of this kind and whether tubes can be successfully treated to restore their full usefulness after the indication of approaching failure has been observed. I should also like to ask him if there is on the market an inexpensive oscillograph for visual use which would permit an operator to detect this condition in advance of failure.

I have tried the oscilloscope and find that it is not adequate for routine use by a station operator. The mirror is not driven synchronously, but is turned with the thumb. A wave without a peak can be made to show a peak. I have also tried an oscillograph of the spring-galvanometer type. Depending upon the tension of the spring, a large number of wave forms can be observed due to mechanical resonance of the vibrator system. I should repeat that a device which would indicate approaching failure of rectifier tubes on street-lighting work will be most valuable to public utility companies.

W. A. Hillebrand: I should like to ask if the use of the mercury arc rectifier is accompanied by any danger from radio interference, particularly in railway service? Is there any danger that arcing between trolley wire and collector will be more likely to set up troublesome oscillations when the circuit is supplied through a rectifier than when the source of supply is a rotating machine?

D. C. Prince: There are two general ways in which series street-lighting rectifiers fail. In one, the rectifiers actually break down at the peak of the voltage wave, due either to poor vacuum or to high temperature in the tank. If the failure be due to too high temperature it can, of course, be guarded against by keeping track of the temperature and reducing it by additional cooling water if it passes the safe value. The other type of failure is due to too perfect a vacuum. As the tube operates, the vacuum becomes more and more perfect until there is nothing to remove charges which accumulate on the glass arms. These charges prevent the current from shifting from one anode to another until a high voltage has been built up, and then the transfer occurs with a considerable shock which may break down some part of the tube or connected apparatus. This kind of breakdown in its incipient state appears as a distortion of the wave, such as is shown in Steinmetzs' "Transient Electric Phenomenon Oscillations," first edition, Fig. 77. This condition can be detected by any sort of oscillograph. When this condition becomes apparent, the tube may be taken out of service and it is customary in some utility properties to impair the vacuum slightly by heating the tube in oil, water, or a small oven, after which the tube can usually be returned to service if the heating has not been overdone.

The question of radio interference has been raised a great many times. As far as we know, a rectifier in good order does not produce any radio interference. However, interference may be caused by the fading condition which produces oscillations, such as shown in Dr. Steinmetz's oscillogram, or by loose contacts. When complaints of radio interference have been made in the past, I believe they have always originated in one or the other of these ways.

Regarding the use of rectifiers on locomotives, I believe that the Westinghouse Company operated a motor car by means of a rectifier over the New Canaan branch of the New Haven railroad. There are naturally difficulties due to vibration, but I do not believe any of them is insurmountable.

As to the voltage limit of mercury arc rectifiers, I believe Mr. Odermatt will corroborate me in saying that his company has in operation a 4000-volt, d-c. railroad system supplied by mercury arc rectifiers. We have experimentally operated at 10,000 volts and there is no apparent theoretical limit, provided the proper temperatures can be maintained.

At present the floor space required by a 600-volt mercury arc rectifier is greater than that required by rotating equipment of the same voltage. At 1500 volts and higher, I believe that the advantage is in favor of the rectifier. It is probable that as rectifiers become more standardized it will be possible to reduce considerably the area which they require.

Vibration Recorder

For Electrically Measuring and Recording Small Mechanical Movements

BY A. V. MERSHON*

Associate, A. I. E. E.

Synopsis.—*The vibration recorder makes it possible to obtain oscillographic measurements and records of rapid mechanical movements as small as five or ten-thousandths of an inch. From two to five millimeters deflection will be obtained on the oscillograph for every one-thousandth of an inch variation of the apparatus undergoing investigation.*

This instrument has been successfully used to record permanent data of vibrations of a rotating turbine wheel, the mechanical movements or whipping of a rotating shaft and transient oil pressures due to an explosion produced by opening up a high-voltage switch in a sealed vessel.

The vibration recorder is a balanced circuit comprising a test coil and a dummy coil, connected to two windings of a differential transformer. When the circuit is balanced there will be zero flux set up in the differential transformer, and the oscillograph vibrator which is connected to a third winding will have a zero deflection. Deflections produced on the oscillograph vibrator are obtained when the circuit is unbalanced by mechanical vibrations in front of the test coil. By suitable calibration curves, these deflections on the oscillograph are used as a measure of the mechanical movements which cause them.

* * * * * *

GENERAL DESCRIPTION

THE vibration recorder was first designed to measure vibrations of rotating turbine wheels and the whipping of rotating shafts. The development of this instrument was necessary because ordinary hand micrometers or indicating devices could not be applied to the moving parts due to their inaccessible location.

Most of the parts in a steam turbine are made of iron or steel; therefore, the fundamental thought was to make use of magnetic flux changes to detect these vibrations. A bridge circuit was constructed having a test coil and a transformer coil as one arm, and a dummy coil and a duplicate transformer coil as the other arm. The dummy coil has an iron core with an adjustable air-gap, and the test coil has an iron core the air-gap of which varies with the vibration or motion to be recorded. Any unbalance of the magnetomotive forces produced by the currents in the two transformer windings gives a flux, which produces a voltage on a third winding connected directly to an oscillograph vibrator. An alternating current of approximately 500 cycles is used to excite this circuit.

The voltage of the internal or test coil is balanced against the external or dummy coil to obtain the greatest sensitivity. For convenience in balancing the voltages of these two coils a transformer was constructed with two windings so arranged that the currents in the two windings oppose one another. A third winding was constructed on this transformer to pick up any differences in the voltages of these two coils. The current of the third transformer winding is calibrated to measure the vibrations, in thousandths of an inch, that take place in front of the core of the test coil.

Fig. 1 shows a graphic representation of the currents in the differential three-winding transformer. When the circuit is balanced the primary currents I_2 and I_3 are equal and 180 electrical degrees out of phase. This produces zero flux in the transformer core. If the air-gap in front of one of the coils is changed, it will unbalance the circuits: I_2 will differ from I_3 as shown in the figure and a flux will be set up in the differential transformer core. The flux in the differential transformer core will generate a voltage in the secondary winding No. 1 of the differential transformer, and produce current I_1 in the oscillograph vibrator circuit. The unbalanced currents may not be exactly 180 electrical degrees out of phase with one another, as shown in the figure, as most of the unbalanced condition is due to the change in the inductive reactance of the circuit. This phase variation does not cause errors, as the same variation occurs during calibration.

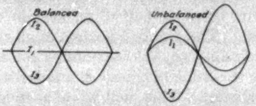

FIG. 1—GRAPHIC REPRESENTATION OF THE 500-CYCLE CURRENTS IN THE DIFFERENTIAL THREE-WINDING TRANSFORMER

In each branch of the vibration-recorder circuit there is a variable inductor, a three-dial resistor with steps of 0.1, 1.0 and 10.0 ohms respectively, a coil and a winding of a transformer, all connected in series. The two branches of the electric micrometer circuit are exact duplicates. The connections are shown in the wiring diagram, Fig. 2. Resistances R_1 and R_2 and inductances L_1 and L_2 are secondary parts of the circuit. Their function is to keep the resistance components and the inductive components of the two arms of the electric micrometer exactly equal so that the currents in each half of the circuit will have the same magnitude and the same phase displacement from the impressed e. m. f.

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Applications. Vibrations as small as five or ten thousandths of an inch have been measured on the periphery of a turbine wheel rotating at 1800 rev. per min. in a steel casing. This was done with the coils exposed to the steam heat and the moisture of the turbine. The whipping of rotating shafts has also been measured. The two above applications were made with an open type coil holder (without a magnetic diaphragm.) Transient oil pressure due to an explosion produced by opening a high-voltage switch in a sealed vessel has been measured with a magnetic diaphragm type of coil holder.

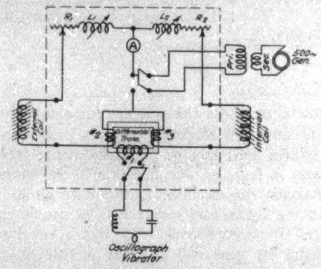

Fig. 2—Connection Diagram for the Vibration Recorder

Investigations of the mechanical motions in various kinds of apparatus require changes in the size of the coils and the design of the coil holders. In the majority of investigations coils one inch in diameter by three-quarters of an inch deep have been found satisfactory. The number of turns required varies widely from 500 to 1500. An iron core one-half of an inch in diameter is generally used. The two coil holders in all cases should be built exactly alike so that the dummy coil and the test coil will have the same magnetic characteristics and the same eddy-current. This will give a rough balance when the air-gaps of the two coils are the same.

A magnetic diaphragm is required in front of the test coil and dummy coil when transient pressures of gases or liquids are being investigated. The diaphragm must be suitably built as a part of the coil holder. In a case where a direct mechanical vibration is produced in a piece of apparatus the coil does not require a diaphragm in front of it. The front of the coil is placed directly opposite a magnetic piece of material which may or may not be a part of the apparatus being tested. The coil is placed so that there is an air-gap between it and the piece of magnetic material in front of it. Any changes in distance between the magnetic material and the coil can be measured by the change in strength of the magnetic field at various air-gaps.

Sensitivity. Changes in the air-gap between the core and the magnetic material of one-thousandth of an inch will produce a deflection on the oscillograph vibrator of from two to five millimeters. The amplitude of the vibration of the apparatus under test is magnified on the oscillograph vibrator 150 times. This magnification ratio is shown in Table I which gives the calibration of the vibration recorder. A magnification of 106 is obtained for small variations in the air-gap, but for larger variations, the magnification is 201.

Accuracy. The vibration recorder will measure accurately any small mechanical movements transient in nature, produced in a piece of apparatus. If this movement is 0.005 in. or greater, an accuracy of five per cent can be obtained.

Instrument Description and Connections to Auxiliary Apparatus

Fig. 3 shows the vibration recorder as it is developed in instrument form. The two halves of the instrument are practically identical. It is approximately 25 in. long, 18 in. wide and 8 in. high. The ammeter shown in the middle is a standard G. E. type P3. The vibration recorder itself is portable although the oscillograph equipment including the 500-cycle generator is usually not constructed with this point in view.

A one-kw., 500-cycle, 110-volt generator is connected externally through a transformer to the upper left hand terminals marked "500-cycle generator." The oscillograph vibrator circuit is also connected externally to the upper right hand terminals marked "Oscillograph Vibrator." In series with the vibrator in this circuit is an inductance coil and a condenser which is set in dull resonance for 500 cycles. Tuning the oscillograph vibrator circuit accomplishes two results; it reduces the impedance to approximately four ohms effective resistance and likewise keeps down all the harmonics in the 500-cycle wave.

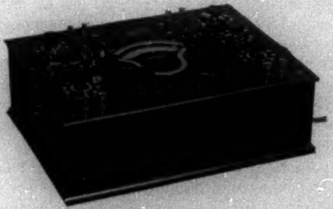

Fig. 3—The Vibration Recorder

Used to measure transient pressures and mechanical vibrations

Before the oscillograph vibrator circuit is tuned, it is very difficult to produce a balanced condition on account of the harmonics interfering. When it is tuned the harmonics disappear and the circuit can easily be balanced so that the zero line on the oscillograph including all the residual deflection is not more than two millimeters wide. The circuit is balanced roughly by setting the air-gap of the external coil approximately the same as that of the internal coil. A fine balance is obtained by moving the dial resistors and the dial inductors until a satisfactory balance is obtained as indicated on the oscillograph vibrator.

CALIBRATION

The internal or test coil is used to obtain a voltage variation due to the vibration to be measured. The external coil is a dummy coil and its function in the circuit is to furnish a voltage to balance the voltage of the internal coil. The voltage of the external coil is of a specified value assumed as a zero from which to measure the amplitude of vibration.

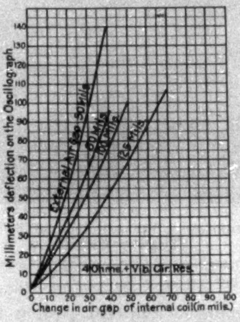

FIG. 4—CALIBRATION CURVES FOR THE VIBRATION-RECORDER CIRCUIT

Showing the effects of various air-gaps between the test coil and the piece of apparatus being investigated

When a calibration is started, the air-gap in front of the external coil must be set approximately the same as that of the internal coil. The air-gap of the external coil is very seldom changed during the test after the balance is once obtained. The apparatus under test furnishes the internal coil with a varying air-gap corresponding to the vibration that we wish to measure. The variations in the air-gap will be recorded by deflections shown on the ground glass of the oscillograph, as illustrated on the calibration curves shown in Figs. 4 and 5. Curves were taken by varying the air-gap of the internal coil while holding constant that of the external coil.

Fig. 4 shows calibration curves for the internal coil for four air-gap settings of the external coil. The air-gap settings should be chosen to correspond with the nature of the test. In order to obtain calibration curve marked "50 mils" the external coil was kept at this constant air-gap and the air-gap of the internal coil was changed. Readings were taken on the oscillograph of the width in millimeters of the bright band produced by the 500-cycle wave and measurements were taken with a suitable thickness gage, in thousandths of an inch, of the corresponding air-gap in front of the internal coil. In adjusting this air-gap either the internal coil or the piece of apparatus under test may be moved.

The sensitivity of the instrument is best, for changes of five or ten-thousandths of an inch, when the width of the bright band during calibration at the smaller lengths of air-gap of the internal coil is greater than the width of the scale of the oscillograph. The total width that can be measured on the ground glass with the zero in the center of the oscillograph scale is approximately 100 mm. For deflections greater than 100 mm., the oscillograph zero is transferred to one side of the scale and the measurement is made from zero to the maximum deflection. In this case the value obtained by measurement is doubled in order to put the results on the same basis as the smaller deflections obtained with the zero in the center of the scale. The upper points of the curves and the tabulations are made on the basis of the full (or "double") deflection.

TABLE I

Calibration of Vibration Recorder for 0.050 in.
(These data are not plotted on a curve)

Mils or thousandths of an inch external coil	Mils or thousandths of an inch internal coil	Millimeters deflection on the oscillograph ground glass or film	Magnification ratio
0.050 in.	0.050 in.	2. mm.	....
0.050 in.	0.040 in.	29. mm.	106.
0.050 in.	0.030 in.	61. mm.	126.
0.050 in.	0.020 in.	102. mm.	161.
0.050 in.	0.010 in.	149. mm.	185.
0.050 in.	0.000 in.	200. mm.	201.

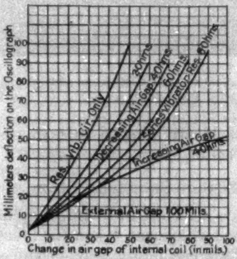

FIG. 5—CALIBRATION CURVES FOR THE VIBRATOR-RECORDER CIRCUIT

Showing the effects of changing the resistance in the oscillograph vibrator circuit

Fig. 5 shows the deflection obtained on the oscillograph vibrator using different resistances in series with the vibrator. These curves are shown to bring out the importance of reducing the impedance of the oscillograph vibrator circuit to a minimum by the use of the 500-cycle dull resonant vibrator circuit.

The curve marked "Decreasing Air-gap 4 ohms," was taken by setting the air-gap of both coils at 100-thousandths of an inch and decreasing the air-gap of the internal coil by successive steps. From the increasing curve and the decreasing curve it can be seen that the circuit is unbalanced when the air-gap is changed either way from 100-thousandths of an inch.

STABILITY OF THE CIRCUIT

The stability of the electric micrometer is good because the calibration is not affected by slight voltage and frequency fluctuations of the 500-cycle generator. Changes in frequency of ± 50 cycles will affect the calibration as the iron losses change and the oscillograph vibrator circuit will not be in resonance.

The stability of the circuit is not disturbed seriously due to the internal coil heating up more than the external coil as this uneven heating can be compensated for by changing R_1 and R_2 at any time during the test. Adding 20 ohms to R_1 and 20 ohms to R_2 will not change the original calibration. The best results are obtained if the electric micrometer circuit and the oscillograph galvanometer field are allowed to heat up about one-half of an hour before using.

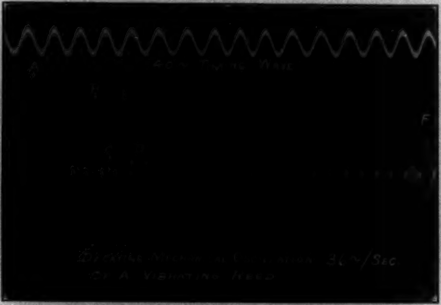

FIG. 6—OSCILLOGRAM SHOWING A SIMPLE MECHANICAL OSCILLATION PRODUCED BY A VIBRATING REED

TESTS RESULTS SHOWN ON OSCILLOGRAMS

Fig. 6 shows a decaying mechanical oscillation of 36 cycles per second which was taken for illustrative purposes only. This oscillation was produced by a vibrating reed clamped to an iron support. Position marked *A* shows the reed pulled over and held 0.044 in. from rest towards the internal or test coil. The reed was first set at 0.125 in. in its rest position. The calibration curve for this film is shown on Fig. 4. Position *B* shows the first cycle after the reed was released and allowed to swing free. The first swing did not return entirely to its original position of 0.044 in. and it, therefore, returned to a position of 0.039 in. representing a total swing of 0.078 in. Position *E* likewise represents a mechanical deflection of 0.036 in. one-half amplitude of the reed. Position *F* represents 0.023 in. one-half amplitude and it is the 12th swing of the reed towards the internal coil which occurred approximately one-third of a second after the reed was released.

Position *B* represents the reed at its nearest position to the internal coil after release and position *C* represents the reed at its greatest distance away from the internal coil in the cycle of which *B* is the beginning.

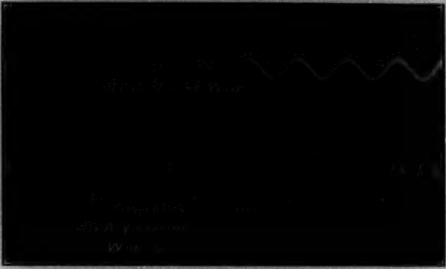

FIG. 7—A RECORD OF A VIBRATION OF AN 8-FT. DIAMETER TURBINE WHEEL

Rotating 1280 rev. per min. and vibrating 0.024 in. in six segments around the periphery

Position *D* represents the reed passing through its rest position or the balanced positions for the electric micrometer circuit. Any simple natural vibrating mass passes through its rest position twice in making a complete cycle.

Fig. 7 is a record of a vibration of an eight-foot diameter turbine wheel rotating 1280 rev. per min. and vibrating 0.024 in. in six segments or nodes around the periphery. The internal or test coil was held in a bracket fastened to a stiff wheel which rotated at the same speed and on the same shaft as the wheel under test. (See Fig. 8.) The stiff wheel ran rigidly at all speeds for which wheels are ordinarily tested and had known vibration characteristics at speeds much higher than that of the wheel being tested. Positions marked with an arrow show every revolution of the wheel. In one revolution there are six different positions shown. These correspond to segments between the six nodes or rest positions of the vibrating wheel. Positions 1, 3, and 5 represent negative segments, or

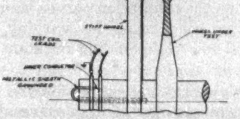

FIG. 8—ARRANGEMENT OF TEST COIL AND HOLDER FOR VIBRATION MEASUREMENTS ON A TURBINE WHEEL

wheel displacements from rest, away from the internal coil. Positions 2, 4 and 6 represent positive segments, or vibration from rest, towards the internal coil.

The deflections under the arrows are larger than the other five intermediate deflections: The reason for this is that the turbine wheel received a slight end thrust or quiver every revolution. This is likewise shown in producing a smaller deflection next to the largest deflection. This quiver in the wheel for every revolution represents approximately two-thousandths of an inch variation. These total deflections are actual measurements of mechanical movements of 0.012 in. or one-half the total amplitude of the vibration of the wheel. The total amplitude of the vibration of the wheel is 0.024 in. The reason this deflection, shown on the oscillogram, Fig. 7, represents one-half the total amplitude of the vibration is that the calibration was taken while the wheel was moved from rest towards the coil. Deflections away from rest do not require a calibration. The mechanical deflections of the wheel on both sides of the rest position are the same.

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Discussion

W. F. Davidson: Some time ago, I had occasion to consider the possibility of carrying on measurements of about the type described by Mr. Mershon and looked into the possibility of several methods, but we seemed to run into difficulties because the vibrations that we wanted to record were in the order of 200 cycles. Modulating a 500-cycle wave with 200 cycles does not give a very satisfactory wave to work on. On the other hand, using frequencies higher than 500 cycles is likely to lead to some circuit difficulties, especially if an oscillograph is to be used. I have wondered whether it might be possible to use frequencies in the order of several thousand cycles and then, through a valve arrangement similar to a radio detector circuit, unscramble the composite current so that the wave can pass through the oscillograph and reproduce truly the original modulating action. It seems to me that there is a possible development of considerable value in this direction.

J. W. Legg: Mr. Mershon's paper proves that the study of vibrations in turbines, and in machinery in general, must be very important or such recording apparatus would not have been developed by several large companies in approximately the same period of time.

Mr. Mershon has perfected one method of recording minute vibrations with the oscillograph. The apparatus described is nicely arranged and has many of the undesirable features of that method eliminated. However, this particular method, of transforming mechanical vibrations into changes in current through an oscillograph vibrator, had so many drawbacks that the writer abandoned this method years ago and chose to push through a design which required no high-frequency alternator for excitation and which gave a true wave record and not a modulated record.

The final design consisted of an inductor-alternator modification which generated a voltage and set up a current in the oscillograph vibrator which was always proportional to the actual vibrations being studied. The calibration was constant over the range of vibrations studied. This apparatus was much more sensitive than that described by Mr. Mershon, and operated over a much greater range. Furthermore it required no 500-cycle motor-generator set nor any other electrical excitation or amplification. A cobalt-steel permanent magnet was the only excitation, and the complete apparatus was no larger than a medium-sized fan motor.

This vibration converter fitted into a cavity in the end of the shaft of a steam turbine or other rotating apparatus. Torsional vibrations of even less than 0.01 deg. and longitudinal vibrations of even less than 0.00001 in., at frequencies even above 2000 cycles, have been recorded. On the same film, within a small fraction of a second, more than a dozen different frequencies have been recorded ranging from 60 cycles to 2200 cycles per second. Such records could never be made by modulating a 500-cycle wave, or even a 5000-cycle wave.

In order to achieve these results, the usual principle of the alternator was modified so that the rotor would tend to seek the position where a slight relative movement between rotor and stator would give the *maximum* voltage generated in the winding, instead of minimum voltage. Also the flux distribution was uniform, each side of this stable position. Both rotor and stator of this special inductor alternator rotated at the same speed as the machine being studied, but one member (usually the outside member) was rigidly fixed to the shaft of the machine being studied, while the other member (usually the inside member, consisting of permanent magnet, pole tips and shaft) was free to rotate at a strictly uniform angular velocity even though the other member vibrated torsionally or longitudinally while it rotated at the same speed.

The special cobalt-steel magnet was magnetized, in place, with 300,000 ampere-turns, and the magnetizing coil (of copper) was allowed to remain on the magnet to have an additional fly-wheel effect.

One end of the permanent magnet had a multiplicity of north poles, each under two teeth of the stator. The stator teeth had a series winding in which was induced the torsional vibration currents. The other end of the permanent magnet had a ring-shaped south pole inside a toroidal winding on the outside member. This winding had radial punchings on each side so that longitudinal vibrations induced currents in this winding proportional to the amplitude of vibration of the machine being studied. The inductive drop in each winding was kept large compared with the ohmic-resistance drop, over the range of frequency to be studied, and hence a calibration *constant* could be used in place of a complicated calibration curve.

The extreme sensitivity, efficiency and compactness of this apparatus made it possible to install several of these vibration converters in different positions in the shafts of the turbines of an ocean liner. Tests were made under actual operating conditions of the vessel during a severe storm, without in any way interfering with its reliability in carrying the usual quota of first-class passengers and freight. The portable oscillograph was located in a first-class cabin at the end of a 250-ft., 12-conductor cable.

In spite of the simplicity of this device, it recorded more minute vibrations in the turbine blades themselves, by reaction through the turbine shaft to the vibration converter, than could be recorded by the apparatus described by Mr. Mershon even though the latter were attached to the blades themselves.

The outstanding advantages of the inductor-alternator vibration-converter scheme over that of the varying inductance, bridge and 500-cycle motor-generator set, may be summarized as follows:

1. Extremely compact (about the size of a fan motor).
2. Fits in a cavity in the end of the shaft of the machine being studied.
3. Requires no wires stretching up into the heart of the apparatus being studied.
4. Will operate on a standard steam turbine running in normal service and yet record the vibration of turbine blades which may be at a dull red temperature in highly super-heated steam.
5. Has actually recorded turbine vibrations of 0.00001 in. at frequencies up to 2200 cycles per second. This is not the limit of possibilities.
6. Has a calibration constant and not the complications of a calibration curve, over the operating range of frequencies.
7. When once determined, this calibration constant did not vary perceptibly even when the converters remained on turbines operating over 12,000 mi. of ocean.
8. Oscillograms are comparatively easy to read even though many different frequencies of vibration are present in a record made in 0.1 sec.
9. Both longitudinal and torsional components of vibration may be recorded simultaneously with but one converter and two oscillograph elements.

A. V. Mershon: In regard to the possibility of measuring mechanical vibrations around 200 cycles per sec., I might speak of Fig. 7 as having a natural mechanical frequency of 120 cycles per sec. What is called approximately 500 cycles in my article is actually around 700 cycles for Fig. 7. We did not try in any case to hold the 500-cycle generator at 500 cycles and I feel sure that the same circuit can be used to measure a 200-cycle mechanical frequency with very good results, and in that case, in order to obtain proper shading, it would be necessary to boost up the frequency of the 500-cycle generator to around 1000 or 2000 cycles. This frequency I do not believe would be harmful to the working of the oscillograph. In this case, as the frequency is boosted up, the losses which occur around the coil would decrease the sensitivity, but not seriously.

The main difference between the vibration converter described by Mr. Legg and the vibration recorder presented in the present paper lies in the results obtained on the oscillograms. The deflections obtained on the oscillograms using the vibration converter are a function of the frequency. The oscillogram deflections obtained by the vibration recorder are true representations of the magnitude of the mechanical vibrations regardless of frequency.

The vibration converter is designed and built to do one class of investigational work and is more analytical than quantitative. The vibration recorder is designed to investigate a different kind of mechanical motion and the results obtained are more quantitative. Each circuit has its own class of investigational work to perform and should be able to perform one class of work better than the other.

The vibration converter cannot be applied at the point where the vibration occurs on turbine blades because a piece of apparatus as heavy as a fan motor is too large. The vibration converter evidently will not work well without the magnetizing coil attached to add weight so that the rotor will have a fly-wheel effect. A heavy vibration converter the size of a fan motor could not be mounted on the turbine blades on account of the mechanical strength required to hold it in place; furthermore if such a scheme of mounting could be used it would damp out the vibration and destroy its natural period. There are many cases in which the weight of a fan motor would change the natural period of a vibrating mass and render the application of a vibration converter impracticable.

The vibration recorder requires a small test coil only 1 in. in diameter by ¾ in. thick which is not attached to the vibrating part. This small coil can be mounted at the periphery of a rotating wheel adjacent to the blade that is vibrating and measure the vibrations without changing the natural period, and it will not damp the vibrating blade. A calibration curve is determined very easily, simply by varying the air-gap a fixed amount and observing the oscillographic deflections.

The vibration recorder has an additional advantage of being able to measure pressure variations. Pressure variations as slow as one per minute can be measured as easily as 200 per sec.

Law of Magnetization

BY S. L. GOKHALE[1]
Member, A. I. E. E.

***Synopsis.**—The purpose of the study, which constitutes the subject matter of this paper, was to investigate the reliability of Frolich's law of magnetization near saturation.*

For the last forty years, Frolich's law has been accepted as the most reliable expression representing the relation of magnetizing force to the magnetic induction produced thereby in ferro-magnetic materials. In the course of the study, which constitutes the subject of this paper, it was found that the law is not as reliable, as it is generally believed to be, particularly for purpose of computation of saturation value. It was also found in the course of the same study that this part of the curve follows the law,

$$B - H = S(1 - b\,e^{-aH}) \quad \text{(equation } \mathbf{30\text{-}2}\text{)}$$

more closely than any other law yet formulated. This law and Frolich's law are both in harmony with Weber's theory of molecular orientation. (See equations **24, 25, 26.***)*

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1. PURPOSE OF THE INVESTIGATION AND PLAN OF PRESENTATION OF RESULTS

THE progress of scientific and industrial development of production, distribution and control of electrical energy depends very largely on the correct knowledge and understanding of the laws representing the relation of electric current to magnetism. During the last hundred years, various attempts have been made to obtain a correct knowledge of these laws. The analytical study and experimental work, which constitutes the subject of this paper, is one more effort toward the same goal; it is limited to a study of the law of magnetization for flux density in the neighborhood of saturation. Stated symbolically, the subject of this investigation is the equation,

$$B = F(H) \qquad (\mathbf{1})$$

and the problem in hand is the determination of the form of the function $F(H)$ and its properties. For reasons to be explained later, the form of the problem is slightly modified in the course of the study. (See § 3, equation **4**; § 9, equation **23**).

The plan of presentation of results, which is followed in this paper is:

Nomenclature and symbols, with illustrative curves. Attention is directed to characteristic peculiarities of curves which occupy an important place in the subsequent argument.

Study of Ewing's conception of intrinsic induction, and of Weber's theory of molecular orientation, as a ground work for formulating a law of magnetization. (§ 3, 4; equations **4, 6**).

Presentation of Frolich's law as a hypothesis in relation to Weber's theory. (§ 5, equation **7**).

Study of different forms of Frolich's law, with a view to formulate some crucial equations, whereby the validity of Frolich's law can be tested. (§ 5, 6, 7, 20; equation **7, 9, 11, 12, 17, 32** and **33**).

Further study of Weber's theory, as groundwork for equation of progress near saturation. (§ 8, 9; equation **22, 23**).

Presentation of Frolich's law, as a hypothetic equation of progress. (§10, equation **24**).

Experimental study of a curve of incremental permeability, and reluctivity. It will be shown that the form of the observation curves does not support Frolich's law, but that on the contrary, it indicates in favor of Lamont's equation of progress. (§ 11, Fig. 3-1, equation **26**).

(Supplement, § 20; Fig. 3-3, -4, equation **32, 33**).

Law of latent induction ($\log \gamma = f - g\,H$), is derived as a corollary to Lamont's equation of progress. (§ 11, equation **29**). This leads to the equation of magnetization, $\mathbf{B} - H = S\,(1 - b\,e^{-aH})$ referred to in the Synopsis. (Equation **30-2**).

Study of saturation value, and of $\beta\,H$, $\beta\,\mu$ and $H\,D$ curves for standard sheet steel. A study of these curves shows, firstly, that the magnetization curve for this material does not follow Frolich's law, and secondly, that for the part of the curve above $H = 300$, it follows the law $\text{Log } \gamma = f - g\,H$. (§ 12, 13, 14).

Study of $H\rho$ curve. The apparent straightness of the $H\,\rho$ curve seems at first sight to support Frolich's law, but a closer study of the curve reveals that this is due to its insensitive character, and proves that it has no evidential value (§ 15, equation **31-2**).

Corroborative evidence from tests on other typical samples (§ 16).

Further corroborative evidence from tests at the Bureau of Standards. (§ 17, Fig. **12-2, -4**).

Practical application of both laws for purpose of extrapolation beyond the limits of test (§18).

Summary of conclusions.

Tables of data and curves.

The scheme of numbering the tables and curves is such that all figures with a common group number refer to tables with the same group number. The sub-numbers have no significance, they are used merely as a convenient mark for subdivision of the group. In view of the difficulties due to the large number of figures necessary to illustrate the various points of the argument, a few curves are selected for publication. Other curves will be available in the form of blue prints, for anyone who desires to study the results further than as given in this paper.

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Necessary references to all curves, including those that are not published, will be found in the tables. References to published curves only are given in the body of the text, each in its proper place.

2. NOMENCLATURE AND SYMBOLS

The following symbols are used in this paper.

B Total induction, or flux density (gausses).

β Intrinsic induction, or flux density, (gausses).

H Magnetizing force, (gilberts per cm: briefly g.).

H Spatial induction, (gausses, See par. 3, note 1).

S Saturation value, *i. e.*, limiting value of β.

γ Latent induction, $(= S - \beta)$. (See §4, 8).

μ Intrinsic permeability $(= \beta/H$, not $= B/H)$.

ρ Intrinsic reluctivity $(= H/\beta)$.

D Distribution ratio $(= \beta/\gamma)$.

μ' Transformation rate $(= - d\,\gamma/d\,H$, or $d\,\beta/d\,H)$. Also called incremental permeability.

ρ' Incremental reluctivity, $(d\,H/d\,\beta)$.

i_1 First inflection of $\beta\mu$ or $H\,\rho$ curves.

i_2 Second inflection of $\beta\mu$ or $H\,\rho$ curves.

a coefficient of permeability $(= \mu/\gamma)$ when used with reference to Frolich's law in its various forms (equations **7**, **8**, **9**, **12**, **16**, etc.).

a Coefficient of incremental permeability $(= \mu'/\gamma)$ when used with reference to Lamont's equation of progress, and other equations derived from it (equation **27**).

σ Coefficient of reluctivity $(= 1/S)$.

α Initial reluctivity $(\alpha = \rho - \sigma\,H)$. This quantity is also defined as $\alpha = 1/a\,S$. (See equations **10** and **11**).

h Coefficient of magnetic hardness $(h = 1/a)$ when used with reference to Frolich's law (see equation **13**).

Curves are designated by joint use of the variables involved, for example, $\beta\,H, \beta\,\mu, H\,\rho$ curves; when one of the variables is logarithmic, a comma is inserted, for example H, log γ curve.

For illustration of these symbols see Fig. 1.

NOTES: The symbol H is used in two senses but this ambiguity does not lead to any inconvenience or misunderstanding, as the context is clear in all cases. The same remark applies to the symbol a.

The symbol μ, in the usual sense of B/H, is not used in this paper.

The symbol ρ, in its original sense of H/B, is not used in this paper.

The numerical figures on the $\beta\mu$ curve refer to the values of H for the corresponding points. The straight line drawn in relation to the $\beta\,\mu$ curve is a hypothetic $\beta\mu$ curve for Frolich's law, that is, the $\beta\,\mu$ curve, which would be represented by this straight line, if the material followed Frolich's law, within the limits indicated. (Fig. 1-1, -2, -3.)

In relation to the reluctivity curve, the straight line passing through the origin is the ideal reluctivity curve, that is, a curve for a hypothetical sample of material having the same saturation value as the test sample, but having no magnetic hardness, so that the material is supposed to be saturated from the start. (Fig. 1-1, -2, -3.)

In studying the various curves, the following characteristics should be noted; their significance will be explained later:

(a) The $\beta\,H$ curve has the appearance of a hyperbola, asymptotic to the saturation line (Fig. 1-1, -2, -3). It will be shown later that the curve is not truly hyperbolic.

(b) For values of H above a certain limit, $(H = 50$ approx.), the $H\,\rho$ curve appears to be almost a perfect straight line (Fig. 1-1, -2, -3). Even below $H = 50$, the curvature is scarcely noticeable.

(c) For corresponding values of H, the $\beta\,\mu$ curve is not straight. It contains three straight sections, *viz.*, regions of Kennelly, Yensen, and saturation. (Fig. 1-1, -2, -3). For further discussion on this point, see §14.

(d) The $H\,D$ curve is not straight; it has the appearance of an exponential curve. The corresponding logarithmic curve is comparatively straight; in the particular case selected for illustration, the logarithmic curve seems to be made up of two separate straight lines, with a seemingly abrupt bend at $H = 370$ (Fig. 1-6).

3. EWING'S CONCEPTION OF INTRINSIC INDUCTION

Prior to the year 1890, the various laws of magnetization propounded by several physicists, were equations of the form $B = F\,(H)$ or other equations derived therefrom. In 1890, Prof. Ewing showed that the induction in all ferro-magnetic material could be conceived as made up of two components, each following its own characteristic law.

$$B = B_s + B_i \tag{2}$$

where

$$B_s = F_1\,(H) = H.$$

and

$$B_i = F_2\,(H)$$

NOTES: B_s is the spatial induction, that is, induction in the space as a magnetic property of the space irrespective of the magnetic character or condition of the material occupying that space. It is supposed to follow the very simple law $B_s = H$. (See Fig. 1-1).

B_i is the intrinsic induction, which depends on, and is an undetermined function of H; it is signified by the symbol β in this paper. (See Fig. 1-1).

B is the resultant induction in the magnetic material. Using the notation of §2, Ewing's conception is,

$$B = H + \beta \tag{3}$$

The problem of determining the law of magnetization is thus reduced to a determination of the equation,

$$\beta = F\,(H) \tag{4}$$

In view of the fact that Ewing's conception is now generally accepted, it is considered necessary to revise the laws of Lamont, Frolich, etc., and regard them as referring to β, although in their original form they referred to B.

4. WEBER'S THEORY OF MAGNETIZATION, AND CONCEPTION OF LATENT FLUX

According to Weber's theory, as modified by Ewing: All magnetic material consists of molecular magnets; the question regarding the cause of magnetism of the molecules is irrelevant for our present purpose, and is therefore left out of consideration in this paper.

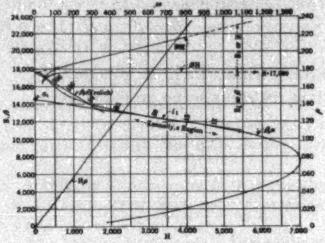

FIG 1-1—SILICON STEEL (SAMPLE NO. R437-6)

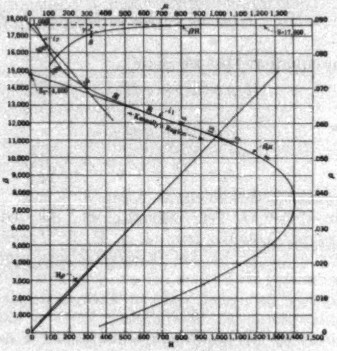

FIG. 1-2—MAGNIFIED SECTION OF FIG. 1-1

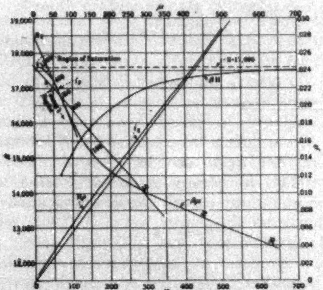

FIG. 1-3—MAGNIFIED SECTION OF FIG. 1-2

The process of magnetization is merely a process of orientation of the molecules.

In the non-magnetized condition the molecules form a large number of small groups, each group making a small but complete magnetic circuit.

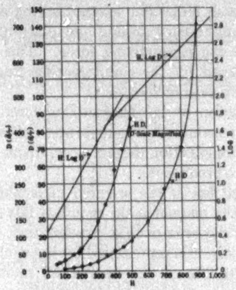

FIG. 1-6—*H D* AND *H* LOG *D* CURVES

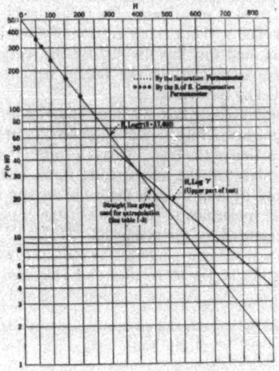

FIG. 1-7—*H* γ CURVE

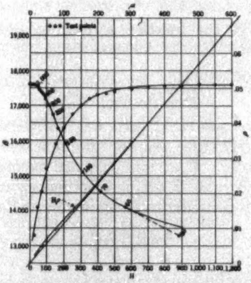

FIG. 1-8—RECONSTRUCTION OF CURVES BY LOGARITHMIC LAW

The molecules in each group are held in their place by magnetic forces, which originate in the molecular magnets of that group, as well as of neighboring groups.

When a magnetizing force is applied to the magnetic material, each molecular magnet acting as a compass needle, tends to turn around and place itself in a line parallel to the magnetizing force. This tendency is partly opposed by the counteracting magnetic forces of the neighboring molecules.

The function of the magnetizing force is therefore merely to determine the configuration of the molecules. For each configuration there is a certain magnitude of H necessary to produce it.

The total flux of the oriented molecules across any section of the magnet is the manifest flux and this flux divided by the area of that cross-section, is the manifest induction; it is represented by the symbol β.

The total flux of the unoriented molecules across the same cross-section is the latent flux; it is made up of two groups of flux lines in opposite directions making an algebraic total of zero lines. The total number of these lines reckoned without consideration of direction is the latent flux, and this flux divided by the sectional area of the whole magnet is the latent induction; it is represented by the symbol γ.

The total intrinsic flux in a magnet across any section is the aggregate of flux lines of all molecules irrespective of the direction of the lines. This is a constant quantity, not dependent on the magnetizing force, being determined only by flux in each molecule and by the total number of molecules in the plane of the cross-section: it is represented by the symbol S.

The function of the magnetizing force is to establish a configuration of the molecules and therefore the distribution of S into its two components β and γ. (See Fig. 1-2).

Weber's theory may therefore be stated analytically by the equations,

$$\beta + \gamma = S \qquad (5)$$

and

$$\beta/\gamma = F\,(H) \qquad (6)$$

the form of the function H being undetermined for the present.

5. FROLICH'S LAW: HYPOTHETIC EQUATION OF DISTRIBUTION RATIO

We have seen that according to Weber's theory, $\beta/\gamma = F\,(H)$, the form of the function being yet undetermined. If at this point we assume that the distribution ratio is not only a function of H, but is directly proportional to H, we arrive at the equation,

$$\begin{aligned}\beta/\gamma &= a\,H \qquad (7)\\ &= H/h\end{aligned}$$

where $h = 1/a$.

or

$$D = a\,H = H/h \qquad (7\text{-}2)$$

This equation may be called Frolich's hypothesis, because it is implied in Frolich's law. It has not been explicitly stated in this form by Dr. Frolich or by any one else. Substituting for γ its equivalent $S - \beta$, we get

$$\beta/\gamma = a\,H$$

$$\therefore \beta/(S-\beta) = a\,H \qquad (8)$$

$$\therefore \qquad \beta = S\,.\,a\,H/(1 + a\,H) \qquad (9)$$

$$= S - S/(1 + a\,H) \qquad (9\text{-}2)$$

This is Frolich's law, (*E. T. Z.* 1886, p. 164).

Equation (**9**) can also be expressed as

$$\beta = \frac{H}{\dfrac{1}{a\,S} + \dfrac{H}{S}}$$

$$= \frac{H}{\alpha + \sigma\,H} \qquad (10)$$

where

$$\alpha = 1/a\,S, \text{ and } \sigma = 1/S.$$

This is another form of Frolich's law (*E. T. Z.*, 1882, p. 71).

From equation (**10**) we obtain,

$$H/\beta = \alpha + \sigma\,H$$

or

$$\rho = \alpha + \sigma\,H \qquad (11)$$

or

$$\rho = (H + h)/S \qquad (11\text{-}2)$$

This equation is known as Kennelly's law. It was formulated first by Prof. Fleming, who derived it as a corollary to Frolich's law, (J. I. E. E. *Trans.*, 1886, p. 569); secondly by Dr. Kennelly, who derived it independently of Frolich's law, empirically from a study of the published data of Ewing, Rowland and Du Bois, (A. I. E. E. TRANS., 1891, p. 503-517); and thirdly by Dr. Steinmetz, who derived it also as an empirical law from the study of data published by Dr. Corsepious, (E. T. Z., 1892, p. 203).

From equation (**8**) we also obtain directly,

$$\beta/H = a\,(S - \beta)$$

or

$$\mu = a\,(S - \beta) \qquad (12)$$

or

$$\mu = \gamma/h \qquad (12\text{-}2)$$

This is Bosanquet's law: (*Electrician* 1886, p. 247). Equation (**12**) may also be derived directly from equation (**11**), by purely algebraic process involving no other equations except the defining equations,

$$\mu = \beta/H$$
$$\rho = H/\beta$$
$$\alpha = 1/a\,S$$

and

$$\sigma = 1/S$$

The relation of equation (**12**) to equation (**11**) is not dependent on their relation to equation (**7**) or (**8**) to any other hypothesis. From this it follows, that if any part of the $H\ \rho$ curve is straight, the corresponding part of the $\beta\ \mu$ curve must also be straight.

Again, starting with equation (**9**)

$$\beta = S\ .\ a\,H/(1 + a\,H)$$

using a symbol $h = \dfrac{1}{a}$ (see equation **7-2**) we get

$$\gamma = S\,h/(H + h) \qquad (\mathbf{13})$$

$$\beta = S\ .\ H/(H + h) \qquad (\mathbf{14})$$

$$= S - S\,h/(H + h) \qquad (\mathbf{15})$$

NOTE: The relation of equations (**7-2**), (**9**), (**11-2**), (**12-2**), (**13**), (**14**), (**16**) and (**17**), can be easily demonstrated geometrically with the help of Fig. 2-1. For example,

$M\,P/P\,N = O\,M/N\,Q$ or $\beta/\gamma = H/h$, equation (**7-2**).

Again $M\,P/O\,S = O\,M/S\,Q$, or $\beta/S = H/(H + h)$, equation (**14**).

Equations (**13**), (**14**) and (**15**) are equations for Frolich's law in its analytical form. They lend themselves easily to computation of β and γ for any value of H. Equation (**15**) is preferred in this paper in the preparation of the several tables. For example see Table II. The several equations given above lead us to expect that

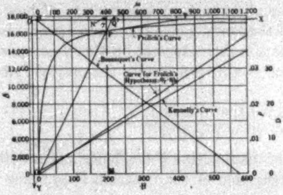

FIG. 2-1—FROLICH'S LAW AND COGNATE CURVES

The $\beta\ H$ curve is an equilateral hyperbola with asymtotes $O'\ X$, $O'\ Y$

$\beta = M\ P$ Then $\beta/H = S/(H + h)$ Frolich's Law
$\gamma = N\ P$ $\beta/H = \gamma/h$ Bosanquet's Law
$H = O\ M = S\ N$ $H/\beta = (H + h)/S$ Kennelly's Law
$S = O\ S$ $\beta/\gamma = H/h$ Frolich's Hypothesis
$h = O'\ S = N\ Q$ $\beta/S = H/(H + h)$
$H + h = O'\ N = N\ T = S\ Q$ $d\beta/d\ H = P\ N/N\ T$
$= \gamma/(H + h)$
$= \gamma^2/S\ .\ h$ Frolich's Equation of Progress

1. If Frolich's law be true for the entire curve above any value of H, the corresponding parts of the $H\ \rho$, $\beta\ \mu$, and $H\ D$ curves should be straight lines, as the corresponding equations are all of the first degree. (See equations **7-2**, **11** and **12**. Fig. 2-1).

2. If Frolich's law be true only between certain limits, and not true above and below that limit, the $H\ \rho$ and $\beta\ \mu$ curves should be straight between those limits, but the $H\ D$ curve should not be straight even between those limits, if the true saturation value determined by measurement be used as a basis of computation, unless the value of S indicated by the straight part of the $H\ \rho$ or $\beta\ \mu$ curve happens to be the true saturation value. No concrete instance of such exceptional coincidence has yet been observed.

3 In no case is it possible to find a particular part of the $H\ \rho$ curve straight unless the corresponding part of the $\beta\ \mu$ curve is also straight.

These three curves, *viz.*, the $\beta\ \mu$, $H\ \rho$ and $H\,D$ curves, are therefore three practical tests whereby the truth of Frolich's law for any part of the $\beta\,H$ curve may be tested; two other practical tests will be described later. (See §6 and 20).

6. FROLICH'S EQUATION OF PROGRESS AND EMERY'S LAW

By differentiation of equation (**12**), we obtain

$$\mu' = \frac{a}{S}\,(S - \beta)^2 \qquad (\mathbf{16})$$

$$= K\,(\,S - \beta)^2 \qquad (\mathbf{17})$$

$$= \gamma^2/S\,h$$

where

$$K = a/S = 1/S\,h.$$

$$\therefore \mu' = \gamma/(H + h) \qquad (\mathbf{17\text{-}2})$$

(See equation **13**).

This is Frolich's equation of progress of magnetization. (*E. T. Z.*, 1886, p. 164). By reintegration of equation (**16**), we get,

$$\beta = S\,(a\,H + c\,S - 1)/(a\,H + c\,S) \qquad (\mathbf{18})$$

or

$$\beta = S - S/(a\,H + c\,S) \qquad (\mathbf{19})$$

where c is an undetermined constant of integration. This is Emery's law. (A. I. E. E. TRANS., 1892, pp. 209, 215). In equation (**18**) or (**19**), if $c = 1/S$, the equation reduces itself to the form

$$\beta = S\ .\ a\,H/(1 + a\,H) \qquad (\mathbf{9})$$

or

$$\beta = S - S\,/(1 + a\,H) \qquad (\mathbf{9\text{-}2})$$

This is Frolich's law.

From Emery's law we have,

$$\beta/\gamma = a\,H + c\,S - 1 \qquad (\mathbf{20})$$

According to Emery's law the distribution ratio expressed as a function of H is a straight line not passing through the origin, except in the particular case where $c\,S - 1 = 0$, in which case Emery's law becomes Frolich's law; in either case it is a straight line. Equation (**16**) is an equation of first degree for μ' and second degree for β. Therefore, if Frolich's law be true, the corresponding curve should be a parabola with the vertex at the point S on the axis of β, and with its directrix parallel to the axis of β.

7. STUDY OF EQUATIONS OF DISTRIBUTION RATIO: (EQUATIONS 7 AND 20)

We have seen that Frolich's law in all its forms, and even Emery's law, involves the hypothesis that the distribution ratio is a linear function of H. Such an assumption has no justification either theoretical or empirical. Fig. 1-6 shows the curve for distribution ratio for a selected sample of silicon steel. This curve is certainly not a straight line. On the theoretical side, we have against it the negative fact that no reason has yet been given to explain why the ratio β/γ should be a simple linear function of H. It is true that there is nothing inherently absurd in the assumption that the ratio of the components β and γ is proportional to magnetizing force: it does not conflict with Weber's theory, but neither is there anything in Weber's theory to justify or even to suggest that assumption; in fact even the form of the general equation (**6**), that is, $\beta / \gamma = F(H)$, does not express the fundamental nature of the phenomenon, which would be better expressed by a differential equation of the form,

$$d\beta / dH = F(\beta, \gamma)$$

although the final result may be expressed in any convenient form, including the form,

$$\beta / \gamma = F(H).$$

NOTE: We have seen that the curve β / γ as a function of H is not straight, but has the appearance of an exponential curve corresponding to the equation $\beta / \gamma = e^H$. (See §2 par. d). If this conjecture be correct, the curve for $\log \beta / \gamma$ should be a straight line. This expectation is found to be partly justified by Fig. 1-6. This point will be discussed more fully later. (See §13 equation **30-4**).

8. EQUATION OF PROGRESS ACCORDING TO WEBER'S THEORY

According to Weber's theory, the values of β and γ for any value of magnetizing force depend on the configuration which has been reached under the influence of that force, and which is therefore determined by the condition of equilibrium between the internal and external magnetic forces, when that configuration is established. The internal forces are those due to the interaction of the magnetic poles of the molecules, from which it follows that the forces which oppose the change of configuration at any step in the process of magnetization are determined by the configuration itself. According to this conception, the process of magnetization is a progressive process, made up of changes of configuration in successive steps, and is at each step accompanied by a transformation of a small part of the latent induction $d\gamma$ into an equal amount of manifest induction $d\beta$, under the influence of the incremental magnetizing force dH. The fundamental law of magnetization should therefore be an equation of the form,

$$d\gamma / dH = -F(\beta, \gamma) \tag{21}$$

or

$$d\beta / dH = F(\beta, \gamma) \tag{22}$$

Equations (**21**) and (**22**) are identical, being merely the negative and positive aspects respectively of the same transformation process. In formulating this conception, the transformation ratio ($d\gamma / dH$) is expressed as a function of β and γ, but not as a function of H; this is because, according to the conception under discussion, the transformation of $d\gamma$ into $d\beta$ is brought about by the operation of the increment dH, not by the whole magnetizing force H. The function of the magnetizing force H is to establish the corresponding configuration, by counteracting the opposing internal forces, and when that configuration has been reached, the function is exhausted. Any further change in the value of β and γ is brought about by the incremental component of force dH, and the magnitude of this component necessary for unit change in γ and β is determined by the component of internal force, which it has to oppose and overcome. This component of internal force is the result of, and therefore some mathematical function of, the configuration itself, that is a function of β and γ; it follows therefore that the rate of transformation $d\gamma / dH$ must be fundamentally a function of β and γ but not of H. But since β and γ are themselves functions of H, it also follows that $d\gamma / dH$ is capable of being expressed as a function of H, although such an equation would not be the fundamental equation.

9. EQUATION OF PROGRESS NEAR THE SATURATION LIMIT

We have seen that $d\gamma / dH$ is a function of β and γ. (See equation **21**). At this point, we can introduce a further simplification by limiting the problem to the part of the curve near saturation. As saturation approaches, the manifest component of flux β becomes practically constant. The corresponding oriented molecules are now practically in the final stage of orientation, their influence in opposing any change of configuration is now practically constant irrespective of how great or how small that influence happens to be; this makes the transformation ratio depend on the γ component of S only, which is now practically the only variable in the function.

Thus we have the equation,

$$d\gamma / dH = -F(\gamma) \tag{23}$$

the form of the function $F(\gamma)$ being yet undetermined; the determination of the function $F(\gamma)$ now becomes the main problem.

10. STUDY OF FROLICH'S EQUATION OF PROGRESS

At this point we might make some assumption as to the form of the function $F(\gamma)$. If we assume $F(\gamma) = K\gamma^2$, we have

$$d\gamma / dH = -K\gamma^2 \tag{24}$$

or

$$d\beta / dH = K(S - \beta)^2 \tag{25}$$

This is Frolich's equation of progress (see equation **17**) wherein K represents the ratio of Frolich's constants, a/S (see §6 equation **16**). The status of Frolich's law depends on the validity of the assumption $d\beta / dH = \dfrac{a(S - \beta)^2}{S}$. On the theoretical side no justification for this assumption has been given by Dr. Frolich or by any one else. On the empirical side, there is no experiment on record to demonstrate that the curve for incremental permeability as a function of β is a parabola such as is represented by the equation in question.

11. FORM OF CURVE FOR INCREMENTAL PERMEABILITY AND EQUATION OF LATENT INDUCTION

In order to ascertain the validity of Frolich's assumption (Equation **16** or **24**), a sample of standard steel in the form of a toroid ring was tested by the differential method. For details of test and result of measurement see Table III, Fig. 3. The curve plotted from test has certainly no resemblance to the parabola representing the equation $d\,\beta/d\,H = \dfrac{a\,(S-\beta)^2}{S}$. Incidentally, we observe that the observation curve though not perfectly straight, is very nearly a straight line for magnetizing forces above $H = 100$, and still more straight above $H = 200$. The test stops at $H = 400$; extrapolation of the curve indicates a saturation value of $S = 20{,}250$. (For true value of S, see next paragraph). We might therefore follow the suggestion of this experiment and assume that the function $F\,(\gamma)$ in equation (**23**), is $a\,\gamma$.

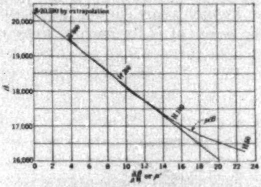

FIG. 3-1—STANDARD SHEET STEEL RING CURVE OF INCREMENTAL PERMEABILITY

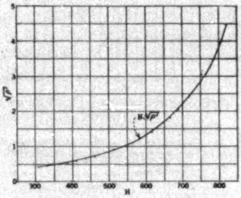

FIG. 3-3—STANDARD SHEET STEEL RING CURVE OF INCREMENTAL RELUCTIVITY H AS A FUNCTION OF $\sqrt{\rho'}$

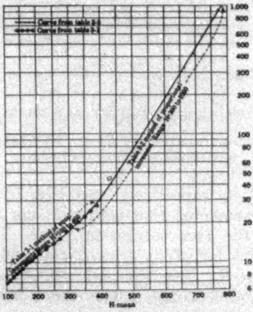

FIG. 3-4—CURVE OF INCREMENTAL RELUCTIVITY H AS A FUNCTION OF LOG ρ'

This leads to the equations:

$$d\,\gamma/d\,H = -\,a\,\gamma \qquad (\mathbf{26})$$

or

$$d\,\beta/d\,H = a\,(S-\beta) \qquad (\mathbf{27})$$

This is Lamont's hypothetical equation of progress.

By integrating either of these we have,

$$\log_e \gamma = C - a\,H \qquad (\mathbf{28})$$

where C is an integration constant

or

$$\log_{10} \gamma = f - g\,H \qquad (\mathbf{29})$$

From equation (**28**), we get

$$\beta = S\,(1 - b\,e^{-aH}) \qquad (\mathbf{30})$$

where $b = e^{\,c}/S$

or

$$B - H = S\,(1 - b\,e^{-aH}) \qquad (\mathbf{30\text{-}2})$$

This is the law of magnetization referred to in the synopsis.

Equation (**30**) is the same as Lamont's equation in form but differs from it in the limitations which are implied in equation (**26**) and (**30**) but not recognized by Lamont, *viz.*, that the equation is true for parts of curve near saturation only (See par. 9). Lamont assumes that the equation is true for all values of H including $H = 0$, which makes $b = \mathbf{1}$; Lamont's equation therefore takes the form,

$$\beta = S\,(1 - e^{-aH}) \qquad (\mathbf{30\text{-}3})$$

This equation has no experimental support.

12. LAW OF MAGNETIZATION FOR STANDARD SHEET STEEL

In order to ascertain the reliability of the extrapolation method based on the assumption $d\,\gamma/d\,H = -\,a\,\gamma$, the ring referred to in the last paragraph was unwound and rewound for higher magnetizing force reaching up to $H = 1000$, if possible. (For details of test and result of measurement, see Table IV, Fig. 4). The sample seems to be saturated at $H = 650$, reaching the saturation value of $S = 20{,}200$, which is in very close agreement with that indicated by extrapolation, *viz.*, 20250 (See Fig. 3). For magnetizing forces above H 300, the observation curve is in very close agreement with the reconstruction curve representing the law, $\log \gamma = f - g\,H$. (see Fig. 4-4); it is not in as good agreement with the law $\beta = S\,a\,H/(1 + a\,H)$. (See Fig. 4-5).

13. CURVE OF DISTRIBUTION RATIO FOR STANDARD SHEET STEEL

According to the equation (**7**), $(\beta/\gamma = a\,H)$, the curve for distribution ratio should have been a straight line. The observation curve is certainly not straight, (Figs. 1-6, 4-1). On the contrary, the curve seems to be exponential, as it ought to be if equation (**30**) represented the true law of magnetization; for

$$\beta = S\,(1 - b\,e^{-aH}) \qquad (\mathbf{30})$$

Then

$$\gamma = S \,.\, b\,e^{-aH}$$

Then

$$\beta/\gamma = (1 - b\,e^{-aH})/b\,e^{-aH}.$$

$$= \frac{1}{b\,e^{-aH}} - 1$$

$$= \frac{1}{b\,e^{-aH}}$$ approximately as saturation approaches,

or

$$D = e^{aH}/b \qquad \textbf{(30-4)}$$

or $\text{Log}_{10}\, D = p\,H + q$ (making due allowance for change of base). The corresponding observation curve is approximately straight showing that the distribution ratio is not a linear function of the magnetizing force as is required by Frolich's law, but nearly an exponential function as is required by the law of equations (**29**) and (**30**).

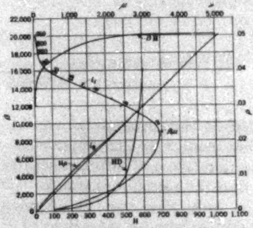

FIG. 4-1—STANDARD SHEET STEEL RING

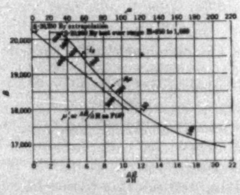

FIG. 4-2—MAGNIFIED SECTION OF $\beta\mu$ CURVE, FIG. 4-1 $\mu'\rho$ CURVE FROM FIG. 3-1 REPRODUCED FOR COMPARISON

14. THE PERMEABILITY AND RELUCTIVITY CURVES

According to Frolich's law, equation (**12**) ($\mu = a\,(S - \beta)$, the $\beta\,\mu$ curve should be a straight line, intersecting the axis of β at the point S. The observation curve does not seem to fulfill this condition, (See Fig. 4-1, -2). For magnetizing forces above $H = 300$ the curve seems to follow very closely the law $\log \gamma = f - g\,H$. The complete $\beta\,\mu$ curve is not straight, but it has three straight regions.

1. First straight region, associated with the first inflection i_1 (Kennelly's region; See Fig. 1-1, 4-1).

2. Second straight region, associated with the second inflection i_2 (Yensen's region; See Fig. 1-3, 4-2).

3. Region of saturation.

The work of Dr. Kennelly was limited mostly to that part of the $\beta\,H$ curve which corresponds to the first straight region, (See Fig. 5-2); and the work of Dr. Yensen was limited to the second straight region, (See Fig. 5-1). From the form of the $\beta\mu$ curve it is obvious that Frolich's law should hold true within the corresponding limits of H for each of these regions, but not for the whole curve above the first or even above the second region. The first and second regions can be represented by the equations, $\mu = a_1\,(S_1 - \beta)$, and $\mu = a_2\,(S_2 - \beta)$ respectively, (See Fig. 1-1, -3), but it must be remembered that neither S_1 nor S_2 represent necessarily the true saturation value. In all cases observed thus far, S_1 is always less, and S_2 always greater, than S. (See Fig. 1-1, -2, -3; $S_1 = 14800$, $S_2 = 18430$, $S = 17600$). The curve for reluctivity as a function of H, *i. e.*, $H\,\rho$ curve, possesses all the characteristics of the $\beta\,\mu$ curve, but they are not so easily noticeable in the $H\,\rho$ curve, (See Fig. 1-1, -2; for points i_1 and i_2, see Fig. 1-3, 4-1, -2). The $H\,\rho$ curve seems to be almost a perfect straight line; in some cases, it seems to resolve itself into two straight lines connected by an apparently abrupt bend; in either case, the straightness is only apparent and misleading.

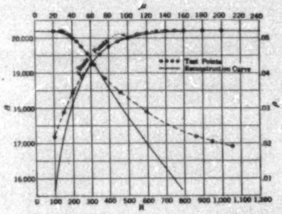

FIG. 4-4—RECONSTRUCTION OF CURVES BY LOG LAW

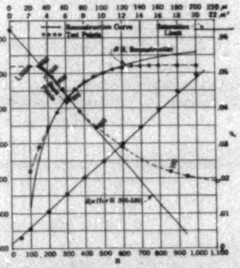

FIG. 4-5—RECONSTRUCTION OF CURVE BY FROLICH'S LAW ON THE BASIS OF YENSEN'S REGION

15. INSENSITIVE CHARACTER OF RELUCTIVITY ($H\ \rho$) CURVE

It has long been recognized that the reluctivity curves are always almost perfectly straight over a wide range although the corresponding parts of the $\beta\ \mu$ curves are not straight. (Kennelly: A. I. E. E., 1891, p. 531). Theoretically, the $\beta\ \mu$ curve and the $H\ \rho$ curve are so related, that if one of them be straight, the other must be straight also; the straightness of the $H\ \rho$ curve over a wide range is therefore anomalous, being mathematically impossible when the corresponding part of the $\beta\ \mu$ curve is not straight. The explanation of the anomaly lies in the insensitive character of the $H\ \rho$ curve in comparison with the $\beta\ \mu$ curve. The insensitive character of the $H\ \rho$ curve is inherent in the form of the function irrespective of the law of reluctivity.

This may be seen from the following analysis:

Let $\rho = \alpha + \sigma H$, and $\mu = a\,(S - \beta)$, be the two forms of Frolich's law, in reference to reluctivity and permeability, where the "constants" are not true constants, but variable functions of H and β. By differentiating the second equation for β we have, by partial differentiation,

$$\frac{d\,\mu}{d\,\beta} = a\left(\frac{d\,s}{d\,\beta}\right) - a + (S - \beta)\,\frac{d\,a}{d\,\beta}$$

By direct differentiation, assuming a and S to be constant,

$$\frac{d\,\mu}{d\,\beta} = -\,a$$

Combining the two equations, we get

$$a\,\frac{d\,s}{d\,\beta} = -\,(S - \beta)\,\frac{d\,a}{d\,\beta}$$

or

$$\frac{d\,a}{d\,\beta} = -\,\frac{a}{\gamma}\cdot\frac{d\,s}{d\,\beta}$$

$$= -\,\frac{a}{\gamma}\cdot\frac{d\,s}{d\,\sigma}\cdot\frac{d\,\sigma}{d\,H}\cdot\frac{d\,H}{d\,\beta}$$

or

$$\frac{d\,\sigma}{d\,H} = -\,\frac{d\,a}{d\,\beta}\,\frac{\gamma}{a}\cdot\frac{d\,\sigma}{d\,s}\cdot\frac{d\,\beta}{d\,H}$$

$$= \frac{d\,a}{d\,\beta}\,\frac{\gamma}{a}\cdot\frac{1}{S^2}\cdot\frac{d\,\beta}{d\,H} \qquad (31)$$

because

$$\sigma = 1/S,\ d\,\sigma/d\,S = -\,1/S^2$$

Let

$$\dot{\rho} = \frac{d\,\sigma}{d\,H},\ \text{bending of the } H\ \rho \text{ curve.}$$

$$\dot{\mu} = \frac{d\,a}{d\,\beta},\ \text{bending of the } \beta\ \mu \text{ curve.}$$

$$\mu' = \frac{d\,\beta}{d\,H},\ \text{bending of the } \beta\ H \text{ curve.}$$

Then

$$\dot{\rho}/\dot{\mu} = \frac{\gamma\cdot\mu'}{a\,S^2}$$

$$= \alpha\,.\,\sigma\,.\,\gamma\,.\,\mu' \qquad \textbf{(31-2)}$$

where

$$\alpha = 1/a\,S. \quad \text{(See par. 5, equation } \textbf{10}\text{).}$$

This equation shows that the $\beta\ \mu$ and $H\ \rho$ curves have the same characteristics of curvature except that the curvature of the $H\ \rho$ curve is less than that of the $\beta\ \mu$ curve in the ratio of α, σ, γ, μ'. As saturation approaches, the factors γ and μ' become very small, which makes $\dot{\rho}$ very small in comparison to $\dot{\mu}$, and which therefore makes the $H\ \rho$ curve appear straight, although the corresponding $\beta\ \mu$ curve might have a very strong curvature; in other words the $H\ \rho$ curve is very insensitive, being merely a progressively flattened copy of the $\beta\ \mu$ curve. The insensitiveness of the reluctivity curve may also be demonstrated graphically by choosing for comparison two samples of magnetic material of nearly identical character. (See Table XII-III). The differences between the two $\beta\ \mu$ curves are then easily noticeable but the corresponding $H\ \rho$ curves are practically indistinguishable, except near the upper end of the curves. Fig. 5-1 is a concrete example of insensitivity of the $H\ \rho$ curve. The data for this curve are taken from an experiment by Dr. Yensen (A. I. E. E. TRANS., 1920, p. 821). In the data as originally published, there appears to be a typographical error, which becomes manifest in the $\beta\ H$ and $\beta\ \mu$ curves, but is scarcely noticeable in the $H\ \rho$ curve.

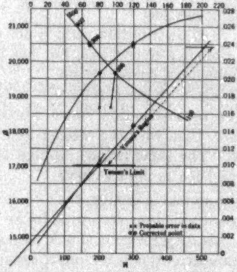

FIG. 5-1—YENSEN'S CURVE FOR 2 *N* *i* 204

16. MAGNETIZATION CURVES FOR MAGNETIC METALS AND BINARY FERRIC ALLOYS

In view of the importance of the conclusions reached in the foregoing pages, it seemed advisable to extend the experiment to a variety of samples. The following list contains a few samples selected for purpose of demonstration; only toroid ring samples are included in this list.

1. Pure Iron, Electrolytic vacuum fused and vacuum annealed.
2. Pure Nickel, 99.5 per cent Nickel.
3. Pure Cobalt, percentage not specified.

NOTE: This sample was used by Mr. J. D. Ball in his well-known experiment: G. E. *Review*, 1916, p. 379.

4. Iron Nickel alloy, 72 per cent Nickel.
5. Iron Cobalt alloy, 20 per cent cobalt.

For details of test and results of measurement see tables and curves in appendix.

A study of the curves shows that:

1. In all cases, except cobalt, the samples are saturated well within the limits of test.

2. In all cases the complete $\beta \mu$ curve above the point of maximum permeability is not straight, but has all the peculiarities mentioned in par. 14, *viz.*, the two inflections and the three straight regions. In the case of cobalt, the curve is incomplete but it demonstrates the first two straight sections.

3. The curve of distribution ratio is not straight, as required by Frolich's hypothesis, but has the appearance of an exponential curve instead.

4. The βH curve by measurement is in closer agreement with the reconstruction curve according to the logarithmic law, than with the reconstruction curve by Frolich's law. The same remark applies to the other derived curves, such as the $\beta \mu$ curve.

5. Below the second inflection, the βH curve does not follow the logarithmic law; according to the theory outlined above, this disagreement should be expected, for the rate of transformation is no longer capable of being represented by the simpler equation $d \gamma / d H = - a \gamma$, but must be a more complex function involving both β and γ. (See equation **21**).

By way of further evidence in support of item (4), I may refer once more to Fig. 1, which has been obtained by the saturation permeameter. In this case also the βH curve is in closer agreement with the logarithmic law than with Frolich's law, (Fig. 1-8). Tests by the saturation permeameter run into several hundred samples. It is obviously not practicable to include them all in this paper and I must therefore limit myself to the bare statement that in all cases the observation curve follows the logarithmic law much more closely than Frolich's law.

17. CORROBORATIVE EVIDENCE FROM TESTS AT THE BUREAU OF STANDARDS

Further evidence in support of the above findings will be found in the data from tests at the Bureau of Standards (See Table XII, Fig. 12). In this case the material under test was a group of samples of electrolytic iron, carefully prepared by Dr. T. D. Yensen. The tests were made at the Bureau of Standards; both the material and the test results are of the highest order of reliability. The curves speak for themselves and need no further explanation.

18. PRACTICAL APPLICATION OF THE LAW

LOG $\gamma = f - g H$

As a practical application of these conclusions, let us complete by extrapolation the incomplete βH curve for cobalt; the whole procedure is as follows:

1. Compute the values of $\Delta \beta / \Delta H$ for successive steps of magnetization for the range $H = 600$ to 1600 (See Table XI-I).

2. Plot $\Delta \beta / \Delta H$ as a function of mean β for the corresponding steps in β.

3. From the points thus plotted, determine by extrapolation the approximate value of S; in the case of cobalt we find $S = 15{,}300$ gausses approximate.

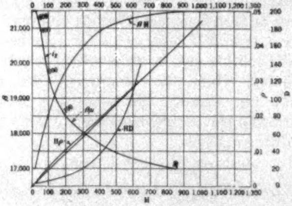

FIG. 12-2—ELECTROLYTIC IRON MAGNIFIED SECTION—TEST BY BUREAU OF STANDARDS

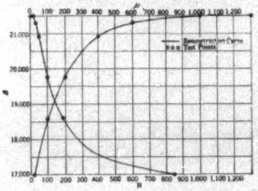

FIG. 12-4—RECONSTRUCTION OF CURVE BY LOG LAW

NOTE: A more satisfactory determination can then be made by method of trial and error.

4. Assuming provisionally the value of S by extrapolation to be exact value, compute the value of γ for the several observation points; plot log γ as a function of H, and see if it is a straight line. (See Table XI-II).

5. If this curve is not straight, try again with a slightly different value of S. In the case of the sample of cobalt, $S = 15{,}300$ gausses, appears to give the best result.

6. From two selected points of this curve compute the value of the constants f and g for the equation, Log $\gamma = f - g H$, (for method of computation, see Table XI-III).

7. The complete βH curve may now be obtained by

extrapolation, by arithmetic computation as shown in Table XI-III and-IV, or graphically as shown by curve sheet. The latter method is much easier and nearly as accurate.

19. SUMMARY OF CONCLUSIONS

We have found that:

1. According to Weber's theory of molecular orientation, the process of magnetization should be expressed fundamentally, by an equation of progress of the form,

$$d\gamma/dH = F_1(\beta, \gamma)$$

and in its ultimate effect by the equation of distribution ratio of the form,

$$\beta/\gamma = F_2(H)$$

2. For the part of the curve near saturation, the equation of progress should be of the simpler form,

$$d\gamma/dH = -F_1(\gamma)$$

3. Frolich's law involves the assumption fundamentally, that,

$$d\gamma/dH = -K\gamma^2$$

and in ultimate effect, that,

$$\beta/\gamma = aH$$

This assumption has no theoretical justification or empirical support.

4. In the case of a toroid ring of standard sheet steel, it was found that for magnetizing forces $H = 200$ to 400, the progress of magnetization followed, approximately, the equation,

$$d\beta/dH = a(S - \beta)$$

5. From the above equation, it follows that as saturation approaches, the magnetization should be expected to follow approximately the equation,

$$\log\gamma = f - gH$$

This expectation has been justified by subsequent verification, (see next item).

6. For magnetizing forces above the second inflection of the permeability curve, the βH curve follows the law $\log\gamma = f - gH$ more closely than any other law yet formulated.

7. The part of the $\beta\mu$ curve above the point of maximum permeability is not straight, but it contains three straight regions, *viz.*,

(a) The region of first inflection (Kennelly's region).
(b) The region of second inflection (Yensen's region).
(c) The saturation region.

8. It follows from this observation, that a magnetization curve does not follow Frolich's law, except in the first two regions mentioned above. The corresponding parts of the βH curve are segments of equilateral hyperbola corresponding to the equations,

$$\beta = S_1 a_1 H/(1 + a_1 H)$$

and

$$\beta = S_2 a_2 H/1 + a_2 H)$$

where S_1 and S_2 are the indicated saturation values corresponding to the two inflection regions. S_1 is always less, and S_2 always greater than the true saturation value.

9. The part of the $H\rho$ curve above the point of minimum reluctivity appears to be very straight. The straightness of the $H\rho$ curve has been generally regarded as a conclusive evidence in favor of Frolich's law, but it has now been demonstrated that the straightness of this curve is only a geometric illusion, being caused by the insensitive character of the reluctivity function. The insensitiveness of the $H\rho$ curve has been proved analytically and demonstrated graphically. It follows therefore that the straightness of the $H\rho$ curve is misleading, and that the curve is therefore not reliable for purpose of drawing any conclusion as to the nature of the law of magnetization or for extrapolation of the βH curve. The $\beta\mu$ curve is a more reliable criterion, and the curvature of a $\beta\mu$ curve between any limits demonstrates conclusively the unreliability of Frolich's law for the corresponding range of the βH curve.

20. INCREMENTAL RELUCTIVITY (SUPPLEMENT)

In paragraph 11 reference has been made to an experiment for determination of the rate of progress, $\Delta\beta/\Delta H$, as a function of β. The test stops at $H = 400\ g$; in view of the great importance of the results, several attempts were made to extend the test to $H = 1000\ g$, but experimental difficulties have thus far proved unsurmountable. By modifying the test procedure, so as to increase H by an increment of constant ratio, (*i. e.*, $\Delta H/H =$ constant), instead of a series of incremental steps of equal magnitude as in Table III-I, it has been possible to extend the test to $H = 1000$, but in this case the result appears in the form of $\Delta\beta/\Delta H$ as a function of H, instead of a function of β. (See Table III-II). It is not therefore possible to use the measurement directly to determine the reliability of either of the two equations.

$$d\beta/dH = Sh(S - \beta)^2 \qquad \text{Frolich's equation } (\mathbf{16})$$

or

$$d\beta/dH = a(S - \beta) \qquad \text{equation } (\mathbf{27})$$

It is possible, however, to use this determination as a criterion of reliability of the two laws in question, by the introduction of a new conception, *viz.*, the incremental reluctivity, $\rho' = dH/d\beta$ as a reciprocal of $\mu' = d\beta/dH$. Starting with Frolich's law, we have

$$\mu' = d\beta/dH = K(S - \beta)^2 \qquad \text{equation } (\mathbf{18})$$

Then

$$\begin{aligned}\rho' &= 1/K(S - \beta)^2 \\ &= K(H + h)^2\end{aligned}$$

$$\because \left\{\begin{aligned} K &= a/S = 1/Sh; && \text{equation } (\mathbf{17}) \\ S - \beta &= Sh/(H + h) && \text{equation } (\mathbf{13}) \end{aligned}\right\}$$

or

$$\sqrt{\rho'} = pH + q \qquad (\mathbf{32})$$

where p and q are abbreviation constants involving S and h. Again starting with the exponential law,

$$\beta = S(1 - be^{-aH}) \qquad \text{equation } (\mathbf{30})$$

We have,

$$d\beta/dH = S\ .\ a\ .\ b\, e^{-aH}$$

$$S\ .\ a\ .\ b\ .\ (dH/d\beta) = e^{aH}$$

$$aH = \log(S\,a\,b) + \log(dH/d\beta)$$

or

$$\log \rho' = p\,H + q \qquad \textbf{(33)}$$

where p and q are abbreviation constants involving a, b, and S. Therefore, if Frolich's law be the true law of magnetization near saturation, the curve for $\sqrt{\rho'}$ as a function of H must be straight for the corresponding values of H; on the other hand if the logarithmic law, $\log \gamma = f - g\,H$, be the true law of magnetization, the curve for $\log \rho'$ as a function of H should be a straight line. In Fig. 3-3, the curve for $\sqrt{\rho'}$ is certainly not straight even in part over any reasonably large range of H, above $H = 300\ g$, which disproves Frolich's law for this range. In Fig. 3-4 the curve for $\log \rho'$ is straight for the range of $H = 400$ to $800\ g$, which supports the logarithmic law for the part of the βH curve near saturation. A careful study of the curve indicates that this curve resolves itself into two straight lines connected by a short transitional curve, and suggests the possibility of a change of constants, f and g. A similar phenomenon was observed in the case of Fig. 1-6, 1-7, (See §2, 6-d). This point is now under investigation, for which further experiments are in progress.

TABLE I

4 per cent Silicon Steel: Sample No. R-437-6 (Ref. G. L. 39705-39, -41)

Note: This is a very unusual sample selected for purpose of illustration only because it combines in a single sample all the characteristic peculiarities that call for a demonstration. The sample is not a representative of the grade of steel to which it nominally belongs. Tests above H-200 are made by Saturation Permeameter and below $H = 200$ by the B of S. method.

Table 1-1; test by the Saturation Permeameter
(Not reliable below $H = 1000$)

H	β	B	μ (β/H)	ρ (H/β)	γ ($S = 17600$)	D (β/γ)	Log D
4000	17,600	21,600	4.40	0.2273			
3000	17,600	20,600	5.87	0.1705			
2500	17,600	20,100	7.04	0.1421			
2000	17,600	19,600	8.80	0.1136			
1500	17,600	19,100	11.73	0.0852			
1300	17,600	18,900	13.54	0.0739			
1200	17,600	18,800	14.66	0.0682			
1100	17,600	18,700	16.00	0.0625			
1000	17,600	18,600	17.60	0.0568	0		
900	17,575	18,475	19.53	0.0512	25	702.	2.896
800	17,550	18,350	21.94	0.0456	50	352.	0.546
700	17,525	18,225	25.03	0.03995	75	234.	0.370
600	17,475	18,075	29.12	0.03434	125	139.7	0.145
500	17,400	17,900	34.8	0.02874	200	87.0	1.940
450	17,350	17,800	38.60	0.02595	250	69.4	0.842
400	17,300	17,700	43.25	0.02312	300	57.7	0.761
350	17,200	17,550	49.15	0.02035	400	43.0	0.633
300	17,000	17,300	56.66	0.01765	600	28.3	0.452
250	16,750	17,000	67.00	0.01493	850	19.7	0.294
200	16,350	16,550	81.75	0.01224	1250		
150	15,800	15,950	105.30	0.00949	1800		
100	15,100	15,200	151.0	0.00662	2500		
50	13,900	13,950	278.0	0.00360	3700		

TABLE I—(*Continued*)

Table 1-2: Test by the B. of S. Compensation Permeameter

H	β	B	μ (β/H)	ρ	γ ($S = 17600$)	D (β/γ)	Log D
400	17,300		43.3	0.02311	300	57.7	1.761
350	17,150		49.0	0.02041	450	38.1	0.581
300	17,000		56.6	0.01765	600	28.3	0.452
250	16,750		67.0	0.01493	850	19.7	0.294
200	16,350		81.75	0.01224	1250	13.15	0.116
150	15,900		106.0	0.00943	1700	9.55	0.980
100	15,200		152.0	0.00658	2400	6.33	0.801
70	14,550		207.8	0.00481	3050	4.77	0.678
50	14,100		282.0	0.00355	3500	4.03	0.605
30	13,300		443.5	0.002254			
20	12,500		625.0	0.00160			
15	11,900		794.0	0.00126			
12	11,400		950.0	0.001053			
10	10,800		1080.0	0.000926			
8	9900		1240.0	0.000808			
7	9250		1320.0	0.000756			
6	8350		1390.0	0.000719			
5.5	7700		1400.0	0.000714			
5.0	7000		1400.0	0.000714			
4.5	6150		1364.0	0.000732			
4.0	5100		1285.0	0.000784			
3.5	3900		1115.0	0.000897			
3.0	2750		916.0	0.00109			
2.5	1700		680.0	0.00147			
2.0	1100		550.0	0.00182			
1.5	400		366.0	0.00375			

TABLE I—(*Continued*)

Table 1-3: Reconstruction of $\beta\,H$ and $\beta\,\mu$ curves according to the law $\log \gamma = f - g\,H$

Note: The values of γ for required values of H are read from the graph γ H (Fig. 1-7), for range of H 50 to 400, and by extrapolation up to H 1200. The value of S taken from test, (Table 1-1).

H	γ	S	β	μ	ρ
30	4080	17,600	13,520	450.7	0.00222
50	3500		14,100	282.0	0.003548
70	3050		14,550	208.0	0.00481
100	2480		15,120	151.2	0.00662
150	1740		15,860	105.7	0.00946
200	1220		16,380	81.9	0.01221
250	870		16,730	66.9	0.01495
300	600		17,000	56.7	0.01764
350	425		17,175	49.1	0.02038
400	300		17,300	43.3	0.02310
450	210		17,390	38.7	0.02585
500	150		17,450	34.9	0.02866
600	70		17,530	29.2	0.03425
700	35		17,565	25.1	0.03984
800	20		17,580	21.9	0.04569
900	10		17,590	19.5	0.0513
1000	0		17,600	17.6	0.0568
1100	0		17,600	16.0	0.0625
1200	0		17,600	14.7	0.06805

TABLE II

Hypothetic curve for Frolich's law and cognate equations

$\beta = S - \gamma = S - S\,h/(H + h)$ Frolich's law (equation 15)

$\mu = S/(H + h) : \rho = (H + h)/S;$

$D = H/h = a\,H$

$d\beta/dH = \gamma/(H + h)$

Note: In the following computations, the value of S is taken from Table 1; the value of h is so selected as to give for $H = 200$ the same value of β as in Table 1.

(1) Determination of constants.

H	β	S	γ	μ	a	h	$S.h$
200	16,350	17,600	1250	81.75	.0654	15.29	269,000

(2) Computation of β and other cognate quantities.

H	$H + h$	$S.h$	γ $= S\,h/(H + h)$	S	β	μ	ρ	D (β/γ)	$d\beta/dH$ $=\gamma/(H + h)$
0	15.3	26,900	17,600	17,600		1150.	0.000869	0.	
10	25.3		10,630		6,970	697.	0.001435	0.656	420
25	40.3		6680		10,920	437.	0.002290	1.635	166.
50	65.3		4120		13,480	270.	0.00371	3.27	63.1
100	115.3		2330		15,270	152.7	0.00655	6.54	20.2
150	165.3		1630		15,970	106.5	0.00939	9.82	9.87
200	215.3		1250		16,350	81.75	0.01225	13.08	5.81
250	265.3		1015		16,585	66.3	0.01508	16.35	3.90
300	315.3		855		16,745	55.8	0.01792	19.63	2.71
400	415.3		650		16,950	42.4	0.02360	26.17	1.565
500	515.3		520		17,080	34.15	0.02920	32.7	1.010
600	615.3		440		17,160	28.6	0.03500	39.25	0.715
700	715.3		375		17,225	24.6	0.04065	45.80	0.526
800	815.3		330		17,270	21.58	0.04635	52.34	0.405
900	915.3		295		17,305	19.23	0.05200	58.9	0.322
1000	1015.3		265		17,335	17.33	0.05770	65.4	0.261

TABLE III

Curve for transformation ratio for standard steel for range $H = 50$ to 400

METHOD OF TEST

1. **Material:** Several rings were punched from a single sheet of standard sheet steel .014 in. From these punchings several rings were made of the same weight, and similarly wound for preliminary test. Two rings were selected having as nearly as possible the same magnetization curve.

2. **Windings:** The two selected rings were now wound for the differential test; the potential coils were wound next to the iron and were of about 500 turns of fine wire uniformly wound, each ring having the same number of turns. The magnetizing coil was made of a cable of twenty conductors, connected in series, with a tap at each joint. This scheme of winding permits a change of magnetizing value from any predetermined value to higher values in small and definite equal steps.

3. **Scheme of wiring:** The two magnetizing coils of the two rings were connected in series; the two potential coils were also connected in series. In all respects the scheme of wiring was the same as the usual scheme for tests on toroids. Two dial switches were provided for control of number of windings. One of the rings was used for measurement and the other for compensation. The potential coil of the compensating ring was provided with a supplementary potential coil of few turns and a fractionizing shunt for exact adjustment of the compensating interlinkage. (For wiring diagram, See Fig. 3-0.)

4. **Test procedure for magnetization curve:** The magnetization coils of the compensating ring and the test ring are tested alone according to the usual procedure. The galvanometer is calibrated to read 100 gausses per millimeter. Assuming that an error in reading the galvanometer cannot exceed half millimeter, it follows that the relative error does not exceed 50 gausses.

5. **Test procedure for differential test:** The galvanometer is calibrated to read a flux density of one gauss per millimeter of deflection. The dial switches of both rings are set at 10 each; the magnetizing current is adjusted to give $H = 50$, 100 or 200. On reversing the magnetizing current, there should be no deflection, that is, if the two rings are exactly alike; but as there is always a slight difference, a small deflection is produced in all cases, which can be eliminated by a careful adjustment of the supplementary coil and fractionizing shunt. The dial switch of the test coil is now set at No. 11. On reversing the current, a deflection is produced which represents the increment of flux density in the test ring. Other details of test procedure are too obvious to need explanation.

6. **Limits of error:** The relative error of any individual point in relation to the whole curve does not exceed 0.5 gauss (corresponding to half millimeter of deflection) and is probably much less for small deflection, which is capable of being read correctly to $^1/_5$ of a millimeter. The whole curve may, however, be in error by any amount not exceeding 50 gausses.

In interpreting the curve, it is permissible to assume the curve to be straight enough for extrapolation, and yet the error in the indicated value of S may be as high as 50 gausses, even though the plotted curve were strictly a straight line.

Note: The original measurements were made in terms of H and apparent B; the true value of B was computed by making correction for space factor.

TABLE III—(*Continued*)

Ring, standard sheet steel 0.014 in.
Diameter 3.5 in., 2.7 in.; mean diameter 3.1 in. = 7.885 cm.
Sectional area 1.245 sq. cm.
Table 3-1: test for incremental permeability

H_t	$H\,a$	ΔH	β mean	$\Delta\beta/\Delta H$
400	380	20	19491.5	3.61
380	360	"	19419.0	4.01
360	340	"	19335.5	4.41
340	320	"	19235.5	4.86
320	300	"	19130.5	5.26
300	280	"	19015.5	5.81
280	260	"	18888.0	6.61
260	240	"	18752.5	7.16
240	220	"	18602.0	7.86
220	200	"	18442.0	8.61
200	190	10	18316.0	9.06
190	180	"	18223.5	9.56
180	170	"	18125.5	9.96
170	160	"	18020.5	10.36
160	150	"	17913.0	11.16
150	140	"	17798.0	11.76
140	130	"	17678.0	12.36
130	120	"	17555.0	13.01
120	110	"	17425.0	13.66
110	100	"	17287.5	14.46
100	95	5	17170.0	15.06
95	90	"	17084.0	15.66
90	85	"	16997.0	16.26
85	80	"	16906.0	16.86
80	75	"	16810.0	17.66
75	70	"	16709.0	18.26
70	65	"	16622.0	19.26
65	60	"	16516.0	20.26
60	55	"	16387.0	21.26
55	50	"	16271.0	22.86

$S = 20250$ by extrapolation. (see Fig. 3-1)

TABLE III-II

Curve for Incremental Reluctivity for Standard Steel for Range $H = 300$ to 1000 g

This test is similar to that of test table 3-1, except that the increment ΔH is in each case one-twentieth part of H. The magnetizing coil of the test ring was made up of two parts of 700 turns, and 35 turns; the first section alone produces the magnetizing force H_1, and the two sections produce the force H_2. The compensating ring has the magnetizing coil of 700 turns, which also gives the magnetizing force H_1. The potential coils are connected as in test 3-1, giving the value ΔB. Each ring is also equipped with a compensative bakelite ring (see table 4-1) which gives the result in terms of $\Delta \beta$ instead of ΔB. The values ρ', log ρ' and $\sqrt{\rho'}$ are obtained by computation.

TABLE III-II—(*Continued*)

Ring same as in Table 3-1
Windings same as in Table 4
Scheme of wiring same as in Fig. 3-0

H_1	H_2	H mean	ΔH	$\Delta \beta$	μ'	ρ'	$\sqrt{\rho'}$
300	315.	307.5	15.	80	5.33	0.1785	0.433
325	341.3	333.1	16.25	79	4.62	0.2055	0.453
350	367.5	359.	17.5	71	4.057	0.2464	0.4967
375	393.7	384.	18.75	67	3.555	0.2813	0.5302
400	420.	410.	20.	59	2.951	0.3422	0.582
425	447.	435.5	21.25	53	2.494	0.4010	0.6334
450	473.	461.	22.5	42	1.867	0.536	0.7322
500	525.	512.5	25.	31	1.240	0.8064	0.898
550	578.	564.	27.5	22	0.800	1.250	1.117
600	630.	615.	30.	15	0.500	2.00	1.414
650	683.	666.	32.5	10	0.3077	3.25	1.802
675	709.	689.	33.75	8	0.237	4.22	2.055
700	735.	718.	35.	6	0.1714	5.834	2.413
750	788.	768.	37.5	4	0.1066	9.38	3.060
800	840.	820.	40.	2	0.0500	20.0	4.470
850	893.	871.	42.5	0	0	inf.	inf.
900	946.	922.	45.	0	0	"	"
950	998.	974.	47.5	0	0	"	"
1000	1050.	1025.	50.	0	0	"	"

Notes:

$H_2 = H_1 \times 1.05$
H mean $= H_1 \times 1.025$
$\Delta H = H_1 \times 0.05$
$\mu' = \Delta \beta / \Delta H$
$\rho' = \Delta H / \Delta \beta$

TABLE III-II—(*Continued*)

Test data in Table 3-1; further computation to determine relation of ρ' to H

H	H a	ΔH	$\Delta \beta / \Delta H$	ρ' $(= \Delta H / \Delta \beta)$	H Mean	$\sqrt{\rho'}$
400	380	20	3.61	0.277	390	0.516
380	360	"	4.01	0.2493	370	0.499
360	340	"	4.41	0.2268	350	0.4765
340	320	"	4.86	0.2059	330	0.454
320	300	"	5.26	0.1890	310	0.435
300	280	"	5.81	0.1721	290	0.415
280	260	"	6.61	0.1513	270	0.389
260	240	"	7.16	0.1396	250	0.3735
240	220	"	7.86	0.1272	230	0.357
220	200	"	8.61	0.1162	210	0.341
200	190	10	9.06	0.1104	195	0.332
190	180	"	9.56	0.1046	185	0.3235
180	170	"	9.96	0.1004	175	0.317
170	160	"	10.36	0.0964	165	0.3105
160	150	"	11.16	0.0896	155	0.2995
150	140	"	11.76	0.0850	145	0.2915
140	130	"	12.36	0.0809	135	0.2845
130	120	"	13.01	0.0768	125	0.277
120	110	"	13.66	0.0732	115	0.271
110	100	"	14.46	0.0691	105	0.263
100	95	5	15.06			
95	90	"	15.66			
90	85	"	16.26			
85	80	"	16.86			
80	75	"	17.66			
75	70	"	18.26			
70	65	"	19.26			
65	60	"	20.26			
60	55	"	21.26			
55	50	"	22.86			

$S = 20{,}250$ by extrapolation (see Fig. 3)

TABLE IV

Magnetization Curve for Standard Steel

The ring used for this test is the same as the test ring in Table 3, Fig. 3. The ring was unwound, and rewound with heavier wire to obtain a magnetizing force of $H = 1000$. In order to eliminate space factor error a compensative ring of bakelite of the same size as the test ring, and wound with the same number of turns of same size wire, was used as a companion ring. The two rings were then treated as a single ring and wound with a common magnetizing coil. During test, the two potential coils of the test ring and the compensative ring are connected in opposition; this arrangement compensates for the entire spatial flux inside of the potential coil of the test ring, and therefore serves the double purpose (1) of eliminating the space factor error and (2) of deducting the spatial flux in the magnetic material. With this arrangement, the galvanometer deflections indicate directly the intrinsic flux density. In testing for saturation value, this method of compensation has the further advantage that the measurement of β is not affected by errors in measurement of the magnetizing current; the adjustment of the current need not be accurate; this permits rapidity of measurement and prevents excessive heating. Without this facility, measurement of saturation value in toroid rings would have been impracticable except in the case of permalloy and other similar material of low magnetic hardness. The following is the result of the test:

TABLE IV—(*Continued*)

Ring Standard Sheet Steel
(Same ring as in Table 3)
Table 4-1: Test Data

H	β	μ	ρ	γ	D $(= \beta/\gamma)$
1000	20,200	20.2	0.0495	0	
900	20,200	22.4	0.04465	0	
800	20,200	25.3	0.0395	0	
750	20.200	26.9	0.03715	0	
700	20,200	28.9	0.03465	0	
650	20,200	31.1	0.03215	0	
600	20,150	33.1	0.02980	50	403
550	20,100	36.5	0.02748	100	201
500	20,000	40.0	0.02500	200	100
450	19,900	44.2	0.02260	300	66.3
400	19,750	49.4	0.02025	450	43.9
350	19,550	55.8	0.01767	650	30.05
300	19,250	64.2	0.01558	950	20.25
250	18,900	75.6	0.01323	1300	14.52
200	18,450	92.3	0.01084	1750	10.55
150	17,900	119.4	0.00838	2300	7.78
100	17,200	172.0	0.00581	3000	5.74
90	17,050	189.5	0.00528	3150	
80	16,900	211.0	0.00474	3300	
60	16,500	275.0	0.00364	3700	
50	16,300	326.0	0.00307	3900	
30	15,700	523.0	0.00191	4500	
20	15,200	760.0	0.001315		
15	14,800	986.0	0.001013		
10	14,200	1420.0	0.000704		
8	13,700	1712.0	0.000584		
6	12,900	2150.0	0.000465		
5	12,000	2400.0	0.0004165		
4	11,250	2817.0	0.0003555		
3	9900	3300.0	0.0003030		
2.5	8600	3440.0	0.0002905		
2.0	6600	3300.0	0.0003030		
1.5	4100	2735.0	0.000367		
1.2	2800	2335.0	0.000429		
1.0	1600	1600.0	0.000625		
0.8	1000	1250.0	0.000800		
0.6	500	833.0	0.001200		

TABLE IV—(*Continued*)

Table 4-2: Reconstruction curve according to Frolich s law on the basis of two points $H = 300$ & 500, (*i. e.* Yensen's region approximate)

(1) Determination of constants S and h.

H	β	μ	$\Delta\mu$	$\Delta\beta$	a	h	$\gamma\ (= h\,\mu)$	S	$S\,h$	$\alpha\ (= h/S)$
500	2000	40	24.17	750	0.03224	31	1240	21,240	659,000	0.00146
300	19,250	64.17					1990	21,240		

(2) Reconstruction of curve.

H	$H + h$	$S\,.\,h$	γ $= S\,h/(H + h)$	S	β	μ	ρ	$d\,\beta/d\,H$ $= \gamma/(H + h)$
0	31							
50	81	65,900	8140	21,240	13,100			101.6
100	131		5040		16,200	162	0.00617	38.5
150	181		3640		17,600			29.1
200	231		2860		18,380			12.37
250	281		2350		18,890			8.37
300	331		1990		19,250			6.01
350	381		1730		19,510			4.54
400	431		1530		19,710			3.55
450	481		1370		19,870			2.85
500	531		1240		20,000			2.335
600	631		1045		20,195			1.66
700	731		900		20,340			1.232
800	831		790		20,450			0.951
900	931		710		20,530			0.762
1000	1031		640		20,600	20.6	0.04855	0.620
1200	1230		535		20,705			0.435
1500	1530		430		20,810			0.281
2000	2031		325		20,915			0.161
3000	3030		220		21,020			0.0703
4000	4040		165		21,075			0.0402

TABLE IV—(*Continued*)

Table 4-3: Reconstruction of curve according to Frolich's law with constants selected for agreement at $H = 200$, and infinity

(1) Determination of constants.

H	β	S	γ	μ	h	$S\,h$
200	18,450	20,200	1750	92.25	18.97	383,000

(2) Determination of β and cognate quantities.

H	$H+h$	$S\,h$	γ	β $(S = 20{,}200)$	μ	ρ	$d\,\beta/d\,H$ $\gamma/(H+h)$
0	18.97	38,300	20,200	0	1064	0.000939	1063
50	69.		5525	14,675	293.	0.00342	86.8
100	119.		3215	16,985	169.7	0.00589	27.0
150	169.		2265	17,935	119.6	0.00836	13.4
200	219.		1750	18,450	92.25	0.01085	7.99
250	269.		1425	18,775	75.1	0.01333	5.30
300	319.		1200	19,000	63.3	0.01579	3.76
350	369.		1037	19,165	54.8	0.01827	2.81
400	419.		914	19,285	48.2	0.02074	2.18
500	519.		728	19,470	38.9	0.02570	1.42
600	619.		619	19,580	32.6	0.03060	1.00
700	719.		533	19,665	28.1	0.03560	0.742
800	819.		468	19,730	24.7	0.04050	0.571
900	919.		417	19,785	22.0	0.04550	0.454
1000	1019.		376	19,825	19.8	0.0504	0.369
1200	1219.		314	19,885			0.258
1500	1519.		252	19,950			0.166
2000	2020.		190	20,010			
3000	3020.		120	20,070			
5000	5020.		74	20,125			

TABLE IV—(*Continued*)

Table 4-4: Reconstruction curve according to equation

$\text{Log}\,\gamma = f - g\,H$

On the basis of agreement for $H = 300$, 500 and infinity

(1) Determination of constants f and g

H	H	β	S	γ	Log γ	Δ Log γ	g	$g\,H$	f
300	200	19,250	20,200	950	2.9775	0.6765	0.003383	1.0149	3.9924
500		20,000		200	2.3010			1.6915	3.9925

(2) Determination of β

H	g	$g\,H$	f	Log γ	γ	S	β	μ	ρ
100	0.003383	0.3383	3.9924	3.654	4510	20,200	15,690	156.9	0.00637
150		0.507		3.485	3055		17,145	114.3	0.00874
200		0.676		3.316	2070		18,130	90.7	0.01103
250		0.846		3.146	1400		18,800	75.2	0.01331
300		1.015		2.977	950		19,250	64.2	0.01558
350		1.185		2.807	640		19,560	55.9	0.01790
400		1.354		2.638	435		19,765	49.4	0.02024
450		1.527		2.465	290		19,910	44.25	0.02260
500		1.692		2.300	200		20,000	40.0	0.02500
550		1.862		2.130	135		20,065	36.5	0.02740
600		2.031		1.961	90		20,110	33.5	0.02985
650		2.200		1.792	60		20,140	31.0	0.03228
700		2.370		1.622	40		20,160	28.8	0.03478
750		2.540		1.452	30		20,170	26.9	0.03720
800		2.705		1.287	20		20,180	25.2	0.03965
900		3.047		0.945	10		20,190	22.4	0.04460
1000		3.382		0.610	0		20,200	20.20	0.04950
1200		4.060		-1.830	0		20,200	16.8	0.0594
inf.							20,200	0	

TABLE V

Insensitiveness of Reluctivity Curve

Table 5-1: Dr. Yensen's Experiment: A. I. E. E., 1920, p. 821

H	β	μ	ρ	
20	16,600	830	0.0012	
100	18,330	183.3	0.0055	
200	18,670	93.35	0.0107	Error in β, probably typographic
300	20,430	68.1	0.0147	Error in β, probably observational
400	21,030	52.6	0.0190	
500	21,310	42.6	0.0235	
200	19,670	98.35	0.01016	Corrected by guess
300	20,500	68.33	0.01463	Corrected by reference to β H curve

Table 5-2: Dr. Kennelly's Curve; A. I. E. E., 1891, p. 509

B	μ $(=B/H)$	H	β	ρ	μ (β/H)	
0	40					
100	50	2.	98	0.0204	49	
420	100	4.2	416	0.01010	99	
720	150	4.8	715	0.006712	149	
1100	200	5.5	1095	0.005028	199	
1500	225	6.6	1493	0.004465	224	
2080	200	10.4	2070	0.005025	199	
2500	150	16.66	2483	0.006708	149	
2960	100	29.6	2930	0.01010	99	
3500	70	50.0	3450	0.01450	69	
3920	50	78.4	3482	0.02040	49	
4500	32	146.0	4354	0.03353	29.8	H, β, ρ, and β/H incorrect
5000	22	227.0	4773	0.04756	21	
4500	32	140.6	4360	0.03225	31	H, β, ρ, and β/H corrected

TABLE VI

Electrolytic Iron: Sample *E-B*

Table 6-1: Test for β H Curve

Method of test same as in Table 4

H	β	μ	ρ	γ	D $(=\beta/\gamma)$	log D
1160	21,000	18.1	0.0552	0		
1000	21,000	21.1	0.0476	0		
900	21,000	23.3	0.0429	0		
850	21,000	24.7	0.0405	0		
800	21,000	26.3	0.0381	0		
750	20,980	28.0	0.03575	20	1050	3.021
700	20,950	30.0	0.0334	30	700	2.845
650	20,900	32.2	0.0311	100	209	2.320
600	20,850	34.7	0.0288	150	139	2.143
550	20,780	37.8	0.0265	220	95	1.978
500	20,650	41.3	0.0242	350	59	1.771
450	20,500	45.6	0.02195	500	41	1.613
400	20,300	50.7	0.0197	700	29	1.462
350	20,000	57.2	0.0175	1000	20	1.301
300	19,700	67.7	0.01525	1300	15.15	1.180
250	19,300	77.2	0.01295	1700	11.35	1.055
200	18,800	94.2	0.01065	2200	8.5	0.930
150	18,300	122.0	0.00820	2700	6.8	0.832
100	17,500	175.0	0.00572	3500	5.0	0.698
50	16,650	333.0	0.00300	4350	3.84	0.585
20	15,900	795.0	0.001258	5100	3.12	0.495
10	15,400	1540.0	0.000649	5600	2.75	0.440
3	14,250	4750.0	0.000211			
1.0	12,500	12500.0	0.0000800			
0.70	11,500	16400.0	0.0000608			
0.60	10,650	17750.0	0.0000563			
0.55	10,000	18200.0	0.0000550			
0.50	9250	18500.0	0.0000541			
0.45	8300	18450.0	0.0000542			
0.40	6950	17400.0	0.0000575			
0.35	5500	15700.0	0.0000636			
0.30	3800	12650.0	0.0000789			
0.25	2550	10200.0	0.0000980			
0.20	1450	7250.0	0.000138			

TABLE VI—(*Continued*)

Table 6-2: Reconstruction of curve (Table 6-1) by the law log $\gamma = f - g\,H$

Note: The values of γ are read from the straight line graph of Fig. 6-3.

H	γ	S	β	μ
50	7800	21,000	13,200	264
100	5500		15,500	155
150	3900		17,100	114
200	2750		18,250	91.25
250	1920		19,180	76.7
300	1350		19,650	65.5
350	960		20,040	57.3
400	680		20,320	50.8
450	480		20,520	45.6
500	340		20,660	41.3
550	240		20,760	37.7
600	170		20,830	34.7
650	120		20,880	32.1
700	85		20,915	29.9
750	60		20,940	27.9
800	40		20,960	26.2
850	30		20,970	24.7
900	20		20,980	23.3
1000	10		20,990	21.0
1100	0		21,000	19.1
1200	0		21,000	

TABLE VII

Nickel Pure: 99.5 per cent Nickel: Ring No. *N-D*

Test 7-1: test for β H curve

H	β	μ	ρ	γ	D (β/γ)
375	5810	15.5	0.0645		
270	5810	21.5	0.0465		
240	5810	24.2	0.0413		
210	5790	27.6	0.03625	20	289.5
180	5750	32.0	0.03122	60	95.8
150	5700	38.0	0.02631	110	51.8
120	5650	47.0	0.02129	160	35.3
105	5600	53.3	0.01875	210	26.68
90	5550	61.6	0.01625	260	21.34
75	5475	73.0	0.01370	325	16.85
60	5350	89.2	0.01121	460	11.63
45	5200	115.5	0.00866	610	8.52
30	5000	166.8	0.00600	810	6.17
15	4600	306.5	0.00326	1210	3.80
7.5	4100	547.0	0.00183	1710	2.41
6.0	3900	650.0	0.00154	1910	2.04
4.5	3600	800.0	0.00125	2210	1.63
3.0	3050	1015.0	0.000986	2760	1.11
2.5	2750	1100.0	0.000910		
2.0	2400	1200.0	0.000833		
1.5	1900	1267.0	0.000790		
1.0	1200	1200.0	0.000833		
0.8	850	1060.0	0.000944		
0.6	450	750.0	0.001333		
0.4	225	560.0	0.001785		
0.2	100	500.0	0.002000		

TABLE VII—(*Continued*)

Table 7-2: Reconstruction of curve (Table 7-1) by the law log $\gamma = f - g\,H$

Note: Values of γ are read from the straight line graph of Fig. 7-3.

H	γ	S	β	μ
20	860	5810	4950	247.5
40	620		5190	129.6
60	440		5370	89.5
80	320		5490	68.8
100	230		5580	55.8
120	160		5650	47.1
140	120		5690	40.6
160	80		5730	35.8
180	60		5750	31.9
200	45		5765	28.8
220	30		5780	26.2
240	20		5790	24.1
260	15		5795	22.3
280	10		5800	20.7
300	0		5810	19.35
320	0		5810	18.15
340	0		5810	17.10
360	0		5810	16.15
380	0		5810	15.3
400	0		5810	14.5

TABLE VIII

Cobalt Pure: (Percentage composition not specified. This ring is the same that was used by Mr. Ball in his well-known experiment published in G. E. *Review*, 1916, p. 379)

H	β	μ	ρ	
1600	13,000	8.13	0.1232	
1400	12,600	9.00	0.1111	
1200	12,100	10.08	0.0992	
1000	11,600	11.60	0.0862	
800	10,950	13.70	0.0730	
700	10,550	15.07	0.0664	
600	10,200	17.00	0.0588	
500	9700	19.40	0.0516	
400	9050	22.63	0.0442	
360	8800	24.45	0.0409	
320	8500	26.55	0.03775	
280	8200	29.30	0.03411	
240	7800	32.5	0.03075	
200	7300	36.5	0.02740	
160	6750	42.2	0.02370	
120	6000	50.0	0.02000	
100	5550	55.5	0.01802	
80	4950	61.8	0.01618	
60	4200	70.0	0.01429	
40	3200	80.0	0.01250	
30	2500	83.3	0.01200	
20	1650	82.5	0.01213	
10	600	60.0	0.01666	
1200	12,130	10.1	0.0989	Corrected by reference to the β H curve
700	10,580	15.11	0.0662	

TABLE IX

Nickel—Iron Alloy. Ring No. R-309

Table 9-1: Test for β H curve

H	β	μ	ρ	γ	β/γ
346	12,250	35.4	0.02825		
277	12,250	44.2	0.02260		
208	12,250	58.9	0.01700		
173	12,210	70.6	0.01416	40	305
138.5	12,170	88.0	0.01136	80	152
104.	12,080	116.1	0.00860	170	71
90.	12,060	134.0	0.00746	190	63.4
79.5	11,900	149.8	0.00668	350	34
69.3	11,730	169.5	0.00590	520	22.5
62.3	11,550	185.5	0.00539	700	16.5
55.4	11,390	206.0	0.00485	860	13.2
48.5	11,080	228.5	0.00438	1170	9.46
41.5	10,690	258	0.00388	1560	6.85
34.6	10,250	296	0.00338	2000	5.12
27.7	9600	348	0.00287	2650	3.43
20.8	8830	425	0.00235	3420	2.58
17.3	8370	484	0.00207	3850	2.17
13.85	7820	564	0.001775	4430	1.76
10.4	7210	693	0.001442	5040	1.43
8.65	6780	784	0.001275		
6.93	6380	921	0.001085		
5.54	6020	1085	0.000921		
4.15	5630	1355	0.000738		
2.77	5120	1850	0.000540		
2.08	4780	2300	0.000435		
1.39	4370	3140	0.000318		
0.693	3730	5380	0.000186		
0.415	2950	7100	0.000143		
0.388	2820	7300	0.000137		
0.346	2610	7540	0.000133		
0.319	2330	7300	0.000137		
0.277	1870	6760	0.000148		
0.208	950	4560	0.000219		
0.139	170	1220	0.000820		

TABLE IX—(*Continued*)

Table 9-2: Reconstruction of curve by the law:

$$\operatorname{Log} \gamma = f - g H$$

Note: Values of γ, are read from the straight line graph of Fig. 9-2.

H	γ	S	β	μ
10	5040	12,250	7210	721
15	4150		8100	540
20	3450		8800	440
30	2350		9900	330
40	1600		10,650	266
50	1100		11,150	223
60	760		11,490	191.5
70	530		11,720	167.4
80	360		11,890	148.6
90	240		12,010	133.4
100	165		12,085	120.8
110	110		12,140	110.4
120	80		12,170	101.4
130	55		12,195	93.8
140	35		12,215	87.3
150	25		12,225	81.6
160	17		12,233	76.2
170	10		12,240	72.0
180	0		12,250	68.1
190	0		12,250	64.5
200	0		12,250	61.2

TABLE X

Cobalt Iron Alloy: 20 per cent Cobalt

Table 10-1: Test for β H curve

H	β	μ	ρ	γ	D
1500	23,200	15.45	0.0646		
1350	23,200	17.18	0.0582		
1200	23,200	19.33	0.0517		
1050	23,200	22.1	0.0453		
900	23,200	25.8	0.0388		
750	23,170	30.9	0.0324	30	772
660	23,130	35.1	0.0285	70	331
600	23,100	38.5	0.0260	100	231
540	23,050	42.7	0.02342	150	153.5
510	23,000	45.1	0.02217	200	115
480	22,950	47.8	0.02092	250	92
450	22,880	50.8	0.01969	320	71.5
420	22,800	54.3	0.01842	400	55.7
390	22,700	58.2	0.01719	500	45.5
360	22,550	62.6	0.01596	650	34.7
330	22,350	67.7	0.01476	850	26.3
300	22,150	73.8	0.01355	1050	21.1
270	21,900	81.2	0.01233	1300	16.7
240	21,650	90.2	0.01109	1550	13.9
225	21,450	95.3	0.01050	1750	12.25
210	21,300	101.4	0.00986	1900	11.20
195	21,150	108.5	0.00922	2050	10.30
180	20,950	116.4	0.00859	2250	9.3
165	20,730	125.6	0.00796	2470	8.4
150	20,550	137.0	0.00730	2650	7.76
135	20,300	150.3	0.00665	2900	7.00
120	20,050	167.0	0.00598	3150	6.37
105	19,800	187.5	0.00530	3400	5.82
90	19,500	217.0	0.004615	3700	
67.5	19,050	283.0	0.0354	4150	
45.0	18,400	409.0	0.002445	4800	
30	17,750	592	0.00169		
24	17,450	727	0.001375		
18	16,950	942	0.001062		
15	16,550	1103	0.000906		
12	15,900	1325	0.000755		
9	14,850	1650	0.000605		
7.5	14,050	1875	0.000534		
6.75	13,350	1975	0.000505		
6.0	12,800	2130	0.000469		
5.4	12,100	2240	0.000446		
4.8	11,300	2355	0.000425		
4.5	10,850	2410	0.000415		
4.2	10,400	2475	0.000404		
3.9	9750	2500	0.000400		
3.6	9200	2555	0.000391		
3.3	8500	2575	0.000388		
3.0	7750	2580	0.000387		
2.7	6800	2520	0.000397		
2.4	6000	2500	0.000400		
2.1	4950	2360	0.000424		
1.8	3800	2110	0.000473		
1.5	2650	1765	0.000566		
1.2	1600	1330	0.000750		
0.9	800	890	0.001125		
0.6	350	585	0.001710		

TABLE X—(*Continued*)

Table 10-2: Reconstruction of curve by the law $\log \gamma = f - g H$

Values of γ read from straight line graph of Fig. 10-2

H	γ	S 23,200	β	μ
.100	5100		18,100	181
150	3450		19,750	131.6
200	2300		20,900	104.6
250	1550		21,650	86.6
300	1050		22,150	73.8
350	700		22,500	64.3
400	470		22,730	56.8
450	320		22,880	50.8
500	210		22,990	45.7
550	140		23,060	41.9
600	95		23,105	38.5
650	65		22,135	35.6
700	45		22,155	33.1
750	30		23,170	30.9
800	20		23,180	28.95
850	13		23,187	27.25
900	10		23,190	25.8
1000	0		23,200	23.2
1100	0		23,200	21.1
1200	0		23,200	19.3

TABLE XI

Cobalt—(Table 8)

Reconstruction by the equation

$\text{Log}\ \gamma = f - g H$

Table 11-1: Approximate determination of S according to equation $d\beta/dH = a\gamma$

H	β	$\Delta \beta$	ΔH	$\Delta \beta / \Delta H$	B Mean
1600	13,000				
		400	200	2.00	12,800
1400	12,600				
		470	200	2.35	12,365
1200	12,130				
		530	200	2.65	11,865
1000	11,600				
		650	200	3.25	11,275
800	10,950				
		370	100	3.70	10,765
700	10,580				
		380	100	3.80	10,340
600	10,200				

$S = 15{,}350$ by extrapolation; see Fig. 11-1

As the points for the curve β/H as functions of β do not form a very good straight line, the extrapolation is a matter of personal judgment.

TABLE XI—(*Continued*)

Table 11-2: Reconstruction of curve by the law $\text{Log}\ \gamma = f - g H$, by graphic method

Values of γ read from graph of Fig. 11-2

H	γ	S	β
100	7600	15,300	7700
200	7000		8300
300	6500		8800
400	6000		9300
500	5500		9800
2000	1700		13,600
2500	1150		14,150
3000	770		14,530
4000	350		14,950
5000	160		15,140
6000	70		15,230
7000	30		15,270
8000	15		15,285
9000	5		15,295
10,000	0		15,300

TABLE XI—(*Continued*)

Table 11_3: Reconstruction of curve by the law $\log \gamma = f - g H$, by the analytical method

Determination of f and g

H	β	S	γ	Log γ	Δ Log γ	g	$g H$	f
600	10,200	15,300	5100	3.7075	0.346	.000346	0.2076	3.9155
1600	13,000		2300	3.3615			0.5536	3.9155

Table 11-4: Reconstruction and extrapolation of the βH curve

H	g	$g H$	f	log γ	γ	S	β	μ
100	0.000346	0.0346	3.9155	3.8805	7600	15,300	7700	77
200		0.0692		3.8459	7010		8290	41.5
300		0.1038		3.8113	6480		8820	29.4
400		0.1384		3.7767	5980		9320	23.3
500		0.1730		3.7421	5522		9780	19.6
600		0.2076		3.7075	5100		10,200	17.0
700		0.2422		3.6729	4710		10,590	15.1
800		0.2768		3.6383	4350		10,950	13.7
1000		0.3460		3.5691	3708		11,592	11.6
1200		0.4152		3.4999	3160		12,140	10.1
1400		0.4844		3.4307	2695		12,605	9.0
1600		0.5538		3.3615	2300		13,000	8.13
2000		0.692		3.2231	1670		13,630	6.82
2500		0.865		3.0501	1125		14,175	5.67
3000		0.6038		2.8771	755		14,545	4.85
4000		1.384		2.5311	340		14,980	3.74
5000		1.73		2.1851	160		15,140	3.03
6000		2.075		1.8401	70		15,230	2.54
7000		2.422		1.4903	30		15,270	2.18
8000		2.768		1.1461	15		15,285	1.91
9000		3.115		1.8001	5		15,300	1.70
10,000		3.460			0		15,300	1.53
12,000		4.152			0		15,300	1.28

TABLE XI—(*Continued*)

Cobalt Pure (Table 8)

Table 11-4: Reconstruction of curve by Frolich's Law on the basis of two points at $H = 1200$ and 1600

(1) Determination of constant h and S.

H	β	μ	$\Delta \mu$	β	h	γ μh	S $(\beta + \gamma)$	γ	$S . h$
1600	13,000	8.125	1.985	870	438.3	3560	16,580	3560	7,260,000
1200	12,130	10.11				4430	16,560	4430	

(2) Reconstruction of βH curve.

H	$H + h$	$S . h$	γ	S	β
		7,260,000		16,560	
100	538		13,500		3060
200	638		11,380		5180
300	738		9840		6720
400	838		8670		7890
500	938		7740		8820
600	1038		7000		9560
700	1138		6380		10,180
800	1238		5860		10,700
1000	1438		5050		11,510
1200	1638		4430		12,130
1400	1838		3950		12,610
1600	2040		3560		13,000
2000	2440		2980		13,580
2500	2940		2470		14,090
3000	3440		2110		14,450
4000	4440		1635		14,925
5000	5440		1335		15,225
7000	7440		975		15,585
10,000	10,440		695		15,865
20,000	20,440		355		16,205
30,000	30,440		240		16,320
40,000	40,440		180		16,380

TABLE XII

Electrolytic Iron

Sample: Yensen-rod 1413-*A*

Test made at the Bureau of Standards

Table 12-1: (Test No. Tem. 41893)

Original data for *H* and *B* received through courtesy of Dr. T. D. Yensen, and computation of β, μ and γ made by S. L. Gokhale

H	B	β	μ	ρ	γ	D
0.2	460		2300.	0.000435		
0.4	4600		11500.	0.000087		
0.5	7300		14600.	0.0000685		
1.0	11,150		11150.	0.0000985		
2.0	14,230		7115.	0.000141		
4.0	15,770		3940.	0.000254		
20.0	17,020	17,000	850.	0.00118	4500	3.78
100.	18,700	18,600	186.	0.00537	2900	6.40
200.	19,960	19,760	99.	0.0101	1740	11.34
400.	21,320	20,920	52.3	0.0191	580	36.1
600.	21,900	21,300	35.5	0.0282	200	105.5
1000.	22,500	21,500	21.5	0.0465	0	
1500.	23,000	21,500	14.3	0.0700	0	
2000.	23,500	21,500	10.7	0.0935	0	
2500.	24,000	21,500	8.6	0.116	0	

TABLE XII—(*Continued*)

Table 12-2: Reconstruction of the curve by the Logarithmic law
Note: Values of γ read from the straight line graph of Fig. 12-3.

H	γ	S	β	μ
20	4500	21,500	17,000	850.
50	3850		17,650	353.
100	2930		18,580	185.8
150	2240		19,260	128.4
200	1710		19,790	98.95
250	1310		20,190	80.4
300	1000		20,500	68.3
350	760		20,740	59.3
400	585		20,915	52.3
450	450		21,050	46.8
500	340		21,160	42.3
600	200		21,300	35.5
700	115		21,385	30.5
800	70		21,430	26.8
900	40		21,460	23.85
1000	20		21,480	21.5
1100	10		21,490	19.5
1200	0		21,500	17.9
1500	0		21,500	14.3
2000	0		21,500	10.7
2500	0		21,500	8.6

TABLE XII—(*Continued*)

Electrolytic Iron (Yensen)

Table 12-3: Comparison of two samples of nearly identical magnetic character

Tests at Bureau of Standards

Test No. Tem. 41893

	Rod No. 1413-*A* (Annealed)				Rod No. 1413-*B* (Unannealed)			
H	B	β	μ	ρ	B	β	μ	ρ
2500	24,000	21,500	8.6	0.1162	24,100	21,600	8.64	0.1157
2000	23,500	21,500	10.7	0.0935	23,600	21,600	10.8	0.0926
1500	23,000	21,500	14.3	0.0700	23,100	21,600	14.4	0.0694
1000	22,500	21,500	21.5	0.0465	22,600	21,600	21.6	0.0463
600	21,900	21,300	35.5	0.02815	22,000	21,400	35.7	0.02800
400	21,320	20,920	52.3	0.01913	21,370	20,970	52.4	0.01910
200	19,960	19,760	99.0	0.01010	19,950	19,750	98.7	0.01013
100	18,700	18,600	186.0	0.00538	18,600	18,500	185.0	0.00541

Discussion

J. R. Craighead: In the study of any characteristic of a material for engineering purposes the first results must be obtained by measurement. From these results an empirical formula is usually developed which should be accepted until fresh data disagree, when the formula should be modified or extended to include all the data.

When knowledge of the characteristic advances sufficiently, a rational formula may be proposed. To be acceptable, the rational formula must not be inconsistent with the existing data and must meet the mental needs of engineers by representing a theory of the variation of the characteristic.

In this case the formula of Frolich with the other developments of it to which Mr. Gokhale has referred, constitutes a long step toward a rational formula. Weber's theory of molecular magnets is at present the fundamental on which magnetic formulas should be based.

Frolich's law is obtainable from Weber's theory by assigning a specific value to the distribution ratio. Mr. Gokhale's tests have shown that this value is not suitable for determining the actual performance in the region near saturation.

Consequently, Mr. Gokhale attempted to find a substitute for that value, and a new formula which would be at least equally in agreement with the Weber theory. He found this in the formula which is quoted in the paper.

Following this step he studied it in comparison with previous formulas and determined it to be a corollary of Lamont's equation, consequently basing it on Weber's theory in the same way that Lamont's equation is based. Thus the formula has a mathematical connection directly back to previous magnetic efforts and represents an effort to connect with the latest data those which were available to those who developed the earlier formulas.

It is not claimed that Mr. Gokhale's law of magnetization near saturation is the final one or that there will never be any more data which will extend it. It is possible that the fundamental Weber theory will be changed, in which case there may be radical changes in the formulas derived from it. But in establishing a step in advance of the formulas that have already been recognized, covering very much more thoroughly a wider range of data, Mr. Gokhale has made a definite advance in the art.

Hans Lippelt: The magnitude of the problem in hand is well illustrated by the fact that the author confines his work to the range of magnetization near saturation, limiting it at the lower end by a magnetizing force of $H = 300$, which is a high value for practical and industrial purposes.

In going over the experimental data given, it should be observed that no attention whatever has been paid to magnetic hysteresis. Small as it may be near saturation, it becomes noticeable and effective as the magnetization curve falls off the saturation value. The majority of the test data start with a high value of *H*, which is receding as the test proceeds. This state of affairs characterizes each of the respective β curves as part of the descending branch of a hysteresis curve. Both branches of the hysteresis curve should be subjected to test and observations recorded and presented.

The establishment of a new law of magnetization necessarily calls for conclusive evidence. With one-half of the evidence left out, the other evidence presented can hardly be called conclusive. This is said without depreciating the great merits of this exemplary piece of research work.

In connection herewith it should be observed that not all the β H curves are shown with their ordinates (or axis of ordinates) starting at the zero line. Fig. 1-8 has the first division at 13,000, and that seems to be the residual magnetism of the sample, being corroborated by Figs. 1-3 and 1-2. It is doubtful whether such a high residual magnetism will be accepted as the basis or starting value for a new law of magnetization. Strictly speaking, the maiden curve of magnetization, which goes through the origin of the system of coordinates, is the only pure representative of the true law of magnetization.

The author, in paragraph 15 of his paper, has explained the insensitive character of the reluctivity curve, and has pointed out that its apparent straightness is no reliable criterion for the course of other correlated curves. I believe it befalls the author to prove that the line of incremental permeability, Fig. 3-1, is inherently sensitive to the curvature of correlated curves. In other words, it should be proved that in spite of other correlated curves (say the β μ curve) possessing a certain *definite* curvature, the curve of incremental permeability is actually a straight line, and not merely the average value of several undulations such as are noticeable in Fig. 3-1.

The justification for such a proof will hardly be denied in view of the fact that the main equations (**29**) and (**30**), which represent one main result of the study, are derived from that line.

We also should not lose sight of the fact that the author of the paper violates his own theory (to a certain extent) when he eliminates β from equation (**21**) and reduces it to the form of (**23**) and (**26**).

Fig. 1-6 shows two straight logarithmic lines. They give rise to the question as to whether the material under test undergoes a distinct molecular change. Such a change may be either gradual and finishing, or occurring abruptly at the point of their intersection. Such a phenomenon would not be unusual with iron, which is known to suffer a change of its elasticity and strength, when under thermal stress. (Breaking strength of steel increases up to 300 deg. cent.)

Another possibility would be that the two straight lines are really the asymptotes of a curve which otherwise runs very close to its asymptotes.

It is conspicuous that no corresponding kink can be detected in the correlated *H D* curve of Fig. 1-6.

That Frolich's law permits a modification which renders it more flexible was shown by myself in a recent paper on the magnetic hysteresis curve.[1]

I believe it would be advantageous for the readers if, in Mr. Gokhale's paper, the method and wiring of the tests were illustrated by diagram.

J. E. Jackson: It is quite gratifying to find that Mr. Gokhale's saturation curve is a straight line, since it was a consideration of Weber's theory two years ago that led O. E. Charlton and myself[2] to predict that the a-c. iron losses would decrease as d-c. excitation was superposed on the core. Weber's theory states that hysteresis loss is due to friction between the molecules or electrons as they reverse their position or polarity under the influence of an alternating magnetomotive force. If this is true, a strong d-c. field should hold the molecules locked in one position so that they cannot reverse, and therefore the hysteresis loss would not appear. Also, the permeability of the specimen should approach that of air. Both of these facts were proved by actual tests; as d-c. excitation was increased the a-c. iron losses decreased.

A. E. Kennelly (communicated after adjournment): The paper is an interesting compendium of the various formulas which have been offered, at different times, to account for the phenomenon of magnetization in the magnetic metals, from the point of view of the physicist, and particularly in regard to the phenomenon of high magnetization, near saturation. It shows that the various formulas of Bosanquet, Emery, Lamont, and the speaker, are much more closely connected with each other than is generally evident or recognized, and also connected with the pioneer work of Frolich, in 1882. It shows, moreover, that none of them is quite satisfactory in the neighborhood of saturation. Near saturation, the paper advocates the formula given in the synopsis, which is a modified form of Lamont's formula.

When we consider the relatively complex and unstable molecular configurations involved in the generally accepted Ewing's theory of magnetization in iron, it would be quite surprising if any single and simple formula satisfied the entire process, from the feeblest to the most powerful magnetization. Near saturation, however, when the last dregs of latent magnetization are being evoked, an exponential relation of the type advocated in the paper seems very reasonable. That is, if we plot the latent induction γ against the magnetizing force $\mathcal{H}$, on arith-log paper, as in Fig. 1-7, we should look for a straight line, or a pair of straight lines, as saturation approaches. This relation should be carefully examined in future, with various samples of magnetic material. The technique of magnetic measurement for advanced values of magnetizing force, however, should be standardized for that purpose. If this straight-line relation between log γ and $\mathcal{H}$ is brought out, the paper will have supplied a valuable contribution.

In regard to the assumption on the fourth page of the paper about latent flux, namely, that "it is made up of two groups of flux lines in opposite directions making an algebraic total of zero lines," it seems to need some proof. The conception of each iron molecule inherently possessing its own bundle of magnetic flux, so that when they are all completely alined, the resulting flux density is determined only by the number of molecules per square cm. of cross-section, is interesting but perhaps unsafe. It seems to go beyond the needs of Ewing's theory.

In regard to my paper of 1891, the ascending straight-line law of metallic or intrinsic reluctivity there pointed out, is indeed based upon the previously published researches of Ewing, Rowland and others, as the paper states. The speaker arrived at the relation, however, from his own tests of samples of dynamo steel, for which the notes and records are still retained. It was found that when the reluctivity ρ was plotted on ordinary squared paper, as ordinates, against magnetizing force $\mathcal{H}$ as abscissas, the graph was always a pair of straight lines, one descending and then the other ascending, with an elbow connecting them, and with the maximum value of the permeability occurring at this elbow. From an engineering point of view, it appears to the speaker that this is still the simplest quantitative relation between the ordinary magnetic phenomena. From a strict physical theory standpoint, however, this pair of straight lines may be only a first approximation to a much more complicated phenomenon. It seemed more convincing, in 1891, to express the geometrical facts in relation to already published researches than in connection with new and unchecked tests.

S. L. Gokhale: I believe that some of the points brought out in the course of the discussion can be best explained by a brief history of the investigation which forms the subject of the paper under discussion.

In January 1913, I was called upon to add to our equipment a simple and reliable method for determination of saturation value of magnetic material used for engineering purposes. Prior to this date and also for some years afterwards, this determination was made by extrapolation according to Kennelly's law on the basis of data for $H = 50$ to 200. By the beginning of 1915 I had succeeded in developing a new method of measuring saturation value, (A. I. E. E. Trans., 1920, p. 819) together with a suitable instrument for that purpose, *viz.*, the saturation permeameter. (See Law of Magnetization, Table I-1.) The new method was easily recognized as the simplest method for this work but its reliability was seriously questioned because the saturation value determined by this method did not agree with the computed value by extrapolation according to Kennelly's law. (See Circular of B. of S. No. 17, p. 36.) When the result of a measurement by a new method conflicts with a law unanimously accepted by the scientific world for nearly forty years, the reliability of the new method should be questioned rather than the law; this was the generally accepted view, and I must confess that I held the same view at first. For about five years I tried unsuccessfully to detect error in the saturation permeameter, until by the beginning of 1920 I began to feel convinced that the permeameter was quite reliable and that the error was probably in the extrapolation method. In April 1920 I expressed this view in the discussion of Dr. Yensen's paper, but I could not convince him at that time. In May 1923 I was able to demonstrate the existence of the second inflection together with the left-handed curvature above that point (Fig. 4-2) by tests on toroid rings; this demonstration indicated the possibility of failure of Kennelly's law for values of β near saturation, although the point was still not proved conclusively. About this time I had also succeeded in obtaining by tests on toroid rings the curve for incremental permeability (Fig. 3-1) which suggested the linear equation of progress (equation **26**), together with the corresponding exponential law of magnetization (equation **30**). I showed these results to Dr. Steinmetz, who seemed to be well convinced by the evidence, but he suggested that a successful demonstration of the phenomenon of saturation would be much more convincing. In September 1923 I succeeded in demonstrating saturation for a toroid ring of standard sheet steel for a range of H = 650 to 1000 (Fig. 4-1.)

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[1] A. I. E. E. Journal, April, 1926, p. 355. See also Discussion A. I. E. E. Journal, August, 1926, p. 770.

[2] Losses in Iron under the Action of Superposed Alternating- and Direct-Current Excitations, by O. E. Charlton and J. E. Jackson, Trans. A. I. E. E., 1925, p. 824.

In May 1924, I showed my notes to Dr. Kennelly and solicited his criticisms. He made many valuable suggestions, the most important one being that I should secure more data on a larger number of test samples tested by approved methods. This advice has been carefully followed; in the preparation of the paper only tests on toroid rings were included for purpose of demonstration.

For purpose of data for the β H curve, the test procedure was the well-known procedure described in standard books. The curve obtained is what Mr. Lippelt calls the "maiden curve," which in every case probably goes through the origin as may be seen from the tables of data. In plotting the curve the lower part of the curve is generally omitted for the simple reason that this paper is limited to a study of the law of magnetization for values of β near saturation; the reason for the preference has already been explained.

In order to develop a sufficiently strong magnetizing force without excessive heating, a form of winding of the magnetizing coil has been developed. (See Fig. 6-0 accompanying this discussion.) This scheme of winding offers the best facilities for cooling; it was found in one case that a test at H = 1000 occupying a period of 15 sec. caused a fall in temperature from 23.4 deg. cent. to 22 deg. cent. instead of a rise, which demonstrates the cooling efficiency of this kind of winding.

Fig. 3-0 herewith gives the scheme of wiring for the test for incremental permeability and Fig. 4-0 herewith, for the β H relationship; the method of test is described fully in the introductory paragraphs in Tables III and IV.

Mr. Lippelt observes that in my investigation "no attention whatever has been paid to magnetic hysteresis." The point is correctly observed; I have concentrated my attention for the present on the form of the normal induction curve near saturation, just as he has concentrated his attention on the form of the hysteresis curve. The field of magnetic phenomena is too vast to be covered fully and satisfactorily by any single explorer; it calls for the collective effort of a large number of physicists, who are expected to divide the field into sections, each one choosing for himself the part he is best fitted to study. Lord Raleigh and Dr. Kennelly made a special study of the magnetization curve below the point of maximum permeability. Dr. Kennelly carried the upper limit of test to H = 90; Dr. Steinmetz carried it to H = 200; Dr. Yensen covered the range H = 200 to 500. When my turn came, I started where my predecessors had stopped and carried the limit to H = 1600 on toroid rings, and to H = 4000 on the saturation permeameter. The problem of hysteresis is certainly very important, and I believe we are all glad to know that Mr. Lippelt has been studying it very carefully, but I fail to see what that problem has to do with the study of the phenomenon of saturation and allied phenomena.

Incidentally, I may mention here that the β H curves in the paper under discussion are normal induction curves; they are not the descending side of the hysteresis loops, as has been assumed by Mr. Lippelt. A large part of his criticism is based on this assumption and needs no further discussion.

With reference to the comparative sensitiveness of the several types of curves, it should be remembered that this peculiarity has no reference to curves expressing the relationship of directly measured variables; it refers to derived variables only. For example, in plotting a H ρ curve the values of H are plotted directly from measurement, but the values of ρ are derived by computation from values of β and H.

The question of sensitiveness has no reference to the β H curve; it is relevant in reference to the H ρ curve; this is also true of the β μ curve because the values of μ are not obtained by direct measurement, but are always derived by computation from

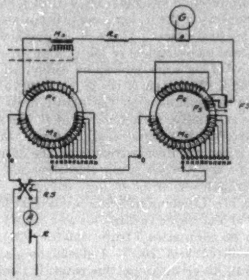

Fig. 3-0—Scheme for Test for Incremental Permeability

Pt	Potential coil for test ring (Sheet iron ring No. 1)
Pc	Potential coil for auxiliary ring (Sheet iron ring No. 2)
Ps	Potential coil for auxiliary (Supplementary)
Fs	Fractionizing shunt
Mt	Magnetizing coil for test ring (Sheet iron ring No. 1)
Mc	Magnetizing coil for auxiliary ring (Sheet iron ring No. 2)
Ms	Standard mutual inductor
G	Galvanometer
R C	Calibrating resistance

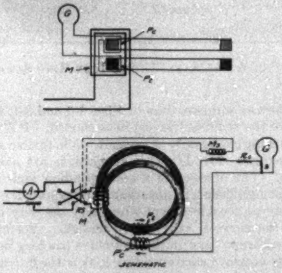

Fig. 4-0—Toroid Ring with Correction for Space Factor

Pt	Potential coil for test (Sheet iron ring)
Pc	Potential coil for compensation (Bakelite ring)
M	Magnetizing winding
G	Galvanometer
Ms	Standard mutual inductor
Rc	Calibrating resistance
RS	Reversing switch

values of β and H. In the case of the $\beta\ \mu'$ curve (Fig. 3-1) the data for β or μ' are both obtained directly, which makes irrelevant any question about the sensitiveness of the $\beta\ \mu'$ curve.

As to the law of magnetization, the function of the straightness of the $\beta\ \mu'$ curve (Fig. 3-1) is purely suggestive; it is not intended to demonstrate the law. The law in its final form is an equation expressing the relation of β to H (equation 30); its legitimate demonstration should consist of an agreement of the $\beta\ H$ curve by observation, with the corresponding curve by computation from the equation in question (see Figs. 1-8, 4-4, 12-4, etc.).

For the same reason the H, log γ curve is not well suited to the purpose of demonstration. There is no method available at present for direct measurement of log γ for any required value of H. The values of log γ are derived by computation from measured values of β. The value of the H, log β curve for purpose of demonstration is dependent on the sensitiveness of the curve, which is quite insensitive for low values of H, and supersensitive for high values of H, as saturation approaches. In view of these facts I have based my demonstration on the agreement of the $\beta\ H$ curve with the corresponding reconstruction curve, rather than on the straightness of the H, log γ curve.

One part of Mr. Lippelt's criticism is not clear to me; he says: "The author of the paper violates his own theory (to a certain extent) when he eliminates β from equation (**21**), and reduces it to the form of (**23**) and (**26**)." I should be glad to give full consideration to his criticism on this point, if I only knew what he meant thereby.

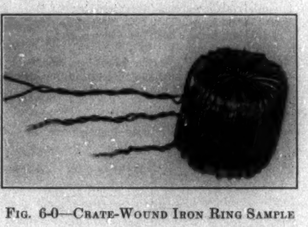

FIG. 6-0—CRATE-WOUND IRON RING SAMPLE

As to the two straight lines in Figs. 1-6 and 3-4, the study of that part of the problem has just been started. It is not possible to say anything confidently at present, while further experiments are in progress. Mr. Lippelt's suggestions are noted for consideration. The absence of a corresponding kink in the $H\ D$ curve is not significant, as none was to be expected. Such kinks are scarcely noticeable except when they are very pronounced, or unless the curve is very nearly straight. For example, the practically abrupt bend in the $H\ \rho$ curve has long been noticed, but nobody notices a corresponding kink in the $\beta\ H$ curve.

With reference to Mr. Lippelt's modification of Frolich's law, that modification does not affect the form of the curve near saturation. The law even in its modified form, therefore, is not expected to give any better agreement with the observation curve than the law in its original form.

With reference to Dr. Kennelly's comments, it is satisfactory to note his agreement on all the main points, except one which will be discussed later.

With reference to latent flux, he takes exception to the assumption that "it is made up of two groups of lines in opposite directions making an algebraic total of zero lines." Perhaps he is right; the assumption may need more evidence than has yet been presented, but I fail to see how that criticism concerns me. All I have done in paragraph 4 of the paper is to present the Weber-Ewing theory in my own words, so as to be able to reduce the theory to an analytical form as represented by equations **5** and **6**. Valid or invalid, it is their theory and my duty was merely to paraphrase it correctly in my own words. It is easy to see that that aspect of the theory to which Dr. Kennelly takes exception is really a part of the original theory as presented by Prof. Ewing. This may be seen from the following extracts from Ewing's Magnetic Induction, Chapter XI, Molecular Theory.

"The Weber molecule is a magnet before it begins to act, and the *amount of magnetism in it need suffer no change*, however widely the magnetism of the bar be altered. Hence Weber's theory explains the process of induction to this extent, that it makes the magnetic change in the bar be brought about by a change in the position of the molecules, and *not by any change in* the quality of the molecules."

"The fact that a definite saturation value is now known to exist adds much probability to Weber's hypothesis."

In article 179, "Amount of Retentiveness Possible Under Molecular Theory," Prof. Ewing takes m = magnetic moment of a single molecule, and treats it as a constant in the integral equations which are functions of the angle of orientation α. It is obvious, therefore, that the constancy of flux in a magnetic molecule is one of the requirements of the Weber-Ewing theory.

Incidentally, I may add that this is also my personal view, but that this consideration is not responsible for my statements in paragraph 4 of the paper, which is merely a presentation of the Weber-Ewing theory, irrespective of my own views on the subject.

On one point it seems we have not been able to reach an agreement; the best I could do is to state the difference as I see it, in explicit terms; I hope that in the course of time one of us or perhaps both of us will have reason to revise the present views, and thus reach an agreement.

Dr. Kennelly's view is that for a range of magnetizing force up to about $H = 90$,

(1) The $H\ \rho$ curve has the form of approximately two straight lines with a short elbow connecting them. (This is Kennelly's law.)

(2) This law was well supported by test data prior to the discovery of the law; later test seems to have confirmed it. Some new tests seem to contradict it, but these are yet unchecked and their evidence is yet unconvincing.

(3) From an engineering point of view, the straight-line relation presented by the $H\ \rho$ curve is the simplest relation, connecting the fundamental quantities H and β.

(4) From the physical theory point of view, the straight line is the first approximation towards a more complicated law, and the straight-line law is therefore quite correct in that limited sense.

My own view is:

(5) The straightness of the $H\ \rho$ curve is not approximate straightness as is generally believed, but only an apparent straightness caused by the insensitive character of the $H\ \rho$ curve.

(6) Assuming that the reliability of Kennelly's law was well proved by the data presented by Dr. Kennelly in the first place, it must be conceded that no new data have been presented by me or by anybody else to refute it.

(7) On the contrary, my own tests have strongly supported the law over a limited range, *viz.*, (a) for the region of first inflection, (b) for the region of second inflection, (c) for the region of saturation.

(8) My objection to Kennelly's law is merely this—that it is not reliable for purpose of computing saturation value. On this point there is no difference of opinion between me and Dr. Kennelly. This objection to Kennelly's law is not an objection to that form of the law, but to Frolich's law in general, of which that law is only one form.

(9) In addition to the above mentioned objection, which holds against all forms of Frolich's law, there is one objection peculiar to the Kennelly form of the law, *viz.*, that the reluctivity

curve is insensitive and misleading. The straightness of the curve is only apparent due to the insensitive character of the curve, and is therefore illusive. The insensitiveness of the reluctivity curve has been demonstrated in the paper analytically by equations 31-2, and graphically by Figs. 5-1 and 5-2. I believed that this evidence of insensitivity should have been quite convincing. Fig. 5-2 was expected to convince Dr. Kennelly in particular, as it represents data (Table V-2) on which he had based his law, instead of what he calls "new and unchecked tests."

(10) Kennelly's law is not the simplest form of Frolich's law. Bosanquet's law, $\mu = a\ \gamma$, is certainly a much simpler form, as may be ascertained by anybody who tries to compute the values of H for any required value of β.

(11) The only reason why Bosanquet's law never became as popular as Kennelly's law is that in that case the failure of the law to represent facts is so obvious as to compel one to reject it. In the case of the failure of the linear law of reluctivity not being so obvious, some of the best scientific minds have been misled by the apparent straightness. For example, in 1891, while expounding the law of reluctivity, Dr. Kennelly had said that the purpose of his paper was to show that while the $\beta\ \mu$ curve is a "complicated curve"—a curve which "you would not like to define by any particular formula"—the "curve which follows from the conception of inverse of permeability is fortunately a very simple one," being made of two straight lines connected by an elbow. In the paper under discussion, I have shown by mathematical reasoning that if for any range of magnetizing force the $H\ \rho$ curve consists of two straight lines, the $\beta\ \mu$ curve for the same range must also consist of two straight lines, and conversely if, as Dr. Kennelly admits, the $\beta\ \mu$ curve be incapable of representation by a pair of straight lines, neither can the $H\ \rho$ curve be so represented. The appearance of straight lines in such a case must be recognized as illusory and misleading, and its value as evidence must be ignored.

The above conclusion (No. 11) is the main point of difference between myself and Dr. Kennelly. The conclusion is not based on any particular tests old or new, checked or unchecked; it follows a mathematical reasoning, based on a study of the relation of $\beta\ \mu$ and $H\ \rho$ curves irrespective of any test data. If, therefore, this conclusion is unacceptable to Dr. Kennelly, it must be only by reason of some fallacy in that reasoning of which I am unconscious and which he has not yet disclosed. I would be very thankful if he makes himself clearer on this point.

Surface Heat Transfer in Electric Machines with Forced Air Flow

BY G. E. LUKE*
Associate, A. I. E. E.

Synopsis.—*Since the insulation of windings in electric machines has comparatively low temperature limits, the problem of cooling these machines with the most economical use of material becomes one of major importance. The design of such machines from a temperature standpoint is usually based on tests of a previously made similar machine or else is of the "cut and try" type where such tests are not available.*

The predetermination of the operating temperature depends a great deal upon the rate at which the heat losses can be liberated from the ventilating surface to some cooling fluid such as air, which is considered in this paper. Some data are available regarding this rate of heat dissipation with forced air convection currents; a comparison of the various results published, however, shows them to be inconsistent. The purpose of this paper is to submit additional information that should be of value to the industry and that will also explain some of the inconsistencies in the past tests.

Extensive experimental tests were made regarding the rate of surface heat transfer with air flows of various velocities for radial and axial ducts. The influence of shape, size, cross-section, condition of surface, mean temperature, and other factors were investigated.

The most important factor found was that this rate of heat transfer was not constant along the air flow path but that its value varied from point to point along the duct. This variation in rate of heat liberated at a constant velocity, in a given duct, is due to changes in the turbulency of air flow along the duct, and results in a rate of heat transfer near the entrance of the duct of about twice that found beyond the entrance where stable flow conditions are found. The above fact explains why the surface heat transfer coefficient of a duct, averaged over the total length, will be an inverse function of the length, and why results as given by experimenters on ducts of various lengths should vary. The above also explains why electric machines having short duct lengths have a capacity in proportion to the surface greater than those with similar ducts of longer air flow paths.

INTRODUCTION

THE main factor that limits the capacity of electric machines is the temperature of the windings. This temperature limit is comparatively low, ranging from approximately 100 deg. cent. to 150 deg. cent., depending upon the class of insulation and the type of machine. Air is used as a cooling medium in the great majority of rotating machines. The heat resulting from the iron and copper losses of the machine is conducted to the ventilating surfaces where it is transferred to the moving air. To conduct this heat through the solid material and to transfer it from a surface to a fluid requires a temperature gradient. Such a flow is shown on Fig. 1 with a radial duct. From the standpoint of heat transfer from a surface, air is one of the poorest of fluids. From 20 to 75 per cent of the temperature rise in rotating machines is due to the gradient necessary to transfer the heat from the surface to the ventilating air. This factor is therefore of considerable importance in the design of an economical machine.

Considerable data in the past have been published concerning heat liberated from surfaces with natural convection currents, but it has been just recently that data applicable to electrical machines with forced air flow have been available. To obtain information regarding the rate at which heat is dissipated with high velocity air flow is difficult since it will depend upon the particular conditions of air flow as well as the mean velocity. Experimental results published by various workers do not agree, therefore, in many cases. The purpose of this paper is to discuss some of the available tests and to submit new tests covering conditions of air flow such as are found in electric machines. The new data presented also will correlate some of the work which offhand seemed to be inconsistent.

COMPARISON OF PUBLISHED DATA

Throughout this paper the coefficient of surface heat transfer will be symbolized (K_v) or W/sq.in./deg. cent. which means watts transferred per square inch of ventilating surface per deg. cent. difference between the surface and mean air temperature flowing in the duct. Thus, in Fig. 1, (K_v) for the particular air flow would depend not upon the minimum air temperature nor upon the mean temperature as given by the curve but upon the integrated mean temperature taking into account the total mass flow. This is the only practical way of defining (K_v) since in any ventilating duct the

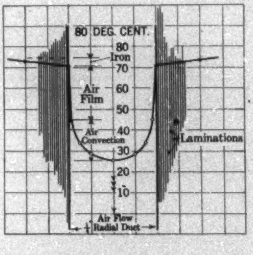

FIG. 1—TEMPERATURE GRADIENT DUE TO HEAT FLOW TO AIR STREAM IN RADIAL DUCT

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air temperatures and velocities vary greatly at any particular point.

This coefficient of surface heat transfer is often called rate of surface heat flow, dissipation constant and erroneously, the emissivity constant. It may also be expressed in other units; the relation of some of these is approximately as follows:

1 B. t. u. per sq. ft. per deg. fahr. per hr. = 0.00366 watts per sq. in. per deg. cent.
1 Calorie per sq. cm. per deg. cent. per sec. = 27.0 watts per sq. in. per deg. cent.
1 Kilo-calorie per sq. m. per deg. cent. per hr. = 0.00075 watts per sq. in. per deg. cent.
1 watt per sq. cm. per deg. cent. = 6.45 watts per sq. in. per deg. cent.

A few of the experimental results published by Nusselt,[1] Dicksee,[2] Rice,[3] and the writer,[4] giving the rate of surface heat transfer for various air velocities, are plotted on Fig. 2. In all of these tests the air velocity referred to is the mean velocity in ft. per min. obtained by dividing the weight of air, in pounds, passing through the duct by the cross-sectional area of the duct in square feet, and by the weight of air in pounds per cubic feet (0.074). This factor (0.074) is the weight of dry air in pounds per cubic foot at 25 deg. cent. and atmospheric pressure of 29.92 inches of mercury. Thus the weight of air flowing will be proportional to the velocity factor.

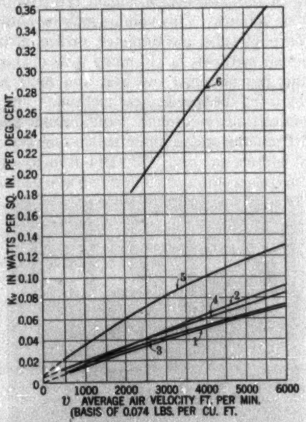

FIG. 2—SURFACE HEAT TRANSFER CONSTANT AGAINST AVERAGE AIR VELOCITY FOR VARIOUS DUCTS OF ATMOSPHERIC PRESSURE

Curve	Author	Type
1	Luke[1]	Smooth circular 1¼ in. Dia. 36 in. long.
2	Luke[1]	Rough circular 1¼ in. Dia. 36 in. long.
3	Nusselt[4]	Smooth circular 0.866 in. dia.
4	Luke[1]	Smooth concentric 25 in. by 26 in. dia. 36 in. long.
5	Dicksee[2]	Smooth concentric 3 in. by $5^3/_8$ in. dia., 7 in. long.
6	Rice[3]	Rough concentric 1.1 in. by 2 in. dia., 5.86 in. long.

The velocities dealt with in this paper result in turbulent air flow. In ducts such as are found in electric machines this turbulent flow will be obtained with velocities above 500 ft. per min. For velocities of 250 ft. per min. and below, a stream line or laminar flow may be found. With such flow the rate of heat transfer will be small and hence should never be used. The range between the lower and upper limits of critical velocity for air in a circular duct are:

$$V_c = \frac{(225 \text{ to } 330)}{d} \left(\frac{T}{273}\right)^{1.75}$$

where

V_c is the critical velocity in ft. min.
T = temperature Kelvin of the air.
d = diameter of duct in inches.

Curves 1, 2, and 4, given by the writer,[4] were with air flowing through ducts such as are found in electric machines and are called axial ducts. Curve 1 applies to a smooth brass tube 1¼ in. in diameter and 36 in. long, with air flowing through it. Curve 2 is for a duct similar to Curve 1 except that the surfaces are rough, since the tube was made by stacking washers of 0.017-in. varnished iron with a 1¼-in. inside diameter. Since the punching and stacking varied a few thousandths of an inch, the inside bore was comparatively rough. This increased the coefficient of friction (f) at least 50 per cent and the heat transfer (K_v) from 20 to 30 per cent. Curve 1 provides a reasonably close check on the data given by Nusselt (Curve 3) which was obtained by using a smooth 0.866-in. diameter tube. Jordan[5] also gives the results of surface heat transfer constants (K_v) with air flow through smooth tubes which are slightly below Curve 1.

Curves 4, 5, and 6 were obtained with the air flowing axially between two concentric cylinders. In the tests made by the writer the cylinders were 25 and 26 inches in diameter and 36 inches long. Due to the radiation loss the values are slightly greater than those indicated by Curve 1. This radiation loss is present in Curves 4, 5, and 6, since the heated surface was the outer surface of the inner cylinder. This radiation loss is approximately 0.003 to 0.005 W/sq.in./deg. cent. and will be independent of the air velocity. Curve 5 by Dicksee[2] was obtained with a much smaller heater cylinder upon which were soldered various numbers of radial copper fins with surfaces parallel to the air flow. The values of the heat liberated are about twice those given for Curve 1. Curve 6 by Rice[3] was plotted in terms of mean velocity instead of maximum velocity by using the mean velocity ratio of 0.85. The values of heat transfer are comparatively large, being about five times as great as those of Curve 1 and about 2½ times as great as those of Curve 5.

From past experience in the cooling of electric ma-

1, 2, 3, and 4. **See Bibliography.**

chines with free convection currents, values of heat transfer from the ventilating surfaces have been obtained that are considerably greater than those given by Curves 1 to 4. The writer has never obtained such high values, however, as those given by Curve 6 and hence additional tests were planned and executed for the purpose of giving more information concerning the variation of this cooling constant (K_v) as influenced by varied air flow conditions.

EXPERIMENTAL

1. *Air Flow Between Two Parallel Flat Plates.* Electric machines are cooled more by radial ventilating ducts in the iron core than by any other method. These ducts range in width from ¼ in. to 1 in., ⅜ in. being the more common figure. The air flow through them is very turbulent, due, not only to the ventilating spacers or fingers and the coils, but also to the changing cross-section for air flow. On large bore machines the change in air velocity due to the radial flow may not be great. The first test made was to imitate such conditions where a minimum rate of heat transfer is to be expected. This test is described in greater detail than is the one following since it is typical of the others in the methods used.

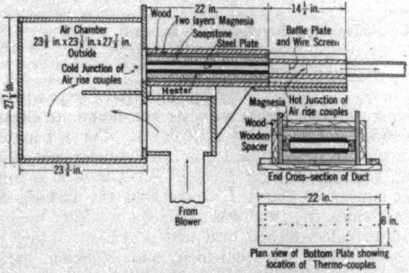
Wood
22 in.
14¼ in.
Two layers Magnesia
Soapstone
Steel Plate
Air Chamber
23⅜ in. x 23⅜ in. x 27⅞ in.
Outside
Cold Junction of Air rise couples
Baffle Plate and Wire Screen
27⅞ in.
Heater
Magnesia
Hot Junction of Air rise couples
Wood
Wooden Spacer
23⅜ in.
End Cross-section of Duct
From Blower
22 in.
8 in.
Plan view of Bottom Plate showing location of Thermo-couples

FIG. 3—APPARATUS FOR DETERMINING THE HEAT LOSS FROM FLAT PARALLEL PLATES

A sketch of the apparatus is shown on Fig. 3 and an illustration of the assembled outfit is shown on Fig. 4. The air was supplied by a centrifugal fan driven by a variable speed d-c. shunt motor. This fan discharged into an expansion chamber. The air then passed through the ventilating duct formed by two hot plates 8 in. wide by 22 in. long, separated a definite distance by proper spacers. The hot discharge air then passed through an outlet duct where its average temperature was measured.

The hot plates from which the heat dissipation was to be measured were composed of ¼-in. smooth steel plates fixed to a soapstone base between which a heater was placed. This heater was made of 0.005-in. by 0.25-in. resistor ribbon and covered practically the entire area. It was insulated with a 0.010-in. mica sheet. On the bottom plate were twenty thermocouples and five were on the top plate. These couples (0.010-in. d. c. c. copper-advance) were soldered in small holes drilled in the plate with the junctions flush with the surface. The leads of the couples were brought out through the soapstone so that no obstruction was placed in the air path. The large number of couples on one plate was required to make a temperature traverse. The plates were thermally insulated on the inactive side by the soapstone 1 in. thick, two sheets of ¼-in. magnesia, and a disk of wood 1 in. thick. In order to calibrate the stray heat loss through this insulation, five stray loss thermocouples were placed between the magnesia plates distributed axially along the duct.

In all tests on heat transfer with forced air flow the accuracy of the data depends upon the true mean air velocity or volume. This volume may be obtained by using the pilot tube, anemometer, orifice meter, and other similar methods, but in many tests of this nature the writer has obtained the best and most accurate results by using the specific heat method. The volume was obtained after thermal equilibrium had been reached from the watts input to the air and the resulting air temperature rise. The watts absorbed by the air equals the total watts input minus the stray loss as given by the calibration curve corresponding to that particular heater temperature. The temperature rise of the air was obtained by five thermocouples with the hot and cold junction distributed in the outlet and inlet air respectively. The accuracy of this method depended upon obtaining the true temperature rise, which necessitated a thorough mixing of the hot air. This is accomplished best as shown by allowing the air to expand on discharge with a baffle placed in the direct path of the high velocity air, forcing the air to change its path. This scheme functioned successfully as shown by temperature traverses of the air with a single couple. The equation used was

FIG. 4—APPARATUS FOR DETERMINING THE RATE OF HEAT TRANSFER FROM FLAT PARALLEL PLATES

$$V = \frac{1.765\, W_a}{\theta_a}$$

where

V = cubic feet of air per minute (25 deg. cent. temperature)

W_a = watts absorbed by the air

θ_a = resulting air temperature rise deg. cent.

The constant (1.765) is the degrees centigrade one cubic foot of air will rise with the storage of 1 watt-minute of energy. This is based upon air as weighing 0.0740 lb. per cubic foot and having a specific heat of 0.2418, all at 25 deg. cent. and at standard atmospheric pressure of 29.92 inches of mercury. Thus a simple and accurate method is obtained which is independent of local variations in humidity and barometric pressure.

The average air velocity (v) through the duct, then, is

$$v = \frac{V}{A}$$

where

v = average air velocity in feet per minute, and

A = cross-sectional area of duct in sq. ft.

The rate of heat loss (K_v) from the surface for any given velocity was determined from the equation

$$K_v = \frac{W_a}{S\left(\theta_s - \dfrac{\theta_a}{2}\right)}$$

where

K_v = surface heat transfer constant in W/sq. in/deg. cent.,

W_a = watts dissipated to ventilating air,

S = ventilating surface of duct in sq. in.,

θ_s = average surface temperature rise of duct above intake air deg. cent., and

θ_a = temperature rise of the outlet air deg. cent.

A stray heat loss curve for each spacing was obtained which gave the watts stray loss for any average plate temperature. This was made by completely closing the ends of the duct so as to minimize loss by convection. The heater input then, when steady conditions had been reached, was the stray loss flowing through the heat insulation. With forced convection the duct was given a definite spacing and air at a constant velocity was forced through. The current through the heaters was adjusted until an average plate temperature of about 80 deg. cent. was reached. When the temperatures became stable all thermocouples and heat inputs were measured. Direct current was used as the heater supply. The voltage dropped across the heater, across a fixed resistance in series with the heater, and the potentials given by the couples were all measured with a Leeds & Northrup type K potentiometer. In this manner tests at various velocities were made for ¼-in., ½-in., ¾-in., and 1-in. spacings, and (K_v) was calculated as previously shown.

The results of these tests are shown on Fig. 5. These curves show a slight decrease in (K_v) with increasing duct spacing. The values of the constant are not materially different from those of Curve 2 in Fig. 2. The average equation for heat loss, as shown, is

$$K_v = 0.020\left(\frac{v}{1000}\right)^{0.80}.$$

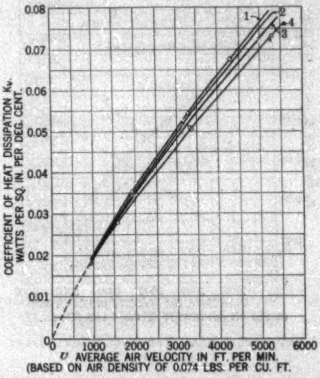

FIG. 5—CURVES FOR FLAT PARALLEL PLATE DUCT

Size duct 8 by 22 in.

Equations of Curves

Curve No. 1; $K_v. = 0.0207\left(\frac{V}{1000}\right)^{0.81}$ (for ¼-in. spacing)

Curve No. 2; $K_v. = 0.0200\left(\frac{V}{1000}\right)^{0.814}$ (for ½-in. spacing).

Curve No. 3; $K_v. = 0.0200\left(\frac{V}{1000}\right)^{0.798}$ (for ¾-in. spacing.)

Curve No. 4; $K_v. = 0.0194\left(\frac{V}{1000}\right)^{0.795}$ (for 1-in. spacing).

In air blast transformers similar ducts between pancake coils are used, the coils being separated by the so-called "wavy" fiber spacers. Such spacers will increase the turbulency of the air flow and should increase (K_v). Results obtained with a ⅜-in.-width duct by the use of these spacers are shown on Fig. 6. The air velocity was calculated from the minimum cross-sectional area between spacers (0.375 in. by 1.25 in.), and the surface was taken as the product of the minimum clear distance between spacers times the straight line length of the duct (1.25 in. by 22 in.). This heat loss coefficient is 40 or 50 per cent greater than that found in the duct without wavy spacers. The coefficient of friction (f) was about 100 per cent greater than that for the duct without spacers. Curve 1, Fig. 6 shows that the turbulency of air flow was practically the same as that in Curve 5 of Fig. 2.

2. *Air Flow in Radial Duct.* The above described tests were on ducts of constant cross-section. In radial ducts, however, the cross-section is changing from point to point along the air path due to the resulting change in the diameter of the duct. In practise, also, such ducts will contain irregular spacers (ventilating fingers) and will be traversed by the conductors. All of these factors will tend to produce very turbulent air flow which should result in a high rate of heat loss.

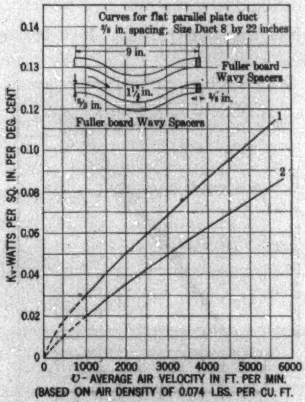

FIG. 6—CURVES FOR FLAT PARALLEL PLATE DUCT 3/8-IN. SPACING:

Size duct 8 by 22 in.

Curve No. 1

3/8-in. duct with 3/8-in. wavy spacers

$$K_v = 0.0298\left(\frac{V}{1000}\right)^{0.772}$$

Curve No. 2

3/8-in. duct without wavy spacers

$$K_v = 0.0203\left(\frac{V}{1000}\right)^{0.824}$$

A sketch showing the cross-section of the apparatus is given on Fig. 7. The duct proper was formed by separating two parallel hot plate disks. These plates were 24 in. in outside diameter and 10 in. in inside diameter. They were made of resistance wire wound uniformly on one side of a 1/8-in. plate; mica was used as insulation with shellac as the binding cement. The heater and plate were likewise cemented to a 1-in. soapstone slab which acted as a heat insulator and also gave rigidity to the heater plate. Two such plates properly spaced were enclosed in a wooden box which acted as an air chamber. Ports were provided for attachment to the fan and discharge duct. Eighteen thermocouples were distributed over the plate and attached as described in the previous test. These were used for obtaining the average plate temperature. The air volume was also measured, as in Test 1, by the specific heat method. As shown inside hoods and baffles were provided so as to correctly measure the air temperatures both on inlet and outlet. The ports to the duct were arranged so that the air direction could be changed to provide either divergence or convergence with respect to the duct (the latter being radially inward toward the center). For determining the stray loss from the back side of the hot plates, the watt loss was calibrated against the temperature gradient across a definite thickness of the heat insulation.

For the first series of tests, (without ventilating fingers), the hot plates were separated by three small strips of iron about 1 in. and $^1/_{16}$ in. thick, with a width equal to the duct spacing. This gave a radial duct free of obstructions. Later, ventilating fingers 7 in. long and made of $^1/_{16}$-in. sheet iron were used as separators; also wooden blocks were uniformly distributed along the outside edge so as to simulate conditions found in practise due to the conductors. The cross-section of these blocks was 0.75 in. by 1.75 in. and the thickness was equal to the duct spacing.

The heat loss constant (K_v) was calculated as before, based upon the average temperature difference $\left(\theta_s - \dfrac{\theta_a}{2}\right)$. The air velocity used in the curves is the average velocity based upon the cross-section in the middle of the duct and hence corresponds to the section where the duct diameter is (24 + 10)/2 or 17 in. When ventilating fingers were used the surface of the fingers was also included.

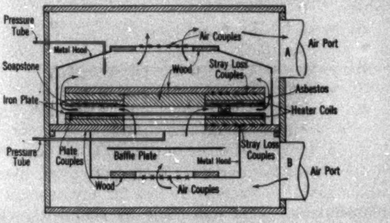

FIG. 7—ARROWS INDICATE THE DIRECTION OF AIR FLOW FOR DIVERGING AIR

The results of tests with diverging flow through the unobstructed radial duct are shown on Fig. 8. The heat loss constant at a definite velocity is greater for the 1-in. ducts than for the 1/4-in. duct which is the reverse of the case given on Fig. 5. The values are also much higher due to greater turbulency of air flow. These values are of the same approximate value as that represented by Curve 5, Fig. 2. The curves of the 3/8-in. or 1/2-

in. ducts, Fig. 8, seem to be out of place. The values given, however, were rechecked several times.

When ventilating fingers and conductors are placed in the duct the curves are as shown on Fig. 9. Since the surfaces of the spacers are also included, the results show that they are almost as effective as the duct surface.

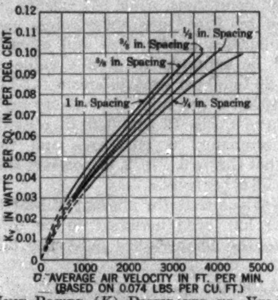

Fig. 8—Unit Power (K) Dissipated vs. Velocity

No fingers or blocks Diverging air
Radial ducts

Many machines are now ventilated with air flow radially inward (converging flow) through a portion of the ducts. The results of such a flow are shown on Fig. 10. It should be noted that the values obtained are lower than those obtained with diverging flow; Curve 2, particularly, is materially lower than the similar curve on Fig. 8. This fact is to be expected since it is known that diverging air flow will be more irregular or turbulent than converging flow with the same axial change in cross-section. Thus the turbulence obtained with Curve 2, Fig. 10, is about the same as that found in Fig. 5 with a constant cross-section since the rate of heat loss is about the same.

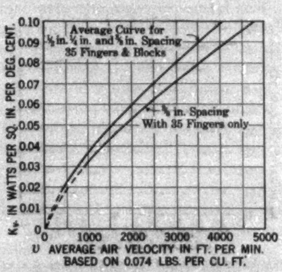

Fig. 9—Unit Power (K) Dissipated vs. Velocity

35 fingers—35 blocks diverging air
Radial duct

3. *Air Flow between Two Concentric Cylinders.* Mr. C. B. Dicksee[2], working in this laboratory, investigated the heat loss from an air cooled gasoline engine cylinder with numerous fins attached. Curve 5, Fig. 2, gives the average results of these tests on a cylinder with 5 to 40 axial metal fins attached. These fins extended to the outer cylinder. The heat loss constant (K_v) was independent of the number of fins used. At the time these tests were made the value of (K_v) was considered high and no reason could be suggested why the heat loss should be any greater than, let us say, Curve 2.

The writer at that time had tests made with the smooth cylinder without fins as shown on Fig. 11. This test equipment was the same as used in the above test. The cylinder was supported as shown with a square orifice air intake. The average temperature of the inner cylinder was obtained with thermocouples as before; (K_v) was calculated as it had been done previously and the air velocity was calculated from the cross-sectional area of the duct.

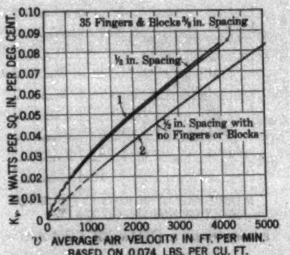

Fig. 10—Unit Power (K) Dissipated vs. Velocity

35 fingers—35 blocks converging air
Radial duct

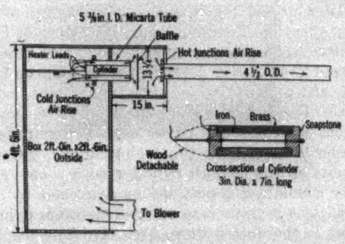

Fig. 11—Diagram of Experimental Apparatus for Determining Heat Loss from a Cylinder with Axial Air Flow

Curve 2, Fig. 12, gives the results of these tests. The values of (K_v) are a little greater than those obtained on the finned cylinder, Curve 1. In comparing this Curve 2, Fig. 12, with Curve 4, Fig. 2, a great difference is observed, although the types of air flow system are about the same; the main difference is the length of duct (7 in. and 36 in.).

It has long been known that when air enters a duct of constant cross-section from a larger chamber a convergence or "vena contracta" of the air stream will take place, and later the effective air stream will expand and gradually assume a stable velocity condition. This convergence near the entrance of the duct and the resulting change in velocity beyond the "vena contracta" will result in turbulence and an increase in heat loss coefficient (K_v). This is evidently the explanation for the high value of (K_v) shown on Curves 1 and 2, Fig. 12. Since the major part of this effect extends over only a few inches, this explains why its influence is small on Curve 4, Fig. 2, with a 36-in. length of duct.

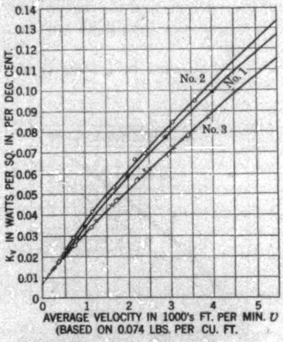

FIG. 12—HEAT LOSS FROM A CYLINDER WITH AXIAL AIR FLOW

About this time Rice[3] published his Curve 6, Fig. 2. This curve, as previously mentioned, gives values of (K_v) about 2½ times as great as those of Curve 1 or 2, Fig. 12, and as Rice states, about five times those of Nusselt.[1] The explanations suggested were that the conditions of air flow and heater surface were different.

In order to investigate this inconsistency, the writer made tests on the smooth cylinder with a converging entrance simulating conditions as described by Rice. This was done by attaching extensions to the cylinders as shown in the dotted lines, Fig. 11. The results of this are given by Curve 3, Fig. 12, and show a decreased heat transfer instead of an increase. This decrease was expected since a converging entrance will result in less air turbulence than a square entrance.

To check the effect of roughness on the rate of heat transfer, the above 3-in. cylinder was closely wound with 0.0155-in. diameter enamel copper wire (0.0394 cm.). This was insulated from the brass tube and the outer exposed enamel was removed with sand paper. The resistance of this outer wire surface was obtained with a wheatstone bridge from which the temperature was calculated. The results of these tests were practically the same as the results of tests made with the smooth cylinder and line up with Curve 3, Fig. 12, within the error of test.

These test conditions, while not exactly the same as those given by Rice,[3] are nevertheless very similar. The dissimilarities due to size and other conditions should not be expected to give a value of (K_v) of more than 20 to 30 per cent difference. The results as given for 4000-ft. per min. air velocity are, however, 0.09 for Curve 3, Fig. 12, and 0.28 Rice's value, Fig. 2. Such a discrepancy is enormous and the writer's opinion is that Curve 6, Fig. 2, is incorrect since it is out of line with all other tests.

In the previous tests the amount of heat transferred (W/sq. in.) has been assumed to be directly proportional to the temperature difference between the surface and the air $\left(\theta_s - \frac{\theta_a}{2}\right)$. In other words, ($K_v$) in W/sq. in. /deg. cent. should be a constant, independent of the surface and air temperature. This is not actually true since (K_v) does vary slightly with temperature. The variation of (K_v) with temperature is rather difficult to determine experimentally due to many other variables which must be controlled. Nusselt,[1] Pohl,[6] Jordan,[5] and Latsko[7] have stated that (K_v) will increase with temperature $\left(\theta_s - \frac{\theta_a}{2}\right)$; Rice[3,8] has calculated the opposite effect.

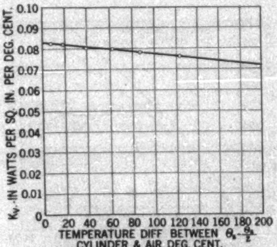

FIG. 13—HEAT LOSS FROM A CYLINDER WITH AXIAL AIR FLOW OF 2800 FT. PER MIN.

(Based on 0.074 lb. per cu. ft.)

Tests on the heat transfer (K_v) from the smooth cylinder with a square entrance at a constant velocity and with variable wall and air temperatures are given n Fig. 13. For a 100-deg. change in $\left(\theta_s - \frac{\theta_a}{2}\right)$ K_v changes from 0.083 to 0.0775, or by a ratio of 1.07.

According to Rice,[3,8] (K_v) varies as ($1/T_{avg.}^n$) where (n) ranges from 0.56 to 1.0 and ($T_{avg.}$) is the average temperature, Kelvin, of the surface and air. A value of (n) of about (0.5) would check the test approximately.

RATE OF HEAT LOSS AS INFLUENCED BY DUCT LENGTH

It was previously suggested that the rate of heat loss due to air flowing through a duct will not be constant for all parts of the duct, even with a constant cross-section.

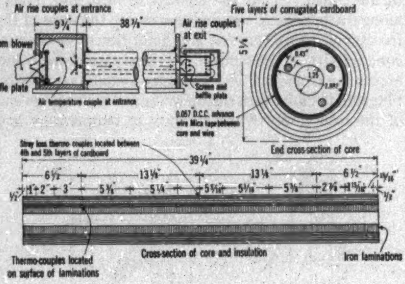
Air rise couples at entrance
9 3/4"
38 7/8"
Air rise couples at exit
Five layers of corrugated cardboard
From blower
Baffle plate
Air temperature couple at entrance
Screen and baffle plate
5 1/8"
0.43"
1.25
2.002"
0.057" D.C.C. advance wire Mica tape between core and wire
End cross-section of core
Stray loss thermo-couples located between 4th and 5th layers of cardboard
39 1/4"
6 1/2"
13 1/8"
13 1/8"
6 1/2"
11/16"
1/2"
1"
2"
3"
5 3/8"
5 1/4"
5 5/16"
5 5/16"
5 3/8"
2 1/8"
1 15/16"
1/2"
Thermo-couples located on surface of laminations
Cross-section of core and insulation
Iron laminations

FIG. 14—APPARATUS FOR DETERMINING RATE OF HEAT DISSIPATION FROM AXIAL DUCTS

It has been observed in machine design that machines with short duct lengths, both radial and axial, can dissipate more heat in proportion to the surface than can large machines with the same type and section of duct but with longer air flow path. A part of this difference was believed to be due to the conditions of air flow. The turbulence caused by the vena contracta at the duct entrance should cause an increase in heat loss comparable to that loss where the stream flow was more uniform. This effect has been masked in many of the past researches due to the difficulty in obtaining the rate of heat transfer over any part of the duct. Thus the work described by Nusselt,[1] Pohl,[6] Rice,[3] Dicksee,[2] and Jordan[5] was all based on an average heat transfer constant taken over the total length of the duct. The writer[4] notices this increased heat loss near the duct entrance but was unable to determine its value accurately because of a variable stray end loss. The Curves 1 and 2, Fig. 2, were, therefore, based upon the average heat loss in the middle 1/3 section of the duct. Curve 4 was based on the average loss over the total duct.

This variation in rate of heat loss with length of duct has been analytically considered by Latzko.[7] He developed a theory of heat transfer as determined by form and dimension with reference to the turbulency of flow. His work was based upon the hydro-dynamical principles of fluid flow. No experimental work, however, was available, and none was submitted to substantiate the analysis.

The following work was planned in order to determine this variation in the rate of surface heat transfer with duct length and also to check Curve 2, Fig. 2, previously submitted.

4. *Air Flow in a Circular Axial Duct.* This air duct used was practically a duplicate of that from which Curve 2, Fig. 2, was obtained. It is called an axial duct since it was made to imitate such ducts found in the punchings of electric machines. The duct was 1¼ in. in diameter and 39¼ in. long. The inside surface was rough since the duct was formed by stacking stampings in the form of washers made from 0.017 in. sheet steel (1 per cent silicon). These stampings were bolted together by three long bolts running the length of the duct. A sketch of the construction and details is given in Fig. 14. An illustration of the assembled test equipment is shown in Fig. 15. The air was fed to the duct from a pressure chamber which in turn was supplied by an adjustable speed motor-driven fan. The air volume flow was accurately determined as previously described by the specific heat method. Special care was taken to secure a true air temperature both on the intake and the outlet. In order to insure a constant heater current and air supply a large storage battery was used as the source of power supply. To obtain the temperature curve along the duct, twelve copper-advance (0.010-in.) thermocouples were soldered to the outside surface as shown in the sketch. The duct was thermally insulated on the outside by five layers of air-cell-corrugated cardboard. To determine the stray loss three themocouples were placed in this insulation so that the temperature drop through the intermediate insulation could be plotted against watts loss.

Since the iron temperature was determined on the outside of the duct it was necessary to calculate the temperature drop through the radial depth of the stampings. This could be done with high accuracy since this temperature drop was only a few per cent of the surface temperature rise. The heater was a uniform layer of 0.057-in. diameter d. c. c. advance wire tightly wound upon the iron core. Since this wire has a practically zero temperature coefficient, the rate of heat generation was constant along the duct for any given current. The heat input in watts, as well as all thermocouples, was read with a Leeds & Northrup type K potentiometer. The accuracy of these readings was evidenced by a maximum error of about 0.2 per cent.

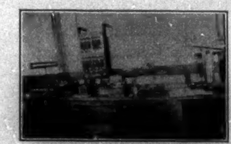

FIG. 15—APPARATUS FOR DETERMINING THE RATE OF HEAT AND TRANSFER FROM AXIAL DUCT

The duct was first tested for stray loss by closing both ends. In this way a curve giving watts loss against temperature gradient in the outer insulation was obtained. With forced convection the fan was driven at a constant speed and the heater input adjusted so as to give a maximum duct temperature of about 85 deg. cent. The average air intake temperature was about 30 deg. cent.

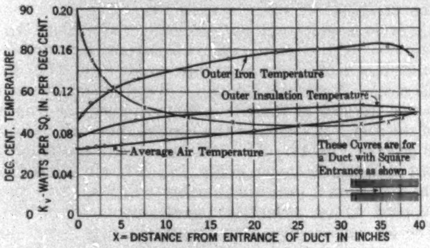

Fig. 16—Heating Curve of a Laminated Axial Duct

Diameter 1¼ in. Length 39¼ in.
503 watts input. Average velocity 5760 ft. per min.

In order to find the rate of heat dissipation to the air at any section of the duct, knowledge was required of the total watts generated by the heater, the stray loss, and the heat conducted to or away from that particular section across the punchings. Since the heater was uniformly wound, the loss in any section is proportional to the length of that section. Fig. 16 gives the curves for a particular run. The stray loss from any section of the duct was proportional to the average temperature drop between the iron and outer insulation temperature of that section. Since the iron core temperature was not constant there is an axial flow of heat across the punching. Due to the high thermal resistance offered to this transverse flow by the various surface drops, however, the rate of heat flow was not great. The unit thermal conductivity was about 0.04 watt-in./sq. in./deg. cent. Thus the amount of heat flow axially along the punchings was proportional to the slope of the iron temperature curve at any point times the thermal conductivity of the punchings. This rate of heat flow across the punchings was determined experimentally by plugging the duct with cotton waste and heating one end of the iron core. When stability of temperature had been reached, a cooling curve along the duct was obtained. Then, knowing the rate of stray heat loss through the outer insulation, the thermal conductivity of the core could be determined from the cooling curve along the duct.

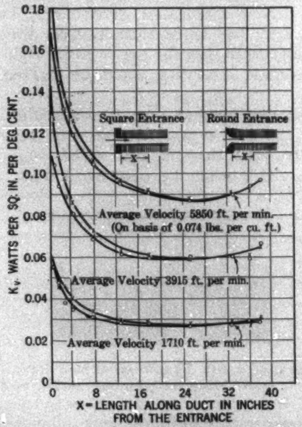

Fig. 17—Rate of Heat Loss from Surface of a Laminated Axial Duct

1¼ in. diameter 39¼ in. long
° These dots are for duct with round entrance
× These dots are for duct with square entrance

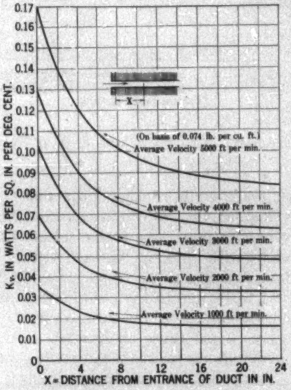

Fig. 18—Rate of Heat Loss from Surface of a Laminated Axial Duct at Various Velocities

These curves are for duct with square entrance as shown
1¼ in. diameter 39¼ in. long

Subtracting the outer stray watts loss and that lost by transverse conduction from the total watts generated in the duct section, the remainder will be the watts dissipated to the air in the duct. Beginning at the entrance of the duct this procedure was performed for various sections of the duct axially. The air temperature curve and the surface heat transfer curve (K_v) were obtained as shown on Fig. 16. The calculations were rather laborious; however, any error, if made, would show up by giving an incorrect heat balance at the end of the calculations.

Tests were made with a sharp or square entrance and with a round entrance as shown on Fig. 17. These values of heat loss (K_v) are the actual values obtained at any particular point along the duct. They verify the previously given theory that the rate of heat loss against duct length will be a variable and should be much higher near the entrance of the duct. For a constant velocity, the loss (K_v) is about twice as high for the first inch of the duct as that found near the middle of the duct. It should be noted, also, that there is a slight increase in (K_v) near the outlet of the duct. This shows an increased turbulence and is verified by static air pressure explorations in that region.

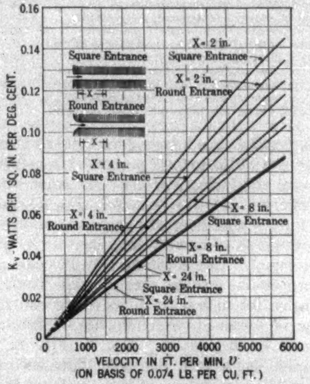

FIG. 19—RATE OF HEAT LOSS FROM SURFACE OF LAMINATED AXIAL DUCT FOR VARIOUS DISTANCES FROM ENTRANCE OF DUCT

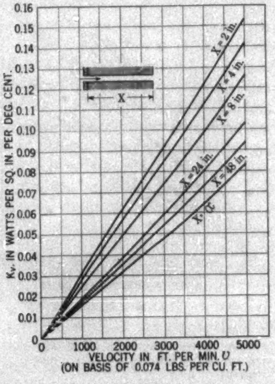

FIG. 20—AVERAGE RATE OF HEAT LOSS FROM SURFACE OF LAMINATED AXIAL DUCTS FOR VARIOUS LENGTHS OF DUCT

These curves are for ducts with square entrance as shown

The influence of the entrance is shown by an increase in (K_v) with a square entrance over that given with a smooth entrance. Near the middle of the duct the two curves are practically the same. These curves with additional ones were re-plotted and are shown on Fig. 18. Fig. 19 gives the rate of heat transfer (K_v) at various points along the duct plotted against velocity. By integrating curves on Fig. 18 for various lengths along the duct, the average rate of heat transfer (K_v) for that length of duct was obtained as shown in Fig. 20.

Since this duct was practically the same as that tested in 1919, Curve 2, Fig. 2, a check on the accuracy was obtained. The curve referred to was calculated on the basis of the average (K_v) over the middle third section of the duct. The curve shown on Fig. 21 gives similar results of the tests made six years later with different apparatus and experimenters. The results check to within one per cent, which is unusually good for this type of work.

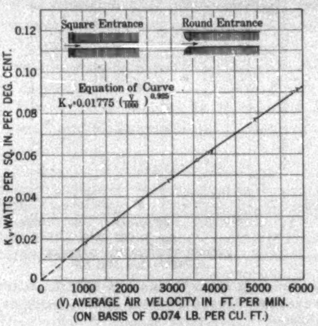

FIG. 21—HEAT TRANSFER COEFFICIENT FROM SURFACE OF AXIAL 1¼-IN. DIAMETER LAMINATED DUCT

Curve based on heat loss and air temperature at center of duct—39¼ in. long

• These dots are for duct with round entrance
× These dots are for duct with square entrance

VARIATION OF HEAT LOSS COEFFICIENT WITH TEMPERATURE

The previous tests were made with an air intake of about 30 deg. cent. and a maximum iron temperature of 85 deg. cent. This gave an average gradient between iron and air in the middle section of the duct of about 35 deg. cent. The effect upon the heat transfer (K_v) of varying this temperature gradient is shown on Fig. 22. This also checks the same effect as given on Fig. 13, the heat transfer varying approximately as $\sqrt{\frac{1}{T_{avg.}}}$ where (T_{avg}) is the average temperature, Kelvin, of the wall and air.

Coefficient of Friction. The static air pressure drop over a distance of 37.4 in. was measured. The equations obtained are:

For square entrance duct $p = 0.068 \left(\frac{v}{1000}\right)^2$

For round entrance duct $p = 0.076 \left(\frac{v}{1000}\right)^2$

where

p = total static air pressure drop in inches of water.

This gives a coefficient of friction (f) of about 0.0108 for an average compared to a value of 0.007 for the smooth duct Curve 1, Fig. 2. Thus there is a definite tendency for the rate of heat transfer to increase with the coefficient of friction as suggested by Reynolds,[9] Stanton,[10] Royds,[11] Rice,[8] and others. It is not a direct proportionality, however, since (f) increased about 54 per cent with an increase in (K_v) of about 25 per cent. The exponent (n) of the velocity factor in the above equation is 2.0; for a smooth tube this ranges from 1.7 to 1.8. This results in the equation of

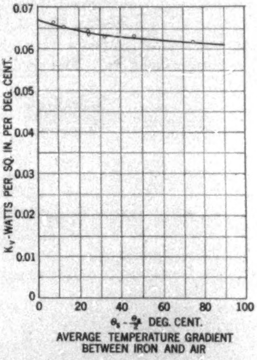

FIG. 22—SURFACE HEAT TRANSFER CONSTANT AS INFLUENCED BY TEMPERATURE GRADIENT OF A MEAN VELOCITY OF 4000 FT. PER MIN.

(On basis of 0.074 lb. per cu. ft.)

$$K_v = A \left(\frac{v}{1000}\right)^n \text{ being}$$

$$K_v = 0.0178 \left(\frac{v}{1000}\right)^{0.93} \text{ for the rough surface}$$

and

$$K_v = 0.0157 \left(\frac{v}{1000}\right)^{0.85} \text{ for the smooth surface.}$$

An increase of both (A) and (n) for an increase in (f) is thus shown.

General Discussion and Conclusions. (a). Expressing the rate of surface heat transfer (K_v) in terms of the average air velocity (v), the following constants for the equation $K_v = A \left(\frac{v}{1000}\right)^n$ are found:

Type of Duct	A	n
Axial smooth surface const. cross-section	0.0157*	0.85
Axial Rough surface const. cross-section	0.0178*	0.93
Axial Annular smooth surface const. cross-section	0.0367	0.75
Rectangular smooth ($^3/_8$ in.) const. cross-section	.0203	0.82
Rectangular smooth ($^3/_8$ in.) (wavy) const. cross-section	.0298	0.77
Radial $^3/_8$ in. diverging flow no fingers variable sect.	.0365	0.77
Radial $^3/_8$ in. diverging flow with fingers variable sect.	.0367	0.72
Radial $^3/_8$ in. converging flow with fingers variable sect.	.0310	0.70
Radial $^3/_8$ in. converging flow no fingers variable sect.	.0204	0.88

*This factor will increase as the duct length decreases.

These results show a wide deviation in values of (A) ranging from 0.0157 to 0.067 and of (n) from 0.70 to 0.93. In general the indications are that when (n) is low the coefficient (A) will be large. The value of (A) of 0.0178 for the axial duct was based on uniform air flow which would exist in very long ducts. As shown in Test 4, Fig. 18, this constant may increase to about double this value for very short ducts. Test 3, Fig. 12, showed that for such short ducts where very turbulent air flow is obtained, the influence of friction upon (K_v) may be negligible. Test 2, Fig. 8, showed that radial ducts with diverging flow and changing cross-section also gave irregular flow and hence high values for (K_v). With converging flow, however, the values of (K_v), Fig. 10, were much lower, even approaching the minimum values found in Curve 1, Fig. 2. The rate of heat transfer (K_v) for Test 1, Fig. 5, with a rectangular duct of constant cross-section is also low and indicates a rather uniform air flow.

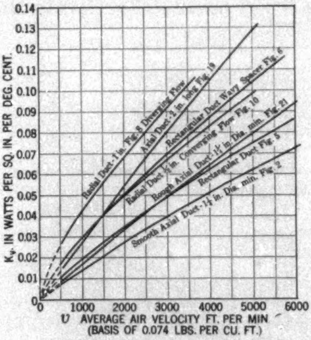

FIG. 23—SUMMARY OF SURFACE HEAT TRANSFER CONSTANT AGAINST AVERAGE AIR VELOCITY FOR VARIOUS DUCTS

Average air temperature intake 25 deg. cent.
Average surface temperature 75 deg. cent.
Atmospheric pressure

A general summary of the variation in (K_v) is given on Fig. 23. It shows that the minimum value is given by the smooth 1¼-in. duct, Fig. 2, with regular flow. The maximum value of (K_v) is found in the radial duct, Fig. 8, with a diverging air flow. This curve is closely approached by axial ducts, Fig. 19, with a 1- to 2-in. duct length.

(b). Test 4, Fig. 18, giving the variation in (K_v) at any point along the duct explains why many researches along this line have been inconsistent. The majority of these tests have been expressed in terms of a (K_v) averaged over the total duct length and since this length varied greatly (K_v) would also vary as shown by Fig. 20. This variable factor of duct length is in principle a check of Latzko's[7] analysis. His equations, however, do not agree in detail with the curves on Fig. 18. According to his calculations, based purely on hydrodynamical relations, the curves on Fig. 18 should reach their minimum value in a length from the entrance much less than shown.

(c). Over the range of temperatures used, 25 to 150 deg. cent., the rate of heat transfer (K_v) at a constant mass air flow decreases slightly with increasing temperature; its approximate relation is: (K_v) varies as $\sqrt{\frac{1}{T_{avg.}}}$ where $(T_{avg.})$ is the average temperature Kelvin of the wall and air. This is in the direction indicated by Rice,[3] Royds, and Campbell,[11] but is opposite to that given by Nusselt,[1] Pohl,[6] and Jordan.[5]

(d). When air flows through ducts of constant cross-section and has reached a stable flow the heat transfer (K_v) will tend to increase with the coefficient of surface friction. When the air flow is irregular, due to conditions such as entrance or irregular cross-section, (K_v) will be large and may be practically unaffected by surface conditions. With the use of baffles the turbulency of the air can be greatly increased with an increase in rate of heat transfer (K_v). The increase in static air pressure drop, however, will be at a greater rate.

(e). A general analysis of surface heat flow constant for a wide range of fluids and gases has been made by Davis,[12] Rice,[3-8] Nusselt,[1] Latzko,[7] McAdams, Frost,[13] and many others. The equations, based upon the physical properties of fluids and upon frictional and hydrodynamical relations, were solved mainly by dimensional analysis. As a general solution, such work is valuable. For specific information regarding the heat transfer covering a definite fluid with definite conditions of flow, temperature, and the like, however, the above general solutions may be seriously in error. The workers listed above have shown that this rate of heat transfer is a function of the physical properties of the fluid such as density, specific heat, viscosity, velocity, thermal conductivity and also a function of the shape and principal dimension of the surface. The tests submitted by the writer have shown that this rate of heat transfer is also a function of the flow lines of the fluid and in this respect becomes a problem of more than a single dimension and at least involves a ratio of principal dimensions. To derive a general solution taking into account the hydrodynamical conditions in addition to the above physical properties seems visionary, at least as regards a practical solution. Thus, for specific information, which is desired by designing engineers of electric machines, experimental results such as submitted on Fig. 23 must be used.

It is also hoped that these specific tests may aid in forming a basis for a better general solution of this surface heat transfer problem covering, let us say, the more restricted field of gas flow which is of great importance in electrical industry.

The writer wishes to express his appreciation of the valuable assistance given in the performance of these tests by Messrs. L. W. Schad, R. R. Sirrs, J. H. Cone, C. G. Veinott, and E. Steinert.

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General Theory of the Auto-Transformer

BY WALTER L. UPSON[1]

Member, A. I. E E.

Synopsis.—*The method of complex quantities lends itself admirably to the development of general theory of the auto-transformer. As worked out, in this instance, it is found desirable to maintain the actual voltage ratio instead of reducing values to an equivalent 1:1 basis. Examples are given of calculations by this method, and also comparison is made with calculated performance of the two-circuit transformer. Certain inherent peculiarities of the auto-transformer are pointed out, being due to the effects of internal resistance and reactance.*

Part II is an application of the theory to transformers without iron and a number of interesting conclusions are brought out.

* * * * *

An auto-transformer may be represented as a single winding on an iron core, as illustrated in Fig. 1. Primary voltage, E_1, is impressed on the winding, and secondary voltage, E_2, is obtained by suitable taps. The primary current, I_1, flows in a portion of the winding of t_1 turns. The rest of the winding carries current, I_2, which is the resultant of the current I_1 and the load current I_L, and it consists of t_2 turns. The total turns of the entire winding are $t = t_1 + t_2$. Ratio of transformation is given by

$$f = \frac{t}{t_2} = \frac{t_1 + t_2}{t_2} = \frac{E_1}{E_2}$$

at no load.

Resistances of windings t_1 and t_2 are r_1 and r_2 respectively, and reactances are x_1 and x_2. Fig. 2 illustrates the vector relations which exist. We may begin with the flux ϕ, drawn vertically upward. This is produced by magnetizing current i_m in phase with it. Ninety deg. ahead of the flux is the core-loss current i_c. i_m and i_c vectorially added give the exciting current I_x. Ninety deg. behind the flux is the e. m. f., e_i, induced in the turns t_2. The load current $I_L = i_L + j\,i_L'$ is drawn at any desired angle with reference to e_i.

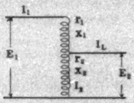

Fig. 1

So far, the vector relations are similar to those of the ordinary two-circuit, constant potential transformer. But in the case of the auto-transformer a departure from similarity occurs at this point. This departure is due to two causes; (1) the current in the turns t_2 is not the load current, and (2) the vectors are given actual values and are not reduced to a 1 : 1 equivalent ratio. This latter consideration is not awkward on account of the fact that auto-transformer ratios do not usually depart very far from 1 : 1, being in that respect quite different from two-circuit transformers.

We now draw $-f\,e_i$, 180 deg. ahead of e_i, and this is that portion of the impressed voltage required to produce e_i in t_2. Similarly, $-\frac{I_L}{f}$ is the load component of primary current, 180 deg. ahead of I_L.

The entire primary current I_1 is the vector sum of I_x and $-\frac{I_L}{f}$. The secondary current, I_2 in t_2, is the vector sum of I_1 and I_L. Secondary voltage, E_2, is obtained by subtracting from e_i the $I_2\,r_2$ and $I_2\,X_2$ drops in turns t_2.

Primary voltage, E_1, is obtained by adding to $-f\,e_i$ the $I_1\,r_1$, $I_1\,x_1$, $I_2\,r_2$, and $I_2\,x_2$ drops, all operations being, of course, vectorial.

The method of complex quantities lends itself readily to this procedure. It is convenient to start with e_i as the zero vector, however, rather than with E_1 or E_2. This makes the use of the method very simple. Certain difficulties which are introduced may be readily provided for. The chief difficulty is in ascribing to the load current a definite, desired phase angle with respect to E_2, as the components of I_L are given with respect to the zero vector e_i. This and other points will appear in the case of examples cited.

The vector equations are as follows:

$$I_1 = I_x - \frac{I_L}{f} = -\frac{i_L}{f} + i_c + j\left(i_m - \frac{i_L'}{f}\right) = i_1 + j\,i_1'$$

$$I_2 = I_1 + I_L = i_1 + i_L + j\,(i_1' + i_L') = i_2 + j\,i_2'$$

$$E_2 = e_i - I_2\,z_2 = e_i - i_2\,r_2 + i_2'\,x_2 - j\,(i_2\,x_2 + i_2'\,r_2) = e_2 + j\,e_2'$$

$$\begin{aligned} E_1 &= -f\,e_i + I_1\,z_1 + I_2\,z_2 \\ &= -f\,e_i + i_1\,r_1 - i_1\,x_1' + i_2\,r_2 - i_2'x_2 \\ &\quad + j\,(i_1\,x_1 + i_1'\,r_1 + i_2\,x_2 + i_2'\,r_2) = e_1 + j\,e_1' \end{aligned}$$

Power input is $P_1 = e_1\,i_1 + e_1'\,i_1'$

Power output is $P_2 = e_2\,i_L + e_2'\,i_L'$

Efficiency is $\eta = \frac{P_2}{P_1}$

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Presented at the Annual Convention of the A. I. E. E., at White Sulphur Springs, W. Va., June 21-25, 1926.

Primary power factor is $$P f_1 = \frac{P_1}{E_1 I_1}$$

Load power factor is $$P f_2 = \frac{P_2}{E_2 I_L}$$

Regulation is given by the equation

$$\text{Per cent regulation} = \frac{E_2 \text{ at no load} - E_2}{E_2}$$

when the primary impressed voltage is constant. All the above relations, however, are based on an assumed constant induced voltage, e_i, and consequently it would seem, judging by common experience with the two-circuit transformer, that the primary voltage will not be the same at load and no load. It is advisable, then, to determine both E_2 and E_1 for the condition of no load, always keeping e_i constant.

The value of E_2 thus obtained should be multiplied by the ratio $$\frac{E_1 \text{ at load}}{E_1 \text{ at no load}}$$

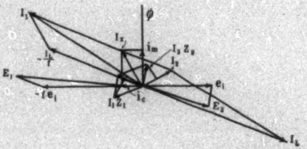

Fig. 2

This reduces E_2 to the value it would have at constant impressed primary voltage. Carrying out this process, equations for the case of $\underset{\cdot}{I}_L = 0$ are as follows:

$$\underset{\cdot}{I}_1 = i_c + j\, i_m = \underset{\cdot}{I}_2$$

$$\begin{aligned} \underset{\cdot}{E}_2 = e_i - i_c z_2 - j\, i_m z_2 &= e_i - i_c r_2 + i_m x_2 \\ &\quad - j\,(i_c x_2 + i_m r_2) \\ &= e_2 + j\, e_2' \end{aligned}$$

$$\begin{aligned} \underset{\cdot}{E}_1 &= -f e_i + i_c (\underset{\cdot}{z}_1 + \underset{\cdot}{z}_2) + j\, i_m (\underset{\cdot}{z}_1 + \underset{\cdot}{z}_2) \\ &= -f e_i + i_c (r_1 + r_2) + j\, i_c (x_1 + x_2) \\ &\quad - i_m (x_1 + x_2) + j\, i_m (r_1 + r_2) \\ &= -f e_i + i_c r_1 + i_c r_2 - i_m x_1 - i_m x_2 \\ &\quad + j\,(i_c x_1 + i_c x_2 + i_m r_1 + i_m r_2) = e_1 + j\, e_1' \end{aligned}$$

$$\text{Per cent regulation} = \frac{K E_2 \text{ (at no load)} - E_2 \text{ (at load)}}{E_2 \text{ (at load)}}$$

where $$K = \frac{E_1 \text{ at load}}{E_1 \text{ at no load}}$$

The value of this constant is either exactly unity or very near to it in practical cases. The difference in voltage between E_1 at load and E_1 at no load is given by the equation

$$\begin{aligned} &\underset{\cdot}{E}_1 \text{ (at load)} - \underset{\cdot}{E}_1 \text{ (at no load)} \\ &= i_L\left(-\frac{r}{f} + r_2\right) + i_L'\left(\frac{x}{f} - x_2\right) \\ &\quad + j\left[i_L\left(-\frac{x}{f} + x_2\right) + i_L'\left(-\frac{r}{f} + r_2\right)\right] \end{aligned}$$

where r and x are total resistance $r_1 + r_2$, and total leakage reactance $x_1 + x_2$, respectively.

This difference reduces to zero in many cases, for example, when $f = 2$, $r_1 = r_2$, and $x_1 = x_2$, which is the case with a uniformly wound coil tapped at its middle point.

Thus, in this case, per cent regulation is simply

$$\frac{E_2 \text{ (at no load)} - E_2 \text{ (at load)}}{E_2 \text{ (at load)}}$$

As an illustration of the whole method we will give values for the calculation of a 50-kva., 60-cycle, 220/110-volt auto-transformer.

Constants, on percentage basis, are as follows:

Induced voltage $= e_i = 1$

$r_1 = r_2 = 0.01$

$x_1 = x_2 = 0.02$

$i_m = 0.025$

$i_c = -0.01$

$f = 2 =$ ratio of transformation

$\underset{\cdot}{I}_L = 1 + j\,0$

Substituting these values into the above equations we get:

$\underset{\cdot}{I}_1 = -0.51 + j\,0.025$

$\underset{\cdot}{I}_2 = 0.49 + j\,0.025$

$\underset{\cdot}{E}_2 = 0.9956 - j\,0.01005$

$\underset{\cdot}{E}_1 = -2.0012 + j\,0.0001$

$P_1 =$ power input $= 1.0206$

$P_2 =$ power output $= 0.9956$

Efficiency $= 0.974$

Primary power factor $= 0.997$

Power factor of load $= \dfrac{0.9956}{0.9956} = 1$

To obtain regulation, we have the following values for the condition of zero loading:

$\underset{\cdot}{I}_1 = \underset{\cdot}{I}_2 = -0.01 + j\,0.025$

$\underset{\cdot}{E}_2 = 1.0006 - j\,0.00005$

$\underset{\cdot}{E}_1 = -2.0012 + j\,0.0001$

$$K = \frac{E_1 \text{ at load}}{E_1 \text{ at no load}} = \frac{2.0012}{2.0012} = 1$$

Therefore,

$$\text{Regulation} = \frac{1.0006 - 0.9956}{0.9956} = \frac{0.005}{0.9956} = 0.005$$

This transformer may be considered as a 1 : 1 two-circuit transformer according to the usual method of calculation. Table I gives comparative performance as auto-transformer and as two-circuit transformer. As transformer of the latter type the constants are:

$e_i = 1, r_1 = r_2 = 0.01, x_1 = x_2 = 0.02, i_m = 0.05$

$i_c = 0.02, \underset{\cdot}{I}_L = 1 + j\,0$

Values of the components of exciting current are seen to be twice those of the auto-transformer. In general, in order that flux and core-loss shall be the same in the two transformers, it is necessary to divide the values for exciting current of the two-circuit machine by the ratio of transformation of the auto-transformer.

TABLE I

	Two-Circuit Trans.	Auto-Trans.
Load Current	$1 + j\,0$	$1 + j\,0$
Efficiency	0.963	0.974
Regulation	0.022	0.005

Comparative efficiencies are as we should expect. Due to reduction in copper losses the auto-transformer shows superiority. In regulation, however, its superiority is far more marked, due largely to the fact that the impedance drop in the secondary winding has a negative effect and tends to raise the secondary voltage instead of lowering it.

Under short circuit, the exciting current ceases to flow in the secondary turns, and is diverted entirely into the short circuit. The short-circuit current may be considered as made up of three parts, (1) that supplied by the secondary turns, which is the induced voltage divided by secondary impedance, (2) the load component of the primary which is equal to (1) multiplied by $\frac{t_2}{t_1}$, and (3) the exciting current, which, at short circuit, is equal to $\frac{t}{t_1}$ times I_x at no load. In the case considered, in which $f = 2$, it is:

$$\underset{\cdot}{I}_2 = \frac{e_i}{r_2 + j\,x_2} = \frac{e_i\;r_2}{z_2^2} - j\,\frac{e_i\,x_2}{z_2^2} = 20 - j\,40$$

$$\begin{aligned}\underset{\cdot}{I}_{sh.\,cir} &= 2\,\underset{\cdot}{I}_2 + 2\,\underset{\cdot}{I}_x \\ &= 40.02 - j\,80.05\end{aligned}$$

where $\underset{\cdot}{I}_x$ is taken as flowing in the same direction as $\underset{\cdot}{I}_2$; that is, $\underset{\cdot}{I}_x = 0.01 - j\,0.025$.

Total short-circuit current is then 89, approximately.

When operated as a two-circuit transformer, the short-circuit current is 22.3, or one-fourth as much as that of the auto-transformer.

It is interesting to notice the conditions as relating to the exciting current with various loadings of the transformer. At no load, its value is the same in both parts of the winding, t_1 and t_2. Exciting ampere-turns are $I_x\,t$. As the load is increased the load impedance offers a path for the exciting current in parallel with t_2, thereby causing a slight increase in the exciting current which flows in t_1 and a slight decrease in that which flows in t_2. No account is taken of this variation for ordinary loading, as it is insignificant. The total exciting ampere-turns remain constant for all loads, producing constant mutual flux and constant induced voltage in the secondary. Voltage induced in t_1 must also be constant. At short circuit, we have maximum exciting current in t_1 and zero exciting current in t_2. In a 2 : 1 transformer the value of this current in t_1 is $2\,I_x$, where I_x is the no-load exciting current.

The total $I_1\,Z_1$ drop in t_1 is approximately equal to e_i, at short circuit, so that the impressed voltage $E_1 = 2\,e_i$. This relationship is similar to that which exists in a two-circuit transformer, where, at short circuit, the voltage drop in the primary is approximately equal to the induced voltage. In this case, however, the equality is brought about by reduction in the value of the induced voltage, which, at short circuit, is only half its no-load value.

Part II

AUTO-TRANSFORMERS WITHOUT IRON[2]

Air-core transformers are in general at a disadvantage owing to the fact that their magnetic fields are produced in a medium of unit permeability. Absence of iron, however, offers some compensating advantages, namely, absence of core-loss and exciting current distortion, and saving in cost of iron and in cost of construction. At sixty cycles, these advantages are far out-weighed by the disadvantages in performance as compared with two-circuit transformers. At higher frequencies the situation becomes more and more favorable for the air-core transformers, until at radio frequencies the use of iron is inadmissible.

With auto-transformers the case is distinctly more favorable for the air-cores at lower frequencies. The same theory applies as with iron-core transformers except that the core-loss component of the exciting current drops out. The constants are, of course, very different and it will be of interest to give some values and results that may be expected in practise. An air-core auto-transformer should be wound for maximum self-inductance of the entire winding and maximum mutual inductance of the two parts, that is, of t_1 and t_2. This maximum mutual inductance will, of course, mean minimum leakage reactance.

An auto-transformer may be designed from either of two points of view as respects operation. It may be designed either to operate at a fixed ratio of transformation or as a compensator with taps to give different voltages over a considerable range of ratio.

Table II gives comparative results to be expected from an air-core compensator with three taps giving voltage ratios of 4 : 3, 4 : 2, and 4 : 1. The same voltage is impressed on the primary in each case, and the same load current is assumed. The constants for the three cases are given in the preceding table.

It is to be noted that for low ratio, the primary turns are relatively heavily loaded while the secondary turns are lightly loaded.

With high ratio, the opposite is true. Therefore, if

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[2] A paper on this subject was presented by the author before Section M of the American Association for the Advancement of Science, December 1916, but was preliminary in character and never has been published.

TABLE II

	A	*B*	*C*
Transformer ratio	4:3	4:2	4:1
Primary resistance, r_1	0.01	0.02	0.03
Secondary resistance, r_2	0.03	0.02	0.01
Primary reactance, x_1	0.1	0.2	0.3
Secondary reactance, x_2	0.3	0.2	0.1
Magnetizing current	0.3	0.3	0.3
Load current	$1-j\ .1$	$1-j\ .1$	$1-j\ .1$
Induced voltage, e_i	1.5	1	0.5
Primary voltage, E_1	2.12	2.12	2.12
Primary current, I_1	0.838	0.61	0.41
Power input	1.5945	1.0642	0.5349
Secondary voltage, E_2	1.575	1.047	0.52
Secondary current, I_2	0.372	0.559	0.783
Power output	1.5833	1.05	0.523
Efficiency	0.992	0.986	0.979
Primary power factor	0.896	0.822	0.614
Load power factor	1 approx	1.	1.
Secondary no-load voltage	1.59	1.06	0.53
Primary no-load voltage	2.12	2.12	2.12
Per cent regulation	0.0095	0.0124	0.0192

the transformer is to be operated continuously at a fixed ratio, the cross-section of wire for primary and secondary should be proportioned to the respective currents to be carried. For a 2 : 1 ratio, the wire should be the same in cross-section for both windings.

In the above table, we see, as we saw with the case of the iron-core transformer, that the impressed voltage is constant regardless of load. The effect of $r\,I$ drop in the primary is exactly neutralized by the $r\,I$ drop in the secondary except for the small and constant drop due to the exciting current. This balance of drops does not obtain if the resistances and reactances are not so proportioned as to accomplish it, but there is always a tendency in that direction. To illustrate this situation we will make comparison of two auto-transformers with the following constants: ratio of transformation, 5 : 4; exciting current, 0.3; load current, $I_L = 1 - j\,.2$; induced voltage, 1; total resistance, 0.04; total reactance, 0.4. The resistances and reactances of the two transformers are differently distributed between primary and secondary, as follows:

TABLE III

	D	*E*
Primary resistance, r_1	0.02	0.003
Primary reactance, x_1	0.2	0.03
Secondary resistance, r_2	0.02	0.037
Secondary reactance, x_2	0.2	0.37
Primary current, I_1	0.922	0.922
Secondary current, I_2	0.328	0.328
Primary voltage, E_1	1.41	1.358
Secondary voltage, E_2	1.05	1.093
No-load primary voltage	1.37	1.37
No-load secondary voltage	1.06	1.115
Regulation constant, K	1.03	0.99
Per cent regulation	0.038	0.011

Transformer *E* is seen to operate at nearly constant impressed voltage, and it is this one in which the wire has been proportioned to the currents to be carried.

Certain inherent features of the performance of air-core transformers are of interest. They are due to the fact that the magnetic field is in a medium of constant permeability and consequently all effects due to magnetic saturation are automatically eliminated.

1. Thus, if the characteristics of an air-core transformer are known for one value of impressed voltage, they may be obtained for any other value. For example, let $E_1 = 100$, and let maximum efficiency occur with a load of 10 amperes, and, let us say, 950 watts. If, now, the voltage is raised to 110, maximum efficiency will occur at $\dfrac{110}{100}$ times $10 = 11$ amperes, and $\left(\dfrac{110}{100}\right)^2$ times $950 = 1150$ watts. One efficiency curve applies to the transformer, whatever the impressed voltage, provided the scale of amperes and watts is changed in the proper ratio.

2. To obtain maximum capacity, it is only necessary to impress such a voltage on the primary as will correspond with maximum permissible current in the windings at full load, or, conversely, the windings of a transformer may be so chosen as to carry maximum permissible current at full load with any specified impressed voltage.

3. Regulation of an air-core auto-transformer is more affected by leakage reactance than by resistance, when operating at high power factor. This fact is quite apparent from a study of the vector diagram. The primary $I\,x$ drop is almost directly in phase with the impressed voltage, instead of being at right angles to it, giving nearly the worst condition for regulation. To offset this, however, the secondary $I\,x$ drop, while considerably out of phase with E_2, is generally so directed as to assist regulation, partially neutralizing the bad effects of primary reactance.

(4). Power factor of the air-core transformer is generally much poorer than that of the iron-core type because the magnetizing current is a comparatively large out-of-phase component of the primary current. This large magnetizing current is not altogether undesirable, however. It causes a greater lead of I_2 with respect to E_2 and reduces the amount of I_2 without proportionally increasing I_1.

In general, it may be said that the air-core auto-transformer offers possibilities at 60 cycles. At higher frequencies, such as 120 or 180 cycles, it becomes of decidedly more than theoretical interest.

Discussion

J. R. Craighead: In calculations on instrument transformers, more exact work is necessary than on power transformers if the results obtained are to be worthy of the accuracy which can be developed in the transformer itself.

One of the main difficulties that we have found in obtaining exact results by calculation is the separation of the primary self-inductance from the secondary self-inductance. It is very easy to obtain a total self-inductance for a transformer but not easy to separate the two components.

We have used various methods and are now able to separate the two quantities with reasonable accuracy on standard transformers. In certain cases where auto-transformers are concerned, there is considerable difficulty in finding a method which will accurately separate the two components.

I should like to ask if Mr. Upson has any suggestions as to how this fundamental separation of the two inductances can be made for purposes of accurate calculation?

In attempting the separation of secondary reactance in current transformers we have calculated ratio and phase angle from exciting current and losses. Successive assumptions were made of secondary reactance until a value was found which brought the result of calculation into agreement with test results. This value of secondary reactance was then assumed to be characteristic of the structure of the type of transformer under consideration. We also built models in which the secondary was made of cored wire so that the inner wire could be used to obtain measurements from which the secondary resistance was excluded.

If we did that with the auto-transformer, would the resulting complex of primary and secondary voltage render the function an unsatisfactory one from which to calculate?

W. L. Upson: I don't think we should have any greater difficulty with the auto-transformer. In general, I think it would be more symmetrical than the other, or may be made so at any rate.

If you are going to fix up an auto-transformer, you can carry the wires themselves right along together so that they will be perfectly symmetrical with respect to one another and that will give you the minimum leakage reactance, which is what you want. In general, that is true; I don't know whether you want it for your current transformers, but I presume that you do. You get better action from an auto-transformer with the minimum reactance and there is no reason why you shouldn't get your leakage reactances by that method in the auto-transformer if you can in the other.

I think we can get the leakage reactances and inductances combined with some fair degree of accuracy. But, to separate them into their two parts and get the correct values for both of them,—the only way I know of doing that, (and it is no good for accurate work) is by some sort of graphical or geometrical study of the system.

I have a feeling, though, that we can make some progress in that kind of determination and that that is about as far as I can go. I don't know how to get at it in any other way.

The Vincent 220-Kv. Transmission Line

Engineering and Construction Features

BY C. B. CARLSON[1] and H. MICHENER[1]

Non-member Associate, A. I. E. E.

***Synopsis.**—This paper deals with the design and construction of the third 220,000-volt line between Los Angeles and Big Creek, giving the results and brief descriptions of the research engineering investigations which were carried out in connection with the design.*

The salient features of the line are:

Terminals—Big Creek No. 3 and Gould Switching Station, near Los Angeles.

Length—223.5 mi.

Route—Direct route with few angles.

Dead-ends—Dead-ends only at station terminals and heavy angles.

Small angles—Small angles up to 11 deg. 40 min., in some cases, turned on suspension insulators.

Conductors—Steel reinforced aluminum. 1,033,500 cir. mils aluminum, 54 strands. 134,000 cir. mils steel, 7 strands. Weight: 1.33 lb. per ft. Diameter: 1.247 in.

Overhead Ground Wire—None.

Towers—Towers of high elastic limit steel. Height of standard towers, 64 ft. to crossarm. Height of anchor tower, 56 ft. to crossarm. Weight of standard tower, 8890 lb. Weight of anchor tower, 13,600 lb.

Tower Footings—Grillage footings in earth. For standard towers, 4 ft. 8 in. sq. at base and 8 ft. deep. Weight, 2040 lb. For anchor towers, 7 ft. 6 in. sq. at base and 9 ft. deep. Weight, 6720 lb.

Tower Extensions—Height 7, 14, or 21 ft. Any of these heights may be placed on any or all corners of the tower and in a few special cases in combination with an additional 14-ft. extension on all corners.

Insulators—10-in. cap and pin having minimum combined electrical and mechanical strength of not less than 18,000 lb.

Suspension insulator assembly: Single string of 13 units where maximum stress in string will not exceed 6000 lb. Double string 13 units in length where maximum load will exceed 6000 lb.

Dead-end assembly: Double string, 15 units in length.

Tie-down assembly: Single string of 14 units.

Shield Rings—Hoops of 3-in. by $^3/_{16}$-in. strap iron.

Bird Guards—Pans and saw-tooth guards on all towers.

Increases in the generator capacity at Big Creek, which are scheduled to be completed in 1928, will require more transmission capacity to deliver the power to Los Angeles. For this reason, a third 220,000-volt line is being built between those points.

As indicated in Fig. 1, which shows the general location, the line starts from Gould, crosses the San Gabriel Mountains where it reaches an elevation of 5609 ft., and then crosses the Antelope Valley. Thereafter it passes over the Tehachipi Mountains at an elevation of 5361 ft., beyond which it crosses the southern end of the San Joaquin Valley, and then stays in the foothills along the eastern edge of that valley until it reaches the mountains at Big Creek where it crosses Pine Ridge at an elevation of 4827 ft.

The first fifteen-mile section through the San Gabriel Mountains is the roughest and most inaccessible part of the route. Through this section it was necessary to construct a road to serve the line,—more than two miles of road for every mile of line. But this heavy road expense is less than the right of way expense would have been for a route paralleling the old Big Creek lines. The remainder of the line is very accessible to already existing lines of travel with a few exceptions where short pieces of road are being constructed

The average number of towers per mi. is 3.94. The normal span in level country for standard towers without extensions is 850 ft. in heavy loading districts and 1000 ft. in light loading districts. As a matter of fact, there are very few, if any, normal spans because of the man-made obstructions in the level country, such as roads, power lines, etc. The span lengths for the whole line range from a minimum of 336 ft. to a maximum of 5191 ft. The two longest spans are the same length. It happens that a change from heavy to light loading occurs at one end of one of these spans, so the conductors are dead-ended there. Otherwise these two long spans are supported on double-string suspension insulators.

LOCATION AND SURVEY

The general route of the line was chosen from a study of maps and from a general knowledge of the country. Reconnaissance along this route was carried out by airplane, automobile, horse, or on foot, the choice between the last three means being made according to the terrain. The whole route was flown over at least once and parts of it several times. In two highly developed sections, airplane photographs were taken.

After the reconnaissance work was done, the line was run with transit and chain. It was so located with regard to topography and man-made improvements that a total of four circuits can be placed side by side with 80-ft. center line separation, but a 200-ft. right of way to accommodate two circuits is all that has been obtained. The line actually run on the ground was the center line of the west tower line. In the rough country where there were no control points, a triangulation system was laid out as a check on the line survey.

The profile of this center line was run, using vertical angles and slope distances, and checking on U. S. G. S. and other bench marks of known elevation wherever such were available. Side slopes were taken at the probable tower locations and at all points where the clearance from one of the outside conductors to the ground would be near or less than the allowable minimum.

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[1] Both of the Southern California Edison Company, Los Angeles, Calif.

Presented at the Pacific Coast Convention of the A. I. E. E., Salt Lake City, Utah, Sept. 6-9, 1926.

With this information on the profile the towers were located by means of a celluloid template. Survey crews took these paper locations into the field and staked out the tower footings, moving the towers along the line if this improved the setting of the towers without decreasing the conductor clearances below the allowable amount. In staking the footing locations, the proper elevations of the footings were determined consistent with the use of the various lengths of leg extensions and downward footing extensions. This information was turned into the office where it was checked in regard to the proper footing elevations and the economic balance between cost of extension and cost of excavation. After checking, it was used for making bills of material for tower and footing extensions.

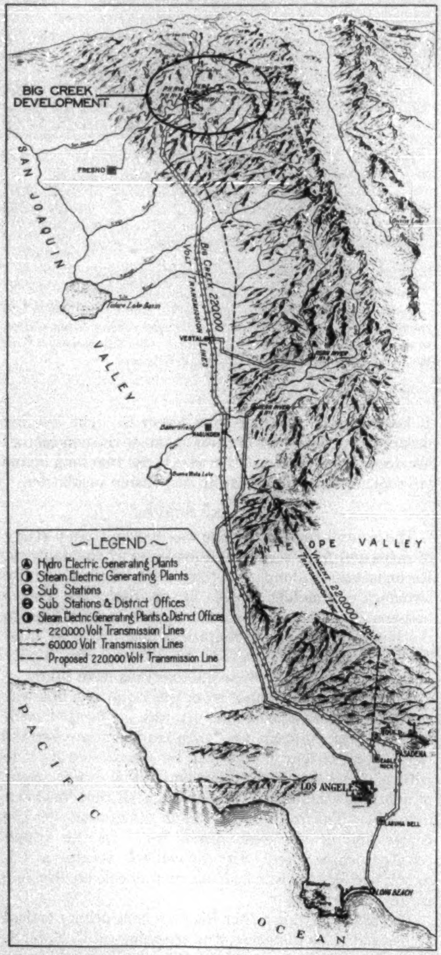
BIG CREEK DEVELOPMENT
SAN JOAQUIN
FRESNO
Tulare Lake Basin
Big Creek 220000 Volt Transmission Lines
VALLEY
VESTAL
Bakersfield
MAGUNDEN
Buena Vista Lake
ANTELOPE VALLEY
Vincent 220000 Volt Transmission Line
LEGEND
Hydro Electric Generating Plants
Steam Electric Generating Plants
Sub Stations
Sub Stations & District Offices
Steam Electric Generating Plants & District Offices
220,000 Volt Transmission Lines
60,000 Volt Transmission Lines
Proposed 220,000 Volt Transmission Line
PACIFIC
OCEAN
GOULD
PASADENA
EAGLE ROCK
LOS ANGELES
LAGUNA BELL
LONG BEACH

Fig. 1—System Map of Southern California Edison Company

Showing route of 220-kv. Vincent transmission line

Telephone Line

A four-wire telephone lead supported on pressure-treated, southern pine poles is being built. It is located mostly along the roads near the transmission line. It will serve the Construction Department until the transmission line is completed, and after that it will be used for the routine operation of the line and as an additional communicational facility between Los Angeles and Big Creek.

Loading Assumptions

The line from Gould to the south end of the San Joaquin Valley and for 10 mi. south from Big Creek No. 3 is in so-called heavy loading territory where it is assumed that the most severe loading conditions will be one-half in. radial thickness of ice at zero deg. fahr., with a wind pressure of six lb. per sq. ft. on the projected area of the ice-covering. These sections are generally above 3000-ft. elevation except the 18 mi. across the Antelope Valley, which has an elevation of 2500 to 2700 ft. where the line crosses but which is subject to high winds and low temperature. The remainder of the line is in light loading territory where the maximum loading conditions are assumed to be no ice at 25 deg. fahr. with a wind pressure of eight lb. per sq. ft. on the projected area of the wire.

Conductor

The preliminary mechanical and electrical studies to determine the most economical conductor, tower height, and conductor tension were described by Messrs. Carlson and Shaw at the Pasadena, 1924 Convention.[2] Since that time the design has been completed. Construction of the necessary roads began in August, 1925, and actual line construction began about January 1st, 1926. It is expected to have the 96 mi. between Gould and Magunden in operation by November, 1926, and the whole line in operating condition by the early months of 1928.

The conductors are steel reinforced aluminum cables having 54 aluminum strands and seven steel strands each 0.1385 in. in diameter. The total diameter is 1.247 in. and the cross-sectional area of the aluminum is 1,033,500 cir. mils. This is equivalent in conductivity to 650,000-cir. mil copper. The weight is 1.33 lb. per ft. The approximate ultimate strengths are 20,000 lb.

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2. *Transmission at 220 Kv. on the Southern California Edison System*, Sections 3-A and 3-B, *Economic Studies of Transmission Line Design*, C. B. Carlson and W. D. Shaw, A. I. E. E. Journal, Vol. 43, Oct. and Nov., 1924, pp. 907 and 1025.

for the core and 36,000 lb. for the composite cable. This is probably the largest cable ever used as a transmission line conductor.

The line was designed for a maximum conductor tension of 12,000 lb. in both heavy and light loading territory, which gave practically the same unit stress as had existed in the old Big Creek lines for 11 years, but when it came to fitting the wire curve to the profile it was found that strict adherence to the 12,000-lb. maximum would give low clearance from conductor to ground in several spans where the topography made it very diffiult to move the towers to get the required clearance. If the 12,000-lb. maximum could not be exceeded, one or more additional towers would be required in these spans and they would be in very costly locations. A slight increase in maximum tension would give ample clearance in these locations at which it was believed that neither maximum wind nor minimum temperature would occur at any time and surely not simultaneously. For these reasons it was decided to allow the maximum tension in these cases to go as high as 13,500 lb. in the heavy loading district and 12,600 lb. in the light loading district, when calculated on the maximum loading assumptions stated above.

CONDUCTOR SPLICES

The conductor splices are being furnished by the manufacturers of the cable and are of the latest type. The main improvement over the old type is in the method of splicing the steel core. Instead of using a twisted McIntyre joint, the two ends of core to be joined are each inserted from opposite ends half-way into a single-bore, soft steel sleeve, and then the sleeve is compressed onto the core. On test the core breaks approximately at its calculated ultimate strength and does not slip out of the sleeve. The use of this compression sleeve on the core permits the use of a single-piece, aluminum compression sleeve for connecting the aluminum strands instead of the two-piece, screwed-together compression joint used in the past. These improvements make better and cheaper joints than those previously used.

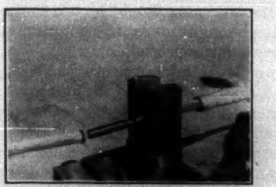

FIG. 2—CONDUCTOR SPLICE BEING MADE

Steel sleeve on core partially compressed. Aluminum sleeve slipped over cable to the right

The steel sleeve and the then exposed parts of the core are given a coating of hot asphaltic compound and then wrapped with a layer of cambric tape after the compression has been made and before the aluminum sleeve has been slipped into place. This is to prevent the black-iron sleeve from starting a rusting reaction which may be communicated to the steel core and eventually destroy it. The tape is to prevent the compound from being wiped onto the inside of the aluminum sleeve as it is slipped into place. Accelerated tests have led to the belief that such a coating will protect the core from rusting at least for the life of the galvanized steel and aluminum wires elsewhere in the line. Fig. 2 shows these splices being made.

GROUND WIRE

No overhead ground wire is being installed, but the towers are designed with sufficient strength to carry one ground wire if it should prove desirable to install it later.

TOWERS

The preliminary economical calculations to which reference was made determined the height of structure necessary as well as the maximum tension which would occur in the cable. The design of tower proper, the extensions, transposition frames, footings, etc., involved types of framing and panel lengths which would prove most economical. Previous experience had shown the desirability of vertical legged extensions rather than continuing along the batter of the main tower.

Panel lengths of eight ft. in the main structure made the height of the tower to the crossarm 64 ft., while the panel length was reduced to seven ft. in the extensions to compensate for the increase in stress caused by change in direction. Extension heights of 7, 14, and 21 ft. were those which seemed to supply the needs of the profile. These extension legs were arranged to permit combinations of any of them on a tower to more economically fit the profile. This latter arrangement has proved useful, as much of the country traversed was very rocky and difficult to excavate.

Special cases required the combination of the 14-ft. and 21-ft. extensions making 35 ft. in all, and in the case of the Tule River Crossing two special 120-ft. towers were used. It was also necessary to supply certain other specialties such as transposition frames, attachments for towers to solid rock, footing extensions where uplift cover resistance was not available, and single leg extensions without bracing to main structure. These were developed to best suit field requirements which were reported in the field notes of the survey parties.

Material for the structures was specified which would have a high elastic limit, permitting the use of higher unit stress value. This value was 22,500 lb. per sq. in. or a factor of safety of two on the elastic limit.

For the light loading district, standard towers were designed to withstand (a) the strain caused by two broken wires having maximum tension (reduced by adding the length of the insulator string to the wire length), plus (b) a wind pressure of 8 lb. per sq. ft. on the wire surface and on 1½ times the area of one tower face, plus (c) the vertical weight of the wire. In the heavy loading district the wind pressure was reduced to 6 lb. per sq. ft., but ½ in. of ice radially was added to the cable. Weight and surface area were calculated on this basis. All loads were assumed as being simultaneous for maximum conditions, which also included the component of the maximum angle, which was 11 deg. 40 sec., to be turned on suspension towers.

The anchor tower was designed for full breast pull, plus wind on line and tower, plus the component of the largest angle on the line, which was 35 deg. 14 sec. The maximum loads were all assumed to be simultaneous.

The footings were designed to use the 30-deg. cone of earth as resistance against uplift and to use a low unit value for bearing as resistance against settlement. They are of steel, soil conditions generally showing no bad effects toward corrosion. It is planned to coat those footings with bitumastic solution where alkali conditions prevail. The general practise in transmission line design of the use of hot dip galvanizing after fabrication and the use of galvanized bolts was maintained for this line. The Southern California Edison Company's experience seems to justify this method as proof against deterioration, but care must be taken to get well coated bolts.

In order to justify the design, tests were conducted on both types of towers and contrary to the usual method of test in which towers are fixed to heavy concrete and steel foundations, the tests were made on the earth footings set to conform to ordinary line conditions. These tests proved that deflections unavoidable in newly tamped earth back fills do not furnish absolute fixed support. This condition in turn sets up secondary stresses which must be recognized in the design. In light framed structures, where there are no contributing elements toward strength such as are found in buildings or like structures, extreme care must be taken to avoid the combination of bending and direct stress. Also, as the connections are all partially fixed, and the ordinary resolution of forces by graphic or analytical methods does not allow for the bending set up in fixed or partially fixed joints, considerable judgment must be exerted in the design of the towers.

It seems well worth while to spend the time on steel details to assure accuracy, as much time and trouble in erection is thereby saved. Also a small saving on parts will result in large aggregate saving due to the great numbers of parts required, but the designer should also guard against leaning too heavily toward saving which would result in the risk of the strength of the structure.

CONDUCTOR STRINGING

The locating of towers on the profiles was done by means of a celluloid template representing sags at 130 deg. fahr. on the basis of 30-ft. clearance for all territory susceptible of agricultural development, and 25-ft. clearance elsewhere. Cross slopes were shown on the profiles and these clearances obtained under the up hill wire.

The 130-deg. sags for the template were calculated on the basis of equal tensions in all spans at 70 deg. fahr. This method was used instead of the more usual method of equal maximum tensions for the following reasons: In this line there are several very long spans which, in heavy loading, are stressed much higher than the other spans. To limit the tension by them would give low values not economical for the majority of spans. By making tensions equal at 70 deg., the desired maximum tension of 12,000 lb. could be attained in most of the spans, and exceeded in only a few. The proper value at 70 deg. was determined from preliminary profiles of representative sections of the line, one value being used for light loading and another for heavy loading.

After towers had been located on the profile, surveying parties made preliminary field stakings from which tower grades were determined and extensions fitted to each location. Then, knowing spans and difference of elevation, sags were calculated and all clearances checked. Special calculations were made to determine clearances over other electric lines, under the assumption of a broken wire in the span adjacent to the crossing span.

From the above field data, the horizontal and vertical loads on each tower were calculated to determine where double strings of suspension insulators were required, where the insulators should be tied down, and also to check conditions at towers where horizontal angles were turned on suspension insulators.

After much of the above work had been done, results from a 1060-ft. test span of the new cable showed that changes must be made in the calculations. Stress-deformation curves proved conclusively that there was a stretch of about 0.04 per cent in the cable after stringing, and that while the first or temporary value of the modulus of elasticity was 7,750,000 lb. per sq. in., the final or permanent value was 9,800,000 lb. per sq. in., this latter value being effective after the cable had been subjected to the maximum tension a number of times. Figs. 4 and 5 show the test span and the recording dynometers.

In order to string the cable under the above conditions and to maintain the 130-deg. clearances obtained from the template layout, the following method was used, all calculations being made on a catenary chart similar to the Thomas Curve:

1. From the 130-deg. sag, calculate the maximum tension (under light or heavy loading conditions, as the case may be) using the permanent modulus.
2. Reduce the stressed length by 0.04 per cent, thereby getting a temporary maximum tension for that loading, assuming no stretch from the increased tension.
3. From this temporary maximum tension, find

sags and tensions at any desired stringing temperature, using the temporary modulus.

From the tensions thus obtained, stringing values for 40 deg., 70 deg., and 100 deg. were chosen such that the resulting maximum tensions would not exceed the values previously mentioned. These stringing values varied for different sections of the line, being dependent on the length and slope of the spans; and care was taken to have small changes in successive values, in order that the difference in tension would cause only a small deflection of the suspension insulators.

Since the line was to be strung to tension by sag measurements, and since the parabolic formula for sag is much simpler than the catenary formula, correction curves were plotted showing the difference between parabolic and catenary sags for various spans and tensions. Then the catenary sag used is equal to $\left(\frac{w\,l^2}{8\,H}\right)$ sec. $\theta + K$, where W = weight per ft., l = span, H = stringing tension at desired temperature, θ = angle of slope of span, and K = correction. Another curve was plotted showing the difference between the stringing sag and the final sag at 130 deg. after the cable had stretched, this latter value being used for determining all final ground clearances.

Two field engineers were assigned to each construction gang so that sags could be checked in at least two spans for each pulling section. The method used was to set a transit a distance below the point of support of the wire equal to the center sag, and sight along a line parallel to the chord of the span. The cable was pulled until it was tangent to this line of sight. Where the sag was less than the height of the tower, a specially designed clamp was used permitting the attachment of the head of the transit to the tower leg. In the longer spans, sag stakes were set at each end of the span, so that, for a fixed height of instrument, the transit was on the required line of sight, and the cable sagged in as before.

Previous to the stringing of the cable there had been considerable discussion in regard to the proper methods of attaching the insulators to the cable. The absence of frequent dead-end connections and the length and slope of the spans made a complicated problem. It was speedily found that the suspension insulators would be vertical in the mountainous sections of the line only at the stringing temperature, and that in some extreme cases, having them vertical, they would not give the required length of wire in each span. The method finally adopted was to attach the suspension clamp vertically below the crossarm in every case, adjusting tension and attaching clamps span by span in the extreme cases to insure having the wire lengths right.

Field experience has justified this method and shown that only in very rare cases is it necessary to insulate the spans separately.

INSULATORS

The insulators are practically the same electrically as those used on the Eagle Bell line. They differ mechanically from those in that all the units are of the high strength type; that is, they have a combined electrical and mechanical strength on short time tests of not less than 18,000 lb. The suspension insulators consist of a single string of 13 units where the maximum load will not exceed 6000 lb. and of two parallel strings 13 units in length where the maximum load will be greater than 6000 lb. The dead-end insulators consist of two parallel strings 15 units in length. Fig. 3 shows these three arrangements. It will be noted that the double suspension and dead-end assemblies use the same yokes and clevises. A single string of 14 units is used in the tie-down insulators where such is required to limit the side swing of the suspension insulators.

About one-half of the units might have been standard 9000-lb. units because they would have given the required factor of safety of three in many of the suspension insulators. But it is very desirable to have units of only one kind in a line, so it was gratifying to find the bid prices such that the additional cost due to using high strength units throughout could be justified on the grounds of simplified construction, maintenance, and store stock.

Extensive long time loading tests were made on sample high strength suspension insulator units submitted by the various manufacturers for the purpose. A description and the results of these tests have been published in the National Electric Light Association 1926 Annual Report of the Overhead Systems Committee, Insulator Research Subcommittee, R. J. C. Wood, Chairman.

Briefly, the tests were as follows:

A frame was made in which four strings of 20 units each were suspended. Each string was composed of four units from each of five manufacturers. These strings were loaded at 9000, 11,000, 13,000, and 15,000 lb. each and the load maintained continuously except as interrupted by broken insulators and, at one time, by a modification in the supporting structure. At irregular intervals the insulators were tested electrically, one at a time. Quick-time, ultimate, mechanical tests also were made on each make of these insulators.

The conclusions drawn from these tests are worthy of quoting from the report; they are:

1. That the quick-time ultimate mechanical strength cannot be relied upon as a criterion upon which to base working load, the relation between the two depending upon either design or material, or both.
2. That more than one manufacturer can supply 10-in. insulators that will apparently sustain from 11,000 to 13,000 lb. dead load for very long periods of time, and it is safe to use them up to 6000 lb. dead load on long-span transmission lines.
3. That incipient mechanical failure occurs at the time of electrical failure under load and will probably

be followed by complete mechanical failure at some subsequent date without increase of load.

4. That there is some indication that at some period during the long time test of 11,000 hours, the average ultimate mechanical strength of all insulators loaded to 9000 and 11,000 lb. decreased some seven or eight per cent. Great caution must be observed, however, in generalizing from tests made upon an aggregate of only 83 insulators.

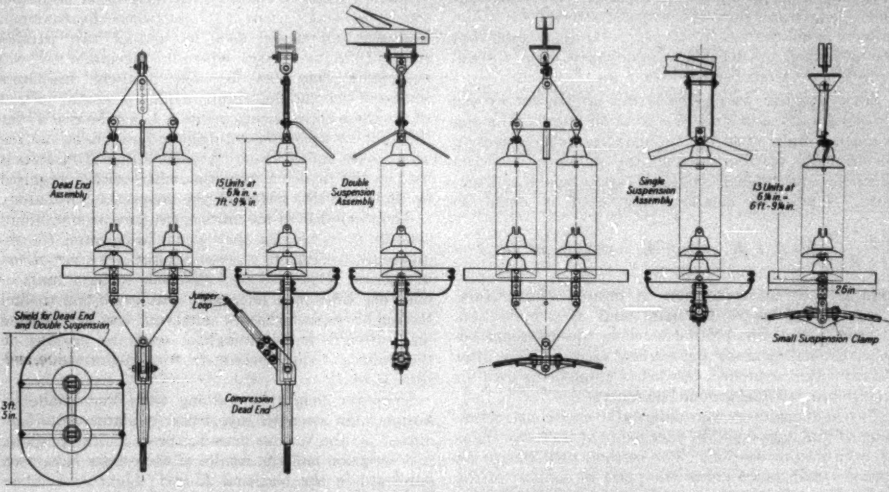
Dead End Assembly
15 Units at 6¼ in. = 7ft.-9¾ in.
Double Suspension Assembly
Single Suspension Assembly
13 Units at 6¼ in. = 6 ft.-9¼ in.
26 in.
Small Suspension Clamp
Jumper Loop
Shield for Dead End and Double Suspension
3 ft. 5 in.
Compression Dead End

FIG. 3—INSULATOR AND INSULATOR HARDWARE ASSEMBLIES

Showing single-string suspension, double-string suspension, and dead-end with compression clamp on cable

INSULATOR HARDWARE

All the insulator hardware is made of forged steel or wrought steel, except the aluminum part of the compression dead-ends and the long suspension clamps which latter are of malleable cast iron. All pins are of chrome-nickle steel.

SHIELD RINGS

The insulator shield rings are practically the same, electrically and also mechanically, so far as over-all dimensions and position on the insulator strings are concerned, as those used on the present 220-kv. lines of this company. However, instead of being of cast aluminum of an inverted U-shaped cross-section, they are each made of a hoop of 3/16-in. by 3-in. steel strap. These are much less expensive than the old shield rings. Laboratory tests do not indicate any particular advantage in favor of the old shape of ring in either voltage grading or flashover characteristics.

DEAD-ENDS

The development (in so far as it had advanced by the summer of 1924) of a light weight, compression, dead-end fixture to minimize the effect on the conductor itself of conductor vibration has been described and illustrated.[3] The main features of this compression dead-end had been worked out simultaneously by the Aluminum Company of America and the Southern California Edison Company. The main differences in

FIG. 4—TEST SPANS FOR DETERMINING AMOUNT OF PERMANENT STRETCH AND MODULUS OF ELASTICITY OF 1,033,500-CIR. MIL STEEL REINFORCED ALUMINUM CABLE

the two designs were that the former anchored the steel core in the steel clevis-socket by pouring melted zinc around the core in the socket while the latter accomplished the same results by means of conical wedges in a conical socket, and that the former transmitted the load to the insulator assembly from the aluminum compression body through aluminum clevis ears cast onto it while the latter accomplished this result by screwing the aluminum body onto the steel clevis-socket. Both of these schemes undoubtedly would have given satisfactory results in service as they did on test but they were not tried because the Aluminum Company put forward an improvement in the method of attaching the steel core to the steel clevis. Instead of a conical socket, a steel tube about six in. long was forged as a part of the clevis. Into this tube the end of the steel core was placed and then the steel tube was compressed using the same press with smaller dies as was used to press the aluminum. Tests proved that the core would break at approximately its calculated ultimate strength and would not pull out of the tube.

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[3] *Transmission at 220 kv. on the Southern California Edison System*, Section 4, *Vibration of Conductors and Overhead Ground Wires*, by J. M. Gaylord, JOURNAL of A. I. E. E., Nov. 1924, p. 1026.

Fig. 5—Recording Dynomometer on Test Spans

The idea of having the load transmitted from the aluminum compression body to the insulator assembly through an aluminum clevis cast as a part of the compression body was accepted, but a change was made in the position of the lug to which the jumper loop fastens which altered the appearance considerably. This change was due to the desire to have the centerline of the jumper loop, where it is attached to the dead-end, pass through the clevis pin of the dead-end and thus minimize the effect that the jumper loop has on the relative movement between the compression dead-end and the conductor immediately adjacent thereto, due to conductor vibration. A test briefly described below had indicated that the jumper loop very appreciably increases this relative movement and that all reasonable measures should be taken to minimize this effect.

On a span having vibrations forced upon it by a vibration machine, measurements of the relative movement of the compression dead-end and the cable were made by means of a pencil attached to one and a strip of paper in a holder attached to the other. With neither shield ring nor jumper loop attached to the dead-end, the mark drawn as the paper moved past the pencil was practically a straight line. With the shield ring but not the jumper loop attached, the waves in the pencil mark had an appreciable amplitude. And with both the shield ring and the jumper loop attached, the amplitude of these waves was several times as great as with the shield ring only; hence the conclusions stated above.

Careful consideration was given to the bolted connection between the jumper loop and the compression dead-end. Such bolted connections have been known to give considerable trouble when the bolts and the lugs are of dissimilar metal. In the case of aluminum lugs and steel bolts, the greater expansion of the aluminum with rise of temperature might cause either or both metals to be stressed beyond the elastic limit. In either case the joint would be loosened upon subsequent cooling. This accounts for the large number of bolts in this connection.

Trouble at Suspension Clamps

About the time the design of the suspension clamps was being considered, it was discovered that many of the aluminum strands were broken at the ends of many of the suspension clamps on the old Big Creek lines. This breakage was due to the repeated bending caused by conductor vibration. The old suspension clamps were divided on the vertical plane through the conductor axis and they ended with a very short radius of curvature on the part where the cable bore as it left the clamp. The conductor was protected by a sheet aluminum liner which hid the broken strands from outside inspection until a considerable number was broken.

Design of Suspension Clamps

There seemed to be three main features which should be embodied in a new suspension clamp to avoid the troubles experienced with the old clamps:

First, a saddle in which the cable will rest entirely outside of the section gripped in the clamp so that the last point of contact between clamp and cable would never be coincident with the end of the gripped section. This would allow the bending of the cable, due to its rise and fall relative to the clamp, to be distributed over an appreciable length of the cable and not be constrained to one particular point.

Second, the radius of curvature of this saddle over which the cable would bend should be sufficiently large to prevent the repeated bending from breaking the cable strands.

Third, the clamp should be suspended in such a way that it would move with the cable as much as possible and thus keep the bending of the cable at the ends of the clamp to a minimum.

Although, at the time of designing this clamp, it was becoming apparent that the vibration in the conductors could be stopped, it was thought advisable to embody all three of these features in the clamp design.

The points of design which would satisfactorily meet these requirements were determined in an order the reverse of that in which the requirements are enumerated.

The third was met by pivoting the clamp on a pin passing through it as close to the lower side of the conductor as possible. The clamp is free to move about this pivot through an angle of 23 and 30 deg. both ways, from the normal position for the short and long clamps respectively.

It was determined that the second requirement was reasonably fulfilled by giving the end portions of the clamp a radius of curvature of 16 in. Of course a longer radius would give easier bending conditions on the cable, but cost must be given consideration. Considerable experimental work was done to determine this radius. Single strands of aluminum wire stressed to unit tensions approximately the average of those expected in the line were bent around curves of various radii a sufficient number of times to break them. From these tests it was determined that if bending around a radius of 16 in., the life of the cable can be expected to be in the order of five times as long as if it were bending about a six-in. radius, which is approximately the radius of the old clamps. This was considered a reasonable balance between increase in cable life and increase in cost of suspension clamps.

The first feature specified required a straight section of clamp in which the cable is gripped with two end sections in each of which the cable merely rests for a part of the length. The length of the gripping section was determined to give the holding power desired. Tests show the first slips to occur at 20,000 and 16,000 lb. for the short and long clamps respectively.

It was decided to have the straight section of the bottom half of the clamp extend one inch beyond the grip of the clamp. At these points the two 16-in. radius end sections begin, the straight section being tangent to the two curved sections. To determine the length to which the end sections should be carried so that the most extreme points of contact between cable and clamp would be an inch inside the outer end of the 16-in. radius curve, one in. having been selected as a sufficient margin of safety, it was necessary to calculate from the profile of the line the various angles at which the cable would leave the clamps. It was found that these angles varied from practically nothing to 22.5 deg. and that a clamp to accommodate all these conditions should be 21.75 in. long. Since there were comparatively few of the large angles, two types of clamps were designed, one for angles up to nine deg. and the other for angles up to 22.5 deg. The former was made with a forged steel body 14.75 in. long and the latter was made of malleable cast iron 21.75 in. long. Both were equipped with cast aluminum liners. There are required for the line 2100 of the short clamps and 477 of the long ones.

CONDUCTOR VIBRATION

The experiences with conductor vibration, the methods of study, the causes, and the means of preventing conductor vibration have been previously described.[4,5,6] Vibration dampers similar to those described by Mr. Stockbridge, but consisting of two cast iron weights attached to a straight and approximately horizontal spring of guy cable, will be used on this line. Judging from the results obtained on the old line, these dampers will entirely eliminate all visible vibration. Fig. 6 shows this type of vibration damper as applied to the old lines.

CONSTRUCTION

Careful plans were made for the construction of the line, particularly with reference to construction equipment, and the results in the field have proved that the expense of this planning was well worth while.

The setting of footings was largely a matter of pick and shovel and powder for the excavation, transit and level for alinement and elevation, and shovel and tamper for back-filling. It pays to do this job well. Fig. 7 shows footings being set.

FIG. 6—CONDUCTOR VIBRATION DAMPER

TOWER ERECTION

When the Eagle Bell 220,000-volt line was built in 1923, several methods of erecting towers were tried, but the method of assembling piece by piece in the erected position was found to be the most economical if the erecting crews are selected and organized with care.

On the Vincent line this method has been modified somewhat in that wherever it is possible to get a line

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[4] *Transmission at 220 kv. at the Southern California Edison Company Sy tem*, Section 4, *Vibration of Conductors and Overhead Ground Wires*, by J. M. Gaylord, JOURNAL of A. I. E. E., Vol. 43, November 1924, p. 1026.

[5] *Notes on the Vibration of Transmission Line Conductors*, by Theodore Varney. Presented at Niagara Falls meeting, A. I. E. E., May, 1926.

[6] Overcoming Vibration in Transmission Cables, by G. H. Stockbridge, *Electrical World*, December 26, 1925, p. 1304.

from a truck the panels of the tower sides are assembled on the ground or horizontally on the top of the completed portion of the tower and raised as a unit. Fig. 8 shows the first side-panel being raised into position on top of a 14-ft. extension. The truck is raising the panel.

In this way a crew of 14 men, including the gang foreman and the truck driver, has been able to erect a standard tower in 4 hours and 15 minutes. This tower was put on top of a 14-ft. extension which had been previously erected. When the work is running smoothly, an average of six hours for a standard tower on easily accessible ground should be maintained by a crew of 14 men and a two-ton truck.

Fig. 7—Setting Tower Footings

Looking southeasterly along line in southern edge of Antelope Valley

Conductor Stringing and Stringing Equipment

The most unsatisfactory part of transmission line construction in the past had been the conductor stringing. Many improvements have been made in the equipment for this work on this line.

The most notable improvement and the one which is paying the greatest dividends is in the stringing sheaves. There was nothing on the market at all adequate for the job. Plain bearings, small wheels, and flat grooves that let the cable flatten and rub on the sides of the blocks seemed to be the rule without exception. It was decided that the friction in the sheaves should be made as low as possible by the use of roller or ball bearings and that the sheave should be made of aluminum with a deep groove having a good fit to the cable and a bottom diameter of 12 in. These specifications led to the design shown in Fig. 9. With these sheaves the length of cable that can be pulled to tension at one time is not limited by the sheave friction. Lengths as long as four miles have been pulled and the tensions in the various spans adjusted with entire satisfaction. On level ground it seems that the limit to the length of pull will be set by the number of sheaves available.

Fig. 8—Erecting Tower on Top of 14-Ft. Extension

Raising first side-panel

The sheave blocks are attached to the bottom of the suspension insulator strings while the conductor is being strung.

For pulling the conductor a 10-ton caterpillar tractor is used. Where the conductors have to be dragged out on the ground, all three are taken at the same time. The pulling of each conductor to final tension is done by the direct pull of the tractor wherever it is possible to find a place for the tractor to travel. The tractor is equipped with a winch, with both drum and spool, with which the pulling is done when the necessary traveling space for a direct pull is not available. The direct pull of this heavy tractor works very nicely and saves much time over any method of using lighter equipment with a winch or blocks and falls. On level ground with the conductor on the ground except at the sheaves, four mi. of one conductor have been pulled up and the tension adjusted to its proper value at three places in the section in 45 min., no splices having passed around the sheaves in this case. Three conductors of a four-mi. section have been pulled up in this way in three hours.

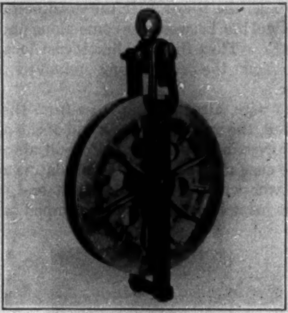

Fig. 9—Stringing Sheave

Telephones are used for communication between the engineers who give the tension orders and the foreman at the tractor end of the section. In some cases twisted pair taps are run to the locations of these men from the telephone line which roughly parallels the transmission line. In other cases, where the telephone line is too far away, a twisted pair is laid along the transmission line from end to end of the pull section. Signaling with flags was tried but was found to be very unsatisfactory for such long pull sections.

As the conductors are pulled to tension, they are anchored to deadmen near the foot of the next tower ahead of the pull section. The come-alongs to hold the two outside wires are placed near the deadmen but the one to hold the center wire is placed very near the sheave in the last tower of the pull section and a steel line is run to the deadman. These come-alongs hold the tension in the line until the cables in the next section to be pulled are spliced on and pulled to tension in the manner described below.

For the outside positions the conductors are not put into the sheaves on the tower where the deadmen are located until the conductors are pulled to sufficient tension to relieve the tension on the come-alongs and the come-alongs are taken off the conductors. This takes place at practically full tension and without appreciably lifting the conductor at the come-alongs from the ground. After the come-alongs are cast off the conductors are placed in the sheaves at that tower.

For the center position the conductor is put through the sheave at the tower near the deadmen and pulled until the steel line between the come-along and the deadmen is slack. Then the steel line is cast loose from the deadmen and the come-along removed from the conductor.

On each wire, after getting loose from the deadman as described, the man at the back end of the pull section tells the foreman at the tractor to pull ahead or back off, as the case may be, until the insulators are vertical on the tower which was at the front end of the last pull, these insulators having been clamped in vertically on the conductor when it was anchored to the deadman. Then the engineer three or four spans farther into the section being pulled tells the man at the tractor to pull ahead or back, as required, until the sag in the span he is watching is right. Then another engineer, who is two or three spans back from the tractor, gives whatever orders are necessary to get his span right. If everything has gone well, no more adjusting need be done and the men at the tractor end put on a come-along and attach it to the deadman. After this is done the conductor can be transferred from the sheaves to the suspension clamps at will.

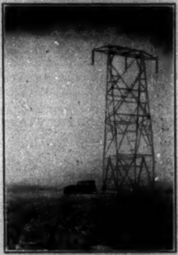

FIG. 10—COMPLETED SECTION OF LINE ON STANDARD TOWERS WITH EXTENSIONS

In rough country, where long and short spans and spans at different elevations are mixed together in the same pull section, some slight modifications of this simple level country procedure are necessary. The longest span, or the span at the lowest elevation, takes more than its share of the proper conductor length for the pull section so it is necessary to let such a span govern the pulling tension until its tension is right and its insulators are clamped to the conductor. Then that span is forgotten and the other spans are given their proper sags, any other spans being treated in this special way if necessary. Of course all the insulators will not hang vertically but there is no way to put up such a line and have all the insulators vertical at all times.

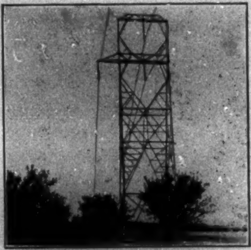

FIG. 11—TRANSPOSITION TOWER ON EXTENSION

COME-ALONGS

The come-alongs used on previous work had many bolts to be tightened in making one fast to the cable. To shorten the time necessary for this, one was designed with only three set screws to be tightened and it is giving entire satisfaction. The body is of cast steel with aluminum liners held to each half by means of counter sunk screws. The pulling sling is of ½-in. steel cable and is zinced into clevis-sockets which attach to the come-along and the middle of the sling passes around a forged eye to which the pulling line may be attached.

Fig. 10 shows a completed section of line on standard towers with extensions. This is looking north across the Antelope Valley.

Fig. 11 shows a transposition tower with an extension. Two such towers are required at each transposition, one with a right-hand and the other with a left-hand lower crossarm. These towers are not set off the center line as has been done in some cases. There are seven complete barrels in the line.

BIRD GUARDS

The towers will be equipped with guards to prevent the hawks and other large birds from causing flashovers by their semi-liquid droppings bridging the air space between the crossarm and the conductor. Experience has shown such protection to be necessary.[7, 8] The center conductor position will be protected by a sheet iron pan approximately four ft. wide and ten ft. long on the lower members of the crossarm. This pan will catch the droppings from birds on the central portion of the tower top, their most favored perch. Birds will be prevented from perching above the outside conductor by saw-tooth guards placed on the crossarm members for a distance of about six ft. from the end of the crossarm. These saw-teeth are three in. long and are made of thin galvanized iron.

CONCLUSION

It is the belief of the authors that the large amount of engineering investigation and careful design work in preparing for this line was decidedly advantageous, not only for this particular line but for the industry in general, parts of which already are profiting by the results of this work.

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[7] Avoiding Flashovers on 220-Kv. Transmission Lines, by G. H. Stockbridge, *Electrical World*, Vol. 85, No. 12, March 12, 1925, p. 611.

[8] *Transmission at 220 Kv. on the Southern California Edison System*, Section 1, *Description of System and Operating Experience*, by H. Michener, JOURNAL of A. I. E. E., October 1924, p. 901.

Unbalanced Conductor Tensions

Tests to Show Their Effects in a Long-Span Transmission Line

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Associate, A. I. E. E. Non-Member

***Synopsis.**—Some interesting studies of the unbalanced tensions produced by heavy ice loadings on long spans supported by suspension insulators were made in the course of designing the rec.ntly completed Wallenpaupack-Siegfried 220-kv. line of the Pennsylvania Power and Light Company. These studies included investigations of the effects produced on suspension insulator construction by a broken conductor. Computations were verified in an approximate way by experiments on a one-mile section of the actual line. For the most part, results are given in diagrammatic form, in sufficient detail to allow both analysis and approximate application to similar problems.*

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GENERAL PROBLEM OF UNBALANCED CONDUCTOR TENSIONS

PREDETERMINING the effect of carrying long spans on suspension insulators has long been a perplexing problem. Span lengths have been gradually increased without any undue operating difficulties until several lines are now in operation with spans averaging well over 1000 ft. Studies for extra high-voltage lines show economies for even longer spans, especially when some of the very high-strength conductors are considered. Such construction requires careful attention to prevent unbalanced conductor tensions,—of no particular importance on shorter spans,—from assuming serious proportions, where combined with long insulator strings and heavy ice loading. In fact, the ice loading appears to be the most important factor.

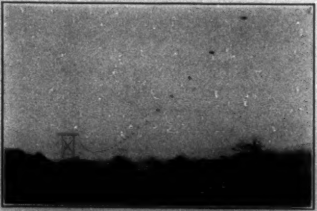

FIG. 1—SPAN LOADED WITH SAND BAGS BETWEEN TOWER NO. 3 AND TOWER NO. 4

The unbalanced loads caused by a broken conductor are also of greater importance with the higher-strength conductors. The usual design for lines carrying comparatively small conductors, making each tower of sufficient strength to support one or more broken conductors, does not seriously affect the cost. The same assumption applied to very high-strength conductors so seriously affects the cost of the entire construction as to make a careful study of the character and amount of the loads caused by a break decidedly important.

In working out the designs for the Wallenpaupack-Siegfried line of the Pennsylvania Power & Light Company, both of these problems assumed considerable importance. The 1100-ft. spans and insulators with an effective mechanical length of over nine ft., located in a very bad sleet country, combined to create rather unusual conditions. Some tests and computations made of these problems on that line are given here in the hope that they may be of assistance in similar studies and in the consideration of extra high-strength conductors.

PROBLEMS ENCOUNTERED ON THE WALLENPAUPACK-SIEGFRIED LINE

As a basis for certain features of design of the Wallenpaupack-Siegfried line, a rather extensive set of calculations was made of the effect of unbalanced conductor tensions. The computed results are summarized in two charts, Fig. 5 showing the unequal tensions caused by a heavy ice load forming on a series of spans of unequal length and Fig. 6, the effect of ice dropping off of all but one or two of a series of spans. Extensive calculations were also made to determine the effect of the longitudinal swing of the insulator string and of the resulting slack thrown into the adjacent spans following a failure of the conductor.

The calculations were intended to obtain results for a few general conditions which could be used as a basis of selecting specific tower locations and as an aid in judging whether special precautions were necessary to meet any particular conditions encountered. For instance, when extra long spans were required over rough country, these calculations gave a logical means of determining how long a span could safely be carried on suspension insulators.

Realizing the importance of a practical verification of the computed results, a set of tests was made as soon as the construction work had reached a stage where it was possible to string a test conductor over a typical section of the actual line. The results of the experiments and a comparison with some of the computed results are shown in the diagrams, Figs. 2, 3, 4, 7, 8 and 9.

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[1] Transmission Line Engineer and Asst. Transmission Line Engineer of the Electric Bond and Share Company, New York City.

Presented at the Pacific Coast Convention of the A. I. E. E., Salt Lake City, Utah, Sept. 6-9, 1926.

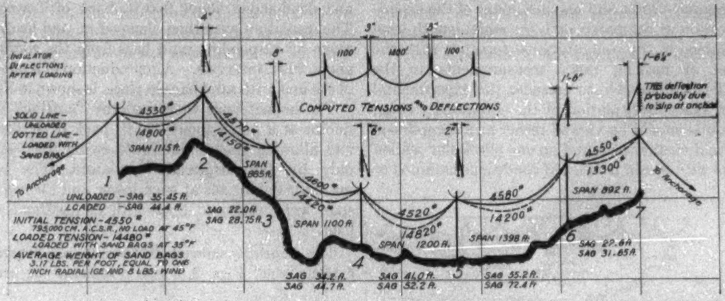

INSULATOR DEFLECTIONS AFTER LOADING
4"
8"
3"
3"
1100'
1200'
1100'
COMPUTED TENSIONS AND DEFLECTIONS
1'-6"
1'-6¼"
THIS deflection probably due to slip at anchor
SOLID LINE - UNLOADED
DOTTED LINE - LOADED WITH SAND BAGS
To Anchorage
4530#
14800#
SPAN 1115 ft.
4570#
14150#
6"
2"
SPAN 885 ft.
4600#
14420#
4520#
14820#
4580#
14200#
4550#
13300#
To Anchorage
1
2
3
4
5
6
7
UNLOADED - SAG 35.45 ft.
LOADED - SAG 44.4 ft.
SAG 22.0 ft.
SAG 28.75 ft.
SPAN 1100 ft.
SPAN 1200 ft.
SPAN 1398 ft.
SPAN 892 ft.
SAG 27.6 ft.
SAG 31.65 ft.
INITIAL TENSION - 4550#
795,000 CM. A.C.S.R., NO LOAD AT 45°F
LOADED TENSION - 14480#
LOADED WITH SAND BAGS AT 35°F
AVERAGE WEIGHT OF SAND BAGS
3.17 LBS. PER FOOT, EQUAL TO ONE INCH RADIAL ICE AND 8 LBS. WIND
SAG 34.2 ft.
SAG 44.7 ft.
SAG 41.0 ft.
SAG 52.2 ft.
SAG 55.2 ft.
SAG 72.4 ft.

FIG. 2—EXPERIMENTAL RESULTS OF ARTIFICIALLY LOADING ALL SPANS OF THE SERIES

795,000-cir. mil steel-reinforced aluminum conductor sagged to 4550 lb. and then artificially loaded with sand bags representing one-in. ice and eight-lb. wind

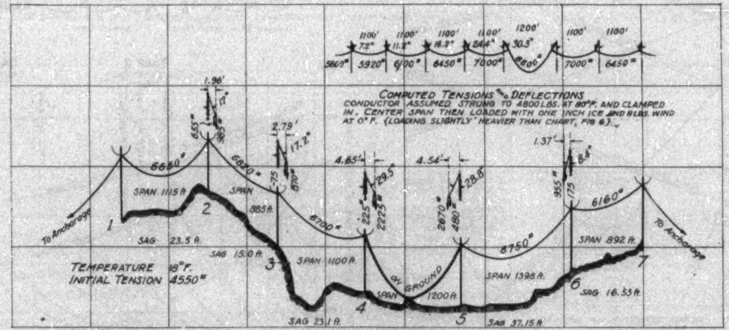

1100'
1100'
1100'
1100'
1200'
1100'
1100'
7.2°
11.2°
16.2°
24.4°
30.3°
560?#
5920#
6100#
6450#
7000#
8200#
7000#
6450#
COMPUTED TENSIONS AND DEFLECTIONS
CONDUCTOR ASSUMED STRUNG TO 4800 LBS. AT 80°F. AND CLAMPED IN. CENTER SPAN THEN LOADED WITH ONE INCH ICE AND 8 LBS. WIND AT 0° F. (LOADING SLIGHTLY HEAVIER THAN CHART, FIG 6).
1.96'
17°
655#
985#
2.79'
17.2°
-75
870#
4.65'
29.5°
225#
2225#
4.54'
28.8°
2670#
480#
1.37'
8.4°
955#
175
6650#
6820#
6700#
6750#
6160#
SPAN 1115 ft.
SPAN 885 ft.
SPAN 1100 ft.
SPAN 1200 ft.
SPAN 1398 ft.
SPAN 892 ft.
ON GROUND
To Anchorage
To Anchorage
1
2
3
4
5
6
7
SAG 23.5 ft.
SAG 15.0 ft.
SAG 23.1 ft.
SAG 37.15 ft.
SAG 16.55 ft.
TEMPERATURE 18°F.
INITIAL TENSION 4550#

FIG. 3—EXPERIMENTAL RESULTS OF ARTIFICIALLY LOADING THE CENTER SPAN OF THE SERIES

795,000-cir. mil steel-reinforced aluminum conductor sagged to 4550. lb. and the center span then loaded with sand bags representing one-in. ice and eight-lb. wind

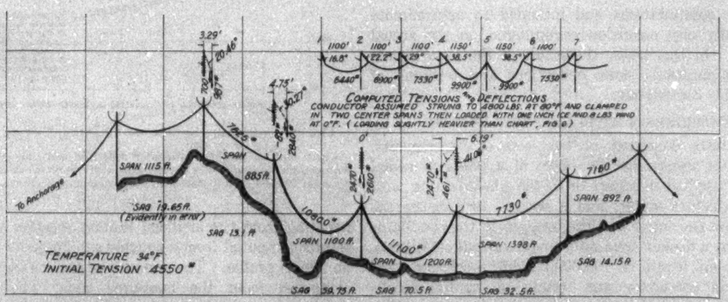

1
2
3
4
5
6
7
1100'
1100'
1100'
1150'
1150'
1100'
16.8°
22.2°
29°
38.5°
38.5°
6440#
6900#
7530#
9900#
9900#
7530#
COMPUTED TENSIONS AND DEFLECTIONS
CONDUCTOR ASSUMED STRUNG TO 4800 LBS. AT 80°F. AND CLAMPED IN. TWO CENTER SPANS THEN LOADED WITH ONE INCH ICE AND 8 LBS WIND AT 0°F. (LOADING SLIGHTLY HEAVIER THAN CHART, FIG 6)
3.29'
20.46°
700#
987#
4.75'
30.27°
-82#
2840#
0'
2470#
2610#
6.19'
41.0°
2470#
461#
7025#
10800#
11100#
7730#
7160#
SPAN 1115 ft.
SPAN 885 ft.
SPAN 1100 ft.
SPAN 1200 ft.
SPAN 1398 ft.
SPAN 892 ft.
To Anchorage
SAG 19.65 ft.
(Evidently in error)
SAG 13.1 ft.
SAG 39.75 ft.
SAG 70.5 ft.
SAG 32.5 ft.
SAG 14.15 ft.
TEMPERATURE 34°F
INITIAL TENSION 4550#

FIG. 4—EXPERIMENTAL RESULT OF ARTIFICALLY LOADING TWO CENTER SPANS OF THE SERIES

795,000-cir. mil. steel-reinforced aluminum conductor sagged to 4350 lb. and the two center spans loaded with sand bags representing one-in. ice and eight-lb. wind

The irregular profile and unequal spans of the experimental section as compared with the equal-length level spans assumed in the computations, together with the difficulties of making exact measurements in the field, make it impossible to consider the experimental results an exact confirmation of the computed results.

The results should be viewed rather as a comparison of the actual conditions found on one particular section of the line with one of a series of conditions assumed to simplify computations and intended to approximate practically any condition encountered on the actual profile. On this basis, the result of the experiments and computations show deflections and tensions of reasonable consistency.

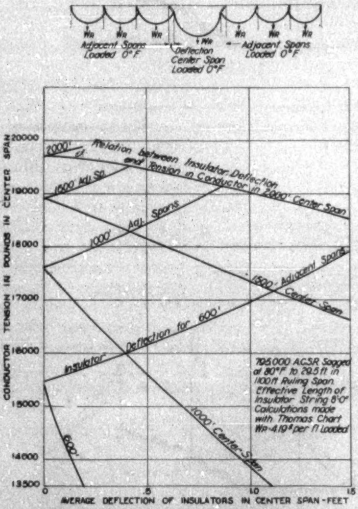


Fig. 5—Computed Effect of Loading A Series of Spans

Chart showing maximum insulator deflection and conductor tension for all spans of a series loaded with one-in. ice and eight-lb. wind at 6 deg. fahr. 795,000-cir. mil steel-reinforced aluminum conductor

Experiments with Artificial Loading

The tests consisted of two sets of experiments; one set to determine the effect of a one-inch radial ice load applied to a part or to the whole section with the conductors intact; and a second, to measure the impact on the tower and the tensions in the conductor caused by a broken conductor under various conditions.

A one-mi. length of 795,000-cir mil. steel-reinforced aluminum conductor was strung on snatch blocks over the section of line shown in the diagrams. The wire was dead-ended to ground pins about 600 ft. ahead of Tower No. 7, sagged to a tension of 4550 lb., and dead-ended about 900 ft. back of Tower No. 1. The insulators were then clamped in, and loading tests made by suspending sand bags from the conductor at about 20-ft. intervals. A reproduction of a photograph of the line, with sand bags in place, is shown in Fig. 1.

The impact tests were made by "cutting" the conductor at a point about 700 ft. back of Tower No. 1, and allowing the conductor to swing free, supported only by the suspension insulators. The resulting insulator deflections at each tower and the sag in each span were measured both with and without artificial load.

The diagrams showing test results are generally self explanatory. Figs. 2, 3 and 4 indicate the conditions resulting from the artificial loading together with the results computed from a roughly similar series of spans on a level profile. The conductor tensions shown were computed from the measured sags. The vertical weights are obtained from the conductor lengths scaled from the profile with the sags drawn in to scale—the weight on an insulator being taken as the weight

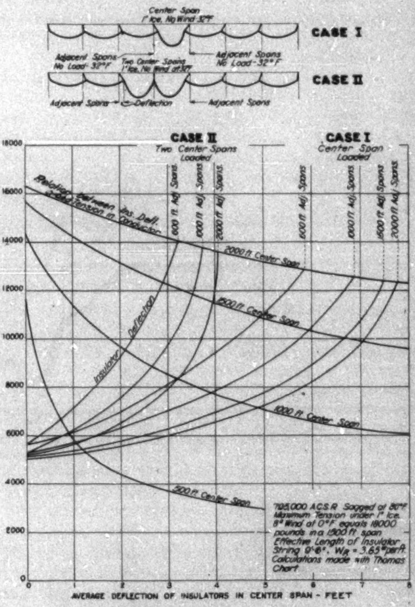
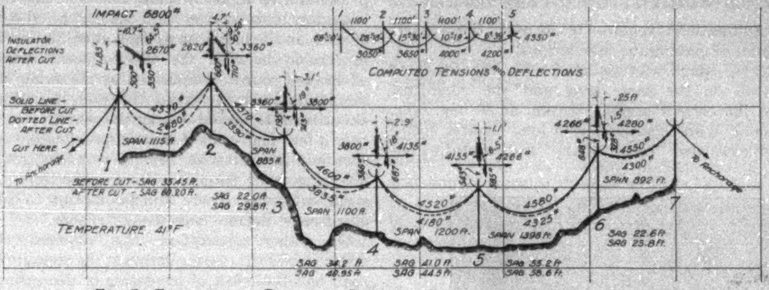


Fig. 6—Computed Effect of Loading Center Spans

Chart showing maximum insulator deflection and conductor tension with one and two spans of a series loaded with one-in. ice at 32 deg. fahr. No wind 795,000 cir-mil. steel-reinforced aluminum conductor of conductor from the insulator to the low point of the sag. All of these quantities show slight discrepancies due to small errors in the sag measurements, but more accurate methods of measurement would not have increased materially the value of the results.

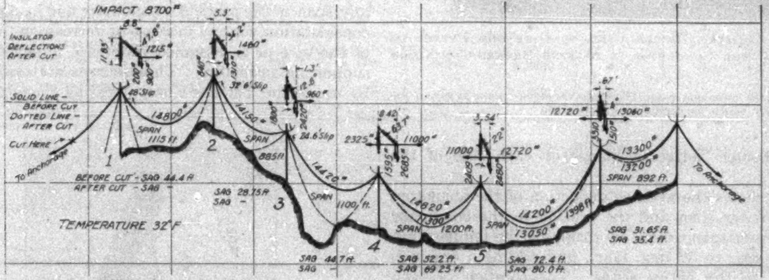
These experiments may be considered as an approxi-



IMPACT 6800#
INSULATOR DEFLECTIONS AFTER CUT
SOLID LINE - BEFORE CUT
DOTTED LINE - AFTER CUT
CUT HERE
To Anchorage
COMPUTED TENSIONS & DEFLECTIONS
SPAN 1115 ft.
SPAN 885 ft.
SPAN 1100 ft.
SPAN 1200 ft.
SPAN 1398 ft.
SPAN 892 ft.
BEFORE CUT - SAG 35.45 ft.
AFTER CUT - SAG 60.20 ft.
SAG 22.0 ft.
SAG 29.8 ft.
SAG 34.2 ft.
SAG 40.95 ft.
SAG 41.0 ft.
SAG 44.5 ft.
SAG 55.2 ft.
SAG 58.6 ft.
SAG 22.6 ft.
SAG 23.8 ft.
TEMPERATURE 41°F
1
2
3
4
5
6
7
To Anchorage

FIG. 7—EXPERIMENTAL RESULT OF BREAKING CONDUCTOR AT NORMAL TENSION

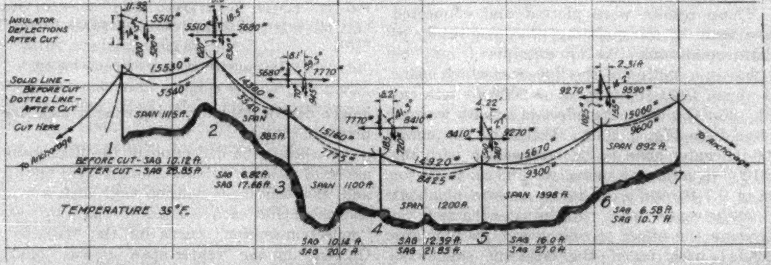
795,000-cir. mil steel-reinforced aluminum conductor "cut" when sagged to 4550-lb. tension. Clamps set to prevent slipping.



IMPACT 8700#
INSULATOR DEFLECTIONS AFTER CUT
SOLID LINE - BEFORE CUT
DOTTED LINE - AFTER CUT
CUT HERE
To Anchorage
SPAN 1115 ft.
SPAN 885 ft.
SPAN 1100 ft.
SPAN 1200 ft.
SPAN 1398 ft.
SPAN 892 ft.
BEFORE CUT - SAG 44.4 ft.
AFTER CUT - SAG -
SAG 28.75 ft.
SAG -
SAG 44.7 ft.
SAG -
SAG 52.2 ft.
SAG 69.23 ft.
SAG 72.4 ft.
SAG 80.0 ft.
SAG 31.65 ft.
SAG 35.4 ft.
TEMPERATURE 32°F
1
2
3
4
5
6
7
To Anchorage

FIG. 8—EXPERIMENTAL RESULT OF BREAKING CONDUCTOR UPON ARTIFICIAL LOAD

795,000-cir. mil steel-reinforced aluminum conductor strung to 4550 lb. and then an artificial load of approximately 3.17 lb. per ft. added equivalent to one-in. and eight-lb. wind. Clamps set to slip at 5000 lb.



IMPACT 15450#
INSULATOR DEFLECTIONS AFTER CUT
SOLID LINE - BEFORE CUT
DOTTED LINE - AFTER CUT
CUT HERE
To Anchorage
SPAN 1115 ft.
SPAN 885 ft.
SPAN 1100 ft.
SPAN 1200 ft.
SPAN 1398 ft.
SPAN 892 ft.
BEFORE CUT - SAG 10.12 ft.
AFTER CUT - SAG 28.85 ft.
SAG 6.82 ft.
SAG 17.68 ft.
SAG 10.14 ft.
SAG 20.0 ft.
SAG 12.39 ft.
SAG 21.85 ft.
SAG 16.0 ft.
SAG 27.0 ft.
SAG 6.58 ft.
SAG 10.7 ft.
TEMPERATURE 35°F.
1
2
3
4
5
6
7
To Anchorage

FIG. 9—EXPERIMENTAL RESULTS OF BREAKING CONDUCTOR AT 15,000-LB. TENSION

795,000-cir. mil steel-reinforced aluminum conductor "cut" when sagged to 15,000-lb. tension without artificial loading. Clamp on Tower No. 1 set with positive grip. Other clamps set to slip.

mate check of one of the many combinations of span lengths given by computation and shown in diagram in Figs. 5 and 6.

EXPLANATION OF CHARTS SHOWING COMPUTED RESULTS

Some explanation is required of the use of these charts and methods of making the calculations.

A series of spans was assumed with the insulator supports all on the same level and the conductor strung to the correct sagging tension at 80 deg. fahr. The series of spans consisted of a long "center span" or, in one case, two "center spans" and several equal-length shorter "adjacent spans" on each side of the center.

FIG. 10—TRIPPING DEVICE USED TO GIVE THE EFFECT OF "CUTTING" THE CONDUCTOR IN MAKING BROKEN-CONDUCTOR TESTS

A pull on the lower arm instantly releases the clamp, shown at right of the illustration, to which the conductor was attached

Fig. 5 shows the average deflection of the insulators on the center span and the tension in the conductor of the center span caused by dropping the temperature from 80 deg. to 32 deg. fahr., and adding a one-inch ice load to the entire series. The solution of each combination of adjacent and center span lengths was made by a series of trials, interpolating between trial results for the correct solution. In making up the chart, a separate set of computations was carried through for each of several combinations of span lengths. These results were plotted and connected by a curve again by interpolating, obtaining results of intermediate conditions. As an example it may be assumed that in a section of the line over which spans averaged about 1000 ft., a span of 2000 ft. was encountered. For this case, the effect of a heavy ice load may be read from Fig. 5, thus: Following the curve for a 2000-ft. center span to its intersection with the curve for 1000-ft. adjacent spans, we find an insulator deflection of 0.7 ft. and a conductor tension of about 19,300 lb. in the center span. In the same manner, we may determine the effect of one center span of any length loaded with ice if the adjacent spans are unloaded from Case I, Fig. 6, or for two center spans loaded from Case II, Fig. 6. Sags may easily be computed from the tension.

Considering the computations involved in solving the various combinations of span lengths shown on Fig. 5, it is evident that the heavy ice loading causes considerably more stress in the long center span than in the shorter adjacent spans, and that this causes the insulators to swing over until the horizontal component of stress in the insulator string is equal to the difference in tension between the two adjacent spans. This effect is, of course, complicated by the swing of successive insulators on each side. However, it was found that about the fifth insulator from the center has a swing of only a few degrees, and that (approximately) the tension in the next span beyond is unchanged. Cases involving very large insulator deflections or more accurate results require this assumption to take effect at a greater distance from the center span.

One other assumption was required—that the deflections of the two insulators at the opposite ends of a span had the effect of increasing or decreasing, as the case might be, the length of the wire in the span by the sum or difference of the insulator deflections. With this assumption, supplementary "deflection-tension" curves were made, showing the change in tension in any span of the particular conductor and loading under consideration, for any increase or decrease in the length of the wire or as assumed difference in swing of the supporting insulators. These curves are easily made by use of the Thomas Chart, treating changes of wire length due to insulator swing exactly as if due to temperature change.

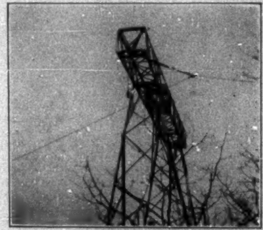


FIG. 11—TOWER NO. 1 AFTER "CUT" SHOWING DUMMY INSULATOR EMBRACING SPRING DYNAMOMETER AND IMPACT DYNAMOMETER

Conductor cut about 700 feet to the left

As a trial, the deflection of the fifth insulator was assumed at some definite amount from one to 10 deg. and the conductor tension and insulator deflection computed for each successive span by use of the "deflection-tension" curve for the "adjacent spans." On reaching the "center span" it was found that the deflection and tension thus computed generally corresponded more or less closely to the relation between deflection and tension shown on the "deflection-tension" curve for the "center span." Two or three trials were generally sufficient to interpolate for the correct results.

The chart shown on Fig. 6 was made in a similar manner.

Incidentally, there is shown in Fig. 2 an interesting comparison between the experimental and computed tension resulting from adding an ice load to the steel-reinforced aluminum conductor. The computed tension resulting from loading a 1100-ft. span, sagged to 4550 lb. with an ice load of 3.17 lb. per ft., is 13,800 lb. The average tension from the artificial loading in the experimental span series was 14,480 lb.

The computations in this case assumed that the steel and aluminum strands, initially of exactly the same length, worked together, each taking load proportional to its respective area and modulus of elasticity up to the elastic limit of the aluminum strands. For tensions greater than this, it was assumed that the steel alone carried the increasing loads.

In this connection, it should be noted that the experiments were made under roughly constant temperature conditions and that while this temperature actually averaged about 35 deg. fahr., the tensions in the conductor corresponded to about 80 deg. fahr. These tensions were used to approximate usual summer conditions. Actually, therefore, the artificial loading experiments correspond to about one-in. radial ice and eight-lb. wind at 80 deg. fahr.

Tests on the Effect of a Broken Conductor

Computed results indicated that the tension in the span adjacent to a break would be very materially reduced by the swing of the insulator. There was, however, a great deal of doubt as to what relation this reduced tension bore to the maximum load thrown on the tower. This maximum load would be applied suddenly and, depending on the momentum developed and the elasticity of the parts absorbing this momentum, the load on the tower might be anywhere from a few per cent to several times greater than the final tension in the span.

The broken-conductor test results are shown in diagram in Figs. 7, 8 and 9. These tests were made to determine the effect of a broken conductor under normal tension, Fig. 7, under maximum ice load, Fig. 8, and under a heavy tension without ice load, Fig. 9. In addition to the insulator deflections and conductor tensions, these diagrams indicate the impact load recorded on the tower next the break. As a check on the impact readings, certain of these tests were repeated. A summary of the results is shown on Table I.

TABLE I

SUMMARY OF IMPACT TESTS

Description of test	Temp. Deg. Fahr.	Wind velocity ft. per min.	Initial tension in conductor, lb.	Final tension in span adjacent to break, lb.	Cylinder length, in.	Impact from calibration curve, lb.	Reading of maximum hand of dynamometer, lb.	See diagram
Conductor sagged to 4550 lb. Clamps tightened with positive grip........	41	410	4550	2680	0.3850	6800	6850	Fig. 7
Conductor sagged to 4550 lb. Clamps tightened with positive grip......	45	..	4550	..	0.3935	6500	..	
Conductor sagged to 4550 lb. Clamps tightened with positive grip.......	45	..	4550	..	0.4028	6100	5800	
Conductor sagged to 4550 lb. Clamps tightened with positive grip........	45	800	4550	..	0.3837	6900	8000	
Conductor sagged to 4560 lb. Clamps set to slip........................	32	..	4500		0.3975	6300	..	
Conductor sagged to 15,000 lb. Clamp set with positive grip......	35	..	15,000	5540	0.423 0.426 0.427	15,450	21,500	Fig. 9
Conductor sagged to 4550 lb., then loaded with sand bags to give a total load of 4.19 lb. per ft. Clamps set to slip......................	32	..	14,500	..	0.4719 0.4716 0.4713	8470	10,000	Fig. 8

The tests of a break under normal tension were made with the suspension clamps all bolted down firmly to prevent any slipping and to obtain actual impact readings. In the test of the conductor with artificial load, the suspension clamps were used as they were assembled in the completed line; that is, to allow slipping under an unbalanced tension of about 5000 lb. The third impact test was made without any superimposed load but under a tension of about 15,000 lb. The conductor was positively gripped at the suspension insulator Tower No. 1 but the remaining clamps were set to slip as before.

The specific values of insulator deflections recorded in the latter tests are therefore of little general value. While quite aside from the scope of this discussion, it might be mentioned that this type of clamp effectively held down the loads on the tower as intended. Furthermore, the results left a serious doubt as to whether a slipping clamp can be so applied to a multi-layer cable of this type as to avoid injury to the strands when slipping takes place.

The impact values given in these tests were measured by an ingenious adaptation of the U. S. Ordnance Department's "Crusher Gage" designed and made by J. B. Thomas, Chief Engineer of the Texas Power & Light Company, and shown in Fig. 12.

The conductor tests were all made with this impact dynamometer used in place of insulator disks on the insulator string of the first tower. The impact dynamometer was attached directly to the suspension clamp. Directly above this was placed an ordinary spring-type dynamometer and the remainder of the "insulator string" consisted of an iron bar of a length and weight to make the whole the same length and about the same weight as the usual string of insulator disks.

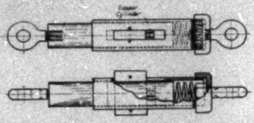


Fig. 12—Impact Dynamometer Used in Measuring Impact on Tower Resulting from a Broken Conductor

A little consideration will show that it is an exceedingly difficult matter to measure the force exerted on a tower by the breaking of a conductor. The *crusher gage* method was devised for measuring the pressures developed in guns. As thus used, the device consists of a small steel cylinder with a light piston so arranged that the force of the explosion will compress a carefully calibrated cylinder of soft copper. The pressure is measured by the reduction in length of the copper cylinder. A description of the uses of the crusher gage is given in the U. S. Bureau of Standards, Technologic Paper, Paper No. 185. After consulting numerous authorities on the subject, it was concluded that even for the comparatively slowly applied loads resulting from these tests, the accuracy was well within practical requirements.

The spring dynamometer which was used in series with the impact dynamometer did not show consistent results. The impact was taken as the reading of the maximum hand. This hand is so arranged as to follow the indicating hand up to its maximum reading and remain at this point. The results indicated that the maximum hand was probably thrown beyond the proper reading as in only the first of the six tests made did the maximum hand approximately check with the crusher gage. It was also found in calibrating the dynamometer, that after tests had been completed, the dynamometer readings were over 1000 lb. lower than before the test.

A small moving-picture camera provided with a telescopic lens was used in the test of the broken conductors, and set up about 300 ft. on one side from the first tower. A time analysis of these pictures should give some interesting information and should be a real aid in solving the mechanics of the problem. Thus far, however, time has not been available for this analysis.

CONCLUSIONS

The results of these experiments and computations were derived only for application to the Wallenpaupack-Siegfried line and in this served as the basis for important decisions.

The results indicated that even with the extraordinarily heavy ice loading adopted for the line, seriously unbalanced longitudinal loads would not result when span lengths were reasonably uniform in length. On the other hand, they indicated that the maximum tension increased quite rapidly for spans materially longer than the average. Consequently an effort was made to equalize span lengths as far as practicable, and spans greater than 1750 ft. were not carried on suspension insulators without a suitable reduction in the sagging tensions.

Curves of Fig. 6 were used as a means of checking the safety of the conductors against coming in contact with the ground under the condition of heavy ice load on one or more adjacent spans, all other spans being unloaded. Two adjacent spans, fully loaded, were assumed as the limiting condition for ground clearance.

Broken conductor tests demonstrated that insulator deflection from the vertical, of magnitude for practical concern, did not extend back more than five towers from the break. Furthermore, these tests demonstrated the ability of the slipping clamp to successfully protect the tower against impact and other stresses from the conductor.

Construction of the 110-Kv. Transmission Line of the Washington Water Power Company

BY LESTER R. GAMBLE[1]

Associate, A. I. E. E.

***Synopsis.**—This paper describes some of the features of design and construction of the 110-kv. transmission lines of The Washington Water Power Compnay.*

A brief outline is given of the loads served and existing interconnections, with other electric utilities.

The general design of single- and double-pole circuits, covering the alinement, the profile, and the structure types, is taken up in detail. Special reference is made to certain long-span construction and to the construction required in heavy frost-loading areas.

The materials used are fully described and several important construction details are discussed. Illustrations are included, showing the form of sag tension curves used for stringing and sagging conductors.

A new type of 110-kv. fuse used in conjunction with an induction type relay is described, the new type of fuse being used on 110-kv. substations of relatively low capacity.

The paper is concluded with a full description of a new method of pole preservation known as the "Cold Treater Dust Method."

GENERAL

THE territory served by The Washington Water Power Company extends 200 miles in an easterly and westerly direction from the Montana state line through northern Idaho into the state of Washington as far west as the Cascade Mountains and 150 miles in a northerly and southerly direction from the Canadian border almost to the south boundary line of Washington.

The industrial and agricultural development in the area described has been very marked during the past few years. In the northern and western parts of the area, large sections of land have been utilized in the growing of orchards. These orchards are irrigated in most cases by electric pumping. In the west and south is a large wheat growing section. Most of this land is non-irrigated but the large annual production of grain has been effective in developing a number of thriving communities. To the east is the Coeur d'Alene Mining District of northern Idaho, where large quantities of both base and precious metals are produced. To the north and south of the mining districts are some of the large lumber producing areas of the Pacific Northwest.

The power supply to the section above described is about centrally located, so that it requires the transmission of large blocks of power over distances of 100 miles or more.

In the year 1917, the first 110-kv. transmission line was built from the Long Lake generating station, southwesterly to Taunton. This line is 113 miles long and was built to serve the Chicago, Milwaukee and St. Paul Railway when their line through the Cascade Mountains to the coast was electrified. After the building of this line, the 110-kv. transmission facilities were rapidly extended, until now they carry power into every section of the area served. The single line diagram shown in Fig. 1 is the existing 110-kv. transmission system. This diagram indicates the industries served, their power requirements and the distances of transmission.

INTERCONNECTION WITH OTHER POWER COMPANIES

The interconnection which exists with the Puget Sound Power & Light Company on the west and the Montana Power Company on the east is shown also in Fig. 1. The connection on the west is made with the Snoqualmie plant through the Chicago, Milwaukee and St. Paul Railway 110-kv. bus. This connection is maintained continuously and successful operation is accomplished. The connection on the east is made, at the present time, through 100 miles of 60-kv. transmission line and a 15,000-kv-a. bank of from 110,000- to 60,000-volt transformers. Continuous parallel operation has not been possible with this connection because the transformer bank has not sufficient capacity to take care of the large swings in power transfer.

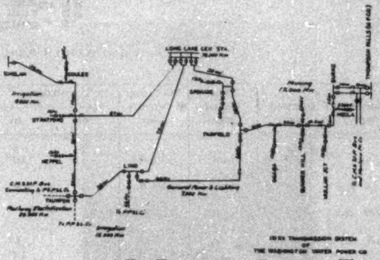


FIG. 1—PLAN OF 110-KV. TRANSMISSION SYSTEM OF WASHINGTON WATER POWER CO.

With the new 110-kv. line now practically completed into the Coeur d'Alene Mining territory, a direct connection with the Montana Power Company will be made, as shown in Fig. 1. A study has been made of this proposed tie and it is found that a maximum of 30,000 kw. can be transmitted. The interconnection

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[1] Assistant Electrical Engineer, Washington Water Power Co , Spokane, Wash.

Presented at the Pacific Coast Convention of the A. I. E. E., Salt Lake City, Utah, September 6-9, 1926.

of the three power companies will represent an electrically connected system, 550 miles in length, reaching across the entire states of Washington, Idaho, and half way into the state of Montana.

DESIGN FEATURES

Voltage regulation and power loss. The voltage regulation and power loss for various kinds and sizes of conductors are determined by means of the Perrine-Baum chart, and the economical size of conductor calculated with due regard to corona limitations. Where stability is a factor, it is also evaluated. All factors established, the kind and size of conductor which will give the most desirable and economical line is chosen. On account of corona loss in the usual altitudes encountered, the minimum size of conductor used is one having a diameter equivalent to seven strands of No. 8 copper. For altitudes above 2800 ft., a diameter equivalent to seven strands of No. 7 copper is specified.

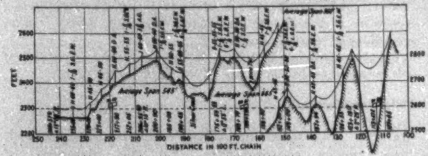


FIG. 2—PROFILE OF 110-KV. TRANSMISSION LINE
Horizontal Configuration: 7-Strand, No. 7 Copper Conductors

Alinement and profile. The data furnished by the survey of a line is sufficient for the drawing up of a plan and profile. (Profile shown in Fig. 2.) The plan is drawn to a scale of 400 ft. to the inch and the profile to a vertical scale of 40 feet to the inch. The sheets are of such size as to cover about two miles of line. The structures are located by means of celluloid sag templates made through the application of the "Melvin-Wynne Sag Tension Charts." The templates represent to scale the approximate catenary which the conductors will take at 100 deg. fahr. and −30 deg. fahr. The 100-deg. template represents the wires in extremely hot weather, when they are sagged to a maximum amount, and is used to determine the location and heights of poles to maintain the specified ground clearances. The 30-deg. template represents the conductors in cold weather when they are sagged to a minimum

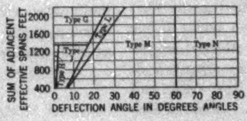


FIG. 3—ANGLE AND GUY CHART FOR 7-STRAND, NO. 7 COPPER CONDUCTORS

Structure type	Normal span	Max. actual span	Max. sum of effective spans	Angles		Side guys		Line guys		Remarks
				from	to	5/16″S.M.	7/16″S.M.	5/16″S.M.	7/16″S.M.	
				0	4°	1 E. W.			4 E. W.	
G	850	1350	2200	4°	30°	1			4 E. W.	Dead end each way
H (*S. A.*)	450	850	1350	0	2°	1				Angle guy
						1 E. W.				Above 650′ spans
H (*D. A.*)	850	1350	2200					2		Next to crossing str.
J (*S. A.*)	450	850	1350	0	6°	1				
				6°	12°	2				
J (*D. A.*)	850	1350	2200	12°	18°	3				
K	850	1350	2200			1 E. W.			4 E. W.	
				6°	12°	2				
				12°	18°	3				
L	450	1350	2200	18°	24°	4				
				24°	31°		3			
				7°	12°	2				
				12°	18°	3				
M	450	1350	2200	18°	24°	4				
				24°	37°		3			
				37°	48°		4			
				48°	60°		5			
N	450	1350	2200			3			3 E. W.	See guy plan dwg. B-1665
O		1800				1 E. W.			6 E. W.	To be used only when approved by engr. office

Assumptions: Guys.
24″ clearance of conductor to structure with 60 mi. wind and no ice.
Guying to withstand tension in all conductors.

amount. This curve is used to check up strain on any of the poles as located.

The height and type of structure at each location is determined by the length of span, the effective spans, the loading conditions, the conflict with other lines and other classes of utilities, and the degree of angle through which the line is turned. Referring to Fig. 2, a notation on the profile *H*-50-55 indicates an *H* type structure, having one pole 50 ft. long and another 55 ft. long; *D A* refers to a double arm; 4$\frac{7}{16}$ *L. G. E. W.* indicates four $\frac{7}{16}$-in. guy wires each way in the line; and 1$\frac{5}{16}$ *S. G. E. W.* indicates one $\frac{5}{16}$-in. side guy each way.

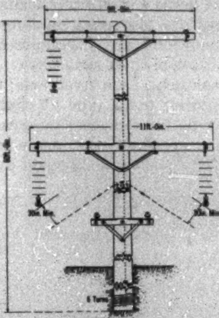


FIG. 4—TYPE "A" SUSPENSION, SINGLE-HOLE STRUCTURE

The proper guying for all structures is obtained from a chart such as shown in Fig. 3. The chart shown is made for seven strands of No. 7 copper. The maximum actual span as referred to in the chart is that one which is limited because of pole strength, conductor spacing and wind pressure. The sum of the effective spans is that span which is limited by crossarm strength. The chart is based on guys being placed at 45 deg. with the structure. Where the angle is decreased, the number of guys is correspondingly increased.

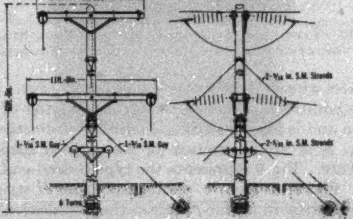


FIG. 5—TYPE "CT" DEAD-END, SINGLE-POLE STRUCTURE

Structure Types. In the development of the 110-kv. transmission structures, the wood pole has been universally used. Two forms of conductor arrangement are provided. One is known as the *L* type configuration, which is used on single-pole lines and the other is the horizontal type of configuration used on double-pole lines. The single-pole structures have their application on the less important lines where conductors are relatively small and where short spans can be maintained.

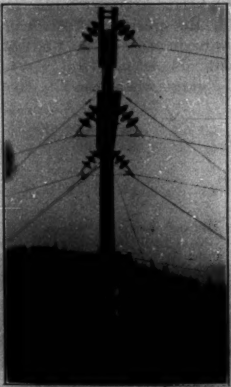


FIG. 6—TYPE "E" SEMI-TENSION, SINGLE-POLE STRUCTURE

FIG. 7—TYPE "A" SUSPENSION, SINGLE-POLE STRUCTURE, SHOWING APPLICATION OF CONDUCTOR WEIGHTS

There are seven types of such structures; the type *A* for normal tangent suspension, the type *B* for very slight angle points, the type *C* and *C T* for semi-tension and dead-ends, and the types *D*, *E* and *F* for large angle points.

Fig. 4 illustrates the *A* type of structure, Fig. 5, the *C T* type of structure, and Figs. 6 and 7 show the actual structures, Fig. 6 being one of the semi-tension type. The semi-tension type of structure is used quite extensively for the purpose of stiffening the line at intervals of approximately one and one-half to two miles in long tangents; also, at points of heavy strain, such as at the top of a ridge where the effective spans are excessive for the suspension structures. This structure is also used in many instances at railroad and telephone crossings. The primary reason for using the semi-tension construction in place of the actual dead-end is to avoid placing insulators in strain position where they are not electrically as good. The use of this structure also permits of the conductors being strung through and facilitates the maintenance of the insulators.

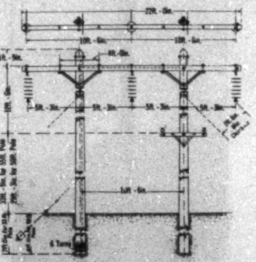


Fig. 8—Type "H" Suspension, Double-Pole Structure

Fig. 7 shows a type *A* suspension structure, demonstrating the use of conductor weights. These weights are used where up-strain occurs in the conductor and where the sum of the negative effective spans is less than 500 ft. Where the sum of the negative effective spans is more than 500 ft., however, a standard, dead-end structure is used. As a general rule, these weights are not used, as the intention is to place structures so that up-strain does not occur. In the few cases in which they are used, they have been necessary because of a slight error in the profile which is discovered when the conductor is installed.

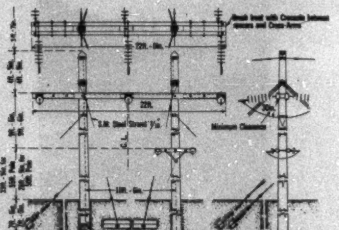


Fig. 9—Type "G", Double-End, Double-Pole Structure

The double-pole structures are used on all lines of any considerable importance and where the size of copper is No. 00 or larger, also where the profile is rough and long, actual, or effective spans are necessary. Such structures are also used on lines where heavy ice-loading conditions are encountered. There are eight distinct types of these structures. The *H* type is used for tangent suspension structures; the *G* type for dead-end structures; the *J* type for slight angle structures; the *K* type for semi-tension structures; the *L*, *M* and *N* types are three-pole, angle structures; and the *O* type is for especially long spans and is used also in places where the effective spans are excessive. There is one additional structure which has no letter of designation and is known as the *transportation structure*.

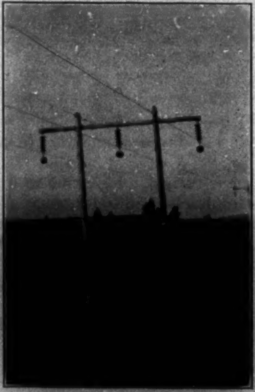
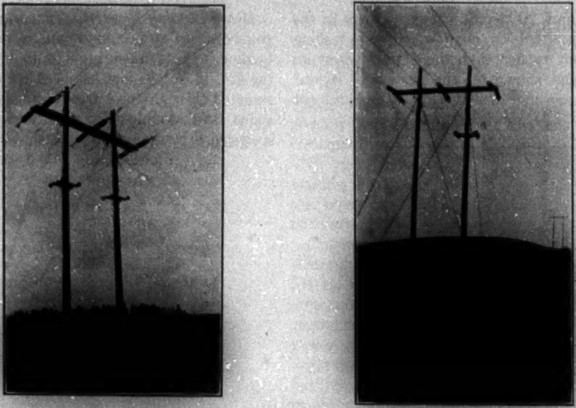


Fig. 10—Type "H" Suspension, Double-Pole Structure Showing Application of Conductor Weights

Fig. 8 is an illustration of the type *H* suspension structure. Fig. 9 represents the type *G* dead-end structure. Fig. 10 is from a photograph of an *H* structure; a slight error in profile required the installation of the weights as shown. Figs. 11 and 12 are taken from photographic reproductions of the type *G* or dead-end structure, and Figs. 13 and 14 show the three-pole structures used in long-span construction. The structure shown in Fig. 14 is supporting a span 2640 ft. in length. In the construction of this span it was necessary to place the telephone on a separate structure. The telephone circuit insulation was equal to that used in the power conductors so that the telephone conductor could be used as a power wire in case of power-wire failure. The bus arrangement between the two structures facilitates connections in case of emergency.



FIGS. 11-12—TYPE "G" DEAD-END, DOUBLE-POLE STRUCTURES

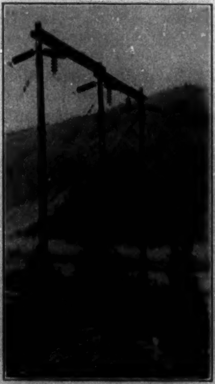


FIG. 13—TYPE "O", THREE-POLE STRUCTURE

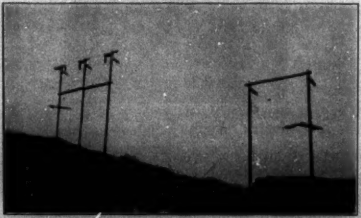


FIG. 14—TYPE "O", THREE-POLE STRUCTURE FOR EXTRA LONG SPANS

Fig. 15 shows the three-pole structure where excessively high poles were required. This span crosses the Columbia River where a 50-ft. clearance over the water was required.

Figs. 16 and 17 show angle structures, types *N* and *M*, respectively.

On a very recently constructed line into the Coeur d'Alene Mining territory, it was necessary to cross the lower end of Coeur d'Alene Lake for a distance of approximately half a mile. The crossing was made through shallow water and the structures placed on piling. Three structures were required in the construction of this crossing. Fig. 18 illustrates the structure

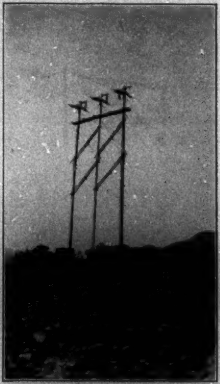


FIG. 15—TYPE "O", THREE-POLE STRUCTURE FOR LONG POLES

used. Figs. 19 and 20 depict the structures in the process of erection. A ½-in. chain was used in lashing the piling together, rather than the 5/16-in. Siemens Martin steel strand shown on the drawing. It was found the chain was more easily applied and did not mutilate the piling when being pulled snug. Two complete wraps were made and the chain fastened to the piling by means of drift pins. The bottom of the lake at the point of crossing was extremely soft, and pilings 65 ft. in length were required.

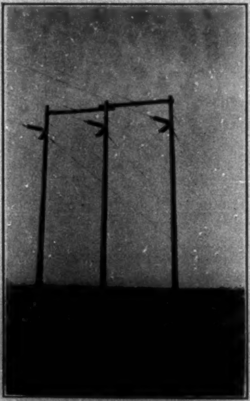


FIG. 16—TYPE "N", THREE-POLE ANGLE STRUCTURE

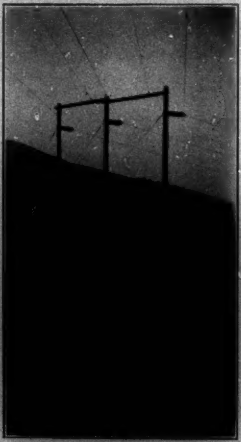
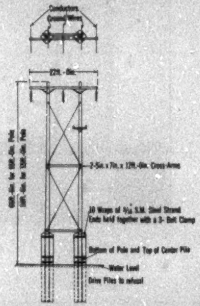


FIG. 17—TYPE "M", THREE-POLE ANGLE STRUCTURE

Heavy Loading Districts. In a number of localities, the lines are subjected to an extremely heavy frost loading. Frost measuring 8 in. in diameter and weighing 2.33 lb. to the foot has been recorded. Lines constructed through these frost belts are built with short spans and double-pole structures. The conductor is sagged excessively. Fig. 21 shows a line in a heavy loading area. The conductor in this line is of No. 000 copper and sagged 18 ft. at 60 deg. The span length is 350 ft. and the sag given is sufficient to prevent the conductor from being over-stressed with a loading of 2½ lb. per foot of frost, and a wind load of one pound per foot.



Conductors
Ground Wires
12ft.-0in.
2-5in. x 7in. x 12ft.-0in. Cross-Arms
10 Wraps of 5/16 S.M. Steel Strand
Ends held together with a 3-Bolt Clamp
Bottom of Pole and Top of Center Pile
Water Level
Drive Piles to refusal

FIG. 18—PILE FOOTINGS FOR STRUCTURES PLACED IN MARSHY OR BOGGY GROUND

FIG. 19—ERECTION OF STRUCTURE ON PILE FOOTING

Clearances. The ground clearances provided are as follows: Over ordinary ground, spans 700 ft. or less—27 ft.; over ordinary ground, spans more than 700 ft.—

29 ft. Over cultivated fields—29 ft. Inside the corporate limits of cities—29 ft. Clearances to guys must be not less than 30 in. The clearances over railroads, telephone circuits and other power lines are such as to be within the requirements of state law.

Telephone Circuits. A telephone circuit is placed on practically all lines, the primary use of such a circuit being for patrol purposes. Such circuits, however, are used also for the transaction of other business. The insulation is of the 13,000-volt class and the conductors are transposed at practically every half mile. Where balance points occur at convenient locations, sectionalizing switches are provided in the telephone circuit. Also the placing of telephone booths at certain advantageous points is arranged.

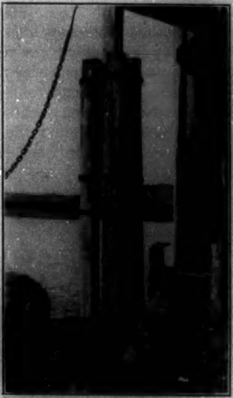


Fig. 20—Pile Footing Showing Chain Lashings

Power Line Transpositions. Transpositions are not placed in the power conductors unless necessary because of interference with foreign telephone or telegraph lines.

Static Wires. Overhead ground wires are installed on practically all lines to insure the service against lightning discharges. The ground wire is also used to give a certain degree of mechanical stability to the line.

One ground wire is installed on single-pole lines at the outer end of the upper crossarm. This wire is grounded at every structure. On two-pole construction, two ground wires are installed, one attached to each pole; these wires are grounded, one on each alternate structure. None of the pole-line hardware is bonded or grounded.

Materials and Construction

Poles. Class *B*, 8-in. top, Western red cedar poles are used for all lines. The pole butts are penetrated and butt-treated by the open-tank, *B* process. Poles treated in this manner have been used almost exclusively until very recently.

The poles on a new line just constructed in the Coeur d'Alene country were not creosoted but were given a special treatment at the time of setting. This treatment is discussed in the last section of this paper.

In view of inability to dig holes to an exact depth, making it very hard to properly line up the structure, the poles in double-pole structures are not framed until after they are set.

Crossarms. The crossarms used on the single-pole structures are 4¾ in. by 5¾ in. by 9 ft. and 4¾ in. by 5¾ in. by 11 ft. The crossarms for the double-pole structures are 5 in. by 7 in. by 22 ft. The telephone crossarms are 3¼ in. by 4¼ in. by 4 ft. 8 in. All power conductor crossarms are provided with 48-in. angle iron braces; the telephone crossarm, with a 28-in. flat brace. To prevent the arms from splitting a ½-in. machine bolt is placed five inches from each end of all crossarms.

Insulators. Insulators of various types consisting of the cap-and-pin, the Jeffery DeWitt and the Hewlett are used. For the suspension strings, six cap-and-pin type, six Hewlett type or five Jeffery DeWitt type, are used. For the strain positions, seven cap-and-pin type, seven Hewlett type, or six Jeffery DeWitt type, are used. As a general rule only the Hewlett type insulator is used in strain.

Wires and Cables. The conductors used are stranded medium hard-drawn copper; aluminum, steel reinforced cable or extra high strength copper-clad cable. The overhead ground wires are 5/16-in., Siemens Martin, seven-strand, galvanized cable. The telephone wires usually consist of No. 6, high-strength, copper-clad wire. The guy wire consists of 5/16-in., and 7/16-in., Siemens Martin, seven-strand, galvanized cable. The power conductors are spliced with two copper splicing sleeves, having two and one-half complete turns each. The ground wire is spliced with one steel sleeve, having three and one-half turns. The telephone wires are spliced with two sleeves, each sleeve having three turns. At the points of attachment to the pole the guy wires

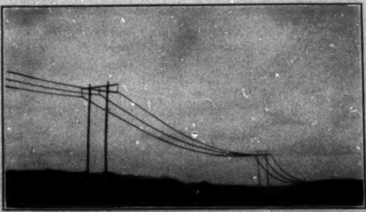


Fig. 21—Line Construction in Extra Heavy Loading Districts. (Note Excessive Amount of Sag in Conductors)

are provided with 4-in. by 8-in. strain plates and guy hooks; standard three-bolt clamps are used, one clamp for 5/16-in. cable and two clamps for the 7/16-in. cable.

Guys. Pressure treated, creosoted ties 7 in. by 8 in. by 4 ft. are used for anchors. The galvanized guy rods are ¾ in. by 8 ft., equipped with one 4-in. by 4-in. square washer and two nuts. Only one ¾-in. rod is used to an anchor. The maximum number of guys in the rod is two 7/16-in. steel cable. A 1-in. thimble is used to take the two 7/16-in. guy wires. In certain locations, where it is impossible to install side guys, the poles are cross-guyed, as shown in Fig. 18. All guy wires are protected by means of a wooden guy guard.

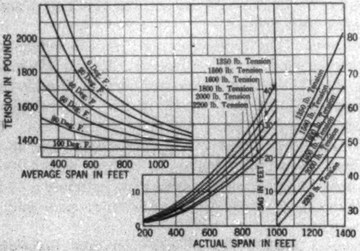


FIG. 22—SAG-TENSION CHART FOR 7-STRAND, NO. 7 COPPER CABLE

Maximum Stress in 7-Strand, No. 7 B & S Cable, 25,000 lb. per sq. in., (2850 lb.) at 15 deg. fahr. with N. E. S. C. medium loading. Modulus of elasticity, 16,000,000; coefficient of expansion, 0.0000096 (Melvin-Wynne Sag-Tension Chart)

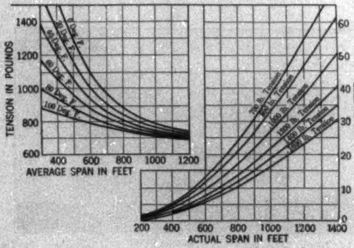


FIG. 23—SAG-TENSION CHART FOR 5/16-IN. SIEMENS MARTIN CABLE

Maximum stress, 40,000 lb. per sq. in., (2430 lb.) at 15 deg. fahr. with N. E. S. C. medium loading. Modulus of elasticity, 29,000,000; coefficient of expansion, 0.0000064 (Thomas Sag-Tension Chart)

Stringing and Sagging. The conductors, which are supported by suspension insulators, are strung through snatch blocks suspended from each insulator support. The snatch blocks have hardwood sheaves and are designed so as to prevent injury to the conductors. No metal grips are used in pulling the conductors, but a 1½-in. rope snub is provided for this purpose. The conductor is entwined through the strands of this rope snub for a distance of four or five feet. A pull on the end of the snub causes the rope strands to firmly grip the wire and makes it possible to pull it into position without injury. This rope snub is commonly termed the "grapevine grip" and is very quickly applied to the conductor.

Dynamometers are used in all cases, but they are used, primarily, for the purpose of giving the conductor the initial pull, which is designated as twice the stringing tension at 100 deg. fahr. The conductor is held at this tension for three minutes and then slacked back to the stringing tension. However, due to the various inaccuracies which may occur in the dynamometer reading, the conductor is never sagged by reading the dynamometer. The proper sag is always obtained by measurement in some convenient span in the section pulled. By the use of thermometers and the sag-tension charts shown in Figs. 22 and 23, the construction forces are able to sag the conductors with a considerable degree of accuracy. Fig. 22 is a chart for seven-strand, No. 7 copper and Fig. 23, a chart for 5/16-in., Siemens Martin steel strand.

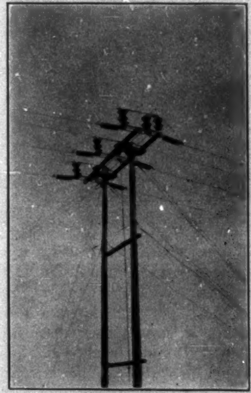


FIG. 24—TWO-POLE STRUCTURE FOR SECTIONALIZING SWITCH

Having given the average span in the section of line being pulled, the actual span where the sag measurement is to be made and the temperature at the time of pulling the wire, the procedure in the use of the sag-tension charts is as follows:

Determine the tension from the curves on the left-hand side of the chart; refer this tension to the tension curves on the right-hand side of the chart and read the sag for the actual span in which the sag is to be measured.

The ground wires are never sagged parallel to the power wires but in accordance with the curves for the material used. In sagging the ground wires, a special dolly, consisting of an iron frame supporting a $\frac{3}{4}$-in. pipe roller and fitted to the top of the crossarm, is used. The device permits the wire to run freely when being pulled in and also avoids injury to the crossarm.

The telephone wires are always sagged parallel to the power conductors.

SWITCHES AND SUBSTATIONS

For the purpose of sectionalizing the transmission lines at certain convenient and advantageous points, a three-phase air break switch is usually installed. The type of structures used in mounting such switches is shown in Figs. 24 and 25. A typical 110-kv. substation is shown in Fig. 26.

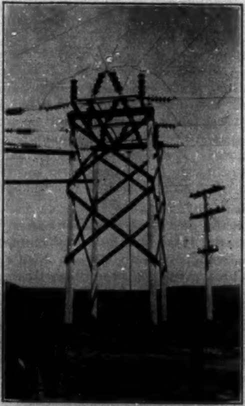


FIG. 25—FOUR-POLE STRUCTURE FOR MOUNTING HEAVY TYPE SECTIONALIZING SWITCH

At the smaller substations, oil circuit breakers are not installed on the 110-kv. side of the transformer bank but fuses are used.

For the 1000- and 1500-kv-a. substations, a satisfactory medium priced 110-kv. fuse which will operate effectively with the relay operation of the balance of the network is not available. The desire has been to have a fuse which will give some selectivity. Such a fuse has been developed recently but as yet it has not been placed in actual service. It is expected that this fuse will be placed at one of the substations where a bank of three 400-kv-a., 110,000- to 13,000-volt transformers will be installed. These transformers are single-bushing transformers with neutral of the Y solidly grounded. The fuse will be of the ordinary open type, designed to give protection from both short circuit and overload, and, at the same time, provide a fuse wire of sufficient size to prevent deterioration due to corona. The fuse is mounted on two stacks of six Jeffery DeWitt post-type insulators. On one post is mounted a small sheet metal cabinet, containing an induction type relay and a suitable clip, into which one end of a micarta tube is secured. A fuse wire of not less than 30-ampere capacity is placed in the tube and secured at the opposite insulator post by a binding screw. At the clip end, a small heating coil, having the same impedance as the relay circuit, is slipped over the lead fuse wire. This heating coil is connected to the tripping circuit of the relay. Under normal operation the main line current passes through the fuse and relay coils to the load. Overload protection is secured by the proper setting of the relay which, when called upon to function, closes its contacts, shunting part of the load current through the heat coil and melting the fuse, thereby opening the 110-kv. circuit. The fuse is equipped with horns so that the arc established is readily extinguished.

COLD TREATER DUST METHOD OF POLE PRESERVATION

The poles which have been used in transmission line construction during the past year have been treated by the "Cold Treater Dust Method." The dust consists of several chemical compounds, obtained in the process of smelting ores. These compounds have varying degrees of solubility, some being highly soluble while others are soluble to a much lesser degree.

The treatment is not applied to the pole before it is set, but always at the time of setting. It may also be applied around poles which have been in the ground for any length of time. The effectiveness of the dust was established after a series of tests covering a period of several years. The tests disclosed the following facts:

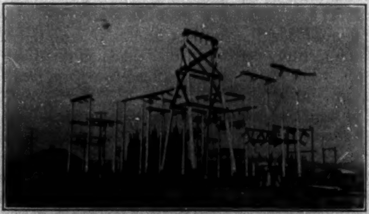


FIG. 26—6000-KV-A. SUBSTATION, 110 KV. TO 60 KV.

1. That the dust absorbs moisture from the surrounding earth and becomes a putty-like mass.

2. That, in all soils except rock, the dust does not readily leach out in the earth.

3. That the highly soluble compounds are soon dissolved and carried up through the outer fibers of the wood by capillary action, and thus immediately head off any decay.

4. That the less soluble compounds act as a storage reservoir, keeping up the supply to the pole in sufficient quantity to prevent decay.

5. That very little or none of the compound is evident in the wood below its point of application.

6. That those portions of wood impregnated with the compound are hard and tough.

7. That the dust must be placed in actual contact with the pole to be effective—any dust out in the earth around the pole is practically useless.

8. That the dust, to be effective, must be placed far enough below the ground surface to get below that point where decay will start.

9. That any rot on the pole at the time of application acts as a cushion or barrier and does not allow the compound to reach the good wood of the pole and thereby stop further rot.

10. That in marshy ground the treater dust is soon dissolved and carried away by the water.

11. That in certain lines built three years ago, in which every alternate pole was treated, there is at this time no evidence of decay and that those poles in the same line which were left untreated show one-fourth to one-half inch of rot.

The following specifications are used in applying the dust:

Seven pounds of treater dust are used at each pole butt. The first application is made by placing one pound of the dust in the bottom of the hole before the pole is set. This is for the purpose of insuring against any heart rot which may develop.

The second application consisting of two pounds of the dust is put around the pole after the hole has been backfilled to approximately midway between the bottom of the hole and a point two feet below the ground line. A narrow groove about ¾ in. wide and one inch deep is made in the earth around the pole with a special tamping bar and this groove is filled with the dust. This application is made as a security against any decay below the two-foot level. It is found that in some territory decay will take place as far down as six feet. The hole is then backfilled to within two feet of the ground surface.

The third and last application is then made at this point (two feet below the ground surface) by tamping a groove around the pole with the same tamping bar as previously used but with groove made two inches deep so as to take four pounds of the dust. The hole is then backfilled and the dirt banked solidly up around the pole.

In marshy ground the dust is not used.

In rock holes the dust is applied in the usual manner but dirt is mixed with the rock used in backfilling so as to fill all crevices between the rocks and thus prevent the sifting away of the dust.

All deadmen for guys are treated with five pounds of the dust placed under and on top of the deadmen.

The cost of the dust ranges from three to five cents a pound. The labor of making the application is relatively small, although the pole setting crew will be forced to lose a little time. Taking everything into consideration, the treatment will possibly cost in the neighborhood of $1.00 per pole.

Discussion

C. R. Higson: I should like to ask Mr. Gamble regarding the effect of this powder on cattle or on a man's hands, or if it is poisonous in case a man should happen to get it in his mouth.

Harold Michener: In the paper, weights are shown at points where there are uplifts due to the structures being low. The author attributes the necessity of these weights to a slight error in profile. In general, for lines over rough country, there will be unavoidable structure locations which, because of being lower than one or both of the adjacent structures, will require either a weight, a tie-down, or a dead-end, to maintain proper clearance between the conductor and the structure under a heavy wind across the line. Usually there will be no actual uplift of the conductors at these structures. The angle to which any unrestrained suspension insulator will swing is the angle the tangent of which is the horizontal wind pressure on the conductor in one-half the sum of the adjacent spans divided by the vertical load on those insulators. The wind pressure on and the weight of the insulators themselves can usually be neglected without appreciable error. If this angle is greater than the allowable clearance will permit, one of the previously mentioned methods of restraining the conductor swing must be employed.

L. R. Gamble: On all the tests we have made in the use of this powder or treater dust, we have never had any cattle or any animal get poisoned. However, the dust is poisonous and to get any of it on the hands, especially if there is an open sore, may cause a little trouble which will necessitate treatment, but such a condition is not dangerous. Dr. Gardner of the Anaconda Copper Mining Co. at Anaconda, Mont., is the chemist who has worked out various uses of this compound and has put it on the market. On his ranch in Montana where he has a number of sheep and cattle he has used this dust for several years in the treating of fence posts and none of his sheep or cattle has in any way been injured by it. In our specifications for the use of dust on poles we specify very definitely that a man should wear gloves when placing the dust around the poles and should also be very careful not to rub his face or eyes while he is applying it. If the dust enters the nose or mouth it is very irritating, and care should be exercised in its handling.

In the design of our lines we use the celluloid sag templets and spot poles from these templets, giving the required clearance to ground. These curves are made up for 100 deg. fahr. and −30 deg. fahr. The −30 deg. fahr. curve is our cold curve and we apply it to the profile of the line and if we find any evidence of upstrain, we immediately re-spot the poles to get away from that upstrain. There is a certain amount of variance in making these sag templets, so we always allow a margin of 4 ft; that is, if we are sagging between two structures and use the cold curve, we put the cold curve between the structures on the two consecutive spans and if the template rests above the pole in the center of these two spans it indicates upstrain. If the cold template rests below the top of the pole there is no upstrain. We usually allow about 4 ft. below the top of the pole in order to play safe. We find that sometimes a little error actually exists in the profile, and 4 ft. is not enough. When we find during construction that there is upstrain it is taken care of by conductor weights. We very rarely find any case in which more upstrain than we can take care of by the use of weights exists. In one instance, however, it was necessary to make a deadend out of the structure.

The Circle Diagram of a Transmission Network

BY FREDERICK E. TERMAN[1]

Associate, A. I. E. E.

***Synopsis.**—The first circle diagrams of the character herein described were published almost simultaneously by Thielemans in Europe and Evans and Sels in this country. The diagram of Evans and Sels is not nearly as complete as Thieleman's, but is more easily constructed because of the mathematical methods employed to determine the circle centers and radii. The present paper is an elaboration of the work done by these two investigators. It coordinates the graphical and mathematical methods of construction and extends their application.*

The principal contributions of this article are incorporated in Tables I and II and in the paragraphs concerning geometrical checks that may be applied to the circle diagram. Tables I and II include formulas for determining the coefficients of a large number of circular loci which have not been heretofore constructed by mathematical methods. Other formulas given in Tables I and II are to be found elsewhere, but generally in a somewhat more complicated, although equivalent, form. The graphical checks that result from the geometrical properties of the circle diagram, as first investigated by Thielemans, have not heretofore been applied to diagrams derived by mathematical computations. By establishing the identity of the Thielemans and Evans and Sels diagrams, it has been possible to utilize the numerous geometrical properties of the diagram that Thielemans has worked out, as well as to make use of other graphical properties. With the aid of the information incorporated in the following paragraphs it is possible to construct a circle diagram on which may be drawn circles representing almost any conceivable locus. This construction is carried out with the aid of computations made from the relatively simple formulas incorporated in Tables I and II. Any errors in the mathematical or graphical work that lead to an incorrect diagram can be simply and quickly uncovered by applying the numerous geometrical checks that are given in the paper. The result is a diagram easy to obtain, almost error proof, and of extreme usefulness.

* * * * *

INTRODUCTION

With the development of longer and longer transmission lines and the growing complexity of power networks, the need has arisen for methods of analysis of transmission circuits that are more extensive and simpler in their application than the usual form of the hyperbolic theory. Using the classical methods of transmission line analysis, a moderately complicated network requires the use of very laborious and tedious mathematics to obtain a solution of even limited application. In the present state of the art the need has arisen for a simple and yet satisfactory way of determining the characteristics of a network composed of the transmission line and the accessory terminal apparatus. This need is being met through the development of the conception of network constants that has been worked out by Evans and Sels,[2] and by the work that has been done on the circle diagram by Thielemans,[3] Evans and Sels, and others.

At present the information available on the circle diagram consists of more or less incomplete and unrelated fragments that have been developed by different writers. There have been two practical methods of attacking the construction of the circle diagram. Thielemans has worked out many of the properties of the circle diagram by purely geometrical considerations and his results are the most complete that have been developed to date. Evans and Sels have developed a diagram based on mathematical considerations and have obtained a result which on the surface appears quite unlike the Thielemans diagram, although fundamentally there is no difference. While the diagram can be approached either from the mathematical or the graphical aspect, the mathematical attack gives the best method of constructing the diagram, being both simpler and more accurate in application.

The purpose of the present paper is, first, to consolidate and expand the work that has already been done upon the circle diagram; second, to develop simpler methods of constructing the circle diagram; and finally, to determine new loci.

EQUATIONS OF A TRANSMITTING NETWORK

Any electrical network to which power is supplied at two terminals, and this power transmitted by the network to another pair of terminals, can always have its characteristics represented by either of the following pairs of equations in which the subscripts *s* and *r* denote sending-end and receiving-end quantities, respectively:

$$E_s = A\,E_r + B\,I_r \qquad \textbf{(1a)}$$

$$I_s = C\,E_r + D\,I \qquad \textbf{(1b)}$$

$$E_r = D\,E_s - B\,I_s \qquad \textbf{(2a)}$$

$$I_r = A\,I_s - C\,E_s \qquad \textbf{(2b)}$$

These two pairs of equations are equivalent to each other, for it is possible to derive the second from the first, but all four equations are useful because each is in a form best suited to handle some particular problem.

The frequency of transmission and the character of the electrical network being used for transmission are taken into account by the four complex quantity coefficients *A*, *B*, *C*, and *D*. A set of such constants, commonly referred to as the *network constants*, can always be determined from a knowledge of the network elements and the frequency. Thus for a long-distance transmission line which has a characteristic impedance Z_0 and a

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[1] Instructor in Electrical Engineering, Stanford University, Calif.

[2] The work of Evans and Sels is to be found in a series of articles in the *Electric Journal*, 1921 volume.

[3] "Diagrams of Transmission Lines," by L. Thielemans, a series of articles published in the 1920-1921 volumes of *Revue Generale de L'Electricitie.*

Presented at the Pacific Coast Convention of the A. I. E. E., Sale Lake City, Utah, Sept. 6-9, 1926.

hyperbolic angle θ, the values of the network constants are:

$$A = D = \text{Cosh}\ \theta$$
$$B = Z_0 \ \text{Sinh}\ \theta$$
$$C = \frac{\text{Sinh}\ \theta}{Z_0}$$

The method of determining the values of network constants is explained in the papers on this subject by Evans and Sels. These authors also give a table of formulas that aid greatly in determining A, B, C, and D for the common networks met in power transmission work, such as a transmission line with terminal transformers, for example.

Only three of the four constants are independent, for it can be shown that in any network the relation $A\,D = 1 + B\,C$ must hold true, which means that when any three constants are known, the fourth can be determined. In the special case of a network that is symmetrical about the center we find that $A = D$, so that for a symmetrical network a knowledge of A and one other constant will suffice to determine all four constants.

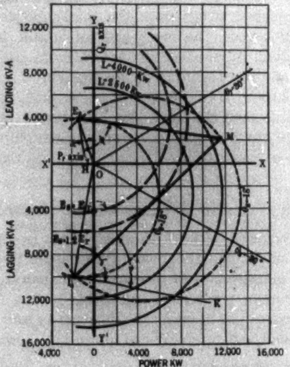


FIG. 1—TYPICAL RECEIVING-END CIRCLE DIAGRAM

Showing loss circles, sending-voltage circles, and sending-end power-factor circles

The conception of network constants is of immense aid in the analysis of transmission circuits. This idea has made it possible to express the over-all characteristics of a transmission system in a simple manner, and to accordingly analyze very complicated systems almost as easily as the simplest problem.

RECEIVING-END CIRCLE DIAGRAM

Construction of the Diagram. Let us first lay out a coordinate system in which the X and Y axes represent received power P_r and received reactive power Q_r, respectively, with *leading values of Q_r considered as positive.* With the aid of equations (**1**) and (**2**) it is possible to compute values of P_r and Q_r which will give a constant generator voltage E_s. When these results are plotted on the $P_r - Q_r$ coordinate system, the result is found to be a circle no matter what constant value of E_s is being investigated *when the receiver voltage E_r is constant.* The only effect of the value of E_s is to vary the radius, so that the locus of points on the $P_r - Q_r$ coordinates which represent constant values of E_s is a family of concentric circles, as shown in Fig. 1. If the investigation is continued, it is found that the loci of points in Fig. 1 which represent constant generator power factor, constant efficiency of transmission, constant sending-end power, etc., are also circles, *provided only that the receiver voltage is kept constant.* A $P_r - Q_r$ coordinate system upon which such circular loci have been placed is called the *receiving-end circle diagram* of the transmission network.

In order that the circle diagram may be of practical use, it is necessary to find some satisfactory way of drawing the various circular loci that may be put on the coordinates in Fig. 1. To draw a circle, one must know the coordinates of the center and the value of the radius measured in terms of the coordinate system. Starting with the equations (**1**) and (**2**), it is possible to derive equations that give the relation of P_r, Q_r and E_r to other quantities, such as generator voltage, generator end power, etc. These all come out as equations of circles, and from these equations we can determine the P_r and Q_r coordinates of the circle centers and the length of the circle radii. The results of such a series of computations are given in Table I which presents the formulas required to compute the center and radius of any kind of circle that one may desire to draw on the $P_r - Q_r$ coordinates. The method used in obtaining the equations in Table I is indicated in Appendix A. In Appendix C will be found a numerical example which shows in a specific way the method of utilizing Table I in constructing the circle diagram.

After Table I has been used to determine the circular loci, and these have been drawn on the $P_r - Q_r$ coordinates, the values of the sending-end voltage, power, power factors, etc., that go with known receiving-end conditions can be determined by inspection from the diagram. It is necessary merely to observe the various circles that pass through the point on the coordinate system that represents the known receiving-end conditions. This indicates the great usefulness of the circle diagram, for the one set of computations required to draw the diagram gives the solution of the transmitting system for all possible conditions.

It is evident from Table I that a great number of families of circular loci may be drawn on one system of coordinates. Not all of these are ordinarily needed in the solution of any particular problem, however, and in any event it is best to draw no more than three or possibly four sets of circles on one coordinate system. When more loci are required it is preferable to divide them between two or more diagrams.

When deciding which loci to draw on a diagram, it is well to keep in mind the information in Table I under the column entitled "Remarks." When representing the characteristics of a transmission system for ordinary purposes it is usually sufficient to draw three families of circles, which can best be sending-end, voltage circles; transmission, power-loss circles; and either sending-end, power factor circles, or sending-end, reactive power circles; drawing each family of circles in colored ink of a distinguishing hue. The data given by these sets of circles will enable other items of importance to be easily determined. Thus the sending-end power can readily be found by adding the power loss to the received power; the efficiency of transmission is the ratio of receiver to sending-end power, etc.

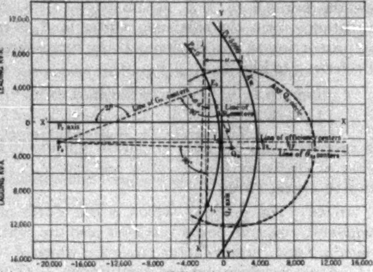


FIG. 2—RECEIVING-END CIRCLE DIAGRAM

Showing approximate location of centers and some of the diagram's geometrical properties

It is worth noting that all of the circle centers and radii can be computed from Table I without involving the network constant C. This is possible because only three of the network constants are independent, so that the constant C can be expressed in terms of A, B, and D.

Procedure to be followed in drawing circle diagram. In drawing the receiving-end circle diagram it is necessary to take the following steps:

1. Lay out a set of $P_r - Q_r$ coordinates to any convenient scale which will cover the range of values to be expected.
2. Decide upon the kinds of loci that will be best suited to give the information that is desired.
3. Compute the centers and radii of the desired circles with the aid of Table I, and draw these circles on the coordinate system that has been laid out. In making these computations it is necessary to exercise care in determining the algebraic signs of the terms involving sines and cosines of angles.
4. Apply as many of the geometrical checks and constructions described below as are desired.

A receiving-end circle diagram can be drawn only for a fixed value of receiving voltage E_r. It is necessary to draw a separate receiving-end diagram for each receiver voltage to be considered. The approximate locations of the various circle centers are shown in Figs. 1, 2, and 3, which represent actual results given by a long transmission line with terminal transformers. The notation of these figures is that of Table I, with the addition that the center of the P_s circles is designated by $P_{s s}$, the center of the E_s circles by $E_{s s}$, etc.

The units involved in the diagram need introduce no confusion. The results given in Table I are based on equations (**1**) and (**2**); thus the units of the circle diagram are the units of these equations. This will ordinarily mean volts to neutral, current in each wire, watts and volt-amperes per phase, transmission loss in watts per phase, etc., as indicated in Table I.

Geometrical properties of the receiving-end circle diagram. A circle diagram drawn following the formulas in Table I must possess certain geometrical properties. While these properties can be used to draw the circular loci, it is more accurate and much quicker to draw the diagram with the aid of Table I, and to use the geometrical properties as checks to verify the correctness of the construction. The only exception to this is in the case of sending-end power factor circles, which are most satisfactorily drawn by the geometrical construction given below. The more important graphical properties of the receiving-end circle diagram follow:

Location of E_s. The angle $X\,O\,E_s$ (see Fig. 3) must equal $(180° + \alpha - \beta)$ when measured in the conventional manner from $O\,X$.

Location of I_s. The angle $X\,O\,I_s$ (see Fig. 3) must equal $(180° - \gamma + \delta)$ when measured backward from $O\,X$.

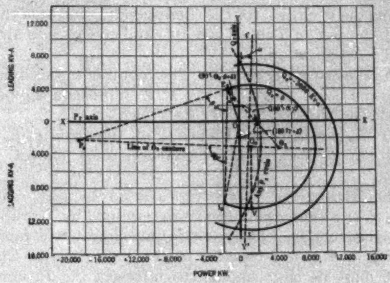


FIG. 3—RECEIVING-END CIRCLE DIAGRAM

Showing approximate locations of centers, and some of the diagram's geometrical properties

Locus of constant receiver power factor. Constant receiver power factor is equivalent to a constant Q_r/P_r, and the locus of this condition is a straight line passing through the origin of the $P_r - Q_r$ coordinates. The angle between this straight line and the axis $O\,X$ is the power-factor angle θ_r. Positive values of θ_r indicate that the received current leads the receiver voltage by the angle θ_r. Fig. 1 shows several constant receiver power-factor loci.

Phase of sending-end voltage and sending-end current. The phase of sending-end voltage with reference to the receiver voltage is found by drawing $E_s H$ in Fig. 1 so that the angle of $O E_s H$ measured backward from $E_s O$ is equal to γ. The true phase of E_s referred to E_r for any point M on the diagram is then the phase of $E_s M$ relative to $E_s H$. This is the angle Φ shown in Fig. 1. The phase of sending-end current with reference to the receiver voltage is found by drawing $I_s K$ backward from $O I_s$ by the angle γ. The phase of I_s referred to receiver voltage for any point M on the diagram is then the phase of $I_s M$ relative to $I_s K$. This is the angle ψ shown in Fig. 1.

Sending power factor circles. If M is some point on the coordinate system in Fig. 1, then the angle $E_s M I_s$ equals $(\theta_s + \beta - \delta)$, where θ_s is the angle by which I_s leads E_s, and therefore is a *positive angle for leading power factors.* The locus of points for constant θ_s is accordingly a circle passing through E_s and I_s, and with its center on the perpendicular bisector of $E_s I_s$. The location of the center on this perpendicular bisector depends upon the power factor being represented, and can be found by laying off the angle $I_s E_s \theta_s$ in Fig. 3 equal to $(90° - \theta_s - \beta + \delta)$ measured in the positive direction from $E_s I_s$. The intersection of $E_s \theta_s$ with the perpendicular bisector is the center desired, and the radius can be readily determined by the fact that the circle must pass through E_s. This geometrical construction offers the only satisfactory method of determining the θ_s circles, for the corresponding formula that might be put in Table I is most cumbersome and awkward in application.

Sending-end power circles. The center of these circles, P_s, is the point where the line $P_s E_s$ intersects the perpendicular bisector of $E_s I_s$, as shown in Fig. 2. The line $P_s E_s$ is determined by laying off the angle $P_s E_s I_s$ equal to $(\beta - \delta)$. This construction is shown in Fig. 2.

The sending-end power represented by any circle with P_s as a center can also be determined by a graphical construction. Draw any convenient circle representing sending-end reactive power (a Q_s circle). This circle must be large enough to intersect the P_s circle that passes through E_s. Draw a line through the two points of intersection (line $j\,k$ in Fig. 2). This line will be parallel to $E_s I_s$. The sending-end power represented by any P_s circle can then be found by noting where this circle intersects the Q_s circle mentioned above. The perpendicular distance from this intersection to $j\,k$ is proportional to the value of P_s being represented. Let this distance be u when measured in terms of the coordinate system. Then in the notation of Table I, the value of P_s that corresponds to u is

$$P_s = \frac{u}{\sin(\beta - \delta)}$$

Transmission loss circles. The center of the loss circles is point L. This is the point where the line parallel to $X\,X'$ and passing through P_s intersects the circle that has P_s as its center and passes through E_s. See Fig. 2. This sending power circle that passes through E_s is the circle for which $P_s = O$. The intersection of this circle with a loss circle must occur at a value of P_r that is the negative of the loss represented by the loss circle forming the intersection. This determines the radii of the loss circles.

Efficiency circles. The centers of the efficiency circles lie on the parallel to $X\,X'$ which passes through P_s. The distance from the efficiency center to P_s is in inverse proportion to the efficiency being represented, so that referring to Fig. 2

$$\frac{\text{Length } P_s\,\eta_y}{\text{Length } P_s\,\eta_x} = \frac{\text{Efficiency } \eta_x}{\text{Efficiency } \eta_y}$$

This property can be used to locate quickly a large number of efficiency centers after one center has been computed from Table I.

Q_s circles. The point Q_s which is the center of the sending-end, reactive power circles is also the center of the sending-end, power-factor circle for which $\theta_s = O$. The Q_s circle representing the condition $Q_s = O$ passes through E_s, and of course coincides with the power-factor circle for $\theta_s = O$. The sending-end, reactive power represented by any Q_s circle can also be determined geometrically. Select a convenient sending-end power circle that intersects the circle for $Q_s = O$ and the other Q_s circles to be investigated. Draw the line $t\,t'$ which joins the points where the circle $Q_s = O$ intersects this P_s circle (see Fig. 3). The value of Q_s represented by any reactive, sending-end power circle can be found by noting where this circle intersects the P_s circle being used in the determination. The perpendicular distance from this intersection to the line $t\,t'$ is proportional to the value of Q_s at the intersection. If this distance is u' (see Fig. 3) when measured in terms of the coordinate system, the value of the Q_s circle that gives this distance is

$$Q_s = \frac{u'}{\cos(\beta - \delta)}$$

The notation used is that given in Table I. Intersections to the right of $t\,t'$ are caused by leading or positive values of Q_s, and intersections to the left of $t\,t'$ represent lagging or negative values of Q_s.

V circles. The center of these circles, the point V in Fig. 3, is where the parallel to $Y\,Y'$ that passes through the point Q_s intersects the sending-end reactive power circle representing $Q_s = O$. This is shown in Fig. 3. These circles must have radii of such values that the V circles intersect the circle $Q_s = O$ at a value of Q_r which is the negative of the value of V at the intersection.

G_s circles. Circles representing different values of

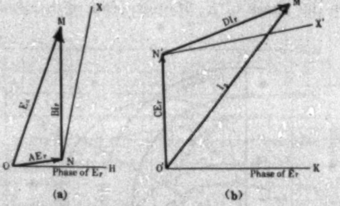
sending-end conductance have their centers lying on the line $E_s P_s$. See Figs. 2 and 3.

B_s circles. Circles representing different values of sending-end susceptance have their centers lying on the line $E_s Q_s$, as shown in Figs. 2 and 3.

In addition to these geometrical properties, the receiving-end diagram must satisfy certain other relations. Thus the P_s circles must intersect the loss circles at a value of P_r such that the received power plus the loss equals the sending power. Similarly, the point of intersection of the P_s circles with the efficiency circles must be at a value of P_r equal to the sending power multiplied by the efficiency. Certain angular relations must also exist between construction lines. These are shown in Fig. 2, and are as follows:

Angle $P_s E_s Q_s = 90$ deg.
Angle $P_s E_s I_s = \beta - \delta$
Angle between $E_s P_s$ and axis $X X' = 180° - 2\beta$

Geometrical basis of the receiving-end diagram. Since the receiving-end diagram is determined by the mathematical properties of equations (**1**) and (**2**), it is possible to plot these equations in vector form and obtain the receiving-end diagram without the use of Table I. This has been done by Thielemans, and gives an alternative method of construction.



(a) (b)

FIG. 4A-B—VECTOR DIAGRAM REPRESENTING EQUATIONS (1A) AND (1B) IN VECTOR FORM

Using the received voltage as the axis of reference, equations (**1a**) and (**1b**) give the vector diagrams shown in Figs. 4A and 4B. The parts $N M$ of Fig. 4A and $N' M'$ of Fig. 4B are both determined by the phase and magnitude of the received current. By suitably choosing the voltage scale in Fig. 4 A with reference to the sending-end current scale in Fig. 4B, it is possible to make $N M$ and $N' M'$ represent received current to the same scale. It is then possible to superimpose $N M$ on $N' M'$, with the result shown in Fig. 5. This is a compound diagram, for the part $O N M$ is a sending-end voltage diagram in which $O H$ represents the phase of E_r, while $O' N M$ is a sending-end current diagram in which $O' K$ represents the phase of E_r. These two triangles are related by the common link $N M$ which simultaneously represents a component of sending voltage, a component of sending-end current, and the received current vector. The phase of the received current is determined by $N X$, the position of $N M$ when the receiver current is in phase with the receiver voltage. The scales in the different parts of Fig. 5 are not the same, but are related to one another by the common part $N M$.

Let us consider Fig. 5 for a constant value of receiver voltage E_r. In this case, points O, O' and N in Fig. 5 are fixed, and the position of point M, which is determined by the received current, is the only variable point on the diagram. Since the receiver voltage is constant, it is possible to make $N X$ and $N Y$ the P_r and Q_r axes of a system of power-reactive power coordinates. This is because the received real and reactive power are proportional to the in-phase and quadrature components of received current, respectively, when the receiver voltage is constant, and it is obvious that these current components are the projections of $N M$ on $N X$ and $N Y$.

Now compare Fig. 5 with Fig. 1. It is evident that these two diagrams are the same, as far as they represent the same things. The points O, O', N, and M in Fig. 5 correspond exactly to E_r, I_s, O, and M in Fig. 1, and the power coordinate systems are identical in the two cases. Thielemans has investigated the geometrical properties of Fig. 5, and his results form the basis of many of the geometrical checks given above for use in verifying the accuracy of the diagram construction.

It is helpful to keep in mind the relation between Figs. 1 and 5. The principal difference is that in Fig. 1 the important points of the diagram are located by plotting on the $P_r - Q_r$ coordinate system, while in Fig. 5 these same points are found to be ends of important vectors in the representation of equations (**1a**) and (**1b**). With Fig. 5 in mind it is easy to see why some of the loci are circles. Thus the sending-voltage vector in this diagram is represented by $O M$, and obviously a constant sending voltage must be for positions of M that lie on a circle having 0 as the center. It is the identity of Figs. 1 and 5 that makes it desirable to plot leading quantities as positive, as explained in Appendix B.

SENDING-END CIRCLE DIAGRAM

Construction of the diagram. Instead of using a system of $P_r - Q_r$ coordinates, as in the receiving-end circle diagram, it is possible to use a set of $P_s - Q_s$ coordinates. When results computed from equations (**1**) and (**2**) on the basis of a *constant generator voltage E_s* are plotted upon this system of coordinates, properties similar to those of the receiving-end diagram are found. Thus the loci of points on the $P_s - Q_s$ coordinate system which represent constant receiver voltage, constant receiver power factor, constant efficiency of transmission, constant receiver power, etc., are all circles, *provided the sending voltage is constant.* Such a diagram is known as the *sending-end circle diagram* of the transmission network. It is found that for every property possessed by the receiving-end diagram there is a corresponding property of the sending-end diagram.

The construction of the sending-end diagram is carried out in a manner exactly similar to that used in obtaining the receiving-end diagram. The only difference is in the formulas used in determining the centers and radii of the circles. The coordinates of the circle centers and the length of the radii can be derived from equations (**1**) and (**2**) in the manner indicated in Appendix A. The results of such derivations are given in Table II, which is used in exactly the same manner as in Table I.

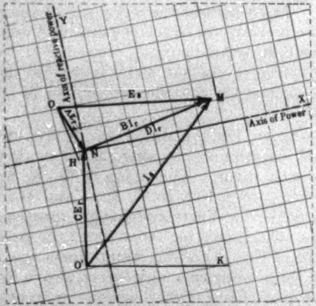


FIG. 5—COMPOUND DIAGRAM RESULTING FROM THE SUPERPOSITION OF FIGS. 4A-B

The difference between the sending-end and receiving-end diagrams is in the coordinate system and in the quantities that are represented by the circular loci. The receiving-end diagram must always be drawn for a constant receiver voltage, and similarly, the sending-end diagram can be drawn only for a constant sending-end voltage. If the sending voltage is changed, it is necessary to draw a new diagram.

Procedure to be followed in drawing the sending-end circle diagram. This procedure is exactly the same as that given under the heading of receiving-end circle diagram, with the exception that the circles are determined with the aid of Table II instead of Table I and that the geometrical properties of the sending-end diagram are somewhat different. The approximate locations of the circle centers are shown in Figs. 6, 7, and 8 which have been drawn to represent a long transmission line with terminal transformers. The notation of these figures is that of Table II with the addition that the center of the P_r circles is labeled P_r, the center of the E_r circles is labeled E_r, etc. As with the receiving-end diagram, care must be taken in using Table II to insure the correct algebraic signs of terms involving sines and cosines.

Geometrical properties of the sending-end circle diagram. The circle diagram drawn with the aid of the formulas in Table II possesses certain geometrical properties. These properties are analagous to those of the receiving-end circle diagram and have the same practical usefulness. The more important graphical characteristics of the sending-end diagram are as follows:

Location of E_r. The angle $X\,O\,E_r$ in Fig. 8 must equal $(\beta - \delta)$ when measured backward from $O\,X$.

Location of I_r. The angle $X\,O\,I_r$ in Fig. 8 must equal $(\gamma - \alpha)$ when measured in the conventional manner from $O\,X$.

Location of constant sending-end power factor. Constant generator power factor is obtained when Q_s/P_s is constant and the locus resulting from this condition is clearly a straight line passing through the origin of coordinates and making an angle with the axis $O\,X$ equal to the power factor angle θ_s. Positive values of this angle indicate that the sending-end current leads the sending-end voltage. Fig. 6 shows several of these loci.

Phase of receiving-end voltage and receiving-end current. The phase of receiving-end voltage referred to the sending voltage is found by drawing $E_r\,H$ in Fig. 6 so that the angle $O\,E_r\,H$ measured backward from $E_r\,O$ is equal to δ. The phase of receiving voltage referred to E_s for any point M on the diagram is then the phase of $E_r\,M$ measured with reference to $E_r\,H$. Similarly the phase of receiving-end current with reference to the sending voltage for any point M on the diagram is the phase of $I_r\,M$ measured with reference to the axis $I_r\,K$. This line is drawn so that the angle $K\,I_r\,O$ is $(180° - \gamma)$ when measured from $I_r\,O$. In Fig. 6 the phases of receiving voltage and current are Φ and ψ, respectively.

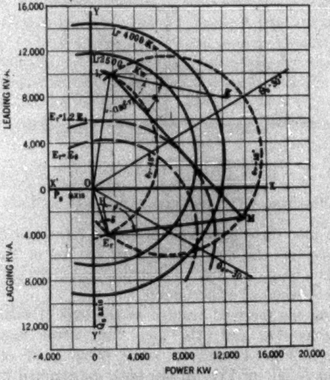
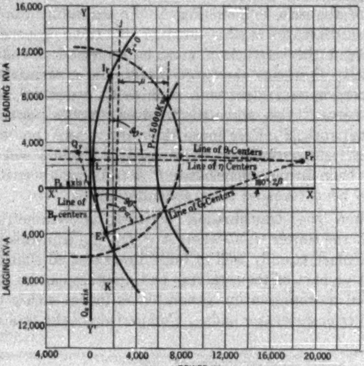


FIG. 6—TYPICAL SENDING-END CIRCLE DIAGRAM

Showing loss circles, receiving-end, power-factor circles, and receiver-voltage circles

Receiving power factor circles. The locus of points on the sending-end diagram which represents constant receiving-end power factor is a circle passing through E_r and I_r and having its center on the perpendicular bisector of the line $E_r I_r$. The location of this center depends upon the power factor being represented and is at the intersection of the perpendicular bisector with a line passing through E_r in such a way that the angle $\theta_r I_r E_r$ is equal to $(90° - \theta_r - \beta + \alpha)$ measured in the conventional direction from $E_r I_r$. This construction is shown in Fig. 8.

Receiving-end power circles. The point P_r is the center of these circles and is located where the line $P_r E_r$ intersects the perpendicular bisector of $E_r I_r$ as shown in Fig. 7. The line $P_r E_r$ is determined by laying off the angle $P_r E_r I_r$ equal to $(\beta - \alpha)$ as is shown in Fig. 8.


16,000
12,000
8,000
4,000
0
4,000
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12,000
LEADING KV-A
LAGGING KV-A
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0
4,000
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12,000
16,000
20,000
POWER KW.
Y
Y'
X'
X
j
$P_r = 0$
I_r
u
$P_r = 5000$ Kw
θ_r
Q_r
L
Line of θ_r Centers
Line of η Centers
P_r
$180° - 2\beta$
P_s axis
Line of B_r centers
Line of Q_r Centers
E_r
K
Q_s axis

Fig. 7—Sending-End Circle Diagram

Showing approximate location of centers and some of the diagram's geometrical properties

The receiving-end power represented by any circle with P_r as the center can be determined by a graphical construction similar to that used in connection with the sending power circles of the receiving-end diagram. Draw any convenient Q_r circle that intersects the P_r circle passing through E_r. Draw a line through the two points of intersection with the P_r circle (line $j\,k$ in Fig. 7). The receiving power represented by any P_r circle can be found by determining the intersection of the power circle with the Q_r circle. The perpendicular distance from this intersection to the line $j\,k$ is proportional to the value of P_r being represented. Let this distance be u when measured in terms of the coordinate system. Then the value of P_r at the intersection having this value of u is

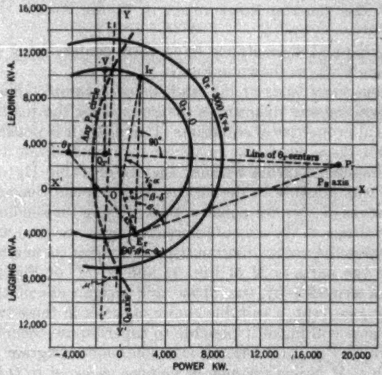
$$P_r = \frac{u}{\sin(\beta - \alpha)}$$

The notation is that of Table II.

Transmission loss circles. The center of these circles is point L, which is the point where the line parallel to XX' and passing through P_r intersects the circle that has P_r as its center and passes through E_r (see Fig. 7). This receiving power circle is the circle for which $P_r = O$. The intersection of this circle with a loss circle must occur at a value of P_s which is equal to the value of loss at the point of intersection.

Efficiency circles. The centers of the efficiency circles lie on the parallel to $X\,X'$ which passes through P_r. The distance from any center to P_r is in direct proportion to the efficiency being represented. This property can be used to locate quickly a large number of efficiency centers after one center has been computed from Table II.

Q_r circles. The point Q_r, which is the center of the receiving-end reactive power circles, is also the center of the receiving-end power factor circle for which $\theta_r = O$. The Q_r circle representing zero received reactive power, of course, and coincides with the circle representing unity receiver power factor.


16,000
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LEADING KV-A.
LAGGING KV-A.
-4,000
0
4,000
8,000
12,000
16,000
20,000
POWER KW.
Y
Y'
X'
X
t
t'
V
I_r
Any P_r circle
$Q_r = 0$
$Q_r = 300$ Kv-a
90°
θ_r
Q_r
Line of θ_r-centers
P_r
P_s axis
O
$\gamma - \alpha$
$\beta - \delta$
E_r
$90° - \theta_r - \alpha - \beta$
u'
Q_s axis

Fig. 8—Sending-end Circle Diagram

Showing approximate location of centers and some of the diagram's geometrical properties

The value of Q_r represented by any receiving-end reactive power circle can be determined by the following geometrical construction. Draw some P_r circle that intersects the circle $Q_r = O$, and the other Q_r circles being investigated. Draw the line $t\,t'$ which joins the points where the circle Q_r intersects this P_r circle (see Fig. 8). The value of Q_r represented by any reactive, receiving-end, power circle can be found by noting where this circle intersects the P_r circle being used in the determination. The perpendicular distance from this intersection to the line $t\,t'$ is proportional to the value of Q_r at the intersection. If this distance is u' (see Fig. 8) when measured in terms of the coordinate system, the value of the Q_r circle at the intersection is

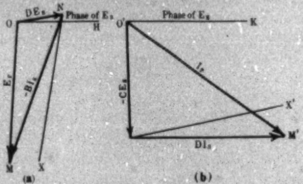
$$Q_r = \frac{u'}{\cos(\beta - \alpha)}$$

The notation used is that given in Table II. Intersections to the right of $t\,t'$ represent positive or leading reactive power, and intersections to the left of $t\,t'$ represent negative or lagging reactive power.

V circles. The point V in Fig. 8 is the center of these circles, and is the point where the parallel to $Y\,Y'$ that passes through the point Q_r intersects the reactive receiving-end power circle representing $Q_r = O$. This is shown in Fig. 8. These circles must have radii of such values that a V circle intersects the circle $Q_r = O$ at a value of Q_s equal to the value of V at the intersection.

G_r circles. Circles representing different values of receiving-end conductance have their centers lying on the line $E_r\,P_r$, as shown in Figs. 7 and 8.

B_r circles. Circles representing different values of receiving-end susceptance have their centers lying on the line $E_r\,Q_r$, as shown in Fig. 7.



FIGS. 9A-B—VECTOR DIAGRAM REPRESENTING EQUATIONS (2A) AND 2(B) RESPECTIVELY IN VECTOR FORM

Geometrical basis of the sending-end diagram. Since the sending-end diagram is determined by the mathematical properties of equations (**1**) and (**2**), it is possible to plot these equations in vector form and obtain the sending-end diagram without the use of Table II, just as it was possible to derive the receiving-end diagram by means of a vector construction. Using the sending-end voltage as the axis of reference, equations (2**a**) and (2**b**) give the vector diagrams shown in Figs. 9A and 9B. Since the parts $N\,M$ of Fig. 9A and $N'\,M'$ of Fig. 9B are determined by the phase and magnitude of the sending current, a suitable choice of scales in these two diagrams will make it possible to superimpose them as shown in Fig. 10. Fig. 10 is a compound diagram in which $O\,M\,N$ is a triangle of voltages referred to $O\,H$, the phase of E_s, while $O'\,N\,M$ is a receiving current triangle with reference to the phase of E_s which is given by $O'\,K$. These two triangles are related by the common link $N\,M$, which simultaneously represents a component of receiving voltage, a component of received current, and also the sending current vector. The phase of the sending current is determined by $N\,X$, which is the position of $N\,M$ when the sending current is in phase with the sending voltage.

Fig. 10 possesses properties similar to those discussed in connection with Fig. 1. When the sending voltage is constant, the points O, O' and N are fixed and it is possible to lay out a system of $P_s - Q_s$ coordinates on the axis $N\,X$ and $N\,Y$. When this is done it is seen that Fig. 10 is exactly the same as Fig. 6, and as far as final results are concerned it makes no difference whether the circle diagram is obtained from this geometrical construction or from results computed with the aid of Table II.

MISCELLANEOUS TOPICS

Relations between sending- and receiving-end diagrams. When the receiver end of the transmission network is supplying power to the network, the received power P_r is negative and the operating point is somewhere to the left of the axis $Y\,Y'$ in Fig. 1. In extending the receiver diagram in this manner to cover generator action at the receiver, it must be remembered that a negative P_r means generated power, and a negative Q_r received is equivalent to a positive Q_r generated, and vice versa. Also, when the receiver supplies power to the network, P_r/P_s is more than unity, and the "efficiency" of transmission to be used in Table I will be more than unity. A negative P_s is to be interpreted as power being received at the generator end.

When the transmission network is symmetrical about the center (*i. e.*, when $A = D$), the transmission takes place equally well in both directions, so that the negative part of the receiving-end diagram can be considered as representing generator action at either the receiver or generator ends. Comparison of the positive part of the sending-end diagram shown in Fig. 6 with the negative part of the receiving-end diagram in Fig. 1 shows that these parts are identical when properly superimposed, as indeed they would have to be when drawn for the same voltages.

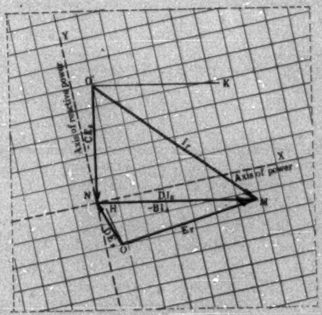


FIG. 10—THE COMPOUND DIAGRAM RESULTING FROM THE SUPERPOSITION OF FIGS. 9A-B

When the transmission network is non-symmetrical, transmission in different directions takes place with somewhat different properties, and we find that the sending-end diagram is similar to the negative part of the receiving-end diagram, but not quite the same because of the difference in direction of transmission.

It is also possible to compare the negative part of the sending-end diagram with the positive part of the receiving-end diagram. The two will be found to be identical when the network is symmetrical, and similar, but somewhat different, when the network is not symmetrical. The details involved in this comparison are quite obvious, but are not of enough importance to warrant consideration here.

The reduced circle diagram. The receiving-end circle diagram must be drawn for a constant E_r, and the sending-end diagram must be constructed for a constant E_s. When neither terminal voltage is constant, it is necessary to use the circle diagram indirectly by drawing the diagram for a certain voltage and then multiplying results obtained from this diagram by appropriate factors to convert the results to those for the voltage desired. Thus a receiving-end diagram drawn for a receiver voltage of one volt can be used to determine conditions that exist for receiver voltage of E_r if all power, reactive power, and loss quantities on the *reduced diagram* are multiplied by E_r^2, and all current quantities on the *reduced diagram* are multiplied by E_r. In the same way it is possible to draw a sending-end diagram reduced to one volt of sending voltage, and obtain results for any value of E_s by multiplying the quantities determined from the reduced diagrams by E_s^2 or E_s, as the case may be.

Evans and Sels diagrams. With the aid of the first line in Table I, one can draw a family of circles representing a constant E_s and various values of E_r, using a $P_r - Q_r$ system of coordinates. The centers and radii of these circles depend on the value of E_r being represented, and the appearance of such a diagram is shown in Fig. 11. Using the first line of Table II in a similar manner, it is possible to obtain Fig. 12, which gives a series of circles representing constant E_r and various values of E_s, drawn on a system of $P_s - Q_s$ coordinates. Such diagrams are very useful in the analysis of certain types of problems, and are called Evans and Sels diagrams, after their originators.

Use of current and admittance coordinate systems. The

TABLE I

CIRCLE COEFFICIENTS OF RECEIVING-END DIAGRAM

Kind of circle	P_r coordinate of center	Q_r coordinate of center	Radius	Remarks*
Sending-end voltage...	$-w = -e_r^2 \frac{a}{b} \text{Cos}(\alpha - \beta)$	$-x = -e_r^2 \frac{a}{b} \text{Sin}(\alpha - \beta)$	$\frac{e_r\, e_s}{b}$	Circles are concentric
Power loss in transmission..............	$-w + m \cos^2 \beta$	$-x - \frac{m}{2} \sin 2\beta$	$\sqrt{m L - m w + m^2 \cos^2 \beta}$	Circles are concentric
Sending-end power factor..................	The sending-end power factor circles can be drawn by a simple geometric construction that is much quicker than the analytic method. See text for graphical method.			
Sending-end current...	$-e_r^2 \frac{c}{d} \text{Cos}(\gamma - \delta)$ $= -w + \frac{e_r^2}{b\,d} \text{Cos}(\beta + \delta)$	$-e_r^2 \frac{c}{d} \text{Sin}(\gamma - \delta)$ $= -x - \frac{e_r^2}{b\,d} \text{Sin}(\beta + \delta)$	$\frac{i_s\, e_r}{d}$	Circles are concentric
Efficiency of transmission................	$-w + \frac{m}{2}\left[\cos 2\beta + \frac{1}{\eta}\right]$	$-x - \frac{m}{2} \text{Sin}\, 2\beta$	$\sqrt{\frac{m^2}{4}\left[1 + \frac{1}{\eta^2} + \frac{4}{\eta}\left(\frac{\text{Cos}\, 2\beta}{2} - \frac{w}{m}\right)\right]}$	Circles are not concentric
Sending-end power....	$-w + m \frac{\cos 2\beta}{2}$	$-x - m \frac{\sin 2\beta}{2}$	$\sqrt{m P_s + \frac{m^2}{4}}$	Center is usually off page
Sending-end reactive volt-amperes.........	$-w - \frac{r}{2} \sin 2\beta$	$-x - \frac{r}{2} \cos 2\beta$	$\sqrt{\frac{r^2}{4} + r Q_s}$	Circles are concentric
Reactive volt-amperes consumed by line.. .	$-w - r \frac{\sin 2\beta}{2}$	$-x + r\, \text{Sin}^2 \beta$	$\sqrt{r V - r x + r^2 \sin^2 \beta}$	Circles are concentric
Receiver current.......	0	0	$i_r\, e_r$	Circles are concentric
Receiver admittance...	0	0	$Y_r\, e_r^2$	Circles are concentric
Sending-end conductance	$-w - k \cos 2\beta$	$-x + k \sin 2\beta$	k	Circles are not concentric
Sending-end susceptance...............	$-w + l \sin 2\beta$	$-x + l \cos 2\beta$	l	Circles are not concentric

Notation

$$\left.\begin{array}{l} A = a\,\underline{/\alpha} \\ B = b\,\underline{/\beta} \\ C = c\,\underline{/\gamma} \\ D = d\,\underline{/\delta} \end{array}\right\} \text{Network constants}$$

P = Power in watts per phase

Q = Reactive volt-amperes per phase

E = Voltage to neutral

I = Current in each wire

G = Conductance to neutral

B_s = Susceptance at sending end

Y = Admittance

$L = P_s - P_r$ = Watts loss in transmission per phase

$\eta = P_r/P_s$ = Efficiency of transmission

$V = Q_s - Q_r$ = Reactive volt-amperes consumed in transmission

$$w = \frac{a}{b}\, e_r^2 \cos(\alpha - \beta); \quad x = \frac{a}{b}\, e_r^2 \sin(\alpha - \beta)$$

$$m = \frac{e_r^2}{b\, d \cos(\delta - \beta)} \quad r = \frac{e_r^2}{b\, d \sin(\delta - \beta)}$$

$$k = \frac{1}{2} \quad \frac{m\, e_r^2}{G_s\, b^2 m - e_r^2}$$

$$l = \frac{1}{2} \quad \frac{r\, e_r^2}{B_s\, b^2 r - e_r^2}$$

Subscript "s" denotes a sending-end quantity.

Subscript "r" denotes a receiving-end quantity.

A positive Q or V denotes leading reactive volt-amperes.

In designating vector quantities a capital letter denotes the vector while the small letter denotes the length of this vector.

*These remarks apply only to the case when the receiver-end voltage is constant.

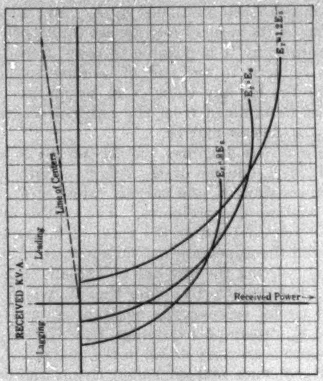


FIG. 11—EVANS AND SELS DIAGRAM DRAWN FOR RECEIVING END

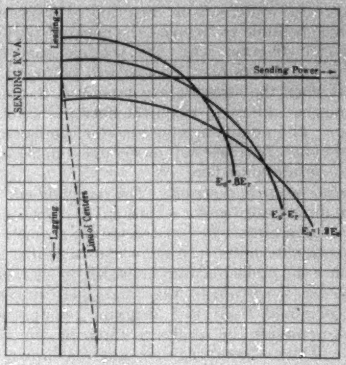


FIG. 12—EVANS AND SELS DIAGRAM DRAWN FOR SENDING END

TABLE II
CIRCLE COEFFICIENTS OF SENDING-END DIAGRAM

Kind of circle	P_s coordinate of center	Q_s coordinate of center	Radius	Remarks*
Receiver voltage	$y = \frac{d}{b} e_s^2 \cos(\delta - \beta)$	$z = \frac{d}{b} e_s^2 \sin(\delta - \beta)$	$\frac{e_s e_r}{b}$	Circles are concentric
Power loss in transmission	$y - n \cos^2 \beta$	$z + \frac{n}{2} (\sin 2\beta)$	$\sqrt{L n - y n + n^2 \cos^2 \beta}$	Circles are concentric
Receiver power factor	The receiving-end power factor circles can be drawn by a simple geometric construction that is much quicker than the analytic method. See text for graphical method.			
Receiver current	$e_s^2 \frac{c}{a} \cos(\gamma - \alpha)$ $= y - \frac{e_s^2}{a b} \cos(\beta + \alpha)$	$e_s^2 \frac{c}{a} \sin(\gamma - \alpha)$ $= z + \frac{e_s^2}{a b} \sin(\beta + \alpha)$	$\frac{e_s i_r}{a}$	Circles are concentric
Efficiency of transmission	$y - \frac{n}{2} (\cos 2\beta + \eta)$	$z + \frac{n}{2} \sin 2\beta$	$\sqrt{\frac{n^2}{4}\left[1 + \eta^2 + 4\eta\left(\frac{\cos 2\beta}{2} - \frac{y}{n}\right)\right]}$	Circles are not concentric
Receiver power	$y - \frac{n \cos 2\beta}{2}$	$z + \frac{n \sin 2\beta}{2}$	$\sqrt{\frac{n^2}{4} - n P_r}$	Center is usually off page
Receiver reactive volt-amperes	$+ y + \frac{s}{2} \sin 2\beta$	$+ z + \frac{s}{2} \cos 2\beta$	$\sqrt{\frac{s^2}{4} - s Q_r}$	Circles are concentric
Reactive volt-amperes consumed by line	$y + \frac{s}{2} \sin 2\beta$	$z - s \sin^2 \beta$	$\sqrt{s V - s z + s^2 \sin^2 \beta}$	Circles are concentric
Sending-end current	0	0	$e_s i_s$	Circles are concentric
Sending-end admittance	0	0	$Y_s e_s^2$	Circles are concentric
Receiver-end conductance	$y - t \cos 2\beta$	$z + t \sin 2\beta$	t	Circles are not concentric
Receiver-end susceptance	$y + u \sin 2\beta$	$z + u \cos 2\beta$	u	Circles are not concentric

Notation

$A = a \angle \alpha$
$B = b \angle \beta$
$C = c \angle \gamma$
$D = d \angle \delta$
} Network constants
P = Power in watts per phase
Q = Reactive volt-amperes per phase
E = Voltage to neutral
I = Current in each wire
G = Conductance to neutral
B_r = Susceptance at receiving end
Y = Admittance
$L = P_s - P_r$ = Watts loss in transmission per phase
$\eta = P_r / P_s$ = Efficiency of transmission
$V = Q_s - Q_r$ = Reactive volt-amperes consumed in transmission

$$y = \frac{d}{b} e_s^2 \cos(\delta - \beta); \quad z = -\frac{d}{b} e_s^2 \sin(\delta - \beta)$$

$$n = \frac{e_s^2}{a b \cos(\alpha - \beta)}; \; s = \frac{e_s^2}{a b \sin(\alpha - \beta)}$$

$$t = \frac{1}{2} \; \frac{n e_s^2}{G_r \, b^2 \, n + e_s^2}$$

$$u = \frac{1}{2} \; \frac{s e_s^2}{B_r \, b^2 s + e_s^2}$$

Subscript "s" denotes a sending-end quantity.
Subscript "r" denotes a receiving-end quantity.
A positive Q or V denotes leading reactive volt-amperes.
In designating vector quantities, a capital letter denotes the vector while the small letter denotes the length of this vector.

*These remarks apply only to the case when the sending-end voltage is constant.

circle diagrams that have been discussed all utilize a coordinate system of power coordinates, but this is not necessary. Thus in the receiving-end diagram one can calibrate the $X X'$ and $Y Y'$ axes in terms of in-phase and quadrature components of received current, or in terms of receiver conductance and susceptance, in place of the $P_r - Q_r$ calibrations. This is possible since, when the receiver voltage is constant, a certain received power is equivalent to a definite received in-phase current and a certain receiver conductance. When the coordinate system is in terms of receiver current components, all centers and radii computed with the aid of Table I must be divided by E_r before plotting on the current coordinates. Table I can be used in locating the circular loci on a system of receiver susceptance-conductance coordinates by dividing the results computed from this Table by E_r^2 before plotting.

The axes $X X'$ and $Y Y'$ of the sending-end circle diagram can be similarly calibrated in terms of in-phase and quadrature components of I_s, or in terms of generator end conductance and susceptance, instead of $P_s - Q_s$ calibrations. Table II can be used to compute circle centers and radii for these cases if one divides the computed results by E_s when current coordinates are used and by E_s^2 for admittance coordinates. After this division, the resulting quantities can be plotted directly on the coordinates.

The geometrical properties of the sending-end and receiving-end diagrams are the same whatever the kind of calibrations that are put on the coordinate axes.

Appendix A

DERIVATION OF THE CIRCLE COEFFICIENTS

The formulas incorporated in Tables I and II have been derived from equations (**1**) and (**2**). Since the formulas desired as expressed in terms of power and reactive power, it is necessary to rearrange equations (**1**) and (**2**) to introduce these quantities. The method of doing this is shown by the following example:

Rewrite equation (**1a**), giving the following:

$$I_r = \frac{E_s}{B} - \frac{A\,E_r}{B} \qquad \textbf{(3)}$$

If the angle by which the sending voltage leads the received voltage is Φ, equation (**3**) can be rewritten to give

$$I_r = \frac{e_s}{b}\,\underline{/\Phi - \beta} - \frac{a}{b}\,e_r\,\underline{/\alpha - \beta} \qquad \textbf{(4)}$$

The notation is that employed in Tables I and II. From (**4**)

Component of I_r in phase with E_r

$$= \frac{e_s}{b}\cos(\phi - \beta) - \frac{a}{b}\,e_r\cos(\alpha - \beta) \qquad \textbf{(5a)}$$

Component of I_r in quadrature with E_r

$$= \frac{e_s}{b}\sin(\phi - \beta) - \frac{a}{b}\,e_r\sin(\alpha - \beta) \qquad \textbf{(5b)}$$

Since power and reactive power are equal to the product of voltage and in-phase and quadrature components of current, respectively, equation (**5**) gives

$$P_r = \frac{e_r\,e_s}{b}\cos(\phi - \beta) - \frac{a}{b}\,e_r^2\cos(\alpha - \beta) \qquad \textbf{(6a)}$$

$$Q_r = \frac{e_r\,e_s}{b}\sin(\phi - \beta) - \frac{a}{b}\,e_r^2\sin(\alpha - \beta) \qquad \textbf{(6b)}$$

It is possible to handle equations (**1b**), (**2a**), and (**2b**) in the same way, with the result:

$$P_r = \frac{i_s\,e_r}{d}\cos(\psi - \delta) - \frac{c}{d}\,e_r^2\cos(\gamma - \delta) \qquad \textbf{(7a)}$$

$$Q_r = \frac{i_r\,e_r}{d}\sin(\psi - \delta) - \frac{c}{d}\,e_r^2\sin(\gamma - \delta) \qquad \textbf{(7b)}$$

$$P_s = \frac{d}{b}\,e_s^2\cos(\delta - \beta) - \frac{e_r\,e_s}{b}\cos(\beta + \Phi) \qquad \textbf{(8a)}$$

$$Q_s = \frac{d}{b}\,e_s^2\sin(\delta - \beta) + \frac{e_r\,e_s}{b}\sin(\beta + \Phi) \qquad \textbf{(8b)}$$

In these equations Φ is the angle by which the sending voltage leads the received voltage, and ψ is the angle by which the sending current leads the received voltage. From equations (**6**), (**7**), and (**8**) it is possible to derive all of the results given in Tables I and II, as well as others not given. In general these solutions are rather lengthy and tedious, although they involve no particular mathematical difficulties. As an example of the method of handling these equations, the derivation of the first line in Table I follows. Since the locus desired is that of generator voltage circles in terms of received voltage, received power, and received reactive power, this locus can be determined by simultaneously solving equations (**6a**) and (**6b**) to eliminate the undesired quantity Φ. Thus, by re-writing, we obtain:

$$P_r + \frac{a}{b}\,e_r^2\cos(\alpha - \beta) = \frac{e_r\,e_s}{b}\cos(\phi - \beta)$$

$$Q_r + \frac{a}{b}\,e_r^2\sin(\alpha - \beta) = \frac{e_r\,e_s}{b}\sin(\phi - \beta)$$

Squaring each of these equations separately and adding the two together gives the result

$$\left\{ P_r + \frac{a}{b}\,e_r^2\cos(\alpha - \beta) \right\}^2 + \left\{ Q_r + \frac{a}{b}\,e_r^2\sin(\alpha - \beta) \right\}^2$$

$$= \frac{e_r\,e_s}{b}\left\{ \sin^2(\phi - \beta) + \cos^2(\phi - \beta) \right\} = \frac{e_r\,e_s}{b}$$

This is the equation of a circle with a radius of $(e_s\, e_r/b)$ and a center at a point having the coordinates

$$P_r = -\frac{a}{b} e_r^2 \cos(\alpha - \beta)$$

$$Q_r = -\frac{a}{b} e_r^2 \sin(\alpha - \beta)$$

Appendix B

THE VECTOR REPRESENTATION OF VOLT-AMPERES

There is no established convention regarding the vector representation of reactive power which can be considered as the definitely correct method when viewed in relation to the usual vector diagram. Volt-amperes depend upon the relative phase of the voltage and current, and this introduces an ambiguity because there is no one correct axis of reference. If one takes the voltage vector as the standard, leading volt-amperes will be plotted in the positive direction because leading volt-amperes are due to current leading the voltage. On the other hand, if the current vector is selected as the reference axis, leading volt-amperes mean that the voltage is lagging behind the current and so must be plotted in a negative way. This makes it clear that reactive volt-amperes may be plotted in either way without violating any of the usual conventions of vector diagrams.

For any particular problem there is ordinarily a best method of representing reactive power, however, although this best method may be different for different problems. In the present paper the convention of considering all leading quantities as positive has been adopted because in vector diagrams of transmission line problems everything is ordinarily referred to the phase of a terminal voltage. Any other axis of reference leads to hopeless confusion, as an inspection of Figs. 4 and 5 or 9 and 10 makes clear. Considerations of convenience require that the vector basis of the circle diagram be based upon either E_r or E_s as the reference axis. This makes it *necessary* to consider leading volt-amperes as positive, and lagging volt-amperes as negative, as far as the graphical derivation of the diagram is concerned, and makes it desirable to do the same also in the mathematically derived diagram in order that the two methods of constructing the circle diagram will give identical results.

Unfortunately, the circle diagrams which have been based solely upon mathematical computations have all drawn leading quantities as negative. The diagrams incorporated in this paper accordingly do not look exactly the same as some similar diagrams published elsewhere. This difference cannot be avoided if the mathematical and geometrical methods of deriving the circle diagram are to be coordinated to give the same result. If the reader prefers to plot lagging quantities as positive and leading quantities as negative, he may do so by changing the signs of the reactive coordinates of the centers that have been computed with the aid of Tables I and II. When it comes to applying the geometrical checks to these reversed coordinates, it is necessary to make a certain number of similar changes in these rules which can be readily worked out if needed.

Appendix C

NUMERICAL EXAMPLE

In order to show in a concrete manner the method of utilizing Tables I and II, the following numerical computation is given. For example: Transmission network has the constants[4]:

$$A = 0.5\ \underline{/\ 8 \text{ deg.}}$$

$$B = 300\ \underline{/\ 80 \text{ deg.}}$$

$$C = 0.00242\ \underline{/\ 91 \text{ deg. } 52 \text{ min.}}$$

$$D = 0.6\ \underline{/\ 12 \text{ deg.}}$$

If $E_r = 50{,}000$, determine the data required to draw E_s and loss circles of the receiving-end circle diagram.

According to the notation in Table I, $a = 0.5$, $\alpha = 8$ deg., etc. From the formulas at the bottom of Table I the secondary constants w, x, and m can be determined, with the result

$$w = 1.288 \times 10^6$$

$$x = -3.962 \times 10^6$$

$$m = 37.08 \times 10^6$$

Substituting in the first line of Table I, it is evident that the P_r coordinate of the center of the E_s circles is -1.288 megawatts, while the Q_r coordinate of the center is $+3.962$ megawatts. The radius of any circle representing E_s volts at the generator is $(166.7\, E_s)$ "watts." Several such circles for different values of E_s are shown in Fig. 1.

To determine the loss circles it is necessary to use the line in Table I that concerns the loss of power in transmission. Using the values of w, x, and m determined above, the P_r coordinate of the center of these circles is -0.17 megawatts, and the Q_r coordinate of the center is -2.38 megawatts. The radius of a circle representing L watts lost in transmission is

$$[\,(37.08 \times 10^6)\, L - 6.30 \times 10^{12}\,]^{\frac{1}{2}} \text{ watts.}$$

Several L circles for different values of L are shown in Fig. 1.

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[4] The values of constants given are about right for a transmission line 300 or 400 miles long, equipped with terminal transformers.

The Space Charge that Surrounds a Conductor in Corona at 60 Cycles

BY JOSEPH S. CARROLL[1] and HARRIS J. RYAN[2]

Synopsis.—*An exploring potential wire was used to locate the radial position and to determine the potential of the space charge that surrounds a conductor in 60-cycle corona. The familiar concentric cylinder set-up was used. The potential of the exploring wire due to the applied voltage and its position in the electric field between the conductor and cylinder was maintained at zero so that the potential on the wire was due only to the presence of the space charge. The cyclic potentials of the exploring wire at various radial distances from the conductor in corona at the center of the cylinder were observed with an electrometer of low capacitance connected through a phase-shifting synchronous contactor to the exploring wire and through its own capacitance to ground. The location of the space charge and the potential it sets up in the electric field in the space surrounding the conductor in corona were thus determined. The paper contains four sections: I—The Problem and Results, II—Description of the Apparatus Used and the Method of Application, III—Discussion, IV—Conclusions.*

* * * * *

I. The Problem and Results

When 60-cycle voltage is applied to a conductor high enough in value to produce corona, a corresponding *space charge* is formed and maintained about the conductor.[3,4,5,6] The *existence and magnitude* of the space charge have been studied heretofore[4,5]. The present studies were made with a potential exploring wire *to determine the radial position* of the space charge with respect to the conductor about which it was formed. The conductor was mounted at the center of a half-in. wire-mesh cylinder (see Fig. 1). Parallel thereto a potential exploring wire was mounted at various radial distances from the conductor, between it and the cylinder. Sixty-cycle corona-forming voltage was applied between the conductor and cylinder. In order that the potential to ground of the potential exploring wire would be due to the presence of the space charge only, the following arrangement was adopted: The high-voltage source-circuit was grounded at a point having the same potential as that of the position that was occupied by the potential wire in the electric field between the conductor in corona and the cylinder. By this strategy an electrometer of low capacitance could be used to measure the potentials of the wire caused only by the presence of space charges. It was thus necessary that the neutral of the high-voltage source-secondary could be changed through all intermediate values from zero voltage-to-neutral on the conductor and maximum voltage-to-neutral on the cylinder to maximum voltage on the conductor and zero voltage on the cylinder. This was accomplished by means of a potentiometer. The isolated secondary of the high-voltage source-transformer was loaded with a water column resistance containing a grounded floating electrode. The electrode could be passed freely from either end to any desired point in the water column.

When a space charge is formed about a conductor in corona a few of the ions constituting the charge go astray and diffuse widely through the air. The voltage is alternating and so is the space charge. Following the application to the conductor of a corona-forming positive voltage crest, the charge is made up of positive ions. The next voltage crest will be negative; it will discharge the positive ions in the space charge and set up in their place a corresponding charge of negative ions. The net result of the diffusion of the alternating positive and negative charges is a small but very real rectifying effect, negative at the start of corona and positive when fully established[7]. This rectifying effect may be due to the differing mobilities[8] of positive and negative ions in the atmosphere.

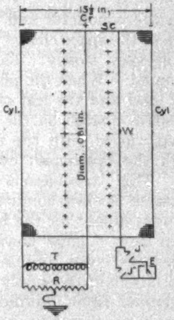

Fig. 1

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[1] Instructor and Graduate Student, Stanford University, California.

[2] Professor of Electrical Engineering, Stanford University, California.

[3] Franklin and McNutt, Lessons in Electricity and Magnetism, 1919 Ed., p. 148.

[4] Ryan and Henline, *Hysteresis Character of Corona Formation*, Trans. A. I. E. E., Vol. XLIII, p. 1118, October 1924.

[5] Hesselmeyer and Kostko, *On the Nature of Corona Loss*, A. I. E. E. Journal, Oct. 1925, p. 1068.

[6] Ryan and Carroll, *On the Nature of Corona Loss*, Discussion. A. I. E. E. Journal, Feb. 1926, p. 175.

Presented at the Pacific Coast Convention of the A. I. E. E., Salt Lake City, Utah, Sept. 6-9, 1926.

[7] J. B. Whitehead and T. Isshiki, Trans. A. I. E. E., Vol. XXXIX. Part II, p. 1091.

[8] R. A. Millikan, The *Electron*, 2nd Ed. (1924), p. 36.

To obtain the cyclic relation of the voltage to the corresponding potential of the wire as affected by the presence of the space charge, a gold leaf electrometer having a small capacitance was connected to the wire by means of two synchronous contactors. The phase of one contactor could be shifted conveniently throughout a complete cycle. It connected the wire to the leaf deflector of the electrometer. The phase of the other contactor was fixed at an instant whereat the voltage applied between the conductor and cylinder was zero and the wire due to the space charge was not changing, or was changing very slowly. Thus the rectifying effect was eliminated. When the phase-changing contactor only was used and the leaf of the electrometer grounded, a positive charge on the electrometer was accumulated that varied with the change of the air currents through the open-mesh cylinder and about the conductor. This troublesome correction was avoided by the use of the two contactors just specified. The 60-cycle power supply for the high-voltage transformer was taken from a large power supply system. As the work progressed some inconsistencies in results were traced to variations in the supply voltage crest and the air density factors. In an intermediate stage of the work the voltage was, therefore, controlled by crest corrected for barometric-temperature variations in lieu of effective values. Other inconsistencies developed and were found to be due to the shifting neutral in the water column and stray capacitances attached to the column and its connections. The remaining inconsistencies were found to be due to the corona-aging effect on the conductor whereby the voltage required to produce a given magnitude of space charge had to be increased. The voltage was increased slowly, of course, but nevertheless rapidly enough to require a corresponding control of the voltage so that in whatever radial position the potential exploring wire was being used and however long the time required to run through an entire series, the space charge had the same value throughout.

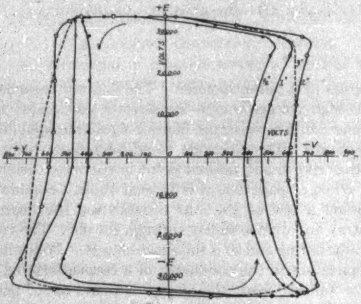

Fig. 2

Corona began when the crest voltage applied between the clean conductor and cylinder reached 32,800. Visual corona was well developed over the wire with fair uniformity by the application of 34,400 crest volts. The corresponding "reading" on the space charge control electrometer was "23." By this control reading "23," the applied voltage was held to produce a constant space charge while the observations for six voltage-potential $(E-V)$ cyclograms were obtained corresponding to six different radial positions of the potential exploring wire. The following radial distances of the potential wire from the surface of the conductor were used, 0.5, 1.0, 2.0, 3.0, 4.5, and 6.0 in. The corresponding cyclograms obtained with the potential wire at these radial positions are reproduced in Figs. 2 and 3.

The character of the cyclograms changed decidedly as the radial distance of the potential wire was less or greater than about 2.8 in. A study of such changes reveals the fact that they are due to the inevitable shift in the space charge radially outward caused by repulsion during the quarter-cycle that followed the last voltage crest and again radially inward caused by attraction during the second quarter-cycle in which the voltage developed the next crest having an opposite sign to that of the potential of the space charge formed by the preceding crest. From inherent dimensional relations, such shift in the radial position of the space charge affects the character of the cyclograms much more when the potential wire is inside the space charge and much less when outside. This change in the form-character of the voltage-potential cyclogram taken from the potential wire as the wire was shifted through the space charge was found to be as good a criterion for the location of the charge as the magnitude of the potential caused by the presence of the charge, which must be maximum when the potential exploring wire is located in its center.

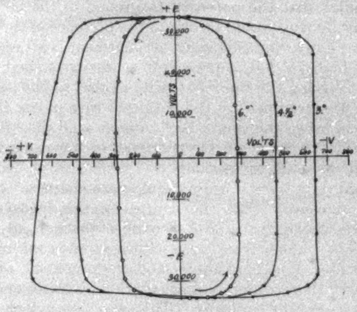

Fig. 3

If the space charge was placed in a fixed radial position as the voltage passes crest and falls below critical value, the voltage potentials due to the direct and reversed space charges would form approximately a rectangular cyclogram having straight parallel potential sides and slightly curvilinear voltage ends. Such was the case for the cyclograms taken from the potential wire when placed outside the space charge, while those taken from the wire mounted inside the space charge had decidedly concave sides due to the outward and inward movement of the space charge.

The understanding hereof is facilitated by the study of the curves in Fig. 4. Curve $I\,V^1$ was located by using the radial positions of the potential wire as abscissas and the corresponding observed potentials V_Q, due to the presence of the alternating space charge, as ordinates, occurring at the instant when the negative voltage crest has been completed. This curve locates a maximum for potentials for the wire placed at a radial distance from the conductor of 2.67 in. As a criterion for understanding the factors that determine the character of Curve $I\,V^1$, Curve $I\,V^2$ was located by commonly adopted theory on the assumption that the entire space charge is uniformly distributed about the conductor at the radial distance of 2.67 in.

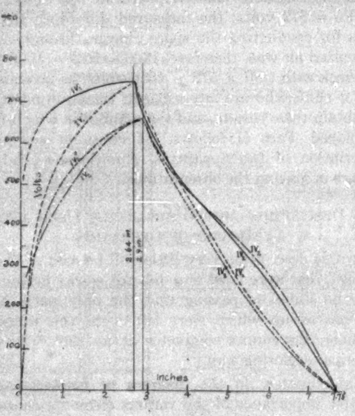

Fig. 4

At positions within the space charge, the potentials were observed to be well above those obtained by calculation on the assumption that this region was free of ions. The differences may be taken as a measure of additional potentials due to the presence of negative charges in such space. The direct inference is that not all of the space charge is located at the radial distance of 2.67 in., and that a small portion thereof is distributed through the intervening space.

At corresponding positions without the space charge, there is a close agreement between potentials by measurement and by calculation at first as the radial distance of the potential wire extends beyond 2.67 in. Beyond four in. the values of the observed potentials due to the presence of the space charge are higher than those by computation. The difference attains a maximum well out toward the surface of the mesh cylinder, let us say at a radial distance of about 6.5 in. This difference clearly indicates that while most of the charge occupies the cylindrical location at a radius of 2.67 in., some of it located in a broad zone at the 6.5-in. central radial position.

A quarter of a cycle, or 1/240 of a second later, the voltage was zero and curve $I\,V_2$ was correspondingly located by the $E - V_Q$ values taken from the cyclograms at that phase. It will be noted that the maximum potential in the charge had dropped from 750 to 666 volts and that the central position of the charge had been repelled from the radial position of 2.67 to 2.90 in. Curve $I\,V_2''$ was computed as the criterion to accompany Curve $I\,V_2$ on the assumption that all of the space charge was located at the radial distance of 2.9 in. from the conductor. A comparison of the two curves, the one observed and the other calculated for the phase whereat the voltage is passing through zero from minus to plus, reveals the fact that the observed potentials through the region within the main space charge at the radial distance of 2.9 in. are now less everywhere than the corresponding values by computation. The inference is that only a small distributed vagrant charge remains in such space and that its sign is positive whereas the sign of such vagrant charge was negative as the voltage was falling from a negative crest.

Radially beyond the 2.9-in. position, corresponding observed and calculated values have scarcely been changed by the migration of the space charge. All of this is in complete agreement with the characteristic difference in the cyclograms taken from the potential wire when located within and without the space charge. The curves show that the cyclograms should have the concave sides when observed within the space charge and straight parallel sides when observed without such charge.

It was shown by Hesselmeyer and Kostko[5] that voltage-charge $(E - Q)$ cyclograms obtained for a line in corona change only slightly in character as the frequency is changed from 60 cycles upwards to 120 and downwards to 10.5 cycles. Substantially the same must be the case for voltage-space charge potentials. It follows, therefore, that in all ordinary study of these phenomena cyclograms, giving the direct relation of voltage and potential due to space charge with time omitted is particularly helpful. However, there are many who are not accustomed to the study of these phenomena without their relation to the times at which they take place. To assist them and for those considerations wherein the time factor is of definite importance, all of the values occurring in the cyclograms are repeated in waves reproduced in Fig. 8, using times for abscissas and voltages or potentials as ordinates. The radial distances of the potential exploring wire are marked on the waves to facilitate their comparison with corresponding cyclograms.

A further study of the voltage-potential cyclograms through the second quarter-cycle after the voltage has passed a negative crest similar to that which was made with the contents of Fig. 4 reveals the fact that at the close of the first half-cycle, just before active ionization is resumed and before the process of reversing the negative space charge has begun, the potentials at the several radial positions within the space charge have again increased and returned nearly to their original values obtained at the beginning of the cycle. There is, however, much less difference between the potentials by measurement and by calculation, showing that the vagrant charges of opposite signs are nearly equal and generally distributed through the region with the space charge. So much for the fact. One may also infer that many positive and negative ions which are vagrant within this region have disappeared by recombination.

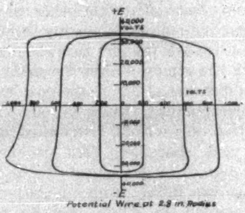

FIG. 5

To check this understanding as to the cause of the change in the form of the voltage-potential ($E - V_Q$) cyclograms taken from the exploring wire when located within and without the main body of the space charge, the potential wire was kept at the 2.8-in. radial distance from the conductor, while three cyclograms were obtained each at a different value of corona-forming voltage. The values were so chosen that one was low enough to place the space charge between the wire and the conductor, another that would place it in the same radial position as the wire, and the third voltage high enough to place the charge well beyond the wire. The first or inner cyclogram thus obtained (Fig. 5) had the flat parallel sides indicating as anticipated that the exploring wire was located radially beyond the space charge; the second or middle cyclogram in Fig. 5 showed but slight departure from that of the preceding rectangular cyclogram; the third or outer cyclogram, Fig. 5, showed a well developed distortion of the rectangular form;—each, therefore, showed characteristics that had been anticipated.

With the potential wire left in the 2.8-in. radial position, the relation between the excess of crest over critical voltage, $E - E_0$, taken at the phase at which the value of the voltage E was zero and the potential of the space charge as given by the potential wire, was found to be *linear* between the limits used in this study, *viz.*, $E_0 = 32{,}800$, and $E = 35{,}200$ volts crest. The relation is given by the equation:

$$(E - E_0) = 2.80\, V_Q \tag{1}$$

Since V_Q is the value of the voltage to which the condenser between the space charge and the cylinder was charged as determined by the exploring wire, it follows that the drop in voltage through the ionized air when it was conducting the space charge to such condenser was

$$(E - E_0) - V_Q = 1.80\, V_Q \tag{2}$$

For example, when the value of E was 34,400, $(E - E_0) = 1600$, the space charge was located 2.8 in. from the conductor and its potential was observed to be $V_Q = 572$ volts; the measured difference that applies for conducting the space charge through 2.8 in. of ionized air was, therefore, $1600 - 572 = 1028$ volts, to check with $1.80 \times 572 = 1030$ volts, as given in (2).

For those who are interested in the equipment used to obtain these results, and the manner in which it was employed, Part II follows. It contains a complete description of the apparatus, connections, and procedure in making the observations.

II. DESCRIPTION OF THE APPARATUS USED AND THE METHOD OF OPERATION

In this description very little will be said of the "99 things" that were tried and did not work; however, it may be stated in passing that the only parts of the original set-up which were left were the wire-mesh cylinder, the copper conductor at its center, and the potential exploring wire.

Unfortunately, in the diagram of connections the relative importance of the various elements cannot be indicated by the amount of space they occupy on the paper. The cylinder of ½-in. mesh was 15.5 in. in diameter and 15 ft. long. It was mounted vertically, the lower end being four ft. above the floor of the laboratory. The No. 12 B. & S. G. copper conductor stretched tightly at the center of the cylinder was supported by hard rubber rods across the ends of the cylinder. Vibrations in the conductor were suppressed by means of silk thread guys in four directions to the cylinder at two points along its length. Both potential wires shown in the diagram were No. 20 B. & S. G. copper. The end supports were hard rubber. To keep the wires free from vibrations, ¼-in. hard rubber rods were extended, through wood blocks on the cylinder, radially inward until they just touched the potential wire.

The source of high voltage was a 60-kv. transformer, the secondary of which had no permanent ground. A high-resistance water column potentiometer was connected across this secondary in such a manner that the ground could be made any place between the two high-voltage terminals of the transformer. This potentiometer consisted of two sections of ¾-in. garden hose each about 12 ft. long. These were mounted vertically on a steel tower just outside the laboratory. The total resistance across the secondary of the transformer remains constant at about 250,000 ohms. With this arrangement the exploring potential wire can always be kept at ground potential due to its position, *i. e.*, the voltage across the right side of the potentiometer is made equal to the voltage between the exploring wire and cylinder; likewise the voltage across the left side is made equal to the voltage between the exploring wire and the conductor at the center of the cylinder. Since the potentiometer is grounded at the center, the exploring wire must therefore be at ground potential due to its position when there is no corona present on the center wire.

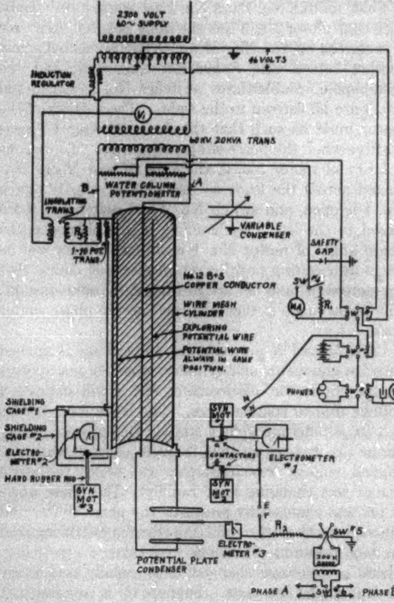

FIG. 6—CONNECTIONS FOR SPACE CHARGE STUDY

The correct position of the ground on the potentiometer is found as follows, after the exploring wire has been moved to the desired position. No one of the three synchronous contactor motors is running, contact (*a*) is left closed, (*b*) is opened, terminals *E* and *F* are connected together and the quick-acting, double-pole double-throw switches No. 5 closed either up or down, also switch No. 6 is open. The voltage on the 60-kv. transformer is slowly raised from zero by means of the tap transformer and induction regulator. Electrometer No. 1 will then indicate the voltage between the exploring wire and ground. The ground on the potentiometer must then be shifted until the electrometer shows zero voltage. Since the electrometer is not sufficiently sensitive at low voltages, a voltage is added to the circuit by closing switch No. 6 to phase *A*. This voltage is in phase with that applied between the No. 12 conductor and cylinder. The voltage used for this purpose is 300, which puts the electrometer leaf up into a sensitive position. If the potentiometer ground is not in the correct position, the exploring wire will have a voltage above ground that will increase the electrometer reading with switch No. 5 in one position and decrease it in the other position. The proper place for the ground on the potentiometer is that which gives the same electrometer reading for both positions of switch No. 5. With a little care, this ground can be set to within one volt in 25,000.

Since the position of this ground must be set below a voltage that will start corona on the conductor, some means must be established whereby it may be known when the position is correct at the operating voltage when the conductor is on corona. An arrangement must also be made to keep watch of this ground to see that it does not change on account of air bubbles in the water column, obstructions in the nozzles, shifting of the chain, or any other cause. To accomplish this, the potential plate condenser was built and connected in parallel with the conductor and cylinder. The condenser consisted of two cast iron disks 12 in. in diameter with amply rounded edges mounted parallel to each other about seven in. apart with a 12-in. aluminum disk as a potential plate ⅛ in. thick arranged to float between the two iron disks by means of a hard rubber rod and screw adjustment. This potential plate is connected to the leaf of electrometer No. 3, the case being connected to ground through the 300-volt transformer. When the position of the ground on the potentiometer is correct as shown by electrometer No. 1, the potential plate is raised or lowered until electrometer No. 3 shows no change on reversing switch No. 5. This means that the potential wire and the potential plate occupy the same electrical positions. Since there is no corona in the vicinity of this potential plate up to the highest voltage used, the correct position of the ground can be maintained by observing electrometer No. 3. This potential plate must obviously be reset each time the exploring wire is moved to another position.

There is another adjustment that must be made before the exploring wire is exactly at ground potential. If the charging current to ground from the apparatus and leads on one side of the high voltage transformer is not equal to the charging current to ground on the other side, the difference between these currents will flow through one side of the potentiometer, thus shifting the phase of the voltage across this side with respect to the other, and making it impossible to find the correct position of the ground. To overcome this, a variable condenser must be connected on the side of the transformer furnishing the least charging current to ground. The test for the right amount of capacitance to be added is made by throwing switch No. 6 to phase *B*, which is in quadrature with respect to phase *A*. The variable condenser is then adjusted until the reading of electrometer No. 1 does not change on reversing switch No. 5. This adjustment must also be made each time the position of the exploring wire is changed, but when the balance is once made it will "stay put" and does not have to be watched. It is necessary when making this condenser balance to follow it up with the correct position of the potentiometer ground by occasionally throwing switch No. 6 to phase *A*. The exploring wire must truly be at ground potential when there is no change in the reading of electrometer No. 1 for phase *A* and phase *B* when switch No. 5 is reversed.

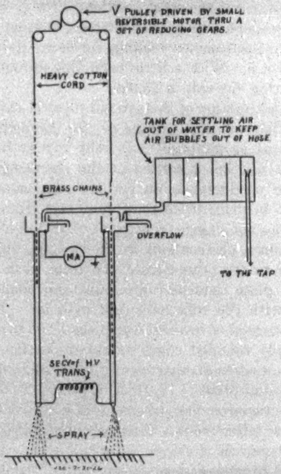
V PULLEY DRIVEN BY SMALL REVERSIBLE MOTOR THRU A SET OF REDUCING GEARS.
HEAVY COTTON CORD
TANK FOR SETTLING AIR OUT OF WATER TO KEEP AIR BUBBLES OUT OF HOSE
BRASS CHAINS
OVERFLOW
MA
+
TO THE TAP
SEC'Y of HV TRANS.
SPRAY

FIG. 7—HIGH-VOLTAGE POTENTIOMETER

With the exploring wire at ground potential due to its position it will show no voltage until corona appears on the conductor; then the voltage it indicates on electrometer No. 1 will be that set up by the charge sent out by corona. The effective value of this voltage can be measured by disconnecting *E* from *F* and grounding *E*. However, we are not only interested in the amount of this voltage but also in its wave form. The latter was determined by means of synchronous contactors and the gold leaf electrometer No. 1.

The three synchronous motors shown in the diagram were four-pole, 1/4-h. p. induction motors, the rotors of which had been slotted for salient poles. The motors were mounted vertically to eliminate end play, ball bearings being used for the upper bearing. The contactors consisted of a hardened steel pin and thin laminated steel spring, contact being made on the center lamination only. Synchronous motor No. 1 was supplied with voltage from a phase shifter consisting of a wound-rotor two-phase induction motor. The phase angle was read on a scale fastened to the frame with a pointer on the rotor shaft. Motors No. 2 and No. 3 were driven from a voltage supply having a fixed phase angle.

First, motors No. 1 and No. 2 are started up. To be sure that motor No. 1 has the same polarity each time it is started up, the clips *M* and *N* are connected across contact (*a*); *E* and *F* are not connected; and the double-pole, double-throw switches No. 1, No. 2 and No. 3 are all thrown to the right. The polarity of the motor must be such that the d-c. voltmeter V_2 reads positive when the phase shifter is on zero. To find the polarity of motor No. 2, the clips *M* and *N* are connected across the leads to electrometer No. 1, switch No. 1 is open, and switch No. 2 and switch No. 3 are closed to the left. By listening in the phones as the phase angle of motor No. 1 is shifted, a click will be heard at a certain position which means of course that contactors (*a*) and (*b*) are hitting at the same instant. The phase angle is then observed on the phase shifter scale.

Clips *M* and *N* are removed and voltage is applied between conductor and cylinder; when corona appears on the conductor, electrometer No. 1 will indicate a voltage due to space charge. The wave form of this voltage is determined by a step-by-step process. Contactor (*b*) was set so that it made contact when the applied voltage was zero; this was where the space charge was changing least rapidly. The phase angle of (*a*) was changed by means of the phase shifter and the angle read on the scale, the electrometer being read for each position. Readings were taken every 10 deg. where the voltage changed most rapidly and every 20 deg. on the flat parts. Suppose (*b*) is contacting at the point (*x*) on the curve taken at six in., and the phase shifter is set on zero; the difference in voltage of these two points on the wave, then, is 330 volts, which is read on the electrometer. The contactor (*a*) is set, say 10 deg. later, by means of the phase shifter and the voltage is found to be 420 and so on. When we get to, say, 240 deg., the voltage on the electrometer is practically zero, so that to get this half of the wave, motor No. 2 is slipped a pole, making the contactor (*b*) shift 180 deg. to (*y*), and then when the contactor (*a*) is on 240 deg. the voltage on the electrometer is 550.

In order to tie-in all the curves taken this way with the voltage wave, the exploring wire is disconnected from the contactors and the clips *M* and *N* are connected across contactor (*a*). Switch No. 4 is thrown to the right, switch No. 1 left, switch No. 2 right, and switch No. 3 left. The phase shifter is then adjusted until there is no sound in the phones, which means that (*a*) is making contact when the voltage across R_1 is zero. The angle is then read on the phase shifter scale.

The potential wire at the left of the conductor, Fig. 6, was used in controlling the voltage. The distance between the conductor and this wire was six in. Motor No. 3 drove a contactor in which the two contacts connected to electrometer No. 2 were 180 electrical deg. apart, both making contact very nearly the instant when the voltage between the conductor and cylinder is zero. This charged the electrometer with the voltage due to the space charge, the plus crest on one side of the electrometer and the minus crest on the other side. Since the voltage due to position amounted to considerably more than the space charge voltage, it would be necessary to keep the contactors hitting exactly when the voltage due to position was zero. Practically, this could not be done with any degree of assurance. To overcome this difficulty, the voltage of this potential wire due to position was bucked out by surrounding the electrometer with a wire screen cage, No. 1, which was raised to the potential of the wire due to position by means of the transformers as shown in the diagram. R_3 was connected across the secondary of the insulating transformer to compensate for the shift in phase angle in the 60-kv. transformer caused by the water column resistance load. To prevent any variation in shielding as the ground on the water column potentiometer was shifted, cage No. 1 was surrounded by a wire screen cage No. 2 which was connected to the wire-mesh cylinder. Electrometer No. 3 was read by means of a telescope and the voltage held so that it read "23" throughout all the runs which gave the data for the curves in Fig. 8.

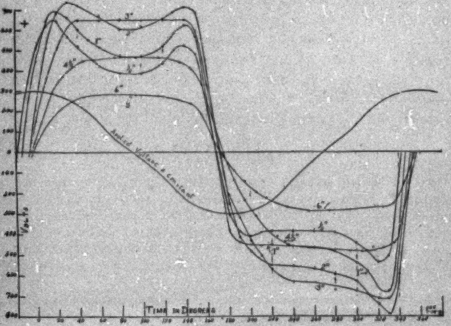

FIG. 8

III. DISCUSSION

From the results of the preceding and present studies, it is now known that corona at all ordinary voltage wave forms, frequencies, and air densities is due essentially to the formation of mixed ions in the air near the conductor where the electric intensity exceeds a certain critical value (30 kv. per cm., or 26.4×10^{-10} coulombs per sq. cm. in air at unit density, $\delta = 1$, which occurs at a temperature of 25 deg. cent. and a barometer of 76 cm.). The mixed ions include only atoms that have lost or gained electrons and have thus become positive or negative ions of low mobilities, 1.36 and 1.83 cm. per sec. per volt per cm., and *free electrons.* The last function as negative ions having mobilities hundreds of times greater and which, when the electric intensity falls to about *one-thirtieth* of that which caused ionization, attach themselves to neutral atoms to form negative ions with their corresponding low mobilities. This electric intensity at which free electrons form negative ions occurred in the present studies at about 1000 volts per cm., or 0.88×10^{-10} coulombs per sq. cm., the air density being near unity. Much experimental work will have to be done to determine the limits through which the electric intensity varies at which the free electrons combine with neutral atoms to form slow moving negative ions.

During copious ionization near the conductor the ions that carry charges having signs unlike the charge on the conductor due to the applied voltage are drawn to contact with the conductor and thus discharged; the ions of the same sign are repelled to a radial distance at which the electric intensity has fallen to the value whereat free electrons can no longer exist, as such, and must form negative ions of low mobility instead. Further details of the action are not as yet clearly defined and much further study is necessary. However, the fact is now known that the presence of free electrons in the midst of mixed ions, in aggregate effect, greatly increases the mobility of all the ions, and when that region of the field is reached in which the free electrons combine with neutral atoms to form negative ions, both positive and negative ions are actuated only by the low mobilities above specified. So low are such mobilities that the further effects they produce at 60 cycles are nearly the same as though they did not move appreciably during the remainder of the half-cycle.

Thus it comes about that electric stress in the region surrounding the conductor, between it and the nearly fixed space charge, can be increased but a small amount, if at all, by raising the applied voltage above the critical value of the voltage. The increase in voltage, $E-E_0$, is consumed in two ways: (1) by the ionic conduction that delivers the space charge at the radial distance above specified, and (2) by the space charge itself. While this understanding of the nature of corona is now clearly indicated by the results of many observers to date, much experimental work will have to be done to know the amount and position of the space charge under most circumstances in order that corresponding formulas can be derived and used reliably for practical purposes.

Something should be said in conclusion in regard to the background of the subject of this paper. There may be many causes of contributing factors that bring about the occasional unexpected flashovers of high voltage insulators that function in air, but among them must always be remembered three factors as follows:

I. Surface conductivity,
II. Surface charger,[9]
III. Space charge.

Surface conduction is a chaotic factor that nevertheless is much better known and understood than the other two. Nishi[9] found that when corona in air is present near an insulator, the surface of the insulator dielectric becomes heavily charged even when the voltage is alternating. There can be little doubt that the space and surface charges are related and that the nature of such relation qualitatively and quantitatively should be known. It is altogether likely that the zone of a comparatively heavy space charge standing well out around a transmission line conductor is a factor that will have to be reckoned with in various practical undertakings such, for example, as the coupling of carrier currents to power transmission lines. It may be that knowledge of the space charges due to the corona on insulator hardware, of surface charge, and of surface leakage will go far toward an understanding of the nature of things that requires the magnitude of tower clearances that practise is now slowly but surely discovering to be necessary.

The dimensional manner for many differing conditions in which the radial distance is related to the crest voltage in the production of space charge, and to the electric intensity of the field wherein the space charge is lodged, should be determined as soon as practicable. In the present study such radial distance was found to be at the average rate of 8.3 kv. (s. w. rms.) per in. and the corresponding electric intensity at the location of the space charge was found to be one kv. per cm. Our reconnaissance has revealed much evidence that the radial distance of the space charge from the conductor, cable, point or other high voltage electrode, is approximately equal to the distance through a needle-gap that the corresponding voltage will discharge and, therefore, approximately at the distance-rate of one in. per 10 kv. (s. w. rms.). It must be remembered, however, that corona phenomena have always a strong tendency toward chaotic behavior and that few things can be taken for granted and that most of them must be determined by exact orderly procedure and measurement.

Until the values thus indicated have been reliably determined, it would be premature to write formulas for the power lost in corona due to the boundaries determined by the space charge and its capacitance. The temptation to do so is there, of course, because the energy in such charge is drawn from the source, is never returned and is, therefore, a measure of the loss produced by corona when due allowance has been made for the ionic conduction by which the space charge was placed in position.

IV. CONCLUSIONS

1. A conductor in corona is surrounded by a free mobile space charge of the same sign as that on the conductor during the preceding voltage crest.

2. The sign of the space charge is reversed by the copious ionization that is produced during each succeeding voltage crest.

3. The mobility of the space charge is so low during interval between voltage crests that it behaves almost as though it were fixed in space about the conductor in corona.

4. The mobility of the mixed ions, positive, negative and free electrons, that reverse the space charge during the voltage crests is much higher than the mobility of the space charge during the interval between voltage crests.

5. For the most part, the space charge has an orderly relation in amount and space position to the voltage applied to the conductor and its dimensions and to the energy lost per cycle in corona at all ordinary frequencies used in the power industry.

Discussion

H. E. Mendenhall: I should like to ask Mr. Carroll one question. Two years ago, Professor Sorenson discussed Doctor Ryan's paper and he had a theory that this corona loss might be caused by the positive ions not traveling as far as the negative ions. I should like to ask if they were able to determine whether or not the space charge is out further when it is negative than it is when positive?

F. O. McMillan: On account of the negative corona forming in tufts on the conductor as shown with the stroboscope by observation of a conductor in a-c. corona, one would expect that the space charge established during the negative half-cycle would also be tufted and not uniformly distributed along the conductor. If this is true then a parallel potential exploring wire such as the one used by Doctor Ryan and Mr. Carroll, would indicate a mean negative space-charge potential smaller in value and located nearer the conductor than the actual tufted space charge. Do the authors consider the diffusion of the space charge to be so great that the distribution of the negative charge is uniform along the conductor?

C. H. Willis (communicated after adjournment): The density of the space charge and the distortion which it produces on the electrostatic field of the conductors, is an exceedingly difficult problem. The method developed by the authors is a very ingenuous attack upon this problem. I think, however, that their method is subject to the following criticisms.

First, the authors assume that their test wire attains the potential of the surrounding space when bombarded by a cloud of ions just as it does when the surrounding air is an almost perfect insulator. It has been found in a number of instances that a test electrode of this type in the presence of an ionic stream accumulates a charge which distorts the electric field around the electrode. This may be a source of considerable error here.

Second, the authors assume that their test electrode follows the potential of the surrounding space with no time lag and therefore in phase with the electric field of the space around the test wire. If this test wire has appreciable capacity to ground or leakage, this assumption would not be justified, since the test wire at times is several hundred volts above ground.

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[9] T. Nishi, *Surface Charges on High Voltage Insulators*, A. I. E. E. JOURNAL, November, 1920.

In Fig. 4 the authors show that one curve is "located by commonly accepted theory, on the assumption that the entire space charge is uniformly distributed about the conductor at the radial distance, 2.67 in.." In that case, this curve should be a portion of a rectangular hyperbola, but the curves given do not appear to be such.

It seems quite surprising that the maximum shown in the curves of Fig. 4 should shift only 0.23 in. in the quarter period following the crest of the voltage wave, and this is particularly unexpected when the center of the space charge layer is found 2.67 in. from the wire almost immediately upon formation; that is, at the crest of the wave.

In some work done at Johns Hopkins University, which I hope will be published soon, the mobility of the ions in the corona space charge was found to be around 7 cm./sec. per volt/cm. In view of this mobility of the ions it seems much more probable that the maximum found in the curves of Fig. 4 represents the inner boundary of the space-charge layer at the instant the impressed voltage passes through zero, or at the instant when the space-charge layer is farthest from the axis of the corona wire. This would indicate that the free charge picked up by the test electrode is the principal factor in the results of this paper.

Jos. S. Carroll and Harris J. Ryan: Replying to Mr. Mendenhall's question: Were we able to determine whether the space charge is or is not further out when negative than when positive? Our results indicate that the difference could not have been very great. We have charted, in rectangular coordinates, the potential-radius relations by means of the requisite values taken from the curves of Fig. 8 at those phases of the cycle wherein the positive and negative voltage crests were just beginning or ending and again when the voltage was zero. As stated in our paper, the point of discontinuity in the potential-radius relation was taken in each case as determining the radial distance to the center of the cylindrical space charge. The results thus obtained are given in the following table:

SPACE CHARGE COMPLETE

Phase (deg.)	Radius (in.)	Condition
40	2.5	+ initial
90	2.9	+ central
140	2.6	+ final
220	2.8	− initial
270	3.0	− central
320	2.5	− final

We hardly feel that much significance can be attached to these radial differences more especially because of the presence of the widely distributed positive space charge caused by the rectifying effect of the conductor and cylinder. When the conductor was positive, the charge due to rectification was repelled by the conductor and the newly formed positive space charge. Such action may well have diminished the radius at the 40-deg. instant with respect to what it might have been in the absence of such charge due to rectification. Again, the converse should be true at the 220-deg. instant when the negative voltage crest had just completed the corresponding negative space charge. Such space charge may have been attracted by the positive charge due to rectification and its radius thus been made greater than would have been the case if the charge due to rectification had not been present. The action being cyclic, one would expect the charge due to rectification to be in its central position when the voltage was zero and that at such instant it would have little or no effect upon the value of the space-charge radius. The corresponding radii for the positive and negative space charges at the zero-voltage instants of 90 and 270 deg. were 2.9 and 3.0 in. respectively.

Mr. McMillan asks: "Do the authors consider the diffusion of the negative space charge to be so great that the distribution of the negative space charge is uniform along the conductor?" For the conductor and voltage used—yes. The voltage used was substantially sufficient to maintain a fixed brush pattern and therefore sufficient to sustain full corona. Had the voltage been just sufficient to begin the formation of one or more brushes in an unstable pattern, the brushes might have been too far apart to form a complete and reasonably complete space-charge cylinder. In that event the wire would have been a form of potential exploring electrode that would have been unsuited for the purpose. When the brushes are too far apart to maintain continuous space charges along the conductor the study would have to be applied to individual brushes and the radial distances to the space charges they set up. When such studies for individual brushes are made, it seems reasonable to us to expect that the radial distance will be found very much the same as for fixed brush patterns. We base this expectation on the fact that approximately 10,000 volts per in., sine-wave effective, are required to produce flashovers, alike for parallel conductors of any size that develop local or full corona in advance of flashover and between pointed conductors in opposing alinement.

Mr. Willis apparently has not made a study of the related papers that preceded the present one[4,5]. In those papers the existence of the space charge that surrounds a conductor in corona was authenticated. It was shown to be the major factor in determining the value of the power loss due to corona. Its cyclic character was observed in a large number of cases. Indirectly its radial distance from the conductor in corona was determined.

When the use of the potential exploring wire was begun, it was largely for the purpose of checking the understanding already gained of the position and cyclic character of the space charge. The difficulties stated by Mr. Willis were anticipated when the work was planned. Some of them were encountered early in the work and the strategy was modified, step by step, until consistent results that checked reasonably well with the known facts were obtained. By test, the insulation of the exploring potential wire-assembly was maintained amply high. By trial in the final set-up, the capacitance of that part of the assembly that extended beyond the corona-forming field was amply low. The conductor and cylinder acted as a limited rectifier and thus constantly maintained a widely diffused positive space charge. To see if the positive charge collecting on the exploring wire would alter the space-charge wave a negative potential was maintained on the exploring wire by means of a kenotron and a very high resistance. The space-charge curve showed only a slight change in form. Various other tests were made, all of which showed that the exploring wire was giving the potential of the space it was in sufficiently accurately for the work.

Regarding the potential curve in Fig. 4 "located by commonly accepted theory," Mr. Willis says "this curve should be a portion of a rectangular hyperbola but the curves given do not appear to be such." Of course not. Charges on concentric cylinders give rise to electrical intensity—radius relations that mathematically speaking are *hyperbolic*, and to potential—radius relations that are logarithmic. As clearly stated, the V series of calculated curves in Fig. 4 are potential curves. They are therefore logarithmic and not hyperbolic.

In conclusion Mr. Willis is apparently convinced that he has found that "the mobility of the ions in the corona space charge was around 7 cm./sec. per volt/cm." and on the basis of that, argues that "the free charge picked up by the test electrode is the principal factor in the results of this (our) paper." To that we reply that the *E-Q* cyclogram is an established fact. It asserts that there must be more than one mobility of the ions in the cyclic production of the space charge,—perhaps a number of mobilities.

Vacuum Switching Experiments
at California Institute of Technology

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Fellow, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*Successful experiments in switching or breaking a circuit in a high vacuum have been made at the California Institute of Technology. This paper is a report on three sets of these experiments which extended over a period of three years. The conclusions drawn from the experiments may be summed up in the statement that vacuum breakers of laboratory type have been successful in breaking circuits and offer a possible solution of the circuit-breaker problem. The results show that switching in vacuum affords the advantages of no pitting of contacts, quick break, the arc always going out on the first half-cycle, small voltage rise across the switch, and small distance of travel necessary for the switch blades.*

Making the vacuum switch practical calls for a solution of the problem of making commercial apparatus with vacuum-tight joints, and the elimination of the use of liquid air with the vacuum pump.

EXPERIMENTS on breaking an electrical circuit in a high vacuum have been made during the last three years at the California Institute of Technology in connection with the study of switching high-voltage, high-power circuits. These experiments were undertaken as a result of the well-known limitations of oil circuit breakers. A large number of tests was made on high-vacuum breakers of laboratory type. Some very promising results were obtained in interrupting large currents.

When these experiments were suggested, the question immediately presenting itself was: Will the vacuum maintain itself at the time the arc is formed between the separating metallic parts of an opening switch?

This doubt was quite generally substantiated by the commonly recognized theory of the electric arc,[1] *viz.*, that the maintenance of an arc is dependent upon the giving out of thermions from hot spots on the electrodes between which the arc is formed, with the attendant vaporization of the metal. If this were true, a large current could not be interrupted in a vacuum because the formation of even a small amount of gas would reduce the vacuum and cause it to become a conducting vacuum rather than an insulating vacuum.

The fact of the matter, however, is that if the vacuum is sufficiently high and all adsorbed gases have been removed from the metal electrodes, very large currents can be broken without formation of enough vapor to maintain an arc.

Dr. R. A. Millikan has shown[2] that, with cold electrodes suitably prepared, millions of volts of potential gradient are required to obtain discharges of any kind between metal surfaces. He has also worked out with much care the conditions necessary for denuding metal surfaces of gases and preventing the impairment of the vacuum through the evolution of gases. A. Janitzky[3] also has reported experiments showing that currents will not flow across the space between cold electrodes in a vacuum provided the electrodes have been completely outgassed.

According to the older theory, it would seem that considerable vapor might be formed in the vacuum on breaking a circuit. P. Charpentier[4] has given the following equation:

$$W = 0.07\,E\,I\,t$$

as the equation for the energy to be dissipated in an oil switch at the time of opening. In this equation, E = voltage, I = current, and t = the time in seconds between the initial separation of the switch contacts and the complete extinguishing of the arc. Charpentier's experiments, and also those made by Swiss engineers in 1915 and 1916, indicate that this energy is used in vaporizing oil at the rate of 46.5 cu. cm. per kw-sec. Some of the tests made on oil switches show the vaporization of smaller amounts of oil per kw-sec. and also show power factors of less than 0.07 across the switch at the time of interruption. Applying Charpentier's equation to a single-pole switch opening a 15,000-volt, 100-ampere, 50-cycle circuit, we find that the switch must dissipate 1.05 kw-sec. if it opens on the first half-cycle. Assuming as an extreme case, for the vacuum switch, all of this energy available to vaporize copper at the switch blades, we find that it would vaporize approximately one-fifth gram. This amount of copper turned into vapor would reduce an insulating vacuum in a container of considerable size to a vacuum which would be conducting for 15,000 volts applied between electrodes extending into the container.

However, the later theories to which reference has been made indicate that such an amount of vapor will not be formed provided the vacuum is high and the electrodes are free from gas.

Therefore, in making the experiments, the prime requisites were to have the electrodes entirely free of adsorbed gases and to obtain a good vacuum. Dr. Millikan was immediately interested in the proposal of the tests and placed at our disposition the facilities and high-vacuum experience of the Norman Bridge Laboratory; also he cooperated in the development of the switch by making many valuable suggestions and by assigning to the work two graduate students of the physics department, H. E. Mendenhall and Russell Otis.

Three switches were developed and tested. The first

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*Both of the California Institute of Technology, Pasadena, Calif.

1. For references, see Bibliography.

Presented at the Pacific Coast Convention of the A. I. E. E., Salt Lake City, Utah, Sept. 6-9, 1926.

switch is shown in Fig. 1. It consists of a glass envelope with two fixed electrodes as shown, separated by one-half inch. These have crescent-shaped contact surfaces, *a* and *b*, as shown. The contact area of each of the fixed terminals is ⅛ sq. in. The circuit is closed by a flat circular copper disk resting upon them with no contact pressure other than the weight of the disk, to which is attached a light plunger. The switch is opened in operation by a solenoid which, when energized, raises the plunger. In interrupting the circuit, the bridging circuit contact is raised ½ in. by the solenoid. This type of construction gives two breaks in series when the switch is open.

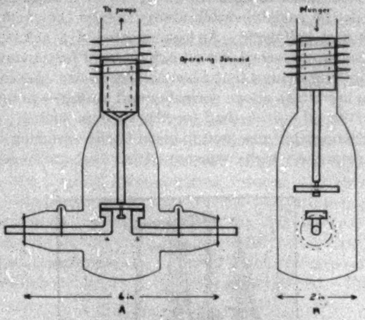

FIG. 1—CROSS-SECTION VIEWS OF SWITCH NO. 1

Vacuum-tight joints between the lead-in conductors and the glass envelope of the switch were easily obtained by means of W. G. Houskeeper's disk seals.[5] This switch was evacuated down to 10^{-6} centimeters of mercury pressure. An initial test was made by using this switch to interrupt currents up to 125 amperes at 110 volts d-c. The results were encouraging, and the switch was connected to an a-c. supply and the test repeated with very satisfactory results, the interruption of current being accomplished with less arcing than occurred when direct current was used.

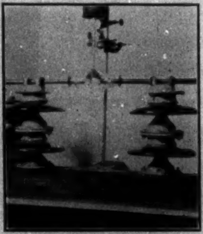

FIG. 2—VACUUM SWITCH NO. 2

The switch was then successively used on a-c. circuits for 220 volts, 2300 volts, and 15,000 volts. The load in every case was a single-phase load connected and disconnected by means of the switch used as a single-pole switch. There was no apparent difference in the operation of the switch at the different voltages with the exception that the switch was not properly designed to guard against arcing over the outside at 15,000 volts. This trouble was eliminated by immersing the switch in oil. When so immersed the switch was operated many times as a single-pole switch to interrupt 100 amperes at 15,000 volts. Every operation was successful.

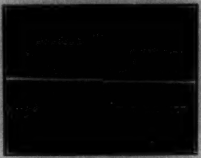

FIG. 3—VACUUM SWITCH INTERRUPTING 272 AMPERES AT 5380 VOLTS

The terminals of this switch, however, were very small and therefore a second switch having terminals with more contact surface and leads of greater carrying capacity was built. Fig. 2 shows switch No. 2. This switch was constructed in the same manner as switch No. 1, but is larger and has better contacts, the bridge being made of spring-copper laminations. When the switch is closed, the edges of the laminations are held against the fixed contacts by the weight of the bridge and its lifting solenoid, the total weight being two pounds. The contact area of each fixed terminal is ¾ sq. in., the distance between the fixed terminals being one inch. In interrupting circuit the bridge is raised one inch.

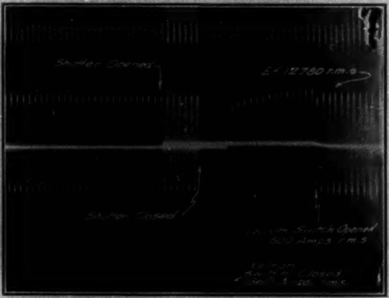

FIG. 4—VACUUM SWITCH INTERRUPTING 600 AMPERES AT 12,780 VOLTS

This switch was given laboratory tests on a 15,000-volt, single-phase circuit providing currents up to 120 amperes at this voltage. The switch was operated as a single-pole switch to open and close this circuit more than 500 times without showing any burning of the switch contacts. It was then sealed off from the vacuum pump and allowed to stand in the laboratory for three months, during which time it was tested occasionally to determine its condition. At the end of the three months the switch was taken to the Torrence substation of the Southern California Edison Company and used to open short circuits made on a synchronous condenser just as the condenser was disconnected from the Edison distribution system. The current was supplied to the switch from the condenser through step-up transformers. The switch repeatedly opened the single-phase short circuit thus provided without any failure to interrupt the circuit or any burning of the switch contacts. Figs. 3 and 4 show oscillograms of switch No. 2 opening 272 amperes at 5380 volts and 600 amperes at 12,780 volts, respectively.

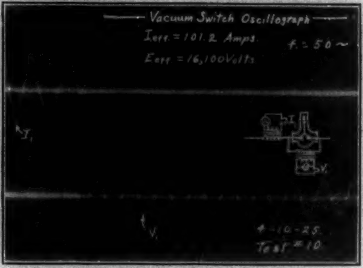

FIG. 5—OSCILLOGRAM SHOWING CURRENT OPENED BY VACUUM SWITCH AND VOLTAGE ACROSS SWITCH AT OPENING

I_{eff} = 101.2 amperes E_{eff} = 16,100 volts f = 50 cycles

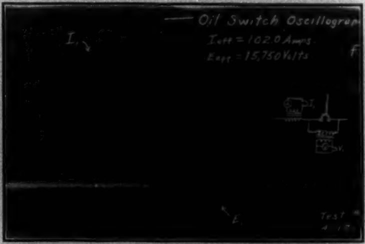

FIG. 6—OSCILLOGRAM SHOWING CURRENT OPENED BY OIL SWITCH AND VOLTAGE ACROSS SWITCH AT OPENING

Note that 12 cycles are between initial contact separation and extinguishing of arc

I_{eff} = 102 amperes E_{eff} = 15,750 volts f = 50 cycles

Figs. 5 and 6 show oscillographic records of vacuum switch No. 2 and a standard make of oil switch opening the same circuit on a load of 100 amperes at 15,000 volts. The tests were made under conditions as nearly identical as possible, and within a few minutes of each other. It will be noted from these graphs that the rise in voltage when the circuit is opened with the oil switch is greater than when the circuit is opened with the vacuum switch. The oil switch in a large number of tests failed to open the circuit on the first half-cycle, while the vacuum switch always opened the circuit on the first half-cycle. An examination of a number of oscillograph records for oil switches and for the vacuum switching showed that when the circuit was opened the rise in voltage above normal circuit voltage was higher for the oil switch than for the vacuum switch. The klydonograph[6] was used in some of the switching tests to record any high-frequency surges that might occur. In no case did the instrument indicate voltages much above normal.

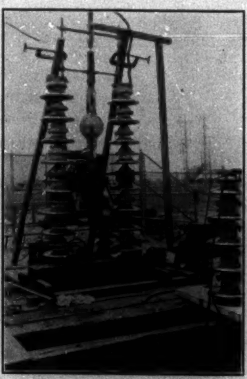

FIG. 7—VACUUM SWITCH NO. 3

Following these tests, switch No. 3 shown in Fig. 7 was constructed. The figure shows the switch in the closed position. Switch No. 3 was constructed primarily to overcome the disadvantage, in switches Nos. 1 and 2, of having all moving parts sealed inside the glass envelope of the switch, a condition requiring the operating solenoid to be kept energized when the switch is open, unless a rather intricate locking mechanism also be installed inside the switch to hold it open.

In switch No. 3, the moving contact is of the bayonet type, the bayonet sliding into a cylindrical socket. The bayonet is a ¾-in. copper rod projecting into the socket when closed so as to give a contact length of one in., the total contact surface obtained in this way being 2.3 sq. in. With this construction, there is only a single break, the contacts opening so as to separate them a distance of one in. when the switch is completely open.

The single break appeared to function as well as the double break used in switches 1 and 2. The switch was operated by a standard switch-operating mechanism borrowed from an oil switch. With such an arrangement, the switch can be left open or closed at will. Vacuum-tight joints for the lead-in conductors of this switch were made by cementing to the glass envelope metal caps attached to the leads and forming a part of the leads.

After some preliminary testing in the laboratory, this switch was taken to the Laguna-Bell substation of the Southern California Edison Company and used as a single-pole switch to open single-phase short circuits on a 30,000-kv-a., synchronous condenser. In performing the tests, the synchronous condenser was brought up to speed on the distribution system of the Edison Company, disconnected from the system and immediately short-circuited through the switch. The condenser used was a 6600-volt, three-phase, Westinghouse condenser which, for the purpose of testing the switch, was connected to the switch through step-up transformers by means of which voltages across the switch as high as 41,500 volts were reached. Fig. 8 shows an oscillogram for this switch interrupting 926 amperes at 41,500 volts.

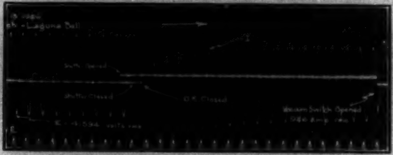

FIG. 8—VACUUM SWITCH OPENING 926 AMPERES AT 41,594 VOLTS

A noticeable feature of the vacuum-switch tests is that every oscillographic record shows that the arc produced at the opening of the switch is extinguished at the end of the first half-cycle after the separation of the contacts. Only the very best oil-switch operations give this result.

The absence of any pitting of switch contacts and the fact that the vacuum is not reduced appreciably when the switch is in operation is evidence that very little of the energy dissipated when the switch is opened is used in vaporizing metal from the contacts.

When the contacts separate, there is a visible arc, just as when a switch is opened in air or oil. The magnitude of this arc, however, is much less than that of an arc made by like values of voltage and current in air or oil. This is to be expected because there is nothing in a vacuum switch to burn or to support combustion, as is the case when a switch is opened in oil or air.

The action of the arc in vacuum also indicates a doubt as to the soundness of the theory that an arc to be maintained must be supported by thermions emanating from hot spots on the electrodes between which the arc is formed. J. Slepian[7] has shown that an arc is probably formed near the surface of metal electrodes by very high temperatures caused by the concentration of electric current in the gas immediately surrounding the electrodes. The experiments at California Institute of Technology show that the vacuum switch, when opened, fails to interrupt an electric circuit if the metal forming the contacts has not been freed of the adsorbed gases; that is, gases adhering to the surface of the metal.

The results of these experiments cannot be taken as conclusive evidence that a new type of electric switch has been developed, because the limits of performance have not been determined and there are many problems relating to details that must be solved to make the switch practical. The switch, however, was never the limiting factor in any of the tests made. There is, therefore, certainly sufficient encouragement to warrant further investigation of the subject for the purpose of determining the fundamentals of switching phenomena, if for no other reason. Also, we have the encouraging fact that many practical devices in use today presented, in the early stages of their development, obstacles which appeared greater than those which these tests indicate.

The authors of this paper are indebted to those already mentioned as having a part in the program, and in addition, to graduate students in the Department of Electrical Engineering, J. J. DeVoe, J. H. Hamilton, and F. C. Lindvall, for assisting in laboratory and field tests; to Julius Pearson and William Clancy for their skill in doing the machine work and glass blowing; and also to the many members of the Southern California Edison Company who made provision for having switches Nos. 2 and 3 tested on the Edison system and helped in making the tests.

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Discussion

Joseph Slepian: I was surprised when I read in Professor Sorensen's paper that he found very little pitting or vaporizing of the electrodes. It seems to me that this, if true, must call for some radical revision in our ideas of electric arcs.

As Professor Sorensen mentions in his paper, the most widely held theory of the arcs requires that the cathode be maintained at a temperature sufficiently high for thermionic emission. I agree with Professor Sorensen that this theory is probably not correct and that a hot cathode is not essential for an arc. It is possible to have a high-current arc with a cold cathode.

Some most convincing evidence on this point published in the *Zeitschrift fur Physik* is due to H. Solt, who caused a 10-ampere arc to move over a copper surface so rapidly that the copper did not get hot at any point.

I have repeated this experiment and I have caused arcs with as high as 20,000 amperes to move so rapidly over a copper ring that there was no visible melting of any kind.

But while it seems that a hot cathode is not necessary it does appear that in order to carry heavy current at low voltage a considerable pressure of gas or vapor of some sort is necessary. This gas or vapor would have to be supplied from the electrodes, and presumably in a successful operation of the switch this metal vapor would condense so rapidly at the zero of current that the high vacuum would be restored sufficiently to withstand the line voltage. But now, however, going by theory and by experience, to carry an arc of several thousand amperes will require a vapor pressure in the order of millimeters, and if the electrodes are to supply this vapor for a full half-cycle, serious consumption of the electrodes will result.

If only a small amount of vapor in fact is formed, it would seem that the discharge must be of some very high-voltage form. I should like therefore to ask Professor Sorensen what the voltage on the arc was in his vacuum-switch experiments.

I should appreciate also some further information concerning the last switch mentioned in his paper, particularly how the moving electrode is brought into the vacuum tube.

D. C. Prince: The oscillograms shown in this paper indicate that in some way current is carried between the electrodes after they have separated until the termination of the half-cycle. It is not necessary to assume that this current is thermionic.

The mechanism used to explain the cathode spot in the mercury rectifier might be applicable to this phenomena. It is briefly as follows: Electrons may be drawn from a relatively cool metal by an electric field of some millions of volts per centimeter. These electrons proceed until they strike neutral gas molecules, which they ionize. The positive ions so formed progress toward the cathode and set up, in this case copper, a high space-charge gradient required to extract further electrons from the cathode material. The positive ions striking the copper surface heat it to the vaporizing point so that copper vapor is given off and is present to be ionized by the electrons.

In order that this action may take place in the extremely short distance required to give the potential gradient of millions of volts per centimeter, the copper vapor would have to have a pressure of the order of atmospheres in the neighborhood of the cathode. Upon the termination of a half-cycle, the cathode spot would go out and there would be no mechanism to establish a cathode spot on the other electrode. Thus conductivity would be interrupted.

P. H. Thomas: Some years ago in the laboratories of the Cooper Hewitt Electric Company, I made rather extensive researches on the behavior of vacuum apparatus passing alternating currents, and arrived at a practical conception of these phenomena which may be of interest in connection with the further work of the authors. The question as to why the arc behaves as it does in a high vacuum is fundamental, of course, in considering what it is likely to do with heavier currents.

My conception of this phenomenon in 1906 is covered in a paper[1] presented before the A. I. E. E., May 31st, 1906, and is briefly as follows:

The electric current in a perfect vacuum is taken to be a stream of electrons, as with the bombardment in an X-ray tube. Once in the perfect vacuum, these electrons pass without resistance and without giving light or generating heat. To release such electrons carrying electric charges from a cold conducting surface in a perfect vacuum would theoretically require an infinite voltage. If, however, it be assumed that the flow of electrons has once been established (as by the well-known method of separating metallic contacts in the vacuum) it is reasonable to suppose that at the point on the cathode surface where the stream issues, an open door, so to speak, may be established in such a way that the assumed perfect surface of the metal is locally changed to give physically a *gradual transition* from a solid state to a gaseous state, then a comparatively small voltage would be sufficient to move electrons from the cathode to the vacuum space in accordance with the observed phenomena. To put it another way, the physical surface tension of the metal, taken in an electrical sense, is punctured at a certain spot by the passage of a certain amount of current and cannot heal until the stream stops, as at the zero point of an a-c. wave. As a matter of fact, if the current from a solid cathode once established in a perfect vacuum be gradually decreased in strength, a point will be reached at a strength of a few amperes at which it will snap out violently. In fact, by using a series inductance in a d-c. circuit, a most excellent Wehnelt interrupter may be obtained, giving exceedingly abrupt high-voltage impulses.

According to this view the cathode spot from which the current issues in a perfect vacuum,—even from a solid electrode,—need not be a *hot* spot and, as a matter of fact, the phenomenon is much more easily explained if it be assumed cold. There is of course a brilliant light spot showing the excitation of the atoms at that point but the actual rise of temperature is probably small. This action is much clearer with mercury cathodes which cannot become heated on account of their power to evaporate but which exhibit the same switch action as the solid cathode.

This does not mean that arcs may not be fed by hot spots on cathodes, in *the presence of gases*. It seems probable that the critical feature of the vacuum switch which easily drops an arc at the zero point of the wave is the *cold* cathode, for such a cathode has no tendency to hold over the zero point, while the reason the arc in air does not drop out easily at the zero point is that the cathode spot is hot enough to emit electrons and stays so over the zero point.

The reason why the vaporization of copper at the cathode does not impare the vacuum would appear to be that copper is naturally a solid at ordinary temperatures and that any vaporized copper would immediately solidify and become inert. I believe that this type of circuit breaker will be very effective up to the point where in operation the vacuum does not deteriorate or the electrodes actually develop hot spots.

As a matter of fact, at the time of writing of the above paper, I operated vacuum switches in the laboratory for the purposes of investigation both with mercury and with solid electrodes, but not with such large quantities of energy nor so comprehensively as the authors.

Some of the specifications of patents on vacuum apparatus filed about this time and assigned to the Cooper Hewitt Electric Company would be of interest in this connection.

R. W. Sorensen: Dr. Slepian asked two or three definite questions. The first two switches were very poorly designed for practical operation; that is, the contacts were poor as judged by switch engineers and the solenoid coils around the neck of the switch, used to raise the plunger, were crude ones built by students inexperienced in the design of solenoids. These solenoids were weak and operated the plunger with a hesitating action. This uncertain action I am sure was the cause of many of the irregularities in the oscillograms made for switches 1 and 2. The poor operating mechanism in these switches was particularly noticeable in closing them. Switches No. 1 and No. 2 were closed by gravity when the solenoid circuit was opened. Such action of course caused the plunger to rebound several times at closing, thus causing an arcover each time the switch was closed, a very poor arrangement for a switching device.

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[1] *Some Fundamental Characteristics of Mercury Vapor Apparatus*, A. I. E. E. TRANS., 1906, p. 601.

We have shown no oscillograms for the switch closing, because there is no question as to its operation on closing. When the switch is closing, the contacts are absolutely cold and the insulation resistance of the vacuum space between switch terminals is so high that no arc can form between terminals when the switch is actually closed. This is frequently the case in an oil switch. Examination of the small switch shows that with all the ill treatment of the contacts for reasons just mentioned, there has been no appreciable pitting of the contacts. Switch No. 1 and also No. 2 has each operated according to our records more than 500 times. There were many additional operations of which no record was made. As stated in the paper, switch No. 3 is unique because in it we have found a means whereby we can bring the operating rods through the switch envelope and at the same time maintain a vacuum-tight envelope. The paper calls attention to the fact that switch No. 2 stood in the laboratory two months, disconnected and sealed off from the vacuum pump, before it was tried out at the Torrence Substation of the Southern California Edison Company.

Also I have used in calculations higher values for the amount of copper to be vaporized by a given amount of energy. The latest international tables of physical constants give 467 kw-sec. of energy required per gram molecule of copper, a much higher figure than the one we had been in the habit of using.

H. E. Mendenhall: Questions have been asked as to the theory of the vacuum switch but our work is in such an early experimental stage that we are not positive as to the exact mechanism of the high-vacuum arc.

Mr. Prince has suggested that the electrons to maintain the vacuum arc may be pulled out of the metal by very high electric fields as in the mercury rectifier. Dr. Slepian has asked what the voltage drop across the arc is. The voltage between the contacts at separation rises momentarily to some value below the maximum value of the voltage wave and then during the time of the arc there is apparently very little drop across the arc (less than in an oil switch). The arc goes out at the end of the first half-cycle, at which time the voltage rises to a value less than 50 per cent above normal, the corresponding value for the oil switch being somewhat higher. The work of Millikan and Eyring has shown that potential gradients as high as 4,000,000 volts per cm. are required to pull electrons out of metals up to 700 deg. cent. It is possible that the first electrons for starting the arc are pulled from the metal in the manner described by them, consuming a considerable portion of the voltage drop representing the energy dissipation in the arc, the balance of the drop going to vaporize and ionize enough copper atoms to establish conditions for thermionic emission for one half-cycle. These conditions disappear with the recombination of the extremely short-lived metal ions and the reverse of the direction of current flow between switch terminals which from the physics point of view are electrodes whose separation distance is increasing during the opening of the switch.

We were very much interested in some work published by Dr. Slepian in which he advocated a rather new theory for the transition from glow to arc discharges. Instead of this transition being due to thermions from within the electrode, he reasoned that temperatures sufficiently high existed in the gas surrounding the electrodes to produce thermal ionization of the gas. From our work, in which we remove as much gas as possible and are thus able to extinguish the arc, we have concluded that the gases within the metal and absorbed on its surface in a layer perhaps only one molecule thick play an essential part in the maintenance of an arc.

We believe that the difference between our results and those mentioned by Dr. Slepian are due to a difference in the degree of vacuum attained in the two cases. We have used the latest high-vacuum technique so that although we have a so-called high vacuum of 10^{-6} mm. of mercury pressure, we have actually removed relatively few gas molecules, but we have increased very materially the mean free path of the molecules or the average distance one molecule will travel before it collides with another molecule. The importance of this factor becomes evident when we consider that an ion or electron may travel across the short distance between the electrodes of the switch in high vacuum without colliding with another molecule and thus ionization of additional gas molecules by collision is not likely to occur.

Mr. Thomas in his discussion assumes it is physically possible to obtain a perfect vacuum but we have already pointed out that there are numerically more gas molecules left in the evacuated chamber than have been removed.

The cathode spot which furnished electrons for one half-cycle becomes the anode when the voltage reverses and the new cathode is a relatively cold cathode so that electron emission from its surface cannot be started without intense bombardment by positive metal ions. We think that these same ions have had time to become ineffective due to recombination since their mean life is of the order of 10^{-8} sec.

We are not yet decided as to how all of the stored energy of the electric and magnetic fields, as roughly evaluated by Charpentier's formula, is dissipated, but our future program includes experiments which should determine better the principles involved in the operation of electric switches.

Stability Characteristics of Alternators

BY O. E. SHIRLEY[1]

Associate, A. I. E. E.

Synopsis.*—During the past few years, stability characteristics of systems using long lines have been discussed at considerable length, but not so much attention has been given to the characteristics of the load. This paper shows that power limits may be reached with very short lines and certain classes of load.*

The characteristics of several classes of load, such as motors of various kinds with constant shaft output, variable impedance loads (synchronous converters for railways), constant impedance, and miscellaneous combinations, are discussed as they affect the stability of the generator. The criterions for the stability of an alternator as developed by this paper are "short-circuit ratio," saturation, power factor of the load, and character of the load.

A series of curves and a formula for the minimum allowable value of short-circuit ratio as a function of saturation and power factor are proposed for general purpose alternators which may be called upon to deliver power to any of the various classes of load. These curves are derived from characteristic curves of typical machines.

It is not intended that these curves shall be used for generators which supply power over long transmission lines, as the characteristics of these lines may require considerably higher values of short-circuit ratio, and the generator must be specially designed to meet the individual requirements.

* * * * *

FOR the past few years, the problem of power limits and stability of alternators has been given considerable attention especially from the standpoint of transmission over lines of lengths approaching the maximum economical distances. The effect of exciter stability, the possibilities of automatic voltage regulators, and compensation for armature reaction by compounding through rectifiers have all been quite fully presented. That there are practical limits for stability in operation for certain classes of load, even with short lines, has been suggested in some papers and discussions, but usually as incidental only to the presentation of other features which were covered much more completely.

The purpose of this paper is to discuss the characteristics of the various classes of load, and to propose a set of curves based on the inherent characteristics of a-c. generators to secure a practical degree of stability with the kinds of load for which this factor must be considered, to secure successful operation. These curves are intended for machines operating under ordinary conditions with comparatively low line drop, and will not be applicable for those operating with very long, high-voltage transmission lines.

In recent years the tendency in the design of alternators has been to decrease the short-circuit ratio, that is, the ratio of field current for normal voltage on open circuit to the field current for rated stator current on short circuit. In many cases this short-circuit ratio may go to extremely low values without serious consequences, aside from poor regulation, but with some kinds of load, an alternator with a short circuit ratio very much below unity may not carry swings appreciably above full load and the voltage will "fade" so that part of the load will be dropped. Such a disturbance may be very puzzling to the operator, as the generator will dip in voltage, dropping a considerable part of the load which caused the instability (due to operation of low-voltage relays), and the voltage will then recover to approximately normal. The machine may then operate as though nothing had happened, unless the maximum load for stability is again exceeded. The fading of voltage, or instability, results from a combination of characteristics of machine, line and load. The principal factors determining stability of the alternator are degree of saturation, power factor of load, and short-circuit ratio. For any particular class and power factor of load, and for alternators with the same degree of saturation, there will be a minimum value of short-circuit ratio which is the lower limit for design of these machines if they are to carry full load with the necessary margin in stability. This margin in stability is necessary to enable the machines to carry the ordinary swings in load successfully.

The degree of saturation in the alternator may be represented by the "saturation factor" as defined in the A. I. E. E. Standards.

Curves showing the proposed minimum values of short-circuit ratio for various lagging power factors and saturation factors are derived in this paper by combining load and generator characteristics.

LOAD CHARACTERISTICS

1. *Constant Power Output.*

a. Induction motors with practically constant shaft output, such as those driving fans, pumps, compressors, direct-current generators, etc.

b. Synchronous motors for same classes of service as "a."

2. *Variable Impedance.*

a. Synchronous converters supplying power to series motors for railway service.

3. *Constant Impedance.*

a. Light and heating.

b. Electric furnaces, welders, etc.

c. Synchronous converters for lighting load.

4. *Miscellaneous.*

Combination of constant power, variable impedance, and constant impedance loads.

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[1] Designing Engineer, General Electric Co., Schenectady, N. Y.

Presented at the Pacific Coast Convention of the A. I. E. E., Salt Lake City, Utah, Sept. 6-9, 1926.

Constant Power Output. Figs. 1A and 1B represent diagrammatically alternators furnishing power to a synchronous motor load and an induction motor load, respectively.

The curves in Fig. 2 show the variation in power factor with voltage for synchronous and induction motors having constant shaft output. The synchronous motor curves are based on adjusting the motor field currents for rated power factors at normal voltage and holding these motor field currents constant for other voltages. The induction motor curve is based on operation at 90 per cent power factor at normal voltage and load. Curves showing variation of current with voltage corresponding to these power-factor curves are given in Fig. 3.

The alternator characteristics are represented by the usual open circuit saturation and synchronous impedance curves. It is possible to design machines with a very wide range of characteristics, but, for the purpose of illustrating the combination of machine and load characteristics, curves for a general purpose alternator with fairly high saturation have been selected.

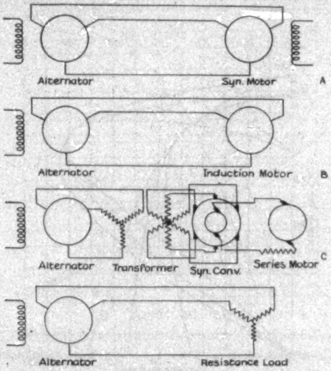

FIG. 1—DIAGRAMS FOR VARIOUS CLASSES OF LOAD

The values of alternator excitation required with constant power output for a number of voltages, above and below normal, were calculated for the different classes of motors represented in Fig. 2, for the stator currents Fig. 3.[2] Refer to Fig. 5 for these curves. They show that excitation decreases slowly with the voltage to a minimum point and then increases again. The minimum excitation and the corresponding voltage determine the limit of steady-state, stable operation. Refer to Fig. 5, Curve *B*. If the generator excitation corresponds to 92 per cent voltage, and the voltage starts to decrease, due to addition of a very small increment of load, the excitation required at a slightly lower voltage will be less than that actually on the machine, and the voltage therefore will be stable. However, if the excitation is decreased to that required for 88 per cent voltage, and the small increment of load is added, the voltage will start to decrease and it will then pass through no value for which the excitation is sufficient. The voltage will continue to drop until the generator and motors pull out of step.

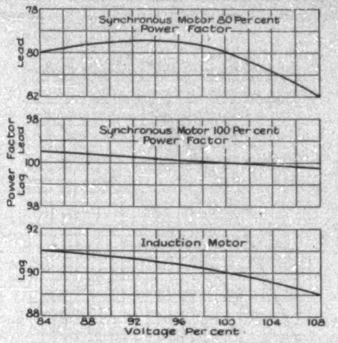

FIG. 2—VARIATION OF POWER FACTOR WITH VOLTAGE. CONSTANT POWER OUTPUT

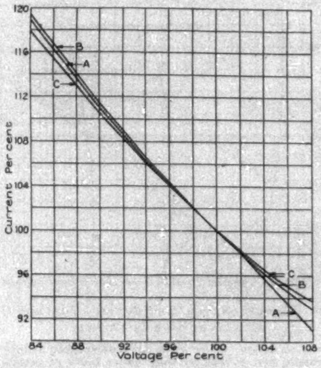

FIG. 3—VARIATION OF ALTERNATOR CURRENT WITH STATOR VOLTAGE. CONSTANT POWER OUTPUT

A. Synchronous motor—80 per cent, power-factor lead at normal voltage
B. Synchronous motor—100 per cent, power factor at normal voltage
C. Induction motor—90 per cent, power-factor lag at normal voltage

The curves in Fig. 5 show quite clearly the behavior of an alternator with motor load having constant shaft output, but the maximum power output of the generator may be determined more simply by a method which has been used in various forms for several years.[3]

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[2] *Reactance*, Appendix C., Doherty and Shirley, A. I. E. E. TRANSACTIONS, Vol. 37, Part 2, p. 1293.

The curve between current and voltage at a given power factor and for a constant excitation current is obtained by calculation or test. These curves, calculated for normal load excitation and at the power factor for normal voltage of each of the classes of motor load in Fig. 3, are shown in Fig. 6. The kv-a. output curves, derived from these curves, are also given in Fig. 6.

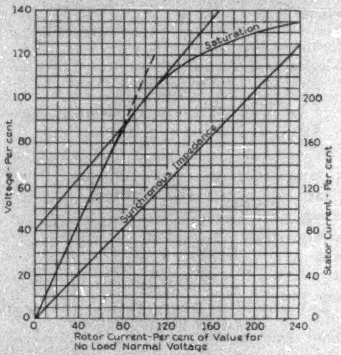

FIG. 4—CHARACTERISTIC CURVES OF ALTERNATOR

Saturation factor—1.65 at normal voltage
Short-circuit ratio—1.04
Stator reactance—18 per cent

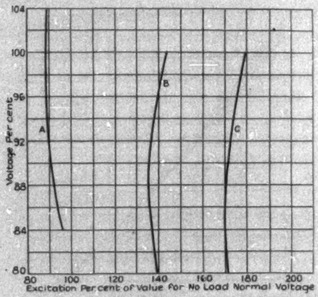

FIG. 5—ALTERNATOR EXCITATION—VOLTAGE CURVES CONSTANT POWER OUTPUT

A. Synchronous motor—80 per cent, power-factor lead at normal voltage
B. Synchronous motor—100 per cent, power factor at normal voltage
C. Induction motor—90 per cent, power-factor lag at normal voltage

The voltages for the maximum kv-a. points in Fig. 6 correspond very closely to the voltages for the minimum excitation points in Fig. 5. The curves do not check exactly, because in Fig. 5 the effect of varying power factor is taken into account, and in Fig. 6, constant power factor is assumed. The difference between the two methods is very small, as will be explained later in connection with Fig. 9.

The behavior of an alternator with increasing load at constant power factor is represented by current-voltage and kv-a.-voltage curves at excitations corresponding to 80-, 100-, 120-, and 140-per cent load at normal voltage (see Fig. 7). The kw. load, expressed as a per cent of the kv-a. rating, is also given by the dotted curves for comparison. These curves show that the maximum kv-a. points occur at voltages increasing up to a certain value and then decreasing, so that the load will not be reached under practical operating conditions where the machine will be unstable at normal voltage. The curves also indicate that alternators with high saturation would operate at very high loads without becoming unstable, but practical experience has shown that it is necessary to maintain a considerable margin in voltage above the minimum point, and in kv-a. below the maximum point.

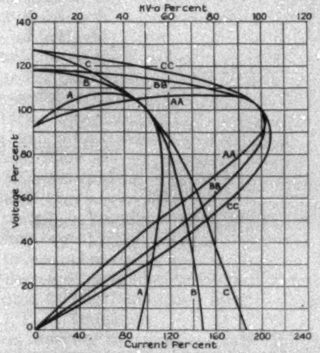

FIG. 6—ALTERNATOR CHARACTERISTICS—CONSTANT EXCITATION

Saturation factor—1.65
Stator reactance—18 per cent
Short-circuit ratio—1.04
Current-voltage curves *A*—*B*—*C*
Kv-a.-voltage curves *A A*—*B B*—*C C*

A—*A A* 80 per cent power-factor lead
B BB 100 per cent power factor
C—*C C* 90 per cent power-factor lad

In Fig. 9, curves *A* and *A A* are calculated for normal load excitation at 90 per cent power-factor lag; curves *B* and *B B*, for 91 per cent power factor; and curves *C* and *C C*, for 89 per cent power factor. The dotted curves, *D* and *D D*, take into account the variation of power factor with voltage for the induction motor represented in Fig. 2. These curves show that the maximum kv-a. for operation at variable power factor, such as that with induction-motor load, is very closely approximated by the curve of kv-a. for normal power factor.

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[3] *Elements of Alternating Currents*, Franklin and Williamson, The MacMillan Company, 1901, p. 124.

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A. I. E. E. Discussion, C. A. Nickle, TRANSACTIONS A. I. E. E., Vol. 43, p. 89.

The curves in Fig. 7 show clearly the importance of keeping up the excitation so as to maintain normal voltage with this class of load. This result may be secured with hand regulation by operating with excitation corresponding to normal load and voltage, which will give a voltage above normal when the load is lower than the machine rating. A regulator is very effective in maintaining stability up to the steady state maximum capacity of the alternator at normal voltage, but if the load exceeds this maximum value, the use of any of the present standard regulators will not maintain stability. If, however, the load conditions are such that the maximum load is exceeded only slightly, stable operation may be obtained by regulating for a voltage a little over normal, provided the machine will not be injured by the higher voltage operation.

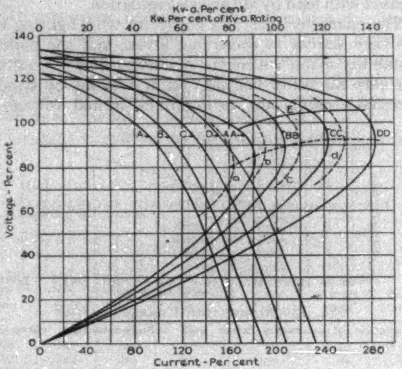

FIG. 7—ALTERNATOR CHARACTERISTICS—CONSTANT EXCITATION

Saturation factor—1.65 *A—A A—a* 80 per cent kv-a. at normal voltage

Stator reactance 18 per cent *B—B B—b* normal kv-a. at normal voltage

Short-circuit ratio 1.04 *C—C C—c* 120 per cent kv-a. at normal voltage

Power factor—90 per cent lag *D—D D—d* 140 per cent kv-a. at normal voltage

E. Kv-a.-voltage curve for 15 per cent margin in voltage above maximum kv-a. points

Current-voltage curves *A, B, C, D*

Kv-a.-voltage curves *A A—B B—C C—D D*

Kw.-voltage curves *a, b, c, d*

The maximum kv-a. values given by the excitation curves do not include any allowance for the effect of the lines and the connecting apparatus, or swings in load, and practical operation will require a margin in the maximum capacity of the alternator to take care of these factors. The required margin is a point that must be determined largely from operating experience.

The more important characteristics of the alternator which affect stability are saturation, relation of no-load field strength to armature reaction, and power factor.

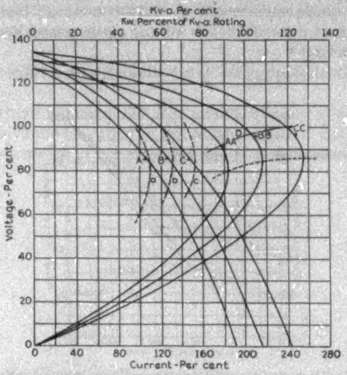

FIG. 8—ALTERNATOR CHARACTERISTICS—CONSTANT EXCITATION

Saturation factor—1.65 *A—A A—a*, 80 per cent kv-a. at normal voltage

Stator reactance 18 per cent *B—B B—b*, normal kv-a. at normal voltage

Short-circuit ratio 1.04 *C—C C—c*, 120 per cent kv-a. at normal voltage

Power factor—60 per cent lag

D. Kv-a.-voltage curve for 15 per cent margin in voltage above maximum kv-a. points

A, B, C current-voltage curves

A A—B B—C C kv-a.-voltage curves

a, b, c kw.-voltage curves

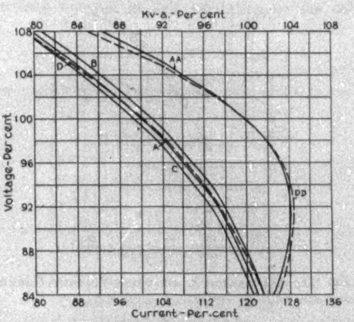

FIG. 9—ALTERNATOR CHARACTERISTICS—CONSTANT EXCITATION. EXCITATION HELD AT VALUE FOR NORMAL KV-A. AT RATED VOLTAGE AND 90 PER CENT POWER FACTOR

Saturation factor 1.65 *A—A A* 90 per cent power factor

Stator reactance 18 per cent *B* 91 per cent power factor

Short-circuit ratio 1.04 *C* 89 per cent power factor

D—D D power factor varying between 89 per cent at 108 per cent voltage and 91 per cent at 84 per cent voltage

A—B—C—C current-voltage curves

A A—D D kv-a.-voltage curves

It is proposed that the saturation factor, as defined by the A. I. E. E. Standards and the short-circuit ratio, be used as the criterion of stability for an alternator at any given power factor. The relation of these two factors for obtaining a consistent margin in stability for synchronous-motor or induction-motor load at different power factors is worked out in a later section of this paper.

The problem of stability with long high-voltage lines and power transmission approaching the maximum line capacity requires a very large increase in stability of the generators and receiving apparatus. A considerable number of papers dealing with this subject has been presented to the Institute, and no attempt is made in this paper to cover this problem.

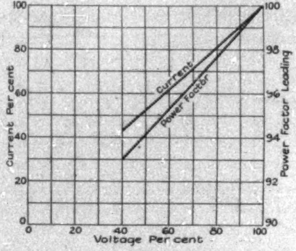

FIG. 10—SYNCHRONOUS CONVERTER CHARACTERISTICS SERIES MOTOR LOAD

D-c. current assumed to vary directly with d-c. voltage.
Curves showing variation of current and power factor on high-voltage side of transformer with voltage on high-voltage side of transformer

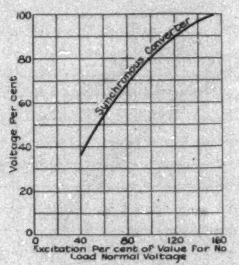

FIG. 11—ALTERNATOR EXCITATION—VOLTAGE CURVES
Synchronous converter with characteristics on Fig. 9

Variable Impedance Load. A synchronous converter furnishing power to series railway motors is an example of a load with quite different characteristics from the one just discussed. For simplicity in working out the curves, the current on the d-c. side of the converter is assumed to vary directly with the voltage. The power factor on the high-voltage side of the transformer will remain quite close to unity due to the converter being self-excited. The current and power-factor curves are drawn as straight lines, and will approximate the actual conditions quite closely. Fig. 10 shows the assumed curves and Fig. 11 is the corresponding excitation-voltage curve. It will be noted that the excitation continues to decrease with the voltage, instead of having a minimum point as with induction or synchronous motor load.

The stability of the alternator is not a practical factor in a design for this class of load. It is therefore possible to use a machine with a short-circuit ratio of unity to carry very high short-time overloads (one to five minutes) on synchronous converters without trouble from instability, provided the alternator field is increased with load by automatic regulation.

The characteristics of a converter with lighting load are very similar to those just given, and the point of instability is not within the operating range of voltage, but very far below normal. It is possible, by hand or automatic regulation, to maintain normal voltage for any excitation within the heating limits of the rotor winding, and to carry any ordinary swings of load.

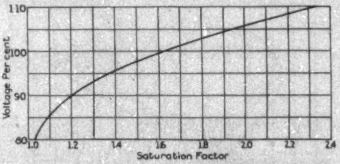

FIG. 12—ALTERNATOR CHARACTERISTICS
Variation of saturation factor with voltage-saturation curve from Fig. 4

Constant Impedance Load. The current in this case will vary directly with the voltage throughout the entire range of load, and the excitation-voltage curve will be almost identical in shape with Fig. 11. With this class of load, the alternator will operate stably at any point on the kv-a.-voltage curve, and the load limit will again be determined by the heating characteristics of the machine.

Alternator Characteristics for the Usual Commercial Load. The usual commercial load is a combination of the various classes just discussed and the selection of the generator with the proper characteristics is largely a matter of judgment. The calculation of excitation voltage curves similar to Fig. 5, for various values of load of the same characteristics as the given load, will show whether conditions of instability will be approached sufficiently to be a factor in the design of the generating equipment.

The kv-a.-voltage curves, Fig. 7, indicate the maximum capacity for load which is practically all Class 1, but if there is a considerable percentage from Classes 2 and 3, the alternator may operate successfully at voltages below the maximum kv-a. point.

This may be illustrated by an example: Assume operation at 90-per cent voltage, Fig. 7. The maximum kv-a. capacity with Class 1 load under this condition is 104-per cent kv-a. as indicated by the maximum point of curve, *BB*, but, with other classes of load, it would be possible to operate at any point on any of the curves. Combinations of all of these different classes of load will give load characteristics, so that stable operation may be obtained somewhat below the maximum kv-a. point on the kv-a. curves. Just how far below the maximum point stable operation can be secured depends upon the percentages of the different kinds of load.

DERIVATION OF CURVES FOR SHORT-CIRCUIT RATIO

The saturation factors for several values of voltage, below and above normal, for the saturation curve in Fig. 4 are plotted as a function of voltage in Fig. 12. The kv-a. values for an operating margin of 15 per cent in voltage, as determined by Figs. 7 and 8, are also shown as a function of voltage in Fig. 13.

The short-circuit ratios for various machine ratings corresponding to the kv-a., and voltage values for 90-per cent power factor from Fig. 13, Curve *A*, are derived as shown in Table I, which also includes the saturation factors for corresponding voltages taken from Fig. 12. Similar data were calculated also for the 60-per cent power factor, Curve *B* in Fig. 13. In Fig. 14, Curves *A* and *B* represent, graphically, the relations of short-circuit ratio and "saturation factor" as determined by these calculations.

TABLE I
CALCULATIONS FOR 90-PER CENT POWER FACTOR

Volts per cent	Kv-a. per cent	Current per cent	Open circuit field current per cent from Fig. 4	Short-circuit field current per cent from Fig. 4	Short-circuit ratio	Sat. factor
100	90	90	100	87	1.15	1.65
103	101	98	105	94	1.12	1.84
105	112	107	109	103	1.06	1.94
106	125	118	110	114	1.03	2.04

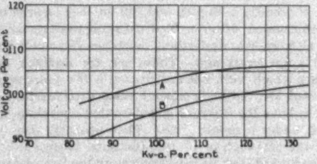

FIG. 13—ALTERNATOR CHARACTERISTICS

Kv-a.-voltage curves for 15-per cent margin in voltage—From Figs. 7 and 8

A 90-per cent power-factor lag
B 60-per cent power-factor lag

The determination by trial of a simple formula to approximate these curves in the working range with reasonable accuracy, resulted in the following equation:

$$\text{Minimum } S\,C\,R = 1.4\sqrt{\frac{\text{Power factor}}{100\,F}}$$

Where minimum $S\,C\,R$ = minimum value of short-circuit ratio for rating to give a safe margin in stability for operation at normal rating and F = saturation factor as defined by the A. I. E. E. Standards 7-59: "The saturation factor of a machine is the ratio of a small percentage increase in field excitation to the corresponding percentage increase in voltage thereby produced. Unless otherwise specified, the saturation factor of a machine refers to the no-load excitation required at rated speed and voltage."

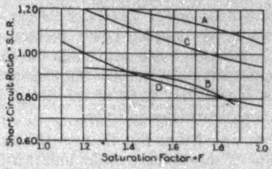

FIG. 14—CURVES FOR RELATION OF SHORT-CIRCUIT RATIO AND SATURATION FACTOR

From Figs. 12 and 13—(See Table I)

A 90-per cent power-factor lag
B 60-per cent power-factor lag

From formula $S\,C\,R = 1.4\sqrt{\frac{\text{Power factor}}{100\,F}}$

C 90-per cent power-factor lag
D 60-per cent power-factor lag

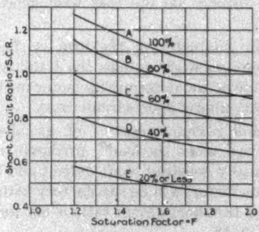

FIG. 15—PROPOSED CURVES FOR MINIMUM VALUE OF SHORT-CIRCUIT RATIO OF ALTERNATORS

A 100 per cent power-factor lag
B 80 per cent power-factor lag
C 60 per cent power-factor lag
D 40 per cent power-factor lag
E 20 per cent power-factor lag

From formula $S\,C\,R = 1.4\sqrt{\frac{\text{Power factor}}{100\,F}}$

The maximum allowable rating for any particular machine is determined by this minimum value of short-circuit ratio.

In Fig. 14, Curves *C* and *D* are derived from this proposed equation, and a comparison with Curves *A* and *B* shows that for all practical purposes the equation will give the proper values of short-circuit ratio in the usual range of values of saturation factor.

Fig. 15 derived from the equation, covers the entire range of lagging power factors and the usual range of saturation factors. The equation will give results approaching zero as the power factor approaches zero, but since this is obviously impractical, the same values are proposed for power factors of 20 per cent or less. The curve covers lagging power factors only and no attempt is made to work out similar curves for leading power factor, since machines of this class are special and will come within the scope of the design of machines for very long transmission lines.

It will be noted that Fig. 15 gives the proposed minimum value of short-circuit ratio, and it is obvious that any machine with a higher short-circuit ratio will be satisfactory from the standpoint of stability.

CONCLUSIONS

Stability of operation of alternator depends on the characteristics of the load as well as on those of the alternator and line.

The minimum allowable value of short-circuit ratio for successful operation is determined by the character of the load, the saturation factor of the alternator, and the power factor. This assumes that the line drop is small enough to be covered by the proposed margin, which will usually be the case with general purpose machines.

When the character of the load is such that the current decreases with voltage, the stability characteristics will not be the determining factor in the selection of the proper value of short-circuit ratio.

The stability characteristics of an alternator for general service, when the load is predominantly motors with practically constant shaft output and line drop is not a serious factor, will be satisfactory with automatic regulation if the short-circuit ratio is not less than that given by Fig. 15.

When the load consists of a few comparatively large motors with swinging load, the proposed curves will give successful operation if the short-circuit ratio at a rating corresponding to the maximum load does not exceed the values indicated by the curves and automatic regulation is used.

Machines designed in accordance with these curves will operate successfully with hand regulation if the increments of load, due to adding more motors, are small enough to prevent excessive drop of voltage before the field rheostats can be adjusted to compensate for the added load.

The proposed values of short-circuit ratio are not applicable to generators for use with very long transmission lines, and the design of these machines is a special problem beyond the field of this paper.

ACKNOWLEDGMENTS

In the preparation of this paper the author desires to express his appreciation of the assistance of Messrs. R. E. Doherty, C. A. Nickle, and R. H. Park.

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Discussion

David Hall: As a designer of electrical machinery, I feel that this paper is valuable to the users of machinery because it points out some of the factors which are more or less concealed. We have specifications covering machines which do not cover the factors presented here and these factors are in many cases as important as the outstanding specifications of temperature and efficiency. We have pointed out in Mr. Shirley's paper certain "red flags" beyond which the designer knows his machine will "lie down." It is very useful to have those red flags placed before the designer.

The designer, as I see it, is a man who after, passing through a long period of designing, stands as near the precipice as he can without falling off. This paper points out that if he goes beyond certain factors he is liable to fall off and it is only through the rare judgment of good designing that he will stay at a respectable distance from the precipice. Those factors are hard to evaluate but the time will come when the user will be able to judge better of what he is buying and will not buy on price but on certain performance. I feel justified in making these remarks because we are all coming to understand machines very much better than we did a few years ago. Today we are reaching heights in designing which were not anticipated a few years ago. When I first read Mr. Shirley's paper I was a little fearful that he intended to have the Institute stipulate that a machine should not be built under a *certain ratio*. That probably does not come within the jurisdiction of the Institute because it wishes to prescribe a yard stick or measuring stick but not to establish the limits of that certain stick to a *certain ratio*. That work rests with the intelligence of the purchaser and the manufacturer.

H. V. Putman: I especially like the method used by Mr. Shirley. He has worked out the stability problem for a few typical cases. From these he draws general conclusions and constructs a formula for a desirable value of short-circuit ratio which will be of great value to all who have to do with the design and application of synchronous machines.

In several of the generator specifications received within the past year or two, by the Westinghouse Company, where the problem of stability has arisen, this company has been asked to guarantee the synchronous reactance of the machines. While it is true that the synchronous reactance is very closely tied up with the short-circuit ratio, it is highly desirable that the short-circuit ratio be the quantity guaranteed because it is much easier to define and is much more easily checked by test. You will note that Mr. Shirley has not mentioned synchronous reactance at all in his discussion. An agreement was recently reached between Westinghouse and General Electric engineers to the effect that in presenting generator propositions the short-circuit ratio should be stated instead of the synchronous reactance, except in special cases.

There is one question I should like to ask Mr. Shirley: Is it not true that if the load on an alternator is increased very suddenly it will carry loads considerably in excess of the values given by the curves shown in Figs. 6, 7 and 8 of the paper, which are all calculated upon the basis of steady-state conditions? I know of several synchronous motors that carry sudden peak loads, continually recurring, from 30 to 50 per cent greater than the steady pull-out torque of the motor. The reason for this is that the peaks take place so suddenly that the armature reactance does not have time to destroy the field flux so that, so far as the peak load is concerned, the machine acts as though it had a much higher short-circuit ratio than it actually has. Would not the same sort of thing, taking place in an alternator, enable it to maintain its voltage under a much higher load than it could under steady-state conditions?

I believe Mr. Shirley's bibliography should include the paper on the same subject by John Strasser, published in the August 1926 issue of the *Electric Journal*. Mr. Strasser has limited his discussion to the stability problem in turbo generators, but reaches practically the same conclusion as Mr. Shirley.

O. E. Shirley: Mr. Hall states that he does not consider it within the realm of the Institute to prescribe values of short-circuit ratio for which a machine should be designed, but that a standard of measuring stability should be given and then the users of the apparatus would be in position to determine the degree of stability required for their particular service. This is certainly quite proper for the larger installations, for which sufficient time can be given to determine the actual operating conditions, but for smaller machines, it may be desirable to establish a standard of stability, so that the purchaser may know that the characteristics of generators, built by different manufacturers, will not differ radically as may be the case if there are no standards.

In the case of important installations where individual characteristics of load and the generators are taken into account it is essential that there should be a common ground for specifying and comparing the stability characteristics of different machines. For these comparative purposes the criteria proposed in this paper should be satisfactory, but there still remain considerable knowledge and experience to be gained before it will be possible to specify the actual values of these factors to meet any given set of operating conditions.

Mr. Putman has brought up the point that the load limit of an a-c. generator for suddenly applied loads of short duration may be appreciably higher than the steady-state value. This appears to be quite reasonable for very short intervals of time, say 1 or 2 seconds, and for any particular machine it should be possible to obtain a curve between load limit and time of applying the load. This curve would start at a maximum value for a load lasting only a fraction of a second, and would decrease quite rapidly down to the steady-state value. The time at which it would practically reach the steady-state value would depend on the rate of change of the field flux and the stored energy of the rotating parts.

Since this paper was sent in for publication, an article, "Stability of A. C. Turbine Generators with Fluctuating Loads" by Mr. John Strasser has appeared in the *Electric Journal* for August 1926, and as suggested by Mr. Putman, this should be added to the bibliography.

Synchronizing Power in Synchronous Machines
Under Steady and Transient Conditions

BY H. V. PUTMAN[1]

Associate, A. I. E. E.

Synopsis.*—The accuracy of all calculations relating to the hunting of synchronous machines connected mechanically to reciprocating apparatus depends largely upon the correctness with which the value of the synchronizing power P_s may be determined. Thus, the calculation of flywheels necessary for the parallel operation of engine driven generators depends entirely on a correct value of P_s. Likewise, the W R^2 which it is necessary to incorporate in the rotors of synchronous motors when driving compressors or pumps can be calculated correctly only when P_s is accurately known.*

So far as the author is aware, no attempt at a complete analysis of the subject has been made heretofore. The object of this paper is to develop a method by which the synchronizing power P_s may be calculated fairly accurately under any condition likely to be met with. The method applies only to synchronous machines of the usual definite pole construction. Blondel's conception of two reactions with some modifications and extensions is followed generally throughout the paper.

* * * * *

SECTION I. INTRODUCTION

SYNCHRONIZING power, P_s, may be defined as the rate at which the input power to a synchronous motor changes with corresponding changes in the angular displacement. The displacement referred to here is not that which actually exists between the rotor and the rotating electrical field in the motor but it is the displacement between the rotor and an imaginary electrical field which corresponds in phase position to the terminal voltage. This will be explained more fully later.

P_s is often called a constant. It would be more correct to say that P_s is constant for a given set of operating conditions because its value changes with the load, the excitation, and with the frequency at which the mechanical load pulsates.

If, for a given synchronous motor operating under constant excitation, a curve is plotted showing the power input as function of the angular displacement, it will appear similar to that shown in Fig. 1.

Now it is obvious that if the mechanical load on the motor changes very slowly, the rate of change of power with displacement is given by the slope of the tangent to the curve at the point corresponding to the given load on the motor. If, however, the mechanical load pulsates up and down rapidly as it does in the case of a compressor, the rate of change of power with the displacement is no longer given by the slope of the tangent to the curve but by the slope of some other line similar to the dotted line in Fig. 1.

The reason for the above phenomenon is not hard to find. A sudden increase in the load on a motor is accompanied by a sudden increase in the armature reaction which tends to buck down, and distort the field flux. It is, however, impossible for the field flux to change immediately, and for the first instant the effect of the armature reaction on the field flux is practically offset by a sudden increase in the field current produced by an induced voltage due to the changing field flux. The result is that the armature reaction has not had time to function at least completely, and the field current has increased or decreased to make up for it. This is precisely what happens in a synchronous motor driving a compressor. The armature reaction functions only partially and the field is kept continually in a transient state. Oscillograms of the field current taken when a machine is hunting often show marked pulsations amounting to a considerable percentage of the average value.

The problem of calculating P_s is thus not simply that of calculating the slope of the tangent line in Fig. 1, but rather the more complicated problem of calculating the slope of the dotted line. In the theoretical analysis which follows, the angular displacement of the machine will be calculated first according to Blondel's theory, then the slope of the tangent line in Fig. 1, and finally that of the dotted line.

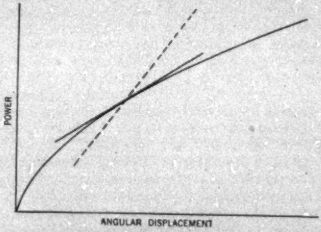

FIG. 1—CURVE SHOWING POWER AS FUNCTION OF DISPLACEMENT UNDER STEADY LOAD

Dotted line represents change of power with displacement under condition of pulsating load.

SECTION II. THEORETICAL TREATMENT

The following symbols will be used throughout the remainder of this paper:

E_i = Induced voltage
E_n = Nominal voltage
E_t = Terminal voltage
E_0 = Impressed voltage

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Presented at the Pacific Coast Convention of the A. I. E. E., Salt Lake City, Utah, Sept. 6-9, 1926.

$\underset{.}{I}$ = Current

r = Resistance

x = Reactance

x_s = Synchronous reactance operating on the component of current at right angles to E_n

x_s' = Synchronous reactance operating on the component of current in phase with E_n

$x_d = x_s - x_s'$

x_e = Effective synchronous reactance, $x < x_e < x_s$

z = Impedance

$z_s = \sqrt{r^2 + x_s^2}$ = Synchronous impedance

$z_s' = \sqrt{r^2 + x_s'^2}$ = Synchronous impedance

m = Coef. of armature reaction. (Ratio of demagnetizing ampere-turns to the no-load excitation ampere-turns)

$\underset{.}{F}_r$ = Resultant field excitation

$\underset{.}{F}_f$ = Applied field excitation

θ = Power-factor angle lagging

δ = Angle between $\underset{.}{I}$ and E_n

ψ = Angular displacement of the rotor

P = Power input

$P_s = \frac{dP}{d\psi}$ = Synchronizing power

K' = Factor of effectiveness of armature reaction

N = No. of turns on field pole

L = Effective inductance of the field

r_f = Effective resistance of the field

K = Coef. of coupling between armature and field

E = D-c. voltage impressed on the field

p = Operator $\frac{d}{dt}$

i = Component of current in phase with E_n

i_1 = Component of current at right angles to E_n

t = Time in sec.

$\Phi \sin \omega t$ = Pulsating flux provided by the demagnetizing component of armature reaction

ω = Frequency of the mechanical pulsation in radians per sec.

i_f = Current in the field circuit

ψ_r = Resultant flux in the field pole

The fundamental equation of all electrical machines is:

$$\underset{.}{E}_i = \underset{.}{E}_t + \underset{.}{I}\,\underset{.}{z} \qquad (1)$$

This equation simply states that the induced voltage is the vector sum of the terminal voltage and the impedance drop.

The induced voltage is proportioned to the flux producing it and lags behind the flux by 90 deg. This is expressed mathematically,[2]

$$\underset{.}{F}_r = j\,c\,\underset{.}{E}_i \qquad (2)$$

where c is the proportionality factor.

The resultant excitation $\underset{.}{F}_r$ is the sum of the armature reaction $\underset{.}{I}\,m$ and the applied excitation $\underset{.}{F}_f$; thus

$$\underset{.}{F}_r = \underset{.}{F}_f + \underset{.}{I}\,m \qquad (3)$$

Substituting (3) in (2) and solving for $\underset{.}{E}_i$,

$$\underset{.}{E}_i = -j\,\frac{\underset{.}{F}_f}{c} - j\,\underset{.}{I}\,\frac{m}{c} \qquad (4)$$

The term $-j\,\frac{\underset{.}{F}_f}{c}$ is defined by most writers as the nominal voltage and denoted by E_n. From (4) it may be seen that it is equal to the induced voltage at no load. Substituting (1) in (4) and replacing $-j\,\frac{\underset{.}{F}_f}{c}$ by $\underset{.}{E}_n$,

$$\underset{.}{E}_t + \underset{.}{I}\,\underset{.}{z} + j\,\underset{.}{I}\,\frac{m}{c} = \underset{.}{E}_n$$

or

$$\underset{.}{E}_t + \underset{.}{I}\left[r + j\left(x + \frac{m}{c}\right)\right] = \underset{.}{E}_n \qquad (6)$$

The term $\left(x + \frac{m}{c}\right)$ is the synchronous reactance, x being the real reactance, and $\frac{m}{c}$ the reactance equivalent of the armature reaction. The complete bracket in equation (6) is, therefore, the synchronous impedance and represented by z_s.

$$\underset{.}{E}_n = \underset{.}{E}_t + \underset{.}{I}\,\underset{.}{z}_s \qquad (7)$$

It is interesting to note the similarity between equations (1) and (7). The nominal voltage corresponds to the induced voltage, and the impedance has become synchronous impedance. Equation (1) applies to all electrical devices while equation (7) is the fundamental equation of, and applies only to, synchronous machines.

For a synchronous motor, equation (7) is more conveniently written

$$-\underset{.}{E}_n = \underset{.}{E}_0 - \underset{.}{I}\,\underset{.}{z}_s \qquad (8)$$

E_0 being the impressed voltage.

The vector diagram for a synchronous motor based on equation (8) is shown in Fig. 2. $-\underset{.}{E}_n$ has been chosen as zero vector, so that vectorially

$$-\underset{.}{E}_n = E_n + 0j$$

It should be noted in connection with Fig. 2 that θ is the power-factor angle lagging, and δ is the angle between $\underset{.}{I}$ and $\underset{.}{E}_n$.

Now it can be shown that the angle between $\underset{.}{E}_n$ and E_0 is the angular displacement of the machine. It is, however, somewhat different from the angular displacement between the rotor and the rotating electrical field.

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[2] The use of a linear relation here between $\underset{.}{F}_r$ and $\underset{.}{E}_i$ does not mean that the effect of saturation in the machine has been neglected. In fact, the proper value of c to be used for a given set of operating conditions must be determined from the saturation curve. c is really the slope of a straight line from the zero point through the operating point on the saturation curve.

It is actually the angular displacement between the rotor and an imaginary rotating electrical field corresponding in phase position to the terminal voltage, while the actual rotating electrical field corresponds in phase position to the induced voltage. There is a distinct advantage in calculating the displacement from the terminal voltage rather than from the induced voltage since the terminal voltage vector always rotates at uniform velocity provided the frequency of the impressed voltage is constant. But in a machine which is hunting, the induced voltage vector or the actual electrical field in the stator does not rotate uniformly because of the reactance, the $I\,x$ drop pulsating as the current pulsates. In problems involving hunting, it is necessary to calculate the variations in the speed of the rotor. This is done by calculating the angular displacement of the machine as function of time but it is evident that the angular displacement must be referred to a uniformly rotating vector. The advantage of referring the angular displacement to the terminal voltage rather than to the induced for all problems concerned with hunting and the calculation of P_s on this basis is thus apparent.

The angular displacement is therefore

$$\psi = (\theta + \delta) \tag{9}$$

The value of ψ corresponding to a given power input at a given power factor is easily determined from equation (**8**) as follows:

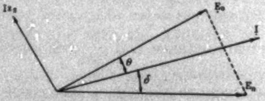

FIG. 2—VECTOR DIAGRAM OF SYNCHRONOUS MOTOR UNDER STEADY LOAD

From Fig. (**2**),

$$\underset{.}{I}\,\underset{.}{z_s} = I\,(\cos\delta + j\sin\delta)\,(r + j\,x_s)$$
$$= I\,[\,(r\cos\delta - x_s\sin\delta) + j\,(x_s'\cos\delta + r\sin\delta)\,] \tag{10}$$

It should be noted that an x_s' is used with the component of current in phase with E_n or that component of the current which is maximum when the armature coils are directly under the centers of the field poles. This is according to Blondel's well-known theory of two reactions and amounts to the same thing as Dr. Berg's[3] method with the exception that Dr. Berg based his equations on the induced voltage as zero vector which introduces a slight error especially at larger loads.

Substituting (**10**) in (**8**),

$$E_n = E_0\,[\,\cos(\delta + \theta) + j\sin(\delta + \theta)\,]$$
$$- I\,[r\cos\delta - x_s\sin\delta) + j\,(x_s'\cos\delta + r\sin\delta)\,] \tag{11}$$

Equation (**11**) may be separated into two equations, one involving only real terms, the other only imaginary. Thus, from the imaginary terms is obtained the equation,

$$E_0\,(\sin\delta\cos\theta + \cos\delta\sin\theta) = I\,(x_s'\cos\delta + r\sin\delta)$$

or

$$\sin\delta\,[E_0\cos\theta - I\,r] = \cos\delta\,[I\,x_s' - E_0\sin\theta] \tag{12}$$

or

$$\tan\delta = \frac{I\,x_s' - E_0\sin\theta}{E_0\cos\theta - I\,r} \tag{13}$$

This equation, which was derived by Blondel[4], gives the value of the angle δ for any load and power factor and consequently the displacement ψ, since $\psi = \delta + \theta$.

Very often, however, the field excitation and consequently the nominal voltage is fixed, and it is desired to find the displacement as function of the power under this condition.[5]

Equation (**12**) may be rewritten:

$$x_s'\,I\cos\delta + r\,I\sin\delta = E_0\sin\psi \tag{14}$$

In a similar manner the real terms of equation (**11**) give:

$$r\,I\cos\delta - x_s\,I\sin\delta = E_0\cos\psi - E_n \tag{15}$$

Equations (**14**) and (**15**) may be solved by determinants for $I\cos\delta$ and $I\sin\delta$ as follows:

$$I\cos\delta = \frac{r\,(E_0\cos\psi - E_n) + x_s\,E_0\sin\psi}{r^2 + x_s\,x_s'} \tag{16}$$

and

$$I\sin\delta = \frac{r\,E_0\sin\psi - x_s'\,(E_0\cos\psi - E_n)}{r^2 + x_s\,x_s'} \tag{17}$$

These are the two components of $\underset{.}{I}$ along and at right angles to E_n, so that vectorially

$$\underset{.}{I} = I\cos\delta + j\,I\sin\delta \tag{18}$$

Also from the vector diagram, Fig. 2,

$$\underset{.}{E_0} = E_0\cos\psi + j\,E_0\sin\psi \tag{19}$$

The input power may now be calculated by telescoping (**18**) and (**19**). Thus:

$$P = E_0\cos\psi\,I\cos\delta + E_0\sin\psi\,I\sin\delta \tag{20}$$

Substituting for $I\cos\delta$ and $I\sin\delta$ from equations (**16**) and (**17**) gives the input power as function of the displacement as follows:

$$P = E_0\cos\psi[r(E_0\cos\psi - E_n) + x_s\,E_0\sin\psi]\frac{1}{(r^2 + x_s x_s')}$$
$$+ E_0\sin\psi[r\,E_0\sin\psi - x_s'(E_0\cos\psi - E_n)\,]\frac{1}{(r^2 + x_s x_s')} \tag{21}$$

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[3] Berg and Upson, "First Course in Electrical Engineering," Chapter XXXV, McGraw Hill & Co., 1916.

[4] "Synchronous Motors and Converters," McGraw Hill & Co., 1913. Blondel's equation involves + signs because it is derived for the alternator instead of the motor.

[5] Incidentally, this is the problem involved in the calculation of P_s with the exception that the field transient must also be considered.

which reduces to

$$P = \frac{E_0^2}{(r^2 + x_s x_s')} \left\{ r - \frac{E_n}{E_0} (r \cos \psi - x_s' \sin \psi) \right.$$
$$\left. + \sin \psi \cos \psi \, (x_s - x_s') \right\} \qquad (22)$$

Equation (**22**) is useful as it gives the input power of a definite pole synchronous motor for a given angular displacement. A close approximation may be made to (**22**) by disregarding the resistance. This gives the somewhat simpler expression,

$$P = \frac{E_0^2}{x_s x_s'} \left\{ \frac{E_n}{E_0} x_s' \sin \psi + \sin \psi \cos \psi (x_s - x_s') \right\} \qquad (23)$$

Differentiating (**22**) with respect to ψ, the slope of the tangent line shown in Fig. 1 is obtained:

$$P_s = \frac{d\,P}{d\,\psi} = \frac{E_0^2}{(r^2 + x_s x_s')} \left[x_d \cos 2\,\psi \right.$$
$$\left. + \frac{E_n}{E_0} (x_s' \cos \psi + r \sin \psi) \right] \qquad (24)$$

where

$$x_d = x_s - x_s'$$

or disregarding resistance terms

$$P_s = \frac{d\,P}{d\,\psi} = E_0^2 \left[\left(\frac{1}{x_s'} - \frac{1}{x_s} \right) \cos 2\,\psi \right.$$
$$\left. + \frac{E_n}{E_0} \frac{1}{x_s} \cos \psi \right] \qquad (25)$$

As pointed out in the beginning of this paper, this is not the P_s to be used in problems of hunting because it does not take into account the field transient. It does, however, represent the lowest limit of the value which P_s would approach if the forced oscillations were to take place very slowly—so slowly that the armature reaction and consequently the synchronous reactance would have time to function completely and there would be no field transient.

The Transient P_s of the Synchronous Motor Neglecting Resistance.[6]

If a machine which has been operating under steady load at displacement ψ is suddenly loaded so that its displacement increases by $\Delta\,\psi$ almost instantaneously, just what happens?

It is at once apparent from the vector diagram, Fig. 3, that the $\underset{\cdot}{I}\, z_s$ vector is increased. This means an increase in the power, current, and armature reaction of the machine. Disregarding the resistance for the present, the $\underset{\cdot}{I}\, z_s$ drop may be considered as made up of two components $i_1\, x_s$ and $i\, x_s'$ according to Blondel's theory of two reactions. These components are shown in Fig. 3. The reactance x_s' which operates on the energy component of current which is i probably acts almost instantaneously. At least it is probable that the cross flux can build up and down as fast as any of the forced oscillations encountered in cases of hunting would require. The main flux, however, requires time to build up or down and so x_s cannot function instantly, or rather that part of x_s which is due to armature reaction cannot function instantly. Hence, the reactance which operates on a sudden increase in the wattless current is some transient or effective reactance which may be denoted by x_e and which lies between x, the real reactance, and x_s, the synchronous reactance, its value depending on the rate of change of the wattless current.

The power given by the motor, corresponding to an increase in the displacement from its average value of ψ to $(\psi + \Delta\,\psi)$, can be calculated as follows under the above assumptions. The method is simply to obtain expressions for $\underset{\cdot}{E}_0$ and $\underset{\cdot}{I}_0$ and telescope the vectors.

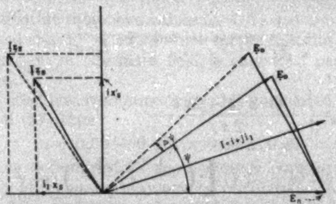

Fig. 3—Vector Diagram of Synchronous Motor Showing Effect of Sudden Increase in Load

From Fig. 3,

$$\underset{\cdot}{E}_0 = E_0 \left[\cos (\psi + \Delta\,\psi) + j \sin (\psi + \Delta\,\psi) \right] \qquad (26)$$

and before the displacement occurred

$$\underset{\cdot}{I}\, z_s = - \left[E_n - E_0 \cos \psi \right] + j\, E_0 \sin \psi \qquad (27)$$

Therefore, disregarding the resistance, the initial value of the current, corresponding to the average displacement ψ, is from (**27**)

$$\underset{\cdot}{I} = \frac{E_0 \sin \psi}{x_s'} + j \frac{E_n - E_0 \cos \psi}{x_s} \qquad (28)$$

The real component of $\underset{\cdot}{I}$ corresponding to the increased displacement $(\psi + \Delta\,\psi)$ is

$$i = \frac{E_0 \sin (\psi + \Delta\,\psi)}{x_s'} \qquad (29)$$

The *increase* in the wattless component is

$$\Delta\, i_1 = \frac{E_0 \left[\cos \psi - \cos (\psi + \Delta\,\psi) \right]}{x_e} \qquad (30)$$

Note that the effective reactance is used here, since, as was shown above, it is the effective reactance which limits an increase in the wattless current. The wattless component of current corresponding to the increased displacement is then

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[6] A complete derivation taking resistance into account is given in the appendix.

$$i_1 = \frac{E_n - E_0 \cos \psi}{x_s} + \frac{E_0}{x_e} \left[\cos \psi - \cos (\psi + \Delta \psi) \right] \tag{31}$$

or

$$i_1 = E_0 \left\{ \left(\frac{1}{x_e} - \frac{1}{x_s} \right) \cos \psi - \frac{1}{x_e} \cos (\psi + \Delta \psi) + \frac{E_n}{E_0 \, x_s} \right\} \tag{32}$$

The power at displacement $(\psi + \Delta \psi)$ is therefore

$$P = \frac{E_0^2 \cos (\psi + \Delta \psi) \sin (\psi + \Delta \psi)}{x_s'} + E_0^2 \sin (\psi + \Delta \psi) \left\{ \left(\frac{1}{x_e} - \frac{1}{x_s} \right) \cos \psi - \frac{1}{x_s} \cos (\psi + \Delta \psi) + \frac{E_n}{E_0 \, x_s} \right\} \tag{33}$$

or

$$P = E_0^2 \left\{ \frac{\sin 2 (\psi + \Delta \psi)}{2 \, x_s'} + \sin (\psi + \Delta \psi) \left[\left(\frac{1}{x_e} - \frac{1}{x_s} \right) \cos \psi + \frac{E_n}{E_0 \, x_s} - \frac{1}{x_e} \cos (\psi + \Delta \psi) \right] \right\} \tag{34}$$

Differentiating P with respect to $\Delta \psi$, we obtain the rate of change of power with respect to an increase in the displacement which is P_s. Thus

$$P_s = E_0^2 \left\{ \left(\frac{1}{x_s'} - \frac{1}{x_e} \right) \cos 2 (\psi + \Delta \psi) + \cos (\psi + \Delta \psi) \left[\left(\frac{1}{x_e} - \frac{1}{x_s} \right) \cos \psi + \frac{E_n}{E_0 \, x_s} \right] \right\} \tag{35}$$

which is P_s at any displacement $(\psi + \Delta \psi)$.

At ψ

$$P_s = E_0^2 \left\{ \left(\frac{1}{x_s'} - \frac{1}{x_e} \right) \cos 2 \psi + \left(\frac{1}{x_e} - \frac{1}{x_s} \right) \cos^2 \psi + \frac{E_n}{E_0 \, x_s} \cos \psi \right\} \tag{36}$$

This is the value of P_s which should be used in all problems of hunting, and is represented by the slope of the dotted line in Fig. 1.

It still remains to calculate the value of x_e to be used in any given case. It has been shown that, in value, x_e lies in between x and $x_s = x + m/c$, where x is the real reactance and m/c is the armature reaction part of the synchronous reactance. Now when a motor is hunting, the demagnetizing component of armature reaction is a pulsating reaction against the field. If the pulsations take place slowly enough, the armature reaction will cause the field flux to pulsate the same amount. If, however, the pulsations are more rapid, the field flux will not have time to follow the pulsations in the armature reaction.

The problem then is merely to find out how much effect the armature reaction has on the field flux. This is a measure of the effectiveness of the m/c term in the synchronous reactance x_s. For very high speed pulsations $m/c = 0$. For very slow speed pulsations $m/c = m/c$, and in general,

$$x_e = x + K' \, m/c \tag{37}$$

where K' is the factor of *effectiveness* of *armature reaction* in destroying and building up the field flux in the main path.

K' may be determined from the following considerations:

Referring to Fig. 4, let the pulsating flux provided by the demagnetizing component of armature reaction be represented by

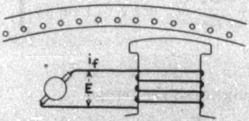

FIG. 4—DIAGRAM ILLUSTRATING PULSATING ARMATURE REACTION

$$\Phi \sin \omega t.$$

where

ω = frequency of the pulsations in radians per sec.

Let

N = No. of field turns per pole.
L = Effective inductance of the field.[7]
r_f = Effective resistance of the field.[7]
K = Coef. of coupling between armature and field.
E = D-c. voltage impressed on the field.

The equation of the field circuit is:

$$E = i_f \, (r_f + p \, L) + N \, K \, \Phi \, \omega \cos \omega \, t. \tag{38}$$

or

$$i_f = \frac{E - N \, K \, \Phi \, \omega \cos \omega \, t}{r_f + p \, L} \tag{39}$$

the permanent solution of which is:

$$i_f = \frac{E}{r_f} - \frac{N \, K \, \Phi \, \omega}{\sqrt{r^2 + L^2 \, \omega^2}} \cos \left(\omega \, t - \tan^{-1} \frac{L \, \omega}{r_f} \right) \tag{40}$$

This is the pulsating field current which exists in a machine when it is hunting. It may be seen from (40) that if ω is small, there is little pulsation in the field current. If, however, ω is larger, there will be larger pulsations.

---

[7] By "effective" resistance and inductance, the writer intends to consider the effect of the amortisseur winding, as will be seen later.

The resultant flux in the field pole can be obtained from the equation,

$$E - i_f r_f = N \frac{d \varphi_r}{d t} \tag{41}$$

or

$$\varphi_r = \frac{1}{N} \int E - i_f r_f \tag{42}$$

From equation (**40**)

$$E - i_f r_f = \frac{N r_f K \Phi \omega}{\sqrt{r_f^2 + L^2 \omega^2}} \cos \left(\omega t - \tan^{-1} \frac{L \omega}{r_f} \right) \tag{43}$$

$$\therefore \varphi_r = \frac{r_f K \Phi}{\sqrt{r_f^2 + L^2 \omega^2}} \sin \left(\omega t - \tan^{-1} \frac{L \omega}{r_f} \right) + C \tag{44}$$

The constant of integration in this case is the steady component of the resultant flux φ_r. From (**44**) it follows that the magnitude of the a-c. component of the resultant flux is

$$\Phi_r = \frac{r_f K \Phi}{\sqrt{r_f^2 + L^2 \omega^2}} \tag{45}$$

Now when ω is very small, the armature reaction is 100 per cent effective and

$$\Phi_r = K \Phi$$

$$\therefore K' = \frac{r_f K \Phi}{\sqrt{r_f^2 + L^2 \omega^2}} \div K \Phi = \frac{r_f}{\sqrt{r_f^2 + L^2 \omega^2}} \tag{46}$$

or

$$K' = \frac{1}{\sqrt{1 + \left(\frac{L}{r_f} \right)^2 \omega^2}} \tag{47}$$

In this formula $\left(\frac{L}{r} \right)$ refers to the field alone and is the time constant which determines the field transient occurring when the field circuit is closed on a constant voltage source. If this time constant is determined by an oscillogram of the starting current in the field, the effect of the amortisseur winding is automatically taken into consideration. K' is thus determined and from it x_e may be calculated by equation (**39**).

SECTION III. EXAMPLES AND CONCLUSIONS

To determine the relative importance of the various factors which affect the value of P_s, consider a typical low-speed synchronous motor, the constants of which are:

$$\left.\begin{array}{ll} x_s = 0.85 & r = 0.06 \\ x_s' = 0.60 & x = 0.30 \\ & m/c = 0.55 \end{array}\right\} \tag{48}$$

$$\frac{L}{r_f} = 0.21 \quad \ldots \text{(from oscillogram of field transient)}$$

Assume that the speed of the above motor is 80 rev. per min. and that it is direct-connected to a single-cylinder, double-acting compressor. A compressor of this type has two peaks in its torque curve every revolution so that the speed of the impressed pulsations is 160 per min. or $\omega = 16.75$ radians per sec.

From (**37**)

$$x_e = x + K' \frac{m}{c} \tag{37}$$

where

$$K' = \frac{1}{\sqrt{1 + \left(\frac{L}{r_f} \right)^2 \omega^2}} \tag{47}$$

Substituting the above values for L/r_f and ω in (**47**), it is found that $K' = 0.273$ which means that the armature reaction in the main field is only 27.3 per cent effective.

The effective reactance may be calculated from equation (**37**) thus:

$$x_e = 0.30 + 0.273 \times 0.55 = 0.45 \tag{48}$$

Having determined all the necessary constants, the behavior of the motor may now be calculated. Assume the machine to be operating at full-load unity power factor so that $\theta = 0$. The angular displacement may be calculated from (**13**).

$$\tan \delta = \frac{0.60 - 0}{1 - 0.06} = 0.638 \tag{49}$$

Since

$$\theta = 0, \quad \psi = \delta = 32.6 \text{ deg.}$$

The nominal voltage at full-load unity power factor may be determined from equation (**15**). Thus

$$E_n = 0.843 - (0.06 \times 0.843 - 0.85 \times 0.54) = 1.26 \tag{50}$$

Resumé of constants necessary for the calculation of P_s at full-load unity power factor, 80 rev. per min., single-cylinder double-acting compressor:

$$\begin{array}{ll} x_s = 0.85 & r = 0.06 \\ x_s' = 0.60 & x_e = 0.45 \\ \psi = 32.6 \text{ deg.} & E_n = 1.26 \end{array} \tag{51}$$

Neglecting resistance, the value of P_s may be obtained by substituting (**51**) in equation (**36**).

$$P_s = \left(\frac{1}{0.60} - \frac{1}{0.45} \right) 0.42 + \left(\frac{1}{0.45} - \frac{1}{0.85} \right) 0.71 + \frac{1.26}{0.85} \times 0.843 = 1.76\text{[8]} \tag{52}$$

Effect of Frequency of Load Pulsations on P_s.

If the motor had been driving a two-cylinder, double-acting compressor, the torque curve of which would have four peaks per revolution, ω would be twice as great and

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[8] This value of P_s is given on a percentage basis. To get it in kw. it must be multiplied by the kw. rating of the motor.

$$K' = 0.141$$

and

$$x_e = 0.378$$

With this value of x_e, P_s would be somewhat greater, thus:

$$P_s = \left(\frac{1}{0.60} - \frac{1}{0.378}\right) 0.42 + \left(\frac{1}{0.378} - \frac{1}{0.85}\right) 0.71 + \frac{1.26}{0.85} \times 0.843 = 1.88 \qquad (53)$$

Thus it is evident that the value of P_s for this case is about 7 per cent greater when the motor is used to drive a two-cylinder, double-acting compressor, than it is when driving a single-cylinder, double-acting machine. In higher speed motors this difference will be still less so that, while P_s is not entirely independent of the nature of the load, it is approximately so, for compressors of the conventional types and speeds.

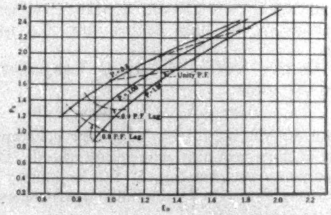

FIG. 5—CURVES SHOWING SYNCHRONIZING POWER, P_s, AS FUNCTION OF THE NOMINAL VOLTAGE, E

Effect of Resistance on P_s.

The effect of the resistance on the value of P_s may be determined by calculating P_s from equation (**20**), given in appendix, which gives

$$P_s = 1.81 \qquad (54)$$

This represents an increase of only 2½ per cent in the value of P_s obtained by neglecting resistance. It should also be noted that 6 per cent resistance is unusually high and hence it may be concluded that for practical work the effect of the resistance may be disregarded.

Effect of Excitation on P_s.

Fig. 5 shows a family of three curves which give P_s as function of the excitation E_n, for 1¼ load, full load, and half load.[9] It may be seen at a glance that the relationship is practically linear for a given load, except at the lower end where the curves droop as the motor approaches the breakdown point. At no load the function would theoretically be a straight line.

The dotted lines are constant power-factor curves and show roughly the power factor which will be obtained for various values of excitation and load.

The half-load line is of particular interest. There is very little tendency for the curve to droop even at the lower extremity. This shows that the motor may be operated at half load at a power factor as poor as 0.7 lagging or even less without danger of the motor falling out of step. In fact, it would be possible to reduce P_s to a value as low as 1.1 at half load by reducing the excitation sufficiently.

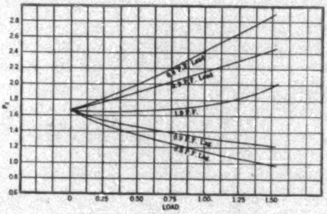

FIG. 6—CURVES SHOWING EFFECT OF CHANGES IN LOAD ON THE VALUE OF THE SYNCHRONIZING POWER, P_s

This fact is made use of in connection with the unloading of two-cylinder, double-acting compressors by removing one connecting rod. Ordinarily the flywheel is so designed that the natural frequency corresponds to about twice the revolutions. This is all right for two-cylinder, double-acting compressors since there is no second harmonic in the torque curve. But if the machine is unloaded there is a large second harmonic and, if severe hunting is to be prevented, the natural frequency must be brought below it. This can be done by reducing P_s and running the motor at a poor lagging power factor. While such operating is not to be recommended as standard practise, it is valuable to know that it may be resorted to in emergency cases.

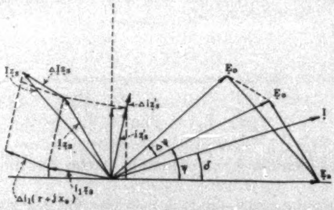

FIG. 7—VECTOR DIAGRAM OF SYNCHRONOUS MOTOR SHOWING EFFECT OF SUDDEN INCREASE IN LOAD. (RESISTANCE TAKEN INTO ACCOUNT)

Effect of Load on P_s.

Fig. 6 shows another family of curves which gives P_s

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[9] The values used in plotting the curves in Figs. 5 and 7 were obtained by substituting (**51**) in equations (**13**), (**15**), and (**36**).

as function of load for various power factors. At a slightly lagging power factor P_s is almost unaffected by the load, while at 0.8 leading power factor there is a decided increase in P_s with load. At 0.8 lagging power factor there is a decided decrease to half load after which the value remains practically the same.

Range of P_s.

In calculating the value of P_s for a number of engine-type, slow-speed synchronous motors, the writer has found that the value is practically always between 1.6 and 2.2 at full load unity power factor. In general, P_s is low in motors of large size and very slow speed, but higher in motors of smaller size and higher speed. Its value depends largely on the real reactance of the machine and on the ratio of the no-load excitation to the full-load armature reaction. A large reactance and low ratio tend to make a low value of P_s.

Appendix

The Transient P_s of the Synchronous Motor Taking Resistance into Account.

It has been shown in Equations (**16**) and (**17**) that under ordinary steady conditions

$$I = i + j\, i_1$$

where

$$i = \frac{r\,(E_0 \cos\psi - E_n) + x_s\, E_0 \sin\psi}{r^2 + x_s x_s'} \qquad (1)$$

and

$$i_1 = \frac{r\, E_0 \sin\psi - x_s'\,(E_0 \cos\psi - E_n)}{r^2 + x_s x_s'} \qquad (2)$$

Now suppose ψ is suddenly increased to $(\psi + \Delta\psi)$. Just as before, there is an increase in the $\underset{\cdot}{I}\,\underset{\cdot}{z_e}$ drop as shown in Fig. 7 which is

$$\Delta\, \underset{\cdot}{I}\,\underset{\cdot}{z_e} = E_0 \{ -[\cos\psi - \cos(\psi + \Delta\psi)] + j\,[\sin(\psi + \Delta\psi) - \sin\psi] \} \qquad (3)$$

This increase in $\underset{\cdot}{I}\,\underset{\cdot}{z_e}$ is made up of two $i\,z$ drops. One is due to the real component of $\underset{\cdot}{I}$, which is

$$\Delta i\,(r + j\, x_s') \qquad (4)$$

since it is assumed that the cross armature reaction flux has time to function. The other $i\,z$ drop is due to the wattless component of current i_1, and is

$$j\, \Delta i_1\,(r + j\, x_e) \qquad (5)$$

where x_e is the effective reactance.

Hence, from the above statement and equations (**4**) and (**5**),

$$\Delta\, \underset{\cdot}{I}\,\underset{\cdot}{z_e} = \Delta i\, r - \Delta i_1\, x_e + j\,(\Delta i\, x_s' + \Delta i_1\, r) \qquad (6)$$

Equating real and imaginary parts of equations (3) and (**6**),

$$\Delta i\, r - \Delta i_1\, x_e = E_0\,[\cos(\psi + \Delta\psi) - \cos\psi] \qquad (7)$$

$$\Delta i\, x_s' + \Delta i_1\, r = E_0\,[\sin(\psi + \Delta\psi) - \sin\psi] \qquad (8)$$

Therefore

$$\Delta i = \frac{E_0}{r^2 + x_s' x_e} \{ x_e\,[\sin(\psi + \Delta\psi) - \sin\psi] + r\,[\cos(\psi + \Delta\psi) - \cos\psi] \} \qquad (9)$$

and

$$\Delta i_1 = \frac{E_0}{r^2 + x_s' x_e} \{ x_s'\,[\cos\psi - \cos(\psi + \Delta\psi)] + r\,[\sin(\psi + \Delta\psi) - \sin\psi] \} \qquad (10)$$

By adding Δi and Δi_1 to the original components of current, the new components are obtained.

$$i = \frac{E_0}{r^2 + x_s x_s'} \left\{ r\left(\cos\psi - \frac{E_n}{E_0}\right) + x_s \sin\psi \right\} + \frac{E_0}{r^2 + x_s' x_e} \{ x_e\,[\sin(\psi + \Delta\psi) - \sin\psi] - r\,[\cos\psi - \cos(\psi + \Delta\psi)] \} \qquad (11)$$

and

$$i_1 = \frac{E_0}{r^2 + x_s x_s'} \left\{ r \sin\psi - x_s'\left(\cos\psi - \frac{E_n}{E_0}\right) \right\} + \frac{E_0}{r^2 + x_s' x_e} \{ x_s'\,[\cos\psi - \cos(\psi + \Delta\psi)] + r\,[\sin(\psi + \Delta\psi) - \sin\psi] \} \qquad (12)$$

The new voltage is:

$$E_0 \{ \cos(\psi + \Delta\psi) + j \sin(\psi + \Delta\psi) \} \qquad (13)$$

Telescoping the current and voltage, the power is obtained.

$$\begin{aligned} P = {} & E_0^2 \cos(\psi + \Delta\psi) \left[\frac{r\left(\cos\psi - \frac{E_n}{E_0}\right) + x_s \sin\psi}{r^2 + x_s x_s'} + \frac{x_e\{\sin(\psi + \Delta\psi) - \sin\psi\} - r\{\cos\psi - \cos(\psi + \Delta\psi)\}}{r^2 + x_s' x_e} \right] \\ & + E_0^2 \sin(\psi + \Delta\psi) \left[\frac{r \sin\psi - x_s'\left(\cos\psi - \frac{E_n}{E_0}\right)}{r^2 + x_s x_s'} + \frac{x_s'\{\cos\psi - \cos(\psi + \Delta\psi)\} + r\{\sin(\psi + \Delta\psi) - \sin\psi\}}{r^2 + x_s' x_e} \right] \end{aligned} \qquad (14)$$

In the above equation let

$$\left. \begin{aligned} & r^2 + x_s' x_e = z_e^2 \\ \text{and} \quad & \\ & r^2 + x_s' x_s = z_0^2 \end{aligned} \right\} \qquad (15)$$

Substituting (**15**) in (**14**) and reducing,

$$\begin{aligned} P = {} & E_0^2 \cos(\psi + \Delta\psi) \left[\frac{-r\,(x_s - x_e)}{z_0^2\, z_e^2} \{ -r \sin\psi + x_s' \cos\psi \} + \frac{1}{z_e^2} \{ x_e \sin(\psi + \Delta\psi) + r \cos(\psi + \Delta\psi) \} - \frac{r\, E_n}{z_0^2 E_0} \right] \\ & + E_0^2 \sin(\psi + \Delta\psi) \left[\frac{x_s'(x_s - x_e)}{z_0^2\, z_e^2} \{ x_s' \cos\psi - r \sin\psi \} \right. \end{aligned}$$

$$+ \frac{1}{z_e^2} \{ r \sin (\psi + \Delta \psi) - x_s' \cos (\psi + \Delta \psi) \}$$

$$+ \frac{x_s'}{z_0^2} \frac{E_n}{E_s} \Bigg] \qquad (16)$$

In Equation (**16**), let

$$C = \frac{(x_s - x_e)}{z_0^2 \, z_e^2} \{ x_s' \cos \psi - r \sin \psi \} \qquad (17)$$

Substituting (**17**) in (**16**),

$$P = E_0^2 \cos (\psi + \Delta \psi) \Bigg[- r\, C + \frac{1}{z_e^2}$$

$$\{ x_s \sin (\psi + \Delta \psi) + r \cos (\psi + \Delta \psi) \} - \frac{r\, E_n}{z_0^2\, E_0} \Bigg]$$

$$+ E_0^2 \sin (\psi + \Delta \psi) \Bigg[x_s' \, C + \frac{1}{z_e^2}$$

$$\{ r \sin (\psi + \Delta \psi) - x_s' \cos (\psi + \Delta \psi) \} + \frac{x_s' \, E_n}{z_0^2\, E_0} \Bigg] \qquad (18)$$

Differentiating P with respect to $\Delta \psi$ to get P_s,

$$P_s = E_0^2 \left\{ \left(C + \frac{E_n}{z_0^2\, E_0} \right) [x_s' \cos (\psi + \Delta \psi) \right.$$

$$\left. + r \sin (\psi + \Delta \psi) + \frac{x_e - x_s'}{z_e^2} \cos 2 (\psi + \Delta \psi) \right\} \qquad (19)$$

At displacement ψ, (**19**) becomes

$$P_0 = E_0^2 \left\{ \left(C + \frac{E_n}{z_0^2\, E_0} \right) [x_s' \cos \psi + r \sin \psi] \right.$$

$$\left. + \frac{x_e - x_s'}{z_e^2} \cos 2 \psi \right\} \qquad (20)$$

which is the required value of P_s, taking resistance into account. In this formula, c is given by equation (**17**), and z_e and z_0 by equation (**15**).

If the resistance terms are disregarded,

$$C = \frac{(x_s - x_e)}{x_s \, x_s' \, x_e} \cos \psi \qquad (21)$$

Substituting (**21**) in equation (**20**) and neglecting r, equation (**20**) degenerates into

$$P_s = E_0^2 \left\{ \left(\frac{1}{x_e} - \frac{1}{x_s} \right) \cos^2 \psi \right.$$

$$\left. + \left(\frac{1}{x_s'} - \frac{1}{x_e} \right) \cos 2 \psi + \frac{E_n}{E_0\, x_s} \cos \psi \right\} \qquad (22)$$

which is the expression obtained previously for the case of no resistance, Eq. (**36**).

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Discussion

F. E. Terman: The results of Mr. Putman's carefully made calculations will be partially upset if the motor is fed by lines and transformers that have considerable impedance. This is because Mr. Putman's derivation assumes a constant voltage at the motor terminals, and this constant terminal voltage is not obtained if the supply lines and transformers have much impedance. In such a case the surging line current causes a varying voltage drop in the line. The effect of the line impedance can be taken into account in Mr. Putman's result by interpreting the motor terminal voltage to be the voltage nearest the motor that is substantially constant during the surging, and interpreting the so-called "real armature impedance" as the actual armature impedance plus the line and transformer impedance between the motor and the point of constant voltage.

Since the supply line characteristics are likely to be beyond the designer's control, the most carefully designed motor cannot be satisfactory under all conditions. It is the old story over again, to the effect that the performance of a piece of electrical apparatus cannot be obtained with 100 per cent accuracy without considering the characteristics of the network with which the apparatus is associated.

F. K. Brainard: A considerable part of Mr. Putman's paper is devoted to the apparent increase in synchronizing power of a synchronous machine as the frequency of the impulses increases. Mr. Putman calculates values of P_s corresponding to each harmonic in the tangential effort curve of mechanical power but unfortunately does not state just how he will use them in obtaining the electrical power pulsation.

Presumably he will use equations similar to those which he derived in his articles which appeared in the *Journal* of the Franklin Institute, May and June, 1924. In the case of a single machine connected to an infinite system, these equations result from the solution of the following differential equation:

$$I \frac{d^2 \psi}{dt^2} + T_d \frac{d \psi}{dt} + G \psi = f(t) \qquad (1)$$

where

ψ = Displacement angle.

I = Moment of inertia of the revolving parts.

T_d = Damping torque per mechanical radian per second change in displacement.

G = Torque per mechanical radian displacement.

$f(t)$ = Torque at the shaft.

This equation is the basis of all previously published work along this line so far as the writer knows, but it does not take account of damping correctly. In fact, the whole phenomenon is somewhat more complicated than this equation indicates, and it has occurred to the writer that it may be the inaccuracy of the solution based on the above equation which causes the discrepancies which Mr. Putman has undoubtedly observed and which he has attempted to take into account by means of a P_s which varies with the frequency.

The trouble is that the damping torque is assumed proportional to $\frac{d\psi}{dt}$ which is the rate of change of the total displacement angle of the machine. It is quite obvious that this cannot be correct since all that the damping can possibly do is to prevent the shifting of the flux relative to the poles. It cannot by any possibility limit the displacement between internal and external voltages resulting from the impedance of the armature. In other words it acts on only a part rather than the whole of the displacement.

It seems to the writer that this part of Mr. Putman's paper is not quite correct especially if he uses the solution of Eq. (1) to compute power pulsations for the following reasons:

1. It involves taking account of damping twice since the effect of the induced currents in the field which Mr. Putman calculates is really damping to a considerable extent and T_d in Eq. (1) is also a damping factor.

2. As stated above, Eq. (1) and anything resulting from its solution is incorrect.

It would seem much more logical and it is certainly much simpler to lump the entire effect of damping into one factor, which will include the effect of the induced currents in the entire field structure, and assume that the damping torque is proportional to the product of this factor (which we will call H) and the rate of change of the flux *relative to the poles.*

This leads to the following formulas from which the pulsations in electrical power can be computed if the analysis of the mechanical power curve is known.

$$Q_n = \sqrt{\frac{G_1^2 + (\omega_n H)^2}{\left[G_1 \Sigma \omega_n^2 I\left(1 + \frac{G_1}{G_2}\right)\right]^2 + \left[\omega_n H - \frac{\omega_n^3 H I}{G_2}\right]^2}} \tag{2}$$

(See derivation at end of discussion).

$$\gamma_n = \tan^{-1}\frac{\omega_n H \Sigma \frac{\omega_n^3 H I}{G_2}}{G_1 - \omega_n^2 I\left(1 + \frac{G_1}{G_2}\right)} - \tan^{-1}\frac{\omega_n H}{G_1} \tag{3}$$

where

Q_n = Ratio of electrical to mechanical power of the nth harmonic in the power curve.

G_1 = Torque per mechanical radian shift of flux across the poles.

G_2 = Torque per mechanical radian displacement between internal and external voltages.

H = Damping torque per mechanical radian per second shift of flux across the poles.

ω_n = 2 π times the frequency of the nth harmonic.

γ_n = Phase diff. between internal and external voltages.

I = Moment of inertia of revolving parts.

In order to compute G_1 and G_2 it is of course necessary to separate the displacement angle ψ into its two parts, *viz:* the displacement between internal and external voltages, and the displacement of the flux relative to the poles.

Perhaps the problem can be most easily understood by reference to the mechanical analogy of the forced oscillation of a weight suspended by a spring. A pulsating force P corresponding to the pulsation in torque required by the compressor results in pulsations in the reaction of the support P_1 which corresponds to the power drawn by the motor from the line. This is the case without damping.

The usual assumption concerning damping mentioned above is that the damping can be represented by a dash-pot in parallel with the entire spring as in Fig. 1 the damping force being proportional to the velocity of W. This leads to Eq. (1) above and is obviously incorrect since, as stated previously, the dampers act on only a part of the displacement.

A more accurate analogy is where the dash-pot is in parallel with the lower part of the spring only as in Fig. 2. The displacement of the upper part of the spring corresponds to the displacement between internal and external voltages while the displacement of the lower part corresponds to the displacement of flux relative to the poles and this later displacement is the part which the dampers restrict. Eq. (2) and (3) give the solution corresponding to this condition.

The apparent increase in synchronizing torque with the frequency of the pulsation can be readily understood from Fig. 2.

As the frequency increases, the displacement of the lower part of the spring decreases, the upper part taking more of the load and hence there is more load per unit total displacement than for low frequencies. If the frequency becomes very large, while the weight is reduced so that the accelerating forces remain finite, the deflection of the lower part of the spring becomes very small, practically the entire deflection occurring in the upper part of the spring.

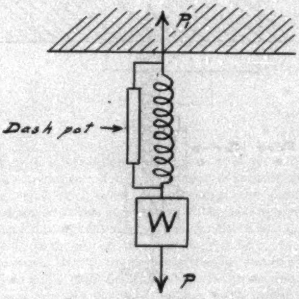

FIG. 1

This is shown by putting a ω_n H infinite in equation (2) which then becomes

$$Q_n = \frac{G_2}{G_2 - \omega_n^2 I}$$

This is the equation for undamped oscillations with resonance at $\omega = \sqrt{\frac{G_2}{I}}$.

If the damping is zero, Eq. (3) becomes

$$Q_n = \frac{G}{G - \omega_n^2 I}$$

Resonance is now at $\omega = \sqrt{\frac{G}{I}}$.

In other words, as the damping or the frequency varies between zero and infinity, the synchronizing torque apparently changes from G to G_2.

This agrees in a general way with Mr. Putman's conclusions and does not require the idea of a P_s which varies with the frequency.

The writer has found also that if T_d in Eq. (1) is calculated from the straight part of the speed-torque curve of the machine in question near synchronism, as mentioned by Mr. Putman in his Franklin Institute articles referred to above, the value of T_d is entirely too great to account for the pulsations observed in operation; but if this same value is used as H in Eq. (2) and (3), the observed results check the calculations much more accurately. The test data which the writer has covering this point are very meager, however, but seem to be correct.

The above method is not beyond criticism since the following assumptions are made which are not strictly correct:

1. The armature losses are neglected.
2. The total displacement, as well as the separate parts of the displacement, are assumed proportional to the torque under steady load over the range of load considered.
3. The direct magnetizing and demagnetizing effect of the current induced in the field winding, which Mr. Putman stresses, is taken account of only as it is included in the damping factor H.

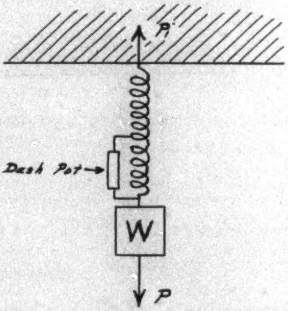

FIG. 2

The first and second assumptions are obviously approximations, but if suitable values of P_s, P_{s1} and P_{s2} are taken, the error from these sources is probably not large.

The error from the third assumption is probably not so very great, since the induced currents in the damper winding have a direct magnetizing or demagnetizing component as well as the current induced in the coils. In fact, the whole effect of the induced currents in both the dampers and the exciting winding on the field poles is to attempt to hold the flux constant in magnitude and fixed in position relative to the poles and so it seems that the error resulting from including the whole effect in the one factor H cannot be so very great.

The following is the derivation of Eqs. (2) and (3):

Let

T = Torque corresponding to mechanical power delivered by the motor which it is assumed can be expressed by a Fourier series of the form $f(t) = a_0 + \Sigma\ [a_n \cos(\omega_n t + \alpha_n)]$.

T_1 = Torque corresponding to the electrical power delivered to the motor which can also be expressed by a Fourier series of the form $f'(t) = b_0 + \Sigma\ [b_n \cos(\omega_n t + B_n)]$.

T_2 = Accelerating torque of revolving parts.

ψ_1 = Displacement of flux relative to the centers of the poles.

ψ_2 = Displacement between internal and external voltages.

$\psi = \psi_1 + \psi_2$ = Displacement of rotor relative to the terminal voltage.

G_1 = Torque per mechanical radian shift of flux across poles.

G_2 = Torque per mechanical radian displacement between internal and external voltages.

$G = \dfrac{1}{\dfrac{1}{G_1} + \dfrac{1}{G_2}}$ = Torque per mechanical radian displacement of rotor relative to terminal voltage.

H = Damping torque per mechanical radian per second shift of flux across the poles.

I = Moment of inertia of revolving parts.

ω_n = 2 π times the frequency of the nth harmonic.

$q_n = \dfrac{b_n}{a_n}$

Then

$$T_1 = H \frac{d\psi_1}{dt} + G_1 \psi_1 \tag{4}$$

Since

$$T_1 = G_2 \psi_2,\ \psi_2 = \frac{H}{G_2}\frac{d\psi_1}{dt} + \frac{G_1}{G_2}\psi_1$$

Then

$$\psi = \psi_1 + \psi_2 = \left(1 + \frac{G_1}{G_2}\right)\psi_1 + \frac{H}{G_2}\frac{d\psi_1}{dt}$$

Hence

$$\frac{d^2\psi}{dt^2} = \left(1 + \frac{G_1}{G_2}\right)\frac{d^2\psi_1}{dt^2} + \frac{H}{G_2}\frac{d^3\psi}{dt^3}$$

$$T_2 = -I\frac{d^2\psi}{dt^2} = -I\left(1 + \frac{G_1}{G_2}\right)\frac{d^2\psi_1}{dt^2} + \frac{H\,I}{G_2}\frac{d^3\psi_1}{dt^3}$$

But

$$T = T_1 - T_2$$

Hence

$$T = \frac{H\,I}{G_2}\frac{d^3\psi_1}{dt^3} + I\left(1 + \frac{G_1}{G_2}\right)\frac{d^2\psi_1}{dt^2} + H\frac{d\psi_1}{dt} + G_1\psi_1$$

Writing this in the operator form, we have

$$\left[\frac{H\,I}{G_2}p^3 + I\left(1 + \frac{G_1}{G_2}\right)p^2 + H\,p + G_1\right]\psi_1 = f(t) \tag{5}$$

Neglecting the transient, the solution for ψ_1 consists of a constant term corresponding to the average torque a_0 plus a variable part corresponding to the series $\Sigma\ [a_n \cos(\omega_n + \alpha_n)]$. The second part represents the hunting due to the harmonics in $f(t)$. Only the second part is of interest since this determines the power pulsations. The transient part of the solution is obtained by letting the right hand member of Eq. (5) = 0 and solving for ψ_1, but this represents oscillations which occur for a short time only after the machine is parallel with the system and so do not interest us.

The permanent hunting is easily obtained by letting $p = j\,\omega_n$ n, being the order of the harmonic in $f(t)$.

Eq. (5) then gives

$$\underset{\cdot}{a}_n = \left\{G_1 - \omega_n^2 I\left(1 + \frac{G_1}{G_2}\right)\right\} + j\left\{\omega_n H - \frac{\omega_n^3 H\,I}{G_2}\right\}$$

or in real quantities

$$a_n = \sqrt{\left\{G_1 - \omega_n^2 I\left(1 + \frac{G_1}{G_2}\right)\right\}^2 + \left\{\omega_n H - \frac{\omega_n^3\ H\ I}{G_2}\right\}^2}$$

Similarly from (4)

$$b_n = \sqrt{G_1^2 + (\omega_n H)^2}$$

Dividing (5) by (4), we have

$$Q_n = \sqrt{\frac{G_1^2 + (\omega_n H)^2}{\left\{ G_1 - \omega_n^2 I\left(1 + \frac{G_1}{G_2}\right) \right\}^2 + \left\{ \omega_n H - \frac{\omega_n^3 \; H \; I}{G_2} \right\}^2}} \qquad (2)$$

The phase lag of b_n with respect to a_n will be the difference between their phase angles, and is

$$\gamma_n = \tan^{-1} \frac{\omega_n H - \frac{\omega_n^3 \; H \; I}{G_2}}{G_1 - \omega_n^2 \; I \left(1 + \frac{G_1}{G_2}\right)} - \tan^{-1} \frac{\omega_n H}{G_1} \qquad (3)$$

C. A. Nickle: Some of the assumptions in this paper are sufficiently in error to affect seriously the correctness of the results. The first part of Section II of this paper deals with this problem and the results are in accord with those presented in a paper[1] by R. E. Doherty and the writer. Our treatment of salient-pole machines under transient conditions, however, does not agree with the treatment just presented.

Equation (**34**), according to the author, is the general equation for power for sustained oscillations. This is not justified because it applies only when x_e is a pure reactance, since this was the assumption under which it was derived. But x_e is not a pure reactance for all conditions of oscillation. It approximates a pure reactance when the resistance of the field winding is either negligible or infinite with respect to its leakage reactance at the oscillation frequency, as may be seen from the equivalent transformer diagram shown in Fig. 3 herewith. In this diagram, x is the armature leakage reactance, x_m is the mutual reactance of the field and armature, x_f is the leakage reactance of the field winding, and r_f/s is the field resistance referred to the armature for a frequency of oscillation s times the electrical frequency of the machine.

Inspection of Fig. 3 shows that as s approaches zero—*i. e.*, as steady-state conditions are approached—r_f/s approaches infinity and the field winding is essentially open-circuited. The impedance is then a pure reactance of value $(x + x_m)$ —*i. e.*, the synchronous reactance in the direct axis. As s becomes larger, r_f/s becomes smaller and may become quite negligible even for rather low values of s. As r_f/s approaches zero, the impedance of Fig. 3 again becomes a pure reactance. It is evident, however, that x_e is not the leakage reactance of the armature as concluded by the author, but is the transient reactance, which may be considerably greater in salient-pole machines.

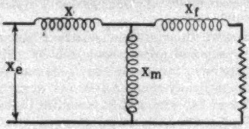

FIG. 3

This may also be interpreted as follows: When a three-phase system of currents of constant amplitude flows in the winding of a three-phase machine, there exists a uniformly rotating sinusoidal m. m. f. of constant amplitude. This m. m. f. rotates at synchronous speed, which is the speed at which the poles are rotating, and by proper time phase of the currents, a at zero power factor, the rotating m. m. f. may be made to exist at every instant directly over the field poles. If each of the three-phase currents pulsates in amplitude at a frequency f, the m. m. f. over the field winding will likewise pulsate at the same frequency. This pulsating m. m. f., acting over the field winding short-circuited through the exciter armature, will induce currents in this winding and we have essentially a transformer with the secondary short-circuited[2]. The magnetic and electric circuits of such a transformer are shown in Fig. 4. In this figure, p_1 is the permeance of the leakage paths of the armature, p_2 is the permeance of the mutual flux path of the field and armature, and p_3 is the permeance of the leakage paths of the field winding. The alternating component of armature m. m. f. is now represented by an alternating current in the winding a, the frequency of this current being the same as the frequency of pulsation of armature m. m. f. in the actual machine. If this frequency is infinitely small, the armature m. m. f. may send flux through the permeance p_1, which corresponds to the permeance of the leakage paths of the armature, and through the permeance p_2, which is the permeance of the mutual path of the armature and field. At this lower limit of frequency, due to the field resistance, flux can be created and destroyed in the field winding without opposition. Negligible flux passes through the field leakage path under this condition, since it is shunted by a path entirely of iron of relatively very great permeance. The reactance of the armature winding is then evidently the synchronous reactance. When, however, the frequency of the current in winding a is high enough so that the resistance of the field winding is negligible with respect to its leakage reactance, the short-circuited field winding effectively prevents any change of flux linking its self. In this case, the permeance encountered by the armature m. m. f. is not only p_1, the true leakage permeance, but also the permeance p_2 and p_3 in series. The total armature flux is correspondingly greater than just the armature leakage flux and the effective reactance is likewise greater than the armature leakage reactance in the same proportion. This reactance, as distinguished from armature leakage reactance, is termed transient reactance. In commercial salient-pole machines, the transient reactance is of the order of 50 per cent higher than the leakage reactance, and the effect of field leakage cannot be neglected as has been done by the author. Thus, the fact that the alternating component of flux linking the

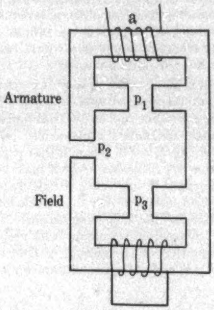

FIG. 4

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[1] *Synchronous Machines*—I, by R. E. Doherty and C. A. Nickle, A. I. E. E. JOURNAL, October 1926, p. 974.

[2] When a machine is oscillating, it delivers or consumes power and a pulsating armature m. m. f. also exists over the quadrature axis. There is, in general, no field winding in this axis, and the flux is always proportional to the current. The reactance in this axis will thus be constant regardless of the frequency of oscillation of the machine—*i. e.*, it will always be x_s'. Although there is no field winding in this axis, there may be an amortisseur winding which acts as a short-circuited secondary. It may, however, be shown that in general the resistance of this winding compared to its leakage reactance is so great at ordinary frequencies of oscillation as to have a negligible influence on the quadrature reactance.

field winding is zero does not mean that armature reaction is entirely ineffective as stated in this paper on the fifth page in the second paragraph. The flux linking the field winding is, therefore, not a correct measure of the effectiveness of armature reaction, and hence the author's expression for the factor K' is invalid.

There is still another point in the calculation of K' which is in error. In deriving the expression for K', the author neglects such time-phase relations which occur, and since this problem differs from the problem of steady-state conditions only in the time element, this is altogether unjustifiable. When the time element is taken into account, the element of damping which exists is readily obtained, as well as a more accurate value of synchronizing power. When the frequency of oscillation is such that the field resistance is neither negligible nor infinite with respect to the field leakage reactance, x_e is not a pure reactance, but an impedance, and Eq. (**34**) does not apply.

In Eq. (**38**), $(N\ K\ \phi\ \omega \cos \omega\ t)$ is the alternating voltage induced in the short-circuited field winding. This voltage is consumed by the resistance and leakage reactance of the winding. On the sixth page, immediately below Eq. (**47**), the method outlined for obtaining L/r is in error, in that it gives the ratio of *exciting* inductance to resistance, which may be several times greater than the proper ratio of *leakage* inductance to resistance. Also, such a method does not take the amortisseur winding into account correctly, for the following reasons: The amortisseur winding is a short-circuited secondary with respect to the field coils, and the equation of the growth of field current when a constant voltage is applied is of the well-known form:

$$i = A + A_1 e^{D_1 t} + A_2 e^{D_2 t}$$

where

$$D_1 = \frac{-(r_1 L_2 + r_2 L_1) + \sqrt{(r_1 L_2 - r_2 L_1)^2 + 4 M^2 r_1 r_2}}{2 (L_1 L_2 - M^2)} \qquad (1)$$

and

$$D_2 = \frac{-(r_1 L_2 + r_2 L_1) - \sqrt{(r_1 L_2 - r_2 L_1)^2 + 4 M^2 r_1 r_2}}{2 (L_1 L_2 - M^2)} \qquad (2)$$

The equation for the field current thus comprises a constant term and two exponential curves, and an oscillogram taken under these conditions cannot be analyzed correctly as a single exponential curve. When the resistance of the secondary is relatively great, D_2 becomes very large and one of the exponential curves in a very short time becomes negligible. The curve then continues as a single exponential, but the value of L/r obtained from this part of the curve is not the correct value to use for the condition where a definite sinusoidal voltage is applied to the field winding.

In the third part of the paper[3], on *Synchronous Machines*, by R. E. Doherty and the writer, our treatment of the present problem is given. When the problem is analyzed with due consideration to the points brought out in this discussion, the expression for synchronizing power becomes:

$$P = P_1 + \frac{a\,(x_s - a) - b^2}{x_s\,[b^2 + (x_s - a)^2]}\ E_0^2 \sin^2 \psi_1 \qquad (3)$$

where P_1 is the synchronizing power under steady-state conditions, ψ_1 is the angular displacement corresponding to the average load, and the factors a and b are functions of the oscillation frequency and may be readily obtained from the constants of the machine. The damping torque per radian per second is also obtained, and is given by:

$$T_d = \frac{b^2 E_0^2 \sin^2 \psi_1}{s\,[b^2 + (x_s - a)^2]} \qquad (4)$$

H. V. Putman: Dr. Terman has pointed out that these calculations assume constant terminal voltage at the machine and if this condition is not fulfilled, the synchronizing power is likely to be very different. This is a point which it is well to remember. I had an interesting experience which illustrates this point clearly. A certain synchronous motor had been directly connected to a particular air compressor. Severe hunting was reported and difficulty was experienced in keeping the motor in step. Investigation showed that the motor was supplied by a diesel-engine-driven generator about 50 per cent larger than the motor. This engine did not have nearly as large a flywheel as it should have had for direct connection to the generator.

Tests showed a current pulsation of about 175 per cent, based on the rated motor current. The pulsation consisted largely of two harmonics, one due to the compressor and the other to the engine.

Calculations showed that to bring the current pulsation down to 60 per cent, five times as much flywheel effect was required on the engine and about twice as much on the motor. It was impossible to add this amount of flywheel effect to the engine, so the trouble was remedied by substituting an induction motor in place of the synchronous motor.

It is not likely a condition as bad as this could occur due to poor regulation of transformers. Such an experience does, however, make one realize that synchronous machines which by themselves may behave quite properly often behave very badly when connected together or to a system having poor regulation.

Mr. Nickle says that my "effective reactance x_e" is not a pure reactance as I assume. Later he says, "However, it is evident that x_e is not the leakage reactance of the armature as concluded by the author, but is the transient reactance which may be considerably greater in salient-pole machines." The term "transient reactance" has been clearly defined by Mr. Doherty and other writers and it is always treated as a pure reactance, so that if my x_e is in reality the transient reactance (which I think it is not) as Mr. Nickle says, then it is at least a pure reactance. Also I did not say that x_e is the leakage reactance of the armature; I said it is the leakage reactance of the armature plus that part of the de-magnetizing armature reactance which is effective under the oscillatory conditions.

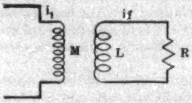

FIG. 5

I am not sure that the equivalent diagram Mr. Nickle gives in his Fig. 3 really represents the present problem. Certainly the field and armature circuits are not directly connected in series. They are only magnetically coupled as shown in my Fig. 5 (this discussion). Before one can justify conclusions based on Mr. Nickle's equivalent, circuit is necessary to prove its equivalence.

Mr. Nickle states that the alternating voltage induced in the short-circuited field winding is consumed by the resistance and leakage reactance of the field winding. I say in the paper that this voltage induced in the field winding is consumed by the resistance and *self*-induction of the field winding. This is an important point as it will make a difference in the value of K'. The point can be demonstrated quite easily by referring to my Fig. 5. In this figure the pulsating current which flows through the armature circuit is represented by i_1. Note that this pulsating armature current flows by virtue of the fact that the compressor to which the motor is connected demands certain pulsating power to drive it. There is only one power line coming into the motor and if the motor is to supply the power required to drive the pulsating compressor load, current must flow through the power line into the motor armature winding. If it does not the motor will stop. Hence, it is evident that this pulsating armature current flows regardless of what goes on in the motor and *this pulsating current is in no way limited or restricted by any of the reactions or reactances within the machine.* It flows because the compressor load demands it.

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[3] *Synchronous Machines*—III. A. I. E. E. Winter Convention, 1927.

This problem is entirely different from the generator short-circuit problem. There the armature current does depend on what goes on inside the machine; in fact it is limited by the combined leakage reactance of armature and field which is the transient reactance, and this is the problem to which Mr. Nickle's diagram is applicable. In Fig. 5, let

L = the self-inductance of the field winding.
R = The resistance of the field.
M = The mutual induction between field and armature.

Now, by Kirchoff's law, let us write the fundamental differential equation of the field circuit. It is obviously

$$0 = i_f\,(R + p\,L) - M\,p\,i_1$$

where

$$p = \frac{d}{dt}$$

The induced voltage produced by the armature current is the term $M\,p\,i_1$ and the field current is

$$i_f = \frac{M\,p\,i_1}{R + p\,L}$$

so that it is easily seen that the pulsating component of the field current is equal to the induced voltage divided by an impedance comprised of the field resistance and the field self-inductance, not the leakage inductance. The field leakage inductance would be

$$(L - M)$$

so that if Mr. Nickle were right, the equation would have to read:

$$i_f = \frac{M\,p\,i_1}{\gamma + p\,(L - M)}$$

which is impossible.

Mr. Nickle also says my calculation of k' is further in error because I have neglected certain time elements which occur, and when these are properly taken into account, it is possible to obtain the damping torque as well as the synchronizing torque. This is not clear to me, but if it is possible to obtain some sort of an expression for the complete torque, (probably a vector expression), one term of which is synchronizing torque and the other damping torque, it would be highly desirable. I shall be very much interested in seeing how Mr. Nickle handles this part of the problem.

Mr. Brainard has submitted a very interesting discussion, explaining why he thinks the synchronizing torque changes with the frequency of the load pulsations. He believes that if the damping is taken into account, properly, in the fundamental equation of the oscillation, this "apparent" change in synchronizing torque is explained without "the idea of a P_s which varies with the frequency."

The equation to which Mr. Brainard refers is $(p^2\,I + p\,T_d + T_s)\,\psi = f\,(t)$. This is his equation (1) except that T_s is used for synchronizing power instead of G. This equation not only has formed the basis of all previous published work along this line, but it has been used successfully for many years in handling problems of oscillation and resonance. Also, many actual comparisons have been made between oscillations calculated from this equation and oscillographic tests which have substantiated, beyond any doubt, its validity within its well-known limitations.

It can be applied to ordinary synchronous machines which, as a rule, have a damping torque of from ½ to 1 per cent of the synchronizing torque. Cases of moderate oscillation can be calculated successfully from it and the magnitude of the oscillation obtained correctly. The shape of the oscillation usually departs somewhat from the oscillographic picture due to the fact that the linear type of differential equation does not apply strictly to the problem. The synchronizing torque is not absolutely proportional to the displacement except for small displacements and the damping torque is not proportional to the rate of change in displacement (either the total displacement or that part of it between the pole and the electrical field) except for very small values of slip. This means that, for a case of very severe oscillation where the magnitude of the oscillation depends to a large extent on the damping, this equation cannot be applied successfully. It will not give the magnitude of the oscillation correctly, but it will at least show if it is very severe and usually this is sufficient. If the oscillation is of such magnitude that it is not objectionable for continued operation, then the magnitude can be calculated very closely.

It should be noted that both these limitations are present in Mr. Brainard's more complicated equation. I say "more complicated" because he is forced to a third-degree equation and has to employ two different values of synchronizing torque. And what is gained by the use of this complicated equation? Nothing, except that the damping torque is made to depend on, (I shall not say proportional to), that part of the displacement between the poles and the electrical field. This is as it should be, but when the proportionality is of such a questionable nature and the whole term is so small anyway, the use of the more complicated equation seems hardly justified.

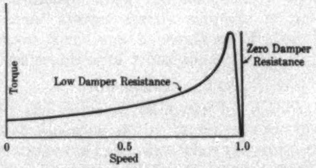

Fig. 6—Diagram Showing How the Speed-Torque Curve of a Motor with Low Damper Resistance Approaches a Straight Line of Finite Height as the Damper Resistance Approaches Zero

Also, the value of resonance $\omega = \sqrt{\frac{G_2}{I}}$, which Mr. Brainard obtains from his equation for the case of infinite damping, has no physical significance. What does infinite damping mean? As Mr. Brainard uses it, it means that the motor develops an infinite torque with an infinitely small slip. This is not only an impossible condition, but it is a condition which cannot even be approached by an actual motor. Even though a motor were made with a damper winding resistance of very small value,—say approaching zero,—the speed-torque curve would approach a vertical line of finite length, not infinite length as Mr. Brainard assumes. (See Fig. 6.) This means that the linear relation between damping torque and slip fails long before anything approaching infinite damping could be obtained, so Mr. Brainard's explanation of a synchronizing torque which apparently changes from G to $G2$ falls through.

I feel very certain that if it is possible to obtain an expression for the complete torque of the motor, including both the damping torque and the synchronizing torque, it will be found that the term representing the synchronizing torque varies with the frequency of the pulsations in a manner independent of the damping torque. Mr. Nickle has suggested the possibility of obtaining such an expression and I hope he will be able to demonstrate this point.

Mr. Brainard says I take damping into account twice, since the effect of the induced currents in the field which I calculate is really damping. I think he does not understand my point of view clearly. When the load increases suddenly on a motor and the angular displacement increases, there are really two voltages induced in the rotor winding, one due entirely to the transformer action between rotor and stator, brought on by the sudden increase in armature current, and the other, a voltage of rotation due to the rotor actually cutting the flux from the stator during the increase in the angular displacement. The effect of the latter voltage is pure damping, while the effect of the former is an increase in the synchronizing power.

Lightning

A Study of Lightning Rods and Cages, With Special Reference to the Protection of Oil Tanks

BY F. W. PEEK, JR.[1]

Fellow, A. I. E. E.

***Synopsis.**—Former papers have discussed the voltage and nature of lightning, lightning voltages on transmission lines, and means of protecting against them, the effect of lightning voltages on insulations, etc. This paper is a further report on this investigation which has been in progress a number of years. Special attention is given here to additional work on the chance of objects being struck; the area protected from direct strokes, by a rod or number of rods, overhead wires, grids, nets, etc.; the effect of nets and cages in reducing induced voltages, etc. This work is particularly applicable to the protection of oil tanks, buildings, magazines, etc.*

INTRODUCTION

THIS paper reports further work which the author has been carrying on for a number of years in an investigation on lightning. The particular phase of the subject treated here is the protection of oil tanks from lightning. Since the data given pertain particularly to a study of direct strokes as well as induced voltages, it can also be applied to the protection of buildings, etc. Former papers have treated of the general subject of lightning, the effect of lightning on transmission lines, line insulation, etc. For more general information and further details as to the methods followed in the investigations, reference should be made to the papers noted below[2].

OIL TANKS AND RESERVOIRS

Oil is frequently stored in very large quantities. This storage is often so great, in fact, that, economically, metal tanks are said not to be feasible. The tanks or reservoirs, which are usually made of re-enforced concrete, are frequently 500 ft. in diameter, but sometimes in oval form as large as 600 by 1200 ft. and 30 ft. deep. Occasionally some of the smaller tanks are of metal. The capacity ranges from seven hundred thousand to three million barrels. A group of tanks makes up a farm.

The tops of the tanks are covered with wood or wood covered with felt or other materials to keep out the sun and to prevent evaporation. Between the roof and the surface of the oil there is an air space which may contain oil vapors. The mixture of air and oil gases may be in the right proportions to be explosive and ignite by a very small spark. Sparks can occur between metal parts on the roof or between wet parts by induction, or by direct strokes, and cause fires or explosions. It is probable that induced voltages can cause fires only when inflammable or explosive gases are present. Direct strokes can set fire to either oil or wood.

Various principles found in the general study of lightning, as well as specific investigations for transmission lines, etc., can be applied to oil-tank and reservoir protection. Work already reported[3] will be outlined here for convenience. Additional work done, specifically on model oil tanks, will be given in greater detail. Although work on models and theoretical work cannot always exactly simulate or anticipate all practical conditions, it should be of great help in solving the problems of protection.

HOW SPARKS OCCUR

Sparks can occur in oil tanks by a *direct stroke*, *electrostatic induction* or by *electromagnetic induction*. A direct stroke can take place when the storm center is over the oil tank and set fire to wooden roofs, ignite oil directly or indirectly, or melt thin, metal plates. Sparks can form between conducting parts by electrostatic induction from storms overhead or at a considerable distance. As already noted, induced sparks can generally cause fires only by the ignition of gases. Electromagnetic induction could cause sparks as a result of heavy currents caused by a direct stroke in the vicinity of the tank.

Direct strokes are by far the most dangerous, but voltages by direct strokes are much less likely to occur than by induction. Induced sparks can occur only between isolated conducting parts or conducting parts in poor contact. These conducting parts could be of metal or water, etc. Trouble by electromagnetic induction is probably the least likely to occur.

INDUCED VOLTAGES

A brief description may be of interest as to how electrostatic induction occurs. Assume the cloud in Fig. 1 to be negatively charged. A well-insulated wire along an equipotential surface takes the potential

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[1] Consulting Engineer, General Electric Co., Pittsfield, Mass.

[2] F. W. Peek, Jr.—The Effect of Transient Voltages on Dielectrics, TRANSACTIONS, A. I. E. E., 1915, Vol. 34, page 1857; 1919 Vol. 38, p. 1137; 1923 Vol. 42, p. 940.

Lightning and Other Transients on Transmission Lines, A. I. E. E., TRANS. 1924, Vol. 43, p. 1205.

F. W. Peek, Jr., High-Voltage Phenomena, *Journal*; Franklin Institute, January, 1924;

Lightning, *Journal*, Franklin Institute, February, 1925.

Presented at the Pacific Coast Convention of the A. I. E. E., Salt Lake City, Utah, Sept. 6-9, 1926.

[3] F. W. Peek, Jr., "High-Voltage Phenomena", *Journal* of Franklin Institute, Jan. 1924. "Lightning", *Journal* of the Franklin Institute, 1925.

of the space in which it is located and becomes (+) on the side nearest the cloud and (−) on the side farthest from the cloud (Fig. 1a). When the cloud discharges, the two charges on the wire go together and the potential of the wire becomes zero. In this case, it is possible for a spark to occur between the insulated wire and some other wire near it but differently located in space, even though the cloud does not discharge. What is usually known as electrostatic induction, however, occurs as follows: Assume the wire in Fig. 1b to be poorly insulated, or grounded; the negative charge leaks away. The wire becomes positively charged, while its potential becomes zero. When the cloud discharges, the bound charge on the wire is released, which causes it to take a potential above ground with a sign opposite to that of the cloud. The wire generally reaches its maximum potential at the instant the cloud reaches zero. The potential that the wire assumes is approximately equal to its height above ground times the voltage gradient or volts per foot, measured vertically. The voltage gradient depends upon the position of the cloud with reference to the wire. The maximum voltage that a wire can assume is equal to 100,000 times its height in feet above ground. This is practically a direct stroke. In general, this voltage is,

$$V = g\,\alpha\,h = G\,h$$

where g is the gradient in volts per foot and h is the height in feet. As shown in Fig. 2, g depends upon the distance of the cloud from the wire. For short wires, α is unity. In the case of long transmission lines, α would be less than unity, its exact value depending upon the rate at which the cloud discharges. This would follow because in slowly discharging clouds the charge would be dispersed over the line for a considerable distance before the cloud became completely discharged. Measured values of $g\,\alpha$, or apparent gradients, G, as high as 50,000 kv/ft., have been obtained on transmission lines. Sparks may occur between wires or other conductors on tanks, due to these induced voltages. Grounded wires near and parallel to the line wire reduce induced voltages; a cage around the wire still further reduces them, while a complete metal cover eliminates them. Ground wires are considered effective on transmission lines if induced voltages are cut in half; this would not be the case of conductors in oil tanks because even 500 volts could cause a tiny spark between metal parts almost in contact. The work on models has shown that tiny sparks can occur between conductors inside of cages even when the mesh is comparatively small. No sparks could be obtained in complete metal tanks. (See table 7). Sparks can occur in metal tanks, however, between plates making

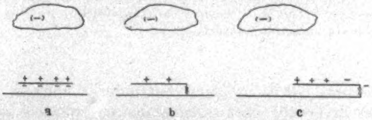
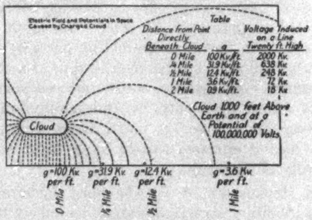


FIG. 1—VOLTAGE INDUCED BY LIGHTNING


Electric Field and Potentials in Space Caused by Charged Cloud
Cloud
Cloud 1000 feet Above Earth and at a Potential of 100,000,000 Volts
g=100 Kv. per ft.
g=31.9 Kv. per ft.
g=12.4 Kv. per ft.
g=3.6 Kv. per ft.
0 Mile
¼ Mile
½ Mile
1 Mile

Table

Distance from Point Directly Beneath Cloud	g	Voltage Induced on a Line Twenty ft. High
0 Mile	100 Kv/ft.	2000 Kv.
¼ Mile	31.9 Kv/ft.	638 Kv.
½ Mile	12.4 Kv/ft.	248 Kv.
1 Mile	3.6 Kv/ft.	72 Kv.
2 Mile	0.9 Kv/ft.	18 Kv.

FIG. 2—ELECTRIC FIELD AND POTENTIAL IN SPACE CAUSED BY CHARGED CLOUDS

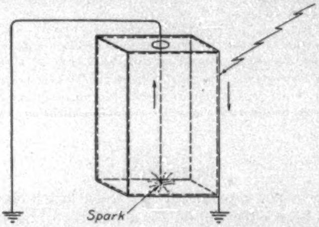


FIG. 3—METHOD OF OBTAINING A SPARK FROM A GROUNDED CONDUCTOR WITHIN A GROUNDED TANK

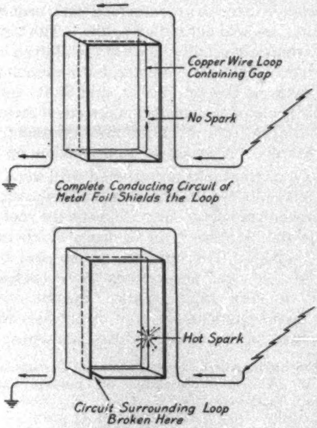

Copper Wire Loop Containing Gap
No Spark
Complete Conducting Circuit of Metal Foil Shields the Loop
Hot Spark
Circuit Surrounding Loop Broken Here

FIG. 4

poor electrical contact or from wires or pipes brought in from the outside and not making good contact with the tank, as in Fig. 3.

Electromagnetic induction resulting from heavy currents flowing in the vicinity of the tank may cause sparks as shown in Fig. 4. In experiments, it was not possible to obtain sparks in the complete metal tank or in the tank with open sides. Sparks were obtained only in the extreme case shown.

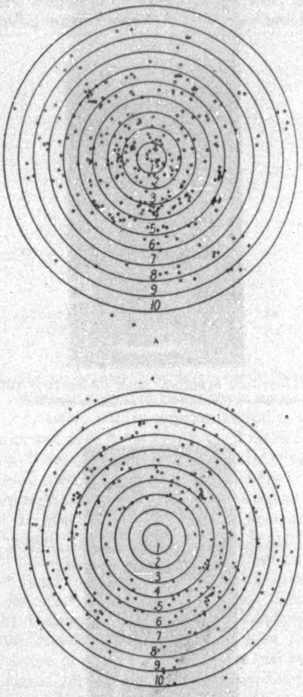


Fig. 5—Target made by Lightning
a. Without Needle b. With ½-in. Needle

Direct Strokes

While the most severe lightning effects are produced by direct strokes, in order to have a direct stroke at a given spot, it is necessary for the cloud to be over that spot at the instant it is charged to sufficient voltage to cause a discharge. Voltages by direct stroke are thus much less likely to occur than voltages by induction since induced voltages can be produced by any storm within a radius of several miles. Sparks by induced voltages cannot occur unless there are conducting parts almost in contact or making poor contact; as wire, nails or wet spots on a wooden roof, etc.

Although the chances of direct strokes are usually small, the effects are so severe that an exhaustive study of protection against them seemed worth while. The general methods followed were similar to those used in previous work. Models were built to scale and subjected to voltage from the lightning generator as well as all other types of voltages. The same general results were obtained from all types of clouds, points, spheres and planes. The hits were recorded by placing paper targets under the tank or other object under study. A hole in the paper recorded the spot struck.

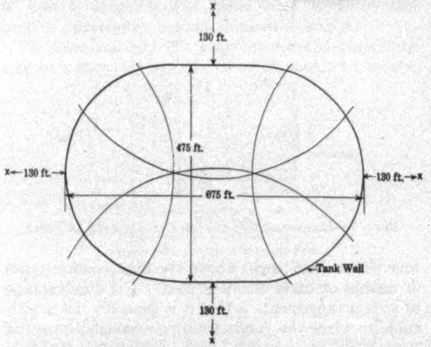


Fig. 6—Diagram Showing Protection of Oil Tank from Direct-Lightning Stroke
Height of roof peak—35 ft.
Crosses denote four 130-ft. poles, 130 ft. from tank wall
Arcs indicate area within the tank wall, protected by each grounded pole

Sample targets are given in Fig. 5. Fig. 5A was made by discharges striking a flat plane; in 5B, there was a rod placed in the center of the target. It will be noted that in this latter case, either the rod was struck or the ground at least four rod lengths away. There was an area immediately around the rod that was not struck. It can be seen that a model building or tank properly located with reference to the rod inside of the protected area would not be struck. It has been found that this same principle applies to a number of rods. For instance, if four rods are arranged at equidistant points around a circle in such a way that their protected areas overlap, a spark never strikes inside of the circle. Thus, a building or tank placed within the circle would be protected from direct hits. It is desirable to place the rods a rod-length away from the tank. Fig. 6 illustrates how this can be applied to an oval tank.

There are other methods of protection which have been investigated and which will be described below. An umbrella frame of conductors, placed over the tank, can be made to take direct hits. Parallel wires strung over the tank would prevent direct hits to the tank provided the wires were not separated more than four times their height above the tank. Other types of meshes or cages could be used. The disadvantage of such arrangements is that it is generally not practicable to place the conductors high enough above the tank to prevent side flashes to the roof. The lightning flash is also brought directly over the tank with the possibility of dropping hot metal, magnetic induction, etc.

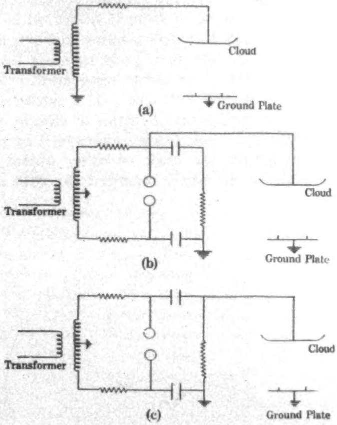


Fig. 7—Lightning Generator Circuit used in Tests

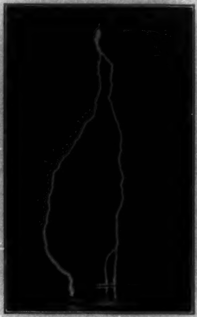


Fig. 8—Lightning Striking Unprotected Oil Tank Without Cover. (Tank Filled with Oil)

Practical Protection

From the investigation it seems that a metal tank offers the only complete protection in oil storage against both direct strokes and induced lightning voltages. The thickness of the metal is not important from the standpoint of induction but from the standpoint of direct strokes must be great enough to prevent melting through. The cover and all other metal parts must be in good electrical contact. This applies especially to parts near together. When explosive or inflammable gases can be eliminated the problem is greatly simplified since it is reduced to the protection of the tank against direct hits. It appears that this can be done by placing pointed rods around the tank. A round tank can be protected by three rods. The method is shown in Fig. 23. No part of the protected area must be a greater distance from some rod than approximately four times the height of the rod. To secure a greater factor of safety it may sometime be advisable to use a ratio of less than four as indi-

Fig. 9—Lightning Striking Rods Protecting Oil Tank without Cover (Tank Filled with Oil)

Fig. 10—Lightning Striking Unprotected Oil Tank with Cover (Tank Filled with Oil)

cated in Fig. 19. It is desirable to locate the rods about a rod's length away from the tank although it is possible that this distance may be as small as half a rod length without trouble. The object of placing the rods in this way is to cause the hits to occur at some distance from the tank and to prevent side flashes. The rods should be grounded to damp earth immediately below. Where the ground resistance is high or uncertain it is probably best to connect the rod to the tank ground as well as to its own ground. Figs. 8, 9, 10 and 11 show tests on protected and unprotected tanks. If guys are used, it is desirable to make them as short as practicable and to attach them to the rod as near the ground as practicable. When there are projections above ground, the height of the rods is increased an amount equal to the height of the highest projection.

Fig. 11—Lightning Striking Rods Protecting Oil Tank with Cover (Tank Filled with Oil)

When inflammable gases are present it is important to reduce or eliminate induced voltages. This can be done by means of a thin metal or conducting roof grounded and preferably extending over the sides. A less degree of protection can be obtained by nets or wires placed on the roof in the same way. When nets are used, the closer the mesh the greater the protection. Less protection would be given by wires or nets in practise than indicated by tests on models. This follows because the inductance of long wires would not permit them to go instantly to zero potential.

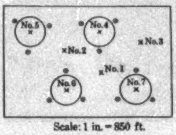
The high degree of protection given by an all-metal tank is probably most nearly approached by a combination of rods to take direct hits and a thin metal or conducting roof and sides to protect against induced voltages. Wires or mesh on or above the roof instead of metal sheets would reduce induced voltages to a less extent. In tests with a uniform field, a considerable variation of the distance of the net above the roof did not materially change its protective value. While these may be theoretical reasons for placing a net a distance above depending upon the size of the mesh, it would appear less expensive and, by tests, as effective to put it directly upon the roof. All metal parts close to the net should be connected to it. If this is difficult or uncertain, it may generally be desirable to raise the net on slats or otherwise to bring it away from such objects. With quarter-inch mesh, the supporting slats need not be more than several inches high. (See induced voltages in Table VII). In working out the protection for any tank, such details are important.

The above may be summarized as follows:

1—An all metal tank offers the most complete *protection against both induction and direct strokes.* Such a tank does not seem to be always economically feasible.

2—When explosive or inflammable gases are not present, it is not necessary generally to provide for protection against induced voltages. *Rods can then be used to protect against direct hits.* Rods do not protect appreciably against induced voltages.

3—When inflammable gases are present protection against induced voltages and direct strokes is necessary. When an all-metal tank is not practicable from the standpoint of cost, the next best thing would be thoroughly bonded and grounded metal roof and sides to protect against induced strokes combined with rods to take direct hits. The roof could be of thin metal, since direct hits would go to the rods.



Scale: 1 in. = 850 ft.
• Denote positions of Grounded Towers
x Denote positions of Storm Center over Farm

Fig. 12—Showing Protection of Oil Tank Farm from Direct-Lightning Strokes

Position of storm center	Hits to rods in per cent of total from cloud	Hits to ground in per cent of total from cloud	Location of hits to ground
No. 1	80%	20%	All hit directly beneath storm center, some distance from tanks
No. 2	30%	70%	All hit directly beneath storm center, some distance from tanks
No. 3	90%	10%	All hit directly beneath storm center, some distance from tanks
Nos. 4, 5, 6, 7	100%	0%	All strokes were to rods, with none to tanks or ground

Height of rods = 1.24 in. (equivalent to 105 ft.)
Height of storm center = 12.4 in. (equivalent to 1050 ft.)
Ratio of cloud height to rod height = 10 : 1
Diameter of tank = 532 ft.
Distance rod to tank = 60 ft.

4—For the same purpose as the metal roof in (3), but less effective, would be the substitution of a wire net. The smaller the mesh the more effective the net. Details are important in placing the net. Other methods of protection can be worked out from the test data given below. The above methods were discussed in detail as an example, because they seemed among the most practicable.

An actual test was made on a model of the tank shown in Fig. 6. All of the hits went to the rods with clouds no higher than ten times the height of the rod.

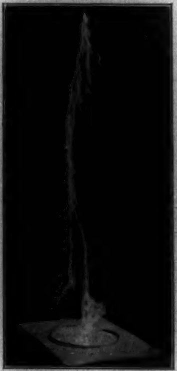


Fig. 13—Sixty-Cycle Discharge to Oil Tank

Fig. 12 shows a section of a tank farm protected by rods. In tests on the model, with the storm center at various positions as indicated, the tanks were never hit.

3. EXPERIMENTAL INVESTIGATION

The arrangements used for studying lightning discharges as well as the lightning generator have been described in former papers[4]. The results obtained in these papers also have a bearing upon the present study. The circuits are shown in a, b and c, Fig. 7. Fig. 7a gives a 60-cycle spark. In Fig. 7b a dielectric field is established over the tank or wires under test. When the sphere-gap discharges, this dielectric field collapses, voltage is induced, and the effect of a cloud discharging in the distance is simulated. When still higher voltages are used, a direct stroke is made to take place to the object under test. How well this arrangement simulates a natural lightning discharge is illustrated in Figs. 8, 9, 10 and 11, which show zig-zag discharges, split strokes, side flashes and branches. The branches which show quite plainly in the photographic negative are partly lost in reproduction. Both unidirectional and oscillatory discharges may be obtained with this circuit. The tests were made in such a way that the cloud was about half the time negative and half the time positive. The circuit in Fig. 7c gives an impulsive discharge. The cloud is "dead" until the discharge takes place. It would simulate one cloud discharging to another and then to ground.

In studying any condition, from fifty to several hundred discharges were usually made. Voltages up to 2,250,000 r. m. s. effective, 60-cycle, 2,000,000 lightning and oscillatory were used.

The cloud shown diagrammatically in Fig. 7 was a horizontal plate 5 ft. x 7½ ft. (152 cm. x 229 cm.) with rounded edges. The height above ground was usually about 43½ in. (110 cm.). Other types of "clouds" such as spheres and points were used. A short pointed rod was usually suspended from the plane to represent the storm center.

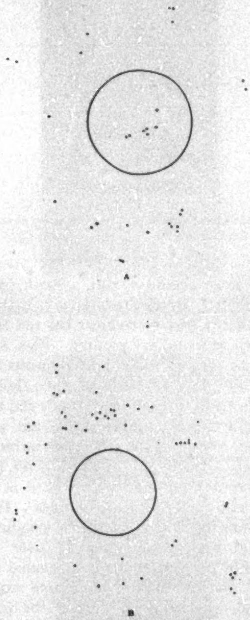


Fig. 14—Target made by Lightning Strokes from Charged Cloud

a. 6¼-in. ciruclar tank, ⅓ in. high
b. 5-in. circular tank, ½ in. high

Division of Hits Around a Tank. These tests were made to determine the division of hits between the ground around the tank, the inside of the tank and the edge of the tank. The tank shown receiving a 60-cycle discharge in Fig. 13 consisted of a circular or elliptical ring, made of a thin metal strip placed on a metal plane. The results of the tests are given in Table I for all types of discharges and various types of "clouds." Fig. 14a is a typical target for a circular tank. A number of strokes went inside of the tank, a number to the edge of the tank and a number to the ground at some distance from the tank. In Fig. 14b the diameter of the tank was reduced while the height was kept the same. In this case, there are no hits inside of the tank. Either the edge of the tank or the ground some distance away was struck. *The tests show that a hit never occurs within a circular tank when the height of the tank is greater than one-tenth the diameter.* The division of hits between the edge and the inside for tanks of different ratios of radius to height is given in Figs. 15 and 16 while complete data will be found in Table I.

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[4] F. W. Peek, Jr., "High Voltage Phenomena", *Journal* of Franklin Institute, Jan. 1924. "Lightning", *Journal* of Franklin Institute, February 1925.

If rods are placed around the tank shown in Fig. 14b,

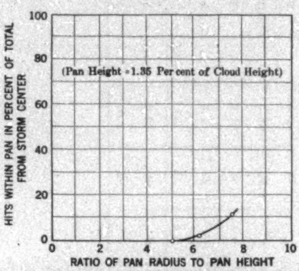

FIG. 15—AREA PROTECTED FROM LIGHTNING BY CIRCULAR METAL PAN OR TANK. HITS WITHIN CIRCULAR PAN; PAN HEIGHT = 1.35 PER CENT OF CLOUD HEIGHT

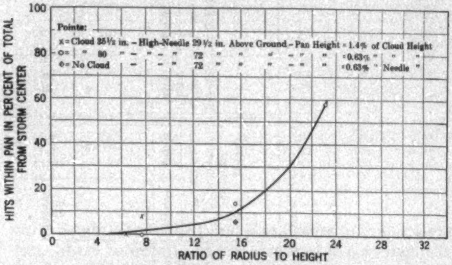

FIG. 16—AREA PROTECTED FROM LIGHTNING BY CIRCULAR PAN OR TANK HITS WITHIN CIRCULAR PAN, 60 CYCLES ONLY

TABLE I

DIVISION OF DIRECT HITS AROUND A METAL RING PLACED ON A METAL PLANE TO SIMULATE A PAN OR TANK

Test arrangement	Test number	Ratio (Radius to height)	Condition of cloud	Cloud height (inches)	Storm center height (inches)	Strokes from storm center	Number of strokes		Hits to ground (as found from holes in paper)		Per cent of total hits inside ring	Distance to nearest hit		Protective ratio Distance to height	
							to ring	to rods	outside ring	inside ring		outside ring	inside ring	outside ring	inside ring
20"x10" x 0.50" ellipse	1	10.0	charged	37	32	415	61	..	10	344	83	..	1.53"	..	3.06
10" x 5" x 0.50" ellipse	2	5.0	"	"	"	286	283	..	3	0	0	3.00"	..	6.0	..
15"x7.50"x0.50"ellipse	3	7.5	"	"	"	111	51	..	3	57	51.3	4.88"	1.50"	9.5	3.0
12.50"x6.25"x0.50" "	4	6.25	"	"	"	136	103	..	3	30	22.0	..	1.62"	..	3.5
7.50" diam. circle (0.50 inch high)	5	7.5	"	"	"	250	207	..	14	29	11.6	2.12"	2.19"	4.25	4.3
6.25" " "	6	6.25	"	"	"	307	276	..	23	8	2.6	2.00"	1.85"	4.0	3.7
5" " "	7	5.0	"	"	"	342	280	..	62	0	0	1.38"	..	2.75	..
15.4" " "	15	15.4	dead	35	29	106	105	..	0	9	6.6	..	2.12"	..	4.25
7.50" " "	16	7.5	"	"	"	100	99	..	0	1	1.0	..	2.62"	..	5.25
15"x7.50"x0.50"ellipse	17	7.5	"	"	"	100	98	..	0	3	2.0	..	1.88"	..	3.75
7.50" diam. circle (0.50 inch high)	18	7.5	60 cycles only	35.5	29.5	50	43	..	3	4	8	1.75"	1.38"	3.5	2.75
6.25" " "	19	6.25	"	"	"	50	44	..	6	0	0	2.50"	..	5.0	..
6.25" " "	19a	6.25	"	"	"	50	44	..	6	0	0	3.25"	..	6.5	..
7.50" " "	20	7.5	"	80	72	50	38	..	12	0	0	3.50"	..	7.0	..
23" " "	21	23.0	"	"	"	50	20	..	1	29	58	..	2.75"	..	5.5
15.4" " "	22	15.4	"	"	"	50	40	..	3	7	14	4.00"	2.50"	8.0	5.0
15.4" " "	23	15.4	"	none	"	51	47	..	1	3	6	6.62"	3.75"	13.2	7.5
Rods placed symmetrically around rings or tanks															
7.50" circular ring-four 0.75". Rods 1.12" out from ring	24	6.5*	charged	37	32	317	113	173	1	30	9.5	..	1.88"	..	2.5
7.50" circular ring-four 0.75" rods touching ring	25	5.0*	"	"	"	309	125	176	5	3	0.98	2.75"	3.38"	3.7	4.5
Rods without tanks															
Four 0.75" rods on 7.50" circle	27	5.0*	"	"	"	349	..	347	2	0	0	2.06"	..	2.75	..
Four 0.75" rods on 15.4" circle	28	10.27*	"	"	"	302	..	18	0	232	77	..	2.38"	..	3.2
Four 0.75" rods on 11" circle	29	7.33*	"	"	"	344	..	265	0	79	22.9	..	2.62"	..	3.5

*Ratio = radius of rod circle divided by rod height.

TABLE II
PROTECTIVE EFFECT OF GROUND WIRE AGAINST DIRECT HITS

Test arrangement	Test number	Ratio (Height of ground wires to height of ring)	Condition of cloud	Cloud height (inches)	Storm center height (inches)	Height of ground wire or metal strip per cent cloud height	Strokes from storm center	Number of strokes		Hits to ground (as found from holes in paper.)		Per cent of total hits inside	Distance to nearest hit		Protective Ratio* (Distance to height)	
								To ring	To ground wire	outside	inside		outside	inside	outside	inside
Ground wires over ring to simulate tank																
23" ring, 0.50" high. Two ground wires at right angles and 0.75" above ground	30	1.5	Charged	37	32	2.02	327	..	306	0	21	6.4	..	2.50" from wire 3.38" from ring	..	3.3 6.8
Same except with ground wires 1" above ground	31	2.0	"	"	"	2.7	318	..	316	(off paper)	1	0.3	..	3.50" from wire	..	3.5
Same except with ground wires 1.25" above ground	32	2.5	"	"	"	3.4	306	..	306	0	0	0	..	..	..	..
Ground wires over ground		Ratio = ½ dist. bet. ground wires / Height of ground wires														
One ground wire 0.75" above ground	33	..	"	"	"	2.02	321	..	294		19	5.9 (to paper)		3.19"		4 2
Two parallel ground wires 0.75" high, 10" apart	34	6.67	"	"	"	"	326	..	233	0	93	28.5	..	2.19"	..	2.0
Same except ground wires 8" apart	45	5.33	"	"	"	"	299	..	248	0	51	17.0	..	1.62"	..	2.2
Same except ground wires 6" apart	46	4.0	"	"	"	"	304	..	282	0	22	7.2	..	1.56"	..	2.1
Same except ground wires 4" apart	47	2.67	"	"	"	"	300	..	294	6	0	0	2.62"	..	3.5	..
Check test with ground wires 6" apart	52	4.0	"	"	"	"	314	..	265	0	49	15.6	..	1.50"	..	2.0
Two parallel 0.75" metal strips grounded 6" apart	54	4.0	"	"	"	"	212	..	210	0	2	1.0	..	2.62"	..	3 5

*Protective ratio = (distance to nearest ground hit from ground wire) divided by height of ground wire.
This ratio is given for hits both outside and inside of the ring and of the parallel wires.

so that a line drawn from the top of a rod to the center of the tank just touches the edge of the tank practically all hits go to the rods.

Area Protected From Direct Hits by Wires and Nets. Fig. 17 shows a typical target of a wire parallel to and connected to ground. Either the wire is hit or the ground some distance from the wire. *For a single wire, the ground is never hit nearer the projection of the wire than about four times its height above ground.*

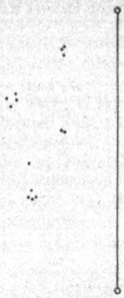

FIG. 17—TARGET MADE BY LIGHTNING STROKES FROM CHARGED CLOUD ONE HORIZONTAL GROUND WIRE, ¼ IN. ABOVE GROUND

A similar test was made on parallel ground wires arranged as in Fig. 18. It was found that the ground between the wires was never hit when the separation of the wires was not greater than about four times their height. Both of these rules, however, are subject to the cloud height as discussed later.

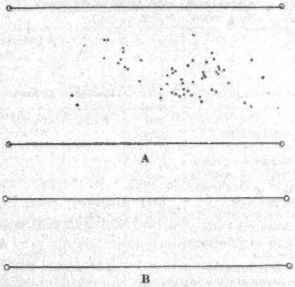

FIG. 18—TARGET MADE BY LIGHTNING FROM CHARGED CLOUD

A. Two parallel ground wires ¼ in. above ground and 8 in. apart.
B. Two parallel ground wires ¼ in. above ground and 4 in. apart

Data on ground wires and ground wires over tanks are given in Tables 2 and 3. In general, it can be seen that they are not especially efficient. There are also several factors not shown by these tests. Since, for large tanks, the wires must be quite long compared to their height above the tank, side flashes to the tank are thus likely to occur. Wires arranged like an umbrella frame would be somewhat more efficient, but a direct hit would be likely to side flash or follow the central insulating pole to the tank. Heavy currents flowing in such wires could produce internal voltages by electromagnetic induction, while hot metal could drop on the tank.

TABLE III

LIGHTNING PROTECTION BY PARALLEL GROUND WIRES

Cloud height = 30 in. (at storm center)
Wire " = 3 in.

Distance between ground wires	Cloud Height / wire height ratio	Ratio of ½ distance between wires to wire height	Percentage division of strokes: to ground (inside wires) per cent	Percentage division of strokes: to wires per cent	Distance to nearest ground hit-in.	Ratio of distance to hit to wire height
24.0"	10:1	4.00:1	50	50	7.88	2.63
18.0"	"	3.00:1	10	90	7.75	2.58
16.5"	"	2.75:1	5	95	8.38	2.79
15.0"	"	2.50:1	5	95	7.00	2.33
13.5"	"	2.25:1	0	100	..	..
Cloud height = 30 in. (at storm center) Wire " = 1.5 in.						
15.0"	20:1	5.00:1	40	60	4.00	2.67
13.5"	"	4.50:1	10	90	5.62	3.74
12.7"	"	4.25:1	10	90	6.00	4.0
12.0"	"	4.00:1	0	100	..	..

Area Protected by Rods. In previous investigations it was found that lightning either struck a rod or the ground some distance from the rod. There was always a protected area around the rod equal to about four times the height of the rod. This is illustrated in Fig. 5. The protective ratio should, however, vary with the height of the cloud for a given rod. The following relation would be expected theoretically:

$$R_p = \sqrt{2 R_c - 1}$$

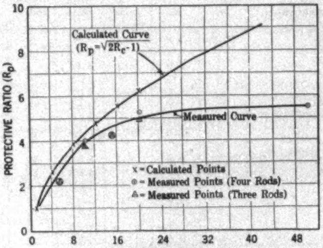

FIG. 19—PROTECTION OF AREAS FROM DIRECT-LIGHTNING STROKES

Curves showing relation between $R\,p$ (Ratio of radius to rod height) or protective ratio, and $R\,c$ (Ratio of cloud height to rod height)

where

R_p = protective ratio

R_c = cloud height divided by rod height or cloud rod ratio.

Actually, however, a ratio much greater than four would not be expected in practise because irregularities would overcome the slight increased distance effect. This is well illustrated in Fig. 19 where the theoretical and measured curves are plotted together.

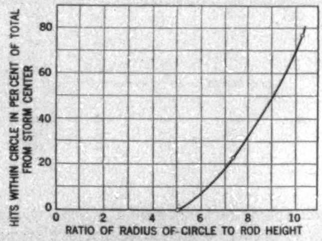

FIG. 20—AREA PROTECTED FROM LIGHTNING BY FOUR VERTICAL RODS

Hits within circles drawn with four rods arranged symmetrically on circumferences. Height of rods, 2.02 per cent of cloud height

TABLE IV

EFFECT OF CLOUD HEIGHT—ROD-HEIGHT RATIO ON AREA PROTECTED BY RODS SYMMETRICALLY ARRANGED AROUND A CIRCLE ON A CONDUCTING PLATE . . . .

Test arrangement	Cloud height (At storm center) in.	Rod height in.	Cloud-rod ratio	* Radius-rod ratio	Percentage Division of strokes To ground (inside rods)	To rods
Three rods	30.0	6.0	5:1	2.5:1	10	90
" "	"	"	"	2.0:1	0	100
" "	"	3.0	10:1	4.0:1	5	95
" "	"	"	"	3.5:1	0	100
" "	"	2.0	15:1	5.0:1	50	50
" "	"	"	"	4.5:1	10	90
" "	"	"	"	4.0:1	0	100
" "	"	1.5	20:1	5.5:1	20	80
" "	"	"	"	5.0:1	0	100
Four "	"	6.0	5:1	3.0:1	30	70
" "	"	"	"	2.5:1	15	85
" "	"	"	"	2.0:1	0	100
" "	"	3.0	10:1	5.0:1	80	20
" "	"	"	"	4.5:1	10	90
" "	"	"	"	4.0:1	0	100
" "	"	2.0	15:1	5.0:1	50	50
" "	"	"	"	4.5:1	10	90
" "	"	"	"	4.0:1	0	100
" "	"	1.5	20:1	6.0:1	30	70
" "	"	"	"	5.5:1	10	90
" "	"	"	"	5.0:1	0	100
Special tests using 25 cm. sphere as cloud. (No needle on sphere).						
Four rods-sphere over center	30.0	3.0	10:1	4.0:1	25	75
Sphere over center	30.0	3.0	10:1	3.5:1	0	100
Four rods—Sphere offset 1/3 from center	30.0	3.0	10:1	4.0:1	0	100
Four rods—Sphere offset ¼ from center	30.0	3.0	10:1	4.0:1	5	95
Four rods—sphere offset 1/6 from center	30.0	3.0	10:1	4.0:1	0	100

*Radius of circle divided by rod height.

It was found that a given area could be protected by placing a number of single rods so that their protective circles or areas overlapped. For example, with a cloud about fifty times the height of a rod the protective ratio should be about five. Four rods were arranged symmetrically about a circle. It was found that no hits took place within the circle when no rod was at a greater distance from the center than five times its height. This is shown in Fig. 20. The data will be found in Table I.

TABLE V

VARIATION OF PROTECTIVE RATIO OR RADIUS—ROD RATIO, WITH CLOUD-HEIGHT ROD-HEIGHT RATIO . . . .

Rods used	Cloud height Rod height Ratio	Ratio of Maximum Radius to Rod Height For complete protection
Four symmetrical rods	50:1	5.00:1
" " "	20:1	5.25:1
" " "	15:1	4.25:1
" " "	10:1	4.00:1
" " "	5:1	2.25:1
Three symmetrical rods	20:1	5.00:1
" " "	15:1	4.25:1
" " "	10:1	3.75:1
" " "	5:1	2.25:1

TABLE VI

AREA PROTECTED BY RODS ARRANGED ON THE AXIS OF AN ELLIPSE

Cloud height = 30 in. (at storm center)
Rod height = 3 in.

Test arrangement	Cloud-rod Ratio	Radius-rod Ratio *	Percentage division of strokes To ground (Inside rods)	To rods
Four rods arranged on metal plate at ends of axes of 7 x 4 ellipse. Storm center over center of ellipse.	10:1	4.60:1	5	95
Same conditions as above		4.05:1	0	100
Same as above except storm center moved 1/3 of distance along major axis	"	"	20	80
Same as previous conditions	"	3.47:1	0	100
Same as previous conditions except metal piece 1/7 height of rods placed along major axis to simulate roof	"	"	0	100
Same as previous test except metal piece made 1/3 of rod height.	"	"	0	100
Same as previous test except metal piece made 7/8 of rod height.	"	"	90 (To roof)	10
Same as previous test except metal piece 1/3 of rod height used. Rod spacing also increased.	"	4.05:1	25 (To roof)	75

*This "Radius-Rod Ratio" is the ratio of the minimum radius of those arcs drawn from each rod position for the given arrangement which will just cover the entire area within the rods to the height of the rods used.

Complete data on the area protected by three or more rods with different cloud heights and different types of clouds are given in Tables 4 and 5.

It may be concluded from these tests that a given area can be protected by arranging a number of rods about it so that no point on the area is at a greater distance from a rod than the protective ratio times the rod height. The protective ratio varies with the cloud-rod ratio or the ratio of the height of the cloud to the height of the rod. The practical protective ratio will usually vary between three and four.

The results in Table VI show the effect of projections within the protected area. It will be noted that, for the given ellipses, complete protection is obtained for a flat plane with the protective ratio of 3.47. This area is still protected when a roof as high as one-third of the rod height is used. The effect of varying the position of the cloud is also illustrated. In this case a protective ratio of four does not give complete protection for all positions of the cloud. The height of the cloud, however, was taken unusually low.

TABLE VII

REDUCTION OF INDUCED VOLTAGES ON MODEL TANKS BY MEANS OF OVERHEAD NETS OR CAGES

Size of mesh used—in.	Clearance to tank—in.	Position of mesh	Induced voltage on tank	Actual induced voltage in per cent of voltage induced of unprotected tank
Cage or net 21 inches square				
..	..	No protection	41.4	100
¼	1.0	Over top and around sides	0(+)	0 (−)
2	"	Over top and around sides	3.2	8
2	"	Around sides only	20.7	50
2	"	Over top only	6.1	14.8
2	"	"	5.5	13.3
2	2.0	"	5.1	12.3
2	4.0	"	5.9	14.3
2	8.0	"	7.4	17.9
Net, 36 in. square over top only				
2	1.0	..	2.4	5.8

Diameter of tank:—17 in. Cloud height:—44.5 in.
Height of tank:— 6 in. Cloud voltage:—372 kv.

It is desirable to protect a tank from all directions of approach of a storm. For this reason it is not well to use less than three rods.

It is desirable to place the rods one rod length away from the tank to prevent side flashes.

Chance of Non-Metallic Objects Being Struck. It is important to know if non-metallic but partly conducting objects such as green trees, wet wood, wet wooden roofs, etc., affect the electrostatic field sufficiently to determine the direction of a stroke as metallic rods do. Tests were made by placing dry wooden poles, wet wooden poles, dry and green branches to simulate trees, etc., under the model cloud. It was found that the green trees or the wooden poles completely wet to ground determined the direction of the stroke in the same way that metal rods did. However, when the trees or wet wooden pieces were struck they were badly shattered and side flashes took place. The dry pieces had practically no effect. Thus a tall dry "building" was not struck but a nearby green tree of less height was. A green tree was generally struck at the top without appreciable effect but with great explosive damage at exit part way down under the branches.

TABLE VIII

LIGHTNING PROTECTION OF OBLONG TANKS

Special tests with arrangement simulating a 1170-ft. x 583-ft. tank, placed on 50 ft. of sand and protected by 140-ft. rods, 110 ft. from edge of tank
Cloud height = 30 in. (at storm center)
Rod height = 3 in.

Test Arrangement	Cloud rod ratio	Radius rod Ratio *	Percentage Division of strokes		Remarks
			To ground (Inside rods)	To rods	
25.0 in. x 12.5 in. lead foil on 1 in. of sand, protected by four 3-in. rods, 2.36 in. from foil edges. Lead foil *ungrounded*, storm center over middle of tank	10:1	3.32:1	0	100	..
Same as previous test, except lead foil *grounded*	"	"	0	100	..
Same as previous test, except storm center *offset* 1/3 along major axis	"	"	0	100	..
Same as previous test, except *rods* on top of sand were *ungrounded*	"	"	0	100	Strokes to rods were afterwards found to have fused sand beneath *ungrounded* rods
Same as previous test, except lead foil, as well as rods was ungrounded	"	"	0	100	Strokes to rods were afterwards found to have fused sand beneath *ungrounded* rods
Same as previous test, except rods and foil were placed on top of grounded metal plate set on sand	"	"	0	100	..

*This "Radius-Rod Ratio" is the ratio of the minimum radius of those arcs drawn from each rod position for the given arrangement which will just cover the entire area within the rods to the height of the rods used.

An examination of the tree without knowledge of what had really happened would give the impression that the hit took place under the branches.

Since wet wood has the same effect in determining the direction of a stroke as metal, it would appear that a metal cover on a wooden roof would not greatly increase the hazard of the tank being struck.

Electrostatic Induction. In previous investigations, it was found impossible to obtain sparks in all-metal tanks unless there were metal parts extending in from the outside and making poor contact with the tank.

Table 7 gives the results of an investigation to determine the reduction of voltage due to over-head grounded nets or cages. It will be seen that a cage of ¼-in. mesh reduced the induced voltages between a model tank and ground practically to zero, while cage of 2-in. mesh reduced induced voltage to 8 per cent of the values without a cage.

TABLE IX

PROTECTION OF ROUND OIL TANK FROM DIRECT LIGHTNING STROKES, BOTH FILLED AND EMPTY, AND WITH VARIOUS TANK COVERINGS AND GROUNDS

Diameter of pan used to simulate tank: 10 in.
Height of pan used to simulate tank : ¼ in.
Height of cloud needle : 30 in. (storm center)
Number of rods used : 3 in.
Height of rods used : 2 in.
Distance of rods to edge of pan : 2 in.

Note: All tests were made with both 60-cycle and oscillatory lightning generator circuits—(see *A* and *B* on Fig. 7. See Figs. 8, 9, 10 and 11.)

Conditions	Results
Test No. 1—*With empty pan*	
No rods—no roofing	All strokes were to edge of pan
3 rods—no roofing	All strokes were to rods
3 rods—dry cardboard roof	All strokes were to rods
3 rods—wet cardboard roof	All strokes were to rods
3 rods—wet cardboard roof (pan insulated)	All strokes were to rods
Test No. 2—*With pan filled with oil*	
No rods—no roofing	All strokes were to edge of pan
3 rods—no roofing	All strokes were to rods
3 rods—dry cardboard roof	All strokes were to rods
3 rods—wet cardboard roof	All strokes were to rods
3 rods—wet cardboard roof (pan insulated)	All strokes were to rods

A net of 2-in. mesh over the tank only reduced the voltages to about 15 per cent. The reduction was practically the same whether the net was very near the top of the tank or a considerable distance above. In the last test a large 2-in. mesh net corresponding more nearly to the size of the cloud was used. Since the edge effect on the tank was removed in this way and the field was practically uniform, some idea of the reduction due to a similar net on a large tank should be given. The voltages were reduced to about 6 per cent of voltages in an unprotected tank. Thus nets do not eliminate induced voltages, but very materially reduce them. It is probable that the hazard is reduced in a greater proportion than the reduction of voltage.

A roof of high-resistance or partly conducting material could take as great a bound charge as a roof of conducting material. Upon release of the charge, however, the high-resistance roof could not discharge instantly. High local voltages would result and such a roof would appear to be a hazard.

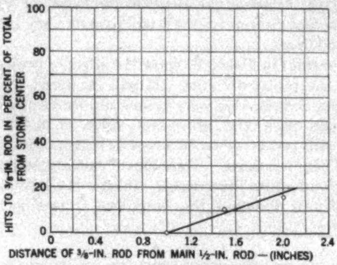

FIG. 21—AREA PROTECTED FROM LIGHTNING BY VERTICAL ROD

Hits to auxiliary rod at various distances from fixed main rod
Height of auxiliary rod = ⅜ in. (1.0 per cent at cloud height)
Height of main rod = ½ in. (1.35 per cent at cloud height)

Tests on Model Tanks and Practical Applications. A number of tests was made on model tanks. A very complete example is illustrated in Figs. 8, 9, 10 and 11, while the results are tabulated in Table 9. In this test a study was made of the effect, with and without oil, in an open and covered tank with both wet and dry roof.

TABLE X

ACTUAL LIGHTNING PROTECTIVE ARRANGEMENTS FOR OIL TANKS

Object being protected	Tank size	Actual ht. of tower ft.	Ht. of tower minus roof ht. ft.	Distance of tower to nearest tank wall-ft.	Actual radius-rod ratio	Effective-radius-rod ratio
					(Using actual tower ht.)	(Using tower ht. minus roof ht.)
Single oblong oil tank	675 x 475 Fig. 6	130	95	130	2.92	4.0
Oil tank	583 x 1177	140	119	110	3.4	4.0
Three-tank farm	One oval two round	150	107	100	2.85	4.0
Four-tank farm	Round and oval	150	107	150	2.85	4.0
Section of tank farm (four tanks)	Fig. 12	105	85	60	3.24	3.83
Large tank farm (thirty tanks)	520 ft. and 430 ft.	150	120	150	3.4	4.0

Further tests were made on models to scale of single tanks, groups of tanks and a large farm. A few of the arrangements are illustrated in Table 10. Table 8 gives results on the effect of ground resistance. The effects of projections on the protective ratio are given in Table 6 and Figs. 21 and 22.

In applying rods to the protection of tanks one of the factors that must be decided upon is the protective ratio to use. By referring to Fig. 19 it will be noted that this ratio depends upon the height of the rod and the cloud. If it is assumed that the minimum thunder cloud height is 1500 ft., the cloud rod ratio is ten for a 150-ft. tower and the protective ratio, four. Weather authorities usually give the cloud heights considerably greater than the above value.

A number of layouts were made of single oval and round tanks as well as of tank farms. (See Table 10.) A height of rod was used that covered the tank with a protective ratio of four. The height of the tallest part of the roof above ground was then added to the rod height. The method is illustrated in Fig. 6 for a single oval tank. Fig. 23 gives the general method. The rods were always placed at least one-half of the rod length away from the tank. All directions of approach were guarded against by using at least three rods per tank. In the tests of these models a cloud rod ratio of ten was used. The layout gave complete protection on the models in all cases. By referring to Table 10 it will be seen that the actual protective ratio used in making the layout, neglecting the effect of the roof, was considerably less than four and, in fact, varied from 2.85 to 3.4. As shown in Table 5, however, the effect of the roof is not so great as the allowance made for it in the layouts. In any event, the actual rod height attained seems about right for the condition assumed. In the above, it is assumed that the top of the rod is brought to a point.

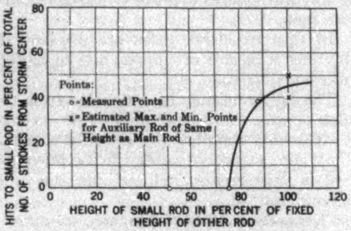

FIG. 22—AREA PROTECTED FROM LIGHTNING BY VERTICAL RODS

Hits to small auxiliary rod of varying height placed near fixed protective rod. Distance between rods equal to twice height of fixed rod. Height at fixed rod = 1.35 per cent of cloud height

Perhaps a more direct method would be to use a actual protective ratio of 3 to 3.5, depending upon the rod height, neglecting the effect of the roof when it is less than one-quarter of the rod height.

4. MISCELLANEOUS FACTORS

It might not be out of place to mention here several factors outside of the scope of this paper.

Any leakage of inflammable gases should be prevented or means taken to prevent flames starting on the outside from going back into the tank. This would apply to vents or to other possible leakage.

It is quite evident that the hazard is increased as the amount of oil stored at a given place is increased. It would thus seem desirable to reduce storage or separate places of storage when practicable.

5. SUMMARY OF RESULTS

Some of the more outstanding results are outlined below.

1. Partially insulated or grounded metal or other conducting parts assume a potential above ground when a cloud discharges. This potential is opposite to that of the cloud and directly proportional to the height of the conductor above ground or equal to the apparent gradient times the height above ground. Sparks may thus pass between isolated metal parts, or between metal parts in poor contact, due to cloud discharges at a considerable distance.

2. Insulated metal parts assume the potential of the space in which they are located. Sparks may occur between two conductors at different heights above ground in the field of a charged cloud. It is not necessary for the cloud to discharge to produce these sparks.

3. One ground wire over a transmission line reduces the induced voltage to about one-half, and two, to about one-third. This is effective on transmission lines. While nets or wires over a tank would reduce induced voltage, to be effective in preventing small sparks in explosive gases the voltages must be reduced almost to zero. Grounded nets, etc., would thus reduce the hazard but would not be complete assurance against trouble.

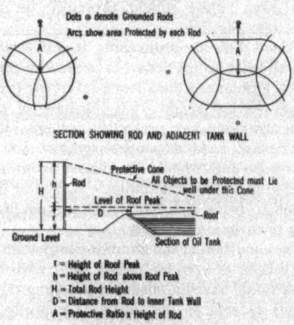
TYPICAL LAYOUTS
Dots ● denote Grounded Rods
Arcs show area Protected by each Rod
A
A
SECTION SHOWING ROD AND ADJACENT TANK WALL
Protective Cone
All Objects to be Protected must Lie well under this Cone
Rod
H
h
r
Level of Roof Peak
D
Roof
Ground Level
Section of Oil Tank
r = Height of Roof Peak
h = Height of Rod above Roof Peak
H = Total Rod Height
D = Distance from Rod to Inner Tank Wall
A = Protective Ratio x Height of Rod

FIG. 23—DIAGRAMMATIC LAYOUT SHOWING PROTECTION AGAINST DIRECT STROKES

4. The only method of preventing internal sparks is a complete metal tank. Induced sparks could not occur in a tank of insulating material, and free from conductors, but such a tank is not practical. In the metal

tank, it is important that all parts be in electrical contact. A slightly insulated pipe or other conductor projecting in from the outside, even if grounded, could cause a spark.

5. Direct hits can produce all of the above effects in an intensified form and, in addition, set oil, wood or other combustible material on fire.

6. Direct hits do not always strike the highest point unless it is 2.5 per cent of the cloud height and directly under the cloud. A rod 1.1 per cent of the cloud height is struck 50 per cent of the time.

7. A direct hit either strikes a rod or the ground several rod lengths away. There is thus a protected area around the rod.

8. A six-foot rod has fifteen times the chance of being struck as a flat area directly under a 1000-ft. cloud.

9. In all protective schemes, the resistance of the earth plays an important part. These herein assume conducting earth.

10. A tank or pan will usually be struck on the edge if the height of the wall is greater than one-tenth of the diameter of the tank, (unless the clouds are very low).

11. A number of rods may be arranged to protect a given area from direct hits. The area to be protected must not be at a greater distance from some rod than the protective ratio times the height of the rod. This protective ratio is approximately four over a wide practical range; it is not actually constant, however, but depends upon the ratio of the cloud height to rod height. Lower values should probably be used in practise. (See Fig. 19).

12. Where there are projections, it is desirable that they be well below a line drawn between the top of the rod and the ground at a distance equal to the protective ratio times the rod height from the base of the rod.

13. In arranging any protective scheme, it is important to guard, so far as possible, against side flashes. For this reason rods should be placed from one-half to one rod length away from tanks. A greater distance would tend to make too many rods or too high rods necessary to cover a given area.

14. The condition of the ground is important. For instance, a layer of non-conducting sand with a water level below would have the effect of increasing the height of the rods as well as the tanks. Generally, a rod should be grounded immediately below, at water level.

15. A grounded wire may be used to protect a given area from direct hits.

16. An area can be protected from direct hits by several parallel ground wires if the ratio of separation to height is not greater than four. One practical difficulty of such protective schemes on large tanks is the possibility of secondary flashes to the roofs as the distance to the roofs would generally be small compared to the distance along the wire to ground. Cages, "spider-webs," over-head nets, umbrella-frames, etc., could also be used for protection against direct hits.

17. Grounded rods, wires, meshes and cages all reduce the induced voltages in metal parts of a tank to a greater or less degree and thus diminish the hazard of small sparks. The mesh of a cage must be quite small to make a great reduction in induced voltages. The net may be on the roof or above.

18. A well bonded metal roof and sides would be more effective than nets, while a complete metal tank would be the only absolute assurance against induced voltages. In case of a metal tank, it is important not to have any loose plates of projections slightly insulated from the tank.

19. In case explosive gases are present, there is probably also a hazard of ignition due to agitation or by sparks due to friction, etc.

20. The following may be concluded from the tests on models:—

1. The only complete protection is a metal tank.
2. Where explosive gases can be prevented, the hazard is very greatly reduced and the protection problem simplified.
3. Nets, cages, wires, rods, etc., all more or less reduce induced voltages. The mesh of a cage or net must be quite small to prevent induced sparks.
4. Rods seem to offer one of the most practical means of protection against direct hits.
5. Rods in combination with a bonded metal roof extending over the sides and grounded seem to be the next best arrangement to a thick all-metal tank. The roof would keep down induced voltages while the rods would prevent any heavy discharges immediately upon the tank. Grounded wires or grounded mesh could be used in place of the sheet metal roof but would be less effective.
6. The condition of the ground and method of making ground connections is important.

Discussion

Joseph Slepian: There is one point that has bothered me about which Mr. Peek may be able to reassure me. The model experiments show quite definitely the existence of a protected area around a lightning rod, but I do not feel so certain of the quantitative applicability of the results to practical cases. If the electrostatic field only is considered, model tests give quantitatively correct results. If, with a set of electrodes, the scale of dimensions is reduced in any ratio we may be sure that the electrostatic field will be faithfully reproduced on the smaller scale. But not so with discharges; there we have another factor coming in. Discharges or sparkovers depend upon the distances between molecules of the gas. It would seem that if the experiment with discharges is to be made on a small scale the distances between molecules of the gas should also be reduced so that absolutely correct model experiments should require molecular distances to be reduced in the same ratio or the pressure of gas to be increased in the same ratio. There is a theorem bearing on this, relating to similarity of discharges given in Townsend's book on "Conduction of Gases." It is that if the dimensions of electrodes are reduced in a given ratio and the gas pressure is increased in the same ratio at the same time, the sparking potential is unchanged. It may be that these model experiments are correct but there is this little doubt. This doubt will be removed if it proves true that the ratio of protected radius to rod height is independent of the scale on which these experiments are made. Very likely Mr. Peek can set that point at rest if he has made tests on different scales and has still obtained the same ratio of protected radius to rod height.

R. W. Sorensen: During the summer of 1926 a number of engineers in Southern California has been trying to find out how to protect oil reservoirs from lightning fires. There were a few strikes of lightning in Southern California early in the year. These few strikes did millions of dollars worth of damage, because some of them set fire to oil stored in reservoirs. Up to the time of the fires of last spring, the oil companies were confident that oil stored in reservoirs represented a small risk for damage which might be caused by lightning. The peculiar coincidence which made three bad oil fires due to lightning occur within a period of about three weeks immediately produced great excitement, which resulted in the opinion that the frequency of lightning storms in Southern California was rapidly increasing. Impelled by this excitement, engineers undertook investigations and laboratory tests to determine the probable danger due to lightning and means for guarding against such danger. Some conclusions have been reached. One is that there are fewer electric storms along the Pacific Coast than in any other place in the United States. Another is, oil in storage presents no special inducement for lightning to strike the tanks containing oil, or in the immediate vicinity of such tanks. Beyond this I feel that as yet we know very little about complete lightning protection.

Mr. Peek, with his equipment and knowledge of high-voltage phenomena, is obviously the one who has the most knowledge concerning laboratory experiments with apparatus constructed to produce in the laboratory electric discharges simulating as nearly as possible those due to lightning. From these tests Mr. Peek has drawn the conclusion that lightning cannot hit within an area circumscribed by a circle having a radius of four rod heights drawn about the rod as a center.

I am in no position to question Mr. Peek's law as to what lightning will do, but I have a record of several thousand shots made from a point to a plane, the shots being recorded on sheets of paper in the manner used by Mr. Peek. These shots were made with direct current obtained from kenetrons without the use of condensers with condensers charged by direct current from kenetrons, with 50-cycle alternating current directly from a transformer and with discharges from a surge generator as used by Mr. Peek. Our surge generator of course had different constants from those of the surge generator used by Mr. Peek. The data to which I refer include tests made with spark-gaps ranging from less than 2 in. to 14 in. in length. Using these gaps connected to the circuits mentioned, we find a very appreciable percentage of the electrical discharge striking the plane surrounding the metal rod within a distance of less than four rods height from the rod. Mr. Peek has suggested that we get strikes within this distance because our gap lengths are short. I see no reason why gaps 5 or 10 in. long should act differently from 30-in. gaps, as used by Mr. Peek. In making these laboratory tests we used rod heights varying from ¼in. to 1 in. The rod height in every case was designated as 150 ft. Using the rod height as the unit, the spark-gap distance for each test was set to represent a cloud height one might expect to find during thunderstorms. The minimum height in each test represented 1000 ft., while the greater number of thunder clouds probably have an average height of 3000 ft. I have data for more than 900 shots made with varying rod heights, and for each rod height for varying gap lengths. From this data I should conclude that for the spark discharges we used there is approximately 43 per cent protection, rather than complete protection, for the area within the four-rods-height circle. Mr. Peek has suggested that our results differ from his because we have circuits which oscillate. Perhaps that is so, but I doubt if we are in possession of sufficient information to declare that no oscillations will ever occur in actual lightning strikes. At any rate, the results we obtained for spark discharges from a point to a plane upon which is mounted a projecting metal rod in our work at Pasadena are not the same as those obtained under the conditions used by Mr. Peek in Pittsfield, Mass.

With regard to the protection of tanks having rods erected around them, we have found that three points around a small metallic tank in the laboratory afforded practically no protection against sparks from a point directly above the center of the circle inclosed by the three rods, most of the discharges going to the space between rods. With sparks up to 14 in. in length the results were approximately the same for all spark lengths. Four rods and six rods around a model tank furnish a considerable degree of protection for the space surrounded by the rods, but give by no means complete immunity.

We have also done some laboratory work to determine the area protected by wires but our time does not permit discussion of the results obtained in these tests. I will therefore close by stating that I do not wish to appear to question Mr. Peek's experimental results in any way, but I do believe that at the present time we are not justified in concluding that Mr. Peek's laboratory tests as reported give complete data as to how an area can be given complete protection by rods spaced according to his four heights rule.

B. F. Howard: I should like to ask Mr. Peek if it is possible—economically possible—to prevent potentials which would cause lightning to build up on clouds in the neighborhood of oil tanks or of wire lines. During this year I have carried out an experiment at Colorado Springs in order to try and prevent two areas of cables being damaged by lightning which heretofore have been so damaged every year. We have had very severe lightning at Colorado Springs this summer and so far we have received no damage due to this cause on these protected cable areas, whereas serious damage by lightning to other cables which were situated around and close to these areas has been experienced. The damage did not appear to be the result of direct strokes but more of induced charges on the wires disruptively discharging to the sheaths on their way to ground at the time of a nearby lightning flash.

The method of protection followed was this: The two cable areas in question are each about half a mile long and the construction which was done several years ago was on the old system of ground wires being placed at every tenth pole. These connected with the cable sheaths and messengers and to ground. The soil is good for making a satisfactory "ground." This is rather unusual, for in Colorado Springs there are often pockets of earth which are not well electrically connected with the main ground. A galvanized wire was run up each pole and connected to the messengers and cables and to an ordinary ground rod driven in at a little distance from the base of the poles. No sharp bends were allowed and the tops of the wires were carried to a height of 6 in. above the poles and each was sharpened to a needle point. The poles for three or four spans on each boundary of the cable areas were included. The idea was that each pole within the areas and several poles around them would act as leakers and so prevent or reduce potentials building up in the nearby clouds sufficiently to prevent lightning discharges from them.

It is a rather bold attempt to try to control lightning, but "the proof of the pudding is in the eating." The fact remains that while cables in other parts nearby have been seriously damaged, and I believe the power people experienced a good deal of disturbance from lightning this summer, the cables in these areas so far have remained undamaged. Of course I may be mistaken, as the effects of lightning are so uncertain and we may find next year that we have had some damage done to these cables, notwithstanding the protective conditions.

M. E. Dice (by letter): Mr. Peek lays down some very definite rules representing the results obtained by using models subjected to "voltage from the lightning generator as well as all other types of voltages." He says the same general results were obtained from all types of clouds,—points, spheres, and planes. The one thing he has not done to make the record complete and the rules applicable is to compare these results with the records of actual lightning strokes. It would be very convincing if he could show historical data from insurance companies and from the many published observations on lightning which would substantiate the laboratory results.

F. W. Peek, Jr: I quite agree with Dr. Slepian's criticism. So long as the problem is purely an electrostatic one, an exact solution can be obtained from a model. The induced voltages on the ground wire, the effect of a net, etc., are purely electrostatic problems. In the study of direct strokes, electrostatic conditions are represented by the model up to the point when the spark occurs. There is reason to believe from a study of lightning strokes that the model also represents actual conditions after the spark occurs. Of course it is not possible to reduce the size of the atoms to correspond to the scale. Tests were made, however, over a wide range of scales with substantially the same results. In other words, for a given arrangement and ratio of rod and cloud height the results were independent of the scale.

Since I have not seen Prof. Sorensen's data, I cannot discuss it in detail nor venture an opinion as to the exact cause of the apparent discrepancies. There are several ways in which it can be explained. For the short sparks used in his tests, high-frequency oscillations are quite likely to occur. Such oscillations change conditions and do not represent the effects caused by an impulsive or highly damped discharge of a lightning stroke or longer spark. On several occasions we have had similar trouble for various reasons but have always been able to determine the cause and eliminate it. Reference to my Fig. 19 will show that the protective ratio varies with the cloud height-rod height ratio. I do not know whether or not Prof. Sorensen made allowance for this.

Since this paper was written, I have extended the tests to include d-c. voltages up to 350 kv. When the cloud is negative, the results are very much the same as in Fig. 19; in fact, the protective value is somewhat better. When the cloud is positive, a curve similar to Fig. 19 is obtained. The protective ratio, however, is less for a given cloud-rod ratio than it is for the negative cloud. For instance, in Fig. 19 a protective ratio of four is given for a cloud-rod ratio of ten. When the cloud is positive the protective ratio of four may not obtain until the cloud-rod ratio is about twenty. Theory and some measurements indicate that the cloud causing the discharge should generally be negative. Also, the lightning voltage just previous to the discharge is generally not steady direct current.

The values given in Fig. 19 are ratios where the rods just do give protection. In using these figures it is, of course, desirable to allow a certain factor of safety.

I doubt if appreciable protection can be obtained by corona discharges as suggested by Mr. Howard. A pine forest should offer the very best kind of a discharger yet lightning frequently strikes trees in such forests.

While it may not be possible to simulate all actual conditions in tests on models, it is felt that the result of such tests should be of very material help in laying out protective schemes. Perhaps the best that can be hoped for in most schemes that are economically feasible is that the hazard may be very greatly reduced.

Engineering Education—Its History and Prospects

BY HENRY H. HENLINE*
Associate, A. I. E. E.

Synopsis.—*A brief history of the development of engineering education in the United States and a consideration of the usual type of curriculum in particular branches of engineering are followed by reasons for the development of a new type of curriculum and a discussion of its principal characteristics.*

The recently adopted curriculum leading to the degree of Bachelor of Arts in Engineering at Stanford University is given in detail.

The ideals discussed in the paper are emphasized by a few selected quotations from statements of leading executives and teachers.

In order to gain a true perspective of the subject of engineering education, one must consider its early history in the United States. The first American colonies were forced by Parliament to limit their production to agriculture and raw materials, and when they made the non-importation agreement in 1774, there appeared an urgent need for skilled workers in all mechanic arts. This situation was relieved by the formation of societies which exerted all possible effort to encourage the useful arts and by prizes offered "for the best achievement in every essential line of industry." After the war, similar activities were made necessary by the fact that England attempted to stop the development of industries by underselling methods.

During the years immediately following the war, many engineering developments were made, notably:

1787 —First flour mill machinery.
1790 —First textile mill driven by water power
1793 —Invention of the cotton gin
1801 —First high-pressure steam engine
1801 —High-capacity double steam pump
1807 —First steamboat
1786-93—Several canals begun

The war of 1812 again made it necessary that American industries manufacture all necessities, and this, together with the fact that much of the soil was becoming exhausted, produced a very urgent demand for scientific information which would lead to greater production in both agriculture and manufacturing.

As a result of this demand, the Rensselaer Polytechnic Institute was established at Troy, New York, in 1824. The curriculum, which was one year in length, contained a great variety of subjects. During the last nine weeks of the year, a study was made of the practical applications of the sciences previously studied. In 1835, instruction in civil engineering was added and students who completed the new curriculum were awarded the degree of Civil Engineer. Following a thorough study of instruction in French technical schools, the Rensselaer curriculum was lengthened in 1849, to three years. The first half of the curriculum was planned so as to lay a general foundation for all engineering, and the last half contained courses designed to allow for specialization in some particular branch.

In much of the early demand for such schools, the need for training as an aid to industrial production was very strongly emphasized, but there was also a rather insistent demand for instruction in science as part of a liberal education.

The Lawrence Scientific School at Harvard and the Sheffield Scientific School at Yale were established in 1847. At the same time, the University of Michigan decided to give a course in civil engineering. No other engineering schools were established before the Civil War[1].

One of the most significant steps ever taken in providing for higher education was the passage by Congress of the Morrill Act in 1862. By this Act, the National Government presented to each state in the Union 30,000 acres of public land for each senator and representative in Congress. The purposes as stated in the Act were, " . . . the endowment, support, and maintenance of at least one college, whose leading object shall be, without excluding other scientific and classical studies, and including military tactics, to teach such branches of learning as are related to agriculture and the mechanical arts, . . . in order to promote the liberal and practical education of the industrial classes in the several pursuits and professions of life."[2]

The results of this Act were extremely gratifying to those who had been appealing for more technical schools. In 1870 there were seventeen such schools as compared with the four established before 1862. The number continued to increase very rapidly, and there were forty-one in 1871, seventy in 1872, and eighty-five in 1880[1]. At the present time there are approximately 130 engineering schools of college grade in the United States.

A number of these early schools were called industrial universities because the greatest demands for them had been based on industrial needs. The idea that manual labor occupied an important place in their training soon became so widespread that many objections to such names arose, and changes were made. For instance, the Illinois Industrial University became the University of Illinois in 1885.[3]

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1. For references see end of paper.

Although the first schools offered civil engineering only, the curricula were soon extended to include several branches of science and engineering. The Massachusetts Institute of Technology, which was established in 1865, offered six four-year curricula as follows:

Civil Engineering
Mechanical Engineering
Mining Engineering
Practical Chemistry
Architecture
General Science

The first curricula of the Illinois Industrial University, established in 1867, were:[4]

- Agriculture
- Polytechnic—including
  - Mechanical Science and Art
  - Civil Engineering
  - Mining and Metallurgy
  - Architecture and Fine Arts
- Military
- Chemistry
- Natural Science

Four years were required for the completion of each. The above examples are fairly representative of curricula offered by all of the early technical schools. The general plan in all cases was to provide training in mathematics, drawing, physics, chemistry, etc., in the earlier parts of the curricula, and to have the work in applied science follow such training according to the plan used in French schools. English and foreign languages were usually included in the curricula.

The number of curricula in engineering increased very rapidly with the extensive development which occurred in the applications of engineering knowledge of all kinds. There are now available curricula in about forty branches of engineering, including aeronautical, agricultural, architectural, automotive, ceramic, chemical, civil, electrical, marine, mechanical, metallurgical, mining, sanitary, etc.

The great increase in the number of curricula offered has been accompanied by numerous changes in their content. The chief tendency has been to include more technical subjects to keep pace with the phenomenal expansion of engineering activities. This expansion itself has made even more necessary than before a very thorough education in the fundamental subjects. Hence, serious overcrowding has resulted.

This has been relieved to some extent in many schools by the growth, from each curriculum in a particular branch of engineering, of several curricula covering the various sub-specialties. This process is only a partial remedy, however, because some of the sub-specialties have grown into branches of great magnitude, and the growth has by no means ended.

The type of curriculum now in most general use is one leading to a Bachelor's Degree in a particular branch of engineering (any one of approximately forty) at the end of four years. Such a curriculum contains a common core of subjects required of all engineering students, and consisting of the following groups of subjects:[5]

Science: Mathematics, chemistry, physics, mechanics
Mechanics Arts: Drawing and shop work
Humanities: English, foreign language, economics, etc.

These foundation courses are followed by the various engineering subjects considered essential in the specialized curricula. As a general average, the student's time during the four years is distributed about as follows[6]:

Languages and humanities	19 per cent
Mathematics and sciences	29 per cent
Engineering subjects	52 per cent

Since the languages and humanities group receives only 20 per cent of the time, and English and foreign languages are usually strongly emphasized, little or no time is spent on certain very important subjects such as biology, economics, geology, history, political science, psychology, business law, etc.

The group of subjects including mathematics and the sciences, principally chemistry and physics, receives strong emphasis in nearly all curricula because these subjects really constitute the foundation of all engineering. These, together with the group mentioned above, are usually given in the first two years.

Drawing and shop work are usually considered very essential subjects, and receive their proper proportion of time. The value of shop work, however, has been seriously questioned during the past few years. Many experiments in the handling of such courses have been tried. In a few schools, the university shops have been operated on a production basis in order that students might have experience as production managers, foremen, machine operators, etc., and thus receive a training designed to enable them to understand shop operations of all kinds. Others have developed cooperative courses with neighboring industries so that shop training as well as many other kinds of training can be obtained under practical industrial conditions. Throughout the period covered by these experiments there has been a considerable strength of opinion that shop work is being allowed to take time which should be allotted to other subjects, and the reasons why all engineering students should develop facility in handling tools have not been clearly shown.

Following these two years of preparation, the latter half of the curriculum is usually nearly filled with engineering subjects. This makes it necessary that each student decide early in his university career which one of the many branches of engineering he is most interested in and best fitted for. The time at which such a decision must be made if he is to graduate with his class depends upon the school chosen. In some it is as early as the beginning of the first year. A large number require this decision at the beginning of the second year, and about one-fourth permit it to be made as late as the beginning of the third year. Very few schools permit greater delay[7].

During the work of the last two years, there is usually a considerable number of courses dealing with the various phases of the specialty chosen. There are also courses in related branches of engineering. Thus a student in civil engineering is usually required to take a course or two in electrical and mechanical engineering, and a student in electrical or mechanical engineering usually takes surveying in the civil engineering department. However, the major portion of the time is spent on the subjects closely related to that chosen for specialization.

In connection with the very thorough investigation of engineering education now being conducted by the Society for the Promotion of Engineering Education, many questionnaires have been used to secure information on all phases of education and related matters. One such questionnaire now being circulated gives ten divisions of electrical engineering, and asks which ones are considered so important that special curricula in them should be offered.

A serious result of the tendency to keep adding technical courses to curricula which are already completely filled is found in many schools in which the requirements for graduation in engineering include a considerable number of units more than required for graduation in other curricula. It is quite common to find this situation made more serious by the fact that there are few if any courses in subjects which prepare men for dealing with people, namely, citizenship, economics, political science, etc., or subjects which provide any cultural value. Many such curricula are extremely rigid, and allow almost no choice of subjects. Therefore, students are expected to follow a certain list of courses with very little thought as to their own interests.

The established order of subjects in most curricula, *i. e.*, languages, chemistry, physics, mathematics, mechanics, drawing, and shop work in the first two years, and engineering subjects in the last two, is often severely criticized because practical applications always follow theoretical principles. Another result of this order of subjects is that the students have very little contact with engineering or engineering faculty members during the first two years. Several institutions have tried the experiment of giving engineering problems throughout the first year. The success has apparently been great, as such work stimulates the students' interest and assists in making the final decision as to the special branch of engineering in which they wish to specialize. However, it seems to be impossible to coordinate theory and practise in a curriculum in such a manner as to meet all objections. Certain fundamental subjects must precede the more advanced subjects, and no Utopian scheme which includes all subjects mixed in proper proportions and arranged in ideal order has been proposed. It seems that the students of a worth while type should be able to maintain their interest through two years of work in fundamental subjects. It is a serious question whether one who cannot do so does not more properly belong in a trade school than in a university.

The investigation now being conducted has shown that about 62 per cent of the students admitted to engineering curricula fail to graduate, and the elimination occurs largely in the first two years[8]. This fact is causing a serious consideration of the factors involved with the hope that the mortality can be reduced. It has been suggested that the adoption of two-year curricula leading to a certificate or diploma, and preparing students definitely for certain kinds of technical employment, might be a satisfactory solution. There are a number of very serious objections to this procedure. Such a two-year curriculum would not provide the most suitable first two years' training for those who wish to complete a four-year curriculum, and it would be difficult or impossible for most institutions to provide facilities and personnel to handle both classes of students. The most serious objection, however, is the fact that this would definitely lead many able men into mediocre technical positions where their future progress would be slow. The failure of a student in mathematics or some of the other subjects placed early in the engineering curricula does perhaps indicate that he cannot be successful in the higher types of technical work, but it by no means indicates that he cannot become a successful man in the business or management side of engineering.

It seems obvious that the greatest need is for some means of determining what each student is best fitted for and the type of intellectual effort in which he is most interested. If each could receive expert assistance in the determination of his strongest natural aptitudes, the number of such eliminations would be greatly reduced. Many men now turned out as failures in the engineering curricula would be highly successful in some field of human endeavor, and this would be a far more satisfactory outcome than would be obtained by guiding all such men into mediocre technical positions.

To sum up the characteristics of most of the curricula now in effect: They are too rigid in that little allowance is made for the interests or initiative of the students when a decision has been made to specialize in a particular branch. This decision must, in most cases, be made before students are mature enough or have had sufficient experience to decide wisely. They devote too small an amount of time to broad education and too much to narrow specialized training.

A number of universities have developed five- or six-year curricula in order that more general subjects might be included without omitting the engineering courses which are considered necessary. In several cases no degree is received until the completion of the entire five- or six-year period.

The cooperative type of education has some well recognized advantages in schools of all grades. The Massachusetts Institute of Technology and the University of Cincinnati are the two outstanding examples in the engineering field. At the former, the cooperative curriculum is five years in length. The first two years are the same as the usual electrical engineering curriculum, but during the last three years, alternate terms, including summer terms, are spent in the industries. The fifth year is devoted to graduate work and research in both the Institute and the companies. Each man has a choice of manufacturing and utility companies, but works in the same company during the three years. His compensation averages about $1500 for this period.[9]

The ideas expressed below, regarding some of the ideals of engineering education, are not those of any one man alone, but represent the aggregate opinion of the author and others. An earnest effort has been made to include the best thought on the subject.

The requirements of engineering education could be determined more definitely if there were a generally accepted definition of engineering. The following definition was given in his president's address in 1908, by Past-President Stott:

> "Engineering—The art of organizing and directing men, and of controlling the forces and materials of nature for the benefit of the human race."[10]

The first part of this definition shows that an important part of education consists of subjects which will enable those men who will become executives to develop more rapidly.

Probably a large majority of undergraduates in engineering schools believe they will be engaged for many years in work primarily technical. The results of the S. P. E. E. investigation show that of the three most recent classes 71 per cent are in work primarily technical, while more than 70 per cent of those in classes out fifteen years or more are in work primarily administrative.[9] Such records certainly indicate clearly that a broad education is more important than training in technical subjects. In 1916 a circular letter was sent to thirty thousand members of the four large engineering societies requesting them to number six groups of qualities headed Character, Judgment, Efficiency, Understanding of Men, Knowledge, and Technique, in the order of importance given them in accounting for engineering success and in considering young men for employment. Of the seven thousand replies received, 94.5 per cent placed the Character group at the top of the list, and about the same number placed Technique at the bottom[11]. Success in engineering obviously depends upon many factors besides technical knowledge and skill. The really successful engineer must be able to coordinate theory, practise, and economics, and to handle men. As shown above, most of the engineering curricula now in effect were planned in the earlier years of engineering education, and the changes made since have consisted principally in the addition of more technical courses. Most of the curricula furnish excellent preparation for certain types of work into which some of the graduates enter. On account of the extremely rapid progress that has been made in many branches of engineering during the past few years, the applications of engineering knowledge are now so many and so diversified in character that any curriculum designed to meet directly certain needs in industry may indeed prepare men in a most excellent manner for those needs, but fail utterly to prepare them for a great range of engineering problems, both executive and technical, which all graduates will be called upon to solve.

The very strong and growing tendency to choose executives from men with engineering training is a force which must be reckoned with. Problems which executives meet are becoming so complex and so closely associated with fundamentals of engineering that some technical knowledge is essential. No one believes the schools can train executives, as ability in this direction depends primarily upon inherent characteristics. However, if engineering graduates are to receive their fair share of such positions, they must be given the broad, general foundation which is absolutely essential.

During the past few years many high executives in some of our largest industries have advocated a broad type of training for engineers. In employing recent graduates, they prefer to obtain men who have had a thorough training in fundamentals and who have not specialized in some small branch of one of the principal types of engineering.

What, then, are the principal characteristics of a satisfactory engineering curriculum? The answer to this question depends upon the types of activity for which the schools attempt to prepare men. In the present stage of development, it seems necessary to recognize the needs of two general groups of students, *viz.*, those who expect to spend their lives in highly technical design or research, and those who will be engaged in commercial, industrial, or administrative phases of engineering. Both groups need a broad foundation consisting of such subjects as English, economics, biology, geology, history, business law, political science, etc., and thorough training in chemistry, physics, mathematics, mechanics, and other subjects which make up the heart of engineering. Such training should be mixed with and followed by courses giving the fundamentals of all of the principal branches of engineering, and there should be a reasonable amount of time available for elective subjects. Thus far there is no serious difference between the wishes of executives in industry and teachers of engineering. It therefore seems that the chief cause of argument is the relatively small group of men who will engage in research and other highly technical phases of engineering. This group must have better opportunities for the development of research ability and for specialized study than can be provided in any four-year curriculum which contains sufficient training in fundamentals. It seems clear that the usual type of four-year curriculum fails to meet the needs of all except those who wish to remain in the specialized divisions of engineering which do not require either a very broad training or an advanced technical education.

Many professional engineers believe a university curriculum should provide broad and thorough training in the fundamentals of engineering, and that considerable emphasis should be placed upon humanistic subjects such as English, economics, sociology, history, etc., not merely on account of their usefulness to the engineer, but also on account of their broadening influence[12]. Another phase of the investigation of engineering education now being conducted has developed the strength of this demand. The opinions of 1931 graduates of the classes of 1919, 1914, 1909, 1904, 1899, 1894, 1889, and 1884 on the principal objectives of engineering education are as follows[13]:

	No.	Per cent
To train broadly for the general needs of the industry	398	20.6
To train for specific needs of specialized divisions of engineering practise	229	11.9
To provide the former type of training for the majority, but provide the latter type for those who desire to spend the additional time required	1304	67.5

The enormous physical plant which has been built up during the last few decades has produced problems never thought of in the early engineering curricula. The country-wide net-words of railroad, telephone, power, and radio systems have brought with them a host of problems in all branches of engineering, ranging by degrees from management, with all of its extremely complicated personnel and technical questions, on the one side to the most advanced scientific research, with its exacting demands in mathematics and the sciences, on the other. The size of the field in which a young engineer finds himself shows clearly the futility of any four-year curriculum of a specialized nature. The only adequate preparedness for such a field is a broad education in the humanities, fundamental sciences, and engineering fundamentals.

The results of the investigation indicate that the most serious criticism of engineering education arises from the lack of training in business and economics[14]. The fact that success depends largely upon a good understanding of those subjects seems to be very generally recognized.

It has been said that engineering graduates of the past have risen to high executive positions and that this indicates that no great changes in curricula are necessary. Many of the questionnaires covering various phases of the investigation contain replies which show that in general engineering alumni think the curricula were not seriously deficient in any important respects except in the lack of business and economics. It must be remembered, however, that the questionnaires were planned to bring out the facts regarding existing curricula, and those who answered them were given no real opportunity to express their opinions of the recent movement toward more liberal engineering curricula.

In 1920 Stanford University put into effect the Lower Division plan which replaces the major department system during the first two years. The principal object is to require more training in fundamental subjects. During this period, the students are registered under the supervision of the Lower Division Committee which is appointed by the President. Each is required to take certain subjects and to choose other subjects from specified groups. Some of the requirements can be met by certain high school subjects. The requirements do not completely fill the first two years, and the remainder of the time can be devoted to electives. The following is a brief summary of the requirements[15]:

Group Requirements

I	Languages and literature and formative art	18 units
II	Natural sciences and mathematics	18 "
III	Social sciences	18 "

Subject Requirements

English composition	6 "
Foreign language	22 units in one, or 15 units in each of two foreign languages
May be anticipated in high school in whole or in part.	
Biological science	9 units
Physics or chemistry	9 "
One of the sciences may be anticipated in the high school.	
American history	9 units
General history	9 "
One of the history requirements may be anticipated in the high school.	
Citizenship	12 units

At the beginning of any quarter, a Lower Division student may designate the department in which he expects to register during the last two years. Those who thus make a tentative choice of major subject are then advised to consult the department regarding the most suitable courses to take as electives during the first two years.

In the autumn of 1924, the President appointed a special committee made up of representatives of all of the engineering departments, and requested that the various phases of work in engineering be considered and recommendations be made to him. That committee presented its report in March 1925. In this it recommended that a School of Engineering be organized, that a more general type of engineering curriculum leading to the degree of A. B. in Engineering be adopted, that the department curricula, with certain modifications, be retained at the option of the departments, and that two-year graduate curricula leading to the degree of Engineer be adopted by those departments which had been requiring only one year. The recommendations were adopted by the University, and Professor Theodore J. Hoover, Executive Head of the Department of Mining and Metallurgy, was appointed Dean. Committees were appointed last October, and the registration of students in the tentative form of general engineering curriculum was begun in January 1926.

A revised form of curriculum has recently been adopted by the Faculty of the School of Engineering, and a copy of it is given below.

FOUR-YEAR CURRICULUM LEADING TO THE DEGREE OF BACHELOR OF ARTS IN ENGINEERING**

FIRST YEAR, TOTAL UNITS, 44

Subject	Course No.	Autumn	Winter	Spring
Foreign Language		3	3	3
Linear Drawing and Lettering	C. E. 1, 2	..	1	1
Chemistry		4	4	4
Co-ordinate Geometry	Math. 10, 11	3	3	..
Calculus	Math. 21	..	..	3
Citizenship	Citizenship 1-3	4	4	4
Total		14	15	15

SECOND YEAR, TOTAL UNITS, 46

Subject	Course No.	Autumn	Winter	Spring
English Composition	Engl. 2a, 2b	3	..	3
Freehand Drawing	M. E. 11	3	..	..
Calculus	Math. 22, 23	3	3	..
Heat and Electricity	Physics 13, 14	..	4	4
Mechanic Arts	M. E. 1, 2 or 3	*..	2	2
Descriptive Geometry	M. E. 10	..	4	..
History		3	3	3
Extemporaneous Speaking	Engl. 7	3	..	..
Elementary Machine Drawing	M. E. 12	..	..	3
Total		15	16	15

THIRD YEAR, TOTAL UNITS, 45

Subject	Course No.	Autumn	Winter	Spring
Theoretical Mechanics	C. E. 30, 31	5	5	..
Hydraulics	C. E. 106	..	..	5
Surveying	C. E. 20	5	..	..
Engineering Geology	Geol. 1a	5	..	..
Electricity in Engineering	E. E. 102, 103	..	3	3
Elementary Accounting	Econ. 3	..	5	..
Principles of Mining	M. & M. 101	..	..	5
Electives		..	2	2
Total		15	15	15

FOURTH YEAR, TOTAL UNITS, 45

Subject	Course No.	Autumn	Winter	Spring
Mechanics of Materials	C. E. 110	5	..	..
Pyrometallurgy of Iron and Steel	M. & M. 105	..	3	..
Exposition	Engl. 131	4	..	..
Business Law	Law 100	4	..	..
Prime Movers	M. E. 123		5 or	..
or Elementary Machine Design	M. E. 113	..	4	..
Engineering Economics	C. E. 130	..	..	3
Human Relations in Business		..	3	3
Electives		2	4 or 5	9
Total		15	15	15

*All these Mechanic Arts Courses are given in the Autumn, Winter, and Spring Quarters. Any two courses may be chosen by the student, limited only by the capacity of the laboratories. Although scheduled for the Winter and Spring Quarters, sufficient registration is desired for the Autumn Quarter to make operation of all three laboratories possible.

**Since the presentation of this paper before the San Francisco Section, minor corrections have been made in this schedule to place it in complete accord with the curriculum appearing in the Stanford University Announcement of Courses for 1926-27.

This curriculum is founded on the belief of many that any considerable amount of specialization in engineering subjects during a four-year course is undesirable. All students should have a good foundation in general or cultural subjects. In addition to this, engineering students must have rather extensive training in chemistry, physics, and mathematics. There is a number of subjects such as mechanics, hydraulics, surveying, geology, business law, etc., which all engineering students should take. Finally, a young man who hopes to become a broad-minded and well-balanced engineer must have some knowledge of the contents of all of the principal branches of engineering, first, in order that he may be able to choose more intelligently the branch he wishes to follow as a specialty, and, second, in order that he may be able to consider all problems in their proper relation to the whole field of engineering. It is desirable also that such a curriculum allow a reasonable amount of time for elective subjects.

The above curriculum is not considered final. It is hoped that improvements can often be made in it. However, we believe it does follow closely the ideals expressed above.

The decision has recently been reached that the A. B. degree will not be awarded in the separate branches of engineering at Stanford after 1929.

The graduate study in each department is to continue over a two-year period leading to the degree of Engineer in the various branches. It is firmly believed that students who complete the six years will be very much better prepared for their life work than are those who take a more specialized course for four years and then one year of graduate work.

The question will often arise as to whether students who complete the four-year curriculum only will be prepared to begin work in certain engineering organizations. It is true that they will not be as well prepared in a certain few special phases of engineering as would the graduates of a more specialized curriculum. However, they will have a much wider range of choice, and need not feel limited to only a few specialties. If they devote their electives to carefully chosen courses, there need be no feeling that they are not prepared to become immediately useful. Future progress should be materially faster due to the broad foundation.

A few quotations from statements of leading executives and engineers will emphasize some of the statements made above.

SECRETARY OF COMMERCE HERBERT C. HOOVER[16]

"There is somewhere to be found a plan of individualism and associational activities that will preserve the initiative, the inventiveness, the individual, the character of man and yet will enable us to socially and economically synchronize this gigantic machine that we have built out of applied sciences. Now, there is no one who could make a better contribution to this than the engineer, but to make that contribution our engineers in the future have got to have a broader and stronger place in our world affairs than they have today. We cannot be turning men out of our universities as we are in many cases today purely mechanical machines devoted to some theory built on applied sciences. If the engineer is going to take his part in this community, is going to give expression to those things that he can express best, he must start with a sense of his public obligations as well as his professional knowledge."

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* * * "but we had better reduce the volume of science and applied science we are pouring into our young men in order to make room for some stimulation of their public relationships, some realization of their public obligations."

F. C. Pratt, Vice-Pres., General Electric Co.[17]

"Voicing my own opinion on this subject, we are not looking to the colleges and technical schools to turn out finished engineers, but we do look to them for a steadily increasing supply of young men who have been thoroughly trained in the fundamental theories of the mathematical and physical sciences, and to the fullest practicable extent in economics and in what are commonly called cultural studies. We believe that, with this ground-work thoroughly prepared, the large industries are in a particularly favorable position to offer exceptional opportunities to young men for gaining practical knowledge and experience along special lines.

"In this connection, I wish to make it quite clear that in the foregoing remarks I am not including those exceptional students who by natural qualifications and inclinations are prepared to pursue post graduate studies in theoretical work in any branch of science or engineering which contributes to the industry."

F. C. Pratt[18]

"My observations also lead me to the conclusion that the percentage of those who fail to attain a reasonable degree of success is greater in the group of men of mediocre ability but narrowly specialized education than in almost any other group coming within my knowledge."

Professor Edward Bennett, University of Wisconsin[19]

"One of the most gratifying developments of recent years has been the recognition on the part of the engineering industries that they do not wish to have the engineering colleges train men for immediate service in their specific fields."

Dean F. L. Bishop, University of Pittsburgh[20]

"There are two factors which enter fundamentally into the life of an engineering student. One is education, the other is training. In the very early days of the engineering school, education was the controlling idea; later, training or specialization became the controlling factor. In other words, we shaped our courses, modified our curriculum, and selected our teachers with the sole purpose of graduating professionally trained engineers.

"The reaction soon became evident. The cry went up that the engineer, although thoroughly trained, was narrowly educated. He lacked the proper perspective of life. He was unable to grasp the economic principles underlying great problems. He was too intent upon design and the solution of specific problems.

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"We must look forward to the time when engineering schools will consider themselves as educating a large body of men who will become effective managers of the industries and who will exert, through their education and training, an important influence on the political and social side of the community.

"If our engineering schools will look upon the education and the training of this large mass of men as their primary object, and delay the specialization and technical training to additional years in the schools or industries, our instruction will be changed to meet this demand so that the emphasis will be placed more on education and less on specilization. Our teachers will not be such highly specialized technical experts but that they will be broadminded educators. The public will show an increasing confidence in our graduates because they will be educated as well as trained."

Dr. F. B. Jewett, Vice-Pres., American Telephone and Telegraph Company[21]

"Consequently I am interested in having those young men who come to us and who are going to be the leaders of our business of the future well grounded in the sciences. First, because we need them in the technical side of our business, and second, because I believe we are going to need them in a larger measure as the recruiting source for the executive directors of our business in years to come."

John Mills, Director of Publication, Bell Telephone Laboratories, Inc.[22]

"After we have answered these intermediate questions, we shall probably agree that we turn to the colleges in the hope of obtaining men of good mental ability and personality, who have acquired habits of thought and study which will enable them to see broadly the business and technical problems of the future, to analyze the factors involved, to arrive objectively and without prejudice at solutions, and, through personality and executive ability give to those solutions weight and effectiveness. We look, I believe, for trained brains in vigorous bodies with pleasant but dynamic personalities; men who may make creative contributions to our respective businesses or arts and, in a sufficient number of cases, develop as capable executives."

Dr. M. I. Pupin, President, A. I. E. E.[23]

"Nothing resists a change so obstinately as the mental attitude of man. The history of science from Archimedes to Newton offers many illustrations of this well-known fact. The change in the mental attitude of our age is one of the greatest achievements of our intellectual renaissance. Less than two generations ago, educational training was expected by many to operate like a penny-in-the-slot machine; that is, learn your lesson and convert your learning into cash without much delay. The so-called practical man who managed our American industries was at that time an ardent advocate of this utilitarian theory. He worshipped the art of making a living. Franklin and Lincoln, my patron saints, had no sympathy with this theory. The art of making a living was not the determining factor in their schooling, but the art of making life worth living was everything to them. They would find no fault with the American college because its diploma does not testify that college graduates are loaded with a knowledge of the art of making a living, provided, however, that they carry with them some definite ideas about the art of making life worth living, not only their own individual life, but also the life of our nation. The expansion of these ideas is the gospel of the American university."

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Discussion

F. E. Terman: The future of engineering education may be profoundly affected by the work which the psychologists are beginning on special interest and special aptitude tests.

The Psychology Department of Stanford has devised an interest test to separate men practising the legal, medical, and engineering professions. This test contains no technical matter, and yet it distinguishes between engineers, doctors and lawyers with about 90 per cent success. That is, if one gives the test to a hundred lawyers, a hundred doctors and a hundred engineers, the interest test will correctly name the professions of about two hundred seventy of the three hundred tested.

The Stanford Psychology Department has devised also a test of scientific aptitude. This test was given to about a dozen graduate students studying electrical engineering and the men were ranked in order of their scientific ability as shown by the test. This ranking was then compared with an independent ranking made by the electrical engineering staff. The two rankings were identical with the exception of one or two students out of the dozen or so ranked.

Thus the one-and-a-half-hour test alone was able to tell about as much regarding the scientific ability of these students as could be discovered in from one to two years' contact by several teachers.

These two tests are illustrative of a movement that the psychologists are just starting, and which appears to have great promise, particularly in vocational guidance and engineering education.

J. H. Johnson: I "enjoyed" one of these courses about fifteen years ago and desire to give my impressions from a study of the suggested course.

The course I pursued was quite similar to the one outlined, except that it was slightly heavier—in that it averaged about eighteen hours where sixteen hours is now suggested. I strongly favor the heavier work.

Rather early in Mr. Henline's paper, he suggests a liberal policy as to electives. I fear the results of such a procedure and believe it much better to offer a "cast-iron" schedule. The student does not like a rigid schedule and in theory I agree with him but practically it appears to be necessary. The average student does not know what he wants. He does not know what courses will best prepare him for his future needs. Conflicts as to time further complicate the problem—to the end that unless a course of study is carefully prepared, the electives end in courses such as typing, glee club, orchestra, or wrestling. For these reasons I object to the rather large amount (about 12 per cent) of electives outlined.

Since courses as given in some colleges are so presented that essential ideas are scattered scantily through a large mass of unimportant elaboration, it appears to me that when it is deemed advisable to give engineering students courses such as economics, business management, and so on, the work should be condensed and reduced to its more important component principles in the same fashion as in his engineering texts. To secure this treatment, it may be necessary to have the work given in the College of Engineering.

Further, as a general thing, I would remind you of the old objection to a five- or six-year engineering course, *i. e.*, the fact that it graduates a student at a correspondingly later and to the employer, less desirable, age.

G. S. Smith: The field of engineering education is being subjected to a great deal of careful scientific research, and already these studies have resulted in the formation of some very definite ideas. So far, it has not been possible to give many of these ideas a fair trial because the general public, upon whom the state universities at least are largely dependent, will seldom sanction a change, especially if it means added expense, until they are educated to realize its value.

However, it is very gratifying that the engineering profession in general is taking such a keen interest in the methods to be used in our engineering colleges. Fortunately, the larger companies are doing more and more to help the engineering colleges to a higher standard. Would it be too much to ask that these larger companies, and in fact the whole engineering profession, help educate the general public to realize these needs? The advances made in engineering education will no doubt prove valuable to education in general.

It is becoming more and more evident that the best engineering professor is not only well versed in the fundamentals of engineering which he must teach, but must also specialize in some branch of that work to the extent that he may be considered authoritative in his specialty. Such a professor will prove an asset to the institution he serves and an inspiration to the students with whom he comes into contact. Here again the larger companies are cooperating in helping faculty members to a higher standing in their profession and offering them opportunities of contact with the practical field. Here also the institution might well offer some incentive for the professor to build up his reputation, and avail himself of the opportunities offered.

The success of many of the ideas which have been put into practise is apparent from Professor Henline's paper. He mentions the courses in general engineering problems now required of engineering freshmen in some colleges. This work has met with fine success at the University of Washington. It gives the student a fair idea of that to which he may look forward and either gives him a good start in his chosen field, or causes him to realize he has been mistaken in his choice.

Those faculty members who have charge of the junior and senior courses might well study and lend their influence toward directing students into the field of work for which they are best fitted. For example, it is often apparent to the professor whether a student is adapted to such fields as research, design, shop or mechanical details, executive work, etc. A little personal advice and aid might stimulate a student to aims which otherwise he may never develop.

That marked advantage is to be gained by having certain required non-engineering courses taught by engineers has been supported by some and questioned by others. While this is seldom possible except in the larger schools, it seems very desirable for the more fundamental subjects, such as engineering mathematics, and at least one course in English. Another alternative that has met with some success is to segregate the engineering classes in such subjects and select some instructor from the faculty of that department to specialize in teaching these classes from the engineering point of view.

R. W. Sorensen: I think it absolutely impractical to make a standard engineering curriculum. I can see no reason why all schools should be alike. Individuals and communities differ from each other, and regardless of our efforts to direct young men along the paths we think they should follow, they seem to do work largely according to their natural abilities. At California Institute of Technology we aim to guide students in the selection of courses and assist them in making efficient use of their time; but we also endeavor so far as possible to give the student the widest latitude as to the manner in which he shall do his work. For several years the classes have been divided into groups according to scholastic standing, the various groups doing work we think best suited to their special abilities. The results of this plan have been wholly good. The Institute faculty has never been satisfied with a one-year course in English for engineering students. When our course was a four-year course, the students had a four-year course in English and associated humanities. The time given to this work was approximately one-fifth of the total time spent on college work. I can report also that our engineering students have not considered these subjects uninteresting or things to be avoided if possible. For example, we have several times had requests from students for additional classes in English and last year we had a request even more surprising than one for additional English—a course in the study of Greek. This was probably due to the fact that Dr. Macarthur, one of our English professors, had so inspired the men with some discussion of Greek Literature as to incite them to a desire to study from the original, methods of Greek thought. Under such conditions I am certain that the course in Greek as given by Dr. Macarthur was a very valuable one. We have always endeavored to make English just English,—not engineering English,—believing that most engineers not only do not write reports well, but cannot expect to learn to write reports well if they endeavor to learn only to write reports rather than learn to make the fundamentals of English and English literature an integral part of the means used by them in expressing ideas.

D. I. Cone: Men of the industry who are not on college faculties have a very great interest in the matter of engineering curricula. If there were no other reason for it the young men being trained would very soon be taking up the work of assisting us in our problems; so we do well to encourage the school to give the best possible training. Just to consider the great variety of topics upon the program of this Convention is to become convinced that a broadly developed curriculum is necessary to fit the student to be appreciative of it. We should, therefore, extend our heartiest interest and our congratulations to the Stanford faculty for undertaking this forward step.

The problem of the fifth and sixth years of training raises a challenge to us of strengthening means for assistance to those students who find difficulty in going on; that is, men who would need to devote a large part of their time to earning their own way. A further problem calling for continued study by the men of the industries and colleges is that of cooperation in bridging the gap between the college life and the ensuing life in industry. It seems clear, however, that the solution of this is not to be found in a specialized curriculum.

H. T. Plumb: Many young men come to me for help in choosing engineering and college courses, and I generally give them four rules as a safe guide for starting out on a college career.

First: During high school and the first three years of college, at least, "study fundamentals."

Second: Avoid early specialization.

Third: Study in school what cannot be learned outside of school.

Fourth: Become an all-around man and do not specialize in any one subject until you have a general knowledge of all. Elect studies that will develop your low spots.

About these four rules I have thought for many years and believe them to be generally good. I think that too much of instruction and not enough of education is attempted in engineering. Education should develop or lead out. All courses should seek first to develop the man and then to add a little knowledge of engineering for him.

The larger engineering firms are coming to demand college men and are willing to teach them the engineering afterward. They are willing to teach their particular form or method of engineering. I think we should not train to be engineers only. That is not the prime object. We have to live and we should educate ourselves and train ourselves so that we can help with the greatest amount of judgment and with the greatest amount of service to our fellow beings.

We are three-sided by nature, and if we neglect the training of any one of these sides, then we reduce the efficiency of the whole.

D. C. Prince: It seems to me that many people have entirely too low an opinion of the college undergraduate. They seem to think that they have to outline everything he should do. It would be interesting if someone would take all the material of the four-year college course and take a good active boy and turn him loose on it. I think he could clean up four years' work in two years.

K. B. Miller: I think we have to admit that the educators are up against a very hard problem and that they are trying to solve it. The most promising thing is that they are trying to solve it in a scientific way. Looking back over my experience as a graduate from one of the technical universities, I have been forced to the conclusion that the average engineering graduate was a pretty lopsided individual, mostly because he had tried to specialize too soon. I do not object to a man having a definite idea of his future, but I think he should be guided and given a chance before he commences some course for which he will not be well adapted. His judgment is not as good as it will be later on and I think I take issue with Mr. Prince on that particular question; not as regards all men, but as regards the rank and file. Most of them do not know what they want to do at the time they enter college. Give a man a pretty good education and do not allow him to specialize too soon. A man will be so much better able to tackle his line of work if he does not specialize too soon in any one thing.

Harold Michener: It seems very important to me that young men should have some choice of instructors. Do not make the curriculum so rigid that those who are bright enough to do so cannot choose the best instructors giving the desired or required courses. Do not, by putting the cultural subjects under instructors in the engineering departments, deprive the engineering students of the opportunity of studying these subjects in outstanding courses under outstanding instructors, such as, in my time, Prof. Gayley's course, "Great Books."

L. N. Robinson: It is gratifying to read in Professor Henline's paper that another positive step is being taken toward the better and broader education of engineers.

One can hardly criticise the curriculum proposed without intimate knowledge of the individual courses. It appears to me, however, that it would be preferable to spread English composition and public speaking over at least the first three years, devoting one unit each of the autumn and winter terms to grammar, rhetoric and themes, and one unit of each of the spring terms to the presentation of prepared speeches, the reading of technical papers written by the student, and extemporaneous speaking.

The course, "Human Relations in Business," appears interesting and I wish Professor Henline would tell us more of the content of this course. Human relations in business can hardly be taught comprehensively without giving a course on human relations in general; and in view of the state of modern society, it appears that it might be desirable to include the subject of domestic relations. It is to be presumed that the course will be offered with a scientific treatment to conform to the methods employed in teaching the theories of physical phenomena, and I wonder what text is to be used?

For a long time it has seemed to me that sooner or later some of the engineering colleges will visualize the importance of teaching the fundamental principles of human relations. Professor Henline alludes to the necessity for this when he ranks the "understanding of men" (often called "ability to handle men") high among the requisites of an engineer, especially if he would attain an executive position. Incidentally, to handle men should not mean manhandling, as too many old-school executives and others believe; it should mean to guide them,—subordinates, superiors and contemporaries. How can that ability be developed?

Now-a-days, we depend almost altogether on the students and graduates learning by merely casual contact and hearsay,—often accompanied by bitter experiences,—what they can and cannot profitably do. Would it not be far more rational to teach them the fundamental principles and give them such laboratory work along these lines while in college as would better prepare them when they graduate, just as we endeavor to equip them with the fundamental laws that govern physical phenomena? Where shall we find these fundamental laws governing human relations?

There is no place where they are so well codified and demonstrated as in the New Testament of the Bible. You may object to my proposing to introduce religion into our colleges. Are we not already putting too much blind faith in our scientific knowledge and adhering too religiously to theoretical fetishes? How many of our so-called scientific laws have stood uncontroverted anywhere nearly as long as the scientifically founded laws of human conduct voiced by Jesus Christ and his disciples? How many have stood half, or even one-tenth, as long?

I hold no brief here for any creed or sect; in fact I would be one of the first to object to introducing into any school a course in blind religion as such. The genuine Christian doctrines, however, as differentiated from sectarian dogmas, are not founded on blind faith. They are rational and demonstrable, permitting of the same scientific methods of analysis and experimentation as the commonest laws of physical phenomena. When they are studied openmindedly by scientific methods and confirmed by experiments and observation, the student will feel fully justified in clinging to them just as religiously as he now clings to Newton's laws and Maxwell's equations despite an Einstein.

Professor Henline's paper advocates a six-year course for engineers. This appears to be predicated on the assumption that 15 units per term is about the permissible maximum. Should we not examine such an assumption very carefully before we cut two years out of every engineer's life? Practically, it amounts to killing him two years before the end of what would otherwise be his useful career. Nevertheless, this assumption is commonly accepted in most colleges, although it seems a reckless waste of students' time and highly conducive to their immorality.

If I am not mistaken, 15 units correspond to 15 hours of classwork and 30 hours of homework per week, with suitable adjustments for drawing and laboratory periods, so that the student is expected to fulfill the requirements of the course by applying 45 hours per week.

After he graduates, the student will be expected to work at least 45, if not 54 or more, hours a week. Furthermore, he will be expected to spend four or five evenings of, say, three hours each, in study and self-betterment, or a minimum of 57 and a maximum of not less than 69 hours. Why train him in laziness at college? On the other hand, any ambitious high-school graduate who is forced by circumstances to go to work, would be expected to work at least 45 hours and probably more per week, attend night school two or three evenings per week and study the intervening evenings. Why should earnest college students do less with their manifold advantages? Is it any wonder our students need automobiles, supper dances and gin to fill up their idle time?

In conclusion, I believe the best of engineering curricula can be condensed very practicably and advantageously to not more than four years of nine months each, giving between 20 and 25 units per term and requiring an average of approximately 60 hours of application per week.

F. O. McMillan: It appears that most persons associated with or interested in engineering education are quite generally convinced that four years is a short time in which to train for an engineering career. These people in general, are also convinced that the fundamental principles of science and engineering and the humanities are of prime importance. Yet with these points conceded, we are still confronted with the problem of training many engineering students who enter the colleges and universities with a desire and a determination to be trained in some specific branch of engineering, such as electrical, mechanical, civil, etc. These students, many times excellent scholars, are unable for financial or other reasons to continue their education longer than four years. How are we to hold the interest of such students in a four-year general engineering course?

Obviously, one way to hold interest in the general engineering course would be for the industries and others employing these engineering graduates to announce publicly that, other conditions being equal, the four-year graduate having a general engineering education will be given preference over the four-year graduate specializing in the branch of engineering for which they are seeking employes. Are the employers of engineering graduates, outside of a few large corporations having well-established training systems, willing to do this? If so the problem is already a long way toward being solved.

Another phase of Professor Henline's paper is that which has to do with the vocational-guidance value of a four-year general engineering course in assisting students to choose their major branch of engineering for the two-year post-graduate course. My personal observation has been that students undecided upon their major course are very largely influenced by the relative personality, enthusuasm and ability of the professors in the various major departments between which the student must choose. This obviously, many times, is detrimental to the best interest of the student because it fails to take into consideration special aptitude and many other points that are vital. For this reason it occurs to me great care would have to be exercised in the presentation of engineering subjects during the four-year general engineering course to avoid the aspect of selling one branch of engineering in competition with another.

G. R. Henninger: One thing that at first, perhaps, reacts negatively upon all of us is the idea of extending courses of engineering study to include more than four years. However, it is of importance to remember that the prime objective of an education is nothing more nor less than to bring the present generation up to date. It is obviously necessary that the individual learn what has gone on before in order to progress logically and economically.

Engineering has branched out in many ways. That of itself makes it essential that more of the engineering student's time be devoted to study. Further in support of the proposed longer courses is the accepted fact that the graduate engineer must have more cultural and commercial training if he is to step into active practise without a serious setback.

The soundest foundation that a young fellow can get, regardless of the nature of the work that he may take up in later life, is a course in engineering fundamentals. Thus at least on two counts I have great regard for the proposed "four-plus-two-year" course developed by Professor Henline, at least as a starter in the right direction. The practising engineer must assist in this.

Regarding the "cast-iron schedule:" Most of my thoughts have already been spoken by others, but I should like to add my urge against too rigid a schedule. It would seem that our educators themselves are in a most advantageous position to give to the individual student something in the way of individual guidance.

Another word of warning should be voiced against the tendency to bring under the control of the engineering department those courses of cultural value that are so badly needed by the student engineer. If liberalizing subjects are brought within the influence of the engineering department they lose at once at least 50 per cent of their broadening value. Perhaps the one outstanding exception to this is mathematics. Theoretical mathematics and the mathematical proficiency that will be most useful to the majority of engineering graduates differ widely. In this one case, therefore, it would be of advantage for the engineering department to have under its influence the study of engineering mathematics.

David Hall: I have come in contact with many students in the Engineering School of the Westinghouse Electric & Mfg. Co. and I have observed that students well grounded in fundamentals easily take the lead. We have a course first on d-c. machinery, then on induction motors, a-c. generators, transformers, control, etc. After the first two or three sessions, one can pick out the leaders among those students. In about two sessions you can pick out the ones best adapted to engineering. The leaders are picked out because they have good fundamental grounding in the studies. They understand the underlying principles. When they get the underlying principles, they can take the subjects and handle them in a masterly manner.

There is one point in this paper where the author says that "70 per cent of the men of those in classes out fifteen years or more are in work primarily administrative, etc." Now, because 70 per cent of these men are in administrative work the author concludes that they should have been trained for administrative work. My conclusion is that the technical education has already trained them for administrative work and the proof lies in the fact that they are selected by their superiors for administrative work.

J. P. Jackson: Engineering education in this country has been developed under the supervision of strong men of brilliant mind. Evidence that a sound foundation has been builded is to be found in the great, beneficent influence wielded by our college-trained engineers in the upbuilding of American industry.

In making changes, therefore, it should be with a view to building further on the existing base, being extremely careful not to cut away any but occasional rotten spots.

With the author, I am convinced that as development proceeds we shall put into our engineering courses more of the humanities. An engineer should be the most useful and influential man in his community. To be so he needs a practical knowledge of government, social customs, of business methods, and of how to handle men. The strengthening of his training along these lines while in college will result in some cutting on the scientific side of his training; but this should be done carefully or a stunted growth will result.

The author states rightly that generally character has been considered a most important element in making a good engineer. When I was Dean of Engineering at Penn State the members of our engineering faculty unanimously decided to give a certain definite proportion of their time to the study of how best to instill the various elements, such as honesty, industry, tolerance, thoroughness, justice, humanity, etc., into the embryo engineers within their care, and also to devote a definite portion of their class time for this purpose. The results as I observed them were eminently helpful, and I believe not only tended to emphasize the importance of sound ethics in the minds of our engineering students, but to increase their enthusiasm and inspiration for their scientific and technical studies. The older professors were generally able to draw their material for this kind of teaching from their own experiences in the engineering field, while the younger ones were expected to confine themselves more specifically to the biographies of great engineers and industrialists.

With regard to the course schedule on the sixth page of the author's paper, permit me to say that I cannot agree comfortably with the use of the B. A. degree for application to a course so substantially scientific as the one he gives. For such a long span of years, courses leading to B. A. degrees have been those which prepare for careers dealing with the fine arts, literature and kindred subjects or professions, that it seems to me to be a distortion of good, sound English language to apply it to such a course as the one given. I should like to know from the author and from the great university which offers the course, what is wrong with the degree, Bachelor of Science?

I believe no one would have serious reason to criticise the fundamentals of the general course on the sixth page. To my mind, it would be far wiser to put the work on citizenship in the fourth instead of the first year. If taught as I conceive it should be to a college man, it is one of the most important subjects of the student's course and requires the brain training of a senior student. I should also use the term "industrial history" instead of "history," not for the purpose of cutting out a suitable review of the political phases of history, but for the distinct purpose of emphasizing the enormous effects of industrial and scientific development on civilization, particularly during the past one hundred years.

I should like to add one more word about this course. If the great institution which is using it continues to offer also the regular engineering undergraduate courses, without question or doubt, there will be a small handful of students enrolled in the general course and a small army will continue to go into the ordinary engineering courses. In the regular engineering courses of great value there is created a certain amount of professional spirit which attracts students and which is not usually so evident in a general course. My own past practise and present belief is that much the same result as desired by the author can be obtained by suitably broadening the existing engineering courses to the point where differentiation is of a comparably slight amount, though I am not at all disposed to the addition of such a general course as he has outlined.

H. H. Henline: I agree fully with Dr. Terman that the interest and scientific aptitude tests which have been developed promise much for the future. We have had the experience of rating in numerical order, two classes of graduate students in electrical engineering using as a basis their scientific aptitude as exhibited in class and laboratory, and comparing our ratings with those made by a graduate student in psychology who had given these classes a special scientific aptitude test developed as part of his own graduate work. Both times our ratings agreed very closely with his. The need for some such means of determining that for which each student is best fitted is very great, and such tests should be given a fair trial.

The fact that the curriculum under discussion contains only about 15 units of work per quarter instead of 18 has been mentioned by Professor Johnson. I can assure him that the majority of the students will find all the work they can do well. He said a "cast-iron" schedule with no electives is better. I cannot agree with this at all. How can we tie men down to a very rigid schedule for four years and hope to find them showing any originality after leaving school? It is better to allow them a little freedom of thought, and give them a chance to strengthen their weak spots. An objection has been raised to this in that many men may choose easy courses. This possibility is, of course, present, but the best that can be done is to give students whatever guidance may be necessary and encourage them to develop their own individualities in the proper directions.

The plan of having English, economics, mathematics, and other such courses for engineering students given in the engineering schools exclusively has been the subject of many arguments. With ideal instructors, perhaps these and other subjects should be given by engineering teachers, but men who know both engineering and the other subjects sufficiently thoroughly and are willing to do such work are rare. The students should by all means have contact with those who are teaching such subjects on account of love for the work, and should not be restricted to teachers who are giving those subjects merely because they are useful tools for engineers. They need the broadening effect as much as the subject matter.

Issue has been taken with the Stanford plan on the apparent assumption that the four-year curriculum will not prepare students thoroughly. We believe they can go out at the completion of it, prepared to enter many types of engineering work and well prepared for the commercial phases of engineering.

The six-year idea has been criticized because it would hold students in school two years longer than the great majority now stay. As just stated, we believe that the four-year curriculum furnishes good preparation for entering actively into the engineering field and those who can follow that with two years of graduate study will have an excellent preparation for their life-work. The first function of a university is to supply the foundation for good citizenship. The second is to train for definite fields of usefulness. The second is important, but it should not be allowed to replace the first.

The question has been raised as to whether we are placing too much emphasis upon training for administrative work and too little on technical education. Records given in the paper indicate clearly that a broad education is of greater importance than training in technical subjects. The plan is to lay the foundation for all life and to build upon it a moderate amount of engineering education. A few courses such as economics, geology, psychology, etc., will awaken students to interests extremely important in later development, with which they might not come into contact for several years if such courses were omitted.

Mr. Robinson has asked for more information on the nature of the course, "Human Relations in Business." This is to be developed and given by the Department of Psychology primarily to meet the needs of engineering students for some training in those fundamental principles governing human relations which one must know if he is to deal successfully with people. It will be given by means of lectures, and adapted to the requirements of the students as well as practicable.

The desire of young men to specialize in some branch of engineering immediately after entering the university is often mentioned as an argument against the more general curriculum. The S. P. E. E. investigation has developed the facts that the great majority of young men entering the engineering schools have chosen particular branches of engineering, and that more than half of those who have made such choice have done so with little or no real information regarding the branches chosen or their fitness for them. It seems necessary to teach many young men that technical subjects are not the only ones needed in an engineering curriculum.

Mr. Jackson has asked what is wrong with the Bachelor of Science degree. Stanford has always given the Bachelor of Arts degree in all departments with the idea that no distinction should be shown between those who graduate in literary subjects and those who graduate in the more specialized subjects, such as engineering. The degree given in all cases is Bachelor of Arts in..............

Citizenship is given during the first year, rather than during the fourth as Mr. Jackson suggested, on account of the desire to place it where it will be most helpful from the standpoint of orientation and the development in the student's minds of a sense of their responsibility as citizens.

Transmission Features of Transcontinental Telephony

BY H. H. NANCE[1] and O. B. JACOBS[1]
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***Synopsis.**—In this paper, the various steps in the establishment of the existing network of transcontinental type circuits and the transmission design considerations are reviewed. The discussion covers the communication channels obtained from transcontinental type facilities and the bands of frequencies employed, and includes carrier-current systems, telephone repeaters and signaling systems. Mention is made of special uses of transcontinental telephone circuits, such as the transmission of program material for broadcasting and the transmission of pictures. Finally, the maintenance methods required to keep the system at full efficiency are outlined.*

IN view of the fact that this convention is being held in Salt Lake City and that the original transcontinental telephone line connects this point with Pacific Coast points and the eastern part of the country including points on the Atlantic Coast, it was suggested to the authors that it would be of interest to present a discussion of the transmission features of transcontinental telephony at this time.

Since considerable information on this subject has been covered by previous papers presented before the Institute, this discussion will be confined to a resumé of the transcontinental type facilities provided for coast-to-coast telephone service and some of the general transmission considerations which are important factors in determining the design of these facilities.

The opening of the first transcontinental line in 1915, between New York and San Francisco, marked a new era in long distance telephony, as this was the first achievement of successful telephone transmission over distances materially in excess of 2000 mi. and demonstrated clearly the practicability of meeting the transmission requirements for a nation-wide telephone service. Previous to that time, New York to Denver represented about the maximum distance for telephone connections and the transmission obtained would not be considered any too good as judged by the standards of today. The circuits for the New York-Denver service had been constructed of copper wire, 165 mils in diameter (435 lb. per mile), and were loaded with 250-milhenry coils spaced about eight miles apart. Repeaters, or amplifiers, however, were not used since methods for applying them to loaded lines, as well as for their use at more than one point in a connnection, had not been developed to a practical extent at that time.

By the time the new line west of Denver was constructed, telephone repeaters of new design and improvements in their application to telephone circuits had been developed so that the difficulty of operating in tandem and also over loaded lines had been overcome. Included in these improvements, which were applied to the new line and also to the existing line from New York to Denver, were new loading coils of a more stable design, very accurately spaced, in order to provide uniform impedance characteristics and balancing networks of simple design for use at repeater points to match or simulate the impedance of the line. The repeaters were located about 500 mi. apart so that on a New York-San Francisco connection, six repeaters were normally in the circuit. The transmission loss in a connection of this kind was about half that in a former New York-Denver connection and about the same as that in a former New York-Chicago connection.

Many new developments have been applied to the transcontinental circuits since the first of these were placed in service and also to other similar circuits throughout the country, which have resulted in a better quality of transmission, including increased over-all volume efficiency[2]. Briefly, the outstanding features of the improved circuits are that they are non-loaded and that the repeaters and the associated equipment have improved transmission characteristics. With the non-loaded circuits, increased speed of propagation and smoother lines are obtained and consequently they can be operated to give better volume without increased echo effect. At the same time, transmission is further improved due to the better attenuation-frequency characteristics. Variations in line attenuation with weather conditions also are considerably reduced.

The transmission improvements in the repeaters and associated equipment consist chiefly of better transmission-frequency and impedance-frequency characteristics, which, together with improved balancing networks, contribute toward better quality of transmission not only from the standpoint of naturalness but also from that of volume efficiency. Due to the higher attenuation of non-loaded lines as compared with loaded lines, a larger number of repeaters is required on a long non-loaded circuit than on a loaded circuit of similar length and therefore the importance of the improvements in the repeaters is correspondingly greater.

Three telephone circuits were provided by the first transcontinental facilities, which consisted of four wires

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[1] Both of the American Telephone and Telegraph Co., New York City.

Presented at the Pacific Coast Convention of the A. I. E. E., Salt Lake City, Utah, Sept. 6-9, 1926.

[2] *Telephone Transmission Over Long Distances,* by H. S. Osborne, TRANS. A. I. E. E., Vol. XLII, 1923, p. 984.

arranged for phantom operation. The circuit layout was planned to provide direct circuits between strategic points along the line to facilitate connections to other trunk routes as well as the handling of the message traffic between large cities on this route. The longest direct circuit set up was a Chicago-San Francisco circuit. There were also direct circuits from New York to Chicago, Chicago to Denver, Denver to San Francisco, Denver to Salt Lake City, Salt Lake City to San Francisco, etc.

For several years these facilities were sufficient to handle the long telephone message traffic connecting the country east of the Rocky Mountains with that to the west, but by 1923 the increase in traffic requirements made it advisable to provide additional facilities. After careful consideration of all factors, it was decided to provide these partly over the direct route between Chicago and Denver and thence over a new route to the south through El Paso and west through Tucson and Phoenix, Ariz. to Los Angeles. A second route from Chicago via Kansas City to Denver was already in existence, so by providing a new route west of Denver, two separate routes were made available from eastern points to the Pacific Coast. This was particularly desirable from the standpoint of service protection, and furthermore, there was an appreciable volume of traffic to Los Angeles and surrounding territory for which it was desirable to provide direct circuits. Toll circuits were already available between San Francisco and Los Angeles so that in times of trouble on either the central or the southern route, the other could be used for connections to both the northern and southern sections of the Pacific Coast.

Following the construction of the line from Denver to Los Angeles, the next steps were to provide trunk routes between New Orleans and Dallas and Dallas and El Paso which connect to other similar routes at New Orleans and Dallas, and to the Denver-Los Angeles route at El Paso. The line across Texas was completed in 1925 and at the present time transcontinental telephone connections may be established over an all southern route.

As a result of further increase in transcontinental traffic requirements, particularly to points in Washington and Oregon now reached over the central route by switching at San Francisco to circuits north along the coast, there is being constructed a direct northern route from Chicago through Minneapolis, Fargo, Bismarck, Billings, Helena, and Spokane, to Seattle. When completed, this will provide a third separate and distinct transcontinental route and will further insure the continuity of telephone service between the east and far west.

There are many other important routes throughout the country carrying circuits of the transcontinental type, as indicated on Fig. 1. Some of the longest direct circuits radiating from Chicago reach to San Francisco, Los Angeles, Denver, Dallas, Atlanta, Washington, New York and Boston, while circuits of similar class radiating from New York terminate in Minneapolis, Milwaukee, St. Louis, Kansas City, New Orleans, Atlanta, West Plam Beach and Havana. Other circuits of corresponding type[3] connect San Francisco with Portland, Salt Lake City, Denver and Los Angeles, while Los Angeles has direct circuits to El Paso and Dallas.

Telephone circuits having repeaters at several intermediate points may be compared to a series of power transmission lines, each one of which receives power from the originating point or a repeater and delivers power to another repeater or to the terminal. In contrast to power transmission lines, however, the sections of a repeatered telephone circuit and the associated equipment are designed with the object of causing power of a complicated nature to be reproduced in form at a distant point, and the fact that none of the original power reaches the far terminal is of no concern since in any event it would be useful only as a means of transmitting intelligible sounds while it would have no appreciable value purely from the power standpoint.

While the application of telephone repeaters to long telephone circuits improves their over-all transmission efficiency, the efficiency from a power transmission standpoint is zero, since none of the original energy passes through a repeater point. It is fortunate that the energy losses do not involve large amounts of power and therefore do not represent an appreciable economic loss from the power standpoint.

In the early days of the telephone, the only method of improving the volume efficiency of a telephone line was to increase the amount of copper; that is, to use wires of larger diameter. The use of metals of higher conductivity than copper was clearly prohibitive because of the cost. But additional increments in copper result in less and less improvement in efficiency so that wire larger than 165 mils in diameter was not used to any important extent.

With the invention and development of the loading coil and its application to open wire lines, it became possible to operate at a higher voltage with a consequent reduction in line losses. With this method, however, leakage losses in wet weather are greatly increased, resulting in considerable variation in efficiency.

The development of efficient amplifying devices and of circuit arrangements for applying them to two-way circuits provided the means for increasing the transmission volume efficiencies of long telephone circuits to a much greater extent than the older methods. Without telephone repeaters but with other parts of a long telephone circuit unchanged, the delivery at the receiving end of the amount of power ordinarily obtained would require startlingly large amounts of power at other points in the circuit. For example, in the case of a San Francisco-New York connection, the amount of power ordinarily applied at San Francisco would be required near Harrisburg, Pa. All of the power introduced ordinarily at all points in the line would be required at a point near Pittsburg. Power sufficient to light two 20-c. p. incandescent lamps would be necessary near Chicago, while the power of a five-kw. radio station would be required near Omaha. The requirements continue to rise rapidly until, near Rawlins, Wyo., a 50,000-kw. generator would have to deliver its entire rated capacity to the circuit, while at San Francisco, something in the order of the estimated total world production of mechanical and electrical power would be needed.

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[3] *Applications of Long Distance Telephony on the Pacific Coast*, by H. W. Hitchcock, TRANS. A. I. E. E., Vol. XLII, 1923, p. 1071.

Let us suppose, however, that a 50,000-kw. generator delivered its entire output to the circuit at San Francisco, and overlook, for the moment, what would happen to the line if any such amount of energy were applied. The power received at New York would be of the order of one five-hundredth of a microwatt, which would have to flow for about 25,000 years in order to equal the energy required to light a 25-watt lamp for one minute.

From this it is evident that the economic solution of the problem of very long distance telephony does not lie in the application of large amounts of power at the circuit terminals, but rather in the use of amplifiers located at suitable intervals along the line.

In the design of telephone circuits, it is necessary to consider the three transmission essentials for easy and natural conversation. These are volume, accurate reproduction, and freedom from disturbance. The factors which tend to impair these qualities are attenuation, distortion and noise. In addition, cross-talk must be so low as to preclude appreciable overhearing of speech over other circuits.

Fortunately it is not necessary, nor even desirable, that all of the energy reaching a telephone transmitter be delivered by the receiver at the other end of the circuit, and the characteristics of the hearing mechanism of the human ear are such that very slight amounts of distortion and noise do not materially affect the intelligibility of received speech energy when the latter is of reasonable magnitude. In designing long telephone circuits the engineer thus has a small range within which to work as regards each of the essential factors for satisfactory transmission.

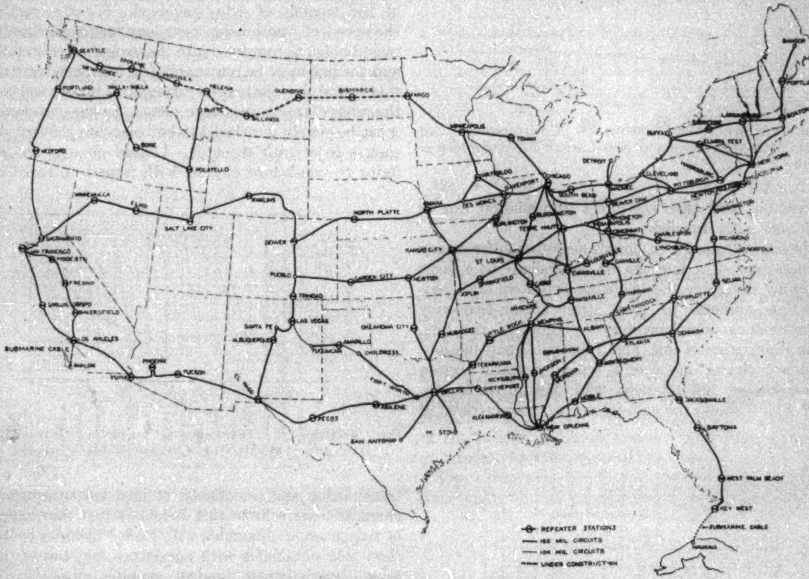
SEATTLE
WENATCHEE
SPOKANE
MISSOULA
PORTLAND
WALLA-WALLA
HELENA
BUTTE
BILLINGS
GLENDIVE
BISMARCK
FARGO
MEDFORD
BOISE
POCATELLO
WINNEMUCCA
ELKO
RAWLINS
SALT LAKE CITY
SACRAMENTO
SAN FRANCISCO
MODESTO
FRESNO
SAN LUIS OBISPO
BAKERSFIELD
LOS ANGELES
SUBMARINE CABLE
AVALON
PHOENIX
YUMA
TUCSON
EL PASO
PECOS
ABILENE
SAN ANTONIO
DENVER
NORTH PLATTE
PUEBLO
GARDEN CITY
NEWTON
TRINIDAD
LAS VEGAS
SANTA FE
ALBUQUERQUE
TUCUMCARI
AMARILLO
CHILDRESS
OKLAHOMA CITY
FORT WORTH
DALLAS
H. STON
MINNEAPOLIS
TOMAH
WATERLOO
DAVENPORT
CHICAGO
DETROIT
SOUTH BEND
BEAVER DAM
OMAHA
DES MOINES
BURLINGTON
BLOOMINGTON
TERRE HAUTE
INDIANAPOLIS
CINCINNATI
KANSAS CITY
ST LOUIS
SPRINGFIELD
JOPLIN
LOUISVILLE
DANVILLE
EVANSVILLE
CAIRO
NASHVILLE
HARRIMAN
CHATTANOOGA
MEMPHIS
LITTLE ROCK
MUSKOGEE
ALBANY
ATLANTA
BIRMINGHAM
MONTGOMERY
TEXARKANA
VICKSBURG
JACKSON
MERIDIAN
SHREVEPORT
MOBILE
ALEXANDRIA
NEW ORLEANS
JACKSONVILLE
DAYTONA
WEST PALM BEACH
KEY WEST
SUBMARINE CABLE
HAVANA
CHARLESTON
LYNCHBURG
RICHMOND
NORFOLK
SELMA
CHARLOTTE
DENMARK
BUFFALO
CLEVELAND
PITTSBURGH
HARRISBURG
NEW YORK
PHILADELPHIA
WASHINGTON
BOSTON
PORTLAND
BANGOR
REPEATER STATIONS
165 MIL CIRCUITS
104 MIL CIRCUITS
UNDER CONSTRUCTION

Fig. 1—Routes of Transcontinental-Type Telephone Circuits in the United States

The attenuation losses in line conductors may be offset largely by the use of repeaters applied at suitable points to give transmission gains. The extent to which such losses can be counteracted in non-loaded open wire circuits arranged for two-way operation is illustrated by the present transcontinental circuits between New York and San Francisco in which the total attenuation is about 165 transmission units[4] and the total repeater gain about 153 transmission units.

Distortion results when too narrow a band of frequencies is transmitted, or when the volume of transmission of part of the frequencies within the range transmitted is materially different from that of another part of the frequency range. Another form of distortion occurs when currents which are reflected from irregularities in a circuit are again reflected by other irregularities and reach the listener as echo currents appreciably later than the direct transmission, due to the longer path traveled.

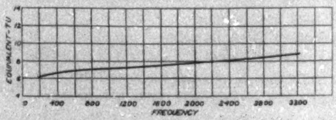

FIG. 2—TRANSMISSION-FREQUENCY CHARACTERISTIC OF 216-MI. REPEATER SECTION OF NON-LOADED 165-MIL PHYSICAL CIRCUIT

Distortion caused by a sloping attenuation-frequency characteristic of a line can be neutralized to a large extent by designing the telephone repeaters and associated equipment to have transmission-frequency characteristics complementary to those of the line. As an illustration of this, Fig. 2 shows the attenuation-frequency characteristic of a typical repeater section of non-loaded 165-mil open wire, 216 mi. in length, while Fig. 3 shows the gain-frequency characteristic of an improved type of telephone repeater which has been adjusted for use with the same section of wire. It will be noted that the attenuation in the line increases with the frequency and that the repeater gain, which corresponds to negative attenuation, also increases at approximately the same rate, so that the result of the combination of the line and the repeater is a transmission-frequency characteristic that is substantially flat. The over-all transmission-frequency characteristic of a long circuit composed of several repeater sections is shown in Fig. 4, which indicates that practically uniform transmission is obtained over the range of frequencies important in speech.

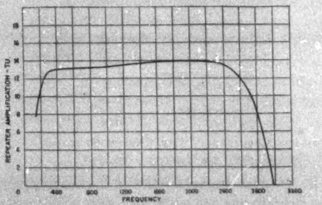

FIG. 3—AMPLIFICATION-FREQUENCY CHARACTERISTIC OF IMPROVED TYPE TWO-WAY TELEPHONE REPEATER

In long repeatered telephone circuits, the time of transmission from one end to the other becomes an important factor in determining the transmission volume efficiency since the seriousness of echo current effects not only is a function of their magnitude compared to the original transmission but also is a function of the amount of delay involved. For this reason the speed of transmission over long telephone circuits must be high as compared with that over shorter circuits, and the line must be reasonably free from irregularities that would give rise to echo currents. Also, throughout the range of frequencies transmitted, the line impedance must be closely matched by the balancing network on each side of each repeater. Loaded circuits are inferior to non-loaded circuits with respect to speed of transmission and smoothness of impedance-frequency characteristics. From this, it follows that the volume of transmission obtainable with loaded facilities is less than that obtainable with non-loaded facilities of the same length, when tandem repeater operation is involved.

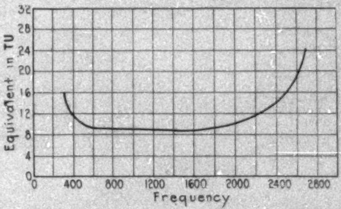

FIG. 4—OVER-ALL TRANSMISSION-FREQUENCY CHARACTERISTIC OF A 1400-MI. NON-LOADED 165-MIL CIRCUIT

Noise and cross-talk are reduced by transposing the wires at frequent intervals throughout the length of the line so that each wire of a circuit will be as nearly as practicable equally exposed to the disturbing influences which exist, at the same time carefully preserving the balance between the impedances to ground of the wires and associated equipment of each circuit[5]. The intensity of the extraneous influences of course should be controlled and kept within reasonable bounds. In designing the transposition layout, it is necessary to take into consideration the effect of each circuit on the line upon each of the other circuits, including the phantom circuits, as well as the effect of neighboring power lines. A single series of transpositions which results in substantially equal exposures of each circuit to every other circuit on the line is called a transposition section. In general, one or more sections of this kind are required for each part of the line that is exposed to different outside influences. The phantom transpositions involve interchanging the positions of the two pairs from which the phantom circuit is derived.

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[4] *The Transmission Unit and Telephone Transmission Reference Systems*, by W. H. Martin, TRANS. A. I. E. E., Vol. XLIII, 1924, p. 797.

[5] *Telephone Circuit Unbalances, Determination of Magnitude and Location*, Ferris and McCurdy, TRANS. A. I. E. E., Vol. XLIII, 1924, p. 1331.

Fig. 5 shows the power at different points in a New York-San Francisco connection, when an arbitrarily assumed power of 1000 microwatts is applied to the line at San Francisco. It will be seen that the power is attenuated at the inputs of many of the repeaters to a value which is of about the same order as that which reaches the New York end. Therefore, any noise induced in the circuit at such points may be as strong when it reaches a terminal of the circuit as it is at the points where it originates. In some places, the transmission level is even lower than at New York, as at Beaver Dam; so that noise introduced at such points may reach New York at greater than its original strength. Thus it is evident that from the noise standpoint, the relative transmission levels at a disturbed point and at the terminals are of particular importance rather than the distances from the disturbed point to the terminals.

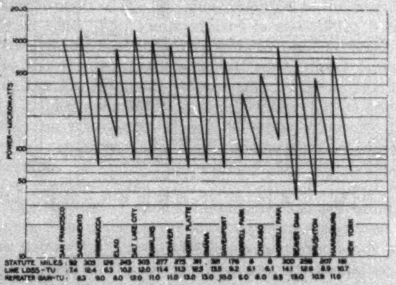

Fig. 5—Power Levels of San Francisco-New York Connection at 1000 Cycles, When Power of 1000 Microwatts is Applied at San Francisco

Besides providing voice-frequency telephone channels, the open wires composing the network of "backbone" telephone circuits are being used to a large extent for superimposed carrier-current systems[6] as well as for providing ordinary grounded telegraph facilities. An example of this is covered by Fig. 6, which shows, schematically, the various communication channels obtained from four wires of a group of transcontinental facilities between Denver and Sacramento. Altogether, there are twenty two-way communication channels operating on these two pairs of conductors, six telephone circuits and fourteen telegraph circuits. A telephone circuit is obtained from each of the two pairs of wires, and a third from the combination of these "side circuits," to form a phantom circuit. The other three telephone circuits are obtained from a carrier-current telephone system superimposed on one of the pairs of wires.

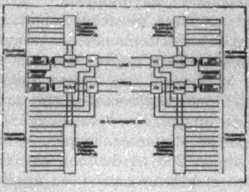

Fig. 6—Communication Channels on Four Wires of Group of Transcontinental Facilities Between Denver and Sacramento

Ten of the telegraph circuits are obtained from a carrier-current telegraph system superimposed on the other pair of wires, and the other four telegraph circuits are direct-current channels derived by ordinary compositing arrangements.

In some cases, a similar group of four wires may have a second carrier telephone system superimposed in place of the carrier telegraph system, while in other cases, a second carrier telegraph system is used in place of the carrier telephone system, according to the requirements for these types of facilities.

Fig. 7 shows the bands of frequencies used at present for communication purposes on typical long open wire circuits. The lowest, from zero to about 80 cycles, is employed for the d-c. telegraph. Each telegraph circuit employs a single wire with ground return, so that two are obtained from each pair of wires.

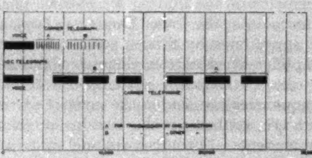

Fig. 7—Frequency Allocations on Typical Transcontinental Facilities

The voice frequencies occupy the next higher band of frequencies, extending to about 3000 cycles. Some circuits of this type are made efficient at frequencies as low as 135 cycles to permit the employment of a current of this frequency for signaling purposes.

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[6] *Carrier-Current Telephony and Telegraphy*, by E. H. Colpitts and O. B. Blackwell, Trans. A. I. E. E., Vol XL, 1921, p. 205.

Above the voice range, each pair of wires may be arranged for superimposed carrier-current operation, either telegraph or telephone. The former utilizes frequencies as high as 10,000 cycles; the latter, as high as 28,000 cycles.

The general use of large wire and long repeater spacings is advantageous in the case of very long circuits, since this results in a smaller number of repeaters, and, in the case of voice frequency circuits, the lesser number of echo current paths permits somewhat better over-all volume efficiencies to be obtained. Carrier systems involve, at the terminals, large investments in apparatus for converting the voice frequencies or telegraph signals, as the case may be, to carrier frequencies, and vice versa. Thus, in general, the longer the distance to be spanned, the lower is the cost of the carrier circuits per mile. The longer or "back-bone" circuits are usually of 165-mil diameter copper, with repeaters spaced from about 200 to 300 mi. apart, and, consequently, carrier systems have been applied to these much more extensively than to wires of smaller gage. In a few of these circuits, however, 128-mil or even 104-mil wire is used through some sections where repeater spacings and other conditions are favorable.

In carrier-current telephone systems, the frequency employed for carrier purposes is modulated by the voice currents, and one of the resulting bands of frequencies is filtered out from the others and transmitted over the line. At the terminals of the systems, the various bands are separated from each other and from the voice-frequency channels by properly designed filters. In the latest systems, six bands of carrier frequencies are utilized, the lower three for transmission in one direction and the other three for transmission in the opposite direction, these being combined at the terminals to form three two-way circuits. The use of separate channels for the two directions of transmission of each circuit permits the use of one-way amplifiers the gains of which are not limited by balance conditions. At repeater points, the lower bands are kept separate from the upper ones by filters, and each one-way repeater amplifies three carrier channels simultaneously.

In the carrier telegraph system, the fundamental carrier frequencies are under the control of telegraph relays and are applied to the line as spurts of high-frequency currents. For a ten-channel system twenty different frequencies are employed, the lower ten for transmission in one direction and the upper ten for transmission in the opposite direction. As in the case of the carrier telephone systems previously mentioned, the group of frequencies transmitted in one direction is kept separate from the opposite bound group at repeater points by means of filters and an entire one-way group of frequencies is amplified by a similar method.

It is necessary, of course, to convert the received carrier telephone or telegraph currents to voice frequencies or d-c. telegraph signals, as the case may be. In the case of the carrier telegraph system the signals are relayed in the form of ordinary direct-current telegraph impulses so that such circuits may be connected to telegraph circuits of other types as desired.

The application of carrier systems employing frequencies as high as 28,000 cycles has brought with it specially difficult problems from the standpoint of cross-talk. With these high frequencies, the transposition spacing requirements become quite stringent and the number of points at which transpositions are necessary is greatly increased. In addition, in toll entrance and intermediate cables, it is necessary usually to arrange the conductors in such a way that those used for carrier systems will not be adjacent to each other.

Frequent reference has been made to telephone repeaters[7] and repeater gains. The amplifying element of a telephone repeater, the vacuum tube, is essentially a one-way device, and for two-way operation it is necessary that the output of the element be prevented from getting back to the input side and thus setting up continuous oscillations known as singing or howling. This is accomplished by the use of two amplifying elements, each with its input connected at a neutral point in the output circuit of the other. The neutral point is obtained in a manner very similar to that employed in duplex telegraphy by dividing the output of each vacuum tube between the line and an artificial line having similar characteristics so that the electrical center of the output circuit remains at a constant potential unaffected by the changing currents in the output circuit itself.

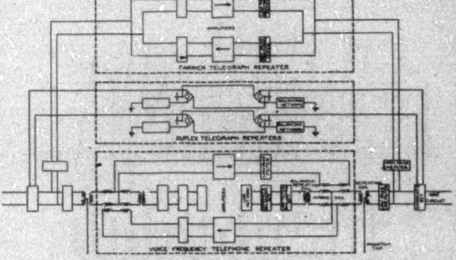
AMPLIFIERS
CARRIER TELEGRAPH REPEATER
DUPLEX TELEGRAPH REPEATERS
AMPLIFIERS
VOICE FREQUENCY TELEPHONE REPEATER
PHANTOM TAP

FIG. 8—SIMPLIFIED EQUIPMENT LAYOUT AT INTERMEDIATE REPEATER STATION ON TRANSCONTINENTAL-TYPE CIRCUITS

Fig. 8 shows the equipment at an intermediate repeater point on one pair of wires of the type used for transcontinental service. The voice-frequency telephone repeater is shown connected to the line in the two directions. Each line, with its associated terminal equipment, is balanced by an artificial line with associated balancing equipment. The hybrid coil located in the electrical center between the line and artificial line is the means by which the transmission in the two directions is separated. Currents amplified by the upper one-way element pass through the so-called third winding of the hybrid coil and set up voltages in the line windings. These voltages are of exactly the same magnitude on both sides of the electrical center of the coil and cause equal currents to flow in the line and artificial lines if their impedances balance each other perfectly. In practise, it is impracticable to obtain an exact balance due to the presence of unavoidable irregularities in the line and its associated equipment, the effect of which is to reflect energy which reaches the other amplifier. For successful operation, however, the average of the two transmission losses represented by the ratio between the reflected energy and the applied energy on each side of the repeater, must be substantially greater than the average of the transmission gains of the two amplifying elements.

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[7] *Telephone Repeaters*, by B. Gherardi and F. B. Jewett, TRANS. A. I. E. E., Vol. XXXVIII, Part II, 1919, p. 1287.

The line repeating coil is shown in the figure with the mid-point of the line side indicated as the phantom circuit tap. The composite set is essentially a filter which allows the voice currents to pass between the line and the telephone repeater, while direct currents and the low-frequency alternating currents involved in d-c. telegraph operation pass between the line and the telegraph repeaters. It will be noted that the telegraph repeaters, also, are arranged for two-way operation involving the employment of balancing networks.

The currents of carrier frequencies are prevented by a filter from reaching the low-frequency equipment but are transmitted easily through a high-frequency path in the filter to the carrier repeater. In the latter, for separating the two directions of transmission, advantage is taken of the fact that the currents in the two directions are of different frequencies so that directional filters can be used to separate them. The repeaters amplify the three one-way channels of a carrier telephone system or the ten one-way channels of a carrier telegraph system simultaneously. In order to avoid interaction or modulation between the various channels, it is essential that the relation between the output current and the input voltage of the vacuum tubes be a straight line function over the energy range employed in the carrier system.

Signal currents, also, must be relayed or amplified on long telephone circuits, just as voice and carrier frequencies are. On some short toll circuits, a signaling current of about 20-cycle frequency is used, the same as that for ringing the bells of subscribers' telephones. On circuits arranged for d-c. telegraph operation, it is impracticable to employ such a low frequency; it is necessary to use a frequency that will be transmitted satisfactorily by the circuit. In actual operation, the signaling channels at the ends of the circuits are arranged usually for 20-cycle operation by means of relays which automatically apply higher frequency signaling currents and receive the incoming signals.

The signaling frequency commonly used on toll circuits of medium length is 135 cycles. With this system, the signals are relayed at least at every other voice-frequency repeater point since the attenuation loss at 135 cycles is greater than that over the main voice-frequency range. In very long circuits, this results in the necessity for operating a train of relays at successive points, which delays the transmission of the signals. To overcome signaling difficulties on such circuits, a system of ringing, employing 1000-cycle currents which are transmitted from end to end of the circuit with the same efficiency as speech currents of that frequency, has been developed and applied. The signaling currents are supplied from a suitable source and are controlled by 20-cycle relays at the terminals. At the receiving end, circuits tuned to the signaling frequency are employed to amplify and convert the signals to currents which operate relays. Interference from speech currents is avoided by interrupting the signaling current at the sending end about twenty times per second, while at the receiving end, the signals pass through a circuit tuned to 20 cycles.

At some points in a long telephone circuit, it is necessary to employ cable, as when passing through a large city. Because of this greater capacity between wires, cable circuits cause much greater attenuation per unit of length than open wire circuits, especially at carrier frequencies. In order to improve the efficiency of the cable circuits they are loaded by means of inductances placed at intervals along the circuit. The inductances and spacing are so chosen as to cause the characteristic impedances of the cable circuits to approximate closely those of the open wire circuits over the range of frequencies transmitted.

Circuits of transcontinental type are used often for special services such as for the transmission of program material to broadcasting stations[8] or to points where such material is desired in connection with a public address system. As this is essentially a one-way service and the programs usually include music, for which the best results are obtained by using a wider range of frequencies than ordinarily is employed for commercial telephone communication, the two-way repeaters in the line are replaced by one-way repeaters and associated equipment for amplifying the currents and equalizing the transmission throughout the wider range of frequencies. Programs are transmitted daily in this manner to a number of broadcasting stations in the eastern half of the country.

Typical of the larger networks that have been set up is that for the inauguration of President Coolidge. Fig. 9 shows the layout arranged for that purpose, which effectively covered the entire country.

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[8] *High Quality Transmission and Reproduction of Speech and Music*, by W. H. Martin and H. Fletcher, TRANS. A. I. E. E., Vol. XLIII, 1924, p. 384.

On occasions, a speaker in one city addresses a meeting in a distant city, the loud speakers at the receiving end being connected to the transmitting telephone by wire. In some cases, such speeches also are broadcast from one or more radio broadcasting stations.

Another use made of transcontinental type facilities is the transmission of pictures or facsimiles of printed or written matter, fingerprints and similar material. The frequencies employed for this purpose are within the voice range and the currents are transmitted in the same way as voice currents. Special apparatus is required, of course, at the transmitting and receiving ends.

The continuity of the many important services routed over the facilities used in making up these long circuits is dependent upon continuous and efficient maintenance methods and performance. Coordination of the work of the different offices is most essential in order to obtain best results especially on the longer direct circuits and on those built up by connecting together several circuits, as there are many variable factors. To assist in obtaining this coordination, one of the terminal offices of each circuit is designated as the controlling office for that circuit and is responsible for the direction and supervision of tests and adjustments required on the circuit as a whole. In addition to the duties in connection with the maintenance of the complete circuit, each office along the circuit is responsible for the proper physical maintenance of the plant in its territory. To assist in this maintenance work, accurate records of the circuit make-up from end to end, including a complete description of the types of equipment and transmission data, are prepared and furnished to the terminal offices and intermediate repeater stations. These records are made on cards of convenient size as illustrated by Table I, which is reproduced from one of three cards containing the data for a Chicago-San Francisco circuit.

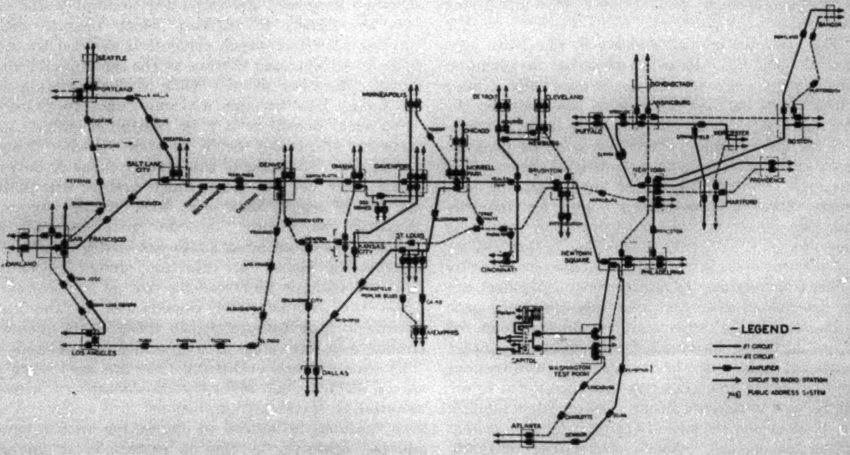
SEATTLE
PORTLAND
SALT LAKE CITY
SAN FRANCISCO
OAKLAND
SAN JOSE
LOS ANGELES
EL PASO
DENVER
OMAHA
DAVENPORT
KANSAS CITY
ST. LOUIS
MEMPHIS
DALLAS
MINNEAPOLIS
CHICAGO
MORRELL PARK
DETROIT
CLEVELAND
NEWBURG
BRUSHTON
CINCINNATI
NEWTOWN SQUARE
PHILADELPHIA
BUFFALO
SCHENECTADY
RENSSELAER
NEW YORK
SPRINGFIELD
WORCESTER
HARTFORD
PROVIDENCE
BOSTON
PORTLAND
BANGOR
CAPITOL
WASHINGTON TEST ROOM
CHARLOTTE
ATLANTA
- LEGEND -
#1 CIRCUIT
#2 CIRCUIT
AMPLIFIER
CIRCUIT TO RADIO STATION
PUBLIC ADDRESS SYSTEM

Fig. 9—Network of Circuits for Transmitting Program Material to Broadcasting Stations—Layout Made Available for the Inauguration of President Coolidge

To insure satisfactory over-all transmission and proper functioning of the circuit, various tests and inspections are made at frequent intervals[9]. Some of the more important maintenance tests are as follows: insulation resistance, loop resistance and resistance balance of the line conductors; measurement of the gain-frequency characteristic as well as the gain at 1000 cycles, and tube tests on all repeaters to detect changes in amplification due to possible changes in the characteristics of the vacuum tubes or repeater equipment; balance tests at each repeater station to check the degree of balance between the line circuit and its balancing network; noise and cross-talk measurements; 1000-cycle transmission measurements and transmission-frequency measurements on the over-all circuit to insure that the circuit equivalent is maintained within proper limits; and finally, over-all talking and signaling tests.

In making many of the measurements, it is necessary to remove the circuit from service. This would result in considerable lost circuit time if each of the stations made the measurements and tests independently. In order to minimize this lost circuit time, it has been found

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[9] *Practises in Telephone Transmission Work*, by W. H. Harden, Trans. A. I. E. E., Vol. XLIII, 1924, p. 1320.

TABLE I.
TOLL-CIRCUIT LAYOUT RECORD

○ E-552

Circuit No. 2 Chicago (E) San Francisco (W)

Control Office Chicago
Meas. At Chicago
Class.......................
Noise Rat'g......

Equiv. in TU	E to W	W to E
Req.	11	11
Meas.	11	11

Circuit order 5785 · Item 42
Date in service 6-1-26
Card issue No. 1 Date 5-22-26

From 1	To 2	Cable or line 3	Pair or pin numbers 4	Size of wire 5	Loading 6	Length 7	Equiv. 8	Toll line equipment On side 9	For SX 10	For other use 11	CX 12	Ringer 13	Misc. 14	Loss (Columns) (9 to 14) 15
C Chicago								75 A				3 A		0.8
D	Morl Park	Chg-MP CA 5	151	19	H–44	7.6	3.6	75 A						0.7
E Morl Park	MH 28	West TE CA	1	13	N	2.2	1.1	75 A			TS		KX	1.1
F P 128	P 328	Chg–Omaha	9–10	165	N	3.8	0.1							
G P 328	P 7415	" "	9–10	165	N	166.3	5.5							
H P 7415	P 7445	" "	9–10	165	N	0.7	0.0							
J P 7445	Davenpt	Aer Cable	9	13	N	0.1	0.0							
K Davenpt	P 7445	" "	5	13	N	0.1	0.0							
L P 7445	P 7557	Chg–Omaha	35–36	165	N	2.1	0.1							
M P 7557	P 7741	" "	25–26	165	N	4.5	0.1							
N P 12976	P 9047	Stl–Davpt	5–6	165	N	83.4	2.8							
P P 9047	Burlington	Aer Cable	5	10	N	0.5	0.2	75 A			TS		KX	1.1
Q Burlington	P 9047	" "	15	13	N	1.0	0.5	75 A			TS		KX	1.1
R P 9047	P 7930	Stl–Davpt	15–16	165	N	29.5	1.0							
S						Total							Total	

Telephone Repeater Data

			Toward W						Toward E						
			Gain or CC step			Singing point				Gain or CC step			Singing point		
Office 16	Type 17	Imp. 18	Req. 19	Meas. 20	Net 21	Req. 22	Meas. 23	Ringing 24	Ringing 25	Req. 26	Meas. 27	Net 28	Req. 29	Meas. 30	Misc. 31
T Chicago	C	N	3		13 P	24									
U Morl Pk	2 A C	N	7.1		17 E	24				7.1		13 P	14		
V Burlington	21 P	N	12		17 F	24				12		17 F	24		
W Kans Cy.	21 P	N	12		17 F	24				12		17 F	24		
Z Newtn.	21 P	N	12		17 E	24				12		17 F	24		

Mileage	O. W.	CA. & O. W
This company	2002.3	2021.2
Other company		644.3
Total		2665.5

Net Equivalent	
Four-wire section	
Two-wire section	
Carrier section	

desirable in the case of long telephone circuits to set aside a definite time for making the periodic tests and to coordinate the work of making the tests under the direction of the controling office. This testing routine has been perfected to such an extent that the circuit need not be kept out of service for more than a few minutes even in the case of the longest circuits.

The measurements of line resistance, insulation resistance, and resistance balance are made by means of Wheatstone Bridge arrangements in the testboards located at each repeater station and sometimes at intermediate points in a long repeater section. The amplification, or gain, given by each repeater is measured by means of a visual reading instrument of the vacuum tube type located at each repeater station. Balance between the line and network circuits in each direction at the various repeater stations is roughly checked by means of the repeater itself. This is done by disconnecting the line and network from one side of the repeater and connecting the two amplifying elements of the repeater in tandem, then increasing the amplification until the repeater circuit begins to oscillate. The sum of the calibrated gains of the two sides of the repeater is a rough measure of the degree of balance existing between the line and network circuits connected to the repeater. Measurements of the over-all equivalent of a circuit are made from the terminal offices by means of transmission measuring sets[10] provided with variable frequency oscillators.

The long telephone circuits in the United States form a network by means of which many important points are connected directly to each other, while practically all large centers may be connected by a very few switches which rarely involve more than three such links. Economic use is made of the wires by the large number of telephone and telegraph channels superimposed on the voice-frequency circuits. The plant composing this network extends over the entire country and close coordination is essential to insure the best results both from the technical and the economic standpoints.

Main arteries of communication have been established in this way which bring the widely separated parts of the country into close touch with one another and the network is being augmented and extended to meet the growth in the traffic as it occurs. These arteries are being developed so that continuous service can be given by means of the protection afforded by alternate routes, and they are well maintained to give efficient service under all the varying conditions encountered.

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10. *Methods for Maintaining the Transmission Efficiency of Telephone Circuits*, by F. H. Best, TRANS. A. I. E. E., Vol. XLIII, 1924, p. 423.

Discussion

J. E. Heller: It might be of interest to point out some applications in the West of the developments described by Mr. Jacobs. The longest circuits in the West include those from San Francisco to Seattle, Denver and Salt Lake City. The Seattle circuits are about 950 mi. in length and are made up of carrier channels to Portland, open wire from Portland to Tacoma and cable from Tacoma to Seattle. A large portion of the toll-circuit relief during 1927 in the Pacific System will be cared for by means of about 10 telephone carrier systems. In the next few weeks an additional circuit consisting of 104 wires will be established between Salt Lake City and San Francisco. This circuit, from a service standpoint, will be practically equivalent to the 165 circuits between San Francisco and Chicago.

Mr. Jacobs mentioned improvements in the quality of the transmission due to better transmission-frequency and impedance-frequency characteristics of repeaters and associated equipment. Quality consists of two components, namely, volume and intelligibility. The volume is a function of the energy transmitted, while intelligibility is a function of the band of frequencies transmitted. The New York-San Francisco connection has a volume equivalent of about 12 T. U. and will transmit frequencies between 300 and 2600 cycles. While it would be possible to increase the volume equivalent and, at the same time, increase the band width above 2600 cycles without materially affecting the quality, the choice of volume equivalent and band width is dictated by the economics of the situation.

In order to stimulate interest in the use of the transcontinental service, public demonstrations were held during 1915 at which a large number of people listened to selected programs transmitted over the transcontinental circuits. These demonstrations were accepted as more or less of a novelty by the majority, who did not give serious consideration to the use of the service. Later, smaller and selected groups were brought together at both ends of the line and members of each group were allowed to talk to friends at the other end of the line while the rest of the group listened. These demonstrations were very successful in showing the practicability of the long-distance service, as is indicated by the growth of the business.

K. B. Miller: The present telephone instruments, transmitter and receiver, are something like 32,000 times more efficient in the air-to-air transmission than was Bell's original centennial experiment which caused such astonishment fifty years ago. That 32,000 times more efficient instrument that we have now does not take into account any difference in the lines. To show what these men have done to the line in the way of improving its efficiency, we have only to recollect that an exact replica of Bell's instrument was used in transcontinental conversation in 1915, so that the difference involves something like a 32,000 ratio (representing the efficiencies of the two instruments) which was overcome entirely by the improvements in the line (by line I mean transmission system between the two terminal points). Looking at it in another way, in order to get the requisite amount of energy from San Francisco to New York without these improved instruments in the line would require enough energy put into the line to annihilate the line absolutely; vaporize it, in fact, in order to get the required amount of energy across the continent. I think those illustrations will show something of what has been done; in fact, the whole tendency lately has been to decrease the conductivity of the line, thus accomplishing very much credit in the amount of copper saved by use of repeaters, loading coils, and similar improvements.

W. G. Rubel: It might be interesting to add that we have a great many circuits throughout the United States which are very similar in type to the ones described, and while not normally employed to connect, directly, points as widely separated as those mentioned, they may be used as sections of built-up long-haul circuits by connecting them together and inserting telephone repeaters at proper intervals. These long built-up circuits are equivalent approximately to the direct transcontinental type circuits, and when once established, the service over them is of high grade. In the case of such circuits an appreciable time is required to build them up and a very large number of pieces of auxiliary apparatus is involved in signaling, in adjusting repeater gains, etc., while establishing through connections.

An ideal arrangement, from the standpoint of traffic at least, would be to have direct circuits from any point to every other point in the country; that is, of course, economically impracticable. That is the trend in the design of circuit layout, however, and we find that when the community of interest between any two localities is such as to indicate the necessity of a direct circuit, such a circuit is provided so that in addition to the long direct circuits discussed in this paper we have a network of shorter circuits radiating from every important center in the country. For example, from Salt Lake City, there are 35 direct toll circuits varying in length from a few miles to 600 or 700 mi. One of these is to Pocatello, Idaho, where a connection may be made through a repeater to a circuit extending from Pocatello to Boise. From Boise, this circuit can be extended to Baker, Oregon, and from there to Spokane, Portland, Seattle and all of the Pacific northwest. We can also build up a circuit from Salt Lake City through Pocatello north to Canadian points, which may in turn be connected either to the East or West at Salt Lake City.

It is by means of circuits of the type discussed in the paper, supplemented by the shorter but none the less important toll circuits, that universal telephone service is now being realized in the Bell System.

Carrier-Current Communication on Submarine Cables

Los Angeles-Catalina Island Telephone Circuits

BY H. W. HITCHCOCK[1]

Associate, A. I. E. E

Synopsis.—*Seven telephone channels and one telegraph channel on one single-conductor deep-sea cable have been made possible by the employment of carrier current on one of the two submarine cables across Catalina Channel. This is the only application of carrier telephony to deep-sea cables; the system is the shortest carrier system (26 mi.) in commercial operation; it provides more separate carrier channels (six) than has been previously attempted; and it differs in other important respects from other systems. This paper describes this carrier-current system.*

* * * * *

IN the commercial application of new developments in the electrical communication art, there are a few places which repeatedly call attention to themselves. Notable among these is Catalina Island, for it is probable that in providing telephone service across the short expanse of water which separates Catalina from the mainland, more novel improvements have been employed than at almost any other point.

The first commercial telephone communication with Catalina Island was established in 1920 when a radio system was placed in operation between Avalon and the mainland, the circuit being extended by wire to Los Angeles. This circuit was in use for several years and featured in a number of transcontinental demonstrations, including the one which was held at the opening of the service to Havana over the Key West-Havana cables.

The system is of considerable interest as it represents the only instance in which radio has been used, in this country at least, to form a portion of a toll telephone system for the general use of the public. That it was reasonably successful is demonstrated by the fact that on some days as many as 183 commercial telephone messages and a large number of telegrams were handled over it. The system also proved to be one of the first popular broadcasting stations and many letters were received from radio fans, often several hundred miles away, telling of some of the amusing conversations which were overheard.

In 1923 the radio was replaced by two single-conductor submarine cables. By that time the demands for service were too great to be met by a single circuit, while the growing interest in radio broadcasting, as well as the increasing interference from ship transmitters, rendered its continued operation very difficult and unsatisfactory. The submarine cables were of the single-conductor, deep-sea type, each providing a single-wire circuit. They are of interest for a number of reasons, chiefly, perhaps, because they represent one of the few instances of deep-sea cable manufacture in this country. From the cable hut at San Pedro, the circuit is extended to the office by means of a special lead-covered cable containing four individually shielded No. 13 B & S gage pairs for the telephone circuits and four 19-gage pairs for the telegraph circuits and other miscellaneous uses. Between the San Pedro office and Los Angeles, the circuit was composed of a No. 19 B & S gage cable phantom. At San Pedro a through-line repeater was inserted in order to secure the desired over-all equivalent between Avalon and Los Angeles. A description of these cables and their laying was given in a paper presented by the writer at the Pacific Coast Convention in 1923 and published in Volume XLII of the TRANSACTIONS.

Although the two circuits provided by the cables represented a great improvement over the previous condition as regards the quality of the service rendered and the number of messages which could be handled, it was realized that they would soon prove inadequate to handle the heavy summer business, for which eight or ten circuits would be required in a relatively short time. To provide for such a large increase by the laying of additional cables was deemed impractical, as the cost would be excessive. Furthermore, in water of this depth—3000 ft. (914 meters)—it is important that cables be laid at least a mile or two apart, so that in the event that trouble develops on one, it can be repaired without disturbing any of the others. For a total distance as short as the width of the Catalina channel—23 nautical mi. (6087 ft. or 1856 meters per nautical mi.)—such a separation between adjacent cables could not be maintained without materially increasing the length of the outer ones with a corresponding increase in their cost and in their transmission equivalents. In view of these facts, it was decided to secure as many more circuits as possible by operating carrier systems over the two cables already in use. This project was actively promoted with the result that on May 15, 1926, six carrier telephone circuits were placed in operation.

The use of carrier in the past few years has increased so rapidly that the mere addition of a new system is,

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[1] Transmission and Protection Engineer, Southern California Tel. and Tel. Co., Los Angeles, Calif.

Presented at the Pacific Coast Convention of the A. I. E. E., Salt Lake City, Utah, Sept. 6-9, 1926.

in itself, of hardly more than passing interest. In this instance, however, there are a number of factors which render the project of particular interest. It is the shortest carrier system—26 mi. (43 km.)—in commercial operation. It is the only application of carrier telephone to deep-sea cables; the system provides more separate channels (six) than has ever before been attempted, while the particular arrangement employed is different in many other important respects from anything which has been used in the past.

In order to better appreciate the reasons for adopting the system finally agreed upon, it may be of interest to review briefly the essential characteristics of carrier systems and the different types which are available. The general principles of carrier-current telephony are described at considerable length in a paper by Messrs. Colpitts and Blackwell which was published in Volume XL of the JOURNAL of the Institute.

Carrier systems may be divided into two general classes, namely, balanced or grouped, depending upon the manner in which the currents in the two directions are prevented from interfering with each other at the terminals. In the balanced system this separation is accomplished by means of a three-winding transformer or hybrid coil together with a balancing artificial line such as is used with a voice-frequency repeater. In the grouped system, different carrier frequencies are used for transmission in the two directions and their separation at the terminals is effected by means of suitable band-pass filters. These two systems are shown diagrammatically in Fig. 1. The balanced system has the advantage that for each channel the same carrier frequency may be used for transmission in both directions so that there may be as many channels as there are separate carrier frequencies. On the other hand, the wire circuit must be very uniform throughout so that the impedance will be very regular over the entire carrier-frequency range, and may be simulated by an artificial line. The line must also be very stable so that the impedance balance, once having been secured, will not be disturbed. Furthermore, as transmission with the same carrier takes place in the two directions, the effect of the cross-talk between systems of the same type is very severe, so that it is usually impractical to operate two of these over-wires which are in close proximity for any considerable distance. The grouped system has the advantage that a balancing line is not required and hence small circuit irregularities are relatively unimportant. Furthermore, the effect of cross-talk is much less severe, so that a number of systems may often be operated over adjacent circuits. One disadvantage is that two carrier frequencies are required for each channel so that fewer circuits can be secured with one system.

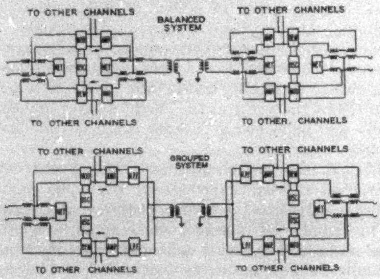

FIG. 1

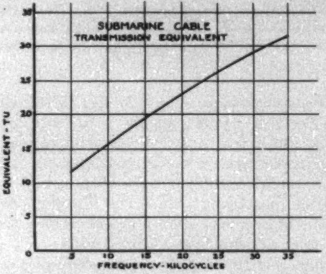

FIG. 2

Carrier systems may also be divided into two classes depending upon the manner in which the carrier current is provided at the receiving end. In the carrier transmission system, the carrier current is supplied by the oscillator at the sending end and is transmitted over the circuit along with one or both of the side bands. In the carrier suppression system, the carrier current itself is not transmitted but is introduced into the receiving equipment from a local source. This latter system is proving to be superior for general carrier purposes because of the advantages which accrue from relieving

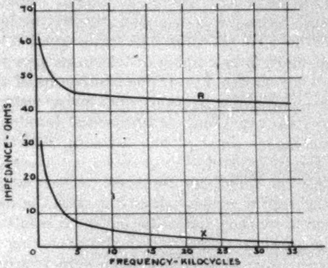

FIG. 3

the line and apparatus from the load of the carrier current.

Turning now to the electrical characteristics of the cables, we find that each one provides a circuit having a transmission equivalent which increases throughout the carrier range but is moderate in magnitude. The impedance, as is to be expected with a uniform, non-loaded cable, is very smooth, and since there is no opportunity for any change in the cable constants, the impedance has practically no variation. The transmission equivalent and the impedance of one of the cables are shown in Figs. 2 and 3 respectively. The cross-talk between the cables is small enough to be entirely negligible, regardless of the type of carrier systems employed.

In view of all the conditions outlined, a balanced system of the carrier suppression type was decided upon. Such a system provides the maximum number of channels per cable, while the usual difficulties of impedance balance and inter-system cross-talk are largely absent due to the unusual characteristics of the cables. The adoption of such a system also made possible the employment of standard units of equipment of the most recent design. The general nature of the system and the arrangement of the component parts is shown diagrammatically in Fig. 4. Fig. 5 is a simplified circuit diagram showing the filters for separating the various circuits at the terminals, together with the balancing arrangement. In Fig. 6 are shown the essential parts of one channel together with the amplifiers

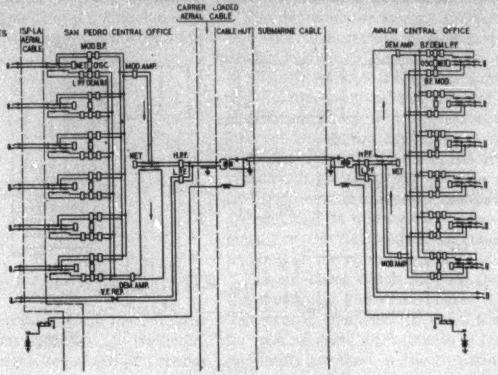
CARRIER LOADED AERIAL CABLE
LOS ANGELES
SP-LA AERIAL CABLE
SAN PEDRO CENTRAL OFFICE
CABLE HUT
SUBMARINE CABLE
AVALON CENTRAL OFFICE
MOD B.F.
NET
OSC
MOD AMP
L.P.F DEM B.F.
NET
H.P.F.
L.P.F.
DEM AMP
V.F. REP
DEM AMP
B.F. DEM L.P.F.
OSC
NET
B.F. MOD
H.P.F.
L.P.F.
NET
MOD AMP

Fig. 4

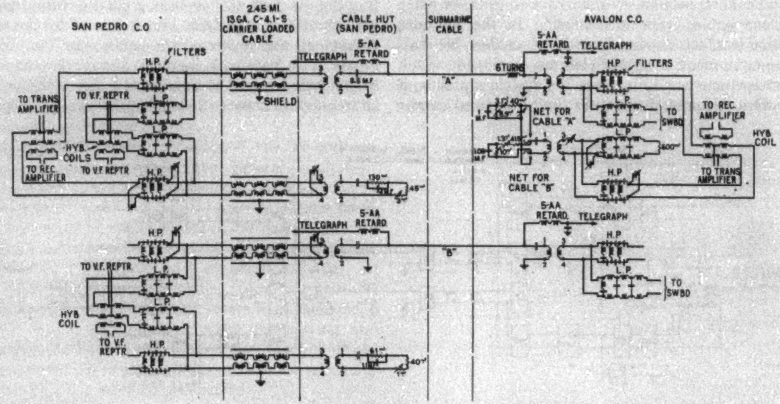
SAN PEDRO C.O.
2.45 MI
13 GA. C-4.1-S
CARRIER LOADED
CABLE
CABLE HUT
(SAN PEDRO)
SUBMARINE
CABLE
AVALON C.O.
FILTERS
H.P.
TO TRANS
AMPLIFIER
TO V.F. REPTR
L.P.
L.P.
HYB.
COILS
TO REC.
AMPLIFIER
TO V.F. REPTR
H.P.
SHIELD
TELEGRAPH
5-AA
RETARD
8.5 M.F.
"A"
130~
45~
5-AA
RETARD
TELEGRAPH
6 TURNS
H.P.
FILTERS
L.P.
TO
SWBD
TO REC.
AMPLIFIER
HYB
COIL
NET FOR
CABLE "A"
1 M.F.
L.P.
600~
NET FOR
CABLE "B"
H.P.
TO TRANS
AMPLIFIER
H.P.
TELEGRAPH
5-AA
RETARD
"B"
5-AA
RETARD
TELEGRAPH
H.P.
L.P.
TO V.F. REPTR
L.P.
TO
SWBD
HYB
COIL
TO V.F.
REPTR
H.P.
40~

Fig. 5

and the hybrid coil which are common to all the channels. For convenience, some of the battery and auxiliary circuits have been omitted in the figure.

At the time the system was under development, it was uncertain that balanced operation of all channels over a single cable would be practicable, so that an alternative arrangement involving substantial four-wire operation over the two cables was provided for. With this arrangement, which is shown diagrammatically in Fig. 7, all transmission in one direction takes place over one cable, while transmission in the opposite direction is effected over the second cable. No balancing equipment or hybrid coils are employed. Such an arrangement would increase the system stability, if such were required, but would limit the total carrier capacity of the two cables to six channels. In the event of the failure of one cable, operation with such a system would be impossible, and it would be necessary, at that time, to revert to the two-wire arrangement as described above, with a possible reduction in the over-all gain or a reduction in the number of operating channels.

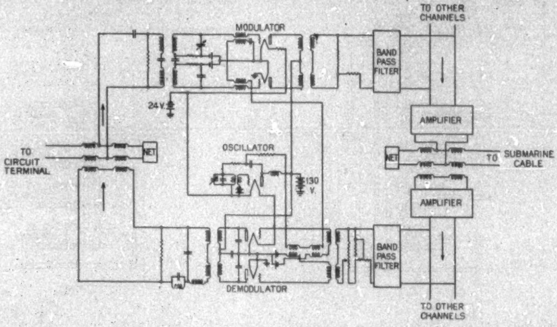
TO OTHER CHANNELS
MODULATOR
BAND PASS FILTER
24 V.
AMPLIFIER
TO CIRCUIT TERMINAL
NET
OSCILLATOR
NET
TO SUBMARINE CABLE
130 V.
AMPLIFIER
BAND PASS FILTER
DEMODULATOR
TO OTHER CHANNELS

Fig. 6

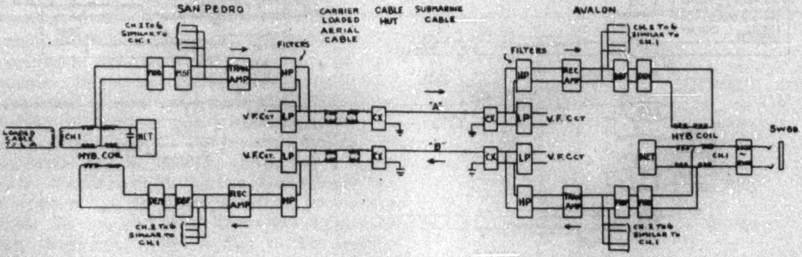
SAN PEDRO
CARRIER LOADED AERIAL CABLE
CABLE HUT
SUBMARINE CABLE
AVALON
CH. 2 TO 6 SIMILAR TO CH. 1
FILTERS
TRAN AMP
HP
FILTERS
REC AMP
CH. 2 TO 6 SIMILAR TO CH. 1
V.F. CCT
LP
CX
"A"
CX
LP
V.F. CCT
LOADED CABLE TO L.A.
CH 1
NET
HYB. COIL
HYB COIL
SWBD
V.F. CCT
LP
CX
"B"
CX
LP
V.F. CCT
NET
CH 1
DEM
REC AMP
HP
HP
TRAN AMP
CH. 2 TO 6 SIMILAR TO CH. 1
CH. 2 TO 6 SIMILAR TO CH. 1

Fig. 7

As may be seen from Fig. 4, a carrier-equipped cable provides a d-c. telegraph circuit, and one voice-frequency, and six carrier-frequency telephone channels. The separation of the various channels is effected by means of electrical filters. Fig. 8 shows the band of frequencies employed for each channel. For the d-c. telegraph this separation is effected at the terminals of the cable as is shown in Fig. 5. The telegraph circuit requires a continuous d-c. path, whereas the telephone channels require the insertion of an inequality ratio insulating transformer at the ends of the cable in order to properly join the 43-ohm grounded cable circuit with the 600-ohm metallic circuit formed by the office equipment and intermediate cable. As this transformer must pass both the voice and carrier channels, it has been designed so as to have a high efficiency for all frequencies between 250 and 30,000 cycles. Separation of the voice-frequency circuit from the carrier system is performed by means of the usual high and low pass filters which are located at the central offices. These filters both have a cut-off frequency of 3000 cycles, the low pass transmitting all frequencies below this value and the high pass transmitting all above it. In the carrier system the transmitting and receiving currents are separated from each other by a hybrid coil and balancing network. Between the output of the six modulators and the common transmitting amplifier, individual band-pass filters are located. Each one of these filters is designed to transmit one of the side bands produced by the modulator associated with it and to suppress all other frequencies. The six receiving currents are separated in a similar manner. Each filter allows current of the proper frequency to pass to the corresponding demodulator and excludes all others. Each demodulator is also provided with a low pass filter which allows the passage of the resulting voice-frequency current but excludes all incidental higher frequencies which might be present and render the circuit noisy. The input of each modulator and the output of the corresponding demodulator are finally joined by means of a voice-frequency hybrid coil and extended to the circuit terminal as a two-wire circuit. At the San Pedro end, each two-wire circuit is extended to Los Angeles over a loaded cable circuit. Phantoms are employed for this purpose as they have a higher cut-off frequency than have the side circuits, with a correspondingly better quality.

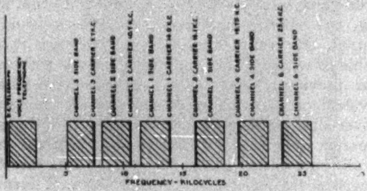

Fig. 8

Concerning the carrier system itself, the two ends are practically identical while the general equipment arrangement for an individual channel is the same in all cases except for the frequency of the band-pass filter. For this reason, a consideration of one channel is sufficient. Each channel is composed of a voice-frequency hybrid coil, a modulator with its band-pass filter, an oscillator, and a demodulator, together with its associated filters. In addition, there is, at each end, a carrier hybrid coil together with transmitting and receiving amplifiers which are common to all channels. The arrangement of this equipment is shown schematically in Fig. 6, as previously indicated.

The modulator the input of which is connected to the center taps of the hybrid coil line windings consists of two vacuum tubes arranged for push-pull operation. The carrier current which is supplied by the oscillator is applied to the two grids by means of a transformer. Such a circuit generates the two side bands but suppresses the carrier. In order that this suppression may be as complete as possible, the small condenser associated with the grid of one of the tubes is made variable and is adjusted until the carrier current in the modulator output is reduced to a minimum. The band-pass filter transmits one of the side bands and suppresses the other, as well as all miscellaneous resultant currents of a higher order which are produced by the modulator. It also prevents the output currents of the other channels from entering the modulator circuit as this would cause a reduction in their efficiency and give rise to undesirable frequencies.

The demodulator is very similar to the modulator. The tube arrangement is substantially the same and carrier current is supplied from the one oscillator. In the demodulator a complete suppression of the carrier is unnecessary as this is accomplished by the low pass output filter. For this reason, the small balancing grid condensers are omitted. In order to adjust the over-all gain of the channel, the demodulator is provided with an adjustable potentiometer graduated in two transmission unit steps, and in addition, fixed pads are provided for making further gain adjustments. The output of the demodulator is connected to the series winding of the voice-frequency hybrid coil.

The oscillator which supplies the carrier current to the modulator and demodulator is of the usual type. The tuning condenser includes a small variable unit for making small adjustments in frequency. Separate oscillators are used at the two ends for each channel, and as these are in no way connected together, it is occasionally necessary to make slight adjustments in order to keep the frequencies at the two ends substantially equal. The oscillators are very stable, however, and such adjustments are seldom required.

The individual channel filters are all of the band-pass type as previously indicated and have a free transmission range of approximately 2500 cycles. Outside this free range they have a high impedance so as not to act as a shunt for the other channels. They are all of substantially the same construction, although the constants of the component parts necessarily vary as the filters for the different channels transmit different frequencies.

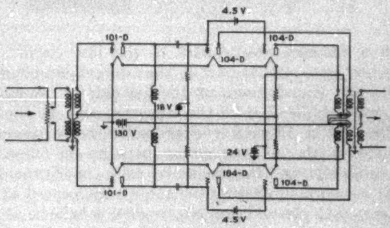

Fig. 9

The transmitting and receiving amplifiers, which are practically identical, are shown schematically in Fig. 9. They consist of two push-pull stages connected in tandem. Each half of the second or output stage consists of two parallel tubes of high output capacity. In this way a comparatively high gain and a large energy output may be secured without overloading. This is very important as these amplifiers are common to all six channels and any tendency to overload would produce objectionable distortion and inter-channel modulation. In order to adjust the over-all gain for the entire system, each amplifier is provided with an input potentiometer.

As has been previously indicated, the transmitting and receiving circuits are joined to the cable by means of a hybrid coil. Probably the most difficult problem encountered in the installation of this system was the securing of an adequate balance. The difficulty of doing this may be better appreciated when it is realized that this balance must cover all frequencies from 3000 to 30,000 cycles, and must have a value of from 30 to 45 T. U., the higher value which represents an impedance unbalance of approximately one per cent being required at the upper frequency. In order to secure such a balance, every part of the line circuit was matched by a similar part in the network circuit. All filters and transformers on the line side of the hybrid coil were duplicated in the network, and on the San Pedro side a 13-gage carrier-loaded cable pair was included in the network circuit between the office and the cable hut, and the inequality ratio transformer and basic network simulating the cable were located at the latter point. In addition to providing a balance within the carrier range, it was necessary at the San Pedro end for the network circuit to balance the cable within the voice-frequency range as a through-line repeater is employed on the voice-frequency circuit. Not only was it necessary to duplicate all parts in the line and network circuits but in addition they were carefully selected and paired so that the two parts associated would have, as nearly as possible, the same electrical characteristics. All wire pairs within the office which appeared in the carrier frequency circuits were individually shielded by means of a grounded metallic covering. The 13-gage carrier-loaded pairs in the cable joining the hut and the office were also individually shielded by means of a lead foil wrapping. This was done in order to preserve the balance and prevent cross-talk with another system which may be placed on the second cable at some future time.

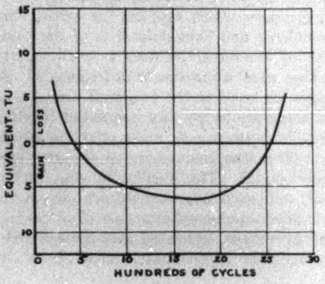

Fig. 10

Although extreme care was exercised in making the refinements described, the balance was still lower than was desired so that small variable auxiliary impedances were inserted at suitably chosen points in the line and network circuits. By the adjustment of these elements, it was found that the balance could be raised to any desired value for any particular channel, but that in so doing, the balance on some of the others would be impaired. By careful adjustment, however, it was possible to secure a balance for all channels within the range previously mentioned. As the transmission equivalent of the cable increases with the frequency, the over-all channel gains must be increased in the same manner in order that all circuits may have the same over-all equivalent. The networks were therefore arranged so that the higher frequencies would have the better balance, as in that way the margin of balance over gain could be made substantially the same for all channels. Since this margin should not be allowed to fall below a fairly definite minimum if the circuit is to have the desired stability, it is evident that the balance which may be secured determines the over-all gain which is possible. In this case the circuit equivalent for all channels between Los Angeles and Avalon was set at five T. U. As the loaded cable between Los Angeles and San Pedro is approximately nine T. U., it may be seen that the carrier system actually introduces a gain and performs the function of a repeater besides increasing the number of circuits. Fig. 10 gives a frequency characteristic of one of the channels which is typical of all of them. Balancing equipment has been provided for both cables as is shown in Fig. 5. With this arrangement, the carrier system may be operated over either cable. The transfer from one cable to the other is so simple that it can be made with practically no traffic interruption.

Signaling over the carrier channels is effected by means of 1000-cycle ringers which are connected to the circuits at the two terminals. As the ringing current is within the voice range, it is transmitted over the regular carrier channel so that no additional signaling equipment is necessary.

In order to insure satisfactory operation, all necessary testing facilities are included. Meters and keys are provided for measuring the voltages of the plate, grid and filament batteries as well as the plate and filament currents of all tubes. Individual rheostats are inserted in all filament circuits for making any adjustments that may be necessary. Alarms are provided to indicate any abnormal condition which might develop on any tube. Thermocouples and artificial lines have been conveniently arranged for checking the efficiency of all units such as the modulators and demodulators.

Jacks are located at suitable points so that any changes which may be necessary can be quickly made.

The general appearance of the carrier system may be seen from Figs. 11 and 12 which show the equipment at San Pedro and Avalon respectively. Fig. 13 is an interior view of the San Pedro cable hut showing the cable terminals, together with the insulating transformers, telegraph composite sets, and basis networks. Referring to the central office equipment, the first bay contains the equipment for two complete channels. At the top are the terminal strips for making all connections with the equipment below. On the next two small panels are mounted the hybrid coils and the other miscellaneous apparatus associated with the voice-frequency ends of the two channels. Below these are the modulator and demodulator band filters which are covered with dust proof cases. Next comes the modulator and demodulator panels for one channel. Below the two jack strips is mounted similar equipment for a second channel but arranged in reverse order. In the upper half of the second bay is located a small panel mounting the carrier hybrid coil and associated equipment. Below this appear the transmitting and receiving amplifiers. The lower half of the bay is similar to the lower half of the first one. In the third bay is mounted all the battery supply and testing apparatus. The first two units contain the battery retard coils. Below these are the alarm relays and auxiliary resistances. Next come the meters for measuring the tube currents and voltages, and below these are the thermocouples and artificial lines for making high-frequency measurements. Below the jack strip are the keys for opening and closing the individual filament circuits used for measuring the plate and filament currents. Alarm lamps are also associated with each of the filament circuits. The fourth bay is similar to the second except that the upper half is vacant. As may be seen from the photographs, the amplifiers appear on the second bay at San Pedro and on the fourth at Avalon. The fifth bay is an exact duplicate of the first.

Fig. 11

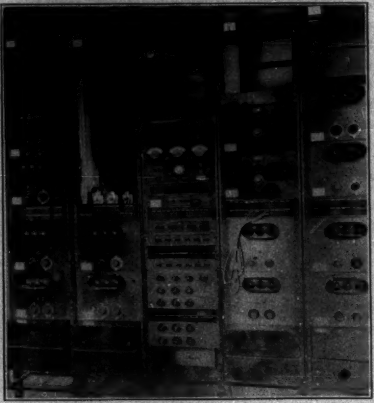
Fig. 12

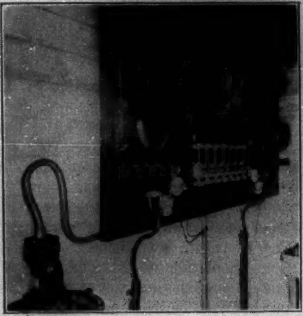
Fig. 13

The new system has now been in successful operation for the past five months. In the light of its performance thus far, we feel assured that when more circuits are required a second system of six channels can be added to the second cable, thus providing a total of fourteen telephones and two telegraph circuits over the two single-conductor cables. Such a circuit group, we believe, will meet the traffic requirements for quite a number of years.

Controlling Insulation Difficulties in the Vicinity of Great Salt Lake

BY B. F. HOWARD[1]

Member, A. I. E. E.

***Synopsis.**—The difficulties encountered in maintaining a sufficiently high degree of line insulation on long communication circuits in the vicinity of Great Salt Lake, Utah, are dealt with in this paper and a review of the study and solution of the problem of controlling the insulation is made.*

Leading up to this is a description of observations on about 40 mi. of line over a part of the Great Salt Lake Desert known as the mud flats west of the Lake. The description also covers observations on other lines north of the Lake.

The object of these studies was to ascertain the variations in the line insulation caused by different weather and temperature conditions in each of these routes. It was necessary also to determine which route would be better for constructing the central transcontinental communication circuits which were about to be built at that time for extending service to provide telephone connection between points on the Atlantic and Pacific coasts.

IN 1913, when it had been decided to construct a transcontinental telephone line in order to connect Atlantic and Pacific coast points, a warning was given by other wire-using companies of difficulties which would be encountered in the Great Salt Lake Desert, owing to very low insulation of the lines under certain weather conditions. These companies were in the communication business and possessed lines which passed through the section.

At certain times, they had experienced complete failure of telegraph lines in that vicinity, due to this low insulation, particularly when heavy fogs prevailed. It was very important to the success of the transcontinental telephone lines that the insulation should be controlled and prevented from falling below a certain minimum. This was particularly true because at that time the most practical way of giving very long distance service was by means of "loaded" circuits. This required high insulation at all times, for if it fell below a certain limit, the qualities of the line for conveying speech would be even worse than had it not been loaded at all.

For a long time these insulation conditions had existed on lines which had been strung over the mud flats west of Salt Lake. There was evidence that there were stretches of line on the route north of the Lake (one of the two routes under consideration) which showed insulation conditions similar to those on the lines situated upon the mud flats. These stretches on the northern route, however, could be avoided by building the lines around them, whereas, in the case of the southern route, it was impossible to avoid crossing the mud flats and salt beds.

It may be well to give here a description of the mud flats and salt beds which have been known for some time as the Great Salt Lake desert. This desert is comprised of an old lake bed probably many hundreds of feet deep consisting of a deep valley filled with mud. It is known with certainty to be over 400 ft., for a chemical company put a drill down at a place where the surface is covered by a large bed of salt and passed through eight ft. of this, 200 ft. of mud, 30 ft. of solid salt, and another 200 ft. of mud. Drilling was stopped at this point, and it is not definitely known what is below.

The salt-bed surface is about eight mi. wide and 20 mi. long, and varies from a few inches to eight ft. in thickness. The mud flats, as they are called, are covered with a salty crust which, in the dry season, is capable of carrying the weight of a horse and has even carried a tractor; but when a hole is broken through this crust, it will rapidly fill with salt water. The mud becomes softer with the depth; where the surface was wet the author has pushed a broom handle down into the mud with ease. The melting snow and rains from the mountains run into these flats, and the surface water, which varies from one inch to several inches in depth, is blown for miles by the wind. Frequently an area that has been covered with water one day will be dry the next for miles, owing to the removal of the water by the wind to a distance of several miles during the night. To say the surface is dry is hardly correct, since except at that time of the year when a salty crust forms it is impossible to walk upon the surface without splashing the mud in all directions.

After storms, salt is sometimes found upon the windows of houses 15 mi. from the Lake in Salt Lake City, where salt-laden moisture has been deposited upon the glass. Telephone leads in the vicinity of Salt Lake City have had their insulation considerably lowered on account of salt and alkali dust being blown upon them.

The track of the Western Pacific Railroad and the lines of two telegraph companies and at the present time the transcontinental telephone lines all pass across this desert. It is understood that the railroad company when building their road laid a floor of planks on the mud and put a gravel bed upon this. In some places, it is necessary to protect this roadway from the action of the drifting water by filling it in with rock.

Investigations made showed that the salt is blown upon the insulators during storms and splashed by the

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Presented at the Pacific Coast Convention of the A. I. E. E., Salt Lake City, Utah, September, 6-9, 1926.

action of rain. This salt forms a crust on the surface, and when this takes place on the interior of the insulators, the insulation of the lines is materially reduced. During fogs which are very prevalent at certain seasons of the year this salt becomes wet and the insulation resistance falls rapidly to a very low value. At such times it has been observed to fall to values as low as 0.09 megohms per mi.

The effect of freezing is interesting. During the tests which were made for the purpose of investigating this subject, it was observed that at sunrise, after sufficient time has elapsed for the insulator coating to thaw, the insulation would drop materially; and then, as the warmth of the sun evaporated the moisture, the insulation would rise again. In one case when the insulation fell to 0.5 megohms per mi. between wires and ground the wind in one hour dried it so that the insulation returned to 4600 megohms per mi.

The southern route known as the Western Pacific Route appeared to be the better for these transcontinental telephone lines when all things were considered, provided the insulation difficulties could be overcome. It had several points of advantage over the northern route. For example:

1. It would shorten the mileage considerably.
2. Less cable would be introduced.
3. A considerably less amount of interference from foreign circuits would be felt.
4. In addition, by shortening the route, there would be less cost for construction and maintenance.

There were favorable climatic conditions on this route from a point of construction and maintenance, and also it was along a through-line of railway which was kept open at all times of the year. The Lincoln Highway, which was not built at that time, was under consideration and quite likely to be constructed by the side of the railroad tracks which passed over the mud flats. This highway has since been built as expected.

The experience of one of those who had given warning of the difficulties which had been encountered in maintaining satisfactory insulation of lines had extended over thirty to thirty-five years. It was therefore decided to make a series of tests in order to find out the cause and, if possible, the remedy for the falling insulation. It was known that this trouble was usually noticed during January, February, and March of each year and chiefly during the night hours. Consequently in January, 1914, an investigation was launched on an experimental basis. The lines chosen for this were those already existing north of the Lake and others to be specially built over the mud flats west of the Lake. The test stations chosen were Kelton in the first case and Arinosa in the second.

The preliminary tests showed how very important it was to have all glass insulators well washed and cleaned before being put in place on the poles. If this were not done, differences of about fifty per cent were observed between the insulations of one line and another which was of similar character.

The observations made were carried out during January, February, and March of 1914, and a series of observations was made and recorded every three hours throughout the twenty-four hours of each day. These comprised insulation tests between wires and between wires and ground, hygrometric, thermometric and barometric observations; and the dew points were calculated.

The charts, Figs. 1 and 2, show typical values of insulation resistance measured between wires and ground upon various circuits under different conditions of temperature and weather. These show clearly the value of having the insulators clean. It was observed that iron-wire lines have a relatively lower insulation than those of copper under ordinary conditions. It is thought that this is due to metallic salts, such as chloride of zinc and iron, or oxides of the latter, being washed onto the arms and the outer surface of the insulators by rain and the action of dripping. The insides of the insulators probably receive these deposits by the action of rain splashing the salts when it strikes the arms around the insulators. These salts when deposited upon the surfaces of the insulators cause leakage which is very marked in damp and foggy

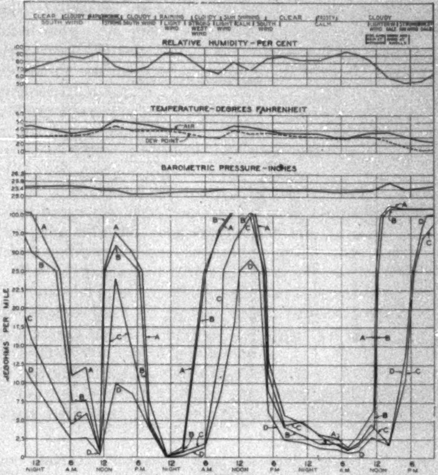

Fig. 1—Chart Showing Line Conditions. Average Insulation of Wires to Ground. Knolls-Arinosa-Wendover Wires that had been Strung upon Insulators for Some Time and Therefore Exposed to Weather

A—Copper circuits—Arinosa to Wendover
B—Iron circuits—Arinosa to Wendover
C—Copper circuits—Arinosa to Knolls
D—Iron circuits—Arinosa to Knolls

weather. As the inside surface of an insulator plays a very important part in its proper functioning, this seems to teach that these surfaces should be as clean as possible at the time of installation. Careless handling and lack of this precaution will often cause unequal leakage on the two sides of a circuit.

The result of this study brought out the following:

That the liability of the insulation of the lines to fall to a dangerously low value was about as great on the route north of the lake as on the southern route west of the lake, and that the solution of the problem of preventing these reductions in insulation was to remove the salt from time to time by washing the insulators. It was therefore decided that this would have to be done in the case of either of the routes under consideration.

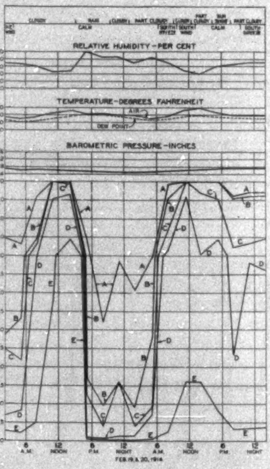

Fig. 2—Chart Showing Line Conditions—Arinosa to Knolls. Average Insulation of Wires to Ground

A—Experimental circuit—No. 8, B. W. G. Copper wires strung upon clean insulators
B—210-lb. Copper wires
C—Iron wire, No. 118
E—Iron wires, No. 19 & No. 1
D—No. 8, B. W. G. Copper wire
} Strung upon insulators that had been in place for some time

It was further decided that, all things being considered, the best engineering solution was to build the transcontinental telephone line which passes through Salt Lake City over a route south of Great Salt Lake. This route, after bending in a northwesterly direction, proceeds westward over the mud flats to the Nevada line and so on towards San Francisco. A portion of this line crosses eight mi. of solid salt and 34 mi. of mud flats, besides a stretch immediately south of the lake which is open to receiving a deposit of salt upon the insulators as it is exposed to storms which blow from the lake.

The transcontinental telephone pole line as constructed on the mud flats consists of 20-ft. by 7-in. Western Cedar poles with 176-ft. spans. The guying on this part of the line is done in all four directions every mile with side guying both ways half-way between. The line crosses the Salt Lake Desert between Clive and Wendover. At locations where the loading coils were placed on the poles, cross pieces were fitted to prevent the poles sinking in the mud owing to the extra weight they had to carry.

It may be interesting to note that it has not been found necessary to apply any preservative to the wooden poles which are set in the mud because of impregnation with salt in solution which preserves the wood to a remarkable extent.

The problem of washing the insulators was a somewhat serious one because, if it were necessary to take the insulators down, it would mean interruption to service and damage to the conductors by undoing and retying the tie wires, and this would be a long and expensive process. The author therefore devised a method by which the insulators could be successfully washed from time to time without climbing the poles or untying the wires. This method consists in spraying the insulators with saturated steam or finely divided hot-water spray. Of course the recognized aging action of steam on glass surfaces might make it undesirable to adopt generally such a method for cleaning insulators, but under the unusual conditions prevailing in this case the method described seemed justified.

The steam is applied through a nozzle at the end of a fishing-rod device which is held under the insulator to be cleaned. The steam is generated in a portable boiler which is heated by an oil fire; see Fig. 3. The temperature is automatically governed so that the steam is kept at about 70- to 90-lb. pressure per sq. in. When an insulator is to be washed, after the nozzle has been placed in position, a valve is opened and the steam forces the water out of the boiler through a pipe leading to the nozzle, and it passes out in the form of a finely divided spray or wet steam. An automatic arrangement of pumps supplies water to the boiler to replace that which is used.

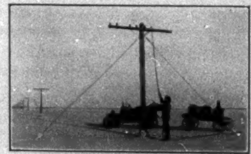

Fig. 3—Washing Insulators with Steam

It is advisable to mention that in this location, economy in the use of water is important, as water is a large item in expense; therefore, the time necessary to wash an insulator has been carefully ascertained as 30 sec. All fresh water has to be carried out to the machine from a considerable distance. It is brought by the railroad, distributed at intervals along the line, and then rolled out in barrels across the mud flats.

During the time when an insulator is being washed, no appreciable disturbance has been noticed in the service taking place over the lines. The rapidity with which salt or alkali dust, both of which are of a stubborn nature, is removed by the steam is astonishing.

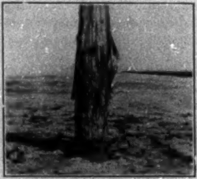

Fig. 4—Pole with Sap Wood Split by Action of Salt

To give an illustration, a broken glass insulator encrusted with salt to the thickness of half an inch was subjected to the spray and in less than a minute after the steam was applied, the glass was completely cleaned.

The steam washing plant is built with wide metal tires and is pulled sometimes by horses and at other times by a Ford car which is also equipped with wide tires. These are fitted on the usual type of demountable rims so that it is easy to change quickly to rubber tires when necessary. Many miles of the mud have been found so soft that it was necessary to lay planks in order to keep the car and the steam-generating plant from sinking. These planks were taken up after the machines had passed over them and relaid in front.

It has been found necessary to wash the insulators at least once every two years but the frequency, of course, is governed by conditions of salt deposit. When the insulation resistance is found to drop to a predetermined amount, it is time to arrange for a washing of the line. While this is in process, the improvement in the insulation of the wires may be observed as the washing proceeds.

The following example of some insulation measurements made in 1923 between a wire and ground shows the effect of washing the insulators.

INSULATION IN MEGOHMS PER MILE

January	March	June	August	October
11.37	32.3	101.6	68.2	637.0

When the insulation had fallen to so low a level in January, when dry, it was decided to wash the insulators as soon as conditions on the mud flats would permit, since in wet weather it would go to about 0.5 megohms per mi. Accordingly they were washed after August. The table shows how the insulation rose from 68.2 megohms per mi. to 637.0 after the washing. Both of these months were comparatively dry months.

The cost of washing the insulators on the mud flats ranges between $500 and $700, according to prevailing conditions.

The accompanying illustration, Fig. 4, shows the action of the salt upon the poles above the ground line. This, at first sight, appears to be rather alarming but it has been observed that the rending action of the salt, which takes place as it crystallizes, affects the sapwood only. On poles which are formed by splitting trees into quarters, only the sapwood which exists on one of the three sides seems to be affected in this manner.

Once since the line was built a rather peculiar experience was recorded. This was at a time when rain had fallen in quantity and was upon the surface of the salt solution on the flats. This was followed by a heavy frost and ice formed in thick layers. The wind broke this into floes, which were forced against the poles and did considerable damage by abrasion. Fig. 5 shows a pole damaged in this way.

The telephone line was built at a distance of 1000 ft. from the railroad. The reason for this was because the engines burn oil as fuel, and a deposit from the products of combustion would form upon the insulators if the line were close to the railroad.

The difficulties of constructing this line can be realized from the description which has been given. Mud boats were used to convey material. The insulators were carefully washed close to the railroad and, before they were carried out to be placed in position, each was placed in a paper bag in order to prevent it being splashed with mud while being carried out to the line over the mud flats.

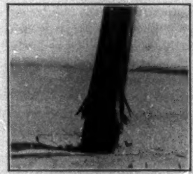

Fig. 5—Pole Cut by Action of Ice

Those in charge of maintenance of lines carrying electric energy, be it either for power or communication purposes, may well consider the advisability of cleaning the insulators from time to time in sections in which they are exposed to receiving a deposit of such substances as tend to lower the insulating qualities of the lines.

After an experience extending over more than twelve years with these transcontinental lines, it is gratifying to find that that portion which passes over the mud flats of the Great Salt Lake Desert is perhaps more free from disturbance due to weather conditions than any other part of this lead which connects the Atlantic and Pacific Coasts. If, however, the insulation of this portion were not kept at a high standard at all times by periodically washing off the accumulation of salts from the insulators, this would doubtless be the worst section from an insulation viewpoint.

Discussion

W. C. Lee: Carrier-current systems are, in general, very sensitive to insulation conditions on the line, especially at frequencies above 10 kilocycles, and the value of washing insulators as a means of maintaining a high degree of insulation resistance west of Salt Lake City is demonstrated by the fact that our carrier telegraph system has operated for five years without a single interruption due to failure of the line to transmit sufficient current.

Of course the insulation resistance does drop in wet weather and during a recent storm it dropped to 0.4 megohm per mi., making necessary a 70-per cent increase in the repeater amplification to maintain a given current level at an average frequency of 20 kilocycles.

Had not a constant program of insulator washing been maintained, it is manifest that the carrier-current system could not have been operated in wet or foggy weather.

N. M. Johnson: In Mr. Howard's paper mention is made of the fact that other wire-using companies had experienced complete failure of telegraph lines in the vicinity of the Great Salt Lake Desert, due to very low insulation, particularly when heavy fogs prevailed. During my five years in Salt Lake, having to do with testing and handling of these lines, I have not found a single case where it was impossible to use either the telephone or telegraph circuits during the period of low insulation. It is true, however, that we have approached the danger point in some cases, especially about the time the insulators are due for washing. The telegraph duplex sets operating on a normal current of 60 mils line current have dropped to as low as 15 mils, but due to the differential type of duplex set in use at Salt Lake which operates on small current margin we have been able at all times to continue operation. Some rather severe cross-talk in the telephone circuits is associated with the low insulation; but in all cases this condition clears up rapidly as the insulation improves. It is, of course, difficult to get any sort of quick test when the lines are weather-crossed and distributed over a rather large territory. Our first test station in this section is at Wendover, about 130 mi. west of Salt Lake. Here we can open the lines and determine an approximate location of the trouble. It is my firm belief that if the insulators are not washed regularly every two years, we shall, as Mr. Howard states, encounter severe difficulties.

A. S. Peters: The difficulties in connection with the construction of the pole line across the flats were quite unusual. Owing to the mud of the flats and the constantly shifting water on the salt beds during all the year except about two months at the end of the summer, it is impracticable to do any extensive work except during this dry season. At this time, because of the heat and lack of precipitation, the salt beds are quite dry and a crust forms on the mud which will often support the workman.

At first, horses were used in connection with the work, but the hoofs would break through the crust, cutting the flesh above them, and the salt of the crust in the wounds soon caused such pains as to render the horses nearly useless.

During the summer the heat is intense, especially on the salt beds, because of the reflection from the crystalized salt. Workmen are compelled to wear colored glasses to protect the eyes. The perfectly level floor of salt or drab-colored mud makes a most monotonous landscape, broken only by a few hills in the distance. The only variations are the frequent mirages. A stray box thrown on the salt beds appears at a distance as a cabin.

B. F. Howard: In regard to Mr. Johnson's comments, I understood him to say that there had been no failure of the lines due to fallen insulation at any time. This would not have been the case had the insulators not been washed, for under certain weather conditions the lines would have undoubtedly been unusable.

Regarding Mr. Lee's remarks about the reason for raising the level of the "carrier," if the insulators had not been washed, I have no doubt that the lines could not have been used under the conditions which he mentioned. If the insulation falls as low as 0.4 megohm per mi., I should say that the insulators should be washed as soon as possible.

Since the time when Mr. Lee's discussion was presented at the convention, it has been ascertained that the low value of insulation alluded to was due to salt having been deposited upon some insulators on the line in a portion of the lead which is immediately south of Salt Lake and situated east of the mud flats. This portion of the lead had not previously been washed. After this was done the insulation of the whole lead rose to a satisfactory figure.

Variable-Voltage Equipment for Electric Power Shovels

BY R. W. McNEILL[1]

Associate, A. I. E. E.

ALTHOUGH the power shovel was developed and brought to a high state of perfection by using steam as a motive power, a constantly increasing number of power shovels is being supplied for electrical operation. These electric power shovels are being used for all sorts of work although they are in particular favor for projects of a permanent nature or for those requiring a considerable period for completion. In some cases the electric power shovel has been adopted when starting these projects, or in other cases it has replaced the steam power shovel because of its superior operating economies. This process of replacement is very active at the present time in connection with open-pit copper and open-pit iron mines. Other extensive uses for electric power shovels are found in "strip" coal mines, quarries with output used in crushed form (such as for blast-furnace purposes), road materials, cement manufacturing steel plants for rehandling ore and slag, and contract work on dams, canals, tunnels, etc.

This widespread use of the electric power shovel has led to a considerable development in connection with this type of machine not only in connection with the electrical equipment but also with regard to general mechanical design. While earlier experiences with electric power shovels had indicated that very good results were obtainable by taking the steam power shovel design and replacing the steam engines and boilers with electrical equipment, later experience has shown that in order to obtain the full advantage of benefits of electrical operation, the shovel must be designed with this point especially in view. Mechanical improvements have taken the form of better gearing, (double helical gears having been substituted for coarse spur gears in many instances), and of better systems of lubrication for gears, bearings, etc.

The latest development in the line of electrical equipment for power shovel operation is characterized by two outstanding features. First is simplicity of control, and second, sturdiness of the entire electrical equipment. Simplicity of control is secured by use of the *variable-voltage* system of motor speed control and durability by designing all the electrical equipment with the same high standards as characterize the sturdy reliable d-c. mill-type motors used for operation of the shovel.

In the development of this latest type of electrical equipment for operation of power shovels, every advantage gained by experience in perfecting the steam power shovel has been utiliized and the equipment has been designed to duplicate the steam power shovel in performance and digging characteristics, for it is a well-recognized fact that the power characteristics of the steam engine are practically ideal for power-shovel operation. The variable-voltage system of motor speed control as developed for electric power shovels employs an individual motor for drive of each principal motion, power being supplied to each motor from an individual generator of such design that the speed torque characteristics of the individual motors closely resemble those of the steam engines used on steam power shovels, as shown in Fig. 1.

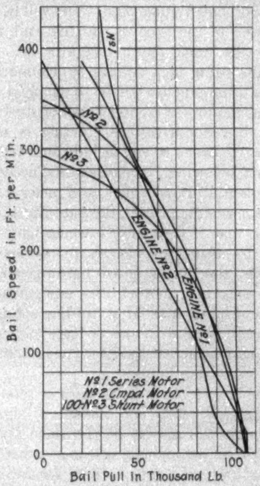

FIG. 1—COMPARISON OF STEAM-ENGINE AND VARIABLE-VOLTAGE-OPERATED MOTOR CHARACTERISTICS FOR POWER SHOVELS

Electrical equipment using the variable-voltage scheme of motor control is adapted to all sizes of shovels from the smallest to the largest, and its principle of operation is the same, regardless of the size of shovel. Where this system is used, a-c. power is usually supplied to the shovel from a central station system. This means that the complete electrical equipment for a shovel using the variable-voltage system will, of necessity, consist of the following principal items in the order of their location with respect to the incoming power line.

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[1] General Engineer, Westinghouse Electric and Mfg. Co., East Pittsburgh, Pa.

Presented at the Pacific Coast Convention of the A. I. E. E., Salt Lake City, Utah, Sept. 6-9, 1926.

1. Starting equipment for a-c. motor.
2. Motor-generator set.
    a. A-c. driving motor.
    b. D-c. generators for supplying power to individual shovel motors.
    c. D-c. generator for furnishing excitation to above d-c. generators, shovel motors and a-c. motor, if same is of synchronous type. (This exciter may be separate-motor-driven, if physical limitations make it impossible to drive same from main set.)
3. Controllers and field resistors.
4. Shovel-driving motors.

Auxiliary equipment may consist of air compressors, magnet valves, solenoid brakes, etc.

Due to differences in size of equipment, type of driving motor, and voltage used, there will be considerable variation in the type of starting equipment furnished for the main motor-generator set. For the larger outfits using high-voltage synchronous driving motors, the usual practise will be to supply a combined synchronous motor and exciter panel of steel clad construction. If reduced-voltage starting is used, this panel or switch structure will include auto-transformer, double-throw oil switch, field rheostats, necessary current and potential transformers, and meter equipment. If full-voltage starting is used, the auto-transformer will be omitted, and if desirable, starting may be made automatic by using an electrically-operated circuit breaker. On the smaller-sized motor-generator sets driven by induction motors, the equipment can be simplified and motor started at full-line voltage by use of an oil circuit breaker in case of high-voltage motors, and by means of an across-the-line type magnetic starter in case of low-voltage motors.

While the size of the motor-generator sets will vary with the different sizes and models of shovels, the general scheme will always be the same and the set will consist of an a-c. driving motor of either the synchronous or induction type, a d-c. generator for the hoist motion, a d-c. generator for the swing motion, a d-c. generator for the thrust motion, and a d-c. exciter. The exciter may or may not be part of the main motor-generator set, depending upon whether or not space limitations will permit the use of a five-unit motor-generator set. Where the exciter is separate-motor driven, it is usual to use a low-voltage driving motor, and, if power supply is at high voltage, to supply transformers for furnishing power to the exciter motor-generator set. Driving motors for the larger shovel motor-generator sets usually are wound so that by a simple change in connections they can be operated on either 2200 or 4000 volts. Smaller sets are wound for 2200, 550, or 440 volts. Lower voltages are not recommended on account of poor line regulation. Either synchronous or induction motors can be designed for full-voltage starting if desirable. In general, this will be the standard method of starting the smaller induction-motor-driven sets.

The individual d-c. generators used for supplying variable voltage to the various motions are of the differentially-compound-wound type, designed with three separate field windings, *viz.*, a separately-excited shunt-field winding, a self-excited shunt-field winding, and a series winding so connected as respects the armature that its effect on total field strength will be subtractive or differential under-load current on the generator. This type of generator will deliver variable-voltage power to the shovel motors, the exact voltage value at any instant being dependent on two factors: first, the value of the separate shunt-field excitation, and second, the value of the load current flowing in the armature and the differential series field winding. Fig. 2 shows voltage characteristics of a typical differential-compound-wound d-c. generator for power-shovel service under different conditions of separate-excited field strength and load currents. These characteristics are practically ideal for power-shovel operation, since by supplying power to the shovel motors from such generators, we are enabled to obtain motor speed torque characteristics closely resembling those of the steam engines used for power-shovel service. Actual practise in the design of these generators proportions the shunt-field windings so as to give the required light-load voltages and proportions the differential series winding so that the total field strength at heavy loads will be only

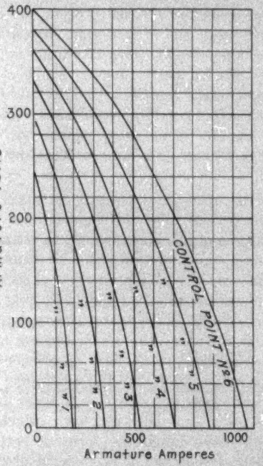

Fig. 2—Volt-Ampere Characteristics of Differential Compound Generator as Used Showing the Effect of Excitation Changes

of such value as to force the maximum required current through the main circuit consisting of motor and generator armatures, series and interpole field windings and connecting cables between the machines.

The motors used for driving the individual motions on variable-voltage equipped electric power shovels are of the well-known d-c. mill type, used so extensively for auxiliary drives in steel mills and on heavy-duty cranes, car dumpers, etc. Open or ventilated type motors are usually used on hoist and swing motions, and totally enclosed motors on the thrust motion. Mill motors have massive cast steel frames split horizontally so that the top half can be readily swung back or lifted off. Widespreading feet, cast integral with the lower half of the frame, insure stability and freedom from vibration when the motor is bolted to a firm foundation. Electrically, the design is of the very highest order. Mica and asbestos insulation is used throughout; all main and commutating field poles are of laminated construction and coils are supported and banded in such a manner that there can be no loosening in service.

There is some variation in the type of field windings employed on d-c. motors used for shovel operation. The swing and thrust motions are always operated either by straight shunt-wound, separately-excited, d-c. motors or by compound-wound motors having separately excited shunt fields. The hoist may be operated either by series-wound motors or by separately-excited, shunt- or compound-wound motors. As compared to the compound-wound or series motor, the straight shunt-wound motor, when controlled by the variable-voltage method, possesses the advantage of no reversal of main circuit connections being necessary to secure reversing of direction of rotation of the motor. As compared to the series motor it has the advantage of positive regenerative braking being easily secured. As compared to the series and compound wound motors it has the disadvantage of not being designed to deliver the same maximum torque for the same current value and its speeds are inherently lower at light loads.

On the swing and thrust motions, the necessity of regenerative braking for quick stopping practically eliminates the series-wound motor, so that choice must be made between the straight, shunt-wound motor and the compound-wound motor. This also applies to the hoist-motion on high-lift shovels which use regenerative braking in lowering the dipper. The main objection to the use of compound-wound motors is that their use complicates the control, as it is necessary to furnish means of reversing the series fields for reversed service. On the smaller motors this is easily accomplished by additional contacts in the small drum controllers used for governing the operation of each motion. But on the larger motors, it is necessary either to add magnetic contactors for reversing these fields or to provide a series exciter, or its equivalent, so that compound motor characteristics can be obtained with shunt-field windings in the motor. These schemes do not eliminate the reversing processes but do reduce the capacity required so that large magnetic contactors are unnecessary. The choice between straight-shunt and compound-wound motors for swing and crowd motions is not very clearly defined; in some cases the extra complications of the compound-wound motor will be justified, while in others the simplicity of control possible with the straight shunt motor will be the deciding factor in determining its use.

On the hoist motion the problem of proper type of motor is further complicated by differences in practise in operation, particularly on low-lift shovels. Some shovel manufacturers prefer to use the motors in lowering, using regenerative braking to obtain speed control, while other manufacturers prefer to lower the dipper on a foot brake without operation of the hoist motor in the reverse direction. If the latter practise is followed, there is little advantage in using the straight shunt or the compound-wound type of motor and advantage can be taken of the superior operating characteristics of the series-wound motor for the hoist motion. If, however, regenerative lowering is needed, a shunt or compound motor is used, control connections being so arranged on the compound wound motor that in lowering, it operates as a straight shunt-wound motor. Comparative speed torque characteristics of shunt, compound and series motors are shown on Fig. 3.

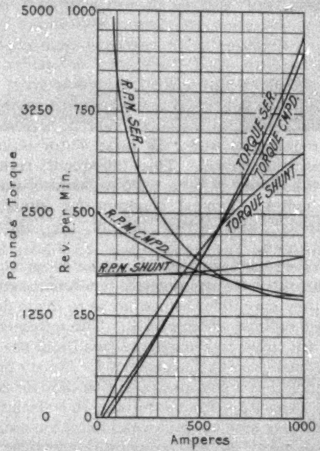

FIG. 3—SPEED-TORQUE CHARACTERISTICS OF 170-H. P., 230-VOLT, D-C. MILL-TYPE MOTORS FOR POWER SHOVEL SERVICE

The control equipment necessary with electric power shovels operated by the variable-voltage system is of the very simplest nature. Actual practise is to use small drum controllers and series resistors connected into the separately excited shunt fields of the individual generators in such a manner that operation of these controllers will direct the flow current and current strength in these fields. Additional contacts are regularly provided in this controller for weakening the self-excited shunt field of the generator in the off position for connection of field discharge resistors and for operation of brake and clutch circuits. Where straight shunt-wound motors are used, these controllers provide for the weakening of the motor shunt field in the off-position and may provide for insertion of resistance into the motor field to secure high light-load speeds. When compound-wound motors are used, it is necessary to furnish not only the field weakening point in the off-position for the separately excited shunt field, but also to furnish additional contacts either for reversing the series field direct through magnetic contactors or by reversing connections to a separate exciter used to secure compound-wound motor characteristics. Where series-wound motors are used on the hoist, it is not necessary to provide for frequent reversals, as the only time that reverse operation is required is when the hoist motor is used for propelling the shovel. This infrequent reversal is usually taken care of by a separate hand-operated reversing switch. Typical diagrams for shunt, compound-wound and series motors are shown on Figs. 1, 2 and 3.

Controllers used with the variable-voltage equipped shovels are all of the same general type and construction though they may differ somewhat as to drum development and method of operation. The currents to be handled by these controllers are small and the design is determined more by the demand for sturdy mechanical construction and ease of operation than for current-carrying capacity. Full revolving shovels are built usually for one-man operation and for this reason there is some demand for a foot-operated controller to be used on one of the motions on these shovels. Standard controllers for variable-voltage equipped shovels are constructed with vertical operating handle. Thumb-operated auxiliary switches for operation of dipper-trip mechanisms, magnet valves, etc., are frequently added to the handle of standard controllers.

While the resistors used with the variable-voltage equipment are of small size and capacity, even on the largest shovels, they are quite an important feature of equipment and it is very essential that they be of ample capacity and sturdy construction. Experience has demonstrated that these resistors should be wound on insulated metal supports and that special care should be taken to insure against connections loosening due to vibration. Practise in method of assembling resistor units varies with shovel design. On one type of shovel it may work out to advantage to assemble resistors all in one unit and provide this assembling with a terminal board which will serve as a distribution center for all control wiring. In other cases, space limitations can best be met by making up the resistors in unit assembly, that is, one unit for hoist generator separate excited field, one unit for hoist generator sets excited field, etc.

Air compressors, when used with shovel equipment, are usually of the industrial type with a-c. driving motor. Air pressure is maintained at a constant value by means of an automatic governor which stops and starts the compressor as the air receiver pressure varies. Compressed air is used for operation of brakes on the swing and thrust motions and for brake and clutch on the hoist motion.

Magnet valves are used to give electrical control of the air brakes, clutches, etc. Two types of valves are used; these are known as "straight" and inverted valves. Both types of valves are of three-way construction and the difference between the two is in the method of operation. The *straight* valve is provided with three openings for pipe connections; one from the air supply, one to the brake or clutch cylinder, and an exhaust opening. With magnet of valve energized, the exhaust opening will be closed, the air inlet will be open, and full air pressure will be exerted against the brake or clutch cylinder. Interruption of magnet circuit closes inlet opening, opens exhaust port, and relieves pressure in brake or clutch cylinder. Operation of "inverted" valve is just the reverse of above, as magnet coil is energized to exhaust brakes or clutch cylinder and circuit broken to apply air pressure. The straight valve is always used on brakes which are of the spring- or gravity-applied, air-released type. The inverted valve is used in connection with air-applied, spring-released clutches where it might be dangerous to use a straight valve as interruption of magnet circuit might be the cause of accident.

Solenoid brakes are used on electric-shovel equipments where compressor equipment is not supplied. On variable-voltage equipments these brakes are of the d-c. type with shunt-wound operating coils designed to operate from the exciter circuit.

While the above has covered only the application of the variable-voltage system to power shovels of the conventional type, the system is equally adaptable to dragline excavators, dipper dredges, and to hoist and swinging motions on all types of cranes, dredges, etc. It provides a simple automatic method of limiting motor torques where serious damage to the mechanical equipment might result if motor torques were not limited, and it also provides a simple automatic method for obtaining high motor torques with low kw.-input to the motor. This last factor is a valuable one where power is purchased on a peak-load basis or where the primary source of power is of limited capacity.

Discussion

P. S. Stevens: Mr. McNeill has mentioned the effect of electric power on the mechanical design of the shovel, such as better gearing, lubrication, etc. I would like to further point out that the variable-voltage control with separately excited motors described by Mr. McNeill is so complete and well suited to shovel operation that it has been possible to simplify the mechanical design of electric shovels without any sacrifice. For example, on two models of medium-sized shovels recently developed, the usual band-type air-operated clutch previously required for disconnecting the engine or motor from the hoisting drum has been omitted. This clutch has been omitted for the same reason also on a large 12-yard Diesel-electric dipper dredge recently put in operation. The elimination of this clutch and its operating mechanism, reduces the care and maintenance of the shovel and is made possible by the quick acceleration and quick and smooth stopping afforded by the separately excited motor control with motor lowering. The motor is actually driven down to a certain extent, so that fast lowering speed is obtained and the shovel operator can graduate his lowering speed or check it any time under perfect control.

On the swing and thrust motions, the variable-voltage control with separately excited motors is so perfect in stopping and plugging that swing and thrust brakes having large heat capacity and requiring air operation are omitted from the two models of shovels mentioned above, being replaced by the usual solenoid-operated motor brakes for holding and infrequent stopping. Hence the complete compressed-air equipment, including compressor, governor, tank, piping and all magnet valves, is eliminated, reducing operating costs and resulting in a more reliable shovel unit.

R. W. McNeill: Since preparing this paper the writer has been fortunate enough to secure graphic meter records showing the performance of a 4½-yard railway shovel, equipped with variable-voltage electrical equipment. This shovel is one of a number which are in operation at a mine of a large western copper-mining company, which has recently gone through the process of converting all of its steam shovels to electric drive.

Electrical equipment on these 4½-yard railway-type shovels consists of a main motor-generator set, an exciter motor-generator set, two hoist motors, a swing motor, a thrust motor, and a motor-driven air compressor. With this equipment is included necessary starting and control equipment for all items. The main motor-generator set consists of a 225-h. p., 80-per cent power factor, 1200-rev. per min., 5000-volt, 3-phase, 60-cycle synchronous motor designed for starting at full line voltage; a 125-kw., 1200-rev. per min., 250-volt, differential compound-wound d-c. generator for operation of the hoist motion of the shovel, and two 30-kw., 1200-rev. per min., 250-volt, differential compound-wound d-c. generators for operation of the swing and thrust motions. The generators are designed to give a maximum voltage of 400 at no-load, and a maximum current under stall conditions of approximately 1000 amperes in the case of the hoist generator and 250 amperes for the swing and thrust generators. The exciter set is of 7.5-kw. capacity at 125 volts and is driven by a 220-volt, 3-phase, 60-cycle motor. The hoist motors are of series type each rated 95 h. p., 230 volts, 440 rev. per min. These motors are operated in parallel from the 125-kw. hoist generator. The swing and thrust motors are of the separately excited shunt-wound type, each rated 50 h. p., 230 volts, 485 rev. per min. The air compressor is operated by a 7.5-h. p., 220-volt, 3-phase, 60-cycle motor. Transformers are used to step down the power for the compressor motor and the motor driving the exciter motor-generator set from 500 volts, the line potential, to 220 volts for the motors.

Fig. 1, herewith, shows power input to this 4½-yard shovel, measured on the 5000-volt line feeding the shovel. The upper curve is made with high paper speed and shows variations in power requirements of the shovel throughout several cycles of operation. The lower curve is made with low paper speed and shows to some extent the percentage of operating time obtained with this shovel equipment. Records kept over a considerable length of time show that the shovels are in operation approximately 60 per cent of the time, the other 40 per cent being taken up with delays due to causes outside of the control of the shovel operator, the principal cause of these delays being waiting for empty cars. The power consumption on this property for shovel service averages approximately 0.2 kw-hr. per ton of material excavated. The amount of material excavated in a given time will depend upon a number of factors; however, average daily loading on this property is approximately 5000 tons per shovel per 8-hr. day. This is roughly equivalent to 3300 cu. yd. per day or 700 cu. yd. per hour of actual digging time.

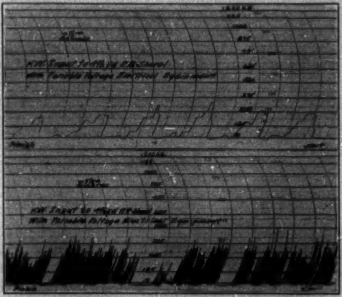
FIG. 1

Fig. 2 shows volt and ampere input to the hoist motors of the shovel over several cycles of operation. I would like to call especial attention to the regularity and uniformity of these charts. The largest area on the voltage chart represents digging areas, while the smaller areas, occurring regularly between the digging areas, represent power requirements for hoisting the loaded dipper during the dumping period.

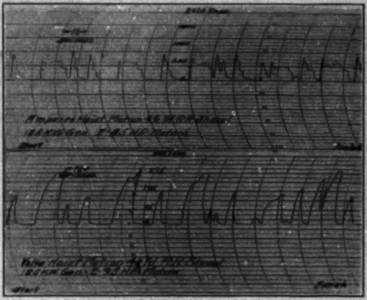
FIG. 2

Similiar areas can be distinguished readily on the ampere curve. It will be noted that neither the current nor the voltage shown on the chart reverses in direction. This is due to the fact that the shovel dipper is lowered by gravity, during which period the hoist motors remain stationary, lowering of the dipper being accomplished by release of a clutch between the hoist drum and the hoisting motors.

Fig. 3 shows volt and ampere charts taken on the swing motion. The same regularity is apparent on this chart as on those taken on the hoist motion with the difference that the current and voltage both reverse in direction, as the swing motor has to furnish power in both directions of swing. The charts as shown on Fig. 3 were made simultaneously and it will be noted that the current reverses at times when the voltage stays in the same direction. This reversal of the current without the voltage reversing gives regenerative braking, which is used to stop the swing motion at the end of travel.

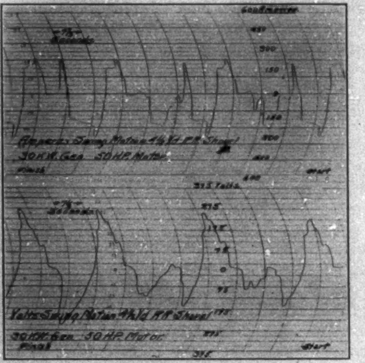

FIG. 3

Fig. 4 shows volt and ampere charts taken on the thrust motion. While there is an appearance of considerable regularity on these charts it is not so pronounced as in the case of the hoist and swing motion as operation of thrust motion does not follow nearly so definite a cycle as is the case on the hoist and swing motion. The operations, however, are more numerous and a snappy thrust motion contributes considerably to the successful operation of the shovel. Plugging or regenerative braking contributes in no small degree to the successful operation of the thrust motion The curves as shown on Fig. 4 were made simultaneously and the action of the current in reversing at plugging can be seen quite clearly.

Mr. Stevens in his discussion mentions that he has found the action of the variable-voltage control with separately excited motors to be so perfect in stopping and plugging that swing and thrust brakes having large heat capacity and requiring air operation are not necessary and that they can be replaced by the usual solenoid-operated brake designed for holding and infrequent stopping. Along the same lines I would like to say that the user of the 4½-yard railway-type shovels, on which the curves presented herewith were taken, has found that on his variable-voltage equipped shovels he obtained improved operation by not using the motor brakes on the swing and thrust motions in ordinary operations. His method of operation is to allow these brakes to remain released during the greater part of his operation. On the swing motion about the only time the brake is used is when the shovel is traveling, at which time the brake is applied to keep the shovel boom from swinging. On the thrust motion the brake is used when it is necessary to thrust the dipper out beyond the end of the shovel boom for dumping, and where digging conditions are very severe. The shovel operator has the option of using the motor brakes if he chooses, as the only modification made in the control equipment to allow operation with brakes "released" was the installation of a small switch to short-circuit the controller contacts used for releasing the brake magnet coil. When this switch is closed, the brake will stay released, and when switch is open the brake will respond to the operation of the controller, that is, it will release when power is applied to the motor and will apply when the controller is brought to the off-position. The advantages of operating without swing or thrust brakes are readily apparent as with the motors free to turn even a small amount, sudden impacts are softened and mechanical strains on the shovel parts decreased.

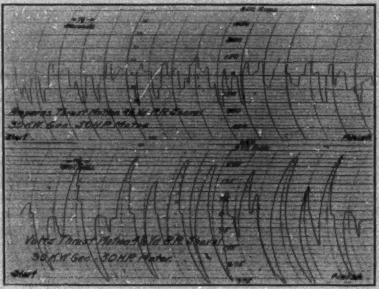

FIG. 4

Recent Progress in Distribution Practise of the Brooklyn Edison Company, Inc.

BY J. F. FAIRMAN[1]
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and
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Synopsis.*—Changes in the transmission and distribution system of the Brooklyn Edison Company during the past four years are briefly described and mention is made of the effect of these changes on the system's efficiency.*

In 1927 a low-voltage, a-c. network will be installed and this will be extended to care for the load growth until it becomes the principal supply system for low-tension energy in Brooklyn.

The plan of development is described in detail. The unique features are:

First, a suitably designed network unit consisting of a three-phase, 500-kv-a., low-loss transformer with an automatic network circuit breaker mounted in the low-tension pothead of the transformer.

Second, the operation of the high-tension feeders without regulators, at 27,000 volts, direct from the generating station.

Third, the ultimate plan of interlacing the high-tension feeders to reduce the overload on transformers and to maintain good voltage regulation with one feeder out of service, and to pick up overload in areas of different load characteristics to obtain the advantage of diversity. It is shown that this plan of development insures better service to the customer from every point of view, makes for safer working conditions on the system, results in general system improvement and involves less total investment as well as a considerable reduction in losses and operating costs.

The appendixes describe tests made to determine whether arcing and short-circuit faults in the network will be self-clearing, the permissible voltage fluctuations on incandescent lamps, and the dissipation of heat from transformer vaults.

* * * * *

THE changes in the transmission and distribution system of the Brooklyn Edison Company during the past four years, leading up to the proposed development of a low-voltage a-c. network, may be summarized as follows:

First, transmission voltages on the 60-cycle system have been increased from 6600 and 13,200 volts to 27,000 volts. This was made necessary by the difficulty of obtaining sufficient duct space in the streets in the vicinity of the new Hudson Avenue generating station to transmit the power away from the station at the lower voltages. Further consideration of the problem indicated that the higher voltage was also very much more economical.

Second, the primary distribution system from the substations has been changed from 2400 volts, two-phase to 2400/4150 volts, three-phase, which increased by 50 per cent the amount of power that could be transmitted over existing distribution feeders and at the same time reduced the losses by 25 per cent. In the substations themselves it was found that with the same buildings, by taking out certain unnecessary parts of the switching equipment, two-phase stations could be converted into three-phase stations of 50 per cent greater capacity with the addition of only a fraction of the equipment normally needed for this extra capacity.

Third, the overloaded d-c. system has been made comfortable and safe by changing over a sufficient amount of load to the a-c. system at points where this could be done most economically and with the least inconvenience to consumers. This plan of changeover was adopted in preference to reinforcing the inefficient d-c. system by adding feeders and mains and building additional d-c. substations at enormous cost. It was recognized frankly that with the art as then developed, short-time interruptions were more likely to occur on an a-c. system than on the d-c. system, but it was felt that careful attention to detail would give results materially as good, and that with the development of a closed low-tension network, an a-c. system would be substantially better than anything that can be developed with direct current.

An interesting result of these changes has been the increase in the over-all system efficiency from a fairly steady value of 78 per cent during the preceding years to 82 per cent at the present time, when all of these changes have been practically completed. This increase in efficiency is illustrated in Fig. 1. The ordinates are computed each month by taking the ratio between the kilowatt-hours sold and the kilowatt-hours generated. This will continue to increase as a greater proportion of the total load is taken on the a-c. system, especially as the network system is extended.

The problem of an a-c. low-voltage network has been studied for the past three years and the first part of this system will be in operation early in the summer of 1927. The network will be extended to take all the increase in load during successive years and will gradually supplant both the present radial a-c. system and the d-c. system.

The network will be entirely underground and will consist of single-conductor mains, solidly connected without fuses at all street intersections. These mains will generally be placed on one side of the street and will consist of three 500,000-cir. mil cables and one 250,000-cir. mil, bare copper neutral conductor. In the case of new construction or extensive revamping of old construction in densely loaded areas, the mains may be run on both sides of the street and the copper section will be 350,000 cir. mils.

Faults developing on these mains will burn clear.

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[1] Both of the Brooklyn Edison Company.

Presented at the New York Regional Meeting of the A. I. E. E., New York, N. Y., Nov. 11-12, 1926.

Experience on d-c. networks, extensive tests conducted by associates of the authors and similar tests conducted by others have definitely proven that failures on a system having 120 volts to ground will always burn clear with the size of cable and the capacity of transformers that will be used. These tests are described briefly in Appendix A.

The network will be three-phase, four-wire with 120 volts between phase wires and neutral and 208 volts between phase wires. These voltages were selected because the company's present standard voltage for lighting is 120 volts.

The matter of motor performance on voltages less than 220 volts has been discussed very extensively and the objections which were raised at first have largely disappeared for the following reasons:

First, all manufacturers guarantee their motors to operate satisfactorily at 10 per cent over or under the rated voltage.

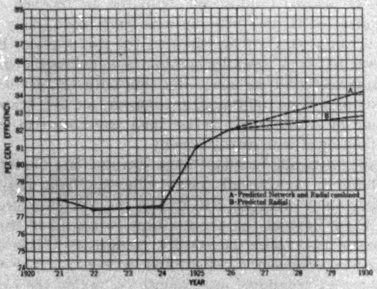

Fig. 1

Second, the large majority of motors are not designed within 10 per cent or sometimes even 20 per cent of their nameplate rating.

Third, on a network system the voltage regulation is much better, inherently, than on any other system, and consequently the voltage delivered to the motor terminals will be more nearly constant at the nominal value.

Fourth, in the majority of applications motors are normally under-loaded, in which case the efficiency and power factor are both better at the lower voltage and any reduction in speed is practically negligible.

Fifth, so far as starting torque is concerned, any desired value may be had, since all the common sizes of motors may be started by throwing them directly on to the line without producing an excessive voltage disturbance on such a network system. Indeed, in special cases it would be permissible to use an inverted compensator to apply greater than rated voltage to the motor terminals for starting.

Perhaps the strongest proof that these objections have been removed is that two of the leading manufacturers have recently instructed their salesmen in the Brooklyn district to quote standard 220-volt motors for use on this 208-volt system and to base performance data on 220 volts when it is requested.

Due to its better inherent regulation, the network system will eliminate the necessity for the rather complex service rules in common use. The principal reason for the adoption of rules restricting motor starting currents and the size of single-phase, low-voltage devices is to prevent excessive voltage fluctuations on combined light and power circuits. In order to determine the proper basis for such rules, in connection with the network system, the tests described in Appendix B were made and the results checked very closely with those obtained by other investigators. From these data, and the constants of the proposed system, it has been calculated that three-phase motors up to 50 h. p. and single-phase loads up to 10 kv-a., at 120 volts may be connected directly to the line at any point on the network without causing excessive voltage fluctuations.

It follows then that service rules can be simplified and reduced to a minimum and this will result in a greater commercial flexibility and improved relations with the consumers.

An objection to the three-phase, four-wire system has been raised in the matter of metering three-wire lighting services. Small three-wire services will be changed to two-wire services whenever the wiring on the customer's premises will permit, and large three-wire services will be changed to four-wire services by splitting up the three-wire circuits on the three sides of the *Y*. Such few three-wire services as necessarily remain can be metered by means of a standard two-element meter. No new services will be made three-wire.

Power will be supplied to the network from transformers located in vaults near street intersections. The transformers will be three-phase, 500-kv-a. and will have a full load loss less than 5 kw. These transformers have been specially designed for this low loss in order that the heat generated by the losses can be dissipated by conduction into the soil and by natural ventilation without exceeding an ambient of 40 deg. cent. in the vault. Experience has indicated that the losses of two 100-kv-a. standard distribution transformers were about all that could be taken care of in the present vault. It was impracticable to increase materially the size of the present vault and it was considered undesirable to use forced ventilation or water cooling. Extensive tests were made to determine accurately the amount of heat which could be dissipated from the standard vaults. These tests are described in Appendix C. The results lead to the conclusion that losses up to 5 kw. could be handled by the present standard vaults.

The connections to the transformer will be made in two potheads, one on either end of the transformer, which will be sealed from the main tank by oil-tight insulating bushings, through which the leads pass. This is for the purpose of making connection in the field without the necessity of opening the transformers. The transformers will be equipped with filter-press connections and oil gages and will be filled with oil and sealed at the factory.

The twelve 500,000-cir. mil cables forming the network mains which proceed in four directions, will be brought out from the bottom and will be connected to three busses inside the pothead. The neutral lead will also be brought out through and solidly grounded to the bottom of the low-tension pothead.

The low-tension pothead will also contain an automatic network circuit breaker which will connect the low-tension transformer leads at the top of the pothead to the three busses at the bottom. The breaker and its auxiliaries will be mounted on a panel which can be readily removed from the pothead. This is for the purpose of permitting quick and easy replacement of a faulty unit and for making repairs and major adjustments in the shop. The circuit breaker will trip out on reversal of power of the order of magnitude of the magnetizing power of the transformer and over a wide range of power factor from the lagging current of the transformer alone to the leading charging current of the cable. It will automatically reclose when the voltage at the low-tension side of the transformer is such as to feed power into the network. The relays are designed to prevent "pumping" of the breakers due to conditions which arise when one breaker fails to open. Heavy fuses on the network side of the breaker form a secondary line of defense to open the circuit in case of a failure in switch operation.

The breaker and the transformer will be furnished as a unit by the manufacturer. The transformer, with potheads attached, will be approximately 10 ft. long, 3½ ft. wide and 7½ ft. high. A special truck is being designed to transport the transformer and to lower it into the vault or to raise it out of the vault in case of necessary replacement. The roof of the vault consists of two removable center sections and two fixed end sections. The center sections of the roof are reinforced concrete slabs. These may be lifted out after the asphalt paving surface has been removed. In each of the fixed end sections there is a 35-in. round manhole. These manholes will be covered with heavy gratings instead of cast iron covers, which will permit a considerable circulation of air. A cross-section sketch showing the transformer in position in the vault is shown in Fig. 2.

The high-tension pothead of the transformer is to be filled with oil and will contain a grounding and short-circuiting switch. Provision will be made for attaching two conductors to each of the high-tension transformer leads in the potheads and for bringing out two three-conductor or six single-conductor cables through the bottom of the pothead. Thus a number of transformers will be connected to a feeder by looping the feeder into and out of the potheads of successive transformers, thereby eliminating *T* joints on the high-tension cable.

The high-tension cable will have a 350,000-cir. mil copper section and will be either single-conductor or three-conductor with the individual insulated conductors covered with a thin metallic ribbon. In either case, the only high potential test required will be a conductor to ground test. It will not be necessary to disconnect the transformers from the cable while applying this test since the high-tension windings will be delta-connected and will be specially insulated for the high potential test. Faults will be located by means of a tracer current method. When it is necessary for work to be done on a feeder or transformer, adequate protection to the workmen is assured by grounding at a transformer on either side of the point at which work is being done, as well as at the station end of the feeder. Further protection from the possibility of back feed from the network may be obtained by manually locking out the low-tension circuit breaker.

Each cable joint will be equipped with a weighted expansion reservoir and the joint and the reservoir will be filled with oil. Extensive experiments have shown this to be a most satisfactory method of preventing failures due to voids which may develop either in the cable or in the joint subsequent to installation. A typical installation is shown in Fig. 3.

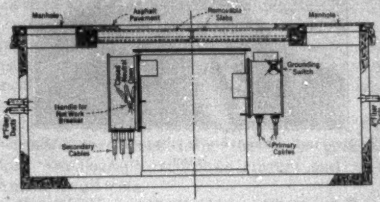

Fig. 2—Network Transformer in Vault

On a radial system of distribution it is necessary to do a great deal of work on live cables in order to reduce the number and duration of outages. This, in the past, has limited the primary distribution voltage to around 2400 volts to ground, but even so it cannot be said that a safe working condition is obtained. On a properly designed network system, a primary feeder and its connected transformers may be taken out of service for an indefinite period without interrupting the service of any consumer. Consequently, since all work can be done on feeders or transformers while out of service, there is no limit to the permissible operating voltage from this viewpoint. Economic considerations then make it desirable to feed into such a network system directly from the generating station.

The high-tension feeders will be supplied from the 27,000-volt bus of the generating station. No feeder regulators will be used. Studies have shown that such a network without regulated feeders will have better voltage regulation than can be obtained on a radial system with regulated feeders. The bus voltage at the generating station will be varied in two steps during the day as is being done at the present time to reduce the range required on the feeder regulators in existing substations. Under normal conditions the voltage on the network will be within the present range on the radial system of plus or minus 3 per cent from 120 volts. In the emergency condition of one feeder out of service, the lowest voltage on the network will be around 110 volts to neutral with the corresponding value of 192 volts between phase wires. These values are based on full load at the time of the annual system peak so that ordinarily, with one feeder out, the voltage conditions will be considerably better. These figures are also based on the initial layout of transformers and will be improved as the transformer spacing decreases with a greater load density.

Fig. 3—Sylphon Reservoir Installation on 27-Kv. Cable Joint

The continuity of service to the consumer on the network system will be greater than that which can be assured on a radial system since a complete interruption to an individual consumer can be caused only by burning off his service or by burning off the network mains in two places, one on either side of his service tap. On the present radial system, interruptions due to secondary troubles constitute only seven per cent of the total interruptions and these would be materially reduced on a network system. Careful attention to detail has made possible the development of a very reliable radial a-c. system. However, there is always a possibility of trouble with transformers, primary fuses and cut-outs, sectionalizing and tie switches, and cable, which will cause a total interruption to one or more consumers until the trouble can be isolated or repaired. The high-tension side of a network system is inherently simpler and hence less subject to trouble, and if a failure does occur, it will not result in an interruption of service to the consumer. If a complete shutdown of the generating station occurs, the network will automatically pick up its load as soon as the power supply is restored.

This system will be started in an area a section of which is shown in Fig. 4. This area happens to have a very simple rectangular street layout and permits a correspondingly ideal layout of secondary mains, transformers, and high-tension feeders. The area will be supplied originally by three feeders and the transformers will be located as indicated in the figure. As the load density increases, additional transformers will be located midway between the first transformers, and three additional feeders will be brought into the area. With further load increases, the same plan will be followed so that ultimately there might be four feeders down each avenue with the successive transformers on different feeders. Under these conditions the outage of one feeder will place an additional load of approximately 25 per cent on the immediately adjacent transformers. Hence the loss of one feeder will not be a serious matter and ample time may be allowed for working on it, or the connected transformers, before putting it back into service.

As the network is extended to cover all of the underground area of the city, the feeders coming through a particular area will have come by different routes and through areas having different kinds of load and will go on by divergent routes to areas of still other load characteristics. In other words, a group of feeders will not pick up their entire load in one area but will pick up load whenever possible in three areas of different characteristics, such as industrial load, office building and department store load, and residential load. By this extensive interlacing of the high-tension feeders, the advantages of diversity between areas of different load characteristics and of the reduction of extra feeder capacity required to carry the load during the outage of a feeder will be realized. The feeders will have a nominal capacity of 15,000 kv-a. each and as many as 40 transformers may be connected to each feeder in the ultimate development of the network.

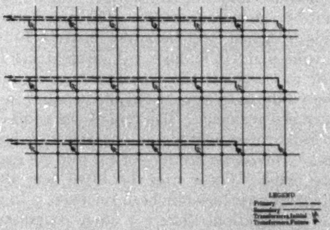

Fig. 4—Typical Layout for Fourth Ave. Network

Preparations are being made for the ultimate development by installing secondary mains of the proper size in all additions to the present radial underground distribution system and by building the new transformer vault wherever additional vaults are necessary. Whenever it is necessary to open the street for extensions or additional duct space, the duct layout for the ultimate network development in that area is installed. This work is justified in the present underground area on account of the relatively short time until the network system will cover the entire area.

In the case of vaults installed on customers' premises and served from the present radial system, the transformers are connected to give 120/208-volt, three-phase, four-wire service. When the network is introduced into the area in which such large customers are located, it will serve as the emergency supply to these customers by being tied in with the secondaries of the existing transformer installations and a network circuit breaker will be connected between the service and these transformers. Thus, part of the present substation capacity will be used as the normal supply to such large customers, part will be used to feed transformers in the street tied in with the network, part will be used to supply high-tension customers, and the remainder will take care of the fringes outside of the underground development where the load density is light and where there is no justification for extending the network.

The reasons for adopting this plan of development are as follows:

First, and most important, is the fact that this system insures even better service to the consumer. Better service means improved continuity of service, improved voltage regulation, a shorter interval of time between the application for service and the actual connection, and minimizing the number of rules and restrictions imposed upon the consumer.

Second, and hardly less important, is the matter of greater safety to workmen on the system. The high-tension cables and transformers will be worked on only when they are out of service and adequately grounded.

Third, since the network system is essentially an underground system wherever the load densities are great enough to justify its development, it becomes possible to improve the appearance of the system by the removal of overhead lines.

Fourth, the savings in investment and operating costs and the increased system efficiency obtained by eliminating the necessity for substations economically justify this development. By 1935, when this system will have been well established, the total investment required will be approximately 10 per cent less than would be required for the present plan of radial development with substations, and the saving in maintenance, operation and losses on the distribution system will be around 35 per cent.

Appendix A

ARCING AND SHORT-CIRCUIT TESTS ON LEAD-COVERED CABLE

The objects of these tests were:

1. To determine whether faults on a 120/208-volt system of the order of magnitude calculated for the proposed alternating-current network will clear themselves.

2. a. To determine whether faults on a 2400/4150-volt system are equivalent in current values in all cases and for practical purposes, to a dead short circuit.

b. To determine whether there is a critical value of current at this voltage above which arcs once established will persist and below which arcs will clear themselves.

Throughout the scope of these tests the physical and magnetic conditions encountered in service were reproduced as closely as possible. Three cables were placed in a 4-in. fiber duct, 16 ft. long, buried 10 in. below the ground, the ends of the duct being plugged with waste to prevent free circulation of air. Voltage was applied to all three cables, the fault being made on one cable only. This was for the purpose of determining whether the fault would injure the remaining cables sufficiently to cause them to fail as well as to keep physical conditions as nearly as possible to those encountered in service.

Power was supplied through a 10,000-kv-a. feeder and transformer direct from the main generating station. Oscillograph records of current and voltage were made for all tests.

Tests on Cables at 120/208 Volts. Tests were made on standard 350,000-cir. mil, 300-volt, 6/64-in. rubber, 1/8-in. lead cable. The fault was placed in the center of the cable by embedding a No. 8 brass screw in the copper and soldering the head to the lead sheath. One leg of a 100-kv-a. transformer was connected to the two ends of the loop and the other leg to the lead sheath. This test was repeated first with three and then with six transformers in parallel; the resultant currents into the fault being 7500 amperes, 13,000 amperes and 19,000 amperes, respectively. In all three tests the fault cleared itself by melting the brass screw without establishing an arc. Similar tests were repeated in an effort to make a lower resistance fault by various methods until a solid lead knob was wiped between the bare conductor and the sheath. The results in all cases were the same; *i. e.*, the fault cleared itself by melting away the lead without establishing an arc.

Copper-to-copper short-circuit tests were also made to determine whether faults of this nature would clear themselves. A copper-to-copper short circuit was made through a 32-ft. loop with a joint in the center of the loop. Tests were made for the maximum (17,000-ampere) and minimum (3000-ampere) current values calculated for the network. In all cases the paraffin in the joint melted but the conductor burned apart, clearing the fault before the joint failed.

Tests on Cables at 2400/4150 Volts. These tests were made on 4/0-, 11/64-in. rubber, ⅛-in. lead, 5000-volt cable. Three cables were placed in one duct and a fault placed on one cable as in the low-voltage tests, all three cables being energized. It was found during the first tests that very soon after the arc was formed the fault spread to all three phases. Since reactors to limit the current in all three phases were not available, subsequent tests were made with only one phase energized.

Tests were made over a range of from 360 amperes to 1750 amperes with the results that in every case the arc continued to burn until it reached the end of the ducts, making it necessary to clear the fault by opening the breaker.

As the result of these tests the following conclusions have been reached:

1. All 120/208-volt faults will clear themselves without damage to other cables in case of ground faults.

2. 2400/150-volt faults, once an arc is established, will continue until the voltage is interrupted at its source.

3. All other cables in a duct on a 2400/4150-volt system where an arcing fault exists will break down, causing a phase-to-phase short circuit.

Results of similar tests which confirm the above are described in *Self-Protecting A-C. Networks* by A. H. Kehoe, TRANS. A. I. E. E., Vol XLIII, p. 850.

Appendix B

PERMISSIBLE VOLTAGE FLUCTUATIONS ON LIGHTING CIRCUITS

The principal reason for the adoption of rules restricting motor starting currents and the size of single-phase, low-voltage devices is to prevent excessive fluctuations in voltage on combined light and power circuits and consequent annoyance to lighting customers. In order to determine a proper basis for such rules in connection with the network system, a number of tests was made to obtain answers to the following questions:

1. What change in voltage occurring instantaneously will produce a noticeable change in illumination of standard lamps?

2. What is the relation between the magnitude and frequency of a periodic voltage change that will produce a noticeable change in illumination?

3. What rate of change will be noticeable?

4. What changes will be objectionable?

5. How are the above affected by different types, ratings and arrangements of lamps?

The conditions under which the tests were made were as follows:

A room was equipped with six suspended outlets so arranged as to allow varying the distance from the lamp to the working plane. Each socket was equipped with an opaque cone shade. The observers were screened from one another and each one indicated independently when he observed a fluctuation and when in his judgment the fluctuation became objectionable. A motor-generator set was used to supply a steady voltage for the lamp circuit. In series with one lead a variable slide wire resistance was inserted, so arranged that it could be switched in and out of the circuit by means of a snap switch or a motor driven commutator. A low reading voltmeter was placed in parallel with this resistor so that the actual voltage drop could be measured.

The first test consisted of switching in and out the resistor at 5-sec. intervals and gradually increasing the drop until all observers reported that they had noticed the fluctuation, then continuing to increase the drop until they had all reported that the fluctuation was objectionable. Twenty-five and 50-watt Mazda B lamps and 75- and 100-watt Mazda C lamps were used. Successive tests were made with the lamps at 18 in. and 72 in. above the observer's reading plane.

The lowest fluctuation observed was a one-volt drop with the 25-watt lamp 72 in. above the reading plane, which gave the lowest illumination intensity. In general, very few of the observers could detect a change smaller than 1½ volts under any of the conditions imposed and some observers had difficulty in detecting a fluctuation smaller than 2¼ volts. It was found that the higher the illumination intensity the greater was the fluctuation in voltage necessary to produce a noticeable change in illumination. The averages of the results for all observers varied from 1.33 volts for the 25-watt lamp at 72 in. to 1.66 volts for the 100-watt lamp at 18 in., which gives an average for all observers under all conditions of 1½ volts.

All observers agreed that the fluctuations became decidedly objectionable when the voltage was varied from four to five volts. As mentioned before, the interval between changes was five sec. and undoubtedly a longer interval would lead to different conclusions as to what fluctuation was objectionable.

The second test consisted of increasing and decreasing the voltage gradually by moving the slide on a resistor by hand at as nearly uniform a rate as possible. It was found that the change could be noticed when the rate of change exceeded 2½ volts per sec. This was checked several times with very uniform results. One lamp only was used in this experiment and one distance from the working plane. The results of the first experiment led us to believe that no great difference would be found by using different sizes of lamps at different distances.

The third test consisted of cutting the resistor in and out by means of a motor-driven commutator, setting the resistor for a certain drop and increasing the speed of the commutator until the flicker disappeared; then decreasing the speed until it reappeared and averaging the results. It was found that no flicker at any frequency could be observed for drops smaller than one volt; also the flicker entirely disappeared for all voltage drops at frequencies above 45 cycles per sec. The results of this test are shown on the curve in Fig. 5.

Before making the above tests, a search was made of the literature and inquiries were made of various individuals and organizations from which sources it was hoped to obtain information on this subject. It seems that a great deal of information is available on the relations between the magnitude and frequency of periodic voltage fluctuations which produce visible flickering of incandescent lamps, but that relatively little work has been done on the problem of determining when non-periodic and infrequent voltage changes become objectionable. The earliest study of this problem which was found was made by Mr. William Kunerth, of Iowa State College, and was published in the August 1915 number of the *Lighting Journal*. C. A. Williams, of the Philadelphia Electric Company, in January 1924 presented a paper on "Voltage Fluctuations and Its Effect Upon Lighting" before the Pennsylvania Electric Association. In the June 1924 number of the A. I. E. E. JOURNAL there appeared an article by A. H. Kehoe, of the United Electric Light & Power Company, on "Underground A-C. Networks." Under Appendix A of this article he describes some tests on the effect of voltage variations on incandescent lamps.

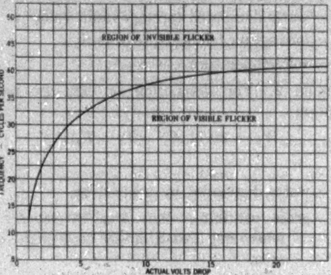

FIG. 5

The information obtained from the tests and from the other sources mentioned seems to indicate:

1. That a sudden change of voltage of less than two volts does not produce a noticeable change in illumination, unless it is periodic.

2. That a sudden change of voltage of more than six volts will probably be objectionable if it occurs more frequently than once or twice an hour.

3. That any noticeable periodic fluctuation or flicker is objectionable.

In connection with the above, two things should be borne in mind:

First, that the values were obtained when the observers were expecting to notice changes; hence they are lower than would have been noticed under ordinary conditions. Second, that all investigators are in much closer agreement as to what changes are noticeable than as to what changes are objectionable. So many factors, psychological and physiological as well as electrical, are involved in the problem of objectionable voltage variations that a more extended study would seem to be warranted. In his article, Mr. Williams emphasizes that we have made only a step toward standardizing the limits of voltage fluctuation for satisfactory service to lighting customers.

Appendix C

HEAT DISSIPATION FROM TRANSFORMER VAULTS

The transformer vaults in use in the streets of Brooklyn were designed to accommodate two 100-kv-a. transformers. These vaults are 4½ ft. wide, 14 ft. long, and 8 ft. deep Access is obtained through a 35-in. manhole located in the center of the roof. The vaults are ventilated by natural circulation of air through banks of three 4-in. fiber ducts at either end and through the openings in the manhole cover. Experience has indicated that this ventilation together with the absorption of heat into the surrounding soil will take care of the losses of two 100-kv-a. transformers under ordinary load conditions the year round without exceeding an ambient temperature of 40 deg. cent. in the vault. Such a vault is shown in section in Fig. 6.

Before deciding upon the type and size of transformers to be used for the network, it was desirable to determine accurately how much heat could be conducted through the walls of the vault into the surrounding soil and how much could be carried away by the circulation of air through the vault using various types of ducts and manhole covers or gratings.

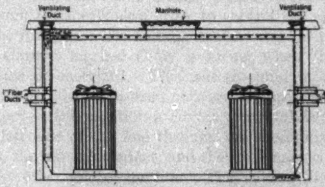

FIG. 6—STANDARD 200-KV-A. TRANSFORMER VAULT

Tests to determine the conduction of heat through the walls of the vault were made in a standard vault with the ventilating ducts closed. Bayonet heaters were placed in two 100-kv-a. transformer tanks filled with oil. These heaters were arranged to give the equivalent losses of different sizes of transformers under various load conditions. Thermocouples were imbedded in the walls of the vault and at distances up to eight feet away from the vault in the earth at various depths. They were used also to measure the temperatures of the oil and air in the vault. Starting with the vault and transformers cold, a constant amount of power was supplied and the temperatures taken at regular intervals until all temperatures became constant. The vents were then opened and temperature readings taken until they again became constant. This was repeated several times with different amounts of power, both for constant loads and for various load cycles. These tests occupied a five-month period, from February to June, so that some data were obtained on the seasonal effect on soil temperatures. From the data obtained during these tests Formulas **1** and **2** were derived.

Further tests to determine the effect of various types and arrangements of ventilating ducts and cover gratings were made using a box built to the dimensions of the vault. The same heating arrangement was used, and air temperatures at various points were measured by means of thermocouples. Air velocities in the inlet ducts were measured as a check on other data. Various constant amounts of power were supplied and readings taken when temperatures became steady. The process was repeated for different sizes of inlet and outlet ducts, both with and without cover gratings. The data thus obtained were used to check Formula **2**.

The constants in the following formulas apply to the size of vault and the ventilating arrangement finally decided upon, which is shown in Fig. 2. A removable canvas tube extending down from one manhole to a point 18 in. above the floor of the vault will lead air into the vault, whence it will flow around the transformer and out of the other manhole.

Formula **1**. For dissipation of heat by conduction through the walls of the vault:

$$W_1 = 95.8\,(T_1 - T_2) + 0.6325\,\epsilon^{0.1(T_1 - T_2)}$$

where

W_1 = watts dissipated through vault walls,

T_1 = average air temperature in vault,

T_2 = soil temperature.

Formula **2**. For dissipation of heat by natural ventilation:

$$W_2 = 30.74\,A^{0.5}\,(T_1 - T_3)^{1.6}$$

where

W_2 = watts dissipated by ventilation,

A = gross ventilator duct area in square feet, where net area is 65 per cent of the gross area,

T_1 = average air temperature in vault,

T_3 = outside air temperature.

Discussion

For discussion of this paper see p. 1220.

Evolution of the Automatic Network Relay

BY JOHN S. PARSONS[1]

Non-member

Synopsis.—*Low-voltage a-c. networks are coming into use as a means of providing a source of power having the dependability of the Edison d-c. network, and at the same time having the efficiency which can be obtained by supplying power from high voltage feeders with transformers located near the point of utilization. As in the case of d-c. networks, it is assumed that all trouble on the network itself will be burned clear without seriously interfering with any of the service. In case of trouble on a primary feeder or in one of its transformers, however, it is necessary to disconnect the feeder from the system. The a-c. network unit has been developed to connect these transformers to the network, and the "brains" of this unit, which is the automatic network relay, is described in this paper. This relay not only opens the network breaker whenever there is trouble in any of the high-tension equipment or when the power feeds back into the high-tension feeder but also recloses the breaker when conditions are restored to normal and the feeder is in condition to supply power to the network. The difficulties which were encountered and the relay characteristics necessary to overcome them are explained briefly.*

The first installation of network units was made in April 1922 on single-phase, three-wire and two-phase, three-wire networks fed by three-phase, 2750-volt primary feeders. Since that time numerous improvements have been made and new designs created to extend the field of application, but the same general scheme of operation has been retained.

The automatic network relay is now past the experimental stage, approximately 1000 of them having been in service for some time. The probable future development of the network relay based upon operating experience to date is pointed out. Attention is called to the close relation existing between the operating characteristics of the automatic network relay and the characteristics of the system on which it is to be installed.

* * * * *

THE advantages of some form of a-c. secondary network distribution system, the chief of which are continuity of service and the ability to use higher voltage primary feeders, have long been recognized. It is only within the last few years, however, that the necessary control and protective apparatus for such a system has been developed. This apparatus, known as the a-c. automatic network unit, consists of an electrically operated carbon circuit breaker controlled by one or more induction type relays. The operating requirements of such a relay are discussed in detail in a companion paper[2] to this one.

It is proposed to give here the history of the development of one type of automatic network relay and to explain how the present network relays meet the various operating requirements.

While one or two installations have been made using reverse power or directional relays, such as are used for transmission line protection, for tripping network breakers when a fault occurs on a primary feeder or in a transformer, they cannot be considered automatic network relay installations. These, however, are of interest in that they have demonstrated that relays of the induction type, which are ordinarily installed in substations where they can be easily and frequently inspected, will operate satisfactorily in man-holes without any change in design. The automatic network relay not only controls the opening but also the closing operation of the network breaker. It is this fact that has made the design of the relay a much more difficult problem than that of most relays. The reverse current opening and automatic synchronized reclosing features which have been embodied in the network relay make the network circuit breaker a reverse power tripped breaker which can be remotely controlled from the station, without the use of pilot wires, merely by opening and closing the primary feeder breaker.

The first network relays were designed and placed in service on single-phase, three-wire lighting and two-phase, three-wire power networks in April 1922. These networks were fed by three-phase, 2750-volt primary feeders from the substation. Since the general scheme employed in these first automatic network units has been followed in future developments, the evolution of the network relay can be more easily followed and understood if a description of it is given at this point.

Fig. 1 shows a schematic diagram of connections for the first single-phase, three-wire network unit. The relay was an induction relay of the double contact type. The single-pole double throw contacts acted through the mechanism relay to control the operation of the breaker. The moving contact was held against the contact marked C by a spring when the relay became de-energized. This was essential in order to be able to start up a dead network. With the moving contact in this position a circuit was established from the left side of the line through the mechanism relay coils, the moving contact, and the stationary contact C to the other side of the line. This caused the core of the mechanism relay to pick up when the transformer was energized and to close the auxiliary switch A. The closing of this switch energized the closing coil, thus closing the breaker. The closing coil circuit was opened by the pallet switch B as soon as the breaker latched in a closed position so that this coil was energized only during the instant the breaker was being closed. In order to open the breaker, the moving contact of the relay had to move from position C to position O. The breaking of the contact at C places a resistor R in series with the mechanism relay coils. The value of this

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[1] Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

[2] "Operating Requirements of the Automatic Network Relay", by W. R. Bullard.

Presented at the New York Regional Meeting of the A. I. E. E., New York, N. Y., November 11-12, 1926.

resistance was such that with it in circuit the mechanism relay core would not pick up, but once picked up it would remain in that position. When the moving contact was made with stationary contact O due to a flow of reverse energy, the mechanism relay coils were shunted, leaving the resistor R connected across the line. The shunting of its coils caused the core of the mechanism relay to drop and to trip the latch that held the breaker closed.

The operating forces in the relay were obtained by means of three sets of coils. The flux produced by the potential coil, shown connected across the secondary of the distribution transformer on the network side of the breaker, combined with the flux of the phasing coils to produce a closing torque in the relay and with the flux of the current coils to produce an opening torque in the relay. It was necessary therefore for the potential coil to be energized at all times in order to secure an electrical torque in the relay. This constituted an important reason for connecting the potential coil on the network side of the switch. When a primary feeder was de-energized at the substation the network breakers on this feeder were opened by their relays due to the flow of reverse energy. Once the breaker was open, the phasing coils, in conjunction with the potential coil, gave a very strong opening torque to keep the moving contact of the relay in position O. If, however, the potential coil had been connected on the transformer side of the breaker, it would have received practically no voltage, since it would have been shunted by the distribution transformer, the impedance of which was very low compared to that of the potential coil. Under this condition, there would have been no torque to keep the moving contact of the relay in position O, and therefore since it would have moved to position C by spring action the synchronized reclosing feature of the relay would have been destroyed. This means that when the primary feeder was next energized the network breaker would have reclosed immediately irrespective of the voltage across it.

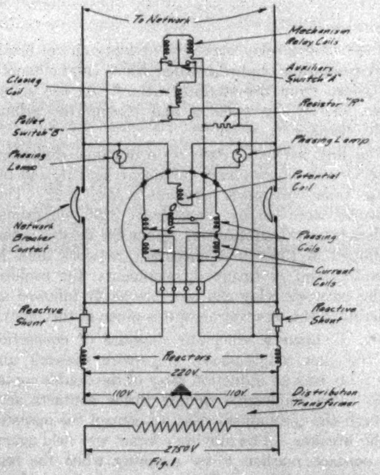

FIG. 1—DIAGRAM OF CONNECTIONS FOR THE FIRST SINGLE-PHASE THREE-WIRE NETWORK UNITS

The design of the phasing coils, one of which was connected across each pole of the network breaker, had to be such that the relay would operate to close the breaker with approximately one volt of the proper phase relation existing across the contacts of the breaker. When the network was energized, however, and the feeder breaker at the substation was open there was full line to ground potential across the open contacts of the network breaker and consequently across each phasing coil of the relay. With the possibility of the transformer voltage being reversed due to an error in making connections, twice normal voltage across each phasing coil may occur. In order to prevent burning out of the phasing coils at these high voltages, a small tungsten filament lamp commonly called the phasing lamp was placed in series with each coil. Since tungsten has a large positive temperature coefficient of resistance, the lamp automatically inserted resistance in the circuit as the voltage across the circuit increased.

When a primary feeder breaker is opened at the substation the transformer connected to that feeder becomes magnetized from the network. This flow of exciting current from the network to the transformer should be sufficient to cause the relay to operate to open the network breaker. The fact that the current coils, acting in conjunction with the potential coil, should produce sufficient torque to operate the relay on this relatively small value of current and must also be capable of being subjected to the full load current of the transformer continuously without overheating, made it necessary to connect them across reactive shunts placed in the secondary leads of the distribution transformer. At very small currents the reactance of the shunts was comparatively high and consequently a large part of the line current passed through the current coils of the relay. Due to the saturation of the magnetic circuits of the shunts, as the line current increased the reactance of the shunts decreased. The percentage of the line current passing through the relay therefore decreased rapidly and in this way the current in the current coils of the relay at full load was limited to a value which would not cause excessive heating.

It is not only essential that the magnitudes of the currents and voltages on which the network relay must operate be considered but their phase positions must also be taken into account in order to secure satisfactory performance. Fig. 2 shows the operating characteristics of the relay for various magnitudes and phase positions of voltage and current. The network voltage, which is the voltage on the network side of the breaker, is used as a reference voltage in plotting these curves since it is the voltage impressed across the potential coil of the relay. The closing Curve No. 1-*A* shows the closing characteristics of the first network relay over the range of phasing voltages on which it ordinarily operated. Lines drawn from the origin to this curve represent in magnitude and phase position voltages across the open network breaker and consequently across the phasing coils of the relay which produced a balanced condition in this relay. Any phasing voltage of such magnitude and phase position that it crossed the closing curve into the zone marked "close" produced a torque in the relay which caused it to operate to close the network breaker. If the phasing voltage did not cross the closing curve, however, the phasing coils produced a torque which caused the moving contact of the relay to move to position *O* and thus prevented the breaker from closing. Lines drawn from the ends of the phasing voltages to the ground potential point of the network voltage vector E_1, which could not be shown in Fig. 2 because of the scale used, represent the voltages of the unloaded transformer.

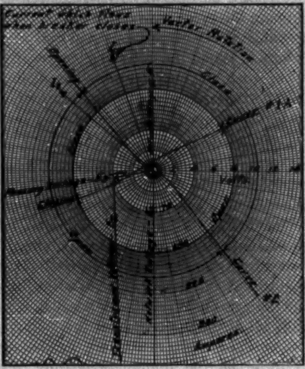

Fig. 2—Curves Showing the Operating Characteristics of the First Network Relays

The opening characteristics of the relay are shown by Curve No. 2, Fig. 2. Lines drawn from the origin to this curve represent in magnitude and phase position, with respect to the network voltage *E*, the line currents that were necessary to produce a balanced condition in the relay. If the line current was increased slightly so that it just crossed the opening curve into the zone marked "open," the relay operated to open the network breaker. Line currents that did not cross the opening curve produced a torque in the relay in the closing direction which held the moving contact against the stationary contact *C*.

The current which flows after the network breaker closes lags the phasing voltage,—that is, the voltage across the breaker before it closed,—by an angle approximately equal to the impedance angle of the system. By impedance angle of the system is meant the angle between the resistance and impedance vectors of the loop consisting of the feeder being connected to the network, together with its regulator and particular transformer being considered, in series with the low tension mains and all parallel transformers, primary feeders, and regulators connected from the same bus to the network. This angle remains almost constant on systems having high impedance transformers and large secondary mains if the spacing of the transformers is not too great.

The relays having operating characteristics as shown in Fig. 2 were installed on a system using two single-phase induction regulators per feeder with their rotors mechanically connected and standard distribution transformers with reactors in the secondary leads which are equivalent to high impedance transformers. With the system impedance angle of approximately 80 deg. the relays operated satisfactorily. The reason for this can be seen by referring again to Fig. 2. The phasing voltage, E_3, which could readily be obtained by having the regulators on the incoming feeder in the buck position, caused the relay to close the network breaker. As soon as the breaker closed, a current, I_1, lagging behind voltage E_3 by about 80 deg., flowed through it. Since this current did not cross the opening curve but fell in the area marked "close," the current coils of the relay produced a closing torque and maintained the breaker in the closed position. Any phasing voltage leading the network voltage which caused the relay to close the breaker caused a current to flow after it closed which produced a closing torque

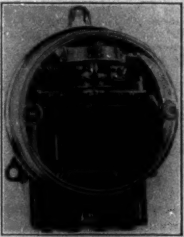

Fig. 3—The First Automatic Network Relay to be used for Controlling Network Units on Three-Phase, Four-Wire Networks

in the relay, because the angle between the opening and closing curves was less than the impedance angle of the system.

On the two-phase, three-wire network previously referred to, a polyphase network relay was used. This relay consisted of two electrically independent elements mechanically connected so as to control one set of single-pole, double throw contacts. The coils of one element were connected to one of the two phases and those of the other were connected to the other phase. Since in this case the normal voltage to ground was 220 volts, two phasing lamps were used in series with each phasing circuit instead of one. Aside from these differences the general scheme and operating characteristics were essentially the same as those described above for the single-phase relay.

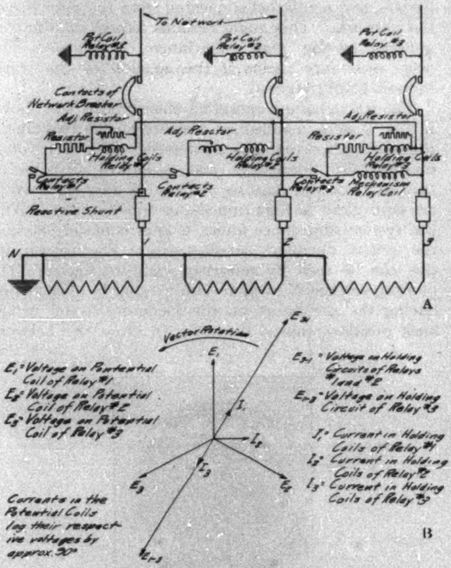
To Network
Pot Coil Relay #1
Pot Coil Relay #2
Pot Coil Relay #3
Contacts of Network Breaker
Adj. Resistor
Resistor
Adj. Reactor
Adj. Resistor
Resistor
Holding Coils Relay #1
Holding Coils Relay #2
Holding Coils Relay #3
Contacts Relay #1
Contacts Relay #2
Contacts Relay #3
Mechanism Relay Coil
Reactive Shunt
1
2
3
N
A
Vector Rotation
E_1
$E_{3\text{-}1}$
E_1 = Voltage on Potential Coil of Relay #1
E_2 = Voltage on Potential Coil of Relay #2
E_3 = Voltage on Potential Coil of Relay #3
$E_{3\text{-}1}$ = Voltage on Holding Circuits of Relays #1 and #2
$E_{1\text{-}3}$ = Voltage on Holding Circuit of Relay #3
I_1
I_2
I_3
E_3
E_2
$E_{1\text{-}3}$
I_1 = Current in Holding Coils of Relay #1
I_2 = Current in Holding Coils of Relay #2
I_3 = Current in Holding Coils of Relay #3
Currents in the Potential Coils lag their respective voltages by approx. 90°
B

FIG. 4—CONNECTIONS FOR THE HOLDING AND POTENTIAL COILS OF THE NETWORK RELAYS USED ON THE FIRST THREE-PHASE, FOUR-WIRE NETWORK UNITS

The next step in the network relay development was the designing of a relay, Fig. 3, which could be used on a three-phase, four-wire network as well as on a single-phase, three-wire network. Because it was found that the time required for the double contact relay to open the network breaker was considerably longer than is desirable in case of a primary cable fault, it was decided to abandon the double contact feature. The use of a single set of contacts made it necessary to place an additional set of coils in the relay on the same poles of the electromagnet as the phasing and current coils. These coils, known as the holding coils, were connected in series with the contacts of the relay and in conjunction with the potential coil, acted to produce a positive torque to hold the contacts closed once they were made. In order to open the contacts of the relay the current coils had to produce sufficient torque to overcome this holding torque. In this way the relay was prevented from tripping the breaker when no current was flowing and the value of reverse current required to open the relay contacts could be adjusted by adjusting the amount of this holding torque. The holding coils, of course, had no effect on the operation of the relay except when the contacts were closed.

The holding coils of the three relays of this type required on a three-phase, four-wire network all had to be connected across one line voltage, as shown in Fig. 4A, since only one set of contacts per relay was used. This figure for simplicity shows a schematic diagram of only the holding and potential circuits of the relays. It may be noted that both relays No. 1 and No. 3 had fixed resistors in series with their holding coils and adjustable resistors in parallel with them. Practically all of the impedance of the holding circuits was in these fixed resistors. Changing the adjustable resistors to adjust the amount of holding torque had practically no effect on the amount of current that flowed in the circuit but merely changed the relative amounts of current in the two parallel paths. Relays No. 1 and No. 3 were identical in every respect except for the polarities of the holding circuits. The polarities of the two had to be different in order to secure the proper direction of torque in the two relays. This can readily be seen by referring to Fig. 4B, wherein it is shown that the three potential coil voltages with which the holding coils had to act to produce a torque were 120 deg. out of phase. If the holding circuit of relay No. 2 had been similar to that of either relay No. 1 or relay No. 3, the current in the holding circuit would have been either in position I_1 or position I_3, either of which would have been approximately 90 deg. out of phase with the potential coil voltage, and no torque would have been produced by the holding coil. From this description it can be seen that all three of the relays were different and had to be mounted in their proper positions on the network unit in order to function correctly.

The other differences between this single contact relay and the double contact relay can best be seen by referring to Fig. 5. The scheme was essentially the same as the one using the double contact relay except for the points already mentioned. There was, however, one other point of sufficient importance to mention here. The single contact relay had its two phasing coils connected in series and in series with the phasing lamp across one pole of the breaker instead of having one phasing coil and lamp across one of the contacts of the breaker as in the previous scheme. The impedance of the phasing circuit of this relay was therefore considerably more than that of the double contact relay. Since the distribution transformer is connected to the network through the phasing circuit when the breaker is open, the impedance of this circuit should be as large as possible so as to keep down the voltage induced in the primary of the transformer, when the feeder is deenergized to a very small value.

By using only the two outer poles of the unit shown in Fig. 5 and blocking the contacts of the No. 2 relay closed or removing the relay altogether and tying together the connections which go to its contacts, the unit can be used on a single-phase, three-wire network. Quite a large number of these units is now operating on such a network.

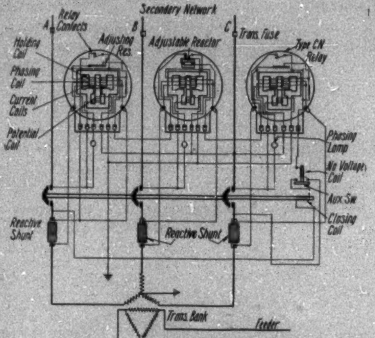
Secondary Network
A
B
C
Relay Contacts
Holding Coil
Adjusting Res.
Adjustable Reactor
Trans. Fuse
Type CN Relay
Phasing Coil
Current Coils
Potential Coil
Phasing Lamp
No Voltage Coil
Aux. Sw.
Closing Coil
Reactive Shunt
Reactive Shunt
Trans. Bank
Feeder

FIG. 5—DIAGRAM OF CONNECTIONS FOR THE FIRST THREE-PHASE, FOUR-WIRE NETWORK UNIT

Since this single contact relay was designed for operation on the system on which the double contact relays were installed, its operating characteristics were made similar to those shown in Fig. 2. Soon after these relays were installed a change was made in the network on which they were placed which decreased the impedance angle of the system. Since the characteristics of the relays were no longer correct for the system on which they were applied, trouble was encountered due to some of them pumping; that is, alternately opening and closing their contacts. This action can best be understood by referring again to Fig. 2. Suppose that for some reason the regulators on one feeder buck the voltage down below that of the other feeders, thus causing a leading current, such as I_3, to flow in the feeder. This would cause the network breaker to open. A voltage E_3 of the proper magnitude and phase position to cause the relays to close their contacts would then exist across the breaker and it would immediately reclose. This action would continue until the unit failed or until conditions on the circuit changed. The same action would result were the primary feeder breaker closed and the phasing voltage E_3 produced across the open network breaker by the feeder regulators and the load on the other feeders. This pumping condition was overcome by changing the operating characteristics of the relays. The opening curve was rotated approximately 30 deg. in the counter-clockwise direction which again made the angle between the opening and closing curves less than the impedance angle of the system. A large number of these relays are now in service and are giving almost perfect performance.

The number of relays required on a fair sized three-phase network is large compared to the number of substation relays required to protect the network. For instance, a six-feeder network having fifteen transformer banks per feeder would require 270 network relays as compared with 24 overcurrent relays in the substation, assuming that both line and ground relays are used. It can be seen from this that the maintenance of the network relays on a system presents a problem which cannot be ignored. It was largely because of this that it was decided to abandon the use of the single-phase, watthour meter case, change the holding circuit so as to make all relays for use either on single-phase, three-wire or three-phase, four-wire systems exactly alike, and make a number of more or less minor changes mostly of a mechanical nature. The relay was completely redesigned and everything possible was done to reduce the amount of maintenance required to an absolute minimum and to facilitate the necessary maintenance. The new network relay is mounted in a very shallow rectangular case with a glass cover. This makes all parts of the

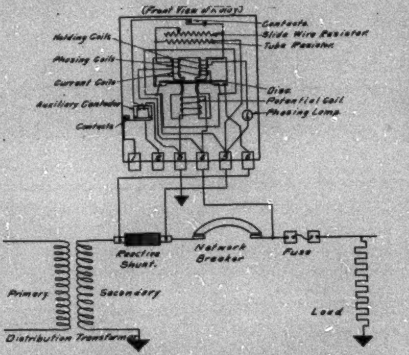
(Front View of Relay)
Contacts
Slide Wire Resistor
Tube Resistor
Holding Coils
Phasing Coils
Current Coils
Disc
Potential Coil
Auxiliary Contactor
Phasing Lamp
Contacts
1
2
3
4
5
6
Reactive Shunt.
Network Breaker
Fuse
Primary
Secondary
Load
Distribution Transformer

FIG. 6—GENERAL ARRANGEMENT OF THE INTERNAL AND EXTERNAL CONNECTIONS OF THE PRESENT NETWORK RELAY

relay easily accessible for inspection. There is much more room in this rectangular case although it does not require any more space for mounting on the panel of the network unit. This has made it possible to mount the phasing lamp, which is really a part of the relay, inside of the relay case where it is less likely to be broken. In all previous designs the phasing lamp or lamps were mounted as separate units on the panel of the breaker.

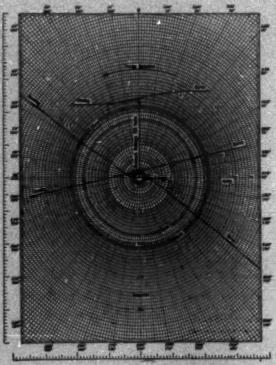

FIG. 7—CURVES SHOWING THE OPERATING CHARACTERISTICS OF A NETWORK RELAY FOR USE ON SYSTEMS WHERE THE EXCITING CURRENT OF THE TRANSFORMERS PREDOMINATES OVER THE CHARGING CURRENT OF THE FEEDER

In order to design the holding circuit so as to make the network relay a strictly single-phase unit it was necessary that the contacts of the relay be double-pole single-throw with the additional requirement that the contact which makes first must also break first. Since no reliable way could be found to accomplish this mechanically, the same result was obtained by connecting the operating coil of a small auxiliary contactor in series with the contacts of the relay, the holding coil, and the series resistor. The holding circuit of the relay is now connected from line to ground on the transformer side of the network breaker. The contacts of the auxiliary contactors which are independent of all other circuits in the relay are used to control the operation of the mechanism relay which in turn controls the operation of the breaker. It has been found possible to arrange the circuits in the relay so as to reduce the number of terminals from eight to six. The arrangement of the circuits in the relay and the way in which they are connected to the network breaker are shown in Fig. 6. This figure shows only one relay and one pole of the network breaker. The other relays, however, are connected to the other poles in exactly the same way. The contacts of the auxiliary contactors in all relays are connected in series with the mechanism relay coil across one of the line voltages. It is necessary that the relay contacts be connected in series both to insure opening the breaker in case of a fault and to prevent pumping because of unbalanced voltage conditions. It will be noted that it is necessary for only one relay to open its contacts to open the breaker but the contacts of all relays must be closed before the breaker will close.

The operating characteristics of the relay, which are very similar to those of the first single contact relay after it was modified, are shown in Fig. 7. These relays are operating satisfactorily on systems having 2750- and 4000-volt feeders and using single-phase induction feeder regulators. Relays having operating characteristics such as those shown in Fig. 7, however, when installed on systems having high-voltage primary feeders, such as 13,200 volts or higher, may not open when the feeder to which they are connected is deenergized at the station. This is because the relays must function to open the breaker on a reverse current which is the vector sum of the feeder charging current and the exciting current of the transformers connected to the feeder. The charging current may predominate and cause the resultant current to take some such position as I_1, Fig. 7, and since it does not cross the opening curve, the current coils of the relays produce a positive closing torque to hold the network breakers closed. On the first network system having 13,200-volt primary feeders the current on which the network relays had to open was almost a 90-deg. leading reversal, and in order to make them function properly it was necessary to change their operating characteristics by rotating both the opening and closing curves in the

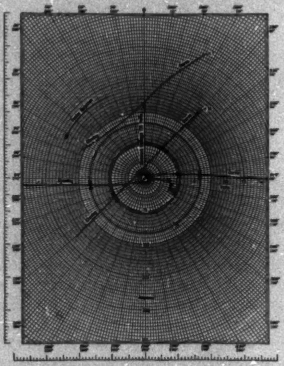

FIG. 8—CURVES SHOWING THE OPERATING CHARACTERISTICS OF A NETWORK RELAY FOR USE ON SYSTEMS WHERE THE CHARGING CURRENT OF THE FEEDER PREDOMINATES OVER THE EXCITING CURRENT OF THE TRANSFORMERS

counter-clockwise direction to the position shown in Fig. 8. Current I_1 on which they had to operate now crosses the opening curve into the zone marked "open" and the relays function as they should. In order to have one relay which could be used on systems having either high- or low-voltage feeders or both, the network relay was modified so that either the characteristics shown in Fig. 7 or those shown in Fig. 8 could be obtained merely by shifting the position of a link on a connector block in the relay.

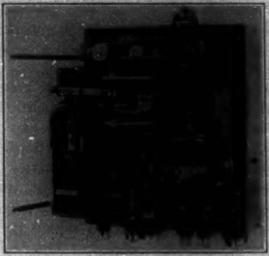

Fig. 9—The Present Automatic Network Relay

The present automatic network relay shown in Fig. 9 is almost identical with the first rectangular case relay previously described except for its operating characteristics. The operating characteristics of the relay are shown in Fig. 10. Two opening curves are provided, instead of using the one curve marked 10, so as to secure maximum sensitivity in the relay and thus insure its opening on small values of reverse current either leading or lagging. When the relay is installed on systems where the transformer magnetizing current predominates over the cable charging current to produce a lagging reversal, opening curve No. 6 should be used. This characteristic is obtained by placing the two-current tap screws in the No. 6 holes of the connector block in the relay. Should the charging current of the primary cable predominate to give a leading reversal, the No. 10 opening curve should be used. Since the angle between the closing and opening curve, regardless of which opening curve is being used, is small, the relay will operate on any system where the phasing voltage leads the network voltage. This is, of course, assuming that the impedance angle of the system is greater than the angle between the opening and closing curves of the relay which will be true in practically all cases. Since, on systems using polyphase regulators or two single-phase regulators connected open-delta, the phasing voltage may lag the network voltage, the relay may pump unless steps are taken to limit the angle of lag. The angle by which the phasing voltage may lag the network voltage in any given case without causing pumping will be determined by the impedance angle of the system. It will be noted that in the present relay the opening curve is rotated without affecting the closing curve. This is desirable as it allows the closing curve to be placed in such a position that the relay will not close its contacts if, when making repairs, the phases on the circuit to which it is connected have been accidentally crossed.

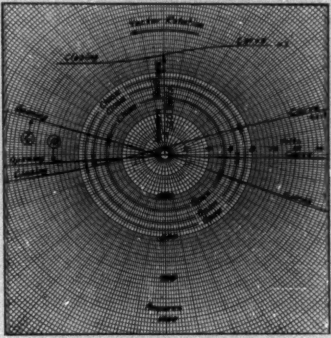

Fig. 10—Curves Showing the Operating Characteristics of the Present Network Relay Which is Applicable to Systems Having Either High- or Low-Voltage Primary Feeders.

Fig. 11—Relay for Controlling Network Unit for use on Two-Phase, Five-Wire or Single-Phase, Three-Wire Networks.

In addition to the adjustment for the opening characteristics, the relay is provided with an over-voltage closing and a reverse current opening adjustment. These adjustments, especially the over-voltage adjustment, should be set as high as can possibly be done and still secure satisfactory operation, because this will increase the stability of the operation of the relay.

The reverse current adjustment is primarily for the purpose of insuring that all relays function alike when opening on small values of reverse current.

A relay was recently developed for use on a two-phase, five-wire network. Two two-pole network breakers will be used with each bank of transformers, one being connected in each phase. The relay just discussed could have been used for this application, but since two relays would have been required to control each breaker and as space was an important item, it was decided to design a relay so that only one would be required to control each breaker. This relay, Fig. 11, commonly referred to as a two-phase re'ay, is really a single-phase device and is very similar in construction and operating characteristics to the present standard network relay. It differs from the standard, however, in that it has two phasing and two current circuits similar to the first double contact network relay. One phasing and one current circuit are connected across each pole of the network breaker.

Experience with the design and operation of network relays all points toward the desirability of having a relay which has operating characteristics that make it applicable to any type of network system. Enough work has been done along this line to show that it is possible to produce such a universal network relay having adjustable operating characteristics. Such a relay would have to be adjusted to fit the characteristics of the system on which it was to be installed. These adjustments, instead of simplifying the relay, will add somewhat to its complication. Because of this, the ideal network relay would be one having fixed characteristics of such a nature as to make it universally applicable to network systems. Such characteristics are theoretically possible and considerable work has already been done which indicates that such a relay is entirely practical.

The automatic network relay is now past the experimental stage, and there are at the present time approximately 1000 network relays, such as have been described in this paper, installed and operating satisfactorily on a number of network systems. From the above discussion it can be seen that the operating characteristics of the automatic network relay and the characteristics of the system on which it is to be installed are very closely related. This relation must be fully understood and appreciated both by the designers of low-voltage, a-c. network systems and the designers of network relays in order to secure the most satisfactory operation of the system as a whole.

Discussion

For discussion of this paper see p. 1220.

Operating Requirements of the Automatic Network Relay

BY W. R. BULLARD[1]

Member, A. I. E. E.

Synopsis.—*This paper describes, in an elementary manner, the various relations between distribution system characteristics and the design characteristics of the relays which are used to provide automatic control for the so-called automatic, low-voltage, a-c. network distribution system.*

In the first part of the paper the general principles of operation of the relays and the limits of magnitude of the various actuating forces are discussed; in the latter part, the relations between distribution system characteristics and the phase angles involved in the operation of the relays are described.

It is shown that relays having simple watthour-meter characteristics are not fully applicable to all network systems of the automatic type, but are generally applicable to those having certain limited characteristics. Relay characteristics which would have practically universal application are described and a brief discussion as to the future trend of development is included.

During the past two or three years several new types of distribution systems classed under the general designation "a-c., low-voltage networks" have been placed in service. Closely connected with the development of the most widely used one of these new types of systems has been the development of a new type of protective apparatus. An individual unit of this apparatus consists essentially of an electrically operated circuit breaker, controlled by one or more relays of a special type, called, for convenience, "network relays." Descriptions of this apparatus and of its application to a-c., low-voltage network distribution are available in the technical literature[2].

The special type of protective apparatus to which reference is made here is automatic in both the opening and the closing operations of the circuit breaker, and the network relay is called upon to supply the control impulses for both operations. The currents and voltages which are used to actuate the relay are related to each other in a somewhat complicated manner and the interrelations of these quantities are dependent upon various physical characteristics of individual a-c. network distribution systems. The design of the relay must therefore be fitted to the physical characteristics of the distribution system in which it is to operate, and, conversely, the design of the distribution system must take into account the limitations of design of the relay. The distribution engineer must be cognizant of the principles involved in the operation of the apparatus and must be thoroughly prepared to cooperate with the manufacturer in fitting together the characteristics of the distribution system and of the network relay, at least until such time as the design of one or both may have become reasonably standardized.

The purpose of this paper is to make available in elementary and condensed form, from the standpoint of the distribution engineer, information concerning the various relations between system characteristics and network relay design, which theoretical considerations and recent operating experience have shown to be important in the successful operation of the system as a whole.

Although the operating requirements of the relay are rather complex, its actual construction and operating adjustments may be quite simple. Details of the construction and operating characteristics of a particular design of network relay have been described in an article by J. S. Parsons[3]. Additional data from the standpoint of the designing engineer will be presented in two companion papers to this one. In this paper, therefore, reference to design details will be avoided so far as possible.

GENERAL PRINCIPLES OF NETWORK RELAY OPERATION

Referring to Fig. 1, the two principal functions of a network relay are to supply the opening impulse of the circuit breaker when conditions are such as to require opening, and to supply the closing impulse when conditions are correct for closing.

The condition which ordinarily requires opening of the breaker is that the flow of energy shall be in the direction from the low-voltage network toward the transformer. Such a condition may be caused by the opening of a feeder circuit breaker at the source of supply or by a short circuit in a transformer or high-voltage circuit. In the first case the reverse flow of energy is caused by the magnetizing losses of the distribution transformers being supplied from the network

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[1] Assistant Engineer, Electric Bond and Share Co., 71 Broadway, New York City.

[2] "The Automatic Alternating-Current Network Protector," G. G. Grissinger, *Electric Journal*, August, 1923.

Underground A-C. Network Distribution for Central Station Systems, A. H. Kehoe, A. I. E. E. Journal, June, 1924.

Study of Underground Distribution Systems for the City of New Orleans, W. R. Bullard, A. I. E. E. Journal, Nov., 1924.

Several Articles on A-C. Network Distribution Systems and Apparatus in the *Electric Journal* for July, 1925.

Presented at the New York Regional Meeting of the A. I. E. E., New York, N. Y., Nov. 11-12, 1926.

[3] "The Automatic Network Relay," J. S. Parsons, *Electric Journal*, July, 1925.

instead of directly from the source of supply as under normal conditions.

The condition which ordinarily requires the closing of the breaker is that the relation between the voltage of the network and that of the transformer banks which are to be connected shall be such as to cause a flow of energy from the transformer toward the network after the closing operation is completed. This flow of energy must not be accompanied, of course, by undesirable cross currents such as would be the case if the phases of the circuit to be connected were twisted (by incorrect connections of the supply conductors) with respect to those of the network.

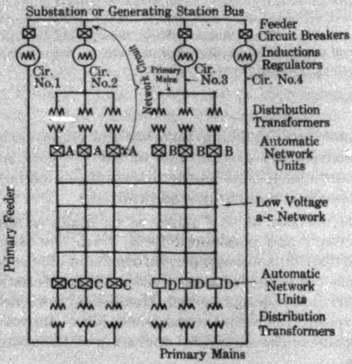
Substation or Generating Station Bus
Feeder Circuit Breakers
Inductions Regulators
Cir. No.1
Cir. No.2
Network Circuit
Primary Mains
Cir. No.3
Cir. No.4
Distribution Transformers
A
A
A
B
B
B
Automatic Network Units
Low Voltage a-c Network
Primary Feeder
C
C
C
D
D
D
Automatic Network Units
Distribution Transformers
Primary Mains

Fig. 1—Diagram Showing the Principal Elements of an Automatic, Low-Voltage, A-C., Network Distribution System

In order to meet the above conditions, the actuating forces of the relay are ordinarily obtained from three local circuits (see Fig. 2); namely, the potential circuit which is connected in shunt to the line conductors on the network side of the circuit breaker and therefore represents network voltage; the current circuit, which obtains current from the line conductors when the circuit breaker is closed and therefore represents line current; and the phasing circuit which is connected from one to the other contact of a single pole of the circuit breaker and therefore represents the voltage difference between network and transformer. The opening operation is controlled by the interaction of the potential and current circuits, while the closing operation is dependent upon the interaction of the potential and phasing circuits. In some designs of network relay, the phasing and current circuits are interconnected so as to utilize a single winding in the relay for the two circuits.

Under one condition,—namely, when the network is dead and is to be made alive by the energizing of one of the primary circuits,—one of the actuating forces, (voltage for the potential circuit) is missing and an independent mechanical force must be supplied to insure the relay contacts being in a position to cause the closing of the circuit breaker. This is usually accomplished by the use of a spring which holds the contacts in the closing position when the electrical circuits of the relay are entirely dead. Since, under normal conditions, a delicate balance must be maintained between opening and closing torques, the force of the spring is neutralized by the action of a local shading circuit when the network is made alive.

Both magnitude and phase relations of the actuating currents and voltages are involved in the operation of the relay. For simplicity, the magnitudes of the forces involved will be discussed separately from the subject of phase relations.

Magnitude of Actuating Forces

In the opening operation there are two extreme limits of current magnitude which may exist in the main line conductors. The lower limit, fixed by the magnetizing current of the transformers together with the charging current of the high-voltage conductors, is involved when it is desired to disconnect a circuit by opening the primary switch at the source of supply. With primary voltages on the order of 2300 or 4000 volts, the reactive component of the transformer magnetizing current, under usual system characteristics, will predominate over that of the line charging current. With primary voltages on the order of 13,200 and above, the reactive component of line charging current will sometimes predominate over that of transformer magnetizing current. Obviously, therefore, there will sometimes exist conditions in which the reactive components of these two currents will neutralize each other, leaving only the energy components of both currents. To make the operation absolutely certain,

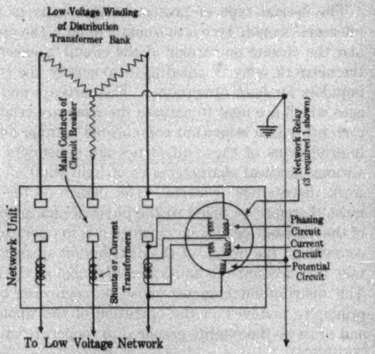
Low Voltage Winding of Distribution Transformer Bank
Main Contacts of Circuit Breaker
Network Relay (3 required 1 shown)
Network Unit
Shunts or Current Transformers
Phasing Circuit
Current Circuit
Potential Circuit
To Low Voltage Network

Fig. 2—Simplified Wiring Diagram of Network Relay Connections for Three-Phase, Four-Wire System. (Three Relays Required—Only One Shown)

therefore, the torque produced by the energy components of transformer magnetizing current and line charging current in combination with network potential should be sufficient to actuate the relay. The value of the sum of the two former components is usually less than one per cent of normal full load current.

On the other hand, the short-circuit current produced by a failure of transformer or primary line insulation must produce the same action in the relay without damaging it mechanically or electrically. Primary line short-circuit currents are limited by the short-circuit impedance of the distribution transformers, but transformer short circuits may involve very high values of current, the worst case being a dead short circuit at the low-voltage terminals of the transformer or at the conductors leading from the transformer to the network protective unit. In this case a large proportion of the full amount of current which the network is capable of supplying from the circuits which are not in trouble may flow through the line conductors of a network unit. The value of this current may be 20 or 30 times normal full-load current, or even more in certain cases. The network relay, therefore, together with its line current transforming device (line shunt or current transformer), is called upon to function properly throughout an enormous range of current in the opening operation. The duty on the relay itself is ordinarily reduced by magnetic saturating characteristics of the line shunt or current transformer.

In the closing operation the relay obtains its torque from the voltage difference between the network and the transformer, in combination with network potential. This voltage difference may be produced by different positions of the primary feeder regulators, if such exist; or may be due to the voltage drop from the source of supply to the network due to the load on the latter; or may be due to a combination of both causes. In the second case the voltage difference is limited by the load and the system impedance, and the value at which the relay must function is therefore dependent upon the system characteristics and upon the percentage of load at which it is desired to connect a feeder to the system. In case feeder regulators are used, any value of voltage difference may be obtained up to the limit imposed by the range of regulation. In any case, however, the voltage difference necessary to operate the relays should be small in order to avoid cross currents and fluctuations of the network voltage immediately after the closing operation. A fair minimum voltage difference necessary to operate the relay, for average systems, is probably one per cent of normal network voltage in phase with the latter.

When the primary circuit is dead and the network alive, practically full line voltage in the opening direction is impressed across the phasing circuit of the relay; or if the primary circuit should be made alive, with its phases twisted with respect to those of the network, more than full line voltage may be impressed across the relay phasing circuit. Thus, in connection with the control of the closing operation, the relay and its phasing circuit must be designed to operate properly under an enormous range of voltage variation. The duty on the relay is ordinarily reduced by the insertion of a small tungsten lamp in the phasing circuit. This lamp limits the range of current in the phasing circuit by virtue of the rapidly increasing resistance of the filament with increasing temperature.

Under normal conditions it is not permissible, either in the closing or the opening operation, for the relay to operate at zero values of voltage in the phasing circuit or current in the current circuit, since this is on the borderline of incorrect operation and provides no factor of safety against "pumping," *i. e.*, opening and closing periodically for an indefinite length of time. There must therefore be a certain amount of torque difference, produced either electrically or mechanically, between the opening and the closing positions of the relay disk. With a double contact relay, one contact for opening and one for closing, this effect can be obtained by the difference in spring tension between the two positions where contact is made. With a single contact relay, an auxiliary circuit is ordinarily used to provide a slight change in torque when the contact is made or broken. This torque difference must be definite and the adjustments should be such that some torque in the opening direction produced by the current circuit is necessary to cause opening action and some torque in the closing direction, produced by the phasing circuit, is necessary to cause closing action.

As mentioned above, since the relay contacts must be in the closing position when the network is dead, a spring is ordinarily used to provide the necessary torque. When the network is alive the force of this spring is neutralized by a local shading circuit arrangement which acts in magnetic combination with the potential circuit. The adjustments are of course ordinarily made to just balance the force of the spring at normal voltage on the network. Since network voltage is not necessarily exactly normal at all times, it is obviously necessary that the value of the torque produced by the shading circuit be such that the operating characteristics of the relay are not materially changed by ordinary variations of network voltage; *i. e.*, it is necessary to insure that enough closing torque at low network voltage will not be produced to cause incorrect closing and that enough opening torque will not be produced by high network voltage to cause incorrect opening. Nevertheless the strength of the spring must be such as to cause positive closing with a dead network. This is a matter of properly proportioning the forces in the design of the relay. In order to provide the proper factor of safety under normal operation, however, a range of probably 20 per cent voltage variation above or below normal (considering average systems) should not cause the relay to operate with the phasing and current circuits de-energized.

PHASE RELATIONS

Relations Between Phasing Voltage and Line Current. In the greater part of the following discussion only one phase of a polyphase system will be considered, since with the use of single-phase relays, the action with respect to each of the phases of the system is, in general, the same as the action would be in an isolated single-phase system.

Consider the conditions shown in Fig. 1 which is a diagrammatical representation of a network system having four primary circuits. Circuits Nos. 1, 2, and 3 are in service and carrying load, both high-voltage circuit breakers and low-voltage network circuit breakers being closed as indicated by the *X*'s in the circuit breaker symbols. Circuit No. 4 has been made alive at the supply station by the closing of the primary circuit breaker, but the network circuit breakers, *D D*, have not yet closed. Assume for the present that the impedance of high-voltage and low-voltage cables is negligible in comparison with the impedance of distribution transformers. This assumption is not strictly justified except in cases of high reactance transformers of reasonably close spacing, connected by low-voltage mains of large cross-section. The error introduced by this assumption, however, can be compensated for by adjustments of the relay, and this will be discussed later.

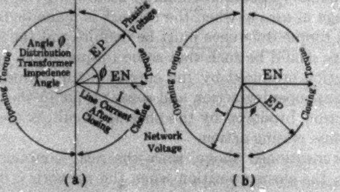

FIG. 3—POLAR DIAGRAM OF PHASE RELATIONS IN A NETWORK RELAY HAVING SIMPLE WATTHOUR-METER CHARACTERISTICS

Under the above conditions a voltage difference will exist between the two sides of the network units, *D D*. This voltage, which, for convenience will be called the phasing voltage, controls the closing operation of the network units and may be produced either by the voltage drop through the transformers of circuits Nos. 1, 2, and 3, or by a difference of settings of regulators on circuits Nos. 1, 2, and 3 as compared with circuit No. 4, or by both voltage drop and regulator settings. Now if this phasing voltage is of such magnitude and phase relation as to cause sufficient closing torque in the network relays, network units *D D* will close. After the closing operation, a current will be set up in circuit No. 4 due to the phasing voltage which existed before the closing. This current will take the form of a circulating current between circuit No. 4 on the one hand and circuits Nos. 1, 2, and 3 in parallel on the other hand, a complete circuit being formed by this combination from the supply bus through circuit No. 4 to the network and back to the supply bus through circuits Nos. 1, 2, and 3 in parallel. The magnitude and phase relations of this current, aside from a possible slight shift in the phase position of network voltage which is ordinarily used as the base of reference, will be determined exactly by the phasing voltage before closing divided by the total impedance around the above mentioned circuit. This relation holds regardless of just what the phasing voltage may be or how it is produced. If it is produced purely by the load current, the circulating current will be subtractive with respect to the load currents existing in circuits Nos. 1, 2, and 3—*i. e.*, will reduce the load currents in these circuits.

If the transformers all have the same impedance phase angle and if, as assumed above, the line conductor impedance is negligible in comparison with the transformer impedance, the line current which flows in each network unit after closing will lag behind the phasing voltage which existed before closing by an angle almost exactly equal to the impedance angle of the transformers

Next, consider Fig. 3 (a) which represents phase-angle conditions for a simple watthour-meter type of relay with controlling currents and voltages impressed either directly upon the relay circuits or through transforming devices which do not change the phase angles. For simplicity, in this diagram relay adjustments for a definite amount of voltage required for closing and a definite amount of current required for opening have been neglected. The network voltage *E N* is the base line of reference. The semicircle to the right of the vertical line represents the range of phase angles of voltage impressed on the phasing circuit of current flowing in the current circuit, throughout which range closing torque will be produced in the relay. The semicircle to the left of the vertical line represents a similar range of phase angles throughout which opening torque will be produced. If voltage drop produced by load current alone is responsible for the voltage impressed upon the phasing circuit before closing, the phase angle of this voltage will ordinarily be somewhat as represented by the vector *E P* in Fig. 3 (a), and if we neglect line conductor impedance, the current after closing will have a position as indicated by the vector *I*, lagging behind the voltage *E P* by an angle equal to the impedance phase angle of the distribution transformers. Both of these vectors give closing torque with the result that the circuit breaker will close and will stay closed, causing the transformer bank to which it is connected to take up its proper share of the system load.

The voltage difference before closing, however, may not be produced entirely by load current. If, for instance, it should be determined mainly by regulator positions, there are possible conditions under which it might take the position indicated by the vector *E P* in Fig. 3 (b). In this case the current after closing would take the position shown by the vector *I*, lagging behind the voltage *E P* by the same angle as before. But in this case the current would fall in the semicircle of opening torque and the result would be that the circuit breaker would close and immediately open again, repeating this operation indefinitely as long as the system conditions remained unchanged. This action, which is called "pumping," would be undesirable not only due to the fact that it might cause objectionable fluctuations of network voltage but also because of the wear and tear on the apparatus itself. In fact, in some designs of network apparatus, this action would cause the burning out of the closing coil of the circuit breaker.

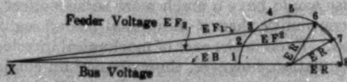

FIG. 4—VECTOR DIAGRAM SHOWING POSSIBLE PHASE RELATIONS OF VOLTAGE DIFFERENCE BETWEEN PRIMARY FEEDERS PRODUCED BY DIFFERENT POSITIONS OF INDIVIDUAL POLYPHASE INDUCTION FEEDER

Phase Position of Phasing Voltage. Fig. 4 represents phase conditions produced by polyphase regulators in individual network feeders. The series regulator voltage is represented by the vector *E R* which revolves with *O* as an axis and which may take any position along the semicircle as indicated, for instance, by the points 1, 2, 3, 4, 5, 6, 7, and 8. The feeder voltage represented by *E F* may take the positions *X*-1, *X*-2, *X*-3, etc., depending upon the position of the series regulator voltage *E R*. The network relay phasing voltage, neglecting voltage drop produced by load current, is the difference between the feeder voltage of the feeders which are in service and that of the feeder that is being connected to the network. This voltage difference may therefore take the positions 1-2, 2-3, 3-4, 4-5, 5-6, etc., depending upon the settings of the regulators in different feeders. In systems having individual feeder regulators of the polyphase type, not mechanically interlocked, therefore, it is possible under conditions of zero load for the phasing voltage to take almost any phase position in the 360-deg. circle of Fig. 3. The position 6-7, for example, is approximately the same as the position of the phasing voltage in Fig. 3 (b), which position would cause pumping in the case of relays with simple watthour-meter characteristics.

The action of single-phase regulators is different from that of polyphase regulators since in the former the phase position of the series voltage is fixed and regulation is accomplished by changes in its magnitude. When each phase of an individual feeder is controlled by a single-phase regulator, therefore, the network relay phasing voltage as determined by regulator positions has a fixed phase relation with respect to the bus voltage. In the case of systems using two single-phase regulators connected open delta, however, the voltage of one of the three phases of a feeder may shift in position if the two regulators do not at all times maintain the same respective amounts of bucking or boosting voltage. Referring to Fig. 5 (a), the triangle *A B C* represents the station bus voltage of a three-phase system. The vectors *E R* represent series regulator voltage. These vectors maintain the same phase positions but may vary in magnitude. The feeder voltage of the phase *B C* may take any position with a point in the line *X Y* as one terminus and a point in the line *M N* as the other terminus. These conditions are represented in a somewhat different manner in Fig. 5 (b). The station bus voltage is represented by the line *O X* and the feeder voltage may take any position from point *O* to any point within the area bounded by the parallelogram. As in the case of polyphase regulators, this system of regulation may, under conditions of zero load, produce a relay phasing voltage of any phase position. As an example, the two positions of feeder voltage shown in Fig. 5 (a) will produce a phasing voltage in approximately the same position as that indicated in Fig. 3 (b). Figs. 1 and 2 represent the position of this phasing voltage in both (a) and (b) of Fig. 5.

Since some load current is nearly always present in a network system, the regulators will hardly ever be the sole determining factor of the phasing voltage. If the voltage difference produced by the regulators, however, is greater in magnitude than that produced by load current, and if the former voltage difference can take any phase position in the 360-deg. range, the vector sum of the two voltage differences can also take any position within the complete circle. During heavy load periods it is unlikely that the voltage difference produced by regulators will be greater than that produced by load

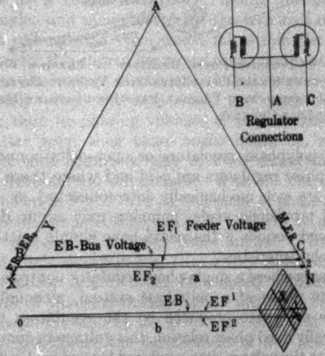

FIG. 5—VECTOR DIAGRAM SHOWING POSSIBLE PHASE RELATIONS OF VOLTAGE DIFFERENCE BETWEEN PRIMARY FEEDERS PRODUCED BY DIFFERENT POSITIONS OF INDUCTION FEEDER REGULATORS CONNECTED OPEN DELTA

current, unless, due to some failure of the control equipment, the regulators should assume considerably different positions. During very light load periods, however, the regulator voltage difference may possibly predominate. Assuming, for instance, that the control equipment of each regulator is set to hold the voltage within a range of two per cent above or below normal, there might easily occur a condition in which the voltage of all but one feeder would be nearly two per cent low and that of the remaining feeder would be nearly two per cent high. This voltage difference of nearly four per cent would be practically equivalent to that produced by between 20 per cent and 40 per cent normal full load in a system having 10 per cent reactance transformers and cables of comparatively small impedance. It sometimes may be difficult to constantly maintain the settings of automatic regulator control equipment within as close limits as two per cent, high or low, and any increase in this value due to wear and tear on the control contacts or other causes will of course cause a proportionate increase in the voltage difference which may be produced by the regulators. Thus in cases where polyphase regulators or open-delta-connected, single-phase regulators are used and where these regulators are not mechanically interconnected or other special provisions made, pumping may occur during light load periods if the relays have simple watthour-meter characteristics.

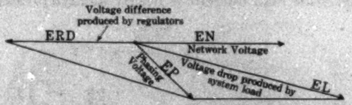
Voltage difference produced by regulators
ERD
EN
Network Voltage
Phasing Voltage
EP
Voltage drop produced by system load
EL

Fig. 6—Possible Phase Relation of Phasing Voltage Produced by Single-Phase Regulator Voltage Difference in Combination with Voltage Drop Produced by Heavily Lagging Load Current

In cases where a single-phase regulator controls each separate phase of a polyphase system, although the voltage difference produced by the regulators has a practically fixed phase relation, this voltage in combination with the voltage drop produced by system load may produce a phasing voltage of varying phase positions. Fig. 6 illustrates a somewhat theoretical case which would cause pumping in a relay of simple watthour-meter characteristics. The voltage drop $E\ L$ is that produced by a load of extremely low power factor lagging. The voltage difference $E\ R\ D$ is produced by a lower regulator setting of the feeder which is being connected than that of the feeders which are in service. The resultant of these two voltages is a phasing voltage $E\ P$ in approximately the same position as indicated in Fig. 3 (b). This is a possible condition although it is not likely to occur in practise. A load current which would be sufficiently lagging to produce the conditions illustrated would in all probability be a local concentrated load and not the general load of the system. Its effect would, therefore, be confined to a very few relays.

So far as regulators are concerned, the phase position of the phasing voltage for any system of regulation can be held in a fixed position merely by mechanically interlocking the rotors of regulators in the same phase of different feeders, or, in the case of open-delta connection, by interlocking the rotors of the two regulators of each feeder. Bus or group regulation might also be used, but some engineers do not consider this permissible because of the question of reliability of service which is involved. A regulator short circuit in this latter case would have a much more serious effect upon service continuity than if individual feeder regulators were used. Even in the case of mechanical interlocking or bus regulation, however, there are certain conditions which may produce a phasing voltage of such phase relations as to cause pumping in relays of simple characteristics. Consider a system having primary voltage sufficiently high to produce a cable charging current correspondingly higher than the transformer magnetizing current. In this case when a station circuit breaker is opened in order to disconnect a network circuit from the system a reverse line current of leading phase position will flow. Thus current will have a phase position somewhat as shown by the vector I in Fig. 3 (b). Now suppose that one or more of the network units is faster in opening than the others. As soon as these units open, their relays will obtain a phasing voltage in the position $E\ P$ of Fig. 3 (b), causing closing torque, and the units will tend to close again. In the meantime the other units may have reached the point of opening, but if they open after the first mentioned units have closed, they will then obtain closing torque in the same manner as the first units, and there may be set up an action similar to that of a see-saw, depending upon the time characteristics of the different units. Conditions of pumping may also be produced in a system of this kind in case one or more of the units should, due to some defect, entirely fail to open when the station breaker is opened.

Relay Phase-Angle Characteristics. In view of the above considerations, it appears that network relays of simple watthour-meter phase-angle characteristics impose certain limitations of design upon the remaining portions of the network system if pumping is to be positively avoided. Before this point is discussed further, however, it may be well to consider briefly what the characteristics of a network relay should be in order to make it universally applicable to network systems of varying characteristics.

Consider first the opening operation. Since the unit may be required to open in case of a small reverse line current either leading or lagging, depending upon whether cable charging current or transformer magnetizing current predominates, and since it may be required to open at very large values of reverse short-circuit current lagging, it is desirable that opening torque be produced by all phase angles of line current within a semicircle extending 90 deg. on each side of the reverse position of network voltage. See Fig. 7 (a).

Next consider the closing operation. In view of the above described relation between phasing voltage and line current, it is desirable that closing torque be produced by all phase angles of phasing voltage within a semicircle generally opposite to the opening sector but shifted in a leading direction with respect to the vertical line which defines the latter by an angle ϕ equal to the impedance angle of the distribution transformer banks. If an angle greater than this is used, troubles from pumping may be encountered due to too great an overlapping of the characteristics in the upper part of the diagram. See Fig. 7a.

In the discussion of the relation between phasing voltage and line current, it was assumed that line conductor impedance is negligible in comparison with transformer impedance. This is only approximately correct for high load density underground systems in which the transformer installations have high reactance,—say 10 per cent. It is not even approximately correct in many systems having transformers of standard impedance. In the latter systems, therefore, unless the line impedance angle is equal to the transformer impedance angle, the phase angle ϕ between phasing voltage and line current is not a fixed quantity but will vary with varying system load, different numbers of feeders in service, etc. In addition to the other advantages of high impedance in the transformer installations, therefore, there is the possible advantage of increased stability of operation of network relays due to the approximately fixed relation between phasing voltage and line current.

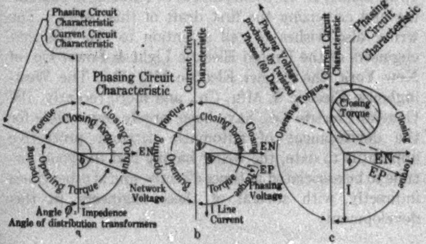
Phasing Circuit Characteristic
Current Circuit Characteristic
Phasing Circuit Characteristic
Torque
Closing Torque
Closing Torque
Opening
Opening Torque
EN
Network Voltage
Angle ϕ = Impedence Angle of distribution transformers
a
Current Circuit Characteristic
Torque
Closing Torque
Closing
Opening
Opening
Torque
Torque
EN
EP
Phasing Voltage
I Line Current
b
Phasing Voltage produced by twisted Phases (60 Deg.)
Opening Torque
Current Circuit Characteristic
Phasing Circuit Characteristic
Closing Torque
Closing
Torque
EN
EP
I
c

Fig. 7—Vector Diagram Illustrating Desirable Phase-Angle Characteristics of Network Relays

In a relay of this kind, reasonably small variations in the system impedance angle and also small differences between the relay phase angle and the basic phase angle of the system are rendered harmless by shifting both the straight line representing closing torque and that representing opening torque away from the zero point of the polar diagram as shown in Fig. 7 (b). This shift corresponds to the relay settings for a definite amount of phasing voltage necessary to close, and a definite amount of line current necessary to open, the diagram, Fig. 7 (a) being for settings representing zero values of these quantities.

So far the requirements for locking the unit open in case of twisted phase relations of line conductors have not been considered. The characteristics shown in Fig. 7 (b) do not entirely conform to the requirements for preventing a closure under twisted phase conditions. For example, if the transformer which is to be connected to the network should have its connections twisted so as to obtain a voltage leading the network voltage by 60 deg., the resultant phasing voltage would fall within the closing sector and if the other relays of the network unit should obtain torque in the same direction the result would be the equivalent of a system short circuit. This possibility would be very remote especially in systems where it is the usual practise to phase out the line conductors before making a connection. Trouble from this source could be entirely prevented, however, by the use of relays having characteristics similar to those shown in Fig. 7 (c). The closing characteristic, instead of being represented by a straight line, is bounded by a curved line which bends back on itself and forms a complete circuit. Phasing voltages of such angles and magnitudes that the terminals of the vectors representing them would fall within the area bounded by this curved line would cause closing action. Voltages the vectors of which would fall outside of, or completely cross, this area would not cause closing action; that is, phasing voltages of large magnitude, no matter what their phase angles might be, would lock the unit open. A relay of these characteristics would have several advantages: With the characteristics shown in Fig. 7 (b) a certain range of phase-angle adjustment would be necessary to conform to distribution systems of different phase angles. With the characteristic shown in Fig. 7 (c) a very wide range of system phase angles could be allowed without the necessity of changing the relay phase angle.

There would also be the advantage that if at some time in the future the development of a-c. networks should require the feeding of a system from separate generating sources and if it should be found practicable to synchronize the generating sources through the network, the synchronizing might be accomplished automatically by relays of this type, assuming that the proper time characteristics could be developed. A design which would produce the characteristics shown in Fig. 7 (c) is theoretically possible with relatively simple construction and somewhat similar characteristics have been produced experimentally.

FUTURE DEVELOPMENT

In the preceding discussion, as well as that which follows, it is of course understood that by "relay characteristics" is meant the combined characteristics of the relay and its associated circuits and transforming devices, since the control of some of the characteristics can be accomplished by altering the constants of the circuits external to the relay.

Relays of the ideal characteristics described above have not yet been given the test of actual service. The design of such a relay, while it need not be unduly complicated, would, of course, partake more of a special nature than that of a simple watthour-meter type of relay or than that of the modified watthour-meter types which are now in commercial production. It is apparent from the above discussion that if relays of simple characteristics are used, certain limitations are placed upon the design of the remainder of the distribution system and provisions must be made in the latter to insure successful operation. For instance, a system having mechanically interlocked regulators, or possibly having single-phase regulators in each phase without mechanical interlocking, and having a primary voltage sufficiently low to insure that the transformer magnetizing current would predominate over the primary cable charging current, could ordinarily be expected to operate without pumping, with relays of simple watthour-meter characteristics. The provision concerning primary voltage might not be considered necessary by some engineers, particularly if the relays were designed so as to require considerably less time for the opening operation than that required for the closing operation. This design characteristic would tend to prevent the see-saw action described above, but would not prevent pumping in case of the failure of one or more units to open when the station breaker is opened. The latter action might not be serious if precautions were taken in the operation of the system to insure that the station breaker would not be left open for any appreciable length of time in case of a "feed back." There are also, of course, other expedients which could be used to minimize the effects of high cable charging current, such, for instance, as the use of reactive loading coils to add lagging current to the primary feeders.

The simple type of relay has the advantage of a probable lower cost, due to the fact that most of the parts used in its construction may be standard for other similar apparatus. It also has the advantage of less complications in the electrical circuits. Neither of these advantages should be given undue importance since the design of the ideal type of relay need not necessarily be excessively complicated. The simple type of relay has the disadvantage that special provisions possibly involving additional cost and operating disadvantages may be required in the design of the remaining portions of the distribution system.

In the future development of this apparatus, therefore, there is a choice to be made between simplicity of relay characteristics involving possible additional cost and operating disadvantages in the remainder of the system, and comparative complexity of relay characteristics involving possible additional cost and complications as regards the relays and associated devices, a compromise solution being the use of simple relays in combination with relays of twisted phase angles or other special characteristics.

It seems reasonable to suppose that the ultimate trend of development will be toward standardization upon either a simple type of relay or upon one of ideal characteristics of universal application. What the trend of this development will be is not apparent from the present state of the art. It will be determined to a large extent by future operating experience, with existing types of relays and network systems, and upon the future development of the latter, such, for instance, as the possible interconnection of separate generating sources through network distribution systems. In the meantime, close cooperation between the relay designing engineer and the distribution engineer is necessary in order to insure that the characteristics of the present types of relays will fit those of the distribution system and vice versa.

In conclusion, the writer gratefully acknowledges the assistance of Messrs. A. H. Kehoe, C. W. Franklin, G. R. Milne, D. K. Blake, and J. S. Parsons, in examining and criticizing the first draft of this paper. The writer also wishes to call attention to the fact that engineers of the United Electric Light & Power Co. of New York, the Palmer Electric Mfg. Co., The Westinghouse Electric & Mfg. Co., and more recently, the General Electric Co., have been largely responsible for the development of automatic network protective apparatus to date, the writer having had the good fortune to be associated at times directly, and at other times indirectly, with several of these engineers in this development.

CONCLUSIONS

1. Due to the combination of a number of different control requirements in one piece of apparatus, the interrelations between the various actuating forces of automatic network relays are somewhat complicated, although the construction and adjustments of the relays themselves may be quite simple.

2. Important factors in the operating requirements as to relay characteristics are the impedance phase angle of the distribution transformers; the phase angle and magnitude of the impedance of primary and secondary line conductors; the relation between the magnitude of line charging current and that of transformer magnetizing current; and the particular type of voltage regulation which is used for the primary feeders.

3. Relays of present commercial design are not universally applicable to all the different physical characteristics of network systems which might be used in practise. There must be close cooperation, therefore, between the distribution engineer and the relay designing engineer in order to insure the correct functioning of the individual distribution system as a whole.

4. A network relay of such characteristics as to be universally applicable to all distribution system characteristics which might ordinarily be encountered would involve somewhat greater complications in design and possibly somewhat greater cost than a relay of simple watthour-meter characteristics. In the future development of network relays, therefore, there is a choice to be made between simple relay characteristics with the corresponding limitations on distribution system design, on the one hand, and universally applicable relay characteristics with the corresponding complications in relay design, on the other hand, a compromise solution being the use of simple relays in combination with relays of special characteristics. The future trend of development will probably be toward one or the other of the first two mentioned solutions. This trend will be determined, to a large extent, by future operating experience with existing types of relays and by the trend of development of network distribution as a whole.

Discussion

For discussion of this paper see p. 1220.

A-C. Network Relay Characteristics

BY D. K. BLAKE[1]

Associate, A. I. E. E.

***Synopsis.**—This paper discusses the required characteristics of relays for use with a low-voltage automatic network circuit breaker. The relay performs the two functions of tripping and reclosing the circuit breaker. The characteristic required for each function is discussed.*

The major part of the paper is given to a discussion of the reclosing function because of the danger of "pumping" caused by cable charging currents and various combinations of induction regulators and transformer connections.

Emphasis is placed on the suitability of relays consisting of watthour-meter characteristics, construction and adjustments, involving only a different current coil and the replacement of the recording mechanism by standard relay contacts.

* * * * *

INTRODUCTION

ABOUT eleven large central station companies have recently installed, or are now installing, low-voltage a-c. networks in areas of high load-densities. With the exception of three companies, all of the networks utilize a low-voltage automatic circuit breaker connected in the transformer secondaries. The major elements of these systems are shown schematically in Fig. 1. It is the purpose of this paper to discuss the functions and required characteristics of the relays for controlling the operation of the breakers.

FUNCTIONS OF THE NETWORK RELAY

Broadly, the relay performs the two functions of tripping on reversals of energy and reclosing when the incoming circuit is energized.

Tripping function. The relay must trip the breaker on short-circuit currents in the reverse direction. It must also trip on the transformer exciting current in the reverse direction in order to permit the disconnecting of a feeder and its transformers from the network by opening the feeder breaker at the substation. The characteristic of the relay must be such that the addition of any amount of cable charging current to the exciting current will not prevent tripping in the reverse direction.

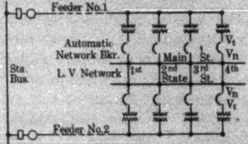

FIG. 1—TYPICAL LOW-VOLTAGE, A-C. NETWORK UTILIZING AUTOMATIC, A-C. NETWORK CIRCUIT BREAKERS

The vector diagram in Fig. 2 indicates a suitable characteristic for the tripping function. The short-circuit current will probably lag at a maximum angle when a short circuit occurs at the high-voltage terminals of the transformer. This angle is 84.3 deg. with a transformer having a one per cent resistance and 10 per cent reactance drop. If the parallel lines $T\ T$ are perpendicular or slant in the leading direction as shown, it is obvious that if the relay will operate on the transformer exciting current it will also operate on any amount of charging current which may be added. The watthour-meter has a similar characteristic.

The tripping function then requires no new untried devices since it is possible to utilize watthour-meter parts and construction to form a simple induction type reverse power relay. The connection shown in Fig. 3 is suitable, being similar to the connection for measuring power. The current transformer and current coil of the relay are designed together to operate over the entire range of reverse current. This range may be from 1.5 amperes to 15,000 amperes. The current transformer has a relatively small number of turns to give sufficient secondary current for relay operation on exciting current and a small amount of iron to limit, by saturation, the secondary current to a safe value for the relay current coil. The time-current characteristics should be such as to require, under short-circuit conditions, not longer than ½ sec. A much shorter time, such as ¼ sec., is preferable.

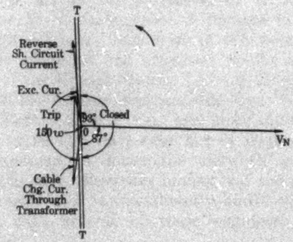
T
Reverse Sh. Circuit Current
Exc. Cur.
Trip
150 w
93°
Closed
0
87°
V_N
Cable Chg. Cur. Through Transformer
T

FIG. 2—VECTOR DIAGRAM OF TRIPPING CHARACTERISTICS

Reclosing function. The network breaker should reclose only when the incoming transformer voltage is of the proper value and phase relation to cause the transformer to take its share of the network load. This requirement does not permit connecting the

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1. Central Station Engineering Dept., General Electric Co., Schenectady, N. Y.

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closing circuit direct to the transformer secondary because after opening the circuit breaker at the substation it would be possible for some network breakers of the particular feeder to open and reclose again before all the breakers opened. The result would probably be a "see-saw" operation of opening and closing. Induction regulators may also cause low voltage on a feeder by being in the buck position when the feeder is energized. Other factors will be discussed later that may cause improper voltage relations. It is obvious, therefore, that the relay should measure the voltage across the open breaker contacts and complete the closing circuit only when the voltage difference is of a certain value in the proper direction.

The voltage difference may be measured by a separate relay, by an extra winding on the reverse power relay, or by the current coil of the reverse power relay. This paper will treat of the latter arrangement (Fig. 4) and the use of a separate phasing relay where the connection shown in Fig. 4 would not be suitable.[2]

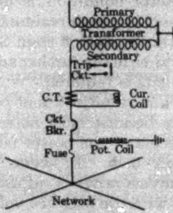

Fig. 3—Relay Connection Diagram For Tripping Function

The circuit of Fig. 4 is similar to Fig. 3 with the addition of an extra relay contact for the closing circuit and the connection of the current coil across the circuit breaker contacts. When the circuit breaker is closed the relay acts as an ordinary reverse power relay and when the circuit breaker is open the relay acts as a voltage direction relay. The tungsten lamp has an approximate resistance of 140 ohms cold and 1400 ohms hot. It therefore prevents short-circuiting the relay coil when the breaker is closed and protects the relay coil when the voltage across the open breaker is high, such as 115 to 230 volts.

The current through the relay coil does not increase directly with the voltage because the resistance of the lamp also increases with the voltage. The high value of the lamp resistance limits the total possible current through the relay coil to a value appreciably below the value of the secondary current corresponding to transformer exciting current. It is obvious, therefore, that the time required for operation is appreciably longer. The impedance of the current transformer at current values passed through the lamp is much greater than the impedance of the relay coil. Thus its connection in parallel does not interfere with the relay operation.

The potential coil is connected on the network side to keep the relay energized when the transformer is de-energized. This arrangement permits a strong torque in the tripping direction with the breaker open insuring against the breaker reclosing as soon as the transformer is energized. The relay must close the closing circuit by a spring when the network, and consequently the potential coil of the relay, is energized. It is also desirable, but not necessary, to have the relay protect against reclosing when phases are crossed or reversed by workmen during installation or repair work. Obviously, such protection would not be obtained if the potential coil is on the transformer side, because it is reversed with the transformer voltage.

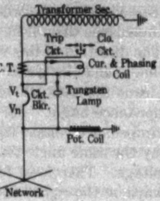

Fig. 4—Relay Connection Diagram For Both Tripping and Reclosing Function

The current coil of the relay has an impedance of less than two ohms. The resistance of the tungsten lamp cold or hot determines that the current through the relay coil is practically in phase with the voltage across the breaker, enabling the relay to operate according to the direction of this voltage as it does according to the direction of the line current. The lines *R R* similar to the lines *T T* in Fig. 2, therefore, may be drawn through the end of vector *V n* as indicated in Fig. 5. The direction and magnitude of the voltage

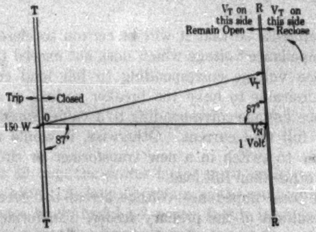

Fig. 5—Vector Diagram For Both Tripping and Reclosing Function

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2. See companion papers by Bullard and Parsons for treatment of relay using an extra phasing winding in the relay.

across the switch are represented by a vector drawn from V_n as its origin to V_t as its terminus. Therefore V_t must go beyond $R\,R$ in order to reclose the breaker. This characteristic obviously protects against reclosing on cross or reverse phases since the voltages would lead or lag the network voltage 120 deg. or 180 deg.

Reclosing where no feeder regulators are used. Where no feeder regulators are used, the difference in voltage that usually exists across the network breaker is that caused by the impedance drop from the substation to the network. The direction of this voltage depends on the impedance angle and the power factor. At unity power factor it leads the network voltage at an angle equal to the impedance angle. For power factor less than unity, this impedance voltage lags its unity power-factor position by the same angle that the current lags the network voltage. Therefore, for lagging power factors, it is always on the right hand side of the lines $R\,R$ of Fig. 5. This impedance voltage will always cause the incoming transformer to carry its share of the load.

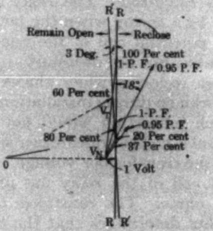


FIG. 6—VECTOR DIAGRAM SHOWING TRANSFORMER IMPEDANCE VOLTAGES USED FOR THE RECLOSING FUNCTION

It is necessary to design the relay with a reclosing characteristic so that it will be certain to close in on the impedance voltage which does not exceed the impedance voltage corresponding to full load current. It is desirable to have the breaker reclose at an impedance voltage corresponding to a value considerably below full load current. Otherwise, it would not be possible to switch in a new transformer or circuit at times other than full load.

The total impedance voltage across the breaker is the resultant of the primary feeder, transformer, and secondary main impedance voltages. It is advisable to use only the transformer impedance voltage in the relay design because it is possible for one breaker to reclose ahead of another, leaving only the transformer impedance voltage for the remaining breaker in cases where two or more transformers are located at the same point or close together. In Fig. 6 is shown the impedance voltage of transformers radiating from the end of V_n. The long vector corresponds to full-load impedance voltage of a high reactance (10 per cent) transformer and the short vector a low reactance transformer. The resistance component of each vector is the same. Two positions are indicated for each vector corresponding to unity power factor and 0.95 power factor.

If the relay is designed with the line $R\,R$ perpendicular and adjusted to close on a component of one volt in phase with the network voltage, it is obvious that approximately full load current at unity power factor is required on the adjacent transformer, either high or low reactance, to give this one volt. If the power factor lags to 0.95, the conditions are considerably improved, the high-reactance transformer closing in on 20 per cent load and the low reactance on 37 per cent load. If the relay is adjusted for ½ volt instead of one volt, these values are reduced to ½ for either the unity or 0.95 power-factor position. Now suppose the line $R\,R$ is advanced about three deg. to the position $R'\,R'$. Then the unity power-factor condition is considerably improved, being reduced from 100 per cent load to 60 per cent for the high-reactance transformer and to 80 per cent for the low-reactance transformer. The low power-factor condition is not changed very much. Notice that the high-reactance transformer is much better for this operation than the low reactance. The line $R\,R$ should be much further advanced if closing in on leading currents is required.

It is probable that an adjustment of about one volt will be satisfactory because most network transformers have high reactance and the power factor is not likely to be higher than 0.95. It seems, therefore, that the relay should have a range of adjustment from ½ to two volts in phase with the network voltage. This may be accomplished by using a device similar to the usual light load adjustment supplied with standard watthour-meters. This adjusting device gives a torque that opposes the closing torque exerted by a spring.

It has been previously explained that the relay will operate on cable charging current in the reverse direction. This charging current does not usually become appreciable until high voltage such as 13,200 volts is used for the primary feeder. Suppose a 13,200-volt feeder has only two transformers feeding the network and that the substation feeder breaker is opened. Then the two relays go to the tripping position. No. 2 breaker of Fig. 7 trips all right but for some reason No. 1 breaker does not trip. The cable may have a charging current of one ampere per phase per mi. and may be five mi. long, giving a total charging current I_c of five amperes or $5 \times 13.2 \times 1.73 = 96.8$ kv-a., which is approximately ⅓ load on a bank of 300 kv-a. This charging current flowing through the transformer causes a voltage to appear across the open breaker No. 2. This voltage $V_n - V_t$ leads the charging current by an angle equal to the impedance angle of the transformer and is about 84.3 deg. for a 10-per cent reactance transformer. Obviously, breaker No. 2 will reclose but will trip again because of the reverse current. It will periodically reclose and trip (called "pumping") as long as breaker No. 1 remains closed or until something happens to breaker No. 2. If the closing circuit of the breaker does not require very much current, voltage fluctuations are not likely to be noticed by the consumers. If the relays are designed so that it takes a much longer time for reclosing than for tripping, it is not possible for a "see-saw" action to take place by some breakers opening before the others.

Trouble from this pumping may be avoided in several ways. The substation operator could stop the pumping by simply reclosing the feeder circuit breaker, for a voltmeter would indicate back feed which means that one or more switches failed to open. Two or three operations of the substation breaker might clear the breaker which stuck. If not, an inspection would be necessary to locate the faulty breaker. The exciting current of the other transformers on the feeder will greatly reduce the charging current flowing through transformer No. 1. Suppose the total transformer capacity on the feeder is 5000 kv-a.; then the exciting current may be approximately two per cent, or 100 kv-a. more than enough to overcome the charging current. If the transformer exciting current is lower than the charging current, it could be increased by connecting a reactor to the feeder at the substation.

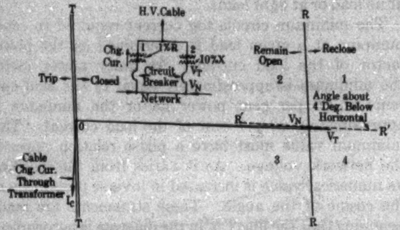


FIG. 7—VECTOR DIAGRAM SHOWING HOW REVERSE CABLE CHARGING CURRENT MAY CAUSE "PUMPING"

The relay characteristics may be modified by changing the design of the relay itself or by adding a separate relay to prevent reclosing on voltages that lag the network voltage more than three deg. to five deg. All the phases of a three-phase system are affected approximately the same by the charging current; therefore it is feasible to employ only one extra relay to prevent reclosing on voltages in the lagging direction. The relay would be identical in construction to the usual relay except that the tripping contact is omitted and a different set of coils designed so as to give the characteristic shown at $R' R'$ in Fig. 7. The phasing coil is connected across the contacts of one pole of the breaker and closes its contacts on any voltage vector in sectors 1 and 2, Fig. 7. The contact of this relay is connected in series with the closing circuit of the standard relay which closes its closing contacts when voltages occur in the sectors 1 and 4. The closing circuit, therefore, is energized only when voltage vectors occur in sector No. 1. The connections of the two relays across one pole of the breaker and the resulting characteristics are shown in Fig. 8.

Case of three single-phase regulators. When the feeder voltage is regulated by individual induction regulators the voltage existing across the contacts of the breakers of the incoming feeder may not be that due to the impedance voltage but may be modified in direction and magnitude by the position of the regulators. The simplest case involving regulators is the three-phase, four-wire feeder regulated by three Y-connected, single-phase regulators and supplying Y-Y connected transformers. In this case the regulators always boost or buck the voltage in phase with the substation voltage. Therefore, with this combination, the incoming transformer voltage is always in phase with the substation voltage. The cable charging current has a negligible effect on the receiving voltage because of the low resistance and reactance of the cable. The substation voltage can lag the network voltage only when the load power-factor angle at the network is greater than the resulting impedance angle of the primary feeder, transformers and network mains. Suppose this resulting impedance angle is 60 deg.; then the power factor must be lower than 50 per cent in order that the substation voltage may lag the network voltage. If the impedance angle is 30 deg. the power factor must be lower than 86.6 per cent. As long, therefore, as the combination of impedance angle and load power factor is such as to keep the substation bus voltage in the leading position as shown in Fig. 5, the regulators cannot set up a condition that will cause pumping. If the regulator bucks the voltage $V t$, Fig. 5, the breaker cannot reclose. If they boost the voltage $V t$, the voltage difference across the breaker cannot cause pumping because the lag of the current set up cannot be over 90 deg. and will be on the closing side of the lines $T T$.

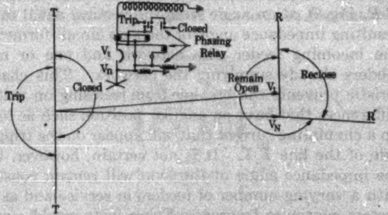


FIG. 8—RELAY CHARACTERISTIC THAT WILL PREVENT "PUMPING"

If the phase angle and power factor are low enough to cause the substation voltage to pass to the lagging position, it is possible for the regulator to buck the voltage $V t$ shown in Fig. 9 to $V' t$ which will cause pumping by setting up a circulating current. In Fig. 9, I_n is the load current the feeder would take if the regulators were not present, I_c the circulating current set up by the regulator running towards the buck position, and I_r the resultant current through the relay which trips the breaker. $V' t$ is just at the point where it will close the breaker, thereby causing pumping. While this condition is theoretically possible, it is believed that practical systems will have an impedance angle and power factor sufficiently high to avoid substation voltages being in the lagging direction. Where this is not true it will be necessary to advance the line $R R$, Fig. 9, at an angle from the vertical equal to the resulting impedance angle of the loop circuit formed by the incoming feeder, the network and one or more feeders already supplying the network. This characteristic prevents the breaker from reclosing on voltage differences that have an angular position such as to set up a circulating current that will appear on the tripping side of the line $T T$. It is not certain, however, that the impedance angle of the loop will remain constant with a varying number of feeders in service and as the system continues to grow. These facts would require that the relay have an adjustable feature to advance or retard the line $R R$, Fig. 9, which is not an easy thing to accomplish without seriously complicating the relay.

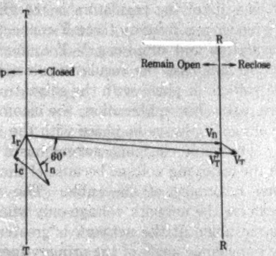


FIG. 9—VECTOR DIAGRAM SHOWING HOW "PUMPING" MAY BE CAUSED BY SINGLE-PHASE REGULATORS IN EACH PHASE

As previously explained, the normal operation of the regulators would cause no trouble. It is only in the rare condition where a regulator runs to full buck by some means that trouble is caused. An inspection of Fig. 9 will show that the usual difference in regulator positions due to contact-making voltmeter adjustments will cause no trouble because the small circulating current set up by the regulators would not be sufficient to subtract from the load current.

Use of polyphase regulators. If the polyphase regulators are used, it is possible for the regulators to cause a voltage difference across the open breaker of the incoming feeder that will lag the network voltage and cause pumping with the relay characteristic shown in Fig. 5. An inspection of Fig. 10 will show how this is possible. It is more convenient in this case to draw the substation voltage V_s as the reference vector. The polyphase regulator voltage is a constant value and changes the feeder voltage by moving from x to y along a circular path. The diagram indicates that one regulator is at the neutral position V_n and the other is appreciably advanced to the boost position V_t. The voltage across the open breaker is $V n$–$V t$ and sets up the circulating current $I c$. This would cause pumping at no load or at light loads.

The minimum circulating current required to cause pumping at a given load is determined by the power factor of the load current. At unity power factor the minimum is approximately equal to the load current; at 80 per cent power factor the minimum is approximately 80 per cent of the load current. This minimum value must have a phase relation opposite the network voltage. As it varies from this position its numerical value is increased in inverse proportion to the cosine of the angle. These statements are made assuming that the line $T T$ in the diagram is perpendicular but since it is advanced about three deg. the values are not exact. If the total impedance of the loop is 20 per cent it will require a voltage difference of about four per cent to equal the load current at 20 per cent load. It is probable that pumping would occur at such loads since with polyphase regulators it is possible for the contact-making voltmeter to cause a phase displacement of four per cent within the limits of its adjustment. As the impedance of the loop increases or the load increases, this danger is eliminated. It would then require some fault in the regulator automatic control equipment to cause pumping.

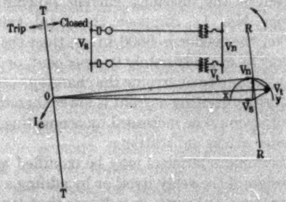


FIG. 10—VECTOR DIAGRAM SHOWING HOW POLYPHASE REGULATORS MAY CAUSE "PUMPING"

Hand control of the regulators would be likely to give trouble from pumping. The mechanical interconnection of the regulators would eliminate pumping caused by regulator displacement. The relay characteristic may be modified so that the switch cannot reclose on voltage differences that will set up a circulating current on the tripping side of the line $T T$ in Fig. 5. The relay characteristic shown in Fig. 8 is suitable. Only one phasing relay is necessary because the regulator affects all phases alike. The breaker cannot close in on a voltage difference that is lagging the network voltage more than three deg. The current lagging this voltage will not exceed 84 deg. on modern systems. Therefore, no phase-angle adjustment is necessary.

Use of two single-phase regulators. When two single-phase regulators are used in open-delta connection it is possible for one of the phase voltages to be displaced by the regulators to either the lag or lead position. A reference to Fig. 11 will explain how this takes place. The equilateral triangle A B C represents the bus voltage and also the line voltage when the regulators are in the neutral position. The normal operation of the regulators increases or decreases the voltage triangle symmetrically. No phase displacement occurs. The voltage range of the regulator in C phase is shown as $c' - x - c''$ and in B phase as $b' - y - b''$. Therefore, phases B and C cannot be displaced by the regulators. The only phase affected is A phase.

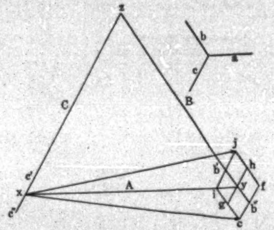


Fig. 11—Vector Diagram Showing how two Single-Phase Regulators Open Delta-Connected may Cause "Pumping"

Suppose the line voltage is impressed on a transformer bank with the high voltage connected delta and the secondary connected Y with the neutral grounded; then the secondary voltages a, b, c correspond in phase with the primary voltages A, B, C. Phases B and C are not displaced by the regulators. Suppose regulator C is in the neutral position and regulator B is run to the full buck position at b''; then voltage A or a is shifted to the lagging direction. Now when regulator C is operated from buck to boost (c' to c'') the end of vector a travels from e to n. When regulator B is in the neutral position and regulator c is varied from c' to c'', the voltage vector moves from g to h. The same thing is true with B regulator in the buck position where a varies from i to j. Obviously, the regulators can be made to cause the secondary voltage a to occur at any point in the diamond shaped area e, t, j, i. The normal operation of the regulators bucks or boosts the secondary voltage in phase with it along the path $i - y - f$.

A three-pole circuit breaker is equipped with three single-pole relays having their tripping contacts connected in parallel and the closing contacts connected in series. If the voltage is low on one phase the breaker cannot reclose. The circuit breaker, therefore, can be reclosed by the regulators only in the diamond-shaped area $y - b'' - f - h$. This is because all the other small areas require one of the regulators to be in the buck position. The foregoing discussion refers to the no-load condition. The impedance voltage is added vectorially to the regulated voltage.

The normal operation of these regulators with the usual possible two per cent difference is not likely to give enough circulating current to cause pumping. In this case a change in position of the regulators causes a more direct change on the voltage applied to the contact-making voltmeter than does the polyphase regulator which depends on the rotation of a voltage vector to produce an effect on the voltmeter. If one regulator is one per cent high and the other one per cent low, the result is a 1.73 per cent voltage to produce a circulating current which cannot cause any trouble unless the load is around 10 per cent and at a very low power factor. Therefore some trouble must develop in the regulator control circuit to cause positions that will produce pumping. The regulators being in the same circuit makes it very convenient to interconnect them mechanically so that they cannot cause a phase displacement. This causes all three phases to be regulated together but the loads of modern network systems are so well balanced that separate phase regulation is not essential.

The characteristics indicated in Fig. 8 may also be used to prevent reclosing on transformer voltages that are in the lagging direction. This characteristic is necessary only on the one phase affected by the regulators. The other two phases can use the characteristic indicated in Fig. 5. It is also possible to utilize a directional voltage relay with a winding on each side of the regulators that will close an alarm circuit when the regulators cause a voltage displacement in the lagging direction. The operator could readjust the regulators by hand control until the alarm circuit is opened. The pumping which might occur in the meantime would do no particular harm.

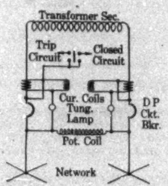
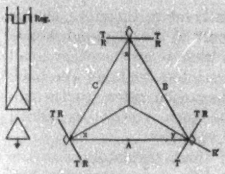
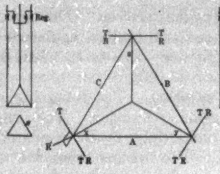


Fig. 12—Connections of one Two-Circuit Relay for Double-Pole Breakers

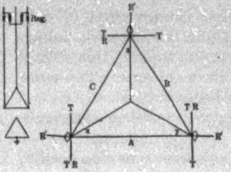
One large eastern company is installing a network utilizing two single-phase, 13,200-volt regulators in open-delta and Scott-connected transformers supplying a two-phase, five-wire network. Both of the phases may be displaced by the regulators. They intend to



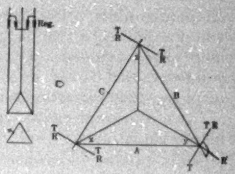
Case Ia



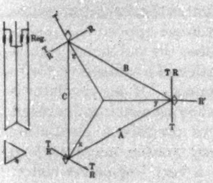
Case Ib



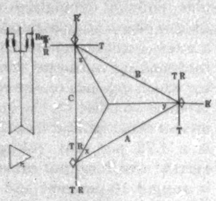
Case Ic



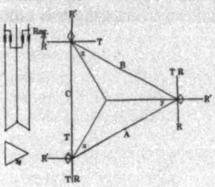
Case Id



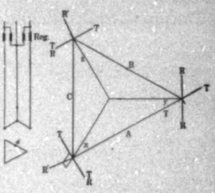
Case IIa



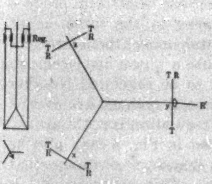
Cases IIb & IId



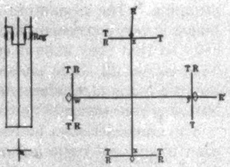
Case IIc



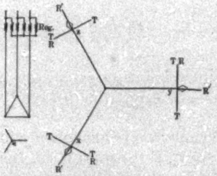
Cases IIe & IIf



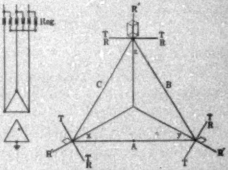
Case IIIa



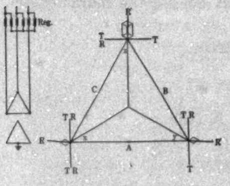
Case IVa



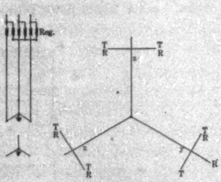
Case Va



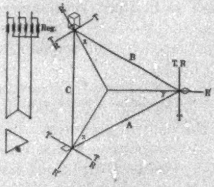
Case VIa



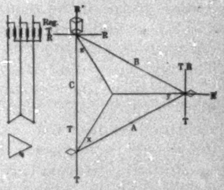
Case VIb



Case VIIa



Case VIIIa



Cases VIIIb & VIIIc

FIGS. Ia TO VIIIc TRANSFORMER AND REGULATOR CONNECTIONS

Case	No. of Single-Phase Regulators	Transformer Connections	Secondary Grounded on—	Number and Type of Relays
Ia	2	Δ–Δ	Phase A	3 Standard 1-Circuit 1 Phasing
Ib	2	Δ–Δ	Phase B	3 Standard 1-Circuit 1 Phasing
Ic	2	Δ–Δ	Phase A	1 Standard 1-Circuit 2 Phasing relays (1 with 2-circuit) 1 Standard 1-Circuit on Phase A
Id	2	Δ–Δ	Phase C	1 Standard 1-Circuit 1 Phasing 1 Standard 2-Circuit on Phase C
IIa	2	Y–Δ	Phase A	3 Standard 1-Circuit 2 Phasing
IIb	2	Y–Δ	Phase A	1 Standard 1-Circuit 2 Special 1-Circuit 2 Phasing
IIc	2	Y–Δ	Phase A	1 Special 2-Circuit 1 Standard 1-Circuit 1 Phasing
IId	2	Y–Δ	Phase A	3 Standard 1-Circuit
IIe	2	Y–Δ	Phase B	1 Standard 1-Circuit 2 Special 1-Circuit 3 Phasing
IIf	2	Y–Δ	Phase B	1 Special 2-Circuit 1 Special 1-Circuit 1 Phasing
IIIa	2	Δ–Y	Neutral	3 Standard 1-Circuit 1 Phasing
IVa	2	Scott	Neutral	2 Standard 2-Circuit 2 Phasing
Va	3	Δ–Y	Neutral	3 Standard 1-Circuit 3 Phasing
VIa	3	Δ–Δ	Phase A	3 Standard 1-Circuit 3 Phasing
VIb	3	Δ–Δ	Phase A	1 Standard 1-Circuit 1 Standard 2-Circuit 1 1-Circuit Phasing 1 2-Circuit Phasing
VIIa	3	Y–Y	Neutral	3 Standard 1-Circuit 1 Phasing
VIIIa	3	Y–Δ	Phase A	3 Standard 1-Circuit 3 Phasing
VIIIb	3	Y–Δ	Phase A	1 Standard 1-Circuit 2 Special 1-Circuit 3 Phasing
VIIIc	3	Y–Δ	Phase A	3 Standard 1-Circuit

mechanically interconnect the regulators to avoid any possibility of phase displacement. The initial installation consists of a double-pole circuit breaker for each transformer. A single relay with two current circuits is used with each circuit breaker. The connections are shown in Fig. 12.

Other combinations of regulators and transformer connections. There are other combinations of regulators and transformer connections possible. The foregoing discussion covers the cases that are now operating or being installed. Appendix A covers an analysis made of combinations likely to be considered. The results are in tabular form because a treatment of each case would require too much space. Enough is included to enable anyone interested in a particular combination to check the factors affecting the relay characteristics.

TABLE I
ANALYSIS OF NETWORK RELAY PERFORMANCE

This table shows the performance of the relays for the cases illustrated in Figs. Ia to VIIIc, inclusive. The left-hand column in the table shows the various conditions which may call for relay action. The columns headed Ia to VIIIc show the performance of the relays. The term "o. k." indicates satisfactory performance. The numerals 1, 2, 3, etc., indicate abnormal performance which is explained for each case under the corresponding numeral in the Explanatory Note at the foot of the table.

	CASE																		
	Ia	Ib	Ic	Id	IIa	IIb	IIc	IId	IIe	IIf	IIIa	IVa	Va	VIa	VIb	VIIa	VIIIa	VIIIb	VIIIc
Short circuit																			
3 ϕ—Normal (or 2 ϕ)	o.k.	o.k.	o.k.	o.k.	o.k.	13	14	13	19	21	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	13	13
3 ϕ—Reverse (or 2 ϕ)	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.
S ϕ—Normal x y	1	1	o.k.	10	1	o.k.	o.k.	o.k.	o.k.	o.k.	1		1	1	o.k.	1	1	o.k.	o.k.
y z (or y w)	1	1	10	1	1	1	15	1	o.k.	o.k.	1	o.k.	1	1	10	1	1	1	1
z x	1	1	1	o.k.	1	16	16	16	1	19	1	o.k.	1	1	1	1	1	16	16
S ϕ—Reverse x y	2	2	o.k.	o.k.	2	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.		2	2	o.k.	2	2	o.k.	o.k.
y z (or y w)	2	2	o.k.	6	2	2	o.k.	2	o.k.	o.k.	o.k.	o.k.	2	2	o.k.	2	2	2	o.k.
z x	2	2	5	o.k.	2	16	16	16	2	o.k.	o.k.	o.k.	2	2	5	2	2	16	16
Line to ground x	1-2-7	o.k.	8	8	1-2-7	7	8	7	o.k.	o.k.	o.k.	8	o.k.	1-2-7	8	o.k.	1-2-7	7	7
y	7	1-2	8	o.k.	7	7	8	7	7	8	o.k.	8	o.k.	7	8	o.k.	7	7	7
z	o.k.	7	o.k.	8	o.k.	o.k.	o.k.	o.k.	7	8	o.k.	8	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.
Reverse exc. cur. trip	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.
Reverse chg. cur. trip	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	17	o.k.	o.k.	17	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.
Reverse chg. cur. reclose	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.
Regulator displacement	3	4	o.k.	o.k.	11	o.k.	o.k.	o.k.	20	o.k.	o.k.	o.k.	o.k.	22	o.k.	o.k.	23	o.k.	o.k.
Phasing-in on impedance voltage	o.k.	o.k.	9	9	o.k.	12	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	9	o.k.	o.k.	12	o.k.
Cross- and reverse-phase protection	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	18	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	o.k.	18

EXPLANATORY NOTE

1. The lagging relay trips on currents lagging greater than 60 deg.

2. The lagging relay will not trip on currents lagging greater than 60 deg.

3. All three phases are affected by the regulators. A phasing relay is required at z. The phasing relay cannot be added at x because the regulators normally add the voltage in the lagging direction, making it impossible to reclose at light loads. Omitting the phasing relay at x makes pumping of the network switches very probable because the voltage is added in the lagging direction. This combination is not acceptable.

4. The voltages at y and z can be added only in one direction by the regulators because of the ground being on B phase. At y the voltage is added in the lagging direction, preventing reclosure or causing pumping as explained in paragraph 3.

5. The relay at z will not trip on short-circuit currents lagging more than 60 deg. It is probable that the x element of the two-circuit relay will have a greater tripping torque than the y element has closing torque which is produced by the load current.

6. Same as paragraph 5 except a different phase is involved.

7. There is a possibility of tripping when faults occur on the network near a transformer because of the auto-transformer action between the two halves of the secondary causing a reversal in the unaffected half of the winding.

8. It is not likely that the two-element relay will trip when faults occur on the network because of the greater torque of the affected element. The saturation of the current transformers renders the operation somewhat uncertain.

9. When the voltage difference on the lagging elements leads more than 60 deg. an opening torque is produced in the disk. This is, however, more than neutralized by the greater closing torque produced by the leading element up to 90 deg. lead at which point the torques are neutralized.

10. The leading element of the two-circuit relay gives a tripping torque for currents lagging more than 30 deg. As long as there is sufficient load to saturate the current transformer of the lagging element there is little danger of tripping.

11. The normal operation of the regulators adds the voltage in the lagging direction at x and z. The phasing relay at z prevents reclosure by boosting the voltage. If the phasing relay at z is omitted then the normal operation of the regulators may cause pumping because of the lagging voltage at x and z.

12. The relay at x will prevent reclosure on voltage differences leading more than 30 deg. if it is connected for phasing. If it is not so connected then the relay at z will prevent reclosure at voltages leading more than 60 deg.

13. The x relay will trip on leading currents greater than 30 deg.

14. The lagging element of the two-circuit relay gives a tripping torque for leading currents greater than 30 deg. but the leading element gives a greater closing torque until 60 deg. at which time the z relay will trip.

15. The same as paragraph 10 except the angle is 60 deg.

16. It is probable that all single-phase loads and shorts will lag enough to function properly.

17. Will not trip on reverse charging currents leading more than 60 deg.

18. The phasing connection across z might hold open for a cross from x to z. An extra phasing relay is necessary at x for holding out on a reverse connection.

19. The x relay will trip on currents leading more than 60 deg.

20. The relay at x will prevent reclosure at voltage differences leading more than 60 deg. if it is connected for phasing. It does not seem necessary to connect this relay for phasing.

21. Same as paragraph 14 except the ground and two-circuit relay are on B phase.

22. The regulators add the voltage in the lagging direction at x. The phasing relay prevents reclosure under normal conditions. Omitting the phasing relay at x may cause pumping under normal conditions.

23. Same as paragraph 22 except that z also is affected.

Such combinations as two single-phase regulators and Y-Y transformers are not included because it is not expected that Y-Y connection would be used on three-wire circuits and on four-wire circuits, the phase without a regulator being unregulated.

CONCLUSIONS

The simple watthour-meter type relay having characteristics similar to Fig. 5 is suitable for the tripping function and for the reclosing function when the incoming transformer voltage leads the network voltage. Under normal operating conditions, the incoming transformer voltage always leads the network voltage with the probable exception of the cases of the polyphase and two single-phase regulators at very light loads. A voltage may be caused in the lagging direction by the failure of regulator control or by reverse charging current when a breaker fails to trip. Failures of breakers and regulator control can be minimized by proper inspection and maintenance. Mechanical interconnection prevents trouble from regulators.

The relay characteristic shown in Fig. 8 prevents reclosing on incoming transformer voltages that lag the network voltage and is, therefore, entirely suitable for the reclosing function and may be used in cases where it is believed that characteristic in Fig. 5 will not be suitable. The network breakers being located at many points in subway vaults makes it desirable that the breaker with the relay equipment be kept at its utmost simplicity. The relay should not be designed to avoid trouble from the regulators, but the regulators should be mechanically interconnected or provided with an alarm device to prevent improper operation of the relays. The induction type relay constructed of watthour-meter parts is the most simple and reliable device known for directional characteristics. When it is realized that such a characteristic as that indicated in Fig. 5 can be obtained with standard watthour-meter construction and adjustments, involving only a different current coil and the replacement of the recording mechanism by standard relay contacts, every reasonable effort should be made to utilize it. Maintenance, testing, and adjustments are simplified. The characteristic of Fig. 8 has the objection of requiring an extra relay which, however, is just as simple in construction and adjustment as the other relays. To incorporate both features in one relay requires an entirely new development without the assurance of obtaining characteristics that are as definite as those of Fig. 8.

The author acknowledges valuable suggestions and criticisms from Messrs. W. R. Bullard and L. F. Kennedy who reviewed the first draft of this paper.

APPENDIX A

Diagrams Ia to VIIIc show the effect of regulator displacement on secondary voltages of the transformers. The diamond or rectangular shaped areas refer to the paths taken by the regulator voltages when they are varied from the neutral to the full boost position. It is not necessarily true, however, that the breaker cannot be reclosed by the regulator voltages being outside the area, because this depends on the relay characteristics and equipment. It simplifies the diagram to omit the other paths. The lines $T\ T$ for tripping and $R\ R$ for reclosing coincide in most cases. It will be observed that the ground on the midpoint of one phase of the delta secondary seriously changes the direction of the normal voltage added by the regulators as far as the reclosing function is concerned. A study of Table I will reveal that the delta-connected secondary is not as simple a proposition for the automatic type of network switch as is the Y-connected secondaries. Single-phase networks must also be given particular attention when the transformer primary is Y-connected. A special phase-angle for reclosing is necessary. The data under the figures apply to the regulator, transformer and relay equipment for the different combinations. In cases Id, IIc, IIf, IVa, and VIb, two element relays similar to Fig. 12 are used on some of the phases.

Discussion

PAPERS ON A-C. DISTRIBUTION NETWORKS

(FAIRMAN AND RIFENBURG, PARSONS, BULLARD, BLAKE)

NEW YORK, N. Y., NOVEMBER 11, 1926

J. C. Parker: I am quite sure from the remarks of Mr. Fairman that he would not want anyone to generalize from his description of what has been the engineering program of the Brooklyn Edison Company into a conclusion that the same medicine is good in all cases.

Some four years ago it was perfectly apparent that pressure of population was forcing and would continue to force an extraordinary growth of the portion of Brooklyn at that time served through converter substations. The sequel has proved fully that predictions made at that time were fully justified. One who gained his impressions of the central portion of Brooklyn in the years 1920-1922 would scarcely recognize the place today.

That meant that an already fairly dense territory which did lend itself to network development was bound to become very, very much more dense in the near future; buildings would be torn down, taking out their equipment, their wiring, and calling for extraordinary increases in the street supply lines to them. Then one had there a unique condition with reference to continuing the expansion of the then system of 25-cycle generation conversion and d-c. distribution, a condition which assuredly could scarcely be expected to find a duplicate in the Loop District of Chicago, in downtown Boston, or in the lower part of Manhattan Island.

That was one thing. For another, the company whose processes Mr. Fairman describes had a relatively small amount of 25-cycle generating equipment, that equipment not of the most efficient type. It was in process of developing a highly efficient 60-cycle generating station to take care of the equally rapidly increasing development in the peripheral sections of Brooklyn. Now, a process of curtailing the d-c. load, of actually diminishing it, would have been almost if not quite unthinkable had it involved the virtual scrapping of a large and highly efficient 25-cycle generating station. Such was not the case in the Brooklyn territory. Easily enough other uses could be found for the 25-cycle generating equipment.

One fact which may be mentioned here is that that equipment—not so good for energy production as the power station being built—was just as good as anything else could have been for peak capacity through the use of a relatively small amount of frequency-converting apparatus.

Again, a system which had a large or preponderant 25-cycle generating capacity, could not anywhere nearly have adapted itself to the program Mr. Fairman has indicated.

Mr. Fairman's and Mr. Rifenburg's paper is not based entirely on prediction, but on a statement of fact accomplished as well.

The experience of the four years with a radial a-c. system has demonstrated an excellent standard of service resultant from the exquisite attention to detail which is absolutely necessary, of course, with any system that is to provide continuity of service for the customers.

The capital savings which should have resulted from the changes indicated in the paper are matters of past experience and of accounting record. The economy in operation indicated in the paper is also up to and inclusive of the year 1926, as a matter of fact.

Again to avoid the danger of generalization, I suggest that one do not extrapolate indefinitely in looking at the efficiency curve in the paper under discussion. I think it would be in the minds of the authors that that curve should flatten out rather speedily in the neighborhood of the year 1930 unless, indeed, they have in mind further and marked improvements in their distribution design which will raise the efficiency ultimately to something in the neighborhood of the upper nineties; that, I believe, they do not expect.

Speaking of the future, may I suggest that discussions of a-c. network problems at the present time seem to concern themselves primarily with what is to be recognized as a *sine qua non* of service; namely, continuity; and that the network development seems to be regarded as something we must have to bolster up our a-c. systems. In so far as the developers of networks stop there, I think they make a mistake and miss at least a half of what these networks will offer in the future.

My own thought, and indeed I believe the plan in the minds of the authors of this paper, is that ultimately the network will accomplish very much more than described here.

Probably the tripping out of a network switch on the relatively small currents involved in the magnetization of transformers will be thrown overboard for things of considerably greater value. Those things, it seems to me, will express themselves in a system which again will become a radial system, but inverted from present practise. If you please, we may consider the network as the center of the system and working backward over the transmission feeders, reach generators which will not be connected together within the power house.

In the attempt to limit switching requirements in our generating stations we have indulged in a semi-isolation of generators. We have got to the point where the rupturing capacity of circuit breakers seems nearly to have come to its limit, and where the further aggregation of generating capacity on one bus seems not sane if indeed it be physically practical.

Now, the development of the network offers a way out. In the extreme, one may consider an individual generator of 160,000- or 200,000-kv-a. capacity with fifteen to twenty feeders of the size described by the authors of the paper, these feeders going directly to the distribution system and tying these larger generators to the distribution network without any use of a generating station bus.

With some necessary details to be worked out and with the routine tripping of feeders from the network by the deliberate imposition on them of regulated short circuits at the generating station, I think such a program as that offers a solution of one the most vexing and perplexing problems of generating station design today.

I venture the prediction that within another decade we shall find that the development of these a-c. networks, initiated for the purpose of procuring the utmost continuity of service to the customers, will have proved itself more valuable in its concomitant effects than in the principal object for which it has been undertaken.

A. H. Kehoe Nearly every presentation on a-c. networks up to the present has set up some extensive area which is to be entirely networked. Since a reasonably dense loading is required before transformers can be connected economically to a single set of mains, the districts where this is practicable seem at first sight to be limited to a few of the larger cities. I do not believe, however, that networks will be restricted to zones of this character. Economical considerations arising from savings possible in other parts of the system are forcing the use of this arrangement in the smaller communities where the increased reliability would otherwise be ignored. In districts using 2300- or 4000-volt distribution from substations that are themselves supplied by lines of 27 kv. or less, it is possible to avoid any additions to the existing substation capacity by supplying new distribution transformer requirements directly at the voltage in use to feed the substations. To do this, a few distribution transformers in several of the small but more densely loaded sections of the community have their secondary lines joined into a network which makes it possible to replace part of the existing capacity with transformers supplied at the higher voltage. All new capacity required can be added in this manner, and the part of the low-voltage (2300/4000) capacity which is released can be used when required in the less densely loaded sections. The economies indicated in the Fairman-Rifenburg paper can be realized in the smaller cities by adopting this so-called "high-voltage infiltration" method even where the supply is overhead. The arrangement results in lower costs than any which are being obtained with the standard radial distribution, and gives greater reliability of service to the more densely loaded districts.

I want to express my belief in what Mr. Parker has described as the ultimate arrangement of the supply systems which will feed our larger distribution networks; but at the same time a word of caution is required, as this method of operation has never been completely demonstrated, to my knowledge, either by calculation or by operation. Those of us who have been studying its possibilities for the past two years are positively hopeful of obtaining as successful results in the supply system as have been indicated here for the distribution system. It is important in selecting network switches and relays to realize the extreme probability of their future use on systems having sources tied only at the load. It is hoped that this subject, which is being considered in one of the committees of a companion organization at present, will have been more fully investigated before another year has passed.

I do not agree with the opinion expressed that the sensitive reverse-energy feature is the point of attack in looking for improved relaying of network switches. The phasing or synchronizing characteristic required up to the present (and these requirements seem to be growing stricter) makes it necessary to have a very delicate relay. So long as this cannot be avoided, I believe it preferable to use a low value of reverse energy.

With the publication of the Fairman-Rifenburg tests, I believe ample data have been presented to be certain of the factors involved in lamp flicker or the elimination of faults on low-voltage, a-c. cables. In making their tests, however, I regret that the authors did not investigate the situation which may occur at induction motors during the time when a single-conductor main is clearing a fault near the motor service. This phase-converting action was discussed at the 1925 Saratoga meeting of the Institute and is emphasized here in order that anyone attempting the expensive procedure of obtaining secondary cable fault behavior will obtain and publish information on the effects of line-to-neutral secondary faults upon loaded polyphase induction motors operating adjacent to the fault.

Mr. Parsons: I should like to ask Mr. Fairman what proportion of the Brooklyn Edison Company has been converted into the new system and what proportion probably will eventually be changed over.

Henry Richter: The last four papers in this session deal mainly with that part of the a-c. network system from the station to the network unit. They apply particularly to two important problems; namely, primary system design (in which I include the distribution transformers) and the automatic network protective device. The third element of every a-c. network is the secondary system, and operating engineers have found it necessary to give this third problem at least as much study as has been required for either of the other two.

There is a decided interrelation between these three subjects. When the relay and network unit developed for one type of network system are considered for other installations where differing arrangements of the secondaries are contemplated, the construction of both relay and unit must often be considerably changed to meet the requirements.

In determining which type of secondary system should be adopted for the network, many cities are choosing that scheme of connection and voltage which fits in best with local conditions at the present time. This is only natural with the network art so new. But it is introducing a complexity in apparatus requirements that threatens to parallel the chaotic lamp and frequency situations of some years ago.

The secondary systems thus far adopted include two-phase and three-phase, combined light and power mains and also separate light and power mains, star- and delta-connected combinations for three-phase, and even differing voltages for the three-phase, four-wire, star-connected, combined light and power scheme.

Should these various secondary systems be adopted extensively and the groups of cities employing each scheme come to require guarantees on satisfactory operation of network and utilization apparatus, three major disadvantages will result.

First, there will be the numerous differing requirements in master relay and network-unit design already mentioned. Second, there will be a great increase in the complexity of manufacturing and applying all the devices connected to the secondary system, affecting not only the consumers but also the operating companies. Third, there will be lost the progress that usually attends concentration of efforts towards improving the best type of system and equipment available.

A number of the operating company engineers intimately interested in a-c. networks are coming to recognize the advisability of standardizing on one secondary scheme of connection and voltage. But each claims his own system is the best and should be the one chosen as the standard. That also is natural.

As a result of numerous requests for information bearing on this problem, an analysis of the situation as it affects the utilization devices and distribution apparatus connected to the secondary mains was started 2½ years ago. It was soon found that the effect of the various schemes on general-purpose motors monopolized the attention of many of the operating engineers who were also working on the problem. On the contrary, there are several other types of equipment each of which is affected just as much as are general-purpose motors.

It was also discovered that the apparatus situation in the future is at least as important as the possibility of having present standards suffice should a given scheme be applied to all existing distribution systems. And, finally, in the effort to make an impartial and complete analysis of the effect of all factors on the electrical industry as a whole, it appeared impossible to disregard the commercial aspects.

This study is scheduled for presentation at the 1927 Winter Convention of the Institute. The object of the paper is to promote widespread discussion of the subject. This, it is hoped, should result in a clarification and should assist those operating engineers who have come to recognize the great benefits that would attend standardization for network secondary systems.

Mr. Bullard suggests standardizing on one type of network relay. Similarly, and without wishing to detract from the specific discussion of the subjects of these papers, I recommend that the same problem as regards secondary systems be given the attention of the Institute along with network relays and network units.

H. R. Searing: The experiences of the United Electric Light & Power Company, of New York City, with network switch and relay operation covering a period from April 1922 to the present time are as follows:

We have 21,209-kv-a. transformer capacity in network operation, 350 switches installed, 253 of which are in automatic operation, with an estimated peak load of 17,000 kv-a.

These feeders serve several different kinds of networks,—some three-phrase, some two-phase, and some single-phase. They are served not only by 2750-volt primary circuits but also by 13,200-volt circuits. We now have a transformer capacity of 2950 kv-a. and a total of 25 switches in service on 13,200-volt feeders. On the three-phase, four-wire system,—that is, the part of the system converted to combined light and power,—there are ten circuits with a transformer capacity of 11,121 kv-a.

The first 13,200-volt bank was cut in April 1, 1926, and the 13,200-volt feeders parallel the 2750-volt network. In this connection, before I go into the operation of relays, I might say that in operating 2750- and 13,200-volt circuits in parallel, we have had no trouble, the 13,200-volt transformers with 10 per cent reactance paralleling the three per cent distribution transformers on the secondary network.

On switch operations from January 1 to September 30, 1926, there was a total of 40,360 switch operations which were obtained by 4838 circuit operations. Ordinarily this is accomplished by dropping circuits over the midnight watch every night, so if there are ten switches on a circuit, there will be ten operations every night. Some circuits cannot be dropped. There may be two-phase power and other lighting load not networked, on the circuit. During the 40,360 operations, there was a total percentage failure to open of 0.263 of one per cent; a percentage failure to close of 0.149 of one per cent, which you will see is a pretty good operating record. The percentage of failure to open is not significant, because, if a switch fails to open, it is backed up by fuses which will blow and clear. The switch which fails to close is a hazard.

In the period from 1922 to date, we have had twenty-four primary faults; that is, faults where a circuit tripped out, which, under the radial system, would have caused an outage to a customer, and of these faults, twenty-one cleared without blowing any network switch fuses. With three faults, one fuse on one switch was blown.

We have had eleven secondary faults. Nine cleared themselves and two cleared and blew the fuses on the switches nearest the fault. We have had one equipment failure and it cleared itself without outage. We have had two operating faults, due to station switches dropping out or being opened by mistake.

One of the gentlemen spoke of the maintenance problem. While it is realized that there are many relays on a network, it is not fair to compare the network relays with the relays on the network feeders as regards maintenance, because backing up that particular feeder relay are relays all the way back to the generating station and each circuit has a certain percentage of the relays back of it to be considered when talking about relay maintenance.

We found it was necessary to breed a new type of repair man; a man who not only could work on relays, but could also adjust the switch and was safe to go into a transformer vault with high-tension equipment. I don't think that the industry is going to find that the ordinary test man or meter tester who maintains generating and substation relays, will be satisfactory for the maintenance of network switches. A mechanic is needed,—a man with mechanical ability as well as test knowledge.

G. R. Milne: I should like to discuss in general the papers on network relay characteristics and in particular the paper by Mr. Blake.

First of all I should like to talk of the operating characteristic of the relay. It is pretty well agreed, now, among those familiar with network development, that the desired opening characteristic is that of a simple watt hour-meter. The method of obtaining that characteristic, however, is not quite so simple; for instance, the statement is made in the paper by Mr. Blake that this requirement can be met by the watthour-meter operating in connection with a saturating current transformer.

Now, it is true that watthour-meters when operated with current transformers of the non-saturating type give this characteristic. Yet, when, in order to protect the network relay from short-circuit currents, it becomes necessary to use a current transformer of the saturating type, there must of necessity be some difference in the curve of opening from that of a true watthour-meter. We are all familiar with the saturating characteristics of iron and, therefore, while the diagrams indicate that the opening characteristic is a perfectly straight line, there must have been, of necessity, some change or variation in the shape of the opening characteristic at low values of current at which the relay is at times required to operate.

To come to the closing characteristic of the relay; one of the fundamental requirements is that when the relay closes, the system conditions should be such that the transformer will feed true power into the network. That is necessary to prevent pumping of the relay—*i. e.*, alternately opening and closing particularly on high-voltage feeders of the order of 13,800 volts or above. Such pumping will occur in openings on 13,800-volt network feeders and such pumping will occur provided the watt-hour closing characteristic is continued.

Of course, there are various devices by which to get around that, and one of the methods suggested is that of closing in the feeder immediately to prevent pumping. That cannot be done if the feeder trips out on a ground fault, and since, in going to higher-voltage cables, we shall probably predominate in single-conductor cables, most of the troubles will be line-to-ground faults, and, therefore, it is not possible for the operator to reclose to prevent the pumping.

Then, to look at the development from a broader angle, two of the previous discussors have indicated that the possible future trend of network development will be to tie the generating sources only through the low-voltage network. When this is done, we must have a relay which is pump-proof, not only throughout limited angles of operation such as we can impose on the relay on our present systems, but we must have a relay which shall be pump-proof throughout the entire 360 deg. which may occur in relay operation. When the sources are out of synchronism, it is possible to have currents and voltages throughout the entire 360 deg. and, therefore, backing up what two of the previous discussors have said, it is necessary not to bridge the subject of what is the most desirable type of characteristic to incorporate in a relay, but get together and decide what are the desirable characteristics not only for the systems we are operating at present, but for the systems as we see them in the future. It is obviously undesirable to get a large amount of equipment on the lines and then decide we want to change the method of operation and find that the limitations originally imposed in the relay prevent it from being used on the new system.

H. C. Forbes: The experience of the New York Edison Company in the design and operation of a two-phase, five-wire a-c. network has brought out some points which are of interest in connection with the papers on distribution.

The network of which I am speaking is somewhat unique in that it is supplied by both 2300-volt, two-phase feeders, and 13,200-volt, three-phase feeders, the latter being connected to the network by means of Scott-connected distribution transformers. This arrangement has enabled the retention of the investment in the 2300-volt equipment and at the same time has permitted the future load growth to utilize the advantages of the higher distribution voltage.

The 13,200-volt feeders have already demonstrated the fact that if one network switch sticks in the closed position when the feeder switch is opened, the remaining switches on the feeder will undergo the pumping action, which was expected and the causes of which are explained in Mr. Blake's paper. In case this happens, however, it is usually possible to remove the feeder from the network by connecting the regulator to the feeder at the substation and allowing its exciting current to flow from the feeder to the regulator. The reverse energy is usually sufficient to trip the switch. In the case of a lower distribution voltage, an intermediate transformation at the substation is usually involved. Here, if a network switch fails to open, it may be possible to remove the feeder from the network by energizing one of the substation transformer banks from the feeder. The procedure would be to put a spare transformer bank on the emergency bus at the substation and excite it from the feeder which is giving the trouble. Unless the relays are badly out of adjustment, the network switches will be then tripped.

I should like to call attention to one point Mr. Blake has mentioned in his paper; namely, that the relays must be designed to meet the worst conditions of lag on short-circuit current which, as he points out, will occur when the fault takes place at the terminals of the transformer. The example which he cites of a transformer having one per cent resistance and 10 per cent reactance, while obviously correct, may be misleading. This is true because the distribution transformers in use are frequently of 10 per cent reactance, but, nevertheless, a portion of this is often external to the transformer in the shape of iron washers slipped on over the leads to the transformer. This type of reactance is effective in obtaining the required distribution of load among the transformers but in case of a short circuit, the iron washers become saturated and will have little effect in determining the phase angle of the short-circuit current. The inherent reactance would be the predominant factor in calculating this phase angle.

Our experience to the present time has indicated that very little mechanical difficulty is to be expected from the switches, but that on the other hand, there is likely to be considerable difficulty in the adjustment and maintenance of the relay. It is to be hoped that closer cooperation between the engineers of the manufacturing companies and the operating companies will result in a simplified design of the relays which will obviate some of these difficulties without sacrificing any of the characteristics the relays are intended to supply.

J. A. Brooks: The design of the Brooklyn Edison Company's low-voltage network, as described in Messrs. Fairman's and Rifenburg's paper, being different in many details from a-c. networks now in operation gave rise to new problems in switch and relay design, the solutions of which while not difficult may be of general interest.

The network transformers being connected delta on the high-voltage side and star with neutral grounded on the low-voltage side causes the direction of power flow on the low-voltage side to be from the network into the transformer in one phase and from the transformer into the network in another phase for single-phase line-to-line or line-to-ground faults on the high-voltage side, the phases referred to on the low-voltage side being taken from line to neutral. Since the present single-phase network relays have potential coils connected from line to neutral on the same phase in which the current coil is connected and since they are necessarily arranged to trip on true power flow due to the wide range in power factor within which the relay must trip the switch, these primary faults would cause not only the network switches which are associated with the particular feeder in trouble to open but would also momentarily open switches not associated with the particular primary feeder in trouble if single-phase relays were used. In order that such misoperation may be avoided, one three-phase network relay will be used per switch instead of three single-phase relays. The use of three-phase relays may in some cases cause the switch to close with one fuse blown or not to open until after the breaker at the generating station opens, but these features are not considered objectionable.

The charging current of the 27,000-volt cables feeding the network will in most cases be greater than the combined exciting currents of the connected transformers which, since no regulators are used, would probably cause pumping similar to that mentioned by Mr. Blake. A separate relay similar to that described in Mr. Blake's paper will be used to overcome this difficulty.

Since the network switch is to be located in the low-voltage pothead of the transformer, the transformer temperature will materially affect the operating temperature of the switch in that the fairly high heat conductivity of the copper connection between the transformer and the switch contacts will cause the switch to have a temperature approximately that of the transformer copper, and the temperature of the air inside the pothead will probably be somewhat higher than it would be otherwise. Under full-load operating conditions the temperature of the switch contacts will be above that permissible for the ordinary type of copper leaf contact, necessitating the use of a type of contact which will retain its contact pressure at these temperatures and will not be seriously affected by oxidation. The relative high temperature of the air inside the pothead makes the use of special relay coils necessary.

There has been some discussion relative to the desirability of synchronizing generating stations through the low-voltage network. This problem, if load other than the network load were taken from the generator station bus, would not only involve suitable closing characteristics of the network relays, as pointed out in Mr. Bullard's paper, but would also involve tripping characteristics differing from the present practise. Under these conditions there would probably be a normal exchange of power between generating stations, the direction of which may change with the hours of the day. Under such conditions it may be necessary to give the relay a trip setting for much greater magnitude of power flow and longer time than is now being used. Where the network transformers have delta primaries with the neutral grounded at the generating station only, the short-circuit currents passing through the network switch for single-phase line-to-ground faults on the primary feeder may not be as great as the float setting necessary to prevent the relay tripping on normal exchange of power between the generating stations. The desirability of attempting to accomplish these added features in the present network relay is questionable, assuming that it can be done.

D. K. Blake: At this time I want to discuss Mr. Fairman's paper along the ideas mentioned by Mr. Parker, Mr. Kehoe, and others. This morning in our discussion we have just one type of network under consideration with these elements: a secondary grid, a protective breaker, a transformer, and the primary cables.

We are thinking these days about an ideal system or a fundamental plan for the system. Some engineers believe some such thing as is being done by the Brooklyn Edison Company is the ultimate ideal.

Now, there is a certain load density—I don't know what it is—at which the low-voltage network is economical. Then the question arises when considering the low-voltage networks as an ideal system—what is the ideal or fundamental plan to take you over that period of time from period of low density up to the low voltage secondary network density?

I believe that the same idea of network grid, protective breaker, transformers, and cables can be applied. In other words, the d-c. Edison network has demonstrated throughout these years that the ideal distribution system is a network. A load can be tapped at any point and the network can be reinforced at any point according to the location of the loads.

The idea I had and have studied to some extent—and other engineers are thinking along the same lines—is that we would have exactly the same kind of network as the low-voltage network except it would be at 4000 volts to take us over the 625-kv-a. per square mile density up to the 4000 kv-a. per sq. mil. density if that is the minimum density for the low-voltage network. In the low-voltage, a-c. network you can burn off a fault on the network mains but at 4000 volts you can't do that. The difference comes here, that you put in at the end of each main where they are interconnected, an oil circuit breaker with plain induction type overload relays and the transformer feeding the interconnected point would have the usual oil circuit breaker overload and reverse power relay.

Then we have a 4000-volt grid like the secondary grid and you start out with certain transformer spacings. As you continue to grow, you decrease the transformer spacing. Your transmission system would be simply a network feeding point, a transformer, and a high-tension cable back to your generating station. This is in harmony with the idea of synchronizing at the load.

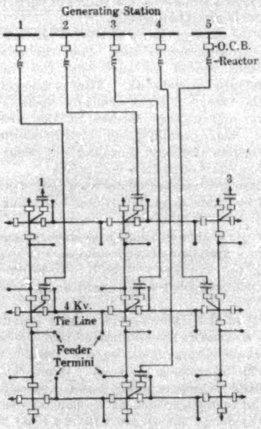


Fig. 1

Fig. 1 shows the connections. The feeding points would be small substations consisting of a three-phase transformer and five truck type panels.

C. A. Corney: I should like to ask Mr. Fairman how his company, in the ultimate development of the network, would take care of large office building or hotel load of the magnitude of 1000 to 1500 kv-a., and also how they would serve a customer whose secondary circuits would require higher than 220 volts for distribution to motors. I would also be interested to hear a little more about how they can do without regulators on the primary circuits and still maintain standard voltages near the generating plants and at some distance from them.

R. K. McMaster (communicated after adjournment): In the discussion following the presentation of the paper the possibility of the new developments having an unfavorable effect on the standardization of equipment was mentioned.

It is possible to build three-phase motors with windings which can be connected parallel-star for 190 volts and series-delta for 220 volts. To promote interchangeability it is suggested that this arrangement be adopted wherever practicable, at least to the extent of providing for a change from either voltage to the other by changing only a minimum number of connections between the coil groups and between these and the terminal leads.

The standard lamp voltage adopted by the National Electric Light Association is 115 volts and the corresponding line-to-line voltage of a four-wire system is 199 volts. The suitability of 190-volt motors for use on 115/199-volt circuits would be the same as that of 110-volt motors on 115-volt circuits or of 220-volt motors on 230-volt circuits. Although 120 volts is a recognized exception to the standard lamp voltage the use of 120/208-volt circuits should be considered as a temporary expedient and not as a standard. Otherwise there would be disadvantageous conditions with regard to lamps and various single-phase devices, without completely justifying the continuance of present standards with regard to motors.

Even where transformers are installed for power only the use of the star-connected grounded-neutral secondary connection has an important advantage from the standpoint of safety. This is that the maximum voltage to ground is comparable to that on lighting circuits. This advantage makes it desirable to contemplate the ultimate elimination of the delta connection of transformer secondaries. Such a change would involve less difficulty if made along with a complete change from 220-volt to 190-volt motors than it would otherwise. A complete change from two-coil to three-coil protection for three-phase motors is also very desirable.

A rating of 190 volts for three-phase motors is preferable to a rating of 199 volts, allowing a somewhat greater combined transformer and line drop for motors than for lamps rated at 115 volts, without involving the question of motor performance at less than normal voltage.

G. J. Newton (by letter): The three-phase, four-wire system of secondary distribution has long been the standard method in many places abroad; several years ago I made an inspection of several systems in England and in Dublin, Ireland. The recent development of a network protector has made it possible to adopt this system in American practise without any radical change in equipment.

While it is most desirable that the network protector be made as near perfect as possible, still the true economy of the system as a whole will never be reached until proper study and care is given to the other parts of the system. Practically all of the existing underground systems show a decided lack of any real engineering ability in their design and consequently the first cost has been out of all reason.

I believe that now, at the beginning of the adoption of this system, is the proper time to work out some system of standards for all of the various parts entering into the system. A study of the various systems that, so far, have been installed in this country shows that each system is following different methods as to the amount of equipment, etc., that is used at feeding points, and unless a systematic method is decided on, the ultimate result will be a large variety of methods.

The system as a whole permits of exceedingly simple arrangement of equipment in vaults; the reduced amount of cable, conduit, equipment and supplying the street lights from the secondary mains all tend to a greater economy if properly designed.

With the exception of the necessity to meet local conditions as to the location of vaults, manholes, etc., it may be said that one standard system would serve practically all cases with slight modifications to meet these particular requirements. Therefore I would strongly suggest that a committee be appointed to make a very thorough study of the general requirements and try and work out some standards to be followed in future work.

Unless something of this nature is done, the result will be fully as bad as the existing conditions where each system has been installed as a unit and no attempt made to standardize any part of the work.

There are many existing systems that can be easily and economically changed to the three-phase, four-wire system if there is a standard method devised and while on these particular systems it may not be economically possible to comply with every particular, still there would be a guide to work by and every effort could be made towards the standard practise approved.

Where entirely new systems were installed, they could, in most cases, follow the standard method entirely; it is only in the changing of existing systems, where present cable and equipment must be used, that any serious difference need be adopted and even then it is believed that a close approximation could be secured by careful study.

From rough estimates that I have made recently I think that a properly designed three-phase, four-wire system can be installed complete for something less than 75 per cent of the cost of the average separate lighting and power system as now installed, and provide fully as much reserve capacity.

The three-phase, four-wire system equipped with the network protectors provides practically the same reliability as the old d-c. system with the added advantage of the economy peculiar to the a-c. system of feeders; if to these advantages is added a reliable method of street light supply and control from the secondary mains you have an ideal economical and efficient system.

Based on over twenty-five years' experience in the design and construction of underground systems, it is my belief that it is possible to develop a standard method of handling most of this work so that the people interested in the work will have some kind of a guide to work by in either changing their present system or installing an entirely new one.

S. B. Hood (communicated after adjournment): In reading over this paper, and also the companion paper by D. K. Blake, the impression is left with me that the a-c. network of the type for which these relays are designed requires a form of relay in which the major part must consist of brains.

I do not question in the least the ability of our designing engineers to develop a form of relay that will be assuredly reliable in functioning under all the multiplicity of conditions that both authors point out as existing, or liable to exist, in the a-c. network. It does seem to me, however, that numerous precision adjustments must be required in the ultimate form of relay in order that it may function properly under every possible condition that may exist locally at the point of installation.

The difficulty, I can foresee, is our inability, as operating engineers, to get maintenance men with brains commensurate with those built into the protective device.

Mr. Bullard mentions that the relay can be materially simplified by modifications in the system or other apparatus used therewith. This is a feature on which I believe a word of caution is opportune at this time. We must be very careful in this development of a new form of distribution system that we do not permit "the tail to wag the dog."

The primary system, where network protectors are used, requires inherently a greater footage of cable for a given load density and area served than does a loop primary system. To this extent at least the network protector form of system is already burdened, although this burden may be nullified through elimination of high-voltage switches in the various transformer vaults.

In the initial development of the network protector type of system a minimum of three primary feeders is essential. This means that at 4000 volts the initial development, to be economical, must be of at least 6000-kv-a. capacity. If 13.2 kv. is the primary voltage used then a minimum initial economic development of not less than 12000 kv-a. is indicated. In many cases such a large initial development is very hard to justify and may lead to retention or enlargement of existing direct-current networks.

With the loop system, either of the type used in Minneapolis, or that in an even simpler form as used in Philadelphia, the initial development may be relatively small and the system can be extended as required very economically. Further, the loop system can be developed successfully with standard protective equipment of moderate cost and proven reliability. The use of pilot wires between vaults assures effective and reliable system protection with none of the uncertainties or refinements which the authors of the two papers under discussion point out. Since transformer vaults in a network are relatively close together, the pilot wire investment is very slight, and more than offset by the lower cost and increased reliability and simplicity of the relays.

During the four years the Minneapolis loop system has been in operation, the service has been almost 100 per cent perfect, and the few partial interruptions that have occurred have all been due to unwarranted functioning of the protective system. In other words, had the system been installed with no protection whatever, in a manner similar to that now usual with direct current networks, we would have had an absolutely clean sheet as to service reliability. I am not advocating elimination of all protective devices, but merely wish to point out that a complicated or super-sensitive protective system will in itself inevitably cause more interruptions than it will save.

Both authors repeatedly bring to the fore the apparent characteristic of the network protector system to "pump" or "see-saw." This characteristic is at least absent in the loop system network.

In Mr. Blake's paper he mentions that "the normal operation of the regulators would cause no trouble. It is only in the rare condition where a regulator runs to full buck by some means that trouble is caused." The standard automatic induction regulator as furnished us today has a device on it that does exactly this every time the feeder is cut out, the object being to prevent picking up a heavy load of cold tungsten filament lamps at full or boosted feeder voltage. Apparently if we are going to avoid the trouble mentioned byMr. Blake we must cut out this device on our standard regulators.

One of the factors in the network protector relay that seems to introduce complications is that of having it function to open the breaker on transformer core loss. Are we not placing too high a value on core loss savings effected by opening part of the network feeders during light load. The actual value of this core loss at best is the fuel value in the energy we are trying to save. No system capacity or other operating costs will be eliminated. In large systems where such an a-c. network would be used, this item is at most less than one-half cent per kilowatt-hour. Taking a transformer vault of 300 kv-a. capacity, the core loss saving by cutting it out one-half the time would only amount to approximately 4000 kw-hr. per annum, with a possible value of $20. Since this saving would have to be spread over two vaults, one being always in service, the actual saving per vault would run around $10 per annum. It must be clear, therefore, that if the building into the protective system of ability to automatically cut out on core loss exceeds much over $100 per vault or equipment it becomes an economic loss.

In so far as ability to kill an entire feeder by the simple opening of the main feeder breaker is concerned, this is of very questionable value since our safety regulations do not permit, and should not permit, men working on a high-voltage circuit with the only protection a switching device that is held open by automatically reclosing control equipment that may function without warning. It follows therefore that in order to work on a network feeder of the type under discussion, every vault on that feeder must be visited and the breakers "blocked" before clearance is given the crew. With the loop system but two vaults need be visited in order to definitely clear any section of the primary loop, and but one vault is deprived of its supply as compared to a possible one-third the total number of vaults in the network protector system.

Unquestionably the manufacturer will solve all the many problems involved in the development of this new protective device if the distribution engineer will cooperate fully with him; but the great question that persists in my mind is—do we really need this development or are we overlooking the use of something that we already have that will do the same work as well or better?

J. F. Fairman: In answer to Mr. Parson's question as to what proportion of the system has been converted into the new system and what proportion will eventually be changed over, I understand that the question refers to the d-c. to a-c., and two-phase to three-phase change-overs, inasmuch as the new network system will not be ready for service until the summer of 1927. In the period from 1922 to 1926, the d-c. load has been reduced by 22 per cent. Two-phase primary feeders have been completely changed over to three-phase and the a-c. system load has increased 280 per cent and, at the present time, is 78 per cent of the total system load. These changes, together with doubling the transmission voltage on the 60-cycle system, account for the 4-per cent increase in system efficiency. By 1935 it is estimated that the network will have been extended to cover approximately two-thirds of the area of Brooklyn and will carry a load of 500,000 kw. The existing 360,000-kw. capacity of the a-c. substations will be used to take care of large customers buying power at 4100 volts, and the remaining overhead area outside of the network zone.

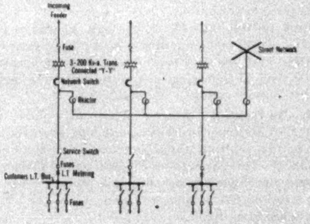


FIG. 2

In answer to Mr. Corney's questions, the accompanying diagram shows the layout which is being used to take care of large office buildings or hotel loads of the magnitude of 1000 kv-a. and above, where the utilization voltage is 120/208. This amounts to a little network in the customer's vault which will be tied into the street network when this is available. These transformers will initially be fed by different 4100-volt feeders from the existing substations and may eventually be supplied by the 27,000-volt network feeders. Each bank will have a capacity of approximately 500 kv-a. consisting of three single-phase transformers or one three-phase transformer, as may be most convenient in each case. The same network switch that is used in the street will be used in these vaults. It will be noted that in the layout shown, each bank and the street network is able to contribute equally in case of an outage of another bank. Thus, the outage of one bank will put on to the street network a demand less than the capacity of one of the units of that system.

In the case of customers requiring a higher utilization voltage, we shall continue our present practise of supplying power at 4100 volts on a high-tension contract, and the customer may arrange his own circuits and his own voltages to suit his convenience. For large customers there would probably be more than one feeder and, as in our practise, we would study the particular case and make our recommendations to the customer as to how he could best accomplish the desired result. Again, such customers might eventually be supplied from one or more of the 27,000-volt network feeders.

With reference to voltage regulation and to the fact that no regulators will be used on the 27,000-volt feeders, I should again call attention to the fact that the drop in these feeders is negligible. The three-phase, 500-kv-a. transformers have an impedance of 5 per cent. The secondary mains have impedances of the order of 5 per cent per 100 ft. The typical city block is approximately 250 x 700 ft. From these figures it may be shown that, with a hand-regulated bus in the generating station, the voltage regulation on the network will be within plus or minus 4 per cent under ordinary conditions.

Without going into detail, I should like to say that we have anticipated Mr. Newton's suggestion and have worked out a plan for street lighting from the network, which we believe will render the same reliable service to the street lamps as we render to the other customers from the network system.

Mr. Hood's statement that the loop system can be installed at lower cost may be true initially but when one considers a system growing as rapidly as the one described, it would be unwise to let the initial costs outweigh the ultimate costs. The system we propose requires an investment somewhat less than the loop system, assuming a primary voltage of 4100 volts. The difference would be greater with higher primary voltages since the cost of the primary switches would be greater.

There would seem to be no justification for the statement that the relays for a loop system are more simple and reliable since both systems use induction-type relays.

We agree with Mr. Hood that savings in core loss would not justify using a relay which would open the network circuit breaker on transformer core loss. It does seem, however, that this characteristic would be most useful on a system of the kind we are proposing, in which there will be between 30 and 40 transformers on each primary feeder in the ultimate development, and these will extend over a distance of several miles. It would be a most laborious process to have to open these switches by hand in case it was desired to work on the feeder or a transformer. We would call attention again to the short-circuiting and grounding switches in the high-tension potheads in the transformers. After the station breaker has been opened and the network switches have opened on magnetizing current it is only necessary to short-circuit and ground the feeder at two adjacent transformers to work safely on the section of feeder between those transformers, or to ground the feeder at the first transformer and the generating station to work safely on that section of the feeder. These grounding switches are of ample capacity to stand the short-circuit current which would flow if any number of the network breakers or the station breaker had not been opened in advance. Further, these grounding switches will be operated from the street outside of the transformer vault, thus assuring further safety to workmen.

G. G. Grissinger: It might be interesting to note that there are a number of network systems throughout the country in successful operation, particularly in points like New Orleans, Memphis, Dallas, and, of course, in New York where it originated, and that in a number of cases on both 4000- and 13,000-volt feeders, primary feeder faults of a serious nature have been cleared satisfactorily.

J. S. Parsons: The theory on which the design of the present network relay is based is fundamentally correct and the development is now well past the experimental stage. Much valuable experience has been gained from the 1000 or more network relays now in service. In view of this experience, I think there will yet be developed a network relay, using the same fundamental principle, which will not place limitations on the design of the network system. It is necessary that such a relay be developed so that the network system can grow to its ideal form and allow the network to be fed from different sources of power and synchronized at the point of load.

W. R. Bullard: In attempting to describe the complex duties which may be imposed upon the prospective apparatus in network systems of the type to which these papers apply, it is very difficult to avoid leaving the impression that the apparatus itself must necessarily be very delicate and complicated. As a matter of fact, in spite of its complex duties, this device really is a relatively simple piece of apparatus. Such matters as phase angles, for instance, are taken care of by simple adjustments in the constants of the relay circuits. These adjustments can be taken care of at the factory, chiefly as a matter of initial design and need not concern the operating man, providing the proper coordination is maintained between the designer of the distribution system and the designer of the relays. The sensitivity necessary to provide opening on magnetizing current of transformers can be provided by an exceedingly simple type of current transformer which gives relatively high values of relay current at low values of line current. The sensitivity necessary for the closing operation is obtained by an even simpler device.

It is only necessary to refer to the figures on percentage failures given in Mr. Searing's discussion to show that this apparatus is making an excellent record as regards performance in actual service. And in this connection I wish to emphasize one point in particular,—namely, that the operating failures that have occurred and are occurring do not affect the service reliability of the system. In other words, the customer never knows about them. This is undoubtedly due to the fact that in this type of system the idea of placing the responsibility upon a number of pieces of apparatus, rather than upon one or two, has been consistently carried out. If one or two switches fail, it is of small importance so far as service is concerned. If a whole primary feeder fails, (as, for instance, might be caused even in a loop system by a regulator short circuit), service will still be maintained.

For some time I have been indirectly in touch with the operation of several systems of this type and know of only two service outages which have occurred. In one of these cases short circuits occurred on both of the only two primary feeders which had then been installed. In the other case, the network was torn apart by the blowing of sectionalizing fuses. The conditions which permitted both of these outages to occur have been or are being corrected,—the first by the installation of additional feeders, and the second by the removal of the sectionalizing fuses. In other words, the service reliability of these systems, in so far as it is related to the full application of this network scheme, has been practically 100 per cent perfect.

The question raised by Mr. Hood concerning the safety to workmen when work is to be done on the high-tension conductors, and a feeder is killed for this purpose by the opening of a station switch, is answered by the fact that the energy necessary for closing each network switch is obtained from the transformer side of the low-voltage circuits. Therefore once the feeder has been killed, it is impossible for any one of the corresponding network switches to close of its own accord until the feeder has first been made alive again by manual operation. However, in order to guard against human errors, it is customary in most systems of this type to short-circuit and ground the primary conductors at one or two different locations when work is to be done. Permanent facilities for doing this are provided at two or three points in each feeder, and the process is therefore no more difficult than clearing the circuit at two points in a loop feeder.

D. K. Blake: I am glad Mr. Milne called attention to the saturation. We do not get a strict measurement of power. But this is an advantage in connection with short circuits because the secondary current lags the primary current, which makes it safe. A highly leading cable charging current in reverse direction would be disadvantageous, but removal of the lag plate would keep it well within the tripping range.

I agree with Mr. Milne regarding the faults to ground and reclosing. I simply had in mind the conditions when a breaker failed to trip during a switching operation made for purposes of working on the feeder or saving the losses.

Mr. Forbes' statement is also true, but in the statement in my paper I had in mind that the reactances in the transformer were inherent; that is, some systems do have 10 per cent inherent reactance and the tripping characteristic must be kept above 84 deg.

The Remote Control of Multiple Street Lighting

BY W. T. DEMPSEY[1]

Member, A. I. E. E.

Synopsis.—This paper describes control systems for multiple street-lighting systems and deals particularly with the systems employed in New York City. The paper is introduced by a brief history of the development of street lighting since 1879 when the series system, the first street-lighting method, was established.

* * * * *

As established in 1879 by the pioneer Charles F. Brush, the generally accepted method of lighting streets and public highways, in so far as the lamp-to-lamp connection and the electrical distributing system are concerned, was by means of the series system which, to this day, is familiar and which down through the years has remained, to a very large extent, the standard method of electrical connection and distribution for supplying service to street lamps.

With the exception of the Borough of Manhattan, in the City of New York, where certain of the streets since 1892 have been lighted in part from the multiple underground commercial network, the series method of service supply has been universally used throughout the country and only within the past several years has any serious attention been directed toward the possibility of lighting streets in congested areas by means of service supplied from the multiple low-tension commercial source. Many installations are in operation with a combination of series and multiple connections, where the lamps themselves are supplied from a multiple service from a 2300-volt primary feeder and a transformer of 115 to 230 volts secondary, the secondary circuit being opened and closed by means of a relay actuated by a nearby series circuit and such relay, in turn, opening and closing a multiple switch on the transformer secondary service supply or by time clock. Also, there are many installations operated from series circuits where a transformer is placed in the circuit for each individual lamp, the primary being energized from the series circuit while the secondary of the transformer is connected in multiple across the lamp terminals, increasing the lamp amperage with consequent improved efficiency. This same combination of circuit is used where a number of series lamps would be connected in a somewhat similar transformer secondary. No doubt the most noteworthy advance in series street lighting was the development some years ago of the constant current transformer, which eliminated the necessity of separate series generator source of supply, all of which indicates, when compared to the original 9.6-ampere series circuit as established by Mr. Brush and which is even standard to this day in many cities, that there has developed over the period of years a flexibility of service supply and control of series street lighting circuits which is advantageous and commendable and which can be applied in many localities where local conditions, for both overhead and underground distribution, are such as to warrant its use.

Previous to November 1892 the limited electrical street lighting of those days in the City of New York was served by means of series overhead lines, but in this year, with the development of the low-tension constant-potential arc lamp, The New York Edison Company entered the field of street lighting.

Today all the street lighting in the Borough of Manhattan is served from a multiple source, that of The United Electric Light and Power Company from a 115- to 230-volt underground main and that of The New York Edison Company from a 120- to 240-volt underground main, the mains in each instance being the same network as used for service supplied to abutting buildings. Also in the Borough of the Bronx, served by The New York Edison Company, the extension of the multiple street-lighting system is progressing from year to year and as in the Borough of Manhattan is operated by the a-c., 60-cycle underground commercial distributing system of 115/230 volts. As of August 1st, 1926, within these two boroughs 70 per cent of the light sources are served from the multiple system, the lamps ranging in size from 25 to 750 watts.

As the growth of the system continued, the lamps were turned on and off by means of a switch in the individual posts, or in many cases groups of posts were turned on and off by a single switch. Also in the lighting of the several parks throughout the city, where a number of lamps was controlled from a single source, each individual circuit was turned on and off by means of time clocks placed in the base of the lamp posts. In 1918, however, the control system as now used was perfected after considerable experimental and development work. Bearing in mind that street lighting is a most important branch of the service rendered by a public utility and particularly so in the congested areas of the larger cities of the country, any system must be so designed as to render the highest degree of reliability. In adopting this method of street-lighting control, therefore, the factors governing the installation are:

First. The controlling circuit must be so designed that outage of a number of street lamps, due to a fault in the control wire or defective relay, is practically eliminated. This is the most important requisite.

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Presented at the New York Regional Meeting of the A. I. E. E., New York, N. Y., Nov. 11-12, 1926.

Second. The system should be flexible as to control, allowing various means to turn the lamps on and off as required, from several sources of supply.

Third. The relays should be so designed as to open and close the circuit with the least possible complication of moving parts; it should be positive in its action and close the lighting circuit by means of gravity. A relay so designed is not dependent on potential for lighting the street lamps but rather to put them out, and when so designed the likelihood of extensive outage is extremely remote, as the lamps will light in case of accidental open circuit of the control wire. This will be seen more readily by referring to wiring diagram, Fig. 1.

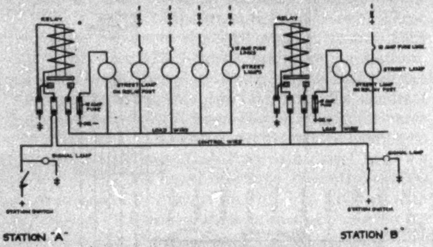
RELAY
STREET LAMP ON RELAY POST
STREET LAMPS
LOAD WIRE
CONTROL WIRE
SIGNAL LAMP
STATION SWITCH
RELAY
STREET LAMP
STREET LAMP ON RELAY POST
LOAD WIRE
SIGNAL LAMP
STATION SWITCH
STATION "A"
STATION "B"

Fig. 1—Wiring Diagram of Remote Control for Multiple Street Lighting
Showing Typical Street Intersections

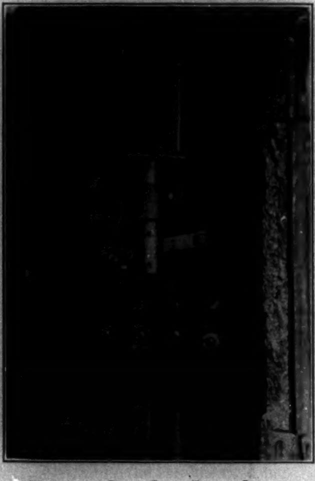

Fig. 2—Street Lamp Post Relay Installation

The system of control as installed by both The New York Edison Company and The United Electric Light and Power Company follows the diagram shown in Fig. 1. The control wire is lead-covered No. 10 or No. 12 B & S rubber insulated and pulled into the distributor ducts occupied by the commercial main cables. These ducts will vary in size from 3 in. to 3½ in. and before installing the street-lighting load or control circuit will contain three cables varying in size from 200,000 to 750,000 cir. mils. The ducts are

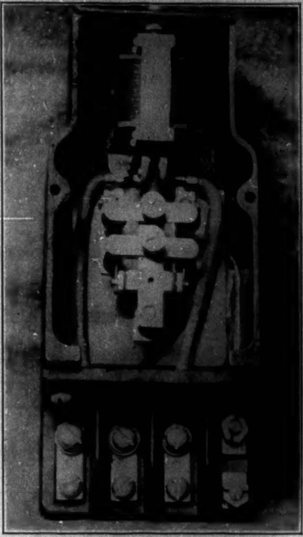

Fig. 3—Relay Assemblage

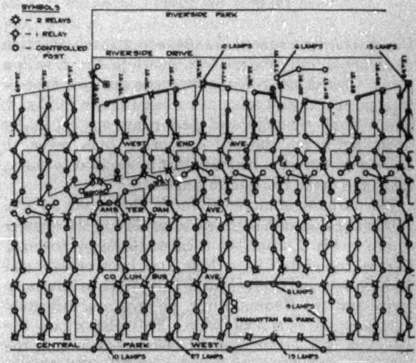
SYMBOLS
– 2 RELAYS
– 1 RELAY
– CONTROLLED POST
RIVERSIDE PARK
RIVERSIDE DRIVE
10 LAMPS
6 LAMPS
15 LAMPS
WEST END AVE.
AMS TER DAM AVE.
CO LUM BUS AVE.
8 LAMPS
4 LAMPS
MANHATTAN SQ. PARK
CENTRAL PARK WEST
10 LAMPS
27 LAMPS
19 LAMPS

Fig. 4—Typical Multiple Street-Lighting Remote Control (Area 64th to 84th St., West Side, New York City)

rodded by means of a 1/8 by 5/8 steel "fish." As shown in Fig. 1, the control switch is closed in Station "B," the relay solenoids being energized, the armature being held in suspension, and the street lamps being out. The signal lamp is lighted in Stations "A" and "B," indicating as an operating detail that the control wire is continuous. The relay connections shown in this diagram can be seen in a photographic reproduction of an actual relay installation, shown in Fig. 2, with a closer view of the relay connections and one type of internal mechanical relay assemblage shown in Fig. 3. The over-all dimensions of these relays are 2¼ in. by 4 in. by 9 in., the coil consuming from seven to ten watts, while the main contact is specified to carry, make and break a load of 25 amperes.

For purposes of more accurately illustrating an area within the City of New York, controlled in the manner shown in Fig. 1, attention is called to Fig. 4, covering that section of the City of New York between 64th and 84th Streets and from Central Park West to Riverside Drive and the Hudson River, an area approximately 1 mi. long and ¾ of a mi. wide. This plan shows the relays as installed at each street intersection and the lamps controlled by these relays, while Fig. 5 is a wiring diagram of the control circuits within this same area.

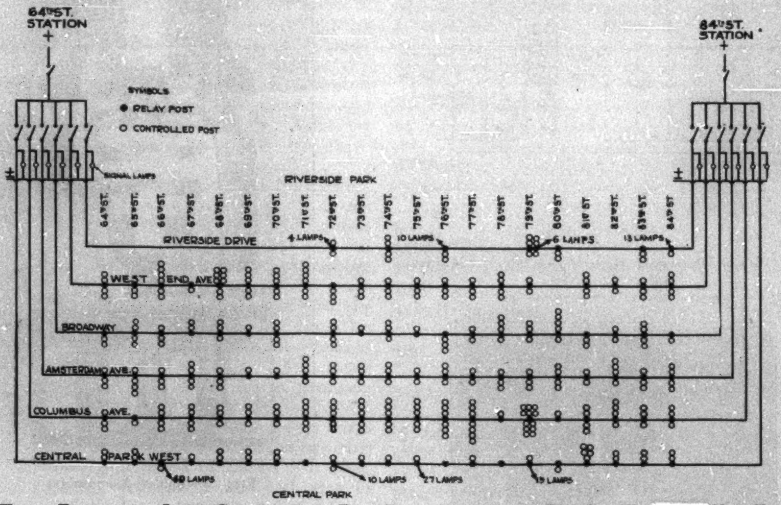
64TH ST.
STATION
84TH ST.
STATION
SYMBOLS
● RELAY POST
○ CONTROLLED POST
SIGNAL LAMPS
RIVERSIDE PARK
64TH ST.
65TH ST.
66TH ST.
67TH ST.
68TH ST.
69TH ST.
70TH ST.
71ST ST.
72ND ST.
73RD ST.
74TH ST.
75TH ST.
76TH ST.
77TH ST.
78TH ST.
79TH ST.
80TH ST.
81ST ST.
82ND ST.
83RD ST.
84TH ST.
RIVERSIDE DRIVE
4 LAMPS
10 LAMPS
6 LAMPS
13 LAMPS
WEST END AVE.
BROADWAY
AMSTERDAM AVE.
COLUMBUS AVE.
CENTRAL PARK WEST
40 LAMPS
10 LAMPS
27 LAMPS
15 LAMPS
CENTRAL PARK

Fig. 5—Wiring Diagram for Lamps Controlled by Each Relay (Area 64th to 84th St., West Side, New York City)

In Fig. 1 it will be noted that the control wire is looped into one relay while teed into the other. This system of connection is used throughout, the loop connection being made on even numbered streets, while the tee connection is made at the odd numbered streets. The object of the loop connection is to provide facilities to enable the breaking up of the control wire in case of necessity, such as testing purposes, into any number of sections that may be required. It also provides a means whereby a particular circuit can be controlled from any street lamp post so connected. The signal lamp shown connected in the control circuits in both Fig. 1 and Fig. 5, is alight when the control wire is alive and indicates continuity of the control wire throughout its entire length.

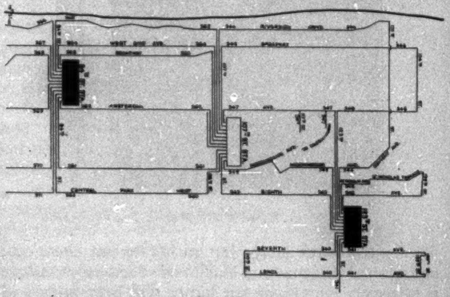

Fig. 6—Section of Remote Control Layout (The New York Edison Company)

Fig. 6 is a section of remote control route map, illustrating in diagram form routes of several control wires from station to station. It shows that control wire will leave one station and terminate in another. Also certain control wires will route out of one station and return to the other station, while in certain other sections the control wire will be teed, the object of such a layout being that there is a check at all times on the continuity of the control wire circuits. Particular substations are selected as operating stations for the turning on and off of the street-lighting system. Those substations shown in the solid squares, such as 84th Street and East 123rd Street, are operating stations, while the blank square as shown by the 107th Street Station is an emergency control point. To illustrate this point further, 84th Street Station is within the area shown in Fig. 6. This station controls all the lighting on the west side of New York from 64th Street to 107th Street, the control board operating 12 control circuits all connected in the manner shown in Fig. 1, so that with the opening or closing of these 12 small switches on the street lighting control board in this station, 1390 lamps are turned on or off within an area approximately 2⅛ mi. long and ¾ of a mi. wide. A further advantage in having either end of a control wire in hand is that it enables operation from both ends, in case of necessity, for subway construction purposes or any changes that may be necessary in the main distributing cables occupying the same duct with the control wire. Also this either-end connection would permit the installation of a time clock for the control of these circuits.

The system of control as used by the United Electric Light and Power Company in the Borough of Manhattan, City of New York, is somewhat different in that there is a relay in each post controlling either the individual lamp or a number of lamps as desired, the control wire being opened and closed by means of a time clock placed in a street lamp post approximately within the center of the control area, with an extension of the control wire into the nearest substation where an indicating lamp is connected to show that the time clock and control circuit are functioning properly.

Again referring to Fig. 1, the connections are identical with the actual photographic reproductions of the terminal block connections in Figs. 2 and 3, where the left-hand terminal is the system neutral, the one to the right being the control circuit, the next being the load wire connection, while the extreme right-hand terminal is a fuse connection on the outside polarity for the particular lamp on the post in which the relay is installed. It will be noted that the make and break of the main circuit is through the system neutral or grounded connection, the outside positive or negative polarity being connected in the individual lamp post through a 15-ampere fuse. For load balancing purposes on the system, the outside polarity of lamps connected to one relay at a certain street intersection will be connected to the negative side of the system while on the next street the lamps will be connected to the positive side of the system. Making and breaking the system neutral on the load wire circuit was done purposely so that at night in case of a relay fault it is only necessary to ground the load wire, at any convenient point, in any one of the several street lamp posts served and the lamps can be lighted in this manner while changes or repairs to the relay can be made conveniently the following day. In leaving the lower terminal block exposed as shown in Figs. 2 and 3 this feature was in mind so that the connection could be quickly and conveniently accessible for change and repair purposes.

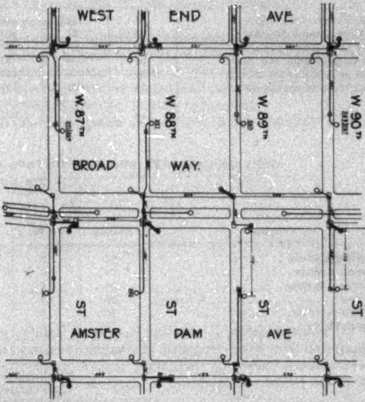

Fig. 7—Actual Layout With Load Wire and Control Wire Running Through Manholes and Handholes

Operating characteristics of the relay shown in Figs. 2 and 3 are as follows:

D-c. relay, with the solenoid cold:

Pick-up voltage	83.5
Drop-out voltage	18.9
Pick-up wattage	3.34

After several hours of operation and a temperature rise in the solenoid:

Pick-up voltage	93.3
Drop-out voltage	23.9
Pick-up wattage	3.63

Fig. 7 shows an actual layout with load wire and control wire running through manholes and handholes, in the subway distributing system, from the substation in 84th Street between Amsterdam Avenue and Broadway through the territory between West 87th Street and 90th Street. This plan also shows the location of each street lamp post with the number of the house in front of which it is located.

Table "A" shows the number of relays and posts controlled within this area on each control wire, the relay posts being 114 in number and the controlled posts 491, or a total number of 605 posts controlled within this area.

Table "B" indicates the various sizes of lamps controlled within this area and it will be noted that the average number of lamps per relay is 5.3.

TABLE A
POSTS IN THE CONTROLLED AREA
64TH STREET TO 84TH STREET WEST SIDE
(The New York Edison Co.)

Circuits	Relay posts	Controlled posts	Total posts
Riverside Drive No. 366	8	52	60
West End Avenue No. 367	20	81	101
Broadway No. 368	20	55	75
Amsterdam Avenue No. 369	21	76	97
Columbus Avenue No. 370	21	95	116
Central Park West No. 371	24	132	156
Totals	114	491	605

TABLE B
SIZES OF LAMPS IN REMOTE-CONTROL AREA 64TH STREET TO 84TH STREET WEST SIDE
(The New York Edison Co.)

Circuits	Relays	Lamps							
		25-W.	100-W.	150-W.	200-W.	300-W.	400-W.	500-W.	750-W.
Central Park West	24	9	59	39	15	9	..	25	..
Columbus Avenue	21	10	8	..	41	16	41	..	..
Amsterdam Avenue	21	9	..	..	35	5	48	..	..
Broadway	20	4	..	..	12	22	34	..	3
West End Avenue	20	8	..	..	50	25	16	2	..
Riverside Drive	8	4	33	..	6	5	15	..	..
Totals	114	44	100	39	159	82	154	27	3

Total number of relays—114
Total number of lamps—608
Average lamps per relay—5.3

TABLE C
REMOTE-CONTROL MULTIPLE STREET LIGHTING AREA
64TH STREET TO 84TH STREET WEST SIDE
(The New York Edison Co.)

Circuits	Total lamps	Total amperes	Kw. load	Control wire	
				amperes	Kw.
Central Park West	156	247.2	29.6	2.00	0.24
Columbus Avenue	116	253.7	30.4	1.35	0.162
Amsterdam Avenue	97	232.7	27.9	0.76	0.091
Broadway	75	189.1	22.7	1.12	0.134
West End Avenue	101	209.1	25.1	1.35	0.162
Riverside Drive	63	102.5	12.3	0.70	0.084
Totals	608	1234.3	148.0	7.28	0.873

Total load controlled—1234.3 amperes
Total load controlled— 148.0 kw.
Control-wire energy — 7.3 amperes
Control-wire energy — 0.87 kw.

Table "C" indicates the total load control in this area and the control wire load, the total load controlled being 1234.3 amperes or 148.1 kw. while the control wire energy required to handle this load is 7.3 amperes or 0.87 kw.

The total annual kw-hr. consumption of the controlling load and street-lighting load is 592,400 kw-hr. street-lighting load controlled by 416 9.7 kw-hr. control wire energy per year within the territory from 64th to 84th Streets, Central Park West to the Hudson River, as previously shown.

The station control board capacity necessary for the operation of this system of street-lighting control is a panel 2 ft. wide and 6 ft. high, on which is mounted one main service switch, shown in Fig. 5, which in turn is connected to a number of small 10-ampere, single-pole, single-throw switches also shown in Fig. 5. The station signal lamps are 10-watt bull's-eye design.

As previously stated, great progress has been made over a period of years in the application of various forms of series lighting circuits and combinations of series and multiple circuits. With the development of the straight multiple control system described above, there has been brought to the art of street lighting an added system of reliability of supply, flexibility of control and distribution.

In the preparation of this paper the author appreciates the cooperation of Mr. A. H. Kehoe, Electrical Engineer, The United Electric Light and Power Company, New York City.

Discussion

A. H. Kehoe: There are a number of systems using the multiple type of street lamp, but the advantages of controlling them by relay has not been generally realized. The subject was intensively studied by the companies supplying Manhattan Island in 1918. Early in 1920 the United Electric Light and Power Company had completely equipped all of its street lamps for relay control of a type as indicated in the paper. This system uses an a-c. relay at every lamp except in the case of park lighting, where an installation's only purpose is for street lighting. With this system, the reliability in operation is very good as a relay failure affects but one lamp.

Several facts were deduced from the cost of making this installation. We found that the relay is not the important element in the cost. The relay used will operate with the smallest conductor which we considered it desirable to use from the standpoint of mechanical strength, this being a lead-covered No. 12 B&S gage. If relays can be made to operate without a special conductor, it will be possible to make a still cheaper installation and yet pay considerably more for the relays. The series system so commonly used requires a large, well-insulated conductor, which makes its cost higher than a multiple-controlled system, provided there is to be other load in the same area; if not, the choice between the two systems will depend upon the particular district to be supplied. The prevailing style of system, of course, has been the series system, and it is important to have the features of the relay-controlled multiple system widely broadcast. It is usually recognized that in densely loaded districts the multiple arrangement may be advantageous, but few realize that many of these advantages hold in the sparsely settled districts (which are now rapidly disappearing on Manhattan). For instance, there is the advantage that the pioneer who comes soon after the street lamps is able to get his service immediately, as it can be had from the street-lamp system.

The combination of all the electric requirements on the single set of mains is the advantage which I feel should be especially emphasized.

N. J. Kelly: In 1923, while the public clamored for the immediate installation of a traffic-control light system on Broadway to relieve the congestion of this heavily laden avenue, the Department of Water Supply, Gas & Electricity delivered some 19 ornamental heavy street-lighting standards to the Police Department, and designed a lamp housing which was attached to the mast arm and had manually-operated switches. These units were erected as a temporary expedient for traffic-control signals and installed on the sidewalks rather than in the street, proper. The idea was new to New York and worked so well that it was decided to "tie in" all units for automatic operation. At this stage, Mr. Dempsey introduced an automatic circuit which was in keeping with the circuit designed for the remote control of multiple street lighting, and met with such success as to be adopted as a standard where automatic signals are required in a d-c. territory.

In this city with its five boroughs comprising an aggregate area of 316 sq. mi., more than 99,000 street-lighting units provide ample illumination to 3400 mi. of streets, avenues and highways. It is therefore of the utmost importance that the control of these units, whether they connect to a multiple or series system, be such as to guarantee maximum service at a minimum cost, and in this Borough of Manhattan, where practically all street lighting is derived from an underground multiple system, trouble has been localized; but continuity of service, which is the governing factor, has been assured.

H. R. Searing: It may be interesting to give some operating results on the multiple street-lighting system of the United Electric Light & Power Company mentioned by Mr. Dempsey. The system has been in operation since 1920. The following table shows the small number of failures of the no-voltage relays to operate. The figures for the year 1925 and the year 1926 up to October 31, are.

GRAVITY-TYPE RELAY

(1242 in service in 1925; 1176 in 1926)

Operation	No. of Operations	Failures	
		Total No.	Per Cent
Drop—p. m.	847,334	83	0.0098
Pick up—a. m.	847,334	15	0.0018

SPRING-TYPE RELAY

(731 in service in 1925; 812 in 1926)

Operation	No. of Operations	Failures	
		Total No.	Per Cent
Drop—p. m.	513,663	29	0.0056
Pick up—a. m.	513,663	5	0.0010

These figures show that relay trouble is insignificant and does not compare with burned-out lamp trouble.

Mr. Dempsey did not point out that the system supply is alternating current and that the time switches which control the circuits are Warren clocks which require no winding.

W. H. Suydam: Local operating conditions dictate to a large degree the type of control and distribution system best suited to a particular street-lighting problem. With low-tension a-c. networks as employed by some power companies, a low-tension multiple network system of street lighting seems to offer a reliable and economical solution. Each block of lamps with this arrangement would be fed as shown in the accompanying illustration.

The control for this type of system narrows down to a selection of either full automatic clock control or pilot control employing relays as described by Mr. Dempsey in his paper. A few desirable time switches which might be employed provided they were located in suitable subway tanks and located in transformer and distribution manholes, are: Sauter, States, Anderson and Warren motor. Of these the Warren motor type seems to show the greatest promise.

Carrier-current control has experienced considerable development during the past few years and may eventually provide a reliable and convenient means of controlling street lamps. The system above mentioned should lend itself readily for conversion to carrier-current control at some future date.

The pilot-wire system as described in Mr. Dempsey's paper is in my opinion simple and reliable. I prefer, however, to work toward extreme simplicity in distribution and total independence of neighboring control apparatus; provided suitable Warren motor clock switches can be obtained this would result in the elimination of the pilot wire.

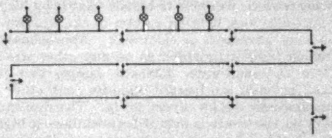

Fig. 1

The New York Company averages 5.3 lamps per control point which is probably due to their peculiar operating conditions. I believe that in connection with a low-tension a-c. network this number could be considerably increased.

It is generally recognized that it is desirable to have lamps go on rather than out in the event of pilot-wire trouble, however; it is also important that they become extinguished promptly at dawn, which would not be the case with the pilot wire until the source of trouble had been located and rectified. On the other hand, with the clock method, all but the particular circuit in trouble would be promptly extinguished. It might be well to note that every alternate clock switch might fail to function without resulting in lamp outage. Two adjacent switch failures would cause one group outage, while trouble with the pilot wire would affect all groups. An advantage often claimed for the pilot-wire arrangement is that it provides a means of checking the continuity of the circuit. While this may be true of the pilot wire, it does not appear that this check can be extended to apply to the relays and lamp circuits. In congested areas I believe the streets are patrolled nightly for lamp burnouts and at such times a defective clock switch would readily be discernible and promptly rectified. Furthermore, experience with Warren motor clocks will enable their further development, resulting in greater reliability. The Warren motor switches might be developed so that resetting could be accomplished without the removal of cover and switches mounted on stab contacts, permitting their ready removal, thus enabling clock substitutions in case of trouble and repairs effected at the shop.

O. F. Haas: The Cleveland Electric Illuminating Company serves some 600 multiple lamps in the residential district of the City of East Cleveland. The house-lighting distribution system is carried along the rear lot lines on wooden poles. The street-lighting feeders are brought down along the side of the pole to the ground and then carried underground along the side lot line to the street lamp at the curb. Time switches are used to turn the lamps on and off, a number being controlled by each switch.

The Minneapolis General Electric Company has standardized on multiple street lighting for the City of Minneapolis, using a system quite similar except that the lamps are turned on and off by a control circuit. Until recently, however, series lamps, rather than multiple lamps, had been used on these multiple circuits. I should like to give just a word of caution in regard to this type of installation, without going into all the technical details involved, which may be obtained from the lamp companies. Where a multiple distribution system is used, multiple lamps should be employed and where a series distribution system is used, series lamps should be employed. Incandescent lamps are designed for a particular service and if they are not used in the manner for which they were designed, satisfactory service will not be obtained.

A new white way lighting installation has just recently been completed on State Street, Chicago. To my mind, one of the most remarkable things about this system is the fact that the Commonwealth Edison Company, by utilizing its duct lines and store-lighting feeders, was able to install what is by far the most outstanding white way lighting system in the country without tearing out any pavement or sidewalks. Approximately three times as much light is provided as on any other street in the world. Two thousand-watt, 4500-c.p. lamps, two per post, are used. The posts are located opposite each other and the energy consumption is 8 kw. every 90 ft. The resultant initial illumination on the street is over 4 foot-candles—a higher level of illumination than obtains in many stores and offices.

L. G. Smith (communicated after adjournment): There is one point which I feel has not been sufficiently stressed. I refer to the subject of maintenance costs, assuming that time switches, relays for use with pilot-wire systems, and Warren-clock time switches, etc., are all equally reliable. There is yet another comparison that should be made in addition to the first costs of the control system; that is, the effect of maintenance costs. In order to keep maintenance costs at a minimum, the control apparatus used should be as simple as possible, because, as a rule, the more complicated a mechanism, the more attention maintenance will require. For this reason, a sturdy relay, simple in design and as fool-proof as possible, would require considerably less maintenance than a time switch.

Moreover, in any multiple system, the essential characteristic of design will be to keep the multiple circuits as short as possible. This necessitates the use of a large number of individual control mechanisms. If time switches are used for each individual circuit, an enormous quantity will be required even in a city of modern size. Assuming a city having about 12 blocks to a mile, each square mile will have about 144 blocks; therefore, a city of approximately 10 sq. mi. would have about 1500 blocks. Assuming that the multiple circuits are so isolated that there is one circuit per block, then 1500 time switches would be required. Comparing this with the use of no time switches and 1500 relays, or a few time switches and 1500 relays, it can be readily seen that there will be a wide difference in the problem of maintenance.

Under certain conditions there seems to be an economy in favor of a multiple system irrespective of whether the control is by pilot wire, a combination of pilot-wire and time-switch control, or a straight time-switch control.

Apparently, the factor that throws the item of economy from the pilot-wire and relay scheme to the straight time-switch system, is the cost of the control wire. Mr. Kehoe has already developed this point in his discussion. However, Mr. Kehoe has emphasized purely the cost of the control wire. There is another and still larger factor in the pilot-wire scheme and that is the cost of the duct. If the distribution system is so laid out that the pilot wire can readily be pulled into a duct where an existing cable is installed, then the cost of the pilot wire is almost negligible due to the fact that the cost of the pilot wire installed is less than 25 per cent of the cost of the duct. Therefore, comparing the economy of the various types of control systems for a multiple street-lighting system, it is necessary to bear in mind the necessity of installing a pilot wire of reasonably low cost in a duct where existing cables are already installed. Otherwise the cost of the pilot wire and relay scheme will be considerably out of line with the cost of an individual time-switch system.

S. B. Hood (communicated after adjournment): This paper is of inestimable value to the industry in that it shows conclusively that street lighting in a large metropolitan area can be successfully and economically carried out without the complete duplication of distribution systems that are involved where series street-lighting circuits are employed.

The description of the New York multiple system is of particular interest to me in that I developed an almost identical system a few years ago for application in Minneapolis, where the bulk of the distribution system is overhead except in a limited congested commercial area. It was not until some months after the first Minneapolis installation was made that I learned of the details of the New York system, as developed by Mr. Dempsey. This duplication of development should serve as additional proof of the practicability of multiple remote-control street lighting.

In the Minneapolis system, where the control is scattered over an area far larger than the underground area in New York, the remote-control system as described in Mr. Dempsey's paper has been enlarged upon by the addition of cascading relays which serve to re-energize control-wire sections. In this manner, the area that can be controlled from any one control station becomes unlimited, thereby making the system even more flexible than any series system. The use of alternating current for the control made necessary the design of a type of relay radically different from that used in the New York system where direct current is used. After considerable experimental development we now have a very simple and reliable a-c. relay that closes by gravity, thereby changing what would be an outage in most systems to a daytime "onage" in both the New York and Minneapolis developments.

With these two installations both operating with an efficiency and proven reliability far in excess of the best of series systems, requiring no duplication of existing investment other than a simple single-wire control circuit, and operating throughout at a voltage not in excess of 120 or 240, there no longer seems any justification for further installation of any form of series circuit which requires dangerously high voltages for its operation, with accompanying hazard to both operating staff and the general public.

It should also be considered that the multiple system operates at the same power factor as the general distribution circuits, usually close to 1.00. Where utility engineers are using every effort to improve the customer's power factor by offering inducement rates for better power factor it seems inconsistent for the same engineers to be purchasing and installing series street-lighting equipment that inherently operates at low power factors, 0.55 to 0.60 being fair averages.

It may be of particular interest to mention that a recent research of the patent situation regarding multiple street lighting showed that Thomas A. Edison developed and patented this remote-control multiple street-lighting system long before series circuits had been commercially developed, so that in adopting the remote-control multiple system we are doing nothing new, but just starting in on the second round of the circle.

W. T. Dempsey: I should like to speak of the points brought forth by Mr. Suydam. In street lighting we have two evils. We may burn lamps in the daytime and we may have them out at night. Personally, I should rather have them burning in the daytime than out at night. Street lighting, we must never forget, is a most important service rendered by a public utility. There is nothing of more importance as a police measure or for traffic reasons.

As to time clocks, they have their place. We have used them for 15 years in turning on and off Central Park, but where we can put a relay in the circuit, we feel there is greater reliability. We have a control circuit in our hands, so to speak, in the station and we are not bothered with adjusting a time clock to take care of the seasonal variations in time.

The number of lamps per relay, 5.3, mentioned by Mr. Suydam, is determined entirely by the size of conductor, allowing for a reasonable drop in voltage in effort to keep the conductor as small as possible. The system has been in very successful operation in New York since 1918 and has been growing steadily all the time. The best point in its favor is that other large cities are installing the same system.

The Lighting of Railway Classification Yards

BY GEORGE T. JOHNSON[1]

Non-Member

Synopsis.—*This paper deals primarily with the illumination of modern large classification yards of the hump type in which the traffic is in one direction. Floodlighting is advocated as the most generally satisfactory system, and various arrangements, to meet the different conditions of yard layout, space available, etc., are described.*

* * * * * *

THE lighting of railway classification yards is a problem of space lighting, the length of yards varying from 3200 ft. to 6500 ft., the width from 400 ft. to 650 ft., and the distance from the hump to the last switch on the ladder, from 1300 ft. to 1800 ft. in our modern yards,—all depending upon the capacity of the yard. The accompanying illustrations show typical layouts of classification yards having four ladder tracks. Ten tracks generally diverge from each ladder track. The space between the ladder tracks is used for the speeder cars which carry the hump riders back to the hump after cars have been brought to rest in the classification yard. In the case of a yard where car retarders are used, this space could be utilized for pole lines carrying a lighting distribution system.

No work other than the classification of cars is performed in the modern classification yard, the air testing and inspection being performed in the departure yard, and consequently there is no necessity for local or intense illumination.

The ladder tracks of classification yards are seldom built on curves; in fact, in yards where retarders are used, the curvature is limited to six deg. Therefore the problem of curved track lighting need not be considered.

There are two adjuncts to a modern classification yard; namely, the receiving yard and the departure yard. No illumination is ordinarily required in these other than that which is necessary for police protection or in cases where short trains are pushed into the receiving yard on the same tracks or diverging tracks, for the storage of previous trains. In some instances, however, where receiving yards are built with considerable curvature, there is an advantage in providing sufficient illumination to enable engine drivers to observe their respective courses.

The tracks connecting the classification and departure yards, covering a distance varying from 400 ft. to 700 ft., require local illumination. The most effective means of providing this illumination is by the angular type of reflector, with the beam of light directed in such a manner as not to affect the vision of the engine driver who is required to cover these tracks and who is frequently unable to obtain the benefit of the locomotive headlight because of the curvature of tracks and on account of switches. With the angular reflectors giving a wide beam, the proper spacing is about 100 ft. for 300-watt lamps at an elevation of 22 ft. above the tracks. A similar type of lighting should be installed at the hump for the benefit of the car riders. The departure yard, as stated above, requires very little illumination other than that necessary for police protection. The inspectors are required to work under cars and between long lines of cars on adjacent tracks and therefore they carry lanterns. Moreover, it would be impractical and expensive to attempt to provide illumination for their benefit, and it would be impossible to show any financial saving or reduction of personal injury hazard.

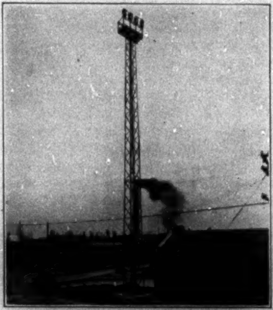

FIG. 1—A 110-FT. FLOODLIGHTING TOWER
In the Markham Yards of the Illinois Central Railroad

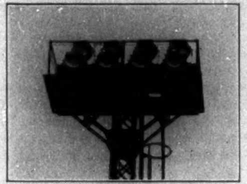

FIG. 2—PLATFORM WITH EIGHT 1000-WATT PROJECTORS
At Cedar Hill Yard of New York, New Haven & Hartford Railroad Co.

Good illumination requires an even distribution of light without glare. In order to obtain this condition, lamps must be so spaced and so placed that there is no interference with one's vision. As space is valuable

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[1] Asst. Elec. Eng., The New York, New Haven & Hartford Railroad Company.

Presented at the New York Regional Meeting of the A. I. E. E., New York, N. Y., Nov. 11-12, 1926.

in a railroad yard, and as the expense of installing and maintaining such an installation is extremely heavy, we have been compelled to seek some other solution of the problem. The method commonly used is the installation of towers 75 ft. to 120 ft. above the base of the rail (Fig. 1), with a number of projectors equipped with 750-watt or 1000-watt lamps mounted on a platform at the top of the tower (see Fig. 2). The protective apparatus and switches are installed at the base of the tower (Fig. 3) in order to avoid all unnecessary maintenance labor. Practically the only necessity for climbing the towers is to change lamps and clean lenses and reflectors, which it is rarely necessary to do more often than every three months in winter and less frequently in summer. It is advisable to place control of the lighting in charge of the hump tower operator. This may be accomplished by installing a small start and stop button in a selected position, the manipulation of which energizes and de-energizes a contactor installed at the lighting tower. Another method is to install time switches at the base of towers which will automatically turn the lamps on and off at a predetermined time. The first method of control enables the tower operator to practise economy in current consumption during slack hours and yet furnishes sufficient light for police purposes. Tower lighting permits adjusting the position of the towers with the result that if space is not available for an ideal location, a change of 100 ft. or even 200 ft. does not materially decrease the efficiency of the installation. There is a possibility (although rarely available in classification yards) of taking advantage of high structures such as coal pockets and bridges for mounting projectors (see Fig. 4).

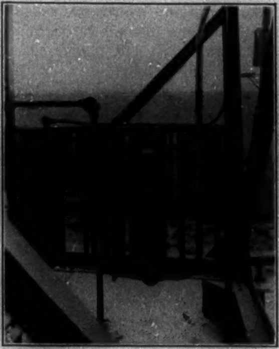

Fig. 3—Typical Switch, Meter and Cutout Installation at Base of Tower

Fig. 4—Projectors Mounted on Coal Pocket

Classification yard lighting may be divided into three general classes or types, each requiring a different solution, as follows:

1. Yards employing hump riders and having hand-operated switches.
2. Yards employing hump riders and equipped with automatic switches.
3. Yards equipped with automatic switches and car retarders.

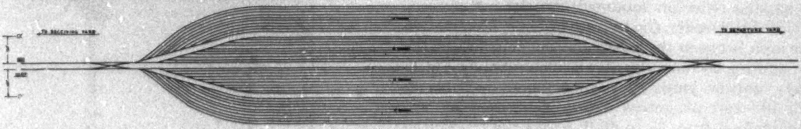

Fig. 5—Typical Lighting With Towers at Hump

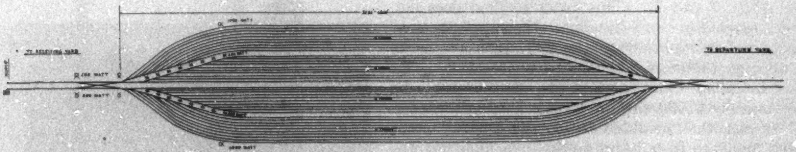

Fig. 6—Modified Lighting System Where Space at Sides of Hump is Scarce

With the first type of yard, extreme care must be exercised to prevent interference with the switchman's vision on account of glare, as this is likely to be the cause of personal injury or accidents to the equipment. If space is available, two towers may be installed, one on each side of the hump and 300 ft. apart, as shown in Fig. 5. If lack of space at the hump makes this impractical, a so-called "modified distribution system" is advisable and the towers may be installed on the outer edges of the classification yard located; 300-watt angular reflectors mounted on poles in the space occupied by speeder tracks will provide illumination in the area between the hump and the point where the tower lighting becomes effective as illustrated by Fig. 6. In very narrow yards it may be found desirable to install 500-watt projectors with a 250-ft. spacing and an elevation of from 35 to 40 ft. above the tracks, the necessary pole line being installed in the speeder car space (Figs. 7 and 8). This latter method does not permit expansion and involves high maintenance costs as well as interference with the operators' vision by the poles; therefore it should not be installed. The cost of installing two towers at the hump amounts to approximately $7000, and involves a low maintenance cost, whereas the "modified distribution system" costs at least $8000 to install and results in a high maintenance cost.

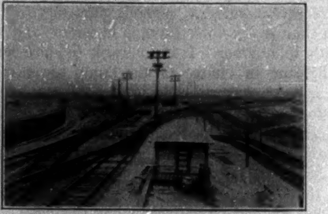

Fig. 7—Projectors on Pole Line in Speeder-Car Space

This is a view of the older installation at Cedar Hill which has been replaced by that shown in Fig. 11

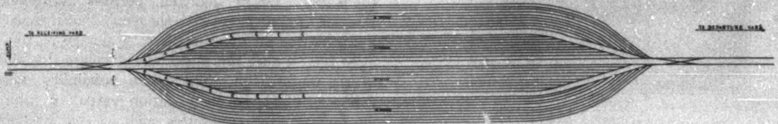

Fig. 8—Typical Lighting of Narrow Yard

500-Watt lamps on 35-ft. wooden poles 250 ft. apart

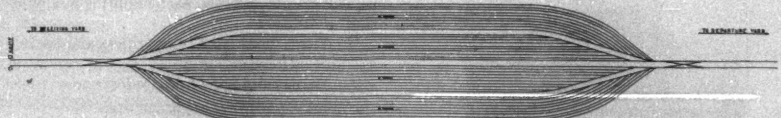

Fig. 9—Typical Lighting of a 3200-ft. Yard With Towers Close to Hump

With the second type of yard it is not necessary to consider the glare as there are no men in the yard and the current of traffic is in the direction of the light beam. One or two towers may be installed in the same relative position as in the first type of yard, or, if space is not available at that point, they may be installed close to the hump, as shown in Figs. 9 and 10. The cost of these towers will be approximately $7000. For a 3200-ft. yard no other lighting is required for the classification yard, an unobstructed view being obtained both from the hump and by the car riders (Fig. 11). The power required for a 40-track yard is approximately 16 kw. The installation and maintenance cost is low. The maximum illumination is at the switches, and the hump rider is enabled to detect track conditions clearly; moreover, in distant parts of the yard, sufficient illumination is provided to permit the observation of low built cars in time to avoid collision with them.

Where yards are of extreme length it will be necessary to place at least two towers approximately 2000 to 3000 ft. from the hump, if possible, (Fig. 12). Where space is not available for this, the towers may be installed on the outer edges of the yard at the same distance from the hump. The wattage requirements for a 6000-ft. yard are from 12 kw. to 16 kw. and decrease pro-

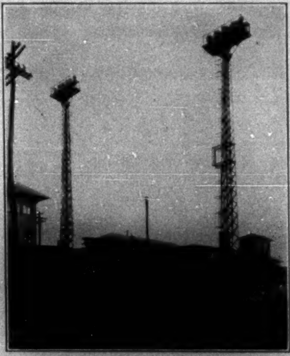

Fig. 10—Two 70-ft. Towers Close to the Hump

Each tower has eight 1000-watt projectors portionately as the yard decreases in length. The increase in installation cost over a 3200-ft. yard will amount to approximately $12,000, making a total installation cost of about $19,000, with a maintenance increase of slightly over 100 per cent. There is some possibility that at some time in the future, lamps of higher wattage may be used in the 5000- and 6000-ft. yards, with a resultant reduction in the number of towers required and a decrease in the installation and maintenance costs, but in considering any such proposition the possibility of interference with the vision of adjacent interests must be taken into account.

FIG. 11—DAY AND NIGHT VIEWS OF YARD LIGHTED BY TOWERS AT THE HUMP

This is Cedar Hill yard of the New York, New Haven & Hartford Railroad. It is a 3200-ft. yard lighted by eight 1000-watt lamps on each of two 70-ft. towers. The pole holding two transformers and the pole beyond, holding two projectors, are part of the old system and will be removed.

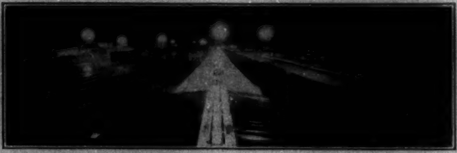

FIG. 13—YARD ILLUMINATED BY FLOODLIGHTING PROJECTORS

This is the Selkirk yard of the New York Central Railroad.

A distribution system similar to that shown in Fig. 8 would cost approximately $15,000, and would result in an increase of maintenance costs as compared with the tower lighting proposition of at least 1000 per cent.

FIG. 14—YARD VIEW SHOWING CAR RETARDERS

This large maintenance cost is due to the maintenance requirements of a large number of poles and projectors. Moreover, the poles would interfere with a clear view of the yard, and would be subject always to destruction on account of derailments with consequent delays in operation. The intensity of illumination is not so great with this type of lighting, although the distribution of light is more even. The only advantage over the tower lighting is in the case of yards so situated that there is a considerable amount of smoke and steam present. This may be considered as a special condition, as yards are usually located at a sufficient distance from engine houses and other smoke producing causes so that this advantage may be eliminated.

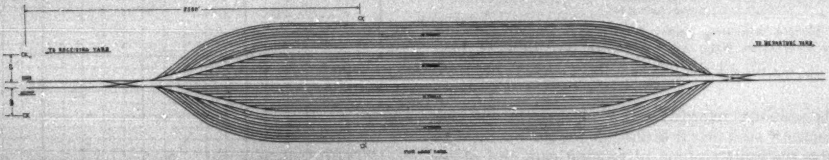

FIG. 12—LIGHTING SCHEME FOR VERY LONG YARD

The third type of yard is similar to the first, but the personal injury hazard is eliminated as no hump riders or switchmen are required in the yard.

The retarder operators are located in secondary towers situated adjacent to the groups of retarders which they control. These towers are in the area of the ladder switches and are at an elevation above the tracks which permits an unobstructed view of the yard. Up to the present time no economical solution of this problem has been worked out. If lighting as recommended for the second type of yard is installed, the glare interferes with the vision of the operators. In this connection it is possible that a visor effect may be obtained by painting the upper part of the tower windows, thereby shutting off from the operators' vision the direct beam of the lights.

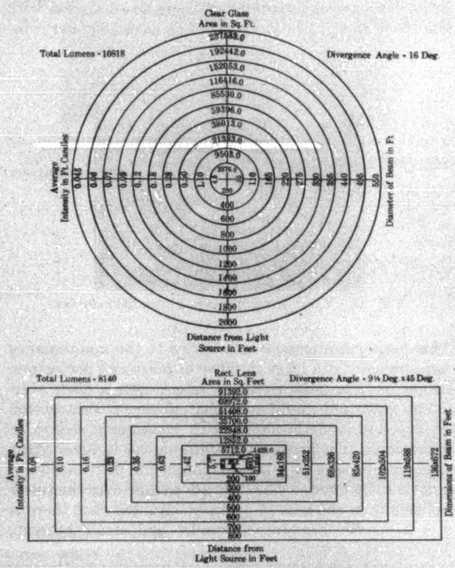
Clear Glass
Area in Sq. Ft.
Total Lumens - 10818
Divergence Angle - 16 Deg.
192442.0
152053.0
116416.0
85530.0
59396.0
38013.0
21383.0
9503.0
2376.0
Average
Intensity in Ft. Candles
0.043
0.07
0.09
0.12
0.16
0.50
1.10
4.4
55
110
165
220
275
330
385
440
495
550
Diameter of Beam in Ft.
200
400
600
800
1000
1200
1400
1600
1800
2000
Distance from Light
Source in Feet
Total Lumens - 8140
Rect. Lens
Area in Sq. Feet
Divergence Angle - 9½ Deg. x45 Deg.
91392.0
69972.0
51408.0
35700.0
22848.0
12852.0
5712.0
1428.0
Average
Intensity in Ft. Candles
0.08
0.10
0.16
0.25
0.35
0.63
1.42
5.7
100
200
300
400
500
600
700
800
34x168
51x252
68x336
85x420
102x504
119x588
136x672
Dimensions of Beam in Feet
Distance from
Light Source in Feet

Fig. 15—Illumination From 1000-Watt Lamp in Floodlight Projector With and Without Rectangular Lens

A 1000-watt lamp with PS-52 bulb was used.

The position of retarders with respect to the operator is frequently such that it is difficult for the operator to observe the exact position of moving cars and of retarders at night on account of shadows (Fig. 14). A possible solution of this problem may be found in "marker lights" so placed that they will indicate when the braking effect on cars is to be applied to the retarders. This solution of the problem would be very economical.

Any system of local illumination covering retarder areas would be expensive and would cause interference with track clearances, which must be avoided.

With the increased experience of retarder operators there will be no necessity for illumination greater than that provided in yards using hump riders and automatic switches, since each operator will know the loading condition of his tracks and will be governed accordingly. The same problem presents itself in lift-bridge operation, where actual practise has demonstrated that the experienced operator knows the position of his bridge at night and does not watch the indication lights, but prefers a marker light so placed that it will not be necessary for him to direct his attention in another direction.

The Committee on Illumination of the Association of Railway Electrical Engineers in November, 1923 reported the following advantages of yard lighting:

1. Speeding up of cars handled in the yard at night time.

2. Reduction in cars damaged by rough handling and collision in the classification yard with consequent reduction in claims, delay in delivery of goods, loss of service of damaged cars, etc.

3. Reduction in losses due to pilfering, on account of more effective policing possible with a well illuminated yard.

4. Improved working conditions and increased safety for employees working in the yard.

The studies and investigations of the Committee on Illumination in conjunction with the lamp and projector engineers, although not as yet completed or published, already have resulted in more efficient units and more economical installations. The problem of lenses deteriorating with age, with resultant decrease of efficiency, has been practically solved. Curves and engineering data are now available (Figs. 15 and 16) which make it possible to plan installations with confidence that the results anticipated will be achieved. The problems of space lighting may be solved in various ways on account

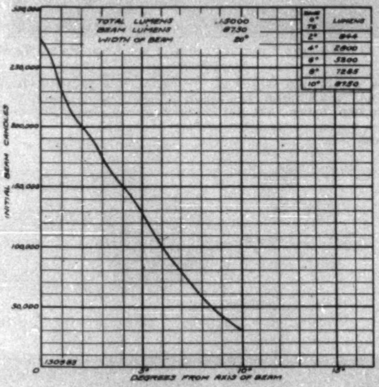

Fig. 16—Illumination Curve of Floodlight Projector With 1000-Watt Lamp

A 115-volt, 1000-watt, Mazda C lamp with PS-52 bulb was used of local conditions, but the fundamental principles are the same in all instances.

Floodlighting is not a "cure-all," but the results are satisfactory to the operating people and provide for them the operating conditions which they have been seeking.

No mention has been made in this discussion of silhouette effect, since it is considered that while theoretically this feature may have some value, at the present time the effect on the vision due to glare more than offsets the benefits. Further studies will be necessary before this type of installation can be recommended.

Discussion

J. A. Summers: One of the difficulties in yard lighting is that space where towers or projectors may be located is valuable. The lighting of the area in the yard itself is very simple, but there are many problems which must be overcome in connection with glare and location of projectors. The intensity may be as low as 0.21 foot-candle. With such low intensities and poor reflection factors from the cars, the problem is not so simple as that of lighting an open space.

If the ends of the cars had white targets across them,—a circle or a band,—it would help the visibility very materially. Such targets could be painted on when the cars are brought in for repairs.

R. W. Cost (communicated after adjournment): The method of yard lighting as outlined by Mr. Johnson is known as "uni-directional" and provides illumination in the direction of traffic primarily to avoid glare to the operators. Another arrangement of projectors known as the "parallel-opposing" system is widely used by railroads in yards employing hump riders and equipped with automatic switches. It consists essentially of opposing projectors banks with four to eight 1000-watt units mounted on towers at least 70 ft. in height and spaced 2500 ft. to 5000 ft. apart. With this system the opposing beams provide more even distribution of light over the entire yard and objects are visible either by direct reflection or in silhouette, depending on the position of the observer in the yard. When traffic moves in the face of projected light there is, of course, the disadvantage of possible glare but it can be minimized if the trainmen wear vizored headgear.

Mr. Johnson states that power for lighting decreases proportionally as the yard decreases in length. As the total wattage or lumen requirements for a yard are usually in proportion to its area, it would also be necessary to take into consideration the yard width as well as its length. The Association of Railway Electrical Engineers, whose Committee on Illumination has made a rather extensive investigation of this subject, advocates the use of 0.1 lumen per sq. ft. of yard. Only the available lumens in each projector beam, which vary from 35 per cent to 45 per cent of the generated lumens, can be used in calculating the total wattage required.

Where a classification yard is built on a curve the number and location of floodlighting projectors will vary somewhat from the arrangement in a straight yard of the same area if light distribution and illumination between car tracks are to be comparable.

G. T. Johnson: With reference to Mr. Cost's discussion, I discussed that in the main body of the paper, and mentioned the fact that I believed it impractical for the men to wear visors. I did not discuss silhouette lighting, as at the present time this has not been developed to a point where we consider it a good practical proposition. It has considerable merit and undoubtedly will be used to some extent in the future when a means has been developed to obviate the glare.

The Induction Lamp, a New Source of Visible and Ultra-Violet Radiation

BY TED E. FOULKE*
Non-Member

***Synopsis.**—This paper deals with the induction lamp, or more generally speaking, the electrodeless electrical discharge. This lamp is a rich source of visible and ultra-violet radiation. The lamp has no electrodes, no conducting filaments and no conducting arc, as ordinarily understood. It operates on the principle of the electrodeless discharge which is produced in a partly exhausted vessel within a rapidly changing electromagnetic field. The discharge consists of visible and invisible radiations. The paper contains a historical introduction, an explanation of the theory of the electrodeless discharge and a description of the results of experiments.*

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HISTORICAL INTRODUCTION

THE first account of the electrodeless discharge was given by Sir J. J. Thomson[1] and describes the glow that is set up in a partly exhausted vessel by the discharge of a condenser through a coplanar coil surrounding it. In his experiments a Wimshurst machine was used to charge the condensers and the discharge took place through the coil and between the spark-gap terminals which consisted of two polished balls in air. This arrangement is clearly shown in Fig. 1, in which it will be observed that there are no electrical connections with the gas within the vessel. The electrodeless discharge, according to Thomson, was explained by the voltage induced in the space surrounded by the coil.

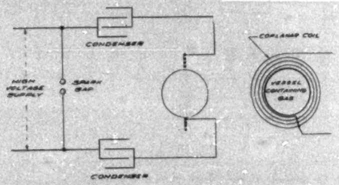

FIG. I—THE ELECTRICAL ARRANGEMENT, AND EDGEWISE VIEW OF COPLANAR COIL SURROUNDING VESSEL IN WHICH ELECTRODELESS DISCHARGE TAKES PLACE

This conclusion was criticized by E. Lecher[2] and A. Steiner[3], who believed that the impedance of the coil would result in a high potential building up across the coil and the electrostatic field produced would be sufficient to cause the glow.

Three experiments performed by Thomson showed beyond reasonable doubt that the electrodeless discharge is a true induction discharge. The first consisted in setting up an electrodeless discharge in a vessel that was completely shielded with a sulphurated blotting paper connected to ground. The second experiment consisted in placing a partition in the vessel that broke up the circular ring discharge into two elliptical ring discharges in the plane of the coil. In the third experiment a steady magnetic field was *superimposed* on the inducing field, which resulted in splitting the ring discharge into two rings tilted at an angle from the normal plane, just as would be expected if the current flowed alternately in one direction and then in the other.

The electrodeless discharge received little attention in technical literature until 1903 when Bergen Davis[4] carried his investigation to the calculation of the voltage gradient set up in the vessel necessary to start the ring discharge. He found that the discharge gradient reached a minimum at a certain pressure and steadily increased as the pressure deviated above or below the value for the minimum. The approximate values found for several gases are tabulated below:

Gas	Min. Volts/Cm.	Pressure Mm.	Bulb Radius
Air	21	0.08	6.0 cm.
CO_2	30	0.06	6.0 cm.
H_2	25	0.17	6.0 cm.
H_e	18.5	0.23	6.0 cm.

The actual values are somewhat affected by frequency, though in a general way the results are very similar to those obtained with the electrode discharge at reduced pressure. The essential difference between the two discharges is found in the pressure required for the minimum discharge voltage. In the electrodeless discharge the critical pressures are consistently lower than those found in the electrode discharge.

The conductivity of the gas was also studied by Davis[5] who observed that the ionization of the gas was fairly high during the period of the discharge. The ionization of the gas and the intensity of the glow were found to be closely related. Observations on several gases indicated that the discharge and ionization did not occur below 0.03 mm. pressure of mercury, or above 6.0 mm., and in the range of 0.6 mm. to 6.0 mm. only a faint discharge and small ionization current were observed.

Several experimenters have since applied the electrodeless discharge to the study of spectra. Among

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1. For all references see bibliography appended.

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these, Kowalski[6] in 1914 found that a bright mercury discharge could be obtained in a vessel that was placed in an oven at 120 deg. cent. The bright discharge was maintained at temperatures below 140 deg. cent., but disappeared at higher temperatures leaving only a weak emerald green glow. Similar experiments were carried out by Robertson[7], who was able to start the discharge at 70 deg. cent. and found that the most intense discharge occurred at 110 deg. cent. He also carried out experiments of the same nature in lithium, sodium, potassium, iodine and hydrogen. Other spectra have been produced by this method. Dunoyer[8] in 1921 studied caesium; Van der Lingen[9] in 1922, the spectra of mercury and cadmium; and a resumé of the subject was made by Miedel[10] in 1924.

The next interesting development was made by L. & E. Block[11] in 1923 on the development of high order spectra from mercury. It was observed that at low spark-gap voltage the discharge showed the normal mercury-arc spectrum, but with increase in spark length and voltage, the normal ring moved toward the center, and a new ring, formed nearest the vessel wall, showed the first spark spectra. Still higher voltage displaced both rings toward the center and a third ring appeared containing lines of the second spark spectrum. Following this same line of experiments, L. & E. Block and Dejarden[12] were able to produce higher order spectra in neon, argon, krypton and xenon. Forte & Ruark[13] have obtained similar results by reducing the pressure at a given exciting voltage. A reduction in pressure increases the free path of the electrons and thus allows higher electron velocities to be gained. In their study of mercury they obtained discharges at room temperature in bulbs exceeding 25 cm. in diameter.

During recent years many carefully interpreted experiments have been carried out to explain the influence of electron velocity on the spectra and ionization of atoms and molecules. Other experiments deal with a mixture of atoms or molecules part of which have become active either by the absorption of radiation of the proper frequency or by the impact with rapidly moving electrons. The correlation of these isolated facts with the experimental data on the electrodeless discharge leads to an understanding of the mechanism of this type of discharge.

THEORY OF THE ELECTRODELESS DISCHARGE

The so-called electrodeless or induction discharge is obtained by inducing a flow of current in a vessel containing a rarefied gas. The most suitable arrangement of the induction coil and discharge vessel is shown in Fig. A, Plate 1.

The phenomenon in this case is to a certain extent similar to that which occurs in the secondary of a high-frequency transformer in which the secondary consists of a closed ring of variable conductance. The essential difference lies in the fact that the induced e. m. f. must exceed a certain critical voltage, known as the ionization potential, before the gas becomes conducting. The conducting or ionized state is due to the impact of electrons with the atoms or molecules of the gas.

To understand the mechanism of the induction discharge we must consider separately the three-fold purpose of the gas. The first is its functioning as a source of the initial electrons; second, its property of becoming ionized when placed in a rapidly changing electromagnetic field; and third, its ability to emit radiation. The first function arises in the capability of all substances, to a certain extent, to absorb the highly penetrating radiation that permeates all space, the atmosphere of our earth, and, at least for a certain distance, all solid bodies. The absorption of this radiation by atoms or molecules, particularly in the

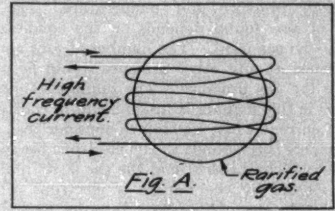
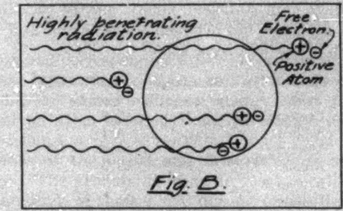
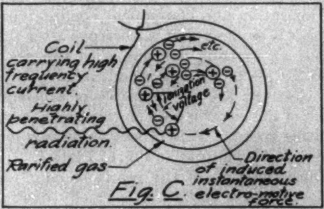


Plate I

A—Experimental Arrangement of Lamp and Coil



B—Illustrating the Random Formation of Ions by the Highly Penetrating Radiation



C—Ideal Example of Ionization by Collision Due to Electrons gaseous state, causes the ejection of one or more electrons from them. The haphazard fashion in which the atoms absorb this radiation and are split up into free electrons and positively charged atoms is shown in Fig. B, Plate 1.

The atom or molecule in this state possesses a residual positive charge and is capable of moving in an electrostatic field; it is then called an ion. It has been found from experiment that about one to ten atoms or molecules are split up in this fashion per sec. per cu. cm. at atmospheric pressure and temperature. The exact number of ions found depends on the condition and nature of the gas used. The number of ions and electrons formed at any other pressure would be proportional to the gas pressure. The number of electrons generated in this manner is insignificant in constituting a flow of current when we stop to consider that a flow of one ampere corresponds to 10^{18} electrons per sec.

The second characteristic of the gas is its ability to become ionized by the impact of electrons with the atoms or molecules of the gas. In order to ionize the gas a given electron must gain a kinetic energy greater than a certain critical minimum energy. The equivalent difference of potential through which an electron must fall to gain sufficient kinetic energy to ionize an atom or molecule, lies between 3.9 volts and 24.5 volts, as shown in the following table:

Atom	Minimum Critical Voltage
Caesium	3.9
Sodium	5.1
Mercury	10.4
Argon	15.5
Nitrogen	16.3
Helium	24.5

Thus, starting with the feeble ionization produced by the penetrating radiation, it is possible by means of the induced electromotive force to increase greatly the state of ionization and consequent conductivity of the gas. For it is evident that n electrons after they have each made an ionizing collision with the gas will increase to $2n$ electrons and n additional positive ions. The $2n$ electrons are then capable of acceleration by the action of the electric field on them and they in turn gain ionizing speed. This process is termed ionization by collision and is pictured in Fig. C, Plate 1.

The third function of the gas is to emit radiation. To be specific, this radiation is chiefly discontinuous in so far as it differs from the continuous band of radiation from incandescent radiators. The characteristic radiation from the atom is of the monochromatic type, that is, it consists of a series of discrete wavelengths, whereas the radiation from molecules, though monochromatic in detail gives rise to bands of radiation that cover large wavelength intervals.

According to modern theory, atoms and molecules possess discrete stationary energy states. The normal energy state is the one in which we find them under ordinary conditions and by the expenditure of energy higher states may be obtained, for instance, by the impact of a rapidly moving electron with an atom or molecule. The transition from a low energy state to a high energy state takes place only when the available energy is equal to, or greater than, the difference between the two energy levels. When the atom returns to the normal state, the energy is given up in the form of radiation of a definite frequency.

A formal explanation of the stationary energy state can best be obtained from Bohr's theory. In the simplest atom, that of hydrogen, Bohr postulates a single electron rotating around a central positive nucleus which, contrary to the electromagnetic theory, does not radiate according to the frequency of rotation. The energy in a definite orbit is equal to the sum of the kinetic and potential energies and is calculated accordingly from simple mechanics. Bohr postulates that only orbits of a definite radius are permissible and so determined that

$$2\pi \times \text{angular momentum} = n\,h$$

He then arrives at the energy in any state n

$$W_n = \frac{2\,\pi^2 m e^4}{h^2}\,\frac{1}{n^2}$$

where

W_n = the energy in the stationary state n
n = assumed integral values 1, 2, 3, etc.
e = the charge on the electron
m = the mass of the electron
h = Plancks constant

If we then let W_a be the energy in the normal state and W_e be the energy in a higher state corresponding to an orbit of larger radius, then the energy required to bring about this transition is

$$\Delta E = (W_a - W_e)$$

After the atom absorbs the energy ΔE and charges to the high energy level, it is in an unstable condition from which it returns in a short interval of time—with the exception of a few cases, in about 10^{-8} seconds. The return to the lower energy level is accomplished by the liberation of energy in the form of monochromatic radiation. The relation between the energy and the frequency of radiation is given by the following relation:

$$\Delta E = (W_e - W_a) = h\,\nu$$

where

ν = the frequency of radiation

Atoms and molecules possess a large number of energy states and once put into the higher energy levels may return to the normal state, that in which they exist under ordinary conditions, directly by emitting radiation of the frequency

$$\nu = \frac{\Delta\,E}{h}$$

or indirectly through a series of intermediate levels.

For example, let W_a and W_e be the initial and final state, respectively, and W_1 and W_2 be two of the intermediate states wherein the atom exists; on returning from the state W_a we would then obtain radiation of frequency.

$$\nu_a - 2 = (W_a - W_2)/h$$
$$\nu_2 - 1 = (W_2 - W_1)/h$$
$$\nu_1 - e = (W_1 - W_e)/h$$

The total energy radiated in the three frequencies is

$$W_a - W_e = h\,(\nu_{a-2} + \nu_{2-1} + \nu_{1-e})$$

In order to gain a better picture of the energy levels in an atom, the levels and wavelengths due to a few transitions between them are plotted in Fig. 2 for the mercury atom.

The normal state is indicated by $1S$ and the other levels represent excited states. The wavelength in angstrom units and the relative intensity are marked on the lines showing transitions that occur. Not all possible transitions occur as will be observed on this diagram. Each atom has its own series of levels and the transitions between them produce spectral lines that are characteristic of the individual atom.

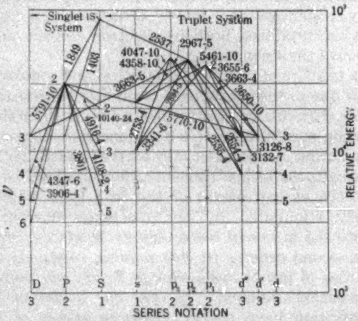


FIG. 2—A FEW ENERGY LEVELS IN NEUTRAL MERCURY

Transitions from high orbit numbers "or high energy states" to low orbit numbers causes emission of monochromatic radiation. It will be observed that the high energy levels actually correspond to low relative energy.

Atoms put into the higher energy levels are said to be in an excited or activated state and are usually called excited atoms. The life of an excited atom is of the order 10^{-8} sec., but in a few cases a return to the normal state does not occur until the atom absorbs additional energy or is disturbed by a collision with a neutral atom. In the latter case the return to the normal state may occur by activating the neutral atom, or both impacting bodies can take up the energy in the form of translational kinetic energy. In either case the originally excited atom returns to the normal state without the emission of radiation. This type of encounter is called a collision of the second kind to distinguish it from the type of impact in which an electron or atom of high kinetic energy transfers this energy to another atom which then becomes excited.

We will now try to show the various factors that govern the character of the electrodeless discharge in a gas or vapor. Let us first of all consider a vessel exhausted under conditions such that a perfect vacuum is attained. When the vessel is placed in a rapidly changing electromagnetic field, an induced e. m. f. will be set up, but as the vessel is free of gas, free electrons will not exist and current cannot flow. The addition of a small quantity of gas causes the spontaneous formation of ions and electrons by the penetrating radiation. Under the action of the induced voltage the positive ions start to move in one direction and the electrons in the other, constituting a flow of current. This current, as was previously pointed out, is insignificant if not further amplified. The current can be increased, however, by ionization by collision. It has been found by experiment that the electrons are the most effective in the ionizing process. The positive ions formed, while not important in the ionization of the gas, do play an important role in their space charge effects. Two conditions are necessary before an electron can ionize. First, the electron must gain a velocity and corresponding kinetic energy great enough to cause ionization of the gas, and second, the electron once having gained sufficient energy must then encounter an atom before it loses its energy, either by going against a reverse potential or by colliding with the walls of the vessel and dissipating its available energy. The probability that an electron will collide with an atom of the gas will depend on the density or pressure of the gas. In a vessel of infinite dimension there exists a frequency of collision with the atoms of the gas and the distance traveled between encounters is called the free path of the electron in the gas. The collision frequency is a statistical problem as the gas is in a state of chaos. The term collision frequency, therefore, must apply to the average number of collisions per second. The reciprocal of the collision frequency must imply the average time between encounters. Multiplying the average velocity by the average time between encounters gives the mean free path of the electrons in the gas, which is found to be inversely proportional to the gas pressure. The mean free paths of electrons in several gases have been calculated from the kinetic theory values at one bar.

Gas	Mean free path of electron at 1 bar[2]
	C
Helium	165. cm.
Hydrogen	108. "
Argon	60. "
Nitrogen	56. "
Mercury	18. "

Therefore, the mean free path at any other pressure is

$$\lambda = \frac{C}{p}$$

where

λ = the mean free path in centimeters
C = the mean free path at one bar
p = the pressure of the gas in bars
1 bar = 0.00075 mm.

In discharge conditions available in the laboratory we must necessarily use vessels of practical size. These may be several centimeters and seldom greater than 30 cm. in diameter. An electron traveling in the largest circle of a 30-cm. bulb would make but one collision per revolution in argon at one bar and in mercury about five collisions at the same pressure. In the small bulb the electron would make, on the average, one collision per ten revolutions in argon at one bar and one collision per three revolutions in mercury at one bar. It has been observed as an experimental fact that the gas does not become highly conducting until the electron makes many encounters per revolution.

The electrons in colliding with atoms of the gas do not lose an appreciable energy to the atoms unless their available energy is greater than that required to cause a transition between two energy levels. There is, however, a small loss due solely to the elastic impact between the two bodies. The electron on colliding with an atom loses on the average a fraction of its energy equal to

$$\theta = \frac{1}{2} \frac{m}{M} E$$

where

θ = the fraction of energy transferred to the atom
E = the kinetic energy of the electron
m = the mass of the electron
M = the mass of the atom

The fraction θ for electron impacts with argon and mercury atoms is of the order 10^{-5}, an insignificant amount per collision. At low pressures this loss does not play any part in the ionization and conduction, but at moderate pressures, of the order of a centimeter pressure of mercury, the loss in energy per centimeter of path traveled is appreciable. At high pressures the loss entailed simply by elastic collisions is so great that an electron cannot gain sufficient energy to excite or ionize the gas, and the latter is in a state of low conductivity.

We see, therefore, that an optimum condition exists wherein the electron makes enough ionizing collisions to bring about a state of high conductivity and excitation, and not enough to cause serious losses due solely to elastic impacts.

Let us now consider the effect of pressure on the degree of excitation and the spectral transitions that occur. When the pressure is low and the mean free path is long, the electrons as a whole will have high velocities and thus will be able to cause the atom to become highly excited. This may mean, according to the Bohr theory, that the valence electron goes into an outer orbit, or that the electron gets beyond the influence of the positive nucleus and this is detached from the parent atom. In the case of mercury the latter will occur when the atom absorbs 10.4 volts energy from the electron. Still higher excitation may occur where the second valence electron is ejected into an outer orbit or so-called higher energy level. As the pressure increases the average electron velocity decreases because of the more frequent loss of energy by excitation and by elastic impacts. At high pressure the energy is controlled mainly by the elastic impacts and the electron reaches a final velocity depending on the voltage gradient and free path.

In strong discharges transition to the higher levels and complete ionization may occur, due to the fact that enormous numbers of excited atoms present in the low energy levels are bombarded by electrons having energy insufficient to ionize directly. The excited atoms are produced both by electron impact and by collision of the second order.

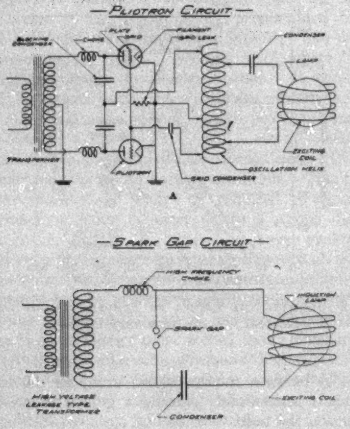
In a mixture of gases the excitation and ionization is usually due to the constituent of greater concentration, or more particularly, shorter mean free path or shorter ionization path, though this by no means indicates that the radiation is characteristic of this constituent. If the constituent of low concentration has lower energy levels, the excited atoms of the first type which are strongly stimulated by the discharge current will activate the atoms of lower energy levels by collision of the second order. In this manner relatively small quantities of gas of lower energy levels will dominate the radiation obtained. In case the atoms of lower concentration have energy levels in excess of those primarily stimulated by the electron current, the spectra of the stimulated atoms only show up in the discharge. These atoms of higher energy level, however, do influence the primary stimulated atoms. This arises in their collision with excited atoms. Though their excitation levels are higher than those of the excited atoms, an impact of the second order may occur wherein the excited atom becomes unexcited and both the previous excited atom and the high energy level atom become atoms of high kinetic energy. These high speed atoms may then collide with other atoms or molecules of the gas, subdividing the kinetic energy until they reach temperature equilibrium.

EXPERIMENTAL METHODS

To carry out this investigation a source of high-frequency current was supplied either from a spark-gap or pliotron converter. The lamp was then placed in a helical coil through which the high-frequency current flowed, and the power dissipated in the lamp and coil was measured by a calorimeter. During the calorimetry period the illumination from the lamp was also measured. The energy distribution and spectrograms were obtained with a spectrograph, thermopile or photographic plate. A thermopile and filters were used to separate the far infra red from the easily obtained spectral region.

Electrical Arrangement. The arrangement of the electrical circuit has been studied with a variety of connections, but all come under two general classes. One class consists of a high voltage transformer 3000 to 15,000 secondary voltage, one or more condensers, 0.001 m. f. to 0.050 m. f., a spark-gap and the helical exciting coil; the other class involves the use of a high voltage, either a-c. or d-c., one or more pliotrons, condensers and helical coil to excite the lamp. Two typical schemes for a-c. lamps are shown in Plate II.

The electrical constants of each general type of circuit are under study at the present time and it has been found that considerable changes in the electrical efficiency can be made by adjustment of the circuit conditions. An example showing the apparatus of the spark converter type necessary to operate efficiently a 3.8-cm. radius mercury argon induction lamp is given in the following table:


—Pliotron Circuit—
Blocking condenser
Choke
Plate
Grid
Filament
Grid leak
Condenser
Lamp
Transformer
Pliotron
Grid condenser
Oscillation helix
Exciting coil
—Spark Gap Circuit—
High frequency choke
Induction lamp
Spark gap
High voltage leakage type transformer
Condenser
Exciting coil

A

B

Plate II

Exciting Helix	—7 turns copper, 4-cm. radius, 1.0-cm. separation
Condenser	—0.005 μ f.
Spark-Gap	—4 G. E. navy type units S. E.-1001 or equivalent
Choke Coil	—30 millihenrys
Transformer	—5500-volt secondary, high-reactance type

Calorimeter. The principle of the calorimeter shown in Fig. 3 is quite simple. It consists of an air-tight thermally insulated box through which air is blown at a constant pressure difference between the intake and exit ports. The walls are blackened with the exception of a small glass window used in making photometric measurements. There are also suitably placed baffles which force the air to circulate around the lamp and box before leaving it. When power is dissipated in the box (for instance, by an induction lamp or the filament of an incandescent lamp), the walls and the gas rise in temperature. It has been observed as an experimental fact that the temperature of the issuing gas measured with an ordinary thermometer (under fixed conditions) will rise with time according to the following empirical relation:

$$t = C\,\epsilon^{\frac{T}{KW}}$$

where

t = Time
T = Temperature
W = Watts input
C, K = Constants

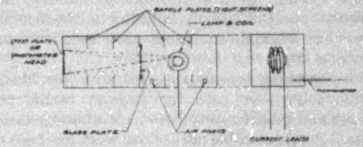

Baffle plates, (light screens)
Lamp & coil
Test plate or photometer head
Thermometer
Glass plate
Air ports
Current leads

Fig. 3—The Combination Calorimeter and Photometer

The form in which we actually use this relation is given below:

$$W = \frac{\log_{10} \epsilon}{K} \; \frac{\Delta T}{\Delta \log_{10} t} = K_1 S$$

where

$$K_1 = \frac{\text{Log}_{10} \epsilon}{K} = \text{the calorimeter constant}$$

$$S = \frac{\Delta T}{\Delta \log_{10} t} = \text{the slope obtained directly from a plot on semi-log paper.}$$

The value of K_1 was determined from the slope S obtained under accurately measured power input. For this purpose either vacuum or gas-filled incandescent lamps were used interchangeably as we found the value of K_1 was apparently unaffected by the type of lamp used. While no attempt has been made to secure an accuracy greater than a few per cent, it can be said with certainty that during a period of over one year, we obtained consistent results under widely different operating conditions.

Photometry. The illumination from the induction lamp was measured during the calorimetry period. (See Fig. 3.) A glass window placed in one end of the calorimeter allowed the light from the lamp to fall on a photometer head or test plate. Either a MacBeth illuminometer or a Sharp-Miller photometer was used to determine the horizontal candle power from the lamp. To obtain consistent results over a long period of time, three carefully seasoned lamps were calibrated against lamps rated at the Bureau of Standards. Our working standards consisted of the three-foot-candle source supplied with the MacBeth illuminometer, a 200-watt type *C* MAZDA lamp run considerably under normal voltage for a 30-foot-candle standard and a 300-watt type *C* MAZDA lamp for a 150-foot-candle field. The working standards were used only a few minutes a day.

The absorbing screens supplied with the MacBeth illuminometer were used for medium candle power readings and rotating sectors for high intensity measurements. A considerable part of our work has been done on colored lights and it seemed best to learn to match light by intensity, irrespective of the color difference. This end, we believe, has been accomplished, for in several cases we were able to check ourselves by placing an absorbing screen in the eye-piece that reduced the color difference without an appreciable change in the intensity match. The errors that did arise, we believe, are small in comparison to the relative changes in intensity that were involved.

Energy Measurements. The distribution of energy in the spectrum of the induction discharge has been determined by two methods. The first method separates the far infra red radiation from the easily obtained spectral region which lies in the range 20,000 Å to 1900. Å. The method consists in the use of a shielded thermopile calibrated to read energy incident on it and suitable filters to separate out certain spectral regions. This method, while simple in principle, is liable to lead to erroneous conclusions because there do not exist any very accurate filters, although from the values of the effective transmission limits approximate values of the total energy in certain regions can be obtained. The second method, while insensitive to the far infra red and possessing insufficient resolution in the range 20,000 Å to 8000 Å, does give fair resolution from 8000 Å to 4000 Å and increasingly better separation as the wavelength decreases to the transmission limit of quartz. In this method a quartz spectrograph is used and a thermopile is so mounted on a carriage that it can be placed at the focus of each spectral line. The instrument used in this work had a working range of 60 mm. between 5700 Å and 2100 Å and the slit width limiting the beam falling on the thermopile was approximately 0.3 mm.

Preparation of the Lamp. The lamp is prepared usually in the form of a spherical bulb. This may consist of soft or hard glass, pyrex, or transparent quartz, depending on the use to which it is to be put. The bulb is exhausted to a pressure of one micron and preferably less. A furnace is then lowered over the bulb and raised to as high a temperature as possible. Soft glass usually can be heated to 400 deg. cent., 702-*P* and pyrex to about 550 deg. cent and quartz nearly to 1000 deg. cent. After a short bake-out of ten minutes or longer, with liquid air on the system, the oven is raised and the lamp is partly cooled. The spectral substance, solid liquid or gas, is now put in the bulb. In the case of the mercury lamp, liquid mercury is either distilled or run into the bulb and then boiled to free it of all water vapor and occluded gases. While the mercury is warm an induction discharge of an extreme intensity is set up in the lamp for several minutes with the vacuum pumps running. This bombarding of the bulb walls is quite the most important treatment as it liberates gas that no amount of heating seems to free. The lamp cannot be prepared without the previous bake-out, as the mercury reacts chemically with the gas liberated, forming oxides, nitrides, etc., which darken the walls of the bulb. It is therefore very important that the residual gas pressure should be below a micron (0.001 mm.) as the nitrogen reacts immediately causing a golden brown film to form on the bulb.

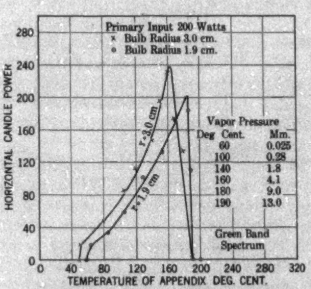


FIG. 4—SHOWING THE EFFECT OF TEMPERATURE OF LIQUID MERCURY ON THE LUMINOUS OUTPUT

It is advisable to torch the contraction and then fill the bulb with the auxiliary starting gas which is ordinarily one of the rare gases. This gas should be particularly free of all ordinary gases, which can be accomplished by running a calcium arc in it. (The pressure of the starting gas will be discussed in another part of the paper.) After filling to the proper pressure, the lamp is sealed off from the exhausting system and "seasoned" as an induction lamp.

The Effect of Vapor Pressure. An induction discharge cannot occur in pure mercury vapor at ordinary room temperature because the pressure of the vapor is too low to sustain a discharge. This is due to the long free path of the electrons in the gas. By raising the temperature of the liquid mercury, the vapor pressure rises until the free path is sufficiently short and discharge occurs. The intensity of the discharge is at first weak but increases with further increase in pressure until a final maximum is reached. A further increase in temperature practically stops the strong discharge and a weak emerald green glow takes its place. Above the maximum critical pressure the electrons make so many collisions with the atoms and molecules of the gas that the loss entailed merely in perfect elastic encounter reduces their velocity and corresponding energy below that which is required to put the gas in a strongly ionized condition. A typical curve showing the effect of temperature on the luminosity is given in Fig. 4.

The critical starting pressure does not appear to be affected much by the induced voltage but is found to be about inversely proportional to the square of the radius of the bulb. The critical maximum pressure depends on both bulb size and induced electromotive force, but to a large extent on the latter. While the variation of maximum critical pressures with electromotive force has as yet not been studied in detail, theoretical considerations suggest that the terminal velocities of the electrons are proportional to X/p, where X and p are the electric intensity and pressure, respectively. Thus if p exceeds X/U, where U is the ionizing voltage, the discharge stops. The conclusion can be drawn, therefore, if this assumption is correct, that the maximum critical pressure is proportional to X.

Now it is known that once a strong discharge is started, the pressure can be carried above the value corresponding to the starting condition. This suggests that the electron energy U required to sustain the ionized condition is lower where a strong degree of excitation exists. We therefore have evidence for believing that under steady operation at high pressures the ionization of the gas occurs not essentially by single impacts but by successive additions of energy to the atoms. Thus in the case of mercury at low pressure, the electron energy must exceed that gained in falling freely through 10.4 volts, whereas at high pressure under strong discharge conditions there exists a high concentration of atoms excited by absorption of 4.9 volts, equivalent energy, and by absorbing 5.5 volts or more from an impinging electron an excited atom becomes ionized.

The Effect of the Bulb Size. Early experiments at low power input indicated that the luminous output of pure mercury induction discharge was proportional to the radius of the discharge bulb and the square of the wattage. Data secured on bulbs of different radii could be approximately expressed by the equation

$$B = c\, r\, w^2$$

where

B = the horizontal candle power

r = the radius of the discharge vessel in cm.

w = the primary input watts

c = a constant = 0.77×10^{-3}

The value of c calculated from the data on hand is tabulated below.

Primary watts	Radius of bulb in cm.				
	2.6	4.6	6.2	7.5	
150	0.87×10^{-3}	1.0×10^{-3}	0.84×10^{-3}	0.86×10^{-3}	
200	0.63	0.71	0.71	0.67	
250	0.83	0.69	0.75	0.69	
300	1.02	0.71	0.65	0.68	
350				0.63	Ave.
	0.84	0.78	0.74	0.71	0.77×10^{-3}

The mercury induction lamps used in securing the above data were heated externally until the discharge occurred. The wattage input was then always kept high enough to maintain sufficient vapor pressure and thus sustain the discharge.

Subsequently the heating was dispensed with. This came about during a study of the effect of the admixture of other gases to mercury vapor. It had been observed that the mercury spectrum appeared in a vacuum system when a low pressure of air was admitted and a spark coil was used to test for leaks. This suggested that a mercury induction lamp could be made self-starting by introducing a low pressure of a permanent gas. When air was used the lamp did operate, but after running a few minutes and then allowing it to cool, it failed to start without external heat. This method was bound to fail because, as we found later on, a chemical reaction forming nitride and oxide of mercury cleaned up the air. This short-lived success, however, was followed by a systematic study of the addition of permanent gases to the mercury lamp, and it was found

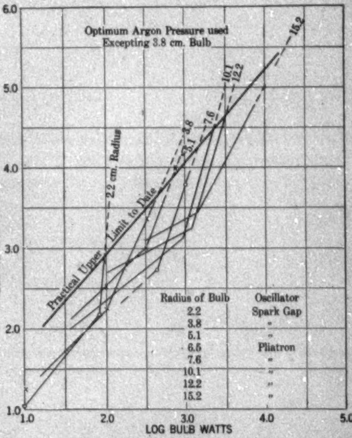


FIG. 5—THE CHANGE IN LUMINOUS INTENSITY OF A MERCURY ARGON LAMP WITH BULB INPUT FOR VARIOUS SIZED BULBS

that small quantities of the rare gases could be admitted to the lamp without impairing the mercury spectrum and at the same time the lamp became permanently self-starting.

After this discovery was made it became a quite simple problem to determine the specific bulb input by calorimetric measurements, and considerable data have been obtained showing the relation between candle power and bulb input for a variety of bulb sizes. A family of curves is shown in Fig. 5.

The data presented are averages of a large number of observations and do not mean to imply exact indications or great accuracy. This set of curves represents mercury in combination with argon gas. Particular attention is drawn to the similarity of the curves and to the two definite slopes that make up each curve. There exists a transition region at the intersection of the two curves, though in general the transition is quite sharp. This latter point is shown in a typical plot shown in Fig. 6.

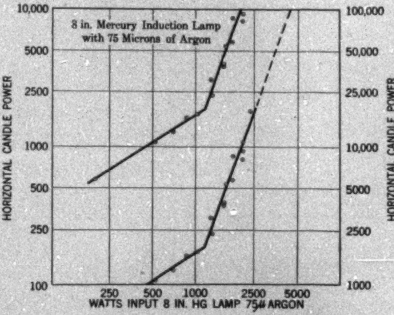


Fig. 6—A Typical "Luminosity-Input" Plot of Data Secured on a 10.1-Cm. Radius, Mercury Argon Induction Lamp

It will be noticed that the transition point shifts to higher wattage with increase in bulb size and suggests that the phenomenon occuring at the transition point is connected with the vapor pressure of the mercury. Assuming that the change occurs at a definite temperature, we would expect to attain the critical temperature when a definite wattage is dissipated per unit area of surface. This assumption leads to the following equation for the critical wattage:

$$W = W_c 4 \pi r^2$$

where

W = the total bulb input at the critical wattage

W_c = the watts per sq. cm.

r = the radius of the bulb

Values of W_c were calculated from existing data.

Radius cm. r	r^2	w	w_c	
2.22	4.49	80	1.41	
3.81	14.5	130	0.71	included
5.10	25.9	316	0.97	
10.1	102.	1000	0.78	
12.2	147.	1270	0.69	
15.2	228.	1860	0.65	
			Avg......0.76	

It will be observed that on the second part of the curves the illumination tends to follow more nearly the square of the wattage as was found in the case of pure mercury, while at lower input, below the critical point, the slope deviates from unity to values less than this as the diameter of the bulb increases.

The upper limit of luminous output has not been determined as yet. There are limitations that occur. In small lamps, high vapor pressure stops the discharge. This, however, can be overcome by inducing higher voltage. The problem here is one of electrical energy transfer to the bulb. If the vapor pressure is reduced by attaching an appendix to the lamp in which the mercury is maintained at a suitable temperature, the power dissipation becomes so high that the glass is softened and punctures. Resort may then be made to quartz bulbs. In large bulbs the temperature of the glass becomes quite serious because the cost of making large quartz bulbs is at present a serious drawback. From practical consideration a line is drawn across the curves showing the upper limits set to date. Our power facilities also limited us in the case of the 12.2-cm. and 15.2-cm. radius bulbs.

Addition of Auxiliary Gas

The auxiliary gas used in the mercury induction lamp seems to play several roles. First in importance is the ability to initiate a discharge, and second is the effect the auxiliary gas atoms have on the mercury atoms when they interact. Observations at low wattage input indicate that the addition of one of the rare gases to mercury vapor increases the luminous intensity for small pressures of the auxiliary gas. A maximum intensity is obtained at a certain pressure, and with further addition of the rare gas the intensity rapidly decreases. This action is plainly shown in Fig. 7.

A spectral examination under these experimental conditions failed to locate any lines other than those due to the mercury vapor. The spectrum of the rare gas did show, however, when the lamp was cold, but disappeared as soon as the bulb became warm as in normal operation. Now it is evident that the added gas in small quantities increased the number of excited mercury atoms (that gave rise to the visible radiation) above the number that would normally occur in the mercury vapor alone. The addition of relatively large quantities of auxiliary gas reduced the number of excited mercury atoms below the normal value in pure mercury.

It is reasonable, therefore, to expect that at low input and low bulb temperature, the pressure of the auxiliary gas will generally be in excess of the pressure of the mercury vapor, and it therefore follows that the initial excitation will take place in the auxiliary gas. Francke and Cario have recently shown that collisions of the second kind very frequently occur, causing a transfer of energy from the excited atom to a neutral atom. In case the neutral atom has energy levels lower than those of the excited atoms, the first atom can become excited and the latter become neutral. If, however, the levels of the neutral atom are above the levels of the excited atom a collision of the second kind between them results in the loss of the excited state with a resulting increase in kinetic energy of both atoms. This translational kinetic energy is then rapidly dissipated in heat as the high speed atoms collide with other atoms of the gas.

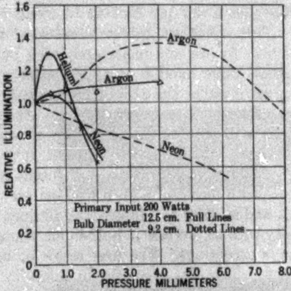


Fig. 7—The Effect on the Luminous Output Caused by Adding Rare Gas to a Mercury Induction Lamp

Thus if we follow this line of reasoning we are led to believe that the mercury atoms become excited to a large extent by collisions of the second kind and therefore the visible radiation depends on the number of excited rare gas atoms stimulated in the discharge. Of course some of the mercury atoms will be excited directly but as the concentration of mercury atoms is undoubtedly low at the lower bulb input and temperature it is reasonable to neglect them for the present. In a previous section it was explained that a maximum degree of excitation and ionization was obtained at a certain pressure and above or below this pressure these factors decrease. Thus in the case of the stimulation of a rare gas we can expect a maximum value at a certain pressure and the number of excited rare gas atoms that collide and transfer their energy to mercury atoms will pass through a maximum. Now some of the mercury atoms excited can in turn collide with neutral argon atoms and if a collision of the second kind occurs the excited state disappears in translational kinetic energy of the two atoms. Thus we see that as the pressure of the auxiliary gas increases, the loss of activated mercury atoms increases in proportion to the pressure, which further decreases the luminous intensity.

Following a similar line of reasoning for the case of high bulb input and corresponding temperatures, where the pressure of mercury vapor would be sufficiently high to become excited directly by electron impacts, it would appear that the auxiliary rare gas atoms would be excited very infrequently, as the electron velocity would be reduced by the ionizing and exciting impacts with the mercury. The few collisions of the second kind between excited argon and neutral mercury, therefore, would be unimportant in comparison to the mercury atoms primarily stimulated by the discharge. The presence of the rare gas, however, would cause a serious drain on excited mercury atoms by second order collisions whereby the excited mercury atoms would collide with the neutral auxiliary gas atoms with a loss of the excited state. The energy, of course, would be dissipated in heat, and we conclude that the addition of a rare gas into a mercury induction lamp which is operated at high bulb input is a decided detriment from the standpoint of luminous output.

This reversal of conditions with small pressures of argon as auxiliary gas is well brought out in Fig. 8 where it will be observed that at low bulb inputs the luminosity is greater (within the range studied) as the pressure is increased, whereas at high wattage, that is, above the critical point, the presence of gas considerably reduces the luminous intensity. Thus we arrive at the conclusion that the critical point determines the change from excitation of mercury by second order collisions to the direct excitation of the mercury by electron impact.

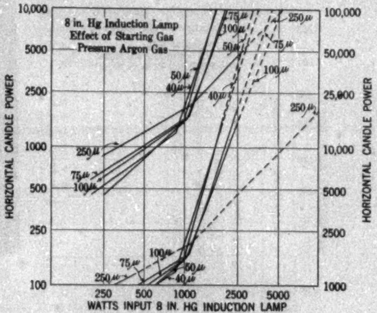


Fig. 8—The Effect of Adding Argon to a 10.1-Cm. Radius, Mercury Induction Lamp

It is evident from the foregoing that in order to obtain the greatest luminous efficiency and output, the lamp should be operated above the critical point with little more than the minimum amount of gas necessary to initiate a discharge.

It has been found in the case of argon and mercury that this pressure varies inversely as the square of the radius of the bulb and is given by the following empirical relation:

$$p = k\, r^{\frac{1}{2}}$$

p = pressure in mm. of pure argon gas

r = the radius of the discharge vessel in cm.

k = 5.7 for argon

ENERGY DISTRIBUTION IN THE MERCURY INDUCTION LAMP

Only a few measurements of the radiation efficiency of the mercury induction lamp by the filter method have been made. The results obtained were found to be quite independent of wattage though there does seem to be a difference due to bulb size. The measurements made are listed below.

Bulb radius cm.	Prim. input watts	Energy ratio 18,000 Å to 1900 Å / total radiation	Energy ratio 3100 Å to 1900 Å / 18,000 Å to 1900 Å
3.8	460	0.234	
3.8	700	0.23	
3.8	960	0.24	
3.8	240		0.30
3.8			0.31
5.1	400	0.32	0.22

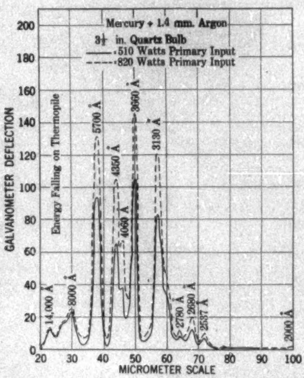


FIG. 9—THE DISTRIBUTION OF ENERGY IN THE RANGE 20,000 Å TO 2000 Å

Quartz actually transmits below 1900 Å, but since the quantity transmitted is relatively small, we have neglected the spectral output at shorter wavelengths except in photographic analysis.

Considerable work has been done on the analysis of the spectrum in the range 20,000 Å to 1900 Å by means of the spectrograph and thermopile called the spectroradiometer. A typical energy distribution curve is shown in Fig. 9.

Detailed studies have been made concerning the behavior of some prominent lines in the mercury spectrum. The lines 5700 Å, 5790 Å and 5461 Å in mercury are chiefly responsible for the high luminosity and these three lines are bulked together in the spectraradiogram at approximately 5700 Å. There are three lines of high energy at about 4350 Å but due to the much lower visibility at this wavelength, these lines only contribute a few per cent to the total luminous output. They do affect, however, the resultant color of the light. At low and moderately high bulb input the strongest wave band is at approximately 3600 Å, next in order is either the 3130 Å group or the 5700 Å group and then the blue line at 4350 Å. At very high bulb input and corresponding vapor pressure, the group at 5700 Å becomes the greatest in intensity. The lines that appear to increase in intensity at the fastest rate with increase in bulb wattage are the lines at 5700 Å and at 2680 Å. In general, all lines increase in intensity with an increase in bulb input, though there are notable exceptions. The first seems to be in the line 2537 Å. This line increases in intensity at low wattage and then falls off in intensity at intermediate bulb watts and finally begins to increase with high bulb wattage. This action may possibly be explained if we consider the action that occurs when the ionization of the atom changes from the single impact type to the double or so-called successive stage type. There are other lines of shorter wavelength that show up only in photography (due to their low intensity) and that are strong only at the low input and particularly low bulb temperature. These lines are recognized by the production of ozone in the air. On first starting, the odor is considerably stronger than after the lamp has been in operation for some time.

The spectral output of the induction lamp will depend of course on the glass used in the bulb. Ordinary glass will cut off all appreciable energy of wavelengths less than 3400 Å, while pyrex has a transmission limit of about 2970 Å in thin samples. Uviol glass transmits the 2537 Å line quite weakly and quartz will show the 1850 Å line though the transmission is not appreciably below 1900 Å.

The spectra photographed with a glass prism and a quartz prism spectrograph are shown in Plate III.

EFFECT OF FREQUENCY

Experiments have been made at different frequencies varying from 50,000 cycles to about 20,000,000 cycles. The electrical conditions seem to vary considerably over this range. For instance, at 50,000 cycles the current circulating in the helix was about 300 amperes before the discharge occurred, whereas at 20,000,000 cycles the circulating current was only a few tenths of an ampere. In general it would be expected that to secure a given ionization voltage, the relation between induced voltage and frequency should follow the equation given below:

$$= 2 \pi f M I \text{ and } \qquad I = \frac{e}{2 \pi f M}$$

where

e = the induced voltage
f = the frequency
M = the mutual induction between the helix and the largest circle inside the bulb
I = the current circulating in the coil

Observations on bulb watts versus luminous output show some variation with frequency, but so far no consistent relation has been found. The data in general included in this paper were secured within the frequency range of 1,000,000 cycles to 3,000,000 cycles.

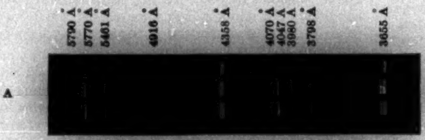
From the standpoint of transfer of energy to the bulb, the frequency is quite important though it seems at present that the variation of specific bulb operation with frequency represents a second order effect. So far we have found that the design of the high-frequency converter for each type of lamp is of first importance from the standpoint of output and over-all efficiency.

Spectral Substance and Uses

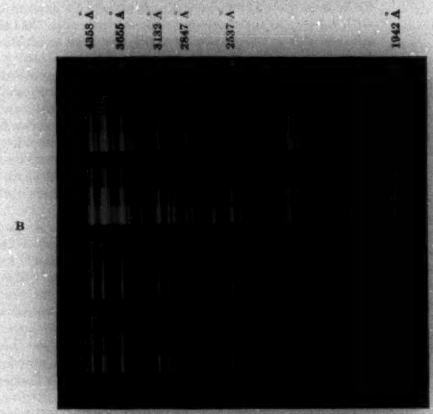
The induction method described in this paper for exciting spectra from atoms and molecules may be used with a variety of substances. For instance, spectra can be obtained without the aid of external heat from over one-third of the elements. Many metallic compounds of low vapor pressure also respond, giving both a line spectrum of the metallic atom and a band spectrum due to the molecule. Space did not allow the discussion of more than one substance, and for this purpose mercury was chosen because of the greater amount of work done on it, but it must be understood that mercury is typical only in so far as it represents the action in the electrodeless discharge. Each substance, whether element or compound, has its own characteristic spectral distribution. By a suitable choice of substance, therefore, radiation of a particular nature may be obtained; thus some of the elements give only single narrow regions of emission in the infra red, while others emit mostly in the visible or ultra-violet, and still others emit very strongly only in the ultra-violet violet region.

No doubt applications for this type of lamp will be found, for its richness of visible, near ultra-violet and far ultra-violet spectra will adapt it to special uses where the initial cost of the auxiliary equipment will not be of prime importance.

In the visible region efficient illumination can be obtained, while the near ultra-violet region can be used



Exposure Time	Bulb Diameter	Auxiliary Gas	Bulb Input
5 sec.	7.6 cm.	Ar	Low
"	"	"	
"	"	"	High



Exposure Time	Bulb Diameter	Auxiliary Gas	Bulb Input
0.5 sec.	1.8 cm.	Vac.	High
2.0 "			
8.0 "	"		"
20.0 "			
0.5	15. cm.	He.	High
2.0			
8.0	"		"
20.0			
0.5	15. cm.	He.	Low
2.0			
8.0	"	"	"
20.0			
0.5	15. cm.	Ar.	Low
2.0			
8.0	"	"	"
20.0			

Plate III
A—Glass Prism Spectrum of Mercury Induction Discharge. B—Quartz Prism Spectrum of the Mercury Induction Discharge

in photography and in fluorescent stage effects. The far ultra-violet region will find its usefulness in therapeutics, sterilization, dye fading, leather processing, promoting photo-chemical action and in the wide open field of research.

CONCLUSION

In concluding this paper I wish to express my appreciation for the guidance given by Dr. Saul Dushman of the General Electric Company, and for the able experimental assistance of Messrs. F. M. Garretson, Jr. and T. J. Radcliffe and of Dr. Carl Eckart in carrying out the experimental work. This work was started by the writer in the Research laboratory of the General Electric Co., at Harrison, N. J.

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Discussion

Saul Dushman: I am going to ask you to take your mind away from the purely material problems of lighting streets, railway yards, etc., and consider, with me, the production of light by the atom. Our present views on the production of light originated with a Danish physicist, Niels Bohr, in the year 1912. So much has been published and written about this theory that I am afraid I will be repeating a great deal of what is probably quite familiar to a number of you. Nevertheless, I am going to present a sort of bird's eye view of the theoretical side of the problem with which Mr. Foulke will deal from a practical side.

Our present views of the theory of light production rest essentially upon the conception of the atom as consisting of a positively charged nucleus surrounded by one or more electrons, the number of electrons being the same as the place of the element in the periodic table. Thus, hydrogen has a unit positive charge on the nucleus and one electron, helium has a charge of two positive units of electricity on the nucleus and two electrons, and so on. In the normal condition of an atom, these electrons are rotating in circular or elliptical orbits about the nucleus.

Now, let us assume that we have a number of hydrogen atoms in one of the two electrode vacuum tubes, with which everyone is familiar nowadays, and we gradually increase the voltage on the plate with respect to the filament from zero to higher values. An examination with a spectroscope would show that at a certain critical voltage, a single line is emitted and that as the voltage is increased, more and more lines appear until, finally, at a certain critical voltage, which is 13.5 volts in the case of hydrogen, we would obtain the complete spectrum of atomic hydrogen.

The explanation of the mechanism by which the spectrum is produced is as follows:

The electrons emitted from the cathode of the two-electrode tube acquire kinetic energy as they move across to the anode. On collision with a hydrogen atom the electrons tend to impart this kinetic energy to the hydrogen atom and they do so if this energy is sufficient to knock the electron in the atom out from its normal orbit and lift it to another orbit which is located at a distance that is four times as great as the radius of the electronic orbit in the normal hydrogen atom. The hydrogen atom, which has collided with the electron, is said to be in an excited condition, and naturally the electron which is now rotating at a much greater distance from the nucleus tends to return to its normal condition. In doing so the same energy which was absorbed from the impinging electron is re-emitted as light of a definite frequency given by the relation

$$\nu = (E_1 - E_2)/h$$

where h is the so-called constant of the quantum theory and $E_1 - E_2$ is the difference in energies of excited and normal atoms, respectively. In the case of an atom like that of hydrogen, the electron can rotate in orbits which have radii whose values are $1, 2^2, 3^2, \ldots n^2$ times that of the electron in the normal orbit. To each of these orbits we can assign a certain amount of energy with respect to the electron in the normal orbit, that is, we can speak of this as the amount of energy required to lift the electron from the normal orbit to the outer one, and the energy required to do this is usually obtained from bombarding electrons, but may also be obtained by collision with high velocity atoms or by collision with other excited atoms. The atom in the excited state tends to return spontaneously to the normal condition and reemits the energy which has been absorbed previously, in the form of radiant energy. Corresponding to transitions from one energy level (or one orbit) to another energy level (or orbit) a monochromatic radiation is emitted whose frequency is given by the equation referred to above.

Of course the whole story is not quite so simple as this; not only do we have circular orbits, but also elliptical orbits as well, and there are certain relations governing the possible transitions that may occur between different classes of orbits. Also, there is an added complication in the case of elliptic orbits, which is due to the fact that the electron moves much more rapidly when it is close to the nucleus. As shown by the theory of relativity, there is an increase in mass of the electron as it approaches the velocity of light, and, therefore, instead of rotating in a closed elliptical orbit, the electron rotates in a precessing orbit. This leads to a fine structure of the spectrum lines.

This picture of the origin of spectral lines as due to the transition of an electron from one energy level in the atom to another is applicable not only to ordinary or visible spectra but also to the infra-red radiations on the one hand, and to ultra violet and x-rays on the other. It also explains a large number of interesting phenomena that occur in gaseous discharges, and has led to a number of new experiments in the past few years, all of which have given additional confirmation of Bohr's point of view.

Thus, the new theory accounts beautifully for certain resonance phenomena observed by Professor R. W. Wood in the case of mercury. As shown in Fig. 2 of Mr. Foulke's paper, the normal level of the valence electron in mercury is known as a 1 *S* level. If mercury atoms are bombarded by electrons having a velocity corresponding to 4.9 volts, the valence electron in the mercury atom is raised to the level which is designated by spectroscopists as a 2 *p* level. As the electron then returns to its normal level, the line of wavelength 2536 Angstrons is emitted. But the same line is also emitted if mercury in an evacuated quartz bulb is illuminated with light of this wavelength, and the explanation is evident from Bohr's theory. Now if in addition to this light, the mercury is also illuminated by light of wavelength 3125, the valence electron is raised to a still higher level known as 3 *d'*, and then it is possible to obtain the lines of wavelengths which correspond to transitions from the higher level to intermediate levels.

As mentioned previously, an atom *A* may be excited not only by collision with electrons, but also by collision with another atom which is in the excited condition. Thus if *B* in the excited condition has more energy available than is required to raise the electron in atom *a* to the next level, it is very probable that under certain favorable conditions, instead of obtaining the radiation which corresponds to the transition from the higher to lower level in *B*, this energy is transferred to *A* and the radiation actually emitted is that corresponding to the transition from higher to lower level in the latter atom. This is known as a collision of the second kind and the phenomenon is of great importance in determining the nature of the light emitted from mixed gases. Mr. Foulke's investigations as described in his paper give many illustrations of this phenomenon.

The whole field of spectroscopy has been completely revolutionized by Bohr's theory, and the influence of this theory on our conceptions of atomic structure has been profound. While a great deal of progress has been made in the development of Bohr's point of view, the whole subject is still in a state of flux and the future will undoubtedly bring forth even more striking conclusions regarding the nature of the electron itself and of the mechanism of the transitions between energy levels.

(In connection with this discussion, a model was shown of a precessing electronic orbit.)

Herman Goodman: Although not an engineer, it has been my good fortune to be associated with them as relates to my work as a physician interested in light in therapeutics. I heard of Mr. Foulke's work during the discussion of my paper on "Light in Medicine and Surgery", which I gave at Spring Lake before the Illuminating Engineering Society.

I have been using high-frequency apparatus to excite a quartz tube, which contained a drop of mercury and some neon gas, in experiments in an attempt to get an emission of more or less unadulterated ultra violet. I have been limited in both the apparatus for excitation, and in the character of the tubes. The reason of the attempt was to get the effect of the ultra violet on the biology of the patient, and to simulate the good effects of the ordinary mercury vapor arc in quartz excited by the ordinary current.

By the high-frequency current mercury vapor bulb it should be possible to arrange an emission more controlled than at present. It is held, for example, that a mercury vapor arc in quartz gives certain biological effects, but everyone knows that the ultra violet which comes from it is in conjunction with zones of visible light and of heat. This is very definite.

From the experiments done, it seems possible that ultimately we will have controlled emission of ultra violet in enough quantity to get effects on the patient. As mentioned, I have gone through some of this work in a crude way. I had no calorimeter nor photometer. It seems to me, though, that from what I have seen here this morning that the ultra violet is strong enough to show its presence but that biologically there would be little effect at 3130.

C. H. Sharp: I want to say a word of appreciation for the wonderfully lucid, simple, and understandable explanation of the modern theory of the production of light by gases that Dr. Dushman has given us.

Mr. Foulke's paper pointed to some remarkably interesting experiments and experimental results. His demonstration of the effects which are produced was most interesting and I am sure that every one here feels very much indebted to him for having made it. Future experimental work along the lines on which Mr. Foulke is working should be of very great promise.

Selby Haar: Is Mr. Foulke's work just an experiment, or has it an immediate commercial aspect?

J. B. Taylor: Is this lamp an induction lamp or something else? Mr. Foulke in his synopsis says "the induction lamp, or, more generally speaking, the electrodeless electrical discharge." Now, is that more generally speaking, or more specifically speaking? It seems to me the latter is the true state of the case. That is only one of the many possible forms of induction lamps.

Also, what possible advantages do we gain by introducing the energy into the glass bulb by induction rather than conduction?

I submit that this lamp is a gaseous lamp more properly. The essence of what we have seen is the gaseous discharge either with or without electrodes. The essence of it is not induction which we have applied one state back in the great majority of all our incandescent lamps. The electrodeless feature is incidental and might be applied to incandescent or arc lamps if you are disposed to it. Incidentally, and to its disadvantage, it is a high-frequency lamp which calls for special apparatus.

If I might ask one specific question, in the calorimeter tests we measure losses not only in the bulb but presumably also in the inducing coil and that leads to the point when you are giving these figures on watts per candle, where should you begin and where should you end? Should we say the bulb is the lamp, or is the coil an essential part of the lamp, or the high-frequency apparatus back of it part of the lamp? It is feasible to determine the losses in the coil alone without the presence of the lamp, or the losses in the coil greatly modified by addition of the lamp, and is it easily feasible to make a separation?

O. H. Ovhler: I should like to know if anything is available concerning possible life of a gaseous container like that through a a period of hours of use and if there are any data to depend on covering total efficiency from input at 60 cycles measured in luminous output.

E. W. Beggs: Can you get a white light from a lamp of this type?

T. E. Foulke: The first point of interest is that brought up by Dr. Goodman is regard to the monochromatic character of the light obtained from the induction discharge. The method of producing spectra which was briefly outlined today, that is, by indirect excitation, has been applied to about thirty pure metals besides mercury. From some of these substances, we obtain strictly monochromatic light. A particular example is a mixture of Argon and phosphorous. The Argon is excited by the discharge and the spectra of phosphorous is obtained. There are two distinct bands of radiation, the first in the neighborhood of 7000 Å and the other at 2537 Å. The location of the emission lines observed in the various substances are sometimes found in the infra-red, others in the visible and still others in the ultra-violet region.

Mr. Selby Haar has asked a question regarding the commercial adaptations. Several fields of application have been considered but at this time it is too early to say what the outcome will be.

In regard to Mr. Taylor's last question concerning the calorimeter measurements, I feel quite safe in saying that the current circulating through the coil causes no appreciable loss under ordinary experimental conditions. This would most certainly not be the case if a small wire were used in construction of the coils.

In regard to Mr. O. H. Ovhler's question regarding the life and the over-all efficiency, in the early stages of the development, the life of these lamps was very erratic and by careful study of the conditions it was found necessary to use pure gas in filling pure vacuum distilled mercury, and the container be baked at a temperature in excess of 400 deg. cent.; furthermore that a discharge should be run in the lamp, while disconnected from the vacuum system, before the final gas was admitted and sealed off when out of use. After these precautions were taken there was very little trouble with blackening, etc. Some of the lamps prepared in this manner were used intermittently for practically the period of a year without any apparent change.

Over-all efficiencies were obtained ranging in the neighborhood from 15 lumens per watt to a value of approximately 40 lumens per watt.

In regard to Mr. E. W. Beggs' question, I believe it is possible to very closely approximate daylight color. Each combination of gas and vapor gives a different tint or color and it is possible to obtain a variety of colors. It is possible that daylight color will best be obtained from metallic compounds. I must state, however, that no attempt has been made to determine whether or not we could produce daylight.

Frequency Measurements with the Cathode Ray Oscillograph

BY FREDERICK J. RASMUSSEN*

Non-member

***Synopsis.**—The cathode ray oscillograph frequency measurement circuit described differs from previous circuits in the use of by-pass condensers and plate leaks which permit connection of the oscillograph to a-c. circuits having large d-c. components and which permit the use of biasing controls for shifting the position of patterns on the screen.*

Reference oscillators are used in conjunction with the frequency standards. They are of a type chosen for their high stability.

The well-known properties of Lissajous' figures are reviewed briefly and then developed more fully for the cases in which only one term of their ratios may be determined from the oscillograph pattern. Following a general discussion of the accuracy of syntonization, there is discussed a detailed method of calibrating oscillators. The patterns used may be interpreted from one term of their ratio.

Interpolation formulas are derived for use in making interpolations on the reference oscillators. The methods of interpolation discussed may be used for intervals up to 0.5 per cent frequency with high degrees of accuracy.

The interpolation formulas are extended further in the development of approximation methods by which any unknown frequency may be quickly determined within a few per cent.

Several special circuits are described for use in frequency measurement work with the cathode ray oscillograph.

The methods and apparatus described are suitable not only for the technical measurements of a development and research nature but are equally adaptable for routine commercial work. The advantages which particularly commend themselves are the rapidity with which such work may be done and the ease with which the average man can learn the work.

With the advance of the telephone and radio broadcasting art and the advent of the radio telephone, the need for more precise measurement and control of the frequency of alternating-current potentials has been increasing. In practise, the accuracy requirements of frequency meters and other frequency measuring devices have become more severe. In the laboratory, rapid and accurate methods of calibrating field standards of frequency, laboratory oscillators and tuning forks are demanded. Resonance methods of measuring electrical constants are applicable only in proportion to the availability and accuracy of frequency measurements. The study of small frequency variations due to extraneous causes is largely dependent upon the relative accuracy of measurements made at infrequent intervals. To meet these needs, J. W. Horton, N. H. Ricker and W. A. Marrison[1] developed a continuously operated, 100-cycle tuning fork from which a source of alternating current is supplied. This system has proved to be an extremely accurate and constant source of frequency. The rate of the 100-cycle fork is maintained to 1 part in 100,000 at all times, and its deviation from 100 cycles may be maintained constant and known to an accuracy considerably better than this. At the same time, circuits were developed for producing and separating the harmonics of the base frequency. By means of these circuits there are produced 1000-cycle and 10,000-cycle sources of alternating current which have the same percentage accuracy as the 100-cycle standard, and may be used when it is desirable to have a higher frequency standard.

Coincidently with the adoption of the above sources of alternating currents as the laboratory standards of frequency, the development of the low-voltage cathode ray oscillograph[2] was completed, and there became available for general laboratory use accurate methods of frequency measurement by means of these two new tools, the extremely accurate source of standard frequencies and a commercial low-voltage cathode ray oscillograph.

The telephone industry uses a very large number of vacuum tube and other types of oscillators. The maintenance of oscillator calibrations in the laboratories and the calibration of field equipment furnish a very considerable amount of frequency measurement work. To take care of this work, frequency calibration has been largely centralized. It is the purpose of this paper to describe a special frequency measurement equipment and the methods of using it. The equipment was designed especially for use in the routine calibration of oscillators, tuning forks, wavemeters and all electrical circuits in which a frequency adjustment or measurement is required. It was also designed for precision measurements incidental to many present developments and necessary in the maintenance of frequency

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[1] For references see end of paper

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substandards of all kinds. Those parts of the equipment which are used in connection with a large part of the work have been installed as a unit; those which are used less frequently have been assembled as portable units which are installed only during use.

The general principles of the circuits and the methods used are in some cases the same or similar to those described in the literature already available on this subject, and the liberty will be taken of including such matter here with references to familiar papers but without attempting to determine original publications.

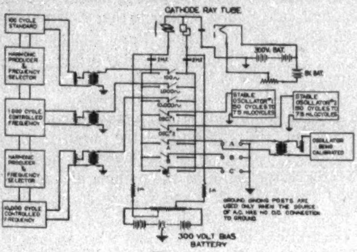
CATHODE RAY TUBE
100 CYCLE STANDARD
HARMONIC PRODUCER & FREQUENCY SELECTOR
1000 CYCLE CONTROLLED FREQUENCY
HARMONIC PRODUCER & FREQUENCY SELECTOR
10,000 CYCLE CONTROLLED FREQUENCY
300V. BAT.
8K BAT
2 M.F.
2 M.F.
100~
1000~
10,000~
OSC. 1
OSC. 2
A
B
C
STABLE OSCILLATOR #1 50 CYCLES TO 75 KILOCYCLES
STABLE OSCILLATOR #2 50 CYCLES TO 75 KILOCYCLES
OSCILLATOR BEING CALIBRATED
1 M
1 M
GROUND BINDING POSTS ARE USED ONLY WHEN THE SOURCE OF A.C. HAS NO D.C. CONNECTION TO GROUND
300 VOLT BIAS BATTERY

Fig. 1

A schematic diagram of the frequency measuring apparatus is given in Fig. 1. Essentially, it consists of the sources of standard frequency, potentiometers for controlling the output of the standards, two stable vacuum tube oscillators, and the cathode ray tube oscillograph with a multiplicity of input terminals for placing frequencies on the oscillograph at will. The alternating-current potentials are applied to either the horizontal or vertical plates of the oscillograph by means of a number of low-capacity, single-pole, double-throw switches. Rather than connect the input frequencies of the oscillograph directly to the plates, they are coupled to the plates through two-microfarad by-pass condensers and a plate leak is added to furnish the correct d-c. potential. This means is used, as it makes the d-c. components of the oscillograph and the source independent. A typical example of its usefulness is in measurements on a circuit where the most available position having a sufficient a-c. potential is at the plate of a vacuum tube. The diagram shows the plate leak going to a potentiometer shunting a 300-volt battery with a grounded center. For most work it is necessary only to have the leak go to ground, in which case the figure on the oscillograph will place itself centrally on the screen. The purpose of the potentiometer is to add to the a-c. potential a d-c. potential which will change the figure's position on the screen.

The vacuum tube oscillators form an important part of the equipment. They are stable oscillators of a type similar to those described by J. W. Horton[3] and are constructed so that under laboratory conditions of constant battery supply and uniform temperature they will maintain a frequency setting to ± 0.01 per cent for long periods of time. The oscillating coils are of the air core toroidal type. The frequency is controlled by changing the oscillating coils and adjusting the associated tuning capacity. The tuning condensers are mica condensers arranged in decade followed by an air condenser, and are calibrated carefully to facilitate interpolation. In order that the frequency settings may be read with a relative precision comparable to the accuracy provided by the frequency standards, the inductance and the number of oscillating coils have been so chosen that any frequency from 35 cycles to 75,000 cycles may be obtained with a capacity setting of not less than 0.015 $\mu f.$ The air condensers are of a type which may be read in increments per scale division of 0.6 $\mu \mu f.$ and 1.0 $\mu \mu f.$ respectively, which permit reading settings at 0.015 $\mu f.$ in increments of ± 0.001 per cent and ± 0.002 per cent frequency, respectively. In order to furnish any desired voltage range for the oscillograph inputs, the oscillator outputs are provided with switching arrangements associated with output transformers of various turn ratios.

LISSAJOUS FIGURES

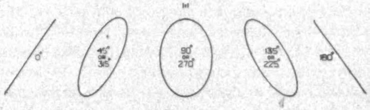
1:1
0°
45° OR 315°
90° OR 270°
135° OR 225°
180°

Fig. 2

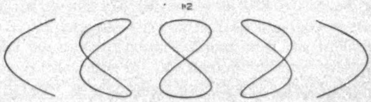

Fig. 3

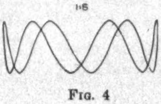

Fig. 4

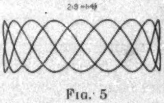

Fig. 5

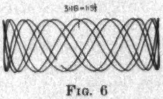

Fig. 6

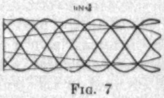

Fig. 7

Although not a permanent part of the equipment, two oscillators having frequency ranges up to 350 and 1500 kilocycles, respectively, are available.

The Use of the Cathode Ray Oscillograph

The cathode ray oscillograph which forms the nucleus of the frequency measurement equipment has for its main purpose the syntonization of alternating potentials. Its usefulness depends upon its property of creating figures of various designs when syntonized frequencies are applied to its plates. Within the limitations of the cathode ray tube used, a figure of some kind always results if two frequencies which are not incommensurate are applied to its plates, the complexity of the figure being less the more simple the ratio between frequencies.

Illustrations of typical and useful figures are given in Figs. 2 to 13. When two sinusoidal alternating potentials having the same frequency are applied to the plates of the cathode ray oscillograph, a straight line is obtained if the phase relation between the potentials is 0 deg. or 180 deg. At other phase relations, an ellipse results, the slope of whose axis depends on the phase relation. This is illustrated by Fig. 2. Fig. 3 shows the figure resulting with various phase relations when the frequency ratio is 2 to 1, and Fig. 4 shows the figure resulting from a ratio of 1 to 6. If these figures are observed on the cathode ray oscillograph when the ratio is not quite commensurate, the figure proceeds from one phase to the next. The movement of the figure makes it appear as if a picture of the higher frequency wave were drawn on a transparent revolving cylinder with its axis in the plane of the oscillograph screen. In the case of a simple ratio of one to an integral number, the ratio may be readily determined by counting the number of waves which appear to be described around the cylinder. Also it is not difficult to determine somewhat more complex ratios as those illustrated in Figs. 5 and 6. When the patterns become more complex than those illustrated, it becomes correspondingly more difficult to determine their ratios. Consequently means are devised to avoid the necessity of actually counting the waves of a pattern to determine frequency ratios.

LISSAJOUS FIGURES

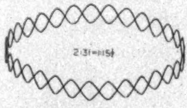

FIG. 8

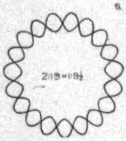

FIG. 9

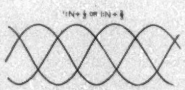

FIG. 10

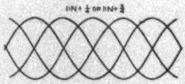

FIG. 11

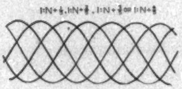

FIG. 12

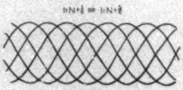

FIG. 13

By various means, it is possible to place the figure upon the oscillograph in such a manner that, referring to the transparent cylinder illusion, the back part of the figure is readily distinguished from that of the front, or in such a manner that one part is absent entirely [1,5]. In the first case, figures similar to that shown in Fig. 7 are obtained; in the second those similar to Figs. 8 to 13. Figs. 10 to 13 are the same as Fig. 8 but are obtained by the use of large a-c. potentials so biased as to bring the desired portion on the screen.

In general calibration work, methods employed are such that only the numerator in the lowest term ratio of the lower frequency to the higher is used. The numerator is spoken of as the number of lines in the figure. Thus, Figs. 2, 3 and 4 show one-line figures. The lowest term ratio in all cases is one to an integer. The illustrations in Figs. 5, 8 and 9 are spoken of as two-line figures. Their ratio is two to an integer and may all be reduced to one over an integer plus one-half. Figs. 6 and 10 are spoken of as three-line figures. Their ratios are one to some integer plus $\frac{1}{3}$ or $\frac{2}{3}$. Thus, if we can count the number of lines, vertical or horizontal, in a given figure, we know that the ratio is $1\colon\left(N + \dfrac{X}{l}\right)$, where N is an unknown integral, l the number of lines counted, and X an integer less than l and not having a factor common to l.

Except for the limitations explained below, it has been found practical with the 100-cycle standard to use figures containing up to 12 lines at audio frequencies, and up to 5 lines through the carrier range. At radio frequencies, it is difficult to use the 100-cycle standard and it usually is not practicable. Against other standards, however, four- and five-line figures are used in this range. In the carrier range, up to 15-line figures have been tried successfully at ratios of over 1:10.

In order to facilitate the use of the more complex ratios, Table I was prepared for use with the 100-cycle standard, but may be used readily with other frequencies. It contains a column giving in order the decimal value times 100 of every fraction down to twelfths, and another giving the denominator of these fractions. An example of its use follows:

Assume that it is desired to set an oscillator at a frequency of 6015 cycles and that the oscillator calibration gives 6000 and 6200 cycles. The frequency, of course, lies between these two points. The 6000 cycles are readily located from the 100-cycle standard by the simple one-line figure. The capacity is now slowly decreased until a figure containing seven lines is obtained. This frequency will be 6000 cycles plus the value 14.29 given in Table I for the seven-line figure next below the one-line figure. Decrease the capacity further and there will obtain a six-line figure, indicating a frequency of 6016.67 cycles. Either of these two frequencies is within 0.05 per cent of the desired frequency. By linear interpolation the desired point may readily be referred to the 100-cycle standard with an accuracy depending on the capacity increment and stability of the oscillator, and the stability of the standard.

The above method can be used universally for obtaining frequencies to 0.01 per cent. If the frequency to be obtained is low, the 100-cycle standard or as low as a 20-cycle frequency may be used. If the frequency is higher, a 1000-cycle standard may be used. As the frequency to be adjusted increases, the number of lines which may be used decreases, but the per cent increment at which observable lines occur may readily be kept below 0.2 per cent above four kilocycles. At frequencies below 500 cycles, syntonization against a multiple frequency adjusted as above is more often the best method.

The apparatus described naturally has its frequency limitations. In general, due to the necessity of holding the higher frequency in syntonization with the lower-frequency oscillator to within a few cycles, the upper frequency limit is fixed by the type of high-frequency source being used. However, using extremely stable oscillators, calibration work has been done to well over two million cycles. At the low-frequency end of the scale the frequency is limited by the ratio of frequencies, the retention of screen fluorescence, and the size of figure. In precision work where relatively large figures are used it is necessary that the beam trace the entire figure ten times per second and if much work is to be done an even higher rate should be used as the flickering of the picture becomes quite tiring. Thus, syntonizing 85 cycles against 40 cycles, a ratio of $1 : 2 + \frac{1}{8}$ would be undesirable as the pattern is retraced only five times per second. In work of relatively low precision, where small figures are used, retracing the picture five times per second is often quite satisfactory.

TABLE I

DECIMAL TABLE FOR USE IN FREQUENCY CALIBRATION WORK ON THE CATHODE RAY OSCILLOGRAPH

Number of lines	$100\frac{X}{e} = f - N(100)$	Number of lines	$100\frac{X}{e} = f - N(100)$	Number of lines	$100\frac{X}{e} = f - N(100)$
1	00.000	11	36.364	10	70.000
12	08.333	8	37.500	7	71.428
11	09.091	5	40.000	11	72.727
10	10.000	12	41.667	4	75.000
9	11.111	7	42.857	9	77.778
8	12.500	9	44.444	5	80.000
7	14.286	11	45.455	11	81.818
6	16.667	2	50.000	6	83.333
11	18.182	11	54.545	7	85.714
5	20.000	9	55.555	8	87.500
9	22.222	7	57.143	9	88.889
4	25.000	12	58.333	10	90.000
11	27.273	5	60.000	11	90.909
7	28.572	8	62.500	12	91.667
10	30.000	11	63.636	1	100.000
3	33.333	3	66.667		

f = frequency observed against 100-cycle standard.

N = any integral.

To use the chart with the 1000-cycle standard, multiply the second column by 10 and change $f - N(100)$ to $f - N(1000)$.

To use with the 10,000-cycle standard multiply by 100 and change to $g - N(10{,}000)$.

Accuracy of Syntonization on the Cathode Ray Oscillograph

The accuracy of syntonization is dependent upon the observation of motion or lack of motion in the figure produced on the screen of the oscillograph. This accuracy for any observation may be checked by watching the figure for a measured period to determine the rate of motion. The best method in routine work of obtaining the required accuracy, however, is to so proportion the potentials that when the figures appear to be standing still the required precision has been attained. For the purpose of this discussion, it is assumed that a spot of light 20 in. from the observer's eyes, moving at a rate of 0.05 in. per second on a blank surface, readily appears to be in motion when observed for only a short interval. Actually, and particularly when a reference mark is used, much slower rates are observed as motion.

Assume that the 100-cycle standard, placed horizontally, is to be used for a series of measurements and that its amplitude is adjusted to make the length from wave to wave of 1000 cycles, placed vertically, one in. Assume further that the approach to syntonization of the two frequencies is such that the waves pass a given reference point at the rate of one per second. It may be shown that the frequency is 999 cycles or 1001 cycles, whence the error in setting the 1000 cycles is ± 0.1 per cent. The rate of motion of the 1000-cycle wave is then one in. per second. If the rate were 0.05 in. per second, which has been assumed to be the slowest movement readily visible as motion, one wave per 20 seconds would pass the reference mark whence the frequency would be 1000.05 cycles or 999.95 cycles. The error of syntonization would be ± 0.005 per cent.

In the preceding discussion one point is left out of consideration; namely, the amplitude of the 1000-cycle figure. If the amplitude of the 1000 cycles is so small that it is practically indistinguishable, it is readily seen that the motion of the figure would have to be considerably faster than assumed in order to be observed. If the amplitude is very large, the lines at the point of observation appear vertical and their motion across the screen is observed very readily. It can be seen, however, that the magnitude of the 1000 cycles should be such that the lines at the point of observation move at least as fast in a vertical direction as in a horizontal direction. Therefore, the slope of the wave at 0 should be 45 deg. or more. This condition exists for a sine wave with a period of one in., when the amplitude is approximately ½ in. Practically, it is most natural to adjust the high-frequency input to the oscillograph to meet this condition. However, it could be checked easily by balancing the frequency against 10,000 cycles and adjusting its input to make the waves of the 10,000 cycles have a 0.1-in. period.

If an accuracy of ± 0.001 per cent instead of ± 0.005 per cent is desired, the amplitude of the 100 cycles should be five times as great. If a precision of only better than ± 0.01 per cent is desired, the figures need be only half as large.

It is very seldom that syntonization is required to an accuracy of better than ± 0.001 per cent. Due to the width of focus, it is necessary to adjust frequency inputs of over 100,000 cycles to give inputs larger than required for this accuracy. Therefore, very little thought of the accuracy of syntonization is necessary when one of the frequencies is larger than 100,000 cycles.

As the frequency values decrease, however, the relation between the amplitude of the figure and the accuracy required for instantaneous readings becomes very important. A simple rule which is used to check the amplitude of the lower frequency used in a measurement follows: When the final observation of a reading requiring syntonization to an accuracy of ± 0.001 per cent is being taken, the amplitude of the lower frequency shall be such that a 100,000-cycle or higher frequency, having an amplitude of 0.1 in., may readily form a distinct figure with it; for ± 0.01 per cent, 10,000 cycles should be used; for ± 0.1 per cent, 1000 cycles.

CALIBRATION OF OSCILLATORS

The method of calibrating an oscillator necessarily varies in accordance with the type of oscillator, the frequency range, the frequency intervals at which the calibrated points are desired, the accuracy and the relative accuracy required. Oscillators in which the accuracy requirements vary from ± 10 per cent to ± 0.1 per cent have been calibrated. On the same oscillators the relative accuracy required has varied from ± 10 per cent to ± 0.01 per cent, the relative accuracy being, of course, a measure of the accuracy of frequency differences which are obtainable between various frequency settings. The type of oscillators for which this calibration equipment is particularly designed is that in which the frequency is continuously variable between a lower and an upper limit. To provide for the various ranges there is usually a coil switch for interchanging the coils of the oscillating circuit and for changing the circuit constants to provide the necessary variations in regenerative coupling and for controlling, on some of the older oscillators, filters for the purification of wave form. Oscillators designed to make measurements throughout the audio frequency range are continuously variable between 50 or 100 cycles and 3000 or 5000 cycles, and their calibration charts call for some 150 frequency points. Another common type is that designed for use in both the audio and carrier frequency ranges. Their frequency is continuously variable from 100 cycles to 50,000 cycles. Their calibrations call for almost 500 frequencies. In order to maintain the required accuracies on these oscillators, it is necessary to calibrate them, in most cases, once every year. Obviously, with so many points on the calibration, it is desirable to provide a very rapid means of calibration. The cathode ray oscillograph method permits the calibration of the first oscillator described to be made by two men in but little over one hour and that of the second in about three or four hours.

In most instances of routine calibration work, the approximate calibration is known from the constants of the frequency-control unit or from the fact that it is recalibration work in which the last previous calibration is available. On most oscillators, decade systems of condenser dials are used which permit reading capacity directly to accuracies of the order of ± 1 per cent, which permits making interpolation calculations over considerable ranges to accuracies approaching ± 0.5 per cent frequency. This of course forms another considerable aid in the calibration work.

To make the calibration, the oscillator is connected to the required source of direct-current supply and the output to one of the input switches of the oscillograph. If the voltage of the oscillator output is not suitable for calibration work on the oscillograph, it is adjusted by means of a step-up transformer, or by means of a resistance network, if the case requires a stepping down of voltage. If the approximate settings of the oscillator are not known, the lowest frequency of a coil position is determined by an approximation method which is given below. From this, the approximate settings of the first frequency required may be calculated from the relation that the capacity of the oscillator setting varies inversely as the square of the frequency, or if the interpolation is less than say 10 per cent frequency, from the more approximate relation that the relative capacity increase equals twice the relative frequency decrease.

Having determined thus the first point, one of the reference oscillators is adjusted against the frequency standard to four times the arithmetic interval of frequency. The second is set at 10 or 20 times the arithmetic frequency interval. The amplitudes of these oscillators and of the oscillator being calibrated are adjusted to give the required precision. To give a clearer conception of the procedure from this point, a definite example will be given.

The oscillator is to be calibrated from 2000 cycles up, in 50-cycle steps. The first reference oscillator is adjusted to 200 cycles. The second is adjusted to 500 cycles. When 2000 cycles is placed against either of these frequencies a one-line figure is obtained. Having adjusted the 2000 cycles with the required precision and recorded its setting, the oscillator being calibrated is set at approximately 2050 cycles by reducing the capacity setting 5 per cent, which is twice 2½ per cent the relative frequency interval at this point*. The air condenser is varied until a four-line figure against the first oscillator appears on the screen. This is adjusted with the required precision. The next point is located in a similar manner, but by the use of a two-line figure, the next by a four-line figure and the next by a one-line figure. When the calibration has proceeded to 2500 cycles, the oscillator being calibrated is checked against the second stable oscillator. If a one-line figure appears, it is assumed that the calibration has been made correctly up to this point. It is to be noticed that as a calibration by this method proceeds the operator has a continuous check at all times. If at any time the frequency is increased by too large an interval, it will be noticed, when checking against the higher frequency oscillator, that a mistake has been made. This, however, is merely an additional check as no mistake would be made if care is taken that the correct number of lines is obtained for each frequency setting.

When making calibrations requiring a high degree of accuracy (± 0.05 per cent), the reference oscillators are accurately checked at this point against one of the standard frequencies. If they have varied between checks, the calibration is repeated after readjusting them. If the accuracy required is not high but the relative accuracy between adjacent points is, the reference oscillators are only readjusted when a coil position or similar change is made on the oscillator being calibrated, or when a large change (± 0.02 per cent) has occurred in the reference oscillator.

The above method of calibration is used entirely except when it calls for the use of reference frequencies below 40 cycles. These cases occur only at frequencies below 300 or 400 cycles and generally are taken care of by one of the two following methods:

By using more complicated figures involving up to twelve lines, or by syntonizing the oscillator against another oscillator which is set progressively to frequencies which are a convenient multiple of the frequencies desired. For instance, in calibrating an oscillator with an accuracy of ± 0.5 per cent in five-cycle steps from 200 cycles to 300 cycles the reference oscillator would be set at 40 cycles and the desired points found using in turn 1-, 8-, 4-, 8-, 2-, 8-, 4-, 8- and 1-line figures. And again, in calibrating an oscillator from 25 cycles to 50 cycles in one-cycle steps, one of the reference oscillators would be set progressively at 250 cycles, 260 cycles, 270 cycles, etc., by the usual method and the oscillator being calibrated syntonized against it, using a ratio of 1:10.

Interpolation and Approximation Methods

As has already been indicated, extensive use is made of mathematical interpolation. The extension of its use is more or less an economic consideration depending upon the time and labor saved and the accuracy limitations involved. The methods used are usually approximation methods, depending for their accuracy upon the relatively small intervals at which known frequency settings on the reference oscillators may be found by means of the oscillograph. In order to use intelligently the various methods of interpolation and approximation, it is necessary to know definitely the limitations of these methods as well as the relative labor involved in their use or omission. To these ends a careful analysis of the constants of the reference oscillators has been made. The exact relation between frequency and capacity in an oscillator involves vacuum tube and circuit constants to form a theoretical relation which is too complicated for general use. However, in the type of oscillators used, it is found that the relation

$$f = \frac{K}{\sqrt{C + C_0}} \tag{1}$$

where f is frequency, C is the condenser setting capacity and C_0, is a capacity correction factor including coil and circuit capacities, holds to very high degrees of accuracy (± 0.001 per cent) over the entire frequency range which may be covered by a single setting of the regenerative coupling. As the circuit constants are used entirely for interpolation work, the formula is used in the form:

$$\frac{f_1}{f_2} = \frac{\sqrt{C_2}}{\sqrt{C_1}} \tag{2}$$

where C_1 is the $(C + C_0)$ corresponding to the frequency f_1. The formula has been studied in this form and found to hold true to ± 0.001 per cent over frequency ranges of ± 15 per cent from f_1. The ± 15 per cent frequency range represents about the largest range which is covered by a single regenerative control position. The C_0 which must be added to the indicated setting of the oscillator may be found by substituting two known settings, and is included as part of the reference oscillator calibrations. This formula is not used, however, as in precision work its use becomes very cumbersome, due to the large amount of squares and square roots to be taken. Instead, it is expanded to the forms:

$$C_2 = C_1 - \frac{2\,C_1\,(f_2 - f_1)}{f_2} + \left[\frac{f_2 - f_1}{f_2}\right]^2 C_1 \tag{3}$$

$$f_2 = f_1 - \frac{f_1\,(C_2 - C_1)}{2\,C_2} - \frac{1}{8}\left[\frac{C_2 - C_1}{C_2}\right]^2 f_1 + \ldots \tag{4}$$

in which the higher order terms may be neglected for small interpolations, giving:

$$C_2 \doteq C_1 - \frac{2\,C_1\,(f_2 - f_1)}{f_2} \tag{5}$$

$$f_2 \doteq f_1 - \frac{f_1\,(C_2 - C_1)}{2\,C_2} \tag{6}$$

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*cf. Formula (5).

These formulas are used extensively in studies where changes of frequency with conditions are being measured. The method consists of syntonizing at any convenient ratio, one of the reference oscillators against the frequency to be studied and noting the reference oscillator setting for each condition. The relative changes are then calculated by the factor $\frac{C_2 - C_1}{2\,C_2}$ from (**6**). If the measurements are made over a short interval of time, the standards of frequency are not used. If the measurements are spread over an extended period of time, the reference oscillator is syntonized at the time of each measurement against a standard of frequency at a ratio to give a setting near those obtained against the frequency under test. The differences in these settings from the first are used to correct the settings against the frequency being studied. For a great deal of the work the settings of the reference oscillator are so chosen that the air condenser difference between settings gives directly the answer in decimal multiples of per cent.

Formulas (**5**) and (**6**) are also used in measuring frequencies and in setting an oscillator at a frequency which may not be syntonized readily against the standards. To facilitate their use and to avoid their misuse, Table IIA was prepared. It gives the errors inherent to the use of the formulas as calculated from the second order term of formula (**3**). It will be seen that an error of only + 0.0002 per cent is introduced by the formulas in a 0.2 per cent interpolation. As points may be found by direct syntonization against the standards at smaller intervals than 0.2 per cent above 4000 cycles and by double syntonization below 4000 cycles, the formula may be extensively used in this connection.

TABLE IIA TABLE IIB

Interpolation errors inherent to the approximate formula: $C_2 \doteqdot C_1 - \frac{2\,C_1\,(f_2 - f_1)}{f_2}$		Interpolation errors inherent to linear interpolation.	
% increase of frequency calculated	% frequency error inherent in calculated setting $\frac{1}{2}\left(\frac{f_2 - f_1}{f_2}\right)^2 \times 100$	% difference in known frequencies	Maximum % frequency error which may be obtained in calculated setting
0.1	+ 0.00005	0.2	− 0.0002
0.2	+ 0.0002	0.5	− 0.0009
0.5	+ 0.0012	1.	− 0.004
1	+ 0.005	2	− 0.014
2	+ 0.02	5	− 0.08
5	+ 0.12	10	− 0.3
10	+ 0.5	20	− 1.0
20	+ 2	40	− 4.0

Linear interpolation between two frequencies, f_1 and f_3, is also used for a large amount of work. Often it is used to replace formulas (**5**) and (**6**) in the adjustment of odd frequencies and in the determination of unknown frequencies. The inherent errors of this method have been calculated also and are given in Table IIB. Linear interpolation expressed as formulas becomes:

$$C_2 = C_3 + (C_1 - C_3)\,\frac{f_3 - f_2}{f_3 - f_1} \qquad (7)$$

$$f_2 = f_3 - (f_3 - f_1)\,\frac{(C_2 - C_3)}{(C_1 - C_3)} \qquad (8)$$

In the measurement of single frequencies, the frequency is known to at least its order in the usual case and to very high accuracy in a considerable proportion of cases. Methods of measuring such a frequency more accurately than it is known readily present themselves. For the few cases where even the order of the frequency is not known, methods of preliminary estimation for determining it to a few per cent are desirable. The first step in such a procedure would be to determine if it is an audio frequency, a radio frequency or intermediate. This may be done by listening to determine if it is an audio frequency. If it is, it can be estimated from its tone more or less closely according to experience. No specific methods of similarly subdividing inaudible frequencies into ranges have been standardized but in all cases of measurement the narrowing of limits is attempted before proceeding further. After such an estimate, the frequency is placed on the oscillograph against one of the reference oscillators. Two general methods are used from this point on. In the first method the reference oscillator is set at a frequency between 20 and 200 times the frequency to be measured. The setting is also so chosen that the capacity setting is between 0.015 μ f and 0.050 μ f. The air condenser is then slowly decreased from a high reading to the first setting which gives a figure the number of whose lines may be determined readily. The reading is noted and the condenser setting further decreased, note being made of the easily identified patterns disclosed in the process. After decreasing the capacity 200 $\mu\,\mu$ f. or more, the next figure to appear is held and the reading noted, together with the pattern obtained. The frequency difference corresponding to these two points is then determined from the calibration or calculated from equation (**6**) in the form

$$\Delta f = f_2 - f_2 \doteqdot f_1\,\frac{C_2 - C_2}{2\,C_2}$$

where f_1 is found from the calibration of the reference oscillator.

f_1 and f_2 may be expressed in terms of the unknown frequency F as follows:

$$f_1 = \left(N + \frac{X_1}{l_1}\right) F$$

$$f_2 = \left(N + \frac{X_2}{l_2}\right) F$$

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whence

$$F = \frac{\Delta f}{\dfrac{X_2}{l_2} - \dfrac{X_1}{l_1}} \qquad (10)$$

in which l is the number of lines in the figure and in which X is known from the order of the figures lying between the two points of observation. For instance, if the observations had been made with five- and three-line figures having a two-line and a five-line figure lying between in that order, it is obvious from Table I that $\dfrac{X_1}{l_1} = 0.4$ and $\dfrac{X_2}{l_2} = 0.666$ are the only two values which satisfy the conditions. It is to be observed that the Ns are not identical if one-line figures are noted while decreasing the capacity. This is taken care of by adding 1 to $\dfrac{X_2}{l_2}$ for every one-line figure noted.

For the measurement of higher frequencies, there is available a similar method in which the unknown high frequency is syntonized against two lower frequencies of a reference oscillator. In this case

$$F = \left(N + \frac{X_1}{l_1}\right) f_1 = \left(N + \frac{X_2}{l_2}\right) f_2$$

from which may be obtained

$$F = \left(\frac{X_2}{l_2} - \frac{X_1}{l_1}\right) \frac{f_2 f_1}{f_1 - f_2} \qquad (11)$$

From equation (**6**) is found

$$\frac{f_2}{f_1 - f_2} \doteq \frac{2\,C_2}{C_2 - C_1}$$

whence

$$F \doteq \left(\frac{X_2}{l_2} - \frac{X_1}{l_1}\right) \cdot \frac{2\,C_2}{C_2 - C_1} \cdot f_2 \qquad (12)$$

In using the methods just described, an accuracy of 2 per cent or 3 per cent is desired. To meet this accuracy, the ratio of $\dfrac{C_2 - C_1}{2\,C_2}$ or of $\dfrac{f_1 - f_2}{f_2}$ must be reasonably small (0.02 or less) in order that the approximation of equation (**6**) will be satisfactory. It is necessary that $(C_2 - C_1)$ may be read to three significant figures and it is necessary to include in C_2, C_0 to the third significant figure of C_2.

Special Circuits

Mention has been made of the fact that various means are available for making the "front" and "back" of the patterns readily distinguishable. There have been devised for this purpose many circuits, some of which have been used in connection with the equipment described in this paper. These auxiliary circuits may be divided roughly into two classes; the first, those in which the beam is caused, by the base frequency and its auxiliary circuit, to traverse the screen much more rapidly in one direction than in the other so that the "back" pattern is spread more than the "front" and even may be so faint as to be invisible, and the second, those in which the "back" of the pattern actually is displaced above or to the side of the "front" pattern.

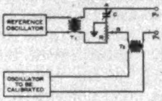

Fig. 14

Those belonging to the first class have been used more often. Perhaps the simplest to use is that of distortion in the auxiliary vacuum tube circuits of the measurement equipment. For instance, in the case of the standard frequencies, by overloading the vacuum tubes of the amplifiers and by using interstage transformers which for any other work would be considered unsatisfactory, sufficient distortion is produced to make the "front" of the picture stand out much more clearly than the "back" of the picture. Consequently, in the larger volume of routine work, this method is used entirely. In precision work, it has a slight advantage in that the "back" pattern is of a larger amplitude than the other and magnifies any lack of syntonization.

One of the simplest methods of separating the "front" from the "back" of the picture is by means of the circuit[4,5] shown in Fig. 14. The reference oscillator output is stepped up to a high voltage by means of transformer, T_1, and placed across a resistance, R, and capacity, C, which are in series and have their common point grounded. The condenser is made variable to facilitate the adjustment of its impedance in accordance with the frequency of the input. In the more useful case the resistance would be in the form of a potentiometer in order that a variable voltage might be taken from it. If, with a small input to T_1, the voltages at A and B are placed on the cathode ray oscillograph, there will result an ellipse, for the voltages

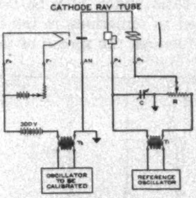

Fig. 15

across the resistance and across the condenser are at right angles to each other. If the secondary of transformer T_2 is placed between B and the oscillograph and the primary is connected to the unknown frequency, there will result on the oscillograph after syntonization a figure such as is illustrated by Fig. 8. If the input to transformer T_1 is increased, the ellipse may be so enlarged that it will lie outside the oscillograph screen. However, by adding a d-c. component by means of the biasing potentiometers of the oscillograph, one portion may be placed at the center of the screen and figures such as those illustrated by Figs. 9 to 13 will result. In this circuit, the separation of the "front" and "back" of the pictures is controlled by the setting of the potentiometer at A and the spread of the pictures by the input to T_1. The condenser is varied with frequency to place an optimum impedance load on the transformer.

Another circuit which often is useful is that shown by Fig. 15[6]. As in the previous circuit, the reference frequency is applied in quadrature to the two plates of the oscillograph. In this instance the values of the resistance and of the condenser impedance are made equal so that, with zero biasing potentials on the oscillograph, a circle appears on the face of the screen. The unknown frequency is then placed in series with the cathode potential by means of transformer T_1. The effect of a change in the cathode potential is to change the sensitivity of the oscillograph, thus enlarging the radius of the circular trace. When the cathode potential is varied by an alternating potential which varies the radius syntonously with the base frequency, a pattern such as shown in Fig. 9 results. This method of separating the "back" and "front" of the picture has found considerable use in radio frequency measurements. However, the circumference of the trace is limited by the circumference of the oscillograph screen which limits the usefulness of this circuit for one of two reasons. The ratio which may be used with such a circuit is limited to something in the order of 1 to 50 and at low frequencies the spread of the unknown frequencies can not be made large enough to obtain high precision.

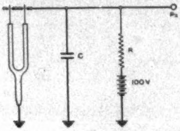

FIG. 16

Several other circuits of limited value have been tried. One which is interesting because it involves the much used charge or discharge curve of a condenser is shown in Fig. 16. It is used for the measurement of the frequency of vibrating contacts. Specifically, it is more often used for the measurement of tuning forks which have driving contacts. The figure merely illustrates one of the numerous ways in which the circuit may be set up. In this instance, when the contact is made, both terminals of the condenser are grounded and the condenser is in a discharged condition. When the contact opens, the condenser is charged at a rate controlled by its value and by the value of the resistance in series with it and some convenient potential. The result is a saw-tooth shaped voltage wave form across the condenser, which is an optimum type of wave shape for frequency measurement. The figure produced by such a system is very similar to that illustrated by Fig. 7, although the "back" lines shown in Fig. 7 are invisible, as the potential of the condenser is reduced to zero instantaneously by the closing of the contact.

FIG. 17—A CATHODE RAY OSCILLOGRAPH EQUIPMENT

Containing an oscillograph, two oscillators, a 300-volt rectifier, and a standard-frequency control panel.

A large number of precision measurements has been made on stroboscopic forks. In this instance, a circuit was used which, while not a circuit relating to the oscillograph, is given to illustrate the wide diversity of measurements and circuits which are used in connection with the frequency measurement equipment. This circuit is constructed by merely placing in the output of one of the reference oscillators a high-ratio transformer, across the secondary of which is connected a neon lamp. The oscillator is then adjusted to twice the frequency of the fork by viewing the neon lamp through the fork and varying the oscillator condensers until the glow is either steady or not seen at all. This measurement gives the frequency of the fork within a fraction of a per cent. If a higher precision is needed, the oscillator is set at approximately 10 to 20 times the frequency of the stroboscopic fork. Its output is increased to the point where the neon tube just discharges. It is now found that the oscillator can be syntonized to the fork to a very much higher precision.

CONCLUSION

The methods and apparatus described were first developed for laboratory precision measurements. They have been in use for several years during which time it has been amply demonstrated that they are suitable not only for the technical measurements of a development and research nature but are equally adaptable for routine commercial work. The advantages which particularly commend themselves are the rapidity with which such work may be done and the ease with which the average man can learn the work.

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Discussion

T. E. Shea: Mr. Rasmussen has discussed the need for accurate and ready measurements of frequency. It is rather difficult to overstress the importance of frequency as an engineering variable in communications work, for the whole transmission problem centers about the signals we want to transmit and the interference we don't want to transmit. Both the signals and the interference are made up of a large number of frequency components and so the vital transmission characteristic of communication networks may be said to be frequency characteristics. We have frequency against impedance, or reflection coefficient; frequency against transmission loss, or attenuation; frequency against phase shift, and so on—always frequency against some other variable.

This is well illustrated in the case of wave filters. It is the function of wave filters to separate bands of frequencies and for economic reasons relatively small frequency intervals can be allowed for filter characteristics—in this case transmission-loss characteristics—to change from free transmission regions to regions of high attenuation or inefficient transmission. Here is where accurate and ready frequency measurements are of great assistance, in insuring that filter characteristics embodied in some physical network are accurately placed along the frequency spectrum so that the signal frequencies pass through as efficiently as possible and the interfering frequencies are barred out.

It is usual in discussing wave filters to stress the large amount of network theory and the extensive calculations involved in their design but one can readily see that an economic balance would hardly be struck if great effort were put into working out theoretical designs and an equal effort were not put into the development of measurement methods so that proper guide posts were not available at all times to indicate how far the theoretical designs had any value. This is, as a matter of fact, an indication of the importance of measurement methods such as Mr. Rasmussen describes—that they are the guide posts to us without which design work would be of little avail.

A Shielded Bridge for Inductive Impedance Measurements at Speech and Carrier Frequencies

BY W. J. SHACKELTON[1]

Associate, A. I. E. E.

Synopsis.*—A shielded, a-c., inductance bridge adapted to the measurement of inductive impedances at frequencies up to 50,000 cycles is described. The bridge comprises a balancing unit and associated standards of inductance and resistance. The balancing unit has resistance ratio arms specially constructed to meet the requirements imposed by the above frequency range. The reference standard makes use of inductance coils of a new type, their cores being of magnetic instead of non-magnetic material as is usually the case. The use of such cores results in coils that are smaller and hence better adapted to assembly in a multiple shielded standard.*

The bridge is completely shielded so as to eliminate, to a high degree, errors due to parasitic capacitance currents. The shielding is also arranged so as to permit the correct measurement of either "grounded" or "balanced-to-ground" impedances. A series of diagrams is shown for the purpose of indicating the function of each part of the shielding system.

Equations expressing the errors resulting from any small residual capacitance unbalances in the resultant bridge network are given and calculations made of the balances required for the desired degree of measurement precision. Test data are presented illustrating a method of experimentally checking the residual shunt and series balances from which it is concluded that the bridge is capable of comparing two equal inductive impedances of large phase angle with an accuracy at the maximum frequency of 0.02 per cent for inductance and 1.0 per cent for resistance.

INTRODUCTION

THE limitations of the ordinary unshielded bridge network as a means of making precise a-c. measurements at speech frequencies were recognized early by telephone engineers. The solution of cross-talk problems arising in connection with the use of cable circuits was found to require an exact knowledge of the capacitive balances existing between such circuits at speech frequencies. For the ready and accurate determination of the capacitances defining these balances, together with their associated conductance values, Dr. G. A. Campbell devised the "shielded balance."[2] This is a bridge network having its parts individually and collectively shielded so as to define exactly the mutual electrostatic reaction of each with respect to all other parts of the electrical system affecting the balance condition.

As a means of more completely treating the cross-talk problems of cable circuits, Dr. Campbell conceived also the very valuable idea of "direct capacity" as distinguished from the "ground" and "mutual capacities" in use up to that time.[3] The shielded balance was found to be especially adapted to the precise measurement of direct capacities employing the substitution method devised by Mr. E. H. Colpitts.[4] Shortly thereafter, with the advent of loading for telephone lines, the same principles of shielding were extended to apply to bridge networks specially arranged for the measurement of the speech-frequency inductance and effective resistance of loading coils. As the successful commercial application of loading required the manufacture of these coils in large numbers to precise requirements, it was quite essential that testing means be available permitting a relatively unskilled tester to determine quickly whether the proper adjustment of the coils had been made. For this purpose the shielded balance has proved to be extremely valuable. More recently the employment of frequencies up to 50,000 cycles for carrier telephone and telegraph purposes has led to the need for correspondingly precise measurements at these higher frequencies. In this field the advantages of the shielded bridge are so great as to make it almost indispensable.

While the fundamental principles of the shielded balance are essentially the same for all impedance measurements, the practical application of shielding to any concrete bridge problem may vary according to the kind and range of impedances to be tested, the frequency range to be covered, and the precision required. It also presents special problems in the design and construction of several of the circuit elements. In this paper there is described a particular form of shielded bridge which has been developed to meet the conditions commonly encountered in the measurement of inductance at speech and carrier frequencies. The facts leading to the detailed construction are discussed and some experimental data given to illustrate the performance of the bridge.

GENERAL FEATURES

A simple schematic diagram of the bridge circuit is shown in Fig. 1. To avoid confusion, no shielding is shown in this diagram. As will be noted, the bridge comprises two equal resistance ratio arms, an adjustable standard of self-inductance, an adjustable resistance standard, a thermocouple milliammeter, two reversing switches, two transformers and two air condensers. Physically, this apparatus is grouped into three separate units, one comprising the standards of inductance, one the resistance standard and the third, the remaining parts of the circuit. The last assembly constitutes what may be

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[1] Development Engineer, Bell Telephone Laboratories, Inc., New York, N. Y.

[2] G. A. Campbell: The Shielded Balance, *Electrical World and Engineer*, April 2, 1904, p. 647.

[3] G. A. Campbell: Measurement of Direct Capacities, *Bell System Technical Journal*, July 1922, p. 18.

[4] See Note 3.

Presented at the New York Regional Meeting of the A. I. E. E., New York, N. Y., Nov. 1-12, 1926.

considered the balance element of the system, by means of which the unknown and standard impedances are compared. Figs. 2 and 3 show the arrangement of the parts in this unit. Fig. 4 illustrates the appearance of the standard inductance unit and Fig. 5 shows how the units are associated when a test is being made. The thermocouple milliammeter indicates the total effective test current applied to the bridge and forms a means of determining when this current has been adjusted to the desired value.

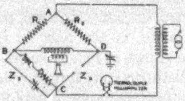

Fig. 1—Schematic Diagram of Bridge Circuit

In operation, the air condensers are first adjusted to produce an initial or zero balance of the residual electrostatic capacitances of the apparatus. Aside from the initial balancing, the operation of the bridge follows the usual practise; that is, the standards of inductance and resistance are alternately adjusted until the balance detector indicates a condition of zero potential difference at every instant between the bridge points to which it is connected. The inductance and resistance values as indicated in the standard arm are then equal (within the precision limits of the bridge) to the corresponding constants of the unknown impedance.

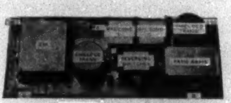

Fig. 2—Balance Element of Shielded Bridge

Rear view of panel removed from case

Fig. 3—Balance Element of Shielded Bridge

Front view of panel and case

Purpose of Shielding

The principal difficulties in attaining a satisfactory degree of precision in inductance measurements at relatively high frequencies by means of unshielded bridges are those due to the presence of residual or stray admittances existing between the bridge parts or from them to ground. All these parts have quite appreciable surface dimensions and when exposed at the usual separations to each other or to ground, have corresponding direct and grounded admittances. Leads to the source of testing current and to the balance detector also introduce rather large admittances. In a bridge intended for rapid operation, the parts subject to manipulation must be arranged compactly and conveniently to the operator. This makes it impracticable to isolate them sufficiently to make the admittance values between these parts and between them and ground (the operator being considered to be at ground potential) negligibly small.

To make the matter more concrete, there is shown in Fig. 6 a schematic diagram with possible positions of some of the more important of these admittances indicated as at C_1, C_2, etc. (With some exceptions, the capacitance components of these stray admittances substantially determine their full effect. In the diagrams and discussion, therefore, the conductance component will be neglected except where its effect is significantly large). The capacitances between the two ratio arm coils, R_1 and R_2, and from each to ground, are shown as being uniformly distributed along the length of the coils symmetrically with respect to each

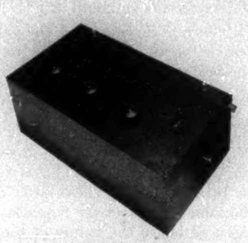

Fig. 4—Inductance Standard

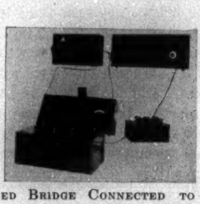

Fig. 5—Shielded Bridge Connected to Vacuum Tube Oscillator and Heterodyne Detector

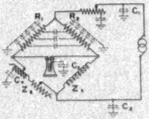

Fig. 6—Bridge Circuit with Stray Admittances

other. If this symmetry is perfect these capacitances do not affect the bridge balance. In practise, however, they will only be approximately so, with the result that the two arms will be somewhat unbalanced to alternating currents, the effect of the unbalance increasing with the frequency. While the ratio arm capacitances can be made fairly small, others such as those indicated at C_1, C_2, C_3, and C_4 will commonly be much larger and hence of greater effect. Capacitances C_1 and C_2 are frequently comparatively large due to the use of long distributing wires, encased in grounded conduit, for supplying the testing current. C_3 may consist chiefly of the ground capacitance of the outer layer of the detector coil winding and C_4 that of dead-end coils of the reference standard, Z_s.

Some of the current flowing along the paths provided by these capacitances will complete its circuit external to the bridge network proper and will not affect the balance; for example, that through capacitances C_1 and C_2 in series. Others, however, will flow unsymmetrically through parts of the bridge circuit; for instance, that through C_1 and C_3 in series and the arm Z_x; also, that through C_2 and C_4 in series, returning through the ratio arm R_1. These latter currents and others of the same sort affect the potential distribution of the bridge and hence the values of the impedances required for balance. Certain of these capacitance currents in the bridge network tend to neutralize or balance the effects of others; for example, that through the arm Z_x due to the series action of capacitances C_1 and C_3 has a balancing effect with respect to that through C_1 and C_4 and the arm Z_s and would be without reaction on the bridge balance if capacitances C_3 and C_4 were exactly symmetrical with respect to the two detector terminals. Such balancing, however, is accidental in nature, seldom satisfactorily complete and, in part, not constant. Even were it made approximately complete for a particular arrangement, the substitution of another detector or the use of another source of testing current would probably destroy the balance. Variable effects would always be present; for example, those due to the changing position of the operator relative to the parts of the circuit or the effects of parallel loads on the supply generator. The distribution and value of the ratio arm ground capacitances described above are functions of the bridge surroundings; hence they are also subject to change if the bridge is moved from place to place. In the bridge being described, however, the shielding used affords a means of definitely fixing and controlling the various intercircuit capacitances. Consequently, such variations cannot take place, balances between the resultant capacitance currents can be made as desired, and the bridge measurements are satisfactorily precise.

SHIELDING SYSTEM USED

It is felt that the merits of the particular shielding system adopted for this bridge can best be brought out by showing, step by step, the reasons for using each of its elements.

The first step is to simplify, for further treatment, the initial residual capacitance network of the unshielded circuit. This is done by providing individual shields for each part of the circuit that it is desired to have function as an independent unit. Such shields can be connected to one of the terminals of the part enclosed and thus there is substituted, from the standpoint of terminal-to-terminal characteristics, a definite and invariable condition in place of that which was previously a function of the relation of the part to its surroundings. For example, as shown in Fig. 7, shields would be placed around the resistance coils forming the ratio arms R_1 and R_2 and connected to the junction point A of the system, one enclosing the elements of the standard impedance Z_s and another around the source of testing current and connected at C; likewise, one around the detector is connected at D. It will readily be seen that these shields localize the effects of the various capacitance currents. Those circulating within the shields have, of course, no effect exterior to the shields, while those flowing between the various shields directly or by way of ground, enter and leave the bridge system at definite points. By themselves, these shields do little good but they are necessary in order to make the next step (the balancing of the capacitances) practicable.

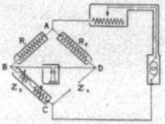

FIG. 7—BRIDGE CIRCUIT WITH LOCAL SHIELDS

Generally it will not be found convenient to shield the current supply apparatus, especially if this is a power driven generator. Also, to promote greater flexibility in respect to testing with a wide range of frequencies, it will often be desirable to substitute one source of current for another and likewise one detector for another. The shielding of this apparatus should therefore be reduced to a minimum. This is readily effected by making both the supply and detector branches of the bridge one of the windings of a transformer. This winding can be electrostatically shielded without affecting its transformer action and then any desirable source of current supply or any type of detector can be magnetically coupled with it.[5] Introducing this change the circuit becomes as shown in Fig. 8. The capacitances of the various shields to ground being still variable, the next step to correct this condition would simply be to add a ground shield around each. At this point, however, it becomes necessary to consider the ground admittance relations of the impedances to be tested.

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[5] U. S. Patent No. 792248, June 13, 1905.

In general, the unknown impedance will have capacitances to ground and the effect of these will be properly included in the measurement only when certain conditions as determined by the nature of the apparatus are fulfilled. From this standpoint the impedances usually encountered are of three general classes: (1) Those having ground admittances negligibly small in comparison with the direct terminal-to-terminal admittance; (2) those having appreciably large admittances to ground approximately balanced with respect to the two test terminals; (3) those having one terminal directly grounded, the other having an appreciably large ground admittance.

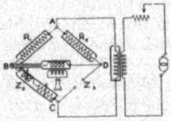

Fig. 8—Bridge Circuit with Shielded Transformers and Local Shields

In measuring apparatus of the first type it is evident that since in connecting it to the bridge circuit no additional ground admittances are introduced, the balance between those previously existing can be made without reference to the test impedance. The connection of an impedance of either of the other types will, however, introduce additional ground admittances into the bridge system, which, unless precautions have been taken, may cause the result to be something other than that which is wanted. In general, the desired test is that which gives the effective impedance applying to the apparatus as it is used. In the case of impedances having balanced admittances to ground, this is the effective value of the direct, terminal-to-terminal impedance as modified by the effect of the two ground admittances acting simply in series with each other. This condition is obtained when equal currents flow in each of these admittances, or, what is equivalent, when the electrical potentials of the terminals are balanced with respect to ground potential. To obtain this condition, when the impedance is being tested, the bridge terminals to which it is connected must likewise be balanced with respect to ground potential; that is, ground potential must be at the mid-point of the unknown impedance arm. If the only admittances to ground of the bridge system are those of the junction points (as is the case in Fig. 8), the potentials of these points with respect to ground are entirely determined by these admittances. To make any two points such as the terminals of the unknown arm have equal potentials to ground, it is sufficient to concentrate all of the ground admittances to these or other equi-potential points and then balance the admittances from each. Referring to Fig. 9, if the testing current is applied at the points *A* and *C*, this condition is realized as shown by concentrating all ground capacitances at junction points *B*, *C* and *D*, and making the sum of the capacitances of junction points *B* and *D* equal to that of junction point *C*. This follows from the fact that when the bridge is balanced, the junctions *B* and *D* are equi-potential points. The mid-point of arm *C D* is now at ground potential. If, however, the testing current is applied at the points *B* and *D*, the equi-potential points are the junctions *A* and *C*, the sum of whose ground admittances would then be made equal to that of *D* and the arm *C D* again balanced with respect to ground potential. In this case there must be no ground admittance from junction *B*. To permit of testing under both conditions, point *A* and all connected conductors are protected with a shield which is then connected to point *C*. Point *B* is likewise enclosed by a shield connected to point *D*. These two main shields then represent the junction points *C* and *D* of the bridge and are fixed with respect to capacitance to ground by a ground shield which may be common to the two.

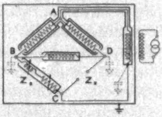

Fig. 9—Shielded Bridge Circuit showing Location of Ground Admittances

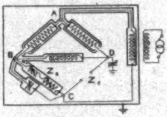

Fig. 10—Shielded Bridge Circuit with Balancing Condensers

There now exist external to the local shields, direct capacitances only between points *A* and *C* and between *B* and *D*, (which do not affect the bridge balance), and from points *C* and *D* to ground. These latter do, of course, affect the balance. Two courses are open. Their effective resultant value shunting the arm *C D* can be determined and allowed for by calculation. Such calculations would involve a considerable amount of labor, however, and can be avoided very simply by providing in the opposite arm an exactly equal shunt capacitance. To permit adjusting the ground capacitances of points *C* and *D*, an adjustable condenser is connected to ground from the point having the lower value. With the apparatus connected as shown this is usually point D. The shielded system then becomes as shown in Fig. 10.

When impedances which in actual service are grounded at one terminal, are to be tested, the matter is much simpler. Then it is necessary only to definitely ground one of the bridge terminals to which the impedance is connected and establish the proper initial capacitance balance of the bridge for this condition. This is readily done by grounding junction point *C* and adjusting the capacitance from *B* to *C* to equal the ground capacitance of *D*. The shielding system may remain the same as in Fig. 10.

In the case of the bridge being described it was desired to have a means of verifying by reversal the degree of balance of the ratio arms and also that of the impedance arms. The bridge is therefore equipped with reversing switches for this purpose. Due to the appreciable effect produced by a relatively small capacitance unbalance arising from factors present only when the arms are in circuit, it is quite important to be able to do this when a high degree of accuracy is desired. To effect the proper reversal, however, certain conditions must be definitely maintained. In reversing the impedance arms none of the inherent bridge admittances should be disturbed; that is, only the unknown impedance of the standard as read should be transferred. On the other hand in reversing the ratio arms not only should the resistance element of these arms be transferred but also all associated shunt admittances. Moreover, in transferring these admittances they must be absolutely unchanged. A further requirement is that the ratio arm reversal must not occasion the shifting of any capacitances shunting the impedance arms. To accomplish these objects a suitable arrangement of shielded switches was worked out and added to the circuit of Fig. 10, the result being as shown in Fig. 11. In this arrangement all capacitances between the various parts of the switches which are subject to change due to physical movement of the switch parts are either short-circuited or connected across opposite bridge points and hence do not affect the bridge balance. The small capacitance C_R between the switch shield and that of the ratio coil R_2 shunts this coil and is not carried with it on reversal. For this reason a corresponding capacitance C_R', shunting the coil R_1, is provided and connected to the opposite point of the switch. This is adjusted by test to equal the value of C_R. The diagram of Fig. 11 represents completely the circuit and shielding used for the balance unit of the bridge.

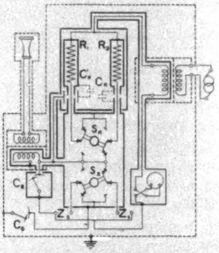

FIG. 11—COMPLETE CIRCUIT DIAGRAM OF BALANCE UNIT WITH SHIELDING

While, from the standpoint of the bridge balance alone, the parts comprising the standard impedance can be shielded with a local and ground shield as shown in Fig. 9, unless the standard has a very limited range, the resulting calibration is exceedingly laborious to make and use. To reduce calibration difficulties, additional shields can be used; this, of course, increasing the cost of construction. In arriving at the proper compromise between these conflicting factors the size and impedance value of the part to be shielded must be considered. This question will therefore be taken up in more detail in the following section.

CONSTRUCTION

The circuit and shielding features discussed so far are of general application to impedance measurements without restriction as to the particular range of values to be tested or frequencies to be used. The physical construction is, however, dependent upon these factors. As initially stated, the bridge is intended for the measurement of audio and carrier frequency inductances. By this term is meant all apparatus having reactance values nearly equal to the respective impedance values. For the purpose of the present discussion, such inductances will be more exactly defined as those having ratios of reactance to resistance of not less than 10 (minimum phase angle of 84 deg., 20 min.). The difference between the reactance and the impedance of any such inductance does not exceed ½ per cent. The impedance values range from about 100 to 10,000 ohms and testing frequencies from 500 to 50,000 cycles.

On the basis of these conditions, the following construction was developed and is used for this bridge.

Ratio Arms. It is desirable from the standpoint of sensitivity of balance to have the ratio arm impedances of approximately the same value as those of the other two arms. Considering the range of impedances to be covered and giving due weight to the values which are of most importance in telephone circuits, a ratio arm resistance of 1000 ohms was selected. The problem then was to construct two 1000-ohm resistances, balanced both as to effective resistance and effective inductance for a frequency range from 500 to 50,000 cycles when subjected to the usual temperature and humidity variations.

Curtis and Grover have discussed the factors affecting the characteristics of a-c. resistances and have suggested forms suitable for general use at frequencies up to 3000 cycles[6]. A 1000-ohm resistance, constructed according to their specifications, is made by winding with a 1/10-mm. diameter, double-silk-covered manganin resistance wire, five 200-ohm bifilar sections on a 1-in. spool of insulating material. These sections are spaced about three mm. apart on the spool and are connected in series to form the 1000-ohm coil. Such a coil, when shellacked, baked and coated with paraffin, was found to be substantially constant in resistance and to have constant phase-angle effects equivalent to shunting capacitances of the order of 10 to 15 m. m. f. for all frequencies up to 3000 cycles. Since individual coils made according to this method may differ in their effective capacitances by as much as five m. m. f., some adjustment of these capacitances (as well as of the resistance) is required in order to make them suitable for use as the required ratio arms. Assuming that this is done by adding to the coil having the lower value a small capacitance of suitable constancy, it may be concluded that two coils so balanced would be suitable for use at frequencies up to 3000 cycles.

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[6] H. L. Curtis and F. W. Grover: Resistance Coils for Alternating Current Work, *Bulletin of the Bureau of Standards*, Vol. 8, No. 3.

In arriving at the requirements for the more extended frequency range of this bridge, the necessary phase-angle balance was first considered. Designating by L_x and R_x the inductance and effective resistance of the impedance being tested, and by L_s and R_s, the corresponding components of the standard impedance required to balance it in a bridge circuit having ratio arms of exactly equal resistances R but shunted by slightly different capacitances, C_1 and C_2, and assuming that the quantities are such that $\omega^2 R^2 C_1^2$ and $\omega^2 R^2 C_2^2$ are small in comparison with unity, the equation for balance is

$$(R_x+j\,w\,L_x)(R-j\,w\,C_1 R^2) = (R_s+j\,w\,L_s)(R-j\,w\,C_2 R^2)$$

which reduces to

$$R_x = R_s + w^2 R\,(C_2 L_s - C_1 L_x) \qquad (1)$$

and

$$L_x = L_s - R\,(C_2 R_s - C_1 R_x) \qquad (2)$$

Neglecting second order effects, these can be written

$$R_x = R_s + w^2 R\,L_x\,(C_2 - C_1) \qquad (3)$$

and

$$L_x = L_s - R\,R_x\,(C_2 - C_1) \qquad (4)$$

If the readings R_s and L_s are taken as the values of the unknown resistance and inductance, respectively, it is evident that errors as given by the last terms of these equations will be present. The percentage errors in the two cases are as follows:

$$\Delta R_x\,(\%) = 100\,\omega^2 R\,(C_2 - C_1)\frac{L_x}{R_x}$$

$$= 100\,\omega\,R\,(C_2 - C_1)\tan\theta \qquad (5)$$

$$\Delta L_x\,(\%) = 100\,R\,(C_2 - C_1)\frac{R_x}{L_x} \qquad (6)$$

For a given capacitance unbalance of the ratio arms, it is seen that the error in inductance is inversely proportional to the time constant $\left(\frac{L}{R}\right)$ of the impedance arm and is independent of the frequency, while the error in resistance is proportional to the frequency and to the ratio of reactance to resistance, that is, to the tangent of the phase angle. The inductance error is, therefore, maximum for the minimum time constant apparatus to be tested. Within the range previously mentioned this occurs when an impedance having the minimum reactance to resistance ratio of 10 is being measured at the minimum frequency of 500 cycles. Under this condition $\frac{R}{L}$ has a value of $\frac{2\,\pi \times 500}{10}$ or approximately 300. The corresponding percentage error in inductance per micro-microfarad of capacitance unbalance is then $300 \times 1000 \times 10^{-12} = 3 \times 10^{-7}$ or 0.00003 per cent. Evidently a very considerable unbalance can be tolerated. In the case of the resistance component, the error is maximum when an unknown impedance having the maximum reactance to resistance ratio is being tested at the maximum frequency. A reactance to resistance ratio of 300 is very rarely exceeded. For this value, the error per micro-microfarad unbalance at a frequency of 50,000 cycles amounts to about 9.5 per cent. Hence, to limit the error from this source to the order of 1 per cent requires a balance of about 0.1 micro-microfarad. It will be appreciated that this is an extremely close balance, the maintenance of which, under the different conditions of temperature and humidity to which the bridge may be subjected, requires careful consideration of the effects of these factors.

The effective phase-angle balance, though discussed above in terms of capacitance only, is, of course, the resultant of the inherent residual inductances and capacitances of the coil windings plus the additional capacitance effects due to the coil shields. The component due to residual magnetic induction is not appreciably affected by temperature or frequency changes. The capacitance component of the winding, however, tends to vary with temperature in accordance with the temperature coefficient of capacitance of the dielectric used for insulating the wire and with frequency to the extent that the capacitance is affected by absorption.

It is common practise to employ silk-insulated wire treated with varnish or wax for purposes of protection against moisture in such coils. In order to obtain data covering the temperature and absorption effects and also the phase-angle characteristics of silk insulation, both untreated and when treated with a number of the more common materials, various samples were constructed and tested as indicated in Table I.

From the standpoint of percentage capacitance change, (reckoning from the minimum temperature and frequency conditions as being those at which initial adjustments would be made), untreated and paraffin-treated silk insulation were found to be appreciably superior to any of the other materials. The change

TABLE I

Capacitance and phase-angle tests between two double-silk-covered No. 38 A. W. G. wires, each 88 in. (224 cm.) long, wound in bifilar fashion on a glass tube one in. (2.54 cm.) in diameter and then treated with various materials. Samples dried before testing.

Test sample number	Material used for treatment	Capacitance, micro-microfarads				Phase angle tangent			
		Temp.—20 deg. cent.		Temp.—45 deg. cent.		Temp.—20 deg. cent.		Temp.—45 deg. cent.	
		1000 ~	50,000 ~	1000 ~	50,000 ~	1000 ~	50,000 ~	1000 ~	50,000 ~
1	None	193.3	188.2	188.7	184.1	0.0083	0.013	0.0076	0.011
2	Paraffin	251.1	243.2	232.1	244.3	0.0095	0.014	0.010	0.011
3	Collodion	238.9	228.0	231.4	222.5	0.016	0.021	0.015	0.018
4	Beeswax compound	258.2	248.7	273.2	265.8	0.013	0.014	0.010	0.013
5	Pyralin	253.1	241.1	258.5	246.6	0.016	0.021	0.018	0.022
6	Insulating varnish	346.4	320.1	361.5	337.0	0.027	0.036	0.030	0.033
7	Shellac	296.9	284.0	314.3	302.4	0.012	0.021	0.015	0.020

due to absorption effect (about three per cent for these two) was considered the more important, as normally variations in temperature would not be very large. As would be expected, the untreated silk had the lowest phase-angle effect and also the smallest capacitance. Assuming that a method of excluding moisture could be devised, it was concluded that an untreated silk-insulated winding would be the best to use, although the paraffin treatment was also considered promising. Discounting the fact that the two ratio coils would change in the same direction though not necessarily by the same amount, it was decided that a satisfactory factor of safety would be provided if it were assumed that the coils might become unbalanced by one-half the observed change in one coil; that is, by about 1½ per cent. In order that such unbalance should not exceed 0.1 micro-microfarad the capacitance of each coil would need to be not more than about 6.0 micro-microfarads. It should be noted that this limit applies to the true inherent capacitance and not to the resultant of the coil capacitance and inductance.

Considering now the variation in resistance over the frequency range of the bridge, it can be shown, following the methods of Curtis and Grover, that the effect of a capacitance of the value noted above on the resistance will not exceed one part in 100,000, which is quite satisfactory. The change in resistance (from the d-c. value) due to energy dissipation in the insulation is, however, somewhat larger than that due to the pure capacitance effect. This change is given to a close approximation by the expression

$$\Delta R = \frac{C \omega R^2 \tan \phi}{3}$$

where C is the total distributed capacitance between the wires of a bifilar winding and ϕ is the phase angle of the capacitance.[7] Clearly both the capacitance and its phase angle should be kept as small as practicable. An obvious and simple way of attaining the first object would be by using the very finest wire available. To do this, however, would in many cases result in excessive heating of the resistance. For bridge tests on telephone apparatus the ratio arm current will rarely exceed 25 milliamperes. The energy to be dissipated in a 1000-ohm coil is then about 0.5 watt, requiring a radiating surface of about 25 sq. cm. for a maximum temperature rise of 10 deg. which is a desirable limit. Since only the outer surface of such a coil is effective in radiating the generated heat the question of the number of layers requires consideration. Other factors being constant, it has been found that of the various possible arrangements that are easily constructed and mounted, a sectionalized, two-layer winding gives minimum capacitance. Hence one-half of the winding is required to have an exposed surface of 25 sq. cm. The gage of wire is then determined as a function of its specific resistance. A resistance alloy having a suitably low temperature coefficient, (such as manganin, advance, etc.) will, on this basis, require that a wire no smaller than No. 38 A. W. G. be used. This is the size of wire used by Curtis and Grover and in the experiment covered by Table I. Using the data of this table, it was calculated that a Curtis and Grover type of coil, except for treatment, would have a change in resistance of not over one part in 50,000. The lower capacitance coils required from the phase-angle standpoint would have even smaller changes.

Summing up, then, the ratio arm coils were to be of approximately 1000-ohm resistance, wound with No. 38 A.W.G. double-silk-insulated manganin, or advance resistance wire, dried, but not impregnated with any moisture-resisting compound; the winding was to be arranged so that the true capacitances would not exceed 6.0 micro-microfarad. Besides being balanced for d-c. resistance, the resultants of their capacitance and inductance values were to be balanced to within 0.1 micro-microfarad. To meet these requirements, the bridge coils were constructed as follows. The spool used is a glass cylinder ¾ in. in diameter. The winding is applied as follows: Starting at one end of the spool, a single strand of the wire is wound on until 14 inductive turns have been applied, giving a resistance of approximately 50 ohms. Then the wire is tied, the direction of winding reversed, and an exactly equal number of turns wound over the first 14, but in an opposite direction. This brings the wire to the point of the beginning. It is again tied, carried parallel to the axis of the spool over this first section and a second section wound. This is continued until ten sections have been applied. A thin sheet of mica is tied in place around the winding and the projecting ends of the wire bared of insulation. The whole is then baked to anneal the wire and dry the insulation. While hot, it is dipped several times in molten asphalt compound until a continuous coating of this moisture-proof material has been formed over the winding and surrounding mica wrapping. Adjustment for resistance balance is made by varying the length of the two-wire ends.

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[7] See Note 6.

The effective reactances of coils made as above are positive before assembly in their shields. The effect of the shield is to increase the capacitance. Table II gives data obtained on the two coils made for the bridge and shows the uniformity of phase-angle difference maintained by these coils over the operating frequency range. Final adjustment for reactance balance is made with the coils in the bridge circuit, a small amount of inductive coiling of the terminal leads sufficing for this purpose. In establishing this balance, use is made of the reversing switch described in the following section. Fig. 12 shows these coils assembled in their shields.

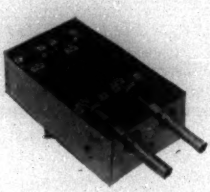

FIG. 12—RATIO ARMS

TABLE I

EFFECTIVE INDUCTANCE OF RATIO ARM COILS

Test Frequency	Microhenrys					
	Before Coating		After Coating		Assembled in Shield	
	Coil A	Coil B	Coil A	Coil B	Coil A	Coil B
1000 cycles	7.4	6.9	6.7	6.3	−1.3	−1.0
50000 cycles	7.4	6.9	6.7	6.2	−1.2	−0.8

Resistance of each coil = 1051.2 ohms

Reversing Switches. As will be noted from the diagram showing the circuit arrangement of the reversing switches, these are required to be completely enclosed in a shield which is connected to the junction point D of the bridge. They must also, of course, be subject to manipulation.

Obviously, then, this shield must be supported in some fashion from the outer enclosing ground shield of the bridge. The admittance between these two shields is a direct shunt on either one-half or all of the impedance arm $C\,D$. While the capacitance component of this admittance can be readily balanced, it is more difficult to balance the conductance component, since the latter varies irregularly with frequency. Consequently, it is desired to make this factor so low that it can ordinarily be neglected. The construction adopted for this purpose is shown in the illustration, Fig. 13, which is a partially assembled view of the two reversing switches, their shield and its supporting brackets. It will be noted that the shield is supported by the brackets by means of small glass rods (four of which are shown). The low phase-angle characteristics of glass make it a favorable material to use, from this standpoint, but from the standpoint of machining into shape suitable for insulating supports, it is not so good. The construction shown, however, adapts it very well to this purpose. One other feature is worthy of special note; that is, the small change in position of any of the switch parts which occurs in effecting a reversal, the only metallic part moving being the small metal segments of the rotating disk.

Transformers. The transformers used for isolating the bridge circuit electrostatically from the source of testing current and from the detector system should have substantially zero external electromagnetic fields. This is to prevent inductive coupling to other parts of the bridge circuit. For this purpose the transformer core is made in toroidal or ring form and the windings, both primary and secondary, are uniformly distributed about its circumference. The wound toroid is also completely enclosed in a sheet iron case.

The winding which is connected to the bridge has an electrostatic shield completely surrounding it for the purpose of concentrating all capacitance currents at one point. Around this localizing shield there is a second or ground shield. These two shields are made of sheet copper approximately No. 30 gage (0.010 in.) in thickness. The inner winding terminal leads are brought out through a small brass tube leading into a terminal chamber which is an extension of the localizing shield. Since the admittance between the localizing and ground shields forms a major part of one of the balanced admittances shunting the impedance arms, it is desirable that the capacitance component be of low value and essential that it be constant. The conductance component should be negligibly small. To attain this end, the shields are separated at definite distances by means of hard rubber rings turned to fit the outer corners of the inner shield and the corresponding inner corners of the enclosing shield. These rings are made of the smallest cross-section consistent with

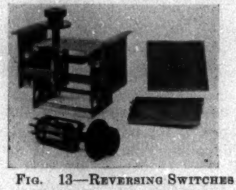

FIG. 13—REVERSING SWITCHES

mechanical strength requirements so as to introduce the minimum amount of solid material into the space between the shields. This minimizes the capacitance and conductance values. These shields must not, of course, be allowed to act as short-circuited secondaries on the transformer which would be the case if they linked conductively with the windings. Each is therefore made in two parts similar to toroidal channels which upon assembly have their overlapping inner circumferences insulated from each other by means of thin mica laminations. Further details of the construction will be evident from a study of Fig. 14.

The windings are, of course, proportioned so as to connect with a reasonable degree of efficiency the associated impedances. For best results two sets of transformers are used to cover the complete frequency range, one from 500 to 5000 cycles and the other from 5000 to 50,000 cycles.

Balancing Condensers and Impedance Arm Balance. It has been brought out previously that two adjustable capacitances are required, one to effect the proper adjustment of the bridge capacitances to ground and the other to balance the residual capacitances shunting one of the impedance arms. Such capacitances are provided in the form of adjustable air condensers, each having a maximum value of about 500 micro-microfarads. The construction used is such as to give a high degree of stability of capacitance combined with low conductance characteristics. The arrangement of these condensers in relation to the other apparatus is shown in Fig. 3.

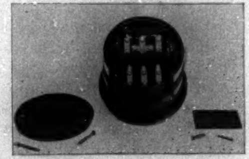

FIG. 14—SHIELDED TRANSFORMER

The effect on the accuracy of the bridge of the degree of balance of the impedance arms obtained by means of the balancing condenser C_b is determined as follows:

Capacitance Shunting the Impedance Arms. The equations giving the equivalent series inductance L' and resistance R' of a reactance of inductance L and resistance R paralleled by a capacitance C are as follows:

$$L' = \frac{L - C\,R^2 - \omega^2\,C\,L^2}{(1 - \omega^2\,C\,L)^2 + \omega^2\,C^2\,R^2}$$

$$R' = \frac{R}{(1 - \omega^2\,C\,L)^2 + \omega^2\,C^2\,R^2}$$

When the bridge is balanced the equivalent series values of each component of the two impedance arms must be equal respectively to each other. If, however, the two arms have different shunting capacitances, it is evident that this equality will be obtained only by making the values of the two inductive branches of the parallel circuit somewhat different from each other. This difference represents the error introduced by the capacitance unbalance. When the values of the shunting capacitances are small, these errors, for the purpose of indicating their order, are sufficiently closely given by the expressions,

$$\Delta\,L_x = \omega^2\,L_x^2\,(C_x - C_s) \qquad \textbf{(7)}$$

and

$$\Delta\,R_x = 2\,\omega^2\,L_x\,R_x\,(C_x - C_s) \qquad \textbf{(8)}$$

where

C_x and C_s are the capacitances shunting the unknown and standard impedance arms, respectively. Reduced to percentages, these expressions become

$$\Delta\,L_x\,(\%) = 100\,\omega^2\,L_x\,(C_x - C_s)$$

and

$$\Delta\,R_x\,(\%) = 200\,\omega^2\,L_x\,(C_x - C_s)$$

and may also be written

$$\Delta\,L_x\,(\%) = 100\,\frac{1}{C_R}\,(C_x - C_s) \qquad \textbf{(9)}$$

and

$$\Delta\,R_x\,(\%) = 200\,\frac{1}{C_R}\,(C_x - C_s) \qquad \textbf{(10)}$$

where C_R is the value of capacitance that would be required for resonance with inductance L_x at the test frequency.

These errors are thus proportional to the ratio of the capacitance unbalance to the resonating capacitance of the inductance under test. Ordinarily, values of the latter factor do not go below about 500 micro-microfarads so that in the worst case a difference in capacitance of 0.1 micro-microfarad corresponds to errors of 0.02 per cent and 0.04 per cent in inductance and resistance respectively.

Shields and Wiring. The shields have sufficient rigidity and are supported so as to maintain a definite and constant space relation to the part shielded and to the other shields. They are also of sufficiently high conductivity to maintain a common definite electrical potential at all points with respect to the part shielded.

The supports of shields or of bridge elements within the shields are as nearly as possible of constant specific inductive capacity, have low dissipative and leakage losses and are restricted to the minimum in number and size consistent with meeting the required rigidity of support.

Interconnecting conductors are shielded within brass tubes of approximately ½-in. diameter, the conductor which is of No. 10 gage copper being supported at the axis of the tube by means of glass beads fitting snugly within the tubes and having holes through which the conductor passes. These beads are located longitudinally on the conductor by means of a small lump of solder placed on each side.

Standards. The impedance standards consist of adjustable self-inductance elements used in series with an adjustable non-inductive resistance. Each self-inductance element consists of a series of inductance coils and a low range inductometer of the Brooks type,[8] arranged in three-decade formation and connected to dial switches by means of which any series combination of the coils can be selected. The inductometer is always in circuit and permits of balancing inductance values that fall between consecutive steps on the dials. Fig. 15 shows, schematically, the connections used for these standards and also the way in which they are shielded. It will be noted that the parts comprising each decade have a shield enclosing them and also all preceding decades of higher value. This makes a rather complicated mechanical arrangement but results in very important advantages from the standpoint of electrical performance. Due to the individual decade shields, each decade has effective values that are entirely independent of the settings of either of the other decades. Hence, once each individual setting of each dial has been calibrated, the value for the standard as a whole for any possible combination is obtained by simple addition of the separate dial values. This saves an immense amount of work in calibrating and also simplifies the reading of the standard. Without these shields the inter-coil and coil-to-ground admittances, at the higher frequencies, are sufficiently large to make the effective impedance of each decade setting depend to an appreciable extent upon the settings of the other decades. Under such conditions a calibration of every combination would be required, and as this calibration would vary with frequency, a correction would be needed for each frequency value used.

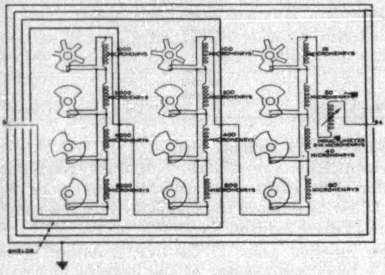

Fig. 15—Circuit Diagram of Shielded Inductance Standard

In a standard of inductance to be used at high frequencies, it is, of course, always desirable in order to minimize capacitance effects to have the inductance coils as small as possible. In the case of a completely shielded decade standard this is even more important on account of the capacitances added by the internesting shields. On the other hand, the coil resistances should be quite small in comparison with their reactances, a requirement which tends to increase the coil dimensions. In addition to the above, such coils should be highly stable in their inductance and effective resistance values with respect to the residual effects of direct and alternating currents and of temperature and humidity changes. Their values should also be of a satisfactory degree of constancy with respect to frequency and value of the testing current.

To meet these varied requirements the coils used in this bridge depart from the air core type ordinarily employed, in that they have a magnetic core of high stability and efficiency. Thus the desired inductance is obtained with a much smaller number of turns in the winding, giving a satisfactorily low resistance even in a coil only a fraction of the size of the equivalent air core coil. An adaptation of the new magnetic material, permalloy, has made this type of inductance standard possible.[9] Their cores consist of finely laminated, high specific resistance permalloy punchings, carefully annealed and assembled to form a toroidal structure whose effective permeability is about forty. On this is wound a sectionalized winding of insulated stranded conductor, the individual strands also being insulated from each other. The wound coils, after adjustment to the value desired, are sealed with moisture-proof compounds in phenol fiber cases. Fig. 16 shows an assembly of the four coils and switch which comprise one decade of the standard. In Table III are given data for typical coils illustrating their performance in respect to the above points.

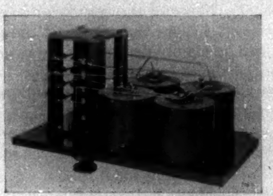

Fig. 16—Coil and Dial Switch Assembly of Typical Inductance Standard Decade

The adjustable, non-inductive resistance is a commercial dial resistance box to which a shield has been added. It has five dials providing a range of 1000 ohms in steps of 0.01 ohm. Its shield is grounded in use, the resistance itself being connected usually between the *C*

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[8] H. B. Brooks and F. C. Weaver: A Variable Self and Mutual Inductor, *Scientific Paper of the Bureau of Standards,* No. 290.

[9] H. D. Arnold and G. W. Elmen, *Franklin Institute Journal,* 195, 1923.

TABLE II
DATA ON COILS FOR INDUCTANCE STANDARDS

	Frequency range	
	500 ~ — 5,000 ~	5,000 ~ — 50,000 ~
Over-all Dimensions		
Diameter of case	6 1/2 in.	3 1/2 in.
Length of case	4 in.	3 1/8 in.
Inductance Characteristics		
Nominal value	0.100 henry	0.010 henry
Change with frequency	+ 0.5% (500 ~ — 5,000 ~)	+ 2.5% (5,000 ~ — 50,000 ~)
Change with current	+ 0.01% per milliampere	+ 0.007% per milliampere
Temperature coefficient	− 0.013% per deg. fahr.	− 0.005% per deg. fahr.
Residual magnetization effect of one ampere d. c.	Less than 0.01%	Less than 0.01%
Resistance Characteristics		
At an alternating current (effective value) of		
2.0 milliamperes	1,000 ~ — 5.2 ohms	10,000 ~ — 3.6 ohms
	3,000 ~ — 8.5 "	30,000 ~ — 13.6 "
	5,000 ~ — 13.0 "	50,000 ~ — 34.0 "
10.0 milliamperes	1,000 ~ — 5.5 "	10,000 ~ — 3.7 "
	3,000 ~ — 9.1 "	30,000 ~ — 13.9 "
	5,000 ~ — 13.9 "	50,000 ~ — 34.7 "
Temperature coefficient	− 0.017% per deg. fahr. at 3,000 ~	− < 0.01% per deg. fahr. at 30,000 ~

corner of the bridge and the inductance standard but in the case of an unknown impedance having a lower resistance than the standard from the C corner to the coil under test.

Performance

As was stated earlier, the operation of the bridge involves an initial balancing of its capacitances. It is then ready for impedance testing which is done by suitably connecting the unknown and standard impedances to the proper terminals and adjusting the latter until a balance of the bridge is obtained. The corresponding constants of the two impedance arms are then taken as being equal. Those of the standards being known, by calibration, it follows that those of the impedance under test can be simply derived. The degree of precision obtained depends upon two major factors, the accuracy of the calibration of the standards and the accuracy of the bridge comparison. The matter of calibration is beyond the scope of this paper and it will be assumed that a suitable calibration of the standards is available.

Due to the construction used, the factors determining the accuracy of the bridge comparison of impedances are reduced to the following:

1. The resistance balance of arms $A\,B$ and $A\,D$.
2. The effective shunt capacitance balance of these arms.
3. The direct capacitance balance of arms $B\,C$ and $C\,D$.
4. The direct conductance balance of arms $B\,C$ and $C\,D$.
5. The series inductance balance of the interior wiring to the impedance terminals of arms $B\,C$ and $C\,D$.

As was explained in the foregoing, two switches are provided for independently reversing the ratio arms ($A\,B$ and $A\,D$) and also the outside connected impedances. These, therefore, afford a very convenient means of checking the above balances of the bridge network. By a suitable choice of the test condition under which the reversals are made, a fairly good approximation of the effect of the separate items can be made. The following series of tests indicates how this was done on one of these bridges.

The junction point C was first grounded. Then, with a telephone receiver as the detector and with a test current having a frequency of 1600 cycles, the setting of the condenser C_b was varied until a balance was obtained. The arms Z_s and Z_x were both open-circuited in this test; hence the capacitances shunting these arms alone determined the balance point. This balance was very sharp, indicating that the shunting conductances were either very small or else accidentally well balanced. Leaving the condenser set at its balance point, there was then connected into one of the impedance arms a toroidal self inductance standard having a nominal inductance of 0.200 henry and an effective resistance of about 50 ohms. In the other arm there was connected a similar standard of the same nominal but of slightly lower actual value in series with a small adjustable inductance and adjustable resistance, each of sufficient range to effect a balance of the corresponding constants. The extension inductance was graduated in steps of one microhenry and the resistance in steps of 0.001 ohm. Balances for the four-combination settings of the reversing switches, S_R and S_Z, were then made, only the extension elements being varied in getting these balances. Readings as given in Table IV were obtained.

TABLE IV

Switch Position		Extension Inductance	Extension Resistance
S_R	S_Z		
Right	Right	124 ± 2 microhenrys	4.00 ± 0.01 ohm
Left	Right	120 " " "	4.00 " " "
Right	Left	124 " " "	3.87 " " "
Left	Left	128 " " "	3.87 " " "

Consideration of these figures led to the following conclusions:

1. The change in inductance balance due to reversal of the ratio arms is not more than eight parts in 200,000 or 0.004 per cent. It has been shown previously that the phase-angle balance of the ratio arms is not critical with respect to inductance readings. Hence, the change in inductance may be considered to be closely indicative of the resistance unbalance of the ratio arms. From the above data it is seen that this does not exceed 0.01 per cent.

2. The change in resistance due to reversal of the ratio arms being within the limits of observational error, the phase angles of the coils themselves are, as nearly as can be determined by this test, exactly balanced.

3. Since the change of inductance balance due to reversal of the impedance arms is no more than that due to the ratio arm reversal, the capacitance balance of the impedance arms is apparently satisfactory. It should be noted, however, that this balance is not critical under these test conditions. (See eq. **9**).

4. Since the resistance balance was appreciably affected by reversal of the impedance arms, it appeared that there was an unbalancing factor present which, if affecting the ratio arms, was not reversed with them but which if present in the impedance arms was reversed. The latter might have been an unbalance of the impedance arm shunt conductances but it was assumed that this unbalance was quite small. On the other hand, as discussed under "Checking of Balances", there are two small inter-shield capacitances shunting the ratio arms which are not reversed by the ratio arm switch and it seemed likely that an unbalance of these capacitances was causing the change in resistance reading. This proved to be the case, as adjustment of the balance of these capacitances for which, as previously noted, provision had been made, resulted in identical resistance readings being obtained for both positions of the impedance arm switch. After this adjustment had been made the previous tests were repeated, resulting in readings as given in Table V.

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TABLE III

Switch Position		Extension Inductance	Extension Resistance
S_R	S_Z		
Right	Right	124 ± 2 microhenrys	3.93 ± 0.01 ohm
Left	Right	120 " " "	3.93 " " "
Right	Left	124 " " "	3.93 " " "
Left	Left	126 " " "	3.93 " " "

As a further check on the performance of this unit, two inductances, each of about 0.01 henry inductance, were compared at two frequencies, 25,000 and 50,000 cycles. Table VI gives the readings obtained in these tests.

TABLE IV

Frequency cycles	Switch Position		Extension inductance	Extension resistance
	S_R	S_Z		
25,000	Right	Right	123 ± 1 microhenrys	5.1 ± 0.1 ohms
	Left	Right	122 " " "	" " " "
	Right	Left	129 " " "	" " " "
	Left	Left	128 " " "	" " " "
50,000	Right	Right	38 " " "	24.2 " " "
	Left	Right	36 " " "	24.2 " " "
	Right	Left	57 " " "	23.8 " " "
	Left	Left	57 " " "	23.8 " " "

At the 25,000-cycle frequency the maximum difference in inductance from the probable correct balance does not exceed ± 5 microhenrys or 0.05 per cent. The resistance balances check to within 0.1 ohm. At 50,000 cycles, the inductance change due to ratio arm reversal is still within ± 0.01 per cent while the resistance change is within 0.1 ohm which would be just under one per cent for a coil of this reactance and a reactance to resistance ratio of 300. This is a critical test of the ratio arm phase-angle balance. Hence it may be concluded that over the entire frequency range the ratio coils meet all balance requirements. The changes in inductance occurring at the higher frequencies when the impedance arms were reversed indicated that the residual capacitance unbalance of these arms was too large. Readjustment of the balancing condenser reduced the changes to less than 0.02 per cent. The difference in resistance balance at the 50,000-cycle frequency indicates that the conductances shunting the impedance arms are not negligible at this frequency. For more accurate results these conductances would require balancing. This would be quite practicable by means of a variable high resistance shunt.

In making each series of tests outlined above, the testing potential applied to the bridge was varied by means of a resistance potentiometer from the lowest value at which a balance could be made to the maximum of the supply oscillator. This was to check the completeness of the shielding and to detect the presence of any coupling with the supply circuit. In no case was there any discernible change in balance produced.

CONCLUSION

A system of electrostatic shielding for a direct reading bridge for the measurement of inductive impedances at frequencies up to 50,000 cycles has been described.

The general considerations defining the balances of the various capacitances which this shielding controls have been discussed and specific requirements derived for a typical range of impedances. The physical construction of a bridge designed to meet these requirements has been described and test data given illustrating its performance. This has shown it to be capable of comparing impedances over the above frequency range with a precision which approximates that ordinarily found in routine direct-current resistance measurements.

Radio Broadcast Coverage of City Areas

BY LLOYD ESPENSCHIED[1]

Member, A. I. E. E.

Synopsis.—*1. Radio broadcasting involves a system of electrical distribution in which dependent relations exist between the transmitting station, the transmitting medium and the receiving station.*

2. The attenuation and fading which attend the spreading out of broadcast waves are considered. The attenuation of overland transmission is shown to be, on the whole, very high and to vary over a wide range depending upon the terrain which is traversed. The distance at which the fading of signals occurs is found to be that at which the normal directly transmitted waves have become greatly attenuated and to depend upon the terrain traversed.

3. A field strength contour map is given of the measured distribution of waves broadcast by station WEAF over the New York metropolitan area. A rough correlation is given between measured field strengths and the serviceability of the reception in yielding high grade reproduction. The range of a station as estimated in terms of year-round reliability is found to be relatively small. It becomes clear that the present radio broadcasting art is upon too low a power level and that higher powered stations are required if reliable year-round reception is to be had at distances as short even as 30 to 50 miles from the transmitting station.

4. The question of the preferred location of a transmitting station with respect to a city area is considered. It is shown that an antenna located upon a tall building may radiate poorly at certain wavelengths and well at others. Surveys are presented of the distribution effected by an experimental transmitting station located in each of several suburban points. The locations are compared upon the basis of the "coverage" of receiving sets which they effect.

5. Finally, there is considered the relation which exists in respect to interference between a plurality of broadcast transmitting stations operating in the same service area. The importance of high selectivity in receiving sets is emphasized and there is given the measured selectivity characteristics for samples of a number of receiving sets.

It is well recognized that the elements which comprise an electrical transmission system are required to function not simply as individual pieces of apparatus, but as integral parts of a whole. In the case of radio broadcasting, the absence of a common control of the two ends makes this over-all "systems" aspect less apparent than it is for wire systems.[2] Nevertheless a definite systems correlation is required between the broadcast transmitting station and each of the receivers served, as will be evident from the following:

1. The transmitter should put into the transmitting medium, without distortion and with the power called for by that medium, all of the wave-band components required and no others.

2. The transmitting medium should be capable of delivering to the receiver an undistorted wave band, reliably and stably, and with sufficient strength to enable the received waves to stand well above the level of the ever present interfering waves.

3. Finally, the receiving set should pass with the necessary volume all of the wave components required to reproduce the program signal and should sharply exclude all others.

The rapid apparatus development borne in by the vacuum tube has brought the art to the point where it is now physically possible to meet quite fully the terminal requirements. The apparatus development, in fact, has outstripped our knowledge of the transmitting medium itself, and we are now in the position of possessing apparatus possibilities without knowing very definitely the limitations and requirements placed upon their use by the intermediate link. Only within the last few years have methods become available for measuring radio transmission and thereby placing it upon a quantitative basis.

Such measuring means have been applied to the study of radio broadcast transmission from certain stations in New York City and in Washington, D. C. The earlier results of this measurement work have already been published. It is the purpose of the present paper to present results of a systematic study which has been made of the coverage which can be effected of the radio broadcast listeners of the New York metropolitan area and in so doing to portray something of the general systems requirements of radio broadcasting.

THE CHARACTER OF RADIO BROADCAST TRANSMISSION

The ideal law for broadcast distribution would be one whereby the transmitted waves are propagated at constant strength over the zone to be served and then fall abruptly to zero at the outer boundary. All receivers within the area would be treated to signals of equal strength and no interference would be caused in territories beyond.

The kind of law which nature has actually given us involves a rapid decadence in the strength of the waves as they are propagated over the service area, and then, instead of a sharp cutoff, a persistance to great distances at field strengths which, although often too low to be generally useful, is sufficient to cause interference in other service areas.

This situation is illustrated in Fig. 1. The upper curve shows the relation between intensity and dis-

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[1] Dept. of Dev. and Research, Am. Tel. and Tel. Co., New York, N. Y.

[2] Some other examples of such a "systems" relationship are given in "Application to Radio of Wire Transmission Engineering," published in the *Proceedings* of the Institute of Radio Engineers, October, 1922.

Presented at the New York Regional Meeting of the A. I. E. E., New York, N. Y., Nov. 11-12, 1926.

tance; the lower portion, the interpretation of this curve in terms of areas of reception. The attenuation traced by the heavy line of the curve is that of the component of the radiation which is propagated directly along the earth's surface. It is this radiation which is ordinarily utilized for reliable broadcast reception. The shaded portions near the outer ends of this curve are intended to indicate the appearance of variations in the signal intensity which occur at the greater distances, particularly at night, and which are known as "fading."

The evidence of recent researches, particularly those made at short wavelengths, indicates that these fading variations are due to radiant energy which has left the earth's surface at the radio transmitter and has been reflected or refracted back to the earth's surface from a conducting stratum in the upper atmosphere. At broadcast frequencies the reflected wave component is observed at night but has not been noticed during the day. At locations close to the transmitting station the effect of the reflected component is negligible as compared with the strength of the directly transmitted waves. At increasing distances the directly transmitted waves die away to very low values and the indirectly transmitted waves begin to show up and appear to become controlling at the longer distances. The fluctuations themselves appear to be due in part, if not entirely, to variations in the reflected waves themselves, resulting perhaps from fluctuations in the conditions of the upper atmosphere.

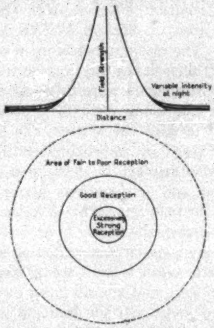
Field Strength
Variable intensity at night
Distance
Area of Fair to Poor Reception
Good Reception
Excessively Strong Reception

Fig. 1—The Attenuation of Broadcast Waves in Reference to the Areas Served

Thus, it seems clear that radio transmission involves wave components of two types: one which delivers directly to the receiving area immediately surrounding a broadcast station, a field capable of giving a reliable high grade reception; and another transmitted through the higher altitudes which permits distant reception but not with the reliability and freedom from interference required of high grade reproduction.

The effects which are actually realized in practise are indicated in a more quantitative manner by the curves of Fig. 2 which are plotted from some measurements made upon WEAF in New York and WCAP in Washington, D. C. They were made at locations in the New York and the Washington areas and at the intermediate points indicated on the curves. The measurements at each of these points are for one day only. They consisted in obtaining continuous graphic records of signal intensity during twenty-minute intervals out of each hour, one interval for each of the two stations. The period of time covered for each set of measurements was that of from one hour before sundown to about three hours after sundown. The time of year was the latter part of May, 1926. The curves are plotted from an analysis of the records in terms of mean field strength. The range of variation due to fading is indicated by the shaded portions of the curves. The day and night fields were found to be roughly the same except for WEAF where there is a material drop in the daytime signal between Baltimore and Washington, shown in the WEAF curve.

Fading was observed to commence somewhere between 50 and 100 mi. from the stations and the range of the fluctuations was found to increase up to the maximum distance observed. That the field of WEAF was found to be practically as strong at Baltimore as at Philadelphia is surprising. The data regarding this point are too meager, however, to enable any very definite conclusions to be drawn. The curves are useful principally in enabling the transition to be followed, in a more quantitative way than is done in Fig. 1, from field strengths capable of giving reliable reception, such, for example, as 10,000 $\mu\ v/m$. (microvolts per meter), to those which characterize the unreliable "distance" reception and are of the order of 100 $\mu\ v/m$.

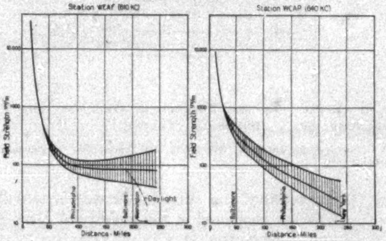

Fig. 2—Results of a Few Measurements upon the Reduction in Field Strength with Distance, Including Distances at Which Fading Occurs

A fact which is of importance to our understanding of these wave phenomena is that "fading," which ordinarily is noticed at distances of the order of 100 mi., may under some conditions become prominent at distances as short as 20 mi. from the transmitting station. Such short-distance fading has been experienced in receiving WEAF in certain parts of Westchester County, New York.[3] It appears to be a case where unusually high attenuation, caused by the tall building area of Manhattan Island, has so greatly weakened the directly transmitted wave as to enable the effect of the indirect wave component to become pronounced at night.

In general, the attenuation suffered by the normal surface-transmitted waves varies over wide limits, depending upon the terrain which is traversed. This is disclosed by the curves of Fig. 3, which show the drop in field strength with distance, for a 5-kw. station, for each of the following conditions:

a. No absorption, the inverse distance curve ($\alpha = 0$),

b. Sea water, for which the absorption is relatively small ($\alpha = 0.0015$),

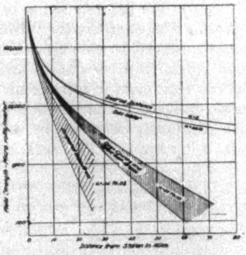

FIG. 3—EFFECT OF THE TERRAIN IN REDUCING THE FIELD STRENGTH OF A BROADCAST TRANSMITTING STATION

(5 kilowatts in antenna, frequency 610 kilocycles, wavelength 492 meters)

c. Open country and suburban areas ($\alpha = 0.02$ to 0.03) as measured in the vicinity of New York and Washington, D. C.,

d. Congested urban areas ($\alpha = 0.04$ to 0.08) as measured for Manhattan Island.

The factor α will be recognized to be the absorption factor of the familiar Austen-Cohen empirical formula, which may be expressed as:

$$e = 0.009 \frac{\sqrt{P}}{d} \cdot \epsilon^{-\frac{\alpha d}{\sqrt{\lambda}}}$$

P = radiated power in watts
d = distance in kilometers
λ = wavelength in kilometers
α = absorption factor
e = in volts per meter

The first term represents the decrease in strength due merely to the spreading out of the waves; the second term, the decrease due to the absorption of the wave energy by the imperfect conductivity of the earth's surface.

The curves given in Fig. 3 are derived from a considerable amount of data taken in the course of field strength surveys of the New York City and Washington, D. C. areas. The results of some of the earlier of these surveys have already been published.[4]

ACTUAL DISTRIBUTION IN NEW YORK CITY

Fig. 4 presents the results of a detailed survey of the field distribution effected over the New York metropolitan area by Station WEAF located at 463 West Street The measurements upon which the plot is based were taken in the daytime during the summer of 1925. Measurements were taken at approximately one-mile intervals along each of a series of circular paths concentric with the station, the radii of which increased in steps of approximately five mi. The distribution was studied in even greater detail close to the station and in locations giving evidence of rapid change in field strength. Ferries were utilized to extend the measurements over bodies of water. Manhattan Island was circumscribed on water by measurements made upon a sight-seeing boat. The land measurements were made in all cases outside of buildings at ground level. In the built-up sections of the city they were taken in the middle of streets and street intersections, and in so far as possible in open places. The plot is based upon over 1000 measurements. While these measurements were taken over a considerable period of time, check measurements proved conditions to have remained quite stable and showed, in fact, little variation from measurements made the previous year. The type of measuring apparatus employed, together with certain of the results obtained in earlier surveys, has already been described.[5]

This plot is actually a simplification of a more detailed one. The number of contour lines has been limited to those of round figures for the sake of clarity. The line marked 10,000, for example, traces the locations at which that field strength was observed and beyond which lower values obtained.

This survey shows strikingly that the terrain over a city like New York is anything but uniform electrically; that the variations in the attenuation which the waves experience in different directions and from one area to another distort the distribution pattern from that which we might imagine from the familiar stone in the pool analogy. It is apparent that this simple analogy will have to be amended by conceiving the pool to be beset by various encumbrances causing high attenuations and reflections, and, in fact, also by the presence of surface ripples to represent the waves foreign to broadcasting which cause interference. Sight should not be lost, however, of the fact that the contour lines of Fig. 4 represent a two-dimensional section of a three-dimensional phenomenon. One should picture the contours as the intersections of the earth's surface with three-dimensional surfaces.

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[3] See "Some Studies in Radio Broadcast Transmission," by Ralph Bown, D. K. Martin and R. K. Potter; *Proceedings*, I. R. E., Feb. 1926.

[4] "Distribution of Radio Waves from Broadcasting Stations over City Districts," by Ralph Bown and G. D. Gillett, published in the *Proceedings* of the Institute of Radio Engineers, August, 1924.

[5] See previous reference; also "Portable Receiving Sets for Measuring Field Strengths at Broadcasting Frequencies," by Axel G. Jensen; *Proceedings*, I. R. E., June, 1926.

The fact previously referred to that the waves transmitted into Westchester County experience high attenuation is shown by the shape of the contour lines. The irregularity of the lines appears to be due to a splitting of the directly transmitted wave by the high building area and the filling in from the sides of wave energy transmitted along the two sides of the peninsula. Although the conditions in Westchester are quite stable during the daytime, they become unstable at night, due, apparently, to the addition of the indirectly transmitted component reflected from above. An experimental study of this interference situation disclosed the fact that the bad quality obtaining at night in certain parts of Westchester was due largely to a rapid frequency modulation of the broadcast transmitter. The frequency fluctuation of the transmitted band apparently caused the direct and indirect transmissions to slip in and out of phase rapidly. The use of a master oscillator control for insuring stability of frequency greatly improved matters, but evidence still remains of what might be called the normal night time fading.

Another interesting effect which stands out in this map is the high attenuation of the wave-front transmitted over Long Island as compared with that which pursues the path of Long Island Sound and that of the ocean front to the south. The field over the eastern half of the island is contributed to by the water-transmitted waves from either side, giving rise to interference patterns similar to those in Westchester County.

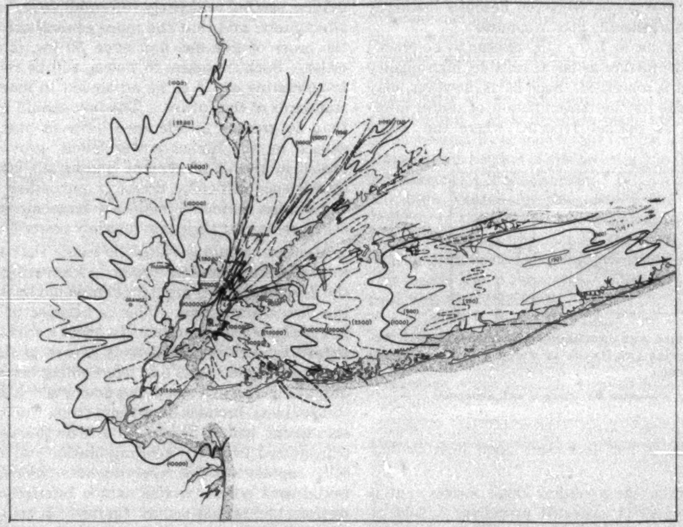

Fig. 4—Field Strength Contour Map of Distribution over the New York Metropolitan Area, Effected by Station WEAF

(5 kilowatts in antenna, frequency 610 kilocycles, wavelength 492 meters)

A question which naturally arises is that of how strong a field, as measured in this way, is required for satisfactory reception. It is too early in the art to answer this question very definitely, for it depends first upon the standard of reception which is assumed, with respect to quality of reproduction and freedom from interference; and second, upon the level of the interference. The interference, both static and man-made, varies widely with time and with location. It is therefore obviously impossible to give anything more than a very general interpretation of the absolute merit of field strength values. Observations made by a number of engineers over a period of several years in the New York City area, having in mind a high standard of quality and of freedom from interference, indicate the following:[6]

1. Field strengths of the order of 50,000 or 100,000 $\mu v/m$. appear to be about as strong as one should ordinarily desire. Fields much stronger than this impose a handicap upon those wishing to receive some other station.
2. Fields between 50,000 and 10,000 $\mu v/m$. represent a very desirable operating level, one which is ordinarily free from interference and which may be expected to give reliable year-round reception, except for occasional interference from nearby thunder storms.
3. From 10,000 to 1000 $\mu v/m$. the results may be said to run from good to fair and even poor at times.
4. Below 1000 $\mu v/m$. reception becomes distinctly unreliable and is generally poor in summer.
5. Fields as low as 100 $\mu v/m$. appear to be practically out of the picture as far as reliable, high quality entertainment is concerned. Such fields, however, may be of some value for the dissemination of useful information such as market reports, where the value of the material is not dependent upon high quality reproduction.

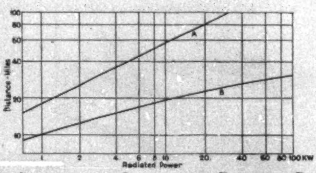

FIG. 5—SHOWING THE INCREASE IN RADIATED POWER REQUIRED TO INCREASE THE RANGE AT WHICH A FIELD OF 10,000 μV/M IS DELIVERED

Curve *A* without absorption and Curve *B* with absorption

It is seen from the preceding three figures that a 5-kw. station may be expected to deliver a field of 10,000 microvolts some 10 to 20 mi. away and a 1000-microvolt field not more than 50 mi. From this it will be evident that the reliable high quality program range of a 5-kw. station is measured in tens of miles rather than hundreds.

HIGHER POWER TRANSMITTING STATIONS REQUIRED

Rough though this interpretation of field strengths is, it indicates clearly the need which exists for the employment of higher transmitting powers. The range goes up with the increase of power disappointingly slowly. Even were no absorption present in the transmitting medium, the range in respect to overcoming interference would increase only as the square root of the increase in power. This is shown in the curve *A* of Fig. 5. It shows that a station which actually radiates five kw. of power would deliver a 10,000 $\mu \frac{v}{m}$ field at about 40 mi., a 20-kw. station the same field at distance 80 mi. Actually, with absorption present, the distances are less. This is shown by the curve *B* which gives the corresponding relations for the absorption observed for suburban and country terrain. To extend the 10,000-microvolt field from some 15 mi. out to 30 mi. would necessitate an increase in the radiated power from about five to 100 kw.

It is apparent from these relations that radio broadcasting is today underpowered; that the common 0.5-kw. station is entirely too small to serve large areas adequately, and that the more general use of powers of the order of five kw. and even 50 kw. is decidedly in order. Such increases in power will be required if the broadcasting art is to be advanced to meet the higher standards of the future. The fact should be recognized that no greater interference between stations will be caused by the higher power levels, providing the increase in power is general among all stations. The interference difficulty arises in particular cases where one station suddenly makes a large increase and the others remain at their previous low power levels.

Mention should perhaps be made that the effect of raising the transmitter power in increasing the level of the *detected* signal is greater than would be inferred from the discussion above. This is because of the square-law action of the detector. In other words, the detector output reflects the increase in power of the carrier as well as the side band. In overcoming interference it is only the increase in side-band power which counts.

The ideal broadcast system from the transmission standpoint would be one in which the carrier is not transmitted from the sending station but is automatically supplied in the receiving sets themselves. This would save power, would reduce interference between stations and would reduce fading. It will be recalled that this system is being used to great advantage in the transatlantic radio telephone development. The practicability of employing it in broadcasting will depend upon receiving set development,—upon the economy with which carrier-generating receiving sets can be made and the ease with which the carrier frequency can be set and maintained with the necessary accuracy.

TRANSMITTING STATION ON TALL BUILDING

The location which naturally suggests itself for a broadcast station intended to serve a city is that of its center. Such a location might be expected to deliver the greatest strength of field to the greatest numbers because of the coincidence between high field strengths and high density of population. The other possibility, of course, is that of placing the station outside of the city, with the object of obtaining a better "get-away" condition, of covering a larger area and of laying down a more uniform, if less strong, field over the city itself. Instances of both of these types of locations readily come to mind. WEAF is a good example of a station located near the center of a large city. The results of a study which has been made upon the effect of moving the station to other possible locations are given below.

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[6] See also the paper by A. N. Goldsmith, "Reduction of Interference in Broadcast Reception," *Proceedings*, I. R. E., October, 1926.

Before coming to this, however, there is another important factor to present and that is the effect of placing the transmitting station upon a tall building. In locating a station near the center of a large city it is natural to select a tall building for the station site. This has been done for a number of stations in various cities. The operation of WEAF, when known as WBAY, was first attempted from the top of the 24-story long-distance telephone building located at 24 Walker Street, New York City. It was found that with the limited wavelength range then open to broadcasting, radiation from the station was relatively poor. Measurements of the field strength delivered to a field laboratory located at Cliffwood, N. J. (on lower New York Bay), were made which gave the results shown in Fig. 6. The radiation was found to be sharply selective with respect to frequency, and to drop to a very low value at 400 meters. This happened to have been the wavelength assigned to the station at the time. When it became possible to shift the station to a longer wavelength, radiation was greatly improved, as indicated by the curve. The study made on this station was the first to disclose the fact that it is possible to have the building too high for the efficient radiation of certain frequencies.

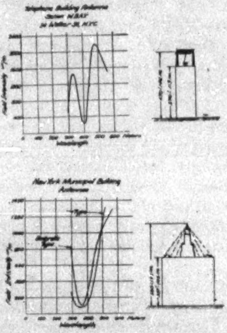


Fig. 6—The Selective Radiation Characteristic of Transmitting Antennas on Tall Buildings

As a result of this work it was possible to predict the probable occurrence of a similar effect in the case of a station which the City of New York desired to establish on the Municipal Building. Temporary antennas were erected and radiation from them measured at Cliffwood, N. J., using a transmitting oscillator of 100 watts. The results of these measurements are given in Fig. 6. The radiation was found to be a minimum in the vicinity of 360 meters, which was very nearly the wavelength which at that time was to have been assigned to this station. The establishment of the station at this location obviously could not be recommended until at a subsequent time when a longer wavelength was made available. The station is now operating on 526 meters, which is seen to be fairly well up on the radiation curve.

In both of these cases experiments were made with a number of different antenna arrangements and with different methods for driving the antenna and effecting the ground connection. None of the modifications, however, materially shifted the frequency of minimum radiation. This minimum occurs when approximately one quarter of the wavelength equals the height of the building. Measurements made upon buildings of lower heights have shown that for the usual broadcast wavelengths heights of the order of 200 ft. are entirely satisfactory. The antenna of WEAF (which has been located for the past several years on the building of the Bell Telephone Laboratories, 463 West Street, New York) and that of WCAP, in Washington, are on buildings which put them at about this height above the street. They both have normal radiation characteristics.

Distribution from Suburban Locations

In order to determine the distribution over New York City which might be effected from locations outside of Manhattan Island, experimental transmitting stations were established at each of several suburban locations. Use was made of an automobile truck equipped with a ½-kw. broadcast transmitter and provided with a transportable mast. The experimental transmitter as set up at Secaucus, N. J., is shown in Fig. 7. Measurements of the field strength

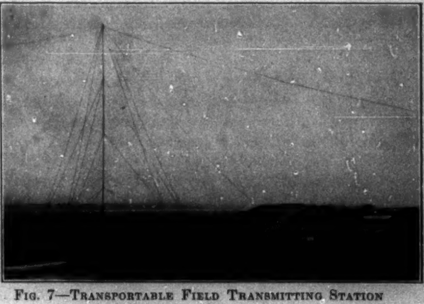
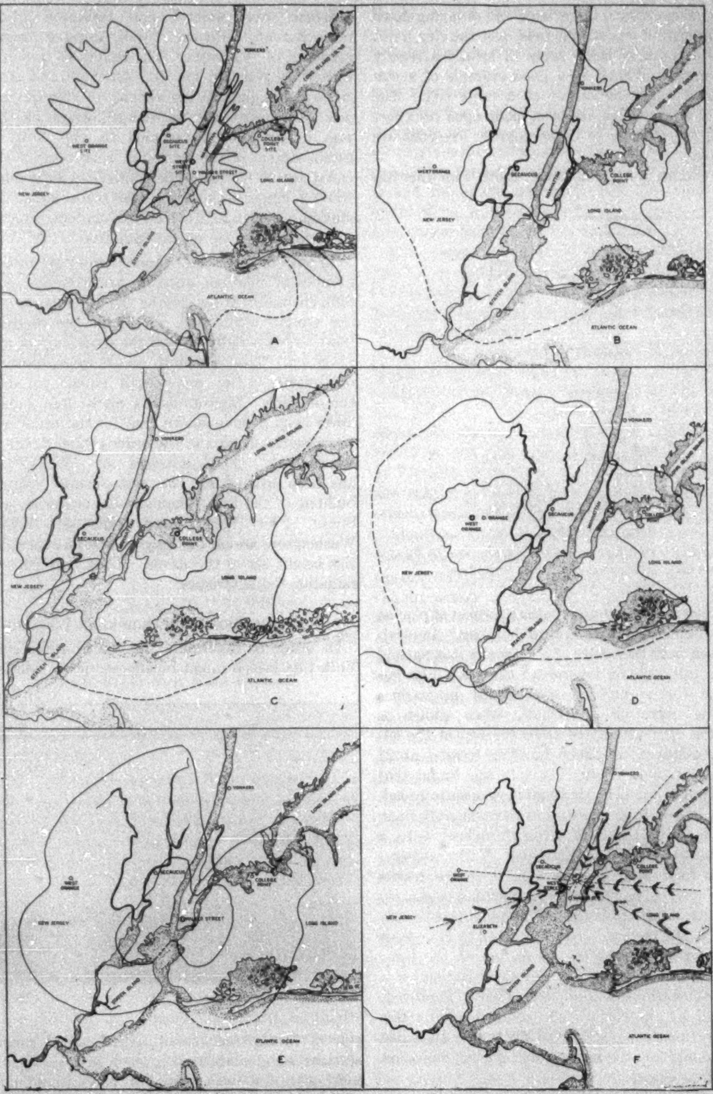


Fig. 7—Transportable Field Transmitting Station


YONKERS
LONG ISLAND SOUND
WEST ORANGE SITE
SECAUCUS SITE
WEST STREET SITE
WALKER STREET SITE
COLLEGE POINT SITE
MANHATTAN
LONG ISLAND
NEW JERSEY
STATEN ISLAND
ATLANTIC OCEAN
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YONKERS
LONG ISLAND SOUND
WEST ORANGE
SECAUCUS
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WEST ORANGE
SECAUCUS
WEST STREET
WALKER STREET
COLLEGE POINT
NEW JERSEY
ELIZABETH
LONG ISLAND
STATEN ISLAND
ATLANTIC OCEAN
F

FIG. 8—EFFECT UPON THE FIELD DISTRIBUTION OF MOVING THE TRANSMITTING STATION TO SUBURBAN LOCATIONS

A—463 West Street, New York City
B—Secaucus, N. J.
C—College Point, Long Island, N. Y.
D—West Orange, N. J.
E—24 Walker Street, New York City
F—Composite figure showing main shadows and center of obstacle

delivered from each of the locations chosen were made over practically the entire metropolitan area. The results of these tests are given in Fig. 8, in comparison with those of transmission from the normal location of WEAF at West Street and from the earlier location at 24 Walker Street. The measured field strengths have been adjusted to correspond to the 5-kw. transmitter of the West Street station.

The smaller irregularities in the West Street curve as compared with the others are due to the greater detail with which these measurements were made. The curves should be compared merely with respect to their major contour characteristics. The inner contour line is for 50,000 $\mu\, v/m$. and the outer line for 10,000 $\mu\, v/m$. Actually, the measurements were made in sufficient detail to enable other contour lines to be drawn, but these have been omitted for the sake of simplicity.

The radiation from Secaucus will be seen to deliver a strong field to Manhattan Island, the most densely populated section, and, in general, to encompass the rest of the city quite well within the 10,000-μv. line. The irregularity in Queens County evidently represents the shadow cast by the tall building area on Manhattan Island.

The distribution effected from the College Point location appears to be generally good. It does not cover the New Jersey suburbs as strongly as might be desired. The shadow cast by the Manhattan Island high buildings lies through Jersey City and lower Newark.

The distribution from the West Orange site appears to be somewhat less favorable. It is not sufficiently close in to deliver with moderate power a very strong field to the center of the population, nor is it sufficiently far out to avoid subjecting a considerable population in the immediate vicinity of the station to an excessive field were high power employed. The indent in the 10,000-μv. line in northern Queens is the shadow of the Manhattan buildings.

The distribution shown for the Walker Street location is seen to be generally similar to that of West Street. The curve presents a smoother appearance than the others because less data were taken in this one of the earlier surveys. The shadows cast to the north and to the south by the two areas of high buildings are prominent. Actually, a close examination of the contour lines reveals a noticeable angular displacement in the Westchester shadow as between Walker Street and West Street, Walker Street transmitting better up the Sound and West Street better up the Hudson. West Street turns out to be somewhat the better of the two.

The last diagram of the series brings together the shadows as determined from the several transmitting sites and shows that they project back to a common general center which locates at approximately 38th Street and Broadway, which corresponds quite well with the center of the up-town tall building center.

Relation Between Wave Distribution and the Distribution of Listeners

The merit of a given distribution pattern obviously depends upon the relation which exists between it and the distribution of the receiving sets themselves. In order to study this relation more closely, the relative distribution of receiving sets was approximated by taking the distribution of residence and apartment house telephones in each of the central office districts of the metropolitan area, excluding the commercial telephones. It was assumed that the receiving set distribution is proportional to that of the telephones. For a given survey the field strength representative of each central office district is known. By assembling the figures for central office areas receiving like field strengths, and by doing this for the whole range of field strengths measured, an accumulative percentage curve may be derived which shows the percentage of the total number of receiving sets included within the contour lines of successively weaker fields.

Curves of this kind for each of the several surveys made are shown in Fig. 9. It will be seen that for field strengths of 10,000 $\mu\, v/m$. and better, the Secaucus and College Point transmitting sites include about 80 per cent of the receivers, that the West Street and West Orange sites include around 70 per cent and Walker Street about 60 per cent. These curves are further analyzed in the chart to the right of the figure to show in each case the proportion of the listeners which may expect to receive

a. less than 10,000 $\mu\, v/m$.
b. between 10,000 and 50,000 $\mu\, v/m$.
c. over 50,000 $\mu\, v/m$.

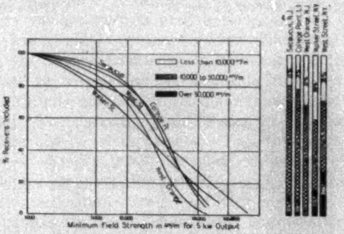

Fig. 9—Percentage of Receiving Sets in Metropolitan Area Included Within Various Strengths of Field for Each of the Transmitting Locations of Fig. 8

It is seen from this that a location to the east or west of Manhattan Island would give a material improvement in uniformity of distribution as compared with a location on Manhattan Island. Had it been possible to include a station on Manhattan Island located farther north than is either West Street or Walker Street and included within the area of high steel buildings, it is probable that the corresponding curves for such a location would show the poorest distribution of the series.

The survey work described above did not go so far as to include a study of the distribution effected from a location well outside of the suburbs. The philosophy of such a location is, of course, that of attempting to encompass within the range of the station a widespread area and of so including the city within the area as to effect a more uniform distribution over it than is possible when transmitting from a location within the city. A theoretical study was made of the distribution to be effected from one such location in the general vicinity of Boonton, N. J., using attenuations obtained in the other surveys. Such a location would be somewhat similar to that of WJZ at Bound Brook, although the distance from Boonton to New York is less. The figures derived upon the basis of a 50-kw. broadcasting station are as follows:

Field Strength	Percentage of Receiving Sets in Metropolitan Area
Below 10,000 μv per m.	10 per cent.
Between 10,000 and 50,000 μv per m.	79 per cent.
More than 50,000 μv per m.	11 per cent.

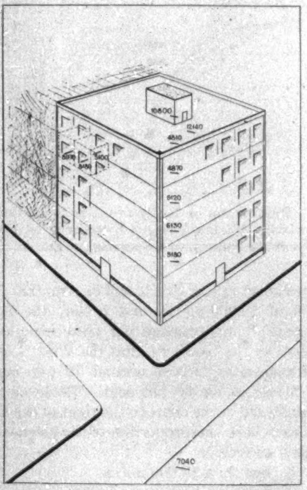

Fig. 10—The Effect of Non-Steel Apartment House Building in Shielding Radio Reception Within it

These figures show a good concentration in the most desirable field strength values. They should be discounted somewhat because they are based upon symmetrical distribution and do not include the effect of irregularities, which an actual survey probably would reveal.

Receiving in Apartment Houses

The surveys described above disclose the field strength distribution as measured generally in the streets and open places. It does not disclose the details of field distribution in the immediate vicinity of a receiver. Perhaps the most difficult situation is that of the large apartment house, particularly where it is desired to receive by means of an indoor antenna.

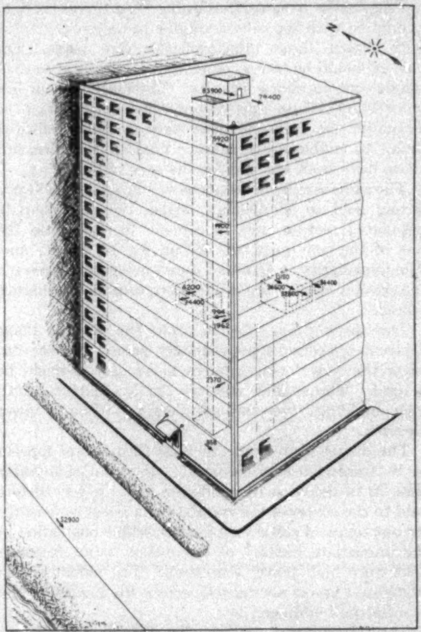

Fig. 11—The Effect of Steel Structure Apartment House Building in Shielding Radio Reception Within it

Two effects are encountered: First, the reduction in signal strength by virtue of the shielding effect of the building; and second, the existence of a relatively high noise level caused mainly by radio-frequency interference from electrical systems within the building.

The results of a few observations upon signal strength reduction within two buildings are presented in Figs. 10 and 11. Fig. 10, for a non-steel building, shows the field to be roughly halved. In the case of the steel structure building depicted in Fig. 11, the interior field is found to be reduced to as low as a few per cent of that outside the building. For outside rooms, the field strength near the window was found to be about eight times that further in the room. Such severe shielding effects obviously call for picking up the wave energy outside the building and conducting it to the receiving sets by wire circuits, preferably by shielded circuits, in order to protect against local interference.

MULTI-STATION OPERATION

The discussion given above has been directed chiefly to the relations which might be called internal to a single-channel radio broadcast system. Actually, of course, broadcasting involves the use of the common transmitting medium for a number of channels. This brings with it the problem of frequency selectivity and raises the question of the capabilities of the various types of radio receiving circuits.

In order to throw some light upon this important factor, measurements have been made upon a sample or two of each of a number of different types of radio receiving circuits. The measurements were made in the laboratory[7], simulating as closely as possible the conditions under which the receiving sets would be used. The curves of Fig. 12 show the reduction which is to be expected in the detected audio-frequency current, were the receiving set tuned to a transmitting station on 900 kc., and the transmitting station then shifted in frequency by the amounts given along the abscissa. In this curve the reduction in current is indicated both as a ratio and in T U, which is a convenient way of indicating power ratios. The relation between T U's and current ratio with a given impedance is indicated in the figure. Thus, for a carrier transmitted 40 kc. off from the one to which the set is tuned, the single-circuit, non-regenerative type of receiver showed a cutoff of only 20 TU, corresponding to an audio-frequency current reduction to 0.1 that of the value at resonance. The curves will be seen to group themselves more or less into three classes in the order of their selectivity merit as follows:

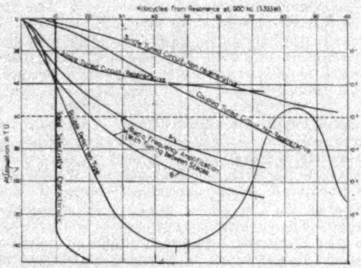

FIG. 12—RECEIVING SET SELECTIVITY CHARACTERISTICS AS MEASURED FROM SAMPLES OF RECEIVERS HAVING DIFFERENT TYPES OF SELECTIVE CIRCUITS

1. The single-tuned circuit (non-regenerative and regenerative), and the combination of two tuned circuits coupled together.
2. Circuits employing radio-frequency amplification with tuned circuits between stages.
3. The double-detection of superheterodyne type of circuit.

The curve for the double-detection type of circuit shows a "come-back" which represents the familiar double-tuning effect. (Incidentally, the admittance of this particular set, which was not a commercial set, needs to be reduced by the use of more selectivity at the radio frequency.)

For comparison purposes there has been added to the figure the curve marked "ideal selectivity characteristic," in accordance with which the receiving set would pass without attenuation all frequencies up to 5000 or 10,000 cycles and would cut off abruptly all frequencies without this band. Attention is first called to the fact that the various circuits attenuate *within* the desired transmission band of five or ten kilocycles. This means the higher frequency components of the side band will be reduced by the amounts indicated (after detection) with corresponding distortions of the reproduction. The distortion will be seen to be greater for the more highly selective sets. This follows from the nature of sharply tuned circuits. Selective circuits, capable of approximating the filter type of characteristic, are to be desired.

In comparing these selectivity characteristics, it is necessary to have in mind the amount of differentiation between the desired and the undesired signal which is necessary for the avoidance of interference. Each of the signals may be considered as fluctuating during the rendition of the program over a considerable range of volume which centers about some average value. The amount of differentiation required between the average values obviously depends upon the range of the fluctuations involved and upon the standard which is assumed with respect to freedom from interference. Experience with loud speaker reproduction indicates that ordinarily a level of the average of the undesired signal 40 TU lower than that of the desired signal, while not giving noticeable interference at times when the desired signal is strong, does permit the undesired signal to "show through" during times when the program rendition is weak. Reducing the undesired signal to 60 TU below the desired signal prevents this interference for the volume ranges which are now commonly transmitted. If the future art brings with it the requirement of following greater swings of volume, a further reduction in the undesired signal may be necessary. The value of 60 TU has been dotted in across the chart of Fig. 12, in order to show readily the frequency separation at which the different selective circuits give this attenuation of the undesired signal. This is upon the basis that the field strengths of the two signals are equal. Inequalities in field strength require that the 60-TU value be increased or decreased by the amount of the inequality as measured in TU.

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[7] The method consists in establishing a small laboratory transmitter and modulating it with a single-frequency tone. The receiving set is tuned to the modulated carrier signal as in practise. The gain or sensitivity of the receiver and its coupling with the transmitter are adjusted to produce normal load upon the detector tube. With the receiving set left at this adjustment the frequency of the radio transmitter is shifted each side of the original single frequency in 10-kc. steps throughout a range of 50 to 100 kc. For each of the offside frequencies the reduction caused in the detector output current is measured, this being an indication of the receiving set selectivity.

The frequency interval which has been recommended by the National Radio Conferences for stations in the same zone is 50 kc. It is evident from the curves that sets equipped with the simpler types of tuned circuits will be subject to some interference between stations thus separated even if the receiver is so favorably situated as to receive equal field strengths from the desired and undesired stations. The selectivity of the other types of receiving circuits is seen to be sufficient to avoid interference under these conditions and allow some margin for overcoming inequalities between the fields. Such inequality becomes great where the attempt is made to receive distant stations through the effect of local stations. Assume, for example, that the listener receives 50,000 microvolts from a local station and 500 microvolts from a distant station to which he desires to listen. There exists a 100 to 1 or 80 TU disadvantage to be overcome. When added to the 60 TU needed for cros-stalk clearance, the total selectivity requirement, as measured in terms of detected audio current, becomes 140 TU, or a current reduction of the order of 10,000,000 to 1. The need for a high degree of selectivity is therefore apparent. The impracticability of receiving distant stations removed in frequency from local stations by any such narrow margin as 10 kc. is also obvious.

The effect of receiving set selectivity in increasing the area over which a station may be received without interference from a second station is illustrated in Fig. 13. The two stations are assumed to be of equal powers so that they deliver equal field strengths to receiving stations along a line midway between them. Receiving sets so located are required to have an amount of selectivity called for by the cros-stalk margin itself, say 60 TU. On the desired-station side of this line the selectivity may be less; this is the region where poorly selective receivers can be employed. On the undesired-station side of the center line the selectivity requirements are greater. The non-interference area is pushed up closer and closer to the undesired station as the receiving set selectivity is improved, as is indicated by areas *A* and *B* of the figure. For example, assume that the selectivity of the receiving set is such as to give a 100-TU cutoff of an undesired station, offset by 50 kilocycles. Sixty TU of this would be required were the two signals of equal strength, so that 40 TU measures the difference by which the undesired signal may be greater than the desired one. The increased area of reception made possible by this additional 40 TU is indicated by that portion of the lower figure which is to the right of the center line and outside of the area *A*. Within the area *A* interference would be suffered. This interference area may be diminished by the use of still greater selectivity. The addition of another 20 TU of selectivity (again as measured in terms of detected audio-frequency current) would reduce the interference area to that within the small area at *B*. The extent to which the selectivity requirement of the receiving set is determined by its location, therefore, is apparent. The conditions which obtain in multi-station areas, such as New York City and Chicago, obviously call for a general use of high selectivity sets.

In locating a new transmitting station it should be possible from a knowledge of the relative field strengths of other stations in the vicinity to predict approximately what the interference area will be for the different types of receiving sets. In this connection there should be recognized the advantage from the interference standpoint which exists in grouping together the broadcast transmitting stations as far as possible in one location, and in equalizing their powers. Such grouping and equalizing would enable the receivers to obtain substantially equal fields from all of the stations and would minimize the selectivity which they are required to possess. While it is impracticable to accomplish this result completely, it is hoped that a better understanding of the interference problem as here outlined and of the mutual advantage to be gained in reducing interference will lead naturally to a better coordination of radio broadcast stations.

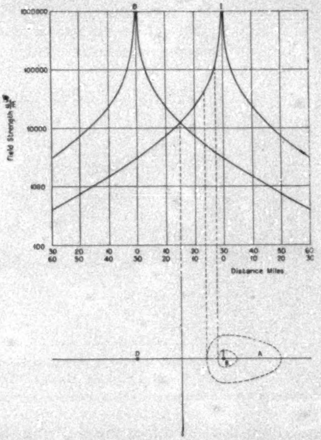

Fig. 13—Showing the Greater Area over Which the More Highly Selective Receiving Sets May Receive a Desired Station D and Exclude an Interfering Station I

Acknowledgment

The data and analyses presented in this paper are the result of the cooperative effort of a number of engineers in both the American Telephone and Telegraph Company and the Bell Telephone Laboratories, Inc. In assembling and presenting the material the writer is acting merely as the spokesman for these development groups. He wishes to acknowledge his indebtedness to his colleagues, particularly to those who have directly participated in the survey work described and assisted in the preparation of the paper, namely, Messrs. D. K. Martin, R. K. Potter, G. D. Gillett and H. B. Coxhead and to Messrs. S. E. Anderson and O. O. Ceccarini of the Bell Telephone Laboratories to whom is due the measurement work upon the radio receiving sets.

Discussion

S. J. Zammataro: The statement was made that it is possible for the ratio of energy in the broadcasting of a symphony orchestra corresponding to the fortissimo and pianissimo parts to be one million to one. Is that provision made in the broadcasting station to control that ratio, and if so, to what extent is it controlled?

J. B. Taylor: This paper shows in a quantitative way what I think many observers must have appreciated in a qualitative way. Most radio fans give the impression that it is perfectly easy at any time to pick out almost any station they want, but I think it is also the experience of those not operating their own sets that when they ask someone else to pick out a particular station at a particular time, for one reason or another, it doesn't come.

This paper shows that the millions of potential listeners who we should like to think are keeping their ears glued to the receivers or sitting in front of the loud speaker enthralled at what is being put out, are a good deal of a myth; that while the audiences are large, tremendously large, compared with what can be placed in an auditorium, they are not, as a rule, in these large figures which are claimed. And this ought to lead to a reappraisal of the extent to which the broadcast message is going and to better engineering judgment in the placing of broadcasting stations, not only in locating one with respect to others, but also reappraisal of the economical size of stations. There shouldn't be a disproportion between the amount of money and energy put into the plant and the amount of program-making energy and artistry which fit into this plant.

If I may attempt to express an opinion on the question raised, "to what extent does the control of the symphony program follow or fail to follow the proper intensity of the music?" I should say that if the intensity of the music ranges from one million to one, (and I have no reason for suggesting anything else), and, as intimated in the paper, the attempt is to keep up with the music or ahead of it, or a little behind it, to the extent of ten thousand to one, perhaps that is the best we can do, but in a typical program perhaps from a good symphony orchestra with a chain of stations, how many operators or supervisors have it within their power to effect this change of intensity? Anywhere, you may be at the mercy of any number from one to six or seven or ten, not, perhaps, counting yourself as the last man in line or the man running the set to which you are listening, and how many of these people have the proper musical judgment to do this? In other words, the musical composer, the conductor, and the skilled musicians are doing their best to give you music with graded intensity, and another lot of men, skilled and trained but perhaps not skilled and trained in music, are doing their best to iron it out to a dead level.

G. T. Croker: I should like to ask if there is a relation between field intensity and distance?

K. B. Lyman: Mr. Taylor pointed out that the control that was exercised on a broadcasting program is under the influence of a number of men. I believe that is not exactly the proper situation. The control is left more or less constant and the difference in energy levels between the maximum volume and the minimum volume is more a function of the ability of the loud speaker to reproduce it and of the noise levels introduced along the line, and that is of the order of ten thousand. If we can assume a reference level, a loud speaker will give 20 to 30 T U above that level, whereas the noise level will be perhaps 20 to 30 T U lower. The noise level will be lower than that, but the sum of noise might come up that far; so, actually, you would get a transmission in your radio system from a tone of about like a whisper to something somewhat louder than the speaking voice, without particular distortion.

R. W. King: Such a formula as Mr. Croker has in mind is, I believe, given in the paper (first column third page).

K. Sreenivasan (communicated after adjournment): The paper is of considerable interest to me, specially from one or two points. The survey that has been carried out cannot but be of great value in the choice of location for broadcast stations in places other than New York as well. The rapid growth of large cities with the enormous increase in the use of reinforced concrete buildings, reaching up to several hundred feet, creates a problem of radio broadcast distribution of considerable complexity.

The measurements of the author within such buildings forcibly impress on the mind the very pronounced shielding effect of steel structure buildings. The figures, 83,900 $\mu v./m.$ on the roof with 858 $\mu v./m.$ on the ground floor, and 52,900 $\mu v./m.$ on the road with 858 $\mu v./m.$ inside the building at the same level as the road, are striking.

The author lays considerable stress on the necessity for increased power in the existing broadcast stations. On the other hand, there are some people who prefer a large number of low power stations to cater to local needs. Economic considerations favor a smaller number of high power stations. The disadvantages of such an arrangement are sufficiently clear. A field strength of 50,000 $\mu v./m.$, not to speak of any higher values, is so large that a receiver situated fairly close to the station and wishing to hear other stations would experience considerable trouble due to jamming, though the set may be highly selective. Secondly, as the author remarks, the field intensity increases relatively slowly with radiated power; the necessity arises, therefore, not so much for an increase of power in the station, as for another station of probably the same or less power. A value of 10,000 $\mu v./m.$ is about the right value for good quality reception without very expensive receiving apparatus. From this point of view, a few stations of 5 to 15 kw. seem to be better for a given area than one or two of 50 or 100 kw., taking the service range to be about 25 to 30 mi.

I am much struck with the field strength curve for WEAF. The intensity seems to remain practically constant after the first 80 mi. right away up to 240 mi. from the transmitter. I wish the author would give us the percentage departure from the theoretical values and carry on a fuller investigation into the curious departure from the normal decay curve.

With regard to the remarks of the author regarding selectivity of receivers, it seems as though reception will not improve by using highly selective receiving circuits. Taking the band of voice frequencies on either side of the radio frequency wave f_r to be about 10 kc., then, for good quality reception, the receiver should exhibit comparative flatness over the range $f_r - 10{,}000$ cycles to $f_r + 10{,}000$ cycles per second. Sharp cut off is not only desirable but essential below $f_r - 10{,}000$ and above $f_r + 10{,}000$. High selectivity within this band will necessarily mean disproportionate reproduction of the various received radio wave components; and this does not mean good quality reception. On the other hand, a fairly wide separation of the radio frequencies of various stations and the use of a moderate number of medium power stations will perhaps be a better solution.

Can the Thermal Capacity of Electric Machines
Be Made a Simple and Practical Element of Rating?

BY A. E. KENNELLY*

PREFACE

THE subject of this paper has been suggested to the U. S. Committee, by the Council of the I. E. C., as suitable for a report at this 1926 meeting, and has been assigned to the writer for preparation. The writer does not claim any authorization from the U. S. National Committee to present its views officially on the subject. He merely submits the paper as expressing personal opinions, aided, however, by the records and reports of a certain group of engineers who have given special attention to the thermal constants of electric machines, and who are mentioned in the following text.

BRIEF ANSWER AND PURPORT OF THE PAPER

It is believed that the thermal constants of electric machines in the variable regime (especially their binary time constants) can, in many cases, be used in a very simple way for practical operating purposes. It is not, however, recommended that such thermal constants should be introduced into the rating of machines at the present time. It is, nevertheless, recommended that these thermal constants should be regarded as useful subsidiary information concerning machines, until such time as engineers may have become more generally familiar with their application and use. All machines do not seem equally subject to their useful application. Attention should be called to the practical service that the purchasing and operating engineer may derive from the thermal constants of machines. It seems likely that, at some future time, after the matter has been more thoroughly investigated by designers, manufacturers, testers, and operators of electric machinery, these thermal constants, including thermal capacity, may be found sufficiently important and reliable to be promoted from the present category of useful subsidiary information, to the category of rating constants of machines.

It should be pointed out that but little of the technical material in this paper is new. It is mainly collected here for convenience of reference.

Thermal Constants of Electric Machines may be divided for convenience into two classes, namely,

1. Steady-regime constants,
2. Variable-regime constants.

Class 1 may be considered to contain the following, for any machine or element of the same (such as field, armature, internal or external part of winding, stator rotor, commutator, bearing, etc.)

a. Hottest-spot maximum safe temperature.

b. Maximum accessible measurable temperature.

c. Ambient temperature, θ_a.

d. Instantaneous temperature rise of machine or element θ.

e. Instantaneous temperature, $\theta + \theta_a$.

f. Ultimate temperature rise under continuous rated load, Θ.

All of the above constants enter either directly or indirectly into the continuous rating of a machine from the thermal standpoint, and need no discussion here.

Class 2 may be considered to contain the following thermal constants for any machine or element:

A Ultimate temperature rise characteristic $\Theta - P$, under different sustained percentages of rated load.

B Time constant or constants of machine or element τ hours or minutes

C Thermal capacity of machine or element k watt-hours per deg. cent. temperature rise above stationary ambient.

D Specific thermal capacity of material in a machine or element—watt-hours per kg. and deg. cent.

E Dissipation constant of machine or element s watts per deg. cent. temperature rise.

The variable-regime thermal constants (*A*) to (*E*) are the main subjects of discussion in this report.

UTILITY OF VARIABLE-REGIME THERMAL CONSTANTS

Characteristic A. The most important of the class 2 here considered is probably *A*, the ultimate temperature rise characteristic $\Theta - P$. This is a characteristic curve connecting the ultimate temperature rise of the machine, presumably at its max. accessible temperature location, for different percentages of rated load. When drawn on logarithm paper, such a curve usually approximates to a straight line over a moderate percentage range of load. Thus, in Fig. 1, the particular machine supposed to be there designated develops an ultimate temperature rise of 40 deg. cent., in a specified element, at continuous rated load. It also develops 50 deg. cent. rise at 1.15 times rated load, and 76 deg. cent. at 1.5 times rated load, if the straight line characteristic *A O C* is adhered to over that range. This straight line rises 1.6 units of ordinates in one unit of abscissa length; so that this straight line represents a temperature rise increasing as the 1.6th power of the load, over the range it may be taken to cover.

An operating engineer, knowing the continuous rating of the machine, can ascertain with the aid of such a characteristic the temperature rise of the element considered, over a certain range of steady loads, from, say, 0.50 to 1.5 times rated load. The maker of the

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*Honorary Secretary of the U. S. National Committee, I.E.C.

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machine may not have made ultimate temperature rise measurements at different steady loads; and therefore may not be prepared to give this $\Theta - P$ characteristic. He will in many cases, however, have secured such a series of three or four different ultimate temperature rises, at different percentages of rated load, on some similar machine of the same dimensions and speed. The purchasing engineer may be able to obtain this auxiliary information from the maker, and so construct a $\Theta - P$ characteristic which is likely to be fairly reliable. The operating engineer may subsequently be able to check this characteristic from the observed behavior of the machine under some particular steady load or loads in service.

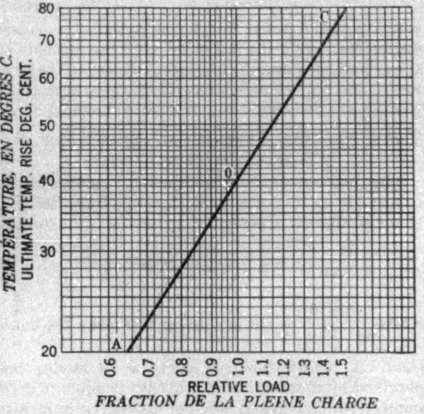

FIG. 1—APPROXIMATE STRAIGHT LINE RELATION BETWEEN ULTIMATE TEMPERATURE RISE AND STEADY LOAD FOR A PARTICULAR DYNAMO MACHINE

Time Constants B. It is well known* that when a machine passes from one steady load to another, at constant ambient temperature, the temperature rise of any element will change, in the simplest case, with time, according to a simple exponential curve or time-constant curve. Owing to complexity of thermal relations among the different elements of a machine, the temperature rise may depart materially from a simple exponential curve; but in many cases it conforms to such a curve sufficiently closely for practical purposes.

Two principal varieties of thermal time-constant present themselves:

1. The exponential time constant τ_ϵ, fundamental in the theory of such curves, but awkward to use.
2. The binary time constant τ_2, suitable for practical use.

In any exponential curve, the exponential time-constant τ_ϵ is that interval of time during which the temperature rise comes within $1/\epsilon$, or 1/2.718, or 36.8 per cent, of the ultimate rise. In a period of 2 τ_ϵ, the rise will come within $1/\epsilon^2$, or about 13 per cent, and in 3 τ_ϵ within $1/\epsilon^3$, or about 5 per cent, of the ultimate rise.

The binary time constant τ_2 is that interval of time during which the temperature rise comes within 1/2, or 50 per cent, of the ultimate rise. In a period of 2 τ_2, the rise will come within $1/2^2$, or 25 per cent, and in 3 τ_2 within $1/2^3$, or 12.5 per cent, of the ultimate rise.

Fig. 2, which is drawn on arith-log paper, shows the attainments and deficiencies at any number up to four exponential or binary time constants. These lines are straight. Thus after four exponential time constants, the attainment is about 0.982, or 98.2 per cent, while the deficiency is 0.018, or 1.8 per cent. Similarly, after the lapse of four binary time constants, the attainment is 0.9375 or 93.75 per cent and the deficency $^1/_{16}$, or 0.0625 or 6.25 per cent. The two time constants are always connected by the relation

$$\tau_2 = 0.69315\,\tau_\epsilon \quad \text{or} \quad \tau_\epsilon = 1.4427\,\tau_2 \text{ hours} \qquad (1)$$

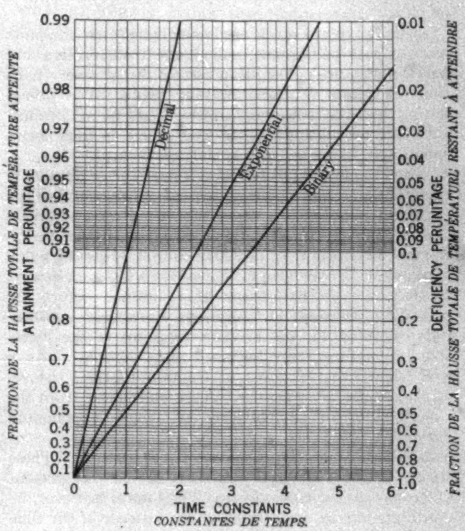

FIG. 2—ATTAINMENTS AND DEFICIENCIES IN TERMS OF THE DIFFERENT TIME CONSTANTS

so that if the exponential time constant is known, the binary constant is always approximately 70 per cent of it.

The binary time constant τ_2 of a modern rotary dynamo machine ordinarily lies between 10 minutes and 40 minutes, and does not vary greatly with the size or rating of the machine. If we take a mean value of, say, 25 minutes, and assume that the temperature rise of the machine element considered follow as time-constant curve, then when the load is suddenly changed from one steady value to another, the characteristic $\Theta - P$ curve enables the corresponding change in ultimate temperature rise to be predicted. Suppose that the change will be ultimately 30 deg. cent. Then in τ_2, or 25 minutes, the rise will be 15 deg. cent., in 2 τ_2, or 50 minutes, it will be $15 + 7.5 = 22.5$ deg. cent., in 3 τ_2, or 75 minutes, it will be $22.5 + 3.75 = 26.25$ deg. cent., and so on. After 6 τ_2, or 150 minutes, the change will be for practical purposes complete.

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*Bibliography 1, 2, 5, 13, 14, 15, 17, 18.

An operating engineer may be able to utilize the time constant of a machine in an emergency. He may want to carry a temporary overload without overheating his machine. He knows that he can carry such an overload during a period of one time constant, without attaining more than half the temperature rise due ultimately to that load, provided that the element of the machine considered has this margin of temperature below the danger point, and that the curve of heating is exponential; also that no other element will rise more quickly.

Although rotary machines have binary time constants usually well below one hour, some transformers and particularly, air-cooled, oil-immersed transformers, have time constants of three or four hours. The period during which a temporary overload can be maintained with due precautions, in an emergency, is thus correspondingly increased with such transformers.

When the temperature rise of a machine or element is exponential, it becomes easy to predict the course of the changing temperature when the $\Theta - P$ characteristic and the binary time constant are given, if the load is changed abruptly from one long continued steady value to another. If the load is changed in a less simple way, so that one thermal transient is started before the preceding transient or transients have been substantially terminated, the predetermination of the corresponding temperature becomes more complex. As is well known to designers of traction motors, the resultant curve of θ against time is obtainable from the superposition of a plurality of single exponential curves.

Significance of the constants C, D and E. These variable-regime thermal constants are not at present of much use by themselves. They may, however, be useful conjointly, in arriving at an estimate of the time constant B. If the time constant can be predetermined with satisfactory precision, a $\theta - t$ test of temperature rise against time becomes unnecessary, provided that it is known that the curve is exponential, and that the ultimate rise is known from the $\Theta - P$ characteristic. The relations between B, C and E are given by the familiar formula:

$$\tau_\epsilon = k/s \qquad \text{hours (2)}$$

or

$$\tau_2 = 0.693\, k/s \qquad \text{hours (3)}$$

The value of k, or the C constant, may be computed theoretically from the weights of the different parts of a machine and their specific heats. In practise, certain empirical constants may have to be used, as in Appendix I. Consequently, a time constant that has been observed, is at present more reliable and satisfactory than a time constant that has been computed; although designers, guided by experience, place confidence in their predetermined time constants.

E. The dissipation constant s is defined by the formula

$$p = \Theta\, s \qquad \text{watts (4)}$$

where p is the power lost in the machine or element during the steady thermal state, after the ultimate rise Θ has been substantially attained. Knowing the power losses, and the ultimate temperature rise corresponding thereto, the constant s can thus be evaluated.

TESTS FOR THE EXPONENTIAL QUALITY OF $\theta - t$ TEMPERATURE RISE CURVES

The time-constant of a machine can only be utilized to the degree of precision within which the temperature rise curve is exponential, according to the formula:

$$\frac{\Theta - \theta}{\Theta} = \epsilon^{-\frac{t}{\tau_\epsilon}} = 2^{-\frac{t}{\tau_2}} = 10^{-\frac{t}{\tau_{10}}} \qquad (5)$$

where $\dfrac{\Theta - \theta}{\Theta}$ is the *deficiency*, θ/Θ being the *attainment*. A temperature rise curve $\theta - t$ having been obtained by observation, it is always possible to determine, from inspection, whether the curve is in satisfactory agreement with (**5**); *i. e.*, whether it admits of time-constant application. It is first necessary that the ambient temperature during the test shall have been kept constant, or that the curve be first corrected accordingly, if it has slightly varied*. The ultimate temperature rise, Θ, for the curve must also be forthcoming, either by noting that the test has been maintained for a time sufficient to develop substantially this ultimate rise, or by extrapolating from the curve as in Appendix II, or from preceding temperature-rise tests as recorded in a $\Theta - P$ characteristic.

The time during which the temperature rise θ changes from 0 to $\Theta/2$ deg. cent. is measured along the time axis of the curve. This should be the binary time constant τ_2. In another such time constant, the further temperature rise should be $\Theta/4$ degrees. In the third such time constant, the further rise should be $\Theta/8$, and so on. If these relations hold satisfactorily, the curve may be regarded as exponential, from an engineering standpoint. Strictly speaking, the theoretical requirements for exponential temperature rise are so severe, that they can probably never be fully complied with in any machine. Nevertheless, it is surprising how many machines develop satisfactory time-constant curves in their tests, although the time constant of one element, such as the field-winding, may be markedly different from that of some other element, say the armature winding in the slots.

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*Bibliography 17.

A second test, but much more severe, of the exponential quality of a $\theta - t$ curve, is to plot the deficiency $(\Theta - \theta)/\Theta$ on inverted arith-log paper. Thus Fig. 9 gives a $\theta - t$ curve for a certain 2000-kv-a. air-cooled transformer, the curve being drawn exponentially to fit the observations, according to the formula

$$(50.5 - \theta)\,/50.5 = \epsilon^{-\frac{t}{3.72}} = 2^{-\frac{}{2.58}}$$

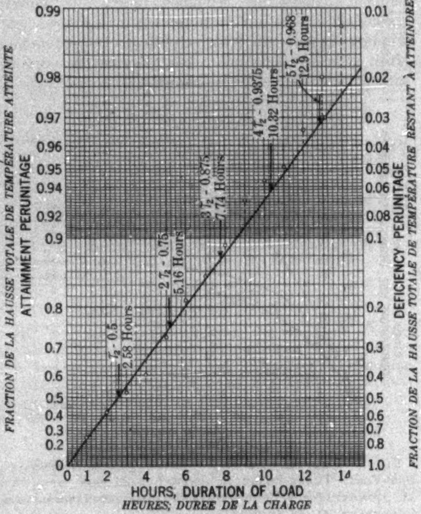

FIG. 3—TEMPERATURE-ELEVATION CURVE OF 2000 KV-A.

Transformer (Fig. 9), drawn on inverted arith-log paper. Straight line, computed exponential rise. Small circles, observed attainments and deficiencies of temperature elevation.

It will be seen that the hourly observations conform to this curve satisfactorily. Fig. 3 gives the corresponding plot of the same observations on arith-log paper. The exponential curve is now a straight line; but the observations appear to depart from it much more widely, especially in the upper parts.

In cases where the temperature rise curve departs so far from an exponential law as not to admit of time-constant reckoning, it may sometimes be dealt with in a relatively simple manner by the use of two independent time-constants. Messrs. V. M. Montsinger* and W. H. Cooney† have shown that when, as in many modern air-cooled, oil-immersed transformers, the windings are thermally separated from the core, but the oil is thermally common to both, the temperature rise of the windings above the top layer of oil follows satisfactorily one exponential curve of, say, 3½ *minutes* binary time constant; whereas the top layer of oil in the transformer follows satisfactorily another exponential curve of, say, 3½ *hours* binary time constant.

Prof. Karapetoff has recently given a theory of the temperature rise of a body containing two thermal elements in partial mutual communication.‡ The solution leads to two independent time constants, associated with two final temperature rises, two thermal capacities k_1, k_2 and dissipations s_1, s_2. By means of appropriate tests, these time-constants may be evaluated. Practical results with the method are not yet forthcoming.

EXPERIMENTAL DATA CONCERNING HEATING CURVES OF MACHINES

Figs. 4 and 5 are taken from curves published by Mr. C. M. Laffoon§ on large turbo alternators, using imbedded temperature detectors in the armature winding. Fig. 4 shows the temperature rise of a 35-megavolt-ampere machine (35 mv-a.), under steady load, with the detectors in contact with the copper armature conductor and within the insulating cover. The curve is drawn exponentially to fit the observations and an ultimate rise of 24 deg. cent. The binary time constant is 20 minutes. The agreement of the observations with the curve is, in general, satisfactory. Fig. 5 shows similar results reported on a 25.9-mv-a. alternator armature using two different positions of imbedded detector; namely, one in contact, as before, with the copper winding, and the other between adjacent coils in the slots; so that an insulating cover separated the

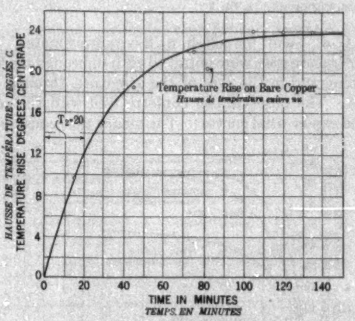

FIG. 4—TEMPERATURE RISE OF A 35,000-KV-A. THREE-PHASE TURBO GENERATOR OPERATING UNDER CONSTANT 75 PER CENT LOAD

Imbedded temperature detectors in contact with the copper winding. Curve computed exponentially to fit the small-circle observations. (C. M. Laffoon, Bibliography 18, Fig. 2.)

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*Bibliography 18, pages 649-651, also 7a.

†Bibliography 19.

‡Bibliography 23.

§Bibliography 18, page 651, Figs. 2 and 3.

detector from the copper. In the upper curve, the agreement with observations is, in general, satisfactory, but there is a wide departure of the observed temperature rises from the exponential lower curve, during the first hour. This departure is attributable to the effects of the interposed layer of insulation between detector and copper. The time constant τ_2 of the upper curve is 26 minutes, and for the lower curve 38 minutes.

Fig. 6 gives three temperature rise curves reported by Mr. G. E. Luke* for a 35-h. p., d-c. railway motor. The temperature rises were obtained with imbedded thermocouples, and in the case of the armature, through slip rings. The observations conform well with the computed exponential curves. The binary time constant of the armature copper was 0.7 hour, that of the field copper, 1.3 hours, and of the field surface 1.4 hours. This shows that the location of the hottest-spot temperature in a machine during a temporary overload, may depend upon the time constants of the different elements.

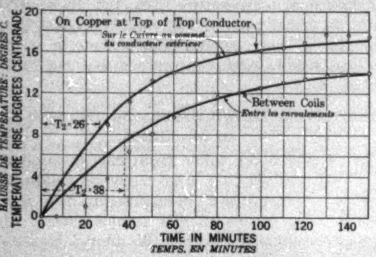

FIG. 5—TEMPERATURE RISE OF A 25,900-KV-A. THREE-PHASE TURBO GENERATOR OPERATING UNDER CONSTANT 75.3 PER CENT LOAD

Exponential curves to fit experimental data. Imbedded temperature detectors. Upper curve for detectors in contact with copper winding. Lower curve for detectors between coils, but outside insulation. (C. M. Laffoon, Bibliography 18, Fig. 3.)

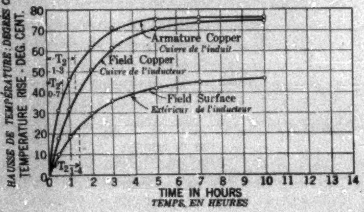

FIG. 6—TEMPERATURE-RISE CURVES OF A 35-H-P., D-C. RAILWAY MOTOR, 300 VOLTS, 35 AMPERES

Temperature by imbedded thermocouples (armature by slip rings). Curves are exponential to fit observations (G. E. Luke, Bibliography 18, Fig. 9.)

In additional temperature rise tests reported by Mr. Luke, the agreement between the test observations and exponential curves was satisfactory in certain cases but unsatisfactory in others, depending upon the relative mutual dependence of the thermal elements in the structure of the machine.

Fig. 7 gives four temperature rise curves reported by M. O. E. Shirley taken in a series of special thermal tests on a 1100-kv-a., 3000-volt, three-phase synchronous motor. This machine was operated as a synchronous condenser, at substantially zero power factor, starting at ambient temperature, under steady excitation. The ambient temperature remained practically uniform throughout. The top curve represents the rotor field-winding temperature, from volt-ampere measurements. Until after the first hour, the observations conform satisfactorily well with the exponential curve of $\tau_2 = 34$ minutes. The maximum rise of this field winding exceeds the usual values on account of the over-excitation for zero power factor. The two R. T. D. curves refer to imbedded resistance temperature detectors. The upper of these curves refers to detectors between stator armature coils in the slots, and the lower to detectors placed between stator armature coils and stator core. Both of these curves are in satisfactory agreement with the observations. The former has a time constant of 19.5 minutes, and the latter 25 minutes. The stator core curve, derived from thermometer readings, is not in good agreement with the observations.

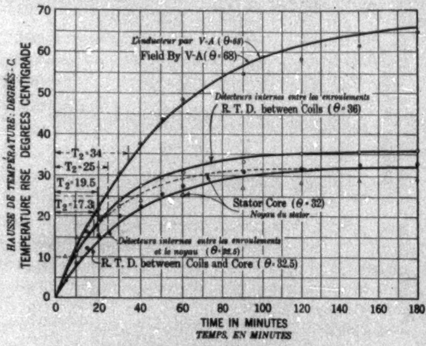

FIG. 7—TEMPERATURE-RISE CURVES FOR A 1100-KV-A., 3000-VOLT, THREE-PHASE SYNCHRONOUS MOTOR OPERATED AT APPROXIMATELY ZERO POWER FACTOR OR SYNCHRONOUS CONDENSER (O. E. SHIRLEY)

Air-cooled, Oil-immersed Transformers. Figs. 9, 10 and 11 give temperature rise curves reported by Mr. W. H. Cooney† from observations on large single-phase transformers tested under conditions of constant load. In these cases the time constant was predetermined by the method outlined in Appendix 1. The curves are exponential with these respective time constants, and conform satisfactorily to the measured temperature rises. They refer to top-oil temperature rise above ambient air. The dissipation constant s in these cases, appears not to have been constant over the temperature range. Fig. 8 gives the relation observed between temperature rise and relative watts loss, as drawn on log paper, for an exponent of 0.8. As a consequence of this variation in s, the time constant varies with the load on the transformer. Thus in Fig. 9 the first part of the curve, for 100 per cent load, during 19 hours, has $\tau_2 = 3.14$ hours, the ultimate temperature rise being 44.4 deg. cent. The second part of the curve, for 125 per cent load, during 11 hours, has $\tau_2 = 2.08$ hours, the ultimate additional temperature rise being 15.1 deg. cent., or 59.5 deg. cent. above ambient.

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*Bibliography 18, page 653, Fig. 9.

†Bibliography 19, Figs. 6, 8 and 10.

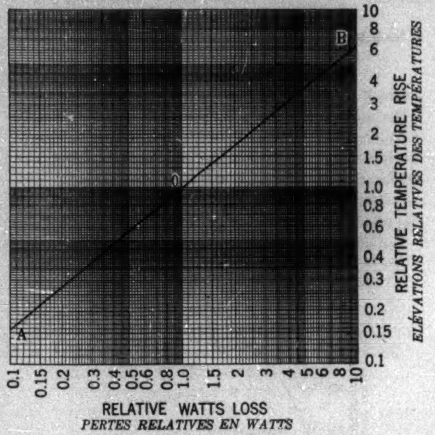

Fig. 8—Approximate Straight-Line Relation on Log Paper Between Relative Temperature Elevation of Top Oil and Relative Loss of Power in Transformer, Referred to the Temperature Elevation at Normal Rated Power Loss

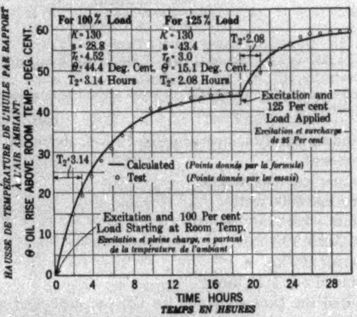

Fig. 9—Temperature Rise of Top Oil Above Ambient Air with Time, in a 75-Kv-a. Transformer, 100 Per Cent Load and Excitation Applied for 19 Hours, Followed by 125 Per Cent Load for 11 Hours More

Fig. 10 gives a corresponding curve of temperature rise in a 2-mv-a. air-cooled transformer, operated for 19 hours under normal load conditions. The thermal capacity is given as 1700 watt-hours per deg. cent., and the dissipation constant 457 watts per deg. cent., so that the exponential time constant τ_e is 3.72 hours, and the binary time constant 2.58.

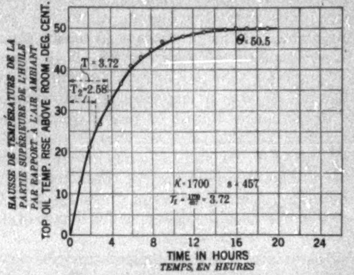

Fig. 10—Temperature Rise of Top Oil in a 2000-Kv-a. Air-Cooled, Oil-Immersed Transformer Starting Under Normal Load and Excitation from Ambient Room Temperature

Exponential curve computed to fit observations. (W. H. Cooney, Bibliography 18, Fig. 8.)

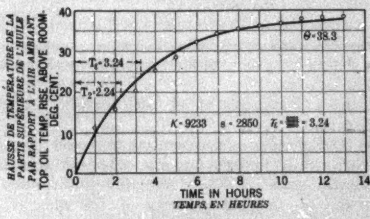

Fig. 11—Temperature Rise of Top Oil in a 12,500-Kv-a., Air-Cooled, Oil-Immersed Transformer, Starting Under Normal Load and Excitation from Ambient Room Temperature

Exponential curve computed to fit observations. (W. H. Cooney, Bibliography, Fig. 10.)

The curves in Fig. 11 refer to the case of a 12.5-mv-a. transformer, with a thermal capacity of 9233 watt hours per deg. cent.

In all of the cases represented in Figs. 9, 10 and 11 the time constants were computed from the dimensions and weights of the transformer elements. The exponential curves were then drawn according to these time constants so as to show the conformity or departure of the observed temperatures from the curves.

CONCLUSIONS

The cases presented in Figs. 4 to 11 indicate that various examples of different types of machines follow temperature rise exponential curves, under steady load, within limits of precision sufficient for many practical purposes. On the other hand, there are various machines which, either in whole or in certain elements, depart materially from such curves. It is, therefore, unsafe to assume that any or every machine, taken at random, will follow an exponential temperature rise curve, unless either a test or record of the machine will establish the fact.

The binary time constant of a machine in association with its ultimate temperature rise characteristic curve, will enable the thermal behavior of the machine to be predicted in the variable regime, under change of steady load.

It is desirable to study the temperature-rise curves of all classes of machines, in order to diffuse a general knowledge of their behavior in the variable regime, this being useful information for the operating engineer.

Appendix I

MONTSINGER-COONEY METHOD OF ESTIMATING THERMAL CAPACITY OF AIR-COOLED OIL-IMMERSED TRANSFORMERS

Data for thermal capacity of transformer from top-oil to exterior ambient temperature.

Sp. heat of copper wire 0.0935
Sp. heat of steel core 0.115
Sp. heat of oil 0.47

Spec. thermal capacity of copper wire 0.109 watt-hr./kg. and deg. cent.

Spec. thermal capacity of steel core 0.134 watt-hr./kg. and deg. cent.

Spec. thermal capacity of oil 0.462 watt-hr./liter per deg. cent.

The ratio of mean oil temperature to top-oil temperature varies with the form and size of the oil tanks, but an average value is 0.86. Applying this ratio, the mean specific thermal capacity of the tank oil is $0.462 \times 0.86 = 0.397$, or in round numbers 0.4 watt-hr./liter deg. cent.

The weight of steel core in a transformer is commonly about 5.5 times that of the copper wire. Applying this ratio to the total metal in the transformer, the average specific thermal capacity of that metal is

$$\frac{0.109 \times 1 + 0.134 \times 5.5}{6.5} = 0.13 \text{ watt-hr./kg. and}$$

deg. cent.

Different parts of the oil tank attain different temperature elevations. Experience shows that a ratio of ⅔ may be applied to the weight of steel in the tank in estimating its specific thermal capacity.

The thermal capacity of the entire transformer from the top oil to the external ambient temperature is then

$$k = 0.13 \text{ (kg. copper + kg. core + } \tfrac{2}{3} \text{ kg. tank)} + 0.4 \text{ (liters oil) watt-hr. per deg. cent.}$$

Data for thermal capacity of winding with respect to top oil:

$$k' = 0.109 \frac{S + s'}{2 S'} \times \text{kg. bare copper wire}$$

watt-hr./deg. cent.

where S is the cross-section of the insulated wire and s' is the cross-section of the bare wire.

This formula for k' takes account of the specific heat of the insulation of the wire as well as that of the wire itself.

EXAMPLE OF TIME-CONSTANT COMPUTATION

Data for 100 per cent load and excitation.

Top-oil steady temperature elevation, 40 deg. cent.

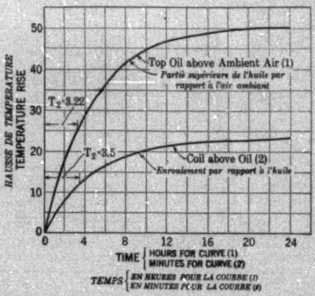

FIG. 12—COMPUTED TEMPERATURE ELEVATION TIME CURVES FOR TRANSFORMER HAVING THE CONSTANTS ANALYSED IN APPENDIX I

Abscissas minutes for curve 2 coil rise above oil, and hours for curve 1 top oil above air

Winding steady temperature elevation, above top oil, 15 deg. cent.

Copper loss 9000 watts.

Core loss 6000 watts.

Weights, steel tank 2040 kg.

Steel core 1900 kg.

Copper 450 kg.

Volume of oil, 3410 liters.

Required the time constant for 125 per cent load and excitation, starting from ambient temperature.

Final copper loss $(1.25)^2 \times 9000$ =	14060 watts
" iron "	6000 "
Total, neglecting changes in resistance,	20060 watts

$$\Theta_0 \textit{ for top oil.} \quad \text{Relative losses } \frac{20060}{15000} = 1.337$$

From Fig. 1, oil rise = 1.26 × 40 deg. cent. = 50.4 deg. cent.

$$k_2 = 0.13\,(450 + 1900 + 2 \times 2040/3) + 0.4 \times 3410$$

$= 1850$ watt-hr./deg. cent.

$s_2 = 20060/50.4 = 398$ watts/deg. cent.

$$\tau_\epsilon = \frac{k_2}{s_2} = \frac{1850}{398} = 4.65 \text{ hours.}$$

$\tau_2 = 0.693 \times 4.65 = 3.22$ hours.

Θ for copper above top oil

$15 \times (1.25)^2 = 23.5$ deg. cent.

$k_\epsilon' = 454 \times 0.109 = 49.5$ watt-hr./deg. cent.

$s' = 14060/23.5 = 508.3$ watts/deg. cent.

$$\tau_\epsilon = \frac{k'}{s'} = \frac{49.5}{508.3} = 0.083 \text{ hour} = 5.0 \text{ min.}$$

$\tau_2 = 0.693 \times 0.0826 = 0.057$ hour $= 3.5$ min.

The exponential curves for θ_0 and θ_c are given in Fig. 12, as computed from the above data, together with their binary constants, to two different scales of time, as abscissas.

The original data from which Appendix I has been prepared, will be found in Bibliography 18 and 19.

Appendix II

FORMULA FOR DERIVING THE ULTIMATE TEMPERATURE RISE FROM AN INCOMPLETE TEMPERATURE RISE—TIME CURVE ASSUMED TO BE EXPONENTIAL.*

$$\Theta = \frac{\theta_1}{2 - (\theta_2/\theta_1)} \quad \text{deg. cent.}$$

where θ_1 is the temperature rise at the end of t_1 hours, and θ_2 is the corresponding temperature rise at the end of $t_2 = 2\,t_1$ hours. It is assumed that $\theta = 0$ at the start, and that the ambient temperature remains constant throughout.

Thus if $\theta_2 = 1.5\,\theta_1$, it will follow that $\Theta = 2\,\theta_1$ and $t_1 = \tau_2$.

FORMULA FOR COMPUTING THE BINARY TIME CONSTANT FROM TWO ORDINATES OF AN EXPONENTIAL TEMPERATURE RISE CURVE TAKEN AT TWO SUCCESSIVE EQUAL INTERVALS OF TIME FROM THE START.

$$\tau_2 = \frac{0.30103\,t_1}{\log\left(\dfrac{\theta_1}{\theta_2 - \theta_1}\right)} \quad \text{hours}$$

where θ_1 is the temperature rise after t_1 hours, and θ_2 the corresponding temperature rise after $t_2 = 2\,t_1$ hours. Thus if $\theta_2 = 1.5\,\theta_1$ $\quad \tau_2 = t_1$

LIST OF SYMBOLS EMPLOYED

ϵ = 2.71828 Napierian base.

θ temp. rise above ambient (the latter assumed as constant) (deg. cent.)

θ_1 temp. rise after time t_1 (deg. cent.)

θ_2 temp. rise after time t_2 (deg. cent.)

θ_a ambient temperature, (deg. cent.)

Θ ultimate temp. rise above ambient, (deg. cent.)

k thermal capacity of machine or element, (watt-hrs. per deg. cent.)

s dissipation constant of machine or element (watts per deg. cent. rise above ambient).

P output of a machine (watts)

p power dissipated steadily in machine or element (watts)

t time elapsed from start of temp. rise test (hours or min.)

τ_ϵ exponential time constant (hours or min.)

τ_2 binary time constant (hours or min.)

τ_{10} decimal time constant (hours or min.)

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Without Pretensions as to Completeness

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Discussion

James Burke: Dr. Kennelly's paper may be considered from four standpoints, namely, the application of his proposed "Binary Time Constant" (B. T. C.)

(a) To scientific research in investigation of heating phenomena in electrical machinery,

(b) As an aid to testing and determination of test rating,

(c) As a factor of rating, in addition to the continuous full-load temperature rating, from which many short time ratings and duty cycle ratings may be determined,

(d) As a means of comparing the capacities of machines for intermittent and short time load applications.

The B. T. C. converts the mathematics of the exponential time constant into simple arithmetic, and permits expediting the slow and tedious procedure otherwise necessary.

(a) The simple B. T. C., although approximating, may not be sufficient for exact analysis or calculations in research or design problems, as there is frequently involved the combination of interrelated masses having different time constants, as for example, in an oil-insulated transformer consisting of windings, core, oil, and case. The B. T. C. can be conveniently used as a starting point, however, and the departure therefrom may be calculated or determined by test.

In all instances which I have investigated, the departures from the simple B. T. C. are in the direction of greater resulting B. T. C. and thus the use of the B. T. C. would result in more conservative rating when time constants are involved.

Under this heading would also come the investigation and determination of which classes of apparatus or parts thereof are subject to simple B. T. C. treatment, and possibly also the formulating of rules for introducing or correcting for departures therefrom.

The I. E. C. has established standards of temperature limits from the standpoint of safety of the insulation of windings, and while it is obvious that different parts of a machine may have different B. T. C., the part with which we are generally concerned in relation to intermittent or short time ratings, is the part with the shortest B. T. C.

For example, in a d-c. series motor having fields and armature with 50 deg. cent. temperature increase on continuous load, the B. T. C. of the armature will be generally shorter than that of the field windings, and the limit of increase above rated continuous load for short time will be determined by the armature B. T. C.

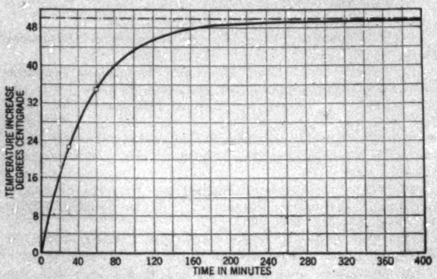

Fig. 1

Further, in most cases, the B. T. C. of the armature end windings is less than that of the windings in the slots, so that the B. T. C. of the end windings will be in such cases the determining factor from the rating standpoint.

The winding or part of winding which has the shortest B. T. C. can be determined by calculation or test, and problems relating to rating concentrated thereon.

(b) In testing machines, the continued operation until maximum temperature is reached is frequently long and expensive. Dr. Kennelly has given in Appendix II:

"Formula for deriving the ultimate temperature rise from an incomplete temperature rise—time curve assumed to be exponential." Also

"Formula for computing the Binary Time Constant from two ordinates of an exponential temperature rise curve taken at two successive equal intervals of time from the start."

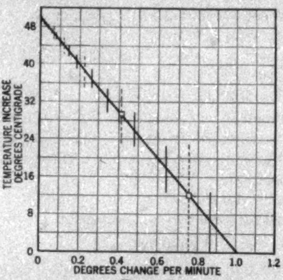

Fig. 2

A simple and quick method, which I found very useful, is illustrated in the following diagrams and curves.

Fig. 1 shows a typical temperature increase curve. It will be noted that two points are indicated by small squares, giving temperature increase at 30 min. and 60 min. respectively, from the commencement of the test. The curve of temperature increase up to 400 min. or nearly seven hours is shown and it will be noted that the temperature is still slightly increasing towards an ultimate of 50 deg. increase. To draw this curve from the two observations shown would require considerable imagination, although it could be calculated from Dr. Kennelly's formula. The simpler way is as follows:

Fig. 2 which is drawn with "Degrees change per minute" instead of "Time in minutes" as in the Fig. 1, has plotted thereon the same two observations referred to in connection with Fig. 1, and indicated by small squares. The dotted vertical lines passing through the small squares represent the increase in temperature during the first 30-min. period and the second 30-min. period. The small squares have been located at the middle of these vertical lines. A straight line drawn diagonally through these two squares indicates the various points which would be observed by the continuation of the test. This diagram, Fig. 2, shows in various positions towards the left of the small squares other dotted vertical lines showing succeeding half-hour periods, and it will be observed that they are all intersected by the diagonal line.

In the same manner there has been shown in Fig. 2 a number of vertical full lines, which are readings at 15-min. periods and which give the same resulting diagonal line. For long periods between individual readings, in relation to the B. T. C., there is a slight correction from the exact intersection of the vertical lines of Fig. 2 which can be readily calculated.

The complete curve of Fig. 1 has been plotted from the diagonal line of Fig. 2, which diagonal line was drawn from the two observations indicated by the small squares taken at the end of the first and second thirty minutes of test. Of course, any other suitable periods would have done as well as the thirty minutes, and the periods do not have to be alike. Without the use of the B. T. C. the procedure for drawing Curve No. 1

from straight line Fig. 2 was to first plot a curve of reciprocals of the values of the straight line, Fig. 2, and then plot the integration of same. This is the manner in which the curve of Fig. 1 has actually been drawn. It is a slow and tedious process.

Dr. Kennelly's B. T. C. is easily found from the straight line Fig. 2 as follows:

The exponential time constant is the height of the diagonal line, which is the maximum temperature increase (namely 50 in this instance) divided by the base, which is the maximum degrees change per minute (namely 1 in this instance), and which therefore gives an E. T. C. of 50 (in this instance). Dr. Kennelly has shown that the B. T. C. is approximately 0.7 of the E. T. C. Therefore, in this instance the B. T. C. is 0.7 of 50, or 35 min. Thus we have

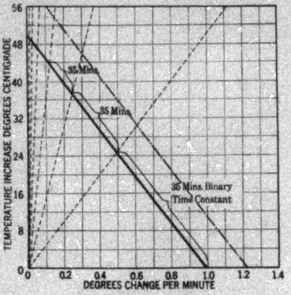

Fig. 3

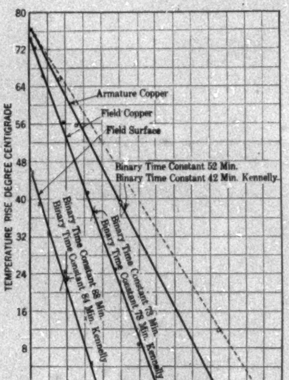

Fig. 4—35-H. P., D-c. Railway Motor (Dr. Kennelly's Fig. 6)

$$\text{B. T. C.} = \frac{\text{Max. temperature increase} \times 0.7}{\text{Max. degrees change per minute}}$$

Having found the B. T. C. (in this instance 35 min.), we can apply it to the problem by dividing the diagonal line of Fig. 2 into two equal parts. This is shown in Fig. 3 in which the diagonal line of Fig. 2 is divided into two equal parts by the radial dotted line. The length of the diagonal line to this radial dotted line correspondence to one B. T. C., in this instance is 35 min. The balance of the diagonal line is again divided into two equal parts, the lower of which corresponds to another B. T. C., in this instance 35 min. Correspondingly, the balance is again subdivided, establishing another B. T. C., (35 min.), and so on for as many B. T. C.'s as may be desired.

If it is desired to plot a curve similar to that shown in Fig. 1, it is easily done from Fig. 3 which gives the increase in temperature at the end of the first 35 min. and each succeeding 35 min. If intermediate points are desired they are easily determined by the subdivision of the B. T. C. into fractional parts, as will be seen in another illustration in Diag. No. 7. The curve will be, of course, identical with the curve of Fig. 1.

A diagram of temperature increase in the form shown in Fig. 3 with the B. T. C. shown thereon, however, is much more useful in treating problems of rating than the conventional form shown in Fig. 1.

This straight line form of Figs. 2 and 3 also permits forming the best conclusion from a number of observations which, as frequently happens, contain some inaccuracies. Knowing that the points of observation should result in a straight line, they can be averaged better than when plotted in the curve form of Fig. 1.

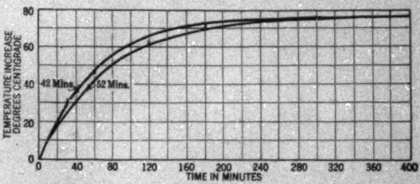

Fig. 5—35-H. P. Armature Copper (From Sheet 4)

Also the B. T. C. can be much more accurately determined from the straight line diagram than from other methods.

Fig. 4 illustrates the foregoing in connection with Dr. Kennelly's paper, Fig. 6—"Temperature Rise Curves of a 35-H. P., D-C. Railway Motor."

The extreme left straight line on Fig. 4 is plotted from the observations on Dr. Kennelly's curve, these being indicated by small crosses. As noted on the chart this straight line gives a B. T. C. of 88 min. for the field surface as compared with Dr. Kennelly's figure of 84 min., showing very close agreement.

The next diagonal line relates to the field copper, the black dots showing the actual observations taken from Dr. Kennelly's curve. This diagonal line gives B. T. C. of 73 min. compared with Dr. Kennelly's 78.

The extreme right dotted line refers to the armature copper and is plotted for the B. T. C. which Dr. Kennelly has concluded, namely 42 min. The observations from his curve are plotted in small squares. The diagonal line to the left of this dotted line is my conclusion from the same evidence, but introducing an additional feature of evidence, and results in a B. T. C. of 52 min. The additional feature of evidence is based on the calculation of the probable size conductor, from which figure and the amperes given in Dr. Kennelly's chart, the current density was obtained, and from this figure the maximum change in degrees per minute at start was calculated. It will be seen that both curves pass through the two points of observation at the maximum of the temperature rise, also that mine favors the average of all the other observations with the exception of the first observation, namely, at 12-deg. temperature increase. The dotted line which represents Dr. Kennelly's conclusion places more reliance on this first observation. My experience has been that the first observation is not as trustworthy as future ones as there are probable errors as to the exact time at which the load was applied, and the influence of partial load before adjusting to the correct full load.

The two conclusions regarding armature copper, however, are much closer together than would be indicated by the appearance of the two lines of Fig. 4 which exaggerate the appearance of the difference. This is shown in Fig. 5 in which both Dr. Kennelly's 42-min. B. T. C. curve and my 52-min. curve are shown, together with the actual points of observation indicated by crosses.

Fig. 6 shows the curve for a 25,000-kv-a. turbo generator for which the observations are given by Mr. C. M. Laffoon in the discussion of a previous paper by Dr. Kennelly on this subject (JOURNAL A. I. E. E., June, 1925). The observations are shown in crosses and indicate a probable falling off of load during the middle of the test, resulting in a reduction of temperature for three readings instead of an increase. These three observations are eliminated in the observations from which the straight line diagram has been drawn. The full straight line shows my conclusions from the evidence, giving an 18-min. B. T. C., and the curve has been drawn accordingly. It will be noted that this curve is in close agreement with the observations indicated by the crosses. The dotted straight line is the result of the curve with which Mr. Laffoon joined the points of observation and which gave a 20-min. B. T. C.

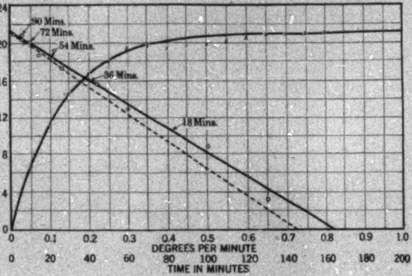

FIG. 6—25,000-KV-A. TURBO GENERATOR (C. M. LAFFOON, JOURNAL A. I. E. E., JUNE, 1925)

Tests made by Burke Electric Company in which special care has been taken for accurate observations during the early part of tests show much closer coincidence with the straight line but the instances given in the foregoing are sufficient for the present discussion.

(c) Fig. 7 illustrates how the B. T. C. can be applied to intermittent and short time ratings. This diagram is drawn for a motor with a 40-deg. temperature increase under continuous full load and with a B. T. C. of 60 min. It will be noted that the diagonal line has been divided into intermediate B. T. C.'s, namely, 15 minutes or ¼, 30 minutes or ½, 45 minutes or ¾ of the first B. T. C; also some subdivisions of the other B. T. C.'s and that the figures and radial lines have been carried out to 300 min. or five B. T. C.'s of 60 min. each.

It will also be noticed that other diagonal lines have been drawn for various overload ratings, based on the extreme assumption that the heating will increase with the amperes square. Dr. Kennelly has shown in his Fig. 3 an exponent of 1.6, compared with the above conclusion of an exponent of 2, and it is probable that most cases will be between these limits, depending upon the location of the copper in relation to other masses, such as armature or stator punchings.

One application of this diagram Fig. 7 is shown in the instance of a 40-deg. rated motor, operated at 25 per cent overload after it has reached its full load temperature increase. Starting at the intersection of the full load diagonal line and the 40-deg. temperature increase, if we trace horizontally until we meet the 25 per cent overload diagonal line, we find that this occurs at approximately 88 min. Following this diagonal line until we reach the 55-deg. cent. increase, which is the customary American guarantee under 25 per cent overload, we find that this occurs at the 180-min. line. Thus a machine with a B. T. C. of 60 min. and 40-deg. continuous rating can carry an overload of 25 per cent without exceeding 55-deg. cent. temperature, for a period of the difference between 180 min. and 88 min. or 92 min., which is approximately 1½ hours. To carry this overload for the customary two hours would require either a greater B. T. C. or a machine with a lower full-load temperature increase. Other applications are obvious.

(d) A motor without any B. T. C. would of course assume its maximum temperature instantaneously, and would therefore have no short time or intermittent overload capacity. The said overload capacity is the result of a motor having B. T. C. It is therefore obvious that in considering motors for service requiring short time or intermittent overload capacity, the B. T. C. becomes a factor of rating, of as great, or perhaps greater, importance than its continuous full-load rating.

It is appreciated that in problems of short time rating, other limits such as mechanical safety factors, commutation, etc., must be taken into consideration. Also it may not be advisable to take advantage of the entire B. T. C. of machines, at first, but to approach this utilization gradually, checking same by tests.

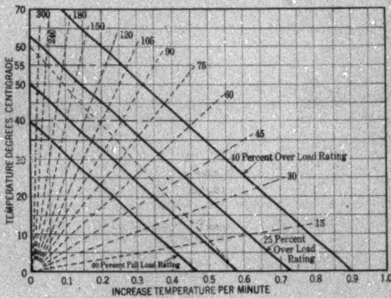

FIG. 7

There is, however, a minimum B. T. C. which exists in practically all instances, and which is based on the following assumptions:

A factor of specific heat to include spec. heat of insulation not less than 1.3,

A factor of hot-spot divided by thermometer reading not less than 1.2.

Binary Time Constant will not be less than

$$\frac{\text{Temp. inc. guaranteed (Thermometer)}}{\text{Amperes per square mm. squared}} \times 3.75$$

With 50-deg. guarantee this becomes

$$\frac{187}{\text{Amperes per square mm. squared}}.$$

Report of the Board of Directors

FOR THE FISCAL YEAR ENDING APRIL 30, 1926

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Forty-second Annual Report, for the fiscal year ending April 30, 1926. A general balance sheet showing the condition of the Institute's finances on April 30, 1926, together with other detailed financial statements, is included herein. The following is a brief summary of the principal activities of the Institute during the year; more detailed information has been published from month to month in the Institute JOURNAL.

Directors' Meetings.—The Board of Directors held seven meetings during the year, six in New York and one at Saratoga Springs, N. Y. The Executive Committee acted upon various matters during intervals between Board Meetings.

Information regarding the more important activities of the Institute which have been under consideration of the Board of Directors, the committees and the various officers, is published each month in the section of the JOURNAL devoted to "Institute Activities."

President's Visits.—President Pupin has made numerous trips during his term of office, speaking before many meetings of the Institute, also various educational and civic organizations. Among the places visited were Chicago, Denver, Kansas City, Pittsburgh, Springfield, Ill., San Francisco, Saratoga Springs, Univ. of California, Univ. of Nevada, Univ. of Colorado and Johns Hopkins Univ.

Other meetings that President Pupin is scheduled to attend in May and June will be held in Boston, Cleveland, Ithaca, Niagara Falls, University of Delaware and White Sulphur Springs.

Meetings.—The general conventions of the Institute for the year were limited to three as follows: Annual, Pacific Coast and Midwinter Conventions. In accordance with the policy of decentralization adopted by the Board, Regional meetings were held in Swampscott, Massachusetts, District No. 1 and in Cleveland, Ohio, District No. 2. Three other Regional meetings are now scheduled.

Annual Meeting.—The Annual Business Meeting was held at Institute headquarters, New York, on May 15, 1925. The Annual Report of the Board of Directors for the fiscal year ending April 30, 1925, was presented. The Tellers Committee made its report upon the election of officers for the administrative year beginning August 1, 1925.

Following the business meeting the subject "Steam Railroad Electrification from the Executive Standpoint," was presented through addresses by six executives from the government, manufacturing and railroad fields. This part of the meeting was held under the auspices of the New York Section. Attendance was about 1000.

Annual Convention.—The Forty-first Annual Convention was held at Saratoga Springs, New York, June 22 to June 26, 1925. Five technical sessions were held at which twenty-eight papers were presented. In addition there were two special sessions devoted to hydroelectric developments and to engineering education. The annual conference of the Sections Committee was held on Monday, occupying both morning and afternoon sessions; forty-three Sections were represented. An elaborate entertainment program was carried out, including golf and tennis tournaments, inspection trips, sightseeing and special features. Attendance was about 900 members and guests.

Pacific Coast Convention.—The Fourteenth Pacific Coast Convention was held at Seattle, Washington, September 15th to 19th, 1925. Seven technical sessions were held at which twenty-eight papers were presented. President Pupin addressed the convention from New York over the transcontinental telephone lines. Registration was 408.

Midwinter Convention.—The Fourteenth Midwinter Convention was held in New York, N. Y., February 8th to 11th, 1926. Thirty-five technical papers were presented at eight sessions. Two lectures, numerous inspections trips and a smoker were also scheduled. Attendance was approximately 1500.

Regional Meetings.—During the year two Regional Meetings or Conventions were held, the first at Swampscott, Massachusetts, District No. 1, May 7, 8 and 9, 1925, at which twenty-one papers were presented at four sessions, registration 370; the second at Cleveland, Ohio, District No. 2, March 18th and 19th, 1926, with eight papers at five sessions; registration 430. Three other districts are scheduled for similar meetings, as follows: Madison, Wisconsin, District No. 5, May 6th and 7th, 1926; Niagara Falls, N. Y., District No. 1, May 26th to 28th, 1926; and New York, N. Y., District No. 3, November 12, 1926. The active cooperation of the Meetings and Papers Committee with the Sections located in the various Districts, acting through District Committees under the supervision of the Vice-Presidents of the Districts, has made all of these Regional Conventions extremely interesting and successful.

Student Conventions.—The steadily increasing number of Student Branches of the Institute has heretofore been the principal sign of the greater interest taken in Institute work by the students registered in electrical engineering in the colleges of the country. That this interest has been greatly augmented recently is indicated by the holding of Student Conventions in Sections of the Institute, planned and carried out by the students under the supervision of Section officers. The Philadelphia Section held its second annual Student Convention at Swarthmore College on March 8, 1926; eight colleges participated. The New York Section held a similar convention on April 23, 1926, participated in by eight colleges in the New York District and the North Eastern District will hold a Student Convention in Boston on May 7, 1926.

Abstracts of the reports of the chairman of many of the Institute committees and delegations are included herein under various headings.

Meetings and Papers Committee.—The work of the Meetings and Papers Committee during the year has consisted principally in arranging the technical sessions of the national conventions and cooperating with the regional meeting committees.

Three national conventions have been held—the Annual Convention, the Pacific Coast Convention and the Midwinter Convention. A Spring Convention was not held in 1926 as the Board of Directors wished to give more opportunity for holding regional meetings during the Spring.

Two regional meetings were held during the year, respectively at Swampscott, Mass., and Cleveland, Ohio. The Committee also has cooperated in arranging two regional meetings in May 1926, respectively at Madison, Wis., and Niagara Falls, N. Y.

The regional meetings appear to be developing very successfully. The quality of papers is practically as high as that of the national convention papers. The subjects presented at each meeting can be confined to those of particular interest in the region where the meeting is held as evidenced by the Cleveland and Madison meetings, or a more elaborate program may be desired as exemplified in the Swampscott and Niagara Falls meetings.

The committee has had the best cooperation from those with whom it has worked, including local convention committees, regional coordinating committees, technical committees and various individuals. This cooperation has been of greatest help in upholding the excellence of technical programs and making successful meetings.

Publication Committee.—The policies adopted by the Directors during the past three or four years regarding the publications of the Institute have been adhered to by the present committee. Monthly meetings have been held by the Committee during the year to pass upon current publication problems as they arise and in numerous cases where quick decisions have been necessary matters have been settled by correspondence between the committee's members. The general rules which were laid down in 1923 for the conduct of the JOURNAL, TRANSACTIONS and pamphlet copies are as follows:

1. That all papers presented at Institute conventions be published in full in pamphlet form for use at the conventions and for distribution to members on request.
2. That papers exceeding about eight pages in length, or almost wholly mathematical, shall not be published in full in the JOURNAL, but that an abridged version to be furnished by the author, not exceeding 4000 words, be published in the JOURNAL with a statement that the complete paper will be furnished without charge in pamphlet form to any member upon request.
3. That mathematical papers be written in two parts: (1) a readable and complete explanatory version of the subject without mathematics, which may be published in the JOURNAL, and (2) a mathematical appendix which will not necessarily be published in the JOURNAL.
4. That the contents of the TRANSACTIONS be selected from the convention and other papers of the year, and that the papers which were abstracted in the JOURNAL be printed in full in the TRANSACTIONS, articles omitted from the TRANSACTIONS to be referred to in the index.

These rulings have been adhered to during the last two years with the exception of abridgments; instead of being limited to about 4000 words they have been in the majority of cases permitted to include about eight pages. During this time the objections and complaints in regard to the JOURNAL have become exceedingly rare and the commendations on the JOURNAL have increased correspondingly so that the Committee feels that the present methods of publication more completely meet the requirements of the membership than ever before. It has been the Committee's policy for the last few years to include in the JOURNAL, either fully or in part, all the papers which have been accepted for publication during the current year.

The date of publication of the Annual TRANSACTIONS will be considerably earlier this year than at any previous time. The make-up for the Annual Volume has been sent to the printer, about 750 pages have already been made up and about 256 pages printed and sent to the bindery. This is as fast as the facilities of our printing office permitted the work to be done.

Sections and Branches.—The rapid growth of the Institute during the last few years has been due largely to the increased activity of the Sections. This pronounced and growing interest has been fostered by the policy of decentralization advocated by the Board of Directors. The Regional Conventions held in Geographical Districts, (see listing under Regional Meetings), have resulted in a development and coordination of Section interests in those Districts. The consideration of Section problems by the Sections Committee Conference during the Annual Convention and by the Sections Committee, the Meetings and Papers Committee and the national Membership Committee has been another factor tending to bind the Sections of the Institute into one body of mutual interests. During the past year two new Sections have been organized at Saskatchewan, Canada and Sharon, Pennsylvania.

Student Branch activity has similarly rapidly increased during the last few years. In order to insure continuity in the work of Branches, the appointment of a Student Branch Counselor from among the electrical engineering faculty of the institution at which each Branch is located, was authorized, and practically all such Counselors have been appointed. Executive Committees of the Counselors have been formed in several Districts to coordinate and further Branch activities of the district. A number of Student Conventions have been held recently, under the supervision of Section and District Committees. New Branches were authorized during the past year at Ohio University, Stevens Institute of Technology, Washington and Lee, Worcester Polytechnic Institute and University of Wyoming.

	For Fiscal Year Ending			
	May 1 1919	May 1 1921	May 1 1923	May 1 1926
SECTION				
Number of Sections	34	42	46	51
Number of Section meetings held	217	303	344	405
Total Attendance	25,837	37,823	46,672	58,959
BRANCHES				
Number of Branches	61	65	68	86
Number of Branch meetings held	156	443	503	714
Attendance	6,441	21,629	26,893	35,270

Standards.—The past year has marked the practical completion of the revision of the Institute standards which has been actively under way during the past three years. The revised standards are published in the form of a number of sections, each section dealing with standards for a particular type of apparatus or a particular branch of the electrical industry. This arrangement has very large advantages which were discussed in the report of the work last year. At the present time, 21 of these sections have been completed, these sections covering the great bulk of the material previously covered in the Institute Standards and also some valuable new material as discussed in last year's report. A number of the other sections are in active preparation.

During the past year the Institute Standards were translated into Russian on the initiative of the Russian engineers. A translation of the revised standards into Spanish is now being carried out under the direction of the Standards Committee. It is thought that this translation will be of great value in the direction of closer relations with Latin America.

A very considerable part of the activities of the Standards Committee during the year has been directed toward the preparation of recommendations to the United States National Committee of the I. E. C. for material to be presented at the April meeting of the I. E. C. in New York. Fifty representatives of the Institute were appointed as delegates or technical advisors to the Advisory Committees of the I. E. C.

During the past year the Standards Committee has devoted a greatly increased amount of time to the consideration of American Engineering Standards Committee work.

The Institute has been appointed sole or joint sponsor for eight projects during the year, including six sections of the revised Institute standards. The Institute has also appointed representatives on additional Sectional Committees for which it is not a sponsor. During the year the American Engineering Standards Committee approved as American standard, the standards on Industrial Control Apparatus and the standards on Instrument Transformers.

A very important step in the development of standardizing work in the electrical field has been taken by the decision to appoint an Electrical Advisory Committee for the American Engineering Standards Committee. This has arisen from the joint study given throughout the year by a Joint Committee of the A. I. E. E. and of the Electrical Manufacturers' Council to the mutual problems involved in electrical standardizing work. This recommendation has been accepted by the American Engineering Standards Committee, who have invited the Institute and the other interested organizations to nominate representatives on this committee.

An interesting step which was taken by the Institute during the year consisted of the appointment of a subcommittee of the Standards Committee on Nomenclature. The function of this Committee is not to define established terms, but to consider the need of additional nomenclature to represent new concepts or to more clearly represent concepts not at present adequately expressed.

The cooperation of the Standards Committee with Technical Committees of the Institute and with other technical organizations has continued to be very close throughout the year. An illustration is the acceptance of the Standards Committee of an invitation to become responsible for the Electrical Measurements Section of the Power Test Codes of the American Society of Mechanical Engineers. Material assistance has been received from the Electrical Machinery Committee of the Institute.

U. S. National Committee of I. E. C.—The past year has been a very active one for the U. S. National Committee, inasmuch as a meeting of the Advisory Committees of the I. E. C. was held in The Hauge, April 16th to 23rd, 1925, and a plenary session of the I. E. C. was held in New York City, April 13-22, 1926. Nine Advisory Committees met at The Hague, and thirteen countries were represented by delegates. Details relative to the work accomplished were outlined in the JOURNAL for June 1925.

The meeting held in New York, April 13th to 22nd, was the first plenary meeting of the Commission ever held in the United States. Over one hundred delegates from sixteen nations attended, accompanied by about twenty ladies. A program of technical sessions, luncheons, dinners, inspection trips and entertainment features was carried out, the foreign delegates being the guests of the U. S. National Committee and of the manufacturing and utility interests. Following the last of the technical sessions, the delegates were taken on an extensive trip covering twelve days and visiting ten cities in this country and Canada. Details of the work accomplished at the Plenary meeting are outlined in the May 1926 JOURNAL.

U. S. National Committee, International Commission on Illumination.—The United States Committee of the Commission is composed of representatives of various organizations interested in illumination, the Institute having three representatives on the committee. Following the plenary session in Geneva, July 1924, the general work of the committee has been proceeding. Measures are now being taken to prepare for the next session which is to be held in the United States in 1927.

Committee on Safety Codes.—The Committee on Safety Codes of the American Institute of Electrical Engineers is made up of representatives of the Institute assigned to cooperate with various standardizing committees in matters relating to National Safety Codes.

During the past year the chief work of the committee has been in connection with the revisions of the National Electrical Code. The Institute has been represented on the Electrical Committee of the National Fire Protection Association and has assisted in the revision of the Code through its representation on that committee and on its various technical subcommittees.

The Institute will be represented by regularly appointed delegates at the annual meeting of the National Fire protection Association, to be held in Atlantic City in May, at which time the recommended changes in the Code will be considered for adoption.

Technical Committees.—Reports of Technical Committees embracing an outline of the year's work and a summary of progress in the industry will be presented at the Annual Convention and printed in the JOURNAL.

Membership.—The present status of the Institute membership in the various grades is indicated in the table below. The results of the Membership Committee's efforts this year, as carried on largely through the local membership committees of the various Sections of the Institute, have been very gratifying. The table contains in the division headed "Additions" a statement of the number of new members elected to each grade of membership during the year.

	Honorary	Fellow	Member	Associate	Total
Membership on April 30, 1925	4	597	2,436	14,282	17,319
Additions:					
Transferred		33	178		
New Members Qualified		5	97	1,976	
Reinstated			8	45	
Total	4	635	2,719	16,303	
Deductions:					
Died		7	16	71	
Resigned		2	36	333	
Transferred			18	193	
Dropped		1	26	800	
Membership, April 30, 1926	4	625	2,623	14,906	18,158
Net increase in Membership during the year					839

Deaths.—The following deaths have occurred during the year.

Fellows: Charles W. Burrows, Willard G. Carlton, Herbert W. Cheney, L. A. Herdt, Harrison P. Reed, James W. Thomas, Albert Winchester.

Members: James A. Brett, Jean Canivet, Charles Chandler, H. H. Clark, Robert C. Cole, Arthur H. Fleet, J. L. Frazier, Albert C. Jewett, Harry B. Marshall, Norman C. Nelson, Harold W. Nichols, Charles T. Perry, J. R. Portnell, B. H. Ryder, Sir David Salomons, Howard A. Stofflet.

Associates: Loyal R. Alden, George Y. Allen, Charles E. Anderson, J. Harvey Anderson, Robert H. Anderton, Ernest J. Bagall, Herbert M. Beechinor, Carl Bendeke, George A. Benney, Angus Black, Edgar I. Bowker, George I. Brown, Harry B. Bullen, Milan R. Bump, Carlos M. Carrillo, L. A. Cavalcanti, Alan C. Crago, D. Inglis Dawbarn, H. E. Driver, Eugen Eichel, Winthrop T. Endicott, Henry C. Forrest, Roger M. Freeman, Charles E. Fritts, Lionel K. Frizzell, Stephen J. Fuller, Charles C. Grandy, Earl Harlan, William C. Hawkins, Glenn L. Henningsen, H. F. Hill, Gustave O. Huebener, Frederick A. Huntress, J. C. B. Ingleby, Arthur W. Jones, Clifton D. Kennedy, Cecil W. King, August H. Kruesi, Edmund Land, Chester W. Lyman, Lawrence J. Marlowe, Amos S. McKee, Harry T. W. McLennan, W. F. Meschenmoser, Otto C. Miller, Orville K. Morris, Rush B. Morrow, John Z. Murphy, A. D. Newmyer, Saitaro Oi, Robert A. Paine, Jr., Kenneth C. Peake, Gordon Philp, E. Hugh Pryce, David C. Rankin, Harry G. Reed, Harry W. Reilly, Leverett E. Rice, J. Paul Rosier, James H. Schrantz, John B. Socci, Alfred C. Thomas, Charles G. M. Thomas, E. M. C. Tigerstedt, Allen A. Tirrill, C. Allen Vallejo, Arthur W. Wickstrand, William H. Wiley, J. P. Wintringham, George H. Wirth, Malcolm Woringer.

Board of Examiners.—The Board of Examiners during the year held fourteen meetings, averaging about three and one-quarter hours. It considered and referred to the Board of Directors a total of

4238 applications for admission or transfer to the higher grades.

APPLICATIONS FOR ADMISSION

Recommended for grade of Associate	2047	
Not recommended	17	2064
Recommended for grade of Member	106	
Not recommended for admission to this grade	30	136
Recommended for grade of Fellow	4	
Not recommended for admission to this grade	4	8
Recommended for enrolment as Students		1750

APPLICATIONS FOR TRANSFER

Recommended for grade of Member	196	
Not recommended for transfer to this grade	23	219
Recommended for grade of Fellow	33	
Not recommended for transfer to this grade	4	37

RECONSIDERATIONS

Previous action reaffirmed	2	

PROPOSALS FOR TRANSFER

Informally Recommended	22	22
Total number of applications considered		4238

Professional Ethics.—During the year the Committee on Code of Principles of Professional Conduct has considered matters coming within its scope and taken appropriate action thereon, including one case in which charges of unprofessional conduct were made against an Associate by a group of members, and after an investigation, under the provisions of the constitution, the Associate complained of was expelled from membership in the Institute by the Board of Directors.

Constitutional Amendments.—On May 15, 1925 the constitutional amendments dealing with election procedure, qualifications for membership, admission, transfer, expulsion, dues, life membership, etc., were adopted, they having been previously accepted by the vote of the membership. The By-laws were similarly revised under date of June 25, 1925.

Scholarships.—The governing bodies of Columbia University have placed at the disposal of the Institute three scholarships in electrical engineering. In consequence, the Institute is now authorized to award a scholarship each year so that there may be one man in each class holding a scholarship on the nomination of the Institute; these scholarships will continue until further notice. Each scholarship pays $350 toward the annual tuition, and reappointment for completion of course is conditioned upon the maintenance of good standing. The student applicant receiving the award in 1925 found it impossible to accept.

Institute Prizes.—At the meeting of the Board of Directors of the Institute of April 16, 1921, recommendations were approved establishing two Institute prizes to be awarded yearly to authors of worthy papers. The prizes consists each of $100. in cash with suitable certificate.

The First Paper Prize for 1924 was awarded to Murray F. Gardner, author of the paper entitled, "Corona Investigation on an Artificial Line." Honorable mention was given to H. C. Hamilton and R. W. Chadbourne for their paper, "The Prediction of Insulation Failures in Transmission Lines," and to J. W. Dodge for "Power Supply for Central Station Auxiliaries" and to S. Aronoff for "Fault-Finding in Power Transmission Lines." The prizes were presented at the Annual Convention, Saratoga Springs.

The Transmission Prize for 1924 was awarded to R. D. Evans and R. C. Bergvall, for their paper entitled, "Experimental Analysis of Stability and Power Limitations." Honorable mention was given to C. F. Harding, for his paper, "Corona Losses between Wires at Extra High Voltage," and to F. W. Peek, Jr. for his paper, "Lightning and Other Transients on Transmission Lines."

A report of a special committee to consider future prizes, and regulations regarding them, was received and adopted by the Board, February 9, 1926. The committee suggested the yearly award of four National Prizes of $100. each and suitable certificate, as follows: 1. Best Paper Prize; 2. First Paper Prize; 3. Best Regional Paper Prize; 4. Best Branch Paper Prize.

The following Regional Prizes may also be awarded yearly in each of the ten Geographical Districts, each prize to consist of $25. and suitable certificate: 1. Best Paper Prize; 2. First Paper Prize; 3. Best Branch Paper Prize. For complete details and conditions of award see the March 1926 JOURNAL.

Edison Medal.—The Edison Medal for 1925 was awarded to Dr. Harris J. Ryan, Stanford University, California, "for his contributions to the science and the art of high-tension transmission of power." The medal will be presented at the Pacific Coast Convention, Salt Lake City, Utah, in September.

John Fritz Medal.—The John Fritz Medal Board of Award, which is composed of representatives of the national societies of Civil, Mining, Mechanical and Electrical Engineers, awarded the 22nd medal to Edward Dean Adams of New York, "for achievement as an engineer, financier, scientist, whose vision, courage and industry made possible the birth at Niagara Falls, of hydroelectric power." The medal was presented at New York, at a meeting held on the evening of March 30th, 1926.

Lamme Gold Medal.—The conditions governing the award of the Lamme Gold Medal are now being prepared by the Committee on Award of Institute Prizes.

A bequest was made by the late B. G. Lamme, to cover the cost of an annual award by the Institute of a gold medal, to a member who has shown meritorious achievement in the development of electrical apparatus.

Commission of Washington Award.—The Washington Award for 1925 was voted to Jonas Waldo Smith, Consulting Engineer for the Board of Water Supply, New York City, "for the rare combination of vision, technical skill, administrative ability and courageous leadership in engineering."

The award is made annually by a committee composed of nine representatives of the Western Society of Engineers and two each from the A. S. C. E., the A. I. M. E., the A. S. M. E. and the A. I. E. E. The award of the medal was established in 1917 by Past-President J. W. Alvord of the Western Society "to be annually presented to an engineer whose work in some special instance, or whose services in general have been noteworthy for their merit in promoting the public good."

Employment Service.—The employment service which the Institute has maintained for many years is now conducted as a cooperative bureau in conjunction with a similar service maintained by the National Societies of Civil, Mining and Mechanical engineers under the title, "Engineering Societies Employment Service." In addition to the main office in the Engineering Societies Building, New York, offices have been opened in Chicago in cooperation with the Western Society of Engineers and in San Francisco under similar arrangements with the California Section, American Chemical Society and Engineers Club of San Francisco. It is hoped to continue this development from year to year as conditions warrant. The Service is supported by the joint contributions of the societies and their individual members who are benefited. As in the past, it consists principally in acting as a medium for bringing together the employer and the employee. In addition to the publication of the "Employment Service Bulletin" in the monthly JOURNALS, weekly subscription bulletins are issued for employers and those seeking positions.

American Engineering Council.—This organization, of which the Institute is one of the constituent bodies, has had another active year. The annual meeting of the Council was held in Washington, D. C., in January 1926, and several meetings of the Administrative Board have been held during the year. The Council sponsored a Conference on Public Works which was held in Washington and was attended by representatives of sixty-three engineering and allied technical organizations; it advocated a bill under which the Department of the Interior will become the Department of Public Works and Domain, and all subdivisions of the federal government of an architectural, engineering, and construction character will be transferred to the reorganized department; it endorsed a bill to increase the salaries of federal judges and instructed the Patents Committee to support the bill; authorized its officers to endeavor to secure a new building for the Patent Office; is active in support of measures to insure a greater degree of street and highway safety; proposed a program of research which includes: waste in agriculture; waste of power in the realm of generation, transmission, and use of power obtained from coal, oil, and gas; reclamation of material wastes, including metals and forest products; the engineering approach to the problem of labor supply; training and employment of the partially incapacitated; industrial fatigue; and integration of industry. This program contemplates the expenditure of approximately $350,000 during the next five years, and on the basis of past experience and a known source of interest it is believed the necessary funds can be obtained. Briefly, the American Engineering Council continues to function as the instrument of the constituent engineering societies through which the engineering profession may contribute toward the solution of the social, economic, and political problems of the day.

National Museum of Engineering and Industry.—The National Museum of Engineering and Industry, incorporated in 1924, and to be under the direction of the Smithsonian Institution, is "a Corporation to establish and maintain a permanent organization and home for commemorating the achievements and perpetuating the records of engineering and industrial progress." Private individuals, corporations and associations have been invited to contribute objects deemed to have historic interest in depicting industrial evolution. The Institute has named two representatives on the Board of Trustees. Similar action has been taken by the national societies of Civil, Mining and Mechanical Engineers.

Muscle Shoals Commission.—On March 27, 1925, President Coolidge named Past-President William McClellan as one member of the Commission of five to report upon the practical method of utilizing Muscle Shoals.

Franklin Medal.—On May 20, 1925 Dr. Elihu Thomson, Past-President, was awarded the Franklin Medal and certificate of honorary membership in the Franklin Institute, "In recognition of his pioneer work in the field of electricity and electrical engineering, and his numerous inventions in these fields."

United Engineering Society.—This Society performs for the national societies of Civil, Mining, Mechanical and Electrical Engineers, certain specific acts which are governed by contracts; the primary function of the United Society being to hold in trust and to administer for these societies the Engineering Societies Building, in which the headquarters of the national societies are located.

Extracts from the annual financial report of the United Engineering Society were published in the March 1926 JOURNAL.

Engineering Societies Library.—The library of the Institute is combined with the libraries of the national societies of Civil, Mining and Mechanical Engineers, administered as the "Engineering Societies Library" under the direction of the Library Board of the United Engineering Society; this board is composed of representatives of each of the four societies referred to above.

In order to place the facilities of the library at the disposal of persons residing at a distance from New York, a Library Service Bureau has been established, and a staff of expert searchers and translators is employed to cover almost any engineering topic, in the following manner: abstracting, translating, bibliographing, statistical searches and reports, searches for patent purposes, copying, preparing reference cards, etc. A lending department is also maintained.

A copy of the annual report of the Engineering Societies Library covering the calender year 1925, extracts from which were published in the April 1926 JOURNAL, may be obtained by applying to Institute headquarters.

Engineering Foundation.—Engineering Foundation is a trust fund established in 1914 by Ambrose Swasey, of Cleveland, Ohio, by gifts to United Engineering Society as a nucleus of a large endowment "for the furtherance of research in science and in engineering, or for the advancement in any other manner of the profession of engineering and the good of mankind." It is administered by the Engineering Foundation Board upon which the Institute and other national engineering societies are represented. The Board is a Department of United Engineering Society.

The Foundation has made appropriations for various research projects and has cooperated in others.

The annual report of the Foundation is available in printed form.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years, and has accepted sponsorship and appointed representatives upon a number of new Sectional Committees of American Engineering Standards Committee. A complete list of representatives is published frequently in the JOURNAL.

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins and Sells, certified public accountants, have audited the books, and their report follows:

Respectfully submitted for the Board of Directors.

F. L. HUTCHINSON,
National Secretary.

New York, May 21, 1926

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS
37 WEST 39TH STREET
NEW YORK

ATLANTA
BALTIMORE
BIRMINGHAM
BOSTON
BUFFALO
CHICAGO
CINCINNATI
CLEVELAND
DALLAS
DENVER
DETROIT
KANSAS CITY
LOS ANGELES
MINNEAPOLIS
NEWARK
NEW ORLEANS
NEW YORK
PHILADELPHIA
PITTSBURGH
PORTLAND
PROVIDENCE
SAINT LOUIS
SALT LAKE CITY
SAN DIEGO
SAN FRANCISCO
SEATTLE
TULSA
WATERTOWN
BERLIN
LONDON
PARIS
SHANGHAI

CANADA - CUBA - MEXICO
DELOITTE, PLENDER, HASKINS & SELLS

May 17, 1926.

American Institute of Electrical Engineers,
33 West 39th Street,
New York.

Dear Sirs:

We have made a general audit of your accounts for the year ended April 30, 1926. We submit herewith, as showing the results of such service, our certificate and the following described exhibits and schedule:

Exhibit "A"—Balance Sheet, April 30, 1926.
Schedule No. 1—Reserve Capital Fund—Securities.
Exhibit "B"—Summary of Income and Profit & Loss for the Year Ended April 30, 1926.

Yours truly,
HASKINS & SELLS

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

CERTIFICATE OF AUDIT

We have made a general audit of the accounts of the American Institute of Electrical Engineers for the year ended April 30, 1926, and

WE HEREBY CERTIFY that the accompanying Balance Sheet properly exhibits the financial condition of the Institute at April 30, 1926, that the Summary of Income and Profit & Loss for the year ended that date is correct, and that the books of the Institute are in agreement therewith.

HASKINS & SELLS

NEW YORK, May 17, 1926.

Exhibit A. **Balance Sheet, April 30, 1926**

ASSETS

Real Estate:			
One-fourth interest in United Engineering Society's Land, Building, and Building Equipment, 25 to 33 West 39th Street (Depreciation carried on books of United Engineering Society)			$491,642.36
Equipment:			
Library—Volumes and Fixtures		$40,545.59	
Works of Art, Paintings, etc		3,001.35	
Office Furniture and Fixtures	$17,315.34		
Less Reserve for Depreciation (including $6,000.00 funded)	12,020.64	5,294.70	
Total Equipment			48,841.64
Working Assets:			
"Transactions," etc		$12,157.06	
Text and Cover Paper		3,891.30	
Badges		1,952.47	
Total Working Assets			18,000.83
Current Assets:			
Cash		$16,056.03	
Accounts Receivable:			
Members—For Dues		18,515.86	
Advertisers		1,835.00	
Miscellaneous		1,669.06	
Accrued Interest on Investments		1,020.80	
Total Current Assets			39,096.75
Funds:			
Reserve Capital Fund:			
Securities—Schedule No. 1	$82,380.63		
Cash	318.07	$82,698.70	
Life Membership Fund:			
Cash	$ 3,545.63		
Chicago, Burlington & Quincy Railroad Company 4% Bonds, 1958, Registered, par Value $5,000.00	4,868.75		
Accrued Interest	33.33	8,447.71	
International Electrical Congress of St. Louis—Library fund			
Cash	$ 718.64		
New York City 4¼% Corporate Stock, 1957, par Value $2,000.00	2,204.05		
New York Telephone Company 4½% Bond, 1939, Registered, par Value $1,000.00	878.75		
Accrued Interest	67.50	3,868.94	
Mailloux Fund:			
Cash	$ 1.39		
New York Telephone Company 4½% Bond, 1939, Registered, par Value $1,000.00	1,000.00		
Accrued Interest	22.50	1,023.89	
Midwinter Convention Fund—Cash		93.67	
Lamme Medal Fund:			
Cash	$ 1,107.00		
American Smelting and Refining Company 5% 1st Mortgage Bond, 1947, Registered, par Value $4,000.00	4,035.00		
Accrued Interest	16.67	5,158.67	
Depreciation of Furniture and Fixtures Fund:			
Cash	$ 987.50		
Cleveland Union Terminals Company 5% Sinking Fund, Gold Bonds, 1973, par Value $5,000.00	5,012.50	6,000.00	
Total Funds			107,291.58
Total			$704,873.16

LIABILITIES

Current Liabilities		
Accounts Payable	$11,297.89	
Dues Received in Advance	2,749.01	
Entrance Fees and Dues Advanced by Applicants for Membership	506.40	
Subscriptions for "Transactions" received in Advance	60.75	
Total Current Liabilities		$ 14,614.05
Fund Reserves (Not Including Depreciation Reserves):		
Reserve Capital Fund	$82,698.70	
Life Membership Fund	8,447.71	
International Electrical Congress of St. Louis—Library Fund	3,868.94	
Mailloux Fund	1,023.89	
Midwinter Convention Fund	93.67	
Lamme Medal Fund	5,158.67	
Total Fund Reserves (Not Including Depreciation Reserves)		$101,291.58
Surplus, Per Exhibit "B"		588,967.53
Total		$704,873.16

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

SUMMARY OF INCOME AND PROFIT & LOSS

FOR THE YEAR ENDED APRIL 30, 1926

EXHIBIT B.

INCOME:			
Dues		*$192,799.16	
Students' Dues		10,366.00	
Entrance Fees		17,214.50	
Transfer Fees		1,575.00	
Advertising		68,605.80	
Journal Subscriptions		7,840.39	
"Transactions" Subscriptions		9,188.50	
Miscellaneous Sales		13,299.19	
Badges Sold	$ 5,105.20		
Less Cost	4,012.87	1,092.33	
Interest on Securities in Reserve Capital Fund		3,424.80	
Interest on Bank Balances		1,491.61	
Total			$326,897.28
EXPENSES:			
Publications:			
Journal	$100,242.95		
"Transactions"	19,132.22		
Year Book	8,062.63		
Miscellaneous	5,218.01	$132,655.81	
Institute Meetings		16,837.66	
Administrative Expenses		52,036.36	
Sections Committee		30,060.76	
Membership Committee		8,643.88	
Standards Committee		5,358.23	
Finance Committee		286.37	
Headquarters Committee		1,175.00	
Code Committee		60.00	
Technical Committees		108.96	
Edison Medal Committee		7.93	
Engineering Education Committee		95.53	
Geographical Districts:			
Traveling Expense:			
Executive Committees	$ 961.34		
Vice-Presidents	215.77		
Best Paper Prize	50.00		
First Paper Prize	25.00	1,252.11	
American Engineering Standards Committee		1,500.00	
International Electrotechnical Commission		517.50	
United States National Committee of International Commission on Illumination		300.00	
President's Special Appropriation		1,774.39	
Board of Directors—Mileage		1,798.14	
National Nominating Committee		1,104.57	
Honorary Secretary		4,000.00	
John Fritz Medal Award		2.00	
First Paper Prize—1925		100.00	
Transmission Prize, 1925		100.00	
Engineering Societies Library—Maintenance		8,000.00	
United Engineering Society Assessment		4,860.00	
American Engineering Council		16,788.50	
Engineering Societies Employment Service		1,635.00	
International Annual Tables		100.00	
International Critical Tables		125.00	
"Transactions" Index		3,080.13	
Total			$294,363.83
NET INCOME			$32,533.45

*Includes $86,590.00 allocated to subscriptions for the JOURNAL.

TOTAL INCOME (FORWARD)		$326,897.28
PROFIT & LOSS CREDITS:		
Adjustment of Inventories:		
Library Volumes and Fixtures	$ 5.74	
"Transactions"	89.25	94.99
GROSS SURPLUS FOR THE YEAR		$ 32,628.44
PROFIT & LOSS CHARGES:		
Uncollectable Dues Written Off	$ 7,282.25	
Uncollectable Advertisers' Accounts Written Off.	140.00	
Provision for Depreciation of Furniture and Fixtures	798.97	
Loss on Sale of Office Furniture	73.80	
Total		8,295.02
SURPLUS FOR THE YEAR		$ 24,333.42
SURPLUS, MAY 1, 1925	$589,487.86	
Less Transferred to Capital Fund Reserve in Accordance with Resolution of Board of Directors	24,853.75	564,634.11
SURPLUS, APRIL 30, 1926		$588,967.53

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

RESERVE CAPITAL FUND—SECURITIES

APRIL 30, 1926

EXHIBIT A.

SCHEDULE NO. 1.

	Par Value	Book Value
The Detroit Edison Company 1st and Refunding, 6% Series "B", Gold Bonds, Due 1940, Registered	$ 5,000.00	$ 5,178.13
The New York Central Railroad Company 5%. Refunding and Improvement Mortgage, Series "C" Bonds, Due 2013, Registered	6,000.00	5,742.50
Chicago, Burlington & Quincy Railroad Company 5%, 1st and Refunding Mortgage, Series "A", Gold Bond, Due 1971, Registered	1,000.00	1,010.00
Great Northern Railroad Company 5½%, General Mortgage, Series "B", Gold Bonds, Due 1952, Registered	10,000.00	9,847.50
Southern Railway Company 5%, 1st Consolidated Mortgage, Gold Bond, Due 1994, Registered	1,000.00	980.00
The Western Electric Company 5% Bonds, Due April 1, 1944, Registered	10,000.00	9,818.75
The American Telephone and Telegraph Company 5%, Gold Debentures, Sinking Fund, Due 1960, Registered	15,000.00	14,625.00
Pacific Gas and Electric Company 5½%, First and Refunding Mortgage, Gold Bonds, Due 1952, Registered	5,000.00	5,137.50
Consolidated Gas Company of New York 5½%, Gold Debentures, Due 1945, Registered	5,000.00	5,187.50
Baltimore & Ohio Railroad Company 4½%, Convertible Gold Bonds, Due 1933, Registered	10,000.00	9,387.50
St. Louis-San Francisco Railway Company 5%, Prior Lien Mortgage, Series "B" Bonds, Due 1950, Registered	6,000.00	5,497.50
American Smelting and Refining Company 5%, First Mortgage, 30-Year Gold Bonds, Due 1947, Registered	5,000.00	5,050.00
Florida East Coast Railway Company 5%, First and Refunding Mortgage, Series "A", Gold Bonds, Due 1974, Registered	5,000.00	4,918.75
Total	$84,000.00	$82,380.63

NOTE: An exchange of securities during the year ended April 30, 1926, resulted in a net reduction of $201.14 in the aggregate book value of the investments of the Reserve Capital Fund, and represents the unamortized premium on Wilmington bonds less profit on Liberty Loan bonds disposed of. The reserve capital fund cash balance of $318.07, shown in the balance sheet, reflects the excess of proceeds from securities disposed of over cost of securities acquired.

Officers A. I. E. E. 1925-1926

PRESIDENT

(Term expires July 31, 1926)
M. I. PUPIN

JUNIOR PAST PRESIDENTS

HARRIS J. RYAN
FARLEY OSGOOD

VICE-PRESIDENTS

HAROLD B. SMITH (District No. 1)
EDWARD BENNETT (District No. 5)
JOHN HARISBERGER (District No. 9)
L. F. MOREHOUSE (District No. 3)
H. W. EALES (District No. 7)
P. M. DOWNING (District No. 8)
HERBERT S. SANDS (District No. 6)
W. E. MITCHELL (District No. 4)
ARTHUR G. PIERCE (District No. 2)
W. P. DOBSON (District No. 10)

MANAGERS

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ERNEST LUNN
G. L. KNIGHT
W. M. McCONAHEY
W. K. VANDERPOEL
H. P. CHARLESWORTH
JOHN B. WHITEHEAD
J. M. BRYANT
E. B. MERRIAM
M. M. FOWLER
H. A. KIDDER
E. C. STONE

NATIONAL TREASURER

GEORGE A. HAMILTON

HONORARY SECRETARY

RALPH W. POPE

NATIONAL SECRETARY

F. L. HUTCHINSON

GENERAL COUNSEL

PARKER & AARON
30 Broad Street, New York

PAST-PRESIDENTS—1884-1925

*Norvin Green, 1884-5-6.
*Franklin L. Pope, 1886-7.
*T. Commerford Martin, 1887-8.
Edward Weston, 1888-9.
Elihu Thomson, 1889-90.
*William A. Anthony. 1890-91.
*Alexander Graham Bell, 1891-2.
Frank Julian Sprague, 1892-3.
*Edwin J. Houston, 1893-4-5.
*Louis Duncan, 1895-6-7.
*Francis Bacon Crocker, 1897-8.
A. E. Kennelly, 1898-1900.
Carl Hering, 1900-1.
*Charles P. Steinmetz, 1901-2.
Charles F. Scott, 1902-3.
Bion J. Arnold, 1903-4.
John W. Lieb, 1904-5.
*Schuyler Skaats Wheeler, 1905-6.
*Samuel Sheldon, 1906-7.
*Henry G. Stott, 1907-8.
Louis A. Ferguson, 1908-9.
Lewis B. Stillwell, 1909-10.
Dugald C. Jackson, 1910-11.
Gano Dunn, 1911-12.
Ralph D. Mershon, 1912-13.
C. O. Mailloux, 1913-14.
Paul M. Lincoln, 1914-15.
John J. Carty, 1915-16.
H. W. Buck, 1916-17.
E. W. Rice, Jr., 1917-18.
Comfort A. Adams, 1918-19.
Calvert Townley, 1919-20.
A. W. Berresford, 1920-21.
William McClellan, 1921-22.
Frank B. Jewett, 1922-23.
Harris J. Ryan, 1923-24.
Farley Osgood, 1924-25.

*Deceased.

LOCAL HONORARY SECRETARIES

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Carroll M. Mauseau, Caixa Postal No. 571, Rio de Janeiro, Brazil, S. A.
Charles le Maistre, 28 Victoria St., London, S. W. 1, England.
A. S. Garfield, 45 Blvd. Beausejour, Paris 16 E, France.
H. P. Gibbs, Tata Sons Ltd., 24 Bruce Road, Bombay—1, India.
Guido Semenza, 39 Via Monte Napoleone, Milan, Italy.
Eiji Aoyagi, Kyoto Imperial University, Kyoto, Japan.
P. H. Powell, Canterbury College, Christchurch, New Zealand.
Axel F. Enstrom, 24a Grefturegatan, Stockholm, Sweden.
W. Elsdon-Dew, P. O. Box 4563, Johannesburg, Transvaal, Africa.

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Farley Osgood, Vice-Chairman
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H. W. Eales, W. K. Vanderpoel

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L. F. Morehouse W. K. Vanderpoel.

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E. E. F. Creighton, J. E. Macdonald, Harold B. Smith.
L. W. W. Morrow
Chairman of Committee on Coordination of Institute Activities, *ex-officio.*
Chairman of Technical committees, *ex-officio.*

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E. B. Meyer,

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(Terms expire July 31, 1926)
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(Terms expire July 31, 1927)
Gano Dunn, Chairman, F. A. Scheffler, W. R. Whitney.
(Terms expire July 31, 1928)
C. C. Chesney, Robert A. Millikan, M. I. Pupin
(Terms expire July 31, 1929)
N. A. Carle, W. C. L. Eglin, John W. Lieb.
(Terms expire July 31, 1930)
George Gibbs, Samuel Insull, Ralph D. Mershon.
Elected by the Board of Directors from its own membership for term of two years.
(Terms expire July 31, 1926)
L. F. Morehouse, Harris J. Ryan, Harold B. Smith.
(Terms expire July 31, 1927)
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L. F. Fuller,
D. H. Gage,
S. P. Grace,
P. J. Howe,
F. H. Kroger,
N. M. Lash,
Ray H. Manson,
R. D. Parker,
H. S. Phelps,
Lieut. Comm. B. B. Ralston
F. A. Raymond,
Chester W. Rice,
J. K. Roosvelt,
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Gano Dunn,
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Farley Osgood,
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R. W. Sorensen,
J. B. Whitehead,
W. R. Whitney,
W. E. Wickenden.

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H. C. Albrecht,
B. F. Bailey,
B. L. Barns,
B. A. Behrend,
A. C. Bunker,
James Burke,
Walter M. Dann,
L. L. Elden,
G. Faccioli,
W. J. Foster,
Harold Goodwin, Jr.,
J. I. Hull,
V. Karapetoff,
A. H. Kehoe,
A. E. Kennelly,
P. M. Lincoln,
A. M. MacCutcheon,
F. D. Newbury,
N. L. Pollard,
R. F. Schuchardt,
C. E. Skinner,
A. Still,
R. B. Williamson.

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Lawrence Addicks,
Arthur N. Anderson,
T. C. Atchison,
Farley G. Clark,
Safford K. Colby,
F. A. J. FitzGerald,
W. F. Hendry,
Walter E. Holland,
F. A. Lidbury,
Wm. A. Moore,
Charles H. Moritz,
Carl G. Schluederberg
Magnus Unger,
J. B. Whitehead,
J. L. Woodbridge,
J. L. McK. Yardley.

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R. E. Doherty,
Herbert Bristol Dwight,
Charles Fortescue,
V. Karapetoff,
A. E. Kennelly,
W. B. Kouwenhoven,
R. A. Millikan,
F. W. Peek, Jr.
Harold Pender,
Chester W. Rice,
J. Slepian,
Harold B. Smith,
J. B. Whitehead.

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H. B. Brooks, Secretary
O. J. Bliss,
Perry A. Borden,
W. M. Bradshaw,
J. R. Craighead,
W. A. Del Mar,
E. D. Doyle,
W. M. Goodwin, Jr.,
C. M. Jansky, Jr.,
W. B. Kouwenhoven,
P. M. Lincoln,
W. M. McConahey
W. J. Mowbrey,
H. A. Perkins,
L. T. Robinson,
Bryon W. St. Clair,
G. A. Sawin,
I. B. Smith,
Roy Wilkins,

APPLICATIONS TO IRON AND STEEL PRODUCTION

F. B. Crosby, Chairman, Morgan Construction Co., 15 Belmont St., Worcester, Mass.
A. C. Cummins,
W. C. Kennedy,
A. G. Pierce,
A. G. Place,
F. O. Schnure,
G. E. Stoltz,
J. D. Wright.

PRODUCTION AND APPLICATION OF LIGHT

Preston S. Millar, Chairman, Electrical Testing Laboratories, 80th St. & East End Ave., New York
W. T. Blackwell,
J. M. Bryant,
W. T. Dempsey,
H. W. Eales,
F. M. Feiker,
F. F. Fowle,
G. C. Hall,
H. H. Higbie,
A. S. McAllister,
G. S. Merrill,
F. H. Murphy,
Charles F. Scott,
B. E. Shackelford,
C. J. Stahl,
G. H. Stickney.

APPLICATIONS TO MARINE WORK

L. C. Brooks, Chairman, Bethlehem Shipbuilding Corp., Quincy, Mass.
H. F. Harvey, Jr., Vice-Chairman
J. S. Jones, Secretary
R. A. Beekman,
J. F. Clinton,
C. S. Gillette,
William Hetherington, Jr.
H. L. Hibbard,
Edward C. Jones,
A. Kennedy, Jr.
M. A. Libbey,
E. B. Merriam,
W. F. Meschenmoser
I. H. Osborne,
Arthur Parker,
G. A. Pierce,
H. M. Southgate,
W. E. Thau,
C. P. Turner,
A. E. Waller,
J. L. Wilson,
R. L. Witham.

APPLICATIONS TO MINING WORK

F. L. Stone, Chairman, General Electric Co., Schenectady, N. Y.
W. C. Adams,
M. C. Benedict,
Graham Bright,
H. W. Eales,
L. C. Ilsley,
G. M. Kennedy,
R. L. Kingsland,
A. B. Kiser,
Carl Lee,
W. H. Lesser,
John A. Malady,
Charles H. Matthews,
D. C. McKeehan,
W. F. Schwedes,
W. A. Thomas,
C. D. Woodward.

GENERAL POWER APPLICATIONS

A. M. MacCutcheon, Chairman, Reliance Electric Engineering Co., 1088 Ivanhoe Road, Cleveland, Ohio
P. H. Adams,
D. H. Braymer,
H. E. Bussey,
R. F. Chamberlain,
C. W. Drake,
E. W. Henderson,
H. D. James,
P. C. Jones,
J. C. Kositzky,
A. C. Lanier,
W. S. Maddocks,
N. L. Mortensen,
H. W. Rogers,
H. L. Smith,
W. H. Timbie,
W. K. Vanderpoel,
A. E. Waller,
W. C. Yates.

POWER GENERATION

Vern E. Alden, Chairman, Consolidated Gas & Electric Company, Baltimore, Md.

H. A. Barre,
E. T. J. Brandon,
H. W. Eales,
Louis Elliott,
N. E. Funk,
C. F. Hirshfeld,
Francis Hodgkinson,
P. Junkersfeld,
H. A. Kidder,
J. T. Lawson,
W. H. Lawrence,
James Lyman,
W. E. Mitchell,
I. E. Moultrop,
J. C. Parker,
M. M. Samuels,
F. A. Scheffler,
R. F. Schuchardt,
A. R. Smith,
Nicholas Stahl,
W. M. White.

POWER TRANSMISSION AND DISTRIBUTION

P. H. Thomas, Chairman, 120 Broadway, New York
P. H. Chase, Vice-Chairman, Philadelphia, Pa.
R. N. Conwell, Vice-Chairman, Newark, N. J.
F. M. Farmer, Secretary, 80th St. & East End Ave., New York

C. G. Adsit,
R. W. Atkinson,
A. O. Austin,
F. G. Baum,
John A. Brundige,
V. Bush,
George F. Chellis,
Wallace S. Clark,
Edith Clarke,
W. H. Cole,
M. T. Crawford,
John C. Damon,
W. A. Del Mar,
Herbert H. Dewey,
L. L. Elden,
R. D. Evans,
C. L. Fortescue,
C. D. Gray,
W. A. Hillebrand,
J. P. Jollyman,
A. H. Kehoe,
C. H. Kraft,
A. H. Lawton,
W. E. Meyer,
W. E. Mitchell,
Clifford R. Oliver,
John C. Parker,
F. W. Peek, Jr.,
G. G. Post,
D. W. Roper,
C. E. Schwenger,
A. E. Silver,
M. L. Sindeband,
H. C. Sutton,
Philip Torchio,
Theodore Varney,
H. S. Warren,
R. J. C. Wood.

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F. L. Hunt, Vice-Chairman, Greenfield, Mass.

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